केन्द्रीय विद्यालय संगठन

Kendriya Vidyalaya Sangathan

गणित

Mathematics

कक्षा/Class: XI

2024-25

विद्यार्थी अध्ययन सामग्री

Student Support Material

<u>संदेश</u>

विद्यालयी शिक्षा में शैक्षिक उत्कृष्टता प्राप्त करना केन्द्रीय विद्यालय संगठन की सर्वोच्च वरीयता है। हमारे विद्यार्थी, शिक्षक एवं शैक्षिक नेतृत्व कर्ता निरंतर उन्नति हेतु प्रयासरत रहते हैं। राष्ट्रीय शिक्षा नीति 2020 के संदर्भ में योग्यता आधारित अधिगम एवं मूल्यांकन संबन्धित उद्देश्यों को प्राप्त करना तथा सीबीएसई के दिशा निर्देशों का पालन, वर्तमान में इस प्रयास को और भी चुनौतीपूर्ण बनाता है।

केन्द्रीय विद्यालय संगठन के पांचों **आंचलिक शिक्षा एवं प्रशिक्षण संस्थान** द्वारा संकलित यह 'विद्यार्थी सहायक सामग्री' इसी दिशा में एक आवश्यक कदम है । यह सहायक सामग्री कक्षा 9 से 12 के विद्यार्थियों के लिए सभी महत्वपूर्ण विषयों पर तैयार की गयी है । केन्द्रीय विद्यालय संगठन की 'विद्यार्थी सहायक सामग्री' अपनी गुणवत्ता एवं परीक्षा संबंधी सामग्री-संकलन की विशेषज्ञता के लिए जानी जाती है और अन्य शिक्षण संस्थान भी इसका उपयोग परीक्षा संबंधी पठन सामग्री की तरह करते रहे हैं । शुभ-आशा एवं विश्वास है कि यह सहायक सामग्री विद्यार्थियों की सहयोगी बनकर सतत मार्गदर्शन करते हुए उन्हें सफलता के लक्ष्य तक पहुंचाएगी ।

शुभाकांक्षा सहित ।

निधि पांडे
आयुक्त, केन्द्रीय विद्यालय संगठन

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INDEX

COURSE STRUCTURE CLASS XI (2024-25)

One Paper — Total Period–240 [35 Minutes each]

Three Hours — **Max Marks: 80**

No.	Units	No. of Periods	Marks
I.	Sets and Functions	60	23
II.	Algebra	50	25
III.	Coordinate Geometry	50	12
IV.	Calculus	40	08
V.	Statistics and Probability	40	12
	Total	240	80
	Internal Assessment		20

*No chapter/unit-wise weightage. Care to be taken to cover all the chapters.

Unit-I: Sets and Functions

1. **Sets** **(20) Periods**

Sets and their representations, Empty set, Finite and Infinite sets, Equal sets, Subsets, Subsets of a set of real numbers especially intervals (with notations). Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set. Properties of Complement.

2. **Relations & Functions** **(20) Periods**

Ordered pairs. Cartesian product of sets. Number of elements in the Cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto R x R x R).Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special type of relation. Pictorial representation of a function, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs. Sum, difference, product and quotients of functions.

3. **Trigonometric Functions** **(20) Periods**

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity $\sin^2 x + \cos^2 x = 1$, for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing *sin (x±y)* and *cos (x±y)* in terms of sin*x*, sin*y*, cos*x* & cos*y* and their simple applications. Deducing identities like the following:

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}, \cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$$

$$cos\alpha - cos\beta = -2sin\frac{1}{2}(\alpha + \beta)sin\frac{1}{2}(\alpha - \beta)$$

Identities related to sin2*x*, cos2*x*, tan2 *x*, sin3*x*, cos3*x* and tan3*x*.

Unit-II: Algebra

1. **Complex Numbers and Quadratic Equations** **(10) Periods**

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve some of the quadratic equations. Algebraic properties of complex numbers. Argand plane

2. **Linear Inequalities** **(10) Periods**

Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line.

+

3. Permutations and Combinations (10) Periods

Fundamental principle of counting. Factorial *n*. (n!) Permutations and combinations, derivation of Formulae for ^nP_r and ^nC_r and their connections, simple applications.

4. Binomial Theorem (10) Periods

Historical perspective, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, simple applications.

5. **Sequence and Series** (10) Periods

Sequence and Series. Arithmetic Mean (A.M.) Geometric Progression (G.P.), general term of a G.P., sum of *n* terms of a G.P., infinite G.P. and its sum, geometric mean (G.M.), relation between A.M. and G.M.

Unit-III: Coordinate Geometry

1. Straight Lines **(15) Periods**

Brief recall of two dimensional geometry from earlier classes. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axis, point -slope form, slope-intercept form, two-point form, intercept form, Distance of a point from a line.

2. Conic Sections **(25) Periods**

Sections of a cone: circles, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.

3. Introduction to Three-dimensional Geometry **(10) Periods**

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points.

Unit-IV: Calculus

1. Limits and Derivatives **(40) Periods**

Derivative introduced as rate of change both as that of distance function and geometrically. Intuitive idea of limit. Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions. Definition of derivative relate it to scope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

Unit-V Statistics and Probability

1. Statistics **(20) Periods**

Measures of Dispersion: Range, Mean deviation, variance and standard deviation of ungrouped/grouped data.

2. Probability **(20) Periods**

Events; occurrence of events, 'not', 'and' and 'or' events, exhaustive events, mutually exclusive events, Axiomatic (set theoretic) probability, connections with other theories of earlier classes. Probability of an event, probability of 'not', 'and' and 'or' events.

MATHEMATICS (Code No. - 041)

QUESTION PAPER DESIGN CLASS - XI

(2024-25)

Time: 3 hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, givingdescriptions, and stating main ideas	44	55
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a differentway.	20	25
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combiningelements in a new pattern or proposing alternative solutions	16	20
	Total	80	100

1. *No chapter wise weightage. Care to be taken to cover all the chapters*
2. *Suitable internal variations may be made for generating various templates keeping the overall weightage to different form of questions and typology of questions same.*

Choice(s):

There will be no overall choice in the question paper. However, 33% internal choices will be given in all the section.

INTERNAL ASSESSMENT	**20 MARKS**
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities	10 Marks

SETS

KEY POINTS:

Set : a set is a well-defined collection of objects

If a is an element of a set A, we say that " a belongs to A" the Greek symbol $\in$ (epsilon) is used to denote the phrase 'belongs to'. Thus, we write a $\in$ A. If 'b' is not an element of a set A, we write b $\notin$ A and read "b does not belong to A".

There are two methods of representing a set :

(i) Roster or tabular form (ii) Set-builder form.

In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within brackets { }. For example, the set of all even positive integers less than 7 is described in roster form as {2, 4, 6}.

In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set. For example, in the set {a, e, i, o, u}, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property. Denoting this set by V, we write V = {x : x is a vowel in English alphabet}

**** Empty Set :** A set which does not contain any element is called the empty set or the null set or the void set. The empty set is denoted by the symbol ϕ or { }.

**** Finite and Infinite Sets :** A set which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

**** Equal Sets :** Two sets A and B are said to be equal if they have exactly the same elements and we write A = B. Otherwise, the sets are said to be unequal and we write A ≠ B.

**** Subsets :** A set A is said to be a subset of a set B if every element of A is also an element of B.

In other words, A $\subset$ B if whenever a $\in$ A, then a $\in$ B. Thus A $\subset$ B if a $\in$ A $\Rightarrow$ a $\in$ B

If A is not a subset of B, we write A $\not\subset$ B.

** Every set A is a subset of itself, i.e., A $\subset$ A.

** ϕ is a subset of every set.

** If A $\subset$ B and A ≠ B , then A is called a proper subset of B and B is called superset of A.

** If a set A has only one element, we call it a singleton set. Thus, { a } is a singleton set.

** Closed Interval : [a , b] = {x : a ≤ x ≤ b} where a,b $\in$ R (Set of Real Numbers)

** Open Interval : (a , b) = { x : a < x < b}

** Semi Closed/Semi open Interval : [a , b) = {x : a ≤ x < b}

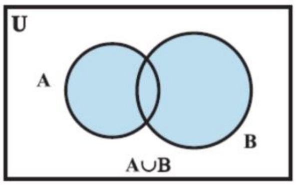
** Semi Open/Semi closed Interval : (a , b] = { x : a < x ≤ b }

**** Power Set :** The collection of all subsets of a set A is called the power set of A. It is denoted by P(A)

If A is a set with n(A) = m, then it can be shown that n [P(A)] = 2^m.

** **Universal Set :** The largest set under consideration is called Universal set.

** **Union of sets :** The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both). In symbols, we write.



$A \cup B = \{ x : x \in A \text{ or } x \in B \}$.

$X \in A \cup B \Rightarrow x \in A \text{ or } x \in B$

$x \notin A \cup B \Rightarrow x \notin A \text{ and } x \notin B$

** **Some Properties of the Operation of Union**

(i) $A \cup B = B \cup A$ (Commutative law)

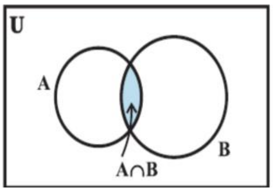
(ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)

(iii) $A \cup \phi = A$ (Law of identity element, ϕ is the identity of $\cup$)

(iv) $A \cup A = A$ (Idempotent law)

(v) $U \cup A = U$ (Law of U)

** **Intersection of sets :** The intersection of two sets A and B is the set of all those elements which belong to bothA and B.

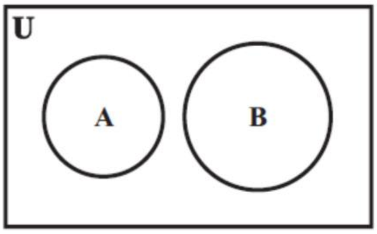


Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$

$x \in A \cap B \Rightarrow x \in A \text{ and } x \in B$

$x \notin A \cap B \Rightarrow x \notin A \text{ or } x \notin B$

** **Disjoint sets :** If A and B are two sets such that $A \cap B = \phi$, then A and B are called disjoint sets.



** **Some Properties of Operation of Intersection**

(i) $A \cap B = B \cap A$ (Commutative law).

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law).

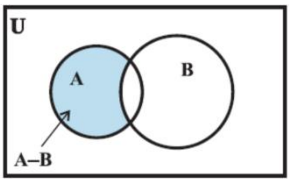
(iii) $\phi \cap A = \phi$, $U \cap A = A$ (Law of ϕ and U).

(iv) $A \cap A = A$ (Idempotent law)

(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law) i. e., $\cap$ distributes over $\cup$

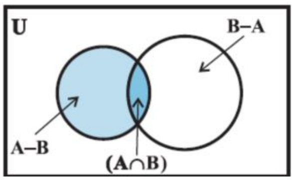
** **Difference of sets :** The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

Symbolically, we write A – B and read as “ A minus B”.

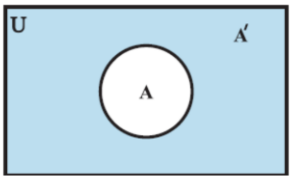


$A - B = \{ x : x \in A \text{ and } x \notin B \}$.

* The sets A – B, A ∩B and B – A are mutually disjoint sets, i.e., the intersection of any of these two sets is the null set.



** **Complement of a Set :** Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write A′ to denote the complement of A with respect to U.



Thus, $A' = \{x : x \in U \text{ and } x \notin A \}$. Obviously $A' = U - A$

** **Some Properties of Complement Sets**

1. Complement laws: (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$

2. De Morgan's law: (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

3. Law of double complementation: $(A')' = A$

4. Laws of empty set and universal set $\phi' = U$ and $U' = \phi$.

MCQS-

1.The number of elements in the Power set P(S) of the set S = {1, 2,3} is:

A. 4 B. 8 C. 2 D. None of these

Answer: B. 8

Explanation: Number of elements in the set S = 3

Number of elements in the power set of set S = {1,2,3} = 2^3= 8

2. Empty set is a ________.

A. Infinite set B. Finite set C. Unknown set D. Universal set

Answer: B. Finite set

Explanation: The cardinality of the empty set is zero, since it has no elements. Hence, the size of the empty set is zero.

3. Cardinality of the power set P(A) of a set A having 'n' number of elements is equal to:

A. n B. 2n C. 2^n D. n^2

Answer: C.

Explanation: The cardinality of the power set is equal to 2^n, where n is the number of elements in a given set.

4. Write X = {1, 4, 9, 16, 25,…} in set builder form.

A. X = {x: x is a prime number} B. X = {x: x is a whole number}

C. X = {x: x is a natural number} D. X = {x: x is a square number}

Answer: D. X = {x: x is a square number}

Explanation: Given,

X = {1, 4, 9, 16, 25,…}

X = $\{1^2, 2^2, 3^2, 4^2, 5^2, \ldots\}$

Therefore,

X = {x: x is a set of square numbers}

Q5.If A={1,2,3,4,5} the number of proper subset is-

A.120 B.30 C.31 D.32

Answer- 31

Explanation-As every set is a subset of itself but not proper subset of itself.

Q6. The group of honest people in a city is

(a) Vois Set (b) Finite Set (c) Infinite Set (d) Not a set

Answer -d

Explanation -As honest people in a set is not a well defined collection , hence it is not a set

Q.7.The set of circles passing through Origin.

A. finite set B.infinite set C.Null set D.None of these

Answer-infinite set

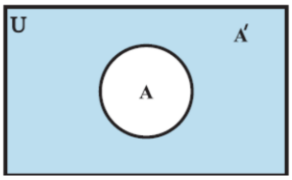
Explanation-Through Origin many circles can pass.

Q.8 The set AUA' IS

A. A   B.A'   C.Ø   D.U

Answer-U

Explanation-



Q.9 Set A and B has 3 and 6 elements. Find the minimum number of elements in AUB-

A.3   B.6   C.9   D.8

Answer-6

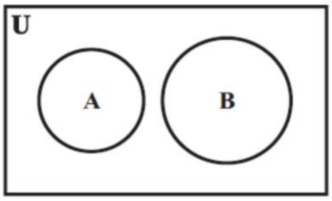
Explanation- If A is a subset of B then AUB=B

Q.10.If A and B are two disjoint sets then A ∩ B =

A. A   B.A'   C.Ø   D.U

Answer=Ø

Explanation-



Assertion Reason Type Questions

DIRECTION: In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as

(a) Both assertion and reason are true, and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true

Q.11 Assertion: The union of two finite sets is always finite.

Reason: The union of two finite sets contains all the elements from both sets, which is a finite collection.

Answer: a) Both assertion and reason are true and reason is the correct explanation of assertion

Q.12.Assertion: The intersection of two finite sets is always finite.

Reason: The intersection of two finite sets contains only the common elements, which is also a finite collection.

Answer:a)Both assertion and reason are true and reason is the correct explanation of assertion

Q.13.Assertion: The power set of a set with 'n' elements contains (2^n) subsets.

Reason: Each element in the power set can either be included or excluded from the original set, resulting in (2^n) possible subsets.

Answer: a) Both assertion and reason are true, and reason is the correct explanation of assertion

Q.14.Assertion: If (A is a subset of B) and (B is a subset of A), then (A = B).

Reason: If every element of set (A) is in set (B) and vice versa, the two sets are equal.

Answer: a) Both assertion and reason are true and reason is the correct explanation of assertion

Q.15.Assertion: The empty set is a subset of every set.

Reason: By definition, the empty set has no elements, so it is a subset of any set.

Answer: a) Both assertion and reason are true and reason is the correct explanation of assertion

Short Answer Type

Q.1 Let X = {1, 2, 3, 4, 5, 6}. If n represent any member of X, express the following as sets:

(i) n + 5 = 8

(ii) n is greater than 4

Solution:

(i) Let B = {x | x ∈ X and x + 5 = 8}

Here, B = {3} as x = 3 ∈ X and 3 + 5 = 8 and there is no other element belonging to X such that x + 5 = 8.

(ii) Let C = {x | x ∈ X, x > 4}

Therefore, C = {5, 6}

2. Write the following sets in the roster form.

(i) A = {x | x is a positive integer less than 10 and 2x – 1 is an odd number}

(ii) C = {x : x^2 + 7x – 8 = 0, x ∈ R}

Solution:

(i) 2x – 1 is always an odd number for all positive integral values of x since 2x is an even number.

In particular, 2x – 1 is an odd number for x = 1, 2, ... , 9.

Therefore, A = {1, 2, 3, 4, 5, 6, 7, 8, 9}

(ii) x^2+ 7x – 8 = 0

(x + 8) (x – 1) = 0

x = – 8 or x = 1

Therefore, C = {– 8, 1}

3. Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}.

Find A′, B′, A′ ∩ B′, A ∪ B and hence show that (A∪B)′ = A′∩ B′.

Solution:

Given,

U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}

A′ = {1, 4, 5, 6}

B′ = {1, 2, 6}.

Hence, A′ ∩ B′ = {1, 6}

Also, A ∪ B = {2, 3, 4, 5}

(A ∪ B)′ = {1, 6}

Therefore, (A∪ B)′ = {1, 6} = A′ ∩ B′

Q.4.Write the solution set of the equation x^2 + x – 2 = 0 in roster form.

Solution-

The given equation can be written as (x – 1) (x + 2) = 0, i. e., x = 1, – 2 Therefore, the solution set of the given equation can be written in roster form as {1, – 2}.

Q.5. Write the set {x : x is a positive integer and x^2 < 40} in the roster form.

Solution-

The required numbers are 1, 2, 3, 4, 5, 6. So, the given set in the roster form is

{1, 2,3, 4, 5, 6}

Q.6.Write the set A = {1, 4, 9, 16, 25, . . . }in set-builder form.

Solution

We may write the set A as A = {x : x is the square of a natural number} Alternatively, we can write A = {x : x = n^2, where n ∈ N}.

Q.7. Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and B = { 2, 3, 5, 7 }. Find A ∩ B and hence show that A ∩ B = B.

Solution

We have A ∩ B = { 2, 3, 5, 7 } = B. We note that B ⊂ A and that A ∩ B = B

Q.8. Let A = { 2, 4, 6, 8} and B = { 6, 8, 10, 12}. Find A ∪ B.

Solution

We have A ∪ B = { 2, 4, 6, 8, 10, 12}.

Q.9.Let A = { a, e, i, o, u } and B = { a, i, u }. Show that A ∪ B = A

Solution

We have, A ∪ B = { a, e, i, o, u } = A.

Q.10. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = {1, 3, 5, 7, 9}. Find A'.

Solution

We note that 2, 4, 6, 8, 10 are the only elements of U which do not belong to A. Hence A' = { 2, 4, 6, 8,10 }.

Case Study Based Questions

Q.1 In a school at Bhubaneswar, students of class XI were forming some sets. Two Students Ankita and Babita form two sets A = {1, 2, 3, 4, 5} and B = {2, 4, 6}.

Based on the above information answer the following:

- Find AUB

- Find A∩B
- Find A-B AND B-A.Are they equal?

Answer- I){1, 2,3,4,5,6}   II){2,4}   II)A-B={1,3,5},B-A={6}….NO

Q.2.A class teacher Mamta Sharma of class XI write three sets A, B and C are such that A = {1, 3, 5,7, 9}, B = {2, 4, 6, 8} and C = {2, 3, 5, 7, 11}.

Answer the following questions which are based on above sets.

(i) Find A ∩ B.

(a) {3, 5, 7}   (b) ϕ

(c) {1, 5, 7}   (d) {2, 5, 7}

(ii) Find A ∩ C

(a) {3, 5, 7}   (b) ϕ

(c) {1, 5, 7}   (d) {3, 4, 7}

(iii) Which of the following is correct for two sets

A and B to be disjoint?

(a) A ∩ B = ϕ   (b) A ∩ B ≠ϕ

(c) A∪B = ϕ   (d) A∪B ≠ϕ

(iv) Which of the following is correct for two sets

A and C to be intersecting?

(a) A ∩ C = ϕ   (b) A ∩ C ≠ϕ

(c) A∪C = ϕ   (d) A∪C ≠ϕ

(v) Write the n[P (B)].

(a) 8   (b) 4   (c) 16   (d) 12

Answers (i) b   (ii) a

(iii) a   (iv) b   (v) 16

Long Answer Type Questions

Q.1.Use the properties of sets to prove that for all the sets A and B, A – (A ∩ B) = A–B

Solution:

A – (A ∩ B) = A ∩ (A ∩ B)′ (since A – B = A ∩ B′)

= A ∩ (A′ ∪ B′) [by De Morgan's law)

= (A∩A′) ∪ (A∩ B′) [by distributive law]

= ϕ ∪ (A ∩ B′)

= A ∩ B′ = A – B

Hence, proved that A – (A ∩ B) = A – B.

Q.2. Two finite sets have m and n elements. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Find the value of m and n.

Solution:

No. of elements in A=m

No. of elements in B=n

According the question

No. of subsets of set A – No. of subsets off set B = 56

$2^m - 2^n = 56$

$2^n(2^{m-n} - 1)$ = 56 because m$> n$

$2^n(2^{m-n} - 1)$ = 8x7

$2^n(2^{m-n} - 1)$ = 2^3x7

By comparing to both sides we get

$2^n = 2^3$, $2^{m-n} - 1 = 7$

n = 3,$2^{m-n} = 7$ +1

$2^{m-n} = 8$

m -n = 3

m = n+3

m=6 because n=3

Q.3. Consider the sets ϕ, A = { 1, 3 }, B = {1, 5, 9}, C = {1, 3, 5, 7, 9}. Insert the symbol ⊂ or ⊄ between each of the following pair of sets: (i) ϕ . . . B (ii) A . . . B (iii) A . . . C (iv) B . . . C

Solution

(i) ϕ ⊂ B as ϕ is a subset of every set.

(ii) A ⊄ B as 3 ∈ A and 3 ∉ B

(iii) A ⊂ C as 1, 3 ∈ A also belongs to C

(iv) B ⊂ C as each element of B is also an element of C.

PRACTICE QUESTIONS

Q1 The number of subsets of a set containing n-elements is

(a) n (b) n^2 (c) 2" (d)2" - 1

Q2 For any two sets A and B, A∩(AUB) =

(a) A (b) B (c)∅ (d)none of these

Q3 If A= {1,3,5,7} B={2,4},then

(a) 4∈A (b) {4} ⊂ A (c) B ⊂ A (d) none of these

Q4 Let A= {x: x∈R, x>4} and B={x: x∈R, x<5}. Then A∩B =

(a) (4,5] (b) (4,5) (c) [4,5) (d) [4,5]

Q5 Let A and B be two sets such that n(A)=16,n(B)=14, n(AUB)=25.Then n(A∩B) =

(a) 30 (b) 50 (c) 5 (d) None of these

Q6 If A={1,2,3,4,5), then the number of proper subsets of A is

(a) 120 (b) 30 (c) 31 (d) 32

Q7 In set builder form empty set is represented by

(a) {} (b)Ø (c){x: x≠x} (d) {x:x=x}

Q.8.For two sets A∪B=A if

(a)B⊂A (b)A⊂B (c)A $\neq$B (d)$A = B$

Q9 In a city 20% of population travel by car,50% travel by bus and 10% travels by both Car and bus. Then percentage of persons travelling neither by car nor bus is:-

(a) 80% (b) 40% (c) 60% (d) 70%

Q.10 Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of second. Then the values of m and n are respectively.

(a) 4,7 (b)7,4 (c) 4,4 (d) 7,7

Answers

1.(c) 2. (a) 3. (d) 4. (b) 5. (c) 6. (c) 7. (c) 8. (a) 9. (b) 10. (b)

DIRECTION: In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as

(a) Both assertion and reason are true, and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true

1. Assertion (A): Set of English alphabets is the universal set for the set of vowels in English alphabets

   Reason (R) : The set of vowels is the subset of consonants in the English alphabets

2. Assertion (A): Let A={1,2,3} and B={1,2,3,4} then $A \subset B$

   Reason (R): A set A is said to be a subset of a set B if every element of A is also an element of B.

3. Assertion (A) : 'The collection of all natural numbers less than 100' is a set.

   Reason (R) : A set is a well-defined collection of distinct objects.

4. Assertion (A) : The set D = {x : x is a prime number which is a divisor of 60}

   Reason(R) : in roster form is{1, 2, 3, 4, 5}.

5. Assertion (A) : The set E = the set of all letters in the word 'TRIGONOMETRY', in the roster form is {T, R, I, G, O, N, M, E, Y}.

Reason (R) In roaster form distinct elements is written ,separated by comma

Answers

1. c 2. a 3. a 4. c 5. a

Short Answer Type Questions

1. Write the set A = {x: x∈ x, x^2 < 20} in roster form.

2. Which of the following sets are empty sets?

A = {x: $x^2 - 3 = 0$ And x is rational}

B = {x ∈ R: 0 < x< 1}

3. Write down all possible subsets of each of the following sets:

(i) {1, {1}}   (ii) {1,2,3}

4. Write the following as intervals:

{x: x ∈ R, −12 < x < −10}

{x: x ∈ R, 3 ≤ x ≤ 4}

5.What Universal Set would you propose for each of the following:

(i)the set of isosceles triangles? (ii) the set of right-angle triangles.

6. Let $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{2,4,6,8\}$ and $B = \{2,3,5,7\}$. $Verify\ that,$

$(i)(A \cup B)' = A' \cap B'$ $(ii)(A \cap B)' = A' \cup B'$.

7. Which of the following sets are finite and which are infinite:

A = $\{x: x\epsilon\ Z\ and\ x^2 - 5x + 6 = 0\}$

$B = \{x: x \epsilon\ Z\ \ and\ \ x^2\ is\ even\}$

C = $\{x : x \epsilon\ Z\ and\ x > -10\}$

8. Let A and B be two sets. Prove that $(A - B) \cup B = A \ if \ and \ only \ if \ B \subset$ A.

9. Let U = {1,2,3,4,5,6,7,8,9}

A = {1,2,3,4}, B = {2,4,6,8}, C = {3,4,5,6}.

Find (i) $(A \cap C)'(ii)(A')'(iii)(B - C)'$

10. Which of the following pairs of sets are equal? Justify your answer

(i)$A = \{x: x \ is \ a \ letter \ of \ the \ word \ "LOYAL"\}$

B = $\{x: x \ is \ a \ letter \ of \ the \ word \ "ALLOY"\}$.

(II) A = $\{x: x \in Z \ and \ x^2 \leq 8\}$

B = $\{x: x \in R, and \ x^2 - 4x + 3 = 0\}$

Answers

1. A = {−4, −3, −2,0,1,2,3,4}

2. 2. A= Empty Set, B= Non – Empty Set

3. (i) ∅, {1}, {{1}}, {1, {1}}

(ii) ∅, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}

4. (−12, −10) [3 , 4]

5. The set of all triangles in plane.

The set of all triangles in plane.

6. verification by given sets

7. A = {2, 3}

So, A is finite set.

B = {… , −6, −4, −2,0,2,4,6, … }

So, B is infinite set.

C= {-9 , -8 ,-7 ,.....}

So C is infinite Set

8. Verification/ Prove Given Set

9.(i) $(A \cap C)'$ = {1,2,5,6,7,8,9}

(ii) $(A')'$ = {1,2,3,4}

(iii) $(B - C)'$ = {1,3,4,5,6,7,9}

10. (i) A$= B$

(II) A$\neq B$

Long Answer Type

1.Three friends were having get together. Suddenly they decided to play with their names using sets. Name of friends were AARTI, CHARVI and AYSHA. They asked each other the following questions.

(i)How letters used for AARTI are written in roster form as a set?

(a) {A, R, T, I} (b) {x: x is a letter of the word AARTI} (c) {A,T,I} (d) none of these

(ii) What is the difference of set of letters of CHARVI and AYSHA?

(a) {C, R, V, I} (b) {C,S,V,I} (c) {C,T,V,I} (d) {C,V,I}

(III)Form a union of sets taking the letters of names of friends.

(a){A,R,T,I,C,H,Y,V,S} (b) {A,R,T,I,C,H,V,} (c) {A,R,C,H,V,Y,S} (d) none of these

Form a set of intersection of sets taking the letters of names of friends.

(a) {A} (b) {A,R,T,I,C,H,V} (c) {A,R,C,H,V,Y,S} (d) none of these

2.For all sets A, B and C. Is (A- B)$\cap (C - B) = (A \cap C) - B$?Justify your answer.

3. A,B and C are subset of universal set U. If A ={2,4,6,8,12,20} , B={3,6,9,12,15},

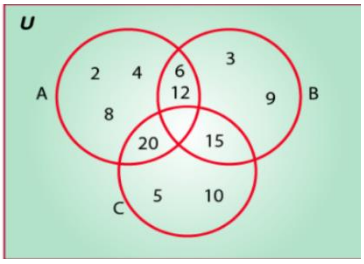
C={ 5,10,15,20} and U is the set of all whole numbers. Draw a Venn diagram showing the relation of U, A,B and C.

Answers

1.(i) a   (ii) a   (iii) a   (iv) a

2. By Verification

3.



RELATIONS AND FUNCTIONS

CARTESIAN PRODUCT OF SETS

- Given two non-empty sets A and B. The cartesian product A x B is the set of all ordered pairs of elements from A and B .
- A x B = { (a, b): a ∈ A and b ∈ B}, where A and B are non-empty sets
- if A =∅ or B =∅, then cartesian product A x B= ∅
- example if A = { 2, 5, 7} and B = { 3, 9} then

A x B = { (2,3), (2,9), (5,3), (5,9), (7,3), (7,9) }
B x A = { (3,2), (3,5), (3,7), (9,3), (9,5), (9,7) }

- if A and B are two sets then

no. of elements in cartesian product = n(A) x n(B)

- If either A or B is infinite set, then A x B is an infinite set

A, B and C are three set
A x (B U C) = (A x B) U (A x C)
A x (B ∩ C) = (A x B)∩ (A x C)
A x (B - C) = (A x B) - (A x C)
A x B = B x A ⇒ A = B
A x B = A x C ⇒ B =C

ORDERED PAIR

- pair of elements in fixed order
- elements can be same or distinct
- example points (3,4), (9,0) representing distinct points in a 2D plane
- in set, if A and B are two sets and a ∈ A and b ∈ B, ordered pair is (a, b)

- if (a, b) = (c, d) ⇔a=c and b=d
- A xA xA ={(a,b,c):a,b,c ∈A }Here (a,b,c) is called an ordered triplet.

RELATION

- Let A and B are two non empty sets. Then relation R ⊆ A x B

The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B. The second element is called the image of the first element.
Domain (R) = {x : (x,y) ∈ R}, Range (R) = {y : (x,y) ∈ R} and codomain = B

- Let A and B are two non empty finite sets consisting of m and n elements respectively.

No. of relation defined = 2^{mn}

FUNCTION

- Function 'f' is subset of relation of two non empty sets A and B such that every element in A has one and only one image in B.

(a, b) ∈ f ⇒ f(a) =b here b is image of a and a is pre image of b

- Domain of f = set A and co-domain of f = set B
- Range of f = set of images of elements in the domain

● A function which has either R or one of its subsets as its range is called a real valued function. Further, if its domain is also either R or a subset of R, it is called a real function.

● Algebra of function : if f: X→ R and g: X→ R are two real functions and k ϵ R, then

○ f ± g : X→ R is defined as (f ± g)(x) = f(x) ± g(x)

○ fg :X→ R is defined as (fg)(x) = f(x)g(x)

○ $\frac{f}{g}$: X – {x : g(x) = 0} → R is defined as $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

○ kf :X→ R is defined as (kf)(x) = kf(x)

ILLUSTRATIVE EXAMPLES

MULTIPLE CHOICE QUESTIONS

Q.1 Let A={1,2,3,4,5} and R be a relation from A to A, R = {(x, y): y = x + 1}. Find the domain.
(a) {1,2,3,4,5} (b) {2,3,4,5}
(c) {1,2,3,4} {d) {1,2,3,4,5,6}

Solution:- (c) Make relation from A to A
Put x=1,2,3,4 we get
y=2,3,4,5
∴R={(1,2),(2,3),(3,4),(4,5)}
Domain {1,2,3,4}, Co-domain {1,2,3,4,5} and Range {2,3,4,5}

Q.2 If set A has 2 elements and set B has 4 elements then how many relations are possible?
(a) 32 (b) 128 (c) 256 (d) 64

Solution:- (c) n(A)=4,n(B)=2
Number of relations from A to B=$2^{n(A)n(B)}$ = 2^8 = 256

Q.3 In a function from set A to set B, every element of set A has___________ image in set B.
(a) one and only one (b) different
(c) same (d) many

Solution:- (b)

Q.4 . f(x) = {$\frac{|x|}{x}$ for x≠0 and 0 for x=0}. Which function is this?
(a) Constant (b) Modulus
(c) Identity (d) Signum function

Solution:- (d)

Q.5 If f(x) = x^3 – (1/x^3), then f(x) + f(1/x) is equal to

(a) $2x^3$ (b) $2/x^3$ (c) 0 (d) 1

Solution:- (c) $f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}}$

$$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3$$

$$f(x) + f\left(\frac{1}{x}\right) = 0$$

Q.6 f(x) = $\sqrt{9-x^2}$. Find the range of the function.

(a) R (b) R^+

(c) [-3,3] (d) [0,3]

Solution:- (d) Given that f(x)= $\sqrt{9-x^2}$

The domain of the given function is $x \in [-3,3]$

$\Rightarrow -3 \le x \le 3$

$\Rightarrow 0 \le x^2 \le 9$

$\Rightarrow -9 \le -x^2 \le 0$

$\Rightarrow 0 \le 9-x^2 \le 9$

$\Rightarrow 0 \le \sqrt{9-x^2} \le 3$

$\Rightarrow 0 \le f(x) \le 3$

The range of the function is [0,3].

Q.7 Let A = {-2, -1, 0} and f(x) = 2x – 3 then the range of f is

(a) {7, -5, -3} (b) {-7, 5, -3} (c) {-7, -5, 3} (d) {-7, -5, -3}

Solution:- (d) f(-2) = 2(-2) -3 = -7

f (-1) = 2(-1) -3 = -5

f(0) = 2(0) -3 = -3

Range = {-7, -5, -3}

Q.8 A function is defined by f(t) = 2t – 5, then the value of f(-3) is

(a) - 11 (b) 11 (c) 1 (d) -1

(b) Solution:- (a) f(-3) = 2(-3) -5

= -6 -5 = -11

Q.9 Let f = {(1, 1), (2, 3), (0, -1), (-1, -3)} be a function from Z to Z defined by f(x) = mx + c. Determine c.

(a) 1 (b) 0 (c) - 1 (d) – 3

Solution:- (c) f(1)=1 gives us m·1+c= m + c =1.

f(2)=3 gives us m·2+c = 2m + c =3

f(0)= −1 gives us m·0+c=−1

$\Rightarrow$c = -1

Q.10 If P X Q is an empty set then which of the following is a null set?

(a) only P (b) only Q

(c) either P or Q (d) both P and Q

Solution :- (c)

ASSERTION AND REASONING

In the following questions a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

1 Assertion (A): If (x+1, y-2) = (3,1), then x = 2 and y =3.

Reason (R): Two ordered pairs are equal, if their corresponding elements are equal.

Solution:- x +1 = 3⇒x=3-1⇒x =2
And y-2 = 1⇒y=1+2⇒y= 3
Reason is the property of ordered pair i.e. (a, b) = (c, d) ⇔ a=c and b=d
Both A and R are true and R is the correct explanation of A.

2 Assertion (A):The cartesian product of two non-empty sets P and Q is denoted as P x Q and P x Q = {(p, q): p ϵ P, q ϵ Q}

Reason (R): If A = {red, blue} and B = {b, c, s}, then A x B = {(red, b), (red, c), (red, s), (blue, b), (blue ,c), (blue, s)}.

Solution:- Assertion (A) states that the cartesian product of two non-empty sets P and Q, denoted P x Q , consists of all ordered pairs ((p, q) where p is an element of P and q is an element of Q . This definition is accurate and corresponds to the formal definition of cartesian product.

Reason (R) provides a valid example that matches the definition of the cartesian product. Both A and R are true and R is the correct explanation of A.

3 Assertion (A): Let A = {1, 2} and B= {3, 4}. Then, number of relations from A to B is 16.

Reason (R): If n(A) = p and n(B) = q, then number of relations in 2^{pq}.

Solution:- Assertion (A) is incorrect. To find the number of relations from set A to set B, we consider all possible subsets of A×B, where A x B denotes the cartesian product of A and B. Given A={1,2} and B={3,4}, the cartesian product A×B consists of 4 ordered pairs: {(1,3),(1,4),(2,3),(2,4)}. Reason (R) provides a general formula

Therefore, while Assertion (A) itself is incorrect in its specific example, Reason (R) provides the correct

4 Assertion (A): f(x) = x^2, f: N to N is a function.

Reason (R): All relations are functions.

Solution**:-** Assertion(A) is correct as each value of x has unique image in f(x)

Reason (R) is false because not all relations are functions; functions are a specific subset of relations where each input is associated with exactly one output.

5) Assertion(A):The domain of the relation R = {(x + 2, x + 4) : x ∈ N, x < 8} is {3, 4, 5, 6, 7, 8,9).

Reason (R): The range of the relation R = {(x + 2, x + 4) : x ∈ N, x < 8} is {1, 2, 3, 4, 5, 6, 7}.

Solution:- on putting the values of x from 1 to 7 in relation we find the domain {3, 4, 5, 6, 7, 8,9) and range ={5, 6, 7, 8, 9, 10, 11}.So, A is true but R is false.

SHORT ANSWER TYPE QUESTIONS

Q.1 Find x and y, if (x+3, 5) = (6, x-y).

Solution:- x+3 = 6 ⇒x = 3
And x – y = 5
⇒x – 5 = y
⇒y= 3 – 5 ⇒y= -2

Q.2 If A = {4, 9, 6} and B = {5, 4, 7, 6}, find A x B and B x A.

Solution:- **A×B**: This is the set of all ordered pairs (a, b) where a ∈ A and b ∈ B
A×B={(4,5),(4,4),(4,7),(4,6),(9,5),(9,4),(9,7),(9,6),(6,5),(6,4),(6,7),(6,6)}
And **B x A** : This is the set of all ordered pairs (b, a) where a ∈ A and b ∈ B
B×A={(5,4),(5,9),(5,6),(4,4),(4,9),(4,6),(7,4),(7,9),(7,6),(6,4),(6,9),(6,6)}.

Q.3 If P ={ 3,5}, then find P x P x P.

Solution:- {((3,3,3),((3,3,5),((3,5,3),((3,5,5),((5,3,3),((5,3,5),((5,5,3),((5,5,5)}.

Q.4 Given R = {(x, y) : x, y ∈ N, $x^2 + y^2 = 25$}. Find the domain and range of R

Solution:- We have $3^2 + 4^2 = 25$ or $4^2 + 3^2 = 25$
So the domain of R is {3,4}. [Values corresponding to x for x being natural number]
And the range is {3,4}. [Values corresponding to y for y being natural number]

Q.5 Redefine the function: f(x) = |x – 1| – |x + 6|

Solution:- Given function is f(x) = |x – 1| – |x + 6|
Redefine of the function is:

$$f(x) = \begin{cases} -x+1+x+6, \ x \le -6 \\ -x+1-x-6, \ -6 \le x < 1 \\ x-1-x-6, \ x \ge 1 \end{cases}$$

$$= \begin{cases} 7, \ x \le -6 \\ -2x-5, \ -6 \le x < 1 \\ -7, \ x \ge 1 \end{cases}$$

The domain of this function is R.

Q.6 The function 't' which maps temperature in degree Celsius in to temperature in degree

Fahrenheit is defined by t(C) = 9C+ 32. Find t(0), t (- 10) and the value of C, when t (C) = 212.

Solution:- For t(0): Substitute C=0 into the function:
t(0)=9·0+32

So, t(0)=32

For t(-10) : Substitute C=-10 into the function:

t(-10)=9(-10)+32

So, t(-10)=-58

For t(C)=212

212= 9C +32

9C = 180

C = 20

Q.7 Determine the domain and range of the relation R = { (a, b): a ϵ N, a ≤ 5, b=4}

Solution:- We have,

R = {(a, b): a ϵ N, a ≤ 5,b=4}

⇒a=1,2,3,4,5 and b = 4

Thus, R = {(1, 4), (2, 4), (3, 4), (4, 4), (5,4)}

Clearly, Domain (R) = {1, 2, 3, 4, 5} and Range (R) = {4}

Q.8 Let A = {1, 2, 3, 4, 5}; B = {2, 3, 6, 7}. Then what is the number of elements in (A × B) ∩ (B × A)?

Solution:- Here, A and B sets have 2 elements in common, so A × B and B × A have 2^2, i.e., 4 elements in common. Hence, n [(A × B) ∩ (B × A)] = 4.

Q.9 The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0,1). Find the set A and the remaining elements of A × A

Solution:- We know that,

If n(A) = p and n(B) = q, then n(A × B) = pq

From the given,

n(A × A) = 9

n(A) × n(A) = 9,

n(A) = 3 ……(i)

The ordered pairs (-1, 0) and (0, 1) are two of the nine elements of A × A.

Therefore, A × A = {(a, a) : a ∈ A}

Hence, -1, 0, 1 are the elements of A. …..(ii)

From (i) and (ii),

A = {-1, 0, 1}

The remaining elements of set A × A are (-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0) and (1, 1).

Q.10 Find the domain of function of f(x) = $\sqrt{16-x^2}$.

Solution:- Consider the given function.

f(x)= $\sqrt{16-x^2}$

For domain under root should not be negative quantity,

$16-x^2 \geq 0$

$16 \geq x^2$

Therefore, x ≤ 4 or x ≥ −4

Then, the domain[−4,4]

LONG ANSWER TYPE QUESTIONS

Q.1 The function f is defined by

$$f(x)=\begin{cases}1-x, & x<0\\ 1, & x=0\\ x+1, & x>0\end{cases}$$

Draw the graph of f(x).

Solution:f(x) = 1 – x, x < 0, this gives

f(– 4) = 1 – (– 4)= 5;

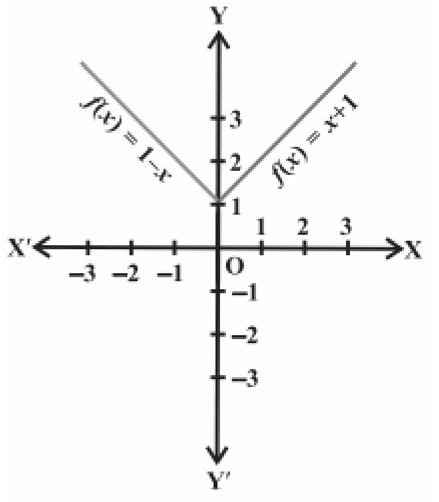
f(– 3) =1 – (– 3) = 4,

f(– 2) = 1 – (– 2)= 3

f(–1) = 1 – (–1) = 2; etc,

Also, f(1) = 2, f (2) = 3, f (3) = 4, f(4) = 5 and so on for f(x) = x + 1, x > 0.

Thus, the graph of f is as shown in the below figure.



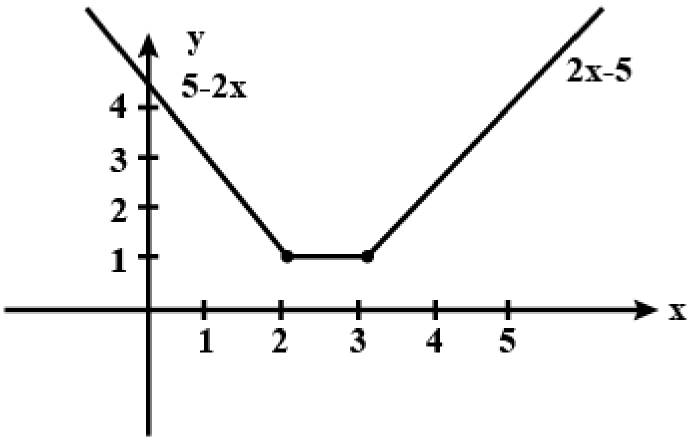
Q.2 Draw the graph of the function f(x) = |x-2|+|x-3|

Solution:- f(x)=|x−2|+|x−3|

When x<2, f(x)=2−x+3−x =5−2x

When 2≤x<3, f(x)=x−2+3−x = 1

When x≥3, f(x)=x−2+x−3 = 2x−5



Q.3 Find the domain and range of the function f(x) = 1/$\sqrt{x-[x]}$

Solution:-We have, f(x)=1/ $\sqrt{x-[x]}$

for domain of f: we know that,
0 ≤ x−[x] ≤ 1,∀ x ϵ R
Also, x−[x]=0 ∀ x ϵ Z
∴0 < x−[x] < 1,∀ x ϵ R−Z
⇒f(x)=1/ $\sqrt{x-[x]}$ ∀ x ϵ R−Z
∴Domain of f(x)=R−Z
For range of f:we have 0 < x−[x] < 1,∀ x ϵ R−Z
⇒0< $\sqrt{x-[x]}$ <1, ∀ x ϵ R−Z
⇒1< 1/ $\sqrt{x-[x]}$ <∞,∀ x ϵ R−Z
⇒1<f(x)<∞, ∀ x ϵ R−Z
∴Range of f(x)=(1,∞)

Q.4 Determine the domain and range of the relation R = {(x, y) : x ϵ N, y ϵ N and x + y = 10}.

Solution:- Given set N = {1, 2, 3,…} A relation R in N is defined as R = {(x, y) : x, y ∈ N, x + y = 10} ∀ x, y ∈ N
So, xRy ⇔ x + y = 10 ⇔ y = 10 – x when x = 1, then y = 10 – 1 = 9 ∈ N then (1, 9) ∈ R
when x = 2, then y = 10 – 2 = 8 ∈ N then (2, 8) ∈ N
when x = 3, then y = 10 – 3 = 7 ∈ N then (3, 7) ∈ R
when x = 4, then y = 10 – 4 = 6 ∈ N then (4, 6) ∈ R
Similarly, (5, 5) ∈ R, (6, 4) ∈ R, (7, 3) ∈ R, (8, 2) ∈ R, (9, 1) ∈ R
R = {(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)}
Domain of R ={1, 2, 3, 4, 5, 6, 7, 8, 9}
Range of R = {9, 8, 7, 6, 5, 4, 3, 2, 1}

Q.5 Assume that A = {1, 2, 3,…,14}. Define a relation R from A to A by R = {(x, y) : 3x – y = 0, such that x, y ∈ A}. Determine and write down its range, domain, and co domain.

Solution:- It is given that the relation R from A to A is given by R = {(x, y): 3x – y = 0, where x, y ∈ A}.It means that R = {(x, y) : 3x = y, where x, y ∈ A}

Hence, R = {(1, 3), (2, 6), (3, 9), (4, 12)}
We know that the domain of R is defined as the set of all first elements of the ordered pairs in the given relation.
Hence, the domain of R = {1, 2, 3, 4}
To determine the co domain, we know that the entire set A is the co domain of the relation R.
Therefore, the co domain of R = A = {1, 2, 3,…,14}
As it is known that, the range of R is defined as the set of all second elements in the relation ordered pair.
Hence, the Range of R is given by = {3, 6, 9, 12}

CASE STUDY BASED QUESTIONS

Q.1 Maths teacher started the lesson Relations and Functions in Class XI. He explained the following topics:

Ordered Pairs: The ordered pair of two elements a and b is denoted by (a, b) : a is first element (or first component) and b is second element (or second component).
Two ordered pairs are equal if their corresponding elements are equal. i.e., (a, b) = (c, d) ⇒ a = c and b = d
Cartesian Product of Two Sets: For two non-empty sets A and B, the cartesian product A x B is the set of all ordered pairs of elements from sets A and B.

In symbolic form, it can be written as A x B= {(a, b) : a ∈ A, b ∈ B.
Based on the above topics, answer the following questions.
(i) If (a – 3, b + 7) = (3, 7), then find the value of a and b
(ii) If (x + 6, y – 2) = (0, 6), then find the value of x and y
(iii) If (x + 2, 4) = (5, 2x + y), then find the value of x and y
(iv) Find x and y, if (x + 3, 5) = (6, 2x + y).
Solution:-
(i) We know that, two ordered pairs are equal, if their corresponding elements are equal.
(a – 3, b + 7) = (3, 7)
⇒ a – 3 = 3 and b + 7 = 7 [equating corresponding elements]
⇒ a = 3 + 3 and b = 7 – 7 ⇒ a= 6 and b = 0
(ii) (x + 6, y – 2) = (0, 6)
⇒ x + 6 = 0 ⇒ x = -6 and y – 2 = 6 ⇒ y = 6 + 2 = 8
(iii) (x + 2, 4) = (5, 2x + y)
⇒ x + 2 =5 ⇒ x = 5 – 2 = 3 and 4 = 2x + y ⇒ 4 = 2 x 3 + y ⇒ y = 4 – 6 = -2
(iv) x + 3 = 6, 2x + y = 5 ⇒ x = 3, y = 1

Q.2 Hanuman Pareek and Pawan Saini are two students of class XIth in a school. The ages of students is represented by f and g be real functions defined by f(x) = $\sqrt{x+4}$ and g(x) = $\sqrt{16-x^2}$. Then answer the some question based on their ages.

(i). Find the domain of f(x) and g(x).
(ii) Find the domain of sum of both ages.
(iii).Find the domain of ($\frac{f}{g}$).
Solution:- (i)for domain of f(x)
$\sqrt{x+4} \geq 0$
⇒ X+4 ≥ 0 ⇒ x ϵ [-4,∞)
for domain of g(x)
$\sqrt{16-x^2} \geq 0 \Rightarrow 16 - x^2 \geq 0 \Rightarrow x^2 \leq 16 \Rightarrow x \ \epsilon\ [-4,4]$
(ii) domain of f(x) ∩ domain of g(x) = [-4,4]
(iii) domain of ($\frac{f}{g}$) = [domain of f(x) ∩ domain of g(x)] – points where g(x) is not defined
= (-4,4)

Q.3 A general election of Lok Sabha is a gigantic exercise. About 969 million people were eligible to vote and voter turnout was about 66%, the highest ever Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2024.

A relation 'R' is defined on I as follows: R = {(V1, V2) :V1, V2 ∈I and both use their voting right in general election – 2024}

(i). Two neighbours X and Y∈ I. X exercised his voting right while Y did not cast her vote in general election – 2024. Which of the following is true?

a. (X,Y) ∈R b. (Y,X) ∈R c. (X,X) ∉R d. (X,Y) ∉R

(ii). Mr.X and his wife 'W'both exercised their voting right in general election -2024, Which of the following is true?

a. both (X,W) and (W,X) ∈ R b. (X,W) ∈ R but (W,X) ∉ R

c. both (X,W) and (W,X) ∉ R d. (W,X) ∈ R but (X,W) ∉

(iii). Three friends x, y and z exercised their voting right in general election-2024, then which of the following is true?

a. (x, x) ∈R, (y, z) ∈ R and (x, z) ∈ R b. (x, y) ∈ R, (y, z) ∈ R and (x, z) ∉ R

c. (x, y) ∈ R, (y, y) ∈R but (z, z) ∉ R d. (x, y) ∉ R, (y, z) ∉ R and (x, z) ∉ R

(iv). Mr. Shyam exercised his voting right in General Election – 2024, then Mr. Shyam is related to which of the following?

a. All those eligible voters who cast their votes

b. Family members of Mr.Shyam

c. All citizens of India

d. Eligible voters of India

Solution :- (i)(d) (ii) (a) (iii) (a) (iv) (a)

Q.4 To make herself self dependent and to earn her living, a college student decided to setup a small scale business of manufacturing hand sanitizers.

She estimated a fixed cost of Rs. 15000 per month and a cost of Rs. 30 per unit to manufacture.
Based on the above information, answer the following questions

(i)Let x units of hand sanitizers are manufactured per month. What is the function of cost ?
(ii)If each unit is sold for Rs. 45, then what is the selling (revenue) function?
(iii)How much units should be manufactured and sold, for break-even (that is, no profit, no loss situation) in a month?
(iv)What is the monthly cost borne by the student, if the student decided to manufacture 1500 units in a month?

Solution:- (i) C(x) = 15000 + 30x

(ii) R(x) =45x

(iii) For break-even, C(x) = R(x)

15000+30x = 45 ⇒ 15x = 15000

⇒ X= 1000

(iv) C(1500) = 15000+ 30(1500) =60000

Q.5 Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by {(a, b): a , b ∈A, b is exactly divisible by a}.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R.

Solution:- A = {1, 2, 3, 4, 6} then R= {(1,1), (1, 2), (1,3), (1,4), (1,6), (2, 4), (2, 6), (2, 2), (3, 3), (4, 4), (3,6), (6,6)}

Domain = {1, 2, 3, 4, 6}

Range = {1, 2, 3, 4, 6}

PRACTICE QUESTIONS
MULTIPLE CHOICE QUESTIONS

Q.1 Express the function f: A→R. $f(x) = x^2 - 1$, where A = { -4, 0, 1, 4} as a set of ordered pairs.

(a) {(-4, 15), (0, -1), (1, 0), (4, 15)}
(b) {(-4, -15), (0, -1), (1, 0), (4, 15)}
(c) {(4, 15), (0, -1), (1, 0), (4, 15)}
(d) {(-4, 15), (0, -1), (1, 0)}

Q.2 The number of relations from A={1,2,3} to B={4,6,8,10} is(a) 4^3 (b) 2^7 (c) 2^{12} (d) 3^4

Q.3 If P X Q has 10 elements then which is not possible?
(a) n(P)=1 and n(Q)=10 (b) n(P)=10 and n(Q)=1 (c) n(P)=2 and n(Q)=5 (d) n(P)=5 and n(Q)=4

Q.4 Let n (A) = m, and n (B) = n. Then the total number of non-empty relations that can be defined from A to B is

(a) m^n (b) $n^m - 1$ (c) mn – 1 (d) $2^{mn} - 1$

Q.5 If f(x) = ax + b, where a and b are integers, f(–1) = – 5 and f(3) = 3, then a and b are equal to (a) a = –3, b = –1 (b) a = 2, b = – 3 (c) a = 0, b = 2 (d) a = 2, b = 3

Q.6 Let R be a relation defined by a relation on N by x + 2y = 8 , the domain of R is
(a){2, 4, 8 } (b) { 2 ,4 ,6 ,8 } (c) { 2, 4 ,6 } (d) { 1 , 2 ,3 ,4 }

Q.7 The domain of the function $f(x) = 1/(x^2 - 3x + 2)$ is
(a) {1, 2} (b) R (c) R – {1, 2} (d) R – {1, -2}

Q.8 If A = {a, b}, B = {c, d}, C = {d, e} then {(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)} is equal to

(a) A∪(B∩C) (b) A∩(B∪C) (c) A×(B∩C) (d) A×(B∪C)

Q.9 Let R be the relation in the set N given by R = {(a, b): a = b – 2, b > 6}. Choose the correct answer.

(a) (2, 4) ∈ R (b) (3, 8) ∈ R (c) (6, 8) ∈ R (d) (8, 7) ∈ R

Q.10 If f(x) = - |x|. Choose the correct option from the following:

(a)Domain is set of negative real numbers (b) Range is set of real numbers
(c)Range is set of all negative integers (d) Range is (-∞, 0]

ANSWER									
Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7	Q.8	Q.9	Q.10
A	c	d	d	b	c	c	d	c	d

ASSERTION AND REASONING

In the following questions a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

1 Assertion (A):If (x,1), (y,2) and (z, 1) are in A x B and n(A)=3, n(B)=2, then A= {x, y, z} and B = {1, 2}. Reason (R):If n(A) = 3 and n(B) = 2, then n(A x B) = 6

2 Assertion (A): Let A = {1, 2, 3, 4, 6}. If R is the relation on A defined by {(a, b): a, b ϵ A, b is exactly divisible by a}.The relation R in Roaster form is {(6, 3), (6, 2), (4, 2)}.

Reason (R):The domain and range of R is {1, 2, 3, 4, 6}.

3 Assertion (A): R= {(1, 2), (3, 4), (4, 5), (5, 6), (6, 6)} is a function. Reason (R): The domain of a real valued function is a subset of real numbers.

4 Assertion (A): Let A={a,b} and B={1,2,3}. The number of relations from A to B is 64.

Reason (R):If n(A)=p and n(B)=q, then the number of relation from A to B is q^p.

5 Assertion (A): If P = {1, 2}, then P × P × P = {(1, 1, 1), (2, 2, 2), (1, 2, 2), (2, 1, 1)}

Reason (R): A × A × A = {(a, b, c) : a, b, c ∈ A}. Here (a, b, c) is called an ordered triplet.

ANSWER				
Q.1	Q.2	Q.3	Q.4	Q.5
a	d	b	c	d

SHORT ANSWER TYPE QUESTIONS

Q.1 If A = {-1, 2, 3 } and B = {1, 3}, then determine
(i) A x B (ii) B x A (c) B x B (iv) A x A

Q.2 If A = {$x : x^2 - 5x + 6 = 0$}; B = {2,4}, and C = {4,5}, find the value of A x (B ∩ C).

Q.3 If A x B = { (a, x), (a, y), (b, x), (b, y) }, then find A and B.

Q.4 Let the function f(x) = x^2 for all x ∈ X, where X = {-2, -1, 0, 1, 2, 3} define f: X → Y. Express the relation f in roaster form. Mention if f is a function.

Q.5 Draw the graph of function f(x) = $\sqrt{4-x^2}$. Also write the domain and range of f(x).

Q.6 If A= { 1, 2, 7}; B= {3, 6, 4} and C= {5, 4}. Prove A x (B ∩ C) = (A x B) ∩ (A x C).

Q.7 Find the domain and range of the real valued function f(x) given by f(x) = $\frac{4-x}{x-4}$.

Q.8 Determine the domain and range of the relation R, where R = { (x, x^3): x is a prime no. less than 10}.

Q.9 Draw the graph of the function f(x) = 5 - |x-2|

Q.10 Calculate the domain and range of f(x) = |2x-3|-3.

	ANSWERS
Q. 1	A × B = {(−1, 1), (−1, 3), (2, 1), (2, 3), (3, 1), (3, 3)} B × A = {(1, −1), (1, 2), (1, 3), (3, −1), (3, 2), (3, 3)} B × B = {(1, 1), (1, 3), (3, 1), (3, 3)} A × A = {(−1, −1), (−1, 2), (−1, 3), (2, −1), (2, 2), (2, 3), (3, −1), (3, 2), (3, 3)}
Q. 2	{(2,4),(3,4)}
Q. 3	A= {a, b} B= {x, y}
Q. 4	{(-2,4), (2,4), (-2,4), (-2,4), (-2,4), (-2,4), (-2,4), (-2,4),
Q. 5	Domain= [-2, 2] and range [0, ꝏ)
Q. 7	Domain= R- {4} and range = {-1}
Q. 8	Domain = {2, 3, 5, 7} and range = {8, 27, 125, 343}
Q. 10	Domain = R and Range = [-3,ꝏ)

LONG ANSWER TYPE QUESTION

Q.1 Find the domain of the function f given by f(x) =$\frac{1}{\sqrt{[x]^2-[x]-6}}$.

Q.2 If a relation R is defined on the set Z of integers as follows $\{a, b\} \in R\ a^2 + b^2 \leq 25$ *find domain range and co-domain*

Q.3 Find the domain and range of the function f(x) = |2x + 1|- |x-3|.

Q.4 Let f: R → R be given by f(x) = x^2 +3. Find {x : f(x) =28} and find the pre-images of 39 and 2 under f.

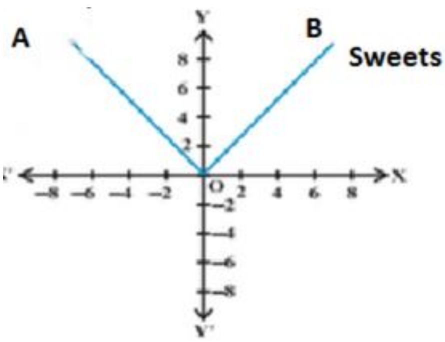
Q.5 Let A = {a, b, c, d}. Examine which of the following relations is a function on A and give reason.

(i) f= {(a, a), (b, c), (c, d), (d, c)}
(ii) g = {(a, c), (b, d), (b, c)}
(iii) h = {(b, c), (d, a), (a, a)}

ANSWERS	
Q. 1	x ϵ (-∞,-2) [4,∞)
Q. 2	Domain= {0,±3,±4,±5}
Q. 3	Domain: R Range: (−∞,−2]∪[7,∞).
Q. 4	{x : f(x)=28}={5,−5} Pre-images of 39 under f are {6,−6}. There are no real numbers x such that f(x)=2.
Q. 5	f and g are function

CASE STUDY BASED QUESTIONS

Q.1 A is the anthills of an ant, at B some sweets are there and ant wants to reach at B. The path traced by an ant is shown in the following graph:



On the basis of the above graph find the following:
(i) When ordinate is 6 then find abscissa
(ii) Which axis is line of symmetry for the graph?
(iii) Write the function for the graph along with domain and range.

Q.2 In a school at Chandigarh, students of class XI were discussing about the relations and functions. Two students Ankita and Babita form two sets A = {1,2, 3, 4, 5} and B = {2, 4, 6}.

Based on the above information answer the following:
(i) Find n(A×B)
(ii) A correspondence of elements from A to B given as {(1, 2), (2, 2), (3, 4), (3, 6), (4, 4), (5,6)}. Is it a function? Justify your answer.
(iii) If the function f: A⟶B such that (a, b) ∈ f and a < b, defined by f = {(1, 2), (x, 4), (2,4), (4, y), (5, 6)}, then find x and y.

Q.3 Method to find set when cartesian product is given

For finding these two sets, we write first element of each ordered pair in first set say A and corresponding second element in second set B (say).
Number of elements in cartesian product of two sets
If there are p elements in set A and q elements in set B, then there will be pq elements in A x B i.e., if n(A) = p and n(B)= q, then n(A x B) = pq
Based on the above two topic, answer the following questions.

(i) If A x B = {(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)}. Then, A and B are

(a) {1, 3, 2}, {a, b} (b) {a, b}, {1, 3, 2} (c) {a, b}, {1, 3} (d) none of these

(ii) If the set A has 3 elements and set B has 4 elements, then the number of elements in A x B is

(a) 3 (b)4 (c) 7 (d) 12

(iii) A and B are two sets given in such a way that A x B contains 6 elements. If three elements of A x B are (1, 3), (2, 5) and (3, 3), then A, B are

(a) {1, 2, 3}, {3, 5} (b) {3, 5}, {1, 2, 3} (c) {1, 2}, {3, 5} (d) {1, 2, 3}{5}

(iv) The cartesian product P x P has 16 elements among which are found (a, 1) and (b,2). Then, the set P is

(a) {a, b} (b) {1,2} (c) {a, b, 1, 2} (d) {a, b, 1, 2, 4}

Q.4 Given set A = {x: x < 5, x ∈ N} and B = {x: 0 ≤ x ≤ 2 , x ∈ Z} .

Based on the above information, answer the following questions:

(i)A × B is a set of ordered pair:

(a) {(1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1),(4, 2)}
(b) {(0, 0), (1, 1), (2, 2)}
(c) {(1, 0), (2, 1), (3, 2), (4, 3)}
(d) {(1, 0), (1, 1), (2, 0), (2, 1), (3, 0), (3, 1), (4, 0), (4, 1)}

(ii) (A ∪ B) × (A ∩ B) is a set of ordered pair:
(a) {(1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1),(4, 2)}
(b) {(0, 1), (0, 2), (2, 2), (1, 1), (1, 2), (2, 1), (3, 1), (3, 2), (4, 1), (4,2)}
(c) {(1, 0), (2, 1), (3, 2), (4, 3)}
(d) {(1, 0), (1, 1), (2, 0), (2, 1), (3, 0), (3, 1), (4, 0), (4, 1)}

(iii) A relation R from set A to B defined by R = {(x, y): x + y = 4, x ∈ A, y ∈ B} as a set of ordered pair is:
(a) {(1, 3), (4, 0), (2, 2)} (b) {(0, 4), (1, 3), (2, 2)}
(c) {(1, 3), (2, 2), (3, 1), (4, 0)} (d) {(2, 2), (3, 1), (4, 0)}

(iv) The range of relation R is:
(a) {1, 2, 3, 4} (b) {0, 1, 2} (c) {2, 3, 4} (d) {0, 1, 2}

Q.5 If A = {x : x ∈ N, x ≤ 2}, B = {x : x ∈ N, 1 < x < 5}, C = {3, 5} find

(i) A x B
(ii) A x C
(iii) B x (A ∩ C)

ANSWERS	
Q.1	(i) {-6, 6} (ii) Y axis (iii) f(x) = |x| , domain = R and range = [0, ∞)
Q.2	(i) 15 (ii) f is not a function (iii)x = 3 and y =6
Q.3	(i) (b) (ii)(d) (iii)(a) (iv)(c)
Q.4	(i)(a) (ii) (b) (iii) (d) (iv) (b)
Q.5	(i) A×B={(1,2),(1,3),(1,4),(2,2),(2,3),(2,4)}.(ii) A×C={(1,3),(1,5),(2,3),(2,5)}.(iii) Ø

TRIGONOMETRIC FUNCTIONS

CONCEPTUAL NOTES

Angles: Angle is a measure of rotation of a given ray about its initial point.

Measurement of an angle.

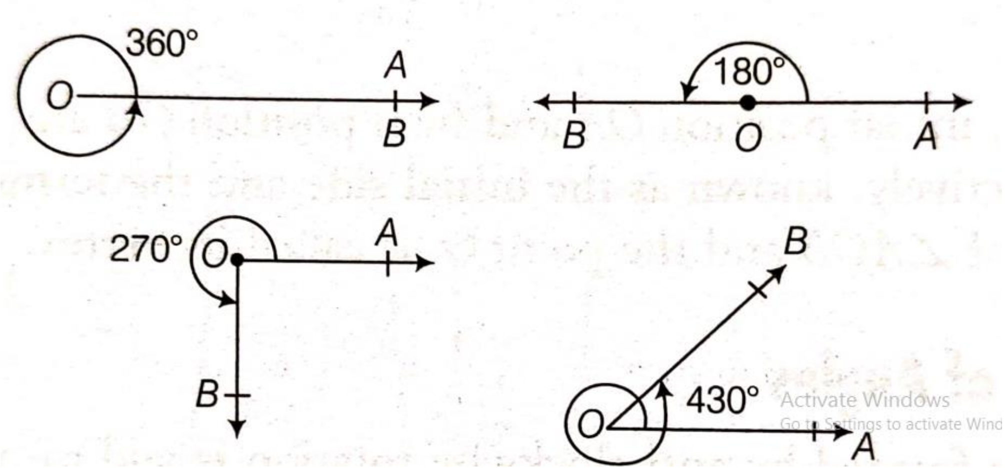
The amount of rotation from the initial side to the terminal side is called the measure of an angle. There are two systems for measuring an angle, which are given below.

Sexagesimal System (Degree Measure)

If a rotation from the initial side to terminal side is $\left[\frac{1}{360}\right]^{th}$ of a revolution, the angle is said to be have measure of one degree written as 1^{o}. A degree is divided into 60 minutes & a minute is divided into 60 second.

1^{o} = 60 minutes = 60′. (iii) 1′ = 60 second = 60′′.

Some of the angles whose angles are $360^{0}, 180^{0}, 270^{0}$ are shown below



Circular System (Radian Measure)

Angle subtended at the center by an arc of length one unit in a circle of radius 1 unit is said to have a measure of 1 radian.

In general a circle of radius r having an arc of length r will subtend an angle of 1 radian at the centre.

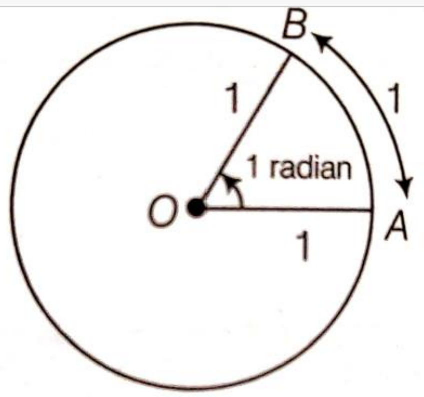
1 radian is written as 1^{c} or 1 rad.

Also a circle of radius r having an arc of length l will subtend an angle $\theta \, radian$

At the centre, where

$$\theta = \frac{l}{r} = \frac{Length of \ Arc}{Radius} \, or \, l = r\theta$$

The figure shows the angle whose measures are 1 radian (1^{c})



Remembering Points
In Sexagesimal System we measure angles in degree, minute & seconds.
1^o = 60 minutes = 60′. (iii) 1′ = 60 second = 60′′.
In circular system, we measure angles in radian.

Relation between Degree & Radian

A circle subtends at its centre an angle whose radian measure is 2π & its degree measures is 360^0.It follows that

$$2\pi\, radian = 360^0 \quad or \quad \pi\, rad = 180^0$$

$$1\, rad = \frac{180^0}{\pi} = 57^0 16' 22''$$ (approx..)

$$1^0 = \frac{\pi}{180} rad = 0.01746\, rad$$

Radian measure = $\frac{\pi}{180} \times Degree\, Measure$	
Degree Measure = $\frac{180}{\pi} \times radian\, Measure$	

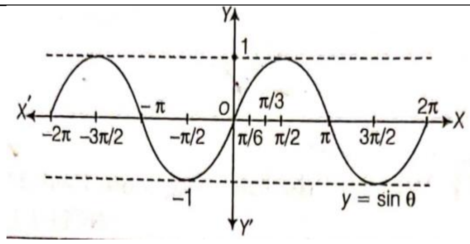
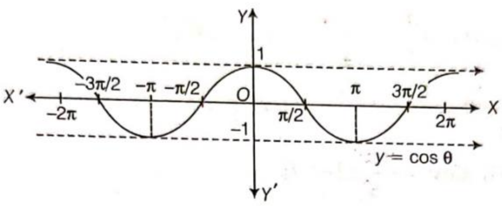
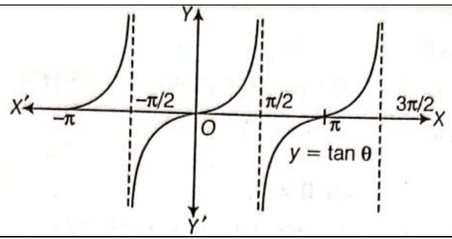
The relation between degree measure & radian measure of some common angles are given in the following table.

