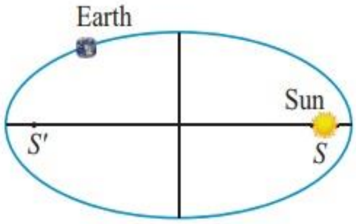
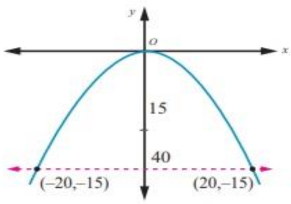
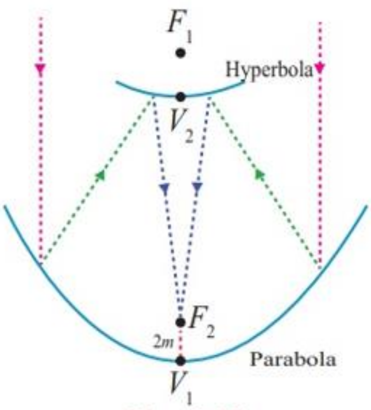
CHAPTER-11

CONIC SECTIONS

03 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.  Fig. 5.56	3
2.	A concrete bridge is designed as a parabolic arch. The road over bridge is 40m long and the maximum height of the arch is 15m . Write the equation of the parabolic arch.  Fig. 5.57	3
3.	The parabolic communication antenna has a focus at 2m distance from the vertex of the antenna. Find the width of the antenna 3m from the vertex.	3
4.	Certain telescopes contain both parabolic mirror and a hyperbolic mirror. In the telescope shown in figure the parabola and hyperbola share focus F_1 which is 14*m* above the vertex of the parabola. The hyperbola's second focus F_2 is 2*m* above the parabola's vertex. The vertex of the hyperbolic mirror is 1*m* below F_1. Position a coordinate system with the origin at the center of the hyperbola and with the foci on the *y* -axis. Then find the equation of the hyperbola.  	3
5.	A semielliptical archway over a one-way road has a height of 3*m* and a width of 12*m* . The truck has a width of 3*m* and a height of 2.7*m* . Will the truck clear the opening of archway ?	3

