तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

KENDRIYA VIDYALAYA SANGATHAN
PATNA REGION

CLASS-X

Based on the latest CBSE Exam pattern

for the session- 2024-25

STUDENT STUDY MATERIAL
(MATHEMATICS)

CHIEF PATRON

Sh. Anurag Bhatnagar

Deputy Commissioner

Kendriya Vidyalaya Sangathan

Patna Region

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PREPARED BY

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1	REAL NUMBERS	SH. BIRENDRA PRASAD, TGT (MATHS), PM SHRI KV PATNA NO.I (KANKAR BAGH) SS
2	POLYNOMIALS	BRAJESH KUMAR SINGH, TGT (MATHS), PM SHRI KV PATNA NO.I (KANKAR BAGH) FS
3	PAIR OF LINEAR EQUATIONS IN TWO VARIABLES	KAMLESH, TGT (MATHS), PM SHRI KV PATNA NO.I (KANKAR BAGH) FS
4	QUADRATIC EQUATIONS	SH. KAMLESH KUMAR, TGT (MATHS), PM SHRI KV NALANDA OF RAJGIR
5	ARITHMETIC PROGRESSIONS	SMT ANKITA, TGT (MATHS), KV DARBHANGA NO.II (ITI)
6	TRIANGLES	MRS SWETA KUMARI, TGT (MATHS), PM SHRI KV GAYA NO.I
7	COORDINATE GEOMETRY	MR.JAY PRAKASH SINGH, TGT (MATHS), PM SHRI KV MOKAM GHAT CRPF
8	INTRODUCTION TO TRIGONOMETRY	SH. ASHARFI PASWAN, TGT (MATHS), KV HAJIPUR
9	SOME APPLICATIONS OF TRIGONOMETRY	SH. BHARAT KUMAR, TGT (MATHS), PM SHRI KV JAMALPUR
10	CIRCLES	SMT SUSHMITA THAKUR, TGT (MATHS), KV (NTPC) KAHAL GAON NTPC
11	AREAS RELATED TO CIRCLES	MR. SHAILENDRA KUMAR, TGT (MATHS), PM SHRI KV KATIHAR
12	SURFACE AREAS AND VOLUMES	MR. VIMAL KUMAR, TGT (MATHS), PM SHRI KV GARHARA
13	STATISTICS	SH. AWADH KISHORE SINGH, TGT (MATHS), KV CRPF JHAPHAN (MUZAFFARPUR)
14	PROBABILITY	MD SOHAIL AZAM, TGT (MATHS), PM SHRI KV MUZAFFARPUR SS
15	SAMPLE PAPER STANDARD (SOLVED)	SH. SUNIL KUMAR, TGT (MATHS), PM SHRI KV PATNA NO.II (BAILEY ROAD) FS
16	SAMPLE PAPER BASIC (SOLVED)	SMT. PRIYANKA KUMARI, TGT (MATHS), KV ECR, SAMASTIPUR
17	SAMPLE PAPER STANDARD (UNSOLVED)	SH. MITHLESH KUMAR, TGT (MATHS), PM SHRI KV NO. 2 GAYA
18	SAMPLE PAPER BASIC (UNSOLVED)	SH. AMIT KUMAR, TGT (MATHS), PM SHRI KV NO. 2 GAYA
19	COMPILED BY	SH. MITHLESH KUMAR, TGT (MATHS), PM SHRI KV NO. 2 GAYA

COURSE STRUCTURE CLASS -X

Units	Unit Name	Marks
I	NUMBER SYSTEMS	06
II	ALGEBRA	20
III	COORDINATE GEOMETRY	06
IV	GEOMETRY	15
V	TRIGONOMETRY	12
VI	MENSURATION	10
VII	STATISTICS & PROBABILTY	11
	Total	**80**

UNIT I: NUMBER SYSTEMS

1. REAL NUMBER

Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of $\sqrt{2}, \sqrt{3}\ \sqrt{5}$

UNIT II: ALGEBRA

1. POLYNOMIALS

Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency.

Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination. Simple situational problems.

3. QUADRATIC EQUATIONS

Standard form of a quadratic equation $ax^2 + bx + c = 0, (a \neq 0)$. Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.

Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. ARITHMETIC PROGRESSIONS

Motivation for studying Arithmetic Progression Derivation of the n^{th} term and sum of the first n terms of A.P. and their application in solving daily life problems.

UNIT III: COORDINATE GEOMETRY

Coordinate Geometry

Review: Concepts of coordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division).

UNIT IV: GEOMETRY

1. TRIANGLES

Definitions, examples, counter examples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.
5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.

2. CIRCLES

Tangent to a circle at, point of contact

1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.

UNIT V: TRIGONOMETRY

1. INTRODUCTION TO TRIGONOMETRY

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0- and 90-. Values of the trigonometric ratios of 30^0, 45^0 and 60^0. Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES

Proof and applications of the identity $sin^2A + cos^2A = 1$. Only simple identities to be given.

3. HEIGHTS AND DISTANCES: Angle of elevation, Angle of Depression

Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30°, 45°, and 60°.

UNIT VI: MENSURATION

1. AREAS RELATED TO CIRCLES

Area of sectors and segments of a circle. Problems based on areas and perimeter / circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only.

2. SURFACE AREAS AND VOLUMES

Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones.

UNIT VII: STATISTICS AND PROBABILITY

1. STATISTICS

Mean, median and mode of grouped data (bimodal situation to be avoided).

2. PROBABILITY

Classical definition of probability. Simple problems on finding the probability of an event.

MATHEMATICS-Standard
QUESTION PAPER DESIGN
CLASS – X (2023-24)

Time: 3 Hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage (approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	43	54
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	19	24
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	18	22
	Total	80	100

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

MATHEMATICS-Basic
QUESTION PAPER DESIGN
CLASS – X (2023-24)

Time: 3Hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage (approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	60	75
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	12	15
3	**Analysing:** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	8	10
	Total	80	100

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

PRESCRIBED BOOKS:

1. Mathematics - Textbook for class IX - NCERT Publication
2. Mathematics - Textbook for class X - NCERT Publication
3. Guidelines for Mathematics Laboratory in Schools, class IX - CBSE Publication
4. Guidelines for Mathematics Laboratory in Schools, class X - CBSE Publication
5. Laboratory Manual - Mathematics, secondary stage - NCERT Publication
6. Mathematics exemplar problems for class IX, NCERT publication.
7. Mathematics exemplar problems for class X, NCERT publication.

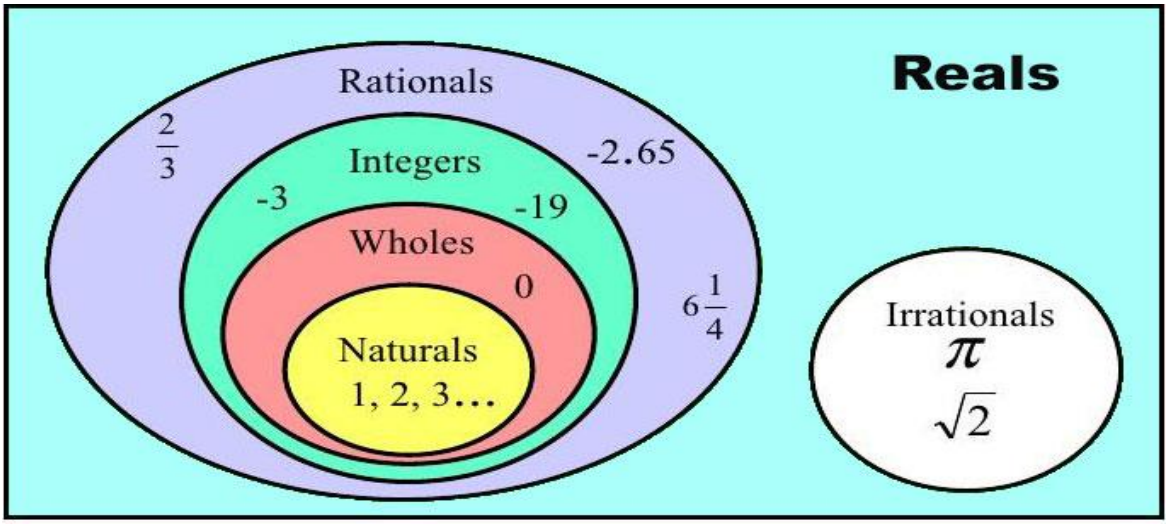
KENDRIYA VIDYALAYA SANGATHAN, PATNA REGION

MATHEMATICS STUDY MATERIAL FOR CLASS X (2024-25)

CONTENTS

Real Numbers

Gist of the lesson:

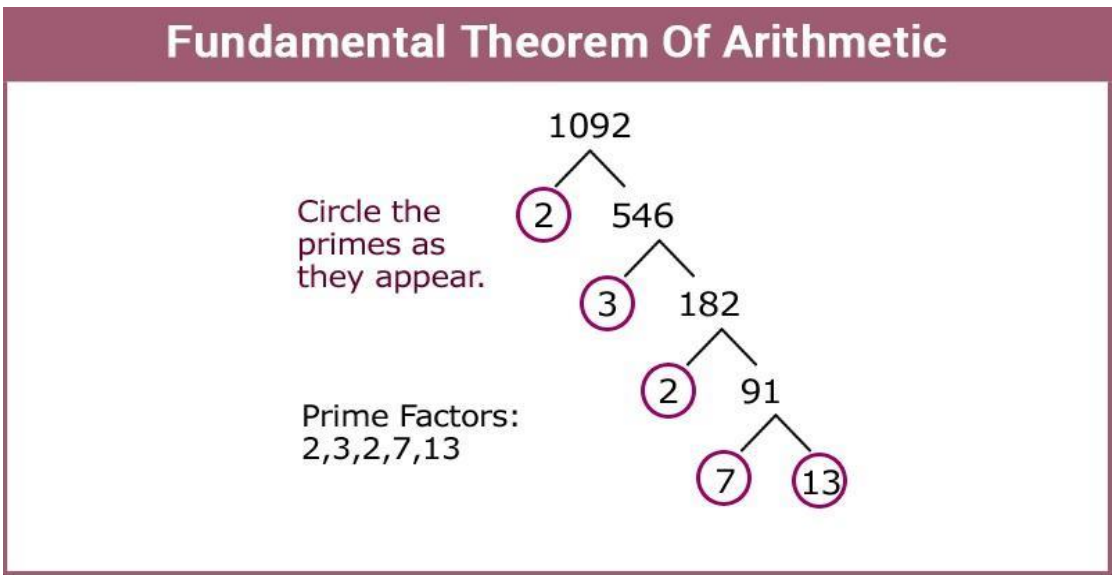


The Fundamental Theorem of Arithmetic

We can factorize each composite number as a product of some prime numbers and of course, this prime factorization of a natural number is unique as the order of the prime factors doesn't matter.

- HCF of given numbers is the highest common factor among all which is also known as GCD i.e. greatest common divisor.
- LCM of given numbers is their least common multiple.
- If we have two positive integers 'm' and 'n' then the property of their HCF and LCM will be:

  HCF (m, n) × LCM (m, n) = m × n.



Rational Numbers

The number 's' is known as a rational number if we can write it in the form of m/n where 'm' and 'n' are integers and n ≠ 0, $\frac{2}{3}$, $\frac{3}{5}$ etc.

Rational numbers can be written in decimal form also which could be either terminating or non-terminating. E.g. $\frac{5}{2}$ = 2.5 (terminating) and $\frac{2}{3} = 0.\overline{6}$ (non-terminating).

Irrational Numbers

The number 's' is called irrational if it cannot be written in the form of m/n, where m and n are integers and n≠0 or in the simplest form, the numbers which are not rational are called irrational numbers. Example - √2, √3 etc.

- If p is a prime number and p divides a^2 , then p is one of the prime factors of a^2 which divides a, where a is a positive integer.
- If p is a positive number and not a perfect square, then $\sqrt{n}$ is definitely an irrational number.
- If p is a prime number, then $\sqrt{p}$ is also an irrational number.

Assertion -Reason based question:

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

1. **Assertion:** For any two positive integers p and q, HCF (p, q) × LCM (p, q) = p × q
Reason: If the HCF of two numbers is 5 and their product is 150, then their LCM is 40.

Basic

Ans: c

2. **Assertion:** if two positive integer m and n are expressible in the form m= pq^3 and n= p^3q^2 where P, q are prime number then HCF (m, n)= pq^2
Reason: HCF is the product of smallest power of each common prime factor in the numbers.

Standard

Ans: a

MCQ type questions:

1. The product of a non-zero rational and an irrational number is Basic
   (a) always irrational (b) always rational
   (c) rational or irrational (d) one
   Ans: a

2. Which of the following is not irrational? Basic
   (a) $(2 – \sqrt{3})2$ (b) $(\sqrt{2} + \sqrt{3})2$
   (c) $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})$ (d)$27\sqrt{7}$
   Ans: c
3. The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is Basic
   (a) 13 (b) 65
   (c) 875 (d) 1750
   Ans: a
4. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3 b$; a, b being prime numbers, then LCM (p, q) is Standard
   (a) ab (b) $a^2 b^2$
   (c) $a^3 b^2$ (d) $a^3 b^3$
   Ans: c
5. If HCF (16, y) = 8 and LCM (16, y) = 48, then the value of y is Standard
   (a) 24 (b) 16
   (c) 8 (d) 48
   Ans: a

VSA (2 Marks) Type Questions:

1. Explain why $3 \times 5 \times 7 + 7$ is a composite number. Basic
   Ans: A composite number is a number that is divisible by another number other than by itself and one. The number 2 is the only even prime number. All other even numbers are composite numbers.
   As per BODMAS rule:
   $3 \times 5 \times 7 + 7$
   $= 105 + 7$
   $= 112.$
   112 is an even number.
   Therefore,112 is a composite number
2. HCF and LCM of two numbers is 9 and 459 respectively. If one of the numbers is 27, find the other number. Basic
   Ans: We know,
   1st number × 2nd number = HCF × LCM
   $\Rightarrow$ 27 × 2nd number = 9 × 459
   $\Rightarrow$ 2nd number = $9 \times 45927 = 153$
3. Find the LCM of 96 and 360 by using fundamental theorem of arithmetic. Basic Ans: 1440
4. Prove that $3 + 2\sqrt{3}$ is an irrational number. Standard
   Ans: Proof
5. Write the smallest number which is divisible by both 306 and 657. Standard
   Ans:22338

SA (3 Marks) type questions:

1. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.
   Ans: No. Basic
   The HCF of two numbers must be a factor of the LCM. Hence the above situation is possible only if 18 is a factor of 380. but we see that it is not a factor of 380.
   So, it is not possible to have two numbers whose HCF is 18 and LCM is 380.
2. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together? Basic
   Ans: $9 = 3^2$, $12 = 2^2 \times 3$, $15 = 3 \times 5$
   LCM = $2^2 \times 3^2 \times 5 = 4 \times 9 \times 5 = 180$ minutes or 3 hours
   They will next toll together after 3 hours.
3. If p is prime number, then prove that √p is an irrational. Basic
   Ans: Proof
4. Prove that $\sqrt{3} + \sqrt{5}$ is irrational. Standard
   Ans: Proof
5. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps? Standard
   Ans: 2520 cm

LA Type Question:

1. In a school, there are two Sections A and B of class X. There are 48 students in Section A and 60 students in Section B. Determine the least number of books required for the library of the school so that the books can be distributed equally among all students of each Section. Basic
   Ans:
   Since the books are to be distributed equally among the students of Section A and Section B. therefore, the number of books must be a multiple of 48 as well as 60.
   Hence, required number of books is the LCM of 48 and 60.
   $48 = 2^4 \times 3$

   $60 = 2^2 \times 3 \times 5$
   LCM = $2^4 \times 3 \times 5 = 16 \times 15 = 240$
   Hence, required number of books is 240.

2	48
2	24
2	12
2	6
	3

2	60
2	30
3	15
	5

2. There are 104 students in class X and 96 students in class IX in a school. In a house examination, the students are to be evenly seated in parallel rows such that no two adjacent rows are of the same class.
   (a) Find the maximum number of parallel rows of each class for the seating arrangement.
   (b) Also, find the number of students of class IX and also of class X in a row.
   (c) What is the objective of the school administration behind such an arrangement?
   Basic

   Solution:
   $104 = 23 \times 13$
   $96 = 25 \times 3$
   HCF = 23 = 8
   (a) Number of rows of students of class X = 1048 = 13
   Number maximum of rows class IX = 968 = 12
   Total number of rows = 13 + 12 = 25
   (b) No. of students of class IX in a row = 8
   No. of students of class X in a row = 8
   (c) The objective of school administration behind such an arrangement is fair and clean examination, so that no student can take help from any other student of his/her class.
3. National Art convention got registrations from students from all parts of the country, of which 60 are interested in music, 84 are interested in dance and 108 students are interested in handicrafts. For optimum cultural exchange, organisers wish to keep them in minimum number of groups such that each group consists of students interested in the same art form and the number of students in each group is the same. Find the number of students in each group. Find the number of groups in each art form. How many rooms are required if each group will be allotted a room? Basic

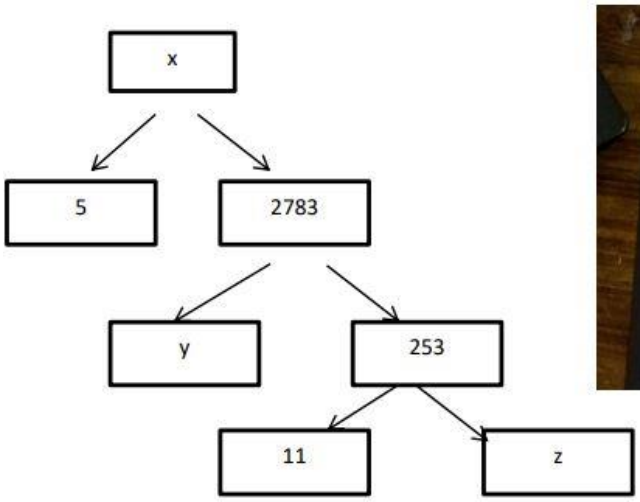
   Ans: No. of students in each group= 12
   No. of group in each art form= 5, 7, 9 respectively
   Total no. of room= 21
4. Show that $(\sqrt{3}+\sqrt{5})^2$ is an irrational number. Standard

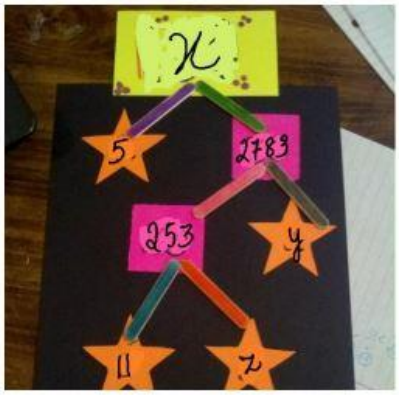
   Ans: Proof
5. Find the largest number that divides 628, 3129 and 15630 to leave remainders 3, 4 and 5 respectively. Standard

   Ans: 625

Case Study based Questions:

1. A Mathematics Exhibition is being conducted in your School and one of your friends is making a model of a factor tree. He has some difficulty and asks for your help in completing a quiz for the audience. Basic





Observe the following factor tree and answer the following:

(i) What will be the value of x?

Answer: 13915

(ii) What will be the value of y?

Answer: 11

(iii) What will be the value of z?

Answer: 23

or

Find the prime factorisation of 13915.

Answer: $5 \times 11^2 \times 23$

2. Rohit Singh is a worker in a petrol pump along with the other co-worker's used to transfer petrol from tanker to storage. On Monday, there were two tankers containing 850 litres bad 680 litres of petrol respectively. Basic

(i) Express 680 as a product of its prime factors.

(ii) What is the maximum capacity of a container which can measure the petrol of either tanker is exact number of time?

(iii) If the product of two positive integers is equal to the product of their HCF and LCM is true then find the LCM (850, 680).

or

Find the product of HCF and LCM of 80,85 and 90.

3. A sweet seller has 420 kaju barfis and 130 badam barfis. She wants to stack them in such a way that each stack has the same number of sweets, and they take up the least area of the tray.

Standard

(i) Find the prime factorisation of 420.

(ii) Find the Prime Factorisation of 130.

(iii) What is the maximum number of barfis that can be placed in each stack for this purpose?

or

A school library has 280 science books and 300 maths books. Students were told to stack these journals in such a way that each stack contains equal number of books. Find the number of stacks of science and maths books. what is the benefit of library in student's life?

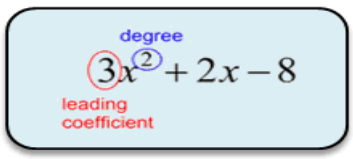
POLYNOMIALS

GIST OF THE TOPIC

- An algebraic expression can have exponents that are **rational numbers.** However, a **polynomial** is an algebraic expression in which the exponent on any variable is a **whole number.**
  $5x^3 + 3x + 1$ is an example of a polynomial. It is an algebraic expression as well polynomial.
  $2x+3\sqrt{x}$ is an algebraic expression, but not a polynomial, since the exponent on x is $1/2$ which is not a whole number.
- General form of a polynomial
  Let x be a variable, n be a positive integer and a_0, a_1,….a_n be constants (real numbers), then,
  $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} \ldots + a_2 x^2 + a_1 x + a_0$
  is called a polynomial in variable x

Degree of a Polynomial

- ***For a polynomial in one variable*** – the **highest exponent** on the **variable** in a polynomial is the **degree** of the polynomial.



Example: The degree of the polynomial $3x^2 + 2x - 8$ is 2, as the highest power of x in the given expression is x^2.

TYPES OF POLYNOMIALS

- Polynomials can be classified based on
  a) Number of terms
  b) Degree of the polynomial.

Types of Polynomials based on the number of terms

- **Monomial** – A polynomial with just one term. Example – $2x$, $6x^2$, $9xy$
- **Binomial** – A polynomial with two terms. Example – $4x^2 + x$, $5x + 4$
- **Trinomial** – A polynomial with three terms. Example – $x^2 + 3x + 4$

Types of Polynomials based on Degree

- Constant polynomial- A polynomial whose degree is zero is called a ***constant polynomial***. For example, 2 is a linear polynomial as it can be expreseed as $2x^0$.
- Linear Polynomial– A polynomial whose degree is one is called a ***linear polynomial***. For example, $2x + 1$ is a linear polynomial.
- Quadratic Polynomial – A polynomial of degree two is called a ***quadratic polynomial***. For example, $3x^2 + 8x + 5$ is a quadratic polynomial.
- Cubic Polynomial – A polynomial of degree three is called a ***cubic polynomial***.
  For example, $2x^3 + 5x^2 + 9x + 15$ is a cubic polynomial.

Type of polynomial	Degree	General Form
Constant	0	$p(x) = a$
Linear	1	$p(x) = ax + b$
Quadratic	2	$p(x) = ax^2 + bx + c$
Cubic	3	$p(x) = ax^3 + bx^2 + cx + d$
Bi-Quadratic	4	$p(x) = ax^4 + bx^3 + cx^2 + dx + e$
Polynomial	n	$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} \ldots + a_2 x^2 + a_1 x + a_0$

Value of Polynomial

Let $p(x)$ is a polynomial in x and k could be any real number, then the value calculated after putting the value $x = k$ in $p(x)$ is the final value of $p(x)$ at $x = k$. This shows that $p(x)$ at $x = k$ is represented by $p\ (k)$.

Zeroes of a Polynomial

A zero of a polynomial $\boldsymbol{p(x)}$ is the value of x for which the value of $p(x)$ is 0. If k is a zero of $p(x)$, then $\boldsymbol{p(k) = 0}$.

For example, consider a polynomial $p(x) = x^2 - 3x + 2$.

When $x = 1$, the value of $p(x)$ will be equal to

$p(1)=1^2 - 3 \times 1 + 2$

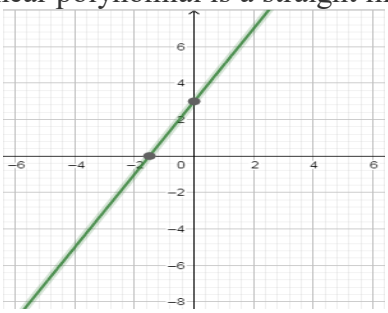
$=1-3+2$

$=0$

Since $p(x) = 0$ at $x = 1$, we say that 1 is a zero of the polynomial $x^2 - 3x + 2$.

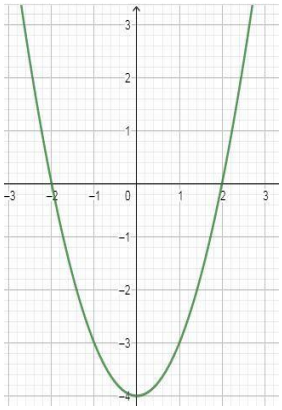
Geometrical Representation of a Linear Polynomial

- The graph of a linear polynomial is a straight line. It cuts the X-axis at exactly one point.



Geometrical Representation of a Quadratic Polynomial

- The graph of a quadratic polynomial is a parabola.
- It looks like a U which either opens upwards or opens downwards depending on the value of $'a'$ in $ax^2 + bx + c$.
- If $'a'$ is positive then parabola opens upwards and if $'a'$ is negative then it opens downwards.
- It can cut the x-axis at 0, 1 or two points.



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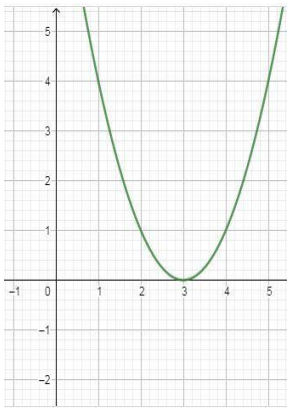

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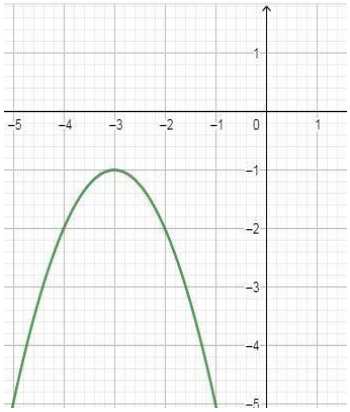
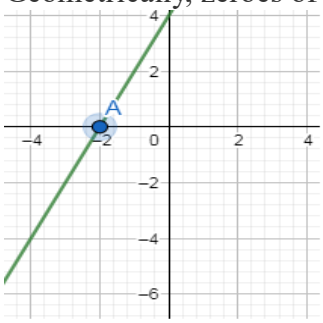
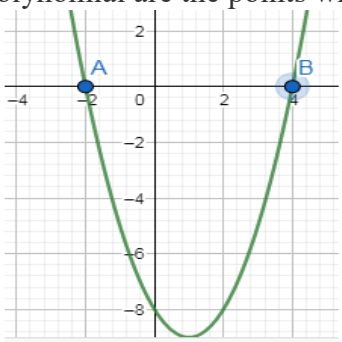


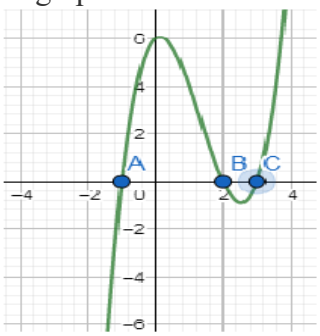
(a<0)

Geometrical Meaning of Zeroes of a Polynomial

Geometrically, zeroes of a polynomial are the points where its graph cuts the x-axis.







(i) One zero (ii) Two zeroes (iii) Three zeroes

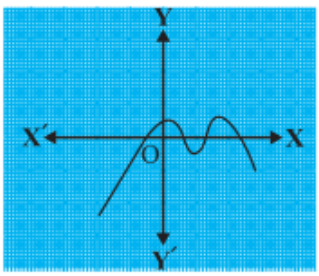
Here A, B and C correspond to the zeros of the polynomial represented by the graphs

Number of Zeroes

In general, a polynomial of degree 'n' has at most 'n'zeroes.

1. A linear polynomial has one zero,
2. A quadratic polynomial has at most two zeroes.
3. A cubic polynomial has at most 3 zeroes.

***For Example*:** For the given graph, find the number of zeroes of $p(x)$.



From the figure, we can see that the graph intersects the x-axis at four points. Therefore, the number of zeroes is 4.

Relationship between the zeroes and the coefficients of a Polynomial.

(i) If α, β are zeroes / roots of $p(x) = ax^2 + bx + c$, then

$$\text{Sum of roots } \alpha+\beta=\frac{-b}{a}\Rightarrow \alpha+\beta=\frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$$

$$\text{Product of roots} = \alpha\beta=\frac{c}{a}\Rightarrow \alpha\beta=\frac{\text{constant term}}{\text{coefficient of } x^2}$$

(ii) If $\alpha, \beta \text{ and } \gamma$ are zeroes / roots of $p(x) = ax^3 + bx^2 + cx + d$

$$\text{Then, sum of roots} = \alpha+\beta+\gamma=\frac{-b}{a}=\frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3}$$

Sum of product of roots taken, two at a time

$$\alpha\beta+\beta\gamma+\alpha\gamma=\frac{c}{a}=\frac{\text{coefficient of } x}{\text{coefficient of } x^3}$$

$$\text{Product of roots} = \frac{-d}{a}\Rightarrow \alpha\beta\gamma=\frac{-(\text{constant term})}{\text{coefficient of } x^3}$$

(iii) If α, β are roots of a quadratic polynomial $p(x)$, then

$$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$\square$ $p(x) = x^2 - (sum\ of\ roots)\,x + product\ of\ roots$

(iv) If $\alpha, \beta \text{ and } \gamma$ are zeroes of a cubic polynomial $P(x)$, then

$$p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma)x - (\alpha\beta\gamma)$$

$\square$ $p(x) = x^3 - (sum\ of\ zeroes)x^2 +$
$(sum\ of\ product\ of\ zeroes\ taken\ two\ at\ a\ time)x - (product\ of\ zeroes)$

MULTIPLE CHOICE QUESTIONS (1 MARK)

1. Which of the following is a polynomial? BASIC

(a) $2x^2 + 3/x - 5$ (b) $-3x^2 + \sqrt{} + 4$

(c) $\sqrt{2}x^3 + \sqrt{3}x^2 + \sqrt{5}x - 3$ (d) $5/x^3 + 2x^2 - 3x + 1/7$

Ans. (c)

Solution: - An expression of the form $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots a_1x + a_0$, $a_0 \neq 0$ is a polynomial of degree n (a natural number).Clearly, in expression given in option (a) has a term containing x^{-1} . So, it is not a polynomial. Expression in option (b) has a term containing $x^{-1/2}$, so it is not a polynomial. The expression in option (d) has a term containing x^{-3} , so it is not a polynomial. Expression in option (c) satisfies the definition of a polynomial.

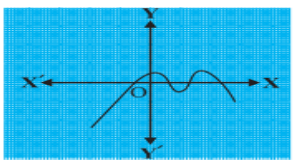
2. If one zero of the quadratic polynomial $kx^2 + 3x + k$ is 2, then the value of k. BASIC

(a) 5/6 (b) -5/6 (c) 6/5 (d) -6/5

Ans. (d)

Solution: - It is given that 2 is a zero of $f(x) = kx^2 + 3x + k$. Therefore, $f(2) = 0 \Rightarrow k \times 2^2 + 3 \times 2 + k = 0 \Rightarrow 5k + 6 = 0 \Rightarrow k = -6/5$

3. The graph of a polynomial f(x) is shown in Fig BASIC



The number of zeroes of f(x) is

(a) 3 (b) 2 (c) 1 (d) 4

Ans. (d)

4. If α and β are the zeros of the polynomial $f(x) = px^2-2x+3p$ and $\alpha + \beta = \alpha\beta$, then the value of p is

(a) -2/3 (b) 2/3 (c) 1/3 (d) -1/3

Ans. (b)

5. If zeroes of the quadratic polynomial $f(x) = (k^2 +4) x^2 + 7x + 4k$ are reciprocal of each other, then the value (s) of k is (are)

(a) 1 (b) -1 (c) 2 (d) -2

Ans. (c)

ASSERTION-REASON BASED QUESTIONS (1 Mark)

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct answer.
(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.
1. Statement-1 (Assertion): 5x + 2 is a linear polynomial. BASIC
Statement-2 (Reason): A polynomial of degree 1 is a linear polynomial.

Ans. (a)

Solution: -we know that definition of degree of a polynomial is same as given in reason.

2. Statement-1(Assertion): A quadratic polynomial having $2+ \sqrt{3}$ and $2-\sqrt{3}$ as its zeroes is given by $f(x) = x^2-4x+1$.
Statement-2 (Reason): Quadratic polynomials whose two zeroes are α and β are given by $f(x) = k (x^2-x (\alpha +\beta) + \alpha\beta)$, where k is any non-zero real number
Ans. (a)

SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. The sum of the zeroes of the given quadratic polynomial $-3x^2 + k$ is BASIC

   Ans. 0

   Solution: - Since polynomial is $-3x^2+0x+k$ a = -3, b = 0, c = k and sum of zeroes = -b/a = 0 / -3 = 0

2. If one zero of the polynomial $x^2 – 4x +1$ is $2+ \sqrt{3}$, then the other zero is

   Ans. $2- \sqrt{3}$ BASIC

Solution: - Let other zero be α, $(2+\sqrt{3}) + \alpha = -b / a = -(-4) / 1 = 4$ $\alpha = 4 – (2 + \sqrt{3}) = 2 - \sqrt{3}$.

3. Find a quadratic polynomial whose zeroes are - 9 and -1/9.

   Ans. $9x^2 + 82x + 9$. BASIC

4. If zeroes of $p(x) = ax^2 + bx + c$ are negative reciprocal of each other, find the relationship between a and c.

   Ans. $a + c = 0$

5. The zeroes of the polynomial $(x-2)^2 + 4$ is

   Ans. No zero.

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

1. If α and β are the zeroes of the polynomial $x^2 - 2x - 15$, then form a quadratic polynomial whose zeroes are 2α and 2β. BASIC

Solution: - As α and β are zeroes of the polynomial $x^2 - 2x - 15$.

$\alpha + \beta = 2$ and $\alpha \times \beta = -15$ Now, $2\alpha + 2\beta = 2 \times 2 = 4$ and $2\alpha \times 2\beta = 4 \times (-15) = -60$

So required quadratic polynomial = $x^2 - 4x - 60$.

2. If one zero of the polynomial $(a^2 + 9) x^2 + 13x + 6a$ is reciprocal of the other, then find the value of a. BASIC

Solution: - Let one of the zeroes of the polynomial be α. Then the other zero will be 1/α. Now, product of zeroes = $\alpha \times 1/\alpha = 1$.

$\Rightarrow 6a / (a^2 + 9) = 1 \Rightarrow a^2 – 6a + 9 = 0 \Rightarrow (a – 3)^2 = 0 \Rightarrow a = 3$.

3. Find the quadratic polynomial if sum and product of whose zeroes are -1 and -20

   respectively. Also find the zeroes of the polynomial so obtained. BASIC

Answer: - Required polynomial is $x^2 + x – 20$. $\alpha = 4$ and $\beta = -5$.

4. If (x-2) is a factor of $x^3 + ax^2 + bx + 16$ and b= 4a, find the value of a and b.

Answer: - a = - 2 and b = - 8

5. If the sum and product of zeroes of the polynomial $p(x) = kx^2 + 2x + 3k$ is equal, then find the value of k.

Answer: - k = -2/3.

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. If the square of the difference of the zeroes of the quadratic polynomial $f(x) = x^2 + px + 15$ is equal to 196, then find the value of p. BASIC

Solution:- Let α and β be the zeroes of the polynomial $f(x) = x^2 + px + 15$.

Then $\alpha+\beta = -p$ and $\alpha\beta = 15$.

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = (-p)^2 - 4 \times 15 = p^2 - 60$.

$\Rightarrow 196 = p^2 - 60 \Rightarrow p = \sqrt{256} = 16$ or -16.

2. If α and β be the zeroes of the polynomial $f(x) = kx^2 + 2x - 15$, such that $\alpha^2 + \beta^2 = 34$. Then find the value of k. BASIC

Solution: - $\alpha^2 + \beta^2 = \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta = (\alpha+\beta)^2 - 2\alpha\beta = 34$

$\alpha+\beta = -2 / k$ and $\alpha\beta = -15 / k$

$(-2 / k)^2 - 2(-15 / k) = 34$

$\Rightarrow (17k + 2)(k - 1) = 0 \Rightarrow k = -2/17$ or 1.

3. Find the zeroes of the quadratic polynomial $f(x) = x^2 - 2\sqrt{a}\,x + (a - b)$. Verify relationship between zeroes and coefficients. BASIC

Answer: - $\alpha = (\sqrt{a} + \sqrt{b})$ and $\beta = (\sqrt{a} - \sqrt{b})$. $\alpha\beta = 2\sqrt{a}$ and $\alpha+\beta = a - b$.

4. If α, β, γ be zeros of polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$. Answer:- $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = 5$.

5. If one zero of the quadratic polynomial $4x^2 - 8kx + 8x - 9$ is negative of the other, then find the zeroes of the polynomial $kx^2 + 3kx + 2$.

Answer: - k = 1. $P(x) = x^2 + 3x + 2$, then the value of x = -2 or - 1.

CASE STUDY BASED QUESTIONS (4 MARKS)

CASE STUDY 1:

The below picture are few natural examples of parabolic shape which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms.




(i) In the standard form of quadratic polynomial $ax^2 + bx + c$, what is the characteristics of a, b, c?

(ii) If the roots of the quadratic polynomial $ax + bx + c$ are equal, then find the nature of discriminant D.

(iii) If α and 1/ α are zeroes of the quadratic polynomial $2x^2 - x + 8k$, then find the value of k.

OR

Is the graph of the polynomial $p(x)= x^2+1$ intersects any axis (by using the concept of D)?

Solution: -

(i) The general form of a quadratic polynomial is ax2 + bx + c, where a, b and c are real numbers such that a ≠ 0.

(ii) If the roots of a quadratic equation are equal, then its discriminant D is equal to zero.

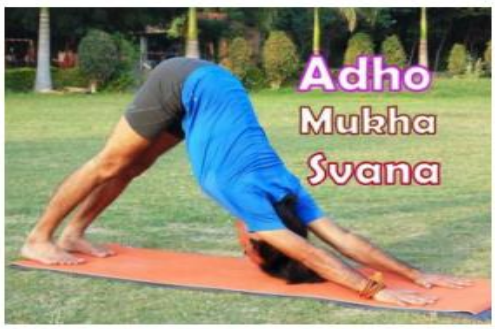
(iii) If α and 1/α are zeros of the quadratic polynomial $2x^2 -x+8k$, then αβ = 8k/2 = α x 1/α = 1
=> k = 1 / 4.

OR,

The discriminant D of the polynomial $p(x) = x^2+0x+1$ is given by D = 0 – 4 = - 4 < 0. So, it has no real zeroes. Consequently, its graph neither touches nor intersects any axis.

CASE STUDY 2:

An asana is a body posture, originally and still a general term for a sitting meditation pose, and later extended in hatha yoga and modern yoga as exercise, to any type of pose or position, adding reclining, standing, inverted, twisting, and balancing poses. In the figure, one can observe that poses can be related to representation of quadratic polynomial.



(i) The shape of the poses shown is_____

(ii) The graph of parabola opens downwards, if _____

(iii) The zeroes of the quadratic polynomial $p(x) = x^2 + 7x + 12$.

OR

Find a quadratic polynomial, the sum of whose zeroes is 0 and one zero is 5.

Solution: -

(i) Parabola

(ii) a < 0

(iii) $p(x) = x^2 + 7x + 12$

$\Rightarrow p(x) = (x + 3)(x + 4)$

$\therefore p(x) = 0 \; if \; x + 3 = 0 \; or \; x + 4 = 0$

$\Rightarrow x = -3 \; or \; x = -4$

∴ – 3 and – 4 are zeros of the p(x).

OR

Let zeroes are α and β.

⇒α + β = Sum of zeroes

⇒α + β = 0 ⇒ 5 + β = 0 ⇒β = –5

Now product of zeroes = αβ = 5 × (–5) = –25

Let polynomial $p(x) = ax^2 + bx + c$

$\therefore \alpha + \beta = 0 = \frac{-b}{a}; \alpha\beta = \frac{c}{a} = -25$

$\therefore a = 1, b = 0, c = -25$

Hence, $p(x) = x^2 - 25$

CASE STUDY 3:

Basketball is played with a spherical ball. Even through an athlete dibbles the ball in sports, a basketball player uses his hand. Usually, the basketball is played indoor on a court made out of wood. The projectile (path traced) of basketball in the form of parabola representing a quadrilateral polynomial.

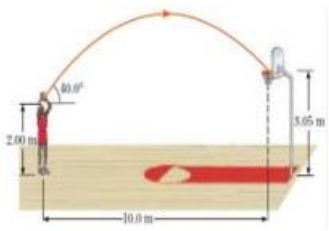


Fig 1

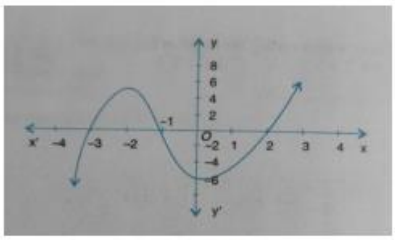


Fig 2

(i) The shape of the path traced shown in fig1, is a ……..

(ii) The polynomial representing the graph shown in fig2 has …… zeroes.

(iii) Write all zeroes of the polynomial representing in the fig2 .

OR

The graph shown in fig2 is represented by the polynomial ….

(a) $x^3 + 2x^2 - 5x - 6$ (b) $x^3 + 2x^2 - 5x + 6$

(c) $x^3 + 2x^2 + 5x - 6$ (d) $x^3 + 2x^2 + 5x + 6$

Solution :-

(i) Ans. parabola

(ii) Ans. 3

(iii) Ans. -3,- 1, 2.

OR,

Ans. (a).

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

An Overview

A pair of linear equations in two variables represents two straight lines in a plane. The solutions to these equations are the points where these lines intersect.

General Form

The general form of a pair of linear equations in two variables is:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

where x and y are variables, and a_1, b_1, c_1, a_2, b_2, and c_2 are constants.