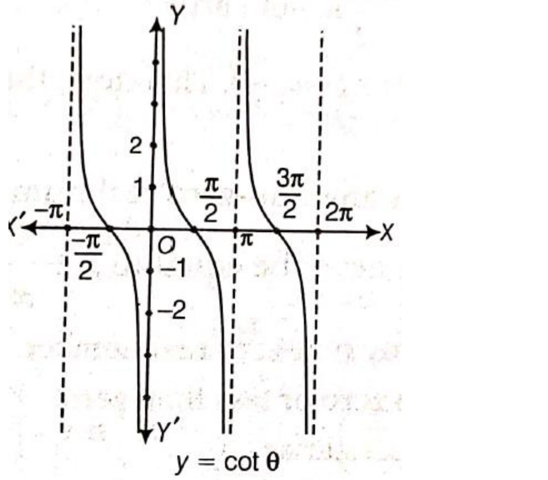
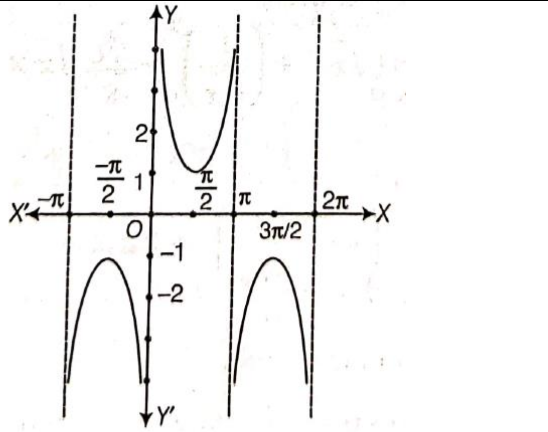
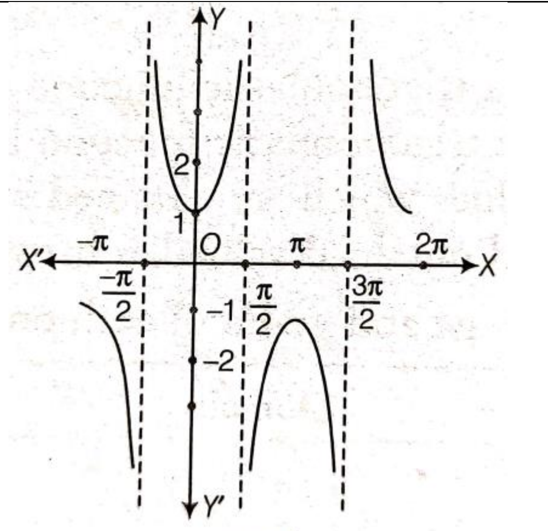
Degree	30^0	45^0	60^0	90^0	180^0	270^0	360^0
Radian	π /6	π /4	π /3	π /2	π	3 π /2	2 π

(iii) Each interior angle of a regular polygon of n sides is equal to $\frac{2n-4}{n}$ right angles.

T-ratios	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	n.d	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

<u>Domain & Range of the Trigonometric Functions.</u>

T Function	**Domain**	**Range**	**Graph**
sin θ	**R**	$[-1,1]$	
cos θ	**R**	$[-1,1]$	
tan θ	$\{x : x \in R \,\&\, x \neq (2n+1)\pi/2, n \in Z\}$	**R**	

cot θ	$\{x : x \in R \,\&\, x \neq n\pi, n \in Z\}$	**R**	 y = cot θ
cosec θ	$\{x : x \in R \,\&\, x \neq n\pi, n \in Z\}$	**R – (-1,1)**	 y = cosec θ
Sec θ	$\{x : x \in R \,\&\, x \neq (2n+1)\pi / 2, n \in Z\}$	**R – (-1,1)**	 y=sec θ

Formulae for T-ratios of Allied Angles:

All T-ratio changes in ($\frac{\pi}{2} \pm \theta$ and $\frac{3\pi}{2} \pm \theta$ while remains unchanged in $\pi \pm \theta$ and $2\pi \pm \theta$.

$\sin\left(\frac{\pi}{2} \pm \theta\right) = \cos\theta$ $\sin\left(\frac{3\pi}{2} \pm \theta\right) = = \cos\theta$ $\frac{\pi}{2}$

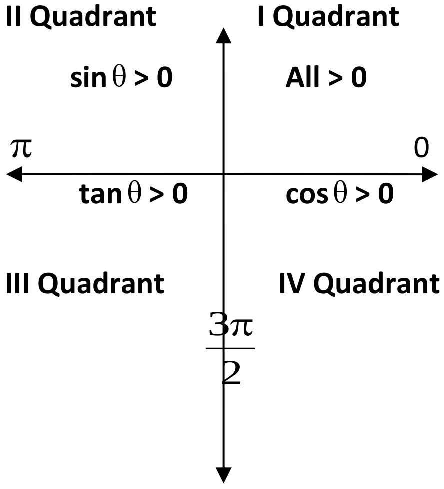
$\cos\left(\frac{\pi}{2} \pm \theta\right) = \mp\sin\theta$ $\cos\left(\frac{3\pi}{2} \pm \theta\right) = \pm\sin\theta$

$\tan\left(\frac{\pi}{2}\pm\theta\right)=\mp\cot\theta$ $\quad\quad$ $\tan\left(\frac{3\pi}{2}\pm\theta\right)=\mp\cot\theta$

$\sin(\pi\pm\theta)=\mp\sin\theta$ $\quad\quad$ $\sin(2\pi\pm\theta)=\pm\sin\theta$

$\cos(\pi\pm\theta)= = \cos\theta$ $\quad\quad$ $\cos(2\pi\pm\theta)= \cos\theta$

$\tan(\pi\pm\theta)= \pm\tan\theta$ $\quad\quad$ $\tan(2\pi\pm\theta)=\pm\tan\theta$



Sum and Difference formule:

Sin (A + B) = sin A cos B + cos A sin B

Sin (A – B) = sin A cos B – cos A sin B

Cos (A + B) = cos A cos B – sin A sin B

Cos (A – B) = cos A cos B + sin A sin B

tan(A + B) = $\frac{\tan A+\tan B}{1-\tan A\tan B}$, tan (A – B) = $\frac{\tan A-\tan B}{1+\tan A\tan B}$. tan $\left(\frac{\pi}{4}+A\right)=\frac{1+\tan A}{1-\tan A}$,

tan $\left(\frac{\pi}{4}-A\right)=\frac{1-\tan A}{1+\tan A}$ $\quad$ cot (A + B) = $\frac{\cot A.\cot B-1}{\cot B+\cot A}$ $\quad$ cot (A – B) = $\frac{\cot A.\cot B+1}{\cot B-\cot A}$

sin(A + B) sin(A – B) = $\sin^2A-\sin^2B = \cos^2B-\cos^2A$

cos(A + B) cos(A – B) = $\cos^2A-\sin^2B=\cos^2B-\sin^2A$

****Formulae for the transformation of a product of two circular functions into algebraic sum of two circular functions and vice-versa.**

2 sinA cos B = sin (A + B) + sin(A – B)

2 cosA sin B = sin (A + B) – sin(A – B)

2 cosA cos B = cos (A + B) + cos(A – B)

2 sinA sin B = cos (A - B) – cos(A + B)

sin C + sin D = 2 sin $\frac{C+D}{2}$ cos $\frac{C-D}{2}$,

sin C – sin D = 2 cos $\frac{C+D}{2}$ sin $\frac{C-D}{2}$.

cos C + cos D = 2 cos $\frac{C+D}{2}$ cos $\frac{C-D}{2}$,

cos C – cos D = – 2 sin $\frac{C+D}{2}$ sin $\frac{C-D}{2}$.

Formulae for T-ratios of multiple and sub-multiple angles :

sin 2A = 2 sin A cos A = $\frac{2\tan A}{1+\tan^2 A}$.

cos 2A = $\cos^2A-\sin^2A$ = $1-2\sin^2A = 2\cos^2A-1$ = $\frac{1-\tan^2 A}{1+\tan^2 A}$

1 + cos2A = $2\cos^2A$ $\quad\quad$ 1 – cos2A = $2\sin^2A$ $\quad\quad$ 1 + cosA = $2\cos^2\frac{A}{2}$

1 – cosA = $2\sin^2\frac{A}{2}$

tan 2A = $\frac{2\tan A}{1-\tan^2 A}$, tan 3A = $\frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$.

sin 3A = 3 sin A – 4 $\sin^3$A, cos 3 A = 4 $\cos^3$A – 3 cos A

sin15° = cos75° = $\frac{\sqrt{3}-1}{2\sqrt{2}}$. & cos15° = sin75° = $\frac{\sqrt{3}+1}{2\sqrt{2}}$,

tan 15° = cot 75° = $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ = 2 – $\sqrt{3}$ & tan 75° = cot 15° = $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ = 2 + $\sqrt{3}$

sin18° = $\frac{\sqrt{5}-1}{4}$ = cos 72° and cos 36° = $\frac{\sqrt{5}+1}{4}$ = sin 54°.

sin36° = $\frac{\sqrt{10-2\sqrt{5}}}{4}$ = cos 54° and cos 18° = $\frac{\sqrt{10+2\sqrt{5}}}{4}$ = sin 72°.

tan $\left(22\frac{1}{2}\right)^o$ = $\sqrt{2}$ – 1 = cot $67\frac{1}{2}^o$ and tan $\left(67\frac{1}{2}\right)^o$ = $\sqrt{2}$ + 1 = cot $\left(22\frac{1}{2}\right)^o$.

General solutions:

$*Sin\theta = 0 \Rightarrow \theta = n\pi,\ n \in Z$

$*\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2},\ n \in Z$

$*\tan\theta = 0 \Rightarrow \theta = n\pi,\ n \in Z$

$*\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha,\ n \in Z$

$*\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha,\ n \in Z$

$*\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha,\ n \in Z$

COMPETENCY BASED SOLVED EXEMPLAR QUESTIONS

MCQs

Examples:

1. The value of tan 1° tan 2° tan 3° … tan 89° is:

(a) 0 (b) 1 (c) ½ (d) Not defined

Correct option: (b) 1

Solution: tan 1° tan 2° tan 3° … tan 89°
= [tan 1° tan 2° … tan 44°] tan 45°[tan (90° – 44°) tan (90° – 43°)… tan (90° – 1°)]
= [tan 1° tan 2° … tan 44°] [cot 44° cot 43°……. cot 1°] × [tan 45°]
= [(tan 1° × cot 1°) (tan 2° × cot 2°) …..(tan 44° × cot 44°)] × [tan 45°]
We know that, tan A × cot A =1 and tan 45° = 1
Hence, the equation becomes as;
= 1 × 1 × 1 × 1 × …× 1= 1 {As 1^n = 1}

2. If α + β = π/4, then the value of (1 + tan α) (1 + tan β) is:

(a) 1 (b) 2 (c) – 2 (d) Not defined

Correct option: (b) 2

Solution: Given, α + β = π/4
Taking "tan" on both sides,
tan(α + β) = tan π/4

We know that, tan(A + B) = (tan A + tan B)/(1 – tan A tan B)
and tan π/4 = 1.
So, (tan α + tan β)/(1 – tan α tan β) = 1
tan α + tan β = 1 – tan α tan β
tan α + tan β + tan α tan β = 1….(i)
(1 + tan α)(1 + tan β) = 1 + tan α + tan β + tan α tan β
= 1 + 1 [From (i)] =2

3. Find the radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm (use π = 22/7).

Solution: Given, Length of the arc = l = 37.4 cm
Central angle = θ = 60° = 60π/180 radian = π/3 radians
We know that,r = l/θ
= (37.4) * (π / 3)= (37.4) / [22 / 7 * 3]= 35.7 cm

4. Find the value of √3 cosec 20° – sec 20°.

Solution: √3 cosec 20° – sec 20°

a. 1 b. 2 c. 3 d. 4

Solutions: $\frac{\sqrt{3}}{\sin 20^0}-\frac{1}{\cos 20^0}=\frac{\sqrt{3}\cos 20^0-\sin 20^0}{\sin 20^0.\cos 20^0}=4\left[\frac{\frac{\sqrt{3}}{2}\cos 20^0-\frac{1}{2}\sin 20^0}{2\sin 20^0.\cos 20^0}\right]$

$= 4\left[\frac{\sin 60^0\cos 20^0-\cos 60^0\sin 20^0}{\sin 40^0}\right]=4\left[\frac{\sin(60^0-20^0)}{\sin 40^0}\right]=4\times 1=4$

5. Find the value of sin75°.

$a.\frac{(\sqrt{3}+1)}{2\sqrt{2}}$ $b.\frac{(\sqrt{3}-1)}{2\sqrt{2}}$ $c.\frac{(\sqrt{3}+1)}{3\sqrt{2}}$ $d.\frac{(\sqrt{3}-1)}{3\sqrt{2}}$

Solutions: Sin(45+30)=sin45.cos30+cos45sin30

$=\frac{(\sqrt{3}+1)}{2\sqrt{2}}$

6. Find degree measure: $\frac{11}{16}$ radian

a. 40.4^0 b. 39.6^0 c. 39.1^0 d. 39.3^0

Solutions : $\frac{11}{16}\left(\frac{180}{\pi}\right)^0$ $=39.3^0$

7. A pendulum of length 14cm swings, find the length of arc described by the pendulum if angle made at the center is 60^0

a. 14.67cm b. 14.76cm c. 14.57cm d. 14.77cm

Solutions: Angle=arc/radius
Length of arc =14.67cm

8. Find the value of $\cos\left(-\frac{\pi}{3}\right)$

a. 1/2 b.-1/2 c. 0 d. 1

Ans: a. 1/2

9. The value of 2 sin75° sin15° is

(a) ½ (b) -1/2 (c) 1 (d) -1

Ans: (a) 1/2

10. If sin y + cosec y = 2, then $\sin^2 y + \text{cosec}^2 y$ is equal to -

(a) 1
(b) 4
(c) 2
(d) None of these

Ans: c) 2

11. The value of cos1° × cos2° × cos3° ………. cos179° is

(a) $\frac{1}{\sqrt{2}}$
(b) 0
(c) 1
(d) -1

Ans: (b) 0

ASSERATION/REASONING TYPE

For Q1 to Q5, a statement of assertion (A) is followed by a statement of reason (R). Choose thecorrect answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

1. Assertion (A): The ratio of the radii of two circles at the centers of which two equal arcs subtend angles of 30° and 70° is 21:10.
Reason (R): Number of radians in an angle subtended at the center of a circle by an arc is equal to the ratio of the length of the arc to the radius of the circle.
Ans: (d) A is false but R is true.

2. Assertion (A): cosec x is negative in third and fourth quadrants.
Reason (R): cot x decreases from 0 to $-\infty$ in first quadrant and increases from 0 to ∞ in third quadrant.
Ans: (c) A is true but R is false.

3. Assertion (A): $(\sin\theta + \cos\theta)^2 = \sin 2\theta$
Reason (R): $\sin^2\theta + \cos^2\theta = 1$
Ans: (a) Both A and R are true and R is the correct explanation of A.

4.Assertion (A): IfA>0,B>0 and $A + B = \pi/3$ then the maximum value of tanA tanB is 1/3
Reason (R): If $a_1 + a_2 + a_3 ..... + a_n = k(constant)\ then the value\ a_1 a_2 a_3 .... a_n\ is\ greatest\ when\ a_1 = a_2 = a_3 ...... = a_n$ Ans: (a) Both A and R are true and R is the correct explanation of A.

As $A + B = \frac{\pi}{3}$ so, $\tan(A + B) = \sqrt{3}$

Or, $\frac{\tan A + \tan B}{1 - \tan A \tan B} = \sqrt{3}$

Or, $\tan A \tan B = 1 - \frac{1}{\sqrt{3}}(\tan A + \tan B)$

$\tan A\ \tan B$ will be maximum if $\tan A + \tan B$ is minimum. But the minimum value of $\tan A + \tan B$ is obtained when $\tan A = \tan B$ or $A = B = \frac{\pi}{6}$

Hence the maximum value of $\tan A \tan B = \tan\frac{\pi}{6}\tan\frac{\pi}{6} = \frac{1}{3}$

5. Assertion (A): tanx+2 tan 2x+4 tan 4x+8 tan 8x+16 cot 16x = cot x

Reason (R): cot x –tan x = 2 cot 2x

Ans- Both A and R are true and R is the correct explanation of A.

LHS= $\tan x + 2\tan 2x + 4\tan 4x + 8\tan 8x + 16\cot 16x$

$= \cot x - (\cot x - \tan x) + 2\tan 2x + 4\tan 4x + 8\tan 8x + 16\cot 16x$

$= cotx - 2(\cot 2x - \tan 2x) + 4\tan 4x + 8\tan 8x + 16\cot 16x$

$= \cot x - 4(\cot 4x - \tan 4x) + 8\tan 8x + 16\cot 16x$

$= \cot x - 8(\cot 8x - \tan 8x) + 16\cot 16x)$

$= \cot x - 16\cot 16x + 16\cot 16x$

$= \cot x = \text{RHS}$

SHORT ANSWER TYPE

1. Solve: $\boldsymbol{tanx + sinx = 0}$

$$tanx + sinx = 0$$

$$sinx\left(1 + \frac{1}{cosx}\right) = 0$$

$$sinx = 0 \quad OR \;\; cosx = -1$$

$$x = n\pi \quad OR \quad x = (2n+1)\pi, \;\; n\epsilon Z$$

2. Show that : $\boldsymbol{\frac{Sin7x - Sin5x}{Cos7x + Cos5x} = tanx}$

$$LHS = \frac{Sin7x - Sin5x}{Cos7x + Cos5x} = \frac{2sin\left(\frac{7x-5x}{2}\right)\cos(\frac{7x+5x}{2})}{2cos\left(\frac{7x+5x}{2}\right)\cos(\frac{7x-5x}{2})}$$

$$= \frac{2\ sinx\ cos6x}{2\ cos6x\ cosx}$$

$$= tanx = RHS$$

3. Find the value of cosec (-1410°)

$$cosec(-1410°) = -cosec(1410°)$$

$$= -cosec(4 * 360 - 30)°$$

= cosec30°=2

4. Find the value of tan15°

$$\tan 15^{\circ} = \tan(45^{\circ} - 30^{\circ}) = \frac{\tan 45 - \tan 30}{1 + \tan 45\ \tan 30} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = 2 - \sqrt{3}$$

5. Find the values of tan ($\boldsymbol{\frac{19\pi}{3}}$)

=$\sqrt{3}$

6. If $\boldsymbol{tanx = \frac{3}{4}, \pi < x < \frac{3\pi}{2}}$, find the values of $\boldsymbol{cos\frac{x}{2}}$, tan $\boldsymbol{\frac{x}{2}}$d $\boldsymbol{sin\frac{x}{2}}$

$$tanx = \frac{3}{4}, \pi < x < \frac{3\pi}{2}$$

$$\frac{2tan\frac{x}{2}}{1 - tan^2\frac{x}{2}} = \frac{3}{4}$$

$$3tan^2\frac{x}{2} + 8tan\frac{x}{2} - 3 = 0$$

$$tan\frac{x}{2} = -3, \frac{1}{3}$$

$\pi < x < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

$tan\frac{x}{2} = -3$ is possible. Find the value of $cos\frac{x}{2}, sin\frac{x}{2}$

7. Prove that cos 4x = $1 - 8sin^2xcos^2x$

1 - 8 sin^2xcos^2x
$= 1 - 2(2\sin x.\cos x)^2$
$=1 - 2sin^2 2x$
= cos4x

8. A horse is tied to a post by a ripe. If the horse moves along a circular path always keeping the rope tight and describes 88 m when it has traced out 72^0 at the centre, find the length of the rope.

Ans-Apply the formula $l = r\theta$ & find the value of the arc, Ans =70cm

9. Show that cos 60^0 + cos 120^0 + cos 240^0 - sin 330^0 = 0

Ans:-Reduce the angles in to acute angles & find the value.

10. Show that: $\sqrt{2+\sqrt{2+2\, cos\, 4x}} = 2\cos x$

Ans: Apply the formula & prove.

CASE STUDIED BASED QUESTIONS

1. If the arcs of same length in two circles subtend angles 65 degrees and 110 degrees at the Centers, find the ratio of their radii.

Ans:- Apply the formula l=r θ , r1:r2=22:13

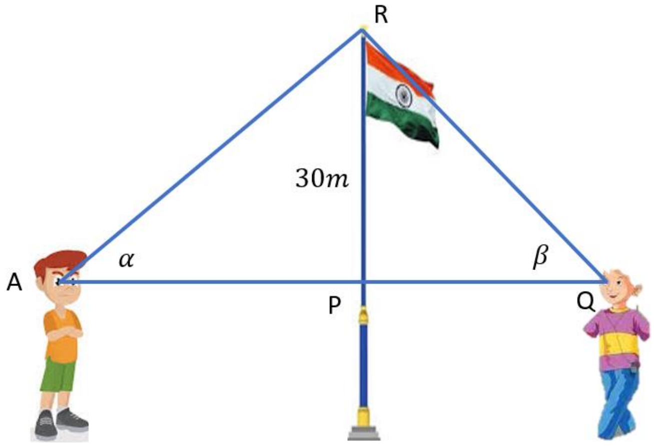
2.

During Durga puja, a mela was organized by the house owner's society Vikram Plaza apartment complexes. Ram Narayan, a 15-year-old boy visited the mela with his younger sister Radha. After enjoying different rides, they decided to ride Giant wheel. Both of them got seated after purchasing tickets from the vendor.
The Giant wheel was moving with a speed of 3km/hr and radius of the Giant wheel was 15m. After two complete rounds, his sister Radha felt vomiting sensation. So, she wanted to get down the wheel immediately. The operator stopped the wheel when she reached the exit point. Look at the figure and find out, how many seconds it takes for Radha to reach the exit point from her present position? ($\pi \approx 3.14$)

(a)31.4 sec (b) 37.68 sec (c) 62.8 sec (d) 45 sec

Ans: (b) 37.68 sec

3. Two students Sai Chandan & Sarada standing on either side of a flag post, observes its top which is 30m at the angles of elevation α & β respectively as shown in the figure. Both are $40\sqrt{3}$ m apart from each other and the distance between Sai Chandan and the flag post is $30\sqrt{3}$ m.



Find the measure of angle ARQ.

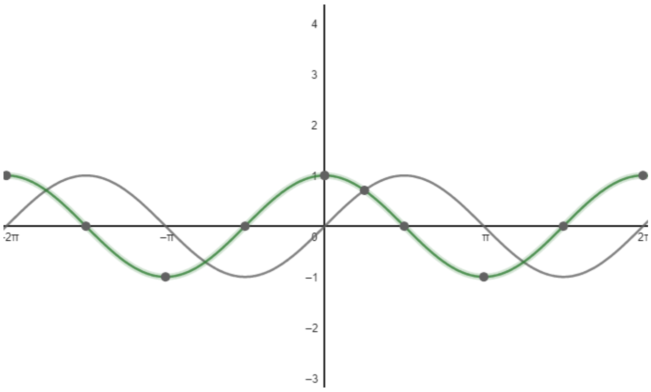
Ans:- $tan\alpha = \frac{1}{\sqrt{3}}$

⇨ $\alpha = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$

⇨ $tan\beta = \sqrt{3} = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

⇨ $angle\ ARQ = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$

4. In a classroom, teacher explains the properties of a particular curve by saying that this particular curve has beautiful ups and downs. It starts at 1 and heads down till π radian and then heads up again. The curve is closely related to sine function and both follow each other exactly $\frac{\pi}{2}$ radians apart as shown in the figure below.



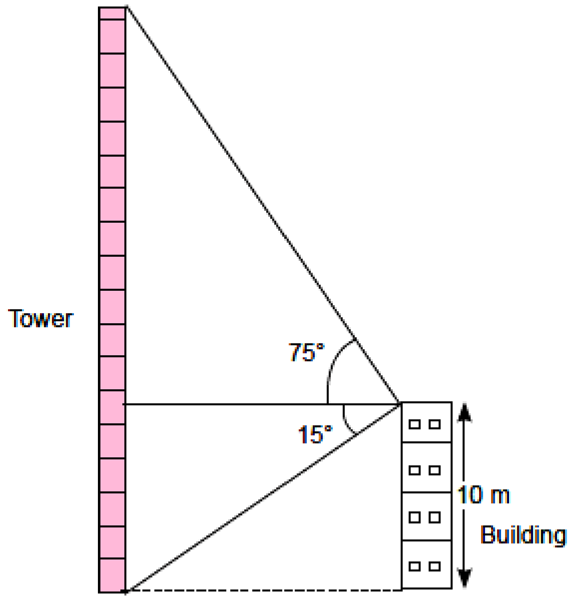
(i) Name the curve about which the teacher is explaining in the classroom.

Ans:- Cos x

ii) Find the domain range of cosine function.

Ans:- Domain R, range=$[-1,1]$

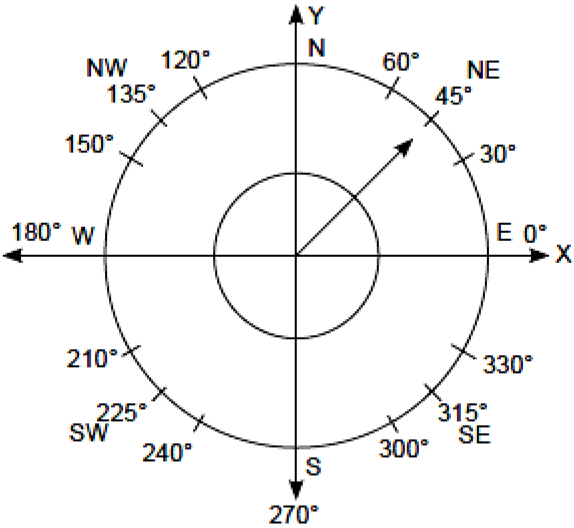
5. From the top of a tower of 10 m high building the angle of elevation of top of a tower is 75° and theangle of depression of foot of the tower is 15°. If the tower and building are on the same horizontal surfaces.



(i) Find the value of tan 15°.
(ii) Find the value of cos 75°.

Ans: (i) By the trigonometry formula, we know, **tan (A – B) = (tan A – tan B) /(1 + tan A tan B)**Therefore, we can write, tan(45 – 30)° = tan 45° – tan 30°/1+tan 45° tan 30°
Now putting the values of tan 45° and tan 30° from the table we get;
tan(45 – 30)° = (1 – 1/√3)/ (1 + 1.1/√3)
tan (15°) = √3 – 1/ √3 + 1
Hence, the value of tan (15°) is √3 – 1/√3 + 1.
(ii) Using the formula for cos(A + B) = cos(A)cos(B)- sin(A)sin(B) we can find the value of cos 75°.
cos(75°) = cos(30° + 45°)
Now, we know that cos 30° = √3/2, cos 45° = √2/2, sin 30° = 1/2, and sin 45° = √2/2 from the special values of trigonometric functions.
cos(75°) = cos(30°)cos(45°) sin(30°)sin(45°)Substituting the known values:
cos(75°) = (√3)/2)(√2)/2) – (1/2)(√2)/2)
Simplifying:
cos(75°) = (√6)/4) – (√2)/4)
Combining the terms:
cos(75°) = (√6 – √2))/4 = (√3-1) / 2√2

6. The below figure shows the compass. The East direction is along the positive X-axis (0° angle) and North direction is along the +ve Y-axis (90° angles). Initially the pointer is pointed towards North-East direction. Pointer is deflected in a magnetic field by some angle.



On the basis of above answer the following.

a. If pointer move in anticlockwise direction by an angle of 90°, then find the value of sine of angle made by pointer from East direction.

b. If pointer moves an angle of 165° from its initial position in anticlockwise direction, then find the value of cosine of angle made by pointer from East direction.

c. If the sine and cosine of angle made by pointer with East direction is $\frac{-1}{\sqrt{2}}$ then find where the pointer pointed?

d. How much angle will pointer move in anticlock wise direction if tangent of angle made by pointer with x-axis is – 1?

Ans: (i) Angle made by pointer with East direction = 45° + 90° = 135°

∴ sin 135° = sin (180° – 45°) = sin 45° = $1/\sqrt{2}$

(ii) Angle made by pointer with East direction = 45° + 165° = 210°

cos 210°= cos (180° + 30°) = – cos 30° = $-\frac{\sqrt{3}}{2}$

(iii) sine and cosine both are – ve so quadrant is III and we known that sin 45° =$1/\sqrt{2}$. Given that,

$$\sin\theta = \frac{-1}{\sqrt{2}}$$

$$\sin\theta = \sin(180^0 + 45^0)$$

⇒ θ = 225°

⇒ South West direction

(iv) If tan θ = – 1

θ = 135° or 315°

Initially the pointer is at 45°.So angle moved by pointer is

= 135° – 45° = 90°Or 315° – 45° = 270°

⇒ 90° or 270°

LONG ANSWER TYPE

1. **Prove that:** $\boldsymbol{sin^4\frac{\pi}{8}+sin^4\frac{3\pi}{8}+sin^4\frac{5\pi}{8}+sin^4\frac{7\pi}{8}=\frac{3}{2}}$.

Ans: $sin^4\frac{\pi}{8}=\left(sin^2\frac{\pi}{8}\right)^2=\left(\frac{1-cos\frac{\pi}{4}}{2}\right)^2=\left(\frac{1-\frac{1}{\sqrt{2}}}{2}\right)^2=\frac{3-\sqrt{2}}{8}$

$$sin^4\frac{3\pi}{8}=\frac{3+\sqrt{2}}{8}$$

$$sin^4\frac{5\pi}{8}=\frac{3+\sqrt{2}}{8}$$

$$sin^4\frac{7\pi}{8}=\frac{3-\sqrt{2}}{8} \qquad \Rightarrow sin^4\frac{\pi}{8}+sin^4\frac{3\pi}{8}+sin^4\frac{5\pi}{8}+sin^4\frac{7\pi}{8}=\frac{3}{2}$$

2. **Prove that** $\frac{\sin 8x\cos x-\sin 6x\cos 3x}{\cos 2x\cos x-\sin 4x\sin 3x}=\tan 2x$

Ans: LHS= $\frac{2\sin 8x\cos x-2\sin 6x\cos 3x}{2\cos 2x\cos x-2\sin 4x\sin 3x}=\frac{(\sin 9x+\sin 7x)-(\sin 9x+\sin 3x)}{(\cos 3x+\cos x)-(\cos x-\cos 7x)}$

$$=\frac{\sin 7x-\sin 3x}{\cos 3x+\cos 7x}=\frac{2\cos 5x\sin 2x}{2\cos 5x\ \cos 2x}=\tan 2x\ =RHS$$

3. **Prove that :** $\boldsymbol{\frac{sin\ 5x-\ 2\ sin\ 3x\ +\ sinx}{cos\ 5x-\ cosx}}$ **= tan x**

ANS- $\frac{sin\ 5x\ +\ sin\ x-2\ sin\ 3x}{cos\ 5x-\ cosx}$

$\frac{2\ sin\ 3x\ cos\ 2x\ -\ 2\ sin\ 3x}{-\ 2\ sin\ 3x\ sin\ 2x}$ = $\frac{\sin 3x(\cos 2x-1)}{-sin\ 3x\ sin\ 2x}$ =tanX=RHS

4. **If sin = 3/5 and cosy = -12/13, where x and y lie in second quadrant, then find the value of sin(x+y) and tan(x-y).**

Ans: cosx =-4/5 , siny = 5 /13
tanx = -3/4 , tany = -5/12
sin(x+y) = -56/65, tan(x-y) =-16/33

5. **Prove that:** $\boldsymbol{\cos 5A=16\ cos^5A-20\ cos^3A+5\cos A}$

$LHS.=cos(3A+2A)=cos\ 3A\,.\,cos\ 2A-sin\ 3A\,.\,sin\ 2A\,.$

$=4\ cos^3A-3\cos A)(2\ cos^2A-1)-(3\sin A-4\ sin^3A)(2\sin A\cos A)$

$=(4\ cos^3A-3\cos A)(2\ cos^2A-1)-(3-4\ sin^2A)(2\sin^2A\cos A)$

$=(4\ cos^3A-3\cos A)(2\ cos^2A-1)-[3-4\ (1-cos^2A)][2\ (1-\cos^2A)\cos A]$

$$=16\ cos^5A-20\ cos^3A+5\cos A$$

6.If α + β = π/4, then the value of (1 + tan α) (1 + tan β) is

Ans: Given, α + β = π/4
Taking “tan” on both sides,
tan(α + β) = tan π/4
We know that, tan(A + B) = (tan A + tan B)/(1 – tan A tan B)
and tan π/4 = 1.
So, (tan α + tan β)/(1 – tan α tan β) = 1
tan α + tan β = 1 – tan α tan β
tan α + tan β + tan α tan β = 1….(i)
(1 + tan α)(1 + tan β) = 1 + tan α + tan β + tan α tan β
= 1 + 1 [From (i)] =2

SELF PRACTICE QUESTIONS

MCQs

Examples:

1. The conversion of $40^0 20'$ into radians is:

a) π Radians b) $\frac{15}{9}$ π radians c) $\frac{121\pi}{540}$ radians d) none of these.

2. If sin= $\frac{\sqrt{3}}{2}$ and cosx= $-\frac{1}{2}$ then x lies in:

a) 1st quadrant b) 2^{nd}quadrant c) 3rdquadrant d) 4^{th}quadrant

3. Value of $\sec^2 x + \cos^2 x$ is:

a) < 0 $b)$ < 1 $c)$ ≥2 d) None of these.

4. If coty = $\frac{7}{24}$ and y lies in the third quadrant then value of cosy-siny is:

a. $\frac{17}{25}$ b. $\frac{16}{25}$ c. $\frac{14}{25}$ d. $\frac{13}{25}$

5. Value of $2\sin^2\frac{\pi}{6} + \mathrm{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$ is :

a. $\frac{1}{2}$ b. $-\frac{1}{2}$ c. 1 d. $\frac{3}{2}$

6. If sin = $\frac{1}{3}$ then value of sin3x is:

a. $\frac{23}{27}$ b. $\frac{-23}{27}$ c. $\sqrt{\frac{23}{27}}$ d. $\frac{4\sqrt{2}}{9}$

7. Value of tan 15^0 is:

a. 0 b. $\frac{1}{2}$ c. $\frac{1}{3}$ d. $2-\sqrt{3}$

8. The minimum value of cosx + sin is:

a. 0 b. 1 c. -1 d. $-\sqrt{2}$

9. Value of tan (-1575^0) is:

a. 1 b. $\frac{1}{2}$ c.0 d. -1

10. The greatest and least values of sin , cosx are respectively

a. 1, -1 b. $\frac{1}{2}, -\frac{1}{2}$ c. $\frac{1}{4}, -\frac{1}{4}$ d.2,-2

Answer key of practice questions

1. C	2. b	3. C	4. a	5. d
6. d	7. d	8. D	9. d	10 b

ASSERATION/REASONING TYPE

For Q1 to Q5, a statement of assertion (A) is followed by a statement of reason (R). Choose thecorrect answer out of the following choices.

a. Both A and R are true and R is the correct explanation of A.
b. Both A and R are true but R is not the correct explanation of A.
c. A is true but R is false.
d. A is false but R is true.

1. **Assertion (A):** Value of $\tan\left(-\frac{11\pi}{4}\right)$ is 1.

Reason (R): $\sin(3\pi+\theta) = -\sin\theta$.

2. **Assertion (A):** Radian measure for the angle 55^0 *is* $\frac{5\pi}{12}$

Reason (R): Radian measure = Degree measure × $\pi/180$

3. **Assertion (A):** The function sin x is negative in third &fourth quadrant.

Reason (R): The function sin is decreasing in the interval $0 \le x \le \pi/2$

4. **Assertion (A):** sin2>sin3

Reason (R): $If\ x, y \in (\pi/2, \pi), x < y, then \sin x > \sin y$

5. **Assertion (A):** The maximum value of $\sin\theta + \cos\theta\ is\ 2$.

Reason (R): The maximum value of $\sin\theta$ is 1 & that of $\cos\theta$ is also 1.

Answer Key

1. b
2. d
3. c
4. a
5. d

SHORT ANSWER TYPE

1. 1 If Cos θ = $-\frac{1}{2}$, π < θ < $\frac{3\pi}{2}$, Evaluate 4 tan ^{2}θ – 3 Cosec ^{2} θ.
2. Show that cos 60^0 + cos 120^0 + cos 240^0 - sin 330^0 = 0
3. Show that $\sqrt{2+\sqrt{2+2\cos 4x}} = 2\ Cosx$
4. Show that $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\left(\frac{x-y}{2}\right)$
5. Show that $\cos 2\theta.\cos\frac{\theta}{2} - \cos 3\theta.\cos\frac{9\theta}{2} = \sin 5\theta.\sin\frac{5\theta}{2}$
6. Show that $\frac{1+Sin\ 2x - Cos\ 2x}{1+Sin\ 2x + Cos\ 2x} = \tan\ x$
7. Show that Cos A. Cos (60 – A). Cos (60 + A) = $\frac{\cos 3A}{4}$
8. If Cos A + Cos B = $\frac{1}{2}$,Sin A + Sin B = $\frac{1}{4}$, Show that tan $\left(\frac{A+B}{2}\right)$ = $\frac{1}{2}$
9. Show that 2 sin^{2}β + 4 cos(α + β) sinα sinβ + cos2 (α + β) = cos2α.
10. Prove that $\sin^2 8x - \sin^2 3x = \sin 11x.\sin 5x$

Answer Key to Short Questions

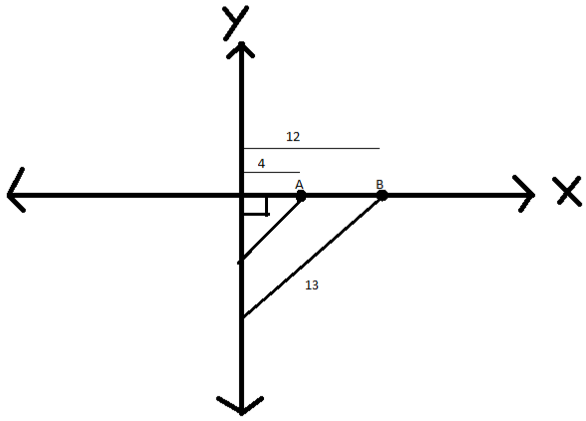
Q1. 11

CASE STUDY BASED QUESTIONS

1. Find in degree the angle through which a pendulum, swings if its length is 50 cm and the tip describes an arc of length 10 cm.

2. If the arcs of same length in two circles subtend angles 65 degrees and 110 degrees at the Centres, find the ratio of their radii.

3.Lalu constructs two right triangles in the fourth quadrants in such a way that the measure of triangle gives cos A = 4/5 and cos B=12/13, where $\frac{3\pi}{2} < A\ and\ B < 2\pi$



Based on above information, answer the following questions.

I)find the value of cos(A+B)

ii)Find the value of sin(A-B)

Iii)Find the value of tan(A+B)

4. The base of a pole of height 20m,the angle of elevation of the top of a tower is 60^0.The pole subtends an angle 30^0 at the top of the tower. Then find the height of the tower.

5. Two poles standing on a horizontal ground are of heights 5m & 10m respectively. The line joining their top makes an angle of 15^0 with ground. Then find the distance (in meter) between the poles.

Answer Key

1. $11\frac{5}{11}$,

2.22:13

3. 33/65,-16/65,-56/33

4. 30m

5. $5(2+\sqrt{3})\,m$

LONG ANSWER TYPE

1. Prove that: $\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) + \cos^2\left(x-\frac{\pi}{3}\right) = \frac{3}{2}$

2. If $\tan x = \frac{5}{12}$ and x lies in 2rd quadrant, find the value of $\sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$

Ans. $\sin\frac{x}{2} = \frac{5}{\sqrt{26}}$, $\cos\frac{x}{2} = \frac{1}{\sqrt{26}}$, $\tan\frac{x}{2} = 5$

3. Show that $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

4. Show that $\frac{(\cos x - \cos 3x)(\sin 8x + \sin 2x)}{(\sin 5x - \sin x)(\cos 4x - \cos 6x)} = 1$

5. Show that Cos20^0 Cos 40^0 Cos 60^0 Cos 80^0 = $\frac{1}{16}$.

Answer Key

2. $\sin\frac{x}{2} = \frac{5}{\sqrt{26}}$, $\cos\frac{x}{2} = \frac{1}{\sqrt{26}}$, $\tan\frac{x}{2} = 5$

COMPLEX NUMBERS AND QUADRATIC EQUATION

CONCEPTUAL NOTES:-

A number consisting of real number and imaginary number is called complex number. A complex number can be defined as a number of the form a+ib,where a and b are real numbers.

The symbol i is used to denote $\sqrt{-1}$ and it is called **iota.**

The complex number is generally denoted by z i.e z = $a + ib$.

In complex number z, a is called the real part and b is called the imaginary part of z and they are denoted as Re(z) and Im(z).

Integral Powers of i

i^1	i^2	i^3	i^4	i^5	i^6
i	-1	$-i$	1	i	-1

Equality of Complex Numbers

Two complex numbers $z_1 = a + ib$,$z_2 = c + id$ are said to be equal, if a = c and b = d.

Operations on Complex Numbers:-

(i) Let $z_1 = a + ib$, $z_2 = c + id$ be two complex numbers, then their addition is defined as

Z = $z_1 + z_2$ =(a+c) + i(b+d)

(ii) Let $z_1 = a + ib$,$z_2 = c + id$ be two complex numbers, then their subtraction is defined as

Z = $z_1 - z_2$ =(a-c) + i(b-d)

(iii) Let $z_1 = a + ib$,$z_2 = c + id$ be two complex numbers, then their multiplication is defined as

$z_1 z_2$ =(ac-bd) + i(ad+bc)

(iv) Let $z_1 = a + ib$, $z_2 = c + id$ be two complex numbers, then their division is defined as

$\frac{z_1}{z_2} = z_1 . z_2^{-1} = z_1 . (\frac{1}{z_2})$

Conjugate of a Complex Number:-

The conjugate of a complex number z, is the complex number, obtained by changing the sign of imaginary part of z. It is denoted by $\underline{z}$.

If z = $a + ib$,then $\underline{z} = a - ib$.

Modulus of Complex Numbers:-

The modulus or Absolute value of a complex number z = $a + ib$ is defined as the non-negative real number $\sqrt{a^2 + b^2}$.It is denoted by $|z| = \sqrt{a^2 + b^2}$.

MCQ QUESTIONS:-

1.If z = 1+i,then the value of z^2 is

(a) 1-i (b) i-1 (c) 2i (d) - 2i

2.If a and b are non-negative real numbers,then the value of $\sqrt{-a}\sqrt{-b}$ is (a)$\sqrt{ab}$ (b) $-\sqrt{ab}$ (c) $i\sqrt{ab}$ (d)$-i\sqrt{ab}$

3.If z = $\frac{1}{i}$,then the value of $|z|$ is

(a) 1 (b) -1 (c) 0 (d) 2

4.Which of the following is true ?

(a) 1-I$< 1+i$ (b) 2i+1$> -2i+1$ (c) 2i > 1 (d) None of these

5.If z= i^9 + i^{19},then z is equal to

(a) 0 + oi (b) 1 + oi (c) 0 + i (d) 0 + 2i

6.If x = $\sqrt{-16}$,then

(a) x = 4i (b) x = 4 (c) x = -4 (d) All of these

7.If z = 5$i(-\frac{3}{5}i)$,then z is equal to

(a) x = 0+ 3i (b) x = 3+ 0i (c) 0+ i (d) 1 + 2i

8.The value of i^{1947} + i^{1948}+ i^{1950} is

(a) 1 (b) -1 (c) 0 (d) i

9.The value of i^{-1097} is

(a) 1 (b) -1 (c) i (d) $-i$

10.If z_1 = 3 + 2 i and z_2 = 2- i ,then $\underline{z_1 + z_2}$ is equal to

(a) $\underline{z_1 z_2}$ (b) $\underline{z_1} + \underline{z_2}$ (c) $\underline{z_1} z_2$ (d) $\frac{z_1}{z_2}$

ASSERTION-REASON BASED QUESTIONS:-

Directions(Q.Nos.11 to 15) In the questions given below are two statements labeled as Assertion(A) and Reason(R) .In the context of the two statements ,which one of the following is correct?

(a)Both A and R are correct ;R is correct explanation of A.

(b)Both A and B are correct ;R is not the correct explanation of A.

(c)A is correct ;R is incorrect

(d)R is correct ;A is incorrect

11 .Assertion(A) If x and y are real numbers and $\frac{(1+i)x-2i}{3+i}$ +$\frac{(2-3i)y+i}{3-i}$ = i,then x=3 and y= -1.

Reason(A) If 2a+3b and $2i$ represent the same complex number,then a = 0, b = $\frac{1}{2}$.

12. Assertion(A) If z is a complex number, then ($\underline{z}^{-1}$)($\underline{z}$) is equal to 4.

Reason(A) The region of the complex plane for which $\left|\frac{z-a}{z+a}\right|$ = 1 $[Re(a) \neq 0]$ is Y-axis.

13. Assertion(A) If $\sqrt{a+ib}$ = x +iy,then $\sqrt{a-ib}$ = x – iy.

Reason(A) A complex number z is said to be purely imaginary. if Re(z) = 0.

14. Assertion(A) The modulus of the complex number z = 2 + 3i is 4.

Reason(A) The arg z = $\frac{\pi}{3}$,Hence the polar form of the complex number 2 + $3i$ is z = 4(cos$\frac{\pi}{3}$ + $i$sin$\frac{\pi}{3}$).

15. Assertion(A) The roots of the quadratic equation x^2-x-6 = 0 are 3 and -2.

Reason(A) The roots are real and distinct as b^2-4ac > 0.

SHORT ANSWER TYPE QUESTIONS:-

1.Find the real value of a for which $3i^3$-2ai^2+(1-a)i +5 is real.

2.For what value of a and b are (1-i)$a + (1 + i)b$ and $1 - 3i$ $equal$?

3.Express in the form of a+ib: $\frac{5+\sqrt{2i}}{1-\sqrt{2i}}$.

4.Express in the form of $a + ib$:

$i^{75} + i^{80} + i^{85}+i^{90}$

5.Evaluate $(1 + i))^6 + (1 - i))^3$.

6.Find the multiplicative inverse of $\sqrt{5} + 3i$.

7.If $a + ib = \left(\frac{1-i}{1+i}\right)^{100}$,then find (a,b).

8.Find the conjugate of $\frac{2-i}{(1-2i)^2}$?

9.Find the modulus of $\frac{1+i}{1-i}$ - $\frac{1-i}{1+i}$.

10.Find the value of $\left|\frac{(1+i)(2+i)}{3+i}\right|$.

CASE STUDY BASED QUESTIONS:-

1.Consider z = $a + ib$ and $\bar{z} = a - ib$,where a and b are real numbers are conjugate of each other.Based on the above information,answer the following questions.

(i) $z_1 = 3 + 2i$, $z_2 = 2 - i$,then $\overline{z_1 + z_2}$ is equal to

(a) $5 - 3i$ (b) $5 - i$ (c) 0 (d) 2

(ii)The conjugate of $(6 + 5i)^2$ is

(a) 11-60i (b) 11+60i (c) $-11 - 60i$ (d) $-11 + 60i$

(iii)If the complex number -3+ $i(x^2$y) and x^2+y+4i,where x and y are real,are conjugate to each other,then the number of ordered pairs(x,y) is

(a) 1 (b) 2 (c) 3 (d) 4

(iv) If $z_1 = x^2 - 7x - 9yi$ $z_2 = y^2 i + 20i - 12$,then the number of ordered pairs(x,y) is

(a) 1 (b) 2 (c) 3 (d) 4

2.A complex number z is purely real if and only if $\bar{z} = z$ and purely imaginary if and only if $\bar{z} = -z$Based on the above information,answer the following questions.

(i) If x and y are real numbers and the complex number $\frac{(2+i)x-i}{4+i}$ + $\frac{(1-i)x+2i}{4i}$ is purely real,the relation between x and y is

(a) 8x +7y = 15   (b) 8x- 17y = 16   (c) 17x – 8y = 15   (d) 17x +8y = 15

(ii)If z be a complex number satisfying $|Re(z)|$+$|Im(z)|$ = 4,then $|z|$ cannot be

(a) $\sqrt{7}$ (b) $\sqrt{\frac{17}{2}}$ (c) $\sqrt{10}$ (d) $\sqrt{8}$

(iii)The smallest positive integer(n) for which $(1+i)^{2n}$ = $(1-i)^{2n}$ is

(a) 2   (b) 3   (c) 1   (d) 4

3.Two complex numbers z_1 = $a+ib$ z_2 = $c+id$ are said to be equal,if a = c and b = d. In other words they have same real parts and imaginary parts. Based on the above information ,answer the following questions.

(i) The real part of the complex number $2i+3i^5$ – $6i^2$.

(a) 2   (b) 3   (c) 5   (d) 6

(ii) If 4x +$i(3x-y) = 3+i(-6)$,where x and y are real numbers ,then the values of x is

(a) $\frac{3}{4}$ (b) $\frac{33}{4}$ (c) 3   (d) $\frac{3}{2}$

(iii)For what values of a and b are $(1-i)a$ *and* $(1+i)b$ *and* $1-3i$ equal?

(a) (2,-1)   (b) (-1,-1)   (c) (2,2)   (d) $(3,-1)$

4.Given two complex numbers z_1 = $1+i$, z_2 = $2-3i$. Based on the above information ,answer the following questions.

(i) The conjugate of $z_1 + z_2$ is

(a) 2+3i (b) 2-3i (c) $3+2i$ (d) $3-2i$

(ii)The modulus of $z_1 - z_2$ is

(a) 15   (b) $\sqrt{15}$ (c) 17   (d) $\sqrt{17}$

(iii)The multiplicative inverse of z_1 is

(a) $\frac{1+i}{2}$ (b) $\frac{1-i}{2}$ (c) $1+i$ (d) $1-i$

5.The following complex numbers are given.

$z_1 = -2i, z_2 = \sqrt{3}+i$ and $z_3 = a+ib$, where a, bϵR. Based on the above information , answer the following questions.

(i) If $\left|\underline{z_1 z_3}\right|$ = 16,find the modulus of z_3

(a) $3\sqrt{2}$ (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) $4\sqrt{3}$

(ii) Given that arg($\frac{z_3}{z_2}$) = $\frac{7\pi}{12}$,determine the argument of z_3

(a) $\frac{3\pi}{4}$ (b) $\frac{3\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{4}$

(iii)The values of and b are

(a) (-4,4)   (b) $(-4,2)$ (c) $(2,4)$ (d) $(-2,2)$

LONG ANSWER TYPE QUESTIONS

1.What is the smallest positive integer n, for which $(1+i)^{2n} = (1-i)^{2n}$?

2.Find the value of z^3-2z^2+3z-4,when $z=1+i$

3.If $z = x+iy$,then prove that $|x|+|y| \leq \sqrt{2|z|}$.

4.Solve for real numbers x and y $(1+i)x^2 + (6+i) = (2+i)y$.

5.Find the value of x^2+y^2 if $x+iy = (1+i)(2+i)(3+i)$.

ANSWER KEY

MCQ QUESTIONS:-

1.(c)

2.(b)

3.(a)

4.(d)

5.(a)

6.(a)

7.(b)

8.(c)

9(d)

10.(b)

ASSERTION-REASON BASED QUESTIONS:-

11.(b)

12.(d)

13.(b)

14.(d)

15.(a)

SHORT ANSWER TYPE QUESTIONS:-

1. a= -2

2.a=2,b=-1

3.$1+2\sqrt{2i}$

4.0

5.$-2-10i$

6.$\frac{\sqrt{5}}{14} - \frac{3}{14}i$

7.(a,b) = (1,0)

8.$\frac{1}{25}$(-2-11i)

9.2

10.2

CASE STUDY BASED QUESTIONS:-

1.(i) b

(ii) a

(iii) b

(iv) d

2.(i) b

(ii) a

(iii) a

3.(i) d

(ii) a

(iii) a

4.(i) c

(ii) d

(iii) b

5.(i) c

(ii) d

(iii) a

LONG ANSWER TYPE QUESTIONS

1. 2

2. -3 +i

3. To prove

4. x = ±2,y = 5

5.100

LINEAR INEQUALITIES

Inequation

A statement involving variables and the sign of inequality viz. >, <, ≥ or ≤ is called an inequation or an inequality.

Numerical Inequalities

Inequalities which do not contain any variable is called numerical inequalities, e.g. 3 < 7, 2 ≥ -1, etc. Literal Inequalities Inequalities which contains variables are called literal inequalities e.g. x – y > 0, x > 5, etc.

Linear Inequation of One Variable

Let a be non-zero real number and x be a variable. Then, inequalities of the form ax + b > 0, ax + b < 0, ax + b ≥ 0 and ax + b ≤ 0 are known as linear inequalities in one variable.

Linear Inequation of Two Variables

Let a, b be non-zero real numbers and x, y be variables. Then, inequation of the form ax + by < c, ax + by > c, ax + by ≤ c and ax + by ≥ c are known as linear inequalities in two variables x and y.

Solution of an Inequality

The value(s) of the variable(s) which makes the inequality a true statement is called its solutions. The set of all solutions of an inequality is called the solution set of the inequality.

Solving Linear Inequations in One Variable

Same number may be added (or subtracted) to both sides of an inequation without changing the sign of inequality.

Both sides of an inequation can be multiplied (or divided) by the same positive real number without changing the sign of inequality.

However, the sign of inequality is reversed when both sides of an inequation are multiplied or divided by a negative number.

Representation of Solution of Linear Inequality in One Variable on a Number Line

To represent the solution of a linear inequality in one variable on a number line. We use the following algorithm.

If the inequality involves „>" or „ we draw an open circle (O) on the number line, which indicates that the number corresponding to the open circle is not included in the solution set

If the inequality involves „≥" or „≤" we draw a dark circle (•) on the number line, which indicates the number corresponding to the dark circle is included in the solution set

MCQ Questions.

1. Mr X has 6 more marks than Mr Y. If they have more than 100 marks. What is the minimum number of marks does Mr Y have?

(a) 46 (b) 47 (c) 48 (d) 49

2. If $|x-2|x-2 \geq 0$, then

a) $x \in [2, \infty)$ (b) $x \in (2, \infty)$ (c) $x \in (-\infty, 2)$ (d) $x \in (-\infty, 2]$

3. The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160 cm, then

(a) breadth > 20 cm (b) length < 20 cm (c) breadth x ≥ 20 cm (d) length ≤ 20 cm

4. If $I\ x + 3I \geq 10$, then

(a) $x \in (-13, 7]$ (b) $x \in (-13, 7]$ (c) $x \in (-\infty, -13] \cup [7, \infty)$ (d) $x \in (-\infty, -13] \cup [7, \infty)$

5. In a garden there are two types of plants Rose & Jasmine with maximum 500 plants.
The ratio of Rose and Jasmine plants are in the ratio 2 : 3. What is the maximum number of Rose plants?

(a) 200 (b) 300 (c) 350 (d) 400

6. A furniture dealer deals in only two items–tables and chairs. He has Rs60,000 to invest and has storage space of at most 100 pieces. A table costs Rs3000 and a chair Rs1000.
Let x be the number of tables and y be the number of chairs that the dealer buys. Which of the following represents the investment constraint/ inequality.

(a) $3000x + 1000y \leq 60000$(b) $3000x + 1000y < 60000$

(c) $3000x + 1000y \geq 60000$(d) $3000x + 1000y > 60000$

7. If $x < 5$, then

(a) $-x < -5$ (b) $-x \leq -5$ (c) $-x > -5$ (d) $-x \geq -5$

8. Given that x, y and b are real numbers and $x < y$, $b < 0$, then

(a) $x/b < y/b$ (b) $x/b \leq y/b$ (c) $x/b > y/b$ (d) $x/b \geq y/b$

9. If $-3x + 17 < -13$, then

(a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$ (c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10)$

10. If x is a real number and $|x| < 3$, then

(a) $x \geq 3$ (b) $-3 < x < 3$ (c) $x \leq -3$ (d) $-3 \leq x \leq 3$

Assertion Reason Questions

Q1.Assertion (A): The minimum value of $2^{sin^2x} + 2^{cos^2x}$ is $\sqrt{2}$ where $x \in R$
Reason (R): A.M $\geq G.M$

1)Both A and R are true and R is the correct explanation of A.

2)Both A and R are true but R is not the correct explanation of A.

3)A is true but R is false.

4)A is false but R is true. Which one is correct

a) 1 (b) 2 (c)3 (d) 4

Q2. Assertion (A) : The inequality $ax + by < 0$ is strict inequality

. Reason(R) : The inequality $ax + b \geq 0$ is slack inequality

1)Both A and R are true and R is the correct explanation of A.

2)Both A and R are true but R is not the correct explanation of A.

3)A is true but R is false.

4)A is false but R is true. Which one is correct

a) 1 (b) 2 (c)3 (d) 4

Q3. Assertion(A) : If $a < b$, $c < 0$, then $a/c < b/c$

Reason(R) : If both sides are divided by the same negative quantity, then the inequality is reversed.

1)Both A and R are true and R is the correct explanation of A.

2)Both A and R are true but R is not the correct explanation of A.

3)A is true but R is false.

4)A is false but R is true. Which one is correct

a) 1 (b) 2 (c)3 (d) 4

Q4. Assertion: The inequality $3x + 2y \geq 5$ is the linear inequality.

Reason : The solution of $5x - 3 < 7$, when x is a real number, is $(-\infty, 2)$

1)Both A and R are true and R is the correct explanation of A.

2)Both A and R are true but R is not the correct explanation of A.

3)A is true but R is false.

4)A is false but R is true. Which one is correct

a) 1 (b) 2 (c)3 (d) 4

Q5. Assertion : Two real numbers or two algebraic expressions related by the symbol $<$, $>$, $\geq$, $\leq$ forms an inequality.

Reason : The inequality $ax + by < 0$ is strict inequality.

1)Both A and R are true and R is the correct explanation of A.

2)Both A and R are true but R is not the correct explanation of A.

3)A is true but R is false.

4)A is false but R is true. Which one is correct

a) 1 (b) 2 (c)3 (d) 4

Short type questions.

Q1. Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

Q2. Abhaya obtained 70 and 75 marks in first two unit tests. Find the number if minimum marks he should get in the third unit test to have an average of at least 60 marks.

Q3. The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 320 cm, then what is the minimum breadth?

Q4. If Kavita's marks in first four examinations are 87, 92, 94 and 95, find the minimum mark in the fifth examination to get grade 'A' in the course.

Q5. State which of the following statements is True or False

(i) If | x| ≤ 4, then x ∈ [– 4, 4] (T/F)

(ii) If | x| > 5, then x ∈ (– ∞, – 5) ∪ [5, ∞) (T/F)

Q6. . Solve 4x+32x-5 < 6 and show the solution on the number line.

Q7. Find the solution set of inequalitiesx2x+1>14.

Q8. . Find all pairs of consecutive odd positive integers, both of which are smaller than 18, such that their sum is more than 20.

Q9. The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 7.2 and 7.8. If the first two pH readings are 7.48 and 7.85, find the range of pH value for the third reading that will result in the acidity level being normal.

Q10. Jitendra, Mahendra and Dharmendra together invest money into a new business. Mahendra gives twice of Jitendra and Dharmendra gives 20000 more than Mahendra. Find the minimum amount of money invested by Mahendra if at least 1 crore needed to start business.

Case based questions.

1. Shweta was teaching "method to solve a linear inequality in one variable" to her daughter.

StepI: Collect all terms involving the variable (x) on one side and constant terms on other side with me help of above rules and then reduce it in the form ax<b or ax$ax \leq b$ or $ax > b$ or $ax \geq b$.

Step II: Divide this inequality by the coefficient of variable (x).This gives the solution set of given inequality.

Step III: Write the solution set.

Based on the above information ,answer the following questions.

I) Find the solution set of 24x<100 , when x is a natural number is?

ii) Find the solution set of -5x+25>0 where x ∈ R

Q2. Aayansh works in a chemical factory .He regularly needs to keep some chemical at different temperatures for storage. Suppose boiling point of one of the chemical is less than 144°F.He knows the relation between Fahrenheit temperature and Celsius temperature as C5=F-325

a)Form the linear inequality from this situation.

b)He needs to keep a solution between 40°C to50°C What could be the range of the temperature in Fahrenheit?

c)He wants to dilute a 500 litres acid solution of 30% concentration by adding water to it . What could be the range of water to be added if he wants acid in solution to keep between 20% and 25% ?

Q3. Mitu is a psychology students and now a days she is learning about intelligence Quotient. She know the result IQ= Mental age/ chronological age× 100

On the basis of the above case answer the following questions

a) What could be the range of mental age if a group of children with chronological age of 15 years have the IQ range as $90 \leq IQ \leq 150$?

b) What could be the range of IQ if a group of children with age of 12 years have the mental age range as $9 \leq MA \leq 15$?

Q4. A company produced cassettes, one cassette Cost Company Rs. 30 and also an additional fixed cost 26000 per week. The company sold each Cassette at Rs. 43. If x is number of cassettes produced and sold by the company in a week. From the following information find

a. The cost function of the company (a) 26000 + 30x (b) 26000 + 43x (c) 30 + 26000x (d) 43 + 26000x

b. The revenue function of the company is

(a) 30x (b) 26000x (c) 43x (d) 13x

c. How many cassettes must be produced by the company in a week to realize some profit?

(a) more than 2000 (b) less than 2000 (c) more than 5000 (d) less than 5000

d. If company incurred an additional cost of Rs. 3 on each cassette per week. How many cassettes must be produced by the company in a week so that there is no profit no loss?

(a) 2000 (b) 5000 (c) 2600 (d) 1000

Q5. Sweta was teaching "method to solve linear inequality in one variable" to her daughter.

Step-I:collect al terms involving the variable (x) on one side and constants on another side with the help of above rules and then reduce it in the form of $ax< b$ or $ax\leq b$ or $ax > b$ or $ax \geq b$.

Step-II: divide this inequality by the coefficient of variable (x). this gives the solution set of given inequality.

Step-III: write the solution set.

Based on the above information, answer the following questions.

i) The solution set of $24x< 100$, when x is a natural number is

a) {1,2,3,4} b) (1,4) c) [1,4] d) none of these

ii) The solution set of 24x< 100 , when x is an integer is

a) {…….-2,-1,0,1,2…….} b) (-∞,4] c) [4,∞) d) none of these

5 long answer type questions

Q1. In drilling world's deepest hole it was found that the temperature T in degree Celsius, x km below the earth's surface was given by T = 30 + 25(x-3), 3 ≤ x ≤ 15. At what depth will the temperature be between 155° C and 205°C.

Q2. Suppose we have been given 200 litres of 10% concentrated H2 SO4 acid. How many litres of 30% conc. H2 SO4 acid should be added so that the concentration of the resulting acid should be more than 15% but less than 20%?

Q3. In an experiment a thermometer only Fahrenheit measurements are shown . Using it a solution of hydrochloric acid is measured which is to be kept between 300 and 350 Celsius. What is the range of temperature in degree Fahrenheit if conversion formula is given by C = 59(F-32) Where C and F represent and degree Celsius and degree Fahrenheit respectively.

Q4. The longest side of a triangle is 3 times the shortest side, and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side.

Q5.A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid must be added to it so that the acid content in the resulting mixture will be more than 15% but less than 18%?

SOLUTION

MCQ

Q1.(c) 48

x + x + 6 > 100 ⇒ 2x > 94 ⇒ x > 47

⇒ Minimum value of x is 48.

Q2. (b) x ∈ (2, ∞)

Q3. (c) 2(3x+x)≥160 => x ≥ 20

Q4. Since x +3 ≥ 10, ⇒ x + 3 ≤ – 10 or x + 3 ≥ 10⇒ x ≤ – 13 or x ≥ 7 ⇒ x ∈ (– ∞, – 13] ∪ [7, ∞)

Q5. 2x + 3x ≤ 500 ⇒ x ≤ 100 ⇒ 2x ≤ 200

Q6. 3000x + 1000y ≤ 60000

Q7. – x > – 5

Q8. $x/b > y/b$

Q9. $x \in (10, \infty)$

Q10. $-3 < x < 3$

Assertion Reason Questions

Q1. (d) 4

Q2. (b) 2

Q3. (d) 4

Q4. (b) 2

Q5. (b) 2

Short answer type questions

Q1. Let x be the smaller of the two positive consecutive even integers, then the other number is(x + 2)

Given $x > 5$ and $x + x + 2 < 23$

$\Rightarrow 2x + 2 < 23$ or $x < 10.5$

Value of x may be 6 , 8 , 10 (even integers)

The pairs may be (6, 8), (8, 10), (10, 12)

Q2. $(70 + 75 + x)/3 \geq 60$

or $145 + x \geq 180$

or $x \geq 35$

Q3.Let breadth be b, so its length $l = 3b$

Thus $2(l + b) \geq 320$

or $2(3b + b) \geq 320$

or $8b \geq 320$

or $b \geq 40$

Therefore, minimum breadth is 40 cm.

Q4. $(87 + 92 + 94 + 95 + x)/5 >= 90$

or $368 + x >= 450$

or $x >= 82$

Kavita should obtain at least 82 marks in the fifth examination.

Q5. (i) T

(ii) F

Q6. $(-\infty,52) \cup (338,\infty)$

Q7. No solution.

Q8. (11,13 , 13,15) and (15,17) .

Q9. Between 6.27 and 8.07.

Q10. Let investment of Jitendra be x.

Therefore, investment of Mahendra is 2x and investment of Dharmendra is 2x +20000.

ATQ,

$\Rightarrow X+2x+2x+20000 \geq 10000000$

$\Rightarrow 5x + 20000 \geq 10000000$

$\Rightarrow 5x \geq 9980000$

$\Rightarrow x \geq 9980000/5$

$\Rightarrow x \geq 1996000$

Hence investment of Mahendra is $2 \times 1996000 = 3992000$

Case based questions

Q1. a) 9.8km& 13.8km

b) $x \leq -1913$

c) (-5,5)

d) d

Q2. a) $x < 144°F$

b) $72 < F < 82$

c) $100 < x < 250$

Q3.a) $90 \leq IQ \leq 150$

=>90 ≤MA15X100≤150

=>90X15100≤MA≤150X15100

=>272≤MA≤452

b) $9 \leq MA \leq 15$

=> $75 \leq IQ \leq 125$

Q4. (a) 26000 + 30x

b) 43x

c) more than 2000

d) 2600

Q5. a) {1,2,3,4}

b) {-3,-2,-1,0,1,2,3}

Long answer type

Q1. $T = 30 + 25(x-3)$, $3 \le x \le 15$

=> $155 < T < 205$

=> $155 < 30 + 25(x-3) < 205$

=>.............

=> $8 < x < 10$

Hence, at the depth 8 to 10 km temperature lies between 155° C and 205°C.

Q2. Let x liters of 30% conc. H_2SO_4acid solution is required to be added. Then Total mixture = (x + 200) litres. Therefore 30% x + 10% of 200 > 15% of (x + 200) and 30% x + 10% of 200 < 20% of (x + 200)

....

or

$200/3 < x < 200$

Q3. $30 < 59(F-32) < 35$

=> $54 < F-32 < 63$

=> $86 < F < 95$

Q4. Let x cm be the length of the shortest side of the triangle.

∴ According to the question, length of the longest side = 3x cm

Length of the third side = (3x – 2) cm

The least perimeter of the triangle = 61 cm (given)

Thus, $x + 3x + (3x – 2)$ cm ≥ 61 cm

= > $7x – 2 \ge 61$

=> $7x \ge 63$

Dividing by 7 on both sides, we get;

=> $7x/7 \ge 63/7$

=> $x \ge 9$

Hence, the minimum length of the shortest side will be 9 cm.

Q5. Let x litres of 30% acid solution be required to be added.

Total mixture = (x + 600) litres

Thus, 30% x + 12% of 600 > 15% of (x + 600)

and

30% x + 12% of 600 < 18% of (x + 600)

⇒ $(30x/100) + (12/100) \times (600) > (15/100)(x + 600)$

And

$(30x/100) + (12/100) \times (600) < (18/100)(x + 600)$

$\Rightarrow 30x + 7200 > 15x + 9000$ and $30x + 7200 < 18x + 10800$

$\Rightarrow 15x > 1800$ and $12x < 3600$

$\Rightarrow x > 120$ and $x < 300$,

i.e., $120 < x < 300$

Hence, the number of litres of the 30% acid solution will have to be more than 120 litres but less than 300 litres.

SELF PRACTICE QUESTIONS

1. Solve $5x < 24$ when $x \in N$.
2. Solve $3x < 11$ when $x \in \mathbb{Z}$.
3. Solve $3 - 2x < 9$ when $x \in \mathbb{R}$.
4. Solve: $\frac{1}{x-4} \leq 0$ .when $X \in$ R
5. Solve: $0 < \frac{-x}{5} < 1$ .when $X \in$ R

Write the solutions in the form of intervals

6 $-3 \leq -3x + 2 < 5$.

7. $2(2x+3) - 10 \leq 6(x-2)$.