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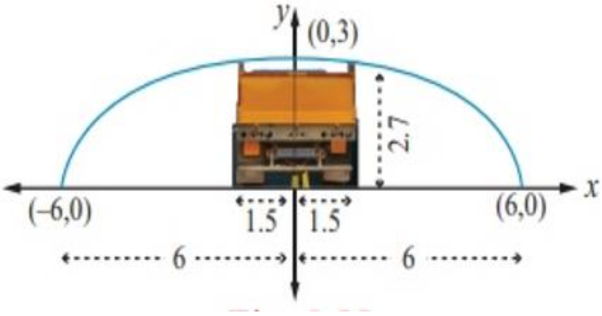
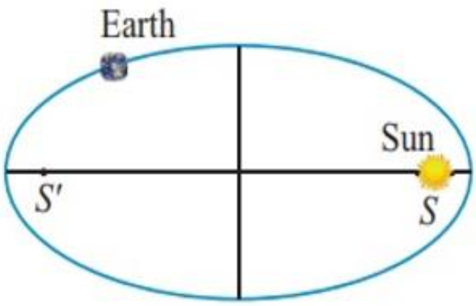
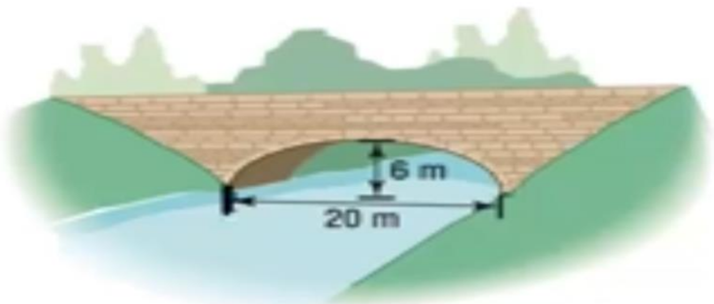
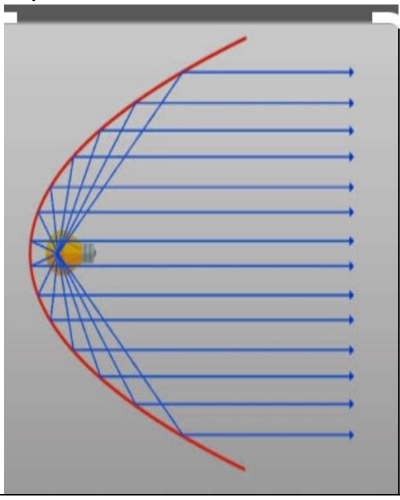
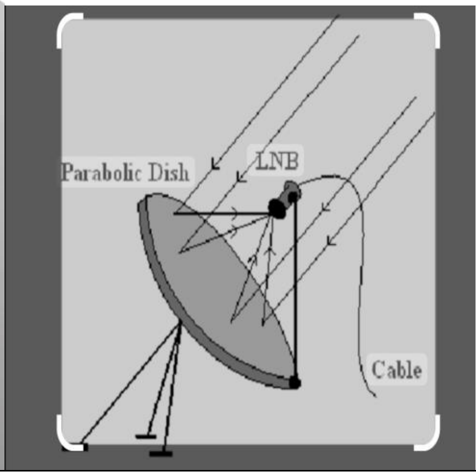
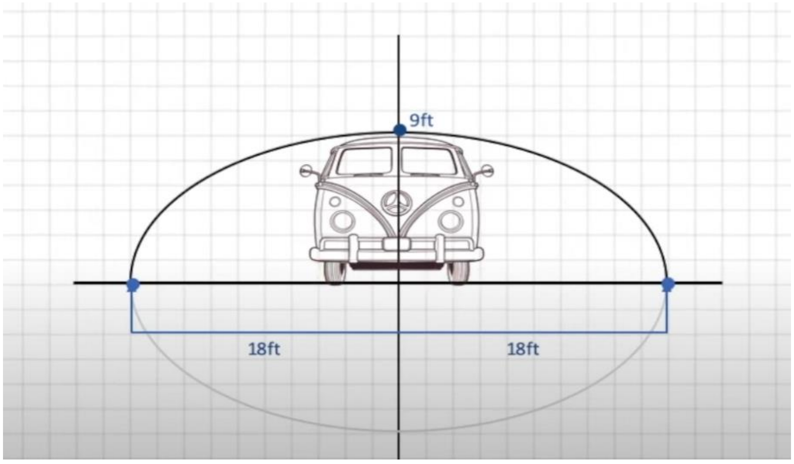
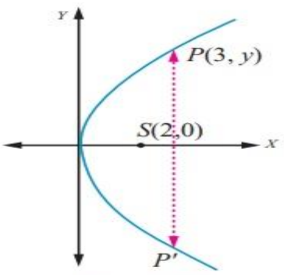
<tr><td></td><td>
</td><td></td></tr>
<tr><td>6.</td><td>The maximum and minimum distances of the Earth from the Sun respectively are 152 $\times10^6$ km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

</td><td>3</td></tr>
<tr><td>7.</td><td>Show that the set of all points such that the difference of their distances from (4, 0) and (– 4, 0) is always equal to 2 represent a hyperbola</td><td>3</td></tr>
<tr><td>8.</td><td>If the latus rectum of an ellipse with axis along x-axis and centre at origin is 10, distance between foci = length of minor axis, then find the equation of the ellipse .</td><td>3</td></tr>
<tr><td>9.</td><td>A bar of given length moves with its extremities on two fixed straight lines at right angles. Prove that Any point of the bar describes an ellipse</td><td>3</td></tr>
<tr><td>10.</td><td>A rod AB of length 15 cm rests in between two coordinate axes in such a way that the end point A lies on x-axis and end point B lies on y-axis. A point P(x, y) is taken on the rod in such a way that AP = 6 cm. Show that the locus of P is an ellipse.</td><td>3</td></tr>
<tr><td>11.</td><td>Find the locus of the mid points of chords of an ellipse drawn through the positive extremity of the minor axis.</td><td>3</td></tr>
<tr><td>12.</td><td>A civil engineer is given a work of renovating a semi-elliptical bridge. This bridge is 20m wide at the base and 6m high at the centre.

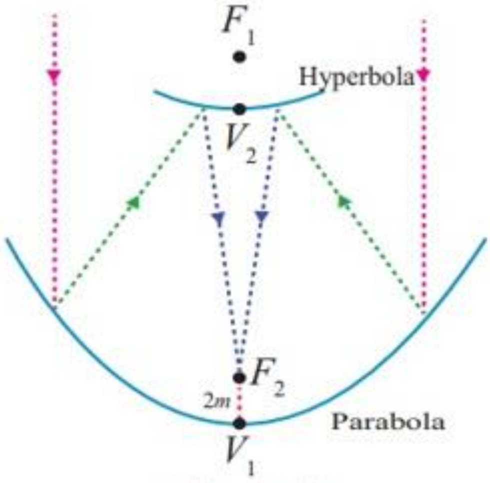
I) What could be the lattus rectum of the elliptical curve showing in the figure?
ii) At what distance from the centre, the height of the bridge would be 2m?

