CHAPTER-15
STATISTICS
04 MARK TYPE QUESTIONS

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A student collected 10 readings and attempted to calculate the mean and variance. Unfortunately, the student mistakenly used a reading of 52 instead of the correct reading 25. The student calculated the mean and variance as 45 and 16 respectively.
Determine the correct mean and variance by considering the mistaken reading of 52 instead of the actual reading 25.

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The weights of coffee in 70 jars is shown in the following table:

Weight Range (grams)	Frequency
200 - 201	13
201 - 202	27
202 - 203	18
203 - 204	10
204 - 205	1
205 - 206	1

Determine variance and standard deviation of the above distribution.

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<tr><td>3.</td><td>

Read the text carefully and answer the questions: For a group of 200 candidates, the mean and the standard deviation of scores were found to be 40 and 15, respectively. Later on, it was discovered that the scores of 43 and 35 were misread as 34 and 53, respectively.

Student	Eng	Hindi	S.St	Science	Maths
Ramu	39	59	84	80	41
Rajitha	79	92	68	38	75
Komala	41	60	38	71	82
Patil	77	77	87	75	42
Pursi	72	65	69	83	67
Gayathri	46	96	53	71	39

1. Find the correct variance.
2. What is the formula of variance.
3. Find the correct mean.
4. Find the sum of correct scores.

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There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test:

Marks	0	1	2	3	4	5
Frequency	$x - 2$	x	x^2	$(x + 1)_2$	$2x$	$x + 1$

where x is a positive integer. Determine the mean and standard deviation of the marks.

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<tr><td>5.</td><td>

The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, then:

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	(a) Find the new mean. (b) Find the new standard deviation of the resulting observations.	
6.	Mean and standard deviation of 100 observations were found to be 40 and 10, respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively, find the correct standard deviation.	4
7.	Find the variance and the standard deviation for the following data: 57, 64, 43, 67, 49, 59, 44, 47, 61, 59	4
8.	Find the mean deviation about the median of the following distribution:	4

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	6	8	14	16	4	2

ANSWERS:

Q. NO	ANSWER	MARKS

1.	Given while calculating the mean and variance of 10 readings, a student wrongly used the reading 52 for the correct reading 25. He obtained the mean and variance as 45 and 16 respectively Now we have to find the correct mean and the variance. As per given criteria, Number of reading, n=10 Mean of the given readings before correction, $\overline{x} = 45$ But we know, $\overline{x} = \frac{\sum x_i}{n}$ Substituting the corresponding values, we get $45 = \frac{\sum x_i}{10}$ $\Rightarrow \sum x_i = 45 \times 10 = 450$ It is said one reading 25 was wrongly taken as 52, So $\sum x_i = 450 - 52 + 25 = 423$ So the correct mean after correction is $\overline{x} = \frac{\sum x_i}{n} = \frac{423}{10} = 42.3$ Also given the variance of the 10 readings is 16 before correction, i.e., $\sigma^2 = 16$ But we know $\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$ Substituting the corresponding values, we get $16 = \frac{\sum x_i^2}{10} - (45)^2$ $\Rightarrow 16 = \frac{\sum x_i^2}{10} - 2025$ $\Rightarrow 16 + 2025 = \frac{\sum x_i^2}{10}$ $\Rightarrow \frac{\sum x_i^2}{10} = 2041$ $\Rightarrow \sum x_i^2 = 20410$ It is said one reading 25 was wrongly taken as 52, so $\Rightarrow \sum x_i^2 = 20410 - (52)^2 + (25)^2$ $\Rightarrow \sum x_i^2 = 20410 - 2704 + 625$ $\Rightarrow \sum x_i^2 = 18331$ So the correct variance after correction is $\sigma^2 = \frac{18331}{10} - \left(\frac{423}{10}\right)^2$ $\sigma^2 = 1833.1 - (42.3)^2 = 1833.1 - 1789.29$ $\sigma^2 = 43.81$ Hence the corrected mean and variance is 42.3 and 43.81 respectively.	

2. Given the weights of coffee in 70 jars

Now we have to find the variance and standard deviation of the distribution

Let us make a table of the given data and append other columns after calculations

Weight (in grams)	Mid-Value (x_i)	Frequency (f_i)	f_ix_i
200 - 201	200.5	13	13×200.5=2606.5
201 - 202	201.5	27	27×201.5=5440.5
202 - 203	202.5	18	18×202.5=3645
203 - 204	203.5	10	10×203.5=2035
204 - 205	204.5	1	1×204.5=204.5
205 - 206	205.5	1	1×205.5=205.5
	Total	N=70	$\Sigma f_ix_i = 14137$