8. Solve: $3x - 5 < x + 1$. Show the solution on the number line.

9.Solve : $5x - 3 \geq 3x - 5$. Show the solution on the number line.

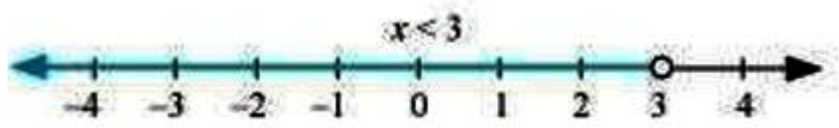
10.If $1/x<0.5$ then find solution set of x, where x is a real number.

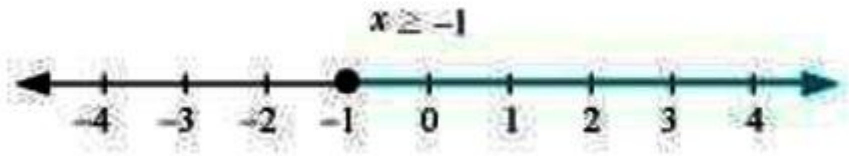
ANSWER

1. $\{1, 2, 3, 4\}$.
2. $\{....., -2, -1, 0, 1, 2, 3\}$.
3. $x > -3$.
4. $x < 4$.
5. $-5 < x < 0$.

6. $\left[1,\frac{5}{3}\right]$

7. $\left(\frac{1}{5},\infty\right)$

8. 

9. 

10. (−∞,0) U (2,∞)

Q1.In the following questions, a statement of Assertion(A)is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d)(A) is false but(R) is true.

. Assertion (A): The solution set of the inequality x-3<2 ,x ∈N is {1,2,3,4,5,6,7,8}.

Reason (R) :Solution set of a inequality in x is set of values of x satisfying the inequality .

Answer :(d)

Q2.In the following questions ,a statement of Assertion(A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but(R) is false.

(d) (A) is false but(R) is true.

Assertion(A) : For x ∈**R**, x<-3 then -5x >15

Reason (R): when both sides are multiplied (or divided) by the same negative number then the sign of inequality reverse.

Answer: (a)

Q3. In the following questions ,a statement of Assertion(A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices.

(a) Both (A) and(R) are true and (R) is the correct explanation of (A).

(b) Both(A)and(R)are true but (R)is not the correct explanation of (A).

(c) (A) is true but(R) is false.

(d) (A) is false but(R) is true.

Assertion(A) : For $x \in R$, and $x < 2$, then $x-2 < 0$

Reason (R): A number can be added or subtracted from both side of inequality without changing the sign of inequality.

Answer: (a)

Q1. Ravi scored 70 and 75 marks in the first two-unit test. Calculate the minimum marks he should get in the third test to have an average of at least 60 marks.

Ans: Assume that x be the marks obtained by Ravi in the third unit test

It is given that the student should have an average of at least 60 marks.

From the given information, we can write the linear inequality as:

$(70+75+x)/3 \geq 60$

Now, simplify the expression:

$\Rightarrow (145 +x) \geq 180$

$\Rightarrow x \geq 180 -145$

$\Rightarrow x \geq 35$

Hence, the student should obtain a minimum of 35 marks to have an average of at least 60 marks.

Q2. 3. The cost and revenue functions of a product are given by $C(x) = 20x + 4000$ and $R(x) = 60x + 2000$, respectively, where x is the number of items produced and sold. How many items must be sold to realise some profit?

Solution:

Given that,

Cost, $C(x) = 20x + 4000$

Revenue, $R(x) = 60x + 2000$

We know that, profit = Revenue – Cost

Now, substitute the given data in the above formula,

Profit = $R(x) – C(x)$

Profit = $(60x + 2000)-(20x + 4000)$

Now, simplify it:

Profit = 60x + 2000 -20x -4000

Profit = 40x – 2000

To earn some profit, 40x – 2000 > 0

⇒40x > 2000

⇒ x>2000/40

⇒ x > 50

Thus, the manufacturer should sell more than 50 items to realise some profit.

Q3. Solve 5x + 6 > 1 when x is real number?

Ans: 5x + 6 > 1

5x > -5

x > -1 ......Dividing by 5

Hence the solution (-1, ∞)

Q4. 50x < 540, where x is the natural number?

Ans: Given: 50x < 540

Dividing both sides by 50

x < 540/50

x < 54/5

Hence the solution set {1,2,3,4,5,6,7,8,9,10}

Q5. Solve the equation, 4x-2 ≤ 6 and 9x+3 ≥ -15.

Ans: 4x-2 ≤ 6 and 9x+3 ≥ -15

First,

4x-2 ≤ 6

2x -1 ≤ 3......divide by 2

2x ≤ 4

x ≤ 2 ....divide 2

Second,

9x+3 ≥ -15

3x+1 ≥ -5 ....Divide by 3

3x ≥ -6

x ≥ -2 ....divide by 3

From both the solutions, -2 ≤ x ≤ 2

Hence the solution [-2, 2]

Q6. Solve the inequalities, $2x-1\leq3$ and $3x+1\geq-5$

Ans: 14We are given equations 2x-1 <3 and 3x+1≥-5, so now solving them we have

2x-1< 3

2x≤ 4

x≤ 2

So the solution set for this is (-infinity, 2]

Now,

3x+1≥-5

3x≥ -6

X -≥2

So the solution set for this is [-2, Infinity)

Now the combined solution set for both the equations will be, [-2.2]

Hence, the solution set is [-2,2]..

Q7. Find all pairs of consecutive odd natural numbers both of which are larger than 10 such that their sum is less than 40.

Ana: Let the two consecutive odd positive integer be xx and x+2x+2.

Both number are smaller than 10 Therefore

$x+2<10$

Adding −2−2 to both sides,

$x<10-2$

$\Rightarrow x<8$

Also sum of the two integers is more than 40.

So,$x+x+2>40$

$\Rightarrow 2x+2>40$

adding −2to both sides,

$2x>40-2$

$\Rightarrow 2x>38$

Divided by 2 both sides

$x>19$

Then the number greater then 10 and less then 19 are the consecutive odd number pair

In (C) (11,13),(13,15) ,(15,17) ,(17,19) is right answer

Q8. solve $-3x + 17 < -13$

Ans: Given,

$-3x + 17 < -13$

Subtracting 17 from both sides,

$-3x + 17 – 17 < -13 – 17$

⇒ $-3x < -30$

⇒ $x > 10$ {since the division by negative number inverts the inequality sign}

⇒ $x \in (10, \infty)$

Q9. Solve: $4x + 3 < 6x +7$

Ans: Given,

$4x + 3 < 6x + 7$

Subtracting 3 from both sides,

$4x + 3 – 3 < 6x + 7 – 3$

⇒ $4x < 6x + 4$

Subtracting 6x from both sides,

$4x – 6x < 6x + 4 – 6x$

⇒ $– 2x < 4$ or

⇒ $x > – 2$ i.e., all the real numbers greater than –2, are the solutions of the given inequality.

Hence, the solution set is $(–2, \infty)$, i.e. $x \in (-2, \infty)$

Q10. Hari obtained 70 and 75 marks in first two unit tests. Find the number if minimum marks he should get in the third unit test to have an average of at least 60 marks.

Ans:

$(70 + 75 + x) \geq 60$

or $145 + x \geq 180$

or $x \geq 35$

Q1. 1. Solve the inequation $3 x + 17 \leq 2 1 – x$.

Q2. 2.Solve the inequality $x+3/ x−2\leq 2$

Q3. Find all pair of consecutive odd integers , both are smaller than 18, such that their sum is more than20.

Q4. Solve $3x-5 < x+1$.Show the solution on number line.

Q5. Solve the inequation$2x+4/x-1\geq 5$

Answer:

Q1. $x\leq −32$. Q2. $x \in (−\infty, 2) \cup [7, \infty)$ Q3. 11 and 13, 13 and 15, 15 and 17

Q4. $x < 2/3$ Q5. $1<x \leq 3$

PERMUTATIONS AND COMBINATIONS

CONCEPTUAL NOTES

The study of permutations and combinations revolves around determining the number of different ways of arranging and selecting objects out of a given number of objects. Before moving to the discussion of permutations and combinations, we require basic knowledge about some symbols and fundamental principle of counting which are given below.

Factorial: The continued product of first n natural number is known as 'n factorial'. It is denoted by n! .

i.e. n! = 1 x 2 x 3 x……………………………….x(n-1) x n

For example,

2! = 2 x 1

3! =3 x 2 x 1

5! = 5 x 4 x 3 x 2 x 1

Fundamental Principle of counting: If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrence of the events in the order is m x n.

- **Permutations:** A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

  The number of permutations of n different objects taken r at a time, where $0 < r \leq n$ and the objects do not

  repeat is $n(n-1)(n-2)\ldots(n-r+1)$, which is denoted by

$$P(n,r) \quad OR \quad {}^nP_r = \frac{n!}{(n-r)!}, 0 \leqslant r \leqslant n$$

- ${}^nP_0 = 1$, ${}^nP_n$=n!
- The number of permutations of n different objects taken r at a time, where repetition is allowed, is n^r.
- The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is $\frac{n!}{p_1!p_2!\ldots p_k!}$.
- The number of permutations of an dissimilar things taken all at a time along a circle is (n -1)!.
- The number of ways of arranging a distinct objects along a circle when clockwise and anticlockwise arrangements are considered alike is $\frac{1}{2}(n-1)$

- The number of ways in which $(m+n)$ different things can be divided into two groups containing m and n things is $\frac{(m+n)!}{m!n!}$.

  Combination of n different objects taken r at a time, denoted by $\quad {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

Important Results:

- ${}^{n}P_{r} = {}^{n}C_{r} r!\,, 0 \leqslant r \leqslant n$
- ${}^{n}C_{0} = 1 = {}^{n}C_{n}$
- ${}^{n}C_{1} = n = {}^{n}C_{n-1}$
- ${}^{n}C_{2} = \frac{n(n-1)}{2} = {}^{n}C_{2}$
- ${}^{n}C_{3} = \frac{n(n-1)(n-3)}{3} = {}^{n}C_{n-3}$
- ${}^{n}C_{r} = {}^{n}C_{s} \Rightarrow r = s \text{ or } r + s = n$
- ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

MULTIPLE CHOICE QUESTIONS

Q1. In a class there are 8 boys and 10 girls. The teacher wants to select a boy and a girl to represent the class in a function. The number of ways in which a teacher can make the selection is

(a) 18

(b) 10^{8}

(c) 8^{10}

(d) 80

Ans (d)

Solution:- The teacher has to select

(i) a boy among 8 boys

(ii) a girl among 10 girls

The first one can be done in 8 ways and second one in 10 ways.

Therefore, by fundamental principle of multiplication, total numbers of ways = 8 × 10 = 80

Q2.The number of ways in which n distinct objects can be put into two different boxes

(a) n^{2}

(b) 2^{n}

(c) 2n

(d) 2 + n

Ans (b)

Solution: Let the two boxes be B_1 and B_2. We have two choice for each of n object. So totalnumber of ways is

$$\underbrace{2 \times 2 \times 2 \ldots \ldots . \times 2}_{n\text{times}} = 2^n$$

Q3. The number of ways in which 8 district toys can be distributed among 4 children is

(a) 4^8

(b) 8^4

(c) ${}^8P_4$

(d) 32

Ans: (a)

Solution: Each toy can be distributed in 4 ways so total number of ways = $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^8$

Q4. How many 5-digits number can be formed using the three digits 0, 1 and Z, is

(a) 3^4

(b) 2×3^4

(c) 3^5

(d) 3×2^4

Ans: (b)

Solution: As 0 cannot be filled at first place and remaining each place can be filled in 3 ways

$$\bar{2} \times \bar{3} \times \bar{3} \times \bar{3} \times \bar{3}$$

So, required number of numbers = 2×3^4

Q5. In how many ways a committee consisting of 3 men and 3 women can be selected from 5 men and 7 women?

(a) 45

(b)4200

(c) 210

(d) 350

Ans (d)

Solution: out of 5 men, 3 can be chosen in ${}^5C_3$ ways. Out of 7 women, 3 can be chosen in ${}^7C_3$ ways

So committee can be chosen ${}^5C_3 \times {}^7C_3 = 350$ ways.

Q6. The number of signals that can be sent by 4 flags of different colours taking one or more at a time is

(a) 64

(b) 24

(c) 16

(d) 74

Ans (a)

Number of signals using one flag = $^4P_1 = 4$

Number of signals using two flag = $^4P_2 = 12$

Number of signals using three flag = $^4P_3 = 24$

Number of signals using four flag = $^4P_4 = 24$

So, total number of signal = $^4P_1 + ^4P_2 + ^4P_3 + ^4P_4$

= 4 + 12 + 24+ 24 = 64

Q7. The number of different seven-digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is

(a) $^7P_2 \cdot 2^5$

(b) $^7C_2 .2^5$

(c) $^7C_2 . 5^2$

(d) $^7p_2 \cdot 2^5$

Ans (b)

Solution: We can choose any two of the seven digits (in seven digits number). This can be done in $^7c_2$ ways. Put 2 in these two digits. The remaining 5 places can be filled using 1 and 3 in 2^5 ways.

So, required number of numbers = $^7C_2 . 2^5$

Q8. If $^{18}c_r = ^{18}c_{r-10}$, then $^{16}c_r$ is equal to

(a) 820

(b) 420

(c) 116

(d) none of these

Ans: (d)

Solution: We have $^{18}c_r = ^{18}c_{r-10}$

$\Rightarrow$ r + r – 10 = 18

$\Rightarrow$ 2r – 10 = 18

If $^nC_a = ^nC_b$ either a=b or a+b = n

$\Rightarrow$ r = 14

$$\therefore \; ^{16}c_r = \; ^{16}c_{14} = \; ^{16}c_2 = \frac{16 \times 15}{2!} = 120$$

Q9.A gentleman has 5 friends to invite. In how many ways can be send invitation cards to them, if he has 2 servants to carry the cards?

(a) 2^5

(b) 5^2

(c) 10

(d) none of these

Ans (a)

Solution: As a card can be sent by any of the two servants, so the number of ways of sending the invitation card to the first friend = 2. Similarly, invitation card can be sent to each of the 5 friends in 2 ways

$\therefore$ Required number of ways = $2 \times 2 \times 2 \times 2 \times 2 = 2^5$

Q10. How many different words, each containing 2 vowels and 2 consonant can be formed with 5 vowel and 17 consonants?

(a) ${}^5c_2 \times {}^{17}c_2$

(b) ${}^5c_2 \times 4!$

(c) ${}^{17}C_2 \times 4!$

(d) ${}^5c_2 \times {}^{17}c_2 \times 4!$

Ans (d)

Solution: There are 5 vowels and 17 consonant. Two vowels out of 5 vowels, 3 consonants out of 17 consonants can be chosen in ${}^5c_2 \times {}^{17}c_2$ ways. Now, we have 4 letters to arrange which can be done in 4! Ways

So total numbers of words = ${}^5c_2 \times {}^{17}c_2 \times 4!$

ASSERTION REASON BASED QUESTIONS

In the following question ,a statement of assestion (A) is followed by a statement of Reason (R).choose the correct answer out of the following choices.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Q1.Assestion (A): $12_{C_2} = 12_{C_{10}}$

Reason (R): Selection of the ɤ distinct thing out of n is equal to the rejection of (n-ɤ) thing out of n.

Ans-(a)

Solution:- Attributing to the reason, selection of 2 distinct thing out of 12,is equal to rejection of (12-2=10) thing out of 12 $\therefore 12_{C_2} = 12_{C_{10}}$

Q2.Assertion (A): 3! +4! = 7!

Reason (R): For any positive integer n, n! = 1×2×3…..×n and 0! =1

Ans-(d)

Solution - (R) is the definition of factorial which is true but assertion is false.

As $3!+4!=(3\times2\times1)+(4\times3\times2\times1)$

$$=6+24=30$$

$$7!=7\times6\times5\times4\times3\times2\times1>30$$

Q3.Assertion (A):-For any natural number $\text{n}, n_{C_n}=1$

Reason (R): $n_{C_r}=\dfrac{n!}{(n-r)!}$

Ans (c)

Solution: $n_{C_r}=\dfrac{n!}{(n-\text{r})!\text{r}!}\therefore n_{C_n}=\dfrac{n!}{(n-n)!n!}=\dfrac{\not{n}!}{\not{n}!}=1$

Q4.Assertion (A): $n_{P_r}=n_{C_r}\cdot\text{r}!\,0<r\leq n$

Reason(R): For each combination of ^nC_r , there are r! permutations.

Ans (a)

Solution: r objects in every combination can be arranged in r! ways. $n_{P_r}=n_{C_r}\cdot\text{r}!\,0<r\leq n$

Q5.Assertion (A):-The number of ways of choosing 4 cards from a pack of cards is 52_{C_4}

Reason (R): permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

Ans (b)

Solution: Selection of 4 card out of 52 can be made in 52_{C_4} ways. And (R) is the definition for permutation

$\therefore$(b) is correct

SHORT ANSWER TYPE QUESTIONS

Q1. How many different words can be formed by using all the letters of the word ALLAHABAD?

Sol :-There are 9 letters in given word ,of which 4 are A'&,2 are L'&. So total number of words is the number of arrangements of 9 things of which 4 are similar of one kind, two are similar of one kind $\therefore$

Total number of words = $\dfrac{9!}{4!2!}$ specific.

Q2. There are 10 points on a circle. How many chords can be drawn by using these points?

Sol:- One chord can be drawn by using two points. So total number of chords = number of ways of choosing two points out of 10.i.e 10_{C_2}

Q3. Find the total number of five-digit numbers that can be formed by using the digits 0,1,2,3, ….,9

Sol:- Total number of five digit numbers $9\times10\times10\times10\times10=90000$

As 0 cannot be used as first place so it has 9 choices and rest all 4 places can be filled in 10 ways.

Q4. At an election, a voter may vote for any number of candidates, not greater than the number to be elected there are 8 candidates and 3are to be elected. If a voter votes for at least one candidate, then find number of ways in which he can vote?

Sol:-The number of ways in which a voter can vote is

$8_{C_1}+8_{C_2}+8_{C_3}=8+28+56=92$

Q5. Find the total number of ways of answering 6 objective type question, each question having 4choies:

Sol:-since each question can be answered in 4 ways ,So total number of ways of answering 6 question is $4\times4\times4\times4\times4\times4 = 4^6$

Q6. There are 3 candidates for a classical ,4 for a social science and 2 for a natural science scholarship In how many ways can these scholarship be awarded?

Sol:-Since, classical scholarship can be awarded to any one of three candidates. So awarding the classical scholarship.

Similarly the social science and natural science scholarship can be awarded in 4 and 2 ways respectively.

So number of ways $3\times4\times2=24$

Q7. A bag contains 4 black and 3 red balls determine the number of ways in which 2 black and 2 red balls can be selected

Sol:-Out of 4 black balls ,2 black balls can be selected in 4_{C_2} ways and out of 3 red balls ,2 red balls can be selected in 3_{C_2} ways

Total number of ways = $4_{C_2}\times3_{C_2}=6\times3=18$

Q8. In how many ways can a student choose programme of 4 courses if 8courses are available and 2 specific courses are compulsory for every student?

Sol:- As 2 courses are compulsory for every student. Therefore, every student has to choose 2 courses out of remaining 6 courses this can be done in $6_{C_2}=15$ ways.

Q9. There are 8 true –false questions in an exam. Find the number of ways in which these questions can be answered

Sol:-Each question cab be answered in two ways . So total number of ways of answering 8question is

$$\underbrace{2 \times 2 \times \ldots \ldots \times 2,}_{8 times} = 2^8 = 256$$

Q10. If there are 10 persons in a room Everybody shakes hands with everybody else How many handshakes will be there?

Sol:-Two hands are required for a handshakes Therefore total number of handshakes = $10_{C_2} = 45$

<u>CASE STUDY BASED QUESTIONS</u>

Q.1 A state cricket authority has to choose a team of 11 members, to do it so the authority ask 2 coaches of a government academy to select the team Members that have experience as well as best performin the last 15 matches. They can make up a team of 11cricketers amongst 15 possible candidates in which 5 players can bowl.

Based on the information answer the following

(i) In how many ways can the final eleven be selected from 15 cricket players?

(ii) In how many ways can the final eleven be selected if exactly 4 bowlers must be included.

(iii)In how many ways can the final eleven be selected if all bowlers must be included.

Solution. (i) $^{15}C_{11}$= 1365

(ii) 4 bowlers can be select by = $^{5}C_{4}$

Remaining 7 players can be select out of 10 (15-5) is = $^{10}C_{7}$

Total no. of ways is = $^{5}C_{4}$,$^{10}C_{7}$ = 600

(iii) all 5 bowlers can be select by = $^{5}C_{5}$

Remaining 6 players can be select out of 10 (15-5) is = $^{10}C_{6}$

Total no. of ways is = $^{5}C_{5}$,$^{10}C_{6}$ = 210

Q 2: In an examination, a question paper consists of 12 questions divided into two parts i.e.,Part land Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part

. Based on the information answer the following

In how many ways can a student select the questions in such a way that

(a) 3 questions from part I and 5 questions from part II

(b) 4 questions from part I and 4 questions from part II

(c) 5 questions from part I and 3 questions from part II

d) In how many ways can a student select the questions

Answer: It is given that the question paper consists of 12 questions divided into two parts - Part I and Part II, containing 5 and 7 questions, respectively.

A student has to attempt 8 questions, selecting at least 3 from each part.

This can be done as follows.

a)3 questions from part I and 5 questions from part II can be selected in$^{5}C_{3}\times{}^{7}C_{5}$ways.

b) 4 questions from part I and 4 questions from part II can be selected in $^{5}C_{4}\times{}^{7}C_{4}$ways.

c) 5 questions from part I and 3 questions from part II can be selected in $^5C_5 \times {}^7C_3$ ways.

d) Thus, required number of ways of selecting questions

$$= {}^5C_3 \times {}^7C_5 + {}^5C_4 \times {}^7C_4 + {}^5C_5 \times {}^7C_3$$

$$= \frac{5!}{2!3!} \times \frac{7!}{2!5!} + \frac{5!}{4!1!} \times \frac{7!}{4!3!} + \frac{5!}{5!0!} \times \frac{7!}{3!4!}$$

$$= 210 + 175 + 35 = 420$$

Q3. The number of different words that can be formed from the letters of the wordINTERMEDIATE such that two vowels never come together is_________.

[Hint: Number of ways of arranging 6 consonants of which two arc alike is $\frac{6!}{2!}$ and number of ways of arranging vowels $= {}^7P_6 \times \frac{1}{3!} \times \frac{1}{2!}$]

Solution. Letters of the word INTERMEDIATE are: Vowels (I, E, E, I, A, E) and consonants (N, T, R, M, D, T)

Now we have to arrange these letters if no two vowels come together.

So, first arrange six consonants in $\frac{6!}{2!}$ ways.

Arrangement of six consonants creates seven gaps.

Six vowels can be arranged in these gaps in ${}^7C_6 \times \frac{6!}{2!3!}$ ways.

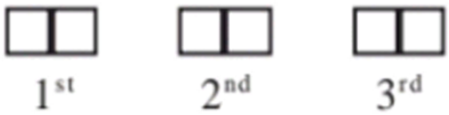
So, total number of words $\frac{6!}{2!} \times {}^7C_6 \times \frac{6!}{2!3!} = 360 \times 7 \times 60 = 151200$

Que4. Four persons entered the lift cabin on the ground floor of 7 floor house suppose each of them can leave the cabin independently at any floor beginning with the first. What is. The total number of ways in which each of the four persons can leave the cabin at any of the six floor?

Solution: suppose that A, B, C, D are 4 persons a can leave the cabin at any of the 6 floors so A can leave the cabin in 6 ways, similarly each of B,C and D can leave the cabin in 6 ways therefore the total number of ways in which each of the 4 persons can leave the cabin at any of the six floor is $6 \times 6 \times 6 \times 6 = 1296$.

Que5.Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in how many ways can they be seated? Find also the number of ways of their seating if all the ladies sit together.

Solution: Let us denote married couples by S_1, S_2, S_3 where each couple is considered to be a single unit as shown in the following figure:



Then the number of ways in which spouces can be seated next to each other is 3! = 6 ways.

Again each couple can be seated in 2! ways. Thus the total number of seating arrangement so that spouces sit next to each other = $3! \times 2! \times 2! \times 2! = 48$.

Again, if three ladies sit together, then necessarily three men must sit together.

Thus, ladies and men can be arranged altogether among themselves in 2! ways.

Therefore, the total number of ways where ladies sit together is $3! \times 3! \times 2! = 144$.

Long Answer Type questions

Q1. In how many ways can final eleven be selected from 15 cricket players' if

(i) there is no restriction

(ii) one of then must be included

(iii) one of them, who is in bad form, must always be excluded

(iv) Two of then being leg spinners, one and only one leg spinner must be included?

Ans. (i) 11 players can be selected out of 15 in ${}^{15}C_{11}$ ways

$= {}^{15}C_{11}$, ways $= \frac{15\times14\times13\times12}{1\times2\times3\times4}$ ways=1365 ways

(ii) Since a particular player must be included, we have to select 10 more out of remaining 14 players.

This can be done in ${}^{14}C_{10}$ ways ${}^{14}C_4$ ways

$= \frac{14\times13\times12\times11}{1\times2\times3\times4}$ ways = 1001 ways

(iii) Since a particular player must be always excluded, we have to choose 11 ways out of remaining 14

This can be done in ${}^{14}C_{11}$ ways $= {}^{14}C_3$ ways

$= \frac{14\times13\times12\times11}{1\times2\times3}$ ways = 364 ways.

(iv) One leg spinner can be chosen out of 2 in ${}^{2}C_1$ ways. Then we have to select 10 more players get of 13 (because second leg spinner can't be included). This can be done in ${}^{13}C_{10}$ ways of choosing 10 players. But these are ${}^{2}C_1$ ways of choosing a leg spinner, therefore, by multiplication principle of counting the required number of ways

$= {}^{2}C_1 \times {}^{13}C_{10}$

$= {}^{2}C_1 \times {}^{13}C_3 = \frac{2}{1}\times\frac{13\times12\times11}{1\times2\times3} = 572$

Q2. A bag contains six white marbles and five red marbles. Find the number of ways in which four marbles can be drawn from the bag, if (i) they can be of any colour (ii) two must be white and two red and (iii) they must all be of the same colour.

Ans: Total number of marbles = 6 white +- 5 red = 11 marbles

(a) If they can be of any colour means we have to select 4 marbles out of 11

$\therefore$ Required number of ways = $^{11}C_4$

(b) Two white marbles can be selected in $^{6}C_2$

Two red marbles can be selected in 5C2 ways.

$\therefore$ Total number of ways = $^{6}C_2 \times {}^{5}C_2 = 15 \times 10 = 150$

(c) If they all must be of same colour,

Four white marbles out of 6 can be selected in $^{6}C_4$ ways.

And 4 red marbles out of 5 can be selected in $^{5}C_4$ ways.

$\therefore$ Required number of ways = $^{6}C_4 + {}^{5}C_4 = 15 + 5 = 20$

Q3. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected, if the team has

(i) no girls

(ii) at least one boy and one girl

(iii) at least three girls

Ans: Number of girls = 4;

Number of boys = 7

We have to select a team of 5 members provided that (i). Team having no girls

Required number of ways = ${}^7C_5 = \frac{7\times6}{2!} = 21$

(ii) Team having at least one boy and one girl

$\therefore$ A Required number of ways

$= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

$= 7\times1 + 21\times4 + 35\times6 + 35\times4 = 7 + 84 + 210 + 140 = 441$

(iii) Team having at least three girls

$\therefore$ Required number of ways $= {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$

$= 4\times21 + 7 = 84 + 7 = 91$

Q4. The number of 6-digit numbers that can be made with the digits 0,1,2,3,4 and 5 so that even digits occupy odd places, is

SOLUTION: There are 3 odd and 3 even places. Three odd places viz. first, third and fifth can be occupied by even digits 0,2 and 4 in 3! ways. But, 0 cannot occupy the first place from the left. Therefore, the number of ways in which 3 odd places can be filled is (3! - 2!). Three even places can be filled in 3! ways. Hence, total number of six digit numbers = (3! - 2!) × 3! = 24.

Q5.The number of words that can be made by down the letters of the word CALCULATE such that each word starts and ends with a consonant, is

SOLUTION: Required number of words

= Number of words which begin and end with C

+ Number of words which begin and end with L

+ Number of words which begin with C and end with L

+ Number of words which begin with L and end with C

+ Number of words which begin with C and end with T

+ Number of words which begin with T and end with C

+ Number of words which begin with L and end with *T*

+ Number of words which begin with T and end with L

$$=\frac{7!}{2!2!}+\frac{7!}{2!2!}+\frac{7!}{2!}+\frac{7!}{2!}+\frac{7!}{2!2!}+\frac{7!}{2!2!}+\frac{7!}{2!2!}+\frac{7!}{2!2!}$$

$$=6\times\frac{7!}{2!2!}+2\times\frac{7!}{2!}=\frac{5\times 7!}{2!}$$

<u>SELF-PRACTICE QUESTIONS</u>

MULTIPLE CHOICE QUESTIONS

Q1. The number of 5-digit number that can be made using the digit 1 and 2 and in which at least one digit is different is

(a) 30 (b) 31

(c) 32 (d) 33

Q2. Total number of 6-digit number in which all the odd digits appear ,is

(a) $\frac{5}{2}\times 6!$ (b) $6!$

(c) $\frac{6!}{2}$ (d) none of these

Q3. The number of all permutations of n different objects taken r at a time when a particular object is never taken in each arrangement, is

(a) ${}^{n-1}C_r \times (r-1)!$

(b) ${}^{n-1}C_{r-1} \times r!$

(c) ${}^{n-1}C_r \times r!$

(d) ${}^{n}C_r \times r!$

Q4. How many diagonals are there in a polygon with n sides?

(a) $\frac{n(n-1)}{2}$

(b) $\frac{n(n-3)}{2}$

(c) $\frac{n(n+1)}{2}$

(d) $\frac{n(n+2)}{2}$

Q5. If ${}^{n}P_r = 720\, {}^{n}C_r$ then the value of r is

(a) 5

(b) 4

(c) 6

(d) 7

Q6. If ${}^{n}C_{12} = {}^{n}C_8$ then n is equal to

(a) 12

(b) 6

(c) 30

(d) 20

Q7. The number of triangles that are formed by choosing the vertices from a set of 12 point ,7 of which lie on same line, is

(a) 15

(b) 175

(c) 185

(d) 105

Q8. The number of ways in which a team of 11 players can be selected from 22 players always including 2 of them and excluding 4 of them is

(a) ${}^{16}C_{11}$

(b) ${}^{16}C_5$

(c) ${}^{16}C_9$

(d) ${}^{20}C_9$

Q9. The total number of five digit numbers with atleast one repeating digit is

(a) 27218

(b) 15120

(c) 90000

(d) 62784

Q10. The number of ways in which n distinct objects can be put into two different boxes is

(a) n^2

(b) 2^n

(c) $2n$

(d) none of these

ASSERTION REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following Choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) id false but (R) is true.

Q1.Assertion: The number of rectangles on a chess board is ${}^8C_2 \times {}^8C_2$

Reason: To from a rectangle, we have to select any two of horizontal line and any two of the vertical line

Q2.Assertion: The number of ways of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$

Reason: The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$

Q3. Assertion: The number of ways in which n different prizes can be distributed among m$(< n)$ person if each is entitled to receive at most (n-1) prizes is $m^n - m$

Reason: Required number of ways=

Total number of ways (m^n)–number of ways in which one gets all prizes (m)

Q4.Assertion; If ${}^{15}C_{3r} = {}^{15}C_{r+3}$,then $r=4$

Reason: ${}^{n}C_{a} = {}^{n}C_{6} \Rightarrow$ either $a=b$ or $a+b=n$

Q5.Assertion; Number of lines formed by joining n points on a circle is $n(n-1)/2$

Reason: $C=(n,3)=n(n-1)/2$

SHORT ANSWER TYPE QUESTIONS

Q1. In a city, all telephone numbers have six digit, the first two digit always being 41 or 42 or 46 or 62 or 64. How many telephone numbers have all six digit distinct?

Q2. Out of 18 points in a plane, no three are in the same line except five points which are collinear find the number of lines that can be formed joining these points

Q3. There are 5 lamps in a hall, each of them can be switched on indepenedtly. Find the number of ways in which hall can be illuminated.

Q4. How many committee of 5 persons (including a chairperson) can be selected form 12 persons?

Q5. The number of ways in which n distinct balls can be put into 3 boxes so that no two boxes remain empty, is __

Q6. A box contains two white, three black and four red balls. Find the number of ways in which three balls can be drawn from the box if atleast one black ball is to be included in the draw.

Q7. Out of all possible permutations of the letters of the word ENDEANOEL, find the number of permutations in which letters A,E,O occur only in odd positions.

Q8. Given 1 2 flags of different colours, how many different signals can be generated if each signalrequires the use of 2 flags, one below the other?

Q9. There are four bus routes between A and B; and three bus routes between B and C. A man cantravel round-trip in number of ways by bus from A to C via B. If he does not want to use a bus route more than once, in how many ways can he make round trip?

Q10. In an examination there are three multiple choice questions and each question has 4 choices.

Find the number of ways in which a student can fail to get all answer correct.

CASE STUDY BASED QUESTIONS

Q1. In a company, CEO wants to establish a new branch. New branch required a committee of 5 members is to be formed out of 6 gents and 4 ladies.

In how many ways this can be done, when

(i) At least two ladies are included?

(ii) II. At most two ladies are included?

Q2. Read the following passage and answer the questions given below.

Every person has Independence thought.

Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,

(i) Do the words start with P

(ii) Do all the vowels always occur together

(iii) Do the vowels never occur together

OR

Do the words begin with I and end in P?

Q3. Read the following passage and answer the question given below.

The longest river of North America is **Mississippi river**

In how many ways can the letters of the word MISSISSIPPI be arranged such that

(i) AIl letters are used

(ii) All I's are together

(iii) All I's are not together

OR

All S 's are not together

Q4. A bag contains six white marbles and five red marbles. four marbles can be drawn from the bag .

Based on the information answer the following .

Find the number of ways in which

(a) They can of any color

(b) Two must be white and two red

(c) They must be all of same color

Q5. Read the following passage and answer the question given below.

In a village, there are 87 families of which 52 families have at most 2 children.

In a rural development program, 20 families are to be helped chosen for assistance. In how many ways can the choice be made in which

a) At least 18 families must have at most 2 children

b) All the families must have at most 2 children

LONG ANSWER TYPE QUESTIONS

Q1. If ${}^{n}C_{r-1} = 36; {}^{n}C_{r} = 84 \text{ and } {}^{n}C_{r+1} = 126$ then Find the value of ${}^{r}C_{2}$

Q2. Find the number of ways in which a miXed double game can be arranged from amongst nine married couples if no husband and wife play in the same game.

Q3. What is the number of ways of choosing four cards from a pack of 52 playing cards? In how many of there

(a) Four cards one of the same suit

(b) Four cards belong to four different suits

(c) Are face cards.

(d) Two are red cards & two are black cards.

(e) Cards are of the same colour?

Q4. If ${}^{n}P_{r} = {}^{n}P_{r+1}$ and ${}^{n}c_{r} = {}^{n}c_{r-1}$ find the value of *n* and *r*

Q5. How many four letter words can be formed using the letters of the letters of the word 'FAILURE' so that

(i) F is included in each word

(ii) F is excluded in each word.

Answer key for self-practice questions

Multiple choice questions

Q1. (a)

Q2. (a)

Q3. (c)

Q4. (b)

Q5. (c)

Q6. (d)

Q7. (c)

Q8. (c)

Q9. (d)

Q10. (b)

Short-answer questions

Q1. 8400	Q2. 144	Q3. 31	Q4. 3960	Q5. 3^n-3
Q6. 64	Q7. 2 x 5!	Q8. 132	Q9. 72	Q4. 63

Assertion-Reason based questions

Q1. (d)	Q2. (b)	Q3. (a)	Q4. (d)	Q5. (c)

Case -Study based questions

Q1. (i) 186 (ii) 186

Q2. (i) 138600 (ii) 16800 (iii) 1646400

Q3. (i) 34650 (ii) 840 (iii) 33810

Q4. (i) 330 (ii) 150 (iii) 20

Q5. ${}^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20}$

Long -answer type questions

Q1. 3

Q2. 3024

Q3. (i) 2860 (ii) $(13)^4$ (iii) 495 (iv) 105625

Q4. n = 3, r = 2

Q5. (i) 480 (ii) 360

BINOMIAL THEOREM

MAIN CONCEPTS AND RESULTS

An algebraic expression containing two terms is called binomial expression

For example, $(2x - y), (x + y), \left(\frac{1}{x} + x\right)$ etc.

The general form of binomial expression is $(x + a)$ and the expansion of $(x + a)^n, n\epsilon N$ is called Binomial theorem.

We have learnt that:

$(x + a)^0 = 1$

$(x + a)^1 = x + a$

$(x + a)^2 = x^2 + 2ax + a^2$

$(x + a)^3 = x^3 + 3x^2a + 3xa^2 + a^3$

$(x + a)^4 = x^4 + 4x^3a + 6x^2a^2 + 4xa^3 + a^4$

We observe that the coefficients in the above expansion follow a particular pattern as

Index of the binomial	Coefficient of various terms
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1

In the above pattern it can be seen that the addition of 1's in the row for the index 1 gives rise to 2 in the row for index 2. The addition of 1,2 and 2,1 in the row for index 2, gives rise to 3 and so on. This can be continued to any index.

Pascal's Triangle

Index	Coefficient
0	1
1	1 ∇ 1
2	1 ∇ 2 ∇ 1
3	1 ∇ 3 ∇ 3 ∇ 1
4	1 ∇ 4 ∇ 6 ∇ 4 ∇ 1

The above structure looks like triangle with 1 at the top vertex and running down the two slanting sides. This array of numbers is known as Pascal's triangle.

Expansion for higher powers of binomial are also possible by using Pascal's triangle but for that we have to find the coefficient for the higher index which is time consuming

Here use of combination to rewrite the numbers of Pascal's triangle.

We know that $C(n, r) = \frac{n!}{r!(n-r)!}$; $0 \le r \le n$, where n is non negative integer.

$$C(n, 0) = C(n, n) = 1$$

Index	Coefficient
0	$\boldsymbol{C(0.0)}$
1	$\boldsymbol{C(1,0)}$ $\boldsymbol{C(1,1)}$
2	$\boldsymbol{C(2,0)}$ $\boldsymbol{C(2,2)}$ $\boldsymbol{C(2,2)}$
3	$\boldsymbol{C(3,0)}$ $\boldsymbol{C(3,1)}$ $\boldsymbol{C(3,2)}$ $\boldsymbol{C(3,3)}$

4	$\boldsymbol{C(4,0)}$	$\boldsymbol{C(4,1)}$	$\boldsymbol{C(4,2)}$	$\boldsymbol{C(4,3)}$	$\boldsymbol{C(4,4)}$

Observing this pattern, we can now write the rows of Pascal's triangle for any index

Binomial theorem for any positive integer n:

$(a+b)^n = C(n.0)a^n + C(n,1)a^{n-1}b + C(n,2)a^{n-2}b^2 + C(n,3)a^{n-3}b^3 + \cdots + C(n,n)b^n$

Some important conclusions from the Binomial Theorem:

i. $(x+a)^n = \sum_{r=0}^{n} C(n,r)x^{n-r}a^r$

Or $(x+a)^n = C(n.0)x^n + C(n,1)x^{n-1}a + C(n,2)x^{n-2}a^2 + C(n,3)x^{n-3}a^3 + \cdots + C(n,n)a^n$

ii. The sum of indices of x and a in each term is n

iii. Since $C(n,r) = C(n,n-r), for\ r = 0,1,2,3,\dots,n$

$$C(n,0) = C(n,n); C(n,1) = C(n,n-1); C(n,2) = C(n,n-2); ..$$

iv. Replacing $a\ by - a$

$$(x-a)^n = C(n.0)x^n - C(n,1)x^{n-1}a + C(n,2)x^{n-2}a^2 - C(n,3)x^{n-3}a^3 + \cdots + (-1)^n C(n,n)a^n$$

i.e $(x-a)^n = \sum_{r=0}^{n}(-1)^n C(n,r)x^{n-r}a^r$

v. putting $x = 1\ and\ a = x$ in the expansion, we get

$$(1+x)^n = C(n.0) + C(n,1)x + C(n,2)x^2 + C(n,3)x^3 + \cdots + C(n,n)x^n$$

$(1+x)^n = \sum_{r=0}^{n} C(n,r)x^r$

vi. putting $x = 1\ and\ a = -x$ in the expansion, we get

$$(1-x)^n = C(n.0) - C(n,1)x + C(n,2)x^2 - C(n,3)x^3 + \cdots + (-1)^n C(n,n)x^n$$

$(1-x)^n = \sum_{r=0}^{n}(-1)^n C(n,r)x^r$

vii. the coefficient of $(r+1)th$ term in the expansion of $(1+x)^n$ is $C(n,r)$

viii. the coefficient of x^r in the expansion of $(1+x)^n$ is $C(n,r)$

ix. if n is odd than $((x+a)^n + (x-a)^n)$ and $((x+a)^n - (x-a)^n)$ both have same number of terms equal to $\left(\frac{n+1}{2}\right)$

if n is even , than $((x+a)^n + (x-a)^n)$ has $\left(\frac{n}{2}+1\right)$ terms and $((x+a)^n - (x-a)^n)$ has $\left(\frac{n}{2}\right)$ terms

Example 1: Which of the following represents the binomial theorem for positive integral indices?

A. $(a+b)^n = a^n + b^n$ B. $(a+b)^n = \sum_{r=0}^{n} C(n,r)a^{n-r}b^r$

C. $(a+b)^n = a^{n+1} + b^{n+1}$ D. none of these

Answer: B

Example2: Which of the following is the correct binomial coefficient?

A. $C(n,r) = \frac{n!}{r!(n-r)!}$ B. $(n,r) = \frac{(n-r)!!}{n!r!}$ C. $(n,r) = \frac{r!(n-r)!}{n!}$ D. $(n,r) = \frac{n!}{r!}$

Answer: A

Example3: What is the value of $C(6,2)$?

A. 10 B. 15 C. 20 D. 30

Answer: B

Example4: Which of the following corresponds to Pascal's triangles coefficients of the expansion $(a+b)^4$

A. 1,3,3,1 B. 1,4,6,4,1 C. 1,5,10,10,5,1 D. 1,2,1

Answer: B

Example5: The expansion $((x+a)^{51}+(x-a)^{51})$ has__________ terms after simplification.

A. 50 B. 52 C. 26 D. none of these

Answer : C

Example6: The total number of terms in the expansion $((x+a)^{100}+(x-a)^{100})$ after simplification is_____________

A. 202 B. 51 C. 50 D. none of these

Answer: B

Example7: Number of terms in the expansion $(1-2x+x^2)^7$ are _______

A. 14 B. 15 C. 16 D. 17

Answer : B

Example 8. The coefficient of x^5 in $(x+3)^9$ is ___________

A. $C(9,4)3^2$ B. $C(9,4)3^3$ C. $C(9,4)3^4$ D. none of these

Answer: C

Assertion –Reason based questions:

In the following questions a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices

a). Both A and R are true and R is correct explanation of A.

b). Both A and R are true but R is not the correct explanation of A.

c). A is true but R is false.

d). A is false but R is true

Example1: Assertion(A) : The sum of the coefficients in the expansion $(x+y)^n$ is 2^n

Reason (R): The binomial coefficients in the expansion of $(x+y)^n$ represent the elements in the nth row of Pascal's triangle.

Answer: A

Example 2: Assertion (A): in the expansion $(x+y)^n$, the coefficient of $x^{n-k}y^k$ is given by $C(n,k)$

Reason(R): The binomial coefficient $C(n,k)$ is equal to $\frac{n!}{k!(n-k)!}$

Answer: A

Example 3: Assertion (A) : Pascal's triangle can be used to find the coefficients in the expansion $(a+b)^5$

Reason (R): The nth

Example 4: Assertion(A) : the number of terms I the expansion of $\{(3x+y)^8-(3x-y)^8\}$ is 4

Reason (R) : if n is even, than $((x+a)^n-(x-a)^n)$ has $\left(\frac{n}{2}\right)$ terms

Answer: A

Example 5: Assertion (A) : the coefficient of the expansions are arranged in an array. This array is called Pascal's triangle.

Reason (R): There are 11 terms in the expansion $\{(3x+4y)^{10}+(3x-4y)^{10}\}$

Answer: B

Short Answer Type Questions:

Example 1: Expand $(2x-3y)^4$ by Binomial theorem

Answer: using

$$(x-a)^n = C(n.0)x^n - C(n,1)x^{n-1}a + C(n,2)x^{n-2}a^2 - C(n,3)x^{n-3}a^3 + \cdots + (-1)^n C(n,n)a^n$$

$=16x^4-96x^3y+216x^2y^2-216xy^3+81y^4$

Example 2: Expand $(x^2+2a\,)^5$ by Binomial theorem.

Answer: using

$$(x+a)^n = C(n.0)x^n + C(n,1)x^{n-1}a + C(n,2)x^{n-2}a^2 + C(n,3)x^{n-3}a^3 + \cdots + C(n,n)a^n$$

$=x^{10}+10x^8a+40x^6a^2+80x^4a^3+80x^2a^4+32a^5$

Example 3: which is larger $(1.01)^{100000}$ or 10000

Answer : $(1.01)^{100000}=(1+01)^{100000}$

$= C(100000{,}0) + C(100000.1)(0.01) + \textit{other positive terms}$

$=1+100000(0.01)+\textit{other positive terms} = 1+10000+\textit{other positive terms}$

> 10000

So, $(1.01)^{100000} > 10000$

Example 4: using binomial theorem prove that 6^n-5n always leaves reminder 1 when divided by 25

Answer : $6^n=(1+5)^n=1+C(n,1)5+C(n,2)5^2+C(n,3)5^3+\cdots+5^n$

$=1+5n+5^2(C(n,2)+C(n,3)5+\cdots+5^{n-2})$

$6^n-5n-1=25k(\,where\ k=\ C(n,2)+C(n,3)5+\cdots+5^{n-2})$

Hence proved.

Long questions

Example-1: find the coefficient of x^5 in the expansion of the product $(1+2x)^6(1-x)^7$

Answer $(1+2x)^6(1-x)^7 = (1+C(6,1)(2x)+C(6,2)4x^2+C(6,3)8x^3+C(6,4)16x^4+C(6,5)32x^5+C(6,6)64x^6)(1-C(7,1)x+C(7,2)x^2-C(7,3)x^3+C(7,4)x^4-C(7,5)x^5+C(7,6)x^6-C(7,7)x^7)$

$=(1+12x+60x^2+160x^3+240x^4+192x^5+64x^6)(1-7x+21x^2-35x^3+35x^4-21x^5+7x^6-x^7)$

Coefficient of x^5 is

$=1(-21)+12(35)+60(-35)+160(21)+240(-7)+192(1)$

$=171$

Example 2: find $(a+b)^4-(a-b)^4$. Hence evaluate $\left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4$

Answer: $(a+b)^4-(a-b)^4 = (C(4,0)a^4+C(4,1)a^3b+C(4,2)a^2b^2+C(4,3)ab^3+C(4,4)b^4) - (C(4,0)a^4-C(4,1)a^3b+C(4,2)a^2b^2-C(4,3)ab^3+C(4,4)b^4)$

$=2(C(4,1)a^3b+C(4,3)ab^3)=2(4a^3b+4ab^3)=8ab(a^2b^2)$

$\left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4$, put $a=\sqrt{3}$ and$b=\sqrt{2}$

$\left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4 = 8\sqrt{3}\sqrt{2}(3+2)=40\sqrt{3}\sqrt{2}=40\sqrt{6}$

Example 3: The sum of coefficients of the 1st three terms of the expansion of $\left(x - \frac{3}{x^2}\right)^m$ is 559, where m is a natural number. Find the number of terms in the expansion.
Answer: Here, coefficient of 1st three terms is $C(m,0), -3C(m,1), 9C(m,2)$+…
According to the question, $C(m,0) - 3C(m,1) + 9C(m,2) = 559$

$$1 - 3m + \frac{9m(m-1)}{2} = 559$$

On solving we get m=12
So, number of terms in the expansion are 13
Example 4: If P be the sum of odd terms and Q that of even terms in the expansion $(a+b)^n$
Prove that

i. $P^2 - Q^2 = (x^2 - a^2)^n$
ii. $4PQ = (x+a)^{2n} - (x-a)^{2n}$

Answer: $(x+a)^n = C(n.0)x^n + C(n,1)x^{n-1}a + C(n,2)x^{n-2}a^2 + C(n,3)x^{n-3}a^3 + \cdots + C(n,n)a^n = t_1 + t_2 + t_3 + \cdots + t_{n+1}$
$= (\boldsymbol{t_1} + \boldsymbol{t_3} + \boldsymbol{t_5} + \cdots) + (\boldsymbol{t_2} + \boldsymbol{t_4} + \boldsymbol{t_6} + \cdots)$

$$(x+a)^n = P + Q - - - - - (i)$$

$$(x-a)^n = C(n.0)x^n - C(n,1)x^{n-1}a + C(n,2)x^{n-2}a^2 - C(n,3)x^{n-3}a^3 + \cdots + (-1)^n C(n,n)a^n$$

$= t_1 - t_2 + t_3 - t_4 + \cdots$
$= (\boldsymbol{t_1} + \boldsymbol{t_3} + \boldsymbol{t_5} + \cdots) - (\boldsymbol{t_2} + \boldsymbol{t_4} + \boldsymbol{t_6} + \cdots)$

$$(x-a)^n = P - Q - - - - - (ii)$$

Multiply (i) with (ii), we get

$$P^2 - Q^2 = (x^2 - a^2)^n$$

by squaring (i) and (ii) and subtract, we get
$4PQ = (x+a)^{2n} - (x-a)^{2n}$

Case Study based question:

1. A bakery sells cookies in packs. Each pack contains either chocolates chips or oatmeal cookies. The owner wants to find out the different combinations of chocolate chip and oatmeal cookies in a pack of 5 cookies

Question 1: How many different combinations of chocolate chip and oatmeal cookies can be made in a pack of 5 cookies?

Answer: The number of combinations corresponds to the coefficients in the expansion $(x+y)^5$, where x represents chocolate chips and y represents oatmeal cookies. The number of different combinations is given by the 5th row of Pascal's triangle 1,5,10,10,5,1. Therefore there are 6 different combinations.

Question 2: What is the probability of having exactly 3 chocolate chip cookies in a pack of 5 cookies?

Answer: The coefficient of x^3y^2 in the expansion $(x+y)^5$ gives the number of ways to have 3 chocolate chip and 2 oatmeal cookies. The coefficient $C(5,3) = 10$. The total number of combinations is $2^5 = 32$. Therefore the probability is $\frac{10}{32}$

Case: A company wants to distribute a total of 10 prizes among its 5 employees. Each employee can receive any number of prizes.

Question 1: How many ways can the company distribute 10 prizes among 5 employees?
Answer: The number of ways to distribute 10 prizes among 5 employees is given by the coefficient of x^{10} in the expansion of $(x + x^2 + x^3 + x^4 + x^5)^5$. Using the binomial theorem, this can be simplified to finding the number of non-negative integer solutions to the equation:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 10,$$

which is given by $\binom{10+5-1}{5-1} = \binom{14}{4} = 1001.$

2.

Case: A company needs to assign 6 projects to 4 employees, where each project must be assigned to exactly one employee.

Question 1: How many ways can the projects be assigned if there are no restrictions?
Answer: The number of ways to assign 6 projects to 4 employees is 4^6.

Question 2: If each employee must get at least one project, how many ways can the projects be assigned?
Answer: This is equivalent to finding the number of surjective functions from 6 projects to 4 employees. The number of such assignments can be found using the principle of inclusion-exclusion, but it's complex for manual computation. In practice, generating functions or advanced combinatorial methods are used.

3.

Case: A garden has a variety of plants. The gardener wants to plant 7 different types of flowers in a row. Each type of flower can be planted in any position.

Question 1: How many different arrangements can be made using 7 different types of flowers?
Answer: The number of different arrangements is $7!$:

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040.$$

Question 2: If the gardener wants to plant 4 roses and 3 tulips, how many different ways can this be done?
Answer: The number of different ways to arrange 4 roses and 3 tulips is given by the binomial coefficient $\binom{7}{4}$:

$$\binom{7}{4} = \frac{7!}{4!3!} = 35.$$

Case: A sports tournament has 8 teams, and each team plays against every other team exactly once.

Question 1: How many matches are played in total?

Answer: The number of matches is given by the combination $\binom{8}{2}$:

$$\binom{8}{2} = \frac{8!}{2!(8-2)!} = 28.$$

Question 2: If each match ends in a win for one team and a loss for the other, how many possible outcomes are there for all the matches?

Answer: Since each match has 2 possible outcomes (win or loss), the total number of outcomes is 2^{28}.

Practice Questions:

1. If P be the sum of odd terms and Q that of even terms in the expansion $(a+b)^n$
   Prove that: $2(P^2+Q^2) = \{(x+a)^{2n} + (x-a)^{2n}\}$
2. If three successive coefficients in the expansion of $(1+x)^n$are 220, 495 and 792, then find n
3. The coefficients of three consecutive terms in the expansion $(1+a)^n$ are in the ratio $1:7:42$. Find n
4. Expand: $(1-x+x^2\ )^4$
5. Expand $\left(x+\frac{1}{x}\right)^5$
6. Compute: 98^5
7. Prove that $\sum_{r=0}^{n} 3^r C(n,r) = 4^n$
8. Find the number of terms in the expansion of $(\ 1-2x+x^2)^7$
9. Evaluate: 101^4

ASSERTION AND REASONING TYPE QUESTIONS

Question 1:

Assertion (A): The sum of the coefficients in the expansion of $(x+y)^n$ is 2^n.

Reason (R): Setting $x=1$ and $y=1$ in the expansion $(x+y)^n$ gives $(1+1)^n$.

A. Both A and R are true, and R is the correct explanation of A.

B. A is true, but R is false.

C. A is false, but R is true.

D. Both A and R are true, but R is not the correct explanation of A.

Question 2:

Assertion (A): In Pascal's Triangle, the number in the 5th row and 3rd column is 10.

Reason (R): The elements of Pascal's Triangle are binomial coefficients.

A. Both A and R are true, but R is not the correct explanation of A.
B. A is false, but R is true.
C. Both A and R are true, and R is the correct explanation of A.
D. A is true, but R is false.

Question 3:

Assertion (A): The coefficient of x^2 in the expansion of $(1+x)^5$ is 10.

Reason (R): The coefficient of x^2 in the expansion of $(1+x)^n$ is given by $\binom{n}{2}$.

A. Both A and R are true, and R is the correct explanation of A.
B. A is false, but R is true.
C. Both A and R are true, but R is not the correct explanation of A.
D. A is true, but R is false.

Question 4:

Assertion (A): The binomial coefficient $\binom{n}{k}$ is equal to the sum of the coefficients of the (k-1)th and kth terms of the previous row in Pascal's Triangle.

Reason (R): Each number in Pascal's Triangle is the sum of the two directly above it.

A. A is false, but R is true.
B. Both A and R are true, but R is not the correct explanation of A.
C. A is true, but R is false.
D. Both A and R are true, and R is the correct explanation of A.

CASE BASED QUESTIONS:

Question 1:

A bakery is making a new layered cake with different combinations of two flavors: chocolate and vanilla. They plan to make a cake with 6 layers. Each layer can be either chocolate or vanilla. How many different ways can they arrange the layers? Use the Binomial Theorem to solve this problem.

Question 2:

A gardener wants to plant a row of 5 different types of flowers. Each type of flower can be either red or yellow. If the gardener wants to know how many ways they can arrange these flowers, calculate the total number of combinations using Pascal's Triangle.

Question 3:

A company is designing a new product line consisting of two features, Feature A and Feature B. They can include up to 8 different variations of these features. If they want to know how many ways they can mix these variations in a product, apply the Binomial Theorem to find the answer.

Question 4:

A sports team has 7 games in a season. Each game can either be a win or a loss. Using Pascal's Triangle, determine the number of different possible outcomes for their season record.

Question 5:

A marketing team is planning a campaign that involves sending out emails. They can choose from 4 different subject lines and 3 different email bodies. If they want to create different combinations of subject lines and email bodies, how many different emails can they create? Use the concept of combinations from the Binomial Theorem to find the answer.

Answers: (PRACTICE QUESTIONS)

Q2	Q3	Q6	Q8	Q9
12	55	9039207968	15	104060401

Answers (Assertion and reason)

Q1	Q2	Q3	Q4
A	C	A	D

Answers (case-based questions)

Q1	Q2	Q3	Q4	Q5
64	32	256	128	12

SEQUENCE AND SERIES

CONCEPTS:

Sequence: A sequence is an arrangement of numbers in a definite order according to some rule. A sequence can also be defined as a function whose domain is the set of natural numbers or some subsets of the set {1, 2, 3....}.

A sequence containing a finite number of terms is called a finite sequence. A sequence is called infinite if it is not a finite sequence.

Series: If a_1, a_2, a_3,....a_n... be a given sequence. Then, the expression

$a_1 + a_2 + a_3 + ... + a_n + ...$

is called a series.

Arithmetic Progression (A.P.) : A sequence in which terms increase or decrease regularly by the same constant.

A sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called arithmetic sequence or arithmetic progression if $a_{n+1} = a_n + d$, $n \in N$, where a_n is called the nth term and the constant term d is called the common difference of the A.P.

The n^{th} term (general term) of the A.P. a, a + d, a + 2d, ... is $a_n = a + (n - 1) d$.

If a, b, c are in A.P. and k($\neq$0) is any constant, then

(i) a + k, b + k, c + k are also in A.P.

(ii) a – k , b – k , c – k are also in A.P.

(iii) ak, bk, ck are also in A.P

(iv) $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in A.P.

If a, a + d, a + 2d, ..., a + (n – 1) d be an A.P. Then l = a + (n – 1) d.

$$\text{Sum to n terms } S_n = \frac{n}{2}[2a + (n-1)d$$

$$= \frac{n}{2}[a + l]$$

Arithmetic mean (A.M.) between two numbers a and b is $\frac{a+b}{2}$.

n arithmetic means between two numbers a and b are $a + \frac{(b-a)}{n+1}, a + \frac{2(b-a)}{n+1}, a + \frac{3(b-a)}{n+1}, \ldots, a + \frac{n(b-a)}{n+1}$.

Sum of n A.M's = n($\frac{a+b}{2}$)

- Three consecutive terms in A.P. are $a - d$, a, $a + d$.
- Four consecutive terms in A.P. are $a - 3d$, $a - d$, $a + d$, $a + 3d$.
- Five consecutive terms in A.P. are $a - 2d$, $a - d$, a, $a + d$, $a + 2d$.

These results can be used if the sum of the terms is given.

In an A.P. the sum of terms equidistant from the beginning and end is constant and equal to the sum of first and last terms.

- m^{th} term from end of an A.P. = $(n - m + 1)^{th}$ term from the beginning.

Geometric Progression (G . P.) :A sequence is said to be a geometric progression or G.P., if the ratio of any term to its preceding term is same throughout.

A sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called geometric progression, if each term is non-zero and

$\frac{a_{k+1}}{a_k} = r(\text{constant})$, for $k \geq 1$.

By taking $a_1 = a$, we obtain a geometric progression, $a, ar, ar^2, ar^3, \ldots$, where a is called the first term and r is called the common ratio of the G.P.

General term of a G .P. = $a_n = ar^{n-1}$.

Sum to n terms of a G .P. = $\frac{a(r^n - 1)}{r - 1}$ if $r > 1$ and $\frac{a(1 - r^n)}{1 - r}$ if $r < 1$.

Sum of terms of an infinite G.P. = $\frac{a}{1 - r}$.

Geometric Mean (G .M.) of two positive numbers a and b is the number is $\sqrt{ab}$.

n geometric means between two numbers a and b are $a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}, a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}, a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}, \ldots a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$.

Product of n G.M's between a and b is $\left(\sqrt{ab}\right)^n$

- Three consecutive terms in G.P. are $\frac{a}{r}, a, ar$.
- Four consecutive terms in G.P. are $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.

❖ Five consecutive terms in G.P. are $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$.

These results can be used if the product of the terms is given.

Harmonic Progression: A series of quantities is said to be in harmonic progression if their reciprocals are in arithmetic progression.

n^{th} term of the H.P. $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \dots$ is $\frac{1}{a+(n-1)d}$.

Harmonic Mean between two quantities a & b is $\frac{2ab}{a+b}$

Relations b/w A(A.M.), G(G.M.) & H(H.M.)

(i) A, G, H are in G.P.

(ii) A, G, H are in descending order of magnitude i.e. A > G > H.

Arithmetico-geometric series : A type of series in which each term is the product of the corresponding terms of an A.P. and a G.P.

$a+(a+d)r+(a+2d)r^2+(a+3d)r^3+\dots$ is an arithmetico-geometric series.

Sum to n terms of the arithmetico-geometric series : $a+(a+d)r+(a+2d)r^2+(a+3d)r^3+\dots$

is $S=\frac{a}{1-r}+\frac{dr(1-r^{n-1})}{(1-r)^2}-\frac{[a+(n-1)d]r^n}{1-r}$

❖ Sum of an Infinite arithmetico-geometric series = $\frac{a}{1-r}+\frac{dr}{(1-r)^2}$.

Some useful results :

$$\sum n = 1+2+3+\dots+n = \frac{n(n+1)}{2}$$

$$\sum n^2 = 1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum n^3 = 1^3+2^3+3^3+\dots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

MCQ's

1) If a, 4, b are in Arithmetic Progression; a, 2, b are in Geometric Progression; then a, 1, b are in

a. A.P
b. G.P
c. H.P
d. None of these

Answer: (c) H.P

Explanation:

Given that a, 4, b are in A.P

Hence, 4-a = b-4

a+b = 8 …(1)

Also, given that a, 2, b are in G.P.

Hence, 2/a = b/2

So, ab = 4…(2)

If a, 1, b are in H.P, then 1 = 2(ab)/(a+b) ..(3)

Now substitute (1) and (2) in (3)

1 = 2(4) /(8)

1 = 8/8

1=1.

Therefore, a, 1, b are in H.P.

2) If "a" is the first term and "r" is the common ratio, then the nth term of a G.P is:

a. ar^n
b. ar^{n-1}
c. $(ar)^{n-1}$
d. None of these

Answer: (b) ar^{n-1}

Explanation:

If "a" is the first term and "r" is the common ratio, the terms of infinite G.P are written as a, ar, ar^2, ar^3, ar^4, …ar^{n-1}.

Hence, the nth term of a G.P is ar^{n-1}.

Therefore, option (b) is the correct answer.

3) If a, b, c are in arithmetic progression, then

a. $b = a+c$
b. $2b = a+c$
c. $b^2 = a+c$
d. $2b^2 = a+c$

Answer: (b) $2b = a+c$

Explanation:

Given that a, b, c are in arithmetic progression.

So, the common difference is $b-a = c-b$

Rearranging the same terms, we get

$b+b = c+a$

$2b = a+c$.

Hence, if a, b, c are in A.P, then $2b = a+c$.

4) The sum of arithmetic progression 2, 5, 8, …, up to 50 terms is

a. 3775
b. 3557
c. 3757
d. 3575

Answer: (a) 3775

Explanation:

Given A.P. = 2, 5, 8, …

We know that the sum of n terms of an A.P is $S_n = (n/2)[2a+(n-1)d]$

Here, a = 2, d = 3 and n=50.

Now, substitute the values in the formula, we get

$S_{50} = (50/2)[2(2)+(50-1)(3)]$

$S_{50} = 25[4+(49)(3)]$

$S_{50} = 25[4+147]$

$S_{50} = 25(151)$

$S_{50} = 3775$.

Hence, the sum of A.P 2, 5, 8, …up to 50 terms is 3775.

5) The 3rd term of G.P is 4. Then the product of the first 5 terms is:

a. 4^3

b. 4^4
c. 4^5
d. None of these

Answer: (c) 4^5

Explanation:

We know that the terms of infinite G.P are written as a, ar, ar^2, ar^3, ar^4, …ar^{n-1}.

Hence, the 3rd term, (i.e) ar^2= 4

Thus, the product of the first 5 terms = $(a)(ar)(ar^2)(ar^3)(ar^4)$

$= a^5r^{10}$

$= (ar^2)^5$

Now, substitute $ar^2 = 4$ in the above form, we get

Product of the first 5 terms = $(4)^5 = 4^5$.

Hence, option (c) 4^5 is the correct answer.

6) Which of the following is an example of a geometric sequence?

a. 1, 2, 3, 4
b. 1, 2, 4, 8
c. 3, 5, 7, 9
d. 9, 20, 21, 28

Answer: (b) 1, 2, 4, 8

Explanation:

Among the options given, option (b) 1, 2, 4, 8 is an example of a geometric sequence.

We know that in a geometric sequence each term is found by multiplying the previous term by a constant.

In option (b) 1, 2, 4, 8, each term is found by multiplying 2 to the previous term. Here, the common ratio is 2.

7) The next term of the given sequence 1, 5, 14, 30, 55, … is

a. 80
b. 90
c. 91
d. 96

Answer: (c) 91

Explanation: The next term in the sequence 1, 5, 14, 30, 55, … is 91.

Ist term = $1^2 = 1$

2nd term = 1^2+2^2 = 1+4 = 5

3rd term = $1^2+2^2+3^2$ = 1+4+9 = 14

4th term = $1^2+2^2+3^2+4^2$ = 1+4+9+16 = 30

5th term = $1^2+2^2+3^2+4^2+5^2$ = 1+4+9+16+25 = 55

6th term = $1^2+2^2+3^2+4^2+5^2+6^2$ = 1+4+9+16+25+36 = 91.

Thus, option (c) is the correct answer.

8) If the nth term of an arithmetic progression is 3n-4, then the 10th term of an A.P is

a. 10
b. 12
c. 22
d. 26

Answer: (d) 26

Explanation:

Given that the nth term of A.P = 3n-4.

To find the 10th term of A.P, substitute n = 10

Therefore, 10th term of A.P = 3(10) -4 = 30-4 = 26.

9) 3, 5, 7, 9 is an example of

a. Arithmetic sequence
b. Geometric sequence
c. Harmonic sequence
d. Fibonacci sequence

Answer: (a) Arithmetic sequence

Explanation: 3, 5, 7, 9 is an example of an arithmetic sequence. In this sequence 3, 5, 7, 9, the difference between each term is 2.

(i.e) 5-3 = 2, 7-5 = 2, 9-7 = 2.

Hence 3, 5, 7, 9 is an arithmetic sequence.

10) The first term of a G.P is 1. The sum of the 3rd and 5th terms is 90. Then the common ratio is:

a. 1
b. 2
c. 3
d. 4

Answer: (c) 3

Explanation:

Given that first term of G.P, a = 1.

The sum of the 3rd and 5th term = 90

(i.e) $ar^2+ar^4 = 90$

Substitute a = 1,

$\Rightarrow r^2+r^4 = 90$

$\Rightarrow r^4 + r^2 - 90 = 0$

$\Rightarrow r^4 + 10r^2 - 9r^2 - 90 = 0$

Now, factorize the above equation,

$\Rightarrow r^2 (r^2+10) - 9 (r^2+10) = 0$

$\Rightarrow (r^2-9)(r^2+10) = 0$

$\Rightarrow r^2 = 9$ or $r^2 = -10$

Here, $r^2= -10$ is not possible, as the square of a number cannot be negative.

So, $r^2 = 9$

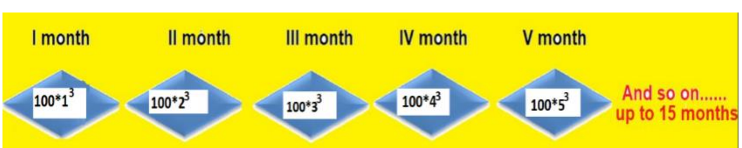
r = 3 or r= -3

Therefore, option (c) 3 is the correct answer.

Case Study Based Questions

1. Question No. 1 to 5 are based on the given text. Read the text carefully and answer the questions:

Ratan wants to open an RD for the marriage of his daughter, He visited the branch of SBI at sector 3, Gurgaon. There he made an agreement with the bank.



According to this agreement, he would deposit₹ $100 \times n^3$ every month (here n =1 to 15). Other terms and conditions areas follows:

1. He has to pay a minimum of six instalments.
2. If he continues the deposit up to 15 months then the bank will pay 20% extra as a bonus.
3. If he breaks the deposit after 6 months then the bank will pay 10% extra as a bonus
4. If he breaks the deposit after 10 months then the bank will pay 15% extra as a bonus.

No other interest will be paid by the bank.

Ques.1 How much amount would be accumulated after 15 months?

a) ₹ 10,00,000

b) ₹ 11,02,500

c) ₹ 15,00,000

d) ₹ 14,40,000

Answer. (d) ₹ 14,40,000

Ques.2 How much total amount would Ratan get after 15 months?

a) ₹ 14,40,000

b) ₹ 13,23,000

c) ₹ 17,28,000

d) ₹ 15,00,000

Answer. (c) ₹ 17,28,000

Ques.3 How much total amount would Ratan get if he breaks the deposit after 10 months?

a) ₹ 3,50,000

b) ₹ 3,23,000

c) ₹ 3,47,875

d) ₹ 3,45,875

Answer. (c) ₹ 3,47,875

Ques.4 How much total amount would Ratan get if he breaks the deposit after 6 months?

a) ₹ 60,000

b) ₹ 50,715

c) ₹ 50,000

d) ₹ 65,875

Answer. (b) ₹ 50,715

Ques.5 How much amount did Ratan pay in the 10th month?

a) ₹ 729,000

b) ₹ 50,715

c) ₹ 1,00,000

d) ₹ 60,000

Answer. (c) ₹ 1,00,000

2. Question No. 6 to 10 are based on the given text. Read the text carefully and answer the questions:

Shamshad Ali buys a scooter for₹ 22,000. He pays ₹ 4,000 cash and agree to pay the balance in annual instalments of ₹ 1000 plus 10% interest on the unpaid amount.