</td><td>3</td></tr>
<tr><td>13.</td><td>The most common modern applications of the parabolic reflector are in</td><td>3</td></tr>
</table>

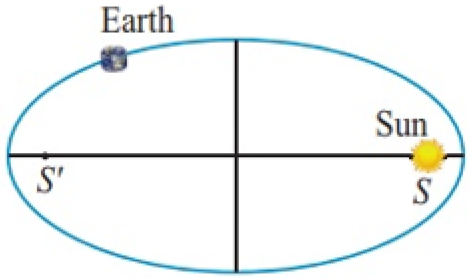
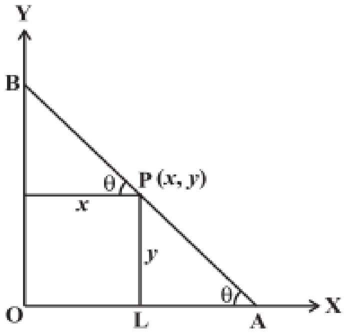
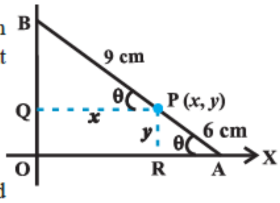
	satellite dishes, reflecting telescopes, radio telescopes, parabolic microphones, solar cookers, and many lighting devices such as spotlights, car headlights, PAR lamps and LED housings. I) If a parabolic reflector is 20 cm in diameter and 5 cm deep, find its focus ii).a parabolic reflector is 18 cm in diameter and 56 cm deep, find the latus rectum. Find the depth when diameter is 12 cm. iii) Also Find find the diameter when depth is  	
14.	Two friend chandan and sharda observe A semielliptical archway over a one-way road has a height of 9 ft and width of 36 ft Your van has a width of 12 ft and height of 8 ft .As it is shown below  (i)What is the equation for this elliptical archway? (ii)What Will your van be able to clear the opening of the archway?	3
15.	Find the equation of the circle passing through (0, 0) and making intercepts a and b on the co – ordinate axes.	3
16.	Find the equation of ellipse whose vertices are $(\pm 13,0)$ and foci are $(\pm 5,0)$.	3
17.	Find eccentricity and foci of hyperbola $2x^2 - 3y^2 = 5$	3
18.	Find the centre and radius of the circle $2x^2 + 2y^2 - x = 0$	3
19.	Find the equation of the circle which passes through the origin and cuts off the intercept ‘a’	3

	and ‘b’ on the coordinate axis.	
20.	Show that the equation $x^2+y^2-6x+4y-36=0$ represents a circle, also find its centre & radius?	3

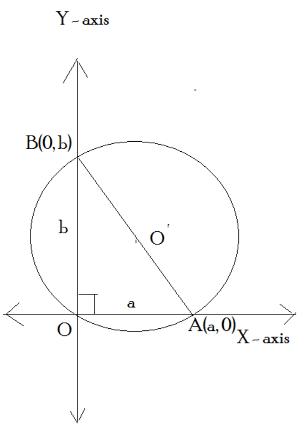
ANSWERS:

Q. NO	ANSWER	MARKS
1.	AS = 94.5×10^6 km, SA ' = 152×10^6 km $a + c = 152\times10^6$ $a - c = 94.5\times10^6$ Subtracting $2c = 57.5\times10^6 = 575\times10^5$ km	1+1+1
2.	From the graph the vertex is at (0, 0) and the parabola is open down Equation of the parabola is $x^2 = -4ay$ (−20, −15) and (20, −15) lie on the parabola $20^2 = -4a(-15)$ $4a = \frac{400}{15}$ $x^2 = \frac{-80}{3}\times y$ Therefore equation is $3x^2 = -80y$ 3.The parabolic communication antenna has a focus at 2m distance from the vertex of the antenna. Find the width of the antenna 3m from the vertex	1+1+1
3.	Let the parabola be $y^2 = 4ax$. Since focus is 2m from the vertex a = 2 Equation of the parabola is $y^2 = 8x$  Fig. 5.58 Let P be a point on the parabola whose x -coordinate is 3m from the vertex P (3, y) $Y^2 = 8\times3$ $y = \sqrt{(8x3)} = 2\sqrt{6}$ The width of the antenna 3m from the vertex is $4\sqrt{6}$	1+1+1
4.	Let V_1 be the vertex of the parabola and V_2 be the vertex of the hyperbola. $\overline{F_1F_2}$ = 14 - 2 = 12m, $2c = 12$, $c = 6$	3