Here mean, $\overline{x} = \frac{\Sigma f_i x_i}{N} = \frac{14137}{70} = 201.9$

So the above table with more columns is as shown below,

Weight (in grams)	Mid-Value (x_i)	Frequency (f_i)	$d_i = x_i - \overline{x}$	f_id_i	$f_id_i^2$
200 - 201	200.5	13	200.5-201.9= -1.4	13×-1.4= -18.2	13×-1.4^2 =25.48
201 - 202	201.5	27	201.5-201.9= -0.4	27×-0.4= -10.8	27×-0.4^2 =4.32
202 - 203	202.5	18	202.5-201.9= 0.6	18×0.6= 10.8	18×0.6^2= 6.48
203 - 204	203.5	10	203.5-201.9= 1.6	10×1.6= 16	10×1.6^2= 25.6
204 - 205	204.5	1	204.5-201.9= 2.6	1×2.6= 2.6	1×2.6^2 =6.76
205 - 206	205.5	1	205.5-201.9= 3.6	1×3.6= 3.6	1×3.6^2 =12.96
	Total	N=70		Σf_id_i=4	$\Sigma f_id_i^2$ =81.6

And we know standard deviation is

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{n} - \left(\frac{\Sigma f_i d_i}{n}\right)^2}$$

Substituting values from above table, we get

$$\sigma = \sqrt{\frac{81.6}{70} - \left(\frac{4}{70}\right)^2}$$

$$\sigma = \sqrt{1.17 - (0.057)^2}$$

$$\sigma = \sqrt{1.17 - 0.003249} = \sqrt{1.17}$$

$\Rightarrow \sigma$=1.08g

And $\sigma^2 = 1.08^2 = 1.17$g

Hence the variance and standard deviation of the distribution are 1.166g and 1.08 respectively.

3. For a group of 200 candidates, the mean and the standard deviation of scores were found to be 40 and 15, respectively. Later on it was discovered that the scores of 43 and 35 were misread as 34 and 53, respectively.

(i) $\mathrm{SD} = \sigma = 15 \Rightarrow$ Variance $= 15^2 = 225$

According to the formula,

	$$\text{Variance } = \left(\frac{1}{n}\sum x_i^2\right) - \left(\frac{1}{n}\sum x_i\right)^2$$ $$\therefore \frac{1}{200}\sum x_1^2 - (40)^2 = 225$$ $$\Rightarrow \frac{1}{200}\sum (x_i)^2 - 1600 = 225$$ $$\Rightarrow \sum (x_i)^2 = 200 \times 1825 = 365000$$ This is an incorrect reading. $$\therefore \text{ Corrected } \sum (x_i)^2 = 365000 - 34^2 - 53^2 + 43^2 + 35^2$$ $$= 365000 - 1156 - 2809 + 1849 + 1225$$ $$= 364109$$ $$\text{Corrected variance } = \left(\frac{1}{n} \times \text{ Corrected } \sum x_i\right) - (\text{ Corrected mean })^2$$ $$= \left(\frac{1}{200} \times 364109\right) - (39.955)^2$$ $$= 1820.545 - 1596.402$$ $$= 224.14$$ Corrected variance $= \left(\frac{1}{n} \times \text{Corrected} \sum x_i\right) - (\text{ Corrected mean })^2 = \left(\frac{1}{200} \times 364109\right) - (39.955)^2$ $= 1820.545 - 1596.402$ $= 224.143$ (ii) The formula of variance is $\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$. $$\text{(iii) Corrected mean } = \frac{\text{Corrected } \sum x_1}{200}$$ $$= \frac{7993}{200}$$ $$= 39.955$$ (iv)We have: $\mathrm{n} = 200, \bar{X} = 40, \sigma = 15$ $\frac{1}{n}\sum x_{\mathrm{i}} = \bar{X}$ $\therefore \frac{1}{200}\sum x_i = 40$ $\Rightarrow \sum x_i = 40 \times 200 = 8000$ Since the score was misread, this sum is incorrect. $$\Rightarrow \text{ Corrected } \sum x_i = 8000 - 34 - 53 + 43 + 35$$ $$= 8000 - 7 = 7993$$	
4.	To find: the mean and standard deviation of the marks. It is given that there are 60 students in the class, so $$\sum f_i = 60$$ $$\Rightarrow (x-2) + x + x^2 + (x+1)^2 + 2x + x + 1 = 60$$ $$\Rightarrow 5x - 1 + x^2 + x^2 + 2x + 1 = 60$$ $$\Rightarrow 2x^2 + 7x = 60$$ $$\Rightarrow 2x^2 + 7x - 60 = 0$$ Splitting the middle term, we get	

