CHAPTER-15

STATISTICS

05 MARK TYPE QUESTIONS

<table>
<tr><th>Q. NO</th><th>QUESTION</th><th>MARK</th></tr>
<tr><td>1.</td><td>A group of individuals' heights were measured and categorized into different height ranges (in centimeters). The number of individuals falling into each height range is recorded as follows:
Height Range (in cms) Frequency
70 - 75 3
75 - 80 4
80 - 85 7
85 - 90 7
90 - 95 15
95 - 100 9
100 - 105 6
105 – 110 6
110 – 115 3</td><td>5</td></tr>
<tr><td>2.</td><td>An age distribution of 100 persons is available, and their ages are categorized into different age ranges. The number of persons falling into each age range is recorded as follows:
Age Range Number
16 - 20 5
21 - 25 6
26 - 30 12
31 - 35 14
36 - 40 26
41 - 45 12
46 - 50 16
51 - 55 9
  </td><td>5</td></tr>
<tr><td>3.</td><td>In a survey of 44 villages of a state, about the use of LPG as a cooking mode, the following information about the families using LPG was obtained.
<table><tr><td>Number of families</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td><td>50-60</td></tr><tr><td>Number of villages</td><td>6</td><td>8</td><td>16</td><td>8</td><td>4</td><td>2</td></tr></table>i. Find the mean deviation about median for the following data.
ii. Do you think more awareness is needed for the villagers to use LPG as a mode of cooking?</td><td>5</td></tr>
<tr><td>4.</td><td>From the frequency distribution consisting of 18 observations, the mean and the standard deviation were found to be 7 and 4, respectively. But on comparison with the original data, it was found that a figure 12 was miscopied as 21 in calculations. Calculate the correct mean and standard deviation.</td><td>5</td></tr>
<tr><td>5.</td><td>The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On checking, it was found that an observation 8 was incorrect.</td><td>5</td></tr>
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	Calculate the correct mean and correct standard deviation in each of the following cases: (a) If the wrong observation is omitted. (b) If it is replaced by 12.	
6.	Life of bulbs produced by two factories A and B are given below: The bulbs of which factory are more consistent from the point of view of length of life?	5
7.	Find the mean , variance and standard deviation using short-cut method.	5
8.	The diameters of circles(in mm) drawn in a design are given below. Calculate the standard deviation and mean diameter of the circles.	5

Length of life(in hours)	Factory A (Number of bulbs)	Factory B (Number of bulbs)
550-650	10	8
650-750	22	60
750-850	52	24
850-950	20	16
950-1050	16	12
	120	120

Height (in cm)	No.of children
70 – 75	3
75 – 80	4
80 – 85	7
85 – 90	7
90 – 95	15
95 – 100	9
100 – 105	6
105 – 110	6
110 – 115	3

Diameters	33 – 36	37 – 40	41 – 44	45 – 48	49 – 52
No.of circles	15	17	21	22	25

ANSWERS:

Q. NO	ANSWER	MARKS
1.		5

Class Interval	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 92.5}{5}$	y_i^2	$f_i y_i$	$f_i y_i^2$
70-75	3	72.5	–4	16	–12	48
75-80	4	77.5	–3	9	–12	36
80-85	7	82.5	–2	4	–14	28
85-90	7	87.5	–1	1	–7	7
90-95	15	92.5	0	0	0	0
95-100	9	97.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	60				6	254

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{9} f_i y_i}{N} \times h = 92.5 + \frac{6}{60} \times 5 = 92.5 + 0.5 = 93$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{9} f_i y_i^2 - \left(\sum_{i=1}^{9} f_i y_i\right)^2\right]$$

$$= \frac{(5)^2}{(60)^2}\left[60 \times 254 - (6)^2\right]$$

$$= \frac{25}{3600}(15204) = 105.58$$

$$\therefore \text{Standard deviation}(\sigma) = \sqrt{105.58} = 10.27$$

2.	The given data is not continuous. Therefore, it has to be converted into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval. The table is formed as follows.	5

Age	Number f_i	Cumulative frequency (c.f.)	Mid-point x_i	$\lvert x_i - \text{Med.} \rvert$	$f_i \lvert x_i - \text{Med.} \rvert$
15.5-20.5	5	5	18	20	100
20.5-25.5	6	11	23	15	90
25.5-30.5	12	23	28	10	120
30.5-35.5	14	37	33	5	70
35.5-40.5	26	63	38	0	0
40.5-45.5	12	75	43	5	60
45.5-50.5	16	91	48	10	160
50.5-55.5	9	100	53	15	135
	100				735

The class interval containing the $\frac{N^{th}}{2}$ or 50^{th} item is 35.5 – 40.5.

Therefore, 35.5 – 40.5 is the median class.

