MATHEMATICS HALF YEARLY PRACTICE PAPER

ANSWER AND SOLUTIONS

SECTION-A

1. Option (1)
194

2. Option (3)
An irrational number

3. Option (2)
$2\sqrt{2}+\sqrt{3}$

4. Option (1)
(iv) Only

5. Option (1)
229

6. Option (1)
100°

7. Option (4)
On Y-axis

8. Option (3)
2

9. Option (3)
$\sqrt{15}\,\text{cm}^2$

10. Option (1)
Axiom 1

11. Option (3)
75°

12. Option (2)
2

13. Option (2)
$\frac{18}{31}$

14. Option (2)
$\frac{1}{9}$

15. Option (2)
$100\sqrt{3}\,\text{m}^2$

16. Option (3)
45°

17. Option (1)
Isosceles

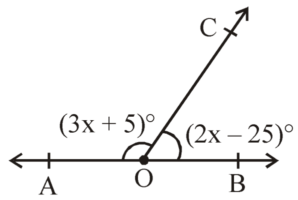
18. Option (2)
1

19. Option (1)
Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

20. Option (2)
Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

SECTION-B

21. AOB will be a straight line, if



$\angle AOC + \angle BOC = 180°$.

$\therefore$ $(3x + 5)° + (2x - 25)° = 180$

$\Rightarrow 5x = 200° \Rightarrow x = 40°$

Hence, x = 40 will make AOB a straight line.

22. $(3\sqrt{5}-5\sqrt{2})(4\sqrt{5}+3\sqrt{2})$

$= 3\sqrt{5}(4\sqrt{5}+3\sqrt{2})-5\sqrt{2}(4\sqrt{5}+3\sqrt{2})$

$= 12 \times 5 + 9\sqrt{10}-20\sqrt{10}-15\times 2$

$= 60 + 9\sqrt{10}-20\sqrt{10}-30$

$= 30 - 11\sqrt{10}$

OR

$$\frac{1}{(27)^{\frac{-1}{3}}}+\frac{1}{(625)^{\frac{-1}{4}}}$$

$= 27^{\frac{1}{3}} + 625^{\frac{1}{4}}$ $\left[\because x^{-a}=\frac{1}{x^{a}}\right]$

$= (3^3)^{\frac{1}{3}} + (5^4)^{\frac{1}{4}}$

$= 3^{3\times\frac{1}{3}} + 5^{4\times\frac{1}{4}}$ $[\because (x^a)^b = x^{a\times b}]$

$= 3 + 5$

$= 8$

23. Draw perpendiculars PL, QM, RN, SU and TV on the X-axis.

(i) The distance P from the Y-axis = OL = 2 units.

The distance of P from the X-axis = LP = 4 units.

Hence, the coordiantes of P are (2, 4)

(ii) The distance of Q from the Y-axis = OM = 4 units.

The distance of Q from the X-axis = MQ = 2 units.

Hence, the coordinates of Q are (4, 2).

(iii) The distance of R from the Y-axis = ON = –2 units.

The distance of R from the X-axis = NR = 3 units.

Hence, the coordinates of R are (–2, 3).

(iv) The distance of S from the Y-axis = OU = 5 units.

The distance of S from the X-axis = US = –3 units.

Hence, the coordinates of S are (5, –3).

(v) The distance of T from the Y-axis = OV = –4 units.

The distance of T from the x-axis = VT = –1 unit.

Hence, the coordinates of T are (–4, –1).

24. We have,

$a^3 - 2\sqrt{2}b^3 = (a)^3 - (\sqrt{2}b)^3$

$= (a-\sqrt{2}b)(a^2+\sqrt{2}ab+2b^2)$

$[\because a^3 - b^3 = (a-b)(a^2+ab+b^2)]$

OR

$x^4 + x^2y^2 + y^4$

Adding x^2y^2 and subtracting x^2y^2 to the given equation

$= x^4 + x^2y^2 + y^4 + x^2y^2 - x^2y^2$

$= x^4 + 2x^2y^2 + y^4 - x^2y^2$

$= (x^2)^2 + 2x^2y^2 + (y^2)^2 - (xy)^2$

Using the identity $(p + q)^2 = p^2 + q^2 + 2pq$

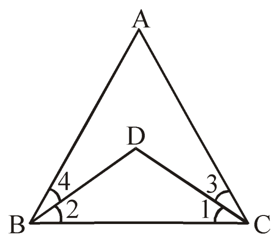
$= (x^2 + y^2)^2 - (xy)^2$

Using the identity $p^2 - q^2 = (p + q)(p - q)$

$= (x^2 + y^2 + xy)(x^2 + y^2 - xy)$

$\therefore x^4 + x^2y^2 + y^4 = (x^2 + y^2 + xy)(x^2 + y^2 - xy)$

25.



