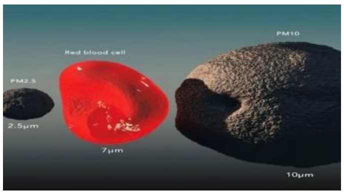
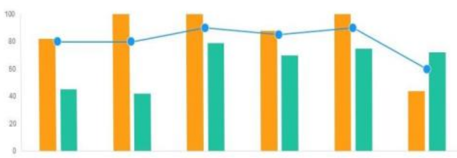
CHAPTER-15

STATISTICS

02 MARK TYPE QUESTIONS

<table>
<tr><th>Q. NO</th><th>QUESTION</th><th>MARK</th></tr>
<tr><td>1.</td><td>An analysis of monthly wages paid to workers in two firms A and B, belonging to the same industry, gives the following results :
Which firm, A or B, shows greater variability in individual wages?
<table>
<tr><th></th><th>FIRM A</th><th>FIRM B</th></tr>
<tr><td>NUMBER OF WORKERS</td><td>897</td><td>468</td></tr>
<tr><td>MEAN AMOUNT OF WAGES(INR)</td><td>6345</td><td>6345</td></tr>
<tr><td>VARIANCE OF DISTRIBUTION OF WAGES</td><td>100</td><td>169</td></tr>
</table></td><td>2</td></tr>
<tr><td>2.</td><td>City A's daily average PM10 (particulate matter with a diameter of 10 micrometers or smaller) levels for a week were: 50, 60, 45, 70, 55, 65, 75 (in µg/m³). City B's corresponding PM10 levels were: 40, 80, 50, 60, 55, 85, 65 (in µg/m³). Which city had greater variability in PM10 levels?

</td><td>2</td></tr>
<tr><td>3.</td><td>A fitness trainer is conducting a study on the performance of two different workout routines. The trainer tracks the number of repetitions completed by participants in each routine. There are two sets of observations, each containing 20 participants. The first set has a mean of 17 repetitions, and the second set has a mean of 22 repetitions. Surprisingly, both sets have the same standard deviation of 5 repetitions.

What would be the standard deviation of the combined set obtained by merging the two sets of observations?</td><td>2</td></tr>
<tr><td>4.</td><td>A collection of 100 items was analyzed, and the statistical properties of the data were observed. The mean of the items is 50, and the standard deviation is 4.


Calculate the sum of all the items and the sum of the squares of the items.</td><td>2</td></tr>
<tr><td>5.</td><td>A set of data points were collected and analyzed. For this distribution, two pieces of information were obtained: (x – 5) = 3 and (x – 5)^2 = 43. It is also known that the total number of items in the dataset is 18.
Calculate the mean and standard deviation for this distribution.</td><td>2</td></tr>
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6. If the mean and standard deviation of 100 observations are 50 and 4 respectively. Find the sum of all the observations and the sum of their squares. — 2

7. Let x_1 , x_2 , x_3 , ..., x_n be n values of a variable X. If these values are changed to $x_1 + a$, $x_2 + a$, ..., $x_n + a$, where a ∈ R, show that the variance remains unchanged. — 2

8. Find the mean, variance and standard deviation for the data: 2, 4, 5, 6, 8, 17 — 2

9. Calculate the mean deviation about the median of the observations: 3011, 2780, 3020, 2354, 3541, 4150, 5000. — 2

10. Find the mean deviation from the mean for the following data: — 2

Classes	0-10	10-20	20-30	30-40	40-50	50-60
Frequencies	6	8	14	16	4	2

11. The average marks scored by Ankur in certain number of tests are 84. He scored 100 marks in his last test. His average score of all these tests is 86, then find the total number of tests he appeared. — 2

12. Find the variance and standard deviation for the following data: 57, 64, 43, 67, 49, 59, 44, 47, 61, 59. — 2

13. The mean weight of 150 students in a certain class is 60 kilograms. The mean weight of boys in the class is 70 kilograms and that of the girls is 55 kilograms, then find the number of boys and girls of the class. — 2

14. The mean of 100 observations is 50 and their standard deviation is 5. Then find the sum of squares of all observations. — 2

15. Mean of 10 items is 17. If an observation 21 is replaced with 12, then what will be the new mean? — 2

16. An analysis of monthly wages paid to the workers of two firms A and B belonging to the same industry gives the following results : — 2

	Firm A	Firm B
No.of wages earners	1000	1200
Mean of monthly wages	Rs.2800	Rs.2800
Variance of the distribution of wages	100	169

In which firm A or B is there greater variability in individual wages ?