Ques.6 Interest paid by Ali on 3rd instalment?

a) ₹ 1500

b) ₹ 1700

c) ₹ 1800

d) ₹ 1600

Answer. (d) ₹ 1600

Explanation: Unpaid amount = ₹ 17000 - ₹ 1000 = 16000 Interest on 2nd instalment = $\frac{16000\times10\times1}{100} = 1600$

Ques.7 Interest paid by Ali on 2nd instalment?

a) ₹ 1500

b) ₹ 1800

c) ₹ 1600

d) ₹ 1700

Answer. Explanation: Unpaid amount = 18000 - 1000 = 17000.

Interest on 2nd instalment= $\frac{17000\times10\times1}{100} = 1700$

Ques.8 Interest paid by Ali on 1st instalment?

a) ₹ 1700

b) ₹ 1900

c) ₹ 1800

d) ₹ 1600

Answer. (c) ₹ 1800

Explanation: Scooter cost = ₹ 22,000 Down payment = ₹ 4,000.

Balance payment = ₹ 18,000

Now, interest on 1st instalment= $\frac{18000\times10\times1}{100} = 1800$

Ques.9 How much will scooter cost him?

a) ₹ 17100

b) ₹ 39100

c) ₹ 22000

d) ₹ 29100

Answer. (b) ₹ 39100

Explanation: Total amount or actual cost = 22000 + 17100 = ₹ 39100.

Ques.10 Total interest paid by Ali is:

a) ₹ 22000

b) ₹ 17100

c) ₹ 25000

d) ₹ 39100

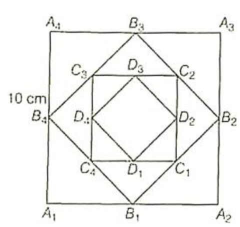
Answer. (b) ₹ 17100

Explanation: Total interest paid by him

= 1800 + 1700 + 1600 + ... + 18 terms

which is an A.P. with a = 1800, d = 1700 - 1800 = -100 Therefore, total interest

3. Question No. 11 to 15 are based on the given text. Read the text carefully and answer the questions: A student of class XI draw a square of side 10 cm. Another student join the mid - point of this square to form new square. Again, the mid - points of the sides of this new square are joined to form another square by another student. This process is continued indefinitely.



Ques.11 The sum of the perimeter of all the square formed is (in cm)

a) 40

b) $40 + 40\sqrt{2}$

c) $80 + 40\sqrt{2}$

d) $40\sqrt{2}$

Answer. (c) $80 + 40\sqrt{2}$

Ques.12 The sum of areas of all the square formed is (in sq cm)

a) 250

b) 300

c) 200

d) 150

Answer. (c) 200

Ques.13 The perimeter of the 7th square is (in cm)

a) $\frac{5}{2}$

b) 10

c) 5

d) 20

Answer. (c) 5

Ques.14 The area of the fifth square is (in sq cm)

a) $\frac{25}{4}$

b) 25

c) 50

d) $\frac{25}{2}$

Answer. (a) $\frac{25}{4}$

Ques.15 The side of fourth square is (in cm)

a) $\frac{\sqrt{5}}{2}$

b) $\sqrt{5}$

c) None of these

d) 5

Answer. (c) None of these

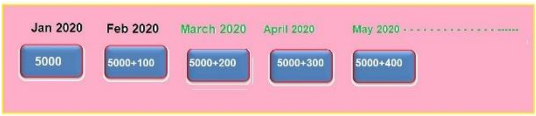
4. Question No. 16 to 20 are based on the given text. Read the text carefully and answer the questions:

A farmer ,Ramgarh, took a bank loan from SBI for repairing his house. But he could not pay the amount on time.

This resulted in the accumulation of interest and the amount to pay reached ₹ 1,00,000.

After a few months, the farmer opened a shop that resulted in enough income and the income increased on a regular basis. So he decided to pay the bank loan in a different manner.

The farmer visited the bank. He made an agreement with the bank that he will start paying the amount of ₹ 1,00,000 without interest from Jan 2020. In January he will pay ₹ 5000 and will increase the payment by ₹ 100 in each month, as shown in the figure.



Ques.16 In how many months will the farmer clear the loan amount?

a) 18

b) 15

c) 16

d) 20

Answer. (a) 18

Ques.17 How much amount he has to pay in last month in rupees?

a) 1500

b) 2000

c) 1400

d) 1800

Answer. (c) 1400

Ques.18 In which month he will pay₹ 6000?

a) 14th

b) 10th

c) 11th

d) 12th

Answer. (c) 11th

Ques.19 How much amount he will pay in 10thmonth in₹ ?

a) 6000

b) 7000

c) 7500

d) 6400

Answer. (d) 6400

Ques.20 How much amount in rupees till 10th month he will have paid?

a) 60000

b) 55000

c) 54500

d) 50000

Answer. (c) 54500

5. Question No. 21 to 25 are based on the given text. Read the text carefully and answer the questions:

Father of Ashok is a builder, He planneda 12 story building in Gurgaon sector 5. For this, he bought a plot of 500 square yards at the rate of₹ 1000 /yard².The builder planned ground floor of 5 m height, first floor of 4.75 m and so on each floor is 0.25 m less than its previous floor.

Ques.21 What is the height of the last floor?

a) 2.5 m

b) 3 m

c) 2.75 m

d) 2.25 m

Answer. (d) 2.25 m

Ques.22 Which floor no is of 3 m height?

a) 10

b) 7

c) 5

d) 9

Answer. (d) 9

Ques.23 What is the total height of the building?

a) 40.5 m

b) 44 m

c) 40 m

d) 43.5

Answer. (d) 43.5

Ques.24 Up to which floor the height is 33 m?

a) 8

b) 7

c) 9

d) 10

Answer. (a) 8

Ques.25 Which floor no. is half in height of ground floor?

a) 10

b) 12

c) 9

d) 11

Answer. (d) 11

<u>Assertion Reasoning Type Questions</u>

Ques.1 Assertion (A): A sequence is said to finite if it has finite no of terms.

Reason (R): The n^{th} term of the sequence: 2, 2, $\frac{8}{3}$, 4..is $\frac{2^n}{n}$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Answer (b) Both A and R are true but R is not the correct explanation of A

Ques.2 Assertion (A): The sum of infinite terms of a geometric progression is given by $S_\infty = \frac{a}{1-r}$, provided $|r| < 1$.

Reason (R): The sum of n terms of Geometric progression is $S_n = \frac{a(r^n-1)}{r-1}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Answer (b) Both A and R are true but R is not the correct explanation of A.

Ques.3 Assertion (A): If the numbers $\frac{-2}{7}$, K, $\frac{-7}{2}$ are in GP, then k = ± 1 .

Reason (R): If a_1 , a_2 , a_3 are in GP, then $\frac{a_2}{a_1} = \frac{a_3}{a_2}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Answer (a) Both A and R are true and R is the correct explanation of A

Ques.4 Assertion (A): If the sum of first two terms of an infinite GP is 5 and each term is three times the sum of the succeeding terms, then the common ratio is $\frac{1}{4}$.

Reason (R): In an AP 3, 6, 9, 12 .... the 10th term is equal to 33.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Answer (c) A is true but R is false

Ques.5 Assertion (A): The sum of first 6 terms of the GP 4, 16, 64, ... is equal to 5460.

Reason (R): Sum of first n terms of the G.P is given by $S_n = \frac{a(r^n-1)}{r-1}$, where a = first term r = common ratio and |r| > 1.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Answer (a) Both A and R are true and R is the correct explanation of A.

Short Answer Type Questions

Ques.1 Find the sum of first 8 terms of the G.P. 10, 5, $\frac{5}{2}$,...

Answer. We have a=10 and r=1/2 and n=8

Now we have to find S_8

so, $S_n = \frac{a(1-r^n)}{1-r}$

$S_8 = \frac{10(1-(\frac{1}{2})^8)}{1-\frac{1}{2}}$

After calculating we have $S_8 = \frac{2550}{128}$.

Ques.2 Find the sum of the geometric series 3 + 6 + 12 + ... + 1536.

Answer. Given series 3 + 6 + 12 + + 1536.

Here we have

r = 6÷3= 2 $a_n = ar^{n-1}$

⇒ 1536 = 3 × 2^{n-1}

⇒ 1536 ÷ 3 = 2^n ÷ 2

⇒ 1536 ÷ 3 × 2 = 2^n

⇒ 1024 = 2^n

⇒ 2^{10} = 2^n (on comparing)

∴ n = 10

Now, a = 3 and r = 2 and n = 10 terms

$s_n = \frac{a(r^n-1)}{r-1}$

Sn = 3 × (1024 - 1)

Sn = 3069

Therefore, the sum of n term of G.P. is 3069

Ques.3 Find three numbers in GP whose sum is 65 and whose product is 3375.

Answer. Let the terms of the given G.P. be

$\frac{a}{r}$, a, and ar.

then, their product of the = 3375

⇒ a^3 = 3375

⇒ a = 15

Similarly, sum of the G.P. = 65

⇒$\frac{a}{r}$+ a + ar = 65 Substituting the value of a

$\frac{15}{r}$+ 15 + 15r = 65

⇒ $15r^2 - 50r + 15 = 0$

⇒ 5(3r2 - 10r + 3) = 0

⇒ 3r2 - 10r + 3 = 0

$(3r-1)(r-3) = 0 \Rightarrow r = \frac{1}{3}\ , r = 3$

for a = 15 and r = 3

Therefore the required three numbers are 5, 15, 45

Ques:4 A GP's 5^{th}, 8^{th} and 11^{th} terms are p, q and s respectively. Prove that $q^2 = ps$

Ans. As a result from the given question, p is the 5^{th} term, $q$is the 8^{th} term and s is the 11^{th} term

We have to prove that $q^2 = ps$. we may express the equation as $a_5 = ar^4 = p$ using the above information________(1)

$a_8 = ar^7 = q \ldots \ldots.(2)$

$a_{11} = ar^{10} = s \ldots \ldots(3)$

when we divide (2) by (1), we get $r^3 = \frac{q}{p} \ldots \ldots.(4)$

when we divide (3) by (2), we get $r^3 = \frac{s}{q} \ldots \ldots.(5)$

so from equations (4) and (5) we get q/p=s/q⇒ $q^2 = ps$.

Ques.5 A man saved Rs. 66000 in 20 years. In each succeeding year after the first year, he saved Rs. 200 more than what he saved in the previous year. How much did he save in the first year?

Sol: Let us assume that the man saved Rs. a in the first year.

In each succeeding year, an increment of Rs. 200 is made. So, it forms an A.P. whose

First term = a, Common difference, d = 200 and n=20

$$\therefore \qquad S_{20} = \frac{20}{2}[2a + (20-1)d]$$

=> 6600 = 2a + 19 x 200 => 2a = 2800

∴a = 1400

Ques.6 The third term of a geometric progression is 4. When the first five terms are multiplied together, what is the result?

Ans: T_3= 4 is deduced from the given question.

⇒ar^2= 4

The product of the first five terms can now be defined as = a.ar.ar2.ar3.ar4.

= a^5r^{10}

=$(ar^2)^5$

= 4^5

As a result, the product of the first five terms is 4^5.

Ques.7 13/6 is the sum of two numbers. An even number of mathematical means are placed between them, and their sum exceeds their number by one. What is the number of inserted means multiplied by two?

Assume a and b are two numbers such that

a + b = 13/6

TakeA_1, A_2,, A_3,,..........A_{2n}, is the arithmetic mean of a and b.

Then, A_1 + A_2 + A_3 +..........+ A_{2n} = 2n(n + 1)/2

⇒ n(a + b) = 13n/6

Given this, the series A_1 + A_2 + A_3 +..........+ A_{2n} = 2n + 1

13n/6 = 2n + 1, so

⇒ n = 6

Ques.8 Show that the sum of (m + n)th and (m – n)th terms of an A.P. is equal to twice the mth term.

Ans: Let a and d represent the A.P.'s initial term and common difference, respectively. It is well known that an A.P.'s nth term is given by $a_n = a + (n-1)d$

∴ $a_{m+n} = a + (m+n-1)d$ and $a_{m-n} = a + (m-n-1)d$

Adding these two ,we have

$$a_{m+n} + a_{m-n} = 2a + (2m-2)d$$
$$= 2\{a + (m-1)d\}$$
$$= 2a_m$$

Hence proved.

Ques.9 Find the sum of integers from 1 to 100 that are divisible by 2 or 5.

Ans: The divisible by 2 integers from 1 to 100 are 2, 4, 6,.... 100.

This results in an A.P. in which the first term and the common difference are both equal to 2.

⇒ 100=2+(n-1)2

⇒ n= 50

So, the sum of integers from 1 to 100 which are divisible by 2 is given as:

2+4+6+…+100 = (50/2)[2(2)+(50-1)(2)]

= (50/2)(4+98)

= 25(102)

= 2550

Divisible by 5, 10,.... 100 integers from 1 to 100.

This generates an A.P. with a common difference of 5 and a first term of 5.

Then, 100= 5+(n-1)5

⇒5n = 100

⇒ n= 100/5

⇒ n= 20

As a result, the total of all divisible by 2 integers from 1 to 100 is:

5+10+15+…+100= (20/2)[2(5)+(20-1)(5)]

= (20/2)(10+95)

= 10(105)

= 1050

As a result, the divisible by both 2 and 5 integers from 1 to 100 are 10, 20,.... 100.

This also creates an Arithmetic Progression. because the first term and the common difference are both equal to ten.

Then, 100= 10+(n-1)10

⇒10n = 100

⇒ n= 100/10

⇒ n= 10

10+20+…+100= (10/2)[2(10)+(10-1)(10)]

= (10/2)(20+90)

= 5(110)

= 550

So, the required sum is:

= 2550+ 1050 – 550

= 3050

As a result, the sum of the divisible by 2 or 5 integers from 1 to 100 is 3050.

Ques.10 Between 1 and 31, m numbers have been introduced in such a way that the resulting sequence is an A.P., with a 7:(m – 1) ratio of 5:9. Calculate the value of m.

Ans: Let assume 1, A1, A2, …., Am, 31 are in A.P.

a = 1, an = 31

a_{m+2} = 31

a_n = a + (n - 1)d

31 = a + (m + 2 - 1)d

d = 30/(m + 1)

Now,A_7 =7 inserted number=a+7d

Similarly,A_{m-1} =a+(m-1)d

$A_7 : A_{m-1}$ = 5:9(given)

$\frac{a+7d}{a+(m-1)d}=\frac{5}{9}$

$\frac{1+7\frac{30}{m+1}}{1+(m-1)\frac{30}{m+1}}=\frac{5}{9}$

On simplification

⇒m = 14

<u>Long Answer Type Questions</u>

Ques.1 :If a, b, and c are in G.P., $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ share a common root then prove that d/a, e/b, and f/c are in A.P.

Ans. (a) A.P

Solution:

Given that a, b, c are in GP (series)

⇒ $b^2 = ac$

⇒ $b^2 – ac = 0$

Hence, $ax^2 + 2bx + c = 0$ have equal roots.

Now the D = $4b^2 – 4ac$

and the root of the above equation is -2b/2a = -b/a

So -b/a is the common root of the above quadratic equation.

Now,

$dx^2 + 2ex + f = 0$

⇒ $d(-b/a)^2 + 2e×(-b/a) + f = 0$

⇒ $db^2 /a^2 – 2be/a + f = 0$

⇒ $d×ac /a^2 – 2be/a + f = 0$

⇒ dc/a – 2be/a + f = 0

⇒ d/a – 2be/ac + f/c = 0

⇒ d/a + f/c = 2be/ac

⇒ d/a + f/c = 2be/b^2

⇒ d/a + f/c = 2e/b

⇒ d/a, e/b, f/c are in arithmetic progression (AP).

Ques.2 :Let the T_r be the r^{th} term of an A.P Series., for r = 1, 2, 3, … If for some positive integers m, n, we have T_m = 1/n and T_n = 1/m, then find the value of T_{mn}.

Ans: Let's assume the first term is a has the common difference of d in the AP

Then, T_m = 1/n

⇒ a + (m-1)d = 1/n ………… (1)

and T_n = 1/m

⇒ a + (n-1)d = 1/m ………. (2)

From equation (1) – (2), we get

(m-1)d – (n-1)d = 1/n – 1/m

⇒ (m-n)d = (m-n)/mn

⇒ d = 1/mn

From equation (1), we get

a + (m-1)/mn = 1/n

⇒ a = 1/n – (m-1)/mn

⇒ a = {m – (m-1)}/mn

⇒ a = {m – m + 1)}/mn

⇒ a = 1/mn

Now, then T_{mn}= 1/mn + (mn-1)/mn

⇒T_{mn} = 1/mn + 1 – 1/mn

⇒T_{mn}= 1

Ques.3: If the sum of the roots of the quadratic equation ax^2 + bx + c = 0 is equal to the sum of the squares of their reciprocals, then a/c, b/a, c/b are in which progression.

Ans: From the given question, the equation is

ax^2 + bx + c = 0

Let p and q are the roots of the above equation.

Now p+q = -b/a

and pq = c/a

Given that, the p + q = $1/p^2 + 1/q^2$, then

⇒ p + q = $(p^2 + q^2)/(p^2 \times q^2)$

⇒ p + q = $\{(p + q)^2 – 2pq\}/(pq)^2$

⇒ -b/a = $\{(-b/a)^2 – 2c/a\}/(c/a)^2$

⇒ $(-b/a)\times(c/a)^2 = \{b^2/a^2 – 2c/a\}$

⇒ $-bc^2/a^3 = \{b^2 – 2ca\}/a^2$

⇒ $-bc^2/a = b^2 – 2ca$

Divide both sides by bc, we get the following

⇒ -c /a = b/c – 2a/b

⇒ 2a/b = b/c + c/a

⇒ b/c, a/b, c/a are in AP

⇒ c/a, a/b, b/c are in AP

$$\Rightarrow \frac{1}{\frac{c}{a}}, \frac{1}{\frac{a}{b}}, \frac{1}{\frac{b}{c}} \, are \ in \ HP$$

⇒ a/c, b/a, c/b are in Harmonic Progression

Ques 4.Three numbers are in A.P.If the sum of these numbers is 27 and the product is 648,find the numbers.

Ans:Let the three numbers are $a - d, a \ \& \ a + d$

$\therefore a - d + a + a + d = 27$ and$(a - d)a(a + d) = 648$

$\therefore 3a = 27$ and $a(a^2 - d^2) = 648$

$\therefore a = 9$ and $9(81 - d^2) = 648$

$$or, (81 - d^2) = 72$$

$$or, d^2 = 9$$

$$\therefore d = \pm 3$$

Hence,the numbers are 6,9 and 12.

Ques.5 Find the sum of integers from 1 to 100 that are divisible by 2 or 5.

Ans: The divisible by 2 integers from 1 to 100 are 2, 4, 6,.... 100.

This results in an A.P. in which the first term and the common difference are both equal to 2.

⇒ 100=2+(n-1)2

⇒ n= 50

So, the sum of integers from 1 to 100 which are divisible by 2 is given as:

2+4+6+…+100 = (50/2)[2(2)+(50-1)(2)]

= (50/2)(4+98)

= 25(102)

= 2550

Divisible by 5, 10,.... 100 integers from 1 to 100.

This generates an A.P. with a common difference of 5 and a first term of 5.

Then, 100= 5+(n-1)5

⇒5n = 100

⇒ n= 100/5

⇒ n= 20

As a result, the total of all divisible by 2 integers from 1 to 100 is:

5+10+15+…+100= (20/2)[2(5)+(20-1)(5)]

= (20/2)(10+95)

= 10(105)

= 1050

As a result, the divisible by both 2 and 5 integers from 1 to 100 are 10, 20,.... 100.

This also creates an Arithmetic Progression. because the first term and the common difference are both equal to ten.

Then, 100= 10+(n-1)10

⇒10n = 100

⇒ n= 100/10

⇒ n= 10

10+20+…+100= (10/2)[2(10)+(10-1)(10)]

= (10/2)(20+90)

= 5(110)

= 550

So, the required sum is:

= 2550+ 1050 – 550

= 3050

As a result, the sum of the divisible by 2 or 5 integers from 1 to 100 is 3050.

Practice Questions

Multiple choice questions

1.The n^{th} term of the sequence $\left\{1,\frac{1}{8},\frac{1}{27},\frac{1}{64}\right\}$ will be

(a)$\frac{1}{n}$ (b)$\frac{1}{n^2}$ (c))$\frac{1}{n^3}$ (d)$\frac{1}{n^3-1}$

2.The sum of five numbers which are in A.P is 50,the third number is

(a)2 (b)5 (c)10 (d)15

3.Let a_1,a_2, a_3, … , a_{40} are in A.P and $a_1 + a_5 + a_{15} + a_{26} + a_{36} + a_{40} = 105$ then sum of that series is

(a)650 (b)700 (c)1400 (d)None of these

4.In an A.P p^{th} term is q and $(p+q)^{th}$ term is 0.Then the q^{th} term will be

(a)$-p$ (b))p (c))$p+q$ (d))$p-q$

5.A G.P of positive numbers is such that each of its term is equal to the sum of the next two terms. Then the common difference is

(a)$\sqrt{5}$ (b))$\frac{\sqrt{5}-1}{2}$ (c)$\frac{\sqrt{5}}{2}$(d)$\frac{\sqrt{5}+1}{2}$

6.The third term of a G.P is 9,then the product of the first five term is

(a)3^9 (b)3^{10}(c)3^{11}(d)3^{12}

7.If 1,$a$and p are in A.P and 1,$g$and p are in G.P then

(a)$1+2a+g^2=0$ (b)$1+2a-g^2=0$(c)$1-2a-g^2=0$ (d)$1-2a+g^2=0$

8.The first term of a G.P is 1.The sum of the 3rd and 5th term is 90,then the common ratio

(a)1 (b)2 (c)3 (d)4

9.The minimum value of $2^{sin^2\theta} + 2^{cos^2\theta}$ is

(a)$\sqrt{2}$ (b))$2\sqrt{2}$ (c)4 (d)2

10.If $a^x = b^y = c^z$, where a, b, c are in G.P and $a, b, c, x, y, z \neq 0$ then x, y, z are in

(a)A.P (b)G.P (c)H.P (d) none of these.

Assertion -Reason based questions:

In the following questions, a statement of Assertion(A) is followed by a statement of Reason (R).Choose the correct answer out of the following choices:

(a)Both (A) and (R) are true and (R) is the correct explanation of (A)

(b)Both (A) and (R) are true but (R) is not the correct explanation of (A)

(c)(A) is true and (R) is false

(d)(A) is false and (R) is true

1. Assertion: If x, 2y and 3z are in A.P. where the distinct numbers x,y and z are in G.P., then the common ratio of the G.P. is$\frac{1}{3}$.

Reason: The G.P relation between x, y and z is $y^2 = xz$

2.Assertion :the sum of odd integers from 1 to 2001 is 1002001

Reason :Then the n^{th} term (general term) of the A.P. is $a_n = a + (n-1)d$.

3. Assertion :The minimum value of $4^x + 4^{1-x}, x \in R$ is 4

Reason: A.M is greater than or equal to G.M

4. Assertion : the products of the corresponding terms of the sequences $a, ar^2, \ldots, ar^{n-1}$ and $A, AR^2, \ldots, AR^{n-1}$form a G.P

Reason:Products of the corresponding terms of two sequences form a G.P

5.Assertion: Six numbers between 3 and 24 such that the resulting sequence is an A.P are 6,9,12,15,18 and 21

Reason : Let $A_1, A_2, \ldots, A_n$ be n numbers between a and b such that a,$A_1, A_2, \ldots, A_n$, b is an A.P then $A_n = a + nd, d =$common difference.

Short answer type questions:

1.If $\frac{x^{n+1}+y^{n+1}}{x^n+y^n}$ is the A.M of $x\ and\ y$,then find the value of n.

2.If 51+53+55+…+t_n=5151 then fint the value of t_n

3.The first and n^{th} term of a G.P are a and b respectively. If p is the product of first n terms then show that $p^2 = (ab)^n$.

4.If $\frac{2}{29}$ and $\frac{2}{41}$ are the 10^{th} and 14^{th} term of a H.P then find t_n.

5.If the $p^{th}, q^{th},\ and\ r^{th}$ term of a G.P are a ,b,c respectively then show that $a^{q-r}.b^{r-p}.c^{p-q} = 1$.

6.Find three in G.P whose sum is 19 and product is 216.

7.In a G.P the first term is 3 and the last term is 48.If each term is twice the previous term then find the total term and summation of that G.P.

8.In an A.P sum of the n terms is $3n^2 + 5n$;then which term will be 152?

9.There are n arithmetic mean between 4 and 31.If 2^{nd} A.M: last A.M=5:14 ,then find the value of n.

10.Insert four geometric mean between $\frac{7}{4}$ and 56.

Long answer type questions

1.Find the sum of the following series upto n terms:

(i).8+.88+.888+…

(ii)1+11+111+…

(iii)1+4+13++40+121+…

(iv)$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\cdots$

2.p^{th}, q^{th} and r^{th}terms of an A.P are P, Q and R respectively. Then show that p(Q-R)+q(R-P)+r(P-Q)=0.

3.If the A.M of two numbers is twice their G.M ,then show that the numbers are in the ratio

$(2+\sqrt{3}):(2-\sqrt{3})$.

4.The sum of n,2n and 3n terms of an A.P are S_1, S_2 and S_3 respectively. Then show that $S_3=3(S_2-S_1)$.

5.If $a_1=2$ and $a_n-a_{n-1}=2n, n\geq 2$ then find the value of $a_1+a_2+a_3+\cdots+a_{20}$.

Case Based Questions

1.India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs.The production of tv sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600in 9th year.

Based on the above information , answer the following questions:

(i)Find the production during the first year.

(ii)Find the production during the 8th year.

(iii)Find the production during first 3 years.

(iv)In which year, the production is 29200.

2.Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of Rs 1,18,000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month

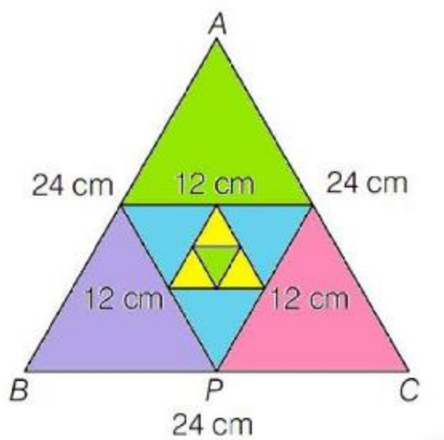
Based on the above story , answer the following:

(i)Find the amount paid by him in 30^{th} instalment .

(ii) What amount paid by him in the 30 instalment?

(iii)What amount does he still have to pay after 30^{th} instalment?

3. In Rangoli competition in school, Preeti made Rangoli in the equilateral shape. Each side of an equilateral triangle is 24 cm. The mid-point of its sides are joined to form another triangle. This process is going continuously infinite.



Based on above information, answer the following questions.

(i) Find the side of the 5th triangle is (in cm)

(ii) Find the sum of perimeter of first 6 triangle is (in cm)

4.Rahul being a plant lover decides to open a nursery and he bought few plants with pots. He wants to place pots in such a way that number of pots in first row is 2,in second row is 4 and in third row is 8 and so on…

Based on the above information answer the following questions:

(i)The constant multiple by which the number of pots is increasing in every row is

(a) 2 (b) 4 (c)8 (d) 1

(ii)The number of pots in 8^{th} row is

(a)156 (b)256 (c)300 (d)456

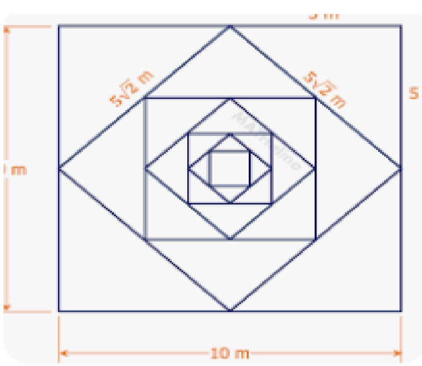
(iii)The difference in number of pots placed in 7^{th} row and 5^{th} row is

(a)86 (b)50 (c)90 (d)96

(iv) Total number of pots upto 10^{th} row is

(a)1046 (b)2046 (c) 1023 (d)1024

5.A student of class xi draw a square of 10 cm. Another student join the mid points of this square to form new square. Again, the mid points of the sides of this new square are joined to form another square by another student. This process is continued indefinitely



Based on the above answer the following questions

(i)what is the measurement of the side of the 4^{th} square?

(ii)what is the area of the 5^{th} square ?

(iii)what is the perimeter of the 7^{th} square?

(iv)Find the sum of the all squared formed.

Answer key:

MCQ:

1.(c) 2(c) 3(b) 4(b) 5(b) 6(b) 7(d) 8(c) 9(b) 10(c)

ASSERTION-REASON TYPE

1(a)2(a)3(a)4c()5(c)

SHORT ANSWER TYPE

1.0

2.151

4.$\frac{2}{3n-1}$

6.4,6,9

7.5&93

8.25th

9.8

10.$\frac{7}{2}$,7,14,28

<u>LONG ANSWER TYPE:</u>

1.(i)$\frac{8n}{9}-\frac{8}{81}(1-10^{-n})$

(ii)$\frac{10}{81}(10^{n}-1)-\frac{n}{9}$

(iii)$\frac{3}{4}(3^{n}-1)-\frac{n}{2}$

(iv)$\frac{n}{2n+1}$

5.3080

CASED BASE QUESTIONS

1.(i)5000 (ii)20400 (iii)21600 (iv)12

2.(i)3900 (ii)73500 (iii)44500

3.(i)1.5 cm (ii)$\frac{567}{4}\ cm$

4.(i)(a) (ii)(b) (iii)(d) (iv)(b)

5.(ii)$\frac{25}{4}$ (iii)5 (iv)200s

<u>**STRAIGHT LINES**</u>

CONCEPTS:

1. Distance formula

Distance between the points P (x_1, y_1) and Q (x_2 , y_2)

PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

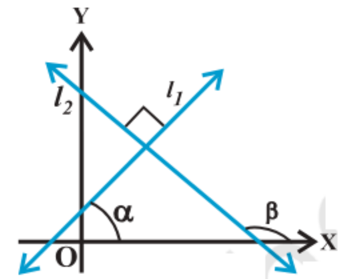
2. The coordinates of a point dividing the line segment joining the points (x_1, y_1) and (x_2-y_2) in the ratio m:n .

i) Internal division : $(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n})$

ii) External division : $(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n})$

3 Mid point formula :

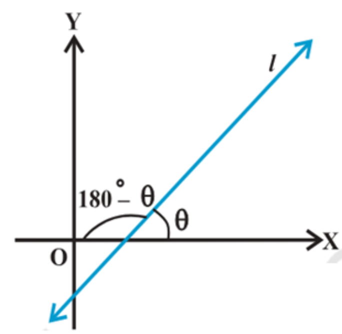
$(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2})$



4 Area of a triangle , whose vertices are (x_1,y_1) (x_2,y_2) and (x_3 , y_3) is

= $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

5 Slope of a line: A line in a coordinate plane forms two angles with the x-axis, which are supplementary. The angle θ made by the line l with positive direction of x-axis and measured anticlockwise is called the inclination of the line. Obviously 0° ≤ θ ≤ 180°

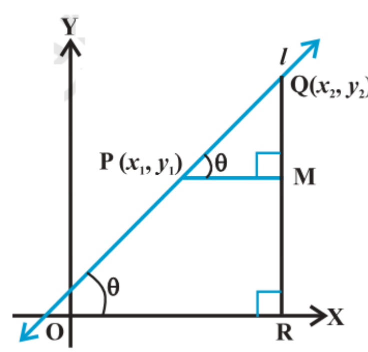


Definition 1 : If θ is the inclination of a line l, then tan θ is called the slope or gradient of the line l. The slope of a line whose inclination is 90° is not defined.

The slope of a line is denoted by m. Thus, m = tan θ, θ ≠ 90° It may be observed that the slope of the x-axis is zero and slope of the y-axis is not defined.

6. Slope of a line when coordinates of any two points on the line are given.

Let P(x_1 ,y_1) and Q(x_2 ,y_2) be two points on non-vertical line l whose inclination is θ. Obviously $x_1 \neq x_2$, otherwise the line will become perpendicular to x-axis and its slope will not be defined. The inclination of the line l may be acute or obtuse.



Therefore slope of line is m = $\frac{y_2-y_1}{x_2-x_1}$

7. Conditions for parallelism and perpendicularity of lines in terms of their slopes in a coordinate plane. suppose that non-vertical lines l_1 and l_2 have slopes m_1 and m_2 , respectively. Let their inclinations be α and β, respectively. If the line l_1 is parallel to l_2 then their inclinations are equal, i.e., α = β, and hence,

$\tan\alpha = \tan\beta$

Therefore $m_1 = m_2$, i.e. their slopes are equal.

Conversely, if the slope of two lines l_1 and l_2 is same, i.e., $m_1 = m_2$. Then $\tan\alpha = \tan\beta$.

By the property of tangent function (between 0° and 180°), $\alpha = \beta$.

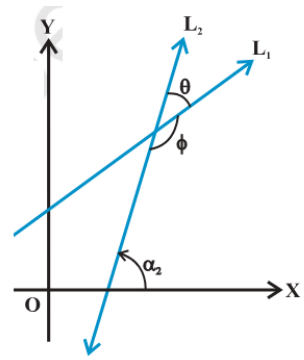
Therefore, the lines are parallel.

If the lines l_1 and l_2 are perpendicular . Then $\beta = \alpha + 90°$.

Therefore, $\tan\beta = \tan(\alpha + 90°) = -\cot\alpha = \frac{1}{tan\alpha}$ i.e. $m_2 = \frac{-1}{m_1}$ or $m_1m_2 = -1$

Conversely, if $m_1m_2 = -1$ i.e. $\tan\alpha\tan\beta = -1$.

Then $\tan\alpha = -\cot\beta = \tan(\beta + 90°)$ or $\tan(\beta - 90°)$



Therefore, α and β differ by 90°. Thus, lines l_1 and l_2 are perpendicular to each other. Hence, two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other. i.e. $m_2 = \frac{-1}{m_1}$ or, $m_1 m_2 = -1$.

8. Angle between two lines.

Case I if $\frac{m_2-m_1}{1+m_1m_2}$ is positive, then tan θ will be positive and tan ϕ will be negative, which means θ will be acute and ϕ will be obtuse.

Case II if $\frac{m_2-m_1}{1+m_1m_2}$ is negative, then tan θ will be negative and tan ϕ will be positive, which means that θ will be obtuse and ϕ will be acute.

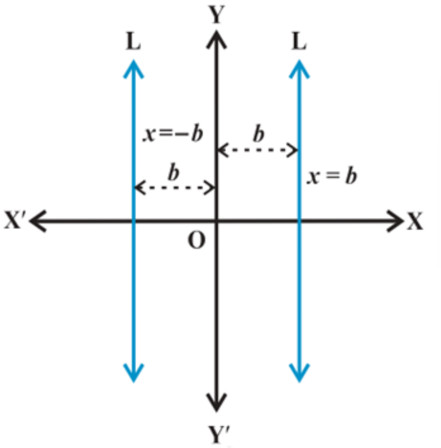
Thus, the acute angle (say θ) between lines L_1 and L_2 with slopes m_1 and m_2 , respectively, is given by tan θ= $\left|\frac{m_2-m_1}{1+m_1m_2}\right|$ as $m_1m_2 + 1 \neq 0$

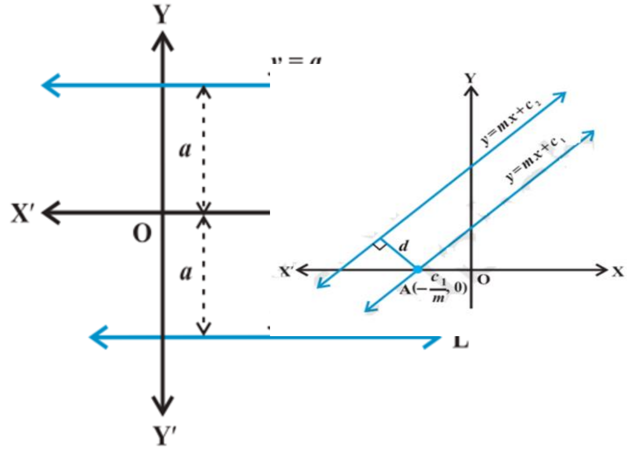
9. Various Forms of the Equation of a Line

i) horizontal and vertical line:

a) $x=\pm a$ which is a equation of line parallel to y-axis

b) $y=\pm a$ which is a equation of line parallel to x-axis



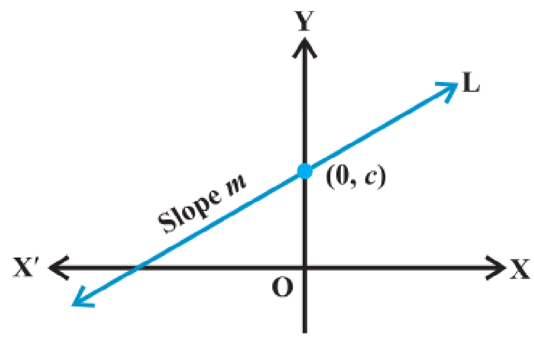


ii. point slope form: Suppose that P(x_0,y_0)

is a fixed point on a non-vertical line L. Whose

slope is m. Let P (x, y) be an arbitrary point on L. Then by the

definition,



the slope of L is given by m=$\frac{y-y_1}{x-x_1}$ and equation of line is y-

y_1=m(x-x_1)

iii. Two-point form: Let the line L passes through two given

points P_1 (x_1 ,y_1) and P_2 (x_2 , y_2).

Let P (x, y) be a general point on L .The three

points P_1 , P_2 and P are collinear. Therefore we

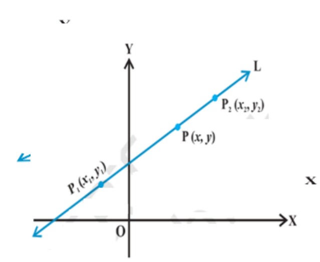
have slope of P_1 P = slope of P_1 P_2 i.e. Thus equation of the line passing through the points (x_1 , y_1) and (x_2 , y_2) is given by

y-y_1 = $\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

iv. Slope-intercept form: Sometimes a line is known to us with its slope and an intercept on one of the axes.

<u>Case I</u>: Suppose a line L with slope m cuts the y-axis at a distance c from the origin. The distance c is called the y-intercept of the line L. Obviously, coordinates of the point where the line meet the y-axis are (0, c). Thus, L has slope m and passes through a fixed point (0, c). Therefore, by point-slope form, the equation of L is

y-c = m(x-0) or y = mx + c



Thus, the point (x, y) on the line with slope m and y-intercept c lies on the line if and only if y = mx +c

Note that the value of c will be positive or negative according as the intercept is made on the positive or negative side of the y-axis, respectively.

<u>Case II:</u> Suppose line L with slope m makes x-intercept d. Then equation of L is

y = m(x – d)

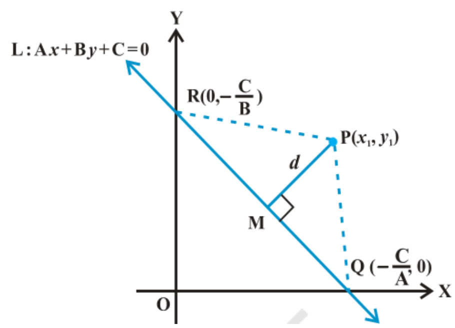
v) Intercept – form: Suppose a line L makes x-intercept a and y-intercept b on the axes. Obviously L meets x-axis at the point (a,0) and y-axis at the point (0, b). By two-point form of the equation of the line, we have

y-0 = $\frac{(b-o)}{(0-a)}(x-a)$.

ay=-bx+ab

ax+by=ab

Thus, equation of the line making intercepts a and b on x-



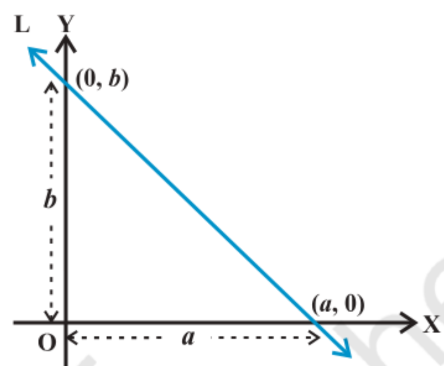
and y-axis, respectively, is

$\frac{x}{a}+\frac{y}{b}=1$

10) Distance of a Point from a Line.

The distance of a point from a line is the length of the perpendicular drawn from the point to the line. Let L :Ax + By + C = 0 be a line, whose distance from the point P(x_1,y_1) is d.

d = $\left|\frac{Ax_1+By_1+C}{A^2+B^2}\right|$



11) Distance between two parallel lines: We know that slopes of two parallel lines are equal. Therefore, two parallel lines can be taken in the form y = mx + c_1 ... (1)

and y = mx + c_2 ... (2)

d=$\left|\frac{c_1-c_2}{m^2+1}\right|$

MCQ

1. If the slope of the line joining the points A(x, 2) and B (6, -8) is − 5/4 , find the value of x.

   A) 2 B) 3 C) -2 D)-3

   Answer :(C)

   If a line passing through (x_1 ,y_1) and (x_2, y_2) then slope of the line is given by :

Slope = $\left(\frac{y_2-y_1}{x_2-x_1}\right)$

Given points are A (x, 2) and B (6,-8) and the slope is -5/4 and

$\frac{-8-2}{6-x}$ = - $\frac{5}{4}$

x = -2

2. Find the value of x so that the line through (3, x) and (2, 7) is parallel to the line through (-1, 4) and (0, 6).

   A) 3 B) 6 C)9 D)-9

   Ans: (C)

We know that for two lines to be parallel, their slope must be the same. The given points are A(3,x),B(2,7) and C(-1,4),D(0,6).

Slope=$(\frac{6-4}{0+1})$ = $(\frac{7-x}{2-3})$

therefore $\frac{2}{1} = \left(\frac{7-x}{-1}\right)$

≫-2=7-x

therefore x=9

3. The slope of a line is not defined when the line is

A) parallel to x – axis B) parallel to y – axis

C) parallel to the line x + y = 0 D) parallel to the line x - y = 0 (B)

4. The slope of any line parallel to X – axis is

A) 1 B) - 1 C) 0 D) not defined (C)

5. The equation of the line through (0, 0) and parallel to the line ax + by + c =0 is (B)

A) ax – by –c = 0 B) ax + by = 0

C) bx + ay = 0 D) bx – ay = 0

6. Slope of a line which cuts off intercepts of equal lengths on the axes is (A)

a) -1 b) 0 c)2 d) 1

Answer Given that equation of a line in intercept form is $\frac{x}{a}+\frac{y}{b}$=1

Since a=b therefore x+y=a

Now slope of the line is -1

7. For specifying a straight line ,how many geometrical parameters should be know

a) 2 b) 3 c) 1 d)4 (A)

8. The equation of a line passing through (1,2) and and perpendicular to the line x+y+1=0 is

a) y-x+1=0 b) y-x-1=0 c) y-x+2=0 d)x+y=3 (D)

given that equation of line is x+y+1=0

now equation of line perpendicular to this line y-x+k=0

since given line passing throughthe point (1,2)

therefore 1-2+ k=0

therefore k=1 now equation of line is y-x+1=0 (A)

9. Slope of a line which makes 45^0 with x-axis is

a) 1 b) 2 c) -2 d) -1

given that $\theta = 45^0$

Then slope of line is m=tan θ=tan45^0 =1 (D)

10. find the angle between a line , whose slope is $\sqrt{3}$ and x-axis is

a) 15^0 b) 30^0 C) 45^0 D) 60^0

Given that slope of line is m= $\sqrt{3}$

Now m= tan θ=$\sqrt{3}$=tan60^0

θ =60^0

` (D)

ASSERTION AND REASON

Read assertion and reason carefully and write correct option for each question

(a) Both A and R are correct, R is the correct explanation of A

(b) *Both A and R are correct, R is not the correct explanation of A*

(c) *A is correct, R is incorrect*

(d) *R is correct, A is incorrect*

1) ASSERTION slope of x-axis is 0

REASON x-axis make zero angle with the positive direction of x-axis

Answer since $\theta = 0$ therefore m=0 and x-axis are parallel to positive direction of x-axis (A)

2) ASSERSION two parallel lines have same slope

REASON any line parallel to x axis have slope 0 (A)

Answer since slope of x-axis is zero and any line parallel to x-axis have same slope

Therefore any line parallel to x-axis have 0

3) ASSERSION product of slope of two perpendicular line is -1

REASON product of slope of x-axis and y-axis is -1

Answer since we know that slope of produt of two perpendicular line is -1 and also we know that x-axis and y-axis is perpendicular therefore product of slope of x-axis and y-axis are -1 . (A)

4) ASSERSION parallel lines have same slope

REASON line P: 2x+ 3y=5 and Q: 6x+ 9y=20 have same slope

Slope of line P= -2/3 and slope of line Q=-2/3 (A)

5) ASSERTION eqn of line Passing through origin is y=mx

REASON line y=mx have intercept on the x-axis is 0 and y-axis is 0

Since assertion and reason are true but R is not correct explanation of A (B)

SHORT ANSWER TYPE QUESTION

1. Show that the line through the points (5, 6) and (2, 3) is parallel to the line through the points (9, -2) and (6, -5)

Answer: We know that for two lines to be parallel, their slope must be the same. Given points are A(5,6),B(2,3) and C(9,-2),D(6,-5)

Slope = $\left(\frac{y_2-y_1}{x_2-x_1}\right)$

$(\frac{3-6}{2-5}) = (\frac{-5+2}{6-9})$

1=1

Hence proved.

2. If a point P(x, y) is equidistant from the points A(6, -1) and B(2, 3), find the relation between x and y.

Answer: Given: Point P(x, y) is equidistant from points A(6, -1) and B(2, 3) i.e., distance of P from A = distance of P from B Squaring both sides,

$\sqrt{(x-6)^2+(y+1)^2}$=$\sqrt{(x-2)^2+(y-3)^2}$

$(x-6)^2 + (y-1)^2 = (x-2)^2 + (y-3)^2$

$x^2-12x+36+y^2-2y-1 = x^2-4x+4+y^2-6y+9$

-12x +36+2y +1 = -4x +4 -6y +9

x-y = 3

Therefore, x – y = 3 is the required relation.

3. Find a point on the x-axis which is equidistant from the points A(7, 6) and B(- 3, 4).

Answer: Let the point on x-axis be P(x, 0). Given: Point P(x, 0) is equidistant from points A(7, 6) and B(-3, 4) i.e., distance of P from A = distance of P from B

$\sqrt{(x-7)^2+36}$ = $\sqrt{(x+3)^2+16}$

Squaring both sides,

$(x-7)^2+36 = (x+3)^2 + 16$

$x^2 - 14x +49+36 = x^2 + 6x+ 9 +16$

-20x = -60

x = 3 Therefore, the point on the x-axis is (3, 0).

4. Show that the points A(-5, 1), B(5, 5) and C(10, 7) are collinear.

Answer: Given: The points are A(-5, 1), B(5, 5) and C(10, 7). Note: Three points are collinear if the sum of lengths of any sides is equal to the length of the third side.

AB = $\sqrt{(5+5)^2+(}5-1)^2$ = $\sqrt{100+16}$

= $2\sqrt{29} units$

BC = $\sqrt{(10-5)^2+(7-5)^2}$ = $\sqrt{25+4}$

= $\sqrt{29} units$

AC = $\sqrt{(10+5)^2+(7-1)^2}$ = $\sqrt{225+4}$

= $3\sqrt{29} units$

From equations 1,2,3 we have AB+BC=AC

Therefore, the three points are collinear.

5. If the three points A (h, k), B(x_1, y_1) and C(x_2, y_2) lie on a line then show that $(h – x_1)(y_2 – y_1) = (k – y_1)(x_2 – x)$.

Answer: For the lines to be in a line, the slope of the adjacent lines should be the same. Given points are A(h,k),B(x_1,y_1) and C(x_2,y_2) So slope of AB = BC = CA

Slope = $(\frac{y_2-y_1}{x_2-x_1})$

Slope of AB = $\frac{y_1-k}{x_1-h}$

Slope of BC = $\frac{y_1-k}{x_1-h}$

Slope of AC = $\frac{y_2-k}{x_2-h}$ =

$\frac{y_1-k}{x_1-h} = \frac{y_1-k}{x_1-h} = \frac{y_2-k}{x_2-h}$

Now Cross multiplying the first two equality,

$(y_1\text{-}k)(x_2\text{-}x_1) = (x_1\text{-}h)(y_2\text{-}y_1)$

$(h\text{-}x_1)(y_2\text{-}y_1) = (k\text{-}y_1)(x_2\text{-}x_1)$

Hence proved.

6. Find the slope of the line which makes an angle of 300 with the positive direction of the y-axis, measured anticlockwise.

Answer: According to the given figure, the angle made by the line from X-axis is 90+30 = 120°

Slope =(y_2-y_1/x_2-x_1)

We also know that slope of a line is equal to tanθ, Where θ = 120° tan (120°) = tan (90°+30°) = -cot (30°) = -√3

Therefor the slope of the given line is -√3.

7. Find the angle between the lines whose slopes are, $\sqrt{3} and \frac{1}{\sqrt{3}}$

Answer: To find out the angle between two lines, the angle is equal to the difference in θ.

The slope of a line = tanθ =($\frac{y_2-y_1}{x_2-x_1}$)

So slope of the first line = $\sqrt{3}$ = tanθ

θ_1 = 60^0

The slope of the second line is = $1/\sqrt{3}$

θ_1 = 30^0

Now the difference between the two lines is θ1-θ2

= 60°-30°

= 30°

8. Find the equation of a line which is equidistant from the lines x = - 2 and x = 6.

Answer: For the equation of line equidistant from both lines, we will find point through which line passes and is equidistant from both line. As any point lying on x = - 2 line is (- 2, 0) and on x = 6 is (6, 0), so mid - point is

$(x,y) = (\frac{-2+6}{2}, \frac{0+0}{2})$

(x, y) = (2,0)

So, equation of line is x = 2.

9. Find the equation of a line passing through the origin and making an angle of 1200 with the positive direction of the x - axis.

Answer: As angle is given so we have to find slope first give by m = tanθ m = tan120°

$m = \tan(180^0 - 60^0) = -\tan 60^0 = -\sqrt{3}$

(tan (180° - θ) is in II quadrant, tanx is negative) Now equation of line passing through origin is given as y = mx

$y = -\sqrt{3}x$

$y + \sqrt{3}x = 0$

so the required equation of line is $y + \sqrt{3}x = 0$

10. Find the slope and the equation of the line passing through the points: (i) (3, - 2) and (- 5, - 7).

Answer: Slope of equation can be calculated using

$m = (\frac{y_2 - y_1}{x_2 - x_1})$

m = 5/8

now using two point formula of the equation of a line

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

y – (-2) = 5/8 (x-3)

8 (y+ 2) = 5 (x-3)

8y + 16 = 5x -15

5x-8y-16-15 = 0

5x-8y -31 = 0

So the required equation of the line is 5x-8y -31 = 0

LONG ANSWER TYPE QUESTION

1. Find the area of the quadrilateral whose vertices are A(-4, 5), B(0, 7), C(5, -5) and D(-4, -2).

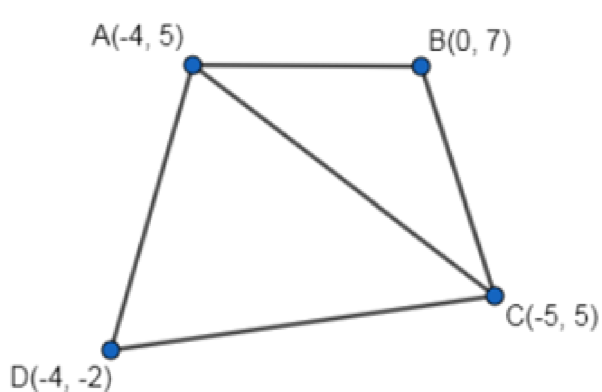
Answer: Given: The vertices of the quadrilateral are A(-4, 5), B(0, 7), C(5, -5) and D(-4, -2).

Area of quadrilateral ABCD = Area of Δ ABC + Area of Δ ADC

area of ABC = $1/2[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

area of quadrilateral ABCD = Area of ABC +Area of ACD

= 1/2 [-4(7+5) + 0 + 5(5-7)]



= 1/2 [-48-10]

= -29

Taking modulus (∵ area is always positive),

Area of Δ ABC = 29 sq. units …..(1)

Area of ACD = 1/2 [-4 (-2 + 5) -4 (-5-5) + 5 (5+2)]

= 31.5 sq. units …..(2)

From 1 and 2,

Area of quadrilateral ABCD = 29 + 31.5

= 60.5 square units.

Therefore, the area of quadrilateral ABCD is 60.5 square units.

2. Find the area of ΔABC, the midpoints of whose sides AB, BC and CA are D(3, -1), E(5, 3) and F(1, -3) respectively.

Answer:

The figure is as shown.

$x_1 + x_2 = 2 \times 3 = 6$ …..(1)

$x_1 + x_3 = 2 \times 1 = 2$ …..(2)

$x_2 + x_3 = 2 \times 5 = 10$ …..(3)

Equation 1 – Equation 2 gives us

$x_2 - x_3 = 4$ …..(4)

Equation 3 + Equation 4,

$2x_2 = 14 \Rightarrow x_2 = 7$

$\therefore x_1 = -1$ and $x_3 = 3$

Similarly,

$y_1 + y_2 = 2 \times -1 = -2$ …..(5)

$y_1 + y_3 = 2 \times -3 = -6$ …..(6)

$y_2 + y_3 = 2 \times 3 = 6$ …..(7)

Equation 5 – Equation 6 gives us

$y_2 - y_3 = 4$ …..(8)

Equation 7 + Equation 8,

$2y_2 = 10 \Rightarrow y_2 = 5$

$\therefore y_1 = -7$ and

$y_3=1$

area of ABC = $1/2[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

= ½ [-1(5-1) + 7(1+7) +3 (-7-5)]

=1/2 (-4+56-36)

= 8

3. Find the coordinates of the point which divides the join of A(-5, 11) and B(4, -7) in the ratio 2 : 7.

Answer: Let P(x, y) be the point that divides the join of A(-5, 11) and B(4, -7) in the ratio 2 : 7

Formula: If $m_1 : m_2$ is the ratio in which the join of two points is divided by another point (x, y), then

$x=\frac{m_1x_2 + m_2x_1}{m_1+m_2}$

$y = \frac{m_1y_2 + m_2y_1}{m_1+m_2}$

Here, $x_1 = -5$, $x_2 = 4$, $y_1 = 11$ and $y_2 = -7$ Substituting,

$x =\frac{2.4+7.-5}{2+7}$

x = -3

$y = \frac{2.-7+7.11}{2+7}$

y=8

Therefore, the coordinates of the point which divided the join of A(-5, 11) and B(4, -7) in the ratio 2 : 7 is (-3, 8).

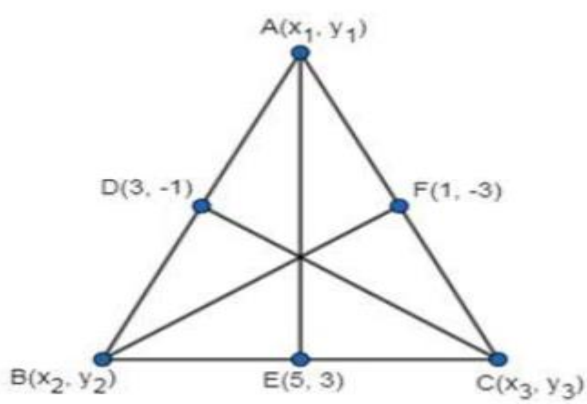
4. Find the ratio in which the x-axis cuts the join of the points A(4, 5) and B(-10, -2). Also, find the point of intersection.

Answer: Let the point which cuts the join of A(4, 5), and B(-10, -2) in the ratio k : 1 be P(x, 0).

Formula: If k : 1 is the ratio in which the join of two points is divided by another point (x, y), then

$x = \frac{kx_2+x_1}{k+1}$

y = $= \frac{ky_2+y_1}{k+1}$



Taking for the y co-ordinate

$0 = \frac{k.(-2)+5}{k+1}$

2k = 5

k=5/2

therefore x = $\frac{\frac{5}{2}(-10)+4}{\frac{5}{2}+1}$

x = -6

Therefore, the ratio in which x-axis cuts the join of the points A(4, 5) and B(-10, -2) is 5 : 2and the point of intersection is (-6, 0).

5. In what ratio is the line segment joining the points A(-4, 2) and B(8, 3) divided by the y-axis? Also, find the point of intersection.

Answer: Let the point which cuts the join of A(-4, 2) and B(8, 3) in the ratio k : 1 be P(0, y)

Formula: If k : 1 is the ratio in which the join of two points are divided by another point

(x, y), then

x = $\frac{kx_2+x_1}{k+1}$

y = = $\frac{ky_2+y_1}{k+1}$

Taking for the x co-ordinate,

0 = $\frac{k.8+(-4)}{k+1}$

8k = 4

K = 1/2

Therefore,

Y = $\frac{\frac{3}{2}+2}{\frac{1}{2}+1}$

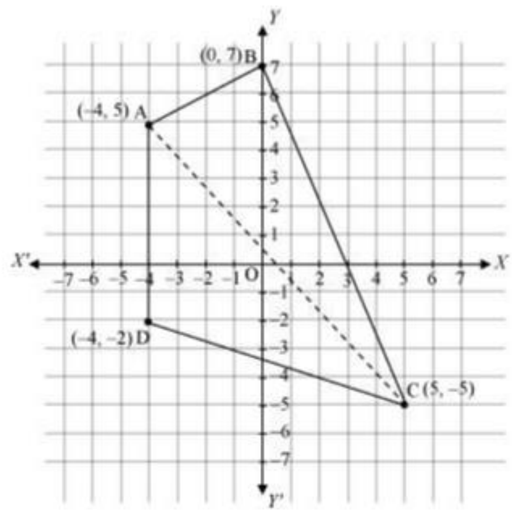
Y = $\frac{7}{3}$

Therefore, the ratio in which the line segment joining the points A(-4, 2) and B(8, 3) divided by the y-axis is 1 : 2 and the point of intersection is $\left(0, \frac{7}{3}\right)$.

CASE BASED QUESTIONS

1. Draw a quadrilateral in the Cartesian plane, whose vertices are (–4, 5), (0, 7), (5, –5) and (– 4, –2). Also, find its area.

Answer: Let ABCD be the given quadrilateral with vertices A (–4, 5), B (0, 7), C (5, –5), and D (–4, – 2). Then, by plotting A, B, C, and D on the Cartesian plane and joining AB, BC, CD, and DA, the given quadrilateral can be drawn as



To find the area of quadrilateral ABCD, we draw one diagonal, say AC.

Accordingly, area (ABCD) = area (ΔABC) + area (ΔACD).

We know that the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Therefore, area of ΔABC

$\frac{1}{2}|-4(7+5)+0(-5-5)+5(5-7)|$ unit^2

$=\frac{1}{2}|-4(12)+5(-2)|$ unit^2

$=\frac{1}{2}|-48-10|$ unit^2

$=\frac{1}{2}|-58|$ unit^2

$=\frac{1}{2}\times 58$ unit^2

= 29 unit^2

Area of ΔACD

$=\frac{1}{2}|-4(5+2)+5(-2-5)+(-4)(5+5)|$ unit^2

$=\frac{1}{2}|-4(-3)+5(-7)+(-4)(10)|$ unit^2

$=\frac{1}{2}|12-35-40|$ unit^2

$=\frac{1}{2}|-63|$ unit^2

$=\frac{-63}{2}$ unit^2

Thus, area (ABCD) $=\left(29+\frac{63}{2}\right)$ unit^2 $=\frac{58+63}{2}$ unit^2 $=\frac{121}{2}$ unit^2

2. Assuming that straight lines work as the plane mirror for a point, find the image of the point (1, 2) in the line $x-3y+4=0$.

Let Q (*h*, *k*) is the image of the point P (1, 2) in the line

$x - 3y + 4 = 0$ ... (1)

Therefore, the line (1) is the perpendicular bisector of line segment PQ.

Hence slope of line PQ = $\frac{-1}{Slope of line x-3y+4=0}$

So that $\frac{k-2}{h-1} = \frac{-1}{\frac{1}{3}} or\ 3h + k = 5$ ... (2)

And the mid-point of PQ, i.e., point$\left(\frac{h+1}{2}, \frac{k+2}{2}\right)$ will satisfy the equation (1) so that

$\frac{h+1}{2} - 3\left(\frac{k+2}{2}\right) + 4 = 0$ or h-3k = -3 ... (3)

Solving (2) and (3) we get, $h = \frac{6}{5} and k = \frac{7}{5}$

Hence, the image of point (1, 2) in the line (1) is, $\left(\frac{6}{5}, \frac{7}{5}\right)$

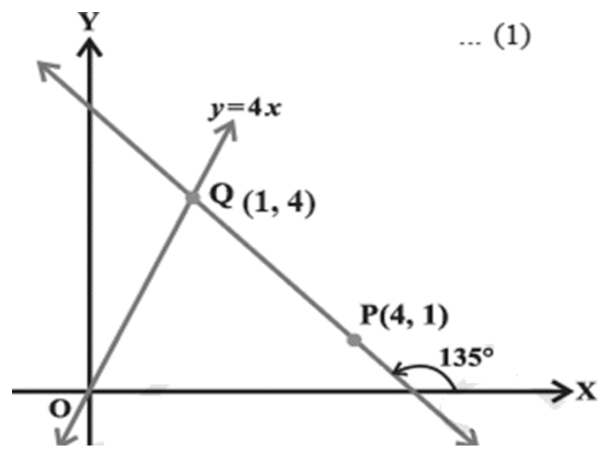
3. Show that the path of a moving point such that its distances from two lines $3x - 2y = 5$ and $3x + 2y = 5$ are equal is a straight line.

Given lines are $3x - 2y = 5$ ... (1) And $3x + 2y = 5$ ... (2) Let (h, k) is any point, whose distances from the lines (1) and (2) are equal. Therefore

$$\frac{|3h-2k-5|}{\sqrt{9+4}} = \frac{|3h+2k-5|}{\sqrt{9+4}} \text{ or } |3h-2k-5| = |3h+2k-5|,$$

which gives $3h - 2k - 5 = 3h + 2k - 5$ or $-(3h - 2k - 5) = 3h + 2k - 5$.

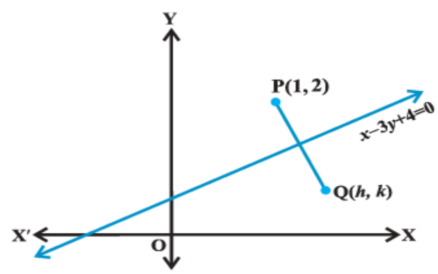
Solving these two relations we get $k = 0$ or $h = 3/5$. Thus, the point (h, k) satisfies the equations $y = 0$ or $x = 3/5$ which represent straight lines. Hence, path of the point equidistant from the lines (1) and (2) is a straight line.



... (1)

4. Find the distance of the line $4x - y = 0$ from the point P (4, 1) measured along the line making an angle of 135° with the positive x-axis.

Given line is $4x - y = 0$ ... (1).



In order to find the distance of the line (1) from the point P (4, 1) along another line, we have to find the point of intersection of both the lines.

For this purpose, we will first find the equation of the second line Slope of second line is tan 135° = –1.

Equation of the line with slope – 1 through the point P (4, 1) is $y - 1 = -1(x - 4)$ or $x + y - 5 = 0$ ... (2)

Solving (1) and (2), we get $x = 1$ and $y = 4$

so that point of intersection of the two lines is Q (1, 4).

Now, distance of line (1) from the point P (4, 1) along the line (2) the distance between the points P (4, 1) and Q (1, 4).

PQ = $\sqrt{(1-4)^2 + (4-1)^2}$

= $\sqrt{(-3)^2 + (3)^2}$

= $\sqrt{9+9}$

= $3\sqrt{2}$

5. The slope of a line is double of the slope of another line. If tangent of the angle between them is 1/3.find the slopes of the lines

Let m_1 and m be the slopes of the two given lines such that m1 = 2m

We know that if θ is the angle between the lines l_1 and l_2 with slopes m_1 and m_2 m^2

Then $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

It is given that the tangent of the angle between two lines is 1/3

$$\frac{1}{3} = \left|\frac{m - 2m}{1 + 2m2}\right|$$

1/3 = $|-m/1+2m^2|$

1/3=-m/1+2m^2 or 1/3=m/1+2m^2

Case 1

$$\Rightarrow \frac{1}{3} = \frac{-m}{1+2m^2}$$
$$\Rightarrow 1+2m^2 = -3m$$
$$\Rightarrow 2m^2+3m+1=0$$
$$\Rightarrow 2m^2+2m+m+1=0$$
$$\Rightarrow 2m(m+1)+1(m+1)=0$$
$$\Rightarrow (m+1)(2m+1)=0$$
$$\Rightarrow m=-1 \text{ or } m=-\frac{1}{2}$$

If m = –1, then the slopes of the lines are –1 and –2.

If m = $-\frac{1}{2}$, then the slopes of the lines are $-\frac{1}{2}$ and –1.

Case 2

$$\frac{1}{3} = \frac{m}{1+2m^2}$$
$$\Rightarrow 2m^2+1=3m$$
$$\Rightarrow 2m^2-3m+1=0$$
$$\Rightarrow 2m^2-2m-m+1=0$$
$$\Rightarrow 2m(m-1)-1(m-1)=0$$
$$\Rightarrow (m-1)(2m-1)=0$$
$$\Rightarrow m=1 \text{ or } m=\frac{1}{2}$$

If m = 1, then the slopes of the lines are 1 and 2.

If m = $\frac{1}{2}$, then the slopes of the lines are $\frac{1}{2}$ and 1.

MCQ

1. The equation of the line joining the points (a , 0) and (0 , b) is

A) ax + by = 0 B) bx + ay = 0

C) x/b + y/a = 1 D) x/a + y/b = 1 (D)

2. The equation of a line through (1 , -2) and which makes equal intercepts on the axes is

A) x + y =1 B) x - y =1 C) x + y +1 =0 D) x - y - 2 =0 (C)

3. A line passes through the point (2 , 2) and is at right angles to the line 3x + y =3 . Its y-intercept is (C)

A) - 4 B) 4/3 C) - 4/3 D) None of these

4. The inclination of the line x – y + 3 = 0 with the positive direction of x-axis is

(A) 45° (B) 135° (C) – 45° (D) –135° (A)

5. The coordinates of the foot of the perpendicular from the point (2, 3) on the line x + y – 11 = 0 are

(A) (–6, 5) (B) (5, 6) (C) (–5, 6) (D) (6, 5) (B)

6. A line passes through P (1, 2) such that its intercept between the axes is bisected at P. The equation of the line is

(A) x + 2y = 5 (B) x – y + 1 = 0

(C) x + y – 3 = 0 (D) 2x + y – 4 = 0 (A)

7. The reflection of the point (4, – 13) about the line 5x + y + 6 = 0 is

(A) (– 1, – 14) (B) (3, 4) (C) (0, 0) (D) (1, 2) (A)

8. Slope of a line which cuts off intercepts of equal lengths on the axes is

(A) – 1 (B) – 0 (C) 2 (D) 3 (A)

9. The equation of the line passing through the point (1, 2) and perpendicular to the line x + y + 1 = 0 is

(A) y – x + 1 = 0 (B) y – x – 1 = 0 (A)

(C) y – x + 2 = 0 (D) y – x – 2 = 0

10. If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is (3, 2), then the equation of the line will be

(A) 2x + 3y = 12 (B) 3x + 2y = 12

(C) 4x – 3y = 6 (D) 5x – 2y = 10 (A)

ASSERTION AND REASON

Read assertion and reason carefully and write correct option for each question

(a) Both A and R are correct, R is the correct explanation of A

(b) Both A and R are correct, R is not the correct explanation of A

(c) A is correct, R is incorrect

(d) R is correct, A is incorrect

1. ASSERTION: Slope of a line passing throught the two point (2,2) and (4,4) is 1
   REASON: A line whose equation is y=x have slope 1 (A)

2. Assertion: Slope of x axis is 0 and slope of y axis is not defined.
   Reason: Slope of y axis is 0 and slope of x axis is not defined. (C)

3. Assertion: Line 3x-4y = 6 have slope 3/4.
   Reason: Line 3x-4y = 6 have x intercept is 2 and y intercept is – 3/2. (A)

4. Assertion: The distance between line 4x+3y=15 and 8x+6y=7 is 23/10.

Reason: the distance between line ax+by = c and ax+by = d is $\left|\frac{c-d}{\sqrt{a^2+b^2}}\right|$ (A)

5. Assertion: Three points are co- linear if they lies in a line. (A)

Reason: Three points are co- linear if the area of triangle is 0.

