Index :

Set, Relation & Functions :

- **Set** : A collection of well-defined objects, i.e obj. follow a given rule or rules.

elements of a set : $x \in A$ (if x satisfy rules defined for a) , $x \notin A$ eg: Set of vowels in the alphabet of Eng. language

*** Some special sets :**

① Finite and infinite sets: Set A is finite no. of elements; we can find exact no. of elements in the set. Otherwise set is infinite.

ex: $\mathbb{Q}$ = Set of all rational no $\Rightarrow \{p/q : p, q \in \mathbb{Z}, q \neq 0\}$

$\mathbb{C}$ = Set of all complex no $\Rightarrow \{x+iy \,;\, x, y \in \mathbb{R}\}$

② Null set : A set which do not have any element.
(empty set)

③ Singleton Set : Have only one element.

④ Power Set : Power set of A is the set of all of its subsets, and is denoted by P(A).

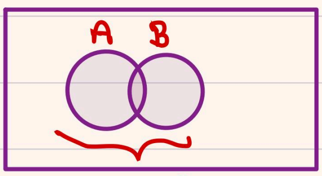
eg : $A = \{4,5,6\}$, $P(A) = \{\{\phi\}, \{4\}, \{5\}, \{6\}, \{4,5\}, \{4,6\}, \{5,6\}, \{4,5,6\}\}$

Note : Null set, A are always elements of P(A).

★★ Thm : If a finite set has n elements, then power set of A has 2^n elements.

*** operations on Sets :**

① Union : '$\cup$' $\Rightarrow \underbrace{\text{either in } A \text{ or } B \text{ or both}}_{\text{"}A \cup B\text{"}}$



shaded Region is $A \cup B$

② **intersection of sets**: Common part of A, B. $\Rightarrow$ "$A \cap B$"

(i) $A \cap B \in \{x : x \in A \text{ and } x \in B\}$

(ii) **intersection of n sets**:

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap A_3 - - - - - A_n = \{x : x \in A_i \text{ for all } i, 1 \leq i \leq n\}$$

③ **Disjoint sets**: Two sets A, B are disjoint if

$\longrightarrow A \cap B = \phi$

Properties of sets $(A \cap B \neq \phi)$:

$n(A) \Rightarrow \underbrace{\text{cardinality of } A}_{\text{no. of elements in set } A}$

① $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

② $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

③ $n(A - B) = n(A) - n(A \cap B)$

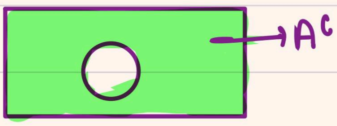
④ $n(B - A) = n(B) - n(A \cap B)$

• **Demorgan's law**: $(A^c = \text{complement of } A)$

(i) $A^c - B^c = B - A$

(ii) $(A \cup B)^c = A^c \cap B^c$

(iii) $(A \cap B)^c = A^c \cup B^c$



④ **Cartesian product of sets**:

let $a \in A, b \in B \Longrightarrow (a, b)$ is an ordered pair.

obviously $(a, b) \neq (b, a)$

$\underbrace{\text{Cartesian product}}$ of 2 sets A & B is defined as set of ordered pairs (a, b).

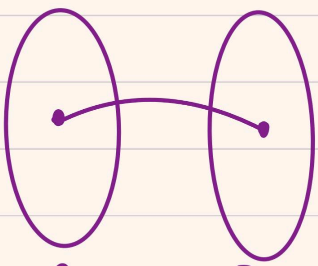
$$A \times B = \{(a, b) ; a \in A, b \in B\}$$

- **Relations**: A Relation R from the set A to set B is a subset of the Cartesian product $A \times B$.

Further if; $(x,y) \in R$, then we say x is Related to y and write this relation as

$x R y$. $R \{(x,y); x \in A, y \in B, x R y\}$

* **Domain & Range of a Relation**: $R: A \longrightarrow B$ i.e $R \subseteq A \times B$

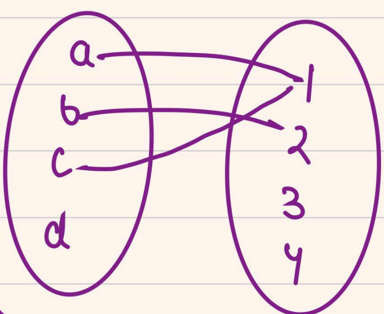


$\underbrace{A}_{\text{domain}}$ $\underbrace{B}_{\text{Range}}$

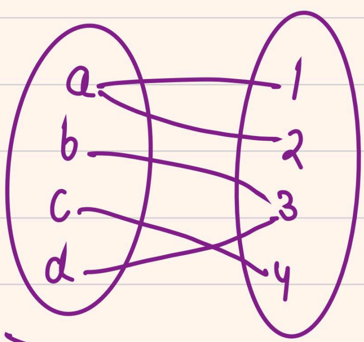
$D \subseteq A$, $\underbrace{R^*}_{\text{Range}} \subseteq B$

** **functions**: A mapping $f: \underset{\text{domain}}{X} \to \underset{\text{co-domain}}{Y}$ is a function if

- each element in the set X has its image in set Y.
- Every element in X should one and only one image.



Not a function
(d does not have an image)



Not a function
(f(a) can't give 2 diff. values)

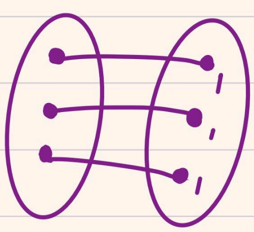
• Algebra of functions :

$f : D_1 \rightarrow R \quad , g : D_2 \rightarrow R$

- $f+g(x) : f(x)+g(x)$ where $D : D_1 \cap D_2$
- $f \cdot g(x) : f(x) \cdot g(x)$ where $D : D_1 \cap D_2$
- $f-g(x) : f(x) - g(x)$ where $D : D_1 \cap D_2$
- $(f/g)(x) = \dfrac{f(x)}{g(x)}$ where $D : D_1 \cap D_2$, $g(x) \neq 0$

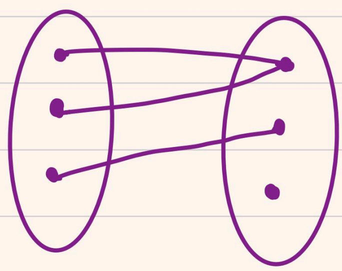
★ Types of function :

• One-One And Many-One functions :



One-One

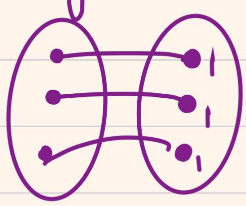
(all elements in domain have distinct img in co-domain)



Many-One

• Onto and Into functions :

Onto : for every element in the co-domain had a pre-img in the domain.



Into : There may be some element in co-domain which don't have a pre-img.

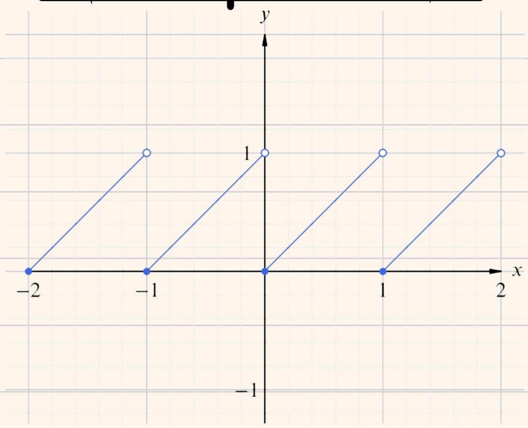
• Odd & Even function :

- $f(x) = f(-x) \implies$ Even $f(x)$
- $f(x) + f(-x) = 0 \implies$ odd $f(x)$.

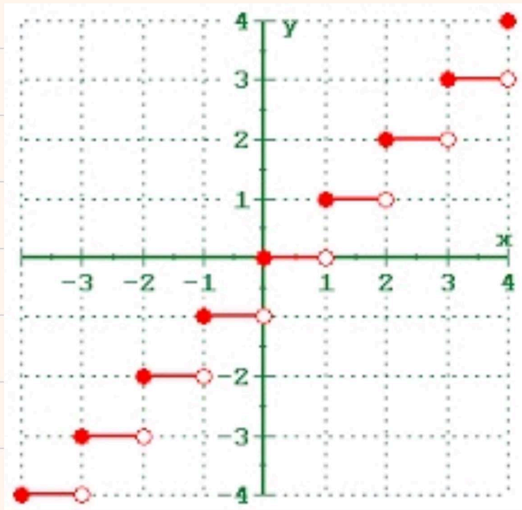
- **Periodic function:**

$\Rightarrow$ $f(x)$ is periodic if : $f(x+T) = f(x)$; T is the period of the function.

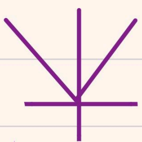
- **Some imp functions:**



$y = \{x\}$

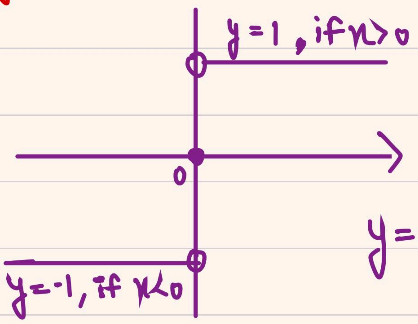


$y = [x]$

- 

$y = |x|$

Signum $f(x)$:



$y = f(x) = \operatorname{sgn} f(x)$

$$f(x) = \begin{cases} 1 & \text{for } x>0 \\ 0 & \text{for } x=0 \\ -1 & \text{for } x<0 \end{cases}$$

$\operatorname{sgn} f(x) \Rightarrow \dfrac{|x|}{x}$, $x \neq 0$, $f(0) = 0$

- **Some rules:**

① $f(xy) = f(x) + f(y) \Longrightarrow f(x) = k \ln x$ or 0

② $f(xy) = f(x) \cdot f(y) \Rightarrow f(x) = x^n$, $n \in \mathbb{R}$

③ $f(x+y) = f(x) \cdot f(y) \Rightarrow f(x) = a^{kx}$

④ $f(x+y) = f(x) + f(y) \Rightarrow f(x) = kx$

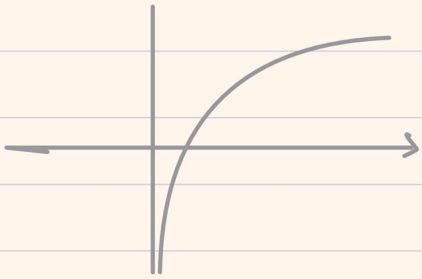
$k \in$ const.

CHEMISTRY CLASS -11 2025-2026 SESSION

Logarithm :

$\Rightarrow b^y = a \Rightarrow \log_b a = y$

($b \in$ base)



$(e = 2.718282)$

- Common logarithm

$\Rightarrow$ Base = 10

- Natural logarithm

$\Rightarrow$ Base = 'e'

- Rules of logarithm :

- Product Rule : $\log_b(mn) = \log_b m + \log_b n$

- Division Rule : $\log_b(m/n) = \log_b m - \log_b n$

- exponential Rule : $\log_b(m^n) = n \log_b m$

- Base changing : $\log_b m = \dfrac{\log_a m}{\log_a b}$, $a \in N$ (Any no.)

- Base switch : $\log_b a = \dfrac{1}{\log_a b}$

- Derivative of log $\Rightarrow f(x) = \log_b(x) \Rightarrow f'(x) = \dfrac{1}{x \log b}$

$\log_b b = 1$

$\log_b 1 = 0$

$\log_b 0 =$ undefined

Sequence & Series

- A.P : $\underbrace{a}_{\text{first term}}$, $d \rightarrow$ common diff. $\Big\}$ $a_n = a + (n-1)d$

In general, any seq- whose nth term is linear is an A.P

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right) = \frac{n}{2}\left(a + \underbrace{l}_{\text{last term}}\right)$$

* Some Tricks :

- $a_1, a_2 \text{-----} a_n$ are in A.P. $\Longrightarrow a_1 + a_n = a_2 + a_{n-1} = \text{-----}$
- If $a_1, a_2, a_3 \text{---} a_n$ in AP $\Longrightarrow a^{a_1}, a^{a_2} \text{----} a^{a_n}$ are in G.P. $(a>0)$
- If $\{t_n\}$ is an A.P, then common difference $\boxed{d = \frac{t_p - t_q}{p-q}}$ $(p, q \in N)$
- 1^{st} diff in A.P, then let $t_n = an^2 + bn + c$, Put $n = 1, 2, 3$ and compare with respective terms to find a, b, c

- **Arithmetic mean :** a, b, c are in AP

  $\downarrow$

  arithmetic mean if : $b = \frac{a+c}{2}$ is AM of a & c.

- if $a_1, a_2 \text{---} a_n$ are n numbers $\Longrightarrow AM = \frac{1}{n}(a_1 + a_2 \text{---} a_n)$

- Geometric Progression (G.P):

- $\underbrace{a}_{\text{first term}}$, $r \to$ Common Ratio $\Longrightarrow$ G.P is written as: $a, ar, ar^2 ----$ } n^{th} term is given by $\boxed{a_n = ar^{n-1}}$

$S_n \Rightarrow \frac{a(r^n-1)}{r-1}$, $r \neq 1$, na $(r=1)$
(sum of n terms)

if $-1 < r < 1$, then sum of infinite G.P $\Rightarrow \frac{a}{1-r}$

* Points to remember: (if $a_1, a_2 --- a_n$ are in G.P)
then,

1) $a_1 a_n = a_2 a_{n-1} = a_3 a_{n-2}$ --- so on.

