CHAPTER-13

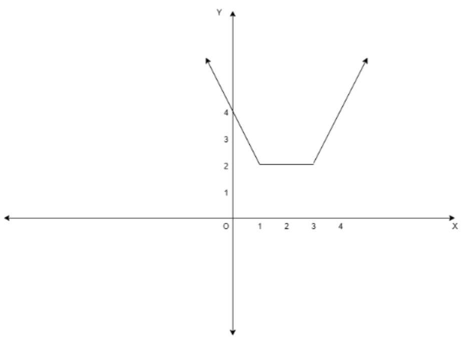
LIMITS & DERIVATIVES

02 MARK TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	If y=1+$\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$ find $\frac{\mathrm{dy}}{\mathrm{dx}}$	2
2.	If y=4x^5-3x+7, find$\frac{\mathrm{dy}}{\mathrm{dx}}$ $at\ x=2$	2
3.	Find the derivative of tan$\sqrt{x}$ with respect to x	2
4.	Find the positive integer n so that $\lim_{x\to 3}\frac{x^n-3^n}{x-3}=108$	2
5.	Evaluate $\lim_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x}$	2
6.	Evaluate $\lim_{x\to\frac{\pi}{6}}\frac{3\sin x-\sqrt{3}\cos x}{6x-\pi}$.	2
7.	If $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, then find the value of k.	2
8.	Differentiate $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$ with respect to x.	2
9.	Write the value of the derivative of $f(x)=\|x-1\|+\|x-3\|$ at $x=2$.	2
10.	If $y=\sqrt{\frac{\sec x-1}{\sec x+1}}$, then find $\frac{dy}{dx}$.	2
11.	Evaluate $\lim_{x\to 0}\frac{sinx^\circ}{x}$	2
12.	If f(x) = 1+ tan x ,then find f'(0) ,using first principle of derivative.	2
13.	**If** f(x) = $\begin{cases} x\ ,\ if\ x\ \geq 2 \\ 1-x, if\ x\ < 2\end{cases}$ then find $\lim_{x\to 2}f(x)$ if exists	2
14.	Let y = $x^{x^{x^{\ldots\ldots\infty}}}$ then find $\frac{dy}{dx}$	2
15.	If y = $x^{2023}+\log_{2023}x$ then find $\frac{dy}{dx}$	2
16.	Evaluate: $\lim_{x\to 0}\frac{\sin(2+x)\ -\ \sin(2-x)}{x}$	2
17.	Find the derivative of f(x) = ax^2 + bx + c, where a, b, and c are non- zero constant, by first principle.	2
18.	Find the derivative of $x^2\cos x$.	2

19.	Evaluate: $\lim_{x\to -2} \frac{\frac{1}{x}+\frac{1}{2}}{x+2}$	2
20.	Evaluate: $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$	2
21.	Discuss the existence of the limit $\lim_{x\to 0} \frac{1}{x}$	2
22.	Evaluate $\lim_{x\to 2} \frac{x^2-5x+6}{x^2-4}$	2
23.	Evaluate $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$	2
24.	Find the derivative of $\sin x$ at x = 0.	2
25.	Find the derivative of $(x^2+1)\cos x$ with respect to x.	2

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\frac{\mathrm{dy}}{\mathrm{dx}} = y$	2
2.	317	2
3.	$\frac{1}{2\sqrt{x}} sec^2\sqrt{x}$	2
4.	n=4	2
5.	2cos2	2
6.	$\lim_{x\to\frac{\pi}{6}} \frac{3\sin x-\sqrt{3}\cos x}{6x-\pi} = \lim_{x\to\frac{\pi}{6}} \frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}\sin x-\frac{1}{2}\cos x\right)}{6\left(x-\frac{\pi}{6}\right)}$ $= \lim_{x\to\frac{\pi}{6}} \frac{\sqrt{3}\sin\left(x-\frac{\pi}{6}\right)}{6\left(x-\frac{\pi}{6}\right)} = \frac{1}{2\sqrt{3}} \lim_{x\to\frac{\pi}{6}} \frac{\sin\left(x-\frac{\pi}{6}\right)}{\left(x-\frac{\pi}{6}\right)} = \frac{1}{2\sqrt{3}}\times 1 = \frac{1}{2\sqrt{3}}$	2
7.	$\lim_{x\to 1}\frac{x^4-1}{x-1} = \lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$ $\Rightarrow \lim_{x\to 1}\frac{\left(x^2+1\right)(x-1)(x+1)}{x-1} = \lim_{x\to k}\frac{(x-k)\left(x^2+xk+k^2\right)}{(x-k)(x+k)}$ $\Rightarrow \lim_{x\to 1}\left(x^2+1\right)(x+1) = \lim_{x\to k}\frac{\left(x^2+xk+k^2\right)}{(x+k)} \Rightarrow 4 = \frac{3k^2}{2k} \Rightarrow k = \frac{8}{3}$	2
8.	$\frac{d}{dx}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = \frac{d}{dx}\left(x+\frac{1}{x}+2\right) = 1-\frac{1}{x^2}$	2
9.	$f(x) = \|x-1\|+\|x-3\|$ $\therefore f(x) = \begin{cases} 4-2x, & for\ x<1 \\ 2, & for\ 1\le x\le 3 \\ 2x-4, & for\ x>3 \end{cases}$ At *x=2*, $f(x)$ is a constant function, hence the derivative of $f(x)$ at *x=2* is zero (0).  	2