SHORT ANSWER TYPE QUESTIONS

1. Find the equation of line intersecting the y-axis at a distance of 2 units above the origin and making an angle of 30^0 with positive direction of the x-axis.

   Answer : $x-\sqrt{3}y+2\sqrt{3} = 0$

2. Find the equation of line when a point R (h, k) divides a line segment between the axes in the ratio 1: 2.

   Answer : 2kx+hy =3hk

3. Find angles between the lines $\sqrt{3}x + y = 1\ and x + \sqrt{3}y = 1$

   Answer : 30^0 or 150^0

4. The line through the points (h, 3) and (4, 1) intersects the line 7x-9y-19=0 at right angle. Find the value of h.

   Answer : 22/9

5. Find the distance between the lines 3x + 4y = 9 and 6x + 8y = 15.

   Answer : 3/10

6. A line which made intercept on the axis are a and b, and p be the perpendicular distance from origin. Then find the value of p^{-2} .

   Answer : $p^{-2}=a^{-2}+b^{-2}$

7. Find the centroid of the triangle whose vertex are A (2, 3) , B (4 - 2) and C(-5 , 1).

   Answer : G(1/3 , 2/3)

8. If the angle between two lines is π /4 and the slope of one of the lines 1⁄2 . Find the slope of the other line.

   Answer : 3 and -1/3

9. Lines through the points (-2 , 6) and (4 , 8) is perpendicular to the line through the points (8 , 12) and (x , 24) find x .

   Answer : x=4

10. Write the equation of the lines for which tan θ = 1⁄2 where θ is the inclination of the line an

    i. y- intercept is – 3/2

    ii. x- intercept is 4

Answer: i) 2y = x-3

Answer : ii) 2y= x-4

LONG ANSWER TYPE QUESTIONS

1. Find the equation of the line passing through the point (2 , 2) and cutting off intercepts on the axes whose sum is 9 .

   Answer : 2x+y-6=0 and x+2y-6=0

2. In a triangle ABC with vertices A(2,3) B(4 , -1) and C (1 , 2) , find the equation and length of altitude from the vertex A .

   Answer : y-x=1

3. Find the equation to the straight line passing through the point of intersection of the lines 5x – 6y – 1 = 0 and 3x + 2y + 5 = 0 and perpendicular to the line 3x – 5y + 11 = 0.

Answer : 5x+3y+8=0

4. The slope of a line is double the slope of another line . If the tangent of the angle between them is 1/3 . Find the slope of the lines .

Answer : -1 and -2 or -1/2 and -1 or 1 and 2 or 1/2 and 1

5. The area of a triangle is 5. Two of its vertices are (2, 1) & (3, – 2). The third vertex lies on y = x + 3. Find the third vertex.

Answer : (7/2 , 13/2) and (-3/2, 3/2)

CASE BASED QUESTIONS

1. The base of an equilateral triangle with side 2a lies along Y-axis such that the midpoint of the base is at the origin. Find vertices of the triangle.

   Answer : (0, a) , (0,-a) ($\pm\sqrt{3}$, 0)

2. The owner of a milk store finds that, he can sell 980 litres of milk each week at Rs 14/litre and 1220 litres of milk each week at Rs 16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs 17/litre?

   Answer: 1340

3. A ray of light passing through the point (1, 2) reflects on the x-axis at point A and the reflected ray passes through the point (5, 3). Find the coordinates of A.

   Answer : (13/5,0)

4. A person standing at the junction (crossing) of two straight paths represented by the equations 2x – 3y + 4 = 0 and 3x + 4y – 5 = 0 wants to reach the path whose equation is 6x – 7y + 8 = 0 in the least time. Find equation of the path that he should follow.

   Answer : 119x+102y =125

5. Find the product of the lengths of the perpendiculars drawn from the points $\left(\sqrt{a^2 - b^2}, 0\right)$and $\left(-\sqrt{a^2 - b^2}, 0\right)$to the line (x/a)cosθ+(y/b)sinθ = 1.

   Answer : b^2

CONIC SECTIONS

CONCEPTS:

CIRCLES

A circle is a locus of a point which moves in a plane such that its distance from a fixed point in that plane is always constant. The fixed point is said to be the centre of the circle and the constant distance is said to be the radius of the circle.

** The equation of a circle with centre (h, k) and the radius r is given by $(x - h)^2 + (y - k)^2 = r^2$.

** The equation of a circle with centre (0, 0) and the radius r is given by $x^2 + y^2 = r^2$.

** General equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ whose centre is $(-g, -f)$ and radius is $\sqrt{g^2 + f^2 - c}$

** Equation of a circle when end points of diameter as $A(x_1, y_1)$, $B(x_2, y_2)$ is given by

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0.$$

** Length of intercepts made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on the X and Y-axes are $2\sqrt{g^2 - c}$ and $2\sqrt{f^2 - c}$.

CONICS

Conic Section or a conic is the locus of a point which moves in such a way that its distance from a fixed point bears a constant ratio to its distance from a fixed line.

The fixed point is called the **focus**, the straight line is called the **directrix** and the constant ratio denoted by **e** is called the **eccentricity of the conics**.

Eccentricity (e) $\quad e = \dfrac{\text{distance between } P(x, y) \text{ \& Focus}}{\text{distance between } P(x, y) \text{ \& Directrix}}$.

If $e = 1$, then conic is a parabola.

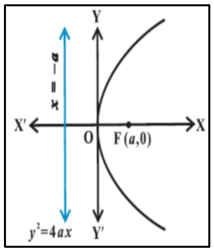
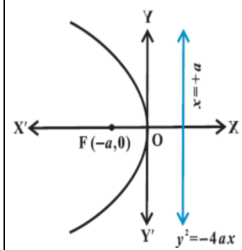
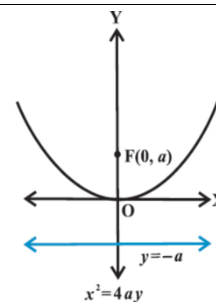
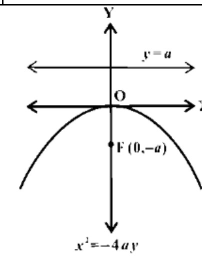
If $e < 1$, then conic is an ellipse.

If $e > 1$, then conic is a hyperbola.

If $e = 0$, then conic is a circle.

PARABOLA

A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point in the plane

S.No		**y^2 = 4ax**	**y^2 = -4ax**	**x^2 = 4ay**	**x^2 = -4ay**
					
1.	Vertex	(0,0)	(0,0)	(0,0)	(0,0)
2.	Focus	(a,0)	(-a,0)	(0,a)	(0,-a)
3	Equation of directrix	x + a = 0	x – a = 0	y + a = 0	y – a = 0
4.	Equation of Axis	(x-axis), y = 0	(x-axis), y = 0	(y-axis), x = 0	(y-axis), x = 0
5.	Length of Latus Rectum	4a	4a	4a	4a
6.	Focal distance of a p(x, y)	x + a	x – a	y + a	y – a

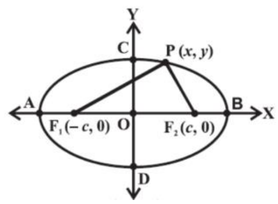
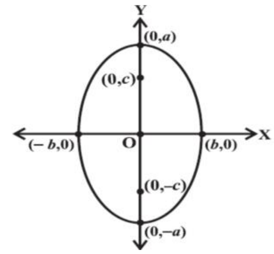
** Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the parabola.

** **Position of a point with respect to a parabola**: The point (x_1, y_1) with respect to the parabola $y^2 = 4ax$ lies

(i) outside , if $y_1^2 - 4ax_1 > 0$ (ii) on it if $y_1^2 - 4ax_1 = 0$ (iii)inside if $y_1^2 - 4ax_1 < 0$.

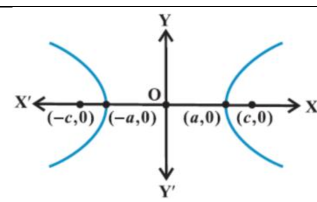
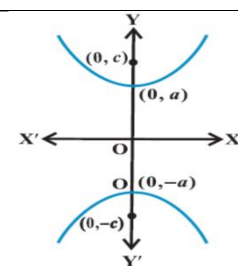
ELLIPSE:

An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.

S. No.			
1.	Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$
2.	Centre	(0 , 0)	(0 , 0)
3.	Vertices	(± a , 0)	(0 , ± a)
4.	Foci	(± ae, 0) or (± c , 0)where $c^2 = a^2 – b^2$	(0 ± ae) or (0 ± c)where $c^2 = a^2 – b^2$
5.	Eccentricity	e = c/a	e = c/a
6.	Length of major axis	2a	2a
7.	Length of minor axis	2b	2b
8.	Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

HYPERBOLA

A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.

S. N			
1	Equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$
2.	Vertices	(± a, 0)	(0, ± a)
3.	Foci	(± ae, 0) or (± c , 0) where $c^2 = a^2 + b^2$	(0 ± ae) or (0 ± ae) where $c^2 = a^2 + b^2$
4.	Eccentricity	e = c/a	e = c/a

5.	Centre	(0, 0)	(0, 0)
6.	Length of Transverse axis	2a	2a
7.	Length of conjugate axis	2b	2b
8.	Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
9.	Equation of Transverse axis	y = 0	x = 0
10.	Equation of conjugate axis	x = 0	y = 0

Competency Based Exemplar Questions

MCQs — Marks

1 The equation $y^2 - x^2 + 2x - 1 = 0$ represents **[1]**

a) a pair of straight lines

b) a circle

c) a parabola

d) an ellipse

2 The length of the latus rectum of the ellipse $3x^2 + y^2 = 12$ is **[1]**

a) 3

b) $\frac{4}{\sqrt{3}}$

c) 4

d) 8

3 The line $2x - y + 4 = 0$ cuts the parabola $y^2 = 8x$ in P and Q. The mid - point of PQ is **[1]**

a) (1, - 2)

b) (1, 2)

c) (- 1, - 2)

d) (- 1, 2)

4 If the parabola $y^2 = 4ax$ passes through the point (3, 2), then the length of its latus rectum is **[1]**

a) 4

b) $\frac{2}{3}$

c) $\frac{4}{3}$

d) $\frac{1}{3}$

5 At what point on the parabola $y^2 = 4x$ the normal makes equal angles with the axes? **[1]**

a) (4, 4)

b) (9, 6)

c) (1, - 2)

d) (4, - 4)

6 The length of latus rectum of an ellipse is one - third of its major axis. Its eccentricity would be **[1]**

a) $\frac{2}{3}$

b) $\sqrt{\frac{2}{3}}$

c) $\frac{1}{\sqrt{3}}$

d) $\frac{1}{\sqrt{2}}$

7 Equation of the hyperbola with eccentricity $\frac{3}{2}$ and foci at (± 2, 0) is **[1]**

a) $\frac{x^2}{9} - \frac{y^2}{9} = \frac{4}{9}$

b) $\frac{x^2}{16} - \frac{y^2}{27} = 1$

c) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

d) $\frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$

8 If (2, 4) and (10, 10) are the ends of a latus rectum of an ellipse with eccentricity $\frac{1}{2}$, then the length of semi - major axis is **[1]**

a) $\frac{40}{3}$

b) $\frac{20}{7}$

c) $\frac{15}{3}$

d) $\frac{20}{3}$

9 The slope of the tangent at the point (a, a) of the circle $x^2 + y^2 = a^2$ is **[1]**

a) - 2

b) - 1

c) 0

d) depends upon h

10 The equation $x^2 + y^2 + 2x - 4y + 5 = 0$ represents. **[1]**

a) A circle of non - zero radius

b) A circle of zero radius

c) A pair of straight lines

d) A point

ASSERTION AND REASONING TYPE QUESTIONS

11 If the distances of foci and vertex of hyperbola from the centre are c and a respectively, then **[1]**

Assertion (A): Eccentricity is always less than 1.

Reason (R): Foci are at a distance of ae from the centre.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

12 **Assertion (A):** The length of major and minor axes of the ellipse $5x^2 + 9y^2 - 54y + 36 = 0$ are 6 and 10, respectively. **[1]**

Reason (R): The equation $5x^2 + 9y^2 - 54y + 36 = 0$ can be expressed as $5x^2 + 9(y - 3)^2 = 45$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

13 **Assertion (A):** The sum of focal distances of a point on the ellipse $9x^2 + 4y^2 - 18x - 24y + 9 = 0$ is 4. **[1]**

Reason (R): The equation $9x^2 + 4y^2 - 18x - 24y + 9 = 0$ can be expressed as $9(x - 1)^2 + 4(y - 3)^2 = 36$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

14 Parabola is symmetric with respect to the axis of the parabola. **[1]**

Assertion (A): If the equation of standard parabola has a term y^2, then the axis of symmetry is along the x - axis.

Reason (R): If the equation of standard parabola has a term x^2, then the axis of symmetry is along the x - axis.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

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15 **Assertion (A):** The circle $x^2 + y^2 + 2ax + c = 0$, $x^2 + y^2 + 2by + c = 0$ touch if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$ **[1]**

Reason (R): The circles with centre C_1, C_2 and radii r_1, r_2 touch each other if $r_1 \pm r_2 = C_1 C_2$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

SHORT ANSWER TYPE QUESTIONS

16 If the distance between the foci of an ellipse is equal to the length of the latus rectum, write the eccentricity of the ellipse. **[2]**

17 The circle $x^2 + y^2 - 2x - 2y + 1 = 0$ is rolled along the positive direction of x - axis and makes one complete roll. Find its equation in new - position. **[2]**

18 Find the lengths of major and minor axes, coordinates of foci, vertices and the eccentricity: $3x^2 + 2y^2 = 6$. **[2]**

19 Find the equation of the hyperbola whose focus is at (5, 2), vertex at (4, 2)andcentre at (3, 2). **[2]**

20 Find the equation of the ellipse whose vertices are ($\pm$ 13, 0) and foci are ($\pm$ 5, 0). **[2]**

21 Find the vertex, focus, axis, directrix and latus - rectum of the following parabolas $y^2 - 4y - 3x + 1 = 0$ **[2]**

22 Write the equation of the parabola with focus (0, 0) and directrix $x + y - 4 = 0$. **[2]**

23 Find the distance between the directrices of the hyperbola $x^2 - y^2 = 8$. **[2]**

24 Find the equation of the ellipse the ends of whose major and minor axes are$(\pm 4,0)$ and $(0, \pm 3)$ respectively. **[2]**

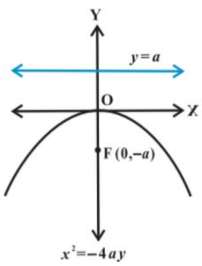
25 ABCD is a square whose side is a; taking AB and AD as axes, prove that the equation of the circle circumscribing the square is $x^2 + y^2 - a(x + y) = 0$. **[2]**

CASE STUDY BASED QUESTIONS

26 **Read the text carefully and answer the questions:** Arun is running in a racecourse note that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m. **[4]**

1. Path traced by Arun represents which type of curve. Find the length of major axis?
2. Find the equation of the curve traced by Arun?
3. Find the eccentricity of path traced by Arun?
4. Find the length of latus rectum for the path traced by Arun.

27 **Read the text carefully and answer the questions:** Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal for India at the Olympics. **[4]**



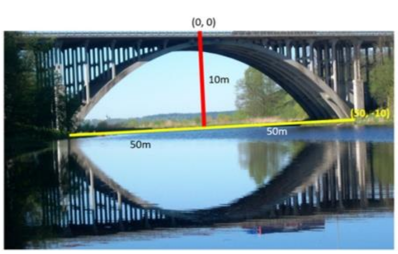
1. Name the shape of path followed by a javelin. If equation of such a curve is given by $x^2 = -16y$, then find the coordinates of foci.

2. Find the equation of directrix and length of latus rectum of parabola $x^2 = -16y$.

3. Find the equation of parabola with Vertex (0,0), passing through (5,2) and symmetric with respect to y - axis and also find equation of directrix.

4. Find the equation of the parabola with focus (2, 0) and directrix x = - 2 and also length of latus rectum.

28 **Read the text carefully and answer the questions:** The girder of a railway bridge is a parabola with its vertex at the highest point, 10 m above the ends. Its span is 100 m. **[4]**



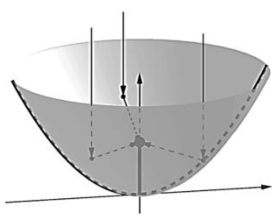
1. Find the coordinates of the focus of the parabola.

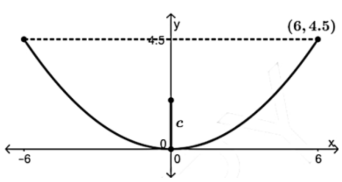
2. Find the equation of girder of bridge and find the length of latus rectum of girder of bridge.

3. Find the height of the bridge at 20m from the mid - point.

4. Find the radius of circle with centre at focus of the parabola and passes through the vertex of parabola.

29 **Read the text carefully and answer the questions:** A satellite dish has a shape called a paraboloid, where each cross section is parabola. Since radio signals (parallel to axis) will bounce off the surface of the dish to the focus, the receiver should be placed at the focus. The dish is 12 ft across, and 4.5 ft deep at the vertex. **[4]**





1. Name the type of curve given in the above paragraph and find the equation of curve?

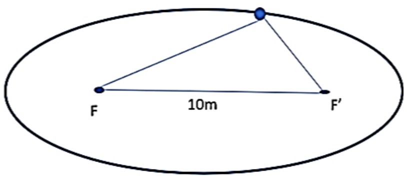
2. Find the equation of parabola whose vertex is (3, 4) and focus is (5, 4).

3. Find the equation of parabola Vertex (0, 0) passing through (2, 3) and axis is along x - axis. and also find the length of latus rectum.

4. Find focus, length of latus rectum and equation of directrix of the parabola $x^2 = 8y$.

30 **Read the text carefully and answer the questions:** A farmer wishes to install 2 hand pumps in his field for watering. **[4]**

The farmer moves in the field while watering in such a way that the sum of distances between the farmer and each handpump is always 26m. Also, the distance between the hand pumps is 10 m.



1. Name the curve traced by farmer and hence find the foci of curve.
2. Find the equation of curve traced by farmer.
3. Find the length of major axis, minor axis and eccentricity of curve along which farmer moves.
4. Find the length of latus rectum.

LONG ANSWER TYPE QUESTIONS

31 Find the equation of the ellipse whose foci are (4, 0) and (- 4, 0), eccentricity = 1/3. **[5]**

32 Find the equation of a circle passing through the points (2, - 6), (6, 4) and (- 3, 1). **[5]**

33 Find the equation of the hyperbola, the length of whose latus rectum is 4 and the eccentricity is 3. **[5]**

34 Find the equation of a circle concentric with the circle x^2 + y $^{$ + 4x + 6y + 11 = 0 and passing through the point (5, 4). **[5]**

35 Find the (i) lengths of major and minor axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of ellipse: $\frac{x^2}{25} + \frac{y^2}{9} = 1$. **[5]**

SOLUTIONS:

MCQs Marks

1 The equation $y^2 - x^2 + 2x - 1 = 0$ represents **[1]**

a) a pair of straight lines

2 The length of the latus rectum of the ellipse $3x^2 + y^2 = 12$ is **[1]**

b) $\frac{4}{\sqrt{3}}$

3 The line $2x - y + 4 = 0$ cuts the parabola $y^2 = 8x$ in P and Q. The mid - point of PQ is **[1]**

d) (- 1, 2)

4 If the parabola $y^2 = 4ax$ passes through the point (3, 2), then the length of its latus rectum is **[1]**

c) $\frac{4}{3}$

5 At what point on the parabola $y^2 = 4x$ the normal makes equal angles with the axes? **[1]**

c) (1, - 2)

6 The length of latus rectum of an ellipse is one - third of its major axis. Its eccentricity would be **[1]**

b) $\sqrt{\frac{2}{3}}$

7 Equation of the hyperbola with eccentricity $\frac{3}{2}$ and foci at (± 2, 0) is **[1]**

d) $\frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$

8 If (2, 4) and (10, 10) are the ends of a latus rectum of an ellipse with eccentricity $\frac{1}{2}$, then the length of semi - major axis is **[1]**

d) $\frac{20}{3}$

9 The slope of the tangent at the point (a, a) of the circle $x^2 + y^2 = a^2$ is **[1]**

b) - 1

10 The equation $x^2 + y^2 + 2x - 4y + 5 = 0$ represents. **[1]**

d) A point

ASSERTION AND REASONING TYPE QUESTIONS

11 If the distances of foci and vertex of hyperbola from the centre are c and a respectively, then **[1]**

Assertion (A): Eccentricity is always less than 1.

Reason (R): Foci are at a distance of ae from the centre.

d) A is false but R is true.

12 **Assertion (A):** The length of major and minor axes of the ellipse $5x^2 + 9y^2 - 54y + 36 = 0$ are 6 and 10, respectively. **[1]**

Reason (R): The equation $5x^2 + 9y^2 - 54y + 36 = 0$ can be expressed as $5x^2 + 9(y - 3)^2 = 45$.

d) A is false but R is true.

13 **Assertion (A):** The sum of focal distances of a point on the ellipse $9x^2 + 4y^2 - 18x - 24y + 9 = 0$ is 4. **[1]**

Reason (R): The equation $9x^2 + 4y^2 - 18x - 24y + 9 = 0$ can be expressed as $9(x - 1)^2 + 4(y - 3)^2 = 36$.

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14 Parabola is symmetric with respect to the axis of the parabola. **[1]**

Assertion (A): If the equation of standard parabola has a term y^2, then the axis of symmetry is along the x - axis.

Reason (R): If the equation of standard parabola has a term x^2, then the axis of symmetry is along the x - axis.

c) A is true but R is false.

15 **Assertion (A):** The circle $x^2 + y^2 + 2ax + c = 0$, $x^2 + y^2 + 2by + c = 0$ touch if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$ **[1]**

Reason (R): The circles with centre C_1, C_2 and radii r_1, r_2 touch each other if $r_1 \pm r_2 = C_1C_2$

a) Both A and R are true and R is the correct explanation of A.

SHORT ANSWER TYPE QUESTIONS

16 If the distance between the foci of an ellipse is equal to the length of the latus rectum, write the eccentricity of the ellipse. **[2]**

Ans:

Given that, the distance between the foci of an ellipse is equal to the length of the latus rectum.

i.e. $\frac{2b^2}{a} = 2ae$

$$\Rightarrow e = \frac{b^2}{a^2}$$

But, $e = \sqrt{1 - \frac{b^2}{a^2}}$

Hence, $e = \sqrt{1 - e}$

Squaring both sides, we get:

$$e^2 + e - 1 = 0$$

$\Rightarrow e = \frac{-1 \pm \sqrt{1+4}}{2}$ (∵ Eccentricity cannot be negative)

$$\Rightarrow e = \frac{\sqrt{5} - 1}{2}$$

17 The circle $x^2 + y^2 - 2x - 2y + 1 = 0$ is rolled along the positive direction of x - axis and makes one complete roll. Find its equation in new - position. **[2]**

Ans:

It is told that the circle is rolled along the positive direction of the x-axis and makes one complete roll.

We know that the complete roll of a circle covers the distance $2\pi r$, where r is the radius of the circle.

The centre of the circle moves $2\pi rn$ the positive direction of the x-axis.

Let the d be the distance moved by the centre on completion of one roll.

$$d = 2\pi r(1)$$
$$\Rightarrow d = 2\pi$$

The new position of the centre is $(1 + d, 1)$ = Centre $= (1 + 2\pi, 1)$

We know that the equation of the circle with centre(p, q) and having radius ' r ' is given by:

$$(x - p)^2 + (y - q)^2 = r^2$$

Now, Equation of circle with centre$(1 + 2\pi, 1)$ and having radius 1 units.

$$\Rightarrow (x - (1 + 2\pi)^2 + (y - 1)^2 = 12$$
$$\Rightarrow (x - 1 - 2\pi)^2 + (y - 1)^2 = 1$$
$$\Rightarrow x^2 - (2 + 4\pi)x + (1 + 2\pi)^2 + y^2 - 2y + 1 = 1$$
$$\Rightarrow x^2 + y^2 - (2 + 4\pi)x - 2y - (1 + 2\pi)^2 = 0$$

The equations of the circles is$x^2 + y^2 - (2 + 4\pi)x - 2y - (1 + 2\pi)^2 = 0$

18 Find the lengths of major and minor axes, coordinates of foci, vertices and the eccentricity: $3x^2 + 2y^2 = 6$. **[2]**

Ans:

We have,

$$3x^2 + 2y^2 = 6$$
$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{3} = 1$$

This equation is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a^2 = 2$ and $b^2 = 3$ i.e. $a = \sqrt{2}$ and $b = \sqrt{3}$.

Clearly, $a < b$, so the major and minor axes of the given ellipse are along y and x-axes respectively.

$\therefore$ Length of the major axis $= 2b = 2\sqrt{3}$

and Length of the minor axis $= 2b = 2\sqrt{2}$

The coordinates of the vertices $= (0, b)$ and $(0, -b) = (0, \sqrt{3})$ and $(0, -\sqrt{3})$

The eccentricity e of the ellipse is $e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{2}{3}} = \frac{1}{\sqrt{3}}$

The coordinates of the foci $= (0,$ be $)$ and $(0, -be) = (0,1)$ and $(0, -1)$.

19 Find the equation of the hyperbola whose focus is at (5, 2), vertex at (4, 2)andcentre at (3, 2). **[2]**

Ans:

Equation of hyperbola with centre (x_0, y_0) is given by;

$$\frac{(x - x_0)^2}{a^2} - \frac{(y - y_0)^2}{b^2} = 1$$

Where, focus $= (ae + x_0, y_0)$

and vertex $= (a + x_0, y_0)$

$\therefore$ ae $= 2$

and $a = 1$

$$b^2 = (2)^2 - a^2$$
$$\Rightarrow b^2 = (2)^2 - (1)^2$$
$$\Rightarrow b^2 = 3$$
$$\Rightarrow \frac{(x - 3)^2}{1} - \frac{(y - 2)^2}{3} = 1$$
$$\Rightarrow 3(x - 3)^2 - (y - 2)^2 = 3$$

20 Find the equation of the ellipse whose vertices are ($\pm$ 13, 0) and foci are ($\pm$ 5, 0). **[2]**

Ans:

Since the vertices are on x-axis, the equation will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

where a is the semi-major axis.

Given that vertices are $(\pm 13,0)$ and foci are $(\pm 5,0)$➔ $> a = 13, c = ae = \pm 5$

Therefore, from the relation $c^2 = a^2 - b^2$, we get

$25 = 169 - b^2$, i.e., $b = 12$

Hence the equation of the ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$

21 Find the vertex, focus, axis, directrix and latus - rectum of the following parabolas **[2]**

$y^2 - 4y - 3x + 1 = 0$

Ans:

21. We are given that:

$$y^2 - 4y - 3x + 1 = 0$$
$$\Rightarrow (y-2)^2 - 4 - 3x + 1 = 0$$
$$\Rightarrow (y-2)^2 = 3(x+1)$$
$$\Rightarrow (y-2)^2 = 3(x-(-1))$$

Let $Y = y - 2$

$$X = x + 1$$

Then, we have:

$$Y^2 = 3X$$

Comparing the given equation with $Y^2 = 4aX$

$$4a = 3 \Rightarrow a = \frac{3}{4}$$

$\therefore$ Vertex $= (X = 0, Y = 0) = (x = -1, y = 2)$

Focus $= (X = a, Y = 0) = \left(x + 1 = \frac{3}{4}, y - 2 = 0\right) = \left(x = \frac{-1}{4}, y = 2\right)$

Equation of the directrix:

$$x = -a$$

i.e. $x+1=\frac{-3}{4} \Rightarrow x=\frac{-7}{4}$

Axis $= Y=0$

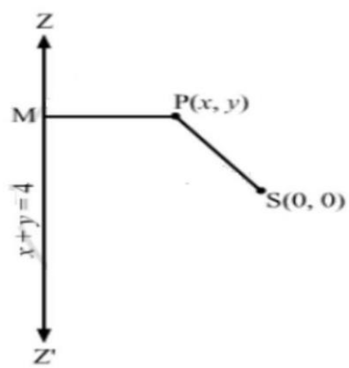
i.e. $y-2=0 \Rightarrow y=2$

Therefore, Length of the latus rectum $= 4a = 3$ units

22 Write the equation of the parabola with focus (0, 0) and directrix x + y - 4 = 0. **[2]**

Ans:

Let $P(x, y)$ be any point on the parabola whose focus is $S(0,0)$ and the directrix is $x+y=4$



Now draw PM perpendicular to $x+y=4$

Therefore, we have: SP = PM

$$\Rightarrow SP^2 = PM^2$$

$$\Rightarrow (x-0)^2+(y-0)^2=\left(\frac{x+y-4}{\sqrt{1+1}}\right)^2$$

$$\Rightarrow x^2+y^2=\left(\frac{x+y-4}{\sqrt{2}}\right)^2$$

$$\Rightarrow 2x^2+2y^2=x^2+y^2+16+2xy-8y-8x$$

$$\Rightarrow x^2+y^2-2xy+8x+8y-16=0$$

This is the required equation of parabola.

23 Find the distance between the directrices of the hyperbola x^2 - y^2 = 8. **[2]**

Ans:

Given equation of the hyperbola is $x^2 - y^2 = 8$.

$$\Rightarrow \frac{x^2}{(2\sqrt{2})^2} - \frac{y^2}{(2\sqrt{2})^2} = 1, a = 2\sqrt{2}, b = 2\sqrt{2}$$

eccentricity, $e = \frac{\sqrt{a^2+b^2}}{a} = \frac{\sqrt{16}}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\therefore$ Distance between the directrices $= \frac{2a}{e}$

$$= \frac{2.2\sqrt{2}}{\sqrt{2}}$$

$= 4$ unis

24 Find the equation of the ellipse the ends of whose major and minor axes are$(\pm 4,0)$ and $(0, \pm 3)$ respectively. **[2]**

Ans:

Given that:

End Points of Major Axis $= (\pm 4,0)$ and End Points of Minor Axis $= (0, \pm 3)$

$\therefore$ The Equation of Ellipse is of the form,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where, a is the semi-major axis and b is the semi-minor axis.

So, $a = 4$ and $b = 3$

Putting the value of a and b in Eq. (i), we get

$$\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$$

25 ABCD is a square whose side is a; taking AB and AD as axes, prove that the equation of the circle circumscribing the square is $x^2 + y^2 - a(x + y) = 0$. **[2]**

Ans:

We are given that $ABCD$ is a square with side 'a' units.

Let AB and AD represent the x-axis and the y-axis, respectively.

Thus, the coordinates of B and D are $(a, 0)$ and $(0, a)$, respectively.

The endpoints of the diameter of the circle circumscribing the square are B and D.

Therefore, equation of the circle circumscribing the square is $(x-a)(x-0)+(y-0)(y-a)=0$

or, $x^2+y^2-a(x+y)=0$

CASE STUDY BASED QUESTIONS

26 Arun is running in a racecourse such that the sum of the distances from the two flag posts from him is always $10m$ and the distance between the flag posts is $8m$. **[4]**

(i) An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant. Hence path traced by Arun is ellipse.

Sum of the distances of the point moving point to the foci is equal to length of major axis $= 10m$

(ii) Given $2a = 10 \& 2c = 8$

$$\Rightarrow a = 5 \& c = 4$$
$$c^2 = a^2 + b^2$$
$$\Rightarrow 16 = 25 + b^2$$
$$\Rightarrow b^2 = 25 - 16 = 9$$

Equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Required equation is $\frac{x^2}{25}+\frac{y^2}{9}=1$

(iii) equation is of given curve is $\frac{x^2}{25}+\frac{y^2}{9}=1$

$a=5, b=3$ and given $2c=8$ hence $c=4$

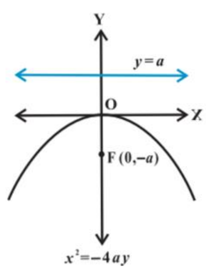
Eccentricity $=\frac{c}{a}=\frac{4}{5}$

(iv) $\frac{x^2}{25}+\frac{y^2}{9}=1$

Hence $a=5$ and $b=3$

Length of latus rectum of ellipse is given by $\frac{2b^2}{a}=\frac{2\times 9}{5}=\frac{18}{5}$

27 Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal for India at the Olympics. **[4]**



(i) The path traced by Javelin is parabola. A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.

compare $x^2=-16y$ with $x^2=-4ay$

$$\Rightarrow -4a=-16$$
$$\Rightarrow a=4$$

coordinates of focus for parabola $x^2=-4ay$ is $(0,-a)$

⇒ coordinates of focus for given parabola is $(0,-4)$

(ii) compare $x^2=-16y$ with $x^2=-4ay$

$$\Rightarrow -4a = -16 \ \Rightarrow a = 4$$

Equation of directrix for parabola $x^2 = -4ay$ is $y = a$

⇒ Equation of directrix for parabola$x^2 = -16y$ is $y = 4$

Length of latus rectum is $4a = 4 \times 4 = 16$

(iii)Equation of parabola with axis along y - axis

$$x^2 = 4ay$$

which passes through $(5,2)$

$$\Rightarrow 25 = 4a \times 2$$

$$\Rightarrow 4a = \frac{25}{2}$$

hence required equation of parabola is

$$x^2 = \frac{25}{2}y$$

$$\Rightarrow 2x^2 = 25y$$

Equation of directrix is $y = -a$

Hence required equation of directrix is $8y + 25 = 0$.

(iv)Since the focus $(2,0)$ lies on the x-axis, the x-axis itself is the axis of the parabola.

Hence the equation of the parabola is of the form either $y^2 = 4ax$ or $y^2 = -4ax$.

Since the directrix is $x = -2$ and the focus is $(2,0)$, the parabola is to be of the form $y^2 = 4ax$ with $a = 2$.

Hence the required equation is $y^2 = 4(2)x = 8x$

length of latus rectum $= 4a = 8$

28 (i) From the diagram equation of parabola is $x^2 = -4ay$ **[4]**

Vertex is $10m$ hight and spam is $100m$

parabola passes through ($50, -10$)

Hence, $50^2 = -4a(-10)$

$$\Rightarrow 2500 = 40a$$

$$\Rightarrow a = \frac{2500}{40} = 62.5$$

Hence coordinates of focus $= (-a, 0) = (-62.5,0)$

(ii) Equation of parabola is $x^2 = -4$ ay and $a = \frac{2500}{40} = 62.5$

Equation is $x^2 = -4\left(\frac{2500}{40}\right)y$

$$\Rightarrow x^2 = -250y$$

Length of latus rectum is $4a = 4 \times 62.5 = 250m$

(iii)Equation parabola $x^2 = -250y$

Coordinates of the point at $20m$ from mid point $= (20, y)$

Substituting in the equation of parabola

$$\Rightarrow 400 = -250y$$

$$\Rightarrow y = \frac{-400}{250} = -1.6$$

height of the bridge $= 10 - 1.6 = 8.4m$

(iv)vertex of parabola is $(0,0)$ and focus is $(0, -62.5)$

$\Rightarrow$ $(0, -62.5)$ is center and $(0,0)$ is on the circle

$\Rightarrow r = 0$ - (-62.5) $= 62.5m$

29 (i) Given curve is a parabola **[4]**

Equation of parabola is $x^2 = 4ay$

It passes through the point $(6,4.5)$

$$\Rightarrow 36 = 4 \times a \times 4.5$$
$$\Rightarrow 36 = 18a$$
$$\Rightarrow a = 2$$

Equation of parabola is $x^2 = 8y$

(ii) Distance between focus and vertex is $= a = \sqrt{(4-4)^2 + (5-3)^2} = 2$

Equation of parabola is $(y-k)^2 = 4a(x-h)$

where (h,k) is vertex

$\Rightarrow$ Equation of parabola with vertex $(3,4) \& a = 2$

$$\Rightarrow (y-4)^2 = 8(x-3)$$

(iii)Equation of parabola with axis along x - axis

$$y^2 = 4ax$$

which passes through $(2,3)$

$$\Rightarrow 9 = 4a \times 2$$
$$\Rightarrow 4a = \frac{9}{2}$$

hence required equation of parabola is

$$y^2 = \frac{9}{2}x$$
$$\Rightarrow 2y^2 = 9x$$

Hence length of latus rectum $= 4a = 4.5$

$$(iv) x^2 = 8y a = 2$$

Focus of parabola is $(0,2)$,length of latus rectum is $4a = 4 \times 2 = 8$

Equation of directrix y $+2 = 0$

30 (i) The curve traced by farmer is ellipse. Because An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant. **[4]**

Two positions of hand pumps are foci Distance between two foci $= 2c = 10$ Hence $c = 5$ Here foci lie on x axis & coordinates of foci $= (\pm c, 0)$

Hence coordinates of foci $= (\pm 5,0)$

(ii) $\frac{x^2}{169} + \frac{y^2}{144} = 1$

Sum of distances from the foci $= 2a$

Sum of distances between the farmer and each hand pump is $= 26 = 2a$

$$\Rightarrow 2a = 26 \Rightarrow a = 13m$$

Distance between the hand pump $= 10m = 2c$

$$\Rightarrow c = 5m$$

$$c^2 = a^2 - b^2$$

$$\Rightarrow 25 = 169 - b^2$$

$$\Rightarrow b^2 = 144$$

Equation is $\frac{x^2}{169} + \frac{y^2}{144} = 1$

(iii)Equation of ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$ comparing with standard equation of ellipse $a = 13, b = 12$ and $c = 5$ (given)

Length of major axis $= 2a = 2 \times 13 = 26$

Length of minor axis $= 2b = 2 \times 12 = 24$

eccentricity $e = \frac{c}{a} = \frac{5}{13}$

(iv) Equation of the ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$ hence $a = 13$ and $b = 12$

length of latus rectum of ellipse is given by $\frac{2b^2}{a} = \frac{2\times144}{13}$

LONG ANSWER TYPE QUESTIONS

31 Find the equation of the ellipse whose foci are (4, 0) and (-4, 0), eccentricity = 1/3. **[5]**

Ans:

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The coordinate of foci are (+ ae, 0) and (-ae, 0).

$\therefore$ ae $= 4[\because$foci $(\pm 4,0)]$

$$\Rightarrow a \times \frac{1}{3} = 4\left[\because e = \frac{1}{3}\right]$$

$$\Rightarrow a = 12$$

$$\Rightarrow a^2 = 144$$

Now, $b^2 = a^2(1 - e^2)$

$$\Rightarrow b^2 = 144\left[1 - \left(\frac{1}{3}\right)^2\right]$$

$$\Rightarrow b^2 = 144\left[1 - \frac{1}{9}\right]$$

$$\Rightarrow b^2 = 144 \times \frac{8}{9}$$

$$\Rightarrow b^2 = 16 \times 8 = 128$$

Substituting, $a^2 = 144$ and $b^2 = 128$ in the above equation of ellipse , we get

$$= \frac{x^2}{144} + \frac{y^2}{128} = 1$$

$$\Rightarrow \frac{1}{16}\left[\frac{x^2}{9}+\frac{y^2}{8}\right]=1$$

$$\Rightarrow \frac{x^2}{9}+\frac{y^2}{8}=16$$

This is the required equation of the ellipse.

32 Find the equation of a circle passing through the points (2, -6), (6, 4) and (-3, 1). **[5]**

Ans:

Let the equation of the circle passing through the given points be

$x^2+y^2+2gx+2fy+c=0$ .... (i)

Since, circle passes through the point$(2,-6)$.

So, put $x=2, y=-6$ in Eq. (i), we get

$$4+36+4g-12f+c=0$$

$\Rightarrow 4g-12f+c=-40$ .....(ii)

Also, circle passes through the point$(6,4)$.

So, put $x=6, y=4$ in Eq. (i), we get

$$36+16+12g+8f+c=0$$

$\Rightarrow 12g+8f+c=-52$...(iii)

Also, circle passes through the point $(-3,1)$.

So, put $x=-3$ and $y=1$ in Eq. (i), we get

$$9+1-6g+2f+c=0$$

$\Rightarrow -6g+2f+c=-10$...(iv)

On subtracting Eq. (iii) from Eq. (ii), we get

$-8g - 20f = 12$ ... (v)

On subtracting Eq. (iv) from Eq. (iii), we get

$18g + 6f = -42$ ... (vi)

On solving Eqs. (v) and (vi) for g and f, we get

$$g = -\frac{32}{13}, f = \frac{5}{13}$$

On putting the values of g and f in Eq. (ii), we get

$$c = -\frac{332}{13}$$

Now, on putting $g = -\frac{32}{13}, f = \frac{5}{13}$ and $c = -\frac{332}{13}$ in Eq. (i), we get

$$x^2 + y^2 - \frac{64}{13}x + \frac{10}{13}y - -\frac{332}{13} = 0$$

$$\Rightarrow 13x^2 + 13y^2 - 64x + 10y - 332 = 0$$

which is the required equation of circle.

33 Find the equation of the hyperbola, the length of whose latus rectum is 4 and the eccentricity is 3. **[5]**

Ans:

Given: The length of latus rectum is 4 , and the eccentricity is 3

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The length of the latus rectum is 4 units.

$\Rightarrow$ length of the latus rectum $= \frac{2b^2}{a} = 4$

$\Rightarrow \frac{2b^2}{a} = 4 \Rightarrow b^2 = 2a$ ... (i)

And also given, the eccentricity, $e = 3$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = 3$$

$\Rightarrow 1 + \frac{b^2}{a^2} = 9$ [Squaring both sides]

$$\Rightarrow \frac{b^2}{a^2} = 8$$

$$\Rightarrow b^2 = 8a^2$$

$\Rightarrow 2a = 8a^2$ [From (i)]

$$\Rightarrow a = \frac{1}{4} \Rightarrow a^2 = \frac{1}{16}$$

From (i) $\Rightarrow b^2 = 2a = 2 \times \frac{1}{4} = \frac{1}{2} \Rightarrow b^2 = \frac{1}{2}$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{\frac{1}{16}} - \frac{y^2}{\frac{1}{2}} = 1$$

$$\Rightarrow 16x^2 - 2y^2 = 1$$

34 Find the equation of a circle concentric with the circle $x^2 + y^2 + 4x + 6y + 11 = 0$ and passing through the point (5, 4). **[5]**

Ans:

34. Here, the equation of circle is $x^2 + y^2 + 4x + 6y + 11 = 0$

$$\Rightarrow (x^2 + 4x) + (y^2 + 6y) = -11$$

On adding 4 and 9 both sides to make perfect squares, we get

$$(x^2 + 4x + 4) + (y^2 + 6y + 9) = -11 + 4 + 9$$

$\Rightarrow (x + 2)^2 + (y + 3)^2 = (\sqrt{2})^2$ ... (i)

Its centre is $(-2, -3)$

The required circle is concentric with circle 1, therefore its centre is $(-2,-3)$.

Since, it passes through $(5,4)$, therefore radius is $r = CP =$

$\sqrt{(5+2)^2+(4+3)^2}$[$\because$ distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$]

$$= \sqrt{49+49} = 7\sqrt{2}$$

Hence, the equation of required circle having centre$(-2,-3)$ and radius $7\sqrt{2}$ is,

$$(x+2)^2+(y+3)^2 = (7\sqrt{2})^2$$

$$\Rightarrow x^2+4x+4+y^2+6y+9 = 98$$

$$\Rightarrow x^2+4x+y^2+6y-85 = 0$$

35 Find the (i) lengths of major and minor axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of ellipse:$\frac{x^2}{25}+\frac{y^2}{9}$ = 1. **[5]**

Ans:

35. Given: $\frac{x^2}{25}+\frac{y^2}{9} = 1$...(i)

Equation	Major Axis					
$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2-b^2$	$(\pm c, 0)$			
$\frac{x^2}{b^2}+\frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2-b^2$	$(0, \pm c)$	$(\pm a, 0)$	$2a$	$2b$

Since, $25 > 9$

So, above equation is of the form.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

Comparing eq. (i) and (ii), we get

$$a^2 = 25 \; and \; b^2 = 9 \;\Rightarrow a = 5 \; and \; b = 3$$

i. To find: Length of major axes

Clearly, $a > b$, therefore the major axes of the ellipse are along x-axes.

∴ Length of major axes $= 2a$

Length of major axes $= 2 \times 5$

ii. To find: Coordinates of the Vertices

Clearly, $a > b$

∴ Coordinate of vertices $= (a, 0)$ and $(-a, 0)$

Coordinate of vertices $= (5,0)$ and $(-5,0)$

iii. To find: Coordinates of the foci

We know that,

Coordinates of foci $= (\pm c, 0)$ where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$$c^2 = a^2 - b^2 = 25 - 9$$

$$c^2 = 16$$

$$c = \sqrt{16}$$

c $= 4$...(ii)

∴ Coordinates of foci $= (\pm a, 0) = (\pm 4,0)$

iv. To find: Eccentricity

We know that,

Eccentricity $= \frac{c}{a} \Rightarrow e = \frac{4}{5}$ [from (ii)]

v. To find: Length of the Latus Rectum We know that,

Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2\times(3)^2}{5}$

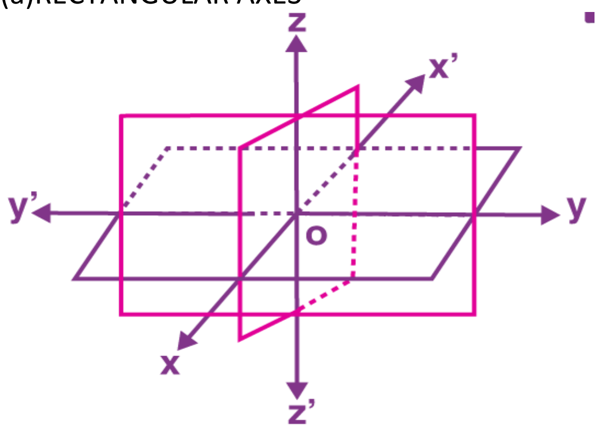
Length of Latus Rectum $= \frac{18}{5}$

INTRODUCTION TO THREE-DIMENSIONAL GEOMETRY

COORDINATE AXES AND COORDINATE PLANE

Let us take three axes in such a way that they form a right-handed system.

(a)RECTANGULAR AXES



Let X'OX, Y'OY and Z'OZ be the three mutually perpendicular straight lines,
(i)The common point O is called Origin.
(ii)X'OX is called the X-axis.
(iii)Y'OY is called the Y-axis.
(iv)Z'OZ is called the Z-axis.
These three, taken together, are called Co-ordinate-axes or simply axes.

(b) CO-ORDINATE PLANES
(i) XOY, the plane containing X and Y axes, is called XY-plane.
(ii) YOZ, the plane containing Y and Z axes, is called YZ-plane.
(iii) ZOX, the plane containing Z and X axes, is called ZX-plane.
These three, taken together, are called co-ordinate planes.

CORDINATE OF A POINT IN SPACE

An arbitrary point P in three-dimensional space is assigned coordinates (x_0,y_0,z_0) provided that
(1)The plane through P parallel to the yz-plane intersects the x-axis at$(x_0,0, 0)$;
(2)The plane through P parallel to the xz-plane intersects the y-axis at $(0,y_0,0)$;
(3)The plane through P parallel to the xy-plane intersects the z-axis at $(0,0,z_0)$.
The space coordinates (x_0,y_0,z_0) are called the Cartesian coordinates of P or simply the rectangular coordinates of P.

SIGN OF CORDINATE OF A POINT

(i)Distances measured upwards XY-plane are taken as +ve and downwards as –ve.
(ii)Distances measured in front of YZ-plane are taken as +ve and back of it as –ve.
(iii)Distances measured to the right of ZX-plane are taken as +ve and left of it as –ve.
The three co-ordinate planes divide the whole space into eight compartments, known as **octants**.

Octant→ CO-ORDINATES ↓	**I** **XOYZ**	**II** **X'OYZ**	**III** **X'OY'Z**	**IV** **XOY'Z**	**V** **XOYZ'**	**VI** **X'OYZ'**	**VII** **X'OY'Z'**	**VIII** **XOY'Z'**
x	+	–	–	+	+	–	–	+
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

DISTANCE FORMULA

The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) and is given by:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The distance of the point (x, y, z) from the origin is given by $\sqrt{x^2 + y^2 + z^2}$.

SECTION FORMULA

(i)The co-ordinates of the point, which divides the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $m_1 : m_2$ are:

$$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}, \frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$$

(ii) The co-ordinates of the mid-point are:

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

CENTROID OF TRIANGLE

The co-ordinates of the centroid of the triangle having vertices; (x_1,y_1,z_1) and (x_2,y_2,z_2) and (x_3,y_3,z_3) are: $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$.

SUMMERY

PROPERTIES	2 DIMESIONAL GEOMETRY	3-DIMENSIONAL GEOMETRY
1.COORDINATE AXIS AND PLANE	Two axes namely lines X'OX, Y'OY are called x-axis,y-axis respectively.	Three axes namely lines X'OX, Y'OY and Z'OZ are called x-axis, y-axis and z-axis. The three axes taken together in pairs determine xy, yz, zx plane, i.e., three coordinate planes.
2.COORDINATE OF A POINT IN SPACE	An arbitrary point P in 2D is assigned co-ordinates as (x_0,y_0)	An arbitrary point P in 3D is assigned co-ordinates as (x_0,y_0,z_0)
3.DISTANCE FORMULA	If $P(x_1,y_1)$ and $Q(x_2,y_2)$ are two points then distance between them are $\sqrt{(x2 - x1)^2 + (y2 - y1)^2}$.	The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) and is given by: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
SECTION FORMULA	The co-ordinates of the point, which divides the line joining the points (x, y) and (p, q) in the ratio m : nare: $\left(\frac{nx+pm}{m+n},\frac{ny+mq}{m+n}\right)$	The co-ordinates of the point, which divides the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $m_1 : m_2$ are: $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2},\frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$
MID-POINT FORMULA	The co-ordinates of the midpoint, of the line joining the points (x, y) and (p, q) are: $\left(\frac{x+p}{2},\frac{y+q}{2}\right)$	The co-ordinates of the mid-point of the line joining the points (x_1, y_1, z_1) and $(x_2, y_2, z_2)$are: $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$
CENTROID OF TRIANGLE	The coordinate of the centroid of the triangle, whose vertices are (x,y),(p,q)and(r,s)are $\left(\frac{x+p+r}{3},\frac{y+q+r}{3}\right)$	The co-ordinates of the centroid of the triangle having vertices; (x_1,y_1,z_1) and (x_2,y_2,z_2) and (x_3,y_3,z_3) are: $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$.

EXEMPLAR QUESTIONS

TYPE-1 (MCQ)

1 .What is the perpendicular distance of the point P (6, 7, 8) from xy-plane?

(A) 8 (B) 7 (C) 6 (D) None of these

Solution:-

(A)Let L be the foot of perpendicular drawn from the point P(6, 7, 8) to the xy-plane and the distance of this foot L from P is z-coordinate of P, i.e., 8 units.

2.What is the locus of a point for which y = 0, z = 0?

(A) equation of x-axis (B) equation of y-axis

(C) equation of z-axis (D) none of these

Solution:-

(A) Locus of the point y = 0, z = 0 is x-axis, since on x-axis both y = 0 and z = 0.

3.L, is the foot of the perpendicular drawn from a point P (3, 4, 5) on the xz plane. What are the coordinates of point L?

(A) (3, 0, 0) (B) (0, 4, 5) (C) (3, 0, 5) (D) (3, 4, 0)

Solution:-

(D) Since L is the foot of perpendicular segment drawn from the point P (3, 4, 5) on the xz-plane. Since the y-coordinates of all points in the xz-plane are zero, coordinate of the foot of perpendicular are (3, 0, 5).

4.The length of the foot of perpendicular drawn from the point P (3, 4, 5) on y-axis is

(A) 10 (B) $(34)^{1/2}$ (C) $(113)^{1/2}$ (D) 52

Solution:-

Let l be the foot of perpendicular from point P on the y-axis. Therefore, its x and z-coordinates are zero, i.e., (0,4,0). Therefore, distance between the points (0,4,0) and (3,4,5) is $(34)^{1/2}$.

<u>Fill in the blanks</u>

5. A line is parallel to xy-plane if all the points on the line have equal _____.

Solution:-A line parallel to xy-plane if all the points on the line have equal z-coordinates.

6. The equation x = b represents a plane parallel to _____ plane.

Solution:-

Since x = 0 represent yz-plane, therefore x = b represent a plane parallel to yz -plane at a unit distance b from the origin.

7.L is the foot of perpendicular drawn from the point P (3, 4, 5) on zx-planes. The coordinates of L are ________.

Solution:-

Since L is the foot of perpendicular from P on the zx-plane, y-coordinate of every point is zero in the zx-plane. Hence, coordinate of L are (3, 0, 5).

Check whether the statements are True or False.

8. The y-axis and z-axis, together determine a plane known as yz-plane.

Solution:- True

9. The point (4, 5, – 6) lies in the VIth octant.

Solution:- False, the point (4, 5, – 6) lies in the Vth octant,

10. The x-axis is the intersection of two planes xy-plane and xz plane.

Solution:- True.

11. Three mutually perpendicular planes divide the space into 8 octants.

Solution:- True.

12. The equation of the plane z = 6 represent a plane parallel to the xy-plane, having a z-intercept of 6 units.

Solution:- True.

13.The equation of the plane x = 0 represent the yz-plane.

Solution :- True.

14. The point on the x-axis with x-coordinate equal to x_0 is written as $(x_0,0, 0)$.

Solution :- True

<u>TYPE – (02) ASSERTION TYPE QUESTION</u>

In the following question a statement of assertion (A) is followed by a statement of Reason (R). Pick the correct option :

(a)Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true and (R) is not the correct explanation of (A).

(c)(A) is true But (R) is false.

(d)(A) is false But (R) is true.

1.ASSERTION-Vertices of triangle are A(2,0,1),B(1,-1,1),C(2,1,-1).Then side AB is $(2)^{1/2}$.

REASON- The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) and is given by:

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Solution:-

(a) Using distance formula we are able to find side AB so it is correct explanation.

2.ASSERTION-If the distance between the points (a,2,1)and (1,-1,1) is 5, then a is

$$\sqrt{16}$$

REASON - The distance between the points (x_1,y_1,z_1) and (x_2,y_2,z_2) and is given by:

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Solution:-

(d) Assertion is false and reason is true.

3.ASSERTION- The locus of a point for which x=0 is yz plane.

REASON – The co-ordinate of a point in yz plane is (x,y,0).

Solution:-

(c)Assertion is true and reason is false. As co-ordinate of point in YZ plane is (0,y,z).

ASSERTION – The point(-2,3,-3) lies in the seventh octant .

REASON – Any arbitrary point of seventh octant have all the coordinate negative.

Solution:-

(b)All the coordinate of seventh octant will be negative , so given co-ordinate will not lies in seventh co-ordinates .Hence it is not the correct explanation of given statement.

5. ASSERTION – Equation of y axis is considered as x=0,z=0.

REASON – Equation of x axis is considered as x=a,y=0,z=2.

Solution:-

(c)Equation of x-axis is x=a, and all other co-ordinates are zero.

TYPE – (03) SHORT ANSWER TYPE

1.Locate the points (i) (2, 3, 4) (ii) (–2, –2, 3) in space.

Solution:-

(i) To locate the point (2, 3, 4) in space, we move 2 units from O along the positive direction of x-axis. Let this point be A(2, 0, 0). From the point A moves 3 units parallel to +ve direction of y-axis. Let this point be B (2, 3, 0). From the point B moves 4 units along positive direction of z-axis. This is our required point be P (2, 3, 4).

(ii)From the origin, move 2 units along the negative direction of x-axis. Let this point be A (–2, 0, 0). From the point A move 2 units parallel to negative direction of y-axis. Let this point be B (–2, –2, 0). From B move 3 units parallel to positive direction of z - axis. This is our required point Q (–2, –2, 3).

2.Let L, M, N be the feet of the perpendiculars drawn from a point P (3, 4, 5)
On the x, y and z-axes respectively. Find the coordinates of L, M and N.

Solution:-

Since L is the foot of perpendicular from P on the x-axis, its y and z co-ordinates are zero. The coordinates of L is (3, 0, 0). Similarly, the coordinates of M and N are (0, 4, 0) and (0, 0, 5), respectively.

3.How far apart are the points (2,0,0)and (-3,0,0)?

Solution :-

We know that to find distance between two points $p(x_1,y_1,z_1)$ and $q(x_2,y_2,z_2)$ is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$. So, putting values of $x_1=2, y_1=0, z_1=0$ and $x_2=-3, y_2=0, z_2=0$ we get $(5^2)^{1/2}$ i.e 5.

4. Write co-ordinate of foot of perpendicular from (3,7,9) on x-axis, y-axis.

Solution:-

On $X-axis$, the co-ordinates are (3, 0, 0)

On $Y-axis$, the co-ordinates are (0, 7, 0)

On $Z-axis$, the co-ordinates are (0, 0, 9)

5. If a parallelo piped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes, then find the length of edges of a Parallele piped.

Solution:-

Length of edges of the parallelepiped are $\sqrt{(5-2)^2+(9-3)^2+(7-5)^2}$.

$\sqrt{3^2+6^2+2^2}$ =$\sqrt{49}$=7

6. Show that the points (5, –1, 1), (7, – 4, 7), (1 – 6, 10) and (–1, – 3, 4) are the vertices of a rhombus.

Solution:-

Let A (5, – 1, 1), B (7, – 4, 7), C(1, – 6, 10) and D (– 1, – 3, 4) be the four points of a quadrilateral. Here

AB = $\sqrt{(5-7)^2+(-1+4)^2+(1-7)^2}$=$\sqrt{4+9+36}$ =$\sqrt{49}$ = 7 ,

BC = $\sqrt{(7-1)^2+(-4+6)^2+(7-10)^2}$ =$\sqrt{36+4+9}$= 7,

CD = $\sqrt{(1+1)^2+(-6+3)^2+(10-4)^2}$=$\sqrt{4+9+36}$= 7

AD=$\sqrt{(5+1)^2+(-1+3)^2+(1-4)^2}$ =$\sqrt{36+4+9}$=7

AB= BC = CD = DA. Therefore, ABCD is a rhombus.

7. Three consecutive vertices of a parallelogram are $\boldsymbol{A}(-\mathbf{6},\ \mathbf{2},\ \mathbf{4})$, $\boldsymbol{B}(\mathbf{2},\ \mathbf{4},\ -\mathbf{8})$ and$\boldsymbol{C}(-\mathbf{2},\ \mathbf{2},\ \mathbf{4})$. Find the co-ordinates of the fourth vertex.

Solution:-

Let the co-ordinates of the fourth vertex be $D(x,\ y,\ z)$ we know that the diagonals of a parallelogram bisect each other

∴ Midpoint of AC = $Midpoint of BD$

⇒ $(\frac{-6-2}{2},\frac{2+2}{2},\frac{4+4}{2})$ = $(\frac{2+x}{2},\frac{4+y}{2},\frac{-8+z}{2})$

⇒ (−4, 2, 4) = $(\frac{2+x}{2},\frac{4+y}{2},\frac{-8+z}{2})$

⇒$\frac{2+x}{2}$=-4,$\frac{4+y}{2}$=2,$\frac{-8+z}{2}$=4

⇒x = −10 , y = 0 ,z = 16.

8. L is the foot of the perpendicular drawn from a point P (3, 4, 5) on the xz plane. What are the coordinates of point L.

Solution:- Since L is the foot of perpendicular segment drawn from the point P (3, 4, 5) on the xz-plane. Since the y-coordinates of all points in the xz-plane are zero, coordinate of the foot of perpendicular are (3, 0, 5).

9.Calculate the perpendicular distance of the point P(6, 7, 8) from the XY – Plane.

Solution :-Assume that A be the foot of perpendicular drawn from the point P (6, 7, 8) to the XY plane and the distance of this foot A from P is the z-coordinate of P, i.e., 8 units.

10.Find the image of:

(i) (-2, 3, 4) in the yz-plane

(ii) (-5, 4, -3) in the xz-plane

(iii) (5, 2, -7) in the xy-plane

(iv) (-5, 0, 3) in the xz-plane

Solution:-

(i) Given: Point is (-2, 3, 4)

To find: the image of the point in yz-plane

Since we need to find its image in yz-plane, a sign of its x-coordinate will change

So, Image of point (-2, 3, 4) is (2, 3, 4)

(ii) Given: Point is (-5, 4, -3)

To find: image of the point in xz-plane

Since we need to find its image in xz-plane, sign of its y-coordinate will change

So, Image of point (-5, 4, -3) is (-5, -4, -3)

(iii) Given: Point is (5, 2, -7)

To find: the image of the point in xy-plane

Since we need to find its image in xy-plane, a sign of its z-coordinate will change

So, Image of point (5, 2, -7) is (5, 2, 7)

(iv) Given: Point is (-5, 0, 3)

To find: image of the point in xz-plane

Since we need to find its image in xz-plane, sign of its y-coordinate will change

So, Image of point (-5, 0, 3) is (-5, 0, 3)

11. Find the ratio in which the line joining (2, 4, 5) and (3, 5, -9) is divided by the yz-plane.

Solution:-

We have points A(2, 4, 5) and B(3, 5, -9)

To find the ratio in which the line joining given points is divided by the yz-plane

line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).

The coordinates of C is given by, $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}, \frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$

x coordinate is always 0 on yz-plane.

Let Point C(0, y, z) and C divides AB in ratio k : 1

Therefore, m_1 = k and m_2 = 1

A(2, 4, 5) and B(3, 5, -9), Coordinates of C using section formula:

$(\frac{3k+2}{k+1}, \frac{5k+4}{k+1}, \frac{-9k+5}{k+1})$

On comparing:

⇒ 3k + 2 = 0(k + 1)

⇒ 3k + 2 = 0

⇒ 3k = −2

Hence, C divides AB externally in ratio 2 : 3.

TYPE (04)- LONG ANSWER QUESTION

1. Using distance formula show that points A(1, −1, 3), B(2, -4, 5) and C(5, −13, 11) are collinear.

Sol. AB = $\sqrt{1 - 2)\text{^}2 + (-1 + 4)\text{^}2 + (3 - 5)\text{^}2}$

= $\sqrt{1 + 9 + 4}$ =$\sqrt{14}$

BC = $\sqrt{(2-5)^2 + (-4+13)^2 + (5-11)^2}$

= $\sqrt{9 + 81 + 36}$ = $\sqrt{126}$= 3$\sqrt{14}$

AC =$\sqrt{(5-1)^2 + (-13+1)^2 + (11-3)^2}$

= $\sqrt{16 + 144 + 64}$ = $\sqrt{214}$= 4$\sqrt{14}$

Since AB+ BC = AC

∴ A, B, C are collinear.

2. A point C with z-coordinate 8 lies on the line segment joining the points A(2,-3,0) and B(2, -3, 10). Find the co-ordinates of C..

Solution:-

Let the co-ordinates of C be (x, y, 8) that divides AB in the ratio m: n

∴$\frac{10m+4n}{m+n}$=8

⇒ 10m + 4n = 8m + 8n

⇒ 2m = 4n

⇒ m = 2n

⇒ m: n = 2: 1

x = $\frac{2*8+1*2}{3}$= $\frac{18}{3}$ = 6 And

y = $\frac{2*0+1*-3}{3}$ = $\frac{-3}{3}$=-1

∴ Co-ordinate of C are (6, −1, 8)

3. What are the coordinates of the vertices of a cube whose edge is 5 units, one of whose vertices coincides with the origin and three edges passing through the origin and coincides with the positive direction of the axes through the origin? .

Solution:- Given, edge of a cube is5unit

∴ Co-ordinates of O = (0, 0, 0)

∴ Co-ordinates of A = (5, 0, 0)

∴ Co-ordinates of G = (0, 5, 0)

∴ Co-ordinates of D = (0, 0, 5)

∴ Co-ordinates of B = (5, 5, 0)

∴ Co-ordinates of F = (5, 5, 5)

∴ Co-ordinates of E = (5, 0, 5)

∴ Co-ordinates of C = (0, 5, 5).

4. Show that the points A(1,3,0), B(−5,5,2), C(−9, −1, 2) and D(−3, −3, 0) are the vertices of a parallelogram ABCD, but it is not a rectangle.

Solution:-

AB = $\sqrt{(1+5)^2+(3-5)^2+(0-2)^2}$ = $\sqrt{36+4+4}$ =$\sqrt{44}$ = $2\sqrt{11}$

BC =$\sqrt{(-5+9)^2+(5+1)^2+(2-2)^2}$ =$\sqrt{16+36+0}$= $\sqrt{52}$ = $2\sqrt{13}$

CD =$\sqrt{(-9+3)^2+(-1+3)^2+(2-0)^2}$ = $\sqrt{36+4+4}$ = $\sqrt{44}$ = $2\sqrt{11}$

AD= $\sqrt{(-3-1)^2+(-3-3)^2+(0-0)^2}$ = $\sqrt{16+81}$ = $2\sqrt{13}$

Also, AC = $\sqrt{(1+9)^2+(3+1)^2+(0-2)^2}$ =$\sqrt{100+16+4}$ =$\sqrt{120}$ = $2\sqrt{30}$

BD =$\sqrt{(-5+3)^2+(5+3)^2+(2-0)^2}$ =$\sqrt{4+64+4}$ = $\sqrt{72}$ = $6\sqrt{2}$ Since AB = CD, BC = AD and AC ≠ BD

∴ The opposite sides are equal and diagonals are unequal, so the given points are the vertices of a parallelogram not rectangle.

5.Determine the point in yz-plane which is equidistant from three points A (2, 0 3) B (0, 3, 2) and C (0, 0, 1).

Solution:-

Since x-coordinate of every point in yz-plane is zero. Let P (0, y,z) be a point on the yz-plane such that PA = PB = PC. Now PA = PB

$\Rightarrow\sqrt{(0-2)^2+(y-0)^2+(z-3)^2} = \sqrt{(0-0)^2+(y-3)^2+(z-2)^2}$

$4+y^2+z^2+9-6z = y^2+9-6y+z^2+4-4z$

i.e. z – 3y = 0 and PB = PC

$\sqrt{(0-0)^2+(y-3)^2+(z-2)^2} = \sqrt{(0-0)^2+(y-0)^2+(z-1)^2}$

$\Rightarrow y^2 + 9 - 6y + z^2 + 4 - 4z = y^2 + z^2 + 1 - 2z$,

i.e. 3y + z = 6 Simplifying the two equating, we get y = 1, z = 3 Here, the coordinate of the point P are (0, 1, 3).

TOPIC (05) – CASE BASED QUESTIONS

1) The Indian coast Guards while patrolling saw a suspicious boat with four men. They were nowhere looking like fisher men the soldiers were closely observing the movement of the boat for an opportunity to seize the boat. They observe the boat is moving along a planar surface . At instant of time theco-ordinate of the position of coast guard helicopter and boat are (2,3,5) and (1,4,2) respectively.

Based on the above information, answer the following:

i)What is the distance between helicopter and boat?

a)$\sqrt{11}$ b)$\sqrt{10}$ c)$\sqrt{12}$ d)$\sqrt{1}$

ii) what is the co-ordinate of (2,3,5) on x axis ?

a)(2,0,0) b)(2,3,0)

c)(0,3,0) d)(0,0,5)

iii) what is the co-ordinate of midpoint of helicopter and boat ?

a)$(\frac{3}{2},\frac{3}{2},\frac{7}{2})$ b)$(\frac{7}{2},\frac{7}{2},\frac{7}{2})$

c)$(\frac{3}{2},\frac{7}{2},\frac{7}{2})$ d)$(\frac{3}{2},\frac{5}{2},\frac{7}{2})$

iv) what is the distance of (1,4,2)from origin?

a)$\sqrt{12}$ b)$\sqrt{21}$ $c)\sqrt{17}$ d)$\sqrt{22}$

Solution:-

We know that to find distance between two points $p(x_1,y_1,z_1)$ and $q(x_2,y_2,z_2)$ is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.

i)(a)Putting the value (x_1,y_1,z_1) as (2,3,5) and (x_2,y_2,z_2) as (1,4,2) then we get $\sqrt{\ }$ $(3-4)^2+(5-2)^2+(2-1)$ $=\sqrt{11}$

ii)(a)co-ordinate on x axis is (2,0,0)

iii)(c) co-ordinates of mid point $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$ is $(\frac{2+1}{2},\frac{3+4}{2},\frac{5+2}{2})$

$= (\frac{3}{2},\frac{7}{2},\frac{7}{2})$

iv)(b) Putting the value (x_1,y_1,z_1) as (0,0,0) and (x_2,y_2,z_2) as (1,4,2) then we have $\sqrt{1^2+4^2+2^2}$ $=\sqrt{21}$

2)In a diamond exhibition , a diamond is covered in cubical glass box having co-ordinate A(0,0,0,),B(1,0,0),C(1,2,0),D(0,2,0),E(0,0,3),F(1,0,3),G(1,2,3)and H(0,2,3).

From the above information, answer the following:

i)(0,0,3) lies on which axis –

a)x-axis b)y-axis

c)z- axis d)origin

ii)what is the length of side AB?

a)$\sqrt{2}$ b)$\sqrt{3}$

$c)\sqrt{4}$ d)$\sqrt{21}$

$iii)$What will be the length of diagonals joining A and C?

$a)\sqrt{5}$ b)$\sqrt{1}$

$c)\sqrt{12}$ d)$\sqrt{2}$

$iv)$In which quadrant the value (1,2,3) lies?

a) I octant b)II octant

c) IV octant d) VI octant

Solution:-

i)© Given point will lie on z axis.

ii)© Using distance formula we get $\sqrt{4}$

iii)(a) Using diatance formula we get $\sqrt{5}$

iv)(a)As all the co-ordinates are positive so it will lie in 1st quadrant.