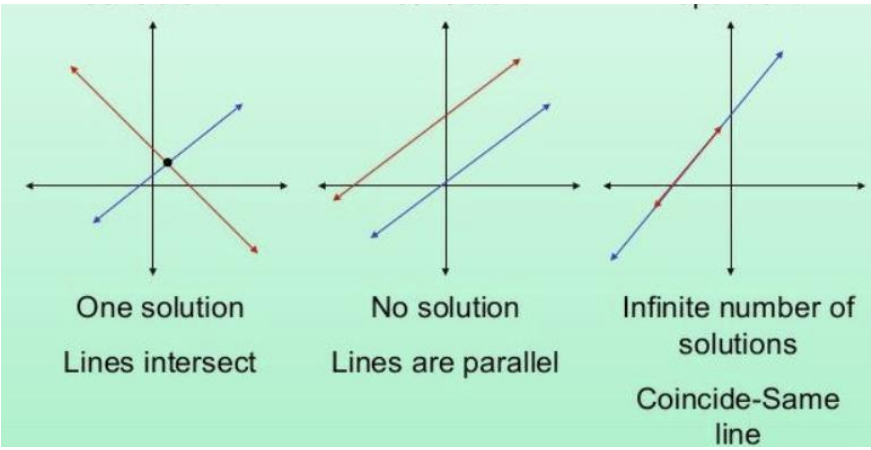
Graphical Representation

The graphical representation of the solutions can be of three types:

1. Intersecting Lines: If the lines intersect at a point, there is a unique solution.

2. Parallel Lines: If the lines are parallel, there is no solution.

3. Coincident Lines: If the lines coincide, there are infinitely many solutions.



Methods of Solving

There are several methods to solve a pair of linear equations:

1. Graphical Method: Plot both equations on a graph to find the intersection point.

2. Substitution Method: Solve one equation for one variable and substitute into the other equation.

3. Elimination Method: Add or subtract the equations to eliminate one variable, then solve for the other.

Example Problems

1. Solve the following pair of linear equations:
   $x + y = 5$
   $x - y = 1$

Solution using Substitution Method:
From the first equation, $y = 5 - x$. Substitute y in the second equation:
   $x - (5 - x) = 1$
   $2x - 5 = 1$
   $2x = 6$
   $x = 3$
Substitute $x = 3$ back into $y = 5 - x$:
   $y = 5 - 3 = 2$
Solution: $x = 3$, $y = 2$.

Consistency and Nature of Solutions

The system is consistent and has a unique solution if $a_1/a_2 \neq b_1/b_2$.
The system is inconsistent if $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.
The system has infinitely many solutions if $a_1/a_2 = b_1/b_2 = c_1/c_2$.

Learnings

Understanding pairs of linear equations in two variables and the methods to solve them is fundamental in algebra and has numerous applications in various fields such as physics, economics, and engineering.

Assertion-Reason based Questions

Question 1:

Assertion (A): The pair of linear equations x + y = 2 and 2x + 2y = 4 have infinitely many solutions.
Reason (R): The given pair of equations are coincident lines. Basic

Answer 1:

Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A). Standard

Question 2:

Assertion (A): The pair of linear equations 3x - y = 1 and 6x - 2y = 3 are inconsistent.
Reason (R): The given pair of equations have no common solution. Basic

Answer 2:

Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A). Standard

Multiple Choice Questions (MCQ)

Question 1:

Which of the following is the condition for a pair of linear equations a1x + b1y + c1 = 0 and a2x + b2y + c2 = 0 to be inconsistent? Standard

A. $a1/a2 \neq b1/b2$
B. $a1/a2 = b1/b2 \neq c1/c2$
C. $a1/a2 = b1/b2 = c1/c2$
D. None of these

Answer 1:

B. $a1/a2 = b1/b2 \neq c1/c2$

Question 2:

For which value of k will the equations 3x + 4y = 5 and 6x + ky = 10 represent parallel lines? Basic

A. 4
B. 8
C. 12
D. 16

Answer 2:

B. 8

Question 3:

If one equation of a pair of dependent linear equations is -5x + 7y = 2, then the second equation can be: Basic

A. 10x - 14y = -4
B. -10x + 14y = 4
C. -10x + 14y = -4
D. 10x - 14y = 4

Answer 3:
C. -10x + 14y = -4

Question 4:
The equations x + 2y = 5 and 2x + 4y = 10 have: Standard
A. A unique solution
B. Exactly two solutions
C. Infinitely many solutions
D. No solution

Answer 4:
C. Infinitely many solutions

Question 5:
The pair of equations y = 0 and y = -7 has: Standard
A. One solution
B. Two solutions
C. Infinitely many solutions
D. No solution

Answer 5:
D. No solution

Very Short Answer Type Questions (2 Marks)

Question 1:
Find the value of k for which the system of equations 3x - y + 8 = 0 and 6x - ky = -16 represents a pair of dependent equations. Standard

Answer 1:
k = 2

Question 2:
For what value of k will the equations 3x + 4y = 5 and 6x + ky = 10 have no solution?
Basic Answer 2:

k = 8

Question 3:

Solve the pair of linear equations: x + y = 7 and x - y = 1. Standard

Answer 3:
x = 4, y = 3

Question 4:
Determine whether the following pair of linear equations is consistent or inconsistent: 2x + 3y = 8 and 4x + 6y = 9. Standard

Answer 4:
The equations are inconsistent.

Question 5:
Find the value of k for which the lines given by the equations $2x - 3y + 7 = 0$ and $kx + 3y - 5 = 0$ are parallel. Basic

Answer 5:
$k = -2$

Short Answer Type Questions (3 Marks)

Question 1:
Solve the following system of equations graphically: $2x + y = 6$ and $2x - y = 2$. Basic

Answer 1:
Graph the equations and find the intersection point. The solution is $x = 2$, $y = 2$. Basic

Question 2:
Solve the pair of equations: $2x + 3y = 9$ and $4x + 6y = 18$. Basic

Answer 2:
The equations are dependent and have infinitely many solutions.

Question 3:
Solve the following system of equations by substitution: $x - y = 3$ and $3x + y = 9$.
Standard

Answer 3:
Substitute $y = x - 3$ into the second equation: $3x + (x - 3) = 9$, which simplifies to $4x - 3 = 9$, giving $x = 3$. Substitute $x = 3$ into $y = x - 3$ to find $y = 0$. Thus, $x = 3$, $y = 0$.

Question 4:
Find the value of 'a' for which the system of equations: $2x + 3y = 7$ and $ax + 6y = 14$ has no solution.
Standard

Answer 4:
$a = 4$

Question 5:
Solve the following pair of equations using the elimination method: $3x + 4y = 10$ and $2x - 2y = 2$.
Standard

Answer 5:
Multiply the second equation by 2: $4x - 4y = 4$. Add the modified second equation to the first equation to get $7x = 14$, giving $x = 2$. Substitute $x = 2$ into $2x - 2y = 2$ to find $y = 1$. Thus, $x = 2$, $y = 1$.

Long Answer Type Questions

Question 1:
Solve the following system of equations by the elimination method and verify the result: $x + 2y = 10$ and $2x + 3y = 20$.

Solve the following system of equations by the elimination method and verify the result: x + 2y = 10 and 2x + 3y = 20. Basic

Answer 1:

Multiply the first equation by 2: 2x + 4y = 20. Subtract the second equation from the modified first equation to get y = 0. Substitute y = 0 into x + 2y = 10 to find x = 10. Thus, x = 10, y = 0.
Verification: Substituting x = 10 and y = 0 into the second equation gives 2(10) + 3(0) = 20, which is correct.

Question 2:

Solve the system of equations: 3x - 5y = -1 and 2x + y = 8 by substitution method.

Basic

Answer 2:

Solve 2x + y = 8 for y: y = 8 - 2x. Substitute this into 3x - 5y = -1 to get 3x - 5(8 - 2x) = -1, which simplifies to 13x = 39, giving x = 3. Substitute x = 3 into y = 8 - 2x to find y = 2. Thus, x = 3, y = 2.

Question 3:

Draw the graph of
2y = 4x – 6; 2x = y + 3 and determine whether this system of linear equations has a unique solution or not. Standard

Answer 3:

$2y = 4x - 6$
$\Rightarrow y = 2x - 3$

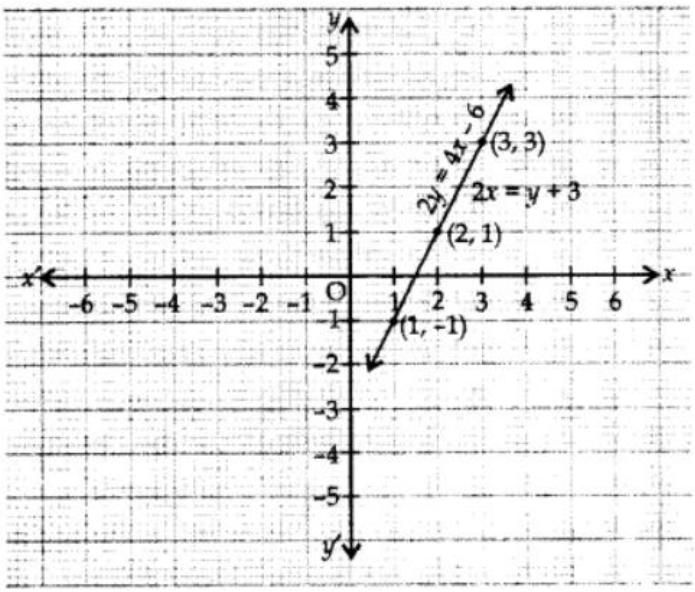
x	1	2	3
y	-1	1	3

(-1, 1), (2, 1), (3, 3)

$2x = y + 3$
$\Rightarrow y = 2x - 3$

x	1	2	3
y	-1	1	3

(1, -1), (2, 1), (3, 3)



Question 4:

Solve the following system of equations graphically: x + y = 7 and x - y = 1 and find the area of the triangle formed by these lines and the x-axis. Basic

Answer 4:

Graph the equations and find the intersection point. The solution is x = 4, y = 3. The points of intersection with the x-axis are (7,0) and (1,0). The area of the triangle is 1/2 x base x height = 1/2 x 6 x3 = 9 square units.

Question 5:

Find the point of intersection of the lines represented by the equations 5x - 2y = 1 and 3x + 4y = 18 using determinants. Basic

Answer 5:

The equations can be written as: 5x - 2y - 1 = 0 and 3x + 4y - 18 = 0. Using Cramer's rule:

$$x = \begin{vmatrix} -1 & -2 \\ -18 & 4 \end{vmatrix} / \begin{vmatrix} 5 & -2 \\ 3 & 4 \end{vmatrix} = -62 / 26 = -31/13$$

$$y = \begin{vmatrix} 5 & -1 \\ 3 & -18 \end{vmatrix} / \begin{vmatrix} 5 & -2 \\ 3 & 4 \end{vmatrix} = 83 / 26 = 83/26$$

Thus, x = -31/13, y = 83/26.

Case Study Based Questions

Case Study 1:

A boat covers 32 km upstream and 36 km downstream in 7 hours. It covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and the speed of the stream. Standard

Answer 1:

Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr. The speed upstream is (x - y) km/hr and downstream is (x + y) km/hr.

We have the system of equations:

$32/(x-y) + 36/(x+y) = 7$

$40/(x-y) + 48/(x+y) = 9$

Solving these equations, we get x = 12 km/hr and y = 4 km/hr.

Case Study 2:

Two trains are moving in opposite directions with speeds of 60 km/hr and 90 km/hr. If they start at the same point, find the time taken by the trains to be 300 km apart. Basic

Answer 2:

Let the time taken be t hours. The relative speed of the trains is (60 + 90) km/hr = 150 km/hr.
We have the equation: 150t = 300
Thus, t = 2 hours.

Case Study 3:

A manufacturer produces two types of toys A and B. The profit on toy A is $5 and on toy B is $7. The manufacturer can produce a maximum of 100 toys. The production of toy B takes twice as long as the production of toy A. If the manufacturer has a maximum of 160 hours for production, how many of each type of toy should be produced to maximize profit? Formulate this problem as a pair of linear equations. Standard

Answer 3:

Let x be the number of toy A produced and y be the number of toy B produced. We have the constraints:
x + y <= 100
x + 2y <= 160
To maximize profit, we need to solve the system of equations and the objective function:
Maximize P = 5x + 7y

QUADRATIC EQUATIONS

GIST OF THE LESSON

Important Concepts :

A quadratic polynomial of the form $ax^2 + bx + c$, where $a \neq 0$ and a, b, c are real numbers, is called a quadratic equation when $ax^2 + bx + c = 0$. Here a and b are the coefficients of x^2 and x respectively and „c" is a constant term. Any value is a solution of a quadratic equation if and only if it satisfies the quadratic equation.

Quadratic formula: The roots, i.e., α and β of a quadratic equation $ax^2 + bx + c = 0$ are given by $x=\frac{-b\pm\sqrt{D}}{2a}$, $b^2 - 4ac = D$ is known as the discriminant and is generally denote D. D helps us to determine the nature of roots for a given quadratic equation.

The rules are: If $D = 0 \Rightarrow$ The roots are Real and Equal. If $D > 0 \Rightarrow$ The two roots are Real and Unequal. If $D < 0 \Rightarrow$ No Real roots exist. If α and β are the roots of the quadratic equation, then Quadratic equation is: $k [x^2 - (\alpha + \beta) x + \alpha\beta]= 0$

or k[x^2 – (sum of roots) x + product of roots = 0

ASSERTION AND REASON

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
(c) If Assertion is correct but Reason is incorrect.
(d) If Assertion is incorrect but Reason is correct

1. Assertion: $(2x - 1)^2 - 4x^2 + 5 = 0$ is not a quadratic equation.
Reason: An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, where a, b, c ∈ R is called a quadratic equation. Basic

Solution:- $4x^2$-4x+1-$4x^2$+5=0

-4x+6=0

So assertion is correct.

Reason is also correct

Correct answer (a)

2Assertion: If one root of the quadratic equation $6x^2 - x - k = 0$ is 2/3, then the value of k is 2.

Reason: The quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has almost two roots. Standard

Solution :-Answer (b)

MCQ QUESTIONS

(1) The quadratic equation X^2 + x – 182 = 0 has degree Basic

(a) 0 (b) 1 (c) 2 (d) 3

ANS:-C(2)

(2) Which of the following are the roots of the quadratic equation x^2-9x+20=0 Basic

(a) 3,4 (b) 4 ,5 (c) 5,6 (d) 0,20

Ans (b) 4,5

(3) The sum of the squares of two consecutive natural numbers is 313. The numbers are

(a) 12, 13 (b) 13,14 (c) 11,12 (d) 14,15

Ans (a)12,13

(4) The equation $2x^2 + kx + 3 = 0$ has two equal roots, then the value of k is

(a) ±√6 (b) ± 4 (c) ±3√2 (d) ±2√6

Ans (d)±2√6

(5) The sum of two numbers is 13 and the sum of their reciprocals is 13/40. What are the two numbers?

a) 7,6 (b) 4,9 (c) 5,8 (d) 9,4

Ans (c) 5,8

VERY SHORT ANSWER TYPE

1. For what value of k, $x^2 + 4x + k$ is a perfect square Basic

Solution

A quadratic expression is a perfect square, if and only if corresponding equation has equal roots.
i.e., D = 0
⇒ 16 – 4k = 0
⇒ k = 4

2. Write the nature of roots if D<0

Soln:- roots are unreal

3. Find the roots by factorization :-x^2-12x+35 Basic

x^2+12x+35=0

x^2+7x+5x+35=0

x(x+7)+5(x+7)=0

(x+5)(x+7)=0x=−5, −7

4. Find the discriminant of the quadratic equation: - $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ Standard

SOLN:-D=9

5. If the roots of the equation $ax^2 + bx + c$ are real and equal, what will be the relation between a, b, c.

ANS:- $b=\pm 2\sqrt{ac}$

SHORT ANSWER TYPE QUESTION

1. If the sum of two natural numbers is 8 and their product is 15, find the numbers Basic

SOLN:-Let one number is x, another number is 8-x

A/Q

(x)(8-x)=15

$8x-x^2=15$

$x^2-8x=15$

$x^2-8x-15=0$

$x^2-5x-3x-15=0$

x(x-3)-3(x-5)=15

if x-3=0 ,x=3

if x-5=0 ,x=5

Numbers are 3, 5

2. Find the value of p so that the quadratic equation px(x – 3) + 9 = 0 has two equal roots Basic

Soln:-

px(x – 3) + 9 = 0

$px^2-3px+9=0$

a=p,b=-3p ,c=9

For equal roots, D=0

$(-3p)^2-4.p.9=0$

$9p^2-36p=0$

9p(p-4)=0

p-4= 0

p=4

3. If the sides of the right angled triangle is x+2, x+1, x then what is the value of x. Basic

Ans x=3

4. Solve the following quadratic equation for x: $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$ Standard

ANS $x=\frac{\sqrt{3}}{4}, \frac{-2}{\sqrt{3}}$

5. Solve the equation: - Standard

Find the roots of the equation $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

Ans. x=1, 2

Long answer type questions

1. A train travels 360km at a uniform speed. If the speed had been 5km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train. Basic

Solution-

Let the speed of train be $x\ km/hr$

distance= 360 km

Speed = $\frac{distance}{time}$

Time = $\frac{360}{x}$

New speed = $(x+5)km/hr$

Time = $\frac{D}{S}$

$$x+5 = \frac{360}{\left(\frac{360}{x}-1\right)}$$

$$(x+5)\left(\frac{360}{x}-1\right) = 360$$

$$(x+5)(360-x) = 360x$$

$$-x^2 - 5x + 1800 = 0$$

$$x^2 + 5x - 1800 = 0$$

$$x^2 + 45x - 40x - 1800 = 0$$

$$x(x+45) - 40(x+45) = 0$$

$$(x+45)(x-40) = 0$$

$$x+45 = 0 \quad , \quad x-40 = 0$$

$$x = -45 \quad , \quad x = 40$$

Speed cannot be negative

Speed of train =40km/hr

2. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹750. We would like to find out the number of toys produced on that day. Basic

Solution:

Let the number of toys produced in a day be x

Then, cost of production of each toy on that day = ₹ $(55-x)$

Total cost of production on that day = $x(55-x)$

According to question,

$$x(55-x) = 750 \Rightarrow 55x - x^2 = 750$$

$$\Rightarrow x^2 - 55x + 750 = 0 \Rightarrow x^2 - 25x - 30x + 750 = 0$$

$$\Rightarrow x(x-25) - 30(x-25) = 0 \Rightarrow (x-30)(x-25) = 0$$

$$\Rightarrow x - 30 = 0 \text{ or } x - 25 = 0$$

$$\Rightarrow x = 30 \text{ or } x = 25$$

Number of toys produced on that day was 25 or 30.

3. Find the consecutive natural numbers, the sum of whose squares is 145. Basic

Ans ; 8 and 9

4. Find the positive value(s) of k for which quadratic equations $x^2+kx+64=0$ and $x^2-8x+k=0$ both will have real and equal roots. Standard

Ans:- k=16

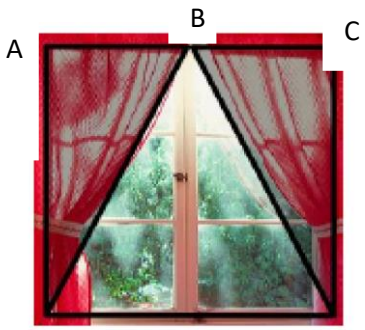
5. If the equation $(1+m^2)x^2+2mcx+c^2-a^2=0$ has equal roots then show that $c^2=a^2(1+m^2)$

Standard

CASE STUDY QUESTION

CASE STUDY 1 (BASIC)

Neelu wants to make the curtains for her window as shown in the figure. The window is in the shape of a rectangle, whose length and the breadth are in the ratio 2 : 3. If the area of the window is 864 square inches and AB =BC .



1 What is the shape of the curtains?

Soln:-right angled triangle

2. Find the dimensions of the window

Area of window=l×b=(2x) ×(3x)=$6x^2$

$6x^2 = 864$

$x^2 = 144$

X=12 inchse

Length =24 inchse, Breadth=36 inchse

3. What will be the perimeter of the window?

Perimeter = 2(24+36) =120 cm

(2) Nidhi and Ria are very close friends. Nidhi's parents own a Toyota Liva. Ria's parents own a Maruti Alto. Both the families decide to go for a picnic to Somnath temple in Gujrat by their own cars. Nidhi's car travels x km/h while Ria's car travels 5 km/h more than Nidhi's car. Nidhi's car uses 4 hrs more than Ria's car in covering 400 km.

1. In an hour, If Nidhi's car travels x km then what will be the distance covered by Ria's car in an hour?

Ans:- (x + 5) km

2. Frame the quadratic equation and solve to find the speed of Nidhi's car?

SOLN:- $\frac{400}{X} - \frac{400}{X+5} = 4$

$400(\frac{1}{X} - \frac{1}{X+5}) = 4$

$400(\frac{X+5-X}{X(X+5)}) = 4$

$400 \times 5 = 4(X)(X+5)$

$2000 = 4(X)(X+5)$

$X^2 + 5X - 500 = 0$

$X^2 + 25X - 20X - 500 = 0$

X=-25,20

X= -25 is not valid value

Speed of Nidhi car=20 km/hr

3.What is ratio of their (Nidhi's and Ria's) cars speed?

Or

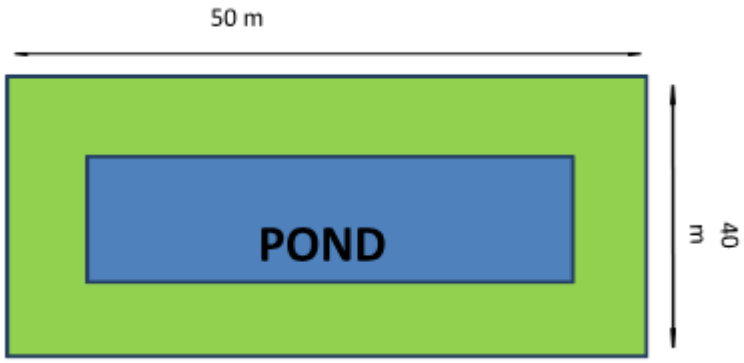
Find value of discriminant of above quadratic equation.

SOLN: - (3) RATIO= 4:5

OR

D=b^2-4ac=5^2-4.1.(-500)=2025

3. In a rectangular park of dimensions 50m X40m, a rectangular pond is constructed so that the green area strip of uniform width surrounding the pond will be 1184 sq.m



1 What is the area of rectangular park?

Answer: - Area= 2000 sq.m

2.If uniform width of strip is(x)m then find the dimensions of pond in terms of x.

Answer: - Length=50-2x, Breadth=40-2x

3. find the area of pond.

Answer:- 816 $square\ meter$

OR

4 Find the length and breadth of pond?

Length=34m , Breadth=24 m

ARITHMETIC PROGRESSION

An Overview
Sequences, Series and Progressions

- A **sequence** is a finite or infinite list of numbers following a specific pattern. For example, 1, 2, 3, 4, 5,… is the sequence, an infinite sequence of natural numbers.
- A **series** is the sum of the elements in the corresponding sequence. For example, 1+2+3+4+5….is the series of natural numbers. Each number in a sequence or a series is called a term.
- A **progression** is a sequence in which the general term can be expressed using a mathematical formula.
- An **arithmetic progression (AP)** is a progression in which the **difference** between two **consecutive** terms is constant.

General Form
The standard form of the arithmetic progression is given by the formula,

a, a + d, a + 2d, a + 3d, a + 4d, ….

The difference between two consecutive terms in an AP (*which is constant*) is the "common difference"(d) of an A.P.

General Term of AP

In Arithmetic progression, a_n is called the general term, where n represents the position of the term in the given sequence.

The nth Term of an AP

The nth term of an A.P is given by $a_n = a+(n-1)d$, where **a** is the first term, **d** is a common difference and **n** is the number of terms.

The Formula for the Sum to n Terms of an AP

The sum to n terms of an A.P is given by: $S_n = n/2(2a+(n-1)d)$

Where **a** is the first term, **d** is the common difference and **n** is the number of terms.
The sum of n terms of an A.P is also given by
$S_n = n/2(a+l)$
Where **a** is the first term, **l** is the last term of the A.P. and **n** is the number of terms.

Sample Questions

Assertion-Reason based Questions

Directions:
(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

(b) If both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
(c) If Assertion is correct but Reason is incorrect.
(d) If Assertion is incorrect but Reason is correct.

Question 1:
Assertion : If S_n is the sum of the first n terms of an A.P., then its nth term an is given by $a_n = S_n - S_{n-1}$.

Reason : The 10th term of the A.P. 5, 8, 11, 14, ……. is 35.
Basic

Answer 1:
(c) If Assertion is correct but Reason is incorrect.

Question 2:
Assertion : The sum of the series with the nth term, $t_n = (9 - 5n)$ is (465), when no. of terms n = 15.
Reason : Given series is in A.P. and sum of n terms of an A.P. is

Answer 2:
(d) If Assertion is incorrect but Reason is correct. Standard

Multiple Choice Questions (MCQ)

Question 1: The missing terms in AP: __, 13, __, 3 are:
(a) 11 and 9 (b) 17 and 9

(c) 18 and 8 (d) 18 and 9 Standard

Answer 1: (c)

Question 2: 11th term of the A.P. -3, -1/2, 2 …. Is
(a) 28 (b) 22 (c) -38 (d) -48 Basic

Answer 2: (b) 22

Question 3: Which term of the A.P. 3, 8, 13, 18, … is 78?
(a) 12^{th} (b) 13^{th} (c) 15^{th} (d) 16th

Answer 3:**(d) 16^{th}** Basic

Question 4: The 21st term of AP whose first two terms are -3 and 4 is:
(a) 17 (b) 137 (c) 143 (d) -143

Answer 4: (b) 137 Standard

Question 5: If 17th term of an A.P. exceeds its 10th term by 7. The common difference is:
Standard
(a) 1 (b) 2 (c) 3 (d) 4

Answer 5:(a) 1

Very Short Answer Type Questions (2 Marks)

Question 1: Is a, 2a, 3a, 4a, … an arithmetic progression? Standard

Answer 1: yes

Question 2:The sequence 28, 22, x, y, 4 is an AP. Find the values of x and y. Basic

Answer 2: The values of x and y are 16 and 10, respectively.

Question 3: Which term of AP 27, 24, 21, … is 0? Basic

Answer 3: n = 10**.** Therefore, the 10th term of AP is 0.

Question 4: Find the sum of first 40 positive integers divisible by 6 Standard

Answer 4: Numbers which are divisible by 6 are 6,12,18,…,240
Using the formula,

$S_n=\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow S_{40}=\frac{40}{2}[12+39(6)]$

$\therefore S_{40}=4920$

Question 5: **Prove that** $a_{m+n}+a_{m-n}=2a_m$**.** Standard

Short Answer Type Questions (3 Marks)

Question 1: How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78? Basic

Answer 1: n = 4 or 13

Question 2: **If the 3rd and the 9th terms of an AP are 4 and -8, respectively, then which term of this AP is zero.** Basic

Answer 2: Hence, 5th term of the given AP is 0.

Question 3: **How many multiples of 4 lie between 10 and 250?** Standard

Answer 3: n = 60

Question 4: **Find the sum of all three-digit numbers which leave remainder 3 when divided by 5.** Standard

Answer 4: S180=99090

Question 5: Ramkali saved Rs 5 in the first week of a year and then increased her weekly saving by Rs 1.75. If in the nth week, her weekly savings become Rs 20.75, find n.
Standard

Answer 5: n = 10

Long Answer Type Questions

Question 1: The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of the first sixteen terms of the AP. Basic

Answer 1:d = 1/2 or -1/2
S16 = 16/2 [2 + (16 – 1)(1/2)] = 8[2 + (15/2)] = 76

S16 = 16/2 [2(5) + (16 – 1)(-1/2)] = 8(5/2) = 20

Question 2: The eighth term of the AP. is half its second term and the eleventh term exceeds one third of the fourth term by 1. Find out the 15th term. Basic

Answer 2: 15th term is 3.
Question 3: An A.P. contains 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find out the A.P. Standard

Answer 3: the AP is a, a + d, a + 2d….

i.e. 3, 7, 11….

Question 4:
Find out the 31st term of the AP. whose 11th term is 38 and the 16th term is 73. Standard

Answer 4: 31st term is 178.

Question 5: The sum of the first five terms of the AP. and the sum of the first seven terms for the same A.P. is 167. If the sum of the first ten terms of the AP. is 235, find out the sum of the first twenty terms. Standard

Answer 5: The sum of the first 20 terms is 970.

Case Study Based Questions

Case Study 1:
India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV

sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year. **Standard**

Based on the above information, answer the following questions:

1. Find the production during first year.

2. Find the production during 8th year.

3. Find the production during first 3 years.

4. In which year, the production is Rs 29,200.

5. Find the difference of the production during 7th year and 4th year.

Answer 1:

1. Rs 5000
2. Production during 8th year is (a+7d) = 5000 + 2(2200) = 20400

3. Production during first 3 year = 5000 + 7200 + 9400 = 21600

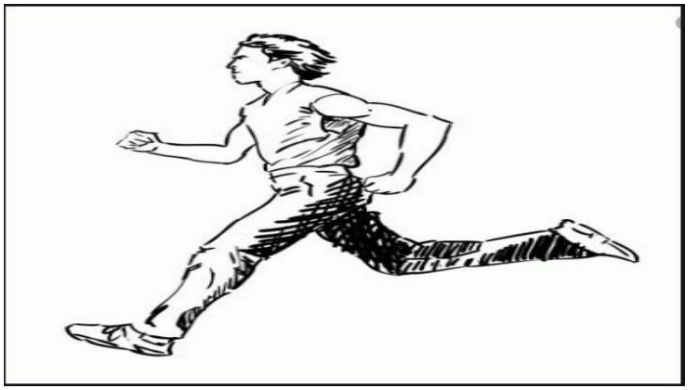
4. N = 12 5.

5. Difference = 18200 - 11600 = 6600

Case Study 2:

Your friend Veer wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds.

Basic



1. Which of the following terms are in AP for the given situation

a) 51,53,55….

b) 51, 49, 47….

c) -51, -53, -55….

d) 51, 55, 59…

2. What is the minimum number of days he needs to practice till his goal is achieved

a) 10 b) 12 c) 11 d) 9

3. Which of the following term is not in the AP of the above given situation

a) 41 b) 30 c) 37 d) 39

4. If nth term of an AP is given by an = 2n + 3 then common difference of an AP is

a) 2 b) 3 c) 5 d) 1

5. The value of x, for which 2x, x+ 10, 3x + 2 are three consecutive terms of an AP

a) 6 b) -6 c) 18 d) -18

Answer 2:
Answer: 1 b) 51, 49, 47….
Answer: 2 c) 11
Answer: 3 b) 30
Answer: 4 a) 2
Answer: 5 a) 6

Case Study 3:
Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of Rs 1,18,000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month, answer the following:

Standard

1. The amount paid by him in 30th instalment is

a) 3900 b) 3500 c) 3700 d) 3600

2. The amount paid by him in the 30 instalments is

a) 37000 b) 73500 c) 75300 d) 75000

3. What amount does he still have to pay offer 30th instalment?

a) 45500 b) 49000 c) 44500 d) 54000

4. If total instalments are 40 then amount paid in the last instalment?

a) 4900 b) 3900 c) 5900 d) 9400

5. The ratio of the 1st instalment to the last instalment is

a) 1:49 b) 10:49 c) 10:39 d) 39:10

Answer 3: Answer:1 a) 3900

Answer: 2 b) 73500

Answer: 3 c) 44500

Answer: 4 a) 4900

Answer:5 b) 10:49

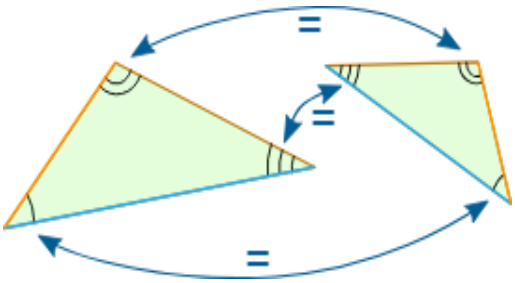
TRIANGLES

GIST OF THE TOPIC

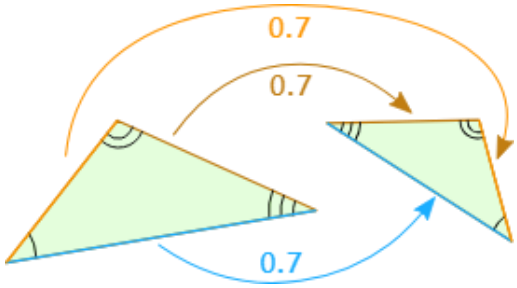
Two triangles are Similar if the only difference is size (and possibly the need to turn or flip one around).

For similar triangles:

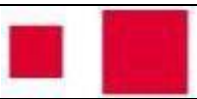
All corresponding angles are equal



All corresponding sides have the same ratio.



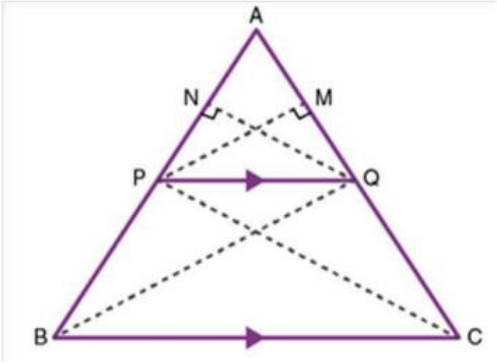
Congruent vs. Similar figures

	Congruent	*Similar*
Angles	Corresponding angles are same.	Corresponding angles are same.
Sides	Corresponding sides are same.	Corresponding sides are proportional.
Example		
Explanation	Both the square have the same angles and same side.	Both the squares have same angles but not the same sides.
Symbols	≅	~

Basic Proportionality Theorem (Thales Theorem)

According to **Thales theorem: -**

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.



Given, ΔABC in which PQ //BC

To prove: $\frac{AP}{PB} = \frac{AQ}{QC}$

Construction: Join BQ and CP

And draw QN⊥AB and also draw PM⊥AC.

Proof

Now the area of ΔAPQ $= \frac{1}{2} \times AP \times QN$

(Since, area of a triangle= $\frac{1}{2}$× Base × Height)

Similarly, area of ΔPBQ $= \frac{1}{2} \times PB \times QN$

area of ΔAPQ $= \frac{1}{2} \times AQ \times PM$

Also, area of ΔQCP $= \frac{1}{2} \times QC \times PM$

Now, if we find the ratio of the area of triangles ΔAPQ and ΔPBQ, we have

Area of $\Delta APQ = \frac{1}{2} \times AP \times QN$

Area of $\Delta PBQ = \frac{1}{2} \times PB \times QN$

$\frac{\text{Area of } \Delta APQ = \frac{1}{2} \times AP \times QN}{\text{Area of } \Delta PBQ = \frac{1}{2} \times PB \times QN} = \frac{AP}{PB}$ ..... (i)

Area of $\Delta APQ = \frac{1}{2} \times AQ \times PM$

Area of $\Delta QCP = \frac{1}{2} \times QC \times PM$

$\frac{\text{Area of } \Delta APQ = \frac{1}{2} \times AQ \times PM}{\text{Area of } \Delta QCP = \frac{1}{2} \times QC \times PM} = \frac{AQ}{QC}$ ..... (ii)

According to the property of triangles, the triangles drawn between the same parallel lines and on the same base have equal areas.

Therefore, we can say that ΔPBQ and QCP have the same area.

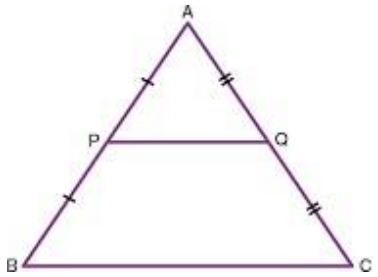
Area of ΔPBQ = area of ΔQCP ..... (iii)

Therefore, from the equations (i), (ii) and (iii) we can say that,

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

Converse of Thales theorem: If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.

If $\frac{AP}{PB} = \frac{AQ}{QC}$ then PQ‖ BC



(AAA **similarity criterion**) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.

IF $\angle A = \angle Q = 40^O$

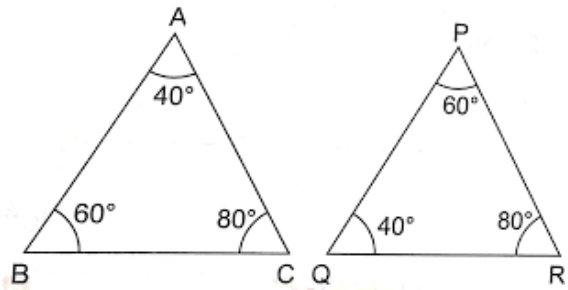
$\angle B = \angle P = 60^O$

$\angle C = \angle R = 80^O$

Then,

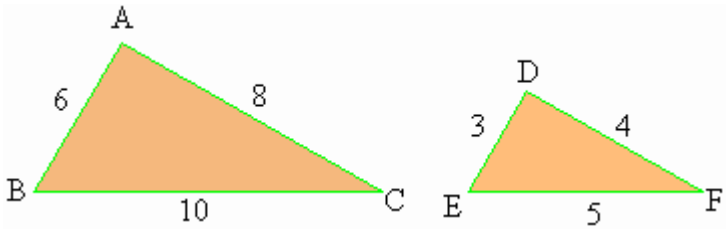
$$\frac{BC}{PR} = \frac{AC}{QR} = \frac{AB}{PQ}$$

and $\Delta ABC \sim \Delta QPR$



(SSS **similarity criterion**) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.

$IF \quad \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{2}{1}$



Then, $\angle A = \angle D$, <C=<F,<B=<E

and $\Delta ABC \sim \Delta DEF$

(SAS similarity criterion)

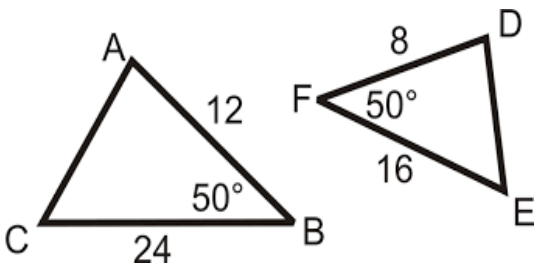
If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.

$\angle B = \angle F = 50^0$

And

$$\frac{AB}{DF} = \frac{12}{8} = \frac{3}{2} \; and \; \frac{BC}{EF} = \frac{24}{16} = \frac{3}{2}$$

And $\Delta ABC \sim \Delta DFE$



MAIN CONCEPTS AND RESULTS

** Two figures having the same shape but not necessarily the same size are called similar figures.

** All the congruent figures are similar but the converse is not true.

** Two polygons of the same number of sides are similar, if

(i) their corresponding angles are equal and

(ii) their corresponding sides are in the same ratio (i.e., proportion).

** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then

the other two sides are divided in the same ratio.

** If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

** If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio

and hence the two triangles are similar (AAA similarity criterion).

** If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle,

then the two triangles are similar (AA similarity criterion).

** If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal

and hence the triangles are similar (SSS similarity criterion).

** If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are

in the same ratio (proportional), then the triangles are similar (SAS similarity criterion).

Triangles
Similarity
(i) Corresponding angles are equal
(ii) Corresponding sides are in the same ratio
ΔABC ~ ΔPQR
Theorems
5. If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.(SAS criterion)
If AB/DE = AC/DF & ∠A = ∠D
then, ΔABC ~ ΔDEF
4. If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.(SSS criterion)
If AB/DE = BC/EF = CA/FD
then, ∠A = ∠D, ∠B = ∠E, ∠C = ∠F
ΔABC~ΔDEF
3. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.(AAA criterion)
If ∠A = ∠D, ∠B = ∠E, ∠C = ∠F
then, AB/DE = BC/EF = AC/DF
ΔABC~ΔDEF
Summary
In ΔABC, let DE || BC. Then,
(i) AD/DB = AE/EC
(ii) AB/DB = AC/EC
(iii) AD/AB = AE/AC
1. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
If, DE || BC
then AD/DB = AE/EC
2. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
If AD/DB = AE/EC
then, DE || BC
Trace the Mind Map
First Level
Second Level
Third Level

MIND MAP

Assertion and reason questions(1Mark)

1. Assertion (A): In ABC, D and E intersects AB and AC respectively, such that DE || BC. If AE = 4cm, EC = 3 cm, BD = 6 cm, then DA = 8 cm
   Reason(R): If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion.

   (Basic)

   (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
   (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
   (c) Assertions (A) is true but reason (R) is false.
   (d) Assertions (A) is false but reason (R) is true.

Ans:- (a) $\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{AD}{6} = \frac{4}{3} \Rightarrow AD = \frac{24}{3} = 8\ cm$

2. Assertion (A): In Triangle POR, L and M are points on sides PQ and PR respectively, such that PL=PM. If <Q= < R, then LM is not parallel to QR.
   Reason(R): If a line is divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

   (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
   (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A). (c) Assertions (A) is true but reason (R) is false.
   (d) Assertions (A) is false but reason (R) is true.
   Ans: - (d)

MULTIPLE CHOICE QUESTIONS (1 mark)

1. If in △ CAB and △ FED, $\frac{AB}{EF} = \frac{BC}{FD} = \frac{AC}{ED}$ then Basic

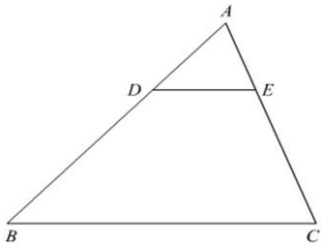
   (A) △ ABC~△ DEF
   (B) △ CAB~△ DEF
   (C) △ ABC~△ EFD
   (D) △ CAB~△ EFD
   Answer: (C) △ ABC~△ EFD

2. If in triangle ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when BASIC

(A) ∠B = ∠E (B) ∠A = ∠D (C) ∠B = ∠D (D) ∠A = ∠F

Answer: (C) ∠B = ∠D

3. △ ABC is an acute angled triangle. DE is drawn parallel to BC. Which of thefollowing are always true?



i) △ABC ~ △ ADE
ii) AD/BD= AE/EC
iii) DE= BC/2
(A) Only (i)
(B) (i) and (ii) only
(C) (i), (ii) and (iii)
(D) (ii) and (iii) only
Answer: (B) (i) and (ii) only

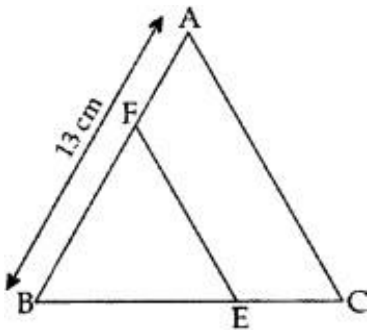
4. If Δ ABC and Δ DEF are similar such that 2AB = DE and BC = 8 cm, then Find EF.
(A) 16 cm (B) 12 cm (C) 8 cm (D) 4 cm
Answer: (A) 16 cm

5. If the ratio of the corresponding sides of two similar triangles is 4:5, then the ratio of corresponding altitudes is:
(A) 5:4 (B) 4:5 (C) 8:10 (D) 16:25
Answer: (B) 4:5

SHORT ANSWER TYPE QUESTIONS (2 marks)

1. In the figure, EF || AC, BC = 12 cm, AB = 13 cm and EC = 3 cm, find AF.

(BASIC)



Answer:
BE = BC – EC = 12 – 3 = 9 cm
Let AF = x cm, then BF = (13 – x) cm
In ΔABC, EF || AC … [Given

$\frac{BF}{FA} = \frac{BE}{EC}$

$\frac{13 - x}{x} = \frac{9}{3} \Rightarrow 3x = 13 - x$ (Thales' theorem)

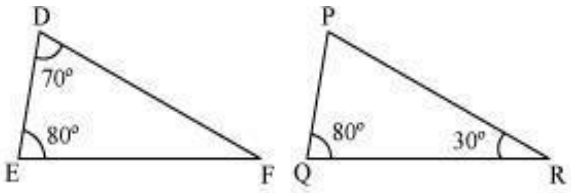
$\Rightarrow 4x = 13$

$\Rightarrow x = 13/4 = 3.25$ cm

2. Write the pair of similar triangles in the symbolic form and also write the similarity criterion used by you for answering the question.