It is known that,

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

Here, l = 35.5, C = 37, f = 26, h = 5, and N = 100

$$\therefore \text{Median} = 35.5 + \frac{50 - 37}{26} \times 5 = 35.5 + \frac{13 \times 5}{26} = 35.5 + 2.5 = 38$$

Thus, mean deviation about the median is given by,

$$\text{M.D.}(M) = \frac{1}{N} \sum_{i=1}^{8} f_i |x_i - M| = \frac{1}{100} \times 735 = 7.35$$

3.						5

Number of families	Mid value (x_i)	Number of villages (f_i)	cf	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
$0-10$	5	6	6	20	120
$10-20$	15	8	14	10	80
$20-30$	25	16	30	0	0
$30-40$	35	8	38	10	80
$40-50$	45	4	42	20	80
$50-60$	55	2	44	30	60

i. Here, $N = 44$

Now, $\frac{N}{2} = \frac{44}{2} = 22$, which, lies in the cumulative frequency of 30, therefore median class is 20-30.

$$\therefore l = 20, f = 16, cf = 14 \text{ and } h = 10$$

$$\therefore \text{ Median (M)} = l + \frac{\frac{N}{2} - cf}{f} \times \text{h}$$

$$= 20 + \frac{22 - 14}{16} \times 10$$

$$= 20 + \frac{8}{16} \times 10 = 20 + 5 = 25$$

$\therefore$ Mean deviation about median $= \frac{\sum_{i=1}^{6} f_i \lvert x_i - M \rvert}{\sum f_i} = \frac{420}{44} = 9.55$

ii. There is a need for awareness among villagers for using LPG as a mode of cooking. Because it will help in keeping the environment clean and will also help in saving of forests.

4.		5

Mean $= 7$

$$\therefore \frac{\sum x_i}{18} = 7 [\because \text{n} = 18]$$

$$\Rightarrow \sum x_i = 18 \times 7 = 126$$

Since, an observation 12 was miscopied as 21

$$\therefore \text{ Correct } \sum x_i = 126 - 21 + 12 = 117$$

$$\text{Hence, true mean } = \frac{\text{Correct } \sum x_i}{18} = \frac{117}{18} = 6.5$$

Also, given variance $= 4^2 = 16$

$$\therefore \frac{\sum x_i^2}{18} - (\text{Mean})^2 = 16$$

$$\Rightarrow \frac{\sum x_i^2}{18} = 16 + (\text{Mean})^2 = 16 + (7)^2$$

$$\Rightarrow \frac{\sum x_i^2}{18} = 16 + 49$$

$$\Rightarrow \sum x_i^2 = 18 \times 65 = 1170$$

But one observation 12 was miscopied as 21

$$\text{Correct } \sum x_i^2 = 1170 - 21^2 + 12^2 = 1170 - 441 + 144 = 873$$

$$\text{Hence, correct variance } = \frac{\text{Correct } \sum x_i^2}{18} - (\text{Correct mean})^2$$

$$= \frac{873}{18} - (6.5)^2 = 48.5 - 42.25 = 6.25$$

$$\therefore \text{ Correct standard deviation } = \sqrt{\text{Correct variance}}$$

$$= \sqrt{6.25} = 2.5$$

5. (i) Given, number of observations n=20

Incorrect mean =10

Incorrect standard deviation =2

$\bar{x} = \frac{1}{n}\sum_{i=1}^{20} x_i$

$$10 = \frac{1}{20}\sum_{i=1}^{20} x_i$$

$$\sum_{i=1}^{20} x_i = 200$$

So, the incorrect sum of observations =200

Correct sum of observation =200−8=192

⇒ Correct mean =Correct sum/19 =192/ 19=10.1

$$\text{S.D } (\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2}$$

$$2 = \sqrt{\frac{1}{20}\ \text{incorrect}\ \sum_{i=1}^{n} x_i^2 - (10)^2}$$

$$4 = \frac{1}{20}\ \text{incorrect}\ \sum_{i=1}^{n} x_i^2 - 100$$

$$\text{incorrect}\ \sum_{i=1}^{n} x_i^2 = 2080$$

$$\text{correct}\ \sum_{i=1}^{n} x_i^2 = \text{incorrect}\ \sum_{i=1}^{n} x_i^2 - (8)^2 = 2080 - 64 = 2016$$

$$\therefore \text{Correct Standard deviation} = \sqrt{\frac{1}{n}\text{correct}\ \sum_{i=1}^{n} x_i^2 - (\text{correct mean})^2}$$

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$= \sqrt{\frac{2016}{19} - (10.1)^2} = \sqrt{106.1 - 102.01} = \sqrt{4.09} = 2.02$