2) $\log a_1, \log a_2$ ---- $\log a_n$ are in AP.

- Geometric mean: $a, \underbrace{b}, c$ are in G.P

Geometric mean if: $b^2 = ac$ | $b = \sqrt{ac}$

- $a_1, a_2 --- a_n$ are +ve numbers $\Longrightarrow$ GM $= (a_1 a_2 a_3 --- a_n)^{1/n}$

- G_1, G_2 ---- G_n are n G.M between a, b then $a, G_1, G_2 -- G_n, b$ are in G.P

$b = ar^{n+1}$, $r = \sqrt[n+1]{\frac{b}{a}}$ $\Longrightarrow G_r = a\left(\sqrt[n+1]{\frac{b}{a}}\right)^r$

Some imp concepts: $a_1, a_2 --- a_n$ be n +ve real no. (all not equal)

$\frac{a_1^m + a_2^m ---- + a_n^m}{n} > \left(\frac{a_1 + a_2 - a_n}{n}\right)^m$, $\frac{a_1^m + a_2^m --- a_n^m}{n} < \left(\frac{a_1 + a_2 --- a_n}{n}\right)^m$

for $m \in R - [0,1]$ if $m \in (0,1)$

H.P (Harmonic Progression):

If $a_1, a_2 \ldots a_n$ are in A.P

$\Downarrow$

$\frac{1}{a_1}, \frac{1}{a_2} \ldots \frac{1}{a_n}$ are in H.P

$a_n = \frac{1}{a+(n-1)d}$ $\quad a = \frac{1}{a_1}, d = \frac{1}{a_2} - \frac{1}{a_1}$

- a, H, b are in HP $\Rightarrow H = \frac{2ab}{a+b}$
- H.M of $a_1, a_2 \ldots a_n$ is given by $\frac{1}{H} = \frac{1}{n}\left(\frac{1}{a_1} + \frac{1}{a_2} \cdots \frac{1}{a_n}\right)$

* Some imp. Results :

- $1+2+3 \ldots n = \frac{n(n+1)}{2}$
- $1^2+2^2 \ldots n^2 = \frac{n(n+1)(2n+1)}{6}$
- $1^3+2^3 \ldots n^3 = \left[\frac{n(n+1)}{2}\right]^2$
- $1+x+x^2+x^3 \ldots = \frac{1}{1-x}, -1<x<1$
- $1+2x+3x^2 \ldots = \frac{1}{(1-x)^2}, -1<x<1$

$\Rightarrow AM = \frac{a_1+a_2 \ldots a_n}{n}$, $GM = (a_1 a_2 \ldots a_n)^{1/n}$, $H = \frac{n}{\left(\frac{1}{a_1}+\frac{1}{a_2} \ldots \frac{1}{a_n}\right)}$

Inequality: $\underbrace{AM \geq GM \geq H.M}$

Equality holds when $a_1 = a_2 = a_3 \ldots a_n$

- **Arithmetico-Geometrico** : 1 A.P & 1 G.P

$$S_n = ab + (a+d)(br) + (a+2d)br^2 \ldots (a+(n-1)d)br^{n-1}$$

$$\underset{n\to\infty}{\Rightarrow} S = \frac{ab}{1-r} + \frac{dbr}{(1-r)^2}$$

Adv. Concepts :

$$e^x = \sum_{n=0}^{\infty}\left(\frac{x^n}{n!}\right) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \text{----} + \frac{x^n}{n!}$$

Put $x=1$, $e = 1 + \frac{1}{1!} + \frac{1}{2!}$ ----∞ — (1)

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

Put $x = -x$ $\Rightarrow e^{-x} = \sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!}$

$\Rightarrow 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!}$ ---∞ — (2)

from (1) & (2)

- $e^{ax} = 1 + ax + \frac{(ax)^2}{2!}$ --- $\frac{(ax)^n}{n!}$

$$\frac{e^x + e^{-x}}{2} = \sum_{n=0}^{\infty}\frac{x^{2n}}{2n!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} \text{---}\infty$$

for $a^x = e^{\underbrace{x \ln a}} \rightarrow ax$

Put this and you can find the sum.

$$\frac{e^x - e^{-x}}{2} = \sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} \text{---}\infty$$

Quadratic Eq:

diff. of roots

$$\boxed{|\alpha-\beta| = \frac{\sqrt{D}}{a}}$$

- ax^2+bx+c (where $a \neq 0$, $a, b, c \in \mathbb{C}$)

α β → Roots of the quadratic eq $(ax^2+bx+c=0)$

→ $\alpha+\beta = \frac{-b}{a}$, $\alpha \times \beta = \frac{c}{a}$ | Discriminant $(D) = b^2-4ac$

Roots are given by: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Eq with roots α, β
$\Rightarrow (x-\alpha)(x-\beta)=0$

$\Rightarrow x^2-(\alpha+\beta)x+\alpha\beta=0$

- Nature of roots :

$D>0$
Real and distinct roots

$D=0$
- Real and equal

$D<0$
- Imaginary and distinct.

**

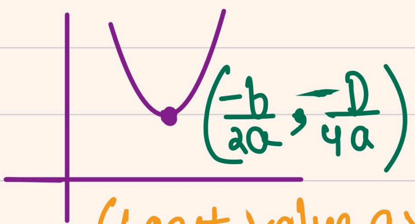
- Transformation of roots : ax^2+bx+c α, β

- for $\frac{1}{\alpha}, \frac{1}{\beta} \Rightarrow x \rightarrow \frac{1}{x}$

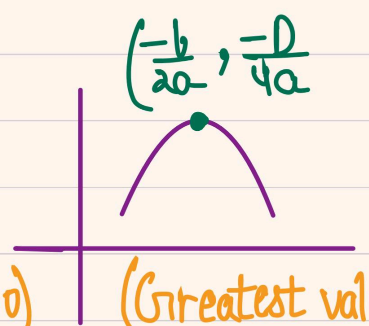
for eg: $a\left(\frac{1}{x^2}\right)+b\left(\frac{1}{x}\right)+c \Rightarrow \underbrace{cx^2+bx+a=0}$

New quadratic with roots $\frac{1}{\alpha}, \frac{1}{\beta}$

- $-\alpha, -\beta \Rightarrow x \rightarrow -x$
- $\alpha+k, \beta+k \Rightarrow x \rightarrow (x-k)$
- $\alpha^n, \beta^n \Rightarrow x \rightarrow x^{1/n}$
- $\alpha^{1/n}, \beta^{1/n} \Rightarrow x \rightarrow x^n$
- $k\alpha, k\beta \Rightarrow x \rightarrow \frac{x}{k}$
- $\frac{\alpha}{k}, \frac{\beta}{k} \Rightarrow x \rightarrow kx$



(Least value, $a>0$)



(Greatest value, $a<0$)

• Concept of Common Root :

eq 1: $ax^2+bx+c=0$, eq 2: $dx^2+ex+f=0$

(Common root $\Rightarrow \alpha$) $\Rightarrow (dc-af)^2=(bf-ce)(ae-cd)$

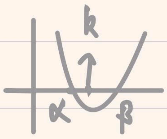
or General formula : $\dfrac{\alpha^2}{bf-ec}=\dfrac{\alpha}{(dc-af)}=\dfrac{1}{ae-db}$

* for both root common: $\dfrac{a}{d}=\dfrac{b}{e}=\dfrac{c}{f}$

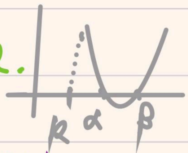
* Location of Roots & its conditions :

① $k\in\mathbb{R}$ and $\alpha<k<\beta$.

→ $f(k)<0$, $D>0$

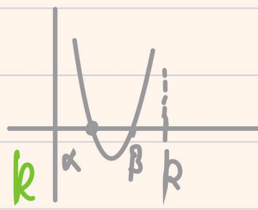


② Both roots greater than k.



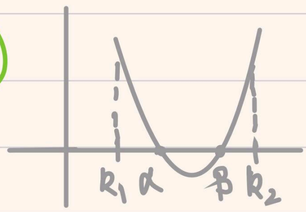
→ $D\geq 0$, $\dfrac{-b}{2a}>k$ and $f(k)>0$

③ Both Roots less than k



$D\geq 0$, $\dfrac{-b}{2a}<k$ and $f(k)>0$

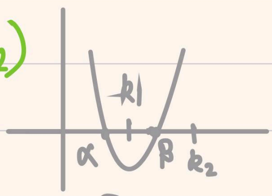
④ $\alpha,\beta\in(k_1,k_2)$



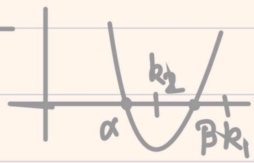
→ $D\geq 0, k_1<\dfrac{-b}{2a}<k_2$, $f(k_1)>0$, $f(k_2)>0$

⑤ Exactly one root lie in (k_1,k_2)

→ $f(k_1)\times f(k_2)<0$



or



Quadratic eq in 2 variable :

$ax^2+2hxy+by^2+2gx+2fy+c$

Condition of resolving it into 2 linear factors:

$\Rightarrow abc+2fgh-af^2-bg^2-ch^2=0$

or $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$

Roots in special Cases :

$(\alpha+\beta<0, \alpha\beta>0)$
$a>0, b>0, c>0$
→ Both roots negative

$(\alpha+\beta<0, \alpha\beta<0)$
$a>0, b>0, c<0$
→ Roots of opp sign
→ mag of −ve Root > mag of +ve root

$(\alpha+\beta>0, \alpha\beta>0)$
$a>0, b<0, c>0$
→ Both roots are +ve

$(\alpha+\beta>0, \alpha\beta<0)$
$a>0, b<0, c<0$
→ Both roots of opp. sign
→ mag of +ve Root > mag of −ve Root

Adv. Section :

- Theory of Equations :

- $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots a_0$ $(a_0, a_1 \ldots a_n \in \mathbb{R}$ & $a_n \neq 0) \Rightarrow p(x) = 0$ has n roots.

A polynomial in x of odd degree has atleast one real root. (it has odd no. of real roots)

- If $x_1, x_2 \ldots x_n$ are roots of $p(x)=0 \Longrightarrow p(x)$ can be written: $p(x) = a_n(x-x_1)\ldots\ldots(x-x_n)$

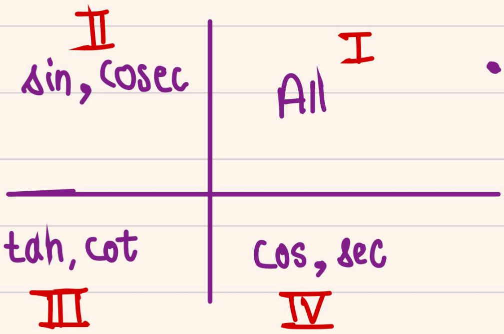
- If eq $f(x) = a_n x^n + a_{n-1} x^{n-1} \ldots + a_0 = 0$ has more than n distinct roots then $f(x)$ is identically zero.

- ★ Imginary roots of a quadratic eq. always occur in conjugate pair.

- Irrational roots of a quadratic eq. always occur in conjugate pair.

Trigonometric Ratios & Identities :

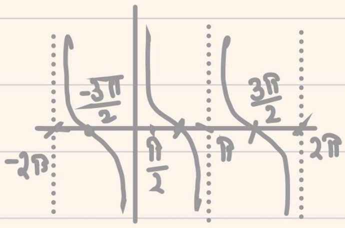
- $180^\circ = \pi$ radians
- $\theta = \frac{l}{r}$



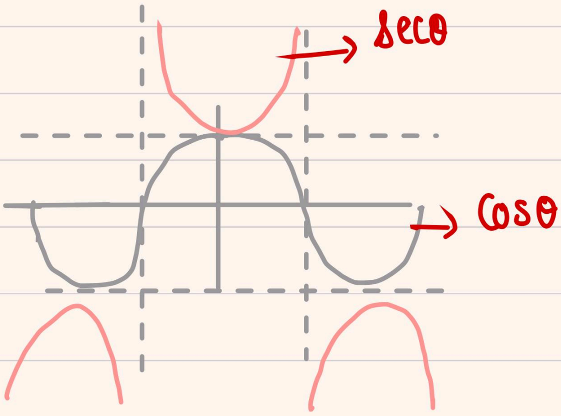
- +ve sign of functions in diff. quad.

(Rest all other -ve)

- Imp graphs :



$\cot\theta$



(simple trick for cosec, sec.)
$\Rightarrow$ Draw sin, cos respectively
* then draw at other ends.