10.	$y=\sqrt{\frac{\sec x-1}{\sec x+1}} \Rightarrow y=\sqrt{\frac{1-\cos x}{1+\cos x}}=\sqrt{\frac{2\sin^2\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}}=\tan\left(\frac{x}{2}\right)$ $\therefore \frac{dy}{dx}=\frac{d}{dx}\tan^2\left(\frac{x}{2}\right)=\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$	2
11.	We know that $x^\circ = \frac{\pi x}{180}$, use this we get the answer $\frac{180}{\pi}$	2
12.	We know that f'(x) = $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ Here x=0 and f(x) = 1+ tan(x) Therefore f'(0) = $\lim_{h\to 0}\frac{f(h)-f(0)}{h} = \lim_{h\to 0}\frac{1+tanh-1}{h}$= $\lim_{h\to 0}\frac{tanh}{h}$=0	2
13.	Limit of this function does not exist since $\lim_{x\to 2^+} f(x)$ =2 $\neq$ $\lim_{x\to 2^-} f(x)$= -1	2
14.	Clearly y = x^y Taking logarithm both side we get logy = y logx Now differentiate both side w.r.t x we get $\frac{1}{y}\frac{dy}{dx}=\frac{y}{x}+logx\ \frac{dy}{dx}$ Simplify above we will get our required answer	2
15.	y = $x^{2023}+\log_{2023}x$ therefore y = $x^{2023}+\frac{\log_e x}{\log_e 2023}$ Therefore $\frac{dy}{dx}$ = 2023 $x^{2022}+\frac{1}{x\log_e 2023}$	2
16.	We have, $$\lim_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x}$$ = $\lim_{x\to 0}\frac{2\cos 2\ \sin x}{x}$ =$x\cos 2\lim_{x\to 0}\frac{\sin x}{x}$ = 2cos 2	2
17.	$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$	2

	$$= \lim_{h\to 0} \frac{a(x+h)+b-(ax+b)}{h}$$ $$= \lim_{h\to 0} \frac{bh}{h}$$ $= b$	
18.	Let $y = x^2 \cos x$ Differentiating both w.r.t x, we ge $\frac{dy}{dx} = \frac{d}{dx}(x^2 \cos x)$ $= x^2(-\sin x) + \cos x\,(2x)$ $= 2x \cos x - x^2 \sin x$	2
19.	$$\lim_{x\to -2} \frac{\frac{1}{x}+\frac{1}{2}}{x+2} = \lim_{x\to -2} \frac{x^{-1}+2^{-1}}{x+2}$$ $$= \lim_{x\to -2} \frac{x^{-1}-(-2^{-1})}{x-(-2)}$$ $$= -1(-2)^{-2}$$ $= \frac{1}{4}$	2
20.	Put $y = 1 + x, so\ that\ y \to 1\ as\ x \to 0$ Then $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} = \lim_{y\to 1} \frac{\sqrt{y}-1}{y-1}$ $$= \lim_{y\to 1} \frac{y^{\frac{1}{2}} - 1^{\frac{1}{2}}}{y-1}$$ $$= \frac{1}{2}$$	2
21.	$LHL = \lim_{x\to 0^-} \frac{1}{x} \to \infty$ $RHL = \lim_{x\to 0^+} \frac{1}{x} \to \infty$ Here, $LHL \neq RHL$ Hence, $\lim_{x\to 0} \frac{1}{x}$ does not exist.	2
22.	When x = 2 the given expression assumes the indeterminate form $\frac{0}{0}$. Therefore, on factorizing the numerator and denominator we get, $\lim_{x\to 2} \frac{x^2-5x+6}{x^2-4} = \lim_{x\to 2} \frac{(x-2)(x-3)}{(x+2)(x-2)} = \lim_{x\to 2} \frac{(x-3)}{(x+2)} = \frac{2-3}{2+2} = \frac{-1}{4}$.	2
23.	When x = 0, the given expression assumes the indeterminate form $\frac{0}{0}$. Therefore, on rationalizing the numerator we get, $\lim_{x\to 0} \frac{(\sqrt{2+x}-\sqrt{2})(\sqrt{2+x}+\sqrt{2})}{x(\sqrt{2+x}+\sqrt{2})} = \lim_{x\to 0} \frac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})} = \lim_{x\to 0} \frac{1}{(\sqrt{2+x}+\sqrt{2})} = \frac{1}{2\sqrt{2}}$.	2

24.	Let $f(x) = \sin x$. Then by using the definition of derivative, we get $f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{\sin h - \sin 0}{h} = \lim_{h \to 0} \frac{\sin h}{h} = 1$	2
25.	$2x \cos x - (x^2 + 1) \sin x$.	2