We have

$\Rightarrow \angle ABC = \angle ACB$ ....(1) [(Given)]

And $\angle 4 = \angle 3$ .... (2) [(Given)]

Now, subtracting (2) from (1), we get

$\angle ABC - \angle 4 = \angle ACB - \angle 3$

(By Euclid's axiom 3, if equals are subtracted from equals, the remainders are equal).

Hence, $\angle 2 = \angle 1$.

$\Rightarrow \angle 1 = \angle 2$

Hence proved

SECTION-C

26. Here, $a = \frac{5}{6}$, $b = \frac{8}{9}$ and n = 5

$$d = \frac{b-a}{n+1} = \frac{\frac{8}{9}-\frac{5}{6}}{5+1} = \frac{\frac{8\times2-5\times3}{18}}{6}$$

$$= \frac{\frac{1}{18}}{6} = \frac{1}{18\times6} = \frac{1}{108}$$

Hence five rational numbers between

$\frac{5}{6}$ and $\frac{8}{9}$ are :

$$a + d = \frac{5}{6} + \frac{1}{108} = \frac{5\times18+1}{108} = \frac{91}{108}$$

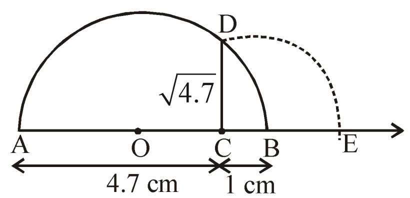
$$a + 2d = \frac{5}{6} + \frac{2}{108} = \frac{5\times18+2}{108} = \frac{92}{108}$$

$$a + 3d = \frac{5}{6} + \frac{3}{108} = \frac{5\times18+3}{108} = \frac{93}{108}$$

$$a + 4d = \frac{5}{6} + \frac{4}{108} = \frac{5\times18+4}{108} = \frac{94}{108}$$

And $a + 5d = \frac{5}{6} + \frac{5}{108} = \frac{5\times18+5}{108} = \frac{95}{108}$

OR



Draw AB = 5.7 cm and draw a semicircle with diameter AB. Mark a point C on AB such that BC = 1 cm.

Draw CD $\perp$ AB to meet the semicircle at D.

Now, with C as centre and CD as radius, draw an arc to intersect the line at point E.

Then, CE represents $\sqrt{4.7}$.

27. $64a^3 - 27b^3 - 144a^2b + 108ab^2$

The expression $64a^3 - 27b^3 - 144a^2b + 108ab^2$ can also be written as

$(4a)^3 - (3b)^3 - 3 \times 4a \times 4a \times 3b + 3 \times 4a \times 3b \times 3b$

$= (4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b)$.

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

with respect to the expression

$(4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b)$,we get

$(4a - 3b)^3 = (4a - 3b)(4a - 3b)(4a - 3b)$

OR

Let $p(x) = x^3 - 23x^2 + 142x - 120$

We shall now look for all the factors of –120. Some of these are

±1, ±2, ±3, ±4, ±5, ±6, ±8, ±10, ±12, ±15, +20, ±24, ±30, ±60

By hit and trial, we find that p(1) = 0.Therefore, x – 1 is a factor of p(x).

Now we see that $x^3 - 23x^2 + 142x - 120 = x^3 - x^2 - 22x^2 + 22x + 120x - 120$

$= x^2(x - 1) - 22x(x - 1) + 120(x - 1)$

$= (x - 1)(x^2 - 22x + 120)$

[Taking(x –1)common]

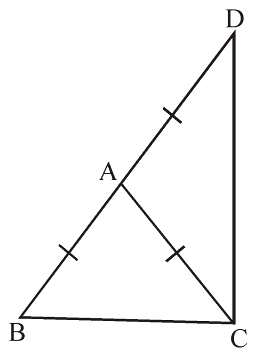
Now $x^2 - 22x + 120$ can be factorised either by splitting the middle term or by using the Factor theorem. By splitting the middle term, we have : $x^2 - 22x + 120 = x^2 - 12x - 10x + 120$

$= x(x - 12) - 10(x - 12)$

$= (x - 12)(x - 10)$

Therefore, $x^3 - 23x^2 + 142x - 120 = (x - 1)(x - 10)(x - 12)$

28.



Given : ΔABC is an isosceles triangle in which AB = AC.

Side BA is produced to D such that AD = AB.

To prove : $\angle BCD$ is a right angle.

