CHAPTER-5

COMPLEX NUMBERS

05 MARK TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	If α and β are different complex numbers with $\lvert\beta\rvert = 1$ then find $\left\lvert\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right\rvert$	5
2.	Find the real number of x and y if (x-iy) (3+5i) is the conjugate of -6-24i	5
3.	If $(x+iy)^3 = u+iv$, then show that $\frac{u}{x}+\frac{v}{y} = 4(x^2-y^2)$	5
4.	If α and β are different complex numbers with $\lvert\beta\rvert = 1$; then find $\left\lvert\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right\rvert$.	5
5.	If $a^2+b^2+c^2 = 1, b+ic = (1+a)z,$ Prove that $\frac{a+ib}{1+c} = \frac{1+iz}{1-iz}$	5
6.	If z_1 *and* z_2 are any two complex numbers, then Prove that $(\lvert z_1\rvert + \lvert z_2\rvert)\left\lvert\frac{z_1}{\lvert z_1\rvert}+\frac{z_2}{\lvert z_2\rvert}\right\rvert \le 2(\lvert z_1\rvert + \lvert z_2\rvert)$	5
7.	Solve the equation $z^2+\lvert z\rvert=0$, where z is a complex number	5
8.	If a+ib=$\frac{(x+i\}^2}{2x^2+1}$, prove that $a^2+b^2=\frac{(x+1\}^2}{(2x^2+1)^2}$.	5
9.	If α *and* β are different complex number with $\lvert\beta\rvert$=1 , find $\left\lvert\frac{\beta-\alpha}{1-\ \bar{\alpha}\beta}\right\rvert$	5
10.	If $\lvert z_1\rvert$=$\lvert\ \lvert z_2\rvert\rvert$=………= $\lvert z_n\rvert$=1, prove that $\lvert z_1+\ z_2 \ldots.+z_n\rvert = \left\lvert\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}+\ \ldots\ldots\ldots+\frac{1}{z_n}\right\rvert$	5

ANSWERS:

Q. NO	ANSWER	MARKS
1.	Let ∝=a+ib,$\beta = x + iy$ It is given that $\|\beta\|$=1 Thau $\sqrt{x^2 + y^2}$=1 $\Rightarrow x^2 + y^2$=1 $\left\| \frac{\beta-\alpha}{1-\bar{\alpha}\beta} \right\| = \left\| \frac{(x+iy)-(a+ib)}{1-(a+ib)(x+iy)} \right\|$ $= \left\| \frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)} \right\|$ $= \left\| \frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)} \right\|$ $= \frac{\|(x-a)+i(y-b)\|}{\|(1-ax-by)+i(bx-ay)\|}$ $= \frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-y)^2}}$ $= \frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$ $= \frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(x^2+y^2)-2ax-2by}}$ $= \frac{\sqrt{a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}}$ =1 Therefore $\left\| \frac{\beta-\alpha}{1-\bar{\alpha}\beta} \right\|$=1	5
2.	Let z=(x-iy)(3+5i) $\Rightarrow z = 3x + 5xi - 3yi - 5yi^2$ $= 3x + 5xi - 3yi + 5y$ $= (3x + 5y) + i(5x - 3y)$ $\bar{z}=(3x + 5y) - i(5x - 3y)$ It is given that $\bar{z} = -6 - 24i$ Thus,$(3x + 5y) - i(5x - 3y)$=$-6 - 24i$ Equating the real and imaginary parts we obtain, 3x+5y=-6 _____ (1) 5x-3y=24 _____ (2) Multiplying equation (1) by 3 and equation (2) by 5 and then adding them we obtain, 9x+15y=-18 25x-15y=1 ――――――――――― 34x=102 $\Rightarrow$ x=3 Putting the value of x in equation (1) we obtain 3(3)+5y=-6 $\Rightarrow$ 9+5y=-6 $\Rightarrow$ 5y=-6-9 $\Rightarrow$ 5y=-15 $\Rightarrow$ y=-3	5

	Thus the value of x and y are 3 and -3 respectively.	
3.	Given, $(x+iy)^3 = u+iv$ $x^3 + (iy)^3 + 3\cdot x\cdot iy(x+iy) = u+iv$ $x^3 + i^3y^3 + 3x^2yi + 3xy^2i^2 = u+iv$ $x^3 - iy^3 + 3x^2yi - 3xy^2 = u+iv$ $(x^3 - 3xy^2) + i(3x^2y - y^3) = u+iv$ On equating real and imaginary parts, we get $u = x^3 - 3xy^2,\ v = 3x^2y - y^3$ $\frac{u}{x} + \frac{v}{y} = \frac{x^3-3xy^2}{x} + \frac{3x^2y-y^3}{y}$ $= \frac{x(x^2-3y^2)}{x} + \frac{y(3x^2-y^2)}{y}$ $= x^2 - 3y^2 + 3x^2 - y^2$ $= 4x^2 - 4y^2$ $= 4(x^2-y^2)$ $\therefore \frac{u}{x} + \frac{v}{y} = 4(x^2-y^2)$ Hence proved.	5

