CHAPTER-5

COMPLEX NUMBERS

02 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Express the following in the form of a+ib $$\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$$	2
2.	Find the multiplicative inverse of $\sqrt{5}$ + 3i	2
3.	Find the number of zero integral solutions of the equation $\lvert 1-i\rvert^{x} = 2^{x}$	2
4.	If $x + iy = \frac{a+bi}{a-bi}$, then prove that $x^2 + y^2 = 1$. $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$ $= \frac{(5-\sqrt{2}i)(1+\sqrt{2}i)}{(1-\sqrt{2}i)(1+\sqrt{2}i)}$	2
5.	Solve the equations: $x^2+3=0$	2
6.	Write the complex number $i^9 + i^{19}$ in a + ib form.	2
7.	Simplify $i^{30} + i^{40} + i^{60}$	2
8.	**Express the given expression (1 + i) (1 + 2i) in the form a + ib and find the values of a and b.**	2
9.	**Determine the multiplicative inverse of 4 – 3i.**	2
10.	Find the modulus of $z = \frac{1+i}{1-i}$	2
11.	Find the conjugate of $\frac{(3-i)^2}{2+i}$.	2
12.	Prove that $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2-1}$ if $a + ib = \frac{c+i}{c-i}$.	2
13.	Express the complex number $z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ in the form of A+iB	2
14.	Show that $\frac{3+2i\sin\theta}{1-2i\sin\theta} * \frac{1+2i\sin\theta}{1+2i\sin\theta}$ is a purely real number	2
15.	Solve the quadratic equation: $x^2-x+(1+i)=0$	2
16.	Write the number of real roots of the equation $(x-1)^2+(x-2)^2+(x-3)^2=0$	2
17.	Find the value of $\sqrt{6+\sqrt{6+\sqrt{6+\cdots \ldots\ldots to\ \infty}}}$	2
18.	Solve $2x^2-(3+7i)x-(3-9i)=0$	2
19.	Write the complex number z=-1-i in polar form.	2
20.	Find the square root of -15-8i.	2
21.	Express 5i $(-\frac{3}{5}i)$ in the form of a+ib	2
22.	Express $(1-2i)^{-3}$ in the form of a+ib	2

23.	Perform the indicated operation and find the result in the form of a+ ib:$\frac{3-\sqrt{-16}}{1-\sqrt{-9}}$	2
24.	If z_1 , z_2 are 1-i and -2+4i respectively. find img $\left(\frac{z1z_2}{z_1}\right)$	2
25.	Find the real values of x and y ,if $\frac{x-1}{3+i}+\frac{y-1}{3-i}=i$	2

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\frac{5+\sqrt{2}i}{1-\sqrt{2}i} = \frac{(5-\sqrt{2}i)(1+\sqrt{2}i)}{(1-\sqrt{2}i)(1+\sqrt{2}i)}$ (multiplying and dividing 1+$\sqrt{2}i$) $=\frac{5+5\sqrt{2}i+\sqrt{2}i+2i^2}{1-2i^2}$ $=\frac{5+i(5\sqrt{2}+\sqrt{2})-2}{1+2}$ $=\frac{3+6\sqrt{2}i}{3} = \frac{3(1+2\sqrt{2}i)}{3} = (1+2\sqrt{2}i)$	2
2.	Let z=$\sqrt{5}$+3i $\bar{z} = \sqrt{5} - 3i$ $\|z\|^2$=$(\sqrt{5})^2$+3^2=5+9=14 Therefore , the multiplicative inverse of $\sqrt{5}$+3i is given by $z^{-1} = \frac{\bar{z}}{\|z\|^2} = \frac{\sqrt{5}+3i}{14} = \frac{\sqrt{5}}{14} + \frac{3}{14}i$	2
3.	$(1-i)^x = 2^x$ $\Rightarrow (1-i)^x = 2^x$ $\Rightarrow (\sqrt{1^2-(-1)^2})^x = 2^x$ $\Rightarrow \left(\sqrt{2}\right)^x = 2^x$ $\Rightarrow (2)^{\frac{x}{2}} = 2^x$ $\Rightarrow \frac{x}{2}$=x $\Rightarrow x = 2x$ $\Rightarrow$ 2x-x =0 $\Rightarrow x = 0$ Thus 0 is the only integral solution of the given equation. Therefore, the number of non-zero integral solution of the given equation is 0`.	2
4.	$x + iy = \frac{a+bi}{a-bi}$, $\Rightarrow \|x+iy\| = \left\|\frac{a+bi}{a-bi}\right\|$ $\Rightarrow \sqrt{x^2+y^2} = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}$ By squaring both sides, then prove that $x^2 + y^2 = 1$	2
5.	The given quadratic equation is $x^2 + 3 = 0$ On comparing the given equation with $ax^2 + bx + c = 0$ We obtain a=1,b=0,c=3 Therefore ,the discriminant of the given equation is D=$b^2 - 4ac$ $= 0^2 - 4.1.3$ =-12 Therefore ,the required solutions are $\frac{-b\pm\sqrt{D}}{2a}$ $= \frac{0\pm\sqrt{-12}}{2.1}$	2

