

**RADIANT P. U. COLLEGE, VIJAYAPUR**  
**DEPARTMENT OF MATHEMATICS**  
**CHAPTERWISE PREVIOUS YEAR BOARD EXAMINATION QUESTIONS**

**1. RELATIONS AND FUNCTIONS:**

**ONE MARK QUESTIONS :**

1. A relation R on a set  $A = \{1,2,3\}$  defined by  $R = \{(1,1), (1,2), (3,3)\}$  is not symmetric . why ?  
(March-2014,Sept-2020)
2. Let \* be a binary operation defined on the rational number Q defined by  $a*b = ab+1$  ,prove that \* is commutative .(July -2014)
3. Let \* be a binary operation on the set of rational numbers by  $a*b = \frac{ab}{4}$  , find the identity element  
(March-2015, July -2017 )
4. Let \* be binary operation on the set of natural numbers given by  $a*b = \text{L.C.M. of } a \text{ and } b$  , find  $5*7$ .  
(July-2015, July-2019)
5. An operation \* on  $\mathbb{Z}^+$  is defined as  $a*b = a-b$  ,  $\forall a,b \in \mathbb{Z}^+$  . Is \* is a binary operation on  $\mathbb{Z}^+$  ?  
(March-2016)
6. An operation \* on  $\mathbb{Z}^+$  is defined as  $a*b = |a-b|$  ,  $\forall a , b \in \mathbb{Z}^+$  . Is \* is a binary operation on  $\mathbb{Z}^+$   
(July-2016, Model paper -02)
7. Let \* be a binary operation on the set of natural numbers given by ,  $a*b = \text{L.C.M. of } a \text{ and } b$  , find  $20*16$ .(March-2017)
8. Define bijective function . (March -2018)
9. The relation R on a set  $A = \{1,2,3\}$  is defined as  $R = \{(1,1), (2,2), (3,3), (1,2), (2,3)\}$  is not transitive why ?  
(July -2018)
10. Define binary operation (March 2019, Model paper 2022)
11. Let \* be a binary operation on the set of natural numbers given by  $a*b = \text{L.C.M. of } a \text{ and } b$  , find  $5*7$ .  
(March - 2020)
12. Define empty relation . (Model paper -1)
13. Give an example of a relation which is symmetric and transitive but not reflexive (Model paper -2022)

**TWO MARKS QUESTIONS :**

1. Verify whether the operation \* defined on Q by  $a*b = \frac{ab}{2}$  is associative or not .  
(March -2014, March -2018, Sept-2020, Model paper -01)
2. On  $\mathbb{R}^+$  is defined by  $a*b = \frac{a+b}{2}$  , verify whether \* is associative (Model paper -01)
3. Show that the function  $f: \mathbb{N} \rightarrow \mathbb{N}$  , given by  $f(1) = f(2) = 1$  and  $f(x) = x - 1$  , for every  $x > 2$  , is onto , but not one-one (July -2014)
4. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 1 + x^2$  , then show that f is neither one-one nor onto . (March -2017)
5. Define binary operation on a set . Verify whether the operation \* defined on Q by  $a*b = ab+1$  a, b  $\in$  Q is binary or not . (July -2018)
6. Show that the function  $f: \mathbb{N} \rightarrow \mathbb{N}$  given by  $f(x) = 2x$  is one-one but not onto . (March -2019)
7. Prove that the greatest integer function  $f: \mathbb{R} \rightarrow \mathbb{R}$  , given by  $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$  , is neither one - one nor onto . Where  $[x]$  denotes greatest integer less than or equal to  $x$  . (Model paper 2022)
8. Show that if  $f: A \rightarrow B$  and  $g: B \rightarrow C$  are one-one , then  $g \circ f: A \rightarrow C$  is also one -one (March 2015)
9. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is given by  $f(x) = (3-x)^{\frac{1}{3}}$  then find  $(f \circ f)(x)$  . (July -2015)
10. If the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  are given by  $f(x) = |x|$  and  $g(x) = [x]$  , where  $[x]$  is greatest integer function find  $f \circ g(-\frac{1}{2})$  and  $g \circ f(-\frac{1}{2})$  (March -2016)
11. Show that if  $f: A \rightarrow B$  and  $g: B \rightarrow C$  are onto , then  $g \circ f: A \rightarrow C$  is also onto . (July -2017)
12. Find  $o f$  : and  $f o g$  , if  $f(x) = 8x^3$  and  $g(x) = x^{\frac{1}{3}}$  (July- 2019) .

### THREE MARKS QUESTIONS :

1. Show that the relation  $R$  in the set  $Z$  of integers given by  $R = \{ (a,b) : 2 \text{ divides } a - b \}$  is an equivalence relation. **(July 2014)**
2. Define whether the relation  $R$  in the set  $A = \{ 1,2,3, \dots, 13,14 \}$  is defined as  $R = \{ (x,y) : 3x - y = 0 \}$  is reflexive, symmetric and transitive **(March 2015, July 2019)**
3. Prove that the relation  $R$  in the set of integers  $Z$  defined by  $R = \{ (x,y) : x - y \text{ is an integer} \}$  is an equivalence relation. **(July 2015, Sept 2020)**
4. Show that the relation  $R$  in the set  $a = \{ x : x \in Z, 0 \leq x \leq 12 \}$  is given by  $R = \{ (a,b) : |a - b| \text{ is a multiple of } 4 \}$  is an equivalence relation. **(March 2016)**
5. Show that the relation  $R$  in set  $A = \{ 1,2,3,4,5 \}$  given by  $R = \{ (a,b) : |a - b| \text{ is even} \}$  is an equivalence relation. **(July 2016, March 2018)**
6. Show that the relation  $R$  in the set  $R$  defined as  $R = \{ (a,b) : a \leq b \}$  is reflexive, transitive but not symmetric. **(March 2017)**
7. Check whether the relation  $R$  defined in the set  $\{1,2,3,4,5\}$  as  $R = \{ (a,b) : b = a + 1 \}$  is reflexive or symmetric. **(July 2017)**
8. Check whether the relation  $R$  in the set  $R = \{ (a,b) : a \leq b^3 \}$  is reflexive, symmetric or transitive **(March 2019)**
9. Show that the relation  $R$  defined in the set  $A$  of all triangles as  $R = \{ (T_1, T_2) : T_1 \text{ is similar to } T_2 \}$  is an equivalence relation. **(March 2020, Model Paper- 01)**
10. Show that the relation  $R$  in the set of all integers  $Z$  defined by  $R = \{ (a,b) : 2 \text{ divides } a - b \}$  is an equivalence relation **(Model Paper -02)**
11. Find  $gof$  and  $fog$  if  $f : R \rightarrow R$  and  $g : R \rightarrow R$  are given by  $f(x) = \cos x$  and  $g(x) = 3x^2$ . Show that  $gof \neq fog$  **(March 2014, July 2016, July 2018, March 2020)**
12. Show that the relation  $R$  on a set  $A = \{ 1,2,3 \}$  is defined as  $R = \{ (1,1), (2,2), (3,3), (1,2), (2,3) \}$  is reflexive but neither symmetric nor transitive. **(Model Paper 2022)**

### FIVE MARKS QUESTIONS :

1. Verify whether the function  $f : N \rightarrow N$  defined by  $f(x) = x^2$  is one-one, onto and bijective **(Model Paper -02)**
2. Check the injectivity and surjectivity of the function  $f : R \rightarrow R$  by  $f(x) = 3 - 4x$ . Is it a bijective function? **(Model Paper -01)**
3. Verify whether the function  $f : A \rightarrow B$ , given by  $f(x) = \frac{x-2}{x-3}$ , where  $A = R - \{3\}$  and  $B = R - \{1\}$  is one-one, onto and bijective? **(Model Paper 2022)**
4. Check the injectivity and surjectivity of the function  $f : R \rightarrow R$ , given by  $f(x) = 1 + x^2$ . Is it bijective? **(Expected)**
5. Show that the function  $f : R^+ \rightarrow R^+$ , defined by  $f(x) = \frac{1}{x}$  is both one-one and onto, where  $R^+$  is the set of all nonzero real numbers. **(Expected)**
6. Prove that the modulus function  $f : R \rightarrow R$ , given by  $f(x) = |x|$ , is neither one-one nor onto **(Expected)**
7. Prove that the function,  $f : N \rightarrow Y$  defined by  $f(x) = 4x + 3$ , where  $Y = \{ y : y = 4x + 3, x \in N \}$  is invertible and also find the inverse of  $f(x)$ . **(March 2014, March -2019)**
8. Prove that the function,  $f : R \rightarrow R$ , given by  $f(x) = 4x + 3$ , is invertible, also find the inverse of  $f(x)$  **(July 2015, July 2017, July 2019, March 2019)**
9. Prove that the function  $f : N \rightarrow Y$  defined by  $f(x) = x^2$ , where  $Y = \{ y : y = x^2, x \in N \}$  is invertible. Also find the inverse of  $f$  **(July 2014)**
10. Let  $R_+$  be the set of all non-negative real numbers. Show that the function  $f : R_+ \rightarrow [4, \infty)$  defined by  $f(x) = x^2 + 4$  is invertible. Also write the inverse of  $f$  **(March 2015, July 2016, March 2018, July 2018)**
11. Let  $f : N \rightarrow R$  be defined by  $f(x) = 4x^2 + 12x + 15$ . Show that  $f : N \rightarrow S$  where  $S$  is the range of the function  $f$  is invertible. Also find the inverse of  $f$  **(March 2016, March 2017, Model Paper 2022)**

## 2. INVERSE TRIGONOMETRIC FUNCTION

### ONE MARK QUESTIONS :

1. Write the domain of the function  $f(x) = \cos^{-1} x$  (March 2014, March 2018, July 2018)
2. Find the principal value of  $\cos^{-1} \left(-\frac{1}{2}\right)$  (July 2014, March 2019, Sept 2020, Model Paper 2022)
3. Write the domain of  $f(x) = \sin^{-1} x$  (July 2016, Model Paper 02)
4. Find the principal value of  $\operatorname{cosec}^{-1}(-\sqrt{2})$  (March -2017)
5. Find the principal value of  $\cot^{-1} \left(-\frac{1}{\sqrt{3}}\right)$  (July 2019)
6. Write the range of  $y = \sec^{-1} x$  (March 2020, Model Paper -01)
7. Write the values of  $x$  for which  $2 \tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2}\right)$  holds (March 2015)
8. Write the values of  $x$  for which  $\tan^{-1} \left(\frac{1}{x}\right) = \cot^{-1} x$  holds (July 2017)
9. Find the value of  $\sin^{-1} \left(\sin \left(\frac{2\pi}{3}\right)\right)$  (July 2015, July 2016)
10. Find the value of  $\cos(\sec^{-1} x + \operatorname{cosec}^{-1} x)$ ,  $|x| \geq 1$  (March 2016)