- $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$
- $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

** Sum and diff. formulae :

- $\sin(A+B) = \sin A\cos B + \cos A\sin B$
- $\cos(A+B) = \cos A\cos B - \sin A\sin B$
- $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$
- $\tan\left(\pi/4 + \theta\right) = \dfrac{1+\tan\theta}{1-\tan\theta}$

- $\sin(A-B) = \sin A\cos B - \cos A\sin B$
- $\cos(A-B) = \cos A\cos B + \sin A\sin B$
- $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$
- $\tan\left(\pi/4 - \theta\right) = \dfrac{1-\tan\theta}{1+\tan\theta}$

$\Rightarrow$ • $\cot(A+B) = \dfrac{\cot A\cot B - 1}{\cot A + \cot B}$

- $\sin(A+B)\times\sin(A-B) = \sin^2 A - \sin^2 B$
- $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

- $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$
- $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$
- $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

- $(\cos A \pm \sin A)^2 = 1 \pm \sin 2A$
- $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A} \quad \left(n \neq n\pi + \frac{\pi}{6}\right)$
- $\sin A/_2 = \pm\sqrt{\dfrac{1-\cos A}{2}}$
- $\cos \dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}}$
- $\tan \dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}}$
- $\sin 3A = 3\sin A - \sin^3 A$
- $\cos 3A = 4\cos^3 A - 3\cos A$

- **C & D formulae:**

- $\sin C + \sin D = 2\sin\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$
- $\sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$
- $\cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$
- $\cos C - \cos D = 2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{D-C}{2}\right)$
- $\tan A \pm \tan B = \dfrac{\sin(A \pm B)}{\cos A \cos B} \quad \left(A \neq n\pi + \frac{\pi}{2}, B \neq m\pi\right)$
- $\cot A \pm \cot B = \dfrac{\sin(B \pm A)}{\sin A \sin B}$
- $\cos A \pm \sin A = \sqrt{2}\sin\left(\frac{\pi}{4} \pm A\right) = \sqrt{2}\cos\left(\frac{\pi}{4} \mp A\right)$
- $\tan A + \cot A = \dfrac{1}{(\sin A \cos A)}$
- $1 + \tan A \tan B = \dfrac{\cos(A-B)}{\cos A \cos B}$
- $1 - \tan A \tan B = \dfrac{\cos(A+B)}{\cos A \cos B}$
- $\cot A - \tan A = 2\cot 2A$
- $\tan A + \cot A = 2\operatorname{cosec} 2A$
- $\sin\left(\frac{A}{2}\right) + \cos\left(\frac{A}{2}\right) = \pm\sqrt{1+\sin A}$
- $\sin\left(\frac{A}{2}\right) - \cos\left(\frac{A}{2}\right) = \pm\sqrt{1-\sin A}$

*
- $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
- $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
- $2\cos A\cos B = \cos(A+B) + \cos(A-B)$
- $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

* **Trigo ratio for some imp angles.:**

- $\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75$
- $\cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ$
- $\sin 18 = \cos 72 = \dfrac{\sqrt{5}-1}{4}$
- $\cos 18^\circ = \sin 72 = \dfrac{1}{4}\sqrt{10+2\sqrt{5}}$
- $\sin 36 = \cos 54^\circ = \dfrac{1}{4}\sqrt{10-2\sqrt{2}}$
- $\cos 36 = \dfrac{1}{4}(\sqrt{5}+1) = \sin 54$

• formula for 3 Angles :

→ $\sin(A+B+C) = \sin A\cos B\cos C + \cos A\sin B\cos C + \cos A\cos B\sin C - \sin A\sin B\sin C$
$= \cos A\cos B\cos C\,(\tan A + \tan B + \tan C - \tan A\tan B\tan C)$

→ $\cos(A+B+C) = \cos A\cos B\cos C - \sin A\sin B\cos C - \sin A\cos B\sin C - \cos A\sin B\sin C$
$= \cos A\cos B\cos C\,(1 - \tan A\tan B - \tan B\tan C - \tan C\tan A)$

→ $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan C\tan A}$

★ • $\sin(60-A)\sin A\sin(60+A) = \dfrac{\sin 3A}{4}$

• $\cos(60-A)\cos A\cos(60+A) = \dfrac{\cos 3A}{4}$

• $\tan(60-A)\tan A\tan(60+A) = \tan 3A$

• conditional identities :

① $A+B+C = 180^\circ$ then,

→ $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$
→ $\sin 2A + \sin 2B - \sin 2C = 4\cos A\cos B\sin C$

• $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)$

• $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

→ $\tan A + \tan B + \tan C = \tan A\tan B\tan C$
→ $\sum \cot A\cot B = 1$

when $A+B+C=\pi$

• $\dfrac{\cos A}{\sin B\sin C} + \dfrac{\cos B}{\sin C\sin A} + \dfrac{\cos C}{\sin A\sin B} = 2$

② $x+y+z = \pi/2$

• $\sin^2 x + \sin^2 y + \sin^2 z = 1 - 2\sin x\sin y\sin z$
• $\cos^2 x + \cos^2 y + \cos^2 z = 2(1 + \sin x\sin y\sin z)$
• $\sin 2x + \sin 2y + \sin 2z = 4\cos x\cos y\cos z$

Adv. Concepts ·

- $-\sqrt{a^2+b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2+b^2}$

- $x^2+\frac{1}{x^2} \geq 2$ (can be used in $\sin^2 x + \frac{1}{\operatorname{cosec}^2 x} \geq 2$ etc.)

- $\sec\theta + \tan\theta = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$
- $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \cot\frac{\theta}{2} = \operatorname{cosec}\theta + \cot\theta$
- $\sec\theta - \tan\theta = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$
- $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \tan\frac{\theta}{2} = \operatorname{cosec}\theta - \cot\theta$

★ $\cos\theta \cos 2\theta \cos 2^2\theta ---- \cos 2^{n-1}\theta = \frac{\sin(2^n\theta)}{2^n \sin\theta}$; $(\theta \neq n\pi)$

★ $\cos A + \cos(A+B) + \cos(A+2B) \text{-----} \cos(A+(n-1)B) = \frac{\sin(nB/2)}{\sin B/2} \cos\left(A+\frac{(n-1)B}{2}\right)$

⇒ Miscellaneous Points :

→ $\tan(A+B+C) = \frac{\sum \tan A - \tan A \tan B \tan C}{1 - \sum \tan A \tan B}$

→ $\tan(A+B) - \tan A - \tan B = \tan A \tan B \tan(A+B)$

→ $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) \text{-----} \sin(\alpha+(n-1)\beta) = \frac{\sin\left(\alpha+\frac{(n-1)}{2}\beta\right)\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$

→ $\cos\alpha + \cos(\alpha+\beta) \text{---------} \cos(\alpha+(n-1)\beta) = \frac{\cos\left(\alpha+\frac{(n-1)}{2}\beta\right)\left(\sin\left(\frac{n\beta}{2}\right)\right)}{\sin\left(\frac{n\beta}{2}\right)}$

$c-\sqrt{a^2+b^2} \leq a\sin x + b\cos x + c \leq c + \sqrt{a^2+b^2}$

Trigonometric Equations :

(Principal Solution :
$\Rightarrow$ solution lying in the interval : $[0, 2\pi]$)

• Periodic functions :

$f(x)$ is said to be Periodic if $\exists T$ s.t $\boxed{f(x+T) = f(x)}$

• $\sin(ax+b)$, $\cos(ax+b)$, $\sec(ax+b)$, $\text{cosec}(ax+b) = \dfrac{2\pi}{a}$ (Period)

• $|\sin(ax+b)|$, $|\cos(ax+b)| \Rightarrow \dfrac{\pi}{a}$ (Period)

• $|\tan(ax+b)|$, $|\cot(ax+b)| = \dfrac{\pi}{2a}$

* Eq. & their General sol. :

eg: $\sin\theta = 0 \Rightarrow \boxed{\theta = n\pi}$ $n \in I$

$f(x)$	0	1	-1	$f(\alpha)$ $\longrightarrow$ eg: ($\sin\theta = \sin\alpha$)	
• $\sin\theta$	$n\pi$	$2n\pi + \frac{\pi}{2}$	$2n\pi - \frac{\pi}{2}$	$n\pi + (-1)^n \alpha$	$\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
• $\cos\theta$	$n\pi + \frac{\pi}{2}$	$2n\pi$	$2n\pi \pm \pi$	$2n\pi \pm \alpha$	
• $\tan\theta$	$n\pi$	$\frac{\pi}{4} + n\pi$	—	$n\pi + \alpha$	

• $\sin^2\theta = \sin^2\alpha \Rightarrow \theta = n\pi \pm \alpha$
• $\cos^2\theta = \cos^2\alpha \Rightarrow \theta = n\pi \pm \alpha$
• $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha$

• Gen sol. for eq : $\boxed{a\cos\theta + b\sin\theta = c}$

$a\cos\theta + b\sin\theta = c$, substitute $a = r\cos\phi$ $b = r\sin\phi$

$$r = \sqrt{a^2+b^2} \qquad \phi = \tan^{-1}\left(\frac{b}{a}\right)$$

On substitution :

$a\cos\theta + b\sin\theta = c \xRightarrow{\text{becomes}} \cos(\theta-\phi) = \frac{c}{r} \Rightarrow \cos(\theta-\phi) = \frac{c}{\sqrt{a^2+b^2}}$

If $c > \sqrt{a^2+b^2}$ then $\Rightarrow$ no solution

If $c < \sqrt{a^2+b^2}$ $\Rightarrow$ take $\frac{|c|}{\sqrt{a^2+b^2}} = \cos\alpha$ $\Rightarrow \cos(\theta-\phi) = \cos\alpha$

$$\theta - \phi = 2n\pi \pm \alpha$$

$$\boxed{\theta = 2n\pi \pm \alpha + \phi}$$

Permutation & Combination :

- <u>Fundamental principle of counting</u> :

→ 'A' job ⟶ m ways
'B' job ⟶ n ways
} Both jobs together in mn ways

- ^nP_r ?

$$\boxed{^nP_r = n(n-1)(n-2)\text{----}(n-r+1) \Rightarrow \frac{n!}{(n-r)!}}$$

- $^nP_n = n!$, $^nP_0 = 1$, $^nP_1 = n$

$^nP_{n-1} = {^nP_n} = n!$

$$\Rightarrow \boxed{^nP_r = n\,{^{n-1}P_{r-1}}}$$

- <u>Permutation of n distinct objects when repetition is allowed</u> ?

⇒ n diff things

each place can be filled in n ways

$$\underbrace{\underline{n} \times \underline{n} \times \underline{n} \times \underline{n} \times \underline{n}}_{r \text{ places}} \Rightarrow n^r.$$

- <u>Arrangement of n things when all are not distinct</u> :

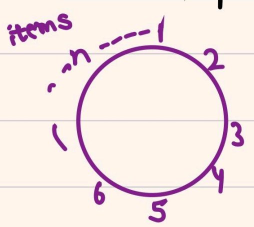
n items , x ⟶ one kind
y ⟶ 2nd Kind
z ⟶ 3rd Kind

$$\boxed{\frac{n!}{x!\,y!\,z!}}$$

$(x+y+z \leq n)$

and rest $n-(x+y+z)$ are all distinct

- <u>Circular permutation</u> :



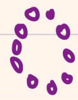
Arrangements ⇒ $(n-1)!$

★ **Special case:** (for circular) n diff. things taken r at a time?

Clock wise & Anticlockwise is taken diff

$\Rightarrow \dfrac{{}^nP_r}{r}$

Clock wise & Anticlockwise are same

eg: Necklace 

$\Rightarrow \dfrac{{}^nP_r}{2r}$

• Combination? ${}^nC_r$ or $C(n, r)$

$${}^nC_r = \frac{n!}{(n-r)!\, r!} \quad (0 \le r \le n) \quad \text{or} \quad {}^nC_r = \frac{{}^nP_r}{r!}$$

Results:

- ${}^nC_0 = {}^nC_n = 1$
- ${}^nC_1 = n$ (as there n ways to select one out of n distinct things)
- ${}^nC_r = {}^nC_{n-r}$
- $n \in$ odd $\Longrightarrow$ greatest value of ${}^nC_r \Rightarrow r = \frac{n+1}{2}$ or $r = \frac{n-1}{2}$
- $n \in$ even $\Longrightarrow$ greatest value of ${}^nC_r \Rightarrow r = n/2$

• Selection of distinct / identical obj?

① Selection from distinct objects:

- Atleast one from them

$${}^nC_1 + {}^nC_2 - - - - + {}^nC_n = 2^n - 1$$

② Selection from identical objects:

- Atleast one out of $a_1 + a_2 - - - - + a_n + k$ objects (where a_1 are of 1^{st} kind ---- a_n are of n^{th} kind & k are distinct)

$\Longrightarrow (a_1+1)(a_2+1) - - - - (a_n+1)\, 2^k - 1$

☆☆ Division of distinct obj. into groups :

(m+n+p things are divided into 3 groups) having m, n, p elements

No. of grouping ⇒ $\dfrac{(m+n+p)!}{m!\,n!\,p!}$

- **In General**, no. of ways in which mn things can be divided equally into 'm' distinct groups when order of elements is imp.

$$\Rightarrow \frac{(mn)!}{(n!)^m}$$

⇒ Division of identical objects into group :

n items ⟶ into r diff groups

empty groups (Allowed) $\Rightarrow {}^{n+r-1}C_{r-1}$

empty groups (Not Allowed) ${}^{n-1}C_{r-1}$

☆ Arrangement in groups :

$\underbrace{\text{(After distribution acc. to above classification)}}_{\text{Just multiply with } n!}$

Adv Concepts :

- $N = 2^a 3^b 5^c 7^d$

Total factors = $(a+1)(b+1)(c+1)(d+1)$

Proper factor = Total factors − 2

- **Distribution :**

$$x = \frac{17!}{4!\,4!\,2!\,2!\,2!\,3!\,\,2!\,3!\,1!}$$ duplicacy.

