CHAPTER-13

LIMITS & DERIVATIVES

04 MARK TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	CASE BASED:  The equation of the path traced by a roller-coaster is given by the polynomial f(x)=a(x+9)(x+1)(x-3).If the roller -coaster crosses y-axis at a point(0,-1),answer the following: (i)Find the value of a (iii) Find f ´(x) at x=1	4
2.	The relation between height of the plant (y in cm) with respect to exposure to sunlight is governed by the equation y=4x-$\frac{1}{2}x^2$ where x is the number of days exposed to sunlight. (i) Find the rate of growth of the plant with respect to sunlight. (ii) What is the rate of growth at x=1	4
3.	Case Based Question-1: In class XI science, the teacher is explaining the concept of derivative and he defined the derivative of a real function $f(x)$ as $f^1(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ and said that this is called the first principle of derivative. But one of the students asked that this is looking like a limit and what they used to call the left-hand and right-hand limits of the above limit. Then the teacher replied that the left-hand limit of $\frac{f(x+h)-f(x)}{h}$ at $h=0$ is called left derivative $Lf^1(x)$ and the right-hand limit is called right derivative $Rf^1(x)$. Based on the above information, answer the following questions. (i)) Verify the function $f(x)=\|x\|$ is differentiable at $x=0$ (ii) Find the derivative of $f(x)=e^x$ by using first principle.	4(2+2)
4.	Case Based Question -2: Given $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots,\quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots,$ and $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$ Based on this information find the following: (i). Evaluate $\lim_{x \to 0} \frac{x - \sin x}{x^3}$. (ii). Evaluate $\lim_{x \to 0} \frac{1-\cos x}{x^2}$.	4 1 1

	(iii). Evaluate $\lim_{x \to 0}\left(\frac{e^{3x}-1}{e^{5x}-1}\right)$. (iv). $\lim_{x \to 0}\left(\frac{e^{x}-1-x}{x^{2}}\right)$	1 1
5.	Mr Amit has a rectangular plot, which is used for growing vegetables. Perimeter of plot is 50 m. Length and width of plot are x m and y m respectively.  Based on the above information, answer the following questions. 1) Area function, A(x) = A) $x^2 – 5$ B) $25x – x^2$ C) $x^2 – 25$ D) 25 – x 2) Derivative of A(x) w.r.t. x, A’(x) = A) 2x B) – 2x C) 25 – 2x D) 2x – 25 3) Value of x for which A’(x) = 0 is A) 25 B) 12.5 C) 5 D) 0 4) Value of A’(x) at x = 12.5 is A) 156.25 B) 250 C) 0 D) 144.25	4
6.	Raj was learning limit of a polynomial function from his tutor Rajesh. His tutor told that a function f is said to be a polynomial function if $f(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots\ldots + a_nx^n$ where a_i’s are real number, n is a whole number and $a_n \neq 0$. Then limit of a polynomial function f(x) = $\lim_{x\to a} f(x)$ = $\lim_{x\to a}(a_0 + a_1x + a_2x^2 + \ldots\ldots\ldots + a_nx^n)$ = $\lim_{x\to a} a_0 + \lim_{x\to a} a_1x + \lim_{x\to a} a_2x^2 + \ldots\ldots\ldots + \lim_{x\to a} a_nx^n$ = $a_0 + a_1a + a_2a^2 + \ldots\ldots\ldots + a_na^n$ = f(a) Based on above information, answer the following questions. 1) $\lim_{x\to -1}(1 + x + x^2 + \ldots\ldots\ldots + x^9)$ is equal to A) 0 B) 1 C) 2 D) 3 2) $\lim_{x\to 5}[x^2(x-1)]$ is equal to A) 10 B) 100 C) 25 D) 125 3) $\lim_{x\to -3}(x^3 + x + 2)$ is equal to A) 28 B) – 28 C) 30 D) – 15 4) $\lim_{x\to 4}(x^4 - x^3)$ is equal to A) 192 B) 180 C) 50 D) 165	4

7.	Differentiate the given function $\frac{\sin x+\cos x}{\sin x-\cos x}$ with respect to x.	4
8.	A function f is said to be a rational function, if $f(x)=\frac{g(x)}{h(x)}$, where g(x) and h(x) are polynomial functions such that h(x)≠ 0. Then $\lim_{x\to a} f(x)=\lim_{x\to a}\frac{g(x)}{h(x)}=\frac{\lim_{x\to a} g(x)}{\lim_{x\to a} h(x)}=\frac{g(a)}{h(a)}$. However, if h(a) =0, then there are two cases arise , i) $g(a)\neq 0$ ii) $g(a)=0$ In first case we say that the limit does not exist. In second case, we can find the limit. Based on above information, answer the following questions. i) $\lim_{x\to -1}\left(\frac{x^{10}+x^5+1}{x-1}\right)$ *is equal to* a) ½ b) - ½ c) 2 d) $\frac{3}{2}$ ii) $\lim_{x\to -1}\frac{(x-1)^2+3x^2}{(x^4+1)^2}$ is equal to a) $\frac{7}{4}$ b) $\frac{6}{5}$ c) $\frac{4}{7}$ d) $\frac{3}{4}$	4