### TWO MARKS QUESTIONS :

1. Write the domain and range of  $\tan^{-1} x$  (Model Paper -2022)
2. Evaluate  $\sin \left[\frac{\pi}{3} - \sin^{-1} \left(-\frac{1}{2}\right)\right]$ . (March 2014)
3. Find the principal value of  $\cot^{-1} \left(-\frac{1}{\sqrt{3}}\right)$  (Model paper -01)
4. Evaluate  $\cos^{-1} \left(\frac{1}{2}\right) + 2 \sin^{-1} \left(\frac{1}{2}\right)$  (Model Paper -02)
5. Find the value of  $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$  (Model Paper -2022)
6. Prove that  $\sin^{-1}(2x\sqrt{1-x^2}) = 2 \sin^{-1} x$  for which  $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$  (March 2015)
7. Prove that  $\sin^{-1}(2x\sqrt{1-x^2}) = 2 \cos^{-1} x$  for which  $-\frac{1}{\sqrt{2}} \leq x \leq 1$  (March 2016, March 2017)
8. Find the principal value of  $\cot^{-1}(-\sqrt{3})$ .
9. Prove that  $\tan^{-1} \left(\frac{2}{11}\right) + \tan^{-1} \left(\frac{7}{24}\right) = \tan^{-1} \left(\frac{1}{2}\right)$  (July - 2014)
10. Write  $\tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$ ,  $0 < x < \pi$  in the simplest form. (July 2015, March 2018)
11. Prove that  $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ ,  $x \in [-1, 1]$  (July 2015, March 2019)
12. Prove that  $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ ,  $x \in \mathbb{R}$  (July 2014)
13. Write the simplest form of  $\tan^{-1} \left[\frac{3 \cos x - 4 \sin x}{4 \cos x + 3 \sin x}\right]$ , if  $\frac{3}{4} \tan x > -1$  (March 2016)
14. Write the simplest form of  $\tan^{-1} \left[\frac{1 - \cos x}{1 + \cos x}\right]$ ,  $0 < x < \pi$  (March 2014, July 2014, July 2018)
15. Write the simplest form of  $\cot^{-1} \left[\frac{1}{\sqrt{x^2-1}}\right]$ ,  $x > 1$  (March 2019, July 2019)
16. Prove that  $3 \sin^{-1} x = \sin^{-1}(3x - 4x^3)$ ,  $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$  (July 2016)
17. Prove that  $\cot^{-1}(-x) = \pi - \cot^{-1} x$ ,  $\forall x \in \mathbb{R}$  (March 2020)
18. Solve the equation  $\tan^{-1} \left(\frac{1-x}{1+x}\right) = \frac{1}{2} \tan^{-1} x$ ,  $x > 0$  (March 2017)
19. Show that  $2 \tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2}\right)$ ,  $x \geq 0$  (July 2017)
20. Find the value of  $\sin^{-1} \left(\sin \left(\frac{3\pi}{5}\right)\right)$  (July 2017, March 2020)
21. Find the value of  $\cos^{-1} \left(\cos \frac{13\pi}{6}\right)$  (July 2018)
22. If  $\sin \left(\sin^{-1} \frac{1}{2} + \cos^{-1} x\right) = 1$ , then find the value of  $x$  (March 2028, Model Paper 2022)



### THREE MARKE QUESTIONS :

1. Prove that  $3 \cos^{-1} x = \cos^{-1}(4x^3 - 3x)$ ,  $x \in [\frac{1}{2}, 1]$  (March 2014)
2. Prove that  $2 \tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{7}) = \tan^{-1}(\frac{31}{17})$  ( March 2015 , March 2018, March 2020 , Model Paper - 2022)
3. Prove that  $\tan^{-1} x + \tan^{-1}(\frac{2x}{1-x^2}) = \tan^{-1}(\frac{3x-x^3}{1-3x^2})$ ,  $|x| < \frac{1}{\sqrt{3}}$  (July 2014)
4. Find the value of  $x$ , if  $\tan^{-1}(\frac{x-1}{x-2}) + \tan^{-1}(\frac{x+1}{x+2}) = \frac{\pi}{4}$  (March 2015)
5. Solve :  $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$  (July 2015 , July 2016 , July 2017 , July 2018 , July 2019)
6. Show that  $\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{2}{11}) + \tan^{-1}(\frac{3}{4}) = \frac{\pi}{2}$  (March 2016 )
7. Write  $\tan^{-1}(\frac{\sqrt{1+x^2}-1}{x})$ ,  $x \neq 0$  in the simplest form (March 2017 )
8. Prove that  $\cos^{-1}(\frac{4}{5}) + \cos^{-1}(\frac{12}{13}) = \cos^{-1}(\frac{33}{65})$  (March 2019)

### 3. MATRICES

#### ONE MARK QUESTIONS :

1. Define scalar matrix ( March 2014 , July 2015 , July 2016 , March 2019 )
2. Define diagonal matrix ( July 2014)
3. Define row matrix (Model Paper 2022)
4. Construct a  $2 \times 2$  matrix  $A = [a_{ij}]$  whose elements are given by  $\frac{1}{2}|-3i + j|$  (March 2015, July 2019 )
5. If  $\begin{bmatrix} x+2 & y-3 \\ 0 & 4 \end{bmatrix}$  is a scalar matrix , find  $x$  and  $y$  . (March 2016)
6. Construct a  $2 \times 2$  matrix  $A = [a_{ij}]$  whose elements are given by  $a_{ij} = \frac{i}{j}$  (March 2017 , July 2017 , March 2018 , Sept 2020 )
7. Construct a  $2 \times 2$  matrix whose elements are given by  $a_{ij} = \frac{(i+j)^2}{2}$  ( Model paper -02)
8. If a matrix has 5 elements , what are the possible orders it can have ? ( July 2018 , March 2020 , Model Paper -01 )
9. Find the values of  $x, y$  and  $z$  if  $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$  ( Model Paper 2022)

#### THREE MARKS QUESTIONS :

1. For any square matrix  $A$  with real entries , prove that  $A + A^1$  is a symmetric matrix and  $A - A^1$  is skew symmetric matrix . ( July 2014)
2. Express  $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$  as a sum of a symmetric and skew -symmetric matrix ( July 2015 )
3. If  $A$  and  $B$  are symmetric matrices of same order , then show that  $AB$  is symmetric if and only if  $AB = BA$  (March 2017 , Model Paper 02 )
4. If  $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$  then show that  $F(x)F(y) = F(x+y)$  (March 2020)
5. For a matrix  $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ , verify that (i)  $A + A^1$  is a symmetric matrix (ii)  $A - A^1$  is a skew symmetric matrix (Model Paper -01)
6. If  $A$  and  $B$  are invertible matrices of the same order , then prove that  $(AB)^{-1} = B^{-1} A^{-1}$  (March 2015 )
7. By using elementary transformation , find the inverse of the matrix  $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$  (March 2015)
8. By using elementary transformation , find the inverse of the matrix  $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$  (March 2016)
9. By using elementary transformation , find the inverse of the matrix  $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$  (July 2016, March 2018 )



10. By using elementary transformation, find the inverse of the matrix  $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

( July -2017, March 2019 , July 2019 )

11. By using elementary transformation, find the inverse of the matrix  $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$  ( July -2018 )

12. By using elementary transformation, find the inverse of the matrix  $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$  (Model Paper 2022)

#### FIVE MARKS QUESTIONS :

1. If  $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$  and  $C = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ . Calculate  $AC$ ,  $BC$  and  $(A+B)C$ . Also verify that  $(A+B)C = AC+BC$  (March 2014)

2. If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$  then prove that  $A^3 - 23A - 40I = O$

(July 2014, March 2015, March 2019 )

3. If  $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$  and  $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$  then compute  $A+B$  and  $B-C$ . Also verify that  $A+(B-C) = (A+B)-C$

(July 2015, March 2020, Model Paper -01)

4. If  $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$ ,  $B = [1 \ 3 \ -6]$ . Verify that  $(AB)' = B' A'$  (March 2016)

5. If  $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ ,  $B = [-1 \ 2 \ 1]$  verify that  $(AB)' = B' A'$  ((July 2018)

6. If  $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ ,  $B = [1 \ 5 \ 7]$ . Verify that  $(AB)' = B' A'$  (Model Paper -02 )

7. If  $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$  and  $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ . Calculate  $AC$ ,  $BC$  and  $(A+B)C$ . Also verify that  $(A+B)C = AC+BC$

(July 2016, July 2017, March 2018, July 2019, Sept 2020 )

8. If  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$  then prove that  $A^3 - 6A^2 + 7A + 2I = O$  (March 2017)

9. If  $a = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$  then verify that  $(A-B)^1 = A^1 - B^1$  and  $(A+B)^1 = A^1 + B^1$  (Model Paper -2022)

#### 4. DETERMINANTS

##### ONE MARK QUESTIONS :

1. If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ , then find  $|2A|$  (March 2014, July 2014)

2. Find the value of  $x$  if  $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$  (March 2015, July 2017, March 2019, July 2019 )

3. Find  $|3A|$ , if  $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$  (July 2015 )

4. Find the value of  $x$  if  $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

(March 2015, July 2018, March 2020, Model Paper -01, Model Paper 2022 )

5. Find the value of  $x$  if  $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$  (July 2016)

6. If  $|A| = 8$  then find  $|AA^1|$  (March 2017, Sept 2020)

7. If  $A$  is an invertible matrix of order  $2 \times 2$  then find  $|A^{-1}|$  (March 2018)

8. If  $A$  is a square matrix and  $\text{adj}(A) = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$  then find  $|A|$ . (Model Paper -02)

