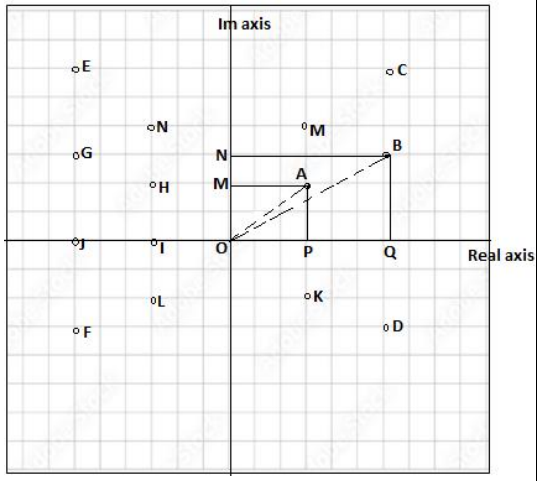
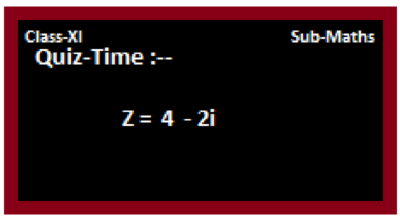
CHAPTER-5

COMPLEX NUMBERS

04 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	If Z_1 = 2-I and Z_2 = 1+i Find $\left\|\frac{z_1+z_2+1}{z_1-z_2+1}\right\|$	4
2.	Find the modulus and argument of the complex number z=-1-i$\sqrt{3}$	4
3.	Anuma and Pritam have created a game on Argand plane. They prepared a board having an Argand plane drawn on it. They have taken two dice of different colors – red & blue. They decided to take the number on red die as real part and that on blue as imaginary part. Now both have thrown that pair of dice and marked the number by each respectively as A & B. Now they are asking each other some questions. Help them out to find the answers …  1. Which complex numbers are represented by the points A & B resp ? i)2+3i, 3+6i ii) 2+6i, 3+2i iii) 3+2i, 6+3i iv) 6+3i, 3+2i 2. Lengths representing modulus of the points A & B resp. are – i)AP,BQ ii) AM, BN iii) OA, OB iv) OM, ON 3. Conjugates of the complex numbers at A & B are respectively represented by the points – i)H,G ii) K,D iii) I, J iv) L, F 4. Multiplicative inverse of the complex number A is – i) $\frac{3}{13}+\frac{2}{13}i$ ii) $\frac{2}{13}+\frac{3}{13}i$ iii) $\frac{2}{13}-\frac{3}{13}i$ iv) $\frac{3}{13}-\frac{2}{13}i$	4
4.	Teacher conducts a quiz on Complex numbers in class-XI. She writes a complex number Z on the board : $Z = 4 - 2i$  She divides the class into groups and asks the following questions on the given complex number to the students. Time allotted for each question is 1 minute. What will be the correct answers for the following questions :	4

	1) What is the value of $\|Z\|$ 2) Find the value of $Z\bar{Z}$ 3) In which quadrant of the Argand plane do Z & $\bar{Z}$ respectively lie? 4) Find Im.$\left(\frac{Z}{\bar{Z}}\right)$.	
5.	Let $z = a + ib$ and $w = \frac{1-iz}{z-i}$. If $\|w\|=1$, then show that z is purely real.	4
6.	If $x + iy = \sqrt[3]{u + iv}$, then show that $\frac{uy+vx}{x^2-y^2} = 4xy$	4
7.	We have i=$\sqrt{-1}$ so we can write the higher powers of i as follows $i^2=-1, i^3=-i, i^4=1, i^5=i, i^6=-1$.......... on the basis of above information, answer the following questions Q(i) The value of $i^{37}$is equal to a) i b) -i c) 1 d) -1 Q(ii) The value of $i^{-30}$is equal to a) i b) -i c) 1 d) -1 Q(iii) The value of $i^{77}$is equal to a) i b) -i c) 1 d) -1 Q(iv) The value of $i^{1000}$is equal to a) i b) -i c) 1 d) -1	4
8.	Two complex numbers Z_1=a+ib and Z_2=c+id are said to be equal if a=c and b=d on the basis of above information, answer the following questions Q(i) If (3a-6)+2ib=-6b+(6+a)I, then the real values of a and b are respectively a) -2,2 b) 3,-3 c)4,2 d) 2,-2 Q(ii) If (2a+2b)+i(b-a)=-4i, then the real values of a and b are respectively a) -2,2 b) 2,-2 c)4,2 d) 2,-3 Q(iii) If $\left(\frac{1-i}{1+i}\right)^{100}$=a+ib then the real values of a and b are respectively a) 1,0 b) 0,-2 c)1,2 d) 2,-3 Q(iv) If $\frac{(1+i)^2}{2-i}$=a+ib then the real values of a + b are respectively a) 5 b) 3/5 c)1,2 d) 2/5	4
9.	Find the real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real	4
10.	If z_1 , z_2 are complex number such that $\frac{2z_1}{3z_2}$ is purely imaginary number then find $\left\|\frac{z_1-z_2}{z_1+z_2}\right\|$	4

