CHAPTER-13

LIMITS & DERIVATIVES

05 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find the values of a and b if $\lim_{x\to 2} f(x) and \lim_{x\to 4} f(x) exists.$ Where f(x)= $\begin{cases} x^2+ax+b, 0\le x<2 \\ 3x+2, 2\le x\le 4 \\ 2ax+5b, 4<x\le 8 \end{cases}$	5
2.	Let f(x)= $\begin{cases} \frac{kcosx}{\pi-2x}, x\ne \frac{\pi}{2} \\ 3, x=\frac{\pi}{2} \end{cases}$ and if $\lim_{x\to\frac{\pi}{2}} f(x)=f(\frac{\pi}{2})$ Find the value of k	5
3.	(i) Evaluate $\lim_{x\to 0}\left(\frac{e^{\tan x}-1-x}{\sin(x^2)}\right)$. (ii) If $\lim_{x\to 0}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then find the values of a and b.	5(3+2)
4.	Differentiate $\tan\sqrt{x}$ with respect to x by using first principle.	5
5.	Let f(x)= $\frac{[x]^2+15[x]+56}{cos(x+7)cos(x+8)}$ where [,] denoted the greatest integer function. Then show that $\lim_{x\to -7} f(x)$ =0	5
6.	Find the derivative of $x^{sinx}+x^{cosx}$	5
7.	Using first principle, show that the differentiation of $\sin x$ with respect to x is $\cos x$?	5
8.	The distance f(t) in meters moved by a particle travelling in a straight line in t seconds is given by $f(t)=t^2+3t+4$. Find the speed of the particle at the end of 2 seconds?	5

ANSWERS:

Q. NO	ANSWER	MARKS
1.	a=3 b=-2	5
2.	k=6	5
3.	(i) $\lim_{x\to 0}\left(\frac{e^{\tan x}-1-x}{\sin(x^2)}\right)=\lim_{x\to 0}\left(\frac{1+\frac{\tan x}{1!}+\frac{\tan^2 x}{2!}+.....-1-x}{\sin(x^2)}\right)$ $=\lim_{x\to 0}\left(\frac{\frac{\tan x}{1!}+\frac{\tan^2 x}{2!}+.....-x}{\sin(x^2)}\right)=\lim_{x\to 0}\left(\frac{x+\frac{x^3}{3}+\frac{2x^5}{15}+.....+\frac{\tan^2 x}{2!}+.....-x}{\sin(x^2)}\right)$ $\lim_{x\to 0}\left(\frac{\frac{x^3}{3}+\frac{2x^5}{15}+.....+\frac{\tan^2 x}{2!}+.....}{\sin(x^2)}\right)=\lim_{x\to 0}\frac{x^2\left(\frac{x}{3}+\frac{2x^2}{15}+.....+\frac{\tan^2 x}{x^2 2!}+.....\right)}{x^2}$ $=\lim_{x\to 0}\left(\frac{x}{3}+\frac{2x^2}{15}+.....+\frac{\tan^2 x}{x^2 2!}+.....\right)=\frac{1}{2!}=\frac{1}{2}$ (ii) $\lim_{x\to 0}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4\Rightarrow\lim_{x\to 0}\left(\frac{x^2+x+1-(x+1)(ax+b)}{x+1}\right)=4$ $\Rightarrow\lim_{x\to 0}\left(\frac{x^2+x+1-(ax^2+bx+ax+b)}{x+1}\right)=4\Rightarrow\lim_{x\to 0}\left(\frac{x^2(1-a)+x(1-a-b)+1-b}{x+1}\right)=4$ Since the limit is a finite value, the deg.of Nr=Deg. Of Dr Hence coefficient of x^2 is $1-a=0\Rightarrow a=1$. And $1-a-b=4\Rightarrow 1-1-b=4\Rightarrow b=-4$.	5
4.	$f(x)=\tan\sqrt{x}$ $f^1(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{\tan\sqrt{x+h}-\tan\sqrt{x}}{h}=\lim_{h\to 0}\frac{\tan\sqrt{x+h}-\tan\sqrt{x}}{x+h-x}$ $=\lim_{h\to 0}\frac{\frac{\sin\sqrt{x+h}}{\cos\sqrt{x+h}}-\frac{\sin\sqrt{x}}{\cos\sqrt{x}}}{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}$ $=\lim_{h\to 0}\frac{\sin\sqrt{x+h}\cos\sqrt{x}-\cos\sqrt{x+h}\sin\sqrt{x}}{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)\cos\sqrt{x+h}.\cos\sqrt{x}}$ $=\lim_{h\to 0}\frac{\sin\left(\sqrt{x+h}-\sqrt{x}\right)}{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)\cos\sqrt{x+h}.\cos\sqrt{x}}$	5

	$=\lim_{h\to 0}\frac{\sin\left(\sqrt{x+h}-\sqrt{x}\right)}{\left(\sqrt{x+h}-\sqrt{x}\right)}\cdot\lim_{h\to 0}\frac{1}{\left(\sqrt{x+h}+\sqrt{x}\right)\cos\sqrt{x+h}.\cos\sqrt{x}}$ $=1\times.\lim_{h\to 0}\frac{1}{\left(\sqrt{x+h}+\sqrt{x}\right)\cos\sqrt{x+h}.\cos\sqrt{x}}=\frac{1}{2\sqrt{x}\cos^2\sqrt{x}}=\frac{1}{2\sqrt{x}}\sec^2\sqrt{x}$	
5.	$\frac{[x]^2+15[x]+56}{cos(x+7)cos(x+8)}=\frac{([x]+7)([x]+8)}{cos(x+7)cos(x+8)}$ Therefore $\lim_{x\to-7^+} f(x)=\frac{0\times1}{cos(0)\times cos(-1)}=0$ $\lim_{x\to-7^-} f(x)=\frac{(-1)\times0}{cos(-1)\times cos(0)}=0$	5
6.	Let u $=x^{sinx}$ and v = $(x)^{cosx}$ Taking logarithm , we get log u= (sinx) logx And log(v) = cosxlogx Differentiating the above functions with respect to x we get $\frac{du}{dx}$=u($\frac{sinx}{x}$ + cosxlogx) ……………….(1) $\frac{dv}{dx}$=v($\frac{cosx}{x} - sinxlog(x)$)…………….(2) Also $\frac{dy}{dx}=\frac{du}{dx}+\frac{du}{dx}$, using algebraic property of derivative function Adding (1) and (2) we get the required answer.	5
7.	This can be proved by first principles,	5
8.	$f(t)=t^2+3t+4$ The speed of the particle at the end of 2 seconds is given by f ’(2). I e the derivative of f (t) at t = 2 We can use first principle here $f'(t)=\lim_{h\to0}\frac{f(2+h)-f(2)}{h}$ $=\lim_{h\to0}\frac{\{(2+h)^2+392+h)+4\}-\{2^2+3\times2+4\}}{h}$ $=\lim_{h\to0}\frac{h^2+7h}{h}=\lim_{h\to0}h+7=7$ Or we can can use the standard result $f'(t)=2t+3$ $\therefore f'(2)=2\times2+3=7.$	5