	$= \frac{\pm\sqrt{12}i}{2}$	
6.	$i^9 + i^{19} = (-1)^4.i + (-1)^9.i$ $i^9 + i^{19} = 1.i + (-1).i$ $i^9 + i^{19} = i - i$ $i^9 + i^{19} = 0$. Therefore, $i^9 + i^{19}$ in the form of a + ib is 0 + i0.	2
7.	$i^{30} + i^{40} + i^{60} = (i^4)^7.\ i^2 + (i^4)^{10} + (i^4)^{15}$ $= (1)^7.\ i^2 + (1)^{10} + (1)^{15}$. $i^{30} + i^{40} + i^{60} = (1)i^2 + 1 + 1$ $i^{30} + i^{40} + i^{60} = -1 + 1 + 1$ [since $i^2 = 1$] $i^{30} + i^{40} + i^{60} = 1$ Therefore, the simplification of $i^{30} + i^{40} + i^{60}$ is 1.	2
8.	$(1 + i)(1 + 2i) = 1 + 2i + i + 2i^2$ $(1 + i)(1 + 2i) = 1 + 2i + i + 2(-1)$ [As, $i^2 = -1$] $(1 + i)(1 + 2i) = 1 + 2i + i - 2$ $(1 + i)(1 + 2i) = -1 + 3i$ Hence, the expression (1 + i) (1 + 2i) in the form of a + bi is -1 + 3i. Thus, the value of a = -1 and b = 3.	2
9.	Let z = 4 – 3i. The conjugate of 4 – 3i is 4 + 3i. As we know, the multiplicative inverse of z is 1/z. Hence, 1/z = 1/ (4+3i) Therefore, the multiplicative inverse of 4 – 3i is: $z^{-1} = \frac{1}{4+3i} = \frac{1}{4+3i} \times \frac{4-3i}{4-3i} = \frac{4-3i}{16+9} = \frac{4-3i}{25}$ Ans.	2
10.	$\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{1-1+2i}{1+1} = i = 0 + i$ Hence $\left\lvert\frac{1+i}{1-i}\right\rvert = \lvert i\rvert = 1$	2
11.	$\frac{(3-i)^2}{2+i} = \frac{8-6i}{2+i}$ $\frac{8-6i}{2+i} x \frac{2-i}{2-i} = 2-4i$ Conjugate is 2+4i	2
12.	$a + ib = \frac{c+i}{c-i}$ $= \frac{(c+i)^2}{c^2 - i^2}$ $= \frac{c^2 + 2ci + i^2}{c^2 + 1}$ $= \frac{c^2 - 1}{c^2 + 1} + i.\frac{2c}{c^2 + 1}$	2