(BASIC)

Answer: $\triangle DEF \sim \triangle PQR$(ASA)



3. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why? **(BASIC)**

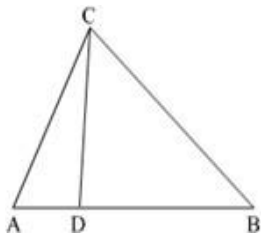
Answer: Let the sides of first triangle be x, y and z.As per given information, sides of second triangle will be $3x$, $3y$ and z'.

Since, the perimeter of other triangle is three times the perimeter of first triangle.

$\Rightarrow 3x + 3y + z' = 3(x + y + z)$

$\Rightarrow 3x + 3y + z' = 3x + 3y + 3z$

$\Rightarrow z' = 3z$



Here, the corresponding three sides of triangle are in proportion.

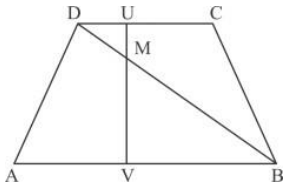
Hence, the two triangles are similar by SSS criteria of similarity.

4. In the given, if $\angle ACB = \angle CDA$, AC = 6 cm and AD = 3 cm, find BD.

Answer: BD = 9 cm

5. In the given Figure,
Prove that $\triangle DMU \sim \triangle BMV$.

Answer: Proof

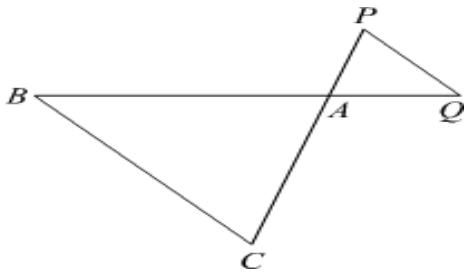


SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. In the given figure, $\triangle ACB \sim \triangle APQ$. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ.

(BASIC)

Answer: Since, $\triangle ACB \sim \triangle APQ$



$$\frac{AC}{AP} = \frac{CB}{PQ} = \frac{AB}{AQ}$$

$$\frac{AC}{2.8} = \frac{8}{4} = \frac{6.5}{AQ}$$

$$\frac{AC}{2.8} = \frac{8}{4} \Rightarrow AC = 5.6 \text{ cm}$$

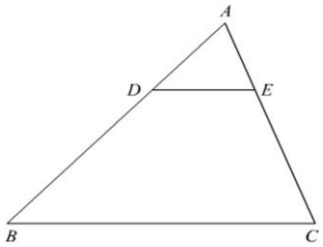
And

$$\frac{6.5}{AQ} = \frac{8}{4} \Rightarrow \text{AQ} = 3.25 \text{ cm}$$

2. In the given figure, DE || BC such that $AE = \frac{1}{4}AC$. If AB = 6 cm, find AD.

(BASIC)

Answer: It is given that, DE || BC & $AE = \frac{1}{4}AC$ and AB = 6 cm



We have to find *AD*.
Since ΔADE ~ ΔABC

$$\frac{AD}{AB} = \frac{AE}{AC}$$

So $\frac{AD}{6} = \frac{1}{4} \Longrightarrow 4$ X AD = 6 $\Longrightarrow$ AD = 6/4 =3/2 $\Longrightarrow$ AD = 1.5 cm

3. Using Basic proportionality theorem, prove that a line drawn through the mid-points ofone side of a triangle parallel to another side bisects the third side.

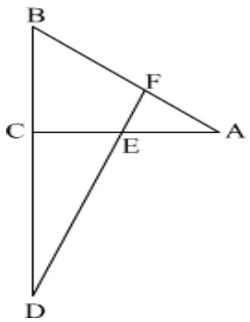
(BASIC)

Answer: Proof

4. In ΔABC, D and E are the points on the sides AB and AC respectively such that DE || BC. If AD = 6x – 7, DB = 4x – 3, AE = 3x – 3, and EC = 2x – 1 then find the value of 'x'.

Answer: 2

5. In the given figure, line segment DF intersect the side AC of a triangle ABC at the point E such that E is the mid-point ofCA and ∠AEF = ∠AFE. Prove that



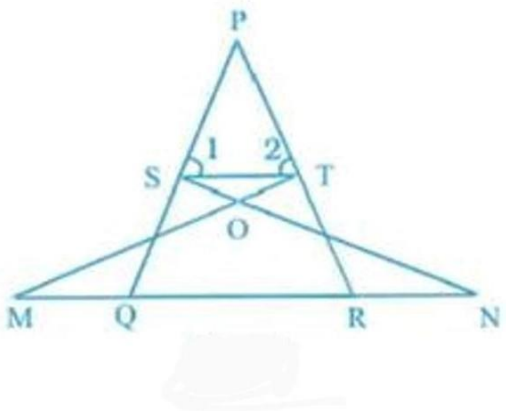
$$\frac{BD}{CD} = \frac{BF}{CE}$$

Answer: Proof

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. If ∠1 =∠2 and Δ NSQ ≅ Δ MTR, then prove that Δ PTS ~ Δ PRQ.

(BASIC)



Answer: Since, Δ NSQ ≅ Δ MTR
SQ = TR (CPCT)------- (i)
We have PS = PT-------(ii)
and SQ = TR---------(iii)
Now divide (ii) by (iii), we have

$$\Rightarrow \frac{PS}{SQ} = \frac{PT}{TR}$$

Add both sides by 1, we have

$$\Rightarrow \frac{PS}{SQ} + 1 = \frac{PT}{TR} + 1$$

$$\Rightarrow \frac{PS + SQ}{SQ} = \frac{PT + TR}{TR}$$

$\Rightarrow \frac{PQ}{SQ} = \frac{PR}{TR}$

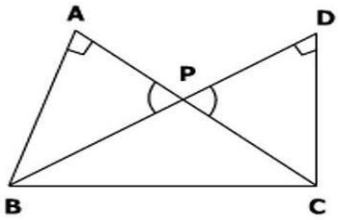
PQ = PR (As SQ = TR from (i))

Now, we can write $\frac{PS}{PQ} = \frac{PT}{PR} and \angle P = \angle P$

Δ PTS ~ Δ PRQ (SAS similar criterion)

2. Two right triangles ABC and DBC are drawn on the same hypotenuse BCand on the same sides of BC. If AC and DB intersect at P, prove that AP x PC = BP x PD.

(**BASIC**)



Answer: To prove: AP x PC = BP x PD.

Proof: In AABP and ADCP

$\angle A = \angle D = 90°$ (given)

$\angle APB = \angle DPC$ (vertically opposite angles)

.. Δ ABP ~ Δ DCP (AA similarity axiom)

.. $\frac{AB}{DC} = \frac{BP}{CP} = \frac{AP}{DP}$ (corresponding sides of similar ▲s are proportional) ..(1)

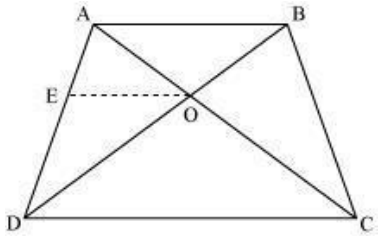
From (1) $\frac{BP}{CP} = \frac{AP}{DP}$

By cross multiplication, BP × DP = AP x PC (proved).

3. The diagonals of a quadrilaterals ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$ show that ABCD is a Trapezium.

(BASIC)

Answer: Proof.



4. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

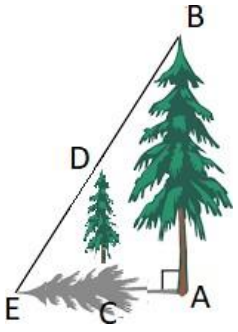
Answer: proof

5. If D, E, F are the mid-points of sides BC, CA and AB respectively of a $\triangle ABC$, then prove $\Delta ABC \sim \Delta DEF$.

Answer: Proof

CASE STUDY QUESTIONS (4 Marks)

1. In Ravi's school garden. Two trees are standing to each other. The bigger tree is 8 m high, casts a shadow of 6m.



(BASIC)

Answer the following questions:

i) If AB and CD are the two trees and AE is the shadow of the longer tress. Then write the symbolic form of similar triangles.

ii) Write the corresponding ratios of heights of the tress and length of their shadows.

iii) If the ratio of the height of two trees is 3:1, then the shadow of the smaller trees is.

iv) Find the distance of point B from E.

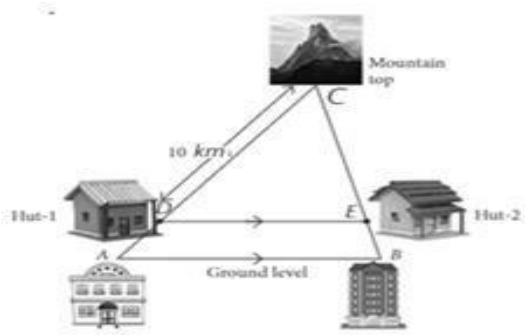
Answer: i) $\Delta EDC \sim \Delta EBA$

ii) $\frac{AB}{DC} = \frac{AE}{EC} = \frac{8}{6} = \frac{4}{3}$

iii) $\frac{AB}{CD} = \frac{6}{EC} \Rightarrow \frac{3}{1} = \frac{6}{CE} \Rightarrow CE = 2$ m

iv) the distance of point B from E = 10 m

2. Mountain Trekking: Two hotels are at the ground level on either side of a mountain. On moving a certain distance towards the top of the mountain two huts are situated as shown in the figure. The ratio between the distance from hotel B to hut-2 and that of hut-2 to mountain top is 3:6. Based on the above information, answer the following questions.



(i) What is the ratio of the distance between mountain top and Hut-1 to the distancebetween Hut-1 and hotel A?

(ii) The distance between the hotel A and hut-1 is_____

(iii) If the horizontal distance between the hut-1 and hut-2 is 8 km, then the distancebetween the two hotels

(iv) If the distance between the mountain top and Hut-2, the distance between thehotel B and hut-2 is_____

Answer:

i) DE//AB, $\Delta ABC \sim \Delta DEC$ (AA – similarity criterion)

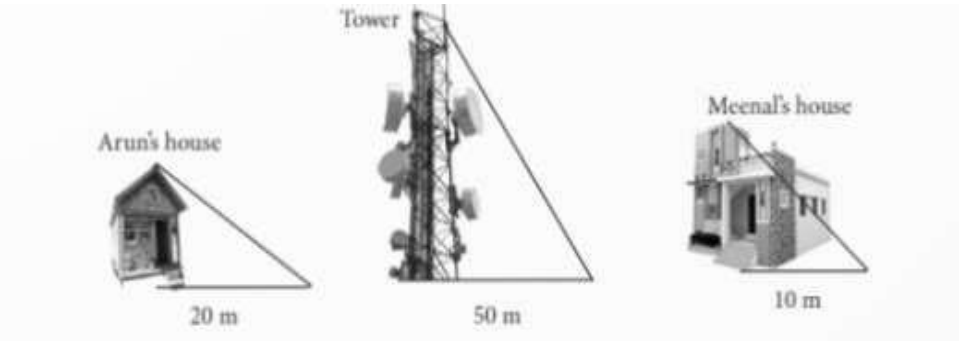
$\frac{DC}{AD} = \frac{CE}{EB} = \frac{6}{3} = \frac{2}{1}$

ii) $\frac{10}{AD} = \frac{2}{1} \Rightarrow AD = 5$ km. (The distance between the hotel A and hut-1 is 5 km)

iii) $\frac{DE}{AB} = \frac{CE}{CB} = \frac{6}{9} \Rightarrow \frac{8}{AB} = \frac{2}{3} \Rightarrow AB = 12$ km. (the distance between the two hotels 12 km)

iv) $\frac{DC}{AD} = \frac{EC}{EB} \Rightarrow \frac{10}{5} = \frac{EC}{2} \Rightarrow EC = 4$ km (the distance between the hotel B and hut-2 is 4 km)

3. Meenal was trying to find the height of tower near her house. She is using the properties ofsimilar triangles. The height of Meenal's house is 20 m. When Meenal's house casts a shadow of 10m long on the ground, at the same time, tower casts a shadow of 50 m long and Arun's housecasts a shadow of 20 m long on the ground as shown below.



Based on the above information, answer the following questions.

(i)What is the height of the tower?

(a) 100m (b) 50m (c) 15m (d) 45m

(ii) What is the height of Arun's house?

(a) 80m (b) 75m (c) 60m (d) 40m

(iii) If tower casts a shadow of 40m, then find the length of shadow of Arun's house

(a) 18m (b) 16m (c) 17m (d) 14m

CO-ORDINATE GEOMETRY

GIST OF THE LESSON

1.**Coordinate Geometry:** co-ordinate geometry is a branch of geometry where the position of the points on the plane is defined with the help of an ordered pair of numbers, known as coordinate of the point.

2. **Coordinate of a point:** In coordinate geometry every point of a plane must be represented by an ordered pair like as (a,b), where first number 'a ' called x- coordinate or abscissa is the distance of the point from the y-axis and second number 'b' called y- coordinate or ordinate is the distance of the point from the x-axis.

3.Distance Formula: The distance between two points $p(x_1 , y_1)$ and $Q(x_2 ,y_2)$ is given by

$PQ= \checkmark[(x_2-x_1)^2 +(y_2-y_1)^2]$

4.Use of Distance Formula:

Collinear points: To show collinearity of three points, we prove that the sum of the distance between two pair of points is equal to the third pair of points.

AB+BC = AC.

5.Section Formula: The coordinates (x,y) of the point which divides the line segment joining the points. (x_1 , y_1) and (x_2 ,y_2) internally in the ratio m:n are given by

$$\left(x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}\right)$$

6. **Coordinate of Mid-point of a Line segment joining Two points (x_1, y_1) and (x_2 ,y_2):** Obviously Mid-point of a line segment divides the line segment in the ratio 1:1.

Therefore the coordinates of mid-point are

$$\left(\frac{1.x_2 + 1.x_1}{1+1}, \frac{1.y_2 + 1.y_1}{1+1}\right) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Assertion and reason questions (BASIC)

The following questions consists of two statements --Assertion (A) and Reason (R). Answer these questions selecting the appropriate option below:

(a) Both A and R are true and R is the correct explanation for A.
(b) Both A and R are true but R is not the correct explanation for A.
(c) A is true but R is false.
(d) A is false but R is true.

1. **Assertion (A):** Point P(0,2) is the point of intersection of y-axis with the the line 3x+2y=4
Reason (R) : The distance of point P(0,2) from x-axis is 2 units.

Solution:- The line 3x+2y=4 cuts y-axis when x=0

3×0+2y=4

2y=4

Y=2

Point p(0,2) is the point of intersection of y-axis with the line 3x+2y=4 . Also the distance of any point p(x,y) from x-axis is its y coordinate.

Distance of the point p(0,2) is 2 units from the x-axis.

Thus, both A and R are true but R is not the correct explanation of A.

Option (b) is correct.

STANDARD

2. **Assertion (A):** If the points A(4,3) and B(x,5) lie on a circle with centre O(2,3), then the value of x is 2.

Reason (R) : Centre of a circle is the Mid-point of each chord of the circle.

Answer – Option (c) is correct.

MULTIPLE CHOICE QUESTIONS

(BASIC)

1. The distance of the point (-6,8) from x-axis is

(a) 6 units (b) -6 units (c) 8 units (d) 10 units

2. If A(3,√3), B(0,0) and C(3,k) are the three vertices of the equilateral triangle ABC, then the value of k is

(a) 2 (b) -3 (c) -√3 (d) -√2

3. The coordinate of the point which is reflection of the point (-3,5) in X-axis are

(a) (3,5) (b) (3,-5) (c) (-3,-5) (d) (-3,5)

STANDARD

4. AD is median of ΔABC with vertices A(5,-6), (B(6,4) and C(0,0), length of AD is

(a)√68 units (b) 2√15 units) (c) √101 units (d) 10 units

5. What is the ratio in which the line segment joining (2,-3) and (5,6) is divided by x-axis

(a) 1:2 (b) 2:1 (c) 2:5 (d) 5:2

Solution (1):- The distance of the point (-6,8) from x-axis is its y coordinate i.e.,8 units.

Option (c) is correct.

Solution (2):- ΔABC is an equilateral triangle

So AB=BC=CA

Or, $AB^2=BC^2$

Or, $(0-3)^2+ (0-\sqrt{3})^2 = (3-0)^2+. (k-0)^2$

Or, $k^2=3$

Or, $k=\pm\sqrt{3}$

Option (c) is correct.

Answer (3):- option (c) is correct.

Answer (4):- option (a) is correct.

Answer (5):- option (a) is correct.

VERY SHORT ANSWER QUESTIONS

(BASIC)

1. Show that the points (-2,3),(8,3) and (6,7) are the vertices of the right angled triangle.

2. If the distance of P(x,y) from A(5,1) and B(-1,5) are equal, then prove that 3x=2y.

3. Find the ratio in which P(4, m) divides the line segment joining the points A(2,3) and B(6-3).

STANDARD

4. If (a,b) is the Mid-point of the line segment joining the points A(10,-6) and B (k,4) and a- 2b=18. Find the value of k and the distance AB.
5. If two adjacent vertices of parallelogram are (3,2) and (-1,0) and the diagonals intersect at (2,-5), then find the coordinates of the other two vertices.

Solution:- (1) Let given points are A(-2,3), B(8,3) and C(6,7)

$AB^2= (8+2)^2+(3-3)^2=100$

$BC^2=(6-8)^2+(7-3)^2=4+16=20$

$AC^2=(6+2)^2+(7-3)^2=64+16=80$

Clearly,$AB^2=BC^2+AC^2$

Therefore ΔABC is a right angle triangle.

Solution:- (2). Here PA = PB

By distance formula,

$(5-x)^2+(1-y)^2$ =. $(-1-x)^2+(5-y)^2$

Or,$(5-x)^2+(1-y)^2 = (1+x)^2+(5-y)^2$

Or,$25-10x+x^2+1-2y+y^2 = 1+2x+ x^2+ 25-10y+y^2$

Or,$-10x-2y=2x-10y$

Or, $8y =12x$

Or,$3x=2y$

Hence proved.

Answer:- (3) Value of m is 0, the point is p(4,0)

Answer:- (4). $2\sqrt{61}$ units

Answer:- (5) Hence coordinates are (1-12) and. (5,-10).

SHORT ANSWER QUESTIONS

(BASIC)

1. If (-5 ,3) and (5,3) are two vertices of an equilateral triangle,then find the coordinates of the third vertex, given that origin lies inside the triangle.
2. Find the coordinates of the point on the x-axis which is equidistant from the points A(2,-5) and B(-2,9).
3. In what ratio does the point (-4, y) divide the line segment joining the points A(-6,10) and B(3,-8) if it lies on AB ? Hence find the value of y.

STANDARD

4. In what ratio does the point (24/11, y) divide the line segment joining the points P(2-,2) and Q(3,7)? Also find the value of y.
5. Find the ratio in which the line segment joining the points A (6,3) and B(-2,-5) is divided by x-axis.

Solution:- (1) Let third vertex be p(x,y) of ΔPAB

PA=PB=AB

Now, PA=PB

Squaring both sides

$PA^2=PB^2$

Or, $(x+5)^2+(y-3)^2 = (x-5)^2+ (y-3)^2$

Or, $x^2+10x+25= x^2-10x +25$

Or, x= 0

Also PB=AB

Squaring both sides

$PB^2=AB^2$

Or, $(x-5)^2+(y-3)^2=10^2$

Or, $(0-5)^2+(y-3)^2=100$

Or, $(y-3)^2= 100-25=75$

Or,$y= 3\pm 5\sqrt{3}$

Therefore coordinates of third vertex is $(0,3\pm 5\sqrt{3})$

Solution: - (2)Let P(x,0) be any point on X-axis

Now P(x,0) is equidistant from the point A(2,-5) and B(-2,9).

AP= BP

Squaring both sides

$AP^2=BP^2$

Or, $(x-2)^2+(0+5)^2= (x+2)^2+ (0-9)^2$

We have $(x-2)^2+25= (x+2)^2 +81$

Or, X= -7

The point on the x-axis equidistant from given points is (-7,0)

Answer: - (3) y=6

Answer: - (4) y=-4/11

Answer: - (5) Required ratio is 3:5.

LONG ANSWER QUESTIONS
(BASIC)

1. (i) Find the ratio in which the y-axis divides the line segment joining the points (6,-4) and (-2,-7). Also find the point of intersection.
   (ii). Show that the points (7,10), (-2,5) and (3-4) are vertices of an isosceles right triangle.
2. Point P divides the line segment joining the points A (2,1) and B (5,-8) such that AP/AB=1/3 . If P lies on the line 2x-y+k=0, find the value of k.
3. Find the centre of a circle passing through the points (6,-6), (3,-7) and (3,3).

4. Show that the points A (1,0) ,B(5,3), C(2,7) and D(-2,4) are the vertices of a rhombus.
5. The base AB of two equilateral triangles ABC and ABC' with side 2a lies along the x-axis, such that the mid point of AB is at the origin. Find coordinates of the vertices C and C' the triangles.

Solution:- (1) (i) Let the point P(0,y) on y-axis divides the line segment AB in k:1

Or, 0= (-2k+6)/k+1

Or, k=3

Ratio is 3:1

Also y= (-7×3+1×-4)/3+1

Or, y= -25/4

Therefore point of intersection is (0,-25/4)

(ii) Let the points beA(7,10), B(-2,5) and C(3,-4).

Now AB=$\sqrt{[(-2-7)^2+(5-10)^2]}$= $\sqrt{106}$

BC= $\sqrt{[(3+2)^2+(-4-5)^2]}$= $\sqrt{106}$

AC =$\sqrt{[(3-7)^2+(-4-10)^2}$=$\sqrt{212}$

AB=BC and AC^2= AB^2+BC^2

Hence ΔABC is isosceles right triangle.

Solution:- (2) Let the point p be (x,y) which divides AB such that

AP/AB=1/3

Or, AB/AP=3/1

Or, AB/AP. -1 = 3/1-1

Or, AP:BP= 1:2

Using section formula,we have

X= (1×5+2×2)/1+2=3

Y= (-8×1+2×1)/1+2 =-2

Therefore coordinate of P be (3,-2)

Now p(3-2) lies on the line 2x-y+ k=0

Or, k=-8

Answer:- (3) (3,-2)

Answer:- (4) To prove

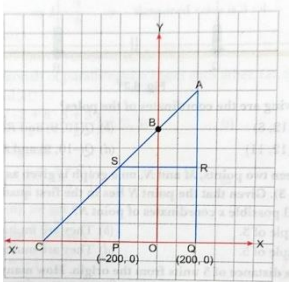
Answer:- (5) C(0,$\sqrt{3}$a), C'(0,-$\sqrt{3}$a)

CASE STUDY BASED QUESTIONS

(BASIC)

1. Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.

Based on the given information, answer the following questions.



(i) Taking O as origin co-ordinates of P are (-200,0) and of Q are (200,0), PQRS being a square,

what are the coordinates of R and S?

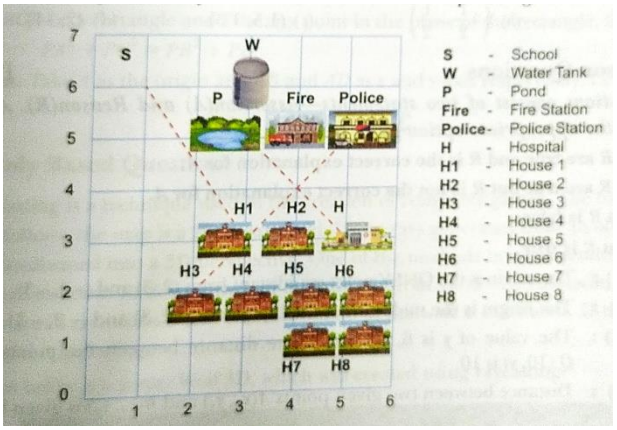
(ii) What is the area of square PQRS?

(iii) If S divides CA in the ratio K:1, what is the value of K, where point A is (200,800)?

STANDARD

2. Shown below is a Town plan on a coordinate grid, where 1 unit=1km

Consider the coordinates of each building to be the point of intersection of the respective grid lines.



Based on the above information answer the following questions

(i) Write the coordinates of hospital.

(ii) What is the distance between the school and House 1 along the path shown?

(iii) Write the y- coordinate of House H2.

Solution:- (1) (i) Since PQRS is a square.

PQ=QR=RS =PS=400

Therefore coordinates of R and S are (200,400) and (-200,400) respectively.

(ii) Area of the square PQRS= $(PQ)^2=(400)^2$

= 160000 sq.units.

(iii). Using section formula

$Y= (ky_2 +y_1)/k+1$

Or, $400=(k\times800+1\times0)/k+1$

Or, K=1

Answer :- (2) (i)(5,3)

(ii)$\sqrt{13}$ km

(iii)3

INTRODUCTION TO TRIGONOMETRY

Trigonometric Ratios of the angle A in a triangle ABC right angled at B are defined as:

$$\text{sine of } \angle A = \sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\text{cosine of } \angle A = \cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\text{tangent of } \angle A = \tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB}$$

$$\text{cosecant of } \angle A = \operatorname{cosec} A = \frac{1}{\sin A} = \frac{AC}{BC}$$

$$\text{secant of } \angle A = \sec A = \frac{1}{\cos A} = \frac{AC}{AB}$$

$$\text{cotangent of } \angle A = \cot A = \frac{1}{\tan A} = \frac{AB}{BC}$$

$$\tan A = \frac{\sin A}{\cos A}, \cot A = \frac{\cos A}{\sin A}$$

The values of trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.

If one trigonometric ratio of an angle is given, the other trigonometric ratios of the anglecan be determined.

Trigonometric ratios of angles: 0°, 30°, 45°, 60° and 90°.

ratio \ angle θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
$\operatorname{cosec}\theta$	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
$\cot\theta$	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A isalways greater than or equal to 1.

Trigonometric identities:

$\sin^2 A + \cos^2 A = 1$

$1 + \tan^2 A = \sec^2 A$

$\cot^2 A + 1 = \operatorname{cosec}^2 A$

MULTIPLE CHOICE QUESTIONS:

1. The value of (sin30° + cos30°) – (sin60° + cos60°) is …..

   (a) −1 (b) 0 (c) 1 (d) 2

   ANS – (b)

2. The value of $\sin^2 30^o + \cos^2 30^o$ is …..

   (a) $\frac{-1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\frac{2}{3}$

   Ans –(c)

3. tan A = ……..

   (a) $\frac{\cos A}{\sqrt{1-\cos^2 A}}$ (b) $\frac{\sec A}{\sqrt{1-\sec^2 A}}$ (c) $\frac{\sin A}{\sqrt{1-\sin^2 A}}$ (d) $\frac{1}{\sqrt{1-\sin^2 A}}$

   Ans – (c)

4. The value of $\frac{\tan 30^o}{\cot 60^o}$ is ……

   (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 1

   Ans – (d)

5. Given that $\sin \alpha = \frac{1}{2}$ and $\cos \beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is ….

   (a) 0^o (b) 30^o (c) 60^o (d) 90^o

   Ans – (d)

SHORT ANSWERS TYPE I (2 MARKS)

1. If tan (A + B) =√3 and tan (A – B) =1/√3, 0° < A + B ≤ 90°; A > B, find A and B.

Ans – A = 45^o B = 15^o

2. If $\sin B = \frac{12}{13}$, then find cot B

Ans – 5/12

3. Prove the identity:

$$\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} = 1 - \sin A \cos A$$

4. If $\sin\theta - \cos\theta = 0$, find the value of $\sin^4\theta + \cos^4\theta$.

Ans – 1/2

5. If $x = a\cos\theta - b\sin\theta$ and $y = a\sin\theta + b\cos\theta$, then prove that $a^2 + b^2 = x^2 + y^2$

SHORT ANSWERS TYPE 2 (3 MARKS)

1. Prove that $\frac{\sin A-\cos A}{\sin A+\cos A} + \frac{\sin A+\cos A}{\sin A-\cos A} = \frac{2}{\sin^2 A - 1}$

2. Prove that $\frac{\tan A+\sec A-1}{\tan A-\sec A+1} = \frac{1+\sin A}{\cos A}$

3. If $\tan A = \frac{a}{b}$, prove that $\frac{a\sin A - b\cos A}{a\sin A + b\cos A} = \frac{a^2-b^2}{a^2+b^2}$

4. If $7\sin^2 A + 3\cos^2 A = 4$, then show that $\tan\theta = \frac{1}{\sqrt{3}}$.

5. Prove that $(\sec A - \cos A)(\cot A + \tan A) = \tan A \sec A$

LONG ANSWER TYPE QUESTION

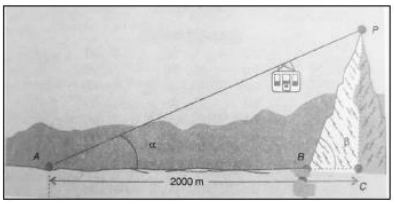
1. If $\tan\theta + \sin\theta = p$; $\tan\theta - \sin\theta = q$; prove that $p^2 - q^2 = 4\sqrt{pq}$

2. If $\sin\theta + \cos\theta = m$ and $\sec\theta + \operatorname{cosec}\theta = n$, then prove that $n(m^2 - 1) = 2m$.

3. If $\frac{1}{\sin\theta - \cos\theta} = \frac{\operatorname{cosec}\theta}{\sqrt{2}}$, prove that $\left(\frac{1}{\sin\theta + \cos\theta}\right)^2 = \frac{\sec^2\theta}{2}$.

CASE STUDY BASED QUESTIONS:

1. A tram transports travellers from the base level at point A to the summit of a mountain chateau at point P. Point A is situated 2000 meters away from point C, which marks themountain's base. If α = 30º and β = 60º, then,



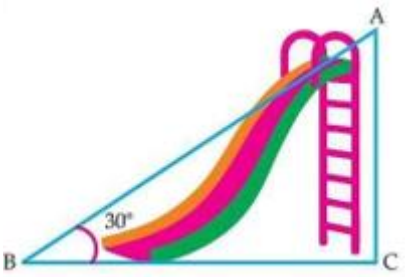
(i) What will be the height of the mountain?
(ii) Assuming the cable is held tight what will be the length of the cable?
(iii) What will be the length of BC?

OR

What will be the distance of point A to the foot of the mountain located at B?

ANSWER KEY: **1.** (i) $\frac{2000}{\sqrt{3}}$ m (ii) $\frac{4000}{\sqrt{3}}$ m (iii) ~~$\frac{2000}{3}$~~ m OR ~~$\frac{4000}{3}$~~ m

2. The local government plans to build a playground slide ina city park specifically designed for children under 12 years old. They envision the top of the slide positioned 4 meters above ground level, inclined at a 30-degree angle relative to the ground.



(i) What is the length of AB?
(ii) What is the value of $\sin^2 30^o + \cos^2 60^o$?
(iii) In the given figure, if AB + BC =25m and AC = 5 m, then what is the value ofBC?

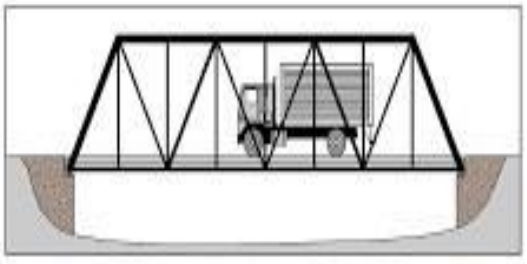
OR

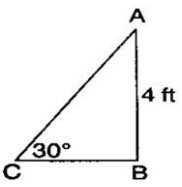
In the given figure, what is the value of (sin B cos A)?

Ans-

2. (i) 8m - (ii) $\frac{1}{2}$ (iii) 12m -OR $\frac{1}{4}$

3. Structural design involves constructing a framework using interconnected triangles. Among the primary engineering structures, trusses play a significant role, particularly inthe construction of bridges and buildings. Trusses are engineered to bear various loads, including human weight, and are constructed solely from elongated, straight elements linked by joints at their endpoints. Each truss system features a repeating triangle as its fundamental unit as shown in the figure below:





(i) If sin A = sin C, what will be the length of BC?

(ii) What is the length of AC?

(iii) If the length of AB doubles, what will become of the length of AC?

OR

What is the length of BC?

Ans

3. (i) 4 ft (ii) 8 ft (iii) doubles the original length OR $4\sqrt{3}$ ft

ASSERTION –REASON TYPE QUESTIONS:

Directions: In the following questions, a statement of Assertion (A) is followed by astatement of Reason (R). Mark the correct choice from the following:

(a) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true. Reason (R) does NOT give correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

Assertion (A) is false but Reason (R) is true.

1. STATEMENT I: ASSERTION (A): For any acute angle θ, the value of sin θ cannot begreater than 1.

STATEMENT II: REASON (R)**:** Hypotenuse is the longest side in any right angledtriangle.

Ans – (a)

2.

STATEMENT I: ASSERTION (A): For $0 \leq \theta \leq 90^0$, sec x + cos x ≥ 2

STATEMENT II: REASON (R): For any x > 0, $x + \frac{1}{x} \geq 2$

Ans – (a)

3.

STATEMENT I: ASSERTION (A): If $\sin \theta + \sin^2\theta = 1$, then $\cos^2\theta + \cos^4\theta = 1$

STATEMENT II: REASON (R)**:** $1 - \sin^2\theta = \cos^2\theta$

Ans – (a)

4.

STATEMENT I: ASSERTION (A): In a right angled triangle, if $\tan\theta = \frac{3}{4}$, the greatest side of the triangle is 5 units.

STATEMENT II: REASON (R): $(\text{greatest side})^2 = (\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

Ans – (a)

5.

STATEMENT I: ASSERTION (A): For $0 < \theta \leq 90^0$, (cosec θ - cot θ) and (cosec θ + cot θ) are reciprocal of each other.

STATEMENT II: REASON (R) : $\cot^2\theta - \text{cosec}^2\theta = 1$

Ans- (c)

SOME APPLICATIONS OF TRIGONOMETRY

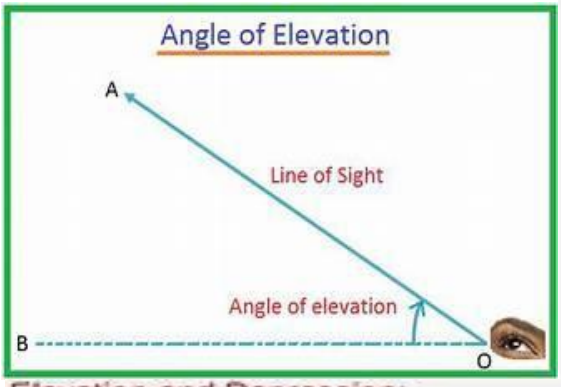
GIST OF THE TOPIC

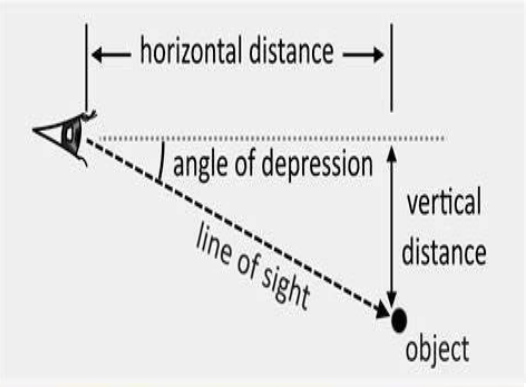
1. The line drawn from the eye of an observer to a point in the object where the person is viewing is called the line of sight.

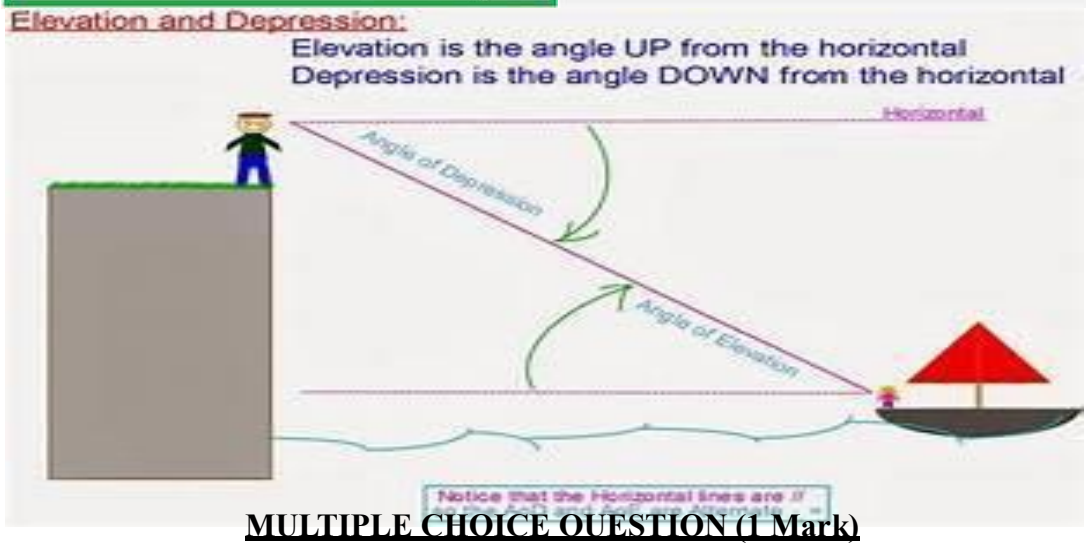
2. The angle formed by the line of sight with the horizontal when the object is above the horizontal level is called the angle of elevation.

3. The angle formed by the line of sight with the horizontal when the object is below the horizontal level is called the angle of depression.

4. The height of an object or distance between distant objects can be determined with the help of trigonometry ratios.

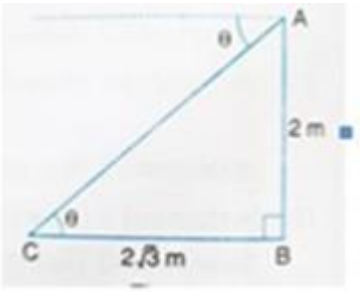






MULTIPLE CHOICE QUESTION (1 Mark)

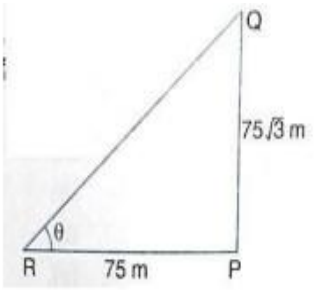
1. In the given figure point C is observed from point A. The angle of depression is



A. 60° B. 30°
C.45° D.75°
(Basic)

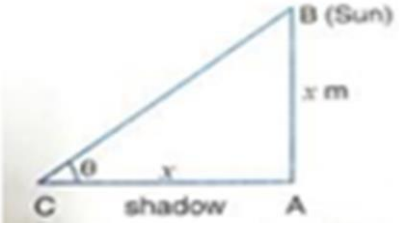
Sol. (B)

2. The measure of the angle of elevation of the top of the tower $75\sqrt{3}$ m high from a point at a distance of 75 m from the foot of the tower in a horizontal plane is



A. 60° B. 30°
C.45° D.90°
(Basic)

3. If the length of the shadow of a vertical pole is equal to its height, the angle of elevation of sun's altitude is



A.45° B.60°

C. 30° D.75° (Basic)

Soln. (A) Let AB be the vertical pole of height x m and let AC be the length of its shadow when the angle of elevation of sun is θ. It is given that AC= x m.

In □ ACB,

Tan θ = AC/AB = x/x =1=tan 45°

=>θ=45°

Sun's altitude is 45°

4. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. The distance of the car from the tower is

a) $50\sqrt{3}$ b)$150\sqrt{3}$ c) $150\sqrt{2}$ d) 75

Ans - (b)

5. The angle of elevation of the top of a tower from a point on the ground 30 m away from the foot of the tower is 30°. The height of the tower is

a) 30 m b)$10\sqrt{3}$ m c) 20 m d)$10\sqrt{2}$ m

Ans - (b

ASSERTION-REASON BASED QUESTIONS (1 MARK)

Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason(R).

Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion(A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c)Assertion (A) is true but reasons (R) is false.
(d) Assertion (A) is false but reasons(R) is true.

1. Assertion: If shadow of pole is $1/\sqrt{3}$ of its height, then the altitude of the sun is 60°

   Reason: If the sun's altitude is 45° , then the shadow of vertical poleis same as height.

(Basic)

Soln. (B)

2. Assertion: The line of sight is the line drawn from the eye of an observer to the point of the object viewed by the observer.

   Reason: Trigonometric ratios are used to find height or length of an object or length of an object or distance between two distant objects

   Answer: (b)both the statement and the reasoning are true, but it is not a correct explanation of the statement.

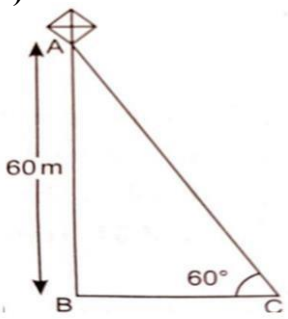
3. Assertion: If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45°

   Reason: According to Pythagoras theorem, h2=p2+b2, where h = hypotenuse, p=perpendicular and b = base

   Answer : (a)

SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string.



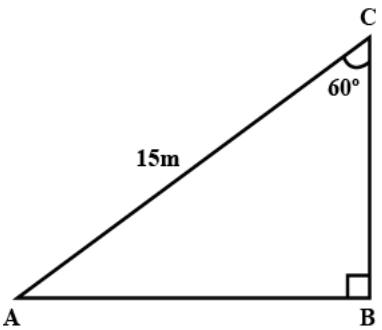
(Basic)

Solution - The height of the kite is 60 m. In Δ ,

AB/AC = sin60°

⟶ AC =40√3 m.

2. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then calculate the height of the wall.



(Basic
)

Solution -Let the length of the ladder be=15m

(hypotenuse)

The angle between the ladder and the wall ∠BCA=60∘

Angle between ladder and the ground

$\angle CAB = 90^o - 60^o = 30^o$

Height of the wall=BC

sin30∘=BC/15⇒1/2=BC/15⇒BC=15/2=7.5m

3. The height of the tower is 10 m. What is the length of its shadow when sun's altitudeis 45°? (Basic)

Solution -Let BC be the length of shadow is x m

Given that: Height of tower is 10 meters and altitude of sun is 45°

Here we have to find length of shadow. So we use trigonometric ratios.

In a triangle ABC ,

$$\Rightarrow \tan C = \frac{AB}{BC}$$
$$\Rightarrow \tan 45^\circ = \frac{AB}{AC}$$
$$\Rightarrow 1 = \frac{10}{x}$$
$$\Rightarrow x = 10$$

Hence the length of shadow is 10 m.