	 The distance of center to the vertex of the hyperbola is $a = 6 - 1 = 5$ $b^2 = c^2 - a^2$ = 36 - 25 = 11. Therefore the equation of the hyperbola is $y^2/25 - x^2/11 = 1$ $\frac{y^2}{25} - \frac{x^2}{11} = 1$	
5.	Since the truck's width is $3m$, to determine the clearance, we must find the height of the archway $1.5m$ from the center. If this height is $2.7m$ or less the truck will not clear the archway. From the diagram $a = 6$ and $b = 3$ yielding the equation of ellipse as $\frac{x^2}{6^2} + \frac{y^2}{3^2} = 1$. The edge of the $3m$ wide truck corresponds to $x = 1.5m$ from center We will find the height of the archway $1.5m$ from the center by substituting $x = 1.5$ and solving for y $\frac{\left(\frac{3}{2}\right)^2}{36} + \frac{y^2}{9} = 1$ $y^2 = 9\left(1 - \frac{9}{144}\right)$ $= \frac{9(135)}{144} = \frac{135}{16}$ $y = \frac{\sqrt{135}}{4}$ $= \frac{11.62}{4}$ $= 2.90$ Thus the height of arch way $1.5m$ from the center is approximately $2.90m$. Since the truck's height is $2.7m$, the truck will clear the archway.	3

6.	$AS = 94.5\times10^6$ km, $SA' = 152 \times10^6$ km $a + c = 152 \times10^6$ $a - c = 94.5\times10^6$ Subtracting $2c = 57.5\times10^6 = 575\times10^5$ km  Distance of the Sun from the other focus is $SS' = 575\times10^5$ km.	3
7.	$\sqrt{(x+4)^2+y^2}-\sqrt{(x-4)^2+y^2}=2$ Solving we will get $\frac{x^2}{1}-\frac{y^2}{4^2-1^2}=1$	3
8.	$\frac{2b^2}{a}=10, 2c=2b \Rightarrow c=b, a^2=b^2+c^2 \Rightarrow a^2=2b^2$ $\frac{a^2}{a}=10 \Rightarrow a=10, b^2=50, so\ equation\ of\ ellipse\ is\ \frac{x^2}{100}+\frac{y^2}{50}=1$	3
9.	 True. Let P (x, y) be any point on the bar such that PA $= a$ and PB $= b$, clearly from the $x =$ OL $= b \cos\theta$ and $y =$ PL $= a \sin\theta$ These give $\frac{x^2}{b^2}+\frac{y^2}{a^2} = 1$, which is an ellipse.	3
10.	**Solution** Let AB be the rod making an angle θ with OX as shown in Fig 10.33 and P (x, y) the point on it such that AP = 6 cm. Since AB = 15 cm, we have PB = 9 cm. From P draw PQ and PR perpendiculars on y-axis and x-axis, respectively.  Fig 10.33	

	From Δ PBQ, $\cos\theta = \frac{x}{9}$ From Δ PRA, $\sin\theta = \frac{y}{6}$ Since $\cos^2\theta + \sin^2\theta = 1$ $\left(\frac{x}{9}\right)^2 + \left(\frac{y}{6}\right)^2 = 1$ or $\frac{x^2}{81} + \frac{y^2}{36} = 1$ Thus the locus of P is an ellipse.	
11.	Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Let the mid point of chord of contact be (h, k) Equation of chord when mid point is given is T = S' $\frac{xx'}{a^2} + \frac{yy'}{b^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ $\frac{xh}{a^2} + \frac{yk}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$ It passes through (0, b) $0 + \frac{bk}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$ $\frac{h^2}{a^2} + \frac{k^2}{b^2} = \frac{k}{b}$ So the required locus is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{y}{b}$ The equation of the parabola takes the form $x^2 = 4ay$. Since it passes through $\left(6, \frac{3}{100}\right)$, we have $(6)^2 = 4a\left(\frac{3}{100}\right)$, i.e., $a = \frac{36\times100}{12} = 300$ m Let AB be the deflection of the beam which is $\frac{1}{100}$ m. Coordinates of B are $(x, \frac{2}{100})$. Therefore $x^2 = 4 \times 300 \times \frac{2}{100} = 24$ i.e. $x = \sqrt{24} = 2\sqrt{6}$ metres	
12.	i) a=18 ,b = 9 x2/324 + y 2/81 = 1 ii)Using above equation x=6 y=? 36+4y^2 =324	3
13.	I) A parabolic reflection with diameter PR = 20 cm and OQ = 5c cm is shown below: Here, vertex of the parabola is (0,0) Let the focus be S (a, 0) Let the equation of the parabola be y ^ 2 = 4ax . Now, PR = 2c cm Rightarrow PQ = 10 cm Since it lies on the parabola sqrt = 4a Also, OQ = 5 cm .. Point P is (5, 10)	3