3)A mobile tower stands at the top of a hill, consider the surface on which tower stands as a plane having points A(0,1,2),B(3,4,5),C(2,4,2) on it. The tower is tied with 3 cables from the points A,C, and C such that it stands vertically on the ground. The peak of the tower is at the points (6,5,9) as shown in figure:

From the above information, answer the following:

i) What is the length of cable A from peak?

a)$\sqrt{101}$ b)$\sqrt{103}$ c)$\sqrt{102}$

d)$\sqrt{106}$

ii)In which quadrant will (3,4,-1) will lie?

a) I octant b)II octant

c) V octant d) VI octant

iii)If base of tower is at origin, then the length of tower will be-

a)$\sqrt{142}$ b)$\sqrt{123}$

c)$\sqrt{161}$ d)$\sqrt{141}$

iv) How far is cable A from cable B?

a)$\sqrt{207}$ b)$\sqrt{72}$

c)$\sqrt{27}$ d)$\sqrt{702}$

Solution:-

i)(a)Using distance formula , we have $\sqrt{101}$

ii)©According to the sign of co-ordinate in octant then this will lie in v octant .

iii)(a)Using distance formula , we have $\sqrt{142}$

iv)©Using distance formula, we have $(27)^{\frac{1}{2}}$

4)Ramesh was playing in triangular garden with is two friend, Vivek and Kundan at the corner of field, having coordinates Ramesh(0,0,0)Vivek (2,4,6),Kundan(0,-2,5).

From the above information, answer the following:

i)What is co-ordinate of mid point of line joining Ramesh and Vivek?

a) $(1,2,1)$b) $(1,2,2)$

c)$(1,2,4)$ d)$(1,2,3)$

ii)The co-ordinate of centriod of the triangle-

a)$(\frac{2}{3},\frac{2}{3},\frac{1}{3})$ c)$(\frac{2}{3},\frac{7}{3},\frac{1}{3})$

b)$(\frac{1}{3},\frac{2}{3},\frac{1}{3})$ d)$(\frac{2}{3},\frac{1}{3},\frac{1}{3})$

iii)Let D be the point of line joining vivek and ramesh such that it divides them in ratio 2:1, the co-ordinate of the point D is

a)$(\frac{2}{3},\frac{4}{3},\frac{6}{3})$ b)$(\frac{2}{3},\frac{2}{3},\frac{1}{3})$

c)$(\frac{2}{3},\frac{5}{3},\frac{1}{3})$ d)$(\frac{2}{3},\frac{0}{3},\frac{-4}{3})$

iv)What is the length of line joining vivek and kundan ?

a)$\sqrt{41}$ b)$\sqrt{72}$

c)$\sqrt{160}$ d)$\sqrt{102}$

Solution:-

i) © Using midpoint formula we have $(\frac{2+0}{2},\frac{4+(-2)}{2},\frac{6+(-5)}{2})=(\frac{2}{2},\frac{4}{2},\frac{6}{2})$=(1,2,3).

ii)(a)Coordinate of centroid is $(\frac{0+2+0}{3},\frac{0+4+(-2)}{3},\frac{0+6+(-5)}{3})=(\frac{2}{3},\frac{2}{3},\frac{1}{3})$

iii)(d) Coordinate of point D is $(\frac{0+2}{3},\frac{4-4}{3},\frac{6-10}{3})=(\frac{2}{3},\frac{0}{3},\frac{-4}{3})$

iv)(a)Using distance formula we have $\sqrt{41}$

5) Kajal is eating with her parents on rectangular table ABCD having co-ordinate A(4,7,8),B(2,3,4),C(-1,-2,1),D(1,2,5) respectively. She has four family member in her family including her and all are sitting at the corner of the table .

From the above information, answer the following:

i)What is the length of the table?

a)6 b)5

c)4 d)7

ii)what is the breadth of the table?

a)$\sqrt{46}$ b)$\sqrt{42}$

c)$\sqrt{40}$ d)$\sqrt{43}$

iii)what is the area of the table?

a)$\sqrt{1588}$ b)$\sqrt{1548}$

c)$\sqrt{1648}$ d)$\sqrt{1528}$

iv)what is the co-ordinate midpoint of AC?

a)$(\frac{2}{3},\frac{5}{2},\frac{6}{2})$ b)$(\frac{3}{2},\frac{4}{2},\frac{6}{2})$

c)$(\frac{2}{2},\frac{5}{2},\frac{1}{2})$ d)$(\frac{3}{2},\frac{5}{2},\frac{7}{2})$

Solution:-

i)(a)Using distance formula , the length of table is 6.

ii)(d)Using distance formula ,the breadth of table is $\sqrt{43}$.

iii)(b)Area of rectangle is $\sqrt{43}\times\sqrt{36}= \sqrt{1548}$.

iv)(d) Midpoint of AC is$(\frac{3}{2},\frac{5}{2},\frac{7}{2})$

SELF PRACTICE QUESTIONS

TYPE (01) – MULTIPLE CHOICE QUESTIONS

1. Write the co-ordinates of the feet of perpendicular from the point (a, b, c) on the co-ordinate axes
2. Find the perpendicular distances of the point P(a, b, c) from the co-ordinate axes.
3. Find the distance between the points A(-1, 3, -4) and B(1, -3, 4).
4. Find the locus of a point, which is equidistant from the point (-1, 2, 3) and (3, 2, 1).
5. Find 'k' so that the distance between the points (7, 1, -3) and (3, 2, 1)be 13 units.

6. Match each item given under the column C1 to its correct answer given under column C2 .

Column C1	**Column C2**
(a) In xy-plane	(i) Ist octant
(b) Point (2, 3,4) lies in the	(ii) yz-plane

(c) Locus of the points having x coordinate 0 is	(iii) z-coordinate is zero
(d) A line is parallel to x-axis if and only	(iv) z-axis
(e) If x = 0, y = 0 taken together will Represent the	(v) plane parallel to xy-plane
(f) z = c represent the plane	vi)) if all the points on the line have equal y and z-coordinates.
(g) Planes x = a,y = b represent the line	(vii) from the point on the respective.
(h) Coordinates of a point are the distances from the origin to the feet of perpendiculars.	(viii) parallel to z - axis
(i) A ball is the solid region in the space enclosed by a	(ix) disc
(j) Region in the plane enclosed by a circle is known as a	(x) sphere

7.Find the co-ordinates of the points, which trisect AB given that A is (4, 2, -6) and B is (10, -16, 6).

8. Square of distance of the point (3, 4, 5) from the origin (0, 0, 0) is

(A) 50 (B) 3 (C) 4 (D) 5 .

9. If the distance between the points (a, 0, 1) and (0, 1, 2) is 27 , then the value of a is

(A) 5 (B) ± 5 (C) – 5 (D) none of these

10. x-axis is the intersection of two planes

(A) xy and xz (B) yz and zx (C) xy and yz

(D) none of these

ANSWER

1.(a,0,0),(0,b,0),(0,0,c)

2.$\sqrt{b^2+c^2}\sqrt{a^2+c^2}$,$\sqrt{b^2+a^2}\sqrt{b^2+c^2}$,$\sqrt{a^2+c^2}\sqrt{b^2+a^2}$

3.$\sqrt{104}$

4.2x-z=0

5.9,-15

6.a-iii

b-i

c-ii

d-vi

e-iv

f-v

g-viii

h-vii

i-x

j-ix

7. (6,-4,-2) (8,-10,2)

8.a

9.b

10.a

TYPE (02) SHORT ANSWER TYPE QUESTION

1. If the origin is the centroid of a triangle ABC having vertices A(a, 1, 3), B(-2, b, -5) and C(4, 7, c), Find the values of a, b and c.
2. Find the co-ordinates of the mid-point of the join of the points A(3, 5, 7) and B(-3, -3, 1).
3. Find the points on the X-axis, which are at a distance of $2\sqrt{6}$units from the point (1, -2, 3).
4. Find the point on the Y-axis, which are at a distance of $5\sqrt{2}$units from the point (3, -2, 5).
5. Write the perpendicular distance of the point (x, y, z) from the three co-ordinate planes.
6. Using distance formula, show that the points A(-3, 2, 4); B((-1, 5, 9) and C(1, 8, 14) are collinear.
7. Show that the points A(0, 1, 2); B(2, -1, 3) and C(1, -3, 1) are vertices of an isosceles triangle.

8. Find the ratio in which yz plane divides the line segment formed by joining the points (-2, 4, 7) and (3, -5, 8).

9. The distance between the points (a, 0, 1) and (0, 1, 2) is $\sqrt{25}$. Find the value of A.

10. Find x so that the point (6, 5, -3) is at a distance of 13 unit from the point (x, -7, 0).

ANSWER

(1)a=2,b=-8,c=2

(2)(0,1,3)

(3)(12,0,0)

(4)(0,2,0) (0,-6,0)

(5) x from YOZ; y from ZOX and z from XOY plane

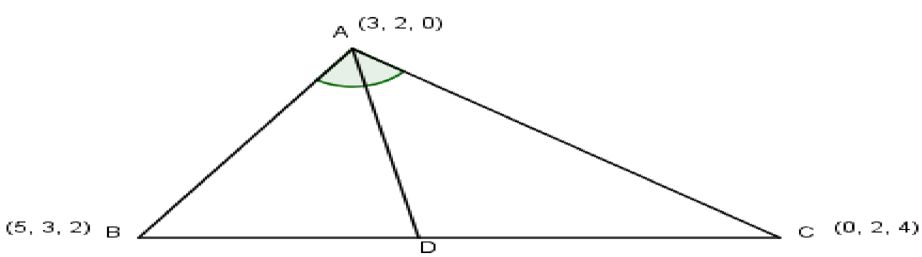
(8)3/2

(9)25

(10)(2-7,0) (10,-7,0)

TYPE-(03) CASE STUDY

1.The points A(3, 2, 0), B(5, 3, 2) and C(0, 2, 4) are the vertices of triangle ABC as given in the figure :



Based on the above information, answer the following:

1. The length of AB is:
   (a) 3 unit
   (b) 4 unit
   (c) 5 unit
   (d) none of these

2. The length of AC is:

(a) 4 unit

(b) 5 unit

(c) 6 unit

(d) none of these

3. If AD is the angle bisector ofA, the D divides BC in:

(a) 1 : 2

(b) 2 : 3

(c) 3 : 4

(d) 4 : 5

4. The co-ordinates of point D are:

(a) $\left(\frac{25}{8},\frac{21}{8},\frac{22}{8}\right)\left(\frac{25}{8},\frac{21}{8},\frac{22}{8}\right)$

(b) $\left(\frac{23}{8},\frac{19}{8},\frac{17}{8}\right)\left(\frac{23}{8},\frac{19}{8},\frac{17}{8}\right)$

(c) (2, 3, 4)

(d) none of these

5. The length of angle bisector AD is:

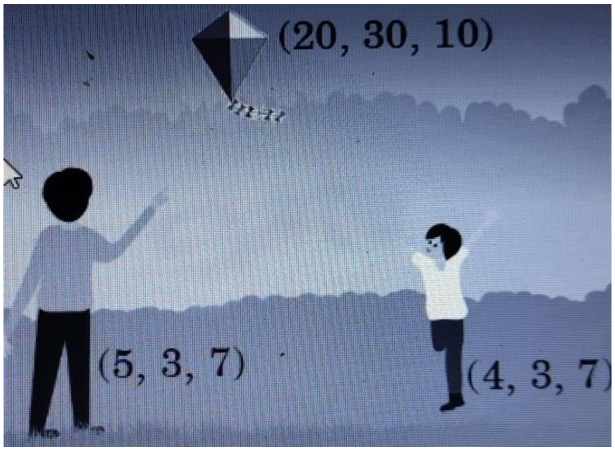
(a) $\frac{1}{4}\sqrt{510}$

(b) $\frac{1}{8}\sqrt{510}$

(c) $\frac{1}{64}\sqrt{510}$

(d) none of these

2. Raj and his father were walking in a large park. They saw a kite flying in the sky. The position of kite, Raj and Raj's father are at (20, 30, 10), (4, 3, 7) and (5, 3, 7) respectively.



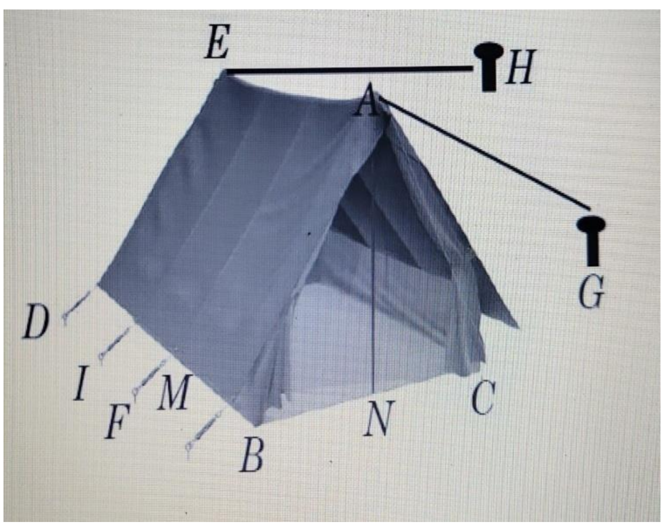
On the basis of above information, answer the following:

(i) Find the distance between Raj and kite.

(ii) Find the form of the co-ordinates of points in the XY-plane.

(iii) If co-ordinates of kite, Raj and Raj's father form a triangle, then find the centroid of it.

3.Deepak and his friends went for camping for 2 or 3 days. There they set up a tent which is triangular in shape. The vertices of the tent are A(4, 5, 9), B(3, 2, 5), C(5, 2, 5), D(–3, 2, –5) and E(–4, 5, –9) respectively. The vertex A is tied up by the rope at the ends F and G and the vertex E is tied up at the ends I and H.



On the basis of above information, answer the following:

(i) If M denotes the position of their bags inside the tent and it is just in middle of the vertices B and D, then find the coordinates of M. (1)

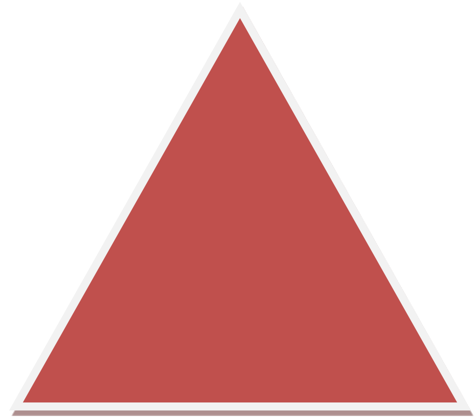
(ii) Find the length AE. (1)

(iii) What is the length of the rope by which E is tied up with D ?

4.Consider a ABC with vertices A(x, y, z), B(a, b, c) and

C(p,q,r). AD, BE and CF are medians of ABC. G is the point of intersection.

Based on the above information, answer the following questions:-



i.) Coordinates of Point D are ?

(a)($\frac{a+p}{2}$,$\frac{b+q}{2}$,$\frac{c+r}{2}$)

(b)($\frac{a+2p}{2}$,$\frac{b+2q}{2}$,$\frac{c+2r}{2}$)

(c)($\frac{x+a+p}{3}$,$\frac{y+b+q}{3}$,$\frac{z+c+r}{3}$)

d) None of these

ii)For ABC, G is

(a) Incentre

(b) Circumcentre

(c) Centroid

(d) Orthocentre

iii)G divides BE in ratio

(a) 1 : 2

(b) 2 : 1

(c) 3 : 1

(d) 1 : 3

iv)A point G divides AD in 2 : 1, the coordinates of G are

(a)($\frac{a+p}{2}$,$\frac{b+q}{2}$,$\frac{c+r}{2}$)

(b)($\frac{a+2p}{2}$,$\frac{b+2q}{2}$,$\frac{c+2r}{2}$)

(c)($\frac{x+a+p}{3}$,$\frac{y+b+q}{3}$,$\frac{z+c+r}{3}$)

(d)($\frac{a+2p}{2}$,$\frac{b+2q}{2}$,$\frac{c+2r}{2}$).

SOLUTION :-

1)(i)a

(ii)b

(iii)d

(iv)a

(v)b

2)(i)$\sqrt{944}$

(ii)(5,3,0),(4,3,0),(20,30,0)

(iii)($\frac{29}{3}$,$\frac{36}{3}$,$\frac{24}{3}$)

3)(i)(3,0,5)

(ii)$\sqrt{388}$

(iii)$\sqrt{26}$

4)(i)a

(ii)c

(iii)b

(iv)c

TYPE (04) – LONG ANSWER QUESTION

1. Find the co-ordinates of the point R, which divides the join of P (0, 0, 0) and Q (4, -1, -2) in the ratio 1 : 2 externally and verify that P is the mid-point of RQ.
2. If A and B are the points (1, 2, 3) and (0, -1, 2) and P is a point such that $AP^2 - BP^2 = 10$. Find the equation of locus of P.
3. The diagonals of a parallelogram meets in the point (1, -2, 3) and the ends of a side are (0, 0, 0) and (2, 4, -3). Find the remaining two vertices of the parallelogram.
4. Find x so that the point (6, 5, -3) is at a distance of 13 unit from the point (x, -7, 0).
5. The mid-point of the sides of a triangle are (1, 5, -1), (0, 4, -2) and (2, 3, 4). Find the vertices of triangle and also find the centroid.

Answers

1) (-4,1,2)
2) 2X+6Y+2Z=0
3) (2,-4,6) (0,-8,9)
4) X=2,10
5) (3,4,5),(-1,6,-7),(1,2,3),(1,4,1/3)

TYPE (05) – ASSERSTION REASONS BASED QUESTION

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

1). Assertion (A) : If a parallelopiped is formed by planes drawn through the points (5, 8, 10) and (3, 6, 8) parallel to the coordinate planes, then the length of diagonal of the parallelopiped is 23.

Reason(R) : If a parallelopiped is formed by planes drawn through the point p(x,y,z) and Q(a,b,c) parallel to the coordinate planes, then the length of the diagonal = $\sqrt{(x-a)^2+(y-b)^2+(z-c)^2}$.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

2). Assertion (A) : The point on x – axis which is equidistant from the points A(3,2,2) and B(5,5,4) is (4,0,0)

Reason (R) : Any point on the x – axis is of the form (x,0,0).

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

3). Assertion (A) : The point (– 4, 5, – 6) lies in the VI octant.

Reason (R) : The three coordinate planes divide the space into eight equal parts known as octants.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

4). Assertion (A) : The points (3, – 1, – 1), (5, – 4, 0), (2, 3, – 2) and (0, 6, – 3) are the vertices of a parallelogram.

Reason (R) : In the parallelogram, both the pairs of opposite sides are equal and diagonals are also same.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

ANSWER

1)(A) Both A and R are true and R is the correct explanation of A.

2)(D) A is false but R is true.

3)(B) Both A and R are true but R is not the correct explanation of A.

4)(C) A is true but R is false.

LIMITS AND DERIVATIVES

CONCEPTS:

Let y = f(x) be a function of x. If at x = a, f(x) takes indeterminate form, then we consider the values of the function which is very near to a. If these values tend to a definite unique number as x tends to a, then the unique number so obtained is called the limit of f(x) at x = a and we write it as $\lim_{x \to a} f(x) = f(a)$.

Left Hand and Right-Hand Limits

If values of the function at the point which are very near to a on the left tends to a definite unique number as x tends to a, then the unique number so obtained is called the left-hand limit of f(x) at x = a, we write it as

$$LHL: \lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a-h)$$

Similarly, right hand limit is

$$RHL: \lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a+h)$$

Existence of Limit

Limit of $f(x)$ exists, if $\boldsymbol{LHL = RHL}$

Some Properties of Limits

Let f and g be two functions such that both $f(x)$and $g(x)$exists, then

(i) $\lim_{x \to a}\{(f+g)(x)\} = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

(ii) $\lim_{x \to a}(kf(x)) = k \lim_{x \to a} f(x)$

(iii) $\lim_{x \to a}[f(x) \,.\, g(x)] = \lim_{x \to a} f(x) \text{x} \lim_{x \to a} g(x)$

(iv) $\lim_{x \to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, where g(x) ≠ 0

Some Standard Limits

(i) $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

(ii) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(iii) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

(iv) $\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a$

(v) $\lim_{x\to 0}\frac{e^x-1}{x}=1$

(vi) $\lim_{x\to a}\frac{log(1+x)}{x}=1$

Derivatives

Suppose f is a real-valued function, then

$f'^{(x)}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is called the derivatives f at x iff $\frac{f(x+h)-f(x)}{h}$ exists finitely.

Fundamental Derivative Rules of Function

(i) $\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}[f(x)]+\frac{d}{dx}[g(x)]$

(ii) $\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}[f(x)]-\frac{d}{dx}[g(x)]$

(iii) $\frac{d}{dx}[f(x).g(x)]=\left[\frac{d}{dx}f(x)\right].g(x)+f(x)\left[\frac{d}{dx}g(x)\right]$

(iv) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{\left[\frac{d}{dx}f(x)\right].g(x)-f(x)\left[\frac{d}{dx}g(x)\right]}{[g(x)]^2}$

Some Standard Derivatives $\frac{d}{dx}(x^n)=nx^{n-1}$

(i) $\frac{d}{dx}(sinx)=cosx$

(ii) $\frac{d}{dx}(\cos x)=-sinx$

(iii) $\frac{d}{dx}(\tan x)=sec^2x$

(iv) $\frac{d}{dx}(\cot x)=-cosec^2x$

(v) $\frac{d}{dx}(\sec x)=\sec x\tan x$

(vi) $\frac{d}{dx}(cosec\ x)=-cosec\ x\cot x$

(vii) $\frac{d}{dx}(a^x)=a^x\ln a$

(viii) $\frac{d}{dx}(e^x)=e^x$

(ix) $\frac{d}{dx}(logx)=\frac{1}{x}$

MCQs

1. $\lim_{x \to 0} \frac{1-\cos x}{\sin x}$ is equal to:

(a) 0 (b) $\frac{2}{3}$ (c) $-\frac{3}{2}$ (d) 1

2. Find the derivative of $\sin^n x$ with respect to x.

(a) $n \sin^{n-1} x . \sin x$ (b) $n \cos^{n-1} x . \sin x$

(c) $(n-1) \sin^n x . \cos x$ (d) $n \sin^{n-1} x . \cos x$

3. Evaluate: $\lim_{x \to 0} \frac{(x+1)^5 - 1}{x}$

(a) 2 (b) 5 (c) 0 (d) 1

4. Evaluate: $\lim_{x \to a} \frac{sin ax}{bx}$

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) $-\frac{a}{b}$ (d)) 0

5. Evaluate: $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

(a) π (b) $\frac{1}{\pi}$ (c) 0 (d) 1

6. Evaluate: $\lim_{x \to \frac{\pi}{2}} \frac{tan 2x}{\left(x - \frac{\pi}{2}\right)}$

(a) 2 (b) 5 (c) 0 (d) π

7. Find the derivative of (sin x. cos x) with respect to x.

a) cos 2x (b) cos x (c) sin 2x (d) sin x

8. Find the derivative of $[x^{-4}. (3 - 4x^{-7})]$ with respect to x.

(a) $12x^{-5} + 44x^{-12}$ (b) $-12x^{-5} + 44x^{12}$

(c) $12x^{-5} + 44x^{12}$ (d) $-12x^{-5} + 44x^{-12}$

9. Find the derivative of (tan x – sec x) with respect to x.

a) $Sec^2 x$ + sec x tan x (b) $Sec^2 x$ – sec x cot x

(c) $Sec^2 x$ – sec x tan x (d) Sec x – sec x tan x

10. Find the derivative of (5 sin x – 7 sec x) with respect to x.

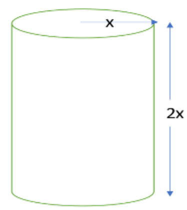
a) 5 cos x – 7 sec x (b) 7 cos x – 5 sec x

(c) 5 cos x +7 sec x tan x (d) 5 cos x – 7 sec x tan x

	ASSERTION REASONING QUESTIONS
1.	**Statement**: f(x) = $sin^2x + \frac{1}{2}cos2x + cot\alpha, then$ f$'$ (x) = 0 **Reason**: Derivative of a constant function is always zero. (a) Both A and R are true and R is the correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true . (e) Both A and R are false.
2.	**Statement**: The derivative of y = $2x - \frac{3}{4}$ w. r .t. x is 2. **Reason**: The derivative of y=cx w. r .t. x is c. (a) Both A and R are true and R is the correct explanation of A . (b) Both A and R are true but R is NOT the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true. (e) Both A and R are false.

3.	**Statement**: The derivative of f(x) = x^3 $w.r.t.\ x$ is x^2 . **Reason**: The derivative of f(x) = x^n $w.r.t.\ x$ is x^{n-1}. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is NOT the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true. (e) Both A and R are false.
4.	**Statement**: $\lim_{x\to 0} \frac{sinax}{bx}$ is $equal\ to\ \frac{a}{b}$. **Reason**: $\lim_{x\to 0} \frac{sinx}{x} = 1$. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is NOT the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true. (e) Both A and R are false.
5.	**Statement**: : $\lim_{x\to 0} \frac{cos2x-1}{cosx-1}$ is $equal\ to$ 4. **Reason**: : $\lim_{x\to 0} \frac{tanx}{x} = 1$ (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is NOT the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true. (e) Both A and R are false.

	SHORT ANSWER TYPE QUESTIONS
1.	Find the derivative of $\frac{1}{x}$ with respect to x.
2.	Find the derivative of $x^3 - 3$ w. r .t. x at x = 10.
3.	Find the derivative of (5 sec x + 7cos x) with respect to x.
4.	Find $f(x)$, where f(x) = $\|x\|$ - 5.
5.	Evaluate : $\lim_{x\to 0}\frac{ax+b}{cx+1}$
6.	Find the derivative of $\frac{x+1}{x-1}$ from first principle.
7.	If the function f(x) satisfies $\frac{f(x)-2}{x^2-1} = \pi$. Evaluate: $f(x)$
8.	Find the derivative of $(x^3 + x^2 + 3)(x - 5)$ with respect to x.
9.	Find $f(x)$, where f(x) = $\begin{cases} x^2 - 1, & x \leq 1 \\ -x^2 - 1, & x > 1 \end{cases}$ at x=1
10.	For the function f(x) = $\frac{x^{100}}{100} + \frac{x^{99}}{99} + \cdots\cdots\cdots + \frac{x^2}{2}$ + x + 1. Prove that: f '(1) = 100 f '(0).

	CASE STUDY BASED QUESTIONS
1.	The shape of a Syntex water tank of a house as shown in the figure.  Radius of its base of tank is ‘x’ m and its height is ‘2x’ m. If total surface area and volume of tank is denoted by A and V respectively then:
(i)	What is the derivative of circumference of its base? (a) π (b) 2 π (c) 3 π (d) 4 π
(ii)	Find $\frac{dA}{dx}$: (a) 12 π x (b) 24 π x (c) 36 π x (d) 48 π x
(iii)	Find $\frac{dV}{dx}$: (a) 2 π (b) 4 π (c) 6 π x^2 (d) 8 π
(iv)	What is the derivative of total surface area of the tank at x = 3: (a) 12 π (b) 24 π (c) 36 π (d) 48 π
(v)	Find the derivative of its volume at x = 2: (a) 6 π (b) 2 π (c) 18 π (d) 24 π
2.	Indeterminate forms of limits. On direct evaluation, if a limit takes the form $\frac{0}{0}, \frac{\infty}{\infty}, 0 \times \infty, \cdots$, we use standard results for evaluating the limits. The below figure shows a few indeterminate forms. $\frac{0}{0}, \infty^0, \frac{\infty}{\infty}$, $0^0, 1^0, 0 \times \infty, \infty - \infty$ Based on the above data, answer any four of the following questions

(i)	Evaluate: $\lim_{x\to 2}\frac{x^6-64}{x-2}$ (a) 0 (b) 80 (c) 192 (d) ∞
(ii)	Evaluate: $\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}$ (a) 0 (b) $\frac{3}{2}$ (c) ∞ (d) 15
(iii)	Evaluate: $\lim_{x\to 0}\frac{\sqrt{1+3x}-\sqrt{1-3x}}{x}$ (a) 0 (b) 1 (c) 3 (d) 6
(iv)	Evaluate: $\lim_{x\to 0}\frac{8^x-2^x}{x}$ (a) 0 (b) log2 (c) log4 (d) log8
(v)	Evaluate: $\lim_{x\to 0}\frac{cos5x-cosx}{x^2}$ (a) 0 (b) -12 (c) 1 (d) 12
3.	Let f(x) be a real function defined as $f(x)=\frac{log(1+ax)-\log(1-bx)}{x}$, $x<0$ $f(x)=5, x=0$ $f(x)=\frac{\sqrt{1+bx}-1}{x}$, $x>0$ Based on the above information, answer the following questions:
(i)	$f(x)$ is (a) a + b (b) a - b (c) b - a (d) -a - b
(ii)	$f(x)$is (a) b (b) $b/2$ (c) $b/3$ (d) $b/4$

(iii)	$f(x) = f(x)$, then a relation between a and b is (a) a+2b=0 (b) 2a-b=0 (c) 2a+b=0 (d) 3a+2b=0
(iv)	The value of b, if $f(x)$ = f (0) is (a) 5 (b) 15 (c) 20 (d) 10
(v)	The values of a and b if $f(x)$ = f (0) = $f(x)$ are (a) -5,5 (b) -5,10 (c) 5,10 (d) 10,15
4.	The derivative of y with respect to x is the change in y with respect to a change in x. The derivative of f(x) at x_o is given by $f'(x_o) = \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_o+\Delta x)-f(x_o)}{\Delta x}$ Based on the above information, answer any four of the following questions:
(i)	Derivative of sin x with respect to x is: (a) sin x (b) cos x (c) -sin x (d) -cos x
(ii)	Derivative of cos x with respect to x is: (a) sin x (b) cos x (c) -sin x (d) -cos x
(iii)	Derivative of tan x with respect to x is : (a) sec^2x (b) $-sec^2x$ (c) $cosec^2x$ (d) $-cosec^2x$
(iv)	If $f(x) = x^{100} - x^{80}$, f′ (1) is………. (a) 0 (b) 20 (c) 51 (d) 101100
(v)	$y = \frac{x}{tanx}$, $then \frac{dy}{dx}$=…………. (a) cos^2x (b) sec^2x (c) $\frac{tanx-secx}{tan^2x}$ (d) $\frac{tanx-x\,sec^2x}{tan^2x}$

	LONG ANSWER TYPE QUESTIONS
1.	Find the derivative of cos (x +1) from first principle.
2.	Suppose f(x) = $\{a + bx, \quad x < 14, \quad x = 1b - ax, \quad x > 1$ and if $f(x) = f(1)$. What are the possible values of a and b?
3.	Find the derivative of $\frac{a}{x^4} - \frac{b}{x^3} + \frac{c}{x^2} + cosx$ w.r.to x.
4.	Find the derivative of (x cos x) from first principle.
5.	If f(x) = $\{ mx^2 + n, \; x < 0; \quad nx + m, \; 0 \le x \le 1; \quad nx^3 + m, \; x > 1$ for what integers m and n does both $f(x)$ and $f(x)$ exist ?

ANSWER KEY AND SOLUTIONS

MCQs

1. a	2. d	3. b	4. a	5. b
6. a	7. a	8. d	9. c	10. d

ASSERTION REASONING QUESTIONS

1.	(b) Both A and R are true but R is NOT the correct explanation of A.
2.	(a) Both A and R are true and R is the correct explanation of A
3.	(d) A is false but R is true
4.	(a) Both A and R are true and R is the correct explanation of A
5.	(b) Both A and R are true but R is NOT the correct explanation of A

SHORT ANSWER TYPE QUESTIONS

1.	$-\frac{1}{x^2}$
2.	300
3.	5 sec x tan x – 7 sin x
4.	0
5.	b
6.	$\frac{-2}{(x-1)^2}$
7.	2
8.	$4x^3 - 12x^2 - 10x + 3$
9.	Limit does not exist at x = 1.
10.	Prove that f '(1) = 100 f '(0)

CASE STUDY BASED QUESTIONS

1.	(i) 2 π	(ii) 12 π x	(i) 6 πx^2	(iv) 36 π	(v) 24 π
2.	(i) c	(ii) b	(iii)c	(iv) c	(v)b
3.	(i) a	(ii) b	(iii) c	(iv) d	(v)b
4.	(i) b	(ii) c	(iii) a	(iv) b	(v)b

LONG ANSWER TYPE QUESTIONS

1.	-Sin (x +1)
2.	a=0, b=4.
3.	$-\frac{4a}{x^5}+\frac{3b}{x^4}$ - $\frac{2c}{x^3}$ - sinx
4.	Cos x - x sin x
5.	For $f(x)$, m = n and for $f(x)$, m and n may have any integral value.

STATISTICS

CONCEPTUAL NOTES:

** Mean ,$\bar{X} = \frac{\Sigma \ x_i}{n}$

$= \frac{\Sigma \ f_i x_i}{\Sigma \ f_i}$

= a +($\frac{\Sigma \ f_i u_i}{\Sigma \ f_i}$) h ,where a is the assumed mean, h is the class width and $u_i = \frac{x_{i-a}}{h}$

**** Median** = ($\frac{n+1}{2}$)th observations (arranged in ascending or descending order) and the number of observations is odd.

= mean of ($\frac{n}{2}$)$th \ and \ \left(\frac{n}{2}+1\right) th$ observations arranged in ascending or descending order & the number of observations is odd.

= l + ($\frac{\frac{n}{2}-cf}{f}$)xh , where, l lower limit of median class, n number of observations, cf cumulative frequency of class preceding the median class, f frequency of median class, h class size.

** Measures of Dispersion: The dispersion or scatter in a data is measured on the basis of the observations and the types of the measure of central tendency, used there. There are following measures of dispersion:

(i) **Range**, (ii) **Quartile deviation**, (iii) **Mean deviation**, (iv) **Standard deviation**.

** **<u>Range:</u>** Range of a series = Maximum value – Minimum value.

** **<u>Mean Deviation</u>** : The mean deviation about a central value „a" is the mean of the absolute values of the deviations of the observations from „a". The mean deviation from „a" is denoted as M.D. (a).

(i)**<u>For ungrouped data</u>**: M.D about mean=$\frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$,where $\bar{x}$ is the mean .

M.D about median =$\frac{1}{n}\sum_{i=1}^{n}|x_i - M|$,where M is the median.

(ii) For grouped frequency distribution:

(a) Discrete frequency distribution:

MD $(\bar{x}) = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{\sum_{i=1}^{n} f_i} = \frac{1}{n}\sum_{i=1}^{n} f_i |x_i - \bar{x}|$

MD $(M) = \frac{1}{n}\sum_{i=1}^{n} f_i |x_i - M|$

(b) Continuous frequency distribution:

MD $(\bar{x}) = \frac{1}{n}\sum_{i=1}^{n} f_i |x_i - \bar{x}|$,$\bar{X}$ = a +($\frac{\Sigma \ f_i u_i}{\Sigma \ f_i}$) h , MD $(M) = \frac{1}{n}\sum_{i=1}^{n} f_i|x_i - M|$,M= l + $(\frac{\frac{n}{2}-cf}{f})$xh

(iii)Variance : Let $x_1, x_2, ..., x_n$ be n observations with $\bar{x}$ is the mean. The variance, denoted $\sigma^2 = \frac{1}{n}(x_i - \bar{x})^2$

(iv)Standard Deviation: If σ^2 is the variance, then σ, is called the standard deviation, is given by $\sigma = \sqrt{\frac{1}{n}(x_i - \bar{x})^2}$

(v)Standard deviation for a discrete frequency distribution is given by

$$\sigma = \sqrt{\frac{1}{N} f_i (x_i - \bar{x})^2}$$

where fi's are the frequencies of xi' s and N = $\sum_{i=1}^{n} f_i$

(vi) Standard deviation of a continuous frequency distribution (grouped data) is given by

$$\sigma = \sqrt{\frac{1}{N} f_i (x_i - \bar{x})^2}$$

where x_i are the midpoints of the classes and f_i their respective frequencies.

<u>MCQs</u>

1. The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is
   (A) 2.23 (B) 2.57 (C) 3.23 (D) 3.57

Solution (B) is the correct answer

M.D. (x) = (4+ 3+ 3+ 3+ 0+ 3+ 2)/7 =2.57

2. Variance of the data 2, 4, 5, 6, 8, 17 is 23.33. Then variance of 4, 8, 10, 12, 16, 34 will be

   (A) 23.23 (B) 25.33 (C) 46.66 (D) 48.66

Solution (C) is the correct answer. When each observation is multiplied by 2, then variance is also multiplied by 2.

1. A set of n values x1, x2, ..., xn has standard deviation σ . The standard deviation of n values x1 + k, x2 + k, ..., xn + k will be
   (A) σ (B) σ + k (C) σ – k (D) kσ

Solution (A) is correct answer. If observation is increased by a constant k, , then standard deviation is unchanged.

2. A quality control team tested the lifetimes of a set of light bulbs to ensure their reliability. The recorded lifetimes (in ours) for 5 bulbs were: 1357, 1090, 1666, 1494, 1623

Calculate the mean deviation (in hours) from the mean lifetime for these light bulbs.

A) 178 B) 179 C) 220 D) 356

Ans. A)178

3. The following information relates to a sample of size 60:

X^2 = 18000, x = 960. The variance is:

(A) 6.63 (B) 16 (C) 22 (D) 44
Ans: 44

6. The average marks obtained by the students in a class are 43. If the average marks obtained by 25 boys are 40 and the average marks obtained by the girl students are 48, then what is the number of girl students in the class?

a) 18 b) 17 c) 20 d) 15

Ans. d)15

7. The sum of 10 items is 12 and the sum of their squares is 18. The standard deviation is

(a)1/5 (b) 2/5 (c) 3/5 (d) 4/5

Ans: (c) 3/5

8. Find the mean deviation about the median for the following data :

3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21.

(a) 5.5 (b) 5.27 (c) 6.27 (d) 6.5

Ans: (b) 5.27

9. When tested the lives (in hours) of 5 bulbs were noted as follows : 1357, 1090, 1666, 1494, 1623. The mean of the lives of 5 bulbs is

(a) 1445 (b) 1446 (c) 1447 (d) 1448

Ans: (b) 1446

10. Compute the variance and standard deviation of the following observations of marks of 5 students of a tutorial group :

Marks out of 25 : 8 , 12, 13 , 15 , 22 .

(a) 13 (b) 15 (c) 15.2 (d) 21.2

Ans: (d) 21.2

Assertion Reasoning

DIRECTION : In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true

11. Assertion (A): Sum of absolute values of Mean of deviations = $\frac{Deviations}{Nu,ber\ of\ observations}$

Reason(R) : Sum of deviations from mean is 1.

Soln: Answer is (C) A is correct and R is incorrect as the sum is zero.

12. Assertion (A):Mean of deviations = $\frac{product\ of\ Deviations}{Nu,ber\ of\ observations}$

Reason(R) :To find the dispersion of values of x from mean ,we take absolute measure of dispersion.

Soln: Answer is (d)

13. Let $x_1, x_2, x_3 \ldots\ x_n\ be\ n\ observations\ and\ let\ \bar{x}$ be the mean and σ^2 be the variance .

Assertion (A): Variance of $2x_1, 2x_2, 2\ x_3 \ldots\ 2x_n$ is $4\sigma^2$

Reason(R) : Arithmetic mean of $2x_1, 2x_2, 2\ x_3 \ldots\ 2x_n$ is $4\bar{x}$.

Soln: Answer is (C) If each observation is multiplied by k, mean gets multiplied by k and variance gets multiply by square of k.

Hence the new mean = $2\bar{x}$ and new variance $=k^2\sigma^2$ ..

So Assertion is true and reason is false.

14. Assertion (A): The range is the difference between two extreme observations of the distribution.

Reason(R) : The variance of a variate X is the arithmetic mean of the squares of all k deviations of X from the arithmetic mean of the observations.

Soln : Answer is (b)

15. Assertion (A):The mean deviation of the data 2, 9, 9, 3, 6, 9,4 from the mean is 2,57 .

Reason(R) : For individual observation Mean Deviation about mean is $\frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$

Soln: Answer is (a)

Mean =n(2+3+9+9+3+6+9+4)/7 =6

MD= (4+3+3+3+0+3+2)/7 = 2.57

SHORT ANSWER QUESTIONS

16. A fitness trainer is conducting a study on the performance of two different workout routines. The trainer tracks the number of repetitions completed by participants in each routine. There are two sets of observations, each containing 20 participants. The first set has a mean of 17 repetitions, and the second set has a mean of 22 repetitions. Surprisingly, both sets have the same standard deviation of 5 repetitions.

What would be the standard deviation of the combined set obtained by merging the two sets of observations?

Ans: To determine the standard deviation of the combined set, we need to consider the concept of weighted averages and their effect on standard deviation.

Calculate the weighted average of the means:

Weighted Mean = (Number of observations in Set 1 * Mean of Set 1 + Number of observations in Set 2 * Mean of Set 2) / Total Number of Observations

Weighted Mean = (20 * 17 + 20 * 22) / 40 = 19.5.

Calculate the variance of the combined set using the formula:

Variance = (Number of observations in Set 1 * Variance of Set 1 + Number of observations in Set 2 * Variance of Set 2) / Total Number of Observations

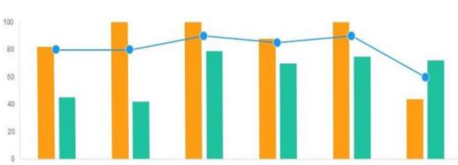
Variance = (20 * 5^2 + 20 * 5^2) / 40 = 25.

Calculate the standard deviation of the combined set:

Standard Deviation = √Variance = √25 = 5.

Therefore, the standard deviation of the combined set obtained by merging the two sets of observations would be 5.

17.A collection of 100 items was analyzed, and the statistical properties of the data were observed. The mean of the items is 50, and the standard deviation is 4.



Calculate the sum of all the items and the sum of the squares of the items.

Ans: To solve this, we can use the formulas for the mean and standard deviation:

Sum of all items = Mean × Number of items = 50 × 100 = 5000.

Sum of squares of items = Variance × (Number of items - 1) + Mean^2 × Number of items

= $(4^2) \times (100 - 1) + 50^2 \times 100 = 159600$.

So, the sum of all the items is 5000, and the sum of the squares of the items is 159600.

18. A set of data points were collected and analyzed. For this distribution, two pieces of information were obtained: (x – 5) = 3 and $(x – 5)^2 = 43$. It is also known that the total number of items in the dataset is 18.

Calculate the mean and standard deviation for this distribution.

Ans: Given that (x - 5) = 3, we can solve for x:

x = 3 + 5 = 8.

Now, let's calculate the mean and standard deviation:

Mean:

The sum of all values (x) can be calculated by multiplying the mean by the number of items: Sum = Mean × Number of items = 8 × 18 = 144.

Variance:

Variance = [(Sum of squares of all values) - (Sum of all values)^2 / Number of items] / (Number of items - 1)

Plugging in the values:

Variance = $[(43) - (144^2 / 18)] / 17 \approx 9$.

Standard Deviation:

Standard Deviation = √Variance = √9 = 3.

So, the mean of the distribution is 8 and the standard deviation is 3.

19. If the mean and standard deviation of 100 observations are 50 and 4 respectively. Find the sum of all the observations and the sum of their squares.

Ans: Let $x_1, x_2, \ldots\ldots x_{100}$ be 100 observations and their mean = $\bar{x}$ and standard deviation = σ

Mean $\bar{x} = \frac{\Sigma x}{n}$ $50 = \frac{\Sigma x}{100}$ Sum of all observations Σx = 5000	$\sigma^2 = \frac{\Sigma x^2}{n} - \bar{x}^2$ $4^2 = \frac{\Sigma x^2}{100} - 50^2$ 1600 = $\Sigma x^2 - 250000$ Sum of their squares Σx^2 = 251600

20. Calculate the mean deviation about the median of the following observations:

38, 70, 48, 34, 63, 42, 55, 44, 53, 47

Ans: Arranging the observations in ascending order :34 , 38 , 42 , 44 , 47 , 48 , 53 , 55 , 63 , 70

Median M = $\frac{47+48}{2}$ = 47.5

Calculation of mean deviation about the median .

x	$\lvert d \rvert = \lvert x - M \rvert$
34	13.5
38	9.5
42	5.5
44	3.5
47	0.5
48	0.5
53	5.5
55	7.5
63	15.5
70	22.5
	$\Sigma \lvert x - M \rvert = 84$

Mean Deviation = $\frac{\Sigma|x-M|}{n} = \frac{84}{10}$ = 8.4

21. Find the mean and variance of first n natural numbers.

Ans: First n natural numbers are = 1 , 2 , 3 , ………… , n

Mean $\underline{x} = \frac{(1+2+3+\cdots+n)}{n} = \frac{1}{n} \cdot \frac{n(n+1)}{2} = \frac{(n+1)}{2}$

Variance $\sigma^2 = \frac{\Sigma x^2}{n} - \underline{x}^2 = \frac{\Sigma n^2}{n} - \left\{\frac{(n+1)}{2}\right\}^2 = \frac{n(n+2)(2n+1)}{6n} - \frac{(n+1)^2}{4} = \frac{(n^2-1)}{12}$

22. Find the variance and standard deviation for the following data: 57, 64, 43, 67, 49, 59, 44, 47, 61, 59.

Ans: Mean($\underline{x}$)=$\frac{57+64+43+67+49+59+61+59+44+47}{10} = \frac{550}{10} = 55$

Variance (σ^2)

$=\frac{\Sigma \quad (x_i-\underline{x})^2}{n}$

$$= \frac{2^2 + 9^2 + 12^2 + 12^2 + 6^2 + 4^2 + 6^2 + 4^2 + 11^2 + 8^2}{10}$$

$$= \frac{662}{10}$$

=66.2

Standard deviation (σ)=$\sqrt{\sigma^2} = \sqrt{66.2} = 8.13$

23. The mean weight of 150 students in a certain class is 60 kilograms. The mean weight of boys in the class is 70 kilograms and that of the girls is 55 kilograms, then find the number of boys and girls of the class.

Ans: Total students in class =150

mean weight=60kg

total weight =150×60=9000kg

Let the total number of boys =x

mean weight of boys =70kg

total weight of boys=70xkg

total number of girls = total students - no. of boys =150−x

mean weight of girls=55kg

total weight of girls =55×(150−x)=55×150−55x=(8250−55x)kg

Total weight = weight of boys + weight of girls

9000=70x+(8250−55x)

9000=70x+8250−55x

9000−8250=70x−55x

750=15x

x=15750

x=50

So number of boys =50

number of girls =150−50=100

24. The mean of 100 observations is 50 and their standard deviation is 5. Then find the sum of squares of all observations.
Ans: $\sum x_i^2 = n \{ \sigma^2 + (\bar{x})^2 \} = 100\ (50^2 + 5^2) = 252500$
25. Mean of 10 items is 17. If an observation 21 is replaced with 12, then what will be the new mean.
Ans: Original sum of all the 10 items

=(mean x number of items)

=17 x 10

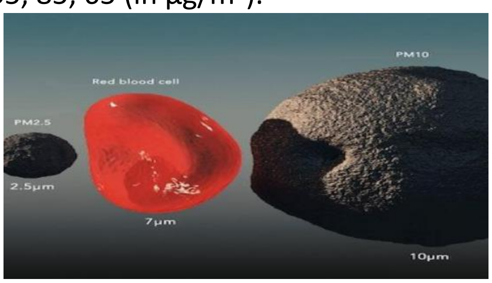
=170

New sum=170 – 21 + 12 =161

New mean = 161/10 =16.1

CASE BASED QUESTIONS

1. City A's daily average PM10 (particulate matter with a diameter of 10 micrometres or smaller) levels for a week were: 50, 60, 45, 70, 55, 65, 75 (in µg/m³). City B's corresponding PM10 levels were: 40, 80, 50, 60, 55, 85, 65 (in µg/m³).



a) What is the range of City A ?
b) What is the range of City B ?
c) Which city had greater variability in PM10 levels?

Soln:

Variability in data can be measured using measures of dispersion such as the range, variance, and standard deviation. In this case, we need to compare the variability of PM10 levels in City A and City B.

City A's PM10 levels: 50, 60, 45, 70, 55, 65, 75

City B's PM10 levels: 40, 80, 50, 60, 55, 85, 65

a) The range is the difference between the maximum and minimum values.

For City A:

Range = 75 (max) - 45 (min) = 30

b) For City B:

Range = 85 (max) - 40 (min) = 45

c) City B has a larger range of PM10 levels, indicating greater variability in its data. Therefore, the correct answer is City B.

2. A fitness trainer is conducting a study on the performance of two different workout routines. The trainer tracks the number of repetitions completed by participants in each routine. There are two sets of observations, each containing 20 participants. The first set has a mean of 17 repetitions, and the second set has a mean of 22 repetitions. Surprisingly, both sets have the same standard deviation of 5 repetitions.

a) Calculate the weighted average of the means.

b) Calculate the variance of the combined set.

c) Calculate the standard deviation of the combined set.

Soln:

a) Weighted Mean = (Number of observations in Set 1 * Mean of Set 1 + Number of observations in Set 2 * Mean of Set 2) / Total Number of Observations

Weighted Mean = (20 * 17 + 20 * 22) / 40 = 19.5.

b) Variance = (Number of observations in Set 1 * Variance of Set 1 + Number of observations in Set 2 * Variance of Set 2) / Total Number of Observations

Variance = (20 * 5^2 + 20 * 5^2) / 40 = 25.

C) Standard Deviation = √Variance = √25 = 5.

3. A group of athletes participated in a practice session for a particular exercise routine over the course of several days. The recorded practice times (in minutes) for each athlete were as follows: 38, 70, 48, 40, 42, 55, 63, 46, 54, 44.
a) Calculate the mean for the athlete's practice time.

a) Calculate the mean deviation about the mean for the athletes' practice times.

b) Calculate the deviations from the mean for each practice time.

c) Calculate the mean of the absolute deviations

Soln: :

a) mean practice time: (38 + 70 + 48 + 40 + 42 + 55 + 63 + 46 + 54 + 44) / 10 = 50.

b) deviations from the mean for each practice time:

Deviations: -12, 20, -2, -10, -8, 5, 13, -4, 4, -6.

c) the absolute values of the deviations: 12, 20, 2, 10, 8, 5, 13, 4, 4, 6.

the mean of the absolute deviations:

= (12 + 20 + 2 + 10 + 8 + 5 + 13 + 4 + 4 + 6) / 10 = 8.4.

The mean deviation about the mean for the athletes' practice times is indeed 8.4.

4. A group of cars went through a series of servicing sessions at a garage. The recorded mileage before each servicing session (in thousands of kilometres) for each car were as follows: 36, 72, 46, 42, 60, 45, 53, 46, 51, 49.

a) Find the median of the observations.

b) Calculate the absolute values of deviations of each mileage from the median.

c)Calculate the mean of the absolute deviations.

Calculate the mean deviation about the median for the recorded mileages of the cars before their servicing sessions.

Soln: a) Arrange the mileages in ascending order: 36, 42, 45, 46, 46, 49, 51, 53, 60, 72.

the median is the average of the fifth sixth values, which is (46 + 49) / 2 = 47.5.

b) deviations from the median for each mileage:

Deviations: -11.5, -5.5, -2.5, -1.5, -1.5, 1.5, 3.5, 5.5, 12.5, 24.5.

absolute values of the deviations: 11.5, 5.5, 2.5, 1.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5.

c)Calculate the mean of the absolute deviations:

= (11.5 + 5.5 + 2.5 + 1.5 + 1.5 + 1.5 + 3.5 + 5.5 + 12.5 + 24.5) / 10 = 10.5.

5. A student collected 10 readings and attempted to calculate the mean and variance. Unfortunately, the student mistakenly used a reading of 52 instead of the correct reading 25. The student calculated the mean and variance as 45 and 16 respectively.

a) Determine the correct mean by considering the mistaken reading of 52 instead of the actual reading 25.
b) Determine the correct variance by considering the mistaken reading of 52 instead of the actual reading 25.

Soln: a) while calculating the mean and variance of 10 readings, a student wrongly used the reading 52 for the correct reading 25. He obtained the mean and variance 45 and 16 respectively.

To find correct mean and variance

N=10

Mean before correction ($\underline{x}$)=45

$$\underline{x} = \frac{\sum_{i=1}^{10} x_i}{N} \Rightarrow \sum_{i=1}^{10} x_i = \underline{x} \times N = 45 \times 10 = 450$$

Correct $\sum_{i=1}^{10} x_i = 450 - 52 + 25 = 423$

Correct mean=$\frac{Correct\ \sum \quad x_i}{N} = \frac{423}{10} = 42.3$

b. given $\sigma^2 = 16$

but $\sigma^2 = \frac{\sum_{i=1}^{10} {x_i}^2}{N} - (\frac{\sum_{i=1}^{10} x_i}{N})^2$

$$\Rightarrow 16 = \frac{\sum_{i=1}^{10} {x_i}^2}{N} - (45)^2 \Rightarrow \frac{\sum_{i=1}^{10} {x_i}^2}{10} = 16 + 2025 = 2041$$

$$\Rightarrow \sum_{i=1}^{10} x_i^2 = 20410$$

One reading 25 was wrongly taken as 52, so

correct $\sum \ x_i^2 = 20410 - 52^2 + 25^2 = 20410 - 2704 + 625 = 18331$

correct $\sigma^2 = \frac{correct \sum \ x_i^2}{N} - (correct\ mean)^2 = \frac{18331}{10} - (42.3)^2 = 1833.1 - 1769.29 = 43.81$

LONG ANSWER TYPE

1. A group of individuals' heights were measured and categorized into different height ranges (in centimetres). Find the variance and standard deviation, if the number of individuals falling into each height range is recorded as follows:

Height	70-75	75 - 80	80 - 85	85 - 90	90 - 95	95-100	100-105	105-110	110-115
frequency	3	4	7	7	15	9	6	6	3

Ans:

CI	f_i	x_i	$y_i = \frac{x_i - 92.5}{5}$	y_i^2	$f_i y_i$	$f_i y_i^2$
70-75	3	72.5	-4	16	-12	48
75-80	4	77.5	-3	9	-12	36
80-85	7	82.5	-2	4	-14	28
85-90	7	87.5	-1	1	-7	7
90-95	15	92.5	0	0	0	0
95-100	9	97.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	$\sum_{i=1}^{n} f_i = 60$				$\sum_{i=1}^{n} f_i y_i = 6$	$\sum_{i=1}^{n} f_i y_i^2 = 254$

Mean $= \underline{x} = A + \frac{\sum_i^n f_i y_i}{N} \times h = 92.5 + \frac{6}{60} \times 5 = 92.5 + 0.5 = 93$

Variance $(\sigma^2) = \frac{h^2}{N^2}\left\{N\sum_{i=1}^{n} f_i y_i^2 - \left(\sum_{i=1}^{n} f_i y_i\right)^2\right\}$

$$= \frac{5^2}{60^2}\{60 \times 254 - (6)^2\}$$

$$= \frac{25}{3600}\{15204\} = 105.58$$

Standard deviation $(\sigma)=\sqrt{105.58} = 10.27$

2. An age distribution of 100 persons is available, and their ages are categorized into different age ranges. The number of persons falling into each age range is recorded as follows:

Find the mean deviation about the median of given data.

age	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55
number	5	6	12	14	26	12	16	9

Ans: The given data is not continuous. Therefore, it has to be converted into continuous frequency distribution.

age	Number(f_i)	c.f	Mid-point(x_i)	$\lvert x_i - Med\rvert$	$f_i\lvert x_i - med\rvert$
15.5-20.5	5	5	18	20	100
20.5-25.5	6	11	23	15	90
25.5-30.5	12	23	28	10	120
30.5-35.5	14	37	33	5	70
35.5-40.5	26	63	38	0	0
40.5-45.5	12	75	43	5	60
45.5-50.5	16	91	48	10	160
50.5-55.5	9	100	53	15	135
	$\sum_{i=1}^{n} f_i = 100$				$\sum_{i=1}^{n} f_i\lvert x_i - med\rvert = 735$

Here, $\frac{N}{2} = 50$, median class is 35.5-40.5

Median $= l + \frac{\frac{N}{2} - C}{f} \times h$

$$l = 35.5, C = 37, f = 26, h = 5 \text{ and } N = 50$$

Median $= 35.5 + \frac{50-37}{26} \times 5 = 38$

MD(M)$= \frac{\sum_{i}^{n} f_i |x_i - med|}{N} = \frac{735}{100} = 7.35$

3. In a survey of 44 villages of a state, about the use of LPG as a cooking mode, the following information about the families using LPG was obtained.

Number of families	0-10	10-20	20-30	30-40	40-50	50-60
Number of villages	6	8	16	8	4	2

i. Find the mean deviation about median for the following data.

Do you think more awareness is needed for the villagers to use LPG as a mode of cooking?

Ans:

Number of families	Mid value (x_i)	Number of villages (f_i)	cf	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
$0 - 10$	5	6	6	20	120
$10 - 20$	15	8	14	10	80
$20 - 30$	25	16	30	0	0
$30 - 40$	35	8	38	10	80
$40 - 50$	45	4	42	20	80
$50 - 60$	55	2	44	30	60

Here, $N = 44$

Now, $\frac{N}{2} = \frac{44}{2} = 22$, which, lies in the cumulative frequency of 30, therefore median class is 20-30

$$l = 20, f = 16, cf = 14 \text{ and } h = 10$$

Median $= l + \frac{\frac{N}{2} - cf}{f} \times h = 20 + \frac{22-14}{16} \times 10 = 20 + 5 = 25$

MD(M)$= \frac{\sum_{i=1}^{n} f_i |x_i - med|}{N} = \frac{420}{44} = 9.55$

ii. There is a need for awareness among villagers for using LPG as a mode of cooking. Because it will help in keeping the environment clean and will also help in saving of forests.

4. From the frequency distribution consisting of 18 observations, the mean and the standard deviation were found to be 7 and 4, respectively. But on comparison with the original data, it was found that a figure 12 was miscopied as 21 in calculations. Calculate the correct mean and standard deviation.

Ans: Mean =7, ($\underline{x} = \frac{\sum_{i=1}^{18} x_i}{N}$)

$$7 = \frac{\sum_{i=1}^{18} x_i}{18} => \sum_{i=1}^{18} x_i = 126$$

Since, an observation 12 was miscopied as 21

So, correct $\sum \quad x_i = 126 - 21 + 12 = 117$

Hence , true mean= $\frac{correct \sum_{i=1}^{18} x_i}{18} = \frac{117}{18} = 6.5$

Also, given variance =$4^2 = 16$

$$\frac{\sum_{1=1}^{18} x^2}{18} - (mean)^2 = 16 \Longrightarrow \frac{\sum_{i=1}^{18} x^2}{18} = 16 + 7^2 \Longrightarrow \sum_{i=1}^{18} x^2 = 18(16 + 49) = 1170$$

But one observation 12 was miscopied as 21

So, correct $\sum \quad {x_i}^2 = 1170 - 21^2 + 12^2 = 1170 - 441 - 144 = 873$

Hence ,true variance =$\frac{correct \sum \quad {x_i}^2}{18} - (true\ mean)^2 = \frac{873}{18} - (6.5)^2 = 48.5 - 42.25 = 6.25$

True S.D=$\sqrt{true\ variance} = \sqrt{6.25} = 2.5$

5. The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On checking, it was found that an observation 8 was incorrect. Calculate the correct mean and correct standard deviation in each of the following cases:

(a) If the wrong observation is omitted.

(b) If it is replaced by 12.

Ans: (i) Given, number of observations n=20
Incorrect mean =10
Incorrect standard deviation =2

$$\underline{x} = \frac{1}{n} \sum_{i=1}^{20} x_i$$

$$10 = \frac{1}{20} \sum_{i=1}^{20} x_i$$

$$\sum_{i=1}^{20} x_i = 200$$

So, the incorrect sum of observations =200
Correct sum of observation =200−8=192

⇒ Correct mean =Correct sum/19 =192/ 19=10.1

S.D (σ) = $\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i{}^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2}$ = $\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i{}^2 - \left(\underline{x}\right)^2}$

2 = $\sqrt{\frac{1}{20}\ incorrect\ \sum_{i=1}^{n} x_i{}^2 - (10)^2}$

4 = $\frac{1}{20}\ incorrect\ \sum_{i=1}^{n} x_i{}^2 - 100$

$$incorrect \sum_{i=1}^{n} x_i{}^2 = 2080$$

$$correct \sum_{i=1}^{n} x_i{}^2 = incorrect \sum_{i=1}^{n} x_i{}^2 - (8)^2 = 2080 - 64 = 2016$$

∴ Correct Standard deviation = $\sqrt{\frac{1}{n} correct\ \sum_{i=1}^{n} x_i{}^2 - (correct\ mean)^2}$

= $\sqrt{\frac{2016}{19} - (10.1)^2} = \sqrt{106.1 - 102.01} = \sqrt{4.09} = 2.02$

(ii)When 8 is replaced by 12
Incorrect sum of observation =200
∴ Correct sum of observations =200−8+12=204
∴ Correct mean =Correct sum /20 =204/20 =10.2

S.D (σ) = $\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i{}^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2}$ = $\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i{}^2 - \left(\underline{x}\right)^2}$

2 = $\sqrt{\frac{1}{20}\ incorrect\ \sum_{i=1}^{n} x_i{}^2 - (10)^2}$

4 = $\frac{1}{20}\ incorrect\ \sum_{i=1}^{n} x_i{}^2 - 100$, $incorrect\ \sum_{i=1}^{n} x_i{}^2 = 2080$

$$correct \sum_{i=1}^{n} x_i{}^2 = incorrect \sum_{i=1}^{n} x_i{}^2 - (8)^2 + 12^2$$

$$= 2080 - 64 + 144 = 2160$$

∴ Correct Standard deviation = $\sqrt{\frac{1}{n} correct\ \sum_{i=1}^{n} \quad x_i{}^2 - (correct\ mean)^2}$

$= \sqrt{\frac{2160}{20} - (10.2)^2} = \sqrt{108 - 104.04} = \sqrt{3.96} = 1.98$

SELF PRACTICE QUESTIONS

1. A group of students participated in a science experiment, and their recorded results (in arbitrary units) were as follows: 6, 5, 9, 13, 12, 8, 10.

Calculate the standard deviation of the recorded results.

(A) 52 B) √ (52/7) C) √ 5 D) 4

2. A group of researchers conducted a study on the growth of a certain type of plant. They collected 100 observations of the plant's height (in centimetres) and found that the mean height was 50 centimetres, and the standard deviation was 5 centimetres.

Calculate the sum of the squares of all the observations.

(A) 50000 B) 250000 C) 252500 D) 255000

3. A meteorological study collected temperature data in degrees Celsius (°C) to analyse variations. The standard deviation of the collected data was found to be 5 °C.

If the temperature data were converted into degrees Fahrenheit (ºF), what would be the variance of the data?

(A) 81 B) 57 C) 36 D) 25

4. A man travels at a speed of 20 km/hour and then returns at a speed of 30 km/hour. His average speed of the whole journey is

a) 25 km/hour b) 24 km/hour c) 24.5 km/hour d) 26 km/hour

5. The mean and standard deviation of 100 items are 50, 5 and that of 150 items are 40, 6 respectively. What is the combined standard deviation of all 250 items?

a) 7.5 b) 7.7 c) 7.3 d) 7.1

6. If the mean of the squares of first n natural numbers is 11, then n is equal to

a) 5 b) 14 c) 11 d) 13

7. Consider the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. If 1 is added to each number, the variance of the numbers so obtained is

a) 3.87 b) 8.25 c) 2.87 d) 6.5

8. Variance of the data 2, 4, 5, 6, 8, 17 is 23.33. Then variance of 4, 8, 10, 12, 16, 34 will be:

(A) 23.23 (B) 25.33 (C) 46.66 (D) 48.66

9. Coefficient of variation of two distributions are 50 and 60, and their arithmetic means are 30 and 25 respectively. Difference of their standard deviation is:

(A) 0 (B) 1 (C) 1.5 (D) 2.5

10. Let a, b, c, d, e be the observations with mean m and standard deviation s. The standard deviation of the observations a + k, b + k, c + k, d + k, e + k is:

(A) s (B) k s (C) s + k (D) s/k

Assertion Reasoning

DIRECTION : In each of the following questions, a statement of Assertion is given followed by a

corresponding statement of Reason just below it. Of the statements, mark the correct answer as

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true

11.Assertion(A) : In order to find the dispersion of values of x from the mean $\underline{x}$, we take absolute measure of dispersion .

Reason(R) :Sum of the deviations from mean is zero.

12. Assertion(A) : The mean deviation about mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is 3.

Reason(R) : The mean deviation about mean for the data38, 70, 48, 40, 42, 55, 63, 46, 54, 44 is 8.5.

13. Assertion(A) :Consider the following data .

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Then the mean deviation about the mean is 6.32 .

Reason(R) : Consider the following data .

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Then the mean deviation about the mean is 15.

14.Assertion(A): The mean deviation about median calculated for series, where variability is very high, cannot be fully relied.

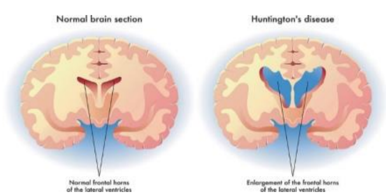
Reason(R): The median is not a representative of central tendency for the series where degree of variability is very high.

15. Assertion(A): The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is 80%.

Reason(R): Mean marks scored by the students of a class is 53. The mean marks of the girls is 55 and the mean marks of the boys is 50. The percentage of girls in the class is 64%.

SHORT ANSWER QUESTIONS

16. The annual incidence rates of Huntington's Disease per 100,000 individuals were recorded over a span of five years. The data is as follows: 4, 7, 8, 9, 10. Calculate the mean deviation about the mean for these rare disease rates



17. A group of athletes participated in a practice session for a particular exercise routine over the course of several days. The recorded practice times (in minutes) for each athlete were as follows: 38, 70, 48, 40, 42, 55, 63, 46, 54, 44.

Calculate the mean deviation about the mean for the athletes' practice times.

18. A group of cars went through a series of servicing sessions at a garage. The recorded mileage before each servicing session (in thousands of kilometres) for each car were as follows: 36, 72, 46, 42, 60, 45, 53, 46, 51, 49.

Calculate the mean deviation about the median for the recorded mileages of the cars before their servicing sessions.

19. The mean and standard deviation of 20 observation is found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in case of the wrong item is omitted.

20. Find the mean and standard deviation of first n terms of an A.P. whose first term is a and the common difference is d.

21. Calculate the mean deviation about median for the daily wages of 12 labours getting wages (in Rs.) 48, 45, 60, 50, 46, 48, 50, 45, 70, 65,47,50

22. The mean life of a sample of 60 bulbs was 650 hours and the standard deviation was 8 hours. A second sample of 80 bulbs has a mean life of 660 hours and standard deviation 7 hours. Find the overall standard deviation.

23. Mean and standard deviation of 100 items are 50 and 4, respectively. Find the sum of all the item and the sum of the squares of the items.

24. The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

25. If a is a positive integer and the frequency distribution has a variance of 160 . Determine the value of .

x	a	$2a$	$3a$	$4a$	$5a$	$6a$
f	2	1	1	1	1	1

CASE BASED QUESTIONS

26. A set of data points were collected and analysed. For this distribution, two pieces of information were obtained: (x – 5) = 3 and $(x – 5)^2$ = 43. It is also known that the total number of items in the dataset is 18.

a) Calculate the mean.

b) Calculate standard deviation for this distribution.

c) Calculate Variance.

27.The weights of coffee in 70 jars is shown in the following table:

Weight Range (grams)	Frequency	Weight Range (grams)	frequency
fre200 – 201	13	203 - 204	10
201 – 202	27	204 - 205	1
202 – 203	18	205 - 206	1

a) Determine the mean.
b) Determine the standard deviation.
c) Find variance.

28.An analysis of monthly wages paid to workers in two firms A and B, belonging to the same industry, gives the following results:

	Firm A	Firm B
Number of workers	897	468
Mean amount of wages(INR)	6345	6345
Variance of distribution of wages	100	169

a) What is the standard deviation of Firm A?

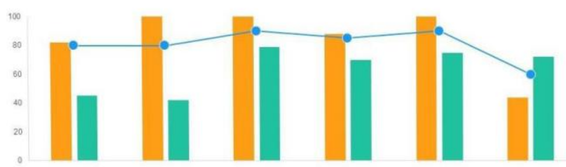
b) What is the standard deviation of Firm B?
c) Which firm, A or B, shows greater variability in individual wages?

29.A meteorological study collected temperature data in degrees Celsius (°C) to analyse variations. The standard deviation of the collected data was found to be 5 °C.

a) If the temperature data were converted into degrees Fahrenheit (ºF), what would be the variance of the data?

(A) 81   B) 57   C) 36   D) 25

30. A collection of 100 items was analysed, and the statistical properties of the data were observed. The mean of the items is 50, and the standard deviation is 4.



a) Calculate the sum of all the items.

b) Calculate the sum of the squares of the items.

c) What is the variance?

To solve this, we can use the formulas for the mean and standard deviation:

Sum of all items = Mean × Number of items = 50 × 100 = 5000.

Sum of squares of items = Variance × (Number of items - 1) + Mean^2 × Number of items = (4^2) × (100 - 1) + 50^2 × 100 = 159600.

So, the sum of all the items is 5000, and the sum of the squares of the items is 159600.

LONG ANSWER TYPE

31 Find the mean , variance and standard deviation using short-cut method.

Height (in cm)	No.of children
70 – 75	3
75 – 80	4
80 – 85	7
85 – 90	7
90 – 95	15
95 – 100	9
100 – 105	6
105 – 110	6
110 – 115	3

32 The diameters of circles (in mm) drawn in a design are given below. Calculate the standard deviation and mean diameter of the circles.