## TWO MARKS QUESTIONS :

- Using determinants find the equation of the line passing through the points (1,2) and (3,6)  
(March 2014, Model Paper 02 , Model Paper 2022)
- Using determinants find the equation of the line passing through the points (3,1) and (9,3) (July 2014)
- Find the value of k if area of the triangle is 4 sq. units and vertices are (-2,0) , (0,4) , (0,k)  
(March 2015, July 2015 , Sept 2020)
- Show that the points (a, b+c) , (b, c+a) and (c, a+b) are collinear by using determinant (March 2016)
- Using determinants find the area of the triangle whose vertices are (3,8) , (-4,2) and (5,1) .  
(July 2016, March 2019)
- Find the value of k if area of the triangle is 4 sq . units and vertices are (k,0) , (4,0) , (0,2)  
(March 2017)
- Using determinants find the area of the triangle whose vertices are (1,0), (6,0) , (4,3) (July 2017)
- Using determinants find the area of the triangle whose vertices are (-2,-3) , (3,2) , (-1,-8)  
(March 2018 , March 2020 , Model Paper 01)
- If the area of the triangle with vertices (2,-6) and (5,4) and (k,4) is 35 sq. units . Find the value of k using determinant method (July 2018 )
- Using determinant find the area of triangle whose vertices are (2,7) , (1,1) (10,8) . (March 2019)

## FOUR MARKS QUESTIONS :

- If  $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$  satisfies the equation  $A^2 - 5A + 7I = O$  , then find the inverse of A using this equation , where I is the identity matrix of order 2 (Model Paper -01)
- If  $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$  , satisfying the equation  $A^2 - 4A + I = O$  , where  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  and  $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$  . Find  $A^{-1}$   
(Model Paper 02 , Model Paper 2022)
- Prove that  $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$  (March 2014)
- Prove that  $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$  (July 2014 , March 2018, July 2018)
- Show that  $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$  (March 2015, July 2015)
- Prove that  $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$  (March 2016)
- Prove that  $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$  (July 2016, Model Paper 2022)
- Prove that  $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$  (March 2017)
- Prove that  $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$  (July 2017)
- Show that  $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$  (March 2019)
- Prove that  $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$  (July -2019)
- Prove that  $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$  (March 2020)

### **FIVE MARKS QUESTIONS :**

1. Solve the following system of linear equations by Matrix method  
 $x - y + z = 4$ ,  $2x + y - 3z = 0$  and  $x + y + z = 2$  (March 2014)
2. Solve the following system of linear equations by Matrix method  
 $x - y + 2z = 7$ ,  $3x + 4y - 5z = -5$  and  $2x - y + 3z = 12$  (July 2014, March 2018)
3. Solve the following system of linear equations by Matrix method  
 $2x + 3y + 3z = 5$ ,  $x - 2y + z = -4$  and  $3x - y - 2z = 3$   
(March 2015, July 2015, March 2020, Model Paper -01, Model Paper 2022)
4. Solve  $4x + 3y + 2z = 60$ ,  $2x + 4y + 6z = 90$  and  $6x + 2y + 3z = 70$  by a matrix method  
(July 2018, Model Paper -02)
5. Solve the following system of linear equations by Matrix method  
 $2x - 3y + 5z = 11$ ,  $3x + 2y - 4z = -5$  and  $x + y - 2z = -3$  (March 2016)
6. Solve the following system of linear equations by Matrix method  
 $3x - 2y + 3z = 8$ ,  $2x + y - z = 1$  and  $4x - 3y + 2z = 4$   
(July 2016, March 2019, July 2019, Sept 2020)
7. Solve the following system of linear equations by Matrix method  
 $x - y + 2z = 1$ ,  $2y - 3z = 1$  and  $3x - 2y + 4z = 2$  (March 2017)
8. Solve the following system of linear equations by Matrix method  
 $x + y + z = 6$ ,  $y + 3z = 11$  and  $x - 2y + z = 0$  (July 2017)

### **5. CONTINUITY AND DIFFERENTIABILITY**

#### **ONE MARK QUESTIONS :**

1. If  $y = \log(\sin x)$ , find  $\frac{dy}{dx}$  (March 2014)
2. Find  $\frac{dy}{dx}$ , if  $y = \cos(1 - x)$  (July 2014)
3. Find  $\frac{dy}{dx}$ , if  $y = \sin(x^2 + 5)$  (March 2015, Model Paper 2022)
4. Find  $\frac{dy}{dx}$ , if  $y = \cos(\sqrt{x})$  (July 2015, March 2017, Model Paper -02)
5. If  $y = a^{\frac{1}{2} \log(\cos x)}$ , find  $\frac{dy}{dx}$  (March 2016)
6. If  $y = \tan(2x + 3)$ , find  $\frac{dy}{dx}$  (July 2016)
7. Find  $\frac{dy}{dx}$ , if  $y = \sin(x^2)$  (July 2017, Sept 2020)
8. If  $y = e^{x^3}$ , find  $\frac{dy}{dx}$  (March 2018)
9. Find  $\frac{dy}{dx}$ , if  $y = \sin(ax + b)$  (July 2018)
10. If  $y = \sin(x^2 + 5)$ , then find  $\frac{dy}{dx}$  (March 2019)
11. If  $y = \cos^{-1}(e^x)$ , find  $\frac{dy}{dx}$  (July - 2019)
12. If  $y = \tan \sqrt{x}$ , find  $\frac{dy}{dx}$  (March 2020, Model Paper -01)
13. If  $y = e^{\cos x}$ , find  $\frac{dy}{dx}$  (Model Paper 2022)



## TWO MARKS QUESTIONS

1. If  $y + \sin y = \cos x$ , find  $\frac{dy}{dx}$  (March 2014)
2. If  $y = x^x$  find  $\frac{dy}{dx}$  (March 2014, March 2020)
3. If  $\sqrt{x} + \sqrt{y} = \sqrt{10}$ , show that  $\frac{dy}{dx} + \sqrt{\frac{y}{x}} = 0$  (July 2015)
4. Find  $\frac{dy}{dx} = (\log x)^{\cos x}$  (July 2014, March 2019)
5. Differentiate  $\left(x + \frac{1}{x}\right)^x$  w.r.t.  $x$  (March 2015)
6. Find  $\frac{dy}{dx}$ , if  $x^2 + xy + y^2 = 100$  (March 2015, March 2018, July 2019, Model Paper 2022)
7. Find  $\frac{dy}{dx}$ , if  $\log_7(\log x)$  (July 2015)
8. If  $x^y = a^x$ , prove that  $\frac{dy}{dx} = \frac{x \log a - y}{x \log x}$  (March 2016)
9. Find  $\frac{dy}{dx}$ , if  $y = x^{\sin x}$ ,  $x > 0$  (July 2016, March 2018, Sept 2020, Model Paper -02)
10. Find  $\frac{dy}{dx}$ , if  $ax + by^2 = \cos y$  (March 2017, March 2019, Sept 2020, Model Paper -02)
11. Differentiate  $(\sin x)^x$  w.r.t.  $x$  (July 2017)
12. Find  $\frac{dy}{dx}$ , if  $2x + 3y = \sin y$  (July 2017)
13. Differentiate  $(\sin x)^{\cos x}$  w.r.t.  $x$  (July 2018)
14. Find  $\frac{dy}{dx}$ , if  $\sin^2 x + \cos^2 y = 1$ , (March 2020, Model Paper -01)
15. Find  $\frac{dy}{dx}$ , if  $y = \cos(\log x + e^x)$  (Model Paper -01)
16. If  $y = x^3 + \tan x$ , then find  $\frac{d^2 y}{dx^2}$  (Model Paper -01)
17. Differentiate  $\cos^{-1}(\sin x)$  w.r.t.  $x$  (Model Paper -02)
18. Find  $\frac{dy}{dx}$ , if  $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$ ,  $0 < x < \frac{1}{\sqrt{2}}$  (July 2015, March 2016, July 2018)
19. Find  $\frac{dy}{dx}$ , if  $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ ,  $0 < x < 1$  (July 2016)
20. If  $x = at^2$  and  $y = 2at$  then find  $\frac{dy}{dx}$  (Model Paper 2022)
21. Differentiate  $\cos(\cos(x^2))$  w.r.t.  $x$  (Model Paper -2022)

## THREE MARKS QUESTIONS

1. If  $x = a(\cos \theta + \theta \sin \theta)$  and  $y = a(\sin \theta - \theta \cos \theta)$  then prove that  $\frac{dy}{dx} = \tan \theta$  (March 2014, Model Paper 2022)
2. If  $x = a(\theta + \sin \theta)$  and  $y = a(1 - \cos \theta)$ , prove that  $\frac{dy}{dx} = \tan\left(\frac{\theta}{2}\right)$  (July 2014, March 2019, July -2019, Model Paper -02)
3. If  $x = \sqrt{a^{\sin^{-1} t}}$  and  $y = \sqrt{a^{\cos^{-1} t}}$  show that  $\frac{dy}{dx} = -\frac{y}{x}$  (March 2015)
4. If  $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$ ,  $y = a \sin t$ , find  $\frac{dy}{dx}$  (July 2015, July 2017)

5. If  $x = a \cos^3 \theta$  and  $y = a \sin^3 \theta$ , prove that  $\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$  (March 2016)
6. If  $x = a(\theta - \sin \theta)$ ,  $y = a(1 + \cos \theta)$  then prove that  $\frac{dy}{dx} = -\cot\left(\frac{\theta}{2}\right)$  (July 2016, July 2018)
7. Find  $\frac{dy}{dx}$ , if  $y = (\log x)^{\cos x}$  (March 2017)
8. Differentiate  $\sin^2 x$  w.r.t.  $e^{\cos x}$  (March 2017)
9. If  $x = \sin t$ ,  $y = \cos 2t$  then prove that  $\frac{dy}{dx} = -4 \sin t$  (March 2018)
10. If  $x = 2at^2$ ,  $y = at^4$  then find  $\frac{dy}{dx}$  (March 2020, Model Paper -01)
11. Find  $\frac{dy}{dx}$ , if  $xy = e^{x-y}$  (Sept 2020, Model Paper -2022)
12. Find  $\frac{dy}{dx}$ , if  $x^y = y^x$  (Model Paper -01)
13. If  $x\sqrt{1+y} + y\sqrt{1+x} = 0$ , for  $-1 < x < 1$  and  $x \neq y$ , prove that  $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$  (Model Paper -02)
14. Verify Rolle's theorem for the function  $y = x^2 + 2$ ,  $[-2, 2]$  (March 2014, March 2018)
15. Verify Rolle's theorem for the function  $f(x) = x^2 + 2x - 8$ ,  $x \in [-4, 2]$  (March 2015, March 2017, March 2019)
16. Verify Mean Value Theorem, if  $f(x) = x^2 - 4x - 3$  in the interval  $[a, b]$ , where  $a = 1$  and  $b = 4$  (July 2014, July 2015, July 2018, July 2019, March 2020, Model Paper 2022)
17. Verify Mean Value Theorem, if  $f(x) = x^3 - 5x^2 - 3x$  in the interval  $[1, 3]$  (March 2016)
18. Verify Mean Value Theorem, if  $f(x) = x^2$  in the interval  $[2, 4]$  (July 2016, July 2017)

