

CHAPTER-8
BINOMIAL THEOREMS
03 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Which is larger $(1.01)^{1000000}$ or 10,000 .	3
2.	Prove that $\sum_{r=0}^n 3^r C_r = 4^n$.	3
3.	Using binomial theorem. Prove that $6^n - 5n$ always leaves remainder 1 when divided by 25 .	3
4.	Find m such that the coefficient of x^2 in expansion of $\left(mx + \frac{m}{x}\right)^6$ is 960	3
5.	If the expansion of $(1 - ax)^n = 1 - 21x + 189x^2 - 945x^3 + \dots$, then find a and n	3
6.	Find the term which is at equal distance from beginning and from end in the expansion of $\left(\frac{x}{3} + \frac{1}{x}\right)^8$	3
7.	Prove by using binomial theorem that $6^n - 5n - 1$ is divisible by 25, for all natural number n .	3
8.	Expand $(a + b)^5$ by using Pascal's triangle.	3
9.	Find the coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$	3
10.	Show that $9^{n+1} - 8n - 9$ is divisible by 64 , whenever n is a positive integer	3
11.	The first three terms in the expansion of a binomial are 1 , 10 and 40..Find the expansion	3
12.	Which is larger $(1.01)^{1000000}$ or 10,000 ?	3
13.	Find the expansion of $(3x^2 - 2ax + 3a^2)^3$ using binomial theorem.	3
14.	Find the coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^{11}$.	3
15.	Find n, in the expansion of $(1 + x)^n$, coefficients of 2nd, 3rd and 4th terms are in A.P. Then n is equal to	3
16.	Find the coefficient of x^5 in binomial expansion of $(1 + 2x)^6(1 - x)^7$	3
17.	What are the last three digits of 27^{26} ?	3
18.	Show that $2^{3n} - 7n - 1$ is divisible by 49, where n is a positive integer.	3

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$(1.01)^{10,00000}$ or 10,000 $(1.01)^{10,00000} = (1 + 0.01)^{10,00000}$ $= {}^{10,00000}C_0 + {}^{10,00000}C_1(0.01) + \text{other +ve term.}$ $= 1 + 10,00000 \times 0.01 + \text{other positive term}$ $= 1 + 10,000$ $= 10,001$ Hence $(1.01)^{10,00000} > 10,000$	3
2.	$(2) \sum_{r=0}^n 3^r {}^nC_r = \sum_{r=0}^n {}^nC_r 3^r$ $= {}^nC_0 + {}^nC_1 \cdot 3 + {}^nC_2 \cdot 3^2 + \dots + {}^nC_n 3^n$ $= (1 + 3)^n$ $= 4^n \text{ (RHS)}$	3
3.	$6^n = (1 + 5)^n$ $= 1 + {}^nC_1 5 + {}^nC_2 5^2 + {}^nC_3 5^3 + \dots + 5^n$ $= 1 + 5n + 5^2 ({}^nC_2 + {}^nC_3 5 + \dots + 5^{n-2})$ $6^n - 5n = 1 + 25 ({}^nC_2 + {}^nC_3 5 + \dots + 5^{n-2})$ $= 1 + 25k \text{ (Hence Proved)}$	3
4.	$\left(mx + \frac{m}{x}\right)^6 = m^6 \left(x + \frac{1}{x}\right)^6$ Then, coefficient of x^2 is $m^6 {}^6C_2$ $m^6 {}^6C_2 = 960$ $m^6 = \frac{960}{15} = 64$ $m = 2$	3
5.	$(1 - ax)^n = 1 - nax + \frac{n(n-1)}{2} a^2 x^2 - \frac{n(n-1)(n-2)}{6} a^3 x^3 \dots,$ Comparing $na = 21$ $\frac{n(n-1)}{2} a^2 = 189$ $n(n-1)a^2 = 378$ Thus, $(n-1)a = 18$ Also $\frac{n}{n-1} = \frac{7}{6}$ $n = 7$ and $a = 3$	3
6.	The term which is at equal distance from beginning and from end is middle term	3