Proof : As ABC is an isosceles triangle

$\angle ABC = \angle ACB$ ...... (1)

[$\angle$s opposite to equal side of a Δ]

AC = AD.....[As given : AB = AC and AD = AB]

In ΔACD,

$\angle CDA = \angle ACD$ .... (2)

[$\angle$s opposite to equal side of a Δ]

Adding equation (1) and (2)

$\angle ABC + \angle CDA = \angle ACB + \angle ACD$

$\angle ABC + \angle CDB = \angle BCD$ ....(3)

In ΔBCD

$\angle BCD + \angle DBC + \angle CDB = 180°$...[Sum of three angles of a triangle]

$\therefore \angle BCD + \angle ABC + \angle CDB = 180°$

$\angle BCD + \angle BCD = 180°$ .... [From (3)]

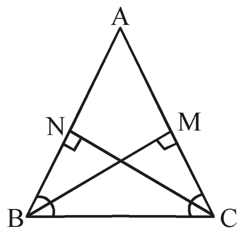
$\therefore 2\angle BCD = 180°$

$\therefore \angle BCD = 90°$

$\therefore \angle BCD$ is a right angle.

Hence proved

29.



In ΔABC, AB = AC (given)

$\angle ABC = \angle ACB$ (angles opposite to equal sides are equal)

In ΔBCM and ΔCBN,

$\angle N = \angle M$ (each = 90°)

$\angle ABC = \angle ACB$ (from above)

BC = BC (common)

$\therefore \Delta BCM \cong \Delta CBN$ (A.A.S. rule of congruency)

$\Rightarrow$ BM = CN (CPCT)

30. (i) I quadrant

(ii) II quadrant

(iii) III quadrant

(iv) IV quadrant

31. In the given equation, we have

$u - 5 = 15$

On adding 5 to both sides, we have

$u - 5 + 5 = 15 + 5$ (on applying Euclid's 2nd axiom)

we get, $u = 20$,

SECTION-D

32. Let a, b, c be the sides of the given triangle and s be its semi-perimeter.

Then, $s = \dfrac{a+b+c}{2}$ ....(i)

$\therefore$ Area of the given triangle =

$\sqrt{s(s-a)(s-b)(s-c)} = \Delta$ say

As per given condition, the sides of the new triangle will be 2a, 2b, and 2c,

So, the semi-perimeter of the new triangle =

$s' = \dfrac{2a+2b+2c}{2} = a+b+c$ ....(ii)

From (i) and (ii), we get

$s' = 2s$

Area of new triangle

$= \sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$

$= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

$= \sqrt{16s(s-a)(s-b)(s-c)}$

$= 4\sqrt{s(s-a)(s-b)(s-c)} = 4\Delta$

The required ratio = $4\Delta : \Delta = 4 : 1$

Therefore the ratio of area of new triangle thus formed and the given triangle is 4 : 1.

OR

Let a = 85 m and b = 154 m

Given that perimeter = 324 m

Perimeter = 2s = 324 m

$\Rightarrow s = \frac{324}{2} = 162$ m

or, a + b + c = 324

$\Rightarrow$ c = 324 – 85 – 154 = 85 m

By Heron's formula, we have :

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{162(162-85)(162-154)(162-85)}$

$= \sqrt{162 \times 77 \times 8 \times 77}$

$= \sqrt{1296 \times 77 \times 77}$

$= \sqrt{36 \times 77 \times 77 \times 36}$

= 36 × 77

= 2772 m^2

33. Given, $a = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $b = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

Here, $a = \frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2})^2 - 1^2}$

$= \frac{(\sqrt{2})^2 + 1 + 2\sqrt{2}}{2-1} = \frac{2+1+2\sqrt{2}}{1} = 3 + 2\sqrt{2}$

$\therefore$ $a = 3 + 2\sqrt{2}$ ....(i)

$b = \frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2 - 1}$

$= \frac{(\sqrt{2})^2 + 1^2 - 2\sqrt{2}}{2-1} = \frac{2+1-2\sqrt{2}}{1} = 3 - 2\sqrt{2}$

$\therefore$ $b = 3 - 2\sqrt{2}$ ....(ii)

From equation (i) and (ii)

$a^2 + b^2 - 4ab$

$= a^2 + b^2 - 2ab - 2ab$

$= (a - b)^2 - 2ab$

$= (4\sqrt{2})^2 - 2(9 - 8)$

= 32 – 2 = 30

34. Let, $p(x) = 2x^3 + 4x^2 - 7ax - 5$ and

$q(x) = 2x^3 + ax^2 - 6x + 3$ be the given polynomials.