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$z_1 = 2 - i, z_2 = 1 + i$ $\left\|\frac{z_1+z_2+1}{z_1-z_2+1}\right\| = \left\|\frac{(2-i)+(1+i)+1}{(2-i)-(1+i)+1}\right\| = \left\|\frac{4}{2-2i}\right\| = \left\|\frac{4}{2(1-i)}\right\|$ $= \left\|\frac{2}{(1-i)} \times \frac{1+i}{1-i}\right\| = \left\|\frac{2(1+i)}{(1)^2-(i)^2}\right\| = \left\|\frac{2(1+i)}{1+1}\right\| = \left\|\frac{2(1+i)}{2}\right\| = \|1+i\|$ $= \sqrt{1^2+1^2} = \sqrt{2}$ Thus $\left\|\frac{z_1+z_2+1}{z_1-z_2+1}\right\|$ is $\sqrt{2}$	4
2.	z=-1-i$\sqrt{3}$ let rcosθ =-1 and rsinθ =-−$\sqrt{3}$ on squaring and adding ,we obtain $(rcos\theta)^2 + (rsin\theta)^2 = (-1)^2 + (-\sqrt{3})^2$ $\Rightarrow r^2 = 4$ $\Rightarrow r = 2$ (conventiationally,r>0) Thus, modulus = 2 Therefore 2cosθ=-1 and 2sinθ=-$\sqrt{3}$ $\Rightarrow$cosθ=$\frac{-1}{2}$ and sinθ=$\frac{-\sqrt{3}}{2}$ Since both the values of sinθ and cosθ are negative and sin$\theta$and cosθ are negative in III quadrant, argument =-$(\pi - \frac{\pi}{3})$ =$\frac{-2\pi}{3}$ Thus, the modulus and argument of the complex number -1-i$\sqrt{3}$ are 2 and $\frac{-2\pi}{3}$ respectively.	4
3.	1. iii) 2. iii) 3. ii) 4. iv)	4
4.	1. $2\sqrt{5}$ 2. 20 3. 4^{th} & 1^{st} 4. - 4/5	4
5.	$$w = \frac{1-iz}{z-i} = \frac{1-i(a+ib)}{a+ib-i} = \frac{(1+b)-ia}{a+i(b-1)}$$ |w|=1 => $\frac{(1+b)-ia}{a+i(b-1)} = 1$ => $\left\|\frac{(1+b)-ia}{a+i(b-1)}\right\| = \|1\|$ => $\sqrt{(1+b)^2+(-a)^2} = \sqrt{a^2+(b-1)^2}$ => b=0 So z is purely real	4

6.	$u = x^3 - 3xy^2,\ v = 3x^2y - y^3$ $\therefore \frac{u}{x} + \frac{v}{y} = \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}$ $= \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y}$ $= x^2 - 3y^2 + 3x^2 - y^2$ $= 4x^2 - 4y^2$ $= 4(x^2 - y^2)$ $\therefore \frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$ $= \frac{uy + vx}{x^2 - y^2} = 4xy$	4
7.	(i)a (ii) b (iii) a (iv) c	4
8.	(i)a (ii) b (iii) a (iv) d	4
9.	Solution: let z =$\frac{3+2isin\theta}{1-2isin\theta}$ On rationalization Z= 3+8isinθ-4sin$^2\theta$/1+4 sin$^2\theta$ Z= 3-4 sin$^2\theta/1 + 4\sin 2\theta$ + 8isin θ/1+4 sin$^2\theta$ im(z)=0 8isin θ/1+4 sin$^2\theta$=0 8isin $\theta = 0$ sin θ=0 $\theta = n\pi,$	4
10.	Given $\frac{2z_1}{3z_2}$ is purely imaginary =$\frac{2z_1}{3z_2}$= μi , $\mu \in R$ =$\frac{z_1}{z_2}$= $\frac{3}{2}$ μi $\left\lvert\frac{z_1-z_2}{z_1+z_2}\right\rvert$= $\left\lvert\frac{z_{1/z_2}-z_2/z_2}{z_{1/z_2}+z_{2/z_2}}\right\rvert$ $\left\lvert\frac{\frac{3}{2}\ \mu i - 1}{\frac{3}{2}\ \mu i + 1}\right\rvert$ $\left\lvert\frac{3\ \mu i-2}{2\mu i+2}\right\rvert$, (Z=a+ib) where $\lvert z\rvert$=$\sqrt{a^2+b^2}$ $\left\lvert\frac{\sqrt{-2^2 + (3\mu)^2}}{\sqrt{2^2 + (3\mu)^2}}\right\rvert$ $\frac{\sqrt{4+9\mu^2}}{\sqrt{4+9\mu^2}}$=1	4