(for duplicate category)

• $x+y+z=29$

$(x\geq 1,\ y\geq 1,\ z\geq 3)$ how to distribute? identical items

$\Rightarrow$ make a new variable

$x+y+z+t=29$

$$\left.\begin{array}{l} x\geq 1 \\ y\geq 2 \\ z\geq 3 \\ t\geq 0 \end{array}\right\} \Rightarrow (x-1)+(y-2)+(z-3)+t = 29-1-2-3$$

$$x+y+z+t=23 \ \Big\}\ x,y,z,t \underset{\sim}{\geq} 0$$

empty grp allowed

$$\Rightarrow {}^{n+r-1}C_{r-1} = {}^{26}C_3$$

• $3x+y+z=24$ (non-ve integral solution)

$y+z=24-3x$

make cases:

$x=0 \quad x=1 \quad ---- \quad x=8$

After this sum would be -ve

$y+z=24 \quad y+z=21 \quad --- \quad y+z=0$

${}^{24+2-1}C_1 + {}^{21-2-1}C_1 + ---- \ {}^{1}C_1 = 25+22+19 --- +1.$

• $x+y+z\leq 10$

$\Rightarrow$ Add dummy variable $w\in[0,\infty)$

$\Rightarrow x+y+z+w=10 \Rightarrow$ Apply empty formula $\left({}^{n+r-1}C_{r-1}\right)$

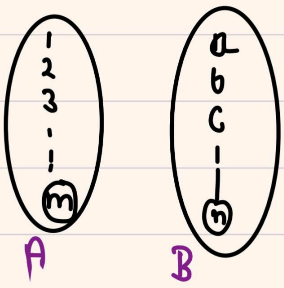
for $x+y+z>10 \Rightarrow$ (Total $-$ less than 10)

• dearrangement formula:

$$n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}---\frac{1}{n!}\right]$$

eg 5 letters, 5 envelope, ways in which letters are sent in wrong envelope?

$$5!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right]=44$$



Total function $(A\rightarrow B)$: n^m

Total one-one $f(x)$: ${}^nP_m$

Many one $f(n)$: $n^m - {}^nP_m$

Onto : Total - Into

$$\Rightarrow n^m - \underbrace{\left[{}^nC_1(n-1)^m - {}^nC_2(n-2)^m + {}^nC_3(n-2)^m ----\right]}_{\text{Into}}$$

• Result: $\boxed{{}^{m+n}C_r = {}^mC_r\,{}^nC_0 + {}^mC_{r-1}\,{}^nC_1 + \_\_\_ + {}^mC_0\,{}^nC_r}$

** (for Rank of a No. in Dictionary?)

$\Rightarrow$ (Recommended to watch YT shortcut videos)

Binomial Theorem :

Imp. Points :

- $(x+a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 \text{-----} {}^nC_n a^n$

$$= \sum_{r=0}^{r=n} {}^nC_r x^{n-r} a^r \Big\} \text{ Total } n+1 \text{ terms.}$$

Binomial coeff: ${}^nC_r = \dfrac{n!}{(n-r)! \times r!}$

- ${}^nC_x = {}^nC_y$, then either $x=y$ or $x+y=n$

- **Greatest Binomial coeff. :**

n ∈ even

$\Rightarrow r = \frac{n}{2}$ i.e ${}^nC_{n/2}$ is max.

n ∈ odd

$\Rightarrow r = \frac{n-1}{2}$ or $\frac{n+1}{2}$

i.e ${}^nC_{\frac{n-1}{2}}$ or ${}^nC_{n+1/2}$

- **General term :**

$T_{r+1} = {}^nC_r x^{n-r} a^r$

- $\dfrac{T_{r+1}}{T_r} = \left(\dfrac{n-r+1}{r}\right)\left(\dfrac{a}{x}\right)$

- **Some relation in Binomial Coeff. :**

$\rightarrow (x+a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 \text{-----} {}^nC_n a^n$

for $a=1$: $\boxed{(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x \text{------} + {}^nC_n x^n}$ — ①

Put $x=1$: $2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 \text{---} + {}^nC_n \longrightarrow$ Sum of coeff.

On diffrentiating ①,

$n(1+x)^{n-1} = {}^nC_1 + 2\,{}^nC_2 x \text{---} n\,{}^nC_n a^{n-1}$ (Put $x=1$) $\Rightarrow \sum_{r=1}^{n} {}^nC_r = n \times 2^{n-1}$

On integrating ①,

Put $x=0$ (to find k)

$$\frac{(1+x)^{n+1}}{(n+1)} = {}^nC_0 x + {}^nC_1 \frac{x^2}{2} + {}^nC_2 \frac{x^3}{3} \text{ ------} + {}^nC_n \frac{x^{n+1}}{n+1} + k$$

Put $x=1$: $${}^nC_0 + \frac{{}^nC_1}{2} + \frac{{}^nC_2}{3} \text{ ----} + \frac{{}^nC_n}{n+1} = \frac{2^{n+1}-1}{n+1}$$

- **Numerically greatest term of Binomial exp. :**

$$(a+x)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1} x \text{ ------} + C_n x^n$$

$$\left|\frac{T_{r+1}}{T_r}\right| = \left|\frac{{}^nC_r}{{}^nC_{r-1}}\right|\left|\frac{x}{a}\right| = \left|\frac{n-r+1}{r}\right|\left|\frac{x}{a}\right|$$

Take $\left|\frac{n-r+1}{r}\right|\left|\frac{x}{a}\right| \geq 1$ $\quad$ $\{\text{As } |T_{r+1}| \geq |T_r|\}$

$r \leq \dfrac{n+1}{1+\left|\frac{a}{x}\right|}$, so greatest term will be T_{r+1} where $r = \left[\dfrac{n+1}{1+\left[\frac{a}{x}\right]}\right]$

- **Binomial theorem for any index :**

n → can be -ve/+ve/ fraction

→ $$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \text{ -------} + \frac{n(n-1)(n-2)\text{ --- }(n-r+1)}{r!} x^r \text{ --- } \infty$$

(valid only if $|x|<1$

- General term of series $(1+x)^{-n} = \dfrac{(-1)^r \times n(n+1)(n+2)\text{ ---- }(n+r-1)}{r!} x^r$
- Gen. term of $(1-x)^{-n} = \dfrac{n(n+1)(n+2)\text{ ----- }(n+r-1)}{r!} x^r$

* Quick & Imp. Results :

- If coeff. of r^{th}, $(r+1)^{th}$, $(r+2)^{th}$ terms in the exp. of $(1+x)^n$ are in

① H.P $\Rightarrow n+(n-2r)^2=0$

② A.P $\Rightarrow n^2-n(4r+1)+4r^2-2=0$

- No. of terms in the exp of $(x_1+x_2 ----+x_r)^n$ is ${}^{(n+r-1)}C_{r-1}$

Adv. Concepts :

- To determine term in the expansion of $\left(x^\alpha \pm \frac{1}{x^\beta}\right)^n$, for x^m occurs in T_{r+1}, then $\boxed{r = \frac{n\alpha-m}{\alpha+\beta}}$

★ Term independent of x, occurs in $T_{r+1} \Rightarrow \boxed{r=\frac{n\alpha}{\alpha+\beta}}$

- Results : $({}^nC_r = C_r)$

① $C_0+C_1+C_2 ---- C_n=2^n$

② $C_0-C_1+C_2-C_3 --(-1)^n C_n = 0$

③ $C_0+C_2+C_4+C_6 ----= C_1+C_3+C_5 --- = 2^{n-1}$

④ $C_0C_r + C_1C_{r+1} + \dots + C_{n-r}C_n = {}^{2n}C_{n-r} = \dfrac{2n!}{(n+r)!\,(n-r)!}$

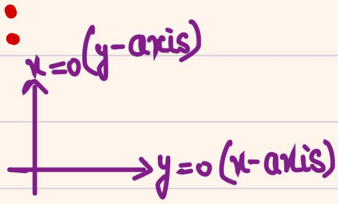
if $r=1$, $C_0C_1 + C_1C_2 + \dots + C_{n-1}C_n = \dfrac{2n!}{(n+1)!\,(n-1)!}$

⑤ $C_1 + 2C_2 + 3C_3 \dots + nC_n = n \times 2^{n-1}$

⑥ $C_1 - 2C_2 + 3C_3 \dots \pm (-1)^n C_n = 0$

⑦ $C_0^2 - C_1^2 + C_2^2 - C_3^2 \dots + (-1)^n C_n^2 = \begin{cases} 0 & n \in \text{odd} \\ (-1)^{n/2}\, {}^{n}C_{n/2} & n \in \text{even} \end{cases}$

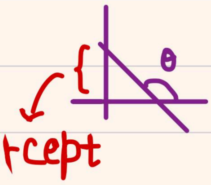
Coordinate Geometry :



• Straight lines :

→ Gen. form : $ax+by+c=0$ —① . if (x_1,y_1) lies on —①, then $ax_1+by_1+c=0$

• Slope:



$\text{slope} = \tan\theta = \frac{y_2-y_1}{x_2-x_1}$, for $ax+by+c=0 \Rightarrow \boxed{m=\frac{-a}{b}}$

(2 Points on line (x_1,y_1), (x_2,y_2))

* Different forms of eq :

① Slope-intercept form : $y = mx+c$

② Point form (x_1,y_1) : $y-y_1 = m(x-x_1)$

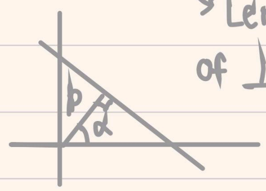
③ Two-Point form : $y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

④ Intercept form : $\frac{x}{a}+\frac{y}{b} = 1$

on x-axis ⇐ a, b → on y-axis

⑤ Normal-Perpendicular form :

$x\cos\alpha + y\sin\alpha = p$ → Length of ⊥



⑥ Parametric form :

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

• Gen form of eq ⟹ Standard form

↳ $ax+by+c=0$ —①

(i) $y = -\frac{a}{b}x - \frac{c}{b}$, $m = -\frac{a}{b}$, intercept $= \frac{-c}{b}$

(ii) $\frac{x}{-c/a} + \frac{y}{-c/b} = 1$, x Intercept $= -c/a$

y " $= -c/b$

(iii) $\frac{-ax}{\sqrt{a^2+b^2}} - \frac{by}{\sqrt{a^2+b^2}} = \frac{c}{\sqrt{a^2+b^2}}$ $(c>0)$ $\cos\alpha = \frac{-a}{\sqrt{a^2+b^2}}$

$p = \frac{c}{\sqrt{a^2+b^2}}$

• (x_1,y_1) , (x_2,y_2)

Put in —①

① Same sign ⇒ On same side

② opp. sign ⇒ on opp. side

• **Angle b/w 2 lines** : θ be then angle , m_1, m_2 slopes of 2 line.

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

for ⊥ lines , $m_1 m_2 = -1$
for // " , $m_1 = m_2$

*** Distance b/w 2 Parallel lines :**

$ax + by + c_1 = 0$, $ax + by + c_2 = 0$

$$\boxed{d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}}$$

*** Line passing through intersection of 2 lines :**

$L_1 : a_1x + b_1y + c_1$, $L_2 : a_2x + b_2y + c_2 = 0$ ⟹ eq. of new line passing :
$L_1 + \lambda L_2 = 0$
↳ obtained by additional info.

*** Eq. of Bisectors of the angle b/w 2 lines :**

⇒ 2 lines : $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ ⟹

Eq. of Bisector : $$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

$\pm$ to be decided by the cases.

Cases :

• If $a_1a_2 + b_1b_2 > 0$
- Take (+) ⇒ obtuse angle Bisector
- Take (−) ⇒ Acute Angle Bisector

• If $a_1a_2 + b_1b_2 < 0$
- take (+) ⇒ Acute angle Bisector
- Take (−) ⇒ Obtuse angle Bisector

(∵ Note: On taking +ve sign = eq. of Bisector containg origin.
On taking −ve sign = eq. of Bisector containing origin.)

Adv. concepts :

- Homogeneous eq. of 2nd degree.

$\Rightarrow$ eq. of $ax^2+2hxy+by^2=0$ represent a pair of st. line through the origin if $h^2 \geq ab$.