(ii) When 8 is replaced by 12

Incorrect sum of observation =200

∴ Correct sum of observations =200−8+12=204

∴ Correct mean =Correct sum /20 =204/20 =10.2

S.D (σ) = $\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2}$

$2 = \sqrt{\frac{1}{20}\ \text{incorrect}\ \sum_{i=1}^{n} x_i^2 - (10)^2}$

$4 = \frac{1}{20}\ \text{incorrect}\ \sum_{i=1}^{n} x_i^2 - 100$

$\text{incorrect}\ \sum_{i=1}^{n} x_i^2 = 2080$

$\text{correct}\ \sum_{i=1}^{n} x_i^2 = \text{incorrect}\ \sum_{i=1}^{n} x_i^2 - (8)^2 + 12^2$

$= 2080 - 64 + 144 = 2160$

∴ Correct Standard deviation = $\sqrt{\frac{1}{n}\text{correct}\ \sum_{i=1}^{n} x_i^2 - (\text{correct mean})^2}$

$= \sqrt{\frac{2160}{20} - (10.2)^2} = \sqrt{108 - 104.04} = \sqrt{3.96} = 1.98$

6. Here h = 100. Let assumed mean, A=800 — 5

We have the following table:

Length of life (in hours)	Mid Value (x_i)	$y_i = \frac{x_i - A}{10}$	Factory A			Factory B		
			f_i	$f_i y_i$	$f_i y_i^2$	f_i	$f_i y_i$	$f_i y_i^2$
550-650	600	-2	10	-20	40	8	-16	32
650-750	700	-1	22	-22	22	60	-60	60
750-850	800	0	52	0	0	24	0	0
850-950	900	1	20	20	20	16	16	16
950-1050	1000	2	16	32	64	12	24	48
			120	10	146	120	-36	156

Factory A:

Mean ($\bar{x}$) $= 800 + \frac{10}{120} \times 100 = 816.67$ hours

S. D. = $\frac{100}{120}\sqrt{120 \times 146 - (10)^2} = 109.98$

Coefficient of variance (C.V.) = $\frac{\text{S.D.}}{\bar{x}} \times 100 = \frac{109.98}{816.67} \times 100 = 13.47$

Factory B:

Mean ($\bar{x}$) $= 800 + \frac{-36}{120} \times 100 = 770$ hours

S. D. = $\frac{100}{120}\sqrt{120 \times 156 - (-36)^2} = 110$

Coefficient of variance (C.V.) = $\frac{\text{S.D.}}{\bar{x}} \times 100 = \frac{110}{770} \times 100 = 14.29$

Clearly C.V. of factory B > C.V. of factory A

i.e. Factory B has more variability

Hence the bulbs of factory A are more consistent.

7.

Height (in cm)	No.of children f	Mid-Value (x)	$u = \frac{x-a}{h}$	u^2	fu	fu^2
70 – 75	3	72.5	-4	16	-12	48
75 – 80	4	77.5	-3	9	-12	36
80 – 85	7	82.5	-2	4	-14	28
85 – 90	7	87.5	-1	1	-7	7
90 – 95	15	92.5	0	0	0	0
95 – 100	9	97.5	1	1	9	9
100 – 105	6	102.5	2	4	12	24
105 – 110	6	107.5	3	9	18	54
110 – 115	3	112.5	4	16	12	48
	$\Sigma f = 60$				$\Sigma fu = 6$	$\Sigma fu^2 = 254$

Mean $\bar{x} = a + \left(\frac{\Sigma fu}{\Sigma f}\right) \times h = 93$ where a = 92.5 and h = 5

Variance = $\left[\frac{\Sigma fu^2}{\Sigma f} - \left(\frac{\Sigma fu}{\Sigma f}\right)^2\right] \times h^2 = 105.58$

Standard Deviation $\sigma = \sqrt{Variance} = \sqrt{105.58} = 10.27$

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8.

Diameters	No.of circles f	Mid-Value (x)	$u = \frac{x-a}{h}$	u^2	fu	fu^2
32.5 – 36.5	15	34.5	-2	4	-30	60
36.5 – 40.5	17	38.5	-1	1	-17	17
40.5 – 44.5	21	42.5	0	0	0	0
44.5 – 48.5	22	46.5	1	1	22	22
48.5 – 52.5	25	50.5	2	4	50	100
	$\Sigma f = 100$				$\Sigma fu = 25$	$\Sigma fu^2 = 199$

Mean $\bar{x} = a + \left(\frac{\Sigma fu}{\Sigma f}\right) \times h = 43.5$ where a = 42.5 and h = 4

Variance = $\left[\frac{\Sigma fu^2}{\Sigma f} - \left(\frac{\Sigma fu}{\Sigma f}\right)^2\right] \times h^2 = 30.84$

Standard Deviation $\sigma = \sqrt{Variance} = \sqrt{30.84} = 5.55$

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