	$\Rightarrow 2x^2 + 15x - 8x - 60 = 0$ $\Rightarrow x(2x + 15) - 4(2x + 15) = 0$ $\Rightarrow (2x + 15)(x - 4) = 0$ $\Rightarrow 2x + 15 = 0$ or $x - 4 = 0$ $\Rightarrow 2x = -15$ or $x = 4$ Given x is a positive rumber, so x can take 4 as the only value. And letthe assumed mean, $a = 3$. Applying the correct formula,the mean and standard deviation of the marks are found to be 2.8 and 1.12 respectively.	
5.	(a) Let the observations be x_1, x_2, x_3, x_4, x_5 and x_6 It is given that mean is 8 and standard deviation is 4. $\Rightarrow$ Mean $(\bar{x})=\frac{x_1+x_2+x_3+x_4+x_5+x_6}{6}=8$ ....(1) If each observation is multiplied by 3 and the resulting observation are y_i, then $y_i=3x_i$, for i=1 to 6 $\therefore$ New mean$(\bar{y})=\frac{y_1+y_2+y_3+y_4+y_5+y_6}{6}$ $=3\left\{\frac{x_1+x_2+x_3+x_4+x_5+x_6}{6}\right\}$ $=3\times 8$ ....[Using (1)] $=24$ (b) Standard deviation $(\sigma)=\sqrt{\frac{1}{n}\sum_{i=1}^{6}(x_i-\bar{x})^2}$ $$4^2 = \frac{1}{6}\sum_{i=1}^{6}(x_i-\bar{x})^2$$ $$\sum_{i=1}^{6}(x_i-\bar{x})^2 = 96 \dots\dots\dots\dots (2)$$ From (1) and (2) it can be observed that $\bar{y} = 3\bar{x}$ $\bar{x} = \frac{1}{3}\bar{y}$ Substituting the values of x_i and $\bar{x}$ in (2) we obtain $$\sum_{i=1}^{6}\left(\frac{1}{3}y_i - \frac{1}{3}\bar{y}\right)^2 = 96$$ $$\sum_{i=1}^{6}(y_i-\bar{y})^2 = 864$$ Therefore, variance of new observation = $(\frac{1}{6}\times 864) = 144$ Hence, the standard deviation of new observations is $\sqrt{144} = 12$	4

6.	S.D(σ) = $\sqrt{\frac{1}{n}\sum x^2 - \left(\frac{1}{n}\sum x\right)^2}$ Given $\bar{x}$ = 40 and S.D=10, N=100 $\frac{1}{n}\sum x = 40$ $\Rightarrow \frac{1}{100}\sum x = 40$ $\Rightarrow \sum x = 4000$ $\therefore \sum x = 4000-30-70+3+27=3930$ $\therefore$ correct mean $=\frac{1}{n}\sum x$ =3930/100=39.30 Given S.D (σ) =10 σ^2=100 $\frac{1}{n}\sum x^2 - \left(\frac{1}{n}\sum x\right)^2 = 100$ $\frac{1}{100}\sum x^2 - (40)^2 = 100$ $\frac{1}{100}\sum x^2 = 100 + 1600$ $\sum x^2 = 100 \times 1700$ $\therefore$ 30 and 70 should be replaced by 3 and 27 S.D (σ) $= \sqrt{\frac{1}{n}\sum x^2 - \left(\frac{1}{n}\sum x\right)^2}$ $= \sqrt{\frac{164938}{100} - \left(\frac{3930}{100}\right)^2}$ $= \sqrt{104.89}$ S.D(σ)=10.241	4
7.	Mean = 550/10 =55 Variance(σ^2) = $(x_i - \mu)^2/n$ $=(2^2+9^2+12^2+12^2+6^2+4^2+6^2+4^2+11^2+8^2)/10$ = 662/10 = 66.2 Therefore, variance(σ^2) = 66.2 Standard Deviation(σ) = $\sqrt{(\sigma^2)} = \sqrt{66.2} = 8.13$	4

8.	(see table and working below)	4

Class	f	cf	Mid-Value x	$\lvert x-M\rvert$	$f\lvert x-M\rvert$
0 – 10	6	6	5	22.86	137.16
10 - 20	8	14	15	12.86	102.88
20 – 30	14	28	25	2.86	40.04
30 – 40	16	44	35	7.14	114.24
40 – 50	4	48	45	17.14	68.56
50 – 60	2	50	55	27.14	54.28
	$\Sigma f = 50$				$\Sigma f\lvert x-M\rvert = 517.16$

$\frac{n}{2} = 25$ lies in the interval 20 – 30 (Median Class). l=20 , f = 14, cf = 14

Median M=l+ $\left(\frac{\frac{n}{2}-cf}{f}\right) \times h$ = 27.86

Mean Deviation about the Median = $\frac{\Sigma f|x-M|}{\Sigma f} = 10.34$