### FIVE MARKS QUESTIONS

1. If  $y = 3e^{2x} + 2e^{3x}$  then prove that  $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$  (March 2014)
2. If  $y = 3\cos(\log x) + 4\sin(\log x)$  then prove that  $x^2 y_2 + xy_1 + y = 0$  (July 2014, July 2016, July 2017, July 2019, Model Paper 2022)
3. If  $y = Ae^{mx} + Be^{nx}$  then prove that  $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$  (March 2015, July 2018, Sept 2020)
4. If  $y = (\tan^{-1} x)^2$  then prove that  $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$  (July 2015, March 2017, March 2018, March 2020, Model Paper -01)
5. If  $y = (\sin^{-1} x)^2$  then prove that  $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$  (March 2016)
6. If  $y = \sin^{-1} x$  then prove that  $(1 - x^2)y_2 - xy_1 = 0$  (March 2019)
7. If  $y = e^{a \cos^{-1} x}$ ,  $-1 \leq x \leq 1$ , then prove that  $(1 - x^2)y_2 - xy_1 - a^2 y = 0$  (Model Paper -02)

### FOUR MARKS QUESTIONS :

1. Find  $k$ ,  $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases}$  is continuous at  $x = \frac{\pi}{2}$  (March 2014, July 2014, March 2017, July 2019, Sept 2020)
2. Find  $k$ , if  $f(x) = \begin{cases} kx + 1 & \text{if } x \leq 5 \\ 3x - 5 & \text{if } x > 5 \end{cases}$  is continuous at  $x = 5$  (March 2015, March 2019, Model Paper 2022)
3. Find the value of 'a' and 'b' such that the function  $f(x) = \begin{cases} 5 & \text{if } x \geq 2 \\ ax + b & \text{if } 2 < x < 10 \\ 21 & \text{if } x \geq 10 \end{cases}$  is continuous function (July 2015, Model Paper -02).

4. Find  $k$ , if  $f(x) = \begin{cases} \frac{1-\cos 2x}{1-\cos x}, & x \neq 0 \\ k, & x = 0 \end{cases}$  is continuous at  $x = 0$  (**March 2016**)
5. Find  $k$ , if  $f(x) = \begin{cases} kx^2 & \text{if } x \leq 2 \\ 3 & \text{if } x > 2 \end{cases}$  is continuous at  $x = 2$  (**July 2016, July 2018**)
6. For what value of  $\lambda$  is the function defined by  $f(x) = \begin{cases} \lambda(x^2 - 2x) & \text{if } x \leq 0 \\ 4x + 1 & \text{if } x > 0 \end{cases}$  continuous at  $x = 0$ ? (**July 2017**)
7. Find the relation between 'a' and 'b' so that the function  $f$  defined by  $f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 1, & \text{if } x > 3 \end{cases}$  is continuous at  $x = 3$  (**March - 2018**)
8. Find  $k$ , if  $f(x) = \begin{cases} kx + 1 & \text{if } x \leq \pi \\ \cos x & \text{if } x > \pi \end{cases}$  is continuous at  $x = \pi$  (**March 2020, Model Paper -01**)

## 6. APPLICATION OF DERIVATIVE :

1. Find the approximate change in the volume  $V$  of a cube of side  $x$  meters caused by increasing the side by 2 % (**March 2014, March 2019**)
2. Approximate  $\sqrt{36.6}$  by using differential (**July 2014, March 2020**)
3. Using differentials, find the approximate value of  $\sqrt{49.5}$  (**July 2015**)
4. Using differentials find the approximate value of  $(25)^{\frac{1}{3}}$  (**March 2016, July 2016**)
5. Find the approximate change in the volume of a cube of side  $x$  meters caused by increasing the side by 3 % (**March 2017**)
6. If the radius of a sphere is measured as 7 cm with an error of 0.02 m, then find the approximate error in calculating its volume. (**July 2018**)
7. Find the slope of the tangent to the curve  $y = \frac{x-1}{x-2}$ ,  $x \neq 2$  at  $x = 10$  (**March 2015**)
8. Find the interval in which the function is given by  $f(x) = 2x^2 - 3x$  is strictly increasing (**July 2016, July 2019**)
9. Find the points on the curve  $\frac{x^2}{4} + \frac{y^2}{25} = 1$  at which the tangents are parallel to  $x$ -axis (**July 2017**)
10. Find the slope of the tangent to the curve  $y = x^3 - x$  at  $x = 2$  (**March 2018, Model Paper -01**)
11. Find the slope of the tangent to the curve  $y = x^3 - x + 1$  at the point  $x$  co-ordinate is 2 (**Model Paper 2022**)
12. Find the interval in which the function  $f$  given by  $f(x) = x^2 - 4x + 6$  is strictly decreasing (**March 2020**)
13. Find the local maximum value of the function  $g(x) = x^2 - 3x$  (**Model Paper -02**)

### THREE MARKS QUESTIONS :

1. Find the intervals in which the function  $f$  given by  $f(x) = x^2 - 4x + 6$  is i) strictly increasing ii) strictly decreasing (**March 2014, Model Paper -01**)
2. Find two positive numbers  $x$  and  $y$  such that  $x + y = 60$  and  $xy^3$  is maximum (**July 2014, March 2017**)
3. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum. (**March 2015, July 2017, July 2018**)
4. Find two numbers whose sum is 24 and whose product is as large as possible (**July 2015, March 2018, Sept 2020**)
5. Find two numbers whose product is 100 and whose sum is minimum (**March 2016**)
6. Find the intervals in which the function  $f$  given by  $f(x) = 2x^3 - 3x^2 - 36x + 7$  is strictly increasing and strictly decreasing. (**March 2019, Model Paper 2022**)
7. Find a point at which the tangent to the curve  $y = \sqrt{4x - 3} - 1$  has its slope  $\frac{2}{3}$  (**July 2019, Model paper -02**)



### FIVE MARKS QUESTIONS :

1. A ladder 24 ft long is leaning against a wall . The bottom of the ladder is pulled along the ground away from the wall , at the rate of 3ft/s . How fast is its height on the wall decreasing when the foot of the ladder is 8ft away from the wall ? **(March 2014 )**
2. A ladder 5m long is leaning against a wall . The bottom of the ladder is pulled along the ground , away from the wall , at the rate of 2cm/ s . How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ? **(July 2016)**
3. The length  $x$  of a rectangle is decreasing at the rate of 5cm /minute and the width  $y$  is increasing at the rate of 4 cm/minute . When  $x = 8$ cm and  $y = 6$  cm , find the rates of change of (a) the perimeter , and (b) the area of the rectangle **(July 2014, March 2017 , Model Paper -2022 )**
4. The length  $x$  of a rectangle is decreasing at the rate of 3 cm /minute and the width  $y$  is increasing at the rate of 2 cm/minute . When  $x = 10$  cm and  $y = 6$  cm , find the rates of change of (a) the perimeter , and (b) the area of the rectangle **(March 2019, March 2016 , Sept 2020, Model Paper -01 )**
5. A particle moves along the curve  $6y = x^3 + 2$  . Find the points on the curve at which  $y$  - coordinate is changing 8 times as fast as  $x$  - coordinate . **(March 2015 , July 2018 , Model Paper -02)**
6. Sand is pouring from a pipe at the rate of  $12 \text{ cm}^3/\text{s}$  . The falling sand forms a cone on the ground in such a way that the height of the cone is always one -sixth of the radius of the base . How fast is the height of the sand cone increasing when the height is 4 cm ?  
**(July 2015 , July 2017 , March 2018 , July 2019 , March 2020 )**

## 7. INTEGRATION

### ONE MARK QUESTIONS :

1. Evaluate :  $\int (\sin x + \cos x) dx$  **(March 2014)**
2. Evaluate :  $\int (2x - 3 \cos x + e^x) dx$  **(July 2014)**
3. Evaluate :  $\int e^x \left( \frac{x-1}{x^2} \right) dx$  **(March 2015)**
4. Evaluate :  $\int \sec x (\sec x + \tan x) dx$  **(July 2015 , July 2018 )**
5. Evaluate :  $\int \operatorname{cosec} x (\operatorname{cosec} x + \cot x) dx$  **(March 2016)**
6. Evaluate :  $\int (2x^2 + e^x) dx$  **(July 2016 , March 2020 , Model Paper -01 , Model Paper 2022 )**
7. Find  $\int \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$  **(March 2017 , Sept 2020 )**
8. Find  $\int \cos 3x dx$  **(July 2017)**
9. Find  $\int \left( \frac{x^3-1}{x^2} \right) dx$  **(March 2018)**
10. Find  $\int (1-x)\sqrt{x} dx$  **(March 2019)**
11. Find  $\int \sec^2(7-4x) dx$  **(July 2019)**
12. Evaluate :  $\int \sqrt{ax+b} dx$  **(Model Paper -02)**
13. Evaluate :  $\int_2^3 \frac{1}{x} dx$  **(Model Paper -2022)**