(i) m_1, m_2 be the slopes,

$$m_1+m_2 = \frac{-2h}{b} \; ; \; m_1 \cdot m_2 = \frac{a}{b}$$

(ii) θ be the angle b/w these lines then : $\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$

(iii) These lines are identical if $h^2=ab$

(iv) These are perpendicular if $a+b=0$

(v) eq. of Bisector b/w these lines $\frac{x^2-y^2}{a-b} = \frac{xy}{h}$

(vi) Product of $\perp$ lst drawn from (x_1, y_1) to the above pair of lines is:

$$\left|\frac{ax_1^2+2hx_1y_1+by_1^2}{\sqrt{(a-b)^2+4h^2}}\right|$$

★★ Foot of $\perp$ & Reflection of a point w.r.t a line :

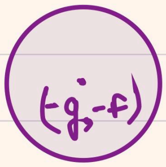
(i) foot (h,k) of the $\perp$ drawn from the point (x_1, y_1) on the line $ax+by+c=0 \implies \frac{h-x_1}{a} = \frac{k-y_1}{b} = -\left(\frac{ax_1-by_1+c}{a^2+b^2}\right)$

(ii) Img (h,k) of the point (x_1, y_1) in the line $ax+by+c=0$ is given by:

$$\frac{h-x_1}{a} = \frac{k-y_1}{b} = -2\left(\frac{ax_1-by_1+c}{a^2+b^2}\right)$$

Circle :

(* Adv concepts are merged within the notes. Kindly Read all the topics)



Centre = $(-g,-f)$
Radius = $\sqrt{g^2+f^2-c}$

*** Parametric Coordinates:**

- for circle $(x-h)^2+(y-k)^2=a^2$

  $x = h + a\cos\theta$, $y = k + a\sin\theta$

- for $x^2+y^2+2gx+2fy+c=0$

  $x = -g + \sqrt{g^2+f^2-c}\,\cos\theta$

  $y = -f + \sqrt{g^2+f^2-c}\,\sin\theta$

Gen form: $x^2+y^2+2gx+2fy+c=0$, where g, f are const.

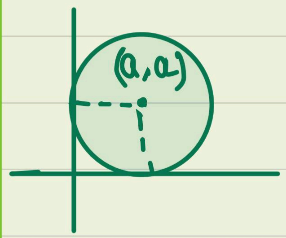
$-g = -\frac{1}{2} \times$ Coeff. of x

$-f = -\frac{1}{2} \times$ coeff of y

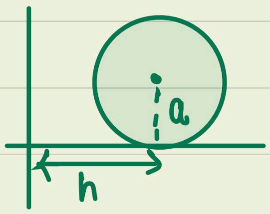
Centre-radius form: $(x-h)^2+(y-k)^2=a^2$
(at (h,k))

Diametric form: $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$
(x_1,y_1) & (x_2,y_2) are the end points of Diameter

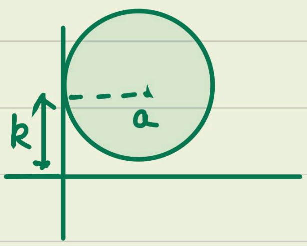
- eq. of circle in special cases :



$(x-a)^2+(y-a)^2=a^2$

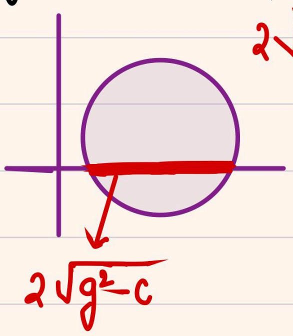


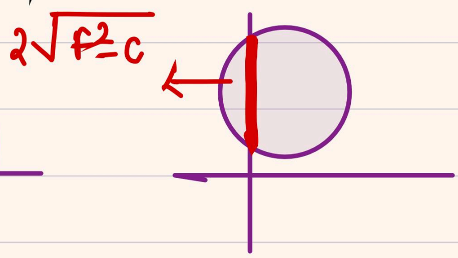
$(x-h)^2+(y-a)^2=a^2$

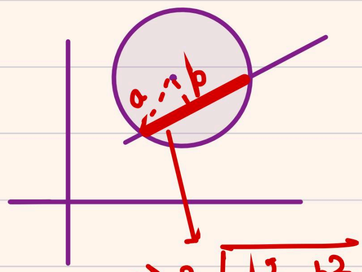


$(x-a)^2+(y-k)^2=a^2$

- Length of intercept :



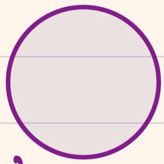




$\Rightarrow 2\sqrt{r^2-p^2}$

($\perp$ distance from centre to line)

• Tangency :



Condition : $\perp$ distance from centre = radius $(p = d)$

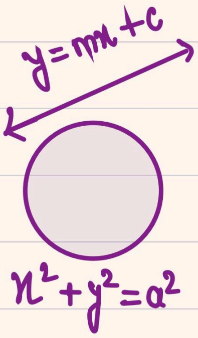
↳ from a point (x_1, y_1) to line $\Rightarrow p = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

for $x^2+y^2=r^2$ (simple circle)

eq. of tangent : $y = mx + c$

$\Rightarrow \boxed{c^2 = a^2(1+m^2)}$

or



$x^2 + (mx+c)^2 = a^2$ (Quadratic)

D>0	D=0	D<0
2 cuts	Tangent	No cut

• eq of tangent : (for $x^2+y^2=a^2$ type) $\Rightarrow y = mx+c$

→ Slope form : $c = \pm a\sqrt{1+m^2}$

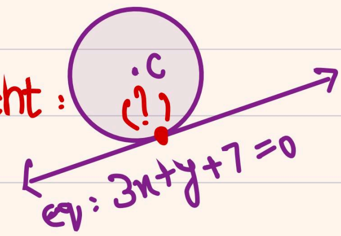
→ Point form : $xx_1 + yy_1 = a^2$

★ for gen Circle : At Point (x_1, y_1)

$\Rightarrow axx_1 + h(xy_1 + yx_1) + byy_1 + g(x+x_1) + f(y+y_1) + c = 0$

eg : $x^2+y^2-4x+6y+8 \longrightarrow xx_1+yy_1-2(x+x_1)+3(y+y_1)+8=0$

* Contact point of the tangent :



Method ① : Compare given tangent eq with Point form.

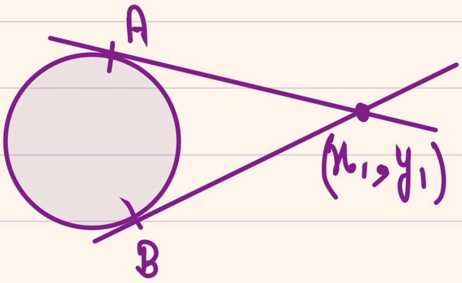
$xx_1 + yy_1 - a^2 = 0 \Longleftrightarrow 3x + y + 7 = 0$

$$\frac{x_1}{3} = \frac{y_1}{1} = \frac{-a^2}{7}$$

Method ② : Find foot of $\perp$ from centre to given eq of line.

- **Normal**: ⊥ to tangent, Passing point is centre of circle, Contact point is same where tangent touch
  - *eq can be found out using above properties*

★ Tangent from a Point:

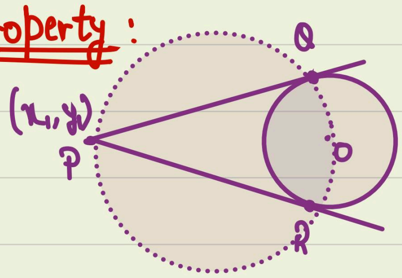


$$\underbrace{mx - y + (y_1 - mx_1) = 0}$$

$$\boxed{y - y_1 = m(x - x_1)}$$

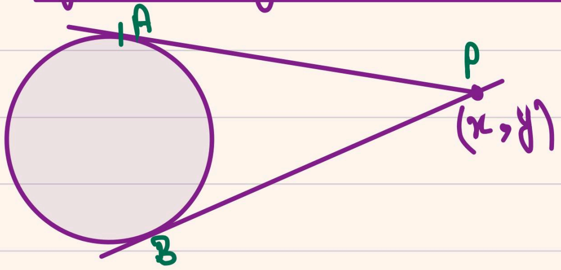
(for a diff. tangent) ← $\boxed{\dfrac{|y_1 - mx_1|}{\sqrt{m^2+1}} = r}$

★ **Property**:



P, Q, O, R are Concyclic

- Length of Tangent drawn from ext. Point?



$PA = PB = \sqrt{\underset{\sim}{S_1}}$

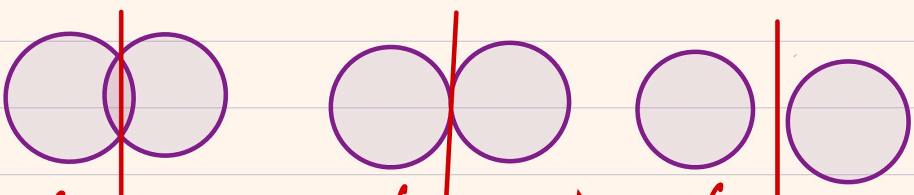
Power of Point of point (x_1, y_1)

★ Power of Point?

A point (x_1, y_1) & circle: $x^2 + y^2 + 2gx + 2fy + c = 0$

$S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

+ve	-ve	0
Point lies outside the Circle	Point lies inside the Circle	Point lies on the circle

- 

(Common chord) (Common Tangent) (Radical Axis)

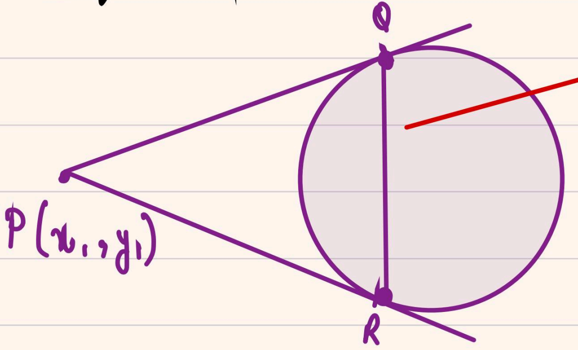
eq: $\underbrace{S_1 - S_1 = 0}$

$\Rightarrow x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

$\Rightarrow x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

$$\boxed{2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0}$$

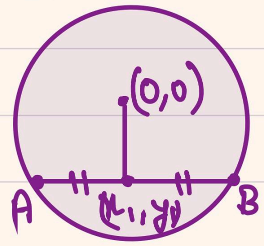
• Eq of chord of Contact :



$\underbrace{\text{eq} \Rightarrow T = 0}$

$\boxed{xx_1 + yy_1 = a^2}$

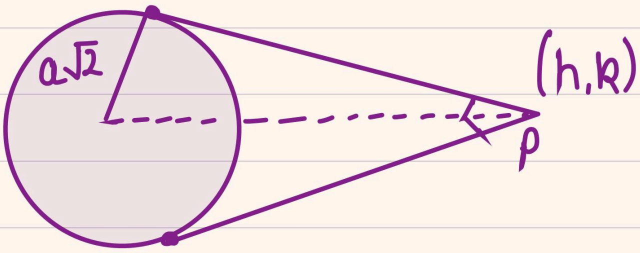
• Chord Bisected at a Point :



eq : $T = S_1$

$\Rightarrow xx_1 + yy_1 - a^2 = x_1^2 + y_1^2 - a^2$

• Director Circle : (Tangent will be always ⊥)

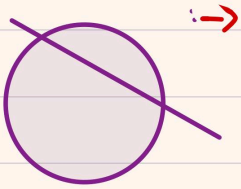


$\boxed{h^2 + k^2 = 2a^2}$ → circle with radius $(a\sqrt{2})$

⇒ family of circles :

• $S = 0$ & $S' = 0$ are 2 intersecting circles ⇒ $\underbrace{S + \lambda S' = 0}_{\text{represent family of circle}}$ $(\lambda \neq -1)$

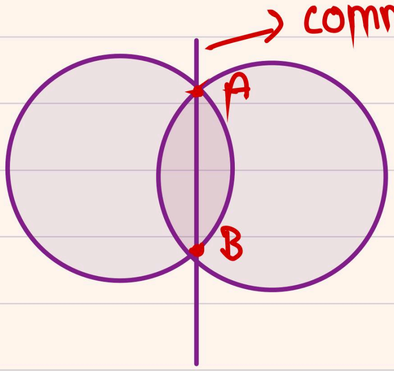
• $S = 0$ & $L = 0$



→ family

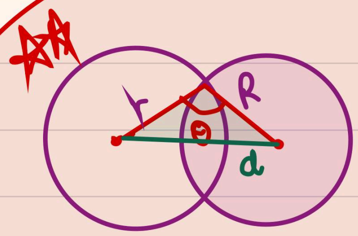
eq : ⇒ $\underbrace{S + \lambda L = 0}$

→ common chord



$AB = 2\sqrt{r^2 - p^2}$

p → ⊥ dist from centre to chord



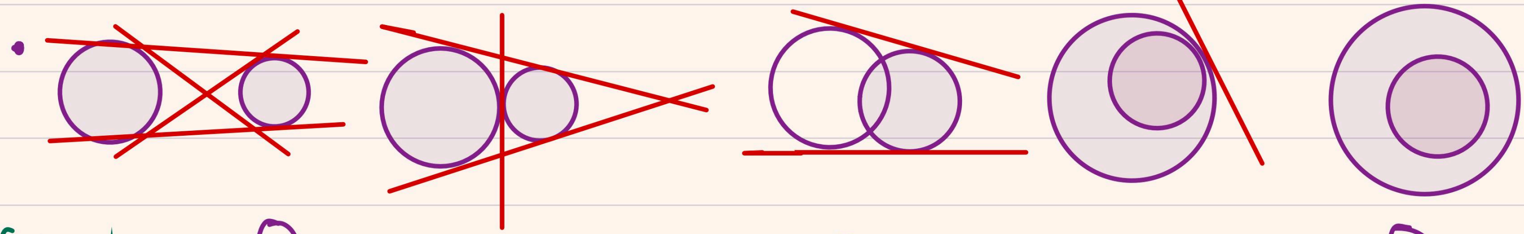
$\cos\theta = \dfrac{r^2 + R^2 - d^2}{2rR}$