4. An observer, 1.7 m tall, is 20√3 m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30°. Find the height of tower.

Answer: 21.7 m

5. The top of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the Centre of line joining their feet, then find x : y.

Answer: 1: 3

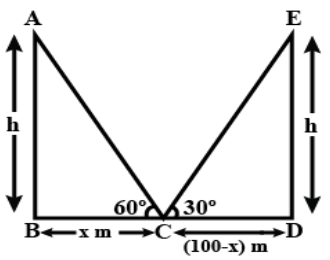
SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Two pillars are of equal height on either side of a road which is 100 m wide. The angles of elevation of the top of the pillars are 60° and 30° at a point on the road between the pillars. Find the position of the point between the pillars and height of each pillar (Basic)

Solution- Let AB and ED be two pillars each of height h metres Let C be a point on the road BD such that BC = x meters

Then CD = (100 - x) meters

Given ∠ACB=60∘ and ∠ECD=30°



In ΔABC,

tan60°=AB/BC

⇒√3=h/x

⇒h=√3x.....(i) In ΔECD, tan30°=ED/CD

⇒1/√3=h/(100−x)

⇒h√3=100−x.....(ii)

Subst. the value of h from (i) in (ii) we get

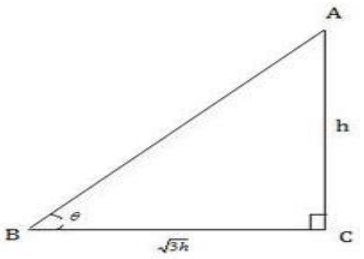
√3.x=(100−x)/√3 ⇒3x=100−x ⇒4x=100
⇒x=25m h=(√ ×25)=25×1.732m=43.3m

The required point is at a distance of 25 m from the pillar B and the height of each pillar is 43.3

m.

2. Find the angle of elevation of the Sun when the shadow of a pole h m high is √3h m long. (Basic)

Solution - Let the angle of elevation of the Sun is θ.



Given, height of pole = h

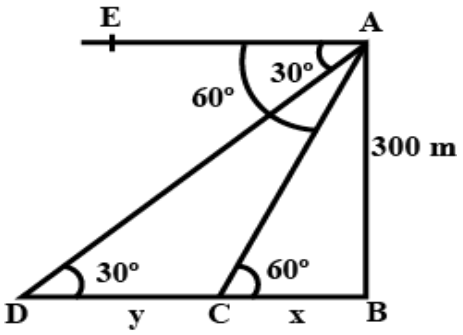
Now, in ΔABC,

tan
θ=AC/BC=h/√3h
tan θ=1/√3=tan 30°
θ=30°

Hence, the angle of elevation of the Sun is 30°.

3. From the top of a 300 meter high light-house, the angles of depression of two ships, which are due south of the observer and in a straight line with its base, are 60° and 30°.Find their distance apart? (Basic)

Solution –



Height of tower= AB = 300m
Let distance between boats= DC = y m
In $\triangle^{le}$ABC, $\tan 60^\circ = \frac{300}{x}$
$\Rightarrow x = \frac{300}{\sqrt{3}}$
$\Rightarrow 100\sqrt{3}$m
In △ABD, $\tan 30^\circ = \frac{300}{x+y}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{300}{x+y}$
$\Rightarrow 100\sqrt{3} + y = 300\sqrt{3}$
$\Rightarrow y = \sqrt{3}(300 - 100) = 200\sqrt{3}m = 346.4m$

4. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. Find the distance between the two towers and also the height of the other tower

   Answer: the required distance and height are $10\sqrt{3}$m and 10 m, respectively.

5. The shadow of a tower standing on a level plane is found to be 50 m longer when Sun's elevation is 30° than when it is 60°. Find the height of the tower

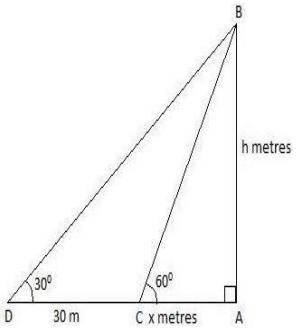
   Answer: the height of the tower is $25\sqrt{3}$m

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60∘. When he moves 30 metres away from the bank, he finds the angle of elevation to be 30∘. Find the height of the tree and the width of the river. [Take $\sqrt{3}=1.732$]

   (Basic)

   Solution:



Let AB be the tree and AC be the river.

Let C and D be the two positions of the person.

Then, $\angle ACB = 60°, \angle ADB = 30°, \angle DAB = 90°$ and CD = 30 m

Let AB = h metres and AC = x metres.

From right ΔCAB, we have

$$\frac{AC}{AB} = \cot 60° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{x}{h} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{h}{\sqrt{3}} \ldots\ldots (i)$$

From right ΔDAB, we have

$$\frac{AD}{AB} = \cot 30° = \sqrt{3}$$

$$\Rightarrow \frac{x+30}{h} = \sqrt{3} \Rightarrow x = \sqrt{3}h - 30 \ldots\ldots (ii)$$

Equating the values of x from (i) and (ii), we get

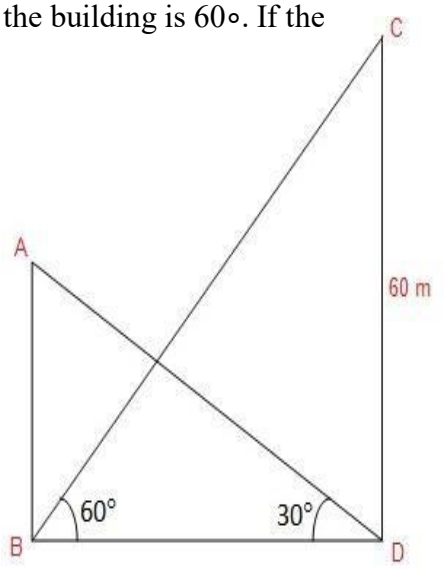
$$\frac{h}{\sqrt{3}} = \sqrt{3}h - 30 \Rightarrow h = 3h - 30\sqrt{3}$$

$\Rightarrow 2h = 30\sqrt{3} \Rightarrow h = 15\sqrt{3} = 15 \times 1.732 = 25.98$

Putting $h = 15\sqrt{3}$ in (i), we get $x = \frac{15\sqrt{3}}{\sqrt{3}} = 15$.

Hence, the height of the tree is 25.98 m and the width of the river is 15 metres.

2. The angle of elevation of the top of a building from the foot of a tower is 30∘. The angle of elevation of the top of the tower from the foot of the building is 60∘. If the tower is 60 m high, find the height of the building.



(Basic)

Solution: Given, height of tower CD = 60 m

Let the height of the building, AB = h
in right angled triangle BDC,

$$\tan 60° = \frac{CD}{BD}$$

$$\sqrt{3} = \frac{60}{BD}$$

$$BD = \frac{60}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}\ m$$

In right triangle ABD

$$\tan 30° = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{20\sqrt{3}}$$

∴ h = 20 m

Thus the height of the building is 20 m.

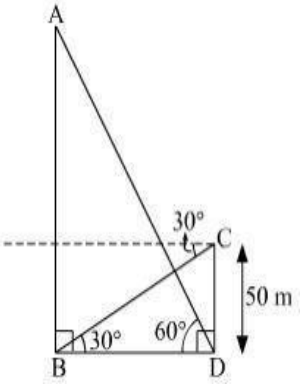
3. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of depression from the top of the tower to the foot of the hill is 30°. If the tower is 50 m high, find the height of the hill. (Basic)

Solution:

Let AB be the hill and CD be the tower.

Angle of elevation of the hill at the foot of the tower

is 60°.i.e., ∠ADB=60° and the angle of depression of the foot of hill from the top of the tower is 30°,



i.e., ∠CBD=30°.

Now in right angled ΔCBD:

tan 30°= $\frac{CD}{BD}$

⇒BD=CD/tan 30°

⇒BD= $\frac{CD}{\frac{1}{\sqrt{3}}}$

⇒BD=50√ 3 m

In right ΔABD:

$\tan 60° = \frac{AB}{CD}$

$\Rightarrow AB = BD \times \tan 60° = 50\sqrt{3} \times \sqrt{3} = 50 \times 3 \text{ m} = 150 \text{ m}$

Hence, the height of the hill is 150 m.

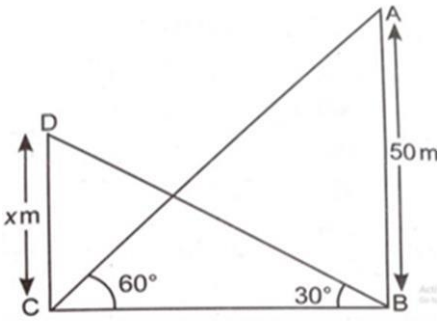
4. A boy whose eye level is 1.3m from the ground, spots a balloon moving with wind in a horizontal line at some height from the ground. The angle of elevation of the balloon from the eyes of the boy at any instant is 60°. After 2 seconds, the angle off elevation reduces to 30°. If the speed of wind at that moment is 29√3 m/s, then find the height of the balloon from the ground.

Answer: 88.3m

5. A man standing on the deck of the ship, which is 16m above the water level, observes the angle of elevation of the top of the clip as 60° and the angle of depression of the base of the cliff as 30°. Calculate the distance of the cliff from the ship and height of the cliff. Answer : 16√3m

CASE STUDY BASED QUESTIONS (4 Marks)

CASE STUDY – 1



The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. The height of the tower is 50m. Observe the given figure and give the answer of the following:

(i) What is the distance between building CD and tower AB? (1)

(ii) What is the distance between root of building

CD and the top of the tower AB? (1) (iii) Find the height of building CD? (2)

OR

Find the distance between root of tower AB and the top of the building CD? (Basic)

Solution: (i) In Right triangle ABC, tan 60° = AB/BC, So BC=50/√3

(ii) In Right triangle ABC, Sin60° = 50/AC So, AC =100/√3

(iii) In Right triangle DCB, tan30° =DC/BC , CD= x = 50/3

OR,

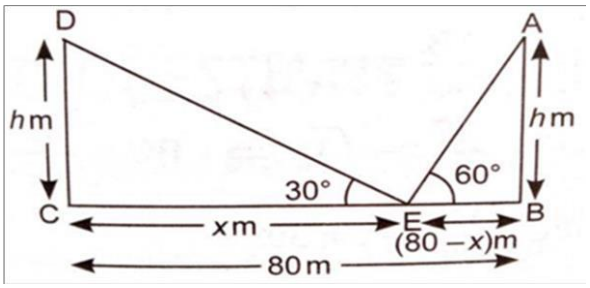
In Right triangle DCB, sin30° = CD/BD So, BD=100/3

CASE STUDY – 2

Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point in between them on the road, the angles of elevation of the top of poles are 600 and 300 respectively. Find the height of the poles and the distances of the point from the poles.

(i) Draw a neat labelled diagram to show the above situation diagrammatically. (1)
(ii) Find the height of the pole? (2)
(iii) Find the distance of the point from the pole? (1)
(Basic)

Solution: (i)



(ii) In Δ,

tan 30° = CD/CE

X =√3h -----(i)

In Δ

tan 60° = AB/BE

_____=√3

h=80√3 - √3x -------(ii)

Solving (i) and (ii), h=20√3 m.

(iii) x=√3h =√3 x 20√3=60 m

And

80-x = 80- 60 = 20 m.

CASE STUDY – 3

A group of students of class x visited India Gate on an education trip. The teacher and students had interest in History as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919.The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 meters) in height.

I. What is the angle of elevation if they are standing at a distance of 42m away from the monument?

II. They want to see the tower at an angle of 60°. So, they want to know the distance where they should stand and hence find the distance.

III. When the altitude of the sun is at 60°, find the height of a vertical tower that casts a shadow of 20m length.

IV. What is the angle of elevation of the sun when the ratio of the height of the tower to its shadow is 1:1

Answer: i. 45°, ii. $14\sqrt{3}$m, iii. $20\sqrt{3}$m, iv.45°

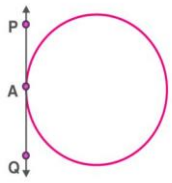
CIRCLES

GIST OF THE LESSON

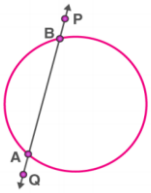
1. The meaning of a tangent to a circle.
2. The tangent to a circle is perpendicular to the radius through the point of contact.
3. The lengths of the two tangents from an external point to a circle are equal.

KEY NOTES

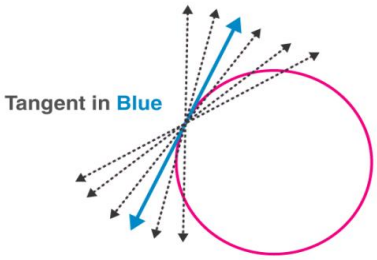
Circle: A circle is a collection of all points in a plane which are at a constant distance from a fixed point.
Centre: The fixed point is called the centre.
Radius: The constant distance from the centre is called the radius.
Chord: A line segment joining any two points on a circle is called a chord.
Diameter: A chord passing through the centre of the circle is called diameter. It is the longest chord.
Tangent: When a line meets the circle at one point or two coinciding The line is known as points, a tangent.



Secant A **secant to a circle** is a line that has two points in common with the circle. It cuts the circle at two points, forming a chord of the circle.



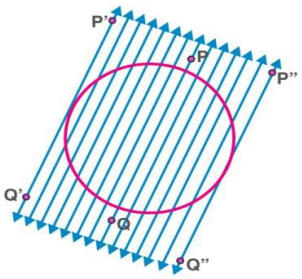
Tangent as a special case of Secant



The tangent to a circle can be seen as a special case of the secant when the two endpoints of its corresponding chord coincide.

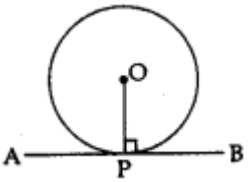
Two parallel tangents at most for a given secant

For every given secant of a circle, there are exactly two tangents which are parallel to it and touch the circle at two diametrically opposite points.

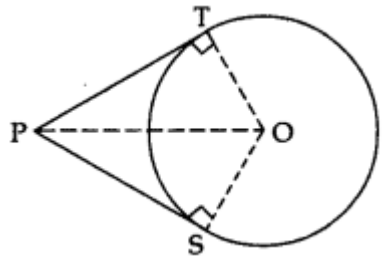


PROPERTIES OF TANGENTS TO THE CIRCLE

1)The tangent to a circle is perpendicular to the radius through the point of contact.
⇒ OP ⊥ AB



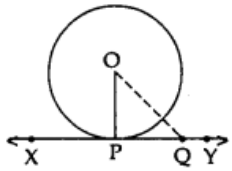
2) The lengths of the two tangents from an external point to a circle are equal.
⇒ AP = PB



<u>3) Length of Tangent Segment</u>
PB and PA are normally called the lengths of tangents from outside point P to the circle.

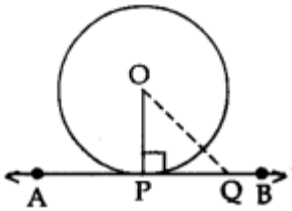
<u>IMPORTANT THEOREMS</u>

Theorem 1: Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.
Given: XY is a tangent at point P to the circle with centre O.
To prove: OP ⊥ XY
Construction: Take a point Q on XY other than P and join OQ
Proof: If point Q lies inside the circle, then XY will become a secant and not a tangent to the circle
OQ > OP



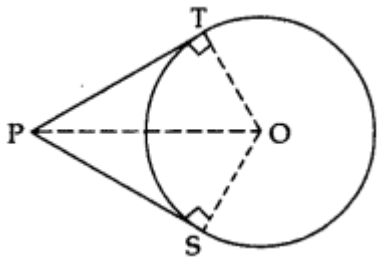
This happens with every point on line XY except point P. OP is the shortest of all the distances of point O the points of XY
OP ⊥ XY …[Shortest side is the perpendicular]

Theorem 2: A line drawn through the endpoint of a radius and perpendicular to it, is tangent to the circle.
Given: A circle C(O, r) and a line APB is perpendicular to OP, where OP is the radius.
To prove: AB is tangent at P.
Construction: Take a point Q on line AB, different from P, and join OQ.
Proof: Since OP ⊥ AB
OP < OQ ⇒ OQ > OP



Point Q lies outside the circle.
Therefore, every point on AB, other than P, lies outside the circle.
This shows that AB meets the circle at point P.
Hence, AP is tangent to the circle at P.

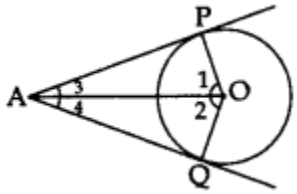
Theorem 3: Prove that the lengths of tangents drawn from an external point to a circle are equal
Given: PT and PS are tangents from an external point P to the circle with centre O.
To prove: PT = PS
Construction: Join O to P, T and S.



Proof: In ΔOTP and ΔOSP.
OT = OS …[radii of the same circle]
OP = OP …[common]
∠OTP = ∠OSP …[each 90°]
ΔOTP = ΔOSP …[R.H.S.]
PT = PS …[c.p.c.t.]

Note: If two tangents are drawn to a circle from an external point, then:

- They subtend equal angles at the centre i.e., ∠1 = ∠2.
- They are equally inclined to the segment joining the centre to that point
- i.e., ∠3 = ∠4.
  ∠OAP = ∠OAQ
- ∠1+∠2+∠3+∠4=180°



QUESTION BANK

MCQ **(BASIC)**

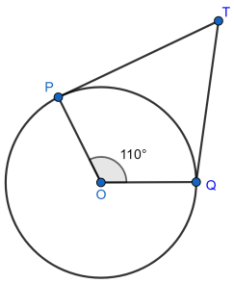
1) If a point is inside the circle, how many tangents can be drawn from that point?
a. 3 b. 1 c. 2 d. 0

2. If two circles are externally and they do not touch, then find the number of common tangents.
a. 4 b. 1 c. 2 d. 3

3. If we draw two tangent at the end of the diameter, then these tangents are always:
a. parallel b. none of these
c. perpendicular d. coincident

4. If TP and TQ are two tangents to a circle with centre O such that ∠POQ = 110°, then ∠PTQ = ?

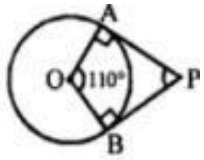
a. 90°
b. 55°
c. 70°
d. 60°



5. At one end of a diameter PQ of a circle of radius 5 cm, tangent XPY is drawn to the circle. The length of chord AB parallel to XY and at a distance of 8 cm from P is
a. 6 cm b. 5 cm c. 7 cm d. 8 cm

ANSWER KEYS MCQ (BASIC)
1. (d) 0
2. (a) 4
3. (a) parallel
4. (c) 70°

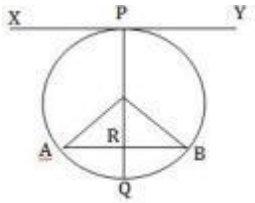
Explanation: In the given figure, PA and PB are the two tangents to the circle with centre O, which subtends ∠AOB = 110°



Now, we have to find the measure ∠APB
OA and OB are the radii of the circle and AP and BP are the tangents
OA ⊥ AP and OB ⊥ BP
∠A = ∠B = 90°
In quadrilateral OAPB,
∠A + ∠B = 90° + 90°= 180°
∠AOB + ∠APB = 180°
⇒ 110° + ∠APB = 180°
⇒ ∠APB = 180° – 110° = 70°

5. (d) 8 cm

Explanation:

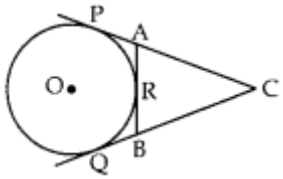


Here, OP = OQ = 5 cm [Radii]
And OR = PR – OP = 8 – 5 = 3 cm
Also, OA = 5 cm [Radius]
Now, in right angled triangle AOR, $OA^2 = OR^2 + AR^2$
AR = 4 cm
Since perpendicular from the centre of a circle to a chord bisects the chord.
∴ AB = AR + BR = 4 + 4 = 8 cm

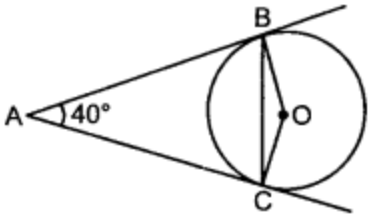
MCQ **(STANDARD)**

1. CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 6 cm then the length of BR is

(a) 6 cm
(b) 5 cm
(c) 4 cm
(d) 3 cm

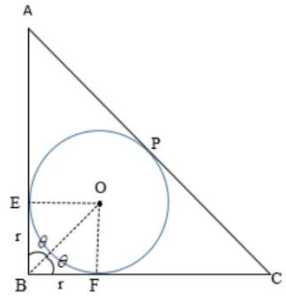


2. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is
(a) 60 cm² (b) 65 cm² (c) 30 cm² (d) 32.5 cm²
3. In the given figure, AB and AC are tangents to the circle with centre O such that ∠BAC = 40°, then ∠BOC is equal to [AI2011]



(a) 40° (b) 50° (c) 140° (d) 150°

4. In the given figure, ABC is a right angled at B such that BC = 6 cm and AB = 8 cm. Find the radius of the circle.



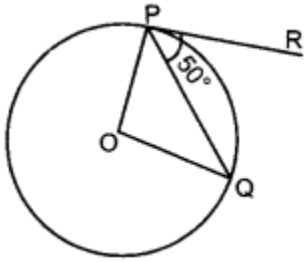
(a) 3 cm (b) 2 cm (c) 4 cm (d) 5 cm

5. A tangent is drawn from a point at a distance of 17 cm of circle C(0, r) of radius 8 cm. The length of its tangent is

(a) 5 cm (b) 9 cm (c) 15 cm (d) 23 cm

OR

In figure if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then ∠POQ is equal to [NCERT Exemplar Problems]

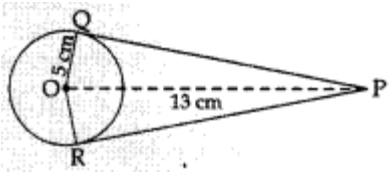


(a) 100° (b) 80° (c) 90° (d) 75°

Answers

1) Answer: b
Explanation: Since BQ = BR …(i) [∵ Tangents drawn from external points are equal]
CQ = CP …[Using (i)]
BC + BQ = 11
⇒ 6 + BR = 11
⇒ BR = 11 – 6 = 5 cm

2) Answer: a
Explaination:



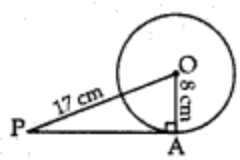
Reason: $OP^2 = OQ^2 + PQ^2$
$169 = 25 + PQ^2$
$PQ^2 = 144$
$PQ = 12$
Area PQOR = ar (AOPQ) + ar (AOPR)
$= 12 \times 12 \times 5 + 12 \times 12 \times 5 = 60\ cm^2$

3) Answer: c
Explanation:
(c) In quadrilateral ABOC
$\angle ABO + \angle BOC + \angle OCA + \angle BAC = 360°$
$\Rightarrow 90° + \angle BOC + 90° + 40° = 360°$
$\Rightarrow \angle BOC = 360° – 220° = 140°$

4) $AC^2=6^2+8^2$ (Since the value of AB and BC are given in the question)
$AC^2=100$
$\Rightarrow AC^2= \sqrt{100}$
$\Rightarrow AC = 10cm$
By the theorem of Tangent to a circle i.e. if two tangents are drawn from an external point of the circle, then they are of equal lengths therefore BE = BF, CP = CF and AP = AE.
By the above statement BE = BF = r
And CP = CF
CP = CF $\Rightarrow\Rightarrow$AC-AP = BC – BF (equation 1)
Now putting the value of BF and BE in the equation 1
AC – AE = BC – BF since AP=AE by the theorem of tangent to a circle
AC – AE = BC – BF (equation 2)
Putting the value of AC, AE, BC and BF in equation 2
10 – (6 – r) = 8 – r
$\Rightarrow\Rightarrow$10 – 6 + r = 8 – r
$\Rightarrow\Rightarrow$2r = 8 – 4
$\Rightarrow\Rightarrow$r = 4/2
$\Rightarrow\Rightarrow$r = 2cm
Therefore the radius of the circle inside the triangle is 2cm.

Note: In these types of questions first draw a rough diagram then find the unknown value of side of the triangle then construct lines from the center of the circle OF and OE that are perpendicular to the line AB and BC also a line OB which bisect the BE and BF then with the help theorem of tangent to a circle show that AP = AE, CP = CF and BE = BF then use the statement CP = CF and put the values of CP and CF and find the value of radius of circle.

5) Answer: c
Explanation:



Reason: In rt ΔOAP, $AP^2 + OA^2 = OP^2$
$\Rightarrow AP^2 + (8)^2 = (17)^2 \Rightarrow AP^2 + 64 = 289$
$\Rightarrow AP^2 = 289 - 64 = 225$
$\therefore AP = \sqrt{225} = 15$ cm

OR

(5)Answer: a
Explaination:
(a) OP ⊥ PR [Y Tangent and radius are ⊥ to each other at the point of contact]
$\angle OPQ = 90° - 50° = 40°$
OP = OQ [Radii]
$\therefore \angle OPQ = \angle OQP = 40°$
In ΔOPQ,
$\Rightarrow \angle POQ + \angle OPQ + \angle OQP = 180°$
$\Rightarrow \angle POQ + 40° + 40° = 180°$
$\angle POQ = 180° - 80° = 100°$.

ASSERTION And REASONING (BASIC)

1) In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

1) **Assertion (A):** From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm then the radius of the circle is 7 cm.

Reason (R): A tangent to a circle is perpendicular to the radius through the point of contact

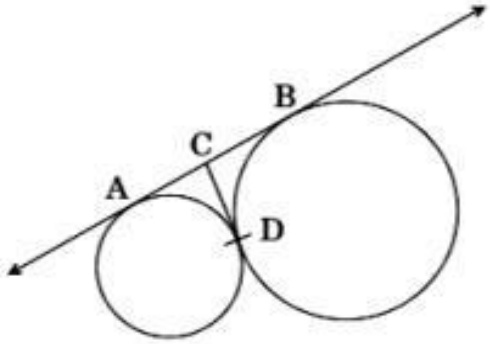
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

2) **Assertion :** If length of a tangent from an external point to a circle is 8 cm, then length of the other tangent from the same point is 8 cm.
Reason : length of the tangents drawn from an external point to a circle are equal.

Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

ASSERTION And REASONING (STANDARD)

1) **Assertion (A):** In the below figure, AB and CD are common tangents to circles which touch each other at D. If AB = 8 cm, then the length of CD is 4cm



Reason (R): A tangent to a circle is perpendicular to the radius through the point of contact

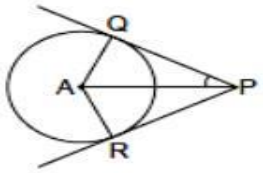
Ans: (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(2) **Assertion** : If in a cyclic quadrilateral, one angle is 40°, then the opposite angle is 140° .
Reason : Sum of opposite angles in a cyclic quadrilateral is equal to 360°

Answer : (c) Assertion (A) is true but reason (R) is false.

VSA (BASIC)

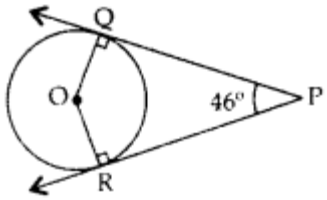
1) In figure, PQ and PR are tangents to a circle with centre A. If∠QPA = 27°, then ∠ QAR equals to



Ans:-
∠QPA = ∠RPA [∵ ΔAQP ≅ ΔARP (RHS congruence rule)]
⇒ ∠RPA = 27°
∴ ∠QPR = ∠QPA + ∠RPA = 27° + 27° = 54°
Now, ∠QAR + ∠AQP + ∠ARP + ∠QPR = 360°
⇒ ∠QAR = 90° + 90° + 54° = 360°
⇒ ∠QAR = 360° – 234° = 126°

Q.2: How many tangents can be drawn from the external point to a circle?
Answer: Two tangents can be drawn from the external point to a circle.

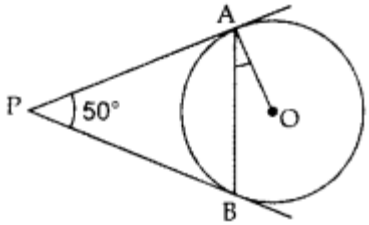
Q.3) In the given figure, PQ and PR are two tangents to a circle with centre O. If ∠QPR = 46°, then calculate ∠QOR.



Solution:
∠OQP = 900
∠ORP = 90°
∠OQP + ∠QPR + ∠ORP + ∠QOR = 360° …[Angle sum property of a quad.
90° + 46° + 90° + ∠QOR = 360°
∠QOR = 360° – 90° – 46° – 90° = 134°

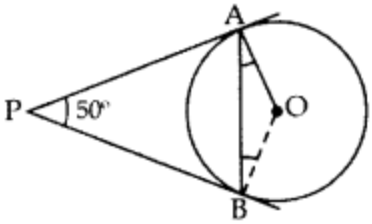
VSA (STANDARD)

Q.1) In the given figure, PA and PB are tangents to the circle with centre O such that



∠APB = 50°. Write the measure of ∠OAB.

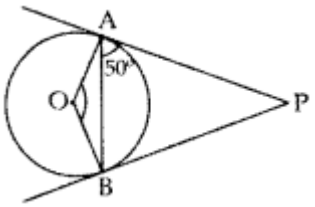
Solution:
PA = PB …[∵ Tangents drawn from external point are equal
∠OAP = ∠OBP = 90°
∠OAB = ∠OBA … [Angles opposite equal sides
∠OAP + ∠AOB + ∠OBP + ∠APB = 360° … [Quadratic rule



90° + ∠AOB + 90° + 50° = 360°
∠AOB = 360° – 230°
= 130°
∠AOB + ∠OAB + ∠OBA = 180° … [Δ rule
130° + 2∠OAB = 180° … [From (i)
2∠OAB = 50°
⇒ ∠OAB = 25°

Q.2) From an external point P, tangents PA and PB are drawn to a circle with centre 0. If ∠PAB = 50°, then find ∠AOB.

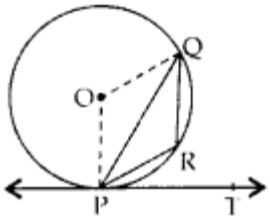
Solution:
PA = PB …[∵ Tangents drawn from external point are equal



∠PBA = ∠PAB = 50° …[Angles equal to opposite sides
In ΔABP, ∠PBA + ∠PAB + ∠APB = 180° …[Angle-sum-property of a Δ
50° + 50° + ∠APB = 180°
∠APB = 180° – 50° – 50° = 80°
In cyclic quadrilateral OAPB
∠AOB + ∠APB = 180° ……[Sum of opposite angles of a cyclic (quadrilateral is 180°
∠AOB + 80o = 180°
∠AOB = 180° – 80° = 100°

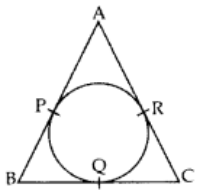
SA (BASIC)

1) In the given figure, PQ is a chord of a circle with centre O and PT is a tangent. If ∠QPT = 60°, find ∠PRQ.

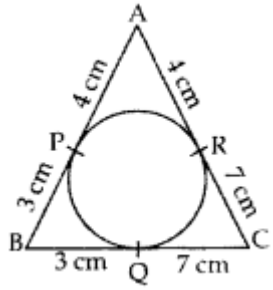


Solution:
PQ is the chord of the circle and PT is tangent.
∴ ∠OPT = 90° …[Tangent is I to the radius through the point of contact
Now ∠QPT = 60° … [Given
∠OPQ = ∠OPT – ∠QPT
⇒ ∠OPQ = 90° – 60° = 30°
In ΔOPQ, OP = OQ
∠OQP = ∠OPQ = 30° … [In a Δ, equal sides have equal ∠s opp. them
Now, ∠OQP + ∠OPQ + ∠POQ = 180°
∴ ∠POQ = 120° …[∠POQ = 180° – (30° + 30°)
⇒ Reflex ∠POQ = 360° – 120° = 240° …[We know that the angle subtended by an arc at the centre of a circle is twice the angle subtended by it at any point on the remaining part of the circle
∴ Reflex ∠POQ = 2∠PRO
⇒ 240° = 2∠PRQ
⇒ ∠PRQ = 240∘/2 = 120°

2) In the given figure, the sides AB, BC and CA of a triangle ABC touch a circle at P, Q and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11 cm, find the length of BC (in cm).

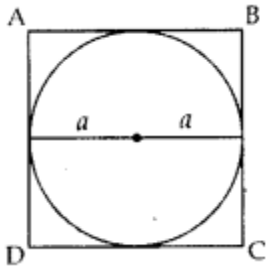


Solution:



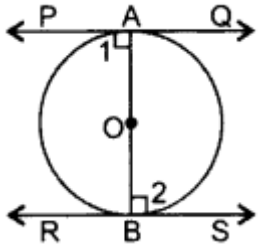
AP = AR = 4 cm
RC = 11 – 4 = 7 cm
RC = QC = 7 cm
BQ = BP = 3 cm
BC = BQ + QC
= 3 + 7 = 10 cm

3) **Find the perimeter (in cm) of a square circumscribing a circle of radius a cm.**
Solution:



Radius = R
AB = a + a = 2a
∴ Perimeter = 4(AB)
= 4(2a)
= 8a cm

4) **Prove that the tangents drawn at the ends of a diameter of a circle are parallel.**
Solution:

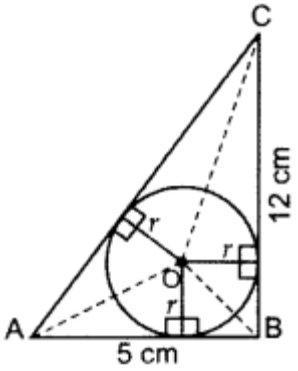


Proof: ∠1 = 90° …(i)
∠2 = 90° …(ii)
∠1 = ∠2 … [From (i) & (ii)
But these are alternate interior angles
∴PQ || RS

SA (STANDARD)

1) **In a right triangle ABC, right-angled at B, BC = 12 cm and AB = 5 cm. Calculate the radius of the circle inscribed in the triangle (in cm).**

Solution:



$AC^2 = AB^2 + BC^2$ …[Pythagoras' theorem

$= (5)^2 + (12)^2$

$AC^2 = 25 + 144$

$AC = \sqrt{169} = 13$ cm

Area of ΔABC = Area of ΔAOB + ar. of ΔBOC + ar. of ΔAOC

$$\frac{5 \times 12}{2} = \frac{AB \times r}{2} + \frac{BC \times r}{2} + \frac{AC \times r}{2}$$

$$\ldots\left[\quad \text{Ar. of } \Delta = \frac{\text{Base} \times \text{Corr. alt.}}{2}\right.$$

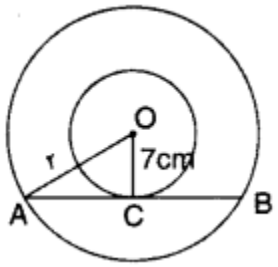
$60 = r(AB + BC + AC)$

$60 = r(5 + 12 + 13)$

$60 = 30r \Rightarrow r = 2$ cm

Q.2) Two concentric circles are of radii 7 cm and r cm respectively, where r > 7. A chord of the larger circle, of length 48 cm, touches the smaller circle. Find the value of r.

Solution:



Given: OC = 7 cm, AB = 48 cm

To find: r = ?

$\angle OCA = 90°$ ..[Tangent is $\perp$ to the radius through the point of contact

$\therefore$ OC $\perp$ AB

AC = 12 (AB) … [$\perp$ from the centre bisects the chord

$\Rightarrow$ AC = 12 (48) = 24 cm

In rt. ΔOCA, $OA^2 = OC^2 + AC^2$ … [Pythagoras' theorem

$r^2 = (7)^2 + (24)^2$

$= 49 + 576 = 625$

$\therefore r = \sqrt{625} = 25$ cm

LA (BASIC)

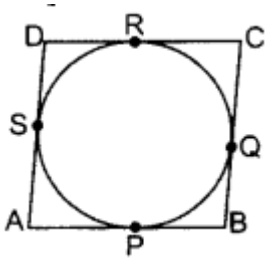
Q.1) Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:

Given. ABCD is a $\|^{gm}$.

To prove. ABCD is a rhombus.

Proof. In $\|^{gm}$, opposite sides are equal



AB = CD

and AD = BC ..(i)

AP = AS …[Tangents drawn from an external point are equal in length

PB = BQ

CR = CO

DR = DS

By adding these tangents,

(AP + PB) + (CR + DR) = AS + BQ + CQ + DS

AB + CD = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

AB + AB = BC + BC … [From (i)

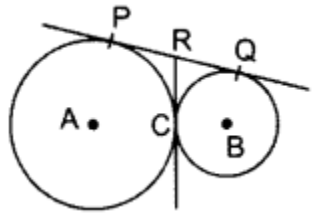
2AB = 2 BC

AB = BC …(ii)

From (i) and (ii), AB = BC = CD = DA

∴ $\|^{gm}$ ABCD is a rhombus.

Q.2) In the figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.



Solution:

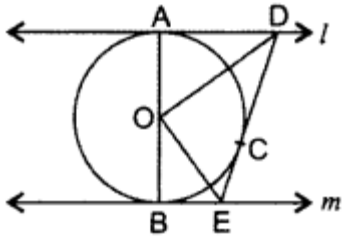
To prove: PR = RQ

Proof: PR = RC … (i)

QR = RC

From (i) and (ii), PR = QR (Hence proved)

Q.3) In the figure, l and m are two parallel tangents to a circle with centre O, touching the circle at A and B respectively. Another tangent at C intersects the line I at D and m

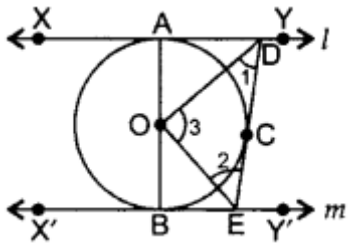


at E. Prove that ∠DOE = 90°.

Solution:

Proof: Let I be XY and m be XY'

∠XDE + ∠X'ED = 180° … [Consecutive interior angles



12XDE + 12∠X'ED =

= 12 (180°)

= ∠1 + ∠2 = 90° …[OD is equally inclined to the tangents

In ΔDOE, ∠1 + ∠2 + 23 = 180° …[Angle-sum-property of a Δ

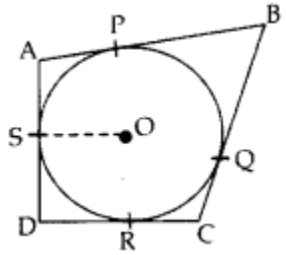
90° + 23 = 180°

⇒ ∠3 = 180° – 90o = 90°

∴ ∠DOE = 90° …(proved)

LA (STANDARD)

Q.1) In the given figure, a circle is inscribed in a quadrilateral ABCD touching its sides AB, BC, CD and AD at P, Q, R and S respectively. If the radius DA of the circle is 10 cm, BC = 38 cm, PB = 27 cm and AD ⊥ CD, then calculate the length of CD

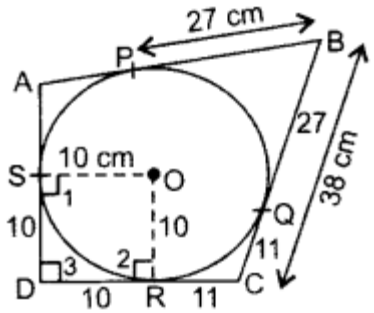


Solution:

Const. Join OR

Proof. ∠1 = ∠2 = 90° … [Tangent is ⊥ to the radius through the point of contact

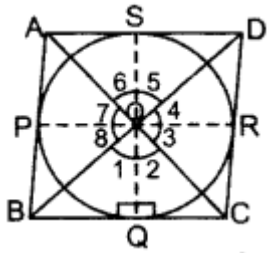
∠3 = 90° …[Given



∴ ORDS is a square.
DR = OS = 10 cm …(i)
BP = BQ = 27 cm …[Tangents drawn from an external point
∴ CQ = 38 – 27 = 11 cm
RC = CO = 11 cm …[Tangents drawn from an external point
DC = DR + RC = 10 + 11 = 21 cm …(From (i) & (ii))

Q.2) Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:
To prove. (i) ∠AOD + ∠BOC = 180°
(ii) ∠AOB + ∠COD = 180°
Proof. In ΔBPO and ΔBQO …[Tangents drawn from an external point are equal

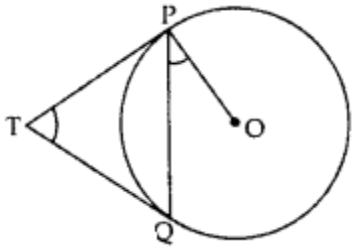


PO = 20 … [radii
BO = BO … [Common
ΔBPO = ΔBQO … [SSS Congruency rule
∠8 = ∠1 …(i) (c.p.c.t.)
Similarly,
∠2 = ∠3, ∠4 = ∠5 and ∠6 = ∠7
∠1 + ∠2 + 23+ 24 + 25 + 26+ 27 + ∠8 = 360° …(Complete angles
∠1 + ∠2 + 22+ 25 + 25 + 26+ ∠6+ ∠1 = 360°
2(∠1 + ∠2 + 25 + 26) = 360°
∠BOC + ∠AOD = 180°…(i) [Proved part I
∠AOB + ∠BOC + ∠COD + ∠DOA = 360° …(Complete angles
∠AOB + ∠COD + 180o = 360° … [From (i)
∴ ∠AOB + ∠COD = 360° – 180o = 180° …(proved)

Q.3) Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2∠OPQ.

Solution:
We are given a circle with centre O, an external point T and two tangents TP and TQ to the

circle, where P, Q are the points of contact (see Figure).



We need to prove that:
$\angle PTQ = 2\angle OPQ$
Let $\angle PTQ = \theta$
Now, TP = TQ ….[∵ Lengths of tangents drawn from an external pt. to a circle are equal
So, TPQ is an isosceles triangle.

$$\therefore \quad \angle TPQ = \angle TQP = \frac{1}{2}(180^\circ - \theta) = 90^\circ - \frac{1}{2}\theta \ldots(i)$$

Also, $\angle OPT = 90^\circ$

...[∵ **Tangent at any point of a circle is ⊥ to the radius through the pt. of contact**

$$\text{So, } \angle OPQ = \angle OPT - \angle TPQ = 90^\circ - \left(90^\circ - \frac{1}{2}\theta\right)$$

...[From (*i*)

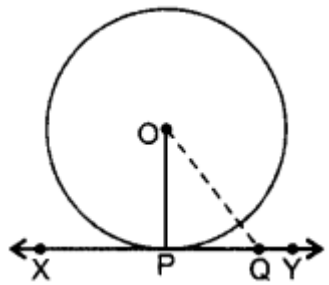
$$\angle OPQ = \frac{1}{2}\theta \quad \Rightarrow \quad \angle OPQ = \frac{1}{2}\angle PTQ$$

$$\therefore \quad \mathbf{\angle PTQ = 2\angle OPQ} \qquad \textbf{...(Proved)}$$

Circles Class 10 Very Important Questions Long Answer (4 Marks)

Q.1) Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Solution:

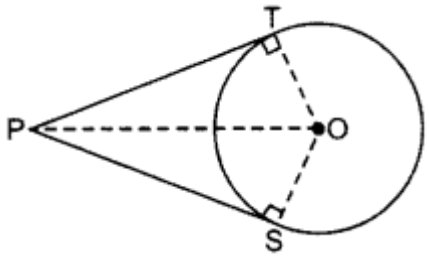


Given: XY is a tangent at point P to the circle with centre O.
To prove: OP ⊥ XY
Const.: Take a point Q on XY other than P and join to OQuestion
Proof: If point Q lies inside the circle, then XY will become a secant and not a tangent to the circle.
∴ OQ > OP
This happens with every point on the line XY except the point P.