## TWO MARKS QUESTIONS

1. Evaluate :  $\int \frac{\sin^2 x}{1+\cos x} dx$  (March 2014)
2. Evaluate :  $\int_1^e \frac{1}{x} dx$  (March 2014)
3. Integrate  $\sin x \sin(\cos x)$  with respect to  $x$  (July 2014)
4. Evaluate :  $\int_0^1 \frac{1}{1+x^2} dx$  (July 2014)
5. Evaluate :  $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$  (March 2015)
6. Evaluate :  $\int \left( \frac{1}{x - \sqrt{x}} \right) dx$  (March 2015, Sept 2020)
7. Evaluate :  $\int \frac{x^2}{1-x^6} dx$  (July 2015)
8. Evaluate :  $\int e^x \left( \frac{1}{x} - \frac{1}{x^2} \right) dx$  (July 2015)
9. Evaluate :  $\int \left( \frac{1}{\sin x \cos^3 x} \right) dx$  (March 2016)
10. Evaluate :  $\int_0^\pi \left[ \sin^2 \left( \frac{x}{2} \right) - \cos^2 \left( \frac{x}{2} \right) \right] dx$  (March 2016, Model Paper -01)
11. Find  $\int x^2 \log x dx$  (July 2016)
12. Evaluate :  $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$  (July 2016)
13. Integrate  $\frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}}$  with respect to  $x$  (March 2017)
14. Evaluate :  $\int_0^3 \frac{1}{4+9x^2} dx$  (March 2017)
15. Evaluate :  $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$  (July 2017)
16. Evaluate :  $\int \left( \frac{x-3}{x-1} \right) e^x dx$  (July 2017)
17. Integrate  $\frac{e^{\tan^{-1} x}}{1+x^2}$  with respect to  $x$  (March 2018)
18. Evaluate :  $\int_2^3 \frac{x}{1+x^2} dx$  (March 2018)
19. Evaluate :  $\int \cos 6x \sqrt{1 + \sin 6x} dx$  (July 2018)
20. Evaluate :  $\int \left( \frac{x e^x}{(x+1)^2} \right) dx$  (July 2018)
21. Find  $\int \frac{1}{\cos^2 x (1 - \tan x)^2} dx$  (March 2019)
22. Find  $\int \sin 2x \cdot \cos 3x dx$  (March 2019)
23. Integrate  $x \sec^2 x$  with respect to  $x$  (July 2019, March 2020, Model Paper -1)
24. Find  $\int \frac{(x^4 - x)^{\frac{1}{4}}}{x^5} dx$  (July 2019)
25. Find  $\int \cot x \log \sin x dx$  (March 2020)
26. Find  $\int e^x \sec x (1 + \tan x) dx$  (Sept 2020)
27. Evaluate :  $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$  (Model Paper -02, Model Paper 2022)
28. Evaluate :  $\int \log_e x dx$  (Model Paper -02)
29. Evaluate :  $\int \left( \frac{(\log x)^2}{x} \right) dx$  (Model Paper -2022)

### THREE MARKS QUESTIONS :

1. Evaluate :  $\int \left( \frac{x^2+1}{(x+1)^2} \right) e^x dx$  (March 2014)
2. Find  $\int \frac{x e^x}{(1+x)^2} dx$  (Model Paper 2022)
3. Evaluate :  $\int \tan^{-1} x dx$  (March 2014)
4. Evaluate :  $\int \sin 3x \cdot \cos 4x dx$  (July 2014 Sept 2020)
5. Integrate  $x^2 e^x$  with respect to  $x$  (July 2014)
6. Evaluate :  $\int x \tan^{-1} x dx$  (March 2015)
7. Evaluate :  $\int \frac{x}{(x+1)(x+2)} dx$  (July 2015, July 2016, July 2017, March 2018, July 2019, Model Paper -02)
8. Evaluate :  $\int \frac{dx}{(x+1)(x+2)}$  (Model Paper -01, Model Paper 2022)
9. Evaluate :  $\int \frac{x}{(x-1)(x-2)} dx$  (March 2016)
10. Integrate  $\frac{2x}{(x^2+1)(x^2+2)}$  with respect to  $x$  (March 2016)
11. Find  $\int e^x \left( \frac{1+\sin x}{1+\cos x} \right) dx$  (July 2016)
12. Evaluate :  $\int \frac{2x}{x^2+3x+2} dx$  (March 2017)
13. Evaluate :  $\int e^x \sin x dx$  (March 2017, March 2018)
14. Evaluate :  $\int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$  (July 2017)
15. Evaluate :  $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$  (July 2018)
16. Find  $\int \frac{dx}{x(x^2+1)}$  (July 2018)
17. Find  $\int x \log x dx$  (March 2019, Sept 2020)
18. Evaluate :  $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$  (March 2019)
19. Find  $\int \frac{x-3}{(x-1)^3} dx$  (March 2020, Model Paper -01)
20. Evaluate :  $\int_0^{\frac{\pi}{2}} \cos^2 x dx$  (March 2020, Model Paper -2022)
21. Evaluate :  $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$  (Model Paper -01)
22. Evaluate :  $\int \tan^4 x dx$  (Model Paper -01)
23. Evaluate :  $\int_0^{\frac{\pi}{4}} \sin 2x dx$  (Model Paper -02)
24. Evaluate :  $\int_0^2 e^x dx$  as the limit of a sum (March 2015, July 2015, Model Paper -2022)
25. Evaluate :  $\int_0^2 (x^2 + 1) dx$  as a limit of a sum (July 2019)

### FIVE MARKS QUESTIONS :

1. Find  $\int \frac{dx}{x^2-a^2}$  and hence evaluate  $\int \frac{dx}{3x^2+13x-10}$ ,  $\int \frac{dx}{4x^2-9}$  (March 2014, Model Paper -02)
2. Find the integral of  $\frac{1}{x^2-a^2}$  and hence evaluate  $\int \frac{dx}{x^2-3^2}$  (March 2019)
3. Find  $\int \frac{1}{\sqrt{x^2+a^2}} dx$  and hence evaluate  $\int \frac{1}{\sqrt{x^2+7}} dx$  (June 2014)
4. Find the integral of  $\frac{1}{\sqrt{x^2-a^2}}$  with respect to  $x$  and hence evaluate  $\int \frac{1}{\sqrt{x^2+6x-7}} dx$  (March 2015)
5. Find the integral of  $\frac{1}{x^2+a^2}$  w.r.t.  $x$  and hence evaluate  $\int \frac{1}{3+2x+x^2} dx$  (March 2016, March 2020, Model Paper 2022)



6. Find the integral of  $\frac{1}{x^2+a^2}$  w.r.t.  $x$  and hence evaluate  $\int \frac{1}{x^2-6x+13} dx$  (March 2018)
7. Find the integral of  $\frac{1}{\sqrt{a^2-x^2}}$  w.r.t.  $x$  and hence evaluate  $\int \frac{dx}{\sqrt{5-4x-x^2}}, \int \frac{1}{\sqrt{9-25x^2}} dx$  (July 2018, Model Paper -01)
8. Find the integral of  $\sqrt{x^2+a^2}$  w.r.t.  $x$  and hence evaluate  $\int \sqrt{x^2+4x+6} dx, \int \sqrt{1+x^2} dx, \int \sqrt{x^2+2x+5} dx$  (July 2015, July 2017, July 2019)
9. Find the integral of  $\sqrt{a^2-x^2}$  w.r.t.  $x$  and hence evaluate  $\int \sqrt{5-x^2+2x} dx$  (July 2016)
10. Find the integral of  $\sqrt{x^2-a^2}$  w.r.t.  $x$  and hence evaluate  $\int \sqrt{x^2-8x+7} dx$  (March 2017)

### SIX MARKS QUESTIONS :

1. Prove that  $\int_0^a f(x) dx = \int_0^a f(a-x) dx$  and hence evaluate (i)  $\int_0^{\frac{\pi}{4}} \log(1+\tan x) dx$   
(ii)  $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$  (iii)  $\int_0^a \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}} dx$  (iv)  $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx$  (v)  $\int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x) dx$   
(March 2014, July 2016, March 2018, March 2019, Sept 2020, Model Paper -01, Model Paper 2022)
2. Prove that  $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$  and hence evaluate  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}} dx$   
(July 2014, July 2015, March 2017, July 2019 Model Paper -02)
3. Prove that  $\int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx & \text{if } f(x) \text{ is even} \\ 0 & \text{if } f(x) \text{ is odd} \end{cases}$  and hence evaluate (i)  $\int_{-1}^1 \sin^5 x \cos^4 x dx$   
(ii)  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^9 x dx$  (March 2015, July 2018, July 2017, March 2020)
4. Prove that  $\int_0^{2a} f(x) dx = \begin{cases} 2 \int_0^a f(x) dx & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$  and hence evaluate  $\int_0^{2\pi} \cos^5 x dx$   
(March 2016)

## 8. APPLICATION OF INTEGRATION:

### THREE MARKS QUESTIONS

1. Find the area of the region bounded by the curve  $y = x^2$  and the line  $y = 4$  (March 2014, March 2018, Sept 2020)
2. Find the area of the region by the curve  $y = x^2$  and the line  $y = 2$  (Model Paper -02)
3. Find the area of the region bounded by the curve  $y^2 = x$  and the line  $x = 1, x = 4$  and the  $x$ -axis in the first quadrant. (July 2014)
4. Find the area of the region bounded by the curve  $y^2 = 4x$  and the line  $x = 3$  (March 2015, March 2019)
5. Find the area of the region bounded by the curve  $y^2 = 4x$  and the line  $y = 2x$ . (July 2015, March 2019)
6. Find the area of the region bounded by  $y^2 = 9x, x = 2, x = 4$  and the  $x$ -axis in the first quadrant. (July 2016, July 2019, Model Paper -2022)
7. Find the area of the region bounded by the curve  $y^2 = 4x, y$ -axis and the line  $y = 3$  (March 2017)
8. Find the area of the region bounded by the curve  $y = \cos x$  between  $x = 0$  and  $x = 2\pi$  (July 2017)
9. Find the area of the parabola  $y^2 = 4ax$  bounded by its latus rectum. (July 2018)
10. Find the area of the region bounded by the curve  $x^2 = 4y, y = 2, y = 4$  and the  $y$ -axis in the first quadrant (March 2020, Model Paper -01)