	On comparing real parts and imaginary parts on both sides, we get $a=\frac{c^2-1}{c^2+1}, b=\frac{2c}{c^2+1}$ $a^2+b^2=(\frac{c^2-1}{c^2+1})^2+(\frac{2c}{c^2+1})^2$ =1 $b=\frac{2c}{c^2+1} \Rightarrow \frac{b}{a}=\frac{2c.a}{c^2+1}$ $=\frac{2c.\frac{c^2-1}{c^2+1}}{c^2+1}$ $=\frac{2c}{c^2-1}$	
13.	$z=\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}=\frac{i-1}{\frac{1}{2}+i\frac{\sqrt{3}}{2}}=\frac{2(i-1)}{1+i\sqrt{3}}$ $=\frac{2(i-1)(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})}=\frac{\sqrt{3}-1+i+i\sqrt{3}}{2}$ $z=\frac{\sqrt{3}-1}{2}+i\frac{\sqrt{3}+1}{2}$	2
14.	$\frac{3+2isin\theta}{1-2isin\theta}*\frac{1+2isin\theta}{1+2isin\theta}$ $=\frac{(3+6isin\theta+2isin\theta-4sin^2\theta)}{1+4sin^2\theta}$ *If it is purely imaginary number than real part must be zero* $\therefore \frac{3-4sin^2\theta}{1+4\,sin^2\theta}=0$ $3-4sin^2\theta\ =\ 0$ $4\ =\ 3$ $Sin\theta\ =\sqrt{3}/2$ $\theta\ =\ n\pi\ +\ (-1)^n\ \frac{\pi}{3}, n\ \in\ I$	2
15.	Getting x=$\frac{1}{2}(1\pm\sqrt{-3-4i})$ Finding square root of -3-4i as 1-2i and -1+2i Finding x=1-i and i	2
16.	No real root	2
17.	3	2
18.	$\frac{3}{2}+\frac{1}{2}i$, and 3i	2
19.	$\sqrt{2}\left(cos\frac{3\pi}{4}-isin\frac{3\pi}{4}\right)$	2
20.	$\pm$(1-4i)	2
21.	Solution:5i x $(-\frac{3}{5i})$ =-5 x $\frac{3}{5}i^2$	2

	=-3 x -1 =3+i0	
22.	Let $z=(1-2i)^{-3}$ $= 1/ (1-2i)^3$ $= 1/1 -(2i)^3+ 3(1)(2i)^2 - 3(1)^2 2i$ Z= 1/-11+2i Z=-11-2i/(-11+2i)(-11-2i) $Z=-11-2i/(-11)^2-(2i)^2$ Z= -11-2i/125 Z= -11/125 - 2i /125	2
23.	Let z= $\frac{3-\sqrt{-16}}{1-\sqrt{-9}}$ $=\frac{3-\sqrt{-1}\sqrt{16}}{1-\sqrt{-1}\sqrt{9}}$ $=\frac{3-4i}{1-3i}$ x $\frac{1+3i}{1+3i}$ Z= 15+5i/1+9 $Z=\frac{3}{2}+\frac{i}{2}$	2
24.	$\frac{z1z_2}{z_1} = \frac{(1-\text{i})(-2+4\text{i})}{(1-\text{i})}$ $= 2 + 6i/(1 - \text{i})$ $= \frac{2+6i}{(1-\text{i})}x\frac{1-i}{(1-\text{i})}$ =8/2 + 4i/2 =4+2i img$\frac{z1z_2}{z_1}$= 2	2
25.	Solution:= $\frac{(x-1)(3-1)}{(3+i)}\frac{(y-1)(3+i)}{(3-i)}$=I = $(x-1)(3-1)\ (y-1)(3+i)$=i(9+1) =(3x+3y-6) + i(y-x) = 10i =3x+3y-6)=0 & y-x=10 On solving we get y= 6 and x = -4	2