4.	Let $\alpha = a + ib$ and $\beta = x + iy$ It is given that, $\lvert\beta\rvert = 1$ $\therefore \sqrt{x^2+y^2} = 1 \Rightarrow x^2+y^2 = 1$ ... (i) $\left\lvert\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right\rvert = \left\lvert\frac{(x+iy)-(a+ib)}{1-(a-ib)(x+iy)}\right\rvert = \left\lvert\frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)}\right\rvert$ $= \left\lvert\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right\rvert$ $= \frac{\lvert(x-a)+i(y-b)\rvert}{\lvert(1-ax-by)+i(bx-ay)\rvert}$ $\left[\left\lvert\frac{z_1}{z_2}\right\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert}\right]$ $= \frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-ay)^2}}$ $= \frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$ $= \frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(y^2+x^2)-2ax-2by}}$ $= \frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}}$ [Using (1)] $=1$ $\therefore \left\lvert\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right\rvert = 1$	5
5.	Given $a^2+b^2+c^2=1 => b^2+c^2=1-a^2$ $\frac{1+iz}{1-iz} = \frac{1+i\frac{b+ic}{1+a}}{1-i\frac{b+ic}{1+a}}$ $= \frac{1+a-c+bi}{1+a-bi+c}$ $= \frac{(1+a-c+bi)(1+a+c+bi)}{(1+a+c-bi)(1+a+c+bi)}$ $= \frac{1+2a+2ib+a^2+2iab-c^2-b^2}{1+2a+2c+a^2+b^2+c^2+2ac}$ $= \frac{1+2a+2b(1+i)+a^2-c^2-b^2}{1+2a+2c+a^2+b^2+c^2+2ac}$ $= \frac{1+2a+2b(1+i)+a^2-c^2-b^2}{1+2a+2c+a^2+b^2+c^2+2ac}$ $= \frac{1+2a+a^2-1+a^2+2b(1+a)i}{1+2a+1+2c+2ac}$ $= \frac{a(1+a)+b(1+a)i}{(1+a)+c(1+a)}$	5

	$=\dfrac{a+bi}{1+c}$	
6.	We have, $(\lvert z_1\rvert+\lvert z_2\rvert)\left\lvert\frac{z_1}{\lvert z_1\rvert}+\frac{z_2}{\lvert z_2\rvert}\right\rvert\le(\lvert z_1\rvert+\lvert z_2\rvert)\left(\left\lvert\frac{z_1}{\lvert z_1\rvert}\right\rvert+\left\lvert\frac{z_2}{\lvert z_2\rvert}\right\rvert\right)$ $\le(\lvert z_1\rvert+\lvert z_2\rvert)\left(\frac{\lvert z_1\rvert}{\lvert z_1\rvert}+\frac{\lvert z_2\rvert}{\lvert z_2\rvert}\right)$ $\le(\lvert z_1\rvert+\lvert z_2\rvert)(1+1)$ $=2(\lvert z_1\rvert+\lvert z_2\rvert)$ Therefore $(\lvert z_1\rvert+\lvert z_2\rvert)\left\lvert\frac{z_1}{\lvert z_1\rvert}+\frac{z_2}{\lvert z_2\rvert}\right\rvert\le 2(\lvert z_1\rvert+\lvert z_2\rvert)$	5
7.	Let z=x+iy then $z^2+\lvert z\rvert=0$ $\Rightarrow(x+iy)^2+\sqrt{x^2+y^2}=0$ $\Rightarrow(x^2-y^2)+\sqrt{x^2+y^2}+2ixy=0$ $\Rightarrow(x^2-y^2)+\sqrt{x^2+y^2}=0$ ……(1)and 2xy=0…….(2) ⇒ x=0 or y=0 caseI when y=0 we gate x=0 from equation 1 there for z=0 caseII when x=0 aftersolving equation 1 we gate y=-1 therefore z=0+I or , z=0-i Hence z=0,i and –i are solutions of $z^2+\lvert z\rvert=0$	5
8.	We have a+ib=$\frac{(x+i\}^2}{2x^2+1}$ …….1 $\Rightarrow \overline{a+ib}=\overline{\frac{(x+i\}^2}{2x^2+1}}$ ⇒ a-ib=$\frac{(x-i\}^2}{2x^2+1}$……………2 Multiplying 1 and 2, we get (a+ib) (a-ib)= $\frac{(x+i\}^2}{2x^2+1}$x$\frac{(x-i\}^2}{2x^2+1}$ ⇒ $a^2+b^2=\frac{(x+1\}^2}{(2x^2+1)^2}$	5
9.	$givn\ \alpha\ and\ \beta$ are different complex number with $\lvert\beta\rvert=1$ $\left\lvert\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right\rvert^2=\left(\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right)\left(\overline{\frac{\beta-\alpha}{1-\overline{\alpha}\beta}}\right)$ $\left\lvert\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right\rvert^2=\left(\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right)\left(\frac{\overline{\beta}-\overline{\alpha}}{\overline{1}-\overline{\overline{\alpha}\beta}}\right)$ $=\left(\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right)\left(\frac{\overline{\beta}-\overline{\alpha}}{\overline{1}-\alpha\ \overline{\overline{\beta}}}\right)$	5

	$= \left(\frac{\beta \bar{\beta} - \bar{\alpha}\beta - \bar{\beta}\alpha + \alpha \bar{\alpha}}{1 - \bar{\alpha}\beta - \bar{\beta}\alpha + \bar{\alpha}\beta \bar{\beta}\alpha} \right)$ $=1$	
10.	Solution: LHS $\lvert z_1 + z_2 + \cdots . + z_n \rvert = \left\lvert \frac{z_1 \overline{z_1}}{\overline{z_1}} + \frac{z_2 \overline{z_2}}{\overline{z_2}} + \ldots \ldots \ldots + \frac{z_n \overline{z_n}}{\overline{z_n}} \right\rvert$ $z\bar{z} = \lvert z \rvert^2 = \left\lvert \frac{\lvert z_1 \rvert^2}{z_1} + \frac{\lvert z_2 \rvert^2}{z_2} + \ldots \ldots \ldots + \frac{\lvert z_n \rvert^2}{\overline{z_n}_n} \right\rvert$ $= \left\lvert \frac{1}{\overline{z_1}} + \frac{1}{\overline{z_2}} + \ldots \ldots \ldots + \frac{1}{\overline{z_n}} \right\rvert$	5