Diameters	33 – 36	37 – 40	41 – 44	45 – 48	49 – 52
No.of circles	15	17	21	22	25

33

A student collected 10 readings and attempted to calculate the mean and variance. Unfortunately, the student mistakenly used a reading of 52 instead of the correct reading 25. The student calculated the mean and variance as 45 and 16 respectively.

Determine the correct mean and variance by considering the mistaken reading of 52 instead of the actual reading 25.

34 The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, then:

(a) Find the new mean.

(b) Find the new standard deviation of the resulting observations.

35 Mean and standard deviation of 100 observations were found to be 40 and 10, respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively, find the correct standard deviation.

KEY FOR SELF PRACTICE QUESTIONS

1	B	11	a	21	5.5	31	93, 105.58 and 10.27
2	C	12	c	22	8.9	32	43.5, 30.84, 5.55
3	A	13	c	23	251600	33	42.3 and 43.81
4	B	14	a	24	3.036	34	24 and 12
5	A	15	c	25	7	35	10.241
6	A	16	1.64	26	3,3,9		
7	B	17	8.4	27	201.9,1.166 ,1.08		
8	C	18	10.5	28	10,13,B		
9	A	19	1.997	29	a		
10	A	20	$\sigma = d\sqrt{\frac{n^2-1}{12}}$	30	5000,15960 0. 971		

PROBABILITY

CONCEPTS:

* **Random Experiments:** An experiment is called random experiment if it satisfies the following two conditions:

(i) It has more than one possible outcome.

(ii) It is not possible to predict the outcome in advance.

** **Outcomes and sample space:** A possible result of a random experiment is called its outcome. The set of all possible outcomes of a random experiment is called the sample space associated with the experiment. Each element of the sample space is called a sample point. Any subset E of a sample space S is called an event.

** **Impossible and Sure Events:** The empty set ϕ and the sample space S describe events. ϕ is called an impossible event and S, i.e., the whole sample space is called the sure event.

** **Compound Event:** If an event has more than one sample point, it is called a Compound event.

** **Complementary Event:** For every event A, there corresponds another event A′ called the complementary event to A. It is also called the event 'not A'.

** **The Event 'A or B':** When the sets A and B are two events associated with a sample space, then 'A ∪ B' is the event 'either A or B or both'. This event 'A ∪ B' is also called 'A or B'.

** **The Event 'A and B':** If A and B are two events, then the set A ∩ B denotes the event 'A and B'.

** **The Event 'A but not B':** the set A – B denotes the event 'A but not B'.

A – B = A ∩ B′

** **Mutually exclusive events:** two events A and B are called mutually exclusive events if the occurrence of any one of them excludes the occurrence of the other event, i.e., if they cannot occur simultaneously. In this case the sets A and B are disjoint i.e. A ∩ B = ϕ.

Exhaustive evnts: if $E_1, E_2, E_3 \dots E_n$ are n events of a sample space S and if $E_1 \cup E_2 \cup E_3 \cup \dots \cup E_n = S$, then $E_1, E_2, E_3 \dots E_n$ are called exhaustive events.

Mutually exclusive and exhaustive events: If $E_I \cap E_J = \emptyset$ for $i \neq j$ and

$E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S$, then $E_1, E_2, E_3 \ldots E_n$ are called Mutually exclusive and exhaustive events

**** Axiomatic Approach to Probability:** Let S be the sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S and range is the interval [0,1] satisfying the following axioms

(i) For any event E, P (E) ≥ 0
(ii) P (S) = 1
(iii) If E and F are mutually exclusive events, then P(E ∪ F) = P(E) + P(F).

From the axiomatic definition of probability, it follows that

(i)0 ≤ P (ωi) ≤ 1 for each ωi ∈ S

(ii) P (ω1) + P (ω2) + ... + P (ωn) = 1

(iii) For any event A, P(A) = Σ P(ωi), ωi ∈ A.

**** Equally likely outcomes:** All outcomes with equal probability.

**** Probability of an event:** For a finite sample space with equally likely outcomes

Probability of an event P(A) =n (A)/ n(S)

where n(A) = number of elements in the set A,

n(S) = number of elements in the set S.

**** Probability of the event 'A or B' :**

P(A or B) = P(A ∪ B) = P(A) + P(B) – P(A∩B).

For mutually exclusive events A and B, we have P(A ∪ B)= P(A) +P(B)

**** Probability of event 'not A'** = P(A') = P(not A) = 1 – P(A).

Example 1: What is the total number of sample spaces when a die is thrown 2 times?

A. 6 B. 12 C. 18 D. 36

Solution: The possible outcomes when a die is thrown are 1, 2, 3, 4, 5, and 6.

Given, a die is thrown two times.

Then, the total number of sample spaces = (6 × 6)=36

Example2: What is the total number of sample spaces when a coin is tossed and a die is thrown?

A. 6 B. 12 C. 8 D.16

Solution: The possible outcomes when a coin is tossed are Head (H) or Tail (T).

The possible outcomes when a die is thrown are 1, 2, 3, 4, 5, and 6.

Then, total number of space = (2 × 6) = 12

Example3: Three identical dice are rolled. What is the probability that the same number will appear on each of them?

A. 1/6 B. 1/36 C. 1/18 D. 3/28

Solution:

Total number of cases = 6^3 = 216

The same number can appear on each of the dice in the following ways:

(1, 1, 1), (2, 2, 2), …………(3, 3, 3)

So, favourable number of cases = 6

Hence, required probability = 6/216 = 1/36

Example4: A bag contains 5 brown and 4 white socks. Ram pulls out two socks. What is the probability that both the socks are of the same colour?

A. 9/20 B. 2/9 C. 3/20 D. 4/9

Solution: Total number of socks = 5 + 4 = 9

Two socks are pulled.

Now, P(Both are same colour) = (5C2 + 4C2)/9C2

= {(5×4)/(2×1) + (4×3)/(2×1)}/{(9×8)/(2×1)}

= {(5×4) + (4×3)/}/{(9×8)

= (5 + 3)/(9×2)

= 8/18

= 4/9

Example5: What is the probability of getting the number 6 at least once in a regular die if it can roll it 6 times?

A. $1 – (5/6)^6$ B. $1 – (1/6)^6$ C. $(5/6)^6$ D. $(1/6)^6$

Solution:

Let A be the event that 6 does not occur at all.

Now, the probability of at least one 6 occurs = 1 – P(A.)

$= 1 – (5/6)^6$

Example6: Events A and B are said to be mutually exclusive if:

A.. P (A U B) = P(A) + P(B) B. P (A ∩ B) = P(A) × P(B) C. P(A U B) = 0

D. None of these

Solution: If A and B are mutually exclusive events,

Then P(A ∩ B) = 0

Now, by the addition theorem,

P(A U B) = P(A) + P(B) – P(A ∩ B)

⇒ P(A U B) = P(A) + P(B)

Example7:A die is rolled. What is the probability that an even number is obtained?

A. ½ B. 2/3 C. ¼ D. 3/4

Solution: When a die is rolled,

total number of outcomes = 6 (1, 2, 3, 4, 5, 6)

Total even number = 3 (2, 4, 6)

So, the probability that an even number is obtained = 3/6 = ½

Example8: What is the probability of selecting a vowel in the word "PROBABILITY"?

A. 2/11 B. 3/11 C. 4/11 D. 5/11

Solution:

In the word "PROBABILITY"

Total no. of letters =11

Number of vowels =4

We can select any of the 4 vowels at random

So, Favourable outcomes =4

∴ Selecting a vowel at random =Favourable outcomes /Total outcomes=4/11

Example9: An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. What is the probability that they are of different colours?

A.⅖ B.1/15 C.8/15 D.4/15

Solution: Given that, the total number of balls = 6 balls

Let A and B be the red and black balls, respectively, The probability that two balls are drawn are different = P (the first ball drawn is red)(the second ball drawn is black)+ P (the first ball drawn is black)P(the second ball drawn is red)

= (2/6)(4/5) + (4/6)(2/5)

=(8/30)+ (8/30)

= 16/30

= 8/15

Example10:20 cards are numbered from 1 to 20. If one card is drawn at random, what is the probability that the number on the card is a prime number?

A. ⅕ B. ⅖ C. ⅗ D. 5

Solution: Let E be the event of getting a prime number.

E = {2, 3, 5, 7, 11, 13, 17, 19}

Hence, P(E) = 8/20 = 2/5

Assertion –Reason based questions:

In the following questions a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices

a). Both A and R are true and R is correct explanation of A.

b). Both A and R are true but R is not the correct explanation of A.

c). A is true but R is false.

d). A is false but R is true

Example1: Assertion : Two dice are thrown simultaneously. There are 11 possible outcomes and each of them has a probability 1/11.

Reason : Probability of an event (E) is defined as

P(E)=Number of favourable outcomes /Total number of possible outcomes

Solution: The correct option is **D.**

Assertion is false but reason is true.

If two dice will be thrown simultaneously then the total number of possible outcomes is 36 and probability of each outcome is not equal to 1/11.

Probability of an event (E) is defined as

P(E)=Number of outcomes favourable to E /Total number of possible outcomes.

For example, probability of getting an even number on a die when a die is thrown

=Number of outcomes favourable to E /Total number of possible outcomes

=3/6

[∵ Here, the number of favourable outcomes will be 3 as 2, 4 and 6 are the possible even numbers]

⇒Assertion: False

⇒Reason : True

Example2: **Assertion:** The probability of a sure event is 1.

Reason: Let E be an event. Then, $0 \leq P(E) \leq 1$.

Solution:(b) Both Assertion and Reason are true and Reason is not a correct explanation of Assertion.

Explanation:

Clearly, Assertion (A) and Reason (R) are both true. But, Reason (R) is not a correct explanation of Assertion (A).

Hence, the correct answer is (b).

Example3: Assertion: In rolling a dice, event A = {1, 3, 5} and event B = {2, 4} are mutually exclusive events.

Reason: In a sample space, two events are mutually exclusive if they do not occur at the same time.

Solution: The correct option is A

Both assertion and reason are true and reason is the correct explanation of assertion.

A={1,3,5} and B={2,4}

Here, $A \cap B = \Phi$

∴ A and B are mutually exclusive events.

Assertion: True

Reason: True and is the correct explanation of assertion.

Example4 Assertion: Two events A and B are such that $P(A)=1/3$ and $P(B)=2/3$ then $P(A\cup B)\geq 2/3$

Reason : If $P(B)>P(A)$, then $P(A\cup B)\geq P(B)$

Solution: the correct option is A Both assertion and reason are true and reason is the correct explanation of assertion.

We know that, $P(B) = P(A)$, then,

$P(B)\leq P(A\cup B)\leq 1$ and $0\leq P(A\cap B)\leq P(A)$

Now $P(B)=2/3, P(A)=1/3$

$\therefore P(B)>P(A)\Rightarrow P(A\cup B)\geq P(B)$

$\Rightarrow P(A\cup B)\geq 2/3$

So, assertion and reason both are true and reason is the correct explanation of assertion.

Example5: Assertion(A) : Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up is $\frac{1}{3}$

Reason (R): let E and F be two events with a random experiment, then $P\left(\frac{E}{F}\right) = \frac{P(E\cap F)}{P(F)}$

Solution: Both A and R are true and R is correct explanation of A

Short Answer Type Questions:

Example 1: If $P(A) = 3/5$. Find P (not A)

Solution: Given that: $P(A) = 3/5$

P(not A) = 1 – P(A)

P (not A) = 1- 3/5

= (5-3)/5

= 2/5

Therefore, P(not A) = 2/5.

Example2: An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. The probability that they are of different colours is

(i) ⅖ (ii) 1/15 (iii) 8/15 (iv) 4/15

Solution: Given that, the total number of balls = 6 balls

Let A and B be the red and black balls respectively,

The probability that two balls drawn, are different = P(the first ball drawn is red)(the second ball drawn is black)+ P(the first ball drawn is black)P(the second ball drawn is red)

= (2/6)(4/5) + (4/6)(2/5)

=(8/30)+ (8/30)

= 16/30

= 8/15

Example3: One die of red colour, one of white colour and one of blue colour are placed in a bag. One die is selected at random and rolled, its colour and the number on its uppermost face is noted. Describe the sample space.

Solution: Let us assume that 1, 2, 3, 4, 5 and 6 are the possible numbers that come when the die is thrown.

And also, assume die of red colour be 'R', die of white colour be 'W', die of blue colour be 'B'.

So, the total number of sample space = (6 × 3) = 18

The sample space of the event is

S={(R,1),(R,2),(R,3),(R,4),(R,5),(R,6),(W,1),(W,2),(W,3),(W,4),(W,5),(W,6) (B,1),(B,2),(B,3),(B,4),(B,5),(B,6)}

Example 4: Three coins are tossed once. Find the probability at most two heads.

Solution: S={ HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}

E={HHT,THH,HTH,HTT,THT,TTH,TTT}

P(E)=7/8

Example 5: Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non-defective (N). Write the sample space of this experiment.

Solution: From the question,

'D' denotes the event that the bulb is defective, and 'N' denotes the event of non-defective bulbs.

Then, Total number of Sample space = 2 × 2 × 2 = 8

Thus, Sample space S = {DDD, DDN, DND, NDD, DNN, NDN, NND, NNN}

Example 6: A die is rolled. Let, E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

Solution:1,2,3,4,5and 6 are the possible outcomes when a die is thrown. So,

S = {1, 2, 3, 4, 5, 6}

As per the conditions given the question

E be the event "die shows 4"

E = {4}

F be the event "die shows even number"

F = {2, 4, 6}

E ∩ F = {4}

∴ E ∩ F≠ϕ ...[because there is a common element in E and F]

Therefore, E and F are not mutually exclusive event

Example 7: Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Solution: Total number of ways by which three letters can be put into three envelopes=

$$3! = 6$$

Derangement of n objects = $n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots+(-1)^n\frac{1}{n!}\right]$

Derangement of 3 objects=3! $\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right]$=6 $\left(1-1+\frac{1}{2}-\frac{1}{6}\right)=2$

P(no letter is in the correct envelope)= $\frac{2}{6}$=$\frac{1}{3}$

P(atleast one is in the correct envelope)= $1-\frac{1}{3}=\frac{2}{3}$

Example 8: If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5 and 7, what is the probability of forming a number divisible by 5 when the digits are repeated?

Solution: Since 4 digit numbers greater than 5000 are formed. The thousand's place digit is either 7 or 5.

The total number of 4 digits number greater than 5000 = 2 × 5 × 5 × 5 − 1 = 249

A number is divisible by 5, if the digit at it's unit place is either 0 or 5.

∴ The total number of 4 − digits numbers greater than 5000 and divsible by 5 = 2 × 5 × 5 × 2 − 1 = 99

∴ P(forming a number which is greater than 5000 and divisible by 5) = 99/249 = 33/83

Example 9: If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5 and 7, what is the probability of forming a number divisible by 5 when the repetitions of digits are not allowed?

Solution: Since 4-digit numbers greater than 5000 are formed. The thousand's place digit is either 7 or 5

The total number of 4-digit number greater than 5000=2X4X3X2=48

A number is divisible by 5, if the digit at its unit place is either 0 or 5

The total number of 4-digits numbers starting with 5 and divisible by 5=1X3X2X1=6

The total number of 4-digits numbers starting with 7 and divisible by 5=1X3X2X2=12

The total number of 4-digits numbers greater than 5000 and divisible by 5=6+12=18

P(forming a number which is greater than 5000 and divisible by 5)= $\frac{18}{48} = \frac{3}{8}$

Example 10: Find the probability that win a hand of 7 cards is drawn from a well shuffled deck of 52 cards, it contains I. All kings. II. 3 kings. III. At least 3 kings

Solution: The total number of possible hands =C(52,7)

i. Number of hands with 4 kings=C(4,4) XC(48,3)

   P(a hand will have 4 king)= $\frac{C(4,4)\times C(48,3)}{C(52,7)} = \frac{1}{7735}$

ii. Number of hands with 3 kings and 4 non-kings= C(4,3)XC(48,4)

   P(a hand with 3 kings) =$\frac{C(4,3)\times C(48,4)}{C(52,7)} = \frac{9}{1547}$

iii. P(atleast 3 king)=P(3 kings or 4 kings)=P(3 kings)+ P(4 kings)= $\frac{9}{1547} + \frac{1}{7735} = \frac{46}{7735}$

Case Study based question:

Example 1: Rahul went to a fair. There he saw in a shop a lottery seller was selling lotteries. He asked the shopkeeper about this lottery game and he got the information that among these 10000 tickets, there are 10 prizes will be awarded. He is willing to know

I.What is the probability of not getting a prize, if he buys one ticket?

II.What is the probability of not getting a prize if he buys 2 tickets?

III. What is the probability of not getting a prize if he buys 10 tickets?

Solution:

i. out of 10000 tickets, one ticket can be chosen in C(1000,1)=10000 ways there are 9990 tickets not containing a prize. Out of these 9990 tickets one can be chosen in C(9990,1)=9990 ways

P(not getting prize) = $\frac{9990}{10000} = \frac{999}{1000}$

ii. out of 10000 tickets, two tickets can be chosen in C(10000,2) ways

as there are 9990 tickets not containing a prize. Out of these 9990 tickets two can be chosen in C(9990,2) ways

P(not getting prize) =$\frac{C(9990,2)}{C(10000,2)}$

iii. out of 10000 tickets, ten tickets can be chosen in C(10000,10) ways

as there are 9990 tickets not containing a prize. Out of these 9990 tickets two can be chosen in C(9990,10) ways

P(not getting prize) =$\frac{C(9990,10)}{C(10000,10)}$

Example 2: Shivnath went to a fair. In the fare he saw a shopkeeper was mixing tickets numbered 1 to 20 thoroughly and asking customers to take out a ticket randomly. He is willing to know

I.What is the probability that drawn ticket number is a multiple of 3?

II. What is the probability that drawn ticket number is a multiple of 7?

III. If drawn ticket number is a multiple of 3 or 7 then he wins. What is the probability of his winning?

Solution: Let S be the sample space associated with the given random experiment. A and B denotes the events getting a ticket bearing a number which is a multiple of 3 and 7 respectively, then $S = \{1,2,3, \ldots 20\}$ $A = \{3,6,9, \ldots 18\}, B = \{7,14\}$

i. P(ticket number is a multiple of 3)=$\frac{6}{20} = \frac{3}{10}$

ii. P(ticket number is a multiple of 7)=$\frac{2}{20} = \frac{1}{10}$

iii. P(winning)= $\frac{3}{10}+\frac{1}{10}=\frac{4}{10}=\frac{2}{5}$

Example 3: In a relay race, there are 5 teams A, B, C, D and E.

I. What is the probability that A, B& C finish first, second and third respectively.

II. What is the probability that A, B & C are first three to finish (in any order).

Solution: If we consider the sample space consisting of all finishing orders in the first three places, we will have $P(5,3) = 60$ sample points, each with probability of $\frac{1}{60}$

i. A,B and C finish first ,second and third respectively. There is only one finishing order for this, i.e ABC

P(A,B and C finish first ,second and third respectively)= $\frac{1}{60}$

ii. A, B and C are the first three finishers. There will be $3! = 6$ arrangements for A, B and C

P(A, B and C are the first three to finish) = $\frac{6}{60}=\frac{1}{10}$

Example 4: One card is drawn from a well-shuffled pack of 52 cards. What is the probability that a card will be

(i) a diamond (ii) Not an ace (iii) a black card (iv) not a diamond

Solution:(i) the probability that a card is a diamond

We know that there are 13 diamond cards in a deck. Therefore, the required probability is:

P(getting a diamond card) = 13/52 = ¼

(ii) the probability that a card is not an ace

We know that there are 4 ace cards in a deck.

Therefore, the required probability is:

P(not getting an ace card) = 1-(4/52)

= 1- (1/13)

=(13-1)/13

= 12/13

(iii) the probability that a card is a black card

We know that there are26 black cards in a deck.

Therefore, the required probability is:

P(getting a black card) = 26/52 = ½

(iv) the probability that a card is not a diamond

We know that there are 13 diamond cards in a deck.

We know that the probability of getting a diamond card is 1/4

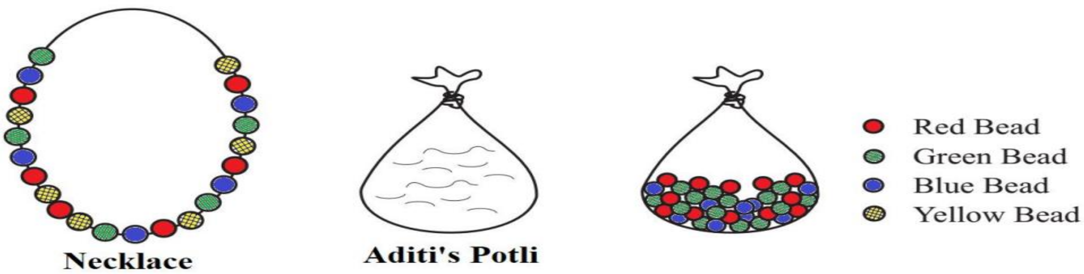
Therefore, the required probability is:

P(not getting a diamond card) = 1- (1/4)

= (4-1)/4

= ¾

Example 5: Aditi runs a handicraft shop in Bapu bazar in Jaipur. She makes beautiful necklaces using colourful beads which she keeps in potli. Today she prepared 19 necklaces but could not make the 20th necklace as she had no yellow beads left. She counted the beads and found that there were 8 red, 6 green and 14 blue beads remaining in her potli. Her little daughter Dulari requested for a bead. Aditi decides to take out one bead from her potli for Dulari.



A. Find the probability that she draws a green bead.

(I) $\frac{3}{11}$ (ii) $\frac{3}{7}$ (iii) $\frac{11}{14}$ (iv) $\frac{3}{14}$

B. Find the probability that the bead drawn by her s not green

(I) $\frac{3}{11}$ (ii) $\frac{3}{7}$ (iii) $\frac{11}{14}$ (iv) $\frac{3}{14}$

C. Find the probability that she draws either a green or a blue bead.

(I) $\frac{5}{7}$ (ii) $\frac{5}{12}$ (iii) $\frac{7}{12}$ (iv) $\frac{3}{14}$

D, Find the probability that she draws neither a red nor a green bead.

(I) $\frac{3}{14}$ (ii) $\frac{1}{3}$ (iii) $\frac{3}{7}$ (iv) $\frac{1}{2}$

Solution:(a) Total number of beads in the Potli = 8+6+14=28

Number of green beads in the Potli= 6

Required probability= 6/28=3/14

(b) Number of beads not green in the Potli= 28-6=22

Required probability= 22/28=11/14

(c) Number of blue and green beads in the Potli= 14+ 6=20

Required probability= 20/28=5/7

(d) Number of beads neither green nor red in the Potli =

Number of blue beads in the Potli=14

Required probability= 14/28=1/2

Long answer type question:

Example1:A couple has two children,

(i) Find the probability that both children are males if it is known that at least one of the children is male.

(ii) Find the probability that both children are females if it is known that the elder child is a female.

Solution: A couple has two children,

Let, the boy be denoted by b & girl be denoted by g

So, S = {(b, b) ,(b, g),(g, b), (g, g)}

To find probability that both children are males, if known that at least one of children is male

Let E : Both children are males
F : At least one child is male

To find P(E|F)

E : Both children are males

E = {(b, b)}

P(E) = 1/4

F : At least one child is male

F = {(b, g), (g, b), (b, b)}

P(F) = 3/4

E ∩ F = {(b, b)}

P(E) = 1/4

F : At least one child is male

F = {(b, g), (g, b), (b, b)}

P(F) = 3/4

E ∩ F = {(b, b)}

P(E ∩ F) = 1/4

P(E|F) = $(P(E \cap F))/(P(F))$

= (1/4)/(3/4)

= 1/3

∴ Required Probability is $\mathbf{1/3}$

(ii) Find the probability that both children are females if it is known that the elder child is a female.

S = {(b, b) ,(b, g),(g, b), (g, g)}

To find the probability that both children are females, if from that the elder child is a female.

Let E : both children are females
F : elder child is a female

To find P(E|F)

E : both children are females

E = {(g, g)}

P(E) = 1/4

F : elder child is a female

F = {(g, b), (g, g)}

P(F) = 2/4=1/2

Also, E ∩ F = {(g, g)}

So, P(E ∩ F) = 1/4

P(E|F) = $(P(E \cap F))/(P(F))$

= (1/4)/(1/2)

= ½

Example2:: An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

A: the sum is greater than 8.

B: 2 occurs on either die

C: the sum is at least 7 and a multiple of 3.

Which pairs of these events are mutually exclusive?

Solution: Given that a pair of dice rolled.

Sample space = S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

n(S) = 36

Event A: The sum is greater than 8

A = {(3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)}

Event B: 2 occurs on either die

B = {(1, 2), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (4, 2), (5, 2), (6, 2)}

Event C: The sum is at least 7 and a multiple of 3

C = {(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)}

Here,

$A \cap B = \Phi$

$B \cap C = \Phi$

$A \cap C \neq \Phi$

Therefore, the pair of events A, B and B, C are mutually exclusive.

Example3:A pack of 50 tickets is numbered from 1 to 50 and is shuffled. Two tickets are drawn at random. Find the probability that (i) both the tickets drawn bear prime numbers (ii) Neither of the tickets drawn bear prime numbers.

Solution: The total number of tickets = 50

Prime numbers from 1 to 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47.

The total number of prime numbers between 1 and 50 is 15.

(i) Probability that both tickets are drawn bears prime numbers:

P(Both tickets bearing prime numbers) = $^{15}C_2$ / ^{50} C_2= 3/35

Hence, the probability that both tickets are drawn bear prime numbers is 3/35.

(ii) Probability that neither of the tickets drawn bears prime numbers:

P (Neither of the tickets bearing prime numbers) = $^{35}C_2$ / $^{50}C_2$= 17/35.

Therefore, the probability that neither of the tickets drawn bears a prime number is 17/35.

Example4:20 cards are numbered from 1 to 20. If one card is drawn at random, what is the probability that the number on the card is:

1. Prime number
2. Odd Number
3. A multiple of 5
4. Not divisible by 3

Solution: Let S be the sample space.

S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

(1) Probability that the card drawn is a prime number:

Let E_1 be the event of getting a prime number.

E_1 = {2, 3, 5, 7, 11, 13, 17, 19}

Hence, $P(E_1)$ = 8/20 = 2/5.

(2) Probability that the card drawn is an odd number:

Let E_2 be the event of getting an odd number.

E_2 = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}

Hence, $P(E_2)$ = 10/20 = 1/2.

(3) Probability that the card drawn is a multiple of 5

Let E_3 be the event of getting a multiple of 5

E_3 = {5, 10, 15, 20}

Hence, $P(E_3)$ = 4/20 = 1/5.

(4) Probability that the card drawn is not divisible by 3:

Let E_4 be the event of getting a number that is not divisible by 3.

E_4 = {1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20}

Hence, $P(E_4) = 14/20 = 7/10$.

Example5: Find the probability that when a hand of 7 cards are drawn from the well-shuffled deck of 52 cards, it contains

(i) all kings and (ii) 3 kings

Solution:

(i) To find the probability that all the cards are kings:

If 7 cards are chosen from the pack of 52 cards

Then the total number of combinations possible is: $^{52}C_7$

$= 52!/[7!\ (52-7)!]$

$= 52!/\ (7!\ 45!)$

Assume that A be the event that all the kings are selected

We know that there are only 4 kings in the pack of 52 cards

Thus, if 7 cards are chosen, 4 kings are chosen out of 4, and 3 should be chosen from the 48 remaining cards.

Therefore, the total number of combinations is:

$n(A) = {}^4C_4 \times {}^{48}C_3$

$= [4!/4!0!\ ] \times [48!/3!(48-3)!]$

$= 1 \times [48!/3!\ 45!]$

$= 48!/3!\ 45!$

Therefore, $P(A) = n(A)/n(S)$

$= [48!/3!\ 45!] \div [52!/\ (7!\ 45!]$

= [48! x 7!] ÷ [3!x 52!]= 1/7735

Therefore, the probability of getting all the 7 cards are kings is 1/7735

(ii) To find the probability that 3 cards are kings:

Assume that B be the event that 3 kings are selected.

Thus, if 7 cards are chosen, 3 kings are chosen out of 4, and 4 cards should be chosen from the 48 remaining cards.

Therefore, the total number of combinations is:

n(B) = ${}^{4}C_{3}$ x${}^{48}C_{4}$

= [4!/3!(4-3)!] x [48!/4!(48-4)!]

= 4 x [48!/4! 44!]

= 48!/3! 45!

Therefore, P(B) = n(B)/n(S)

= [4 x48!/4! 44!] ÷[52!/ (7! 45!]

= 9/1547

Therefore, the probability of getting 3 kings is 9/1547

Practice Questions

Multiple choice questions:

Q.1 What is thetotal number of elements in sample spaces when a coin is tossed and a die is thrown?

a)12 b)10 c)11 d)13

Q.2 A bag contains 5 brown and 4 white socks. Ram pulls out two socks. What is the probability that both the socks are of the same colour?

a)4/5 b)4/9 c)4/3 d)4/7

Q.3: What is the probability of selecting a vowel in the word "ZIET"?

a)1 b)2 c) 0.5 d) None of the above

Q.4 An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. What is the probability that they are of different colours?

a)4/15 b)2/15 c)1/15 d)8/15

Q.5What is the total number ofsamplespaceswhenadieisthrown2 times?

a)6 b)12 c)18 d)36

Q.6Three identical dice are rolled. What is the probability that the same number will appear on each of them?

a)1/6 b)1/36 c)1/18 d)3/28

Q.7An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. What is the probability that they are of different colour?

a)2/15 b)1/15 c)8/15 d)4/15

Q.8 If A and B are two mutually exclusive and exhaustive events,

then P(A) + P(B) =

a) 0 b) 0.5 c) 0.25 d)1

Q.9 If A, B and C are three mutually exclusive and exhaustive events of an

experiment such that 3P(A) = 2P(B) = P(C), then P(A) equals to:

a)2/11 b) 3/11 c) 4/11 d)6/11

Q.10 If P(A) = 0.2,P(B) = 0.3, and $P(A \cap B) = 0.1$. Then $P(A \cup B)$

equal to:

a)1/2 b) 2/5 c) 5/6 d)4/5

Assertion –Reason based questions:

In the following questions a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices

a). Both A and R are true and R is correct explanation of A.

b). Both A and R are true but R is not the correct explanation of A.

c). A is true but R is false.

d). A is false but R is true.

Q.1 **Assertion (A):** Probability of getting a head in a toss of an unbiased coin is 1/2.

Reason (R): In a simultaneous toss of two coins, the probability of getting 'no tails' is 1/4.

Q.2**Assertion (A):** In tossing a coin, the exhaustive number of cases is 2.

Reason (R): If a pair of dice is thrown, then the exhaustive number of cases is 6x6

Q.3**Assertion:** If a box contains $\mathbf{5}$ white, $\mathbf{2}$ red and 4 black marbles, then the probability drawing a white marble from the box is $\mathbf{5/11}$.

Reason: $\mathbf{0} \leq \boldsymbol{P}(\boldsymbol{E}) \leq \mathbf{1}$, where E is any event.

Q.4**Assertion :** Two dice are thrown simultaneously. There are 11 possible outcomes and each of them has a probability 1/11

Reason : Probability of an event E is defined as P(E) =Number of favourable outcomes/total number of outcomes

Q.5 **Assertion:** If a die is thrown, the probability of getting a number less than 3 and greater than 2 is zero.

Reason: Probability of an impossible event is zero.

Short Answer Type Questions:

Q.1 A young man visits a hospital for medical check-up. The probability that he has lungs problem is 0.45, heart problem is 0.29 and either lungs or heart problem is 0.47. What is the probability that he has both types of problems?

lungs as well as heart? Out of 1000 persons how many are expected to have both types of problems?

Q.2The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Q.3What is the probability that a leap year selected at random will contain 53 Sundays?

Q.4A bag contains 5 green and 7 red balls. Two balls are drawn at random. What is the probability that one is green and the other is red?

Q.5 4 cards are drawn from a well-shuffled deck 52 cards. What is the probability of obtaining 1 diamond and 3 spades?

Q.6 A card drawn from a pack of 52 cards.

(i) How many points are there in the sample space?

(ii) Calculate the probability that card is an ace of spade.

(iii) Calculate the probability that the card is

(a) an ace (b) a black card

Q.7Given P(A) = 3/5 and P(B) = 1/5, find P(A or B) if A and B are mutually exclusive events.

Q.8An experiment involves rolling a pair of dice and recording the number that comes up. Describe the following events:

A : the sum is greater than 8,

B : 2 occurs on either die,

C : the sum is at least 7 and a multiple of 3; which

pairs of these events are mutually exclusive?

Q.9The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English Examination is 0.75. What is the probability of passing Hindi Examination?

Q.10A and B are two events such that P(A) = 0.54, P(B) = 0.69 and P(A ∩ B) = 0.35. Find

i. P(A ∪ B) II. P(A ′) III. P(A ′ ∩ B ′) IV. P(A ∩ B ′)

i.22/25 ii.0.46 iii.3/25 iv. 19/100

Case Study based question:

Read the following passage and answer the questions given below

Q.1 In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. One of these students is selected at random.

I. Find the probability that the student opted for NCC or NSS

ii. Find the probability that the student has opted NSS but not NCC.

Q.2 One school decided to organize the sports day. In one of the events of relay

race students of class 11 participated.

In the relay race there are five teams A, B, C, D and E.

On the basis of above information answer the following:

(a) What is the probability that A, B and C finish first, second and

third respectively. Answer 1/60

(b) What is the probability that A, B and C are first three to finish (in

any order) 1/10

(Assume that all finishing orders are equally likely).

Q.3Two students Rohan and Soham appeared in an examination. The probability that Rohan will qualify the examination is 0.05 and that Soham will qualify the examination is 0.10. The 97 probabilities that both will qualify the examination is 0.02. Based on the above information, answer the following questions:

(i) Find the probability that both Rohan and Soham will not qualify the examination.

(ii) Find the probability that at least one of them will not qualify the examination.

(iii) only one of them will qualify the exam

Q.4 20 cards are numbered from 1 to 20. One card is drawn at random what is the prob. that the number on the card drawn is (i) A prime Number (ii) An odd Number (iii) A multiple of 5 OR (iii) Not divisible by 3

Q.5 Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students what is the probability that

(a) You both enter the same section

(b) You both enter the different section.

Long answer type question:

Q.1. A fair coin is tossed four times, and a person win ₹ 1 for each head and lose ₹ 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Q.2.In a town, there are 6000 people of which 1200 are over 50 years old and 2000 are females. It is said that 30% of females are over 50 years. Find the probability that an individual chosen randomly from the town is either female or over 50 years.

Q.3. There are 20 cards that are numbered from 1 to 20. If a card is withdrawn randomly, then find the probability that a number on the card will be: (i) Multiple of 4 (ii) Even number (iii) Not divided by 5 (iv) Prime Number

Q.4. If an entrance exam that is graded based on two exams, the probability of chosen at random, students clearing the 1st exam is 0.8 and the probability of passing the 2nd exam is 0.7. The probability of clearing at least one of them is 0. 95. Find the probability of clearing both.

Q.5. A card has been drawn from a well-shuffled deck of 52 cards. What will be the probability that a card will be an (i) Diamond (ii) Black card (iii) Not an ace (iv) Not a diamond

ANSWERS

Multiple choice questions:

1.(a) 2.(b) 3.(c) 4.(d) 5.(d) 6.(b) 7.(c) 8.(d) 9.(a) 10.(b)

Short Answer Type Questions:

1. 0.27, 270
2. 0.65
3. 2/7
4. 35/66
5. 286/20825
6. (i)52,(ii)1/52,(iii) a)1/13, b)1/2
7. 4/5
8. A & B, A &C
9. 0.65
10. (i)22/25(ii)0.46(iii)3/25(iv) 19/100

Assertion –Reason based questions:1.(a) 2.(b) 3.(b) 4.(d) 5.(a)

Case Study based question:1. i) 19/30 ii)4/30

2. i) 1/60 ii) 1/10

3. i) 0.87 ii)0.98 iii)0.11

4.i) 2/5 ii)1/2 iii)1/5 OR 7/10

5.i) 17/33 (ii) 16/33

LONG ANSWER TYPE QUESTION

1. 1/16

2. 13/20

3. (i) ¼ (ii) ½ (iii) 4/5 (iv) 2/5

4. 0.55

5.(i) ¼ (ii) ½ (iii) 12/13 (iv) 3/4

Class XI – Mathematics
SAMPLE PAPER -I[BLUE PRINT]

CHAPTERS	1M	2 M	3 M	5 M	4 M	Marks for each unit
Sets	**3**	1				
Relations & Functions	2	1	1			**23M**
Trigonometric Functions	4		1	1		
Complex Numbers and Quadratic Equations		1				
Linear Inequalities	1					
Permutations and Combinations			1		1	**25M**
Binomial Theorem	1			1		
Sequence and Series	1			1		
Straight Lines	1		1			
Conic Sections	1				1	**12M**
Introduction to Three-dimensional Geometry	1	1				
Limits and Derivatives	2	1	1			**08M**
Statistics				1		
Probability	3		1		**1**	**12M**
Total Marks	**20 M**	**10 M**	**18 M**	**20 M**	**12 M**	**80 M**

SAMPLE PAPER -I

Maximum Marks: 80 **Time Allowed: 3 hours**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

Q.1) Derivative of the function $f(x) = sin^2x$ is

a) cos^2x b) $cos2x$ c) $-cos2x$ d) $sin2x$

Q.2) S= {x:x is a positive multiple of 3 less than 100}
P= {x: x is a prime no. less than 20},
then **n(s)+n(p)** is
a) 34 b)41 c)33 d)30

Q.3) Two finite sets A and B are such that A ⊂B, then which of the following is not correct?
(a) A ∪ B = B (b) A ∩ B = A (c) A – B = ∅ (d) B – A = ∅

Q.4) Which of the following is not equal to (a) $cos^2x - sin^2x$
(b) $1 - 2sin^2x$
(c) $1 - 2cos^2x$
(d) $\frac{1-tan^2x}{1+tan^2x}$Q.5) . the locus of a point for which y = 0, z = 0?
a) equation of z-axis
b) equation of y-axis
c) equation of x-axis
d) All of these

Q.6) The domain of real function f(x) = $\frac{x^2+2x+1}{x^2-x-6}$ is given by
a) $R - \{3, -2\}$ b)$R - \{-3,2\}$ c)$R - \{-3, -2\}$ d)$R - (3, -2)$

Q.7) Two dice are thrown simultaneously. The probability of obtaining total score of seven is
a) $\frac{5}{36}$ b)$\frac{6}{36}$ c)$\frac{7}{36}$ d)$\frac{8}{36}$

Q.8) . If – 3x + 17 < – 13, then
(a) x ∈ (10, ∞)

(b) $x \in [10, \infty)$
(c) $x \in (-\infty, 10]$
(d) $x \in [-10, 10)$

Q.9) If $sin \sin \theta + \theta = 2$ then $sin^2\theta + cosec^2\theta$ is equal to …
(a) 1 b) -1 c) 2 d) -2

Q.10) Range of real function f(x) = $-|x|$ is
(a) $(0,\infty)$ (b) $(-\infty,0)$ (c) R (d) None of these

Q.11) *Number of proper subset of a set A of order n is equal to:*
(a) n (b) $2^n - 1$ (c) 2^n (d) n^2

Q.12) A line cutting of intercepts -3 on Y-axis and slope of the line is $\frac{3}{5}$, then equation of line is
(a) $5y-3x+15=0$ (b) $3y-5x+15=0$ (c) $5y-3x-15=0$ (d) None of these

Q.13. The value of sin (45° + θ) – cos (45° – θ) is
(a) 2 cosθ
(b) 2 sinθ
(c) 1
(d) 0

Q.14) Radius of circle $x^2 + y^2 - 4x - 4y - 5 = 0$, is

(a) $\sqrt{13}$ (b) 7 (c) 4 (d) 5

Q.15) In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, the probability of the student has opted NSS but not NCC
(a) $\frac{19}{30}$ (b) $\frac{11}{30}$ (c) $\frac{2}{15}$ (d) None of these

Q.16) . What is the value of $\frac{y^2-4}{y-2}$
(a) 2
(b) 4
(c) 0
(d) cannot be evaluated

Q.17) One card is drawn from a pack of 52 cards. The probability that it is the card of a King or spade is

(a) $\frac{1}{26}$ (b) $\frac{3}{26}$ (c) $\frac{4}{13}$ (d) $\frac{3}{13}$

Q.18) Value of **cosec (-1410^0)** is

(a) 2 (b) 1 (c) -2 (d) -1

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19 If a, b ∈ R and n ∈N, then

$$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}b^2 + \dots\dots\dots + {}^nC_n b^n$$

Q.20) Assertion (A): If 16 and 9 be A.M and G.M respectively between two +ve numbers, then numbers are $16 \pm 5\sqrt{7}$

Reason (R): If A and G A.M and G.M respectively between two +ve numbers, then numbers are $\boldsymbol{A} \pm \sqrt{(\boldsymbol{A}+\boldsymbol{G})(\boldsymbol{A}-\boldsymbol{G})}$

SECTION B

Q.21) If $f(x) = x - \frac{1}{x}$, prove that $\left(f(x)\right)^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$

OR

Let A = {9,10,11,12,13} and let f: A→N be defined by f (n) = the highest prime factor of n. Find the range of f.

Q.22) If $x = -1 + i$, then find the value of $x^2 + 2x - 1$

OR

Express the following in $a + ib$ form:

$(i^{18} + i^{-25})^3$

Q.23) Evaluate the following Limit:

$\frac{1-cos2x}{x^2}$

Q.24) Show that the points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.

Q.25) If A = {1,2,3,4,5,6} and B = {2,4,6,8} then find that (i) AUB (ii) A-B.

SECTION C

Q.26) Find the derivative of $sinx$ by first principle.

OR

Find derivative of following function w.r.t. x

$$f(x) = \frac{3x+4}{5x^2-7x+9}$$

Q.27) Let $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \epsilon R\right\}$ be a function from R into R. Determine the range of f.

OR

Find the domain of the function f(x) $= \frac{x^2+2x+1}{x^2-8x+12}$

Q.28) In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers.

OR

In how many ways can the letters of the word "ASSASSINATION" be arranged so that all the S's are together?

Q.29) How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Q.30) If p and q are the lengths of perpendiculars from the origin to the lines $xcos\theta - ysin\theta = kcos2\theta$ and $xsec\theta + ycosec\theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$.

Q.31 Find the value of $tan\ tan\frac{\pi}{8}$.

SECTION D

Q.32) Find the mean and the standard deviation for the following data:

classes	$30-40$	$40-50$	$50-60$	$60-70$	$70-80$	$80-90$	$90-100$
Frequency	3	7	12	15	8	3	2

Q.33) Prove that

If $tan\ \tan x = \frac{3}{4}$, $\pi \le x \le \frac{3\pi}{2}$ then find the values of $\frac{x}{2}$,$cos\ \cos\frac{x}{2}$,$tan\ \tan\frac{x}{2}$

Q.34) If S be the sum, P be the product and R be the sum of reciprocals of n terms of a G.P. Prove that: $P^2R^n = S^n$.

OR

Sum of two numbers is six times their geometric mean, show that the numbers are in the ratio $(3+2\sqrt{2})$: $(3-2\sqrt{2})$.

Q.35) Using binomial theorem, find $(x+1)^6+(x-1)^6$. Hence, by using it find the value of $\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6$.

SECTION E

(CASE STUDY BASED QUESTIONS)

This section comprises of 3 case-study/passage-based question of 4 marks each. First case study problems have 3 sub parts (i), (ii) and (iii) of marks 1, 1 and 2 respectively. Second case-study has four sub parts (i), (ii), (iii), (iv) each carrying 1 mark. Third case-study problem has two sub parts (i) and (ii) of two marks each.

Q.36) During the Mathematics class, A teacher clears the concept of permutations and combinations to the 11^{th}class students. After the class was over he asks the students some more questions.

On the basis of the information given above answer the following:-

(a) Find the number of arrangements of the letters of the word INDEPENDENCE.

(b) In How many of these do the words begin with I and end in P.

(c) In How many of these do all the vowels never occur together.

OR

In How many of these do all the four E's do not occur together

Q.37) On her winter vacations, Ayesha visits four cities (Delhi, Mumbai, Goa and Bangalore) in random order.

On the basis of the information given above answer the following

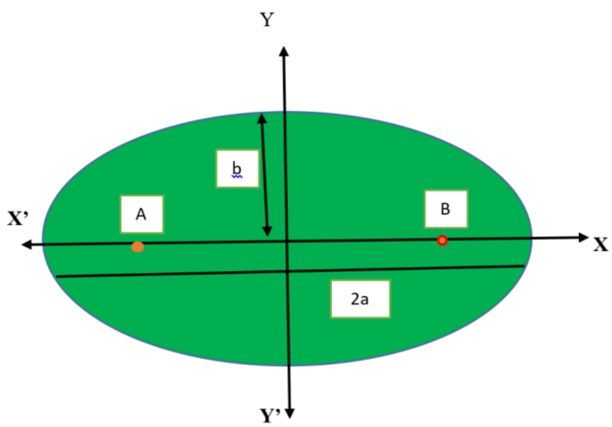
(a) What is the probability that she visits Delhi before Goa and Goa before Mumbai?

OR

What is the probability that she visits Delhi First and Mumbai last?

(b) What is the probability that she visits Delhi just before Mumbai?

Q.38) A colonel running a race course observes that sum of the distances of two flag posts A and B from him is always 10 m and distance between the flag posts is 8 m.



Now answer the following questions:

i. Find the value of b?

ii. Find the locus (equation) of the path traced out by colonel?

MARKING SCHEME
Class XI
Mathematics (Code – 041)

Q.NO	Answer	Marks
1.	D	1
2.	B	1
3.	D	1
4.	C	1
5.	C	1
6.	A	1
7.	B	1
8.	A	1
9.	C	1
10.	D	1
11.	B	1
12.	A	1
13.	D	1
14.	A	1
15.	C	1
16.	B	1
17.	C	1
18.	A	1
19.	C	1
20.	C	1
21.	To write $f\left(\frac{1}{x}\right) = \frac{1}{x} - x$, and $f(x^3) = x^3 - \frac{1}{x^3}$, and for proving $\left(f(x)\right)^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$ **OR** To write f = $\{(9,3), (10,5)(11,11)(12,3)(13,13)\}$,	½ 1½ 1½

	And to write range of f = {3,5,11,13}	½
22.	Simplifying and getting answer as -3 **OR** Getting $-(1+i)^3$ And to get the correct solution as **2-2i**.	2 1 1
23.	$\frac{2sin^2x}{x^2}$ $\frac{sinx}{x}.\frac{sinx}{x})$ 2.1.1=2	1 ½ ½
24.	Find PQ=$\sqrt{14}$, QR = 2$\sqrt{14}$, PR = 3$\sqrt{14}$ PQ+QR = PR Hence, points P,Q,R are collinear	1 1
25.	$A = \{0,-1,1\}$ and $B = \{-2,-1,1\}$ (i) $A \cup B = \{0,-1,1,-2\}$ (ii) $A \cap B = \{-1,1\}$	1 ½ ½
26.	Let f(x) = $sin\ sinx$=> f\`(x) = $\frac{sin(x+h)-\sin(x)}{h}$ (by using first principle) =$\frac{2cos\left(\frac{2x+h}{2}\right).\sin(\frac{h}{2})}{h}$ = $cos\left(\frac{2x+h}{2}\right)$ X $\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$ = $cos(x)$ X 1 = $cos(x)$ **OR** For applying quotient rule of derivative correctly. And for getting the correct answer	1 ½ 1 ½ 1½ 1½
27.	Here $f(x)=\frac{x^2}{1+x^2}$ $\Rightarrow$ $x=\pm\sqrt{\frac{y}{1-y}}$ Now, x will be real if $\frac{y}{1-y}\geq 0$ $\Rightarrow$ $\frac{y}{y-1}\leq 0$ $\Rightarrow$ $0\leq y<1$ $\Rightarrow$ $y\in[0,1)$ $\therefore$ Range of $f(x)=[0,1)$ **OR**	1 1 1

	$f(x)$ is a rational function of x. $f(x)$ assumes real values of all x except for those values of x for which $x^2 - 8x + 12 = 0 \Rightarrow (x-6)(x-2) = 0 \Rightarrow x = 2, 6$ ∴ Domain of function = R – {2, 6}	3
28.	Total number of players =17 It is given that only 5 players can bowl, ∴ Number of bowlers =5 ∴ Number of other players =17-5 =12 A team of 11 players can consist of exactly 4 bowlers and 7 other players. Exactly 4 bowlers can be selected out of 5 bowlers 5_{c_4} ways, 7 other players can be selected out of 12 other players in 12_{c_7}. Hence , a team of 11 players consisting if exactly 4 bowlers and 7 other players can be selected out of 5 bowlers and 12 other players in $5_{c_4} \times 12_{c_7}$ ways $=5_{c_1} \times 12_{c_5}$ ways=3960 ways. OR Find total no of letters taking all S together Use correct formula Calculate correct answer = 151200	1½ ½ 1 1 1 1 1
29.	Let x litres of water can be added to 1125 litres of 45% of boric acid solution, then A.P.Q., 25% of (x+1125) < 45% of 1125 < 30% of (x+1125). On solving the above inequations and to get the answer as More than 562.5 litres but less than 900 litres.	 1½ 1½
30.	31. $p =$ length of perpendicular from origin to the line x cosθ -y sin θ = k cos2θ $p = \frac{\lvert 0-0-k cos cos2\theta \rvert}{1} = \lvert k\ cos\ cos2\theta \rvert$ q =length of perpendicular from origin to the line x secθ + y cosecθ= k =length of perpendicular from origin to the line $x\ sin\ sin\theta + y$ $cos\ cos\theta = k\ sin \sin\theta\ \ cos \cos\theta$ $q = \frac{\lvert 0-0-\frac{k}{2} sin sin2\theta \rvert}{1} = \left\lvert \frac{k}{2} sin\ sin2\theta \right\rvert$ so $p^2 + 4q^2 = k^2(1) = k^2$	 1 1 1

31.

$tan\ \tan(2\theta) = \frac{(\theta)}{1-\theta}$

$\tan\left(\frac{\pi}{4}\right) = \frac{\left(\frac{\pi}{8}\right)}{1-\frac{\pi}{8}}$ $\left(\theta = \frac{\pi}{8}\right)$

$1 = \frac{2x}{1-x^2}$, $\left(x = tan\ \tan\left(\frac{\pi}{8}\right)\right)$

So, $x^2 + 2x - 1 = 0$

So, $x = \frac{-2\pm\sqrt{4+4}}{2} = \frac{-2\pm2\sqrt{2}}{2}$

So, $x = -1 \pm \sqrt{2}$ but $\frac{\pi}{8} \in$ I quadrant

So, $x = tan\ \tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1$

32..

CLASS	FREQUENCY (f_i)	MID-POINT(x_i)	$f_i \cdot x_i$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
30-40	3	35	105	729	2187
40-50	7	45	315	289	2023
50-60	12	55	660	49	588
60-70	15	65	975	9	135
70-80	8	75	600	169	1352
80-90	3	85	255	529	1587
90-100	2	95	190	1089	2178
TOTAL	50		3100		10050

MEAN = $\sqrt{\frac{\Sigma\ f_i x_i}{\Sigma\ f_i}} = \frac{3100}{50} = 62$ 2

S.D.= $\sqrt{\frac{\Sigma\ f_i(x_i - \underline{x})^2}{\Sigma\ f_i}} = \sqrt{\frac{10050}{50}} = 14.18$ 3

33.	$\cos x = -\frac{1}{\sqrt{1+tan^2 x}} = -\frac{4}{5}\ as\ \pi < x < \frac{3\pi}{2}$ Again $\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \Rightarrow \quad \cos x < 0\ and \quad \sin x > 0$ $\cos\frac{x}{2} = -\sqrt{\frac{1+\ \ cos\, x}{2}} = -\frac{1}{\sqrt{10}},$ $= \sqrt{\frac{1 - cos\, x}{2}} = \frac{3}{\sqrt{10}}$, $\tan\frac{x}{2} = \frac{sin\ \frac{x}{2}}{cos\ \frac{x}{2}} = -3$	1 1 1 1 1
34.	Let the terms of G.P. are a, ar, ar^2,… ar^{n-1} So, S = $\frac{a(r^n-1)}{r-1}$ ………(1) & P= a . ar.ar^2…. ar^{n-1} = a^n . $r^{1+2+\ldots(n-1)}$ = $a^n . r^{\frac{n(n-1)}{2}}$ …..(2) R = $\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \cdots \frac{1}{ar^{n-1}} = \frac{\frac{1}{a}\left(1-\frac{1}{r^n}\right)}{1-\frac{1}{r}}$ ……………(3) For reaching the answer as $P^2 . R^n = S^n$ **OR** Let the two numbers are a and b. So, a + b = $6\sqrt{ab}$ $\frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$=>$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$ (By C and D) ⇨ $\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{2}{1}$ ⇨ $\frac{(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})} = \frac{\sqrt{2}}{1}$ Again applying C & D to get $\frac{\sqrt{a}}{\sqrt{b}} = \frac{(\sqrt{2}+1)}{(\sqrt{2}-1)}$ Finally, for getting the answer as $\frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$	1 1 1 2 1 2 2

35.	Using binomial theorem, the expressions $(x + 1)^6$ and $(x - 1)^6$ can be expressed as	
	$(x + 1)^6 = {}^6C_0\, x^6 + {}^6C_1\, x^5 + {}^6C_2\, x^4 + {}^6C_3\, x^3 + {}^6C_4\, x^2 + {}^6C_5\, x + {}^6C_6$	1
	$(x - 1)^6 = {}^6C_0\, x^6 - {}^6C_1\, x^5 + {}^6C_2\, x^4 - {}^6C_3\, x^3 + {}^6C_4\, x^2 - {}^6C_5\, x + {}^6C_6$	
	Now, $(x + 1)^6 - (x - 1)^6 = {}^6C_0\, x^6 + {}^6C_1\, x^5 + {}^6C_2\, x^4 + {}^6C_3\, x^3 + {}^6C_4\, x^2 + {}^6C_5\, x + {}^6C_6 - [{}^6C_0\, x^6 - {}^6C_1\, x^5 + {}^6C_2\, x^4 - {}^6C_3\, x^3 + {}^6C_4\, x^2 - {}^6C_5\, x + {}^6C_6]$	1
	$= 2\, [{}^6C_0\, x^6 + {}^6C_2\, x^4 + {}^6C_4\, x^2 + {}^6C_6]$	
	$= 2\, [x^6 + 15x^4 + 15x^2 + 1]$	
	Now by substituting $x = \sqrt{2}$, we get	
	$(\sqrt{2} + 1)^6 - (\sqrt{2} - 1)^6 = 2\, [(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1]$	1
	$= 2\, (8 + 15 \times 4 + 15 \times 2 + 1)$	
	$= 2\, (8 + 60 + 30 + 1)$	
	$= 2\, (99)$	1
	$= 198$	1
36	1. In the word INDEPENDENCE there are 12 letters of which 3 are N's, 4 are E's and 2 are D's	
	(i) Total number of arrangements $= \frac{12!}{3!4!2!} = 1663200$	1
	(ii) Number of words begin with I an end with P $= \frac{10!}{3!4!2!} = 12600$	1
	(iii) Number of Words in which vowels never occur together = Total number of arrangements – Number of arrangements in which vowels occur together	1
	$= 1663200\, \frac{8!}{3!2!} \times \frac{5!}{4!1!}$	
	$= 1663200 - 16800 = 1646400$	1
	OR	
	Number of words in which all the four E's do not occur together	
		1

	= Total number of arrangements – Number of arrangements in which all the four E's occur together $= 1663200 - \frac{9!}{3!2!} = 1663200 - 30240 = 1632960$	1
37	1. Here n(s)= 24 (i) $E_1 = \{DBGM, DGBM, DGMB, BDGM\}$ $P(E_1) = \frac{4}{24} = \frac{1}{6}$ OR $E_2 = \{DBGM, DGBM\}$ $P(E_2) = \frac{2}{24} = \frac{1}{12}$ (ii) $E_3 = \{DMGB, DMBG, GBDM, GDMB, BGDM, BDMG\}$ $P(E_3) = \frac{6}{24} = \frac{1}{4}$	1 1 1 1 1
38	(I) PA + PB = 2a = 10 (Hence, locus is ellipse) => a = 5cm Now, 2c = 8 => c = 4cm But $c^2 = a^2 - b^2$ ⇨ $b^2 = a^2 - c^2 = 25 - 16 = 9$ ⇨ b = 3cm (II) Equation of required locus is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{25} + \frac{y^2}{9} = 1$	2 2

SAMPLE PAPER –II

BLUE PRINT

SNO	UNIT/TOPIC	SECTION'A'		SECTION'B'	SECTIO N'C'	SECTIO N'D' (LA)	SECTI ON'E' (CBQ)	TOTAL	UNIT WISE MAR KING
		1M	1 M	2 M	3 M	5 M	4 M		
1	SETS	2(2)		1(2)	1(3)			3(4)	19 M
2	RELATIONS AND FUNCTIONS	2(2)				1(5)		3(7)	
3	TRIGONOMETRIC FUNCTIONS	2(1)		1(2)		1(5)		3(8)	
4	COMPLEX NUMBERS	2(2)		1(2)				3(4)	27 M
5	LINEAR INEQUALITIES	1(1)			1(3)			2(4)	
6	PERMUTATIONS AND COMBINATIONS	1(1)	1(1)				1(4)	3(6)	
7	BINOMIALTHEOREM	1(1)			1(3)			2(4)	
8	SEQUENCE AND SERIES			1(2)		1(5)		4(9)	
9	STRAIGHT LINES				1(3)		1(4)	3(8)	13 M
10	CONIC SECTION	1(1)			1(3)			2(4)	
11	3-DIMENSIONAL GEOMETRY	1(1)						1(1)	
12	LIMITS AND DERIVATIVES	2(2)	1(1)			1(5)		5(11)	11M
13	STATISTICS			1(2)	1(3)			2(5)	10 M
14	PROBABILITY	3(1)					1(4)	2(5)	
	TOTAL	18(18)	2(2)	5(10)	6(18)	4(20)	3(12)	38(80)	80

SAMPLE PAPER-02

TIME= 3HRS CLASS = XI M.M 80

General Instructions-

i. the question paper contains 5 sections- A, B, C, D and E. each section is compulsoryhowever, there are internal choices in some questions.
ii. section A has 18 MCQ and 2 A &R type questions of 1 mark each.
iii. Section B has 5 VSA type questions of 2 marks each.
iv. Section C has 6 SA type questions of 3 marks each.
v. Section D has 4 LA type questions of 5 marks each.
vi. Section E has 3 Source Based / Case based / Passage based/ integrated units of assessments (4marks each) with sub parts.

SECTION A

1. n(A)= m, and n(B) = n. Then the total number of non-empty relations that can be definedfrom A to B is

(a) m^n (b) n^m (c) $mn - 1$ (d) $2^{mn} - 1$

2. If A = {1,3,5, {2, 4 }}, then which of the following is correct ?

(a) {2} ⊂ A (b) {2,3} ⊂ A (c) {2, 4} ⊂ A (d) {{2, 4 }} ⊂ A

3. Find the range of the relation R = $\left\{(x, y): y = x + \frac{6}{x}; where\ y \in N\ and\ x < 6\right\}$

(a) $\{1,2,3,4,5,6\}$ (b)$\{1,2,3,\}$ (c) $\{5,7\}$ (d)$\{5, 7 , 9\}$

4. ∅∩A = …………………..

(a) A (b) ∅ (c) U (d) ∅′

5. Find the value of $\sin \frac{31\pi}{3}$

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) not defined (d) 1

6. i^{243} *is equl to*

(a) -1 (b) 1 (c) i (d) -i

7. The number of non-zero integral solutions of the equation $|1 - i|^x$ =2 is

(a) 1 (b) 2 (c) -1 (d) -2

8. If $-\frac{3}{4}x \leq -3$, then which of the following is correct?

(a) $x \geq 4$ (b) $x \geq -4$ (c) $x \leq -4$ (d) $x \leq 4$

9. 6 boys and 6 girls sit in a row at random. The probability that all the girls sit together is______

(a) 1/432 (b) 12/ 431 (c) 1/132 (d) none of these

Answer- c

10. The total number of terms in the expansion of $(x + a)^{51} - (x - a)^{51}$ after simplification is ?

(a) 102 (b) 26 (c) 25 (d) 0

11. A box contains 10 good articles and 6 with defects. One item is drawn at random. The probability that it is either good or has a defect is__________

(a) 64/64 (b) 49/64 (c) 40/64 (d) 24/64

12. If A.M and G.M of two positive numbers are 10 and 8 respectively., then the numbers are.(a) 20 and 64 (b) 100 and 16 (c) 4 and 16 (d) 10 and 64

13. If $\tan x = -1/\sqrt{5}$ and x lies in 4th quadrant, then the value of $\cos x$ is __________

(a) $\sqrt{5}/\sqrt{6}$ (b) $2/\sqrt{6}$ (c) $1/2$ (d) $1/\sqrt{6}$

14. Find the equation of ellipse with focus at $(\pm 5, 0)$ and length of major axis 26.

(a) $\frac{x^2}{169} + \frac{y^2}{144} = 1$ (b) $\frac{x^2}{169} - \frac{y^2}{144} = 1$ (c) $\frac{y^2}{169} + \frac{x^2}{144} = 1$ (d) $\frac{x^2}{144} - \frac{y^2}{169} = 1$

15. Let L be the foot of perpendicular from a point P(3, 4, 6) on the XY plane. Then coordinates of L are

(a) (3, 0, 0) (b) (-3, 4, 0) (c) (0, 0, 6) (d) (3, 4, 0)

16. The derivative of 99x at x = 0 is

(a) 99 (b) 0 (c) 1 (d) 100

17. What is the probability of drawing a red king from a pack of 52 playing cards?(a) 30/52 (b) 1/13 (c) ½ (d) 1/26

18. $\lim_{x \to 5} \frac{x^2-9x+20}{x^2-6x+5}$

(a) 1 (b) 4 (c) not defined (d)1/4

<u>ASSERTION-REASON BASED QUESTIONS</u>

In Questions number 19 and 20, a statement of Assertion (A) is followed by a statement ofReason (R). Choose the correct answer out of the following choices.

(a) If both (A) and (R) are true and (R) is the correct explanation of (A)
(b) If both (A) and (R) are true and (R) is not the correct explanation of (A)
(c) if (A) is true and (R) is false.
(d) If (A) is false (R) is true.

19. Assertion (A) : 7! = 42X5!

Reason (R): n! = n(n – 1)!

20. Assertion (A) : $\lim_{x\to 3}\frac{x^3-27}{x-3} = 27$

Res on (R): $\lim_{x\to a}\frac{x^a-a^n}{x-a} = a^n$

SECTION-B

21. If A={x:x∈R,x *is the root of the equation* x^3-x =0},

B={x:x∈R,x *is the root of* $x^3+2x^2-x-2=0$} ,Then find the values of (i)$A∪B$(ii)$A∩B$

21. Provethat

$\tan 2023x-\tan 2022x-\tan x=\tan 2023x .\tan 2022x .\tan x$

OR

If tan(A+B)=p and tan(A–B)=q t hen prove that

tan2A = $\frac{p+q}{1-pq}$

23.If$(x+iy)^3=u+iv$ then show that

$\frac{u}{x}+\frac{v}{y}$= 4(x^2-y^2)

24.Find the sum of n terms of the following series 7 +77+777+7777+…….

25.Mean and standard deviation of 100 observations are 50 and 4 respectively. Find the sum ofall the observations and the sum of the squares of the observations.

OR

Given that $\overline{x}$ is the mean and σ^2 is the variance of n observations x_1, x_2, x_3 … …. . x_n . *Prove that the mean and variance of the observations* $ax_1, ax_2, ax_3,$ … … ax_n $(a \neq 0)$ are a$\overline{x}$ and $\alpha^2\sigma^2$

SECTION-C

26. IQ of a person is given the formula$Q = \frac{MA}{CA}X100$; where MA is Mental age and CA is Chronological

age. If 80 ≤ IQ ≤ 140 for a group of 12-year children, find the range of mental age.

27.Find the coefficient of a^4 in the product$(1+2a)^4(2-a)^5$using binomialtheorem.

28.If p and q are the lengths of perpendiculars from the origin to the lines

$x\cos\theta - y\sin\theta = k\cos 2\theta$ and $x\sec\theta + y\,\mathrm{cosec}\,\theta = k$, respectively, prove that $p^2+4q^2=k^2$.

29.Find the equation of the circle which passes through the point (1,1) and centre lies at thepoint of intersection of lines (x+y=4) & (x-y =0)

OR

If the eccentricity of the ellipse is 5/8 and distance between its foci is 10. Find the equationof ellipse.

30.For sets A,B and C using properties of sets prove that

$$A-(B\cup C)=(A-B)\cap(A-C)$$

OR

For any two sets A and B , prove that $A\cup B = A\cap B \Rightarrow A = B$

31 Find the mean deviation about the median for the following data:

x_i	3	6	9	12	13	15	21	22
f_i	3	4	5	2	4	5	4	3

SECTION-D

32.Find the domain and range of 1 / (2−sin $sin 3x$).

33.If tanx = 3/4 , π<x<3π/2 then find the values of sin(x/2) , cos(x/2) and tan(x/2).

OR

Prove that sin10°sin30°sin50°sin70° = 1/16.

34.If a and b are the roots of $x^2-3x+p = 0$ and c and d are roots of $x^2-12x+q = 0$ where a, b, c and d form a GP. Then prove that $\frac{q+p}{q-p}=\frac{17}{15}$

35. i. find the derivative of $\frac{sin^2x}{1+cosx}$ w.r.t x 2 marks

ii. derivative of xSinx by first principle method. 3 marks

SECTION-E

36.During the mathematics class, a teacher clears the concept of permutations and combinations to the 11^{th} class students. After the class was over he asks students somemore questions:

On the basis of the information given above, answer the following

i. Find the number of arrangements of the letters of the word "INDEPENDENCE". 1mark

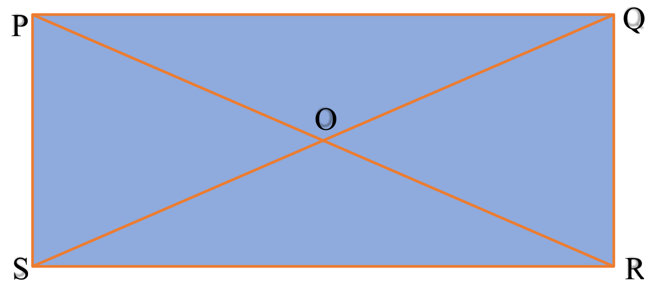
ii. In how many of these do the words begin with "I" and end with "P". 1mark

iii. In how many of these do all the vowels never occur together. 2marks

OR

In how many of these do all the four E's do not occur together. 2marks

37.For an EMC project student need rectangular sheets, therefore they made eco-friendly rectangular sheets PQRS from the paper-waste such that on the cartesian plane. Equationof QR is (3x+4y = 12) and point P is (2,4) while point R is (16/5,a)



38.On her winter vacations, Aisha visits 4 cities (Delhi, Mumbai, Goa and Bangalore) in randomorder.