OP is the shortest of all the distances of the point O to the points of XY
∴ OP ⊥ XY … [Shortest side is ⊥

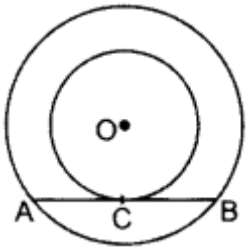
Q.2) Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution:
Given: PT and PS are tangents from an external point P to the circle with centre O.

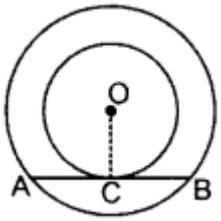


To prove: PT = PS
Const.: Join O to P,
T & S
Proof: In ΔOTP and
ΔOSP,
OT = OS …[radii of same circle
OP = OP …[circle
∠OTP – ∠OSP …[Each 90°
∴ AOTP = AOSP …[R.H.S
PT = PS …[c.p.c.t

Q,3) In the figure, the chord AB of the larger of the two concentric circles, with centre

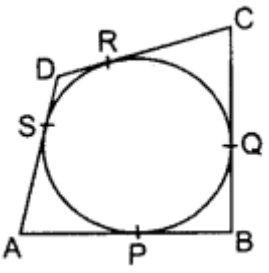


O, touches the smaller circle at C. Prove that AC = CB.
Solution:



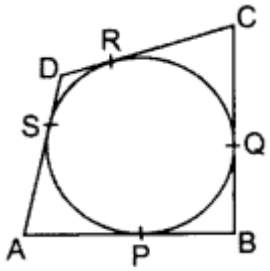
Const.: Join OC
Proof: AB is a tangent to smaller circle and OC is a radius.
∴ ∠OCB = 90° … above theorem
In the larger circle, AB is a chord and OC ⊥ AB.
∴ AC = CB … [⊥ from the centre bisects the chord.

Q.4) In figure, a quadrilateral ABCD is drawn to circum- DA scribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q,



RA and S respectively. Prove that: AB + CD = BC + DA

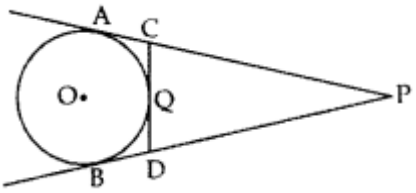
Solution:



AP = AS ……(i) (Tangents drawn from an external point are equal in length
BP = BO …(ii)
CR = CQ ….(iii)
DR = DS ..(iv)
By adding (i) to (iv)
(AP + BP) + (CR + DR) = AS + BQ + CQ + DS
AB + CD = (BQ + CQ) + (AS + DS)
∴ AB + CD = BC + AD (Hence proved)

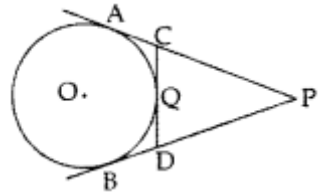
sides of an equilateral A are equal

Q.5) In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Question If PA = 12 cm, QC = QD = 3 cm,



then find PC + PD.

Solution:

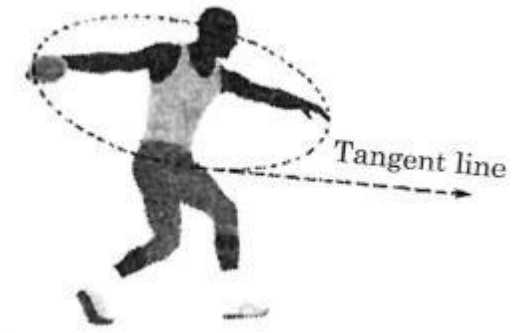


PA = PB = 12 cm …(i)
QC = AC = 3 cm …(ii)
QD = BD = 3 cm …(iii)
To find: PC + PD

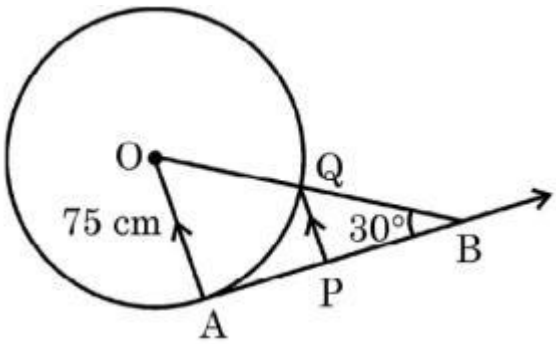
= (PA – AC) + (PB – BD)
= (12 – 3) + (12 – 3) … [From (i), (ii) & (iii)
= 9 + 9 = 18 cm

CASE STUDY 1:- **(STANDARD)**

The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.



In the given figure, AB is one such tangent to a circle of radius 75 cm. Point O is centre of the circle and ∠ABO = 30°. PQ is parallel to OA.



Based on above information:

(i) Find the length of **AB**.
(ii) Find the length of ***OB***.
(iii) Find the length of AP
OR
Find the length of PQ

Ans: (i) $\tan 30^0$ =75/AB

$\Rightarrow AB = 75\sqrt{3}$

(ii) $\sin 30^0 = 1/2$

OA/OB=1/2

$\Rightarrow$ OB = 150 cm

(iii) AP = $\frac{75\sqrt{3}}{2}$ cm.

OR

$QB = 150 - 75 = 75cm$

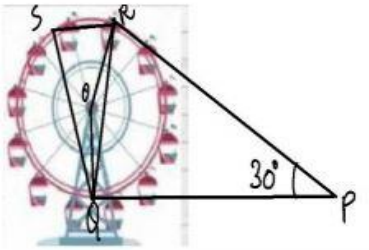
$Now, \Delta BQP \sim \Delta BOA$

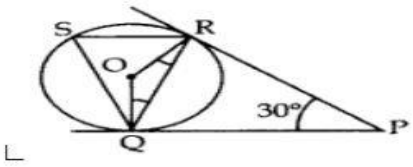
PQ=75/2

CASE STUDY 2: (STANDARD)

A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger-carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity.

After taking a ride in Ferris wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride . She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.





1. In the given figure find Angle ROQ

a) 60^0 b) 100^0 c) 150^0 d) 90^0

Answer: c) 150

2. Find Angle RQP

a) 75^0 b) 60^0 c) 30^0 d) 90^0

Answer: a) 75

3. Find Angle RSQ

a) 60^0 b) 75^0 c) 100^0 d) 30^0

Answer: b) 75^0

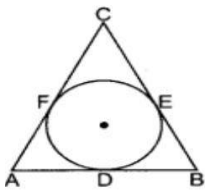
4. Find Angle ORP

a) 90^0 b) 70^0 c) 100^0 d) 60^0

Answer: a) 90^0

CASE STUDY 1: **(BASIC)**

Varun has been selected by his School to design logo for Sports Day T-shirts for students and staff . The logo design is as given in the figure and he is working on the fonts and different colours according to the theme. In given figure, a circle with centre O is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively.



1. Find the length of AD

a) 7 b) 8 c) 5 d) 9

Answer: a) 7

2. Find the Length of BE

a) 8 b) 5 c) 2 d) 9

Answer: b) 5

3. Find the length of CF

a) 9 b) 5 c) 2 d) 3

Answer: d) 3

4. If radius of the circle is 4cm, Find the area of ΔOAB

a) 20 b) 36 c) 24 d) 48

Answer: c) 24

5. Find area of ∆ABC

a) 50 b) 60 c) 100 d) 90

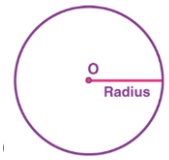
Answer: b) 60

Areas related to circles

Basic concepts-

Circles

A circle is a collection of all such points of a plane, which are at constant distance from a fixed point of that plane. Fixed point "O" is called centre and constant distance OA is called radius of the circle.



A

Perimeter and Area of a circle

Perimeter of a circle is generally known as its circumference. Since the ratio of circumference "C" of a circle to its diameter always remain constant and this constant is denoted by Greek letter π (Pie).

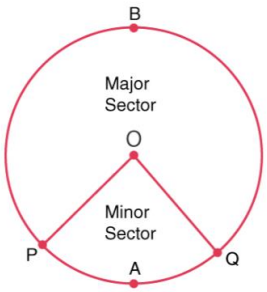
Therefore, Circumference/Diameter = $\pi \Longrightarrow C/2r = \pi$
Hence, $C = 2\pi r$, where circumference =C, radius =r
Area of circle=- πr^2
Area of semi-circle = $\pi r^2/2$
Area of Quadrant circle = $\pi r^2/4$

Sector of circle

The portion of the circular region enclosed by two radii and corresponding arc is called sector of the circle. The figure shown below $\angle POQ$ is called angle of the sector and OPAQ is called sector.

Minor sector and Major sector

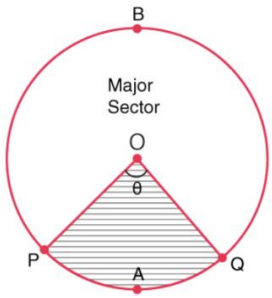
A sector in which angle of sector is less than 180° is called minor sector while sector having angle more than 180° is called Major sector



Area of a sector

Let OPAQ be a sector of a circle with centre "O", radius r and angle of sector ɵ. Now we assume circular region to be a sector forming an angle of 360°. By applying the unitary method, we can derive formula for area of sector as follows-

When the sector is 360°, area - πr^2
When the sector is 1°. area - $\pi r^2/360^o$
When the angle of sector is ɵ°, area - $\pi r^2 ɵ/360^o$
where r is the radius and ɵ is angle of sector in degree measure



Length of an Arc of sector of angle "ɵ" –

By applying unitary method and taking the whole length of the circle as 2πr, we can obtain the results as follows:

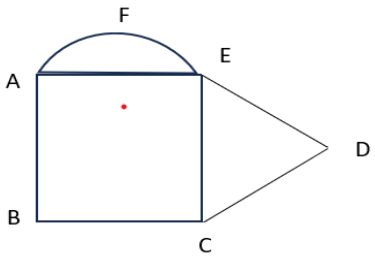
Length of an arc of a sector of angle ɵ(*l*) = $\frac{ɵ}{360}$× 2πr

= 2πr ɵ/360°, where r is radius and ɵ is the angle corresponding sector.

Area of combination of plane figures –

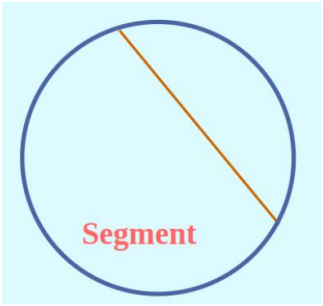
We can find the area by adding the area of all such figures, one by one with the combination of which given figure is formed.
For example, suppose we have to find the area of plane *ABCDEFA* given alongside.

Area of figure *ABCDEFA* = area of square *ABCE* + ar (ΔDCE) +Area of semicircle *AFE*



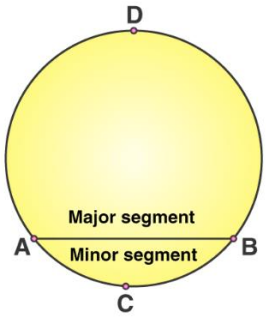
Segment-

The portion of the circular region enclosed between a chord and the corresponding arc is called a segment of the circle.

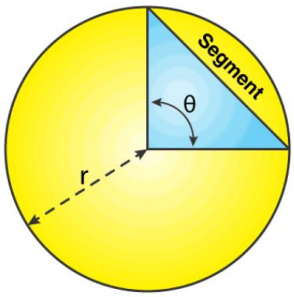


Minor Segment and major Segment-

Below shown is major segment and minor segment.



Area of Segment- -



$\mathbf{A = \frac{1}{2} x\ (\theta - sin\ \theta)\ x\ r^2}$

Formula To Calculate Area of a Segment of a Circle	
Area of a Segment in Radians	$A = (½) \times r^2 (\theta - \text{Sin } \theta)$
Area of a Segment in Degrees	$A = (½) \times r^2 \times [(\pi/180)\ \theta - \sin \theta]$

Multiple Choice Questions

1. What is the area of semi- circle of diameter “d”?

a) $\frac{1}{16}\times\pi d^2$ c) $\frac{1}{8}\times\pi d^2$

b) $\frac{1}{4}\times\pi d^2$ d) $\frac{1}{2}\times\pi d^2$

Solution- c) area of semicircle = ½ πr^2 = ½ $\pi(d/2)^2$
= 1/8 πd^2

2. OACB is a quadrant of a circle with centre O and radius 7 cm where ACB is the arc. Then the perimeter of the quadrant is –

a) 15cm c) 50cm
b) 25cm d) 44cm

Solution- c) length of arc = $2\pi r \times \theta/360^o = 2 \times 22/7 \times 90^o/360^o$
= 11cm

3. If a bicycle wheel makes 5000 revolutions in moving 11 km, then the diameter of the wheel is-

a) 65cm c) 70cm
b) 35cm d) 50cm

Solution- c) let the radius of bicycle wheel be r cm.
ATQ,
$5000 \times 2\pi r = 1100000$
$2r = 1100000/5000 \times 7/22 = 70cm$

4. The area of Quadrant of a circle where the circumference of the circle is 176m,is

a) $2464m^2$ c) $616m^2$
b) $1232m^2$ d) $308m^2$

Solution- c) circumference = 176m, $2\pi r = 176$
$r = 28m$, area of quadrant of circle= ¼ πr^2
$= 1/4 \times 22/7 \times (28)^2$
$= 616m^2$

5. The minute hand of a clock is 84cm long. The distance covered by tip of minute hand from 10:10 am to 10:25 am is-

a) 44cm c) 132cm
b) 88cm d) 176cm

Solution-c) distance covered by tip of minute hand from 10:10 am to 10:25 am
$\theta/360^\circ \times 2\pi r = 90^\circ/360^\circ \times 2 \times 22/7 \times 84 = 132cm$

6. The area of square that can be inscribed in a circle of area 1408/7 cm^2 is-

   a) 321cm^2 c) 128cm^2
   b) 642cm^2 d) 256cm^2

Solution- c) area of circle = 1408/7 cm^2

$\pi r^2 = 1408/7$, r= 8cm

now, diagonal of inscribed square, d = 2r =16cm

$d = \sqrt{2}\, s$ (s- side of square)

$16 = \sqrt{2}s$, $s^2 = 128$

Hence, area of square =128 cm^2

7. if the perimeter of a circle is half to that of a square, then the ratio of area of the circle to the area of the square is

   a) 22:7 c) 7:11
   b) 11:7 d) 7:22

Solution-d) perimeter of circle= 1/2×perimeter of square

$2\pi r = 1/2 \times 4s$, $r/s = 1/\pi$

Area of circle/area of square= $\pi r^2/s^2$

= 1/22/7

Required ratio = 7/22

Very Short Answer Question

1. A piece of wire 22cm long is bent into the form of an arc of a circle subtending an angle of 60^o at its centre. Find the radius of the circle. (use $\pi = 22/7$)

Solution-let the radius be, r, arc AB=22cm

$60^\circ/360^\circ \times 2\pi r = 22$

r = 21cm

2. The perimeter of a sector of a circle of radius 5.2cm is 16.4cm. Find area of the sector.

Answer – 15.6 cm^2

3. The minute hand of clock is 12cm long. Find the area of the face of the clock described by minute hand in 35 minutes.

Solution- angle described by minute hand in 1 min=6^o, so angle described by minute Hand in 35 min = 210^o, area = $210^o/360^o \times \pi (12)^2$

= 264^2

4. The outer and inner diameter of a circular ring are 34cm and 32cm respectively, then find the area of the ring.

Answer- 33π cm^2

Short Answer Type Question

1. Find the area of the minor and the major sectors a circle with radius 6cm, if the angle subtended by the minor arc at the centre is 60°. (use π= 3.14)

Solution – area of a circle = πr^2 =3.14 × $(6)^2$ = 113.04cm^2

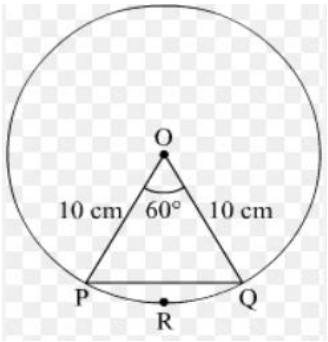
Area of minor sector = $\pi r^2 \times \theta/360°$

= 113.04 × 60°/360° =18.84cm^2

Hence, area of major sector= 113.04 – 18.84

= 94.2cm^2

2. If a chord of a circle of radius 10cm subtends an angle of 60° at the centre of the circle, find the area of corresponding minor segment of the circle. (use π – 3.14 and √3- 1.73).



Solution- area of sector OPRQ = $\pi r^2 \times \theta/360°$

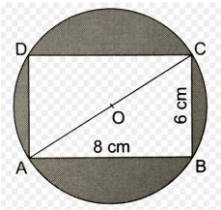
$= 3.14 \times (10)^2 \times 60°/360° = 53\frac{1}{3}$ cm^2

In ΔPOQ PO = OQ , $\angle POQ = \angle OQP$ = 60°, =10cm

Area of ΔPOQ = √3/4 × $(10)^2$= 43.25cm

Area of minor segment = 52.33 -43.25 = 9.08cm^2

3. Find the area of the shaded region of given figure, if ABCD is a rectangle with sides 8cm and 6cm and O is the centre of the circle. (take π – 3.14).



Solution- O is the centre of circle, AC is diameter,

ΔADC, $\angle D$=90, $AC^2 = AD^2 + CD^2$

= $(6)^2 + (8)^2$ =100

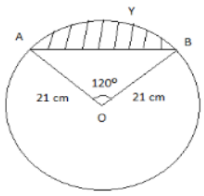
AC^2 = √100 = 10cm, radius = 5cm

Area of rectangle = (6) × (8) = 48cm^2

Area of shaded region = area of circle- area of rectangle

= 78.5 – 48 = 30.5cm^2

4. Find the area of the segment shown in figure, if radius of the circle is 21cm and $\angle AOB$ = 120° (use π= 22/7).



Solution- Given angle of arc = 120 degrees

radius of circle = 21 cm

Area of circle = πr^2

=3.14*21*21

=1386 cm2

Area of Arc= (angle of arc/ 360) *area of circle

= (120 /360) *1386

=1386/3

=462 cm2

Area of triangle = a*b*sin x / 2

=21*21*sin (120) /2

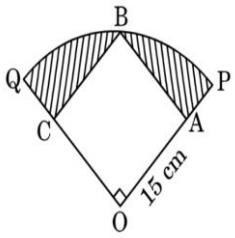
=190.96 cm2

Area of segment = area of arc - area of triangle

=462 - 190.96

=271.04 cm2

5. In the figure, a square OABC is inscribed in a quadrant OPBQ. if OA =15cm, find the area of the shaded region. (use π= 3.14)



Solution- Join OB.

We know, as ∠OAB = 90° (angle of a square)

Given, OA = 20 cm

Thus, OA = AB = 15 cm [Sides of a square]

∴ Using <u>Pythagoras theorem</u>

$OB^2 = OA^2 + AB^2$

$= (15\ cm)^2 + (15\ cm)^2$

$OB = \sqrt{2} \times (15cm)^{2}$

$= 15\sqrt{2}$ cm

Therefore, <u>radius</u> of the quadrant, r = OB = $15\sqrt{2}$ cm

Area of quadrant OPBQ = 90°/360° × πr^2

= 1/4 × 3.14 × (15√2 cm)^2

= 1/4 × 3.14 × 225× 2 cm^2

= 353.24cm^2

Area of square OABC = (side)^2

= (OA)^2

= (15 cm)^2

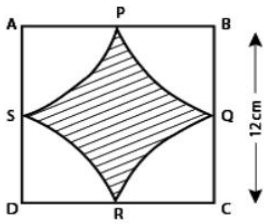
= 225cm^2

Area of the shaded region = Area of quadrant OPBQ - Area of square OABC

= 353.24 cm^2 - 225 cm^2

= 128.24 cm^2

6. Find the area of the shaded region in the figure where arcs drawn with the centre A, B, C and D intersects in the pairs at the mid-points P, Q, R and S of the sides AB,BC,CD and DA respectively of a square ABCD Of side 12cm.



Solution- Given side of the square BC=12cm

Since, Q is the midpoint of BC

Radius =BQ=122=6cm

Area of quadrant BPQ=$\pi r24=62\pi4=113.044cm^2$

Area of four quadrants =4×113.044=113.04cm^2

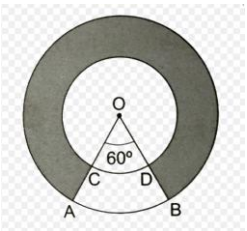
Area of square =122=144cm^2

∴ Area of shaded region = Area of square − Area of four quadrants

=144−113.04

∴ Area of shaded region =30.96cm2

7. In the given Figure, two concentric circles with centre O have radii 21cm and 42cm. if $\angle AOB$= 60^o, find the area of the shaded region.



Solution- Radius of bigger circle = 42 cm

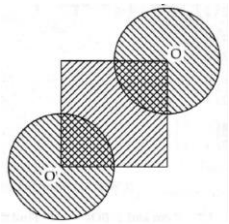
Area of major sector in bigger circle = θ/360πr^2

Area of major sector in bigger circle = 300/360 × π (42)^2

Area of major sector in bigger circle = 4620 cm^2
Radius of smaller circle = 21cm
Area of major sector = $\theta/360 \times \pi r^2$
Area of major sector = $300/360 \times \pi (21)^2$
Area of major sector = 1155 cm^2
Area of shaded portion = 4620−1155 = 3465 cm^2

Long Answer Type Questions

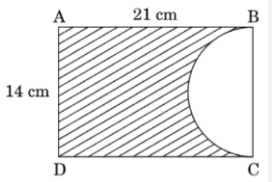
1. In the given figure, the sides of a square is 28cm and radius of each circle is half of the length of the side of the square where O and O' are centres of the circles. Find the area of the shaded region.



Solution- Side of square 28cm

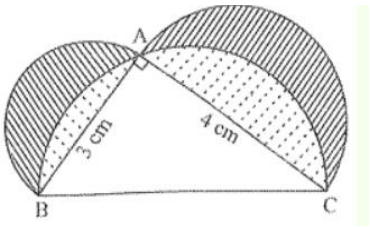
The area of square $28\times28=784cm^2$
The radius of circle is 14cm
The area of circle is $=22/7\times14\times14$
$\Rightarrow44\times14=616cm^2$
The overlapped area is $90°/360°\times616=154cm^2$
The resultant area of circle is $616-154=462cm^2$
The total area is $784+462+462=1708cm^2$

2. In the given figure ABCD is a rectangle of dimension 21cm × 14cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.



Solution- Area of shaded region = Area of rectangle - Area of semi-circle
$= l \times b - \frac{1}{2} \pi r^2 = 21 \times 14 - 1/2 \times 22/7 \times (7) \times (7)$
$=294-77=217\ cm^2$
Perimeter of shaded region
$= 2l+b+\pi r=2\times21+14+22/7\times7=42+14+22=78$ cm

3. In the given figure,ΔABC is a right-angled triangle in which $\angle A$ is 90°, Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region.



Solution- $BC^2=AB^2+BC^2$

$=(3)^2+(4)^2$

$=9+16=25$

$BC=\sqrt{25}=5$ cm

Area of semi-circle with diameter $BC=1/2\pi r^2$

$=1/2\times\pi(5/2)^2=25/8\pi$ cm^2

Area of semi-circle with diameter $AB=1/2\pi r^2$

$=1/2\times\pi(3/2)^2=9/8\pi$ cm^2

Area of semi-circle with diameter $AC=1/2\pi r^2$

$=1/2\times\pi(4/2)^2=16/8\pi$ cm^2

Area of semi-circle with diameter $AC=1/2\pi r^2$

Area of rt $\Delta BAC=1/2\times AB\times AC$

$=1/2\times3\times4=6$ cm^2

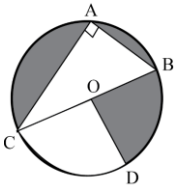
Area of dotted region $=(25/8\pi-6)$cm^2

Area of shaded region $=16/8\pi+9/8\pi-(25/8\pi-6)$

$=16/8\pi+9/8\pi-25/8\pi+6$

$=6$ cm^2

4. In the given figure O is the centre of the circle with AC = 24cm, AB = 7cm and $\angle BOD = 90°$,find the area of the shaded region.



Solution- In right triangle ABC

$BC^2=AB^2+AC^2=72+242=49+576=625$

$\therefore BC^2=625\Rightarrow BC=25$

So,

Area of a circle $=\pi r^2=22/7\times(12.5)^2$

$=491.07$ cm^2

Also,

Area of triangle $ABC=1/2\times(AC\times AB)$

$=1/2\times(24\times7)=84\ cm^2$

Since, $\angle BOD=90°$ and $\angle COD=\angle BOD-90°$ (Linear pair)

So, $\angle COD=90°$

$OC=BC/2=25/2=12.5$ cm

So, sector OCDO is a quadrant.

Area of sector $OCDO=1/4\times(\pi r^2)$

$=1/4\times22/7\times12.5\times12.5=122.7\ cm^2$.

Area of the shaded region = area of the circle - area of triangle ABC – area of sector OCDO.

$=491.07-84-122.7=284.37cm^2$

So, the area of the shaded region is approx. 284.37 cm^2

5. Two circles touch internally. The sum of their areas is 116 πcm^2 and the distance between their centre is 6 cm. find the radii of the circle.

Solution- Sum of area of two circles $=116\times\pi$ cm2

Distance between the centres of two circles = 6 cm

Since the two circles are touching internally, the distance between the centres of the two circles = (Radius of bigger circle) - (Radius of smaller circle)

Let r =radius of bigger circle

Then radius of small circle = r - 6

Area of big circle = 3.14 × r × r

and area of small circle $=116\times(\pi)$....(given)

$=(\pi)\times(r-6)2=(\pi4)\times(r2-12r+36)$

Sum of area of two circles $=(\pi)\times r^2+[(\pi/4\times r^2-12r+36)]$

$=(\pi)\times(2r^2-12r+36)$

Dividing both side of equation by $2\times(\pi)$ we get

$r^2-6r-40=0$

by factorising left hand side of the equation we get:

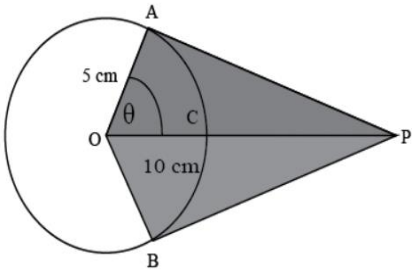
(r - 10)(r + 4) = 0

Therefore r = 10, or r=-4

As radius cannot be negative, the radius of bigger circle = 10 cm

And the radius of smaller circle = 10 - 6 = 4 cm

6. An elastic belt is put around a rim in a pulley of radius 5cm.Find one point C on the belt, the elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10cm away from the point O find the length of the belt that is still in contact with the pulley. Also find the area of shaded region. (use π – 3.14 and √3- 1.73).



Solution - In right angle AOP,

Cos θ = Base/hypotenuse=AO/OP=5/10=1/2

⇒θ=60°

∠AOB=2θ=2×60°=120°.

Length of ADB=(360°−120°)/360°×2πr= 240°/360°×(2×3.14×5)=20.93 cm.

Hence, length of belt is 20.93 cm.

In right angle OAP,

$OP^2=AO^2+AP^2$

$10^2=5^2+AP^2$

$AP^2=75$

AP=5√3 cm.

Area of triangle OAP=1/2×(AO×AP)=1/2×(5×5√3)=25√32 cm^2.

Area quadrilateral AOBP=2×area of triangle OAP=2×25√32=25√3=43.25cm^2.

Also,

Area of sector OACB=120°/360°×(3.14×5×5)=26.16 cm2.

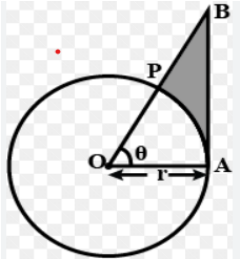
So,

Area of shaded region = Area of quadrilateral AOBP - Area of sector OACB

= (43.25−26.16) cm^2=17.09 cm^2.

Hence, area of shaded region is equal to 17.09 cm2.

7. In figure, is shown a sector OAP of a circle with centre O, containing ∠ɵ.AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is r [tanɵ + secɵ + πɵ/180 – 1].



Solution- From the given figure we have, in Δ OAB,

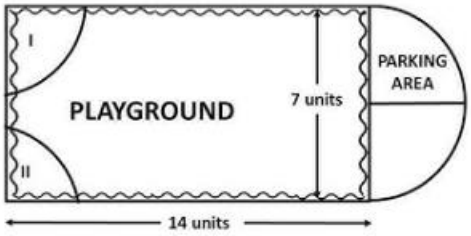
Tan θ=AB/OA⇒AB=r tan θ

[Since OA=r] .... (1)

And cos θ=OA/OB⇒OB=r sec θ..... (2).
Now PB= OB−OP= r sec θ−r. .... (3). [Using (2)].
Again πθ/180°= AP/OA
or, AP=rπθ/180°.....(4).
Now perimeter of the shaded region is
AB+PB+AP=r [sec θ + tan θ+πθ/180°−1]. [Using (1), (3) and (4)].

Case Based Question

1. Governing council of a local public development authority of Dehradun decided to build an adventurous playground on the top of hill, which will have adequate space for parking.



After survey, it was decided to build rectangular, with a semi-circle area allotted for Parking at one end of the playground. The length n breadth of the rectangular play Ground are 14 units and 7 units respectively. There are two quadrants of radius 2 Units on one side of special seats.

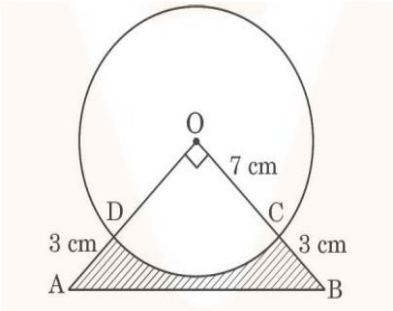
Based on the above information answer the following questions:

a) What is the total perimeter of the parking area
b) What is the total area of parking and the two quadrants
c) Find the cost of fencing the playground and parking area at the rate of Rs 2 per unit.

Solution- a) Radius of semi-circle, r = 7/2 units
Perimeter of parking area = πr + 2r = 22/7 × 7/2 + 2 × 7/2 = 18 units
b) area of parking = ½ πr^2 = ½ × 22/7 × 7/2 × 7/2 = 77/4 sq units
area of two quadrant = 2 × ¼ πR^2 = ½ × 22/7 × 2^2 =44/7 sq units
area of parking +area of two quadrant = 77/4 + 44/7 = 715/28 sq units
c) Perimeter = πr +2 × (length + Breadth)
= 22/7 × 7/2 + 2 × (14+7) = 53units
Total cost of fencing = (2 × 53) = 106.00

2. In a annual function of school, the organizers wanted to give a cash prize along with a memento to their best students. Each memento is made as shown in figure and its base ABCD is shown from front side. Rate of silver plating is Rs 20 per cm^2.
Based on above information, answer the following question-

   a) What is the area of Quadrant ODCO?
   b) Find the area of ΔAOB
   c) What is the length of arc CD



Solution- a) We have, ɵ = 90°, radius (r) = 7cm
Hence, area of quadrant ODCO = ɵ /360 × πr^2 = 90/360 × 22/7 × 7 × 7=77/2cm^2
b) ΔAOB is aright angled triangle with size OA= OB= 7+3=10cm
area of ΔAOB = ½ OB × OA = ½ × 10 × 10 = 50cm^2
c) Length of arc CD = ɵ/360 × 2πr = 90/360 × 2 × 22/7 × 7= ¼ × 44= 11cm

Assertion-Reason Questions

The following questions consist of two statements – Assertion(A) and Reason (R). Answer these questions selecting the appropriate option given below:

a) Both A and R are true and R is correct explanation for A.
b) Both A and R are true but R is not correct explanation for A
c) A is true R is false.
d) A is false but R is true.

1. **Assertion –** If the circumference of two circles is in ratio 2:3 then ratio of their areas are 4:9.
   Reason – The circumference of a circle is 2πr and its area is πr^2.

   Answer – (a)

2. **Assertion** – In a circle of radius 6cm, if the angle of sector is 60° then the area of sector is $18\frac{6}{7}$
   Reason- area of circle with radius r is πr^2
   Answer- (b)
3. **Assertion-** if the circumference of a circle is 176cm, then its radius is 28cm
   Reason - circumference = 2π × Radius of a circle.

   Answer – (a)

SURFACE AREAS AND VOLUMES

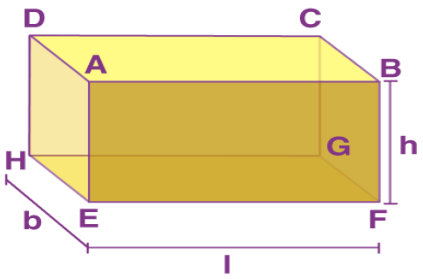
GIST OF THE CHAPTER

The concept of surface area and volume for Class 10 is provided here. In this article, we are going to discuss the surface area and volume for different solid shapes such as the cube, cuboid, cone, cylinder, and so on. The surface area can be generally classified into Lateral Surface Area (LSA), Total Surface Area (TSA), and Curved Surface Area (CSA). Here, let us discuss the surface area formulas and volume formulas for different three-dimensional shapes in detail. In this chapter, the combination of different solid shapes can be studied. Also, the procedure to find the volume and its surface area in detail.

Surface Area and Volume of Cuboid

A cuboid is a region covered by its six rectangular faces. The surface area of a cuboid is equal to the sum of the areas of its six rectangular faces.

Surface Area of the Cuboid



TSA (cuboid) = 2(l × b) + 2(b × h) + 2(l × h) = 2(lb + bh + lh)

The lateral surface area of the cuboid = Area of face AEHD + Area of face BFGC + Area of face ABFE + Area of face DHGC

LSA (cuboid) = 2(b × h) + 2(l × h) = 2h (l + b)

Length of diagonal of a cuboid =√ (l^2 +b^2 +h^2)

Volume of a Cuboid

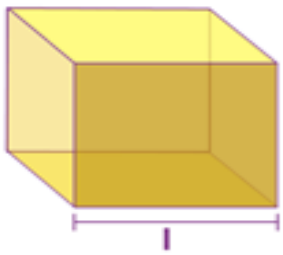
The volume of a cuboid is the space occupied within its six rectangular faces.

Volume of a cuboid = (base area) × height = (lb)h = lbh

Surface Area and Volume of Cube

A cube is a three-dimensional solid that has six square faces, twelve edges and eight vertices.

Surface Area of Cube



TSA of Cube = 6 × area of Square = $6l^2$ square units.

Similarly, the Lateral surface area of cube = 2(l × l + l × l) = $4l^2$

Note: Diagonal of a cube =√3l

Volume of a Cube

Volume of a cube = base area × height

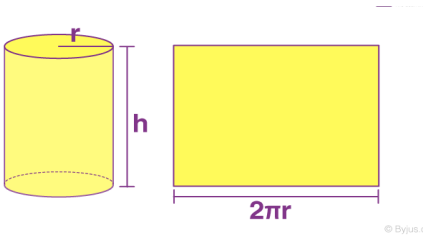
Since all dimensions of a cube are identical, volume = l^3

Surface Area and Volume of Cylinder

A cylinder is a solid shape that has two circular bases connected with each other through a lateral surface. Thus, there are three faces, two circular and one lateral, of a cylinder. Based on these dimensions, we can find the surface area and volume of a cylinder.

Surface Area of Cylinder

Take a cylinder of base radius r and height h units. The curved surface of this cylinder, if opened along the diameter (d = 2r) of the circular base can be transformed into a rectangle of length 2πr and height h units.



Transformation of a Cylinder into a rectangle.

CSA of a cylinder of base radius r and height h = 2π × r × h

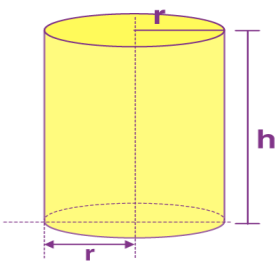
TSA of a cylinder of base radius r and height h = 2π × r × h + area of two circular bases

=2π × r × h + $2\pi r^2$

=2πr (h + r)

Volume of a Cylinder

Volume of a cylinder = Base area × height = (πr^2) × h = πr^2h

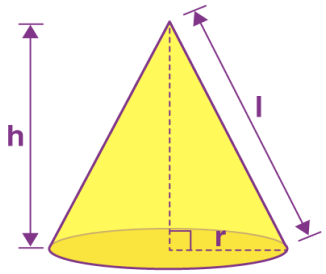


Surface Area and Volume of Right Circular Cone

A cone is a 3D shape that has one circular base and narrows smoothly from the base to a point called the vertex.

Surface Area of Cone

Consider a right circular cone with slant length *l*, radius *r* and height *h*.



one with base radius r and height h

CSA of right circular cone = πrl
TSA = CSA + area of base = πrl + πr^2 = πr(l + r)

Volume of a Right Circular Cone

The volume of a Right circular cone is 1/3 times that of a cylinder of the same height and base.
In other words, 3 cones make a cylinder of the same height and base.
The volume of a Right circular cone = (1/3) πr^2h
Where '*r*' is the radius of the base and '*h*' is the height of the cone.

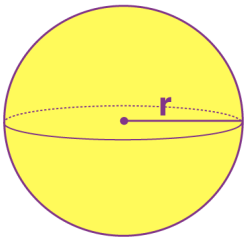
Surface Area and Volume of Sphere

A sphere is a solid that is round in shape, and the points on its surface are equidistant from the center.

Surface Area of Sphere

For a sphere of radius *r*

Curved Surface Area (CSA) = Total Surface Area (TSA) = $4\pi r^2$

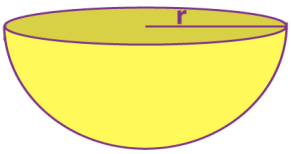


Volume of Sphere

- The volume of a sphere of radius *r* = $(4/3)\pi r^3$

Surface Area and Volume of Hemisphere

A hemisphere is a shape that is half of the sphere and has one flat surface. The other side of the hemisphere is shaped as a circular bowl.



Surface Area of Hemisphere

We know that the CSA of a sphere = $4\pi r^2$.

A hemisphere is half of a sphere.
∴ CSA of a hemisphere of radius *r* = $2\pi r^2$
Total Surface Area = curved surface area + area of the base circle
TSA = $3\pi r^2$

Volume of Hemisphere

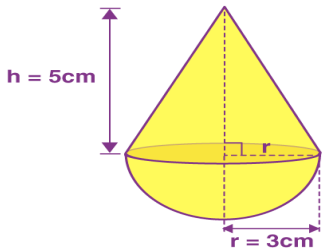
The volume (V) of a hemisphere will be half of that of a sphere.
∴

the volume of the hemisphere of radius r = (2/3) πr^3

Surface Area and Volume of Combination of Solids

Volume of Combined Solids

The volume of complex objects can be simplified by visualising them as a combination of shapes of known solids.
Example: A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 3 cm and the height of the cone is equal to 5 cm.
This can be visualised as follows



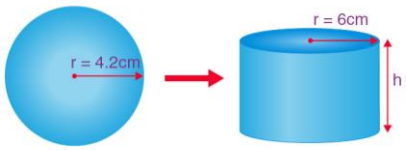
Volume of combined solids
V (solid) = V (Cone) + V (hemisphere)

Conversion of Solid from One Shape to Another

When a solid is converted into another solid of a different shape (by melting or casting), the volume remains constant.

Example:

A metallic sphere with a radius of 4.2 cm is melted and recast in the shape of a cylinder with a radius of 6 cm. Determine the cylinder's height.



Sphere's volume = Cylinder's volume.

Surface Areas and Volumes Formulas – Summary

Shape	Parameters	Surface Area (Square units)	Volume (Cubic units)
Cuboid	Length = l Breadth = b Height = h	TSA = 2(lb + bh + lh) LSA = 2h(l + b)	V = l × b × h
Cube	Length = Breadth = Height = l	TSA = $6l^2$ LSA = $4l^2$	V = l^3
Cylinder	Radius = r Height = h	CSA = 2π × r × h TSA = 2πr(h + r)	V = $\pi r^2 h$
Cone	Radius = r Height = h Slant Height = l	CSA = πrl TSA = πr(l + r)	V = $(1/3)\pi r^2 h$
Sphere	Radius = r	CSA = TSA = $4\pi r^2$	V = $(4/3)\pi r^3$
Hemisphere	Radius = r	CSA = $2\pi r^2$ TSA = $3\pi r^2$	V = $(2/3)\pi r^3$

Assertion Reason Questions

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:
(a) Both Assertion (A) & Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) & Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

A

1. **Assertion:** Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is 3872 cm^2.

Reason: If r be the radius and h be the height of the cylinder, then total surface area = ($2\pi rh + 2\pi r^2$).

Answer: (a) (BASIC)

2. **Assertion:** If a ball is in the shape of a sphere has a surface area of 221.76cm^2 , then its diameter is 8.4 cm.

   Reason: If the radius of the sphere be r, then surface area, S = 4πr^2, i.e., r = √(S/4π)

   Answer: (a) (STANDARD)

B

1. **Assertion:** No. of spherical balls that can be made out of a solid cube of lead whose edge is 44 cm, each ball being 4 cm. in diameter, is 2541

   Reason: Number of balls = Volume of one ball / Volume of lead

Solution:- Number of Balls = volume of cube / Volume of sphere

= 44 x 44 x 44/$\frac{4}{3}$ x $\frac{22}{7}$x2x2x2 =2541

So, A is true but R is false

Answer: (c) (BASIC)

2. **Assertion:** If the volumes of two spheres are in the ratio 27:8.Then their surface areas are in the ratio 3:2.

   Reason: Volumes of the sphere = $\frac{4}{3}\pi r^3$ and its surface area = 4πr^2

   Solution:- Ratio of volumes = $\frac{4}{3}\pi R^3 / \frac{4}{3}\pi r^3 = \frac{27}{8}$

   Or $R^3/r^3 = \frac{27}{8}$

   Or R/r = 3/2

   Ratio of surface areas = $4\pi R^2/4\pi r^2 = R^2/r^2 = (\frac{3}{2})^2$ = 9/4

So, A is false but R is true.