### **FIVE MARKS QUESTIONS :**

1. Find the area of the region bounded by two parabolas  $y = x^2$  and  $y^2 = x$ . **(March 2014)**
2. Find the area of the region enclosed between two circles  $x^2 + y^2 = 4$  and  $(x - 2)^2 + y^2 = 4$  **(July 2014)**
3. Using integration find the area of triangular region whose sides have the equations  $y = 2x + 1$ ,  $y = 3x + 1$  and  $x = 4$  **(March 2015, March 2017)**
4. Find the area of the region bounded by the curves  $(x - 1)^2 + y^2 = 1$  and  $x^2 + y^2 = 1$  using integration method. **(July 2015)**
5. Using integration find the area of the region bounded by the triangle whose vertices are  $(1,0)$ ,  $(2,2)$  and  $(3,1)$ . **(March 2016, March 2018)**
6. Find the smaller area enclosed by the circle  $x^2 + y^2 = 4$  and the line  $x + y = 2$  **(July 2017, March 2019)**
7. Using integration find the area of the region in the first quadrant enclosed by the  $x$ -axis, the line  $y = x$  and the circle  $x^2 + y^2 = 32$  **(July 2019)**
8. Using method of integration, find the area enclosed by the circle  $x^2 + y^2 = a^2$  **(Model Paper -02, Model Paper 2022)**
9. Find the area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $a > b$ ) by method of integration and hence find the area of an ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$  **(July 2016, July 2018, Model Paper -02)**
10. Using the method of integration, find the area of the smaller region bounded by the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and the line  $\frac{x}{3} + \frac{y}{2} = 1$  **(March 2020)**
11. Find the area enclosed by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the ordinates  $x = 0$  and  $x = ae$ , where  $b^2 = a^2(1 - e^2)$  and  $e < 1$ , using integration **(Expected)**
12. Find the area of the smaller part of the circle  $x^2 + y^2 = a^2$  cut off by the line  $x = \frac{a}{\sqrt{2}}$  **(Expected)**
13. Find the area of the region bounded by the parabola  $y = x^2$  and  $y = |x|$  **(Expected)**

### **9. DIFFERENTIAL EQUATIONS :**

#### **TWO MARKS QUESTIONS :**

1. Find the order and degree of the differential equation  $xy \left( \frac{d^2y}{dx^2} \right) + x \left( \frac{dy}{dx} \right)^2 - y \frac{dy}{dx} = 0$  **(March 2014)**
2. Find the order and degree of the differential equation  $\left( \frac{d^3y}{dx^3} \right)^3 + \left( \frac{d^2y}{dx^2} \right)^3 + \frac{dy}{dx} + y = 0$  **(July 2014)**
3. Find the order and degree of the differential equation  $\left( \frac{d^2y}{dx^2} \right)^3 + \left( \frac{dy}{dx} \right)^2 + \sin \left( \frac{dy}{dx} \right) + 1 = 0$  **(March 2015, March 2020)**
4. Find the order and degree of the differential equation  $\frac{d^2y}{dx^2} + \cos \left( \frac{dy}{dx} \right) = 0$  **(July 2015, March 2019)**
5. Find the order and degree of the differential equation  $\frac{d^4y}{dx^4} + \sin \left( \frac{d^3y}{dx^3} \right) = 0$  **(March 2016, July 2017)**
6. Find the order and degree of the differential equation  $\frac{d^3y}{dx^3} + 2 \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$  **(July 2016)**
7. Find the order and degree of the differential equation  $\left( \frac{dy}{dx} \right)^2 + \frac{dy}{dx} - \sin^2 y = 0$  **(March 2017)**
8. Find the order and degree of the differential equation  $\left( \frac{d^3y}{dx^3} \right)^3 + \left( \frac{d^2y}{dx^2} \right)^3 + \left( \frac{dy}{dx} \right)^4 + y^5 = 0$  **(March 2018)**
9. Find the order and degree of the differential equation  $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$  **(July 2018)**
10. Find the order and degree of the differential equation  $y''' + y^2 + e^{y'} = 0$  **(July 2019)**
11. Find the order and degree of the differential equation  $y''' + y'' + y' = 0$  **(Model Paper -01)**
12. Find the order and degree of the differential equation  $y' + y = e^x$  **(Model Paper 2022)**

### THREE MARKS QUESTIONS :

1. Form the differential equation representing the family of curves  $y = mx$ , where  $m$  is arbitrary constant. (March 2014)
2. Form the differential equation, if the family of circles touching the  $x$ -axis at the origin. (July 2014)
3. Form the differential equation representing the family of curves  $y = a \sin(x + b)$ , where  $a$  and  $b$  are arbitrary constants. (July 2015, March 2018, July 2018, Model Paper 2022)
4. Form the differential equation of the family of curves  $y = a e^{3x} + b e^{-2x}$  by eliminating arbitrary constants  $a$  and  $b$ . (March 2019, July 2019)
5. Form the differential equation representing the family of curves  $\frac{x}{a} + \frac{y}{b} = 1$ , where  $a$  and  $b$  are arbitrary constants. (July 2016)
6. Form the differential equation of the family of circles having the centre on  $y$ -axis and radius 3 units. (March 2017)
7. Find the equation of the curve passing through the point (1,1), given that the slope of the tangent to the curve at any point is  $\frac{x}{y}$ . (March 2017)
8. Find the equation of the curve passing through the point (-2,3), given that the slope of the tangent to the curve at any point is  $\frac{2x}{y^2}$ . (July 2017, March 2020)
9. Solve the differential equation  $\frac{dy}{dx} = e^{x+y}$  (Model Paper -01)
10. Solve:  $y \log y \, dx - x \, dy = 0$  (Model Paper -02)
11. Find the general solution of the differential equation  $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$  (Model Paper 2022)

### FIVE MARKS QUESTIONS :

1. Solve differential equations,  $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$  (July 2014)
2. Solve the differential equation,  $\frac{dy}{dx} + (\sec x)y = \tan x$  (March 2015, March 2019)
3. Find the general solution of differential equation,  $x \frac{dy}{dx} + 2y = x^2$   
(July 2016, July 2018, Model Paper -01, Model Paper 2022)
4. Find the general solution of differential equation  $\cos^2 x \frac{dy}{dx} + y = \tan x$  (March 2017, Sept 2020)
5. Find the general solution of the differential equation,  $x \frac{dy}{dx} + 2y = x^2 \log x$   
(March 2017, March 2020, Model Paper -01)
6. Find the general solution of the differential equation  $e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$   
(March 2014)
7. Solve the differential equation  $y \, dx + (x - ye^y) \, dy = 0$  (March 2017)
8. Find the general solution of the differential equation  $y \, dx - (x + 2y^2) \, dy = 0$  (July 2017)
9. Find the general solution of the differential equation  $(x + y) \frac{dy}{dx} = 1$  (July 2019)



## 10. VECTOR ALGEBRA :

### ONE MARK QUESTIONS

1. Find the direction cosines of the vector  $\hat{i} + 2\hat{j} + 3\hat{k}$  (March 2014)
2. Define unit vector . (July 2014)
3. Define negative of a vector . (March 2015, July 2018, March 2020, Model Paper -01)
4. Show that  $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$  and  $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$  are collinear (July 2015)
5. If vectors  $\vec{AB} = 2\hat{i} - \hat{j} + \hat{k}$  and  $\vec{OB} = 3\hat{i} - 4\hat{j} + 4\hat{k}$ , find the position vector  $\vec{OA}$  (March 2016)
6. Find a unit vector in the direction of vector  $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$  (July 2016)
7. Define collinear vectors . (March 2017)
8. Find a unit vector in the direction of vector  $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$  (July 2017, March 2018)
9. Find a value of  $x$  for which  $x(\hat{i} + \hat{j} + \hat{k})$  is a unit vector . (March 2019)
10. If  $\vec{a} = \frac{1}{\sqrt{14}}(2\hat{i} + 3\hat{j} + \hat{k})$ , then find the direction cosines of  $\vec{a}$ . (July 2019)
11. If the vectors  $2\hat{i} + 3\hat{j} - 6\hat{k}$  and  $4\hat{i} - m\hat{j} - 12\hat{k}$  are parallel, find the value of  $m$ . (Sept 2020)
12. Find the vector components of the vector with initial point (2,1) and terminal point (-5,7). (Model Paper -02)
13. Find a unit vector in the direction of vector  $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ . (Model Paper 2022)
14. Write the two different vectors have same magnitude . (Model Paper 2022)

### TWO MARKS QUESTIONS :

1. If  $\vec{a}$  is a unit vector and  $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$  then find  $|\vec{x}|$ . (March 2014, July 2017, July 2019)
2. Find the area of the parallelogram whose adjacent sides are  $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$  and  $\vec{b} = \hat{i} - \hat{j} + \hat{k}$  (March 2014, March 2018, July 2019, Sept 2020)
3. Find the area of the parallelogram whose adjacent sides are  $\vec{a} = \hat{i} + \hat{j} - \hat{k}$  and  $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ . (July 2014, July 2017)
4. Obtain the projection of the vectors  $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$  on the vector  $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$  (July 2014)
5. Find  $|\vec{a}|$  and  $|\vec{b}|$ , if  $(\vec{x} + \vec{a}) \cdot (\vec{x} - \vec{a}) = 8$  and  $|\vec{a}| = 8|\vec{b}|$  (March 2015, March 2019, Sept 2020)
6. Find the area of the parallelogram whose adjacent side are  $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$  and  $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$  (March 2015, July 2016, March 2017, July 2018, March 2020, Model Paper -01, Model Paper 2020)
7. Find the projection of  $\hat{i} + 3\hat{j} + 7\hat{k}$  on  $7\hat{i} - \hat{j} + 8\hat{k}$  (July 2015, March 2018, March 2020, Model Paper -01)
8. Find the projection of  $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$  on  $\vec{b} = 2\hat{i} + 3\hat{j} + 2\hat{k}$  (July 2018, Model Paper 2022)
9. If two vectors  $\vec{a}$  and  $\vec{b}$  such that  $|\vec{a}| = 3$ ,  $|\vec{b}| = \frac{\sqrt{2}}{3}$  and  $|\vec{a} \times \vec{b}| = 1$ . Find the angle between  $\vec{a}$  and  $\vec{b}$  (July 2015)
10. If  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$  then prove that  $\vec{a}$  and  $\vec{b}$  are perpendicular . (March 2016)
11. If two vectors  $\vec{a}$  and  $\vec{b}$  such that  $|\vec{a}| = 2$ ,  $|\vec{b}| = 3$  and  $\vec{a} \cdot \vec{b} = 4$  find  $|\vec{a} - \vec{b}|$  (March 2016)
12. Find the angle between the following vectors  $\vec{a} = \hat{i} + \hat{j} - \hat{k}$  and  $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ . (July 2016)
13. Find the position vector of the point R which divides the join of the points P and Q with position vectors are  $\hat{i} + 2\hat{j} - \hat{k}$  and  $-\hat{i} + \hat{j} + \hat{k}$  in the ratio 2:1 internally and externally (March 2017)
14. Find a vector in the direction of a vector  $\vec{a} = \hat{i} - 2\hat{j}$  that has magnitude 7 units (Model Paper -02)
15. Show that the vector  $\hat{i} + \hat{j} + \hat{k}$  is equally inclined to the positive direction of the axes . (Model Paper -02)