Condition of orthogonality $(\theta = 90°)$

$\Rightarrow 2g_1g_2 + 2f_1f_2 = c_1 + c_2$

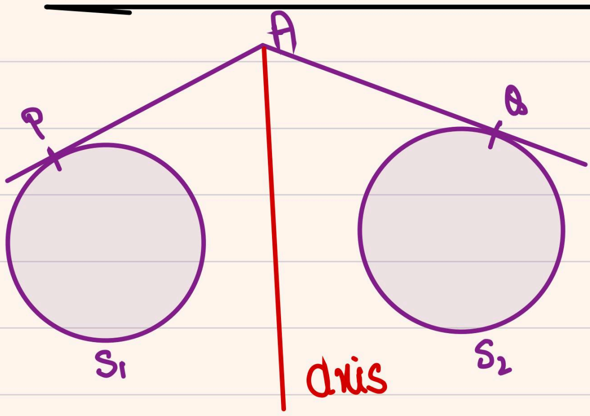
• Identification of type of circle :

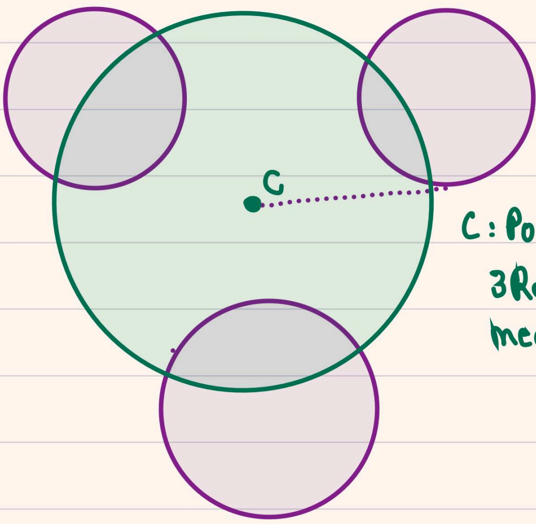
(C_1C_2 = dist b/w 2 centres)
r_1, r_2 are radius



Common Tangents :	4	3	2	1	0
Conditions :	$C_1C_2 > r_1+r_2$	$C_1C_2 = r_1+r_2$	$\lvert r_1-r_2\rvert < C_1C_2 < r_1+r_2$	$C_1C_2 = \lvert r_1-r_2\rvert$	$C_1C_2 < \lvert r_1-r_2\rvert$

• Radical Axis & Radical Centre :





C : Point where 3 Radical axis meet

eg: $S_1 - S_2 = 0$

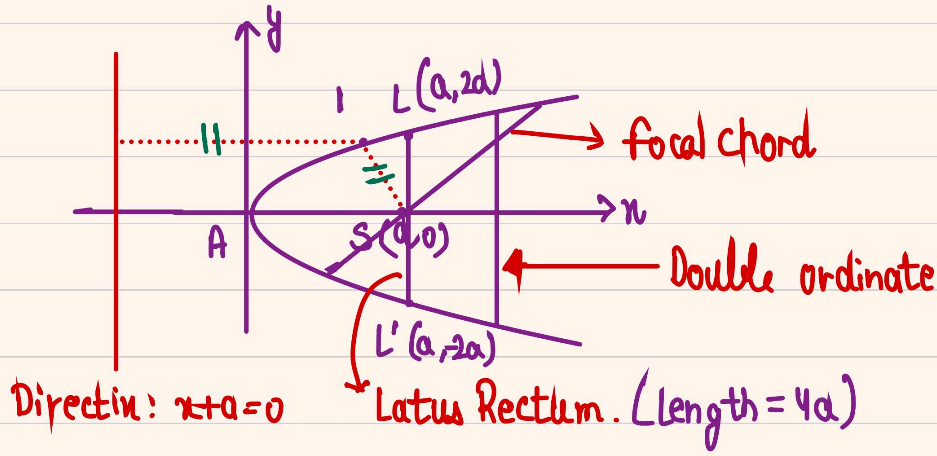
Properties:

→ It bisects every common tangent

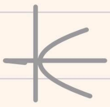
→ It's ⊥ to line joining the centres.

→ If 2 circle intersect a third circle orthogonally, then their radical axis will pass through the centre of third circle.

Parabola :

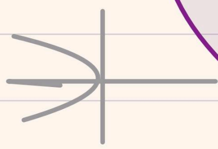


Understanding parametric coordinates :

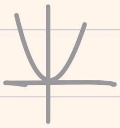


$(at^2, 2at)$ $\quad$ $(-at^2, 2at)$

$y^2 = 4ax \xrightarrow{x \to -x} y^2 = -4ax$

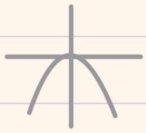


$\downarrow x \to y$



$x^2 = 4ay \xrightarrow{y \to -y} x^2 = -4ay$

$(2at, at^2)$ $\quad$ $(2at, -at^2)$

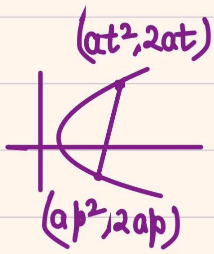


for shifted parabola: at (h,k) vertex.

$\left(at^2 + h,\ at^2 + k\right)$

• Key concepts :

① Eq of chord :



• $(t+p)y = 2x + 2atp$: eq

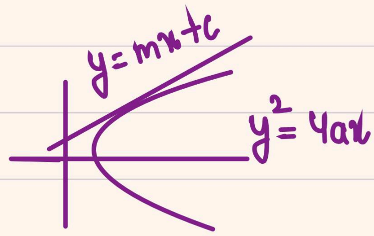
• slope : $m = \dfrac{2}{t+p}$

(Condition for focal chord : $tp = -1$)

Length of focal chord : Result

$\underbrace{a\left(t + \frac{1}{t}\right)^2}_{\text{length of F.C}} \geq \underbrace{4a}_{\text{Latus Rectum}}$

② Tangent :



• Can do intersection $D = 0$

(Put linear eq into eq of Parabola and a quadratic will form)

• $C = \dfrac{a}{m}$

Const. in St. line

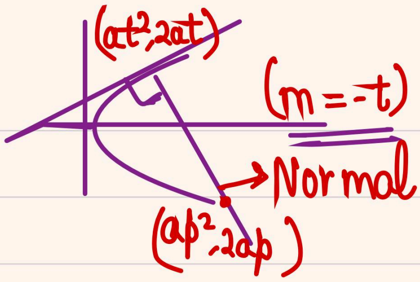
* Tangent from directrix are ⊥ to each other.

• eq : ① $y = mx + \dfrac{a}{m}$

② $yy_1 = 2a(x + x_1)$, at pt. (x_1, y_1)

③ $yt = x + at^2$ $\quad$ ($m = 1/t$: slope)

③ Normal:



Condition:

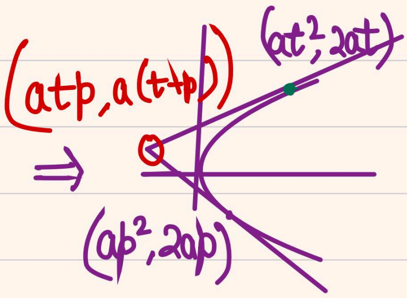
- $p = -t - \frac{2}{t}$
- $-t = \frac{2}{t+p}$
- $y = -tx + 2at + at^3$
- $y = mx - 2am - am^3$

$(c = -2am - am^3)$ → $(y = mx + c)$

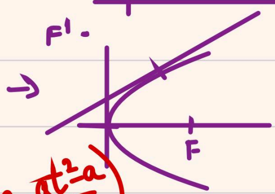
* Condition for Normal to meet parabola again: $tp = 2$

Imp. Results:

- intersection of 2 tangent ⇒

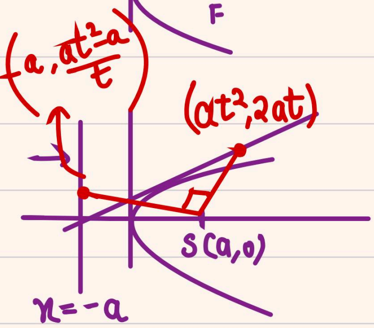


- Properties of Parabola:



⇒ (Reflection of focus in tangent will be on Directrix)

$x = -a$

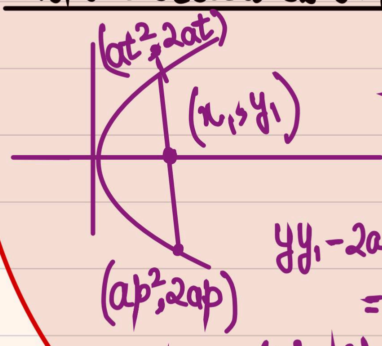


→ In $y = ax^2 + bx + c$

⇓

$LR = \frac{1}{|a|}$

Chord Bisected as a point:



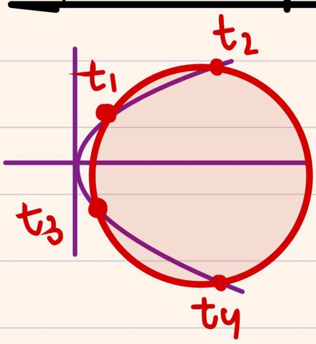
$T = S'$

$yy_1 - 2a(x + x_1) = y_1^2 - 4x_1$

$x_1 = \frac{a(t^2 + p^2)}{2}$

$y_1 = \frac{2a(t+p)}{2}$

⇒ Circle and parabola:

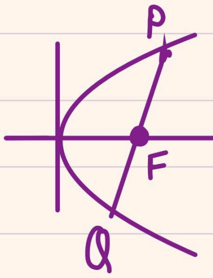


→ $t_1 + t_2 + t_3 + t_4 = 0$

for t_1, t_2, t_3, t_4,

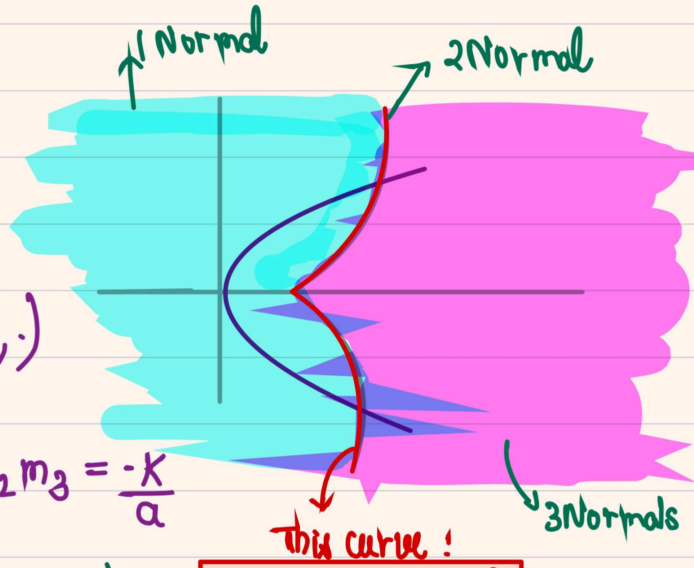
Put $(at^2, 2at) \underset{\text{in}}{\Rightarrow} x^2 + y^2 + 2gx + 2fy + c = 0$

Adv. Concepts :

- 

$$\frac{1}{a} = \frac{1}{PF} + \frac{1}{FQ}$$

- 3 Normals of Parabola :



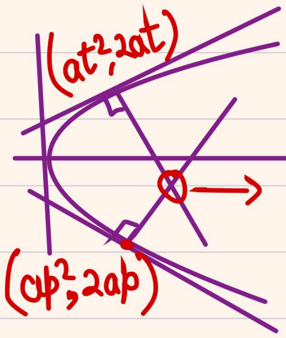
$$\boxed{y^2 = \frac{4}{27a}(x-2a)^3}$$

→ $am^3 + (2a-h)m + k = 0$ (slope giving eq.)