	Thus, $T_5 = 8C_4 \left(\frac{x}{3}\right)^4 \left(\frac{1}{x}\right)^4 = \frac{70}{81}$	
7.	$(1+x)^n = C(n,0) + C(n,1)x + C(n,2)x^2 + \dots + C(n,n)x^n$ So, $(1+x)^n - C(n,1)x - 1 = C(n,2)x^2 + \dots + C(n,n)x^n$ Putting $x=5$, we get $6^n - 5n - 1$ is divisible by 25.	3
8.	By using Pascal triangle, the coefficients are 1,5,10,10,5,1 $(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + b^5$	3
9.	$(1+x)(1-x)^n = (1+x)\left\{1 - nx + \frac{n(n-1)x^2}{2} - \dots + (-1)^n x^n\right\}$ So, coefficient of $x^n = (-1)^n C(n,n) + (-1)^{n-1} C(n,n-1) = (-1)^n (1-n)$	3
10.	$(1+a)^m = {}^m C_0 + {}^m C_1 a + {}^m C_2 a^2 + \dots + {}^m C_m a^m$ putting $a=8$ and $m=n+1$ we get $(1+8)^{n+1} = {}^{n+1} C_0 + {}^{n+1} C_1 8 + {}^{n+1} C_2 8^2 + \dots + {}^{n+1} C_{n+1} 8^{n+1}$ $9^{n+1} = 9 + 8n + 64 \left[{}^{n+1} C_2 + {}^{n+1} C_3 8 + \dots + {}^{n+1} C_{n+1} 8^{n-1} \right]$ So $9^{n+1} - 8n - 9 = 64k$ implies the expression is divisible by 64	3
11.	Let the expansion be $(a+b)^n$, $(1+2)^5$ $a^n = 1$(i) $na^{n-1}b = 10$(ii) $\frac{n(n-1)}{2} a^{n-2} b^2 = 40$ on simplifying $a=1$, $b=2$	3
12.	Splitting 1.01 and using binomial theorem to write the first few terms we have $(1.01)^{1000000} = (1+0.01)^{1000000} = 1 + 10000 + \text{other positive terms}$ Which is greater than 10000	3
13.	$(27a^6 - 54ax^3 + 117a^2x^4 - 116a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6)$	3
14.	990	3
15.	7	3
16.	Using binomial theorem we will expand both the terms. We know that, $(x+y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1}y + {}^nC_2 x^{n-2}y^2 + {}^nC_3 x^{n-3}y^3 + \dots + {}^nC_r x^{n-r}y^r + \dots + {}^nC_0 x^0 y^n$ Applying the formula we get, $(1+2x)^6 (1-x)^7 = (1 + 6C_1(2x) + 6C_2(2x)^2 + 6C_3(2x)^3 + 6C_4(2x)^4 + 6C_5(2x)^5 + 6C_6(2x)^6)$ $(1 - 7C_1x + 7C_2(x)^2 - 7C_3(x)^3 + 7C_4(x)^4 - 7C_5(x)^5 + 7C_6(x)^6 - 7C_7(x)^7)$ $= (1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6) (1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7)$ Clearly, it can be determined that the coefficient of x^5 is	3

	$\Rightarrow 1 * (-21) + 12 * 35 + 60(-35) + 160 * 21 + 240 * (-7) + 192 * 1$ $\Rightarrow 171$ <p>Therefore, the coefficient of x^5 in $(1 + 2x)^6 (1 - x)^7$ is 171.</p>	
17.	<p>In order to determine the same, the value 27^{26} needs to be reduced into the form $(730 - 1)^n$. After using simple binomial expansion, the digits can be obtained.</p> <p>Now, we have, $27^2 = 729$</p> <p>Thus,</p> $27^{26} = (729)^{13} = (730 - 1)^{13}$ $= {}^{13}C_0 (730)^{13} - {}^{13}C_1 (730)^{12} + {}^{13}C_2 (730)^{11} - \dots - {}^{13}C_{10} (730)^3 + {}^{13}C_{11} (730)^2 - {}^{13}C_{12} (730) + 1$ $= 1000m + [(13 \times 12)/2] \times (14)2 - (13) \times (730) + 1$ <p>Herein, we can say that 'm' is a positive integer</p> <p>Thus,</p> $= 1000m + 15288 - 9490 = 1000m + 5799$ <p>Therefore, the last three digits of 27^{26} are 799.</p>	3
18.	<p>Given $2^{3n} - 7n - 1 = 8^n - 7n - 1$</p> $= (1+7)^n - 7n - 1$ $= {}^nC_0 + {}^nC_1 7 + {}^nC_2 7^2 + \dots + {}^nC_n 7^n - 7n - 1$ $= 1 + 7n + {}^nC_2 7^2 + \dots + {}^nC_n 7^n - 7n - 1$ $= {}^nC_2 7^2 + \dots + {}^nC_n 7^n$ $= 49({}^nC_2 + \dots + {}^nC_n 7^{n-2})$ <p>Which is divisible by 49</p>	3