ANSWERS:

Q. NO	ANSWER	MARKS
1.	(i)$\frac{1}{27}$ (ii)$\frac{-4}{27}$	4
2.	(i) 4-x (ii)3	4
3.	(i) $f(x)=\begin{cases} x, & x\geq 0 \\ -x, & x<0 \end{cases}$ $\therefore Lf^1(0)=\frac{d}{dx}(-x)=-1$ and $\therefore Rf^1(0)=\frac{d}{dx}x=1$ Hence derivative of $f(x)=\|x\|$ at $x=0$ does not exist. (ii) $f^1(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{e^{x+h}-e^x}{h}=\lim_{h\to 0}\frac{e^x.e^h-e^x}{h}$ $=\lim_{h\to 0}\frac{e^x(e^h-1)}{h}=\lim_{h\to 0}e^x\lim_{h\to 0}\frac{(e^h-1)}{h}=e^x\times 1=e^x$	4
4.	(i) $\lim_{x\to 0}\frac{x-\sin x}{x^3}=\lim_{x\to 0}\frac{x-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-....\right)}{x^3}=$ $\lim_{x\to 0}\frac{x-x+\frac{x^3}{3!}-\frac{x^5}{5!}+....}{x^3}=\lim_{x\to 0}\frac{x^3\left(\frac{1}{3!}-\frac{x^2}{5!}+....\right)}{x^3}=\lim_{x\to 0}\left(\frac{1}{3!}-\frac{x^2}{5!}+....\right)=\frac{1}{3!}=\frac{1}{6}$ (ii) $\lim_{x\to 0}\frac{1-\cos x}{x^2}=\lim_{x\to 0}\left(\frac{1-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-...\right)}{x^2}\right)$ $=\lim_{x\to 0}\left(\frac{1-1+\frac{x^2}{2!}-\frac{x^4}{4!}+........}{x^2}\right)=\lim_{x\to 0}\frac{x^2\left(\frac{1}{2!}-\frac{x^2}{4!}+...\right)}{x^2}$ $=\lim_{x\to 0}\left(\frac{1}{2!}-\frac{x^2}{4!}+...\right)=\frac{1}{2!}=\frac{1}{2}$ (iii) $\lim_{x\to 0}\left(\frac{e^{3x}-1}{e^{5x}-1}\right)=\lim_{x\to 0}\left(\frac{3.\frac{e^{3x}-1}{3x}}{5.\frac{e^{5x}-1}{5x}}\right)=\frac{3.\lim_{x\to 0}\left(\frac{e^{3x}-1}{3x}\right)}{5.\lim_{x\to 0}\left(\frac{e^{5x}-1}{5x}\right)}=\frac{3\times 1}{5\times 1}=\frac{3}{5}$	4

	(iv) $\lim_{x\to 0}\left(\frac{e^x-1-x}{x^2}\right)=\lim_{x\to 0}\left(\frac{1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+......-1-x}{x^2}\right)$ $=\lim_{x\to 0}\left(\frac{\frac{x^2}{2!}+\frac{x^3}{3!}+......}{x^2}\right)=\lim_{x\to 0}x^2\left(\frac{\frac{1}{2!}+\frac{x}{3!}+......}{x^2}\right)=\lim_{x\to 0}\left(\frac{1}{2!}+\frac{x}{3!}+......\right)=\frac{1}{2!}=\frac{1}{2}$	
5.	(1)B (2)C (3) B (4)C	4
6.	(1)A (2) B (3) B (4) A	4
7.	Using quotient rule, $\frac{d}{dx}\left(\frac{\sin x+\cos x}{\sin x-\cos x}\right)$ $=\frac{(\sin x-\cos x).\frac{d}{dx}(\sin x+\cos x)-(\sin x+\cos x).\frac{d}{dx}(\sin x-\cos x)}{(\sin x-\cos x)^2}$ $=\frac{(\sin x-\cos x)(\cos x-\sin x)-(\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2}$ $=\frac{-\left[(\sin x-\cos x)^2+(\sin x+\cos x)^2\right]}{(\sin x-\cos x)^2}$ $=\frac{-2(\sin^2 x+\cos^2 x)}{(\sin^2 x+\cos^2 x-2\sin x\cos x)}$ we get f ’(x) = $\frac{-2}{1-\sin 2x}$.	4
8.	i) b ii) a	4