	10 ^ 2 = 4 (a) 5 100 = 20a a = 5 Focus is S(5, 0) which is same as point Q ii) diameter is 12.0 then I will be its depth 224/9 iii) If depth is 2 m then (2,y) lies on parabola Y^2=81/56 Y=9/2√7	
14.	i) a = 20,b = 6 $x^2/400 + y^2/36 = 1$ Required latus rectum=$2b^2/a$ =3.6m ii) $x^2/400 + y^2/36 = 1$ Let (p, 2) lie on ellipse. $P^2/400+4/36=1$ P=40√2 /3m.	3
15.	Let the circle cuts X –axis at point A and Y – axis at point B. Since the circle makes intercepts as a and b on the co-ordinate axes.  Therefore, co-ordinate of points A and B are (a, 0) and (0, b) Since angle in semi circle is 90^0 Equation of circle is (x – a)(x – 0) + (y – 0)(y – b) Or, $x^2 + y^2 - ax - by = 0$	3
16.	The vertices (± 13,0) lies on X –axis, therefore the equation will be of the form $x^2/a^2 + y^2/b^2 = 1$ Now vertices = (± 13,0) = (± a, 0)	3

	So, a = 13 Now foci = $(\pm 5,0) = (\pm ae, 0)$ So, ae = 5 Now $b^2 = a^2(1 – e^2) = 169 – 25 = 144$ So, $x^2/169 + y^2/144 = 1$	
17.	Given, $2x^2 – 3y^2 = 5$ So, $x^2/(5/2) – y^2/(5/3) = 1$ Here, $a^2 = 5/2$, $b^2 = 5/3$ So, $b^2 = a^2(e^2 – 1)$ $5/3 = 5/2(e^2 – 1)$ So, $e^2 = 5/3$ Foci = $(\pm ae, 0)$ = $(\pm \frac{5}{\sqrt{6}}, 0)$	3
18.	Given that, the circle equation is $2x^2 + 2y^2 – x = 0$ This can be written as: $\Rightarrow (2x^2-x) + y^2 = 0$ $\Rightarrow 2\{[x^2 – (x/2)] +y^2\} = 0$ $\Rightarrow \{ x^2 – 2x(¼) + (¼)^2\} +y^2 – (¼)^2 = 0$ Now, simplify the above form, we get $\Rightarrow (x- (¼))^2 + (y-0)^2 = (¼)^2$ The above equation is of the form $(x – h)^2 + (y – k)^2= r^2$ Therefore, by comparing the general form and the equation obtained, we can say h= ¼ , k = 0, and r = ¼.	3
19.	OA = a and OB = b Which implies OC = a/2 and OD = b/2 Therefore h = a/2 and k = b/2 Therefore centre of the circle is (a/2, b/2) Now, OE = r (radius) $\sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$ = r	3

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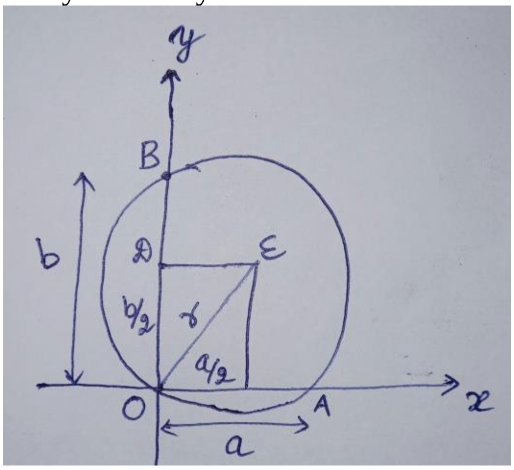
$r^2 = \frac{a^2}{4} + \frac{b^2}{4}$
Now, equation of circle is given by
$(x - h)^2 + (y - k)^2 = r^2$
$(x - a/2)^2 + (y - b/2)^2 = r^2$
$x^2 + y^2 - ax - by = 0$



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It is of the form $x^2+y^2+2gx+2fy+c=0$

Where $2g =-6$, $2f=4$ & $c=-36$

$\therefore$ $g =-3$, $f=2$ & $c=-36$
Thus, center of the circle is $(-g,-f)=(3,-2)$
Radius of the circle is $\sqrt{g^2 + f^2 - c} = \sqrt{9 + 4 + 36}$
=7units

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