Answer: (d) (STANDARD)

MCQ

BASIC

1. If a cone is cut parallel to the base of it by a plane in two parts, then the shape of the top of the cone will be a:

(a) Sphere

(b) Cube

(c) Cone itself

(d) Cylinder

Answer: **(c) Cone itself**

Explanation: If we cut a cone into two parts parallel to the base, then the shape of the upper part remains the same.

2. **If we change the shape of an object from a sphere to a cylinder, then the volume of cylinder will**

(a) Increase

(b) Decrease

(c) Remains unchanged

(d) Doubles

Answer: **(c) Remains unchanged**

Explanation: If we change the shape of a three-dimensional object, the volume of the new shape will be same.

3. **A cylindrical pencil sharpened at one edge is the combination of**

(a) a cone and a cylinder

(b) frustum of a cone and a cylinder

(c) a hemisphere and a cylinder

(d) two cylinders

Answer: **(a) a cone and a cylinder**

STANDARD

4. A solid piece of iron in the form of a cuboid of dimensions 49 cm × 33 cm × 24 cm, is moulded to form a solid sphere. The radius of the sphere is

(a) 21 cm

(b) 23 cm

(c) 25 cm

(d) 19 cm

Answer: **(a) 21 cm**

5. The number of shots each having diameter 3 cm can be made from a cuboidal lead solid of dimensions 9 cm × 11 cm × 12 cm is approximately equal to

(a) 84

(b) 90

(c) 92

(d) 80

Answer: **(a) 84**

VERY SHORT ANSWER QUESTIONS

BASIC

1. A cone of maximum size is carved out from a cube of edge 14 cm. Find the surface area of the remaining solid after the cone is carved out.(competency based)

Solution

One will have a diameter and height of 14cm

∴ Radius =7cm

Short height =$\sqrt{14^2 + 7^2}$

=$\sqrt{245}$=l

Surface area =πr^2+πrl

=π (49) +π (7) $\sqrt{245}$=7π (7+$\sqrt{245}$)

Surface area of remaining solid = Total surface area of cube - Area of circle where cone carved out + curved surface area of cone.

=14×14×6−π×7^2+ (π×7×7π (7+$\sqrt{245}$)) =1366.344cm^3

2. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}cm^3$. Find the height of the cylinder.

Solution

Let Radius and height =a cm.
Volume of a Cylinder =πR^2h=22/7×a×a×a=$25\frac{1}{7}cm^3$

a^3=176/22=8cm^3

a=2cm

3. **If the total surface area of a solid hemisphere is $462cm^2$, find its volume.**

Answer: - V =718.67cm^3 (Approximately)

STANDARD

4. A well of diameter 4 m is dug 21 m deep. The earth taken out of it, has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment.

Answer :- (h=4m)

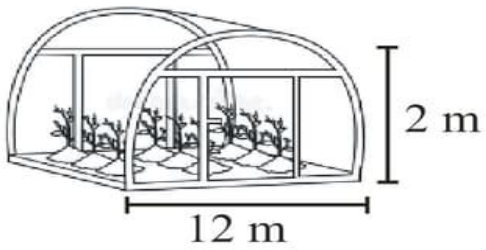
5. A solid is in the form of a cylinder with hemispherical ends. Total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the volume of the solid.

(Answer 641.6cm^3)

SHORT ANSWER QUESTIONS

BASIC

1. Dinesh is building a greenhouse in his farm as shown below. The base of the greenhouse is circular having a diameter of 12 m and it has a hemispherical dome on top. (Note: The image is not to scale.) How much will it cost him to cover the walls and top of the greenhouse with transparent plastic, if the plastic sheet costs Rs.77 per sq. m? Show your steps. (Note: Take π=22/7) (Competency based)



Solution:

r=12/2=6 m

h=2 m

CSA of top of the greenhouse=$2\pi r^2$

= 2 x π x 6^2 m^2

CSA of wall=2πrh

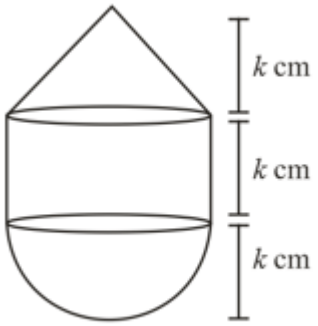
= 2 x π x 6 x 2 m^2

Total surface area to be covered with plastic=72π + 24π= 96π m^2

Cost of plastic to cover the whole greenhouse=96π x Rs.77

= Rs.23232

2. Shown below is a solid made of a cone, a cylinder and a hemisphere.



(Note: The figure is not to scale.)

Prove that the total volume of the solid is twice the volume of the cylinder. (Competency based)

Solution:

Volume of cylinder= πr^2h

Volume of cone= 1/3πr^2h

Volume of hemisphere= 2/3πr^3

Therefore, we get:

Volume of cylinder= πk^2×k= πk^3 cm^3

Volume of cone= 1/3π×k^2×k=1/3πk^3 cm^3

Volume of hemisphere= 2/3πk^3cm^3

Total volume of solid= πk^3 +1/3πk^3 + 2/3πk^3cm^3

$= 2\pi k^3 cm^3$ ... (i)

Volume of cylinder= $\pi k^3\ cm^3$ ... (ii)

Comparing both (i) and (ii),

We get:

The volume of Solid= 2×volume of cylinder

3. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed.

(Answer: $855 cm^2$)

STANDARD

4. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by 329 cm. Find the diameter of the cylindrical vessel.

(Answer: Diameter = 18cm)

5. A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. (Use π=22/7)

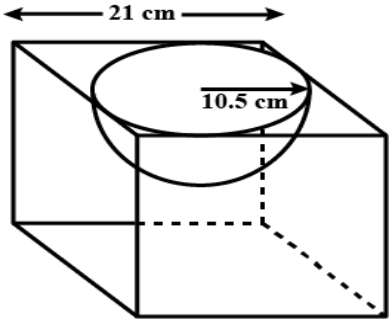
(Answer: h=2cm)

LONG ANSWER QUESTIONS

BASIC

1. **A hemispherical depression is cut out from one face of a cubical wooden block of edge 21cm, such that the diameter of the hemisphere is equal to the edge of the cube. Determine the volume and total surface area of the remaining block.**
   (Competency based)

Solution:



Length of edge of the cube a=21 cm

Diameter of hemisphere d=a=21 cm

Hence, radius of the hemisphere r=d/2=21/2=10.5 cm

Surface area of cube=$6l^2$

Curved surface area of hemisphere =$2\pi r^2$

Area of base of hemisphere=πr^2

Total surface area of remaining block = surface area of cube + surface area of hemisphere - area of base of hemisphere

=$6r^2+2\pi r^2-\pi r^2$

=$6r^2+\pi r^2$

= $6\times (21)^2 +22/7\times (10.5)_2\ cm^2$

= $(6\times441+22/7\times110.25)\ cm^2$

= $(2646+346.5)\ cm^2$

Total surface area of remaining block=2992.5 cm^2

Volume of the cube=l^3

Volume of hemisphere=$2/3\pi cm^3$

Hence,

Volume of remaining block = volume of cube - volume of hemisphere

=$l^3-2/3\pi r^3$

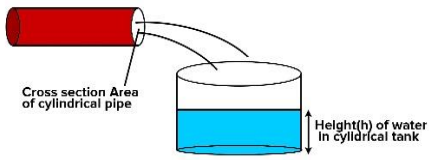
= $((21)^3-2/3\times22/7\times(10.5)^3)cm^3$

= $(9261-2/3\times22/7\times1157.625)cm^3$

= $(9261-2425.5)cm^3$

2. Water is flowing through a cylindrical pipe, with an internal diameter of 2cm, into a cylindrical tank with a base radius of 40cm, at the rate of 0.4m/s. Determine the rise in the level of water In the tank in half hours. (Competency based)

Solution:



Volume of water flowing through pipe in 1 secs

$=\pi R^2 H=\pi\times 1^2\times 0.4\times 100 cm^3$

Volume of water flowing through pipe in 30mins=30×60secs

$=\pi\times 1^2\times 0.4\times 100\times 30\times 60$

Volume of cylindrical tank in 30mins

$\pi\times 40^2\times h=\pi r^2 h$

$\Rightarrow\pi\times 40^2\times h=\pi\times 1^2\times 0.4\times 100\times 30\times 60$

$\Rightarrow h=\pi\times 1^2\times 0.4\times 100\times 30\times 60/\pi\times 40^2$

$=0.4\times 100\times 30\times 60/40\times 40=40\times 30\times 60/40\times 40=30\times 60/40=45cm$

3. A tent is in the shape of a right circular cylinder up to a height of 3 m and conical above it. The total height of the tent is 13.5 m and the radius of its base is 14 m. Find the cost of cloth required to make the tent at the rate of Rs 80 per square metre. [Take $\pi=22/7$.]

Answer: Rs. 82720

STANDARD

4. A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone formed. (Choose value of π as found appropriate).

Answer: 52.75cm^2

5. Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tank will rise by 21 cm.

Answer: 2 Hrs.

CASE STUDY BASED QUESTIONS

BASIC

1. The word 'circus' has the same root as 'circle'. In a closed circular area, various entertainment acts including human skill and animal training are presented before the crowd. A circus tent is cylindrical up to a height of 8 m and conical above it. The diameter of the base is 28 m and total height of tent is 18.5 m.

Based on the above information, answer the following questions:

(i) **Find slant height of the conical part.**

Answer:

l =$\sqrt{r^2 + h^2}$ = 17.5 cm

(ii) **Determine the floor area of the tent.**
Answer:
A = πr^2 = 22/7 x 14 x 14 = 616 m^2

(iii) **Find the area of the cloth used for making the tent.**
Answer:
Area of the cloth = CSA of cone + CSA of cylinder
= πrl + $2\pi r$h
= $\pi r(l +$ 2h) = 22/7 x 14 x 16 = 1474 m^2

OR

Find the total volume of air inside an empty tent.

Answer: H = 18.5m - 8m = 10.5m

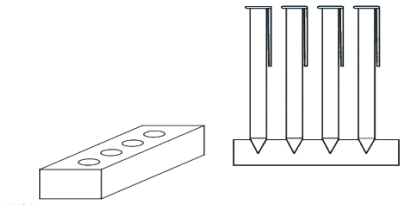
V = Volume of the cone + Volume of the cylinder

V = $\frac{1}{3}\pi r^2 H$ + $\pi r^2 h$ = $\pi r^2(\frac{1}{3}H$ + h)

= 22/7 x 14 x14(10.5/3 + 8)

= 616 x 11.5 cm^3 = 7084cm^3

2. A carpenter used to make and sell different kinds of wooden pen stands like rectangular, cuboidal, cylindrical, and conical. Aarav went to his shop and asked him to make a pen stand as explained below. Pen stand must be of the cuboidal shape with three conical depressions, which can hold 4 pens. The dimensions of the cuboidal part must be 15 cm x 10 cm x 3.5 cm and the radius and depth of each conical depression must be 0.5 cm and 1.4 cm respectively.



(i) Find the the volume of the cuboidal part.

Answer: Volume of cuboid = length × width × height
=15×10×3.5=525 cm^3

(ii)Find the total volume of conical depressions.

Answer: Volume of cone
=1/3πr2h
=1/3×22/7×0.5^2×1.4
=1130 cm^3

(iii)Find the volume of the wood used in the entire stand.

Answer: Dimensions of cuboid =15cm×10cm×3.5cm, radius of cone =0.5 cm, depth of cone = 1.4 cm
Volume of cuboid = length × width × height
=15×10×3.5=525 cm^3
Volume of cone
=1/3πr2h
=1/3×22/7×0.5^2×1.4
=1130 cm^3
Volume of wood = Volume of cuboid – 4 × Volume of cone
=525−4×1130
=525−2215
=523.53

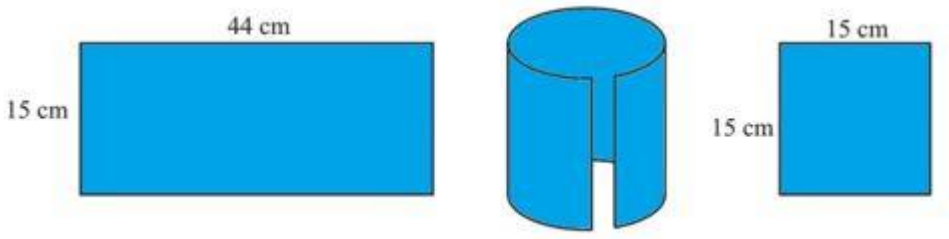
OR

If the cost of wood used is Rs 5 per cm^3, then find the total cost of making the pen stand.

Answer: Cost of making pen stand = Rs. 5 x 523.53 = Rs.2617.65

STANDARD

3. Mohan lives in Hyderabad in Telangana. Those were very hot days of May. He thought that if we human beings need so much of water to drink, won't the birds also be thirsty. He decided to prepare a vessel to provide water for birds. He found a flexible blue coloured plastic rectangular sheet 44 cm × 15 cm. He rolled it along its length and joined the two opposite ends using a tape. He wanted to have a circular base for this cylinder and searched for another sheet. He found a square sheet 15 cm × 15 cm. He got a circular sheet just equal to the base of the cylinder cut from it.



(i)Find the curved surface area of the cylinder formed. ($660 cm^2$)

(ii) Find the area of the circular base required for the cylinder. ($154 cm^2$)

(iii)How much will be the area of square sheet left unused after removing the circular base of the cylinder from it? ($71 cm^2$)

OR

Find the volume of water that can be filled in the cylinder. (2310ml)

STATISTICS

GIST OF THE LESSON:

Statistics is one of the parts of mathematics in which we study about the collecting, organizing, analyzing, interpreting and presenting data. Statistics is very helpful in real life situations as it is easy to understand if we represent a data in a particular number which represents all numbers. This number is called the **measure of central tendency**. Some of the central tendencies commonly in use are -

Mean: It is the average of "n" numbers, which is calculated by dividing the sum of all the numbers by n.

The mean $\bar{x}$ of n values $x_1, x_2, x_3, \ldots\ldots x_n$ is given by $\bar{x} = \frac{x_1+x_2+x_2+\cdots+x_n}{n}$

Median: If we arrange the numbers in an ascending or descending order then the middle number of the series will be median. If the number of series is even then the median will be the average of two middle numbers.

If n is odd then the median is $\left(\frac{n+1}{2}\right)^{th}$ observation.

If the n is even then the median is the average of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n+1}{2}\right)^{th}$ observation.

Mode: The number which appears most frequently in the series then it is said to be the mode of n numbers.

Mean of Grouped Data (Without Class Interval):

If the data is organized in such a way that there is no class interval then we can calculate the mean by $\bar{x} = \frac{f_1x_1+f_2x_2+\cdots+f_nx_n}{f_1+f_2+\cdots+f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i}$

where, $x_1, x_2, x_3, \ldots\ldots x_n$ are the observations

$f_1, f_2, f_3, \ldots\ldots f_n$ are the respective frequencies of the given observations.

Mean of Grouped Data (With Class-Interval):

When the data is grouped in the form of class interval then the mean can be calculated by three methods.

1. Direct Method

In this method, we use a midpoint which represents the whole class. It is called the **class mark**. It is the average of the upper limit and the lower limit.

Class Mark= $\frac{upper\ class\ limit - lower\ class\ limit}{2}$

Or, $\bar{x} = \frac{\sum f_ix_i}{\sum f_i}$

2. Deviation or Assumed Mean Method:

If we have to calculate the large numbers then we can use this method to make our calculations easy. In this method, we choose one of the x's as **assumed mean** and let it as "a". Then we find the deviation which is the difference of assumed mean and each of the x. The rest of the method is the same as the direct method.

$$\bar{x} = a + \frac{\sum f_id_i}{\sum f_i}$$

a is the assumed mean and $d_i = x_i - a$ are the deviations of x_i from a for each i

3. Step Deviation Method:

In this method, we divide the values of d with a number "h" to make our calculations easier.

$\bar{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times h$, where a is the assumed mean, h is the class size and $u_i = \frac{x_i-a}{h}$

Mode of Grouped Data:

In the ungrouped data the most frequently occurring no. is the mode of the sequence, but in the grouped data we can find the class interval only which has the maximum frequency number i.e**. the modal class.**

The value of mode in that modal class is calculated by

Mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

l = lower class limit of the modal class

h = class interval /size

f_1 =frequency of the modal class

f_0 =frequency of the preceding class

f_2 = frequency of the succeeding class

Median of Grouped Data:

To find the median of a grouped data, we need to find the cumulative frequency and n/2, then we have to find the median class, which is the class of the cumulative frequency near or greater than the value of n/2.

Cumulative Frequency is calculated by adding the frequencies of all the classes preceding the given class.

Then substitute the values in the formula

Median = $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

where l = lower limit of median class

n = no. of observations

cf = cumulative frequency of the class preceding to the median class

f = frequency of the median class

h = size of class

Remark: The empirical relation between the three measures of central tendency is

3 Median = Mode + 2 Mean

M.C.Q.(1 MARK)

(Q:1) The arithmetic mean of x,x+3,x+6, x+9and x+12 is

(a) X+6 (b) x+5 (c) x+7 (d) x+5

Ans (a)

(Q:2) If the value of each observations of a statistical data consisting of n observations is increase by 3 then the mean of the data

(a) Remain unchanged (b) increases by 3 (c) increases by 6 (d) increases by 3n

Ans (b)

(Q:3) If for adata , Mean :Median = 9:8 , then Median : Mode =

(a) 8:9 (b) 4:3 (c) 7:6 (d) 5:6

Ans (b)

(Q:4) If the difference of mode and median of a data is 24, then the difference of median and mean is

(a) 12 (b) 24 (c) 8 (d) 36

Ans (a)

(Q:5) Which of the following is not a measure of central tendency?

(a) Mean (b) Median (c) Mode (d) Standard deviation

Ans (d)

ASSERTION –REASON BASED QUESTIONS (1 MARK)

Each of the following examples contains STATEMENT -1 (Assertion) and STATEMENT -2 (Reason) and has following four choices (a), (b) ,(c)and (d) ,only one of which is the correct answer.Mark the correct choice.

(a) Statement -1 and Statement2are True, Statement-2 is a correct explanation for statement -1
(b) Statement-1and Statement-2are True ;Statement -2is not a correct explanation for Statement -1
(c) Statement -1 is True ;Statement- 2 is False.
(d) Statement -1 is False , Statement -2 is True.

(Q:1) Statement -1 (A) The mean of first fifty nine natural numbers is 30

Statement -2 ® : The sum of first n natural numbers is $\frac{n(n+1)}{2}$

Ans (a)

Explanation : 1,2,3,……(n-1) ,n is an A,P, with first term =1, kast term n and common difference =1

1+2+3+4+……+ (n-1) +n = $\frac{n(n+1)}{2}$

So, Statement -2 is true.

Let x= be the mean of first fifty nine natural numbers . Then

X= $\frac{1+2+3+\cdots\ldots\ldots+59}{59}$ = $\frac{1}{59}\left[\frac{59(59+1)}{2}\right]$ =30

So Statement -1 is also True ,Also statement -2 is correct explanation for statement -1.

Hence, option (a) is correct.

(Q:2) Statement -1 (A) If the difference of the mode and median of a data is 24. Then the difference of the median and the mean is 12.

Statement -2 (R) For a moderately asymmetric distribution ,mode ,median and mean are connected by the relation: Mode = 2 Median -3 Mean

Ans (c)

Explanation: It is given that : Mode –Median =24

We know that Mode –Median = 2(Median -Mean)

24 = 2(Median -Mean)

24 = Median –Mean =12

So, Statement -1 is True.

For a moderately asymmetric distribution, the relationship between Mode, Median and Mean is

Mode =3 Median - 2 Mean.

So, Statement 2 is not true. Hence, Option (c) is correct.

SHORT ANSWER TYPE : 1 QUESTION (2 MARKS)

(Q:1) If the mean of the following distribution is 8, find the value of p.

x	2	4	6	10	P+5
y	3	2	3	1	2

Ans : p=18

(Q:2) In a school 85 boys and 35 girls appeared in public examination. The mean marks of boys was found to be 40%,where as the mean marks of girls was 60% Determine the average marks percentage of the school. Ans : 45.83

(Q:3) 20 years ago when my parents got married , their average age was 23 years.now the average age of my my family consisting of myself and my parents is 34 years. What is my present age? Ans : x=16

(Q:4) Thje mean of a set of a number is x. if each number is multiplied by k, then find the mean of the new set. Ans : kx

(Q:5) The mean of the following distribution is 18. Find the frequency f of the class 19-21

Class Interval	Frequency
11-13	3
13-15	6
15-17	9
17-19	13
19-21	f
21-23	5
23-25	4

Ans : f=8

SHORT ANSER TYPE :2 QUESTIONS (3 MARKS)

(Q:1) The mean of the following frequency distribution is 25. Find the value of f.

Class	Frequency
0-10	5
10-20	18
20-30	15
30-40	F
40-50	6

(The Q:2) Find the mode of the following distribution:

Marks	Number of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

Ans:35.56

(Q:3) The table below shows the salaries of 280 persons. Calculate the median salary of the data.

Salary (in thousand)	No. of persons
5-10	49
10-15	153
15-20	63
20-25	15
25-30	6
30-35	7
35-40	4
40-45	2
40-50	1

Ans:13.42

(Q:4) Find the median of the following data

Class-Interval	Frequency
130-139	4
140-149	9
150-159	18
160-169	28
170-179	24
180-189	10

190-198	**7**

Ans:166.3

(Q:5) Find the mean of the following frequency distribution:

Class Interval	Frequency
0-10	**4**
10-20	**4**
20-30	**7**
30-40	**10**
40-50	**12**
50-60	**8**
60-70	**5**
Total	**50**

Ans:38.2

LONG ANSWER TYPE QUESTIONS (5 MARKS)

(Q:1) The ages of employees in two factories A and B are given below:

Age of employees (in years)	No.of employees in factory A	No.of employees in factory B
20-30	5	8
30-40	26	40
40-50	78	58
50-60	104	90
60-70	98	83

Find the modal age of employees in factory A and factory B

For factory A, max. frequency = 104.

$\therefore$ Modal class = 50 – 60.

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

Here, $f_1 = 104, f_0 = 78, f_2 = 98, h = 10, l = 50$

$$\text{Mode} = 50 + \left[\frac{104 - 78}{2 \times 104 - 78 - 98}\right] \times 10$$

$$= 50 + \frac{26}{32} \times 10 = 50 + 8.125 = 58.125$$

For factory B, max. frequency = 90.

$\therefore$ Modal class = 50 – 60.

$$\text{Mode} = 50 + \left(\frac{90 - 58}{2 \times 90 - 58 - 83}\right) \times 10$$

$$= 50 + \frac{32 \times 10}{39} = 50 + 8.205$$

$$= 58.205$$

(Q:2) Find the value of f from the following data.If it's mode is 65.

Class	Frequency
0-20	6
20-40	8
40-60	F
60-80	12
80-100	6
100-120	5

Where frequency 6,8, f, and 12 are in ascending order.

Mode = 65

Modal class = 60 – 80 as its frequency is 12.

$l = 60, f_1 = 12, f_0 = f, f_2 = 6, h = 20.$

(Q:3) Find mean , median and mode of the following data:

Class	Frequency
0-20	6
20-40	8
40-60	10
60-80	12
80-100	6
100-120	5
120-140	3

23.

Classes	Frequency	frequency	
0 – 20	6	6	
20 – 40	8	14	
40 – 60	10	24	
60 – 80	12	36	← Median Class
80 – 100	6	42	
100 – 120	5	47	
120 – 140	3	50	
	$n = 50$		

$\therefore \quad \frac{n}{2} = 25$

Median class = (60 – 80)

$l = 60, f = 12, cf = 24, h = 20.$

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 60 + \frac{25 - 24}{12} \times 20$$

$$= 60 + \frac{1 \times 5}{3} = \frac{180 + 5}{3} = \frac{185}{3}$$

$$= 61.6$$

Modal class = (60 – 80) as its frequency is 12

$h = 20, l = 60, f_1 = 12, f_0 = 10, f_2 = 6.$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 60 + \frac{12 - 10}{2 \times 12 - 10 - 6} \times 20$$

$$= 60 + \frac{2}{8} \times 20 = 65$$

Now, Mode = 3 Median – 2 Mean

65 = 3 (61.6) – 2 Mean

2 Mean = 184.8 – 65

2 Mean = 119.8

$\Rightarrow$ Mean = $\frac{119.8}{2} = 59.9$

$\therefore$ Mean = 59.9; Median = 61.6; Mode = 65

(Q:4) If the median of the following frequency distribution is 32.5.Find the value of f1 and f2

Classes	Frequency
0-10	F1
10-20	5
20-30	9
30-40	12
40-50	F2
50-60	3
60-70	2

Class interval	Frequency	Cumulative frequency
$0-10$	f_1	f_1
$10-20$	5	$5+f_1$
$20-30$	9	$14+f_1$
$30-40$	12	$26+f_1$
$40-50$	f_2	$26+f_1+f_2$
$50-60$	3	$29+f_1+f_2$
$60-70$	2	$31+f_1+f_2$
Total	**40**	

$$\text{Median} = 32.5 \Rightarrow \text{median class is } 30-40.$$

$$\text{Now } 32.5 = 30 + \frac{10}{12}(20 - 14 - f_1) \Rightarrow f_1 = 3$$

$$\text{Also } 31 + f_1 + f_2 = 40 \Rightarrow f_2 = 6$$

(Q:5) An incomplete distribution is given below:

Variable	Frequency
10-20	12
20-30	30
30-40	X
40-50	65
50-60	Y
60-70	25
70-80	18

You are given that the median value is 46 and the total number of items is 230.

(1) Using the median formula and fill up missing frequencies

(2) Calculate the A.M.of the completed distribution.

C.I.	Frequency	Cumulative frequency
10 – 20	12	12
20 – 30	30	42
30 – 40	x	$42 + x$
40 – 50	65	$107 + x$
50 – 60	y	$107 + x + y$
60 – 70	25	$132 + x + y$
70 – 80	18	$150 + x + y$
	230	

Given $150 + x + y = 230 \Rightarrow x + y = 80$

$\therefore$ Using formula Median $= 46 \Rightarrow$ Median class is 40 – 50.

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

Here,

$$l = 40,\ cf = 42 + x,\ f = 65,\ h = 10,\ n = 230$$

$$46 = 40 + \left\{\frac{\frac{230}{2} - (42 + x)}{65}\right\} \times 10$$

$$\Rightarrow \quad 6 = \frac{115 - 42 - x}{65} \times 10$$

$$\Rightarrow \quad \frac{6 \times 65}{10} = 73 - x$$

$$\Rightarrow \quad \frac{390}{10} = 73 - x$$

$$\Rightarrow \quad 39 = 73 - x \Rightarrow x = 73 - 39 = 34$$

$$x = 34$$

Putting in (*i*), we get

$$x + y = 80 \Rightarrow 34 + y = 80$$

$$\Rightarrow \quad y = 80 - 34 = 46$$

For mean, $a = 45$, $h = 10$

Class interval	Frequency f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
10 – 20	12	15	–3	–36
20 – 30	30	25	–2	–60
30 – 40	34	35	–1	–34
40 – 50	65	45	0	0
50 – 60	46	55	1	46
60 – 70	25	65	2	50
70 – 80	18	75	3	54
	$\Sigma f_i = 230$			$\Sigma f_i u_i = 20$

$$\bar{x} = a + \frac{1}{\Sigma f_i} \times (\Sigma f_i u_i) \times h = 45 + \frac{1}{230} \times 20 \times 10 = 45.8695 = 45.87$$

CASE STUDY BASED QUESTIONS(4-MARKS)

(Q:1) Dengue is caused by the dengue virus .which is transmitted to humans through the bite of infected Aedes mosquitoes. The following table s shows thebage distribution of case admitted during a dayin two different hospitals

Table :1 **Table :2**

Age (in years)	No.of cases
5-15	6
15-25	11
25-35	21
35-45	23
45-55	14
55-65	5

Age (in years)	No.of cases
5-15	8
15-25	16
25-35	10
35-45	42
45-55	24
55-65	12

Refe to table 1:

(1) Find the average age for which maximum cases occurred. (Ans 16.82)
(2) Find the upper limit of modal class. (Ans: 45)
(3) Find the mean of the given data. (Ans : 35.375)

OR

Refer to table 2
Find the median of the given data. (Ans 40.23)

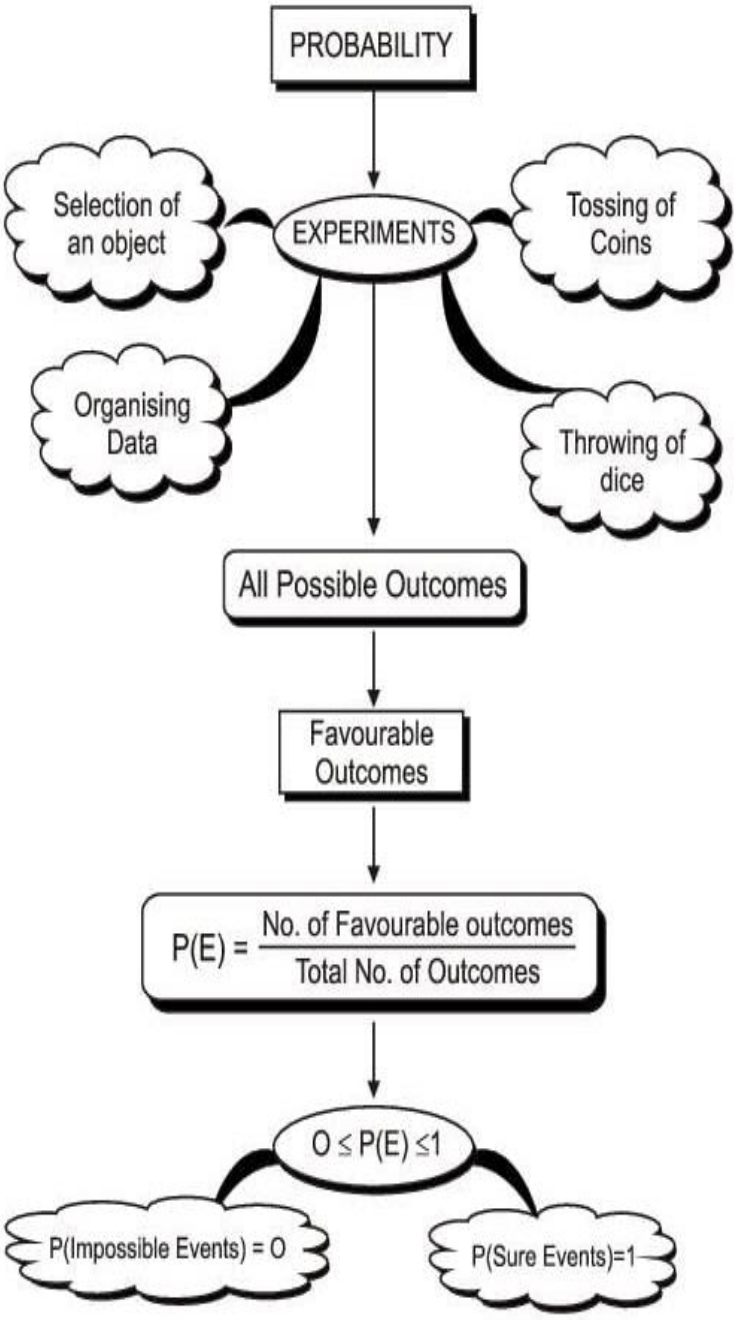
(Q:2) During the annual sports meet in a school,all the athlets were very enthusiastic, They all wanted ito be the winner so that their house could stand first . The instructor noted down the time taken by a group of students to complete a certain race .The data recorded is given below

Time (in seconds)	Number of students
0-20	1
20-40	4
40-60	3
60-80	7
80-100	5

Based on the above , answer the following questions.

1. **What is the classmark of the model class? (Ans: 70)**
2. **Find the median class of the given data (Ans: 60-80)**
3. **Find the mode of the given data. (Ans :73.33)**

CHAPTER-15
PROBABILITY
MIND MAP



IMPORTANT NOTES OF PROBABILITY

Probability: It is the numerical measurement of the degree of certainty.

Formula used to find probability:

$$P(E) = \frac{\text{Number of Outcomes Favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

- If P(E) = 1, then it is called a 'Certain Event'.
- If P(E) = 0, then it is called an 'Impossible Event'.
- The probability of an event E is a number P(E) such that: $0 \leq P(E) \leq 1$.
- An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.
- For any event E, P(E) + P(*not E*) = 1, E and *[not E]* are called complementary events.

Sum of Probabilities

The **sum** of the probabilities of all the **elementary events** of an experiment is **one**.Example: take the coin-tossing experiment

P(Heads) + P(Tails)

= (1/2)+ (1/2) =1

Impossible Event:

An event that has **no chance of occurring** is called an **Impossible event**,

i.e. P(E) = 0.

E.g., The probability of getting a 7 on a throwing a die is 0. As 7 can never be an outcome of this trial.

Sure event:

An event that has a **100% probability** of occurrence is called a **sure event**.

E.g., What is the probability that a number obtained after throwing a die is less than 7?So, P(E) = P(Getting a number

less than 7) = 6/6= 1

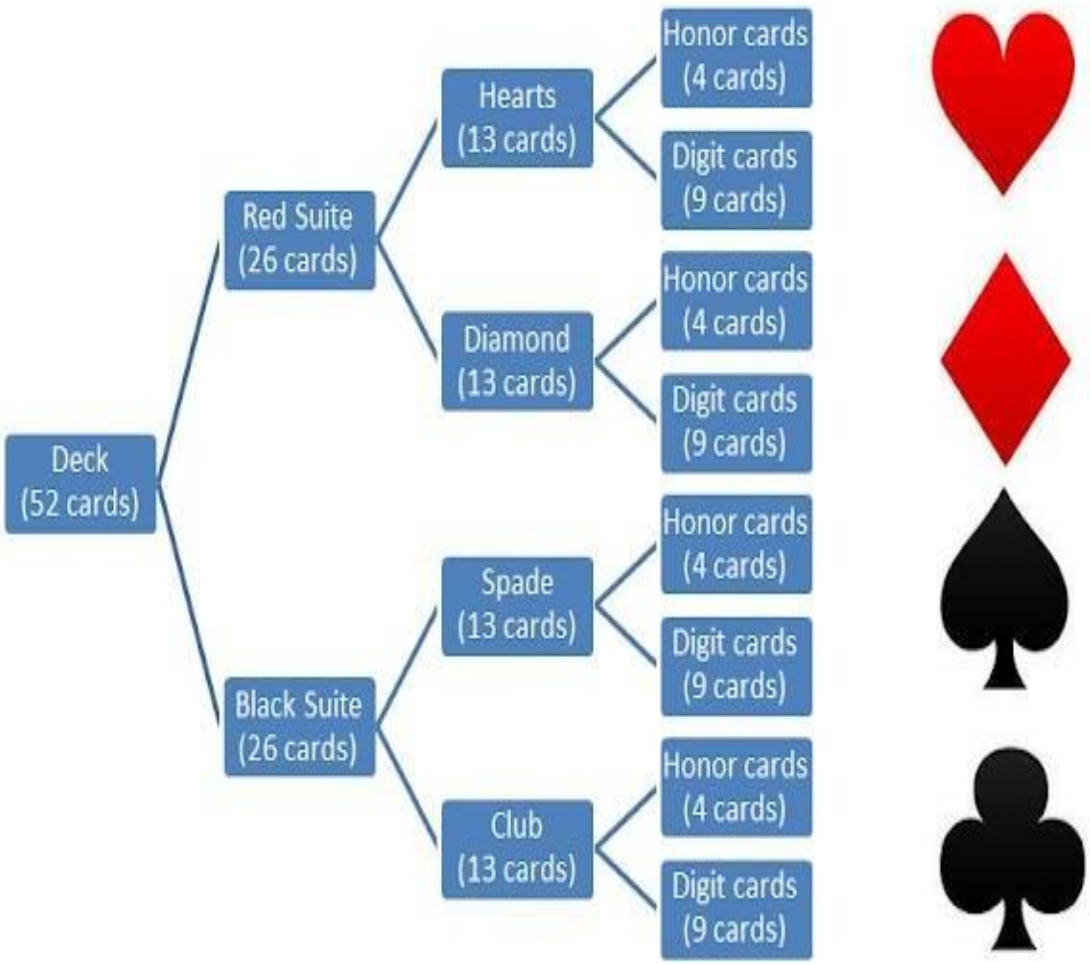
Range of Probability of an event:

Probability of an event lies between 0 and 1, where 0 probability means the event to be an impossible one and probability of 1 indicates a certain event.

i.e. $0 \leq P(E) \leq 1$.

Important points:-

- **Coin:** A coin has two faces termed a Head(H) and Tail(T). **If we toss a coin, it would be HEAD orTAIL, only two outcomes.**
- **Dice:** A dice is a small cube that has between one to six spots or numbers on its sides, which isused in games. **A DICE has six outcomes numbering 1 to 6.**
- **Cards:** A pack of playing cards (52 CARDS) consists of four suits called Hearts, Spades,Diamonds, and Clubs. Each suite consists of 13 cards.



Assertion and Reason

Maths Basic

Q.1. **Assertion:** The probability of getting a prime number when a die is thrown once is 2/3.

Reason: Prime numbers on a die are 2, 3, 5.

Answer: prime no. on die is 2, 3, 5

Total outcome = 6

Probability = 3/6 = ½

Correct option d

Maths Standard

Q2. Assertion: If a box contains 5 white, 2 red and 4 black marbles, then the probability of not drawing a white marble from the box is 5/11.

Reason: $P(\bar{E}) = 1 - P(E)$, where E is any event.

Answer: d

SECTION A - MCQ QUESTIONS:1 MARKS

Maths Basic

1. The probability of event equal to zero is called;
(a) Unsure event (b) Sure Event (c) Impossible event (d) Independent event
Answer: (c) Impossible event

2. The probability that cannot exist among the following:

(a) ⅔ (b) -1.5 (c) 15% (d) 0.7

Answer: (b) -1.5

3. If P(E) = 0.07, then what is the probability of 'not E'?

(a) 0.93 (b) 0.95 (c) 0.89 (d) 0.90
Answer: (a) 0.93

Maths Standard

4. A bag has 3 red balls and 5 green balls. If we take a ball from the bag, then what is the probability of getting red balls only?

(a) 3 (b) 8 (c) 3/8 (d)

8/3Answer: (c) 3/8

5. A bag has 5 white marbles, 8 red marbles and 4 purple marbles. If we take a marble randomly,then what is the probability of not getting purple marble?

(a) 0.5 (b) 0.66 (c) 0.08 (d) 0.77

Answer: (d) 0.77

SECTION B : VERY SHORT ANSWER I (VSA I): 1 MARK QUESTIONS

Maths Basic

Q1 A die is thrown once. Find the probability of getting Aa prime number,

Answer:- prime no. on die is 2, 3, 5

Total outcome = 6

Probability = 3/6 = ½

Q2. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red ?

Answer: Total ball 5 + 3 = 8

Probability of getting red ball = 3/8

Q3. A card is selected at random from a well shuffled deck of 52 playing cards. The probability of its being a face card

Maths Standard

Q1. A die is thrown once. Find the probability of getting

(i) a number lying between 2 and 6; (ii) an odd number.

Q2. One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will be an ace

SECTION B : SHORT ANSWER I (SA I): 2 MARK QUESTIONS

Maths Basic

Q. 1: Two dice are thrown at the same time. Find the probability of getting

(i) the same number on both dice.

(ii) different numbers on both dice.

Solution: Given that, Two dice are thrown at the same time.So, the total number of possible outcomes $n(S) = 6^2 = 36$

(i) Getting the same number on both dice:

Let A be the event of getting the same

number on both dice. Possible outcomes are

(1,1), (2,2), (3, 3), (4, 4), (5, 5) and (6, 6).

Number of possible outcomes = n(A) = 6

Hence, the required probability =P(A) = n(A)/n(S)

= 6/36= 1/6

(ii) Getting a different number on both dice.

Let B be the event of getting a different number on both dice.

Number of possible outcomes n(B) = 36 – Number of possible outcomes for the same numberon both dice

= 36 – 6 = 30

Hence, the required probability = P(B) = n(B)/n(S)

= 30/36

= 5/6

Q. 2: A bag contains a red ball, a blue ball and a yellow ball, all the balls being of the same size. Kritika takes out a ball from the bag without looking into it. What is the probability thatshe takes out the (i) yellow ball?(ii) red ball?

Solution: Kritika takes out a ball from the bag without looking into it. So, it is equally likelythat she takes out any one of them from the bag.

Let Y be the event 'the ball taken out is yellow', B be the event 'the ball taken out is blue', andR be the event 'the ball taken out is red'.

The number of possible outcomes = Number of balls in the bag = n(S) = 3.

(i) The number of outcomes favourable to the event Y = n(Y) = 1.

So, P(Y) = n(Y)/n(S) =1/3

Similarly, (ii) P(R) = 1/3

Q.3: One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability thatthe card will(i) be an ace,(ii) not be an ace.

Solution: Well-shuffling ensures equally likely outcomes.

(i) Card drawn is an ace There are 4 aces in a deck.

Let E be the event ‘the card is an ace’.

The number of outcomes favourable to E = n(E) = 4

The number of possible outcomes = Total number of

cards = n(S) = 52Therefore, P(E) = n(E)/n(S) = 4/52 =

1/13

(ii) Card drawn is not an ace

Let F be the event ‘card drawn is not an ace’.

The number of outcomes favourable to the event

F = n(F) = 52 – 4 = 48Therefore, P(F) =

n(F)/n(S) = 48/52 = 12/13

Maths Standard

Q1.A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be:

(i) red? (ii) white? (iii) not green?

Q2. Five cards – then ten, jack, queen, king and ace of diamonds, are well-shuffled with their facedownwards. One card is then picked up at random.

(i) What is the probability that the card is the queen?

(ii) If the queen is drawn and put aside, what is the probability that

the second card picked up is (a) an ace? (b) a queen?

SECTION C : LONG ANSWER I (LA I): 5 MARK QUESTIONS

Maths Basic

Q1. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at randomfrom the box, find the probability that it bears

(i) a two-digit number

(ii) a perfect square number

answer:Total number of discs = 90

Total number of 2 digit numbers between 1 to 90 = 81

Total number of perfect square numbers between1 to 90 are 1, 4, 9, 16, 25, 36, 49, 64, 81 = 9

Total numbers that are divisible by 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90 = 18

(i) Probability of getting a two digit number = Number of possible outcomes/Total number of favourable outcomes

= 81/90

= 9/10

(ii) Probability of getting a perfect square number = Number of possible outcomes/Total number of favorable outcomes

= 9/90

= 1/10

Q2. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that

(i) She will buy it ? (ii) She will not buy it ?

Solution:

We use the basic formula of probability and favourable outcomes.