### THREE MARKS QUESTIONS :

1. If  $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j}$  such that  $\vec{a} + \lambda \vec{b}$  is perpendicular to  $\vec{c}$  then find  $\lambda$  (March 2014)
2. If two vectors  $\vec{a}$  and  $\vec{b}$  such that  $|\vec{a}| = 2$ ,  $|\vec{b}| = 3$  and  $\vec{a} \cdot \vec{b} = 4$  find  $|\vec{a} - \vec{b}|$  (July 2014)
3. Find a unit vector perpendicular to each of the vector  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  where  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$  (July 2014, July 2015, March 2019, Model Paper -02)
4. Show that the position vector of a point P, which divides the line joining the points A and B having position vectors  $\vec{a}$  and  $\vec{b}$  internally in the ratio  $m:n$  is  $\frac{m\vec{b} + n\vec{a}}{m+n}$  (March 2015, July 2016, July 2017, March 2018, July 2019, Sept 2019, Sept 202 Model Paper -02)
5. Find the sine of the angle between the vectors  $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$  and  $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$  (March 2016)
6. If three vectors  $\vec{a}, \vec{b}, \vec{c}$  are such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$  then evaluate  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$  if  $|\vec{a}| = 1$ ,  $|\vec{b}| = 4$ ,  $|\vec{c}| = 2$  (March 2017)
7. If three vectors  $\vec{a}, \vec{b}, \vec{c}$  are unit vectors such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$  then find  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$  (Model Paper -2022)
8. Find a unit vector perpendicular to each of the vector  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  where  $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$  (July 2018, March 2020)
9. If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are three vectors such that  $|\vec{a}| = 3$ ,  $|\vec{b}| = 4$ ,  $|\vec{c}| = 5$  and each vector is perpendicular to sum of the other two vectors then find  $|\vec{a} + \vec{b} + \vec{c}|$ . (Model Paper -01)
10. Show that the points A( $-2\hat{i} + 3\hat{j} + 5\hat{k}$ ), B( $\hat{i} + 2\hat{j} + 3\hat{k}$ ) and C( $7\hat{i} - \hat{k}$ ) are collinear (Model Paper -01)
11. Prove that  $[\vec{a}, \vec{b}, \vec{c} + \vec{d}] = [\vec{a}, \vec{b}, \vec{c}] + [\vec{a}, \vec{b}, \vec{d}]$  (March 2014)
12. Prove that  $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2 [\vec{a}, \vec{b}, \vec{c}]$  (July 2016, July 2018)
13. Show that the four points with position  $4\hat{i} + 8\hat{j} + 12\hat{k}$ ,  $2\hat{i} + 4\hat{j} + 6\hat{k}$ ,  $3\hat{i} + 5\hat{j} + 4\hat{k}$  and  $5\hat{i} + 8\hat{j} + 5\hat{k}$  are coplanar (March 2015, March 2019, July 2019)
14. Find  $\lambda$ , if  $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$ ,  $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$  and  $\vec{c} = \lambda\hat{i} + 7\hat{j} + 3\hat{k}$  are co planarity (July 2015)
15. Find  $x$ , such that the four points A(3,2,1), B(4,  $x$ , 5), C(4,2,-2) and D(6,5,-1) are co planar. (March 2017, July 2017, March 2018, March 2020, Model Paper 2022)
16. For any three vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$  prove that  $\vec{a} - \vec{b}$ ,  $\vec{b} - \vec{c}$  and  $\vec{c} - \vec{a}$  are coplanar (March 2016)

## 11. THREE DIMENSIONAL GEOMETRY :

### ONE MARK QUESTIONS :

1. Find the equation of the plane with intercept 4 on  $z$  - axis and parallel to  $XOY$  plane . (March 2014)
2. If a line makes angle  $90^\circ$  ,  $60^\circ$  and  $30^\circ$  with positive direction of  $x$  ,  $y$  and  $z$  axis respectively . Find its direction cosines . (July 2014, March 2018, March 2020)
3. Find the direction cosines of  $x$  -axis . (March 2015, Model Paper 2022)
4. Find the intercepts cut off by the plane  $2x + y - z = 5$  (July 2015 , July 2019 )
5. Find the distance of a point  $(-6, 0, 0)$  to the plane  $2x - 3y + 6z = 2$  (March 2016)
6. Find the direction cosines of  $z$  -axis . (July 2016)
7. Find the direction cosines of a line which makes equal angles with co- ordinate axes . (March 2017)
8. Find the direction cosines of  $y$  - axis (July 2017)
9. The Cartesian equation of a line is  $\frac{x-5}{3} = \frac{y-4}{7} = \frac{z-6}{2}$  . Write its vector form . (July 2018)
10. If a line makes angle  $90^\circ$  ,  $135^\circ$  and  $45^\circ$  with positive direction of  $x$  ,  $y$  and  $z$  -axis respectively . Find its direction cosines . (March 2019 , Sept 2020)
11. If a line has direction ratios  $2$  ,  $-1$  ,  $-2$  determine its direction cosines . (March 2020, Model Paper -01 )
12. Find the distance of the plane  $3x - 4y + 12z - 3 = 0$  from the origin . (Model paper -02)

### TWO MARKS QUESTIONS :

1. Find the equation of the plane through the line of intersection of the planes  $3x - y + 2z - 4 = 0$  and  $x + y + z - 2 = 0$  and passing through  $(2, 2, 1)$  (July 2014)
2. Find the vector equation of the line , passing through the points  $(-1, 0, 2)$  and  $(3, 4, 6)$  . (July 2015)
3. Find the vector equation of the line passing through the points  $(3, -2, -5)$  and  $(3, -2, 6)$  . (March 2017)
4. Find the vector equation of the line , passing through the points  $(3, -2, -5)$  and  $(3, -2, 6)$  (Model Paper -02)
5. Find the Cartesian equation of the line parallel to  $y$  -axis and passing through the point  $(1, 1, 1)$  (March 2016)
6. Show that the lines  $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$  and  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  are perpendicular to each other . (July 2016)
7. Find the distance of a point  $(3, -2, 1)$  to the plane  $2x - y + 2z + 3 = 0$  (March 2019)
8. Find the distance of a point  $(-6, 0, 0)$  to the plane  $2x - 3y + 6z - 2 = 0$  (Model Paper 2022)
9. Find the equation of a plane with intercepts  $2, 3$  and  $4$  on  $x$  ,  $y$  and  $z$  axis respectively ( March 2020, Model Paper - 01 )
10. Find the angle between pair of lines  $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$  and  $\vec{r} = \hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$  (March 2014 , March 2015)
11. Find the angle between pair of lines  $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$  and  $\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + \hat{k})$  (July 2017)
12. Find the angle between the planes whose vector equations are  $\vec{r} \cdot (2\hat{i} + 2\hat{j} + 6\hat{k}) = 5$  and  $\vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$  (March 2018)
13. Find the angle between the line  $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$  and the plane  $10x + 2y - 11z = 3$  (July 2018)
14. Find the angle between the pair of lines  $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$  and  $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$  (July 2019)
15. Find the intercepts cut off by the plane  $2x + y - z = 5$  (Model Paper 2022 )



### THREE MARKS QUESTIONS :

1. Find the distance of a point  $(2,5,-3)$  from the plane  $\hat{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$ . (March 2014)
2. Find the shortest distance between the lines  $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$  and  $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$ . (July 2014, July 2019)
3. Find the equation of the plane through the line of intersection of the planes  $3x - y + 2z - 4 = 0$  and  $x + y + z - 2 = 0$  and passing through the point  $(2,2,1)$  (March 2015, March 2018, July 2018, March 2020, Model Paper -01)
4. Find the shortest distance between the lines  $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$  and  $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ . (Model Paper 2022)
5. Find the distance between the lines whose vector equations are  $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$  and  $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$  (July 2015, March 2016, Model Paper -02)
6. Find the equation of the line, passing through the points  $(-1,0,2)$  and  $(3,4,6)$  in both vector and Cartesian form. (July 2016)
7. Find the shortest distance between the lines  $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$  and  $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ . (March 2017, Sept 2020)
8. Find the vector and Cartesian equations of the plane which passes through the point  $(5,2,-4)$  and perpendicular to the line with direction ratios  $2,3,-1$ . (July 2017)
9. Find the vector equation of the plane passing through the points  $R(2,5,-3)$ ,  $S(-2,-3,5)$  and  $T(5,3,-3)$ . (March 2019)

### FIVE MARKS QUESTIONS :

1. Derive the equation of a plane in Normal form in both vector and Cartesian form. (March 2014, July 2016, July 2019, Model Paper -02)
2. Derive the equation of the line in space, passing through a point and parallel to a vector both in vector and Cartesian form. (July 2014, March 2015, March 2019, March 2020, Model Paper -01)
3. Derive the equation of a plane passing through a point and perpendicular to a vector in both vector and Cartesian form. (July 2015, March 2016, March 2017, Model Paper 2022)
4. Derive the equation of the line in space, passing through two points both in vector and Cartesian forms (July 2017, March 2018, July 2018, Sept 2020)

## 12. LINEAR PROGRAMMING PROBLEM:

### ONE MARK QUESTIONS :

1. Define feasible region in a LPP  
(March 2014 , March 2015 , March 2016 , March 2017 , July 2019 , Model Paper 2022)
2. Define linear objective function in LPP (July 2014 , March 2019 , Model Paper -02)
3. Define optimal solution in LPP  
(July 2015 , July 2016 , July 2017 , March 2018 , July 2018, March 2020 , Sept 2020, Model Paper -01 )