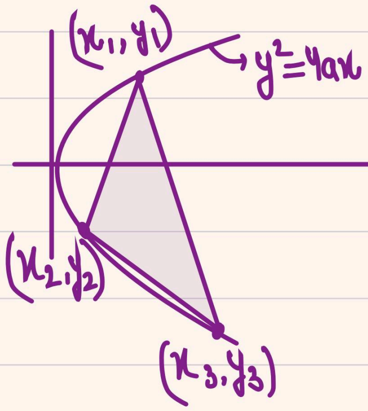
- $\sum_{i=1}^{3} m_i = 0$, • $\sum m_1 m_2 = \frac{2a-h}{a}$, • $m_1 m_2 m_3 = \frac{-k}{a}$

(No. of normal that can be drawn depends on the region where the point lies)

- intersections of normals:

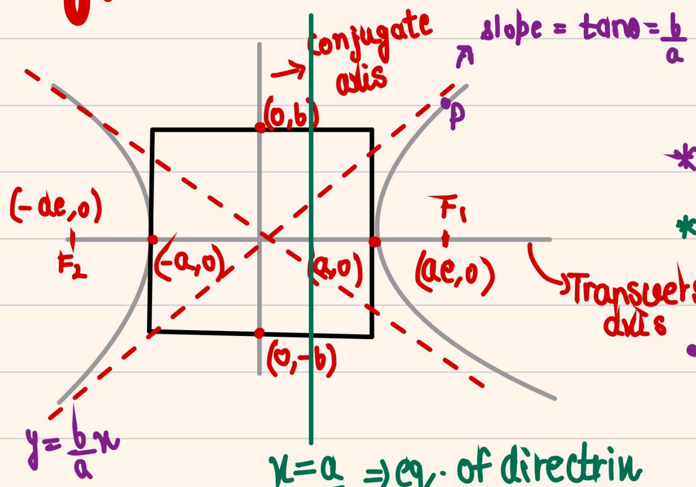


→ $(2a + a(t^2+p^2+tp), -atp(t+p))$

- 

$$\boxed{\text{Ar. of } \Delta = \frac{1}{8a}\left|(y_1-y_2)(y_2-y_3)(y_3-y_1)\right|}$$

Hyperbola :



$x=\frac{a}{e} \Rightarrow$ eq. of directrix

Condition :

* $|PF_1 - PF_2| = 2a$
* $b^2 = a^2(e^2-1)$, $e=\sqrt{\frac{a^2+b^2}{a^2}}$, $LR = \frac{2b^2}{a}$, $\frac{PF}{PM}=e$

• $(x,y) \in \underbrace{(a\sec\theta,\ b\tan\theta)}_{\text{Parametric coord.}}$

• **Eq. of chord :**

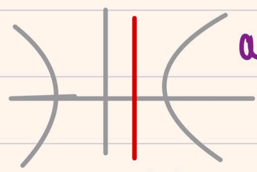
• $\frac{x}{a}\cos\left(\frac{\theta_1-\theta_2}{2}\right) - \frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right) = \cos\left(\frac{\theta_1+\theta_2}{2}\right)$

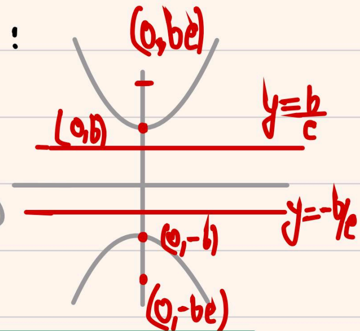
* Condition for a chord to be focal chord

$\Rightarrow \tan\left(\frac{\theta_1}{2}\right)\tan\left(\frac{\theta_2}{2}\right) = \underbrace{\frac{1-e}{1+e}}_{\text{for Right}}$ or $\underbrace{\frac{1+e}{1-e}}_{\text{for left}}$

• **Conjugate hyperbola :**

$x \leftrightarrow y$
$a \leftrightarrow b$





$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$ $\Rightarrow$ $\boxed{\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1}$

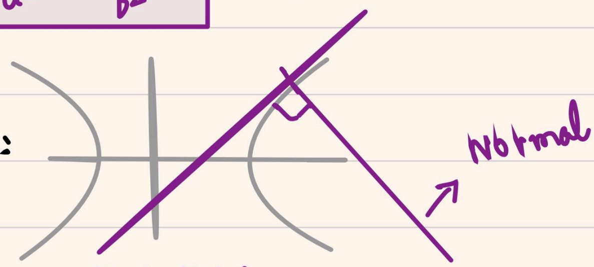
• $b^2 = a^2(e^2-1)$ • $a^2 = b^2(e^2-1)$

• $LR = \frac{2b^2}{a}$ • $LR = \frac{2a^2}{b}$

• **Eq. of chord of Contact :**

$T=0 \Rightarrow \boxed{\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1}$

• **Tangent :**



Conditions :

• $\boxed{c^2 = a^2m^2 - b^2}$

eq :

• Point form : $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

• slope form : $y = mx \pm \sqrt{a^2m^2 - b^2}$

• Parametric form : $\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$

Point of intersection of 2 tangents :-

$(x_1, y_1) \equiv \left(\frac{a\cos\left(\frac{\theta_1-\theta_2}{2}\right)}{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}, \frac{b\cos\left(\frac{\theta_1-\theta_2}{2}\right)}{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}\right)$

• **Normal :**

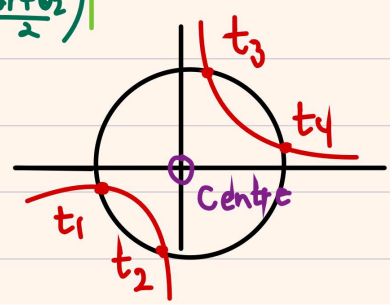
Eq :

• Point form : $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$

• slope form : $y = mx \pm \frac{a^2+b^2}{\sqrt{\frac{a^2}{m^2} - b^2}}$

• Parametric form : $ax\cos\theta + by\cot\theta = a^2 + b^2$

• **Circle & hyperbola :**



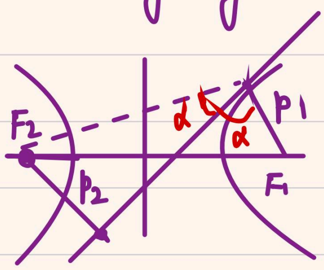
for H : $xy = c^2$

• $t_1 t_2 t_3 t_4 = -1$, Centre $\equiv \left(\frac{c\Sigma t}{4}, \frac{c\Sigma \frac{1}{t}}{4}\right)$

• **Some imp Properties:**

$\ast$ Eq of chord of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ bisected at point $(x_1,y_1) \Rightarrow \boxed{T=S_1 \Rightarrow \frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1=\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1}$

→ Incoming ray from one focus striking the hyperbola, passes through other focus.

→ 

$p_1p_2=b^2$

→ eccentricities of hyperbola & its conjugate:

$$\boxed{\frac{1}{e_1^2}+\frac{1}{e_2^2}=1}$$

Adv. Section !

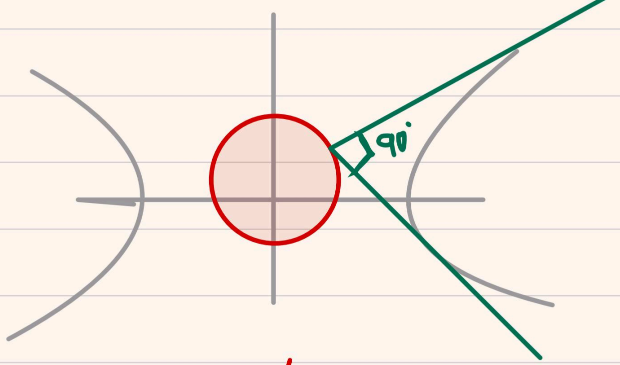
• Combined eq of pairs of tangent from (x_1,y_1) of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Longrightarrow \boxed{SS_1=T^2}$

• **Director Circle**: Locus of points of intersection of $\perp$ tangents

→ $y=mx+\sqrt{a^2m^2-b^2}$

↳ make quadratic in m

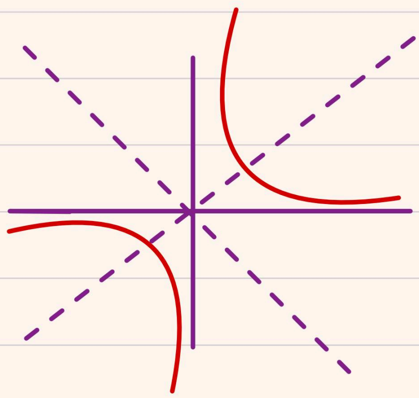
then Put $m_1m_2=-1$



• **Rectangular hyperbola**: $e=\sqrt{2}$

H: $x^2-y^2=a^2 \longrightarrow xy=c^2$

• $\left(c=\frac{a^2}{2}\right)$



focus on $y=x$

• foci: • $(c\sqrt{2},c\sqrt{2})$, $(-c\sqrt{2},-c\sqrt{2})$, $\underset{\text{Latus Rectum}}{LR}=2\sqrt{2}c$, Parametric $\Rightarrow \left(ct,\frac{c}{t}\right)$

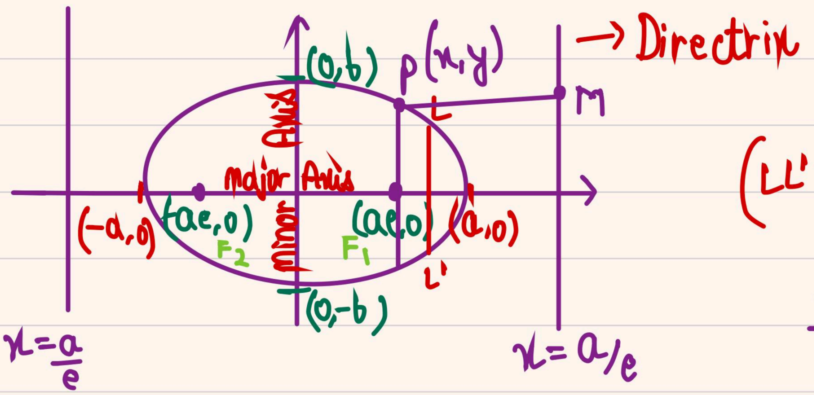
• Tangent eq: Point form: $\frac{1}{2}(xy_1+yx_1)=c^2$

Parametric: $\frac{x}{t}+yt=2c$

• Normal eq: $xx_1-y_1y_2=x_1^2-y_1^2$

Ellipse :

- Standard form of ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a>b)$



$PF_1 : a - ex$

$PF_2 : a + ex$

$(LL' \Rightarrow \text{Latus Rectum}) \Rightarrow \frac{2b^2}{a}$

$\frac{PF}{PM} = e$

$PF_1 + PF_2 > F_1F_2 \longrightarrow$ ellipse

Power of Point?

$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 \begin{cases} >0 \rightarrow \text{outside} \\ =0 \rightarrow \text{on ellipse} \\ <0 \rightarrow \text{inside} \end{cases}$

- **Conditions :**

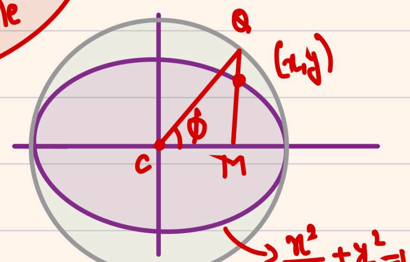
e : eccentricity

$\rightarrow b^2 = a^2(1-e^2)$

$e = \sqrt{\frac{a^2 - b^2}{a^2}}$

ϕ = eccentric angle

- **Parametric coordinates:** $(x = a\cos\theta,\ y = b\sin\theta) \Rightarrow$



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

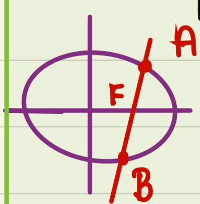
$x^2 + y^2 = a^2$

Auxillary Circle : Circle taking major axis as diameter.

Quick check

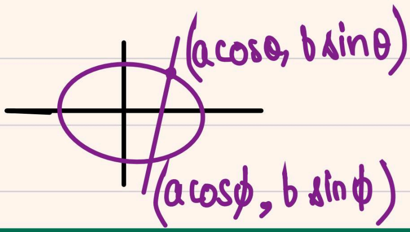
Write eq. of auxillary circle from ch. circles?
(Hint: end points $(a,0)$, $(-a,0)$)

Adv. Concept.



$\Rightarrow \underbrace{AF,\ \frac{b^2}{a},\ FB}_{\text{are in H.P.}}$

Eq. of chord :



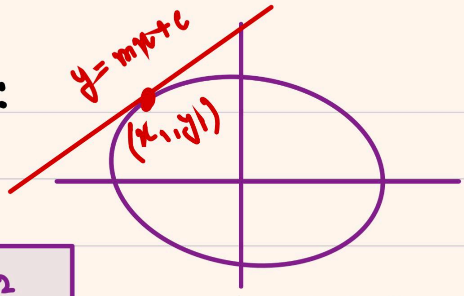
eq: $\boxed{\frac{x}{a}\cos\left(\frac{\theta+\phi}{2}\right) + \frac{y}{b}\sin\left(\frac{\theta+\phi}{2}\right) = \cos\left(\frac{\theta-\phi}{2}\right)}$

Condition for focal chord?