Total number of ball pens = 144

Number of defective ball pens = 20

Number of good ball pens = 144 - 20 = 124

(i) Probability that she will buy it = Number of possible outcomes/Total number of favourable outcomes

= 124/144

= 31/36

(ii) Probability that she will not buy it = Number of possible outcomes/Total number of favourable outcomes

= 20/144

= 5/36

Q3. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. Whatis the number of rotten apples in the heap?

Maths Standard

Q1. A die is thrown twice. What is the probability that
(i) 5 will not come up either time?
(ii) 5 will come up at least once?
[Hint : Throwing a die twice and throwing two dice simultaneously are treated as the same experiment]

Q2. 5 cards are drawn successively from a well-shuffled pack of 52 cards with replacement. Determine the probability that (i) all the five cards should be spades? (ii) only 3 cards should be spades? (iii) none of the cards is a spade?

CASE STUDY QUESTION

Maths Basic

Q1. Rahul and Ravi planned to play Business (board game) in which they were supposed to use twodice.

(i). Ravi got first chance to roll the dice. What is the probability that he got the sum of the twonumbers appearing on the top face of the dice is 8?

(ii). Rahul got next chance. What is the probability that he got the sum of the two

numbers appearingon the top face of the dice is 13?

(iii). Now it was Ravi's turn. He rolled the dice. What is the probability that he got the sum of the twonumbers appearing on the top face of the dice is less than or equal to 12?

Answer: i). 5/36 ii).0 iii) 3.1

Q2. On a weekend Rani was playing cards with her family. The deck has 52 cards. If herbrother drew one card.

(i). Find the probability of getting a king of red colour.

(ii). Find the probability of getting a face card.

(iii). Find the probability of getting a queen of diamond?

Answer: i) 1/26 **ii)** 3/13 iii) **1/52**

Maths Standard

of weather for the month of June. The given table shows the probabilities of forecast of different days

DAYS	SUNNY	CLOUDY	PARTIALLY CLOUDY	RAINY
PROBABILITY	1/2	X	1/5	Y

i) The number of sunny days in June is?

ii) If the number of cloudy days in June is 5 , then x =

iii) The probability that the day is not rainy

iv) find the probability of partially cloudy?

Answer i) 15 ii) 1/10iii) 13/15 iv) 6

KENDRIYA VIDYALAYA SANGATHAN, PATNA REGION

SAMPLE PAPER -1 (2024-25)

BLUE PRINT

CLASS: X MAX.MARKS: 80

SUBJECT: MATHEMATICS(STANDARD) TIME: 3 Hours.

S.No	LESSON	VSA 1m	SA -I 2m	SA-II 3m	LA 5m	Case Based 4m	TOTAL
1	Real Numbers	1	1	1			6
2	Polynomials	1				1	5
3	Pair of linear equations in two variables	1			1		6
4	Quadratic Equations	1		1			4
5	Arithmetic Progression	2		1			5
6	Triangles	3			1		8
7	Coordinate Geometry	2	2				6
8	Introduction to Trigonometry	2	1	1			7
9	Some Applications of Trigonometry				1		5
10	Circles	2	1	1			7
11	Areas related to Circles				1		5
12	Surface Areas and Volumes	1				1	5
13	Statistics	2		1			5
14	Probability	2				1	6
		1x20=20	2x5=10	3x6=18	5x4=20	4x3=12	80(38)

KENDRIYA VIDYALAYA SANGATHAN, PATNA REGION

SET -1(2024-25)

CLASS: X MAX.MARKS: 80

SUBJECT: MATHEMATICS (STANDARD) TIME: 3 Hours.

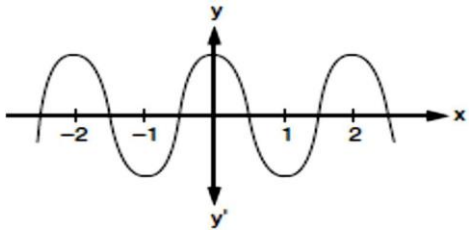
General Instructions:

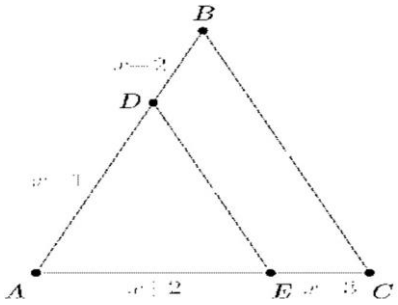
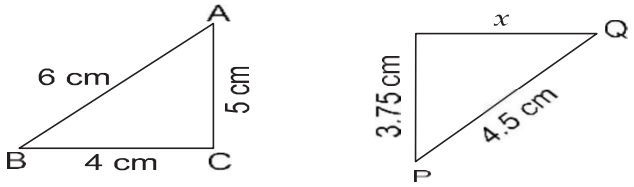
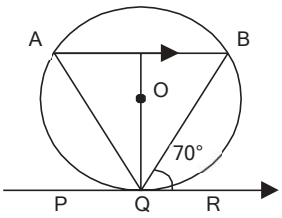
(i) This question paper contains **38** questions. **All** questions are **compulsory.**
(ii) This question paper is divided into **five** sections - **A,B,C,D and E.**
(iii) In **Section A**, Questions number **1** to **18** are multiple choice questions (**MCQ**) and question number **19** and **20** are **Assertion-Reason** based questions of **1** mark each.
(iv) In **Section B**, Questions number **21** to **25** are very short answer (**SA-1**) type questions, carrying **2** marks each.
(v) In **Section C**, Questions number **26** to **31** are short answer (**SA-II**) type questions, carrying **3** marks each.
(vi) In **Section D**, Questions number **32** to **35** are long answer (**LA**) type questions, carrying **5** marks each.
(vii) In **Section E**, Questions number **36** to **38** are case based questions carrying **4** marks each.Internal choice is provided in 2 marks questions in each case study.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in section B, 2 questions in section C, 2 questions in section D and 3 questions in section E.
(ix) Draw neat diagrams wherever required. Take π = 22/7 wherever required, if not stated.
(x) Use of calculators is **not** allowed.

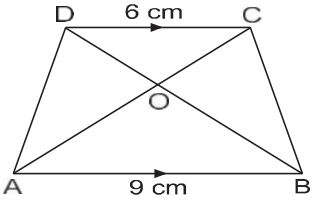
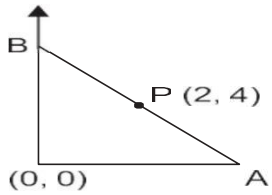
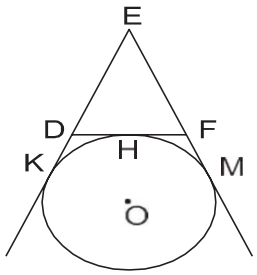
SECTION - A

This section comprises multiple choice questions (MCQs) of 1 mark each.

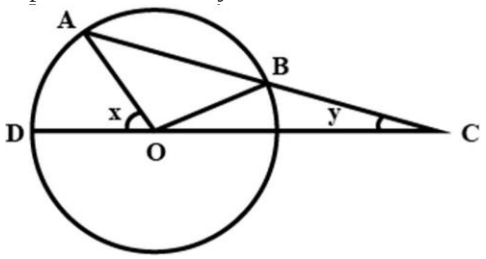
1	The LCM and HCF of two rational numbers are equal ,then the numbers must be (a) prime (b) co-prime (c) composite (d) equal	1

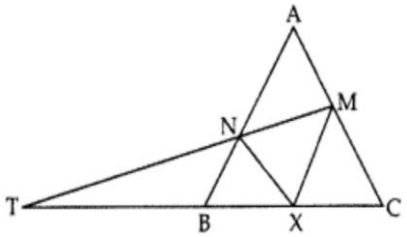
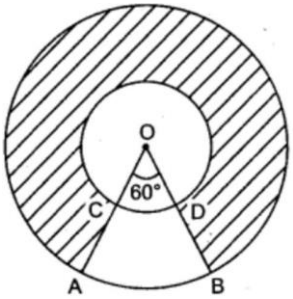
2	The graph of $y = f(x)$, where $f(x)$ is a polynomial in x is given below. Find the number of zeroes lying between –2 to 0.  (a) 0 (b) 1 (c) 2 (d) none of these	1
3	The system of equations $2x + y = 3$ and $4x + 2y = 5$ has	1

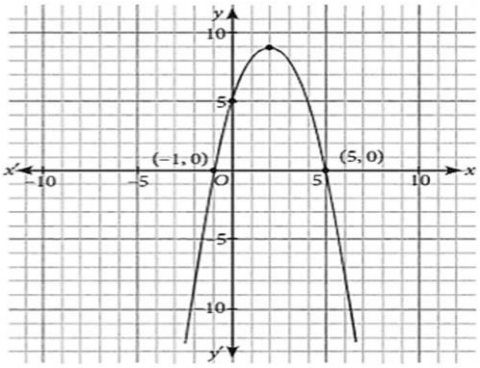
	(a) no solution (b) 1 solution (c) 2 solutions (d) infinite solutions	
4	If no roots of the equation $x^2 - p\,x + 1 = 0$ is real, then (a) $p > 2$ (b) $p < -2$ (c) $p = 2$ (d) $-2 < p < 2$	1
5	In the given figure DE ‖ BC, then **x** equals  (b) 1 (b) 0.5 (c) -2 (d) -0.5	1
6	ABC ~ PQR. The value of *x* is  (a) 2.5cm (b) 3cm (c) 2.75cm d) 3.5cm	1
7	If PQR is a tangent to a circle at Q whose centre is O, ABis a chord parallel to PR and ∠BQR = 70°, then ∠AQB is equal to  (a) 25^0 (b) 50^0 (c)30^0 (d) 40^0	1
8	The measure(s) of central tendency that would be best suited to determine the consumer item in demand is (a) mean (*b*) median (*c*) mode (*d*) mean and median	1
9	The perpendicular bisector of the line segment joining the points A(2, 3) and B(5, 6) cuts the y-axis at (a) (8, 0) (b) (0, 8) (c) (0, – 8) (d) (0, 7)	1

10	In a ABC right angled at B, the value of sin (A+C) is (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$	1
11	In trapezium ABCD, if AB DC, AB = 9 cm, DC = 6 cm and BD = 12 cm, then BO is equal to  (a) 7.4cm (b) 7cm (c)7.2cm (d) 7.5cm	1
12	If a, b, c, d and e are in AP, then the value of a – 4b + 6c – 4d + e is (a) 0 (b) 1 (c) 2 (d) 3	1
13	The nth term of an AP whose sum of n terms is Sn, (a) $S_n + S_{n+1}$ (*b*) $S_n - S_{n-1}$ (*c*) $S_n + S_{n-1}$ (*d*) $S_n - S_{n+1}$	1
14	In the figure given alongside, point P(2, 4) is the mid-point of line segment AB, then the coordinates of A and B respectively are  (*a*) A(0, 4), B(8, 0) (*b*) A(8, 0), B(0, 4) (*c*) A(4, 0), B(0, 8) (*d*) A(2, 6), B(6, 2)	1
15	In the figure, a circle touches the side DF of EDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then perimeter of EDF (in cm) is  (a) 18 (b) 13.5 (c)12 (d) 9	1
16	The empirical relationship between the three central tendencies is	1

	(a) Mode = 2median – 3 mean (b) Mode = median – 2 mean (c Mode = 2 median – mean (d) Mode = 3 median – 2 mean	
17	If a coin is tossed three times, then the probability of getting at most 2 heads is (a) $\frac{5}{8}$ (b) $\frac{3}{8}$ (c) $\frac{7}{8}$ (d) $\frac{3}{4}$	1
18	A bag contains 5 red balls and *n* green balls. If the probability of drawing a green ball is three times that of a red ball, then the value of *n* is (a) 18 (b) 15 (c) 10 (d) 20	1
	Direction for question numbers 19 and 20 : A statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.	
19	Assertion(A) : The value of $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0 = 1$ Reason (R) : $\sin 90^0 = 1$ and $\cos 90^0 = 0$ (a) Both Assertion(A) and Reason(R) are true and the Reason(R) is the correct explanation of Assertion(A). (b) Both Assertion(A) and Reason(R) are true but the Reason(R) is **not** the correct explanation of Assertion(A). (c) Assertion(A) is true, but Reason(R) is false. (d) Assertion(A) is false, but Reason(R) is true.	1
20	Assertion(A) : A cone, a hemisphere and a cylinder stand on equal bases and have same height, then their volumes are in ratio 1:2:3. Reason (R) : $\frac{1}{3}\pi r^2 x\ r : \frac{2}{3}\pi r^3 : \pi r^2 x\ r$ (a) Both Assertion(A) and Reason(R) are true and the Reason(R) is the correct explanation of Assertion(A). (b) Both Assertion(A) and Reason(R) are true but the Reason(R) is **not** the correct explanation of Assertion(A). (c) Assertion(A) is true, but Reason(R) is false. (d) Assertion(A) is false, but Reason(R) is true.	1
	SECTION B ***This section comprises of 5 very short answer type questions of 2 marks each***	
21	Explain why 17 X 11 X 7 + 11 is a composite number.	2
22	If $\tan\theta + \cot\theta = 2$ find the value of $\sqrt{\tan^2\theta + \sec^2\theta}$ OR Evaluate : $\frac{\mathbf{cos\,45^\circ}}{\mathbf{sec\,30^\circ + cosec\,30^\circ}}$	2

23	Find the value of x if A(6, 1), B(8, 2), C(9, 4) and D(x, 3) are the vertices of a parallelogram ABCD.	2
24	Find the value of x if the distance between the points P(-2, 3) and Q (x, 7) is $4\sqrt{5}$ units. OR Find the coordinates of the point which divides the join of A(4,-3) and (8, 5) in the ratio 3:1 .	2
25	Prove that the tangents drawn from an external point to a circle are equal.	2
	SECTION C ***This section comprises of 6 short answer type questions of 3 marks each***	
26	Prove that $\sqrt{3}$ is an irrational number.	3
27	A teacher on attempting to arrange the students for a mass drill in the form of a solid square, found that 24 students were left over. When he increased the size of the square by one student, he found he was short of 25 students. Find the number of students.	3
28	The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of numbers of the houses preceding the house markedx is equal to the sum of the houses following it. Find this value of x. OR If the 9^{th} term of an AP is zero, prove that its 29^{th} term is twice its 19^{th} term.	3
29	Prove that (1+cotA – cosec A) (1+tanA+secA) = 2	3
30	In the given figure, AB is the chord of a circle with centre O. AB is produced to C such that BC = OB. CO is joined and produced to meet the circle in D. If ∠ACD = y° and ∠AOD = x°, prove that x° = 3y°.  OR Prove that the parallelogram circumscribing a circle is a rhombus.	3
31	The mean of the following frequency distribution is 50. Find the missing frequencies m and n. Class | 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 | Total Frequency | 17 | $f1$ | 32 | f 2 | 19 | 120	3
	SECTION D ***This section comprises of 4 long answer type questions of 5 marks each***	

32	Solve graphically : $2x + 3y = 12$ $x - y = 1$ Find the coordinates of the vertices of the triangle formed by the two straight lines and the y- axis. **OR** One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II]	5
33	Prove that if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio. **OR** Any point X is taken on the side BC of a triangle ABC and XM, XN are drawn parallel to BA & CA meeting CA &BA at M and N respectively. MN meets BC produced in T. Prove that: $TX^2 = TB \times TC$.  	5
34	Two pillars of equal height stand on either side of a roadway which is 150m wide. From a point on the roadway between the pillars, the angles of elevation of the top of the pillars are 60^0 and 30^0 respectively. Find the height of the pillars and the distance of the point from the pillars.	5
35	In figure, two concentric circles with centre O, have radii 21 cm and 42 cm. If $\angle AOB = 60°$, find the area of the shaded region.  	5
	SECTION E ***This section comprises of 3 case study-based questions of 4 marks each***	

36	A construction company got the contract of making speed humps on roads. Speed humps are parabolic in shape and prevents over speeding, minimise accidents and gives a chance for pedestrians to cross the road. The mathematical representation of a speed hump is shown in the given graph.   Based on the above information, answer the following questions. (i) Name the type of polynomial represented by the graph. (ii) Find the zeroes of the polynomial represented by the graph. (iii) Write the expression of the polynomial represented by the graph OR Give the expression of the polynomial if its zeroes were 2 and -3	4
37	All the students were excited about the coming festival Diwali. To reduce the air pollution, the government decided to ban bursting of crackers. To cheer them up, their math teacher asked to decorate the classroom by hanging models of crackers/rockets. The radius and height of the cylindrical part of rocket is 3 cm and 10 cm respectively and the radius and slant height of the conical part of rocket is 5cm and 13cm respectively.   Based on the above information, answer the following questions. (i) Name the combination of solids used to make these rockets. (ii) How much paper is used to cover the conical part. (iii)What is the total height of the rocket model? OR How much paper is used to cover the cylindrical part	4

38 In the month of May, the weather forecast department gives the prediction of weather for the month of June .The given table shows the ***probabilities*** of forecast of different days :

Days	Sunny	Cloudy	Partially cloudy	Rainy
Probability	$\frac{1}{2}$	x	$\frac{1}{5}$	y

Consider the forecast is 100% correct for June.

Use the above information to answer the questions that follow .(consider each question independent of each other)

(i) Find the number of sunny days in June.

(ii) If the number of cloudy days in June is 5 , then find the value of x

(iii) If the number of rainy days in June is 6 and the sum of x and y is $\frac{3}{10}$, then find the number of cloudy days in June.

OR

If the number of cloudy days in June is 3 , then find the probability that the day is not rainy.

KENDRIYA VIDYALAYA SANGATHAN, PATNA REGION

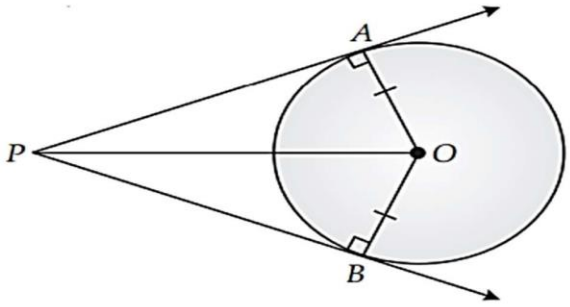
Set -1 (2024-25)

MARKING SCHEME MATHEMATICS (STANDARD)CLASS X

Q - No.	**SECTION - A**	Marks
1	(d)	1
2	(c)	1
3	(a)	1
4	(d)	1
5	(d)	1
6	(b)	1
7	(d)	1
8	(c)	1
9	(b) 8	1
10	(c) 1	1
11	(c)	1
12	(a)	1
13	(b)	1
14	(c)	1
15	(a)	1
16	(d)	1
17	(c)	1
18	(b)	1
19	(b)	1
20	(a)	1
	SECTION - B	
21	17 X 11 X 7 + 11 = 11 (17 X 7 +1) = 11 X 120 The given number has more than two factors so it's a composite number.	1 1
22	$\tan\theta + \cot\theta = 2$ squaring both sides ,we get $\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 4$ $\tan^2\theta + \cot^2\theta + 2 = 4$ $\tan^2\theta + \cot^2\theta = 4 - 2 = 2$ OR Putting the value of each trigonometric ratios, we get $\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} = \frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$ $= \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} \times \frac{2\sqrt{2}-2\sqrt{6}}{2\sqrt{2}-2\sqrt{6}} = \frac{2\sqrt{6}-2\sqrt{18}}{(2\sqrt{2})^2-(2\sqrt{6})^2} = \frac{2\sqrt{6}-6\sqrt{2}}{8-24} = \frac{-2(3\sqrt{2}-\sqrt{6})}{-16}$ $= \frac{3\sqrt{2}-\sqrt{6}}{8}$	1 1 1 1

23	diagonals of a parallelogram bisect each other. midpoint of AC = midpoint of BD Now, [(8 + x)/2, (2 + 7)/2] = [15/2, 5/2] (8 + x)/2 = 15/2 8 + x = 15 x = 15 – 8 =7	1 1 1
24	$PQ^2 = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ $80 = (x + 2)^2 + (7- 3)^2$ $x^2 + 4x - 60 = 0$ solving we get , x = 6 , x = -10 OR Let P(x, y) be the required point. Using the section formula $P(x, y) = \left(\frac{m \cdot x_2 + m_2 x_1}{m_1 + m_2}, \frac{m \cdot y_2 + m_2 y_1}{m_1 + m_2}\right)$, we get $x = \frac{3(8) + 1(4)}{3 + 1} = \frac{24 + 4}{4} = \frac{28}{4} = 7,$ $y = \frac{3(5) + 1(-3)}{3 + 1} = \frac{15 - 3}{4} = \frac{12}{4} = 3$ P(x, y) = (7, 3) is the required point.	
25		1 1

Given : AP and BP are tangents of circle having centre O.



To prove : $AP = BP$

Construction Join OP, AO and BO.

Proof : ΔOAP and ΔOBP

$OA = OB$ (Radius of circle)

$OP = OP$ (Common side)

$\angle OAP = \angle OBP = 90°$

(Radius $\perp$ to the tangent)

$\therefore \quad \Delta OAP \cong \Delta OBP$

(RHS congruency rule)

$\therefore \quad AP = BP$ (cpct)

Hence proved.

SECTION – C

26

Assume that $\sqrt{3}$ is a rational number.

Then, $\sqrt{3} = \frac{a}{b}$, where a, b are integers, ½

$b \neq 0$ and HCF $(a, b) = 1$ ...(i)

$\Rightarrow \sqrt{3}b = a$

$\Rightarrow \sqrt{3}b^2 = a^2$ (squaring both sides)

$\Rightarrow a^2$ is dividible by 3 ½

$\Rightarrow$ a is divisible by 3 ...(ii)

$\Rightarrow a = 3c$, where c is an integer.

$\therefore \sqrt{3}b = 3c$

$\Rightarrow 3b^2 = 9c^2$

$\Rightarrow b^2 = 3c^2$

$\Rightarrow b^2$ is divisible by 3 1

$\Rightarrow$ b is divisible by 3 ...(iii)

From (ii) and (iii), we get

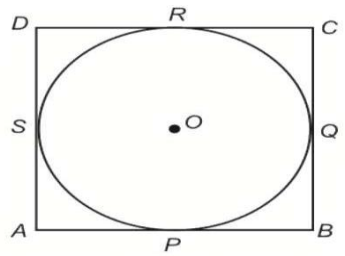
3 is the common factor of a and b which contradicts (i). Thus, our assumption was wrong.

$\sqrt{3}$ is an irrational number. 1

27	**Ans:** Let the side of the square be x. No. of students = $x^2 + 24$ New side = x + 1 No. of students = $(x + 1)^2 - 25$ APQ $\Rightarrow x^2 + 24 = (x + 1)^2 - 25$ $\Rightarrow x^2 + 24 = x^2 + 2x + 1 - 25$ $\Rightarrow 2x = 48$ $\Rightarrow x = 24$ $\therefore$ side of square = 24 No. of students = 576 + 24 = 600	1 1 1
28	Here, $a = 1$, $d = 1$ and $x = 49$ $\therefore$ Sum of the numbers of the houses preceding the house numbered $x = S_{x-1}$. According to the question, we have $S_{x-1} = S_{49} - S_x$ $\Rightarrow \frac{x-1}{2}[2a + (x - 1 - 1)d] = \frac{49}{2}[2a + 48d] - \frac{x}{2}[2a + (x - 1)d]$ $\Rightarrow \frac{x-1}{2}[2a + (x - 2)d] = \frac{49}{2}[2a + 48d] - \frac{x}{2}[2a + (x - 1)d]$ $\Rightarrow x - 1[2a + (x - 2)d] = 49[2a + 48d] - x[2a + xd - d]$ Substituting the value of a and d, we get: $(x - 1)(2 + x - 2) = 49(2 + 48) - x(2 + x - 1)$ $\Rightarrow (x - 1)x = 49 \times 50 - x(x + 1)$ $\Rightarrow x(x - 1) + x(x + 1) = 49 \times 50$ $\Rightarrow 2x^2 = 2450 \Rightarrow x^2 = \frac{2450}{2}$ $= 1225 = (35)^2$ $\Rightarrow x = \mathbf{35}$ OR	1 1 1

	Let a and d respectively be the first term and common difference of the AP. Given $a_9 = 0$ So, a + (9-1)d = 0 a+8d=0 a= -8d Now, 29th term = a+28d =-8d+28d = 20d = 2 x 10d = 2(-8d + 18d) =2(a+18d) = 2 x 19th term Thus, the 29th term of the AP is twice the 19th term.	
29	$LHS = (1+\cot A - \operatorname{cosec} A)(1+\tan A+\sec A)$ $=\left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$ $=\left(\frac{\sin A+\cos A-1}{\sin}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$ $=\frac{(\sin A+\cos A)^2-1}{\sin A\cos A}$ $=\frac{1+2\sin A\cos A-1}{\sin A\cos A}$ $\left[\because \sin^2 A+\cos^2 A=1\right]$ $=2.$	1 1
30	It is given that AB is a chord of a circle with centre O and AB is produced to C such that BC = OB We know that ∠BOC and ∠BCO form isosceles triangle ∠BOC = ∠BCO = y° **Given that,** **BC = OB** **⇒ ∠OCB = ∠BOC = y°.** **In ΔOBC,** **Exteíioí ∠OBA = ∠BOC +∠OCB** **= y° + y° = 2y°** **Now, OA = OB (Radii of same ciícle)** **⇒ ∠OAB = ∠OBA = 2y°** **In ΔAOC, Exteíioí ∠AOD = ∠OAC + ∠OCA** **= 2y° + y° = 3y°** **But, ∠AOD = x° (Given that)** **∴ x° = 3y°**	1 1 1

OR



Given ABCD is $||^{gm}$ such that its sides touch a circle with centre O. 1

\ AB = CD and AB || CD,

AD = BC and AD || BC

Now, P, Q, R and S are the touching point of both the circle and the $||^{gm}$

We know that, tangents to a circle form an exterior point are equal to length.

∴ AP = AS [tangents form point A] ...(1)

BP = BQ [tangents form point B] ...(2)

CR = CQ [tangents form point C] ...(3) 1

DR = DS [tangents form point D] ...(4)

On adding (1), (2), (3) and (4), we get.

AP + BP + CR + DR = AS + BQ + CQ + DS

Þ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

Þ AB + CD = AD + BC

Þ AB + AB = BC + BC [∵ ABCD is a $||^{gm}$ \ AB = CD and AD = BC]

Þ 2AB = 2BC

Þ AB = BC 1

Therefore, AB = BC implies

AB = BC = CD = AD

Hence, ABCD is a rhombus.

31

Class	Mid-point (x_i)	Frequency (f_i)	x_if_i
0 – 20	$\frac{0+20}{2} = 10$	17	170
20–40	$\frac{20+40}{2} = 30$	f_1	$30f_1$
40–60	$\frac{40+60}{2} = 50$	32	1600
60–80	$\frac{60+80}{2} = 70$	f_2	$70f_2$
80–100	$\frac{80+1000}{2} = 90$	19	1710
Total		$\sum f_i = 120 = 68 + f_1 + f_2$	$\sum f_ix_i = 3480 + 30f_1 + 70f_2$

1

$17 + f_1 + 32 + f_2 + 19 = 120$ 1

$\Rightarrow 68 + f_1 + f_2 = 120$

$\Rightarrow f_1 + f_2 = 52$

or, $f_2 = 52 - f_1$ ...(ii)

By putting the value of f 2 in (i), we get:

$2520 = 30f_1 + 70(52 - f_1)$

$\Rightarrow 2520 = 30f_1 + 3640 - 70f_1$

$\Rightarrow 40f_1 = 1120$

$\Rightarrow f_1 = 28$ 1

Substituting the value in (ii), we get:

$f_2 = 52 - f_1 = 52 - 28 = 24$

SECTION – D

32

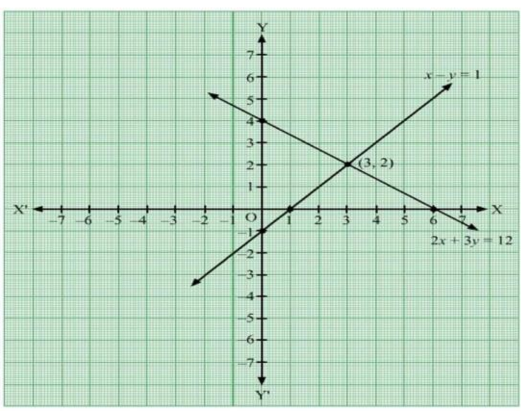
(1) $2x + 3y = 12$

x	0	6	3
y	4	0	2

1

$x - y = 1$

x	0	1	3
y	−1	0	2



2

Solution: x=3 & y=2

The vertices of the triangle are (-1,0) , (3,2) and (0,4)

OR

Let the first friend have ₹ x 1

And the second friend has ₹ y

Using the information given in the question,

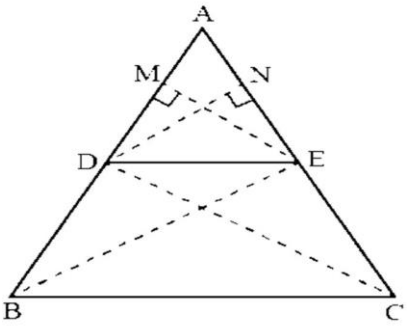
Condition 1: When second friend gives ₹ 100 to first friend; 1

x + 100 = 2 (y - 100) 1

x + 100 = 2y - 200

x - 2y = - 300 .... (1)

Condition 2: When first friend gives ₹ 10 to second friend;

	$y + 10 = 6 (x - 10)$ $y + 10 = 6x - 60$ $6x - y = 70$ ....(2) Multiplying equation (2) by 2, we obtain $12x - 2y = 140$ ....(3) Subtracting equation (1) from equation (3), we obtain $12x - 2y - (x - 2y) = 140 - (- 300)$ $11x = 440$ $x = 440/11$ $x = 40$ Substituting $x = 40$ in equation (1), we obtain $40 - 2y = - 300$ $2y = 40 + 300$ $y = 340/2$ $y = 170$ Therefore, the first friend has ₹ 40, and the second friend has ₹ 170 with them.	1 1 1 1 1
33	Given, In ΔABC, $DE \| BC$  To prove: $\frac{AD}{DB} = \frac{AE}{EC}$ Construction : Draw $EM \perp AB$ and $DN \perp AC$. Join B to E and C to D Proof: In ΔADE and ΔBDE $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times DB \times EM} = \frac{AD}{DB}$ . . . (i) [Area of $\Delta = \frac{1}{2} \times$ base × corresponding altitude] [Δs on the same base and between the same parallel sides are equal in area]	1 ½ ½ 1 1

	OR In triangle TXM, XM ‖ AB & XN ‖ AC,	1 1 1 1 1 1
34	Let AB and CD be two pillars ,each of height hmetres. Let P be a point on the road such that AP=xm. Then,CP = (150 – x)m In triangle PAB , we have $\tan 60° = \frac{AB}{AP}$ $= \sqrt{3} = \frac{h}{x}$ $= \sqrt{3}x = h$.....1 In triangle PCD , we have $\tan 30° = \frac{CD}{CP}$ $= \frac{1}{\sqrt{3}} = \frac{h}{150 - x}$ $= h\sqrt{3} = 150 - x$.....2 Eliminating h between eq. 1 and 2, we get $3x = 150 - x$ $= x = 37.5$	1 1 1 1

	Substituting x = 37.5 in eq.1 we get , h = 64.95 Thus the required point is at the distance of 37.5 m from the first pillar and 112.5 m from the second pillar. The height of the pillars is 64.95 m So the height of the pillars is **64.95m** and the distance of the point from the pillars is **37.5m**.	1
35	*Solution :* *Radius of inner circle* $(r) = 21cm$ *Radius of outer circle* $(R) = 42cm$ *Area of between circles* $= \pi(R^2 - r^2)$ $= \frac{22}{7}(42^2 - 21^2)$ $= \frac{22}{7} \times 1323$ $= 4158cm^2$ *Area ABCD = Area AOB − Area COD* $= \frac{\Theta}{360^0}\pi R^2 - \frac{\Theta}{360^0}\pi r^2$ $= \frac{\Theta}{360^0}\pi(R^2 - r^2)$ $= \frac{60^0}{360^0}\pi(42^2 - 21^2)$ $= \frac{1}{6} \times \frac{22}{7} \times 1323$ $= 693cm^2$ *Area of shaded region* *= Area of between circles − Area ABCDD* $= 4158 - 693 = 3465cm^2$	1 1 1 1 1
	SECTION -E	
36	(i) Quadratic (ii) Zeroes are -1 and 5 (iii) $x^2 - 4x - 5$ OR $x^2 + x - 6$.	1 1 2
37	(i) cylinder and cones (ii) 204 cm^2 approx	1 1

	(iii) 22cm Or 188.4 cm^2 z	2
38	(i) 15 (ii) 1/6 (iii) 3 (iv) 4/5	1 1 2
	~~	

Sample Question Paper SET -1 (2024-25)

Class – X

Basic Mathematics (241)

Time Allowed: 3 Hrs Maximum Marks: 80

General Instructions:

1. **This Question Paper has 5 Sections A, B, C, D, and E.**
2. Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. **Section B has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.**
4. Section C has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.
5. **Section D has 4 Long Answer (LA) type questions carrying 5 marks each.**
6. Section E has 3 sourced based/Case Based/passage based/integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. **All Questions are compulsory. However, an internal choice in 2 Qs of 2 marks, 2 Qs of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.**
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

SECTION A

1. If two positive integers a and b are written as a = x^3y^2 and b = xy^3; where x, y areprime numbers, then HCF (a,b) is:

   a) xy b) xy^2 c) x^3y^3 d) x^2y^2

2. The LCM of smallest two digit composite number and smallest composite number is:a) 12 b) 4 c) 20 d) 44

3. If x = 3 is one of the roots of the quadratic equation $x^2 - 2kx - 6 = 0$, then the value ofk is

   a) $-\frac{1}{2}$ b) $\frac{1}{2}$ c) 3 d) 2

4. The pair of equations y = 0 and y = -7 has:

   a) one solution b) two solutions c) infinitely many solutions d) no solution

5. Value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is :

   a) 0 only b) 4 c) 8 only d) 0,8

6. The distance of the point(3, 5) from x-axis is k units, then k equals:

a) 3 b) 4 c) 5 d) 8

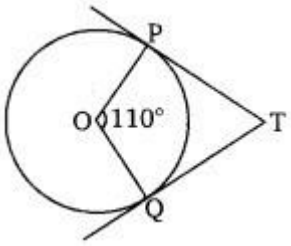
7. If in Δ ABC and Δ PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then:

   a) ΔPQR ~ΔCAB b) ΔPQR ~ΔABC c) ΔCBA ~ΔPQR d) ΔBCA ~ΔPQR

8. Which of the following is NOT a similarity criterion of traingles?

   a) AA b) SAS c) AAA d) RHS

9. In figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to



   (a) 60° (b) 70° (c) 80° (d) 90°

10. If cosA = $\frac{4}{5}$ then tan A is :

    (a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{1}{8}$

11. If the height of the tower is equal to the length of its shadow, then the angle of elevation of the sun is _____

    a) 30° b) 45° c) 60° d) 90°

12. $(1 - \cos^2 A)$ is equal to

    a) $\sin^2 A$ b) $\tan^2 A$ c) $1 - \sin^2 A$ d) $\sec^2 A$

13. The radius of a circle is same as the side of a square. Their perimeters are in the ratio

    a) 1 : 1 b) $2 : \pi$ c) $\pi : 2$ d) $\sqrt{\pi} : 2$

14. The area of the circle is 154cm^2. The radius of the circle is

    a) 7cm b) 14cm c) 3.5cm d) 17.5cm

15. When a dice is thrown once, the probability of getting an even number less than 4 is

    a) 1/4 b) 0 c) 1/2 d) 1/6

16. For the following distribution:

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

The lower limit of modal class is:

a) 15 b) 20 c) 10 d) 5

17. A rectangular sheet of paper 40cm x 22cm, is rolled to form a hollow cylinder of height 40cm. The radius of the cylinder (in cm) is :

a) 3.5 b) 7 c) $\frac{80}{7}$ d) 5

18. Consider the following frequency distribution:

Class	0-6	6-12	12-18	18-24	24-30
Frequency	12	10	15	8	11

The median class is:

a) 6-12 b) 12-18 c) 18-24 d) 24-30

19. Assertion (A): The point (0, 4) lies on y-axis.

Reason(R): The x-coordinate of a point on y-axis is zero

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertions (A) is true but reason (R) is false.

(d) Assertions (A) is false but reason (R) is true.

20. Assertion (A): The HCF of two numbers is 5 and their product is 150. Then their LCM is 40.

Reason(R): For any two positive integers a and b, HCF (a, b) x LCM (a, b) = a x b.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertions (A) is true but reason (R) is false.

(d) Assertions (A) is false but reason (R) is true.

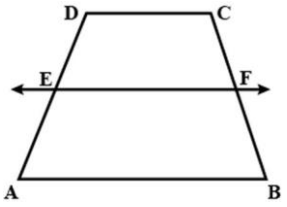
SECTION B

21. Find whether the following pair of linear equations is consistent or inconsistent:

$3x + 2y = 8$

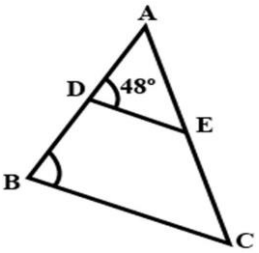
$6x - 4y = 9$

22. In the given figure, if ABCD is a trapezium in which AB ∥ CD ∥ EF, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$



OR

In figure, if AD = 6cm, DB = 9cm, AE = 8cm and EC = 12cm and ∠ADE = 48°. Find ∠ABC.



23. The length of a tangent from a point A at distance 5cm from the centre of the circle is 4cm. Find the radius of the circle.

24. Evaluate: $\sin^2 60^\circ + 2\tan 45^\circ - \cos^2 30^\circ$

25. Find the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 40cm and 9cm.

OR

A chord of a circle of radius 10cm subtends a right angle at the centre. Find the area ofminor segment. (Use $\pi = 3.14$)

SECTION C

26. Prove that $\sqrt{3}$ is an irrational number.

27. Find the zeroes of the quadratic polynomial $4s^2 - 4s + 1$ and verify the relationship between the zeroes and the coefficients.

28. The coach of a cricket team buys 4 bats and 1 ball for Rs. 2050. Later, she buys 3 batsand 2

and 2 balls for ₹1600. Find the cost of each bat and each ball.

OR

A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

29. A circle touches all the four sides of quadrilateral ABCD. Prove that AB + CD = AD + BC.

30. Prove that

$$(\text{cosec } \theta - \cot \theta)^2 = \frac{1-\cos}{1+\cos\theta}$$

OR

Prove that sec A (1 – sin A) (sec A + tan A) = 1.

31. A bag contains 6 red, 4 black and some white balls.

(i) Find the number of white balls in the bag if the probability of drawing a white ball is $\frac{1}{3}$.

(ii) How many red balls should be removed from the bag for the probability of drawing a white ball to be $\frac{1}{2}$?

SECTION D

32. A train travels 360km at a uniform speed. If the speed had been 5km/h more, it wouldhave taken 1 hour less for the same journey. Find the speed of the train.

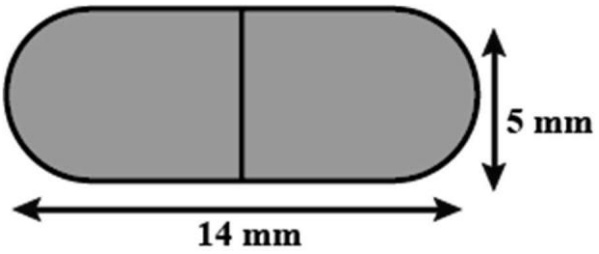
OR

A motor boat whose speed is 18km/h in still water takes 1 hour more to go 24km upstream than to return downstream to the same spot. Find the speed of the stream.

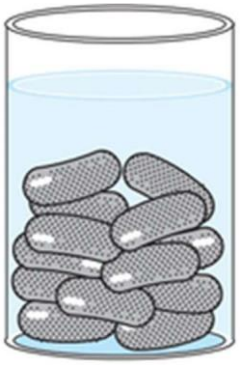
33. Prove that If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

In ΔPQR, S and T are points on PQ and PR respectively. $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.

34. A medicine capsule is in the shape of a cylinder with two hemispheres stuck at each of its ends. The length of the entire capsule is 14mm and the diameter of the capsule is 5mm. Find its surface area.



OR



A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like cylinder with two hemispherical ends with length 5cm and diameter 2.8cm.

35. The following table gives the distribution of the life time of 400 neon lamps:

Life time (in hours)	Number of lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the average life time of a lamp.

SECTION E

36. CASE STUDY 1

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher qualityproduction runs. The production of TV sets in a factory increases uniformly by a fixed number every year. Itproduced 16000 sets in 6th year and 22600 in 9th year.

1) In which year, the production is 29,200 sets?

2) Find the production in the 8^{th} year.

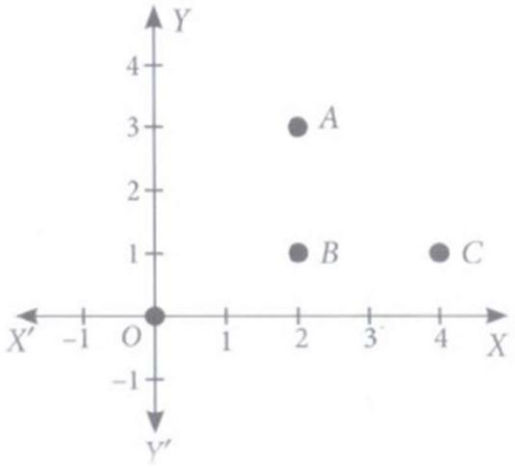
OR

Find the production in first 3 years.

3) Find the difference of the production in 7th year and 4th year.

37. CASE STUDY 2

Alia and Shagun are friends living on the same street in Patel Nagar. Shagun's house is at the intersection of one street with another street on which there is a library. They both study in the same school and that is not far from Shagun's house. Suppose the school is situated at the point O, i.e., the origin, Alia's house is at A. Shagun's house is at B and library is at C. Based on the above information, answer the following questions.



(i) How far is Alia's house from Shagun's house?

(ii) How far is the library from Shagun's house?

(iii) Show that for Shagun, school is farther compared to Alia's house and library.

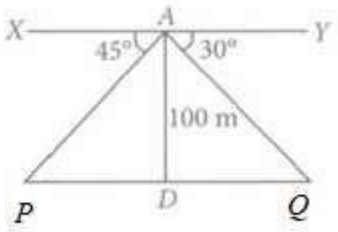
OR

Show that Alia's house, shagun's house and library for an isosceles right triangle.

38. CASE STUDY 3

A boy is standing on the top of light house. He observed that boat P and boat Q are

approaching the light house from opposite directions. He finds that angle of depression of boat P is 45° and angle of depression of boat Q is 30°. He also knows that height of the light house is 100 m.



Based on the above information, answer the following questions.

