

CHAPTER-13
LIMITS & DERIVATIVES
03 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{x}$	3
2.	If the function $f(x)$ satisfies $\lim_{x \rightarrow 1} \frac{f(x) - 2}{x^2 - 1} = \pi$, evaluate $\lim_{x \rightarrow 1} f(x)$	3
3.	Differentiate $\frac{\sec x + \tan x}{\sec x - \tan x}$ with respect to x .	3
4.	If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$, then show that $\frac{dy}{dx} = y$.	3
5.	Find the derivative of $(1-x)(2-x)(3-x)\dots(n-x)$ at $x = 1$.	3
6.	Let $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{where } x \neq \frac{\pi}{2} \\ 3, & \text{where } x = \frac{\pi}{2} \end{cases}$ and if $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$, then find the value of k .	3
7.	Let $f(x) = \begin{cases} 1, & \text{for } x < 0 \\ 1 + \sin x, & \text{for } x \geq 0 \end{cases}$ using first principle check $f'(0)$ exists or not.	3
8.	Find $\lim_{x \rightarrow 0} \frac{x \cos x - \log(1+x)}{x^2}$	3
9.	Find the derivative of $\frac{x^2}{x^2 + 2x + 2023}$	3
10.	Evaluate: $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$	3
11.	Find the derivative of $\frac{x^5 - \cos x}{\sin x}$	3
12.	Find the derivative of $\frac{x + \cos x}{\tan x}$	3
13.	Suppose $f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$ and if $\lim_{x \rightarrow 1} f(x) = f(1)$. What are the possible values of a	3

	and b?	
14.	Find the derivative of expression $1+x+x^2+x^3+\dots+x^{50}$ for $x=1$.	3
15.	If $y = \left[\sin \frac{x}{2} + \cos \frac{x}{2} \right]^2$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$.	3

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\frac{1}{2}$	3
2.	2	3
3.	$\frac{2\cos x}{(1 - \sin x)^2}$	3
4.	$y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \Rightarrow \frac{dy}{dx} = \frac{d}{dx} \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right)$ $= 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \frac{4x^3}{4!} + \dots = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = y$ $\therefore \frac{dy}{dx} = y$	3
5.	$\frac{d}{dx} [(1-x)(2-x)(3-x)\dots(n-x)]$ $= (-1)(2-x)(3-x)\dots(n-x) +$ $(-1)(1-x)(3-x)\dots(n-x) + \dots + (-1)(1-x)(2-x)(3-x)\dots[(n-1)-x]$ $\frac{d}{dx} [(1-x)(2-x)(3-x)\dots(n-x)]_{x=1}$ $= (-1)(2-1)(3-1)\dots(n-1) + 0 + \dots + 0$ $= (-1)(n-1)!$	3
6.	$\lim_{x \rightarrow \frac{\pi}{2}} f(x) = \lim_{x \rightarrow \frac{\pi}{2}} \frac{k \sin\left(\frac{\pi}{2} - x\right)}{2\left \frac{\pi}{2} - x\right } = \frac{k}{2} \times 1 = \frac{k}{2}$ But given $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right) \Rightarrow \frac{k}{2} = 3 \Rightarrow k = 6$	3
7.	$\lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} = 1 \text{ but } \lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} = 0$ Hence $f'(0)$ does not exist.	3
8.	We know that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ And $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$ Use the above series we get the answer $\frac{1}{2}$	3

9.	This is into the form $\frac{u(x)}{v(x)}$, use derivative property $\frac{u(x)}{v(x)}$ we get the required answer	3
10.	$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{\sin x (\frac{1}{\cos x} - 1)}{\sin^3 x}$ $= \lim_{x \rightarrow 0} \frac{1 - \cos x}{\cos x \sin^2 x}$ $= \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{\cos x (4 \sin^2 \frac{x}{2} \cos^2 \frac{x}{2})}$ $= \frac{1}{2}$	3
11.	Let $f(x) = \frac{x^5 - \cos x}{\sin x}$ Differentiating w.r.t x we get $f'(x) = \frac{(5x^4 + \sin x) \sin x - (x^5 - \cos x) \cos x}{\sin^2 x}$ $= \frac{-x^5 \cos x + 5x^4 \sin x + 1}{\sin^2 x}$	3
12.	Let $f(x) = \frac{x + \cos x}{\tan x}$ Differentiating w.r.t x we get $f'(x) = \frac{(1 - \sin x) \tan x - (x + \cos x) \sec^2 x}{\tan^2 x}$	3
13.	We have $\lim_{x \rightarrow 1} f(x) = f(1) \Leftrightarrow \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$ $\Leftrightarrow \lim_{x \rightarrow 1^-} f(x) = f(1)$ and $\lim_{x \rightarrow 1^+} f(x) = f(1)$ $\Leftrightarrow \lim_{h \rightarrow 0} f(1 - h) = 4$ and $\lim_{h \rightarrow 0} f(1 + h) = 4$ $\Leftrightarrow \lim_{h \rightarrow 0} \{a + b(1 - h)\} = 4$ and $\lim_{h \rightarrow 0} \{b - a(1 + h)\} = 4$ $\Leftrightarrow a + b = 4$ and $b - a = 4$ $\Leftrightarrow a = 0$ and $b = 4$.	3
14.	By the power rule of differentiation we get, $f'(x) = 1 + 2x + 3x^2 + 4x^3 + \dots + 50x^{49}$ By substituting $x=1$ in the above derivative we get, $f'(1) = 1 + 2 + 3 + \dots + 50$ $= \frac{50(50+1)}{2}$	3

	=25×51 =1275	
15.	$\frac{\sqrt{3}}{2}$.	3