→ for FC to be on Right side

- $\tan(\theta/2) \times \tan(\phi/2) = \frac{e-1}{e+1}$
- $\tan\left(\frac{\theta}{2}\right) \times \tan(\phi/2) = \frac{e+1}{e-1}$

• Tangency :



• $\boxed{c^2 = a^2m^2 + b^2}$

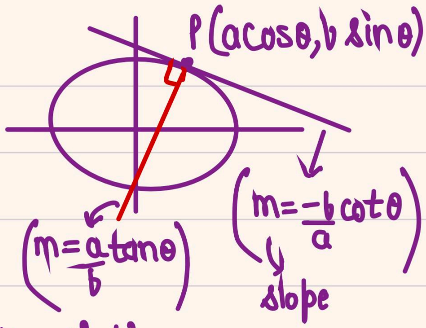
Point of Contact

Eq: $\boxed{(x_1, y_1) \equiv \left(\frac{\pm a^2 m}{\sqrt{a^2m^2+b^2}}, \frac{\mp b^2}{\sqrt{a^2m^2+b^2}} \right)}$

→ Point form : $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

→ Parametric : $\frac{x\cos\theta}{a} + \frac{y}{b}\sin\theta = 1$

• Normal :



Eq:

• Point form : $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$

• Parametric : $ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2 - b^2$

• Slope form : $y = mx + \frac{(a^2-b^2)}{\sqrt{\frac{a^2}{m^2}+b^2}}$

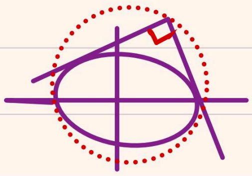
• Chord of contact : (T = 0)

$\Rightarrow \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

• Chord bisected at a Point : $(T = S_1)$

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$

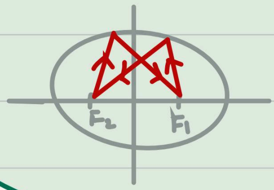
• Director Circle :



For

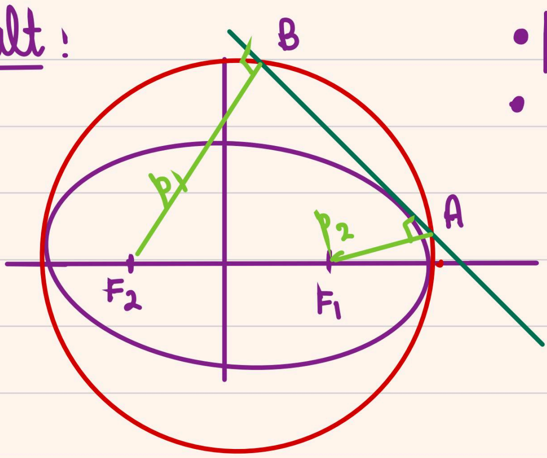
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \implies$ Eq: $\boxed{x^2 + y^2 = a^2 + b^2}$

* Reflection property :



Adv. Concepts :

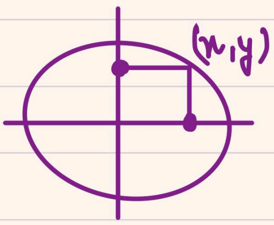
• Result :



• $p_1 \times p_2 = b^2$

• Points A, B lie on auxillary circle.

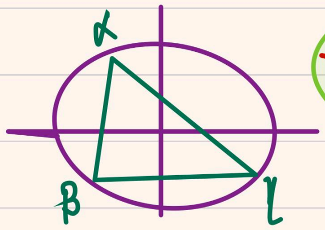
- **Another Definition of ellipse :**



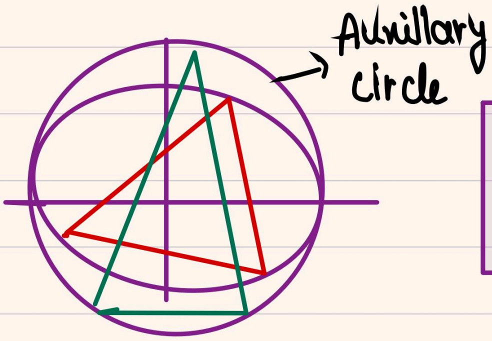
$$\Rightarrow \frac{(\text{Dist. from } L_1)^2}{a^2} + \frac{(\text{Dist. from } L_2)^2}{b^2} = 1$$

$\uparrow$ minor Axis $\quad\quad$ $\uparrow$ major Axis

* (L_1, L_2 are $\perp$).

- 

***Result:** Max area of Δ inscribed in an ellipse $\Rightarrow$ Equilateral Δ

- 

$$\boxed{\frac{Ar_1}{Ar_2} = \frac{b}{a}}$$

- **Point of intersection 2 tangents :**

eq T_1: $\frac{x}{a}\cos\theta_1 + \frac{y}{b}\sin\theta_1 = 1$ & $\frac{x}{a}\cos\theta_2 + \frac{y}{b}\sin\theta_2 = 1$

at Point $(a\cos\theta_1, b\sin\theta_1)$ & $(a\cos\theta_2, b\sin\theta_2)$

$$\underbrace{(x_1, y_1)}_{\text{Point of interaction}} \equiv \left(\frac{a\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}, \frac{b\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)} \right)$$

- If normal at four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ and $S(x_4, y_4)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are Concurrent.

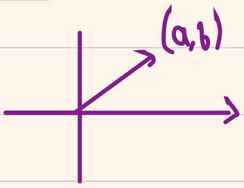
then, $$(x_1 + x_2 + x_3 + x_4)\left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4}\right) = 4$$

(Same concept is applicable for hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.)

Complex No's :

- $a+ib$

Real part   Img. part



- $i=\sqrt{-1}$, $i^2=-1$, $i^3=-i$, $i^4=1$

- $\underbrace{a+ib = c+id}_{a=c,\ b=d}$

$i^{46}=?$

Trick:

- simply divide power by 4 and Take the remainder

$\Rightarrow \frac{46}{4} \Rightarrow 2 \Rightarrow i^2=-1$

- **Operations on Complex:**

$(z=x+iy)$

- $z_1 \pm z_2 = (x_1+x_2) \pm i(y_1+y_2)$
- $z_1 \times z_2 = (x^2-y^2)+i2(xy)$
- $\frac{z_1}{z_2}$ = Rationalize the complex & then multiply
- Conjugate $\Rightarrow \bar{z} = x-iy$
- Modulus $\Rightarrow \underbrace{|z|}_{\text{magnitude}} = \sqrt{x^2+y^2}$

- **Polar form of Complex:**

$z=x+iy \Rightarrow (x=r\cos\theta , y=r\sin\theta)$

$\theta \in$ amplitude or argument

$|x+iy| = r = \sqrt{x^2+y^2}$

- $z_1\times z_2 = r_1r_2(\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2))$
- $\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2))$

☆ Imp properties:

- $\bar{\bar{z}} = z$
- $z+\bar{z} = 2Re(z)$, $z-\bar{z} = 2iIm(z)$
- $\overline{z_1\pm z_2} = \bar{z}_1 \pm \bar{z}_2$, $\overline{z_1\times z_2} = \bar{z}_1\times\bar{z}_2$, $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$
- $|z|=0 \Rightarrow z=0+0i$, $|z_1\pm z_2| \leq |z_1|+|z_2|$
- $|z| = |\bar{z}| = |-z| = |-\bar{z}|$, $|z_1\times z_2| = |z_1|\times|z_2|$

$\left(\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right)$

- $z\times\bar{z} = |z|^2$

★★ Sq. Root of Complex :

Q: $\underbrace{\sqrt{1+4\sqrt{3}i} = x+iy}$

Sq. & compare $\Longrightarrow$ $1+4\sqrt{3}i = x^2-y^2 + i(2xy)$

$x = \pm 2 \;,\; y = \pm\sqrt{3}$

($z = \bar{z} \Rightarrow z$ is Real

$z = -\bar{z} \Rightarrow z$ is imaginary)

Trick ☺

$\omega^{14}? \Rightarrow$ divide by 3 & take remainder

$\frac{14}{3} \Rightarrow 2 \Rightarrow \omega^2$

- **Cube root of unity :**

$x = (1)^{1/3}? \Rightarrow x^3 - 1 = 0$

$x = 1 \;,\; \omega = \frac{-1+i\sqrt{3}}{2} \;,\; \omega^2 = \frac{-1-i\sqrt{3}}{2}$

- $1+\omega+\omega^2 = 0$
- $1 \cdot \omega \cdot \omega^2 = 1$
- $\omega^3 = 1$
- $\omega^2 = \frac{1}{\omega}$

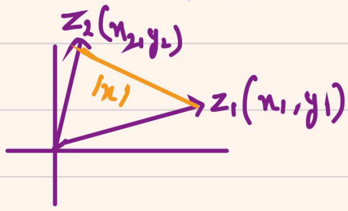
★★ Imp. formulae :

- $a^2+b^2+c^2-ab-bc-ca$
- $a^3+b^3 = (a+b)(a+b\omega)(a+b\omega^2)$
- $a^3-b^3 = (a-b)(a-b\omega)(a-b\omega^2)$
- $a^2+b^2 = (a+ib)(a-ib)$

dist b/w 2 complex?

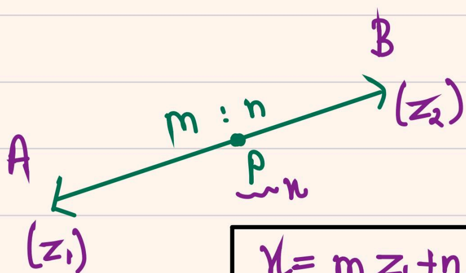
z_1, z_2

$|x| = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$



★ Adv Section :

- $|z_1 \pm z_2| \geq |z_1| - |z_2|$



$$\boxed{x = \frac{mz_1+nz_2}{m+n}}$$

- $1+\omega^n+\omega^{2n} = \begin{cases} 3 & , n \in 3k \\ 0 & , \text{else} \end{cases}$

★★ Euler form :

$$\left.\begin{aligned} e^{i\theta} &= \cos\theta + i\sin\theta \\ e^{-i\theta} &= \cos\theta - i\sin\theta \end{aligned}\right\}$$

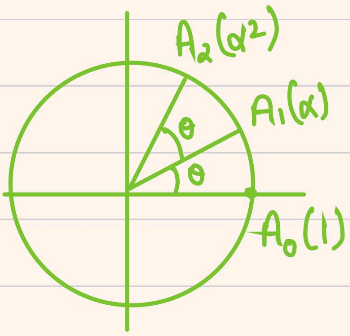
• n^{th} Root of unity?

$$x = (1)^{1/n} = \left(\cos(2k\pi) + i\sin(2k\pi)\right)^{1/n}$$

root ← $\alpha = \cos\left(\frac{2\pi k}{n}\right) + i\sin\left(\frac{2\pi k}{n}\right)$, $k \in 1, 2, --- n$

n^{th} roots $\Rightarrow 1, \alpha, \alpha^2 ------ \alpha^{n-1}$ are in G.P

$\sum_{i=0}^{n-1} \alpha_i = 0$, Product of n^{th} roots $= (-1)^{n+1}$



• Eq. of circle at centre z_0:

- $|z_1 - z_0| = a$
- Gen form: $\underbrace{zz' + az' + a'z + b = 0}$

Centre : $-a$, Radius $= \sqrt{aa' - b}$

Statistics:

- **Arithmetic mean ($\bar{x}$):** (i) $\frac{\sum x_i}{n}$

  (ii) $\frac{1}{N}\sum f_i x_i$ ($f_i \Rightarrow$ frequency)

Properties:

→ sum of all deviations from arithmetic mean zero.

$$\sum_{i=1}^{n}(x_i - \bar{x}) = 0$$

→ if each value is inc./dec. by K AM also inc./dec by K.

→ $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$

items in x_1 sequence.

- **Geometric mean:** $(x_1 x_2 \ldots x_n)^{1/n}$

  for discrete: $\left(x_1^{f_1} x_2^{f_2} \ldots x_n^{f_n}\right)^{1/N}$, where $N = \sum_{i=1}^{n} f_i$

- **Harmonic Mean:**

  $\Rightarrow \frac{1}{H} = \frac{1}{n}\sum \frac{1}{x_i}$

★★ $A.M \geq G.M \geq H.M$

if $x_1, x_2 \ldots x_n > 0$

Median	Mode
$\Rightarrow \frac{n+1}{2}$ th item $n \in$ odd (total no. of terms)	(item having highest frequency)
$\Rightarrow$ Avg of $\frac{n}{2}$ and $\frac{n+2}{2}$ $n \in$ even	$\Rightarrow$ simple count which items is occurring maximum times.

$$\boxed{\text{Mode} = 3\,\text{median} - 2\,\text{mean}}$$

- **Deviation:** (degree of scatteredness)

• Mean deviation : (about A)

(it is least when taken about Median)

$$M.D = \frac{1}{n}\sum |x_i - A|$$

* Standard deviation : (S.D)

$$\underbrace{\sigma^2}_{\text{Variance}} = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2 \quad \text{or} \quad \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2$$

$$\left(S.D = \pm\sqrt{\text{variance}} \right)$$

$$\Downarrow$$

$$\sigma$$

(two samples of sizes n_1, n_2 with $\bar{x}_1$ & $\bar{x}_2$ as their means & $(\sigma_1$ & $\sigma_2)$ as their S.D)

$$\underbrace{\sigma^2}_{\text{Combined variance}} = \frac{n_1(\sigma_1^2+d_1^2) + n_2(\sigma_2^2+d_2^2)}{n_1+n_2}$$

$$d_1 = \bar{x}_1 - \bar{x}, \quad d_2 = \bar{x}_2 - \bar{x}$$

($\bar{x}$ is combined mean)

• Coeff. of variance : $\frac{\sigma}{X} \times 100$

Properties of variance :

• If $x_i \to x_i + a \implies$ variance is unchanged

• $x_i \to a x_i \implies$ new v. $= a^2$ (old v.)

• $x_i \to a + b x_i \implies$ new v $= b^2$ (old v) ; $b \neq 0$