Now,

When p(x) is divided by (x + 1), remainder = y

$y = p(-1)$

$y = 2(-1)^3 + 4(-1)^2 - 7a(-1) - 5$

$y = -2 + 4 + 7a - 5$

$y = -3 + 7a$

And, when q(x) is divided by (x – 1), remainder = z

$z = q(1)$

$z = 2(1)^3 + a(1)^2 - 6(1) + 3$

$z = 2 + a - 6 + 3$

$z = a - 1$

Substituting the values of y and z we have

$y - 3z = 16$

$-3 + 7a - 3(a - 1) = 16$

$-3 + 7a - 3a + 3 = 16$

$4a = 16$

$a = 4$

OR

LHS = $(a + b + c)^3 - a^3 - b^3 - c^3$

$= (a + b + c)^2(a + b + c) - a^3 - b^3 - c^3$

$= (a^2 + b^2 + c^2 + 2ab + 2bc + 2ca)(a + b + c) - a^3 - b^3 - c^3$

$= a^3 + a^2b + a^2c + ab^2 + b^3 + b^2c + ac^2 + bc^2 + c^3 + 2a^2b + 2ab^2 + 2abc + 2abc + 2b^2c + 2bc^2 + 2a^2c + 2abc + 2ac^2 - a^3 - b^3 - c^3$

$= 3a^2b + 3a^2c + 3ab^2 + 3bc^2 + 3ac^2 + 3b^2c + 6abc$

$= 3(a^2b + a^2c + ab^2 + bc^2 + ac^2 + b^2c + 2abc)$

$= 3[a^2(b + c) + a(b^2 + c^2 + 2bc) + bc(b + c)]$

$= 3[a^2(b + c) + a(b + c)^2 + bc(b + c)]$

$= 3(b + c)(a^2 + ab + ac + bc)$

$= 3(b + c)[b(a + c) + a(a + c)]$

$= 3(a + b)(b + c)(c + a)$

= RHS

Hence proved

35. Since OP and OQ bisects angles AOC and BOC respectively, therefore

$\therefore \angle AOC = 2\angle POC$ ....(i)

and $\angle COB = 2\angle COQ$ ....(ii)

Adding (i) and (ii), we get

$\angle AOC + \angle COB = 2\angle POC + 2\angle COQ$

$\Rightarrow \angle AOC + \angle COB = 2(\angle POC + \angle COQ)$

$\Rightarrow \angle AOC + \angle COB = 2\angle POQ$

$\Rightarrow \angle AOC + \angle COB = 2 \times 90°$

$[\because OP \perp OQ, \angle POQ = 90°]$

$\Rightarrow \angle AOC$ and $\angle COB$ are linear pair of angles.

$\Rightarrow$ AOB is a straight line

$\Rightarrow$ Point A, O and B are collinear.

SECTION-E

36. (i) The degree of polynomial in one variable is the highest power in the algebraic expression. The degree of the equation is 2.

(ii) Given, $P(x) = -3x^2 + 24x + 12$

$P(5) = -3(5)^2 + 24 \times 5 + 12$

$= -75 + 120 + 12$

$= -75 + 132 = 57$ m

So, the total height of the projectile is 57 m.

OR

Given, $P(x) = -3x^2 + 24x + 12$

$P(4) = -3(4)^2 + 24 \times 4 + 12$

$= -24 + 96 + 12$

$= -24 + 108 = 84$ m

So, the total height of the projectile is 84 m.

(iii) Given, $p(x) = 3x^2 - 2x - 4$

$p(2) = 3(2)^2 - 2(2) - 4$

$= 12 - 4 - 4 = 12 - 8 = 4$

37. (i) A pentagon is a closed-shaped figure which has five sides.

(ii) In 2nd quadrant x-coordinate is negative and y-coordinate is positive.

(iii) Point lying on Y-axis is S(0,2).

OR

Coordinates of points lying on X-axis are P(–1, 0) and Q(1, 0)

38. (i) Let x cm be the length of equal sides of the isosceles triangle.

So, $x + x + 4 = 20$

$2x + 4 = 20$

$2x = 20 - 4$

$2x = 16$

$x = \frac{16}{2} = 8$ cm

(ii) Required semi perimeter $= \frac{\text{Perimeter}}{2} = \frac{20}{2}$

$= 10$ cm

(iii) Since, semi perimeter, s = 10 cm

Thus, area of the triangle

$\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{10(10-8)(10-8)(10-4)}$

$= \sqrt{10(2)(2)(6)} = 4\sqrt{15}\text{cm}^2$

OR

Let the sides of a triangle are

$a = 3x, b = 5x, c = 7x$

then $a + b + c = 300$

$3x + 5x + 7x = 300$

$15x = 300$

$x = 20$

So, $a = 60, b = 100, c = 140$

$s = \frac{a+b+c}{2} = \frac{300}{2} = 150$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{150(150-60)(150-100)(150-140)}$

$= \sqrt{150 \times 90 \times 50 \times 10} = 1500\sqrt{3}\,\text{m}^2$