(i) What is the measure of ∠APD?

(ii) If ∠YAQ = 30°, then ∠AQD is also 30°, Why?

(iii) Find length of PD

OR

Find length of D

Basic Mathematics SET 1 (241)
Marking Scheme
2024-25

Section A

1) (b) xy^2	1
2) (c) 20	1
3) (b) ½	1
4) (d) No Solution	1
5) (d) 0,8	1
6) (c) 5 Unit	1
7) (a) $\Delta PQR \sim \Delta CAB$	1
8) (d) RHS	1
9) (b) 70°	1
10) (b) ¾	1
11) (b) 45°	1
12) (a) $\sin^2 A$	1
13) (c) $\pi:2$	1
14) (a) 7 cm	1
15) (d) $\frac{1}{6}$	1
16) (a) 15	1
17) (a) 3.5 cm	1
18) (b) 12-18	1
19) (a) Both assertion and reason are true and reason is the correct explanation of assertion.	1
20) (d) Assertion (A) is false but reason(R) is true.	1

SECTION B

21) $3x+2y = 8$

$6x- 4y = 9$

$a_1=3, \quad b_1=2, \quad c_1 = 8$

$a_2=6, \quad b_2=-4, \quad c_2 = 9$ 1

$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2} \qquad \frac{b_1}{b_2} = \frac{2}{-4} = \frac{-1}{2} \qquad \frac{c_1}{c_2} = \frac{8}{9}$ 1/2

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The given pair of linear equations are consistent. 1/2

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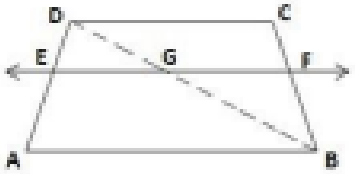
22) Given:- AB II CD II EF

To prove:- $\frac{AE}{ED} = \frac{BF}{FC}$



Construction:- Join BD to 1/2

intersect EF at G.

Proof:- in Δ ABD

EG II AB (EF II AB)

$\frac{AE}{ED} = \frac{BG}{GD}$ (by BPT)_____(1) 1/2

In ΔDBC

GF II CD (EF II CD)

$\frac{BF}{FC} = \frac{BG}{GD}$ (by BPT)_____(2) 1/2

from (1) & (2)

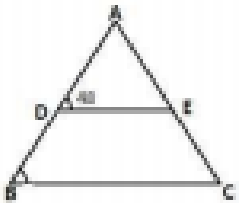
$\frac{AE}{ED} = \frac{BF}{FC}$ 1/2

OR

Given AD=6cm, DB=9cm

AE=8cm, EC=12cm, ∠ADE=48

To find:- ∠ABC=?



Proof:

In ΔABC

$\frac{AD}{DB} = \frac{6}{9} = \frac{2}{3}$ .....(1)

$\frac{AE}{EC} = \frac{8}{12} = \frac{2}{3}$.....(2)

From (1) & (2) 1

$\frac{AD}{DB} = \frac{AE}{EC}$

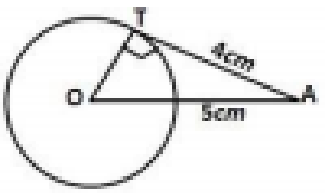
DE II BC (Converse of BPT)

∠ADE=∠ABC (Corresponding angles)

→ ∠ABC=48° 1

23) In Δ OTA, ∠OTA = 90°

By Pythagoras theorem



$OA^2 = OT^2 + AT^2$ 1/2

$(5)^2 = OT^2 + (4)^2$

$25-16 = OT^2$

$9 = OT^2$ 1/2

OT=3cm

radius of circle = 3cm. 1

24) $\sin^2 60° + 2\tan 45° - \cos^2 30°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$ 1

$= \frac{3}{4} + 2 - \frac{3}{4}$

$= 2$ 1

25) Area of the circle= sum of areas of 2 circles

$\pi R^2 = \pi(40)^2 + \pi(9)^2$ 1/2

$\pi R^2 = \pi \times (40^2 + 9^2)$ 1/2

$R^2 = 1600 + 81$

$R^2 = 1681$

$R = 41\ cm.$ 1/2

$Diameter\ of\ given\ circle = 41 \times 2 = 82cm$ 1/2

OR

radius of circle = 10cm, $\theta = 90°$

Area of minor segment = $\frac{\theta}{360°}\pi r^2$ - Area of Δ

$= \frac{\theta}{360°} \times \pi\ r^2 - \frac{1}{2} \times b \times h$ 1/2

$= \frac{90°}{360°} \times 3.14 \times 10 \times 10 - \frac{1}{2} \times 10 \times 10$ 1/2

$= \frac{314}{4} - 50$

$= 78.5-50 = 28.5\ cm^2$ 1/2

Area of minor segment = $28.5\ cm^2$ 1/2

Section C

26) Let us assume that $\sqrt{3}$ be a rational number

$\sqrt{3} = \frac{a}{b}$ where a and b are co-prime. 1

squaring both the sides

$(\sqrt{3})^2 = (\frac{a}{b})^2$ 1/2

$3 = \frac{a^2}{b^2} \rightarrow a^2 = 3b^2$

a^2 is divisible by 3 so a is also divisible by 3_____(1)

let $a = 3c$ for any integer c.

$(3c)^2 = 3b^2$ 1/2

$9c^2 = 3b^2$

$b^2 = 3c^2$

since b^2 is divisible by 3 so, b is also divisible by 3 _____(2)

From (1) & (2) we can say that 3 in a factor of a and b 1/2

which is contradicting the fact that a and b are co- prime.

Thus, our assumption that $\sqrt{3}$ is a rational number is wrong.

Hence, $\sqrt{3}$ is an irrational number. 1/2

27) $P(S) = 4S^2 - 4S + 1$

$4S^2 - 2S - 2S + 1 = 0$

$2S(2S-1) - 1(2S-1) = 0$

$(2S-1)(2S-1) = 0$

$S = ½ \quad S = ½$ 1

$a = 4 \quad b = -4 \quad c = 1 \quad \propto = ½ \quad \beta = ½$

$\propto + \beta = \frac{-b}{a}, \qquad \propto \beta = \frac{c}{a}$

$\frac{1}{2} + \frac{1}{2} = \frac{-4}{4}, \qquad \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$ 1

$\frac{1+1}{2} = \frac{+4}{4}, \qquad \frac{1}{4} = \frac{1}{4}$

$\frac{2}{2} = 1$

$1 = 1$ 1

28) Let cost of one bat be Rs x

Let cost of one ball be Rs y 1/2

ATQ

$4x + 1y = 2050$_____(1)
$3x + 2y = 1600$_____(2) 1/2

from (1) $4x + 1y = 2050$

$y = 2050 - 4x$ 1/2

Substite value of y in (2)

$3x + 2(2050 - 4x) = 1600$

$3x + 4100 - 8x = 1600$

$-5x = -2500$

$x = 500$ 1/2

Substiture value of x in (1)

$4x + 1y = 2050$

$4(500) + y = 2050$

$2000 + y = 2050$

$y = 50$ 1/2

Hence

Cost of one bat = Rs. 500 1/2

Cost of one ball = Rs. 50

OR

Let the fixed charge for first 3 days= Rs. x

And additional charge after 3 days= Rs. y 1/2

ATQ

$x + 4y = 27$ ----------(1)

$x + 2y = 21$ ----------(2) 1/2

Subtract eq^n (2) from (1)

$2y = 6$

$y = 3$ 1

Substitute value of y in (2)

$x + 2(3) = 21$

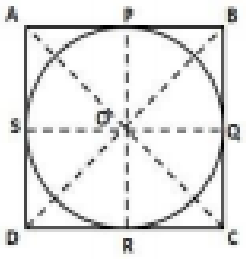
$x = 21 - 6$

$x = 15$ 1

Fixed charge= Rs. 15

Additional charge per day = Rs. 3

29) Given circle touching sides of ABCD at P,Q,R and S



To prove- AB+CD=AD+BC

Proof- 1

AP=AS-------(1) tangents from an external point

PB=BQ-------(2) to a circle are equal in length

DR=DS ------ (3)

CR=CQ -------(4) 1

Adding eq^n (1),(2),(3) & (4)

AP+BP+DR+CR=AS+DS+BQ+CQ

AB+DC=AD+BC 1

30) $(cosec\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

LHS=$(cosec\theta - cot\theta)^2$

$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$ 1/2

$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$ 1/2

$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$ 1

$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta}$ =RHS 1

LHS = RHS, Hence Proved

OR

$\sec A\,(1-\sin A)(\sec A+\tan A)=1$

$LHS=\frac{1}{\cos A}(1-\sin A)\,(\frac{1}{\cos A}+\frac{\sin A}{\cos A})$ 1

$=\frac{(1-\sin A)}{\cos A}\frac{(1+\sin A)}{\cos A}$

$=\frac{(1-\sin A)(1+\sin A)}{\cos^2 A}$

$=\frac{1-\sin^2 A}{\cos^2 A}$ $(1-\sin^2 A=\cos^2 A)$ 1

$=\frac{\cos^2 A}{\cos^2 A}$

= 1 = RHS 1

LHS=RHS. Hence Proved

31) (i) Red balls= 6 , Black balls = 4 , White balls = x

P(white ball) = $\frac{x}{10+x}=\frac{1}{3}$ 1

$\rightarrow 3x = 10 + x \rightarrow x = 5$ white balls 1/2

(ii) Let y red balls be removed, black balls = 4, white balls = 5

P(white balls)=$\frac{5}{(6-y)+4+5}=\frac{1}{2}$ 1

$\rightarrow \frac{5}{15-y}=\frac{1}{2} \rightarrow 10 = 15 - y \rightarrow y = 5$ ½

So 5 balls should be removed.

Section D

32) Let the speed of train be $x\ km/hr$ 1/2

distance= 360 km

Speed = $\frac{\text{distance}}{\text{time}}$

Time = $\frac{360}{x}$ 1/2

New speed = $(x+5)km/hr$

Time = $\frac{D}{S}$

$x+5=\frac{360}{(\frac{360}{x}-1)}$ 1

$(x+5)\,(\frac{360}{x}-1)=360$

$(x+5)(360-x) = 360x$

$-x^2 - 5x + 1800 = 0$

$x^2 + 5x - 1800 = 0$ 1

$x^2 + 45x - 40x - 1800 = 0$

$x(x+45) - 40(x+45) = 0$

$(x+45)(x-40) = 0$ 1

$x + 45 = 0$, $x - 40 = 0$

$x = -45$, $x = 40$

Speed cannot be negative

Speed of train =40km/hr 1

OR

Let the speed of the stream=xkm/hr 1/2

Speed of boat= $18\ km/hr$

Upstream speed= $(18-x)km/hr$

Downstream speed=$(18+x)km/hr$ 1/2

Time taken (upstream)=$\frac{24}{(18-x)}$

Time taken (downstream)=$\frac{24}{(18+x)}$

ATQ

$\frac{24}{(18-x)} = \frac{24}{(18+x)} + 1$ 1

$\frac{24}{(18-x)} - \frac{24}{(18+x)} = 1$

$24(18+x) - 24(18-x) = (18-x)(18+x)$

$24(18+x-18+x) = (18)^2 - x^2$

$24(2x) = 324 - x^2$

$48x - 324 + x^2 = 0$

$x^2 + 48x - 324 = 0$ 1

$x^2 - 6x + 54x - 324 = 0$

$x(x-6) + 54(x-6) = 0$

$(x-6)(x+54) = 0$ 1

$x - 6 = 0$, $x + 54 = 0$

$x = 6$, $x = -54$

Speed cannot be negative 1

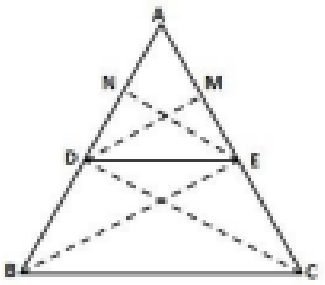
Speed of stream=$6km/hr$

33) Given ΔABC , DE || BC

To prove $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: join BE and CD 1/2

Draw DM $\perp$ AC and EN $\perp$ AB



Proof: $Area\ of\ \Delta ADE = \frac{1}{2}$ x b x h

$= \frac{1}{2}$ x AD x EN----------(1)

Area $(\Delta DBE) = \frac{1}{2}$ x DB x EN-------(2)

Divide eq^n (1) by (2)

$\frac{ar\ \Delta ADE}{ar\ \Delta DBE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB}$----------(3) 1

area $\Delta ADE = \frac{1}{2}$ x AE x DM --------(4)

area $\Delta DEC = \frac{1}{2}$ x EC x DM ------- (5)

Divide eq^(4) by (5)

$\frac{ar\,\Delta ADE}{ar\,\Delta DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC}$ ----------(6) 1

ΔBDE and ΔDEC are on the same base DE and between same parallel lines BC and DE

$\therefore area\ (\Delta DBE) = ar\ (DEC)$

hence

$\frac{ar(\Delta ADE)}{ar\ (\Delta DBE} = \frac{ar(\Delta ADE)}{ar\ (\Delta DEC)}$ [LHS of (3) =RHS of (6)]

$\frac{AD}{DB} = \frac{AE}{EC}$ [RHS of (3) = RHS of (6) 1/2

Since $\frac{PS}{SQ} = \frac{PT}{TR} \therefore ST \parallel QR$ $(by\ converse\ of\ BPT)$

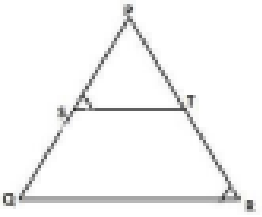
∠PST = ∠PQR (Corresponding angles) 1

But ∠PST = ∠PRQ (given)

∠PQR = ∠PRQ

PR = PQ (sides opposite to equal angles are equal

Hence ΔPQR is isosceles. 1



34) Diameter of cylinder and hemisphere = 5mm radius, (r) = $\frac{5}{2}$

Total length = 14mm

Height of cylinder = 14 - 5 = 9mm 1

CSA of cylinder = 2πrh

$= 2 \times \frac{22}{7} \times \frac{5}{2} \times 9$

$= \frac{990}{7}$ mm² 1

CSA of hemispheres = $2\pi r^2$

$= 2 \times \frac{22}{7} \times (\frac{5}{2})^2$

$= \frac{275}{7}$ mm² 1

CSA of 2 hemispheres = $2 \times \frac{275}{7}$

$= \frac{550}{7}$ mm² 1

Total area of capsule = $\frac{990}{7} + \frac{550}{7}$

$= \frac{1540}{7}$

= 220 mm² 1

OR

Diameter of cylinder = 2.8 cm

radius of cylinder = $\frac{2.8}{2}$ = 1.4 cm

radius of cylinder = *radius* of hemisphere = 1.4 cm

Height of cylinder = 5-2.8 1

= 2.2 cm

Volume of 1 Gulab jamun = vol. of cylinder + 2 x vol. of hemisphere

$= \pi r^2 h + 2 \times \frac{2}{3} \pi r^3$ 1

$\frac{22}{7} \times (1.4)^2 \times 2.2 + 2 \times \frac{2}{3} \times \frac{22}{7} \times (1.4)^3$

= 13.55 + 11.50

$= 25.05\ cm^3$ 1

volume of 45 Gulab jamun = 45 x25.05

syrup in 45 Gulab jamun = 30% x 45 x 25.05

$= \frac{30}{100} \times 45 \times 25.05$ 1

= 338.175 cm^3

≈ 338 cm^3

35)

Life time (in hours)	Number of lamps(f)	Mid x	d	fd
1500-2000	14	1750	-1500	-21000
2000-2500	56	2250	-1000	-56000
2500-3000	60	2750	-500	-30000
3000-3500	86	3250	0	0
3500-4000	74	3750	500	37000
4000-4500	62	4250	1000	62000
4500-5000	48	4750	1500	72000
	400			64000

2

Mean = $a + \frac{\Sigma fd}{\Sigma f}$ 1/2

a = 3250 1/2

Mean $= 3250 + \frac{64000}{400}$ 1

$= 3250 + 160$

$= 3410$

Average life of lamp is 3410 hr 1

Section E

36) $a_6 = 16000$ $a_9 = 22600$

$a+5d=16000$ ------(1)

$a+8d=22600$ ----------(2)

substitute a = 1600 -5d from (1)

16000-5d + 8d = 22600

3d = 22600-16000

3d=6600

$d = \frac{6600}{3} = 2200$

a = 16000-5(2200)

a = 16000-11000

a = 5000

(i) $a_n = 29200$, a = 5000, d = 2200

$a_n = a + (n-1)d$

$29200 = 5000 + (n-1)2200$ 1/2

29200-5000 = 2200n-2200

24200+2200=2200n

26400=2200n

$n = \frac{264}{22}$

n=12 1/2

in 12th year the production was Rs 29200

(ii) n=8, a=5000, d=2200

$a_n = a + (n-1)d$ 1/2

$= 5000+(8-1)2200$ 1/2

$= 5000+7 \times 2200$

$= 5000+15400$ 1/2

$= 20400$

The production during 8th year is = 20400 1/2

OR

n = 3, a = 5000, d = 2200

$s_n = \frac{n}{2}[2a + (n-1)d]$ 1/2

$= \frac{3}{2}[2(5000) + (3-1)\ 2200]$

$S_3 = \frac{3}{2}(10000 + 2 \times 2200)$ 1/2

$= \frac{3}{2}(10000 + 4400)$ 1/2

$= 3 \times 7200$

$= 21600$ 1/2

The production during first 3 year is 21600

(iii) $a_4 = a+3d$

$= 5000 + 3(2200)$

$= 5000 + 6600$

$= 11600$ 1/2

$a_7 = a+6d$

$= 5000 + 6 \times 2200$

$= 5000 + 13200$

$= 18200$

$a_7 - a_4 = 18200-11600 = 6600$ 1/2

---

37) coordinates of A (2, 3) Alia's house

coordinates of B (2, 1) Shagun's house

coordinates of C (4,1) Library

(i) $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2-2)^2 + (1-3)^2}$ 1/2

$= \sqrt{(0^2 + (-2)^2}$

$AB = \sqrt{0+4} = \sqrt{4} = 2$ units 1/2

Alia's house from shagun's house is 2 units

(ii) C(4,1), B (2,1)

$CB = \sqrt{(2-4)^2 + (1-1)^2}$ 1/2

$= \sqrt{(-2)^2 + 0^2}$

$= \sqrt{4+0} = \sqrt{4} = 2$ unit 1/2

(iii) O(0,0), B(2,1)

$OB = \sqrt{(2-0)^2 + (1-0)^2}$

$= \sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$ units 1

Distance between Alia's house and Shagun's house, AB = 2 units

Distance between Library and Shagun's house, CB = 2 units 1/2

OB is greater than AB and CB, 1/2

For shagun, school [O] is farther than Alia's house [A] and Library [C]

OR

C (4, 1), A(2, 3)

$CA = \sqrt{(2-4)^2 + (3-1)^2}$

$= \sqrt{(-2)^2 + 2^2}$+ $= \sqrt{4+4}$ $= \sqrt{8}$

$= 2\sqrt{2}$ units $AC^2 = 8$ 1

Distance between Alia's house and Shagun's house, AB = 2 units

Distance between Library and Shagun's house, CB = 2 units 1/2

$AB^2 + BC^2 = 2^2 + 2^2 = 4 + 4 = 8 = AC^2$ ½

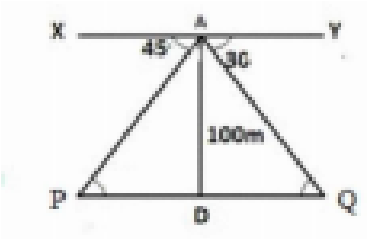
Therefore A, B and C form an isosceles right triangle.

38)

(i) XY ‖ PQ and AP is transversal.

∠APD = ∠PAX (alternative interior angles) 1/2

∠APD=45° 1/2



(ii) Since XY || PQ and AQ is a transversal

so alternate interior angles are equal

hence ∠YAQ = ∠AQD=30° 1/2

(iii) In ΔADP, $\theta = 45°$

$\tan\theta = \frac{P}{B}$

$\tan 45° = \frac{100}{PD}$ 1/2

PD=100 m

1/2

Boat P is 100 m from the light house 1

OR

In ΔADQ, $\theta = 30°$

$\tan\theta = \frac{P}{B}$ 1/2

$\tan 30 = \frac{100}{DQ}$

$\frac{1}{\sqrt{3}} = \frac{100}{DQ}$ 1/2

$DQ = 100\sqrt{3}$ m 1

KENDRIYA VIDYALAYA SANGATHAN, PATNA REGION

SAMPLE PAPER (2024-25)

CLASS-X

SUBJECT-MATHEMATICS STANDARD (CODE-041)

Time allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains five sections A , B , C , D , E
2.Section A has 18 MCQs and 02 Assertion – Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) type questions of 5 marks each.
6. Section E has 3 Case study type questions of 4 marks each with sub parts.
7. All questions are compulsory. However, an internal choice in 2 questions of 5 marks. 2 Qs os 3 marks and 2 Qs of 2 marks has been provided. An internal choice has been provided in the 2 marks Qs of Section E.

SECTION A

(This section comprises of MULTIPLE CHOICE QUESTIONs type questions (MCQ) of 1 marks each)

1. The LCM of smallest two-digit composite number and smallest composite number is :
   a) 12 b) 4 c) 20 d) 44
2. The total number of factors of a prime number is
   a) 1 b) 2 c) 3 d) 4

3 . The LCM of two prime numbers p and q (p > q) is 221.Find the value of 3p – q

a) 4 b) 28 c) 38 d) 48

3. The graph of a polynomial P(x) cuts the X- axis at 3 points and touches it at 2 other points. The number of zeroes of P(x) is :
   a) 4 b) 2 c) 3 d) 5

5 . If 5 is a zero of the quadratic polynomial , $x^2 – kx -15$, then the value of k is

a) -2 b) 2 c) 1 d) -1

6. If α and β are the roots of $4x^2 + 3x + 7 = 0$, then the value of $\frac{1}{\alpha}+\frac{1}{\beta}$ is

a) 3/7 b) - 3/7 c) 7/ 3 d) -7/ 3

7. The value of k for which the lines $5x + 7y = 3$ and $15x + 21y = k$ coincide is :

a) 9 b) 5 c) 7 d) 18

8. The common difference of an AP, whose n^{th} term is $a_n = 3n+ 7$ is

a) 3 b) 2 c) 0 d) 4

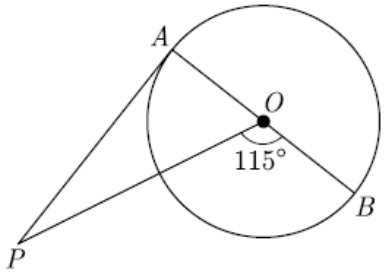
9. In a ΔABC, D and E are points on the sides AB and AC respectively such that DE II BC If AD=6cm, DB= 9cm and AE=8cm, find AC

a) 12 b) 8 c) 10 d) 20

10. Two concentric circles are of radii 10 cm and 8 cm, then the length of the chord of the larger circle which touches the smaller circle is

(a) 6 cm (b) 12 cm (c) 18 cm (d) 9 cm

11. In the given figure, PA is a tangent from an external point P to a circle with centre O. If ∠POB = 115°, then value of∠ APO is



(a) 25° (b) 20° (c) 30° (d) 65°

12. The value of θ for which cos (10° + θ) = sin 30° , is

(a) 50° (b) 90° (c) 30° (d) 45°

13. The two side AB and BC of right triangle ABC are in the ratio 1 : 3 . What will be the value of sin C ?

(a)$\frac{3}{10}$ (b)$\frac{1}{\sqrt{10}}$ (c) $\frac{1}{3}$ (d) 1

14. The area of the square that can be inscribed in a circle in a circle of radius 8cm is :

a) 256cm^2 b) 64 cm^2 c) 128 cm^2 d) 32 cm^2

15. Two right circular cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1 . What is the ratio of their volumes ?

a) 9 : 1 b) 1:3 c) 2 ; 3 d) 3 : 1

16 . For a frequency distribution, mean, median and mode are connected by the relation

(a) Mode = 3 Mean – 2 median (b) Mode = 2 median - 3 Mean

(c) Mode = 3median -2 Mean (d) Mode = 3 median + 2 Mean

17. If P(E) = 0.007, then what is the probability of 'not E'?

(a) 0.93 (b) 0.095 (c) 0.890 (d) 0.993

18. If two dice are thrown, the probability of getting sum as 3 will be

(a) $\frac{1}{18}$ (b)$\frac{2}{3}$ (c)$\frac{5}{36}$ (d)$\frac{1}{36}$

Assertion reason based questions

Instructions for answering assertion and reason based questions. You have to choose option

19. **Assertion:** D and E are points on sides AB and AC of triangle ABC such that AD=(7x-4) cm , AE=(5x-2) cm, DB=(3x+4) cm and EC= 3x cm. if DE || BC, then x=4

Reason: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points then the other two sides are divided in the same ratio.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true

20. **Assertion :** If Sn is the sum of the first n terms of an A.P., then its nth term an is given by an = Sn – S_{n-1} .

Reason : The 10th term of the A.P. 5, 8, 11, 14, ……. is 35

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true

Section B

(This section comprises of very short answer type questions (VSA) of 2 marks each)

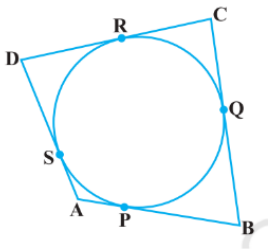
21. Find the value of k such that the polynomials x^2 – (k + 6) x+ 2(2k -1) has sum of its zeros equal to half of their product.

22. In what ratio does the point ($\frac{24}{11}$, y) divide the line segment joining the point P (2, -2) and Q (3, 7) ? Also find the value of y.

23. D is a point on the side BC of a triangle ABC such that ∠ADC= ∠BAC. Show that CA^2 =CB.CD

24. A quadrilateral ABCD is drawn to circumscribe a circle as in figure.

Prove that AB + CD = AD + BC.



OR

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

25. If $\sin\theta + \cos\theta = \sqrt{2}$, then prove that $\tan\theta + \cot\theta = 2$.

OR

Prove that : $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$.

Section C

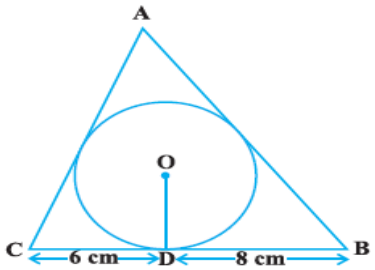
(This section comprises of very short answer type questions (SA) of 3 marks each)

26. Prove that $\sqrt{5}$ is an irrational number.

27.Sum of the areas of two squares is 157 m^2. If the sum of their perimeters is 68 m. Find the sides of two squares.

28. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6cm respectively .

Find the sides AB and AC

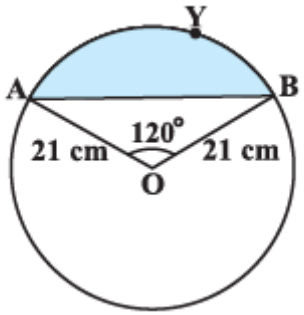


29. Show that: $\tan^4\theta + \tan^2\theta = sec^4\theta - sec^2\theta$

30. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre.
Find: (i) the length of the arc (ii) area of the segment formed by the corresponding chord

OR

Find the area of the segment AYB shown in below figure, if radius of the circle is 21 cm and $\angle AOB = 120°$.



31. One card is drawn from a well-shuffled pack of 52 playing cards. Find the probability of getting

i) A king of black colour

ii) Either a red card or a Jack

iii) Not a face card

OR

Two dice are thrown at the same time. Find the probability of getting

(i) Even number on both dice

(ii) The sum of the two numbers appearing on the top of the dice is 7.

(iii) The sum of the two numbers appearing on the top of the dice is prime number.

Section D

(This section comprises of long answer type questions (LA) of 5 marks each)

32. A train covered a certain distance at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time and if the train were slower by 6 km/h , it would have taken 6 hours more than the scheduled time. Find the length of journey.

OR

Draw the graph of 2x + y = 6 and 2x – y +2 = 0. Shade the region bounded by these lines and x- axis. Find the area of the shaded region.

33. A solid toy is in the form of a hemisphere surmounted by a right circular cone of same radius. The height of the cone is 10cm and the radius of the base is 7cm. Determine the volume of the toy. Also find the area of the coloured sheet required to cover the toy.(Use $\pi = \frac{22}{7}$ and $\sqrt{149} = 12.2$)

34. The angles of depression of the top and the bottom of an 8 m tall building from the top of a multistoried building are 30°and 45°, respectively. Find the height of the multistory building and the distance between the
two buildings.

OR

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

35. The median of the following data is 525. Find the values of *x* and *y*, if the total frequency is 100.

Class Interval	Frequency
0 - 100	2
100 - 200	5
200 - 300	x
300 - 400	12
400 - 500	17
500 - 600	20
600 - 700	y
700 - 800	9
800 - 900	7
900 - 1000	4

SECTION E

(This section comprises of 3 case – study / passage based questions of 4 marks each with two sub sections.

First two case study questions have three sub questions of marks 1 , 1 , 2 respectively. The third case study question has two sub questions of 2 marks each.)

36. In the month of April to June 2022, the exports of passenger cars from India increased by 26% in the corresponding quarter of 2021–22, as per a report. A car manufacturing company planned to produce 1800 cars in 4th year and 2600 cars in 8th year. Assuming that the production increases uniformly by a fixed number every year.

above information answer the following questions.

Based on the above information answer the following questions.

(i) Find the production in the 1st year. (1 mark)

(ii) Find the production in the 12th year. (1 mark)

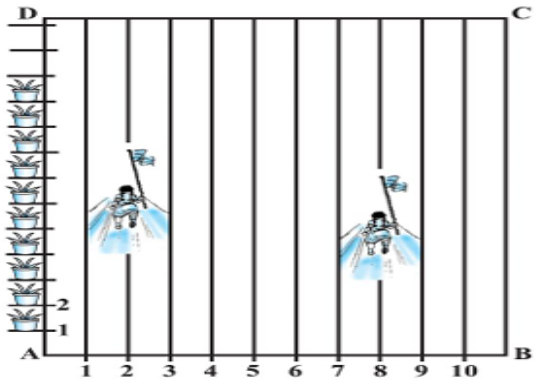
(iii) Find the total production in first 10 years. (2 marks)

OR

(iii) In how many years will the total production reach 31200 cars? (2 marks)

37. In order to conduct sports day activities in your school, lines have been drawn with chalk powder at a distance of 1 m each in a rectangular shaped ground ABCD. 100 flower pots have been placed at the distance of 1 m from each other along AD, as shown in the following figure. Niharika runs (1 /4)th distance AD on the 2nd line and posts a green Flag. Preet runs (1/ 5) th distance AD on the eighth line

and posts are red flags. Taking A as the origin AB along x-axis and AD along y-axis, answer the following questions:

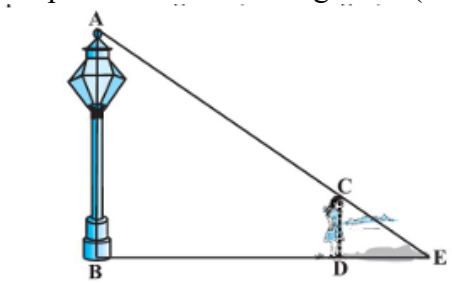


(i) Find the coordinates of the green flag. (1 mark)
(ii) Find the distance between the two flags. (1 mark)
(iii) If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag? (2 marks)

OR

(iii) If Joy has to post a flag at one fourth distance from the green flag, in the line segment joining the green and red flags, then where should he post his flag? (2 marks)

38. On one day, a poor girl of height 90 cm is looking for a lamp-post for completing her homework as in her area power is not there and she finds the same at some distance away from her home. After completing the homework, she is walking away from the base of a lamp-post at a speed of 1.2 m/s. The lamp is 3.6 m above the ground (see below figure).



(i) Find her distance from the base of the lamp post after 4 seconds. (2 marks)
(ii) Find the length of her shadow after 4 seconds. (2 marks)
OR
(ii) Find the ratio AC : CE. (2 marks)

KENDRIYA VIDYALAYA SANGATHAN PATNA REGION

SAMPLE PAPER (2024-25)

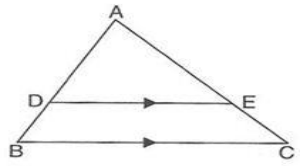
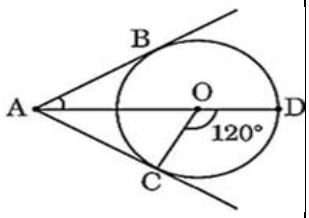
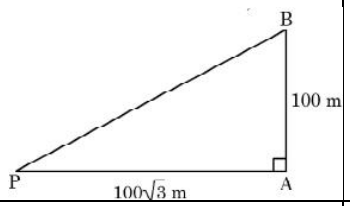
CLASS 10 – MATHEMATICS BASIC (241)

Time Allowed : 180 mins **Maximum Marks :** 80

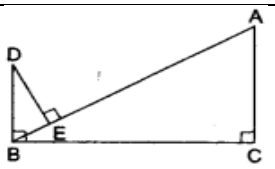
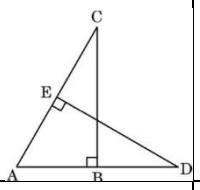
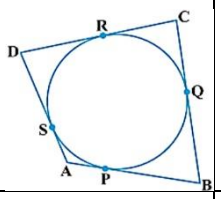
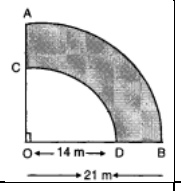
General Instructions:

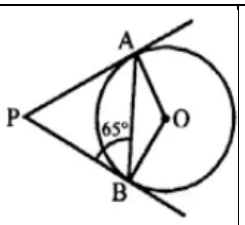
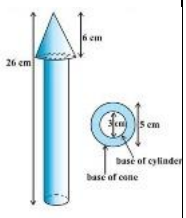
1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub - parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take$\pi = \frac{22}{7}$ wherever required if not stated.

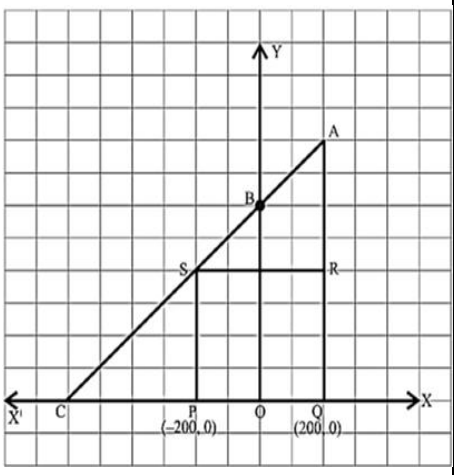
	Section A	
1	If p and q are co - prime numbers, then p^2 and q^2 are a) even b) coprime c) not coprime d) odd	**[1]**
2	If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a = a) 1 b) 2 c) 4 d) 3	**[1]**
3	$(x^2 + 1)^2 - x^2 = 0$ has a) two real roots b) no real roots c) one real root. d) four real roots	**[1]**
4	If (- 3, 2) is a solution of the linear equation 5x + 3 ky = 3, then the value of k is _____. a) 5 b) 6 c) 2 d) 3	**[1]**
5	If$\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is a) $\frac{1}{2}$ b) - 2 c) $\frac{1}{4}$ d) 2	**[1]**
6	The points (- 4, 0), (4, 0) and (0, 3) are the vertices of a:	**[1]**

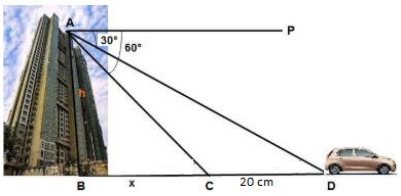
	a) isoscelestriangle b) scalenetriangle c) equilateraltriangle d) right triangle	
7	If△ ABC ~ △ DEF and ∠ A = 47 ^{o} , ∠ E = 83 ^{o} , then ∠ C is equal: a) 50^o b) 130^o c) 83^o d) 47^o	**[1]**
8	In the given figure,$DE \mid \mid BC$. AB= 15, cm, BD = 6cm, AC = 25 cm, then AE is equal to a) 15 cm. b) 18 cm. c) 20 cm. d) 10 cm. 	**[1]**
9	In the given figure, AC and AB are tangents to a circle centered at O. If∠ COD = 120 ^{o} , then ∠ BAO is equal to: a) 30^o b) 45^o c) 90^o d) 60^o 	**[1]**
10	9sec^2 A - 9tan ^{2} A = a) 8 b) 9 c) 0 d) 1	**[1]**
11	The angle subtended by a vertical pole of height 100 m at a point on the ground $100\sqrt{3}$ m from the base is, has measure of a) 45^o b) 90^o c) 60^o d) 30^o 	**[1]**
12	If cos A =$\frac{\sqrt{3}}{2}$, 0 ^{o} < A < 90 ^{o} , then A is equal to a) 30^o b) 1 c) $\frac{\sqrt{3}}{2}$ d) 60^o	**[1]**
13	Find the area of a sector of angle p (in degrees) of a circle with radius R. a) $\frac{p}{360} \times \pi R^2$ b) $\frac{p}{180} \times \pi R^2$	**[1]**

	c) $\frac{p}{30} \times \pi R^2$ d) $\frac{p}{120} \times \pi R^2 l$	
14	In a circle of radius 21 cm, an arc subtends an angle of 60^o at the centre. The area of the sector formed by the arc is: a) 231cm^2 b) 250 cm^2 c) 220 cm^2 d) 200 cm^2	**[1]**
15	The probability that a non leap year selected at random will have 53 Sundays is a) $\frac{1}{7}$ b) $\frac{2}{7}$ c) $\frac{4}{7}$ d) $\frac{3}{7}$	**[1]**
16	In a data, if l = 60, h = 15, f_1 = 16, f_0 = 6, f_2 = 6, then the mode is a) 67.5 b) 72 c) 60 d) 62	**[1]**
17	If a cone is cut into two parts by a horizontal plane passing through the mid - point of its axis, the ratio of the volumes of the upper part and the cone is a) 1 : 2 b) 1 : 4 c) 1 : 6 d) 1 : 8	**[1]**
18	The mean of 2, 7, 6 and x is 5 and the mean of 18, 1, 6, x and y is 10. What is the value of y? a) 30 b) 10 c) 5 d) 20	**[1]**
19	**Assertion (A):** Distance of point (a, b) from origin is $\sqrt{b^2 - a^2}$ **Reason (R):** Distance of point (x, y) from origin is $\sqrt{(x-0)^2 + (y-0)^2}$ a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A. c) A is true but R is false. d) A is false but R is true.	**[1]**
20	**Assertion (A):** If a number x is divided by y(x, y) (both x and y are positive) then remainder will be less than x. **Reason (R):** Dividend = Divisor × Quotient + Remainder. a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A. c) A is true but R is false. d) A is false but R is true.	**[1]**

	Section B	
21	On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether thepair of linear equations are consistent, or inconsistent: $\frac{3}{2}x + \frac{5}{3}y = 7$, 9 *x* - 10 *y* = 14	**[2]**
22	In the given figure,$DB \perp BC$, $DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$  **OR** In the given figure, ABC and AED are two right triangles, right angled at B and E respectively. Prove that: 1. $\triangle ABC \sim \Delta AED$ 2. AB× AD = AC × AE 	**[2]**
23	A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC 	**[2]**
24	Prove that:$\frac{\tan A}{(1+\tan^2 A)^2} + \frac{\cot A}{(1+\cot^2 A)^2}$ = sin A cos A	**[2]**
25	To warm ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned.(use$\pi = 3.14$) **OR** ABCD is a flower bed. If OA =21 m and OC = 14 m, find the area of the bed. 	**[2]**
	Section C	
26	Provethat$\sqrt{5}$ is irrational.	**[3]**
27	Find a quadratic polynomial, the sum and product of whose zeroes are$\frac{1}{4}$ and - 1, respectively.	**[3]**
28	Solve the pair of linear equations$\sqrt{2}x - \sqrt{3}y = 0$ and $\sqrt{3}x - \sqrt{8}y = 0$ by substitution method. **OR** The sum of digits of a two digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number.	**[3]**

29	In the given figure, PA and PB are two tangents from an external point P to a circle with centre O. If∠ PBA = 65°, find ∠ OAB and ∠ APB.  	**[3]**
30	Prove that$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$, using identity $sec^2\theta = 1 + tan^2\theta$. **OR** If tan A = n tanB and sin A = m sinB, then prove that cos^2 A = $\frac{m^2-1}{n^2-1}$	**[3]**
31	Cards marked with numbers 1,3,5,..., 101 are placed in a bag and mixed thoroughly. A card is drawn at random from the bag. Find the probability that the number on the drawn cards is 1. less than 19, 2. a prime number less than 20.	**[3]**
	Section D	
32	The hypotenuse (in cm) of a right angled triangle is 6 cm more than twice the length of the shortest side. If the length of third side is 6 cm less than thrice the length of shortest side, then find the dimensions of the triangle. **OR** A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If, the total cost of production on that day was₹ 90, find the number of articles produced and the cost of each article.	**[5]**
33	Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio.	**[5]**
34	A student was asked to make a model shaped like a cylinder with two cones attached to its ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its total length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model. **OR** A wooden toy rocket is in the shape of a cone mounted on a cylinder as shown in given below figure. The height of the entire rocket is 26 cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Takeπ =3.14)  	**[5]**
35	The table below gives the percentage distribution of female teachers in the primary schools of rural areas of various states and union territories (U.T.) of India. Find the mean	**[5]**

	percentage of female teachers by all the three methods discussed in this section. **Percentage of female teachers** | 15 - 25 | 25 - 35 | 35 - 45 | 45 - 55 | 55 - 65 | 65 - 75 | 75 - 85 **Number of states/U.T.** | 6 | 11 | 7 | 4 | 4 | 2 | 1	
	Section E	
36	**Read the text carefully and answer the questions:** In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?  1. Find the terms of AP formed in above situation. 2. What is the total distance the competitor has to run? **OR** 3. Find the distance covered by competitor in order to put 5^{th} potato in the bucket. 4. Find distance cover after 4 potato drop in the bucket?	**[4]**
37	**Read the text carefully and answer the questions:** Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.  1. Taking O as origin, coordinates of P are (- 200, 0) and of Q are (200, 0). PQRS being a square, what are the coordinates of R and S? 2. What is the area of square PQRS? **OR** 3. What is the length of diagonal PR in square PQRS? 4. If S divides CA in the ratio K : 1, what is the value of K, where point A is (200, 800)?	**[4]**
38	**Read the text carefully and answer the questions:** Vijay lives in a flatina multi - story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drivesfrom his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60^o . After accelerating 20 m from point C, Vijay stops at point D to buy ice cream andthe angle of depression changed to 30 ^{o} .	**[4]**



1. Find the value of x.
2. Find the height of the building AB.

   OR
3. Find the distance between top of the building and a car at position C?
4. Find the distance between top of the building and a car at position D?