17. Find the mean and variance of first n natural numbers. — 2

18. Calculate the mean deviation about the median of the following observations : — 2

38 , 70 , 48 , 34 , 63 , 42 , 55 , 44 , 53 , 47

19.	If the mean and standard deviation of 100 observations are 50 and 4 respectively. Find the sum of all the observations and the sum of their squares.	2
20.	If for a distribution of 18 observations $\sum(x-5)=3$ and $\sum(x-5)^2=43$ Find the Mean and Standard Deviation.	2

ANSWERS:

Q. NO	ANSWER	MARKS
1.	Variance of the distribution of wages in firm A = 100 ∴ Standard deviation of the distribution of wages in firm A (($\sigma 1$) = $\sqrt{100}$ =10 Variance of the distribution of wages in firm = 169 ∴ Standard deviation of the distribution of wages in firm A (($\sigma 2$)=$\sqrt{169}$= 13 The mean of monthly wages of both the firms is same i.e., 6345. Therefore, the firm with greater standard deviation will have more variability. Thus, firm B has greater variability in the individual wages.	2
2.	Variability in data can be measured using measures of dispersion such as the range, variance, and standard deviation. In this case, we need to compare the variability of PM10 levels in City A and City B. City A's PM10 levels: 50, 60, 45, 70, 55, 65, 75 City B's PM10 levels: 40, 80, 50, 60, 55, 85, 65 To determine which city has greater variability, we can look at the range of PM10 levels in each city. The range is the difference between the maximum and minimum values. For City A: Range = 75 (max) - 45 (min) = 30 For City B: Range = 85 (max) - 40 (min) = 45 City B has a larger range of PM10 levels, indicating greater variability in its data. Therefore, the correct answer is City B.	2
3.	To determine the standard deviation of the combined set, we need to consider the concept of weighted averages and their effect on standard deviation. Calculate the weighted average of the means: Weighted Mean = (Number of observations in Set 1 * Mean of Set 1 + Number of observations in Set 2 * Mean of Set 2) / Total Number of Observations Weighted Mean = (20 * 17 + 20 * 22) / 40 = 19.5.	2

	Calculate the variance of the combined set using the formula: Variance = (Number of observations in Set 1 * Variance of Set 1 + Number of observations in Set 2 * Variance of Set 2) / Total Number of Observations Variance = (20 * 5^2 + 20 * 5^2) / 40 = 25. Calculate the standard deviation of the combined set: Standard Deviation = √Variance = √25 = 5. Therefore, the standard deviation of the combined set obtained by merging the two sets of observations would be 5	
4.	To solve this, we can use the formulas for the mean and standard deviation: Sum of all items = Mean × Number of items = 50 × 100 = 5000. Sum of squares of items = Variance × (Number of items - 1) + Mean^2 × Number of items = (4^2) × (100 - 1) + 50^2 × 100 = 159600. So, the sum of all the items is 5000, and the sum of the squares of the items is 159600.	2
5.	Given that (x - 5) = 3, we can solve for x: x = 3 + 5 = 8. Now, let's calculate the mean and standard deviation: Mean: The sum of all values (x) can be calculated by multiplying the mean by the number of items: Sum = Mean × Number of items = 8 × 18 = 144. Variance: Variance = [(Sum of squares of all values) - (Sum of all values)^2 / Number of items] / (Number of items - 1) Plugging in the values: Variance = [(43) - (144^2 / 18)] / 17 ≈ 9. Standard Deviation: Standard Deviation = √Variance = √9 = 3. So, the mean of the distribution is 8 and the standard deviation is 3.	2
6.	Sum of all the observations is 5000 Sum of their squares is 251600	2
7.	Let $u_i = x_i + a, i = 1,2, \ldots, n$ be the n values of variable U. Then,	2