### SIX MARKS QUESTIONS :

1. Maximize and Minimize  $Z = x + 2y$  . Subject to the constraints :  $x + 2y \geq 100$  ,  $2x - y \leq 0$  ,  $2x + y \leq 200$  ,  $x, y \geq 0$  by graphical method . (March 2014, July 2014, Model Paper -02 )
2. Maximize and Minimize  $Z = 3x + 9y$  , Subject to the constraints:  $x + 3y \leq 60$  ,  $x + y \geq 10$  ,  $x \leq y$  ,  $x, y \geq 0$  (July 2016, March 2018 )
3. Maximize and Minimize  $Z = 5x + 10y$  , Subject to the constraints :  $x + 2y \leq 120$  ,  $x + y \geq 60$  ,  $x - 2y \geq 0$  ,  $x \geq 0$  ,  $y \geq 0$  (March 2016 , March 2019 )
4. Maximize and Minimize  $Z = 600x + 400y$  , Subject to the constraints ,  $x + 2y \leq 12$  ,  $4x + 5y \leq 12$  ,  $4x + 5y \leq 20$  and  $x \geq 0$  ,  $y \geq 0$  by graphically (March 2017 )
5. Maximize and Minimize  $Z = -3x + 4y$  , subject to the constraints ,  $x + y \leq 8$  ,  $3x + 2y \leq 12$  ,  $x \geq 0$  ,  $y \geq 0$  (July 2017 )
6. Maximize and Minimize  $Z = 10500x + 9000y$  , subject to the constraints ,  $x + y \leq 50$  ,  $20x + 10y \leq 800$  ,  $x \geq 0$  ,  $y \geq 0$  (July 2018 )
7. Maximize and Minimize the following  $Z = 4x + y$  , Subject to the constraints :  $x + y \leq 50$  ,  $3x + y \leq 90$  ,  $x \geq 0$  ,  $y \geq 0$  (March 2020, Model Paper -01 )
8. Maximize and Minimize the following :  $Z = 3x + 2y$  , subject to the constraints  $x + 2y \leq 10$  ,  $3x + y \leq 15$  ,  $x \geq 0$  ,  $y \geq 0$  (Sept 2020, Model Paper -2022 )
9. A manufacturing company makes two models A and B of a product . Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing . Each piece of model B requires 12 labour hours for fabricating and 3 labour hour for finishing . For fabricating and finishing , the maximum labour hours available are 180 and 30 respectively . The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of model B . How many pieces of model A and model B should be manufactured per week to realize a maximum profit ? What is the maximum profit per week ? (March 2015 )
10. One kind of cake requires 200g of flour and 25g of fat , and another kind of cake requires 100g of flour and 50g of fat . Find the maximum number of cakes which can made from 5 kg of flour and 1kg of fat assuming that there is no shortage of the other ingredients used in making the cakes .  
(July 2015)
11. A factory manufactures two types of screws , A and B . Each type of screw requires the use of two machines, an automatic and a hand operated . It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A , while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B . Each machine is available for at the most 4 hours on any day . The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs 10 . Assuming that he can sell all the screws he manufactures , how many packages of each type should the factory owner produce in a day in order to maximize his profit ? Determine the maximum profit (July 2019 )

## 13. PROBABILITY

### ONE MARK QUESTIONS :

1. If  $P(A) = \frac{4}{5}$ ,  $P(B/A) = \frac{2}{5}$ , find  $P(A \cap B)$  (March 2014)
2. If  $P(A) = 0.6$ ,  $P(B) = 0.3$ ,  $P(A \cap B) = 0.2$ , then find  $P(A/B)$  (June 2014)
3. If  $P(A) = \frac{3}{5}$ ,  $P(B) = \frac{1}{5}$  then find  $P(A \cap B)$  if A and B are independent events.  
(March 2015, March 2017, July 2019, March 2020)
4. If  $P(A) = 0.8$ ,  $P(B) = 0.5$ ,  $P(B/A) = 0.4$  then find  $P(A \cap B)$ . (June 2015, March 2016, July 2017)
5. If  $P(A) = \frac{7}{13}$ ,  $P(B) = \frac{9}{13}$  and  $P(A \cap B) = \frac{4}{13}$  then find  $P(A/B)$   
(July 2016, March 2018, Model Paper -01)
6. If  $P(B) = 0.5$  and  $P(A \cap B) = 0.32$  then find  $P(A/B)$  (July 2018, Model Paper -2022)
7. If  $P(E) = 0.6$ ,  $P(F) = 0.3$  and  $P(E \cap F) = 0.2$  then find  $P(F/E)$  (March 2019, Sept 2020)
8. If F is an event of a sample space S of an experiment then find  $P(S/F)$ . (Model Paper -02)

### TWO MARKS QUESTIONS :

1. A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F is the 'the number appearing is even', then find whether E and F are independent. (July 2014)
2. If A and B are two independent events then prove that the probability of occurrence of atleast one of A and B is given by  $1 - P(A')P(B')$ . (July 2017)
3. Probability of solving a specific problem independently by A and B are  $\frac{1}{2}$  and  $\frac{1}{3}$  respectively. If both try to solve the problem independently, find the probability that exactly one of them solves the problem. (March 2019)
4. Two cards drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black. (Model Paper -02)
5. Find the probability distribution of number of heads in two tosses of a coin.  
(March 2014, July 2016, March 2017)
6. Find the probability distribution of the number of tails in simultaneous tosses of three coins.  
(July 2015)
7. Let X denotes the number of hours you study during a randomly selected school day. The probability that X can take the values of x, has the following form, where k is some constant.

$$P(X=x) = \begin{cases} 0.1 & \text{if } x = 0 \\ kx & \text{if } x = 1 \text{ or } x = 2 \\ k(5-x) & \text{if } x = 3 \text{ or } x = 4 \\ 0 & \text{otherwise} \end{cases}$$

find the value of 'k' (March 2015)

8. The random variable X has a probability distribution P(X) of the following form where k is some number :

$$P(X=x) = \begin{cases} k, & \text{if } x=0 \\ 2k, & \text{if } x=1 \\ 3k, & \text{if } x=2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of 'k' and  $P(X \leq 2)$

(March 2016, July 2018, Model Paper 2022)



9. A random variable X has the following probability distribution :

X	0	1	2	3	4
P(X)	0.1	k	2k	3k	5

Determine (i)  $k$  (ii)  $P(X \geq 2)$  ( March 2018 , March 2020 )

10. The probability distribution of random variable X is as follows

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Find the expectation of X . (July 2019 )

### THREE MARKS QUESTIONS :

1. A die is tossed thrice . Find the probability of getting an odd number at least once .  
(March 2014 , July 2016 , )
2. Bag I contains 3 red and 4 black balls and bag II contains 5 red and 6 black balls . One ball is drawn at random from one of the bag and it is found to be red . Find the probability that it was drawn from bag II (July 2014 , July 2015 , July 2019 , Sept 2020 , Model Paper -02 )
3. An insurance company insured 2000 scooter drivers , 4000 car drivers and 6000 truck drivers . The probability of an accidents are 0.01 , 0.03 and 0.15 respectively . One of the insured persons meets with an accident . What is the probability that he is a scooter driver ? (March 2015 , March 2019 )
4. Given three identical boxes , I, II and III , each containing two coins . In box I, both coins are gold coins , in box II , both are silver coins and in box III , there is one gold coin and one silver coin . A person chooses a box at random and takes out a coin . If the coin is gold , what is the probability that the other coin in the box is also gold ? (March 2016)
5. Given that the two numbers appearing on throwing two dice are different . Find the probability of the event " the sum of numbers on the dice is 4 " . (March 2017)
6. A man is known to speak truth 3 out of 4 times . He throws a die and reports that it is a six . Find the probability that it is actually a six . (July 2017 , March 2020 , Model Paper -01 , Model Paper 2022)
7. A bag contains 4 red and 4 black balls , another bag contains 2 red and 6 black balls . One of the two bags is selected at random and a ball is drawn from the bag which is found to be red . Find the probability that it was drawn from first bag . (March 2018)
8. A man is known to speak truth 4 out of 5 times . He tossed a coin and reports that it is head . Find the probability that it is actually head . (July 2018)

### FIVE MARKS QUESTIONS :

1. Probability of solving specific problem independently by A and B are  $\frac{1}{2}$  and  $\frac{1}{3}$  respectively . If both try to solve the problem independently , find the probability that (i) problem is solved (ii) exactly one of them solves the problem . (Model Paper -01 , Model Paper 2022 )
2. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls . Find the probability that (i) Both balls are red (ii) First ball is black and second is red (iii) One of them is black and other is red . (Model Paper -02 )
3. A die is thrown . If E is the event ' the number appearing is a multiple of 3 ' and F be the event ' the number appearing is even ' then find whether E and F are independent ? (Expected )
4. Given two independent events A and B such that  $P(A) = 0.3$  and  $P(B) = 0.6$  find (i)  $P(A \text{ and } B)$  (ii)  $P(A \text{ or } B)$  (iii)  $P(A \text{ and not } B)$  . (Expected )

5. A fair coin and unbiased die are tossed . Let A be the event ' head appears on the coin ' and B be the event ' 3 on the die ' . Check whether A and B are independent events or not ? **(Expected )**
6. A person buys a lottery ticket in 50 lotteries , in each of which his chance of winning a prize is  $\frac{1}{100}$  .  
What is the probability that he will win a prize a) at least once b) exactly once  
**(March 2014 , July 2018, March 2020, Model Paper 2022 )**
7. Probability that a student is not a swimmer is  $\frac{1}{5}$  . Find the probability that out of 5 students , (i) at least four are swimmers and (ii) at most three are swimmers . **(March 2016 )**
8. If a fair coin is tossed 10 times , find the probability of (i) exactly six heads (ii) at least six heads  
**(July 2014, July 2016, July 2017 , March 2018 )**
9. A die is thrown 6 times . If " getting an odd number " is success , what is the probability of  
(i) 5 success (ii) at least 5 success (iii) at most 5 success . **(March 2015 , July 2019)**
10. Five cards are drawn successively with replacement from a well shuffled deck of 52 cards . What is the probability that (i) all the five cards are spades ? (ii) only 3 cards are spades ? (iii) none is a spade ?  
**(July 2015, March 2019)**
11. The probability that a bulb produced by a factory will fuse after 150 days of use 0.05 , find the probability that out of 5 such bulbs (i) none (ii) not more than one (iii) more than one will fuse after 150 days of use . **(March 2017 )**