On the basis of above information answer the following:

iv. What is the probability that she visits Delhi before Goa and Goa before Mumbai? 2marks

OR

What is the probability that she visits Delhi first and Mumbai last? 2marks

v. What is the probability that she visits Delhi just before Mumbai? 2marks

MATHEMATICS (041) - Sample Paper 02

Time – 3 hours **Class XI** **Maximum Marks - 80**

MARKING SCHEME

1	d	6	D	11	a	16	a
2	d	7	B	12	c	17	d
3	c	8	A	13	a	18	d
4	b	9	C	14	a	19	a
5	b	10	B	15	d	20	c

21	21. A={0,-1,1} and B={-2,-1,1} i. AUB={0,-1,1,-2} ii. $A \cap B = \{-1,1\}$	1 1
22	$\tan x = \tan(2023x - 2022x)$ $\tan(2023x - 2022x) = \frac{tan2023x - tan2022x}{1+\tan 2023 \tan 2022x}$ $\tan x(\ 1 + \tan 2023x.\tan 2022x) = \tan 2023x - \tan 2022x$ $\tan 2023\text{x} \tan2022\text{x}\tan \text{x} = \tan 2023\text{x} - \tan2022\text{x} - \tan \text{x}$ OR tan 2A=tan$\{(A+B)+(A-B)\} tan[(A+B)+(A-B)] =$ $\tan 2A = \frac{tan(A+B)+tan(A+B)}{1-tan(A+B)tan(A+B)}$ $=\frac{p+q}{1-pq}$	1 1 1 1
23	$(x\ + iy)3^3\ = u + iv$ $(x^3 - 3xy^2)\ +\ i\ (3x^2 - y^3)\ = u + iv$ $x(x^2 - 3y^{2)}\ = u\ and\ y(3x^2 - y^2) = v$ $\frac{u}{x} = x^2 - 3y^2 and \frac{v}{y} = 3x^2 - y^2$	1 1

24	S= 7+77+777+7777+…………n terms =7(1+11+111+11111+………. n terms) $=\frac{7}{9}(9+99+999+9999+\cdots\ldots\ldots n\ terms)$ $=\frac{7}{9}[(10+10^2+10^3+10^4+\cdots\ldots\ldots\ldots n\ terms)-(1+1+1+\cdots\ldots n\ terms)]$ $= \frac{7}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]=\frac{7}{9}\left[\frac{10(10^n-1)}{9}-n\right]$	1 1
25	Mean=$\frac{\sum x_i}{100}=50\ gives\ \sum x_i=5000$ S.D=$\sqrt{\frac{\sum_{i=1}^{n} x^2{}_i}{100}-50^2}=4$ $\sum_{i=1}^{n} x^2{}_i=251600$ OR New observation $ax_1, ax_2, ax_3, \ldots\ldots\ldots\ldots ax_n (a\neq 0)$ New Mean = $\underline{y}=\frac{\sum y_i}{n}=\frac{\sum ax_i}{n}=a\times\frac{\sum x_i}{n}=\overline{ax}$ $(x_{i-x)^2}n=a2\sigma2$	1 1
26	Mean=$\frac{\sum x_i}{100}=50\ gives\ \sum x_i=5000$ SD= $\sqrt{\frac{\sum x_I^2}{100}-50^2}=4\ gives\ \sum x_i^2=251600$ OR New Mean = $\underline{y}=\frac{\sum y_i}{n}=\frac{\sum ax_i}{n}=a\times\frac{\sum x_i}{n}=\overline{ax}$ New Variance = $\sigma\frac{2}{1}=\frac{\sum (y_i-y)^2}{n}=\frac{\sum (ax-ax)^2}{n}=a^2\times(x_{i-x)^2}n=$ $a2\sigma2$	1 1
27	IQ =$\frac{MA}{CA}\times 100=\frac{MA}{12}\times 100=\frac{25}{3}MA$ $80\leq\frac{25}{3}MA\leq 140$ $80\times\frac{3}{25}\leq MA\leq 140\times\frac{3}{25}$ $9.6\leq IQ\leq 16.8$	1 1 1
28	27. $(1+2a)^4=1+8a+24a^2+32a^3+16a^4$ $(2-a)^5=32-80a+80a^2-40a^3+10a^4-a^5$ =Coefficient of a^4 in $(1+2a)^4(2-a)^5$ =$(10a^4)$ + 8a($-40a^3$) + $24a^2(80a^2)$ + 32 a^3(-80 a)+$16a^4$(32) =$-438a^4$	1 1 1
29	point of intersection of x + y =4 and x – y = 0 is (4,0)	1

	Radius = $\sqrt{10}$ Equation of circle is $(x-4)^2+(y)^2=10$ OR 2c = 10 gives c=5 and e=$\frac{5}{8}$ $\frac{c}{a} = \frac{5}{8}$ gives a=8 Now $b^2+c^2=a^2$ $gives\ b^2=39$ So equation of ellipse is $\frac{x^2}{64}$ + $\frac{y^2}{39}$ =1	1 1 1 1 1
30	$A-(B\cup C)=A\cap(B\cup C)'$ - $\Rightarrow A\cap(B'\cap C')$ $\Rightarrow (A\cap B')\cap(A\cap C')$ $\Rightarrow (A-B)\cap(A-C)$ OR Let $A=B$. Then $A\cap B=A\ and\ A\cup B=A$ $\Rightarrow A\cup B=A\cap B$ (1) Conversely, if $A\cup B=A\cap B$ Let , $x\epsilon A\Rightarrow x\epsilon A\cup B$ $\Rightarrow x\epsilon A\cap B$ $\Rightarrow x\epsilon A\ and\ x\epsilon B$ $\Rightarrow x\epsilon B$ So, $A\subset B$ Let $y\epsilon B\Rightarrow yA\cup B$ $\Rightarrow y\epsilon A\cap B$ $\Rightarrow y\epsilon A\ and\ y\epsilon B$ $\Rightarrow y\epsilon A$ So, $B\subset A$ Therefore A=B	1 1 1 1 1 1
31	Make table of column x_i , f_i , c_if_i , $\lvert x_i-M\rvert$, $f_i\lvert x_i-M\rvert$ Median = $\frac{15th\ 0bs+16th\ obs}{2}$ = 13 Mean deviation about median =$\frac{\sum\ f_i\lvert x_i-M\rvert}{\sum\ f_i}$ = 4.97(appr.).	2 1
32	y = $\frac{1}{2-sin3x}$ Since -1≤ $sin3x\ \le 1$ 1≥ $-sin\sin 3x\ge -1$ 1≤ $2-sin3x\ \le 3$.	1 1

	So domain of f = R $1 \leq 2 - sin3x \leq 3.$ $1 \geq \geq \frac{1}{3}$ Range of f $= \left[\frac{1}{3}, 1\right]$	1 1 1
33	. cosx = $-\frac{1}{\sqrt{1+tan^2 x}} = -\frac{4}{5}$ as $\Pi < x < \frac{3\Pi}{2}$ Cosx< 0 , $and\ sinx > 0$ Find $\cos\frac{x}{2} = -\frac{1}{\sqrt{10}}$, $\sin\frac{x}{2} = \frac{3}{\sqrt{10}}$, $\tan\frac{x}{2} = -3$ OR Sin 10° Sin 30° Sin 50° Sin 70° $= \frac{1}{2}$ (Sin 50° Sin 10°) Sin 70° $= \frac{1}{4}$ (2 Sin 50° Sin 10°) Sin 70° $= \frac{1}{4}$ (cos40° - cos60°) Sin 70° $= \frac{1}{8}[\sin(40° + 70°) + \sin(70° - 40°)] - \frac{1}{8}\sin 70°.$ =. $\frac{1}{8}$ (sin110° $+\frac{1}{2}$)- cos60°) - $\frac{1}{8}$ sin (180° -110°) $= \frac{1}{8}sin110° + \frac{1}{16} - \frac{1}{8}sin110°$ $= \frac{1}{16}$	1 1 1+1+1 1 1 1 1 1
34	Let b = ar. ,c = ar^2 , d = ar^3 ,	1

	Given a + b = 3 , ab = p , c + d = 12 , cd = q a (1 + r) = 3 , ar^2(1 + r) = 12 on dividing we get r = 2 & a = 1 so $\frac{q+p}{q-p}$ =. $\frac{17}{15}$	1 1 1 1
35	(1) $\frac{d}{dx}\left\{\frac{sin^2x}{1+cosx}\right\}$ = $\frac{(1+cosx\ )2\ sinx\ cosx-sin^2x(-sinx)}{(1+cosx\ )^2}$ 1 =$\frac{(1+cosx\ )\ sin2x+sin^3x}{(1+cosx\ )^2}$ (2) f'(x) = $\frac{f(x+h)-f(x)}{h}$ = $\frac{(x+h)sin(x+h)-xsinx}{h}$ $\frac{x\ sinx(coshcosh-1\ )}{h}$ + $\frac{sinh}{h}$ + = x cosx + sin x	1 1 1 1 1
36	In the word INDEPENDENCE there are 12 letters of which 3 are N's, are 4 are E's and 2 are D's (i) Total number of arrangements =$\frac{12!}{3!4!2!} = 1663200$ (ii) Number of words begin with I an end with P =$\frac{10!}{3!4!2!} = 12600$ (iii) Number of Words in which vowels never occur together = Total number of arrangements- Number of arrangements in which vowels occur together = 1663200-$\frac{8!}{3!2!} \times \frac{5!}{4!1!}$ =1663200-16800=1646400 **OR**	1 1 1 1

	Number of words in which all the four E's do not occur together= total no. of arrangements – no. of arrangements in which all the four E's = 1663200- 9!/3!2! = 1663200- 30240 = 1632960	1
37	(A) Slope of PS= slope of QR= -3/4 Equation of PS is y-4= -3/4(x-2) 4y – 16 = -3x+6 3x + 4y = 22 (B) 3(16/5)+4a=12 gives a =3/5 (C) Equation of PQ is 4x-3y= -4 Coordinates of QR(4/5,12/5) PQ=2 & QR=3 Area of Rectangle= 6sq.unit **OR** Equation of PQ is 4x-3y= -4 Coordinates of QR(4/5,12/5) PQ=2 & QR=3 Perimeter= 10 units.	1 1 1 1 1 1
38	Here n(s) = 24 (A) E1= (DBGM, DGBM, DGMB, BDGM) P(E1) = 4/24= 1/6 **OR** E2 = (DBGM, DGBM) P(E2) = 2/24=1/12 (B) E3= (DMGB, DMBG,GBDM,GDMB,BGDM,BDMG) P(E3) = 6/24=1/4	1 1 1 1 1 1

BLUE PRINT

SAMPLE PAPER-III

Blue print-Class XI th Mathematics Sample paper-3 Session 2024-25										
UNIT	TOPICS	1 M	A AND R	2 M	3 M	5 M	CASE STUDY	TOTAL	TOTAL	MAX
Sets and Functions	Sets	1		1	1			6	23	23
	Relations and Functions	1		1	1			6		
	Trigonometric Functions	1	1			1	1	11		
Algebra	Complex Numbers	2						2	25	25
	Linear Inequalities	2						2		
	Permutations and Combinations	2				1	1	11		
	Binomial Theorem	1						1		
	Sequences and Series	2		1		1		9		
Coordinate Geometry	Straight Lines	2	1		1			6	12	12
	Conic Sections	2		1				4		
	Introduction of Three-Dimensional Geometry			1				2		
Calculate	Limits and derivatives	2			2			8	8	8
Statics and Probability	Statistics					1		5	12	12
	Probability				1		1	7		
		18	2	5	6	4	3	80	80	80

Sample Paper -3

Maximum Marks: 80 **Time Allowed: 3 hours**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION-A

1.If A and B are two sets then A $\cap(A\cup B)$ equals

(a)A (b)B (c)$\emptyset$ (d) A $\cap$B

2 If A ={x:x^2-5x+6=0} ,B = {2,4} , C ={4,5} then A X (B $\cap$ C) is

(a) {(2,4),(3,4)} (b) {(4,2),(4,3)} (c) {(2,4),(3,4)(4,4)} (d) {(2,2),(3,3),(4,4),(5,5)}

3.I f(x)= x^3-1/x^3, then f(x)+f(1/x) is equal to

(a)2x^3 (b) 2/ x^3 (c) 0 (d) 1

4.The conjugate of complex number is 1/i-1, then the complex number is

(a)-1/i+1 (b) 1/ i-1 (c) -1/i-1 (d) 1/i+1

5.If x,y∈R, then x+iy is not purely imaginary is

(a)x=0 (b)y=0 (c) x≠0 (d) y≠0

6. If |x+1|/x+1 >0, x ∈R then __________

(a)x∈ (-1,∞] (b) x∈ (-1,∞) (c) x∈ (-∞,-1) (d) x∈ (-∞,-1]

7. If 1/x-2 ≤ 0, xϵR , then _______ $\frac{2x-6}{5} \leq 0$

(a) $(-\infty,-3)$ (b) x∈ (-∞,2) (c) (-∞, 3] (d) (-∞,2)U (3,∞)

8.The number of signals that can be sent by 6 flags of different colors of taking one or more at a time is

(a)63 (b)1956 (c)720 (d)21

9.The number of permutations of n different object taken r at a time when repetition are allowed is

(a) ^nP_r (b)n! (c) n^r (d) r^n

10.The middle term in the explanation of $(2x^2/3+3/2x^2)^{10}$ is

(a) 251 (b)252 (c) 250 (d) none of these

11. A sequence may be defined as

(a)relation whose range ⊂ **Natural number**

(b) as function whole range ⊂ N

(C) function whole domain ⊂ N

(d) progression having real values

12.The third term of G.P.is 4 If first term is 1 then the product of first three term is

(a)2 (b)4 (c)8 (d) none of these

13.If the line x/a+ y/b=1 passes through the point (2,-3)and (4,-5), then(a,b) is

(a)(1,1) (b)(-1,1) (c) (1,-1) (d) (-1,-1)

14.The inclination of the line x-y+3=0 with the positive direction of x- axis is

(a)45° (b)135° (c)-45° (d)-135°

15.The equation of the circle which passes through the point(4,5) and has it centre at (2,2) is

(a)$x^2+y^2-4x-4y-5=0$

(b) $x^2+y^2+4x+4y+5=0$

(c) $x^2+y^2-4x+4y+5=0$

(d)none of these

16.If the parabola $y^2 = 4x$ passes through the point (3,2) then the length of its latus rectum is

(a)2/3 (b)4/3 (c)1/3 (d)4

17. $\lim_{x\to 0} \frac{|\sin x|}{x}$ is_____________

(a)1 (b)-1 (c) 0 (d) does not exist

18.If $F(x)=(1+x+x^2/2+ x^3/3+\ldots\ldots\ldots\ldots+x^{1000})/1000$,then f(-1) is equal to

(a)1 (b)-1 (c) 0 (d) does not exist

19. Directions: Each of these questions contains two statements : **Assertion (A**) and **Reason (R**). Each of these questions has four alternative choices, any one of which is the correct answer. You have to select one of the codes (A), (B), (C) and (D) given below.

(A) A is true, R is true; R is correct explanation for A.
(B) A is true, R is true; R is not correct explanation for A.
(C) A is true; R is false.
(D) A is false; R is true.

A:The value of $f(x) = 3cos\sqrt{x+2}$ lie in the interval $[-3,3]$

R:The value of cos x lie in the interval $[-1,1]$

20.A: The tangent of the angle between the lines x/a + y/b =1 and x/a – y/b = 1 is $2ab/a^2$-b^2

B: : The tangent of the angle between the lines with clops m_1 and m_2 is given by

$$\tan\theta = \left|\frac{m_1+m_2}{1-m_1m_2}\right|$$

SECTION- B

21.Are the following pair of sets equal? Give reasons.

(i) A = {2, 3}, B = {x : x is solution of $x^2 + 5x + 6 = 0$}

(ii) A = { x : x is a letter in the word FOLLOW} B = { y : y is a letter in the word WOLF}

22.Find the domain and range of the mentioned below real valued function.

$f(x) = \frac{1}{\sqrt{16-x^2}}$

$f(x) = |x|$

23.Find three numbers are in GP, whose sum is 13 and the sum of whose squares is 91?

24.If the vertex of a parabola is the point (-3, 0) and the directrix is the line x + 5 = 0, then find its equation.

25.Find the coordinates of a point on y-axis which are at a distance of 5√2 units from the point P(3, -2, 5)

SECTION-C

26.Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second. The values of m and n are respectively

27.The cartesian product A × A has 9 elements among which are found (-1,0) and (0, 1). Find the set A and the remaining elements of A × A.

28.Find equation of the line through the point (0, 2) making an angle of $\frac{\pi}{3}$with the positive x- axis. Also, find the equation of line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

29.Find $\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

30.Find the derivative of $\frac{x^5 - \cos x}{\sin x}$

31. In a certain lottery 10, 000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy (a) one ticket (b) two tickets (c) 10 tickets?

SECTION-D

32.Prove that : $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

OR

Prove that : $\cot x . \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

33. A group consists of 4 girls and 7 boys. In how many on a team of 5 members be selected, if the team has

(i) No girl.

(ii) at least one boy and one girl.

(iii) at least three girls.

34.If the pth and qth terms of a G.P. are q and p, respectively, then show that (p + q)th term is $(q^p/pq)^{1/p-q}$

35. Calculate the mean deviation about median age for the age distribution of 100 persons given below :

Age	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55
Number	5	6	12	14	26	12	16	9

SECTION-E

36. For any real number x, $(\sqrt{x})^2$ is not equal to $\pm$x but $\sqrt{x^2} = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Based on the above information, answer the following questions;

i. if $\frac{\pi}{2} < x < 3\pi/2$, then $\sqrt{1 - sin^2 x}$ is equal to

a)cosx b) –cos x c) ±cosx d) none of these

ii. if π< x< 2π, then $\sqrt{1 - cos^2 x}$ is equal to

a)sinx b)-sinx c) ±sinx d) none of these

iii .if $\pi < x < 2\pi$ then $\sqrt{\frac{1+COS\,X}{1-COS\,X}}$=?

(a) $\frac{\sin x}{1-\cos x}$ (b) $\frac{-\sin x}{1-\cos x}$ (c)$\frac{1+cosx}{sinx}$ $\frac{1+cosx}{cosx}$

iv. if $\pi < x < \frac{3\pi}{2}$,then $\sqrt{\frac{1+COS\,X}{1-COS\,X}} + \sqrt{\frac{1-COS\,X}{1+COS\,X}}$ is equal to

(a)2cosecx (b)-cosecx (c)cosecx (d)-2cosecx

37. Five couples were invited in a tea party. they were asked to sit on one side of a long table. based on the above information, answer the following questions:

i. find the number of ways in which five couples can be seated.

ii. find the number of ways in which all males sit together and all females sit together.

iii.find the number of ways in which no two females sit together.

iv.find the number of ways in which all females are never together.

38.A team of medical students doing their internship have to assist during surgeries at a city hospital. The probabilities of surgeries at a city hospital, the probabilities of surgeries rated as very complex, complex, routine, simple and very simple are respectively 0.15,0.20,0.31,0.26,0.08. based on the above information, answer the following questions;

Find the probabilities that a particular surgery will be rated

a. Complex or very complex
b. Neither very complex nor very simple
c. Routine or complex
d. Routine or simple

Answer key

1	A	6	B	11	C	16	B
2	A	7	C	12	C	17	D
3	C	8	B	12	D	18	c
4	A	9	C	14	A	19	B
5	C	10	B	15	A	20	C

21	i. No ii. yes	1 1
22	(I) Domain $(-4,4)$, range $x \geq \frac{1}{4}$ (ii) domain = R ,range, $x \geq 0$	1 1
23	9,3,1 or 1,3,9	1+1
24	$x^2 = 8(x+3)$	1+1
25	$(0,2,0) and\ (0,-6,0)$	1+1
26	Correct formula $m = 7\ and\ n = 4$	1 1+1
27	. A ={-1,0,1} The remaining element of set A×A are (-1,1) ,(0,-1),(0,0), (1,-1),(1,0)and (1,1)	1 2
28	Calculation, $\sqrt{3}$ +y-2=0 and $\sqrt{3}$+y+2=0	1 1+1

29	Using limit, finding LHL and RHL Limit does not exist at x=0	2 1
30	Using proper derivative formula Finding correct derivative of each part $\frac{-x^5\cos x+5x^4\sin x+1}{(\sin x)^2}$	1 1 1
31	.(a)$\frac{999}{1000}$ (b) $\frac{9990c2}{10000c2}$ (c) $\frac{9990c10}{10000C10}$	1 1 1
32	$\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}$ $=\frac{(\cos 4x+\cos 2x)+\cos 3x}{(\sin 4x+\sin 2x)+\sin 3x}=\frac{2\cos 3x\cos x+\cos 3x}{2\sin 3x\cos x+\sin 3x}$ $=\frac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)}$ =cot 3x Or Use of cot 3x =cot(2x+x) Write the formula Simplification Obtaining result	1+1 1+1 1 1 1 2 1
33	i. 21 ii. 441 iii. 91	1 2 2
34	Obtain p th term Obtaining q th term Obtaining (p+q) th term Proving	1 2 2
35	Obtaining table Getting correct values Putting correct formula 7.35	2 1 1 1
36	36. (i) b (iii) b (iv) (iii)b	1 1

	(v) (iv) d	1 1
37	.(i) 10 ! (ii) $2 \times 5! \times 5!$ (iii) $6!$ (iv) $10! - 6!$	1 1 1 1
38	(i) 0.35 (ii) 0.77 (iii) 0.51 (iv) 0.57	1 1 1 1

SAMPLE PAPER-4

SNO	UNIT/TOPIC	SECTION‘A’ (OBJECTIVETYPE)	SECTION ‘B’ (VSA)	SECTION‘ C’ (SA)	SECTION’ D’ (LA)	SECTION‘ E’ (CBQ)	TOTAL	UNITWISE MARKING
		MCQ (1MarkEach)	(2Marks Each)	(3Marks Each)	(5Marks Each)	(4Marks Each)		
1	SETS	3(3)	1(2)				4(5)	23 M
2	RELATIONS AND FUNCTIONS	2(2)	1(2)	1(3)			4(7)	
3	TRIGONOMETRIC FUNCTIONS	3(3)		1(3)	1(5)		5(11)	
4	COMPLEX NUMBERS	1(1)		1(3)			2(4)	27 M
5	LINEAR INEQUALITIES	1(1)			1(5)		2(6)	
6	PERMUTATIONS AND COMBINATIONS		1(2)			1(4)	3(7)	
7	BINOMIALTHEOREM	2(2)					2(2)	
8	SEQUENCE AND SERIES		1(2)		1(5)		3(8)	
9	STRAIGHT LINES	1(1)	1(2)	1(3)			3(6)	12M
10	CONIC SECTION	1(1)				1(4)	2(5)	
11	3 DIMENSIONAL GEOMETRY	1(1)					1(1)	
12	LIMITS AND DERIVATIVES	2(2)		2(6)			4(8)	8 M
13	STATISTICS				1(5)		1(5)	10 M
14	PROBABILITY	3(1)				1(4)	2(5)	

SAMPLE PAPER – 4

Time: 3 Hours **SESSION 2024 – 25** **M.M. 80**

General Instructions

- *This question paper contains five sections viz. Section A, Section B, Section C, Section D and Section E.*
- *Section A : 20 objective type questions 1 mark each.*
- *Section B : 5 very short answer type questions 2 marks each*
- *Section C : 6 short answer type questions 3 marks each.*
- *Section D : 4 short answer type questions 5 marks each.*
- *Section E : 3 Case Based Question of 4 marks each.*
- *All questions are compulsory. Internal choice are given in two question of 2 marks, two question of 3 marks and two questions of 5 marks. Attempt any one.*
- *Write question number carefully before attempting it.*
- *Attempt Objective type questions in first page of the answer script.*
- *The question paper has 6 printed pages.*

Section A (Objective Type Questions) 1 Mark Each

Q No	Question	Value point
1	Which of the following statement is false: **(A)A – B = A ∩ B'** **(B) (A – B) = A – (A ∩ B)** (C) A – B = A – B' (D) A – B = (A ∪ B) – B	1
2	Let {1, 2, {3, 4}, 5}, which of the following statement is correct: **(A) {3, 4} ⊂ A** **(B) {{3, 4}} ∈ A** **(C) {{3, 4}} ⊂ A** **(D) {1, 2, 5} ∈ A**	1
3	Let R be the relation defined in ℕ defined by R = $\{(1+x, 1+x^2): x \le 5, x \in N\}$. Which of the following statement is false: (A) R = {(2, 2), (3, 5), (4, 10), (5, 17), (6, 25)} (B) Domain of R = {2, 3, 4, 5, 6} (C) Range of R = {2, 5, 10, 17, 26} (D) (B) and (C) are true	1
4	If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is: (A) 2^9 (B) 9^2 (C) 18 (D) $2^9 - 1$	1

5	The value of $\frac{1-tan^2 15°}{1+tan^2 15°}$ is __________ (A) 1 (B) √3 (C) $\frac{\sqrt{3}}{2}$ (D) 2	**1**
6	. If $sin\theta + cos\theta = 1$, then the value of $sin2\theta$ is: **(A) 1** **(B) √3** **(C) 0** **(D) – 1**	**1**
7	If a complex number lies in second quadrant in the argand plane, then its conjugate lies in ________quadrant. (A) second (B) third (C) first (D) fourth	**1**
8	6 boys and 6 girls sit in a row at random. The probability that all the girls sit together is______ (a) 1/432 (b) 12/ 431 (c) 1/132 (d) none of these	**1**
9	The total number of terms in the expansion of $(x + a)^{51} - (x - a)^{51}$ after simplification is ___ (A) 102 (B) 104 (C) 26 (D) none of these	**1**
10	Solution of $-8 \leq 5x - 3$, for $x\epsilon R$ is ________ (A) (– 1, 2) (B) [– 1, ∞) (C) [– 1, 2] (D) {– 1, 2}	**1**
11	The largest coefficient in the expansion of $(a + b)^{18}$is: (A) $18C18$ (B) $18C12$ (C) $18C9$ (D) $18C6$	**1**
12	A box contains 10 good articles and 6 with defects. One item is drawn at random. The probability that it is either good or has a defect is__________ (a) 64/64 (b) 49/64 (c) 40/64 (d) 24/64	**1**
13	A line cutting off intercept – 3 from the y – axis and the tangent of angle to x – axis is 3/5, then its equation is:	**1**

	(A) $5y - 3x + 15 = 0$ (B) $3y - 5x + 15 = 0$ (C) $5y - 3x - 15 = 0$ (D) none of these	
14	For hyperbola $\frac{x^2}{cos^2\alpha} - \frac{y^2}{sin^2\alpha} = 1$ the absolute value of ae is: (A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 1 (D) 0	**1**
15	If L is the foot of perpendicular drawn from the point P (3, 4, 5) on the xy – plane, then the coordinates of L are: (A) (3, 0, 0) (B) (0, 4, 5) (C) (3, 0, 5) (D) none of these	**1**
16	$\lim_{x \to 5} \frac{e^x - e^5}{x-5}$ is equal to______ A) e^{-5} (B) 1 (C) e^5 (D) $5e$	**1**
17	Two students Ajay and Vijay appeared in an examination. The probability that Ajay will qualify the examination is 0.05 and that Vijay will qualify the examination is 0.10 . The probability that both will qualify the examination is 0.02. The probability at least one of them not will not qualify the examination is A) 0.13 (B) 0.87 (C) 0.02 (D) 0.98	
18	If $y = sinx + \cos x$ then value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ is is: **(A) 1 (B)$\frac{\sqrt{3}}{2}$ (c) ½ (D)$\frac{\sqrt{3}-1}{2}$**	**1**
	Assertion and Reason questions	
	Directions: Each of these questions contains two statements : **Assertion (A**) and **Reason (R**). Each of these questions has four alternative choices, any one of which is the correct answer. You have to select one of the codes (A), (B), (C) and (D) given below. (E) A is true, R is true; R is correct explanation for A. (F) A is true, R is true; R is not correct explanation for A. (G) A is true; R is false. (H) A is false; R is true.	
19	**Assertion (A):** If A = $\{8^n - 7n - 1 : n \in \mathbb{N}\}$ and B = $\{49n - 49 : n \in \mathbb{N}\}$ then A and B are equal sets.	**1**

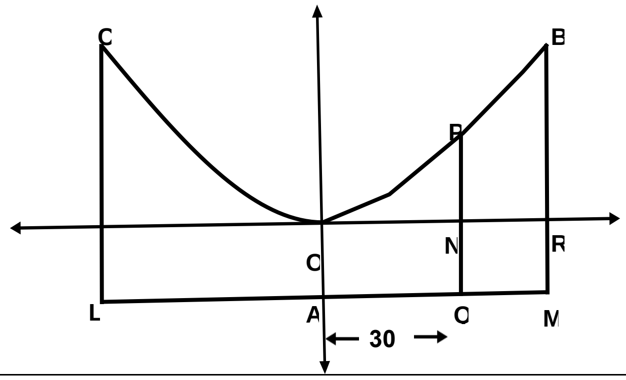
	Reason (R): Two sets A and B are equal sets if A and B have exactly the same elements.	
20	**Assertion (A)** : $tan3xtan2xtanx = tan3x - tan2x - tanx$ **Reason (R) :** $\tan(A-B) = \frac{tanA - tanB}{1 - tanA\, tanB}$	**1**
	Section B (Very Short Answer Type Questions) 2 Marks Each	
21	If A and B are two sets such that A $\cap$ B $\neq \phi$, then draw the Venn diagram to represent (A – B) $\cup$ (B – A).	**2**
22	Let $f = \{(1,1),(2,3),(0,-1),(-1,-3)\}$ be a function defined by $f(x) = ax + b$ for some integers a and b, then find $f(x)$.	**2**
23	If 5th , 8 th and 11th term of a G.P. are p, q and r respectively. Show that $q^2 = ps$ **OR** If 4th, 10th and 16th term of a G.P. are x, y and z respectively. Prove that x, y, z are in G.P.	**2**
24	If a line joining two points A (2, 0) and B (3, 1) is rotated about A in anti-clock wise direction through an angle of 15°. Find the equation of line in new position.	**2**
	<u>OR</u> Find the equation of the line which passes through the point (1, – 2) cuts off equal intercepts from axes.	
25	The number lock of suitcase has 4 wheels, each labelled with ten digits i.e. from 0 to 9. The lock opens with a sequence of four digits with no repeat. What is the probability of a person getting the right sequence to open the suitcase?	**2**
	Section C (Short Answer Type Questions) 3 Marks Each	

26	Let R be the relation from ℤ to ℤ defined by R = { (a, b) : a – b is divisible by 2, a, b ∈ Z}. Show that: (i) (a, a) ∈ R for all a ∈ Z (ii) (a, b) ∈ R ⟹ (b, a) ∈ R (iii) (a, b) ∈ R and (b, c) ∈ R ⟹ (a, c) ∈ R. **OR** Find domain of the function $f(x) = \sqrt{x^2 - 4}$	3
27	Prove that : $\frac{\sin 4x+\sin 3x+\sin 2x}{\cos 4x+\cos 3x+\cos 2x} = \tan 3x$	3
28	If α and β are different complex numbers with $\|\beta\| = 1$ then find value of $\left\|\frac{\alpha-\beta}{1-\alpha\bar{\beta}}\right\|$ Or	3
	If $x + iy = (a + ib)^3, x, y, a, b\varepsilon R$. Show that $\frac{x}{a} - \frac{y}{b} = -2(a^2 + b^2)$	
29	Find the equations of lines through (3, 2) which makes an angle of 45o with the line $x - 2y - 3 = 0$.	3
30	Evaluate: $\lim_{x\to 2} \frac{\sqrt{x^2+1}-\sqrt{5}}{x-2}$	3
31	Find the derivative of xcosx by first principle.	3
	Section D (Long Answer Type Questions) 5 Marks Each	
32	Prove that (1+cos$\frac{\pi}{8}$)(1+cos$\frac{3\pi}{8}$) (1+cos$\frac{5\pi}{8}$) (1+cos$\frac{7\pi}{8}$) = $\frac{1}{8}$	5
	OR Prove that: $sin^2x + sin^2(x + \frac{\pi}{3}) + sin^2(x - \frac{\pi}{3}) = \frac{3}{2}$	
33	1. A solution of 8% boric acid is to be diluted by adding 2% boric acid solution to it. The resulting The resulting mixture to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of 2% solution be added to the 8% solution.	5
34	If A and G be A.M. and G.M. respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$	5
35	Find the mean and variance for the following frequency distribution.	5

Class	00 - 30	00 - 30	00 - 30	00 - 30	00 - 30	00 - 30	00 - 30
Frequency	2		5	10	3	5	2

	OR The mean of the 5 observation is 4.4 and their variance is 8.24. If three of the observations are1, 2, 6, find the other two observations.	
	Section E (Case Based Questions) 4 Marks Each	
36	For a debate competition in a school the house master of Subhash House enrolled 4 girls and 7 boys. From these 11 students the house master have to select 5 participants for the competition. In how many ways can the house master select when 1.The team consists no girl (1) 2.The team consists at least three girls. (1) 3.The team consists at least one girl and one boy. (2)	
37	In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of the student is selected at random, then find the probability that 1.The student opted for NCC or NSS. (1) 2.The student opted neither NCC nor NSS. (1) 3.The student has opted NSS but not NCC.(2)	
38	A suspension bridge is a type of bridge in which the deck is hung below suspension cables on vertical suspenders. The basic structural component of a suspension bridge system includes stiffening girders/trusses, the main suspension cables, main towers and the anchorages for the cable at each end of the bridge. The main cables are suspended between towers and are finally connected to the anchorage or the bridge itself, and vertical suspenders carry the weight of the deck and the traffic load on it. Like other cable supported bridge, the super structure of suspension bridges is constructed without false work as the cable erection method is used. The main cables carrying members us the main cables, which are tension members made of high – strength steel. The whole cross – section of the main cable is highly efficient in carrying the loads and bucking is not  problem.	

The main cables of a suspension bridge are hang in the form of a parabola between towers of height 30 metre above the roadways and are 200 metre apart. If the supporting cable is 5 metre above the roadway at the centre of the bridge.



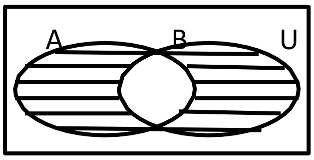
Based on above information answer the following:

1.Find the equation of main cable. (2)

2.If the supporting cable PQ is 30 m from the mid – way find its length. (2)

MARKING SCHEME

SAMPLE PAPER 4

Q. No.	Value Point/ Correct answer	Marks
1.	C	
2.	C	
3.	A	
4.	A	
5.	C	
6.	C	
7.	B	
8.	C	
9.	C	
10.	B	
11.	C	
12.	A	
13.	A	
14.	C	
15.	D	
16.	C	
17.	B	
18.	D	
19.	A	
20.	B	
21.	 for showing sets A, B and U For correct shading	1 1
22.	$f(1) = 1 \Longrightarrow a + b = 1$ $f(0) = -1 \Longrightarrow b = -1$ $a = 2$ $f(x) = 2x - 1$	½ ½ ½ ½
23.	Let a be the first term and r be the common ratio of the G.P. $\therefore ar^4 = p,\ ar^7 = q, \quad ar^{10} = s$ $ps = ar^4 \times ar^{10} = a^2r^{14}$ $\Longrightarrow ps = (ar^7)^2$ $\Longrightarrow ps = q^2$ OR Let a be the first term and r be the common ratio of the G.P. $\therefore ar^3 = x,\ ar^9 = y, \quad ar^{15} = z$ $xz = ar^3 \times ar^{15} = a^2r^{18}$ $\Longrightarrow xz = (ar^9)^2$ $\Longrightarrow xz = y^2$ $\Longrightarrow x, y, z$ are in GP	½ ½ ½ ½ ½ ½ ½ ½

24	Slope of line AB=$\frac{1-0}{3-1} = 1, \theta = 45°$ Inclination of line in new position=$45° + 15° = 60°$	1

	Slope of line in new position=$tan 45° = \sqrt{3}$ Equation of line in new position :$y - \sqrt{3}x + 2\sqrt{3} = 0$ OR Let the line be $\frac{x}{a} + \frac{y}{b}$=1 It passes through the point $(1, -2)$ $\frac{1}{a} + \frac{-2}{a} = 1$ $a = -1$ Equation of the line is $x + y + 1 = 0$	1 1 1
25.	Total number of outcomes = ${}^{10}C_4 \times 4!$ OR ${}^{10}P_4$ = 5040 No. of favorable outcomes = 1 P(the lock will open)= $\frac{1}{5040}$	½ ½ ½ ½
26.	(i) Since $a - a = 0$ which is divisible by 2 Hence $(a, a) \in R$ (ii) $(a, b) \in R$ $\Rightarrow a - b$ is divisible by 2 $\Rightarrow -(b - a)$ is divisible by 2 $\Rightarrow b - a$ is divisible by 2 $\Rightarrow (b, a) \in R$ (iii) $(a, b) \in R \Rightarrow a - b$ is divisible by 2 $(b, c) \in R \Rightarrow b - c$ is divisible by 2 On adding above two result we get $a - b + b - c$ is divisible by 2 $\Rightarrow a - c$ is divisible by 2 $\Rightarrow (a, c) \in R$ OR For real function f $x^2 - 4 \geq 0$ $\Rightarrow (x + 2)(x - 2) \geq 0$ $\Rightarrow$ either $x \leq -2$ or $x \geq 2$ $\Rightarrow D_f = (-\infty, -2] \cup [2, \infty)$	1 1 1 1 1 1
27.	LHS:$\frac{(\sin 4x+\sin 2x)+\sin 3x}{(\cos 4x+\cos 2x)+\cos 3x}$ $= \frac{2\cos x \sin 3x + \sin 3x}{2os\ 3x \cos x + \cos 3x)} = \frac{sin 3x(2\cos x + 1)}{\cos 3x(\cos x + 1)}$ $=\frac{\sin 3x}{\cos 3x} = \tan 3x = RHS$	½ 1 ½ 1

28	$\left\|\frac{\overline{\beta-\alpha}}{1-\bar{\alpha}\beta}\right\|=\left\|\frac{(\beta-\alpha)\bar{\beta}}{(1-\bar{\alpha}\beta)\bar{\beta}}\right\|$ $= \left\|\frac{(\beta-\alpha)\bar{\beta}}{\bar{\beta}-\alpha\beta\bar{\beta}}\right\|$, we know that $z\bar{z} = \|z\|^2 = 1$ and $\overline{z_1 - z_2}=\overline{z_1} - \overline{z_2}$ $\|\bar{\beta}\| = 1$ OR $x + iy = (a + ib)^3$	1 2 1

	$\Rightarrow x+iy=a^3+3a^2ib+3ai^2b^2+i^3b^3$ $=(a^3-3ab^2)+i(3a^2-b^3)$ $\Rightarrow x+iy=a(a^2-3b^2)+b(3a^2-b^2)$ Comparing real and imaginary parts $\frac{x}{a}=a^2-3b^2 \quad and \quad \frac{y}{b}=3a^2-b^2$ $\frac{x}{a}-\frac{y}{b}=a^2-3b^2-3a^2+b^2$ $=-2a^2-2b^2$ $=-2(a^2+b^2)$	1 1
29.	Let slope of required line be m Slope of given line is $\frac{1}{2}$ $tan45° = \frac{m-\frac{1}{2}}{1+\frac{m}{2}}$ $\frac{2m-1}{2+m}=\pm 1$ For positive sign, $m=3$ For negative sign $m=-\frac{1}{3}$ Equations of lines are $3x-y=7$ $x+3y=9$	1 1 ½ ½
30.	$\lim_{x\to 2}\frac{\sqrt{x^2+1}-\sqrt{5}}{x-2}$ $=\lim_{x\to 2}\frac{\sqrt{x^2+1}-\sqrt{5}}{x-2}\times\frac{\sqrt{x^2+1}+\sqrt{5}}{\sqrt{x^2+1}+\sqrt{5}}$ $=\lim_{x\to 2}\frac{x^2+1-5}{(x-2)(\sqrt{x^2+1}+\sqrt{5})}$ $=\lim_{x\to 2}\frac{x^2-4}{x-2)(\sqrt{x^2+1}+\sqrt{5})}$ $=\lim_{x\to 2}\frac{x+2}{\sqrt{x^2+1}+\sqrt{5}}=\frac{4}{2\sqrt{5}}$	1 1 1
31.	Let $f(x)=xcosx$ $\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)\cos(x+h)-xcos\ x}{h}$ $=\lim_{h\to 0}\frac{x\cos(x+h)-xcos\ x+hcos\ (x+h)}{h}=\lim_{h\to 0}\frac{x\cos(x+h)-xcos\ x}{h}+\lim_{h\to 0}\frac{hcos(x+h)}{h}$ $=x\lim_{h\to 0}\frac{\cos(x+h)-cos\ x}{h}+\cos x=x\lim_{h\to 0}\frac{-2\sin\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}+\cos x$ $=-xsin\ x+\cos x$	1 1 1

32.	LHS: $(1+\cos\frac{\pi}{8})(1+\cos\frac{3\pi}{8})(1+\cos(\pi-\frac{3\pi}{8}))(1+\cos(\pi-\frac{\pi}{8}))$ $=\left(1+\cos\frac{\pi}{8}\right)(1+\cos\frac{3\pi}{8})(1-\cos\frac{3\pi}{8})\left(1-\cos\frac{\pi}{8}\right)$ $=\left(1-cos^2\frac{\pi}{8}\right)\left(1-cos^2\frac{3\pi}{8}\right)$ $=sin^2\frac{\pi}{8}\,sin^2\frac{3\pi}{8}$ $=\frac{1}{4}\left(1-\cos\frac{\pi}{4}\right)(1-\cos\frac{3\pi}{4})$ $=\frac{1}{4}(1-\frac{1}{\sqrt{2}})(1+\frac{1}{\sqrt{2}})$ $=\frac{1}{8}$ OR LHS: $\frac{1-co\ 2x}{2}+\frac{1-\cos 2\left(x+\frac{\pi}{3}\right)}{2}+\frac{1-\cos 2\left(x-\frac{\pi}{3}\right)}{2}$ $=\frac{1}{2}(3-\left(\cos 2x+\cos(2x+\frac{2\pi}{3}\right)+\cos(2x-\frac{2\pi}{3}))$ $=\frac{1}{2}\left(3-\left(\cos 2+2\cos x\cos\frac{2\pi}{3}\right)\right)$ $=\frac{1}{2}\left(3-\left(\cos 2x-2\cos 2x\times\frac{1}{2}\right)\right)$ $=\frac{1}{2}(3-\underline{0})=\frac{3}{2}$	1 1 1 1 1 1 1 1 1 1

33.	Let x litres of 2% boric acid solution be added to the 8% solution Therefore total quantity of solution = x + 640 litres $\therefore 2\%\ of\ x+8\%\ of\ 640>4\%\ of\ (640+x)$ $and\ 2\%\ of\ x+8\%\ of\ 640<6\%\ of\ (640+x)$ $or\ \frac{2x}{100}+\frac{8}{100}\times 640>\frac{4}{100}\times(640+x)$ $\Rightarrow x<1280$ $\frac{2x}{100}+\frac{8}{100}\times 640<\frac{6}{100}\times(640+x)$ $\Rightarrow x>320$ $\therefore 320<x<1280$ We have to add more than 320 litres but less than 1280 litres of 2% solution to fulfil the given condition.	½ 1 ½ 1 1 ½ ½
34.	Let two positive numbers be $a\ and\ b$ $A=\frac{a+b}{2}\ ,G=\sqrt{ab}$ $\frac{A}{G}=\frac{a+b}{2\sqrt{ab}}$ Applying Componendo and Dividendo $\frac{A+G}{A-G}=\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}$	1

$\frac{A+G}{A-G} = \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2}$

$\frac{\sqrt{A+G}}{\sqrt{A-G}} = \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}$

Applying componendo and dividend — 1

$\frac{\sqrt{A+G}+\sqrt{A-G}}{\sqrt{A+G}-\sqrt{A-G}} = \frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}}$

$\frac{\sqrt{A+G}+\sqrt{A-G}}{\sqrt{A+G}-\sqrt{A-G}} = \frac{2\sqrt{a}}{2\sqrt{b}}$

Squaring both sides — 1

$\frac{2A+2\sqrt{A+G}\sqrt{A-G}}{2A-2\sqrt{A+G}\sqrt{A-G}} = \frac{a}{b}$

$\frac{A+\sqrt{A+G}\sqrt{A-G}}{A-\sqrt{A+G}\sqrt{A-G}} = \frac{a}{b}$

Let $a = k\left(A + \sqrt{A+G}\sqrt{A-G}\right)$

$b = k(\mathrm{A} - \sqrt{A+G}\sqrt{A-G})$ — 1

Put values of a and b in $G = \sqrt{ab}$

$G = \sqrt{k\left(A + \sqrt{A+G}\sqrt{A-G}\right)k\left(\mathrm{A} - \sqrt{A+G}\sqrt{A-G}\right)}$

$G = k\sqrt{A^2 - A^2 + G^2}$ — 1

We get $k = 1$

Therefore, numbers are $A \pm \sqrt{A+G}\sqrt{A-G})$

35.

Class	Freq.	Class Mark	$y_i = \frac{x_i - 105}{30}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0 – 30	2	15	−3	9	−6	18
30 – 60	3	45	−2	4	−6	12
60 – 90	5	75	−1	1	−5	5
90 – 120	10	105	0	0	0	0
120 – 150	3	135	1	1	3	3
150 – 180	5	165	2	4	10	20
180 – 210	2	195	3	9	6	18
Total	30				2	76

2

$\bar{x} = A + \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \times h$ — 1

$\bar{x} = 105 + \frac{2}{30} \times 30 = 107$

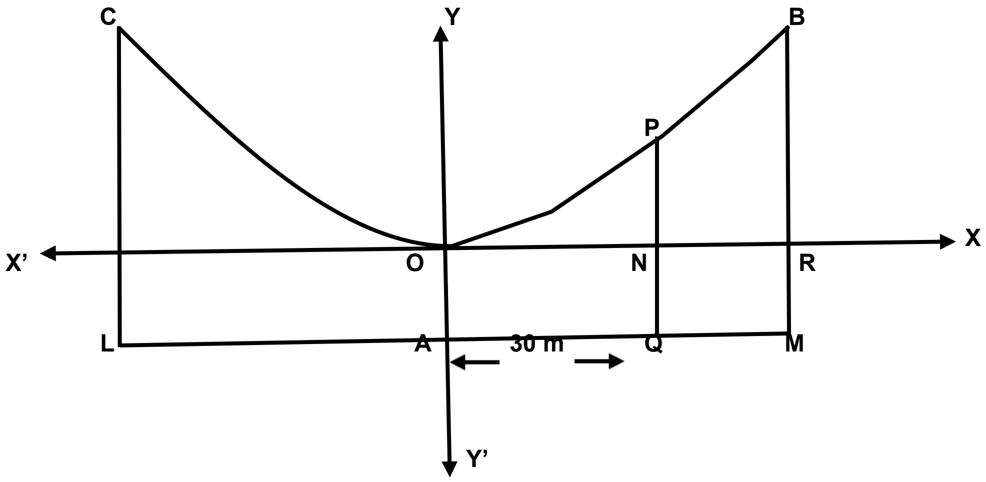
Var$=\sigma^2 = \frac{h^2}{N^2}\{N\sum_{i=1}^{n} f_i {y_i}^2 - (\sum_{i=1}^{n} f_i y_i)^2\}$ — 1

$\sigma^2 = \frac{30^2}{30^2}(30 \times 76 - 2^2)$ — 1

$\sigma^2 = 2280 - 4 = 2276$

OR

	Let other two numbers be x and y. Mean $(\bar{x}) = \frac{1+2+6+x+y}{5}$ $x + y = 13 - - - (1)$ Variance =8.24 $=\frac{1}{n^2}(n\sum_{i=1}^{n} x^2 - (\sum_{i=1}^{n} x)^2)$ $=\frac{1}{25}(5(1^2 + 2^2 + 6^2 + x^2 + y^2) - 22^2)$ $206 = 5(41 + x^2 + y^2) - 484$ $x^2 + y^2 = 97 - - - (2)$ From (1) and (2) and using $(x + y)^2 = x^2 + y^2 + 2xy$ $13^2 = 97 + 2xy$ $2xy = 72$ $(x - y)^2 = x^2 + y^2 - 2xy$ $(x - y)^2 = 97 - 72$ $x - y = \pm 5 - - - (3)$ On solving (1) and (3) Numbers are 4 and 9	½ ½ 1 ½ 11/2 1
36.	36.1 Since the team will not include no girl ∴ selection be made from boys only ∴ number of selections = ${}^7C_5 = 21$ 36.2 To select at least 3 girls, 3 or 4 girls can be selected ∴ required number of selections ${}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1 = 91$ 36.3 When at least 1 boy and 1 girl be selected Number of selections = ${}^4C_1 \times {}^7C_4 + {}^4C_2 \times {}^7C_3 + {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$ = 140 + 210 + 84 + 7 = 441	1 1 1 1
37	A : Event that the student opted NCC. B : Event that the student opted NSS. $\therefore P(A) = \frac{30}{60}, \quad P(B) = \frac{32}{60}, P(A \cap B) = \frac{24}{60}$ 37.1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= \frac{30}{60} + \frac{32}{60} - \frac{24}{60} = \frac{19}{30}$ 37.2 P(student opted neither NCC nor NSS) $= P(A' \cap B') = P((A \cup B)')$ $= 1 - P(A \cup B)$ $= 1 - \frac{19}{30} == \frac{11}{30}$ 37.3 P(student opted NSS but not NCC) $= P(B - A)$ $= P(B) - P(A \cap B) = \frac{32}{60} - \frac{24}{60} = \frac{2}{15}$ — — —	1 1 1 1

38		
	38.1 Let equation of main cable be $x^2 = 4ay$ OA = NQ = BM = 5 m BR = BM – RM = 30 – 5 = 25 m Hence coordinates of B (100, 25) B lies on the parabola $100^2 = 4a \times 25$ $a = 100$ Therefore equation of main cable $x^2 = 400y$	1 1
	38.2 Let PN = h Therefore coordinates of P (30, h) P(30, h) lies on parabola $\therefore 30^2 = 400h$ $h = \frac{9}{4}$=2.25 $\therefore$ PQ = 2.25 + 5 = 7.25 m Ans: Length of supporting cable 30 m from the midway is 7.25 m	 1 1

BLUE PRINT SAMPLE PAPER 5

SNO	UNIT/TOPIC	SECTION'A' (OBJECTIVETYPE)		SECTION 'B' (VSA)	SECTION'C' (SA)	SECTION' D' (LA)	SECTION' E' (CBQ)	TOTAL	UNITWISE MARKING
		MCQ (1MarkEach)	A&R (1MarkEach)	(2Marks Each)	(3Marks Each)	(5Marks Each)	(4Marks Each)		
1	SETS	1(1)				1(5)	1(4)	3(10)	23 M
2	RELATIONSANDFUNCTIONS	2(2)		2(2)				4(6)	
3	TRIGONOMETRICFUNCTIONS	1(1)	1(1)			1(5)		3(7)	
4	COMPLEXNUMBERS	1(1)		1(2)				3(3)	25M
5	LINEAR INEQUALITIES	1(1)			1(3)			2(4)	
6	PERMUTATIONS AND COMBINATIONS				1(3)		1(4)	2(7)	
7	BINOMIALTHEOREM	2(2)		1(2)				3(4)	
8	SEQUENCEAND SERIES	2(2)				1(5)		3(7)	
9	STRAIGHTLINES	1(1)			1(3)			2(4)	12 M
10	CONIC SECTION	2(2)			1(3)			3(5)	
11	3DIMENSIONALGEOMETRY				1(3)			1(3)	
12	LIMITSANDDERIVATIVES	2(2)	1(1)	1(2)	1(3)			5(8)	08M
13	STATISTICS	1(1)				1(5)		2(6)	12M
14	PROBABILITY	2(2)					1(4)	3(6)	
	TOTAL	18(18)	2(2)	5(10)	6(18)	4(20)	3(12)	36(80)	80

SAMPLE PAPER - 5

Time: 3 hours **Maximum marks 80**

General Instructions:

a) The question paper contains- five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
b) Section A has 18 MCQ's and 02 Assertion- Reason based questions of 1 mark each.
c) Section B has 5 very short Answer (VSA)-type questions of 2 mark each.
d) Section C has 6 Short Answer (SA)-type questions of 3 marks each.
e) Section D has 4 Long Answer (LA)-type questions of 5 mark each.
f) Section E has 3 source based/ case based/ passage/ integrated units of assessment of 4 marks each with sub-parts.

Section-A

1.	If Let n(A)=m, n(B)=n, then the total number of non-empty relations that can be defined from A to B is: (A)m^n (B)n^m- 1 (C)2^{mn} -1 (D)None of these	1
2.	A line passes through (2,2) and is perpendicular to the $3x + y = 3$.Find its y-intercept. (A)4/3 (B) 1 (C) 1/3 (D) 2/3	1
3.	**The domain of the function $f(x) = \frac{x}{x^2+3x+2}$ is ___________** (a) [-2, -1] (b) R – {1, 2} (c) R – {-1, -2} (d) R – {2}	1
4.	If |x+3|≥10 then (a) x ∈ (-13,7] (b) x ∈(-10,7] (c) x∈ (-∞, -7) ∪ $[13, \infty$) (d) x ∈ (-∞, $-13]$ ∪ $[7, \infty$)	1
5.	One card is drawn from a pack of 52 cards. The probability that it is the card of king or spade is______________ (a)$\frac{4}{13}$ (b$\frac{1}{13}$ (c)$\frac{17}{52}$ (d) $\frac{1}{26}$	1
6.	The standard deviation of the observation 5,5,5,5,5 is- (a) 0 (b) 5 (c)25 (d) 20	1
7.	If 8x+i(2x-y) = 3-8i and x , y ∈ R , then values of x and y are (a) x=3/8,y=35/4 (b) x=-3/8,y=35/4 (c) x=3/8,y=-35/4 (d) x=-3/8,y=-35/4	1
8.	If f(x) = $x^3 - \frac{1}{x^3}$then the value of f(x)+ f($\frac{1}{x}$) is equal to (a) $\frac{2}{x^3}$ (b) $2x^3$ (c) 1 (d)0	
9.	If z=$\frac{1+7i}{(2-i)^2}$ then these $\|z\|$ equals to ___________ (a)$\sqrt{2}$ (b)$\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) 2	1
10.	The total number of terms in the expansion of $(x + a)^{51}$-$(x - a)^{51}$ (a) 23 (b) 24 (c) 26 (d) 28	1
11	A pair of die is rolled , the probability of getting a total of 6: (a) 4/36 (b) 5/36 (c) 5/18 (d) 1/18	1
12	The third term of the expansion $(1 - 2x)^5$ is a)$-40x^2$ b) $40x^2$ c) 40x d) none of these	1
13	**Which of the following is an example of a geometric sequence?** (a) 1,2,3,4 (b) 1,2,4,8 (c) 3,5,7,9 (d)9,20,21,28	1
14	$\lim_{x\to a} \frac{x^7-a^7}{x-a} = 7$ the the value of a is : (a) 1 (b) -1 (c) 0 (d)±1	1

15	What is the angle through which a pendulum swings if its length is 75 cm and the tip describe an arc of length 10cm (a) 2/15 radian (b) 3/15 radian (c) 1/15 radian (d) 4/15radian	1
16	The length of transverse axis hyperbola is the distance between the ….. (a) Two vertices (b) two foci (c) vertex and the origin (d) focus and the vertex	1
17	The derivative of 1/(1-x) w.r.t. x is : (a) $1/(1-x)^2$ (b) $-1/(1-x)^2$ (c) $2/(1-x)^2$ (d)none of these	1
18	Find the equations of the directrix & the axis of the parabola ⇒$3x^2$=8y (a) 3y-4=0, x=0 (c) 3x-2=0, x=0 (b) 3y-4x=0 (d) none of these	1
	Q. No 19 and 20 based on statement of Assertion (A) is followed by a statement of Reason (R). choose the correct answer out of the following choices. A) Both A and R are true and statement R is the correct explanation for A B) Both A and R are true and but R is the not correct explanation for A. C) A is true but R is false D) A is false but R is true	
19	**Assertion(A)** :If the third term of a G.P is 4,then the product of its first five term is 4^5. **Reason (R)** : The product of first five terms of a G.P is given as $a(ar)(ar^2)(ar^3)(ar^4)$	1
20	**Assertion (A)** : The value of sin(-690°) cos(-300°) + cos (-750°)sin (-240°) = 1 **Reason (R)** : The value of sin and cos is negative in third and fourth quadrant respectively.	1
	SECTION:B	
21	Let A={1,2} and B={3,4} Write A×B.How many subsets will A×B Have? **OR** Let $A = \{x: x = 6n, n \in N\}$, $B = \{x: x = 9n, n \in N\}$.find $A \cap B$	2
22	Prove that $\sum_{r=0}^{n} 3^r\, {}^nCr$ = 4^n	2
23	Express $(1-2i)^{-3}$ in the standard form. Also find its conjugate. **OR** If $\frac{a-ib}{a+ib} = \frac{1+i}{1-i}$, then find the value a+b.	2
24	Write the value of $\lim_{x\to 0} \frac{\sin 5x}{\sin 7x}$	2
25	Draw the graph of f(x) = I x-2I.	2
	SECTION:C	
26	Find the centroid of a triangle, mid-points of whose sides are D(1,2,–3), E(3,0,1) F(−1,1,−4)	3
27	Find the derivative of $\frac{4x+5sinx}{3x+7cosx}$	3
28	Solve the following system of inequalities: $2(2x+3)-10<6(x-2), \frac{2x-3}{4}+6 \geq 4+\frac{4x}{3}$	3

29. Find the equations of the straight lines which pass through the origin and trisect the portion of the straight line $2x + 3y = 6$ which intercepted between the axes. **(3)**

OR

Points A and B have coordinates $(3,2), (7,6)$ respectively. Find

(i) The equation of right bisector of the segment AB.

(ii) The value of p if $(-2, p)$ lies on it.

30. Find the point in XY-plane Which is equidistant from the points $(2,0,3), (0,3,2)$ and $(0,0,1)$. **(3)**

31. A committee of 7 has to be formed from 9 boys and 4 girls. in how many ways this can be done when the committee consists of at least 3 girls. **(3)**

i. exactly 3 girls?
ii. at least 3 girls?
iii. almost 3 gilrs?

OR

If sin A + sin B=p , cos A + cos B =q , find cos(A-B)

SECTION:D

32. Prove that: $cos^2 x + cos^2\left(x + \frac{\pi}{3}\right) + cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$ **(5)**

OR

If cos$(\alpha + \beta)$ =$\frac{4}{5}$ and sin$(\alpha - \beta)$ =$\frac{5}{13}$ and 0$\leq \alpha, \beta \leq \frac{\pi}{4}$then find tan2 α .

33. For sets A, B and C using properties of sets , prove that: **(5)**

(i) $A - (B \cup C) = (A - B) \cap (A - C)$

(ii) $A - (B - C) = (A - B) \cup (A \cap C)$

34. Find three numbers in G.P. whose sum is 13 and the sum of whose square is 91. **(5)**

OR

If S be the sum ,P be the product and R be the sum of reciprocals of n terms of a G.P. then prove that $(\frac{S}{R})^n$ =P^2

35. Find the variance and standard deviation for the following distribution: **(5)**

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
frequency	3	7	12	15	8	3	2

SECTION:E

[This section comprises of 3 case- study/passage based questions of 4 marks each

36	A Mathematics teacher of class XI write three sets A, B and C are such that A= { x∈ N | x is an even number , x ≤10 } B = { y∈ N | y is prime number , y≤ 10 } C = {z ∈ N | z is divisible by 3 , z ≤ 10 } Based on the above information answer the following  i) Among the set A, B and C which set contain maximum number of elements ii) Write the number of element present in (A U C) iii) Write the set B in tabular form. iv) Write the set(A-B) in tabular form.	1 1 1 1
37	Each zone is allotted a specific non-zero digit which is to be used as first digit of alltelephone numbers of that zone.Based on the above information, answer the following questions. Q . (i) How many different telephone numbers are there in each zone, if the digit on first places not used again? Q. (ii) How many different telephone numbers are there in the city, if there is no restriction? Q (iii) How many different telephone numbers are there in each zone with all digits distinct? 	1 1 2

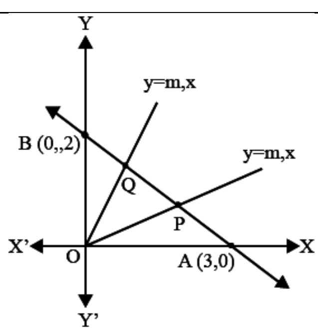
38	Some friends went on a picnic and after moving round and having light snacks, they sat down and started playing with a pair of dice each time they throw the dice they asked other friends about the chance of getting the event which they have thought. You also try to help them out.  (i) Find the probability of getting a number less than 3 on first dice. (ii) Find the probability of getting an even total of numbers on two dice. (iii) Find the probability of getting a number multiple of 3 on both dice. **OR** (III) Find the probability of getting a doublet.	 1 1 2

MARKING SCHEME [SAMPLE PAPER 5]

Q.No.	ANSWERS	STEP MARKS
1	(C)	1
2	(A)	1
3	(C)	1
4	(D)	1
5	(A)	1
6	(A)	1
7	(B)	1
8	(D)	1
9	(D)	1
10	(C)	1
11	(B)	1
12	(B)	1
13	(B)	1
14	(D)	1
15	(A)	1
16	(A)	1
17	(A)	1
18	(D)	1
19	(A)	1
20	(C)	1

Section-B

21	A×B=$\{(1,3),(1,4),(2,3),(2,4)\}$ Since, No. Of subsets of A×B is 2^4=16 **OR** $A=\{6,12,18,24,\ldots\}$ and $B=\{9,18,27,36,\ldots\}$ $A\cap B=\{18,36,54,\ldots\}$	(1) (1) (1) (1)
22	$3^0C(n,0)+3^1C(n,1)+3^2C(n,2)+\cdots+3^nC(n,n)$=$(1+3)^n=4^n$	(1+1)
23	=$1/(1-2i)^3$=1/(-11+2i) ……=$\frac{-11}{125}-\frac{2}{125}$i Conjugate is $\frac{-11}{125}+\frac{2}{125}$i **OR** By rationalising both sides $\frac{a^2-b^2}{a^2+b^2}-\frac{2ab}{a^2+b^2}i$ = 0+i ⇒ $a^2-b^2=0 \rightarrow a=\pm b$ But $-\frac{2ab}{a^2+b^2}$=1 $=> a=-b =>$a+b=0	(1) (1/2) (1/2) (1) (1/2) (1/2)
24	$\lim\limits_{x\to 0}\frac{\sin 5x}{\sin 7x}$ $=\lim\limits_{x\to 0}\frac{sin5x}{5x}\times 5x\times\frac{7x}{\sin 7x}\times\frac{1}{7x}$ $\lim\limits_{x\to 0}\frac{\sin 5x}{\sin 7x}=\frac{5}{7}\left(\lim\limits_{x\to 0}\frac{\sin x}{x}=1\right)$	(1) (1)
25	f(x)= $\{x-2,\ x\geq 1\ 2-x,\ x<1$ for Correct graph	(1) (1)

26	$A(x_1, y_1, z_1), B(x_2, y_2, z_2) and\ C(x_3, y_3, z_3)$ D is the mid-point of BC $\frac{x_2+x_3}{2} = 1, \frac{y_2+y_3}{2} = 2, \frac{z_2+z_3}{2} = -3$ $x_2 + x_3 = 2, y_2 + y_3 = 4, z_2 + z_3 = -6$ --------(i) E is the mid-point of CA So, $x_1 + x_3 = 6, y_1 + y_3 = 0, z_1 + z_3 = 2$ -------(ii) F is the mid-point of AB So, $x_1 + x_2 = -2, y_1 + y_2 = 2, z_1 + z_2 = -8$ -------(iii) By adding the three equations , we get $x_1 + x_2 + x_3 = 3, y_1 + y_2 + y_2 = 3, z_1 + z_2 + z_3 = -6$ Coordinate of centroid(1,1,−2)	1 1 1
27	The given function is f(x) = (4x + 5 sin x) / (3x + 7 cos x) Its derivative using the is, $\frac{d}{dx}$(f(x)) = $\frac{(3x + 7\cos x)\frac{d}{dx}(4x + 5\sin x) - (4x + 5\sin x)\frac{d}{dx}(3x + 7\cos x)}{(3x + 7\cos x)2}$ = $\frac{35 + 15x\cos x + 28x\sin x - 15\sin x}{(3x + 7\cos x)2}$ [As $\cos^2 x + \sin^2 x = 1$]	1 2
28	solving 1st inequality x>4 and 2nd equality x≤3/2 (1) Solution impossible (1)	(1) (1) (1)
29	 A(3,0) and B(0,2) The line $2x + 3y = 6$ trisect by points P and Q Therefore , AP=PQ=QB P(2,2,3) and Q(1,4,3)	1

	Clearly ,P and Q lie on line $y = m_1 x$ and $y = m_2 x$ respectively. $y = \frac{1}{3}x$ and $y = \frac{4}{3}x$ So, $x - 3y = 0$ and $4x - 3y = 0$ **OR** (I) the mid point of AB=(5,4) Line passes through mid-point of AB is perpendicular to AB So,(slope of AB)(slope of line passes through mid point of AB)=-1 slope of AB=1, slope of line passes through mid point of AB=-1 equation of right bisector of AB is $y - 4 = -1(x - 5)$ $x + y = 9$ (ii) $(-2, p)$ lies on $x + y = 9$ There fore $p = 11$	1 1 1 1 1
30	Let the Point in XY Plane (x, y,o) is equidistant from A, B & C PB=PC ⇒ y= 2. PA=PC ⇒ x= 3. The point is A(3,2,0)	(1/2) (1) (1) (1/2)
31	No. of ways of selection=C(9,4)XC(4,3)=504 No. of ways of selection=C(4,3)XC(9,4)+C(4,4)XC(9,3) =504+84=588 We have to select at most 3 girls. So, number of ways of selection =C(4,0)XC(9,7)+C(4,1)XC(9,6)+C(4,2)XC(9,5)+C(4,3)XC(9,4)+C(4,4)XC(9,3)= 36+336+756+504=1632	1 1 1

32.	= ½ $(2\cos^2 x+2\cos^2(x+\pi/3)+2\cos^2(x-\pi/3))$ = $(1/2)(1+\cos 2x + 1+ \cos(2x+2\pi/3) + 1+ \cos(2x-2\pi/3))$ =$(1/2)(3+ \cos 2x + \cos(2x+2\pi/3) + \cos(2x-2\pi/3))$ =$(1/2)(3+\cos 2x+2\cos 2x.\cos 2\pi/3)$ = 3/2 **OR** $tan\ tan(A+B)=\frac{3}{4}, tan\ tan(A-B)=\frac{5}{12}$ $tan\ tan2A = tan\ \tan((A+B)+(A-B))$ $tan\ tan2A=\frac{tan\ tan(A+B)+tan\ \tan(A-B)}{1-tan\ tan(A+B)\ tan\ \tan(A-B)}$ $=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}$ $=\frac{56}{33}$	(1) (1) (1) (1) (1) (1) $(1\frac{1}{2})$ (1) $(1\frac{1}{2})$
33.	$(i)\ A-(B\cup C)=A\cap(B\cup C)'$ $=A\cap(B'\cap C')=(A\cap B')\cap(A\cap C')$ $=(A-B)\cap(A-C)$ (II) $A-(B-C)=A-(B\cap C')$ $=A\cap(B\cap C')'=A\cap(B'\cup C)=(A\cap B')\cup(A\cap C)$ $=(A-B)\cup(A\cap C)$	(1) (1) (1/2) (1) (1) (1/2)

34	**Numbers** in G.P are $a, ar\ and\ ar^2$ Given sum =13 $a + ar + ar^2 = 13$ ---------(i) $a^2 + a^2r^2 + a^2r^4 = 91$ $a^2(1 + r^2 + r^4) = 91$ $a^2(1 - r + r^2)a(1 + r + r^2) = 91$ $a(1 - r + r^2)a(1 + r + r^2) = 91$ $a(1 - r + r^2) = \frac{91}{13} = 7$ ------(ii) By Dividing (i) by (ii), we get $\frac{a+ar+ar^2}{a(1-r+r^2)} = \frac{13}{7}$ $7(1 + r + r^2) = 13(1 - r + r^2)$ $6r^2 - 20r + 6 = 0$ $(3r - 1)(r - 3) = 0$ $r = 3\ or\ r = \frac{1}{3}$ When, $r = 3, a = 1$ When, $r = \frac{1}{3}$, $a = 9$ So, the numbers are 1,3 and 9 or 9,3 and 1 **OR** S =Sum=$a + ar + ar^2 + \cdots n\ terms$ = $\frac{a(r^n-1)}{(r-1)}$ P= Product =$a.ar.ar^2 \ldots n\ terms$ = $a^n r^{\frac{n(n-1)}{2}}$ R=Reciprocal=$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \cdots n\ terms = \frac{\frac{1}{a}(1-(\frac{1}{r})^n)}{\left(1-\frac{1}{r}\right)}$ = = $\frac{1}{ar^{n-1}}\{\frac{r^n-1}{r-1}\}$ $(S/R)^n = (a^2r^{n-1})^n$ =.......=.P^2	 (1) (1) (1) (1) (1) (1) (1) (1) (2)

35		(3)

Class	Frequency (f_i)	Mid point (x_i)	$f_i x_i$	$(x_i - \underline{x})^2$	$f_i (x_i - \underline{x})^2$
30-40	3	35	105	729	2187
40-50	7	45	315	289	2023
50-60	12	55	660	49	588
60-70	15	65	975	9	135
70-80	8	75	600	169	1352
80-90	3	85	255	529	1587
90-100	2	95	190	1089	2178
	50		3100		10050

	Mean = 62	(½)
	Standard Deviation = σ = $\sqrt{201}$ = 14.18	(1)
	Variance = $\sigma^2 = 201$	(½)

SECTION-E

36	i) A set contain maximum number of element ii) n (A U C) =7 iii) B ={ 2,3,5,7 }. iv) (A-B) = { 4,6,8,10 }	(1) (1) (1) (1)
37	i) 10×9^6 different telephone numbers are their in each zone,if the digit on the first place is not used again. If zero is not to be used on the first place then number of ways are 9^7. different telephone numbers are there in the city, if there is no restriction. ii) Total number of ways are 604,800.	(1) (1) (2)
38	(1,1)(1,2)(1,3)(1,4)(1,5)(1,6) (2,1)(2,2)(2,3)(2,4)(2,5)(2,6) (3,1)(3,2)(3,3)(3,4)(3,5)(3,6) (4,1)(4,2)(4,3)(4,4)(4,5)(4,6) (5,1)(5,2)(5,3)(5,4)(5,5)(5,6) (6,1)(6,2)(6,3)(6,4)(6,5)(6,6) n(E) – 36 (I) Events for less than 3 on the first dice (1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,2)(2,3)(2,4)(2,5)(2,6) n (E)-12 Probability - 12/36. =1/3 (II)n(E) (1,1)(1,3)(1,5)(2,2)(2,4)(2,6)(3,1)(3,3)(3,5)(4,2)(4,4)(4,6)(5,1)(5,3)(5,5)(6,2)(6,4)(6,6) n(E)- 18 Probability - 18/36 = ½ (iii) 3,3)(3,6)(6,3)(6,6) n(E)- 4 probability - 4/36 = 1/9 Or n(E)- 6 probability =6/36=1/6	1 1 2

तत् त्वं पूषन् अपावृणु

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