	$\bar{U}=\frac{1}{n}\sum_{i=1}^{n} u_i=\frac{1}{n}\sum_{i=1}^{n}(\mathrm{x_i}+\mathrm{a})=\frac{1}{n}\left\{\sum_{i=1}^{n} x_i+na\right\}=\frac{1}{n}\sum_{i=1}^{n}\mathrm{x_i}+\mathrm{a}=\bar{X}+\mathrm{a}$ $\therefore \mathrm{u_i}-\bar{U}=(\mathrm{x_i}+\mathrm{a})-(\bar{X}+\mathrm{a})=\mathrm{x_i}-\bar{X}, \mathrm{i}=1,2,\ldots,\mathrm{n}$ $\Rightarrow \sum_{i=1}^{n}(u_i-\bar{U})=\sum_{i=1}^{n}(x_i-\bar{X})$ $\Rightarrow \frac{1}{n}\sum_{i=1}^{n}(u_i-\bar{U})^2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{X})^2$ $\Rightarrow \mathrm{Var(U)}=\mathrm{Var(X)}$	
8.	Mean=7, Variance=23.33, Standard Deviation=4.8	2
9.	M. D. (M) = 649.428	2
10.	Mean =27 M.D. (Mean)= 10.24	2
11.	$x_1 + x_2 + x_3 + \ldots + x_n = 84x$ $\frac{84x+100}{x+1} = 86$ x=7 Total number of test 7+1=8	2
12.	Mean($\bar{x}$)=$\frac{57+64+43+67+49+59+61+59+44+47}{10}=\frac{550}{10}=55$ Variance (σ^2) $=\frac{\sum(x_i-\bar{x})^2}{n}$ $=\frac{2^2+9^2+12^2+12^2+6^2+4^2+6^2+4^2+11^2+8^2}{10}$ $=\frac{662}{10}$ =66.2 Standard deviation(σ)=$\sqrt{\sigma^2}=\sqrt{66.2}=8.13$	2
13.	Total students in class =150 mean weight=60kg total weight =150×60=9000kg Let the total number of boys =x mean weight of boys =70kg total weight of boys=70xkg total number of girls = total students - no. of boys =150−x mean weight of girls=55kg total weight of girls =55×(150−x)=55×150−55x=(8250−55x)kg Total weight = weight of boys + weight of girls 9000=70x+(8250−55x) 9000=70x+8250−55x 9000−8250=70x−55x 750=15x x=15750 x=50	2

So number of boys =50
number of girls =150−50=100

14. $\sum x_i^2 = n\{\sigma^2 + (\bar{x})^2\} = 100\,(50^2 + 5^2) = 252500$ 2

15. Original sum of all the 10 items 2
=(mean x number of items)
=17 x 10
=170
New sum=170 – 21 + 12 =161
New mean = 161/10 =16.1

16. We observe that the average monthly wages in both the firms is same i.e.Rs.2800. Therefore, the firm with greater variance will have more variability. Thus, firm B has greater variability in individual wages. 2

17. First n natural numbers are = 1 , 2 , 3 , ………… , n 2

Mean $\bar{x} = \frac{(1+2+3+\cdots+n)}{n} = \frac{1}{n} \cdot \frac{n(n+1)}{2} = \frac{(n+1)}{2}$

Variance $\sigma^2 = \frac{\Sigma x^2}{n} - \bar{x}^2 = \frac{\Sigma n^2}{n} - \left\{\frac{(n+1)}{2}\right\}^2 = \frac{n(n+2)(2n+1)}{6n} - \frac{(n+1)^2}{4} = \frac{(n^2-1)}{12}$

18. Arranging the observations in ascending order :34 , 38 , 42 , 44 , 47 , 48 , 53 , 55 , 63 , 70 2

Median M = $\frac{47+48}{2}$ = 47.5

Calculation of mean deviation about the median .

x	$\lvert d \rvert = \lvert x - M \rvert$
34	13.5
38	9.5
42	5.5
44	3.5
47	0.5
48	0.5
53	5.5
55	7.5
63	15.5
70	22.5
	$\Sigma\lvert x - M \rvert = 84$

Mean Deviation = $\frac{\Sigma\lvert x-M \rvert}{n} = \frac{84}{10}$ = 8.4

19. Let $x_1, x_2, \ldots\ldots x_{100}$ be 100 observations and their mean = $\bar{x}$ and standard deviation = σ 2

Mean $\bar{x} = \frac{\Sigma x}{n}$ $50 = \frac{\Sigma x}{100}$ Sum of all observations Σx = 5000	$\sigma^2 = \frac{\Sigma x^2}{n} - \bar{x}^2$ $4^2 = \frac{\Sigma x^2}{100} - 50^2$ $1600 = \Sigma x^2 - 250000$ Sum of their squares $\Sigma x^2 = 251600$

20.	$$\sum_{i=1}^{18}(x-5)=3$$ $$\sum_{i=1}^{18}x-\sum_{i=1}^{18}5=3$$ $$\sum_{i=1}^{18}x-5\times 18=3$$ $$\sum_{i=1}^{18}x=93$$ Mean = $\frac{\sum x}{n}$ = $\frac{93}{18}$ = 5.17	$$\sum_{i=1}^{18}(x-5)^2=43$$ $$\sum_{i=1}^{18}x^2-10\sum_{i=1}^{18}x+\sum_{i=1}^{18}25=43$$ $$\sum_{i=1}^{18}x^2-10\times 93+25\times 18=43$$ $$\sum_{i=1}^{18}x^2=523$$ Standard Deviation = $\sqrt{\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2}$ = $\sqrt{\frac{523}{18}-\left(\frac{93}{18}\right)^2}$ 1.536	2