केन्द्रीय विद्यालय संगठन, नई दिल्ली

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तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

अध्ययन सामग्री / STUDY MATERIAL

कक्षा / Class : XII

विषय/ Subject : गणित / MATHS

संकलन द्वारा : राजीव रंजन , प्रशिक्षण सहायक, गणित

Compiled by: Rajiv Ranjan, TA, Maths

स्थल : शिक्षा एवं प्रशिक्षण का आंचलिक संस्थान , चंडीगढ़

Venue- Zonal Institute of Education and Research, Chandigarh

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MATHEMATICS (Code No. 041)
CLASS XII (2023-24)
COURSE STRUCTURE

One Paper **Max Marks: 80**

No.	Units	Marks
I	Relations and Functions	08
II	Algebra	10
III	Calculus	35
IV	Vectors and Three - Dimensional Geometry	14
V	Linear Programming	05
VI	Probability	08
	Total	**80**
	Internal Assessment	**20**

Unit-I: Relations and Functions

1. Relations and Functions
Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. Inverse Trigonometric Functions
Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

Unit-II: Algebra

1. Matrices
Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. On-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants
Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. Continuity and Differentiability
Continuity and differentiability, chain rule, derivative of inverse trigonometric functions, $like$ $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$, derivative of implicit functions. Concept of exponential and logarithmic functions. Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives
Applications of derivatives: rate of change of bodies, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3.Integrals Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, only simple integrals of the type $\int \frac{dx}{x^2 \pm a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{1}{\sqrt{a^2 - x^2}} dx$, $\int \frac{dx}{ax^2 + bx + c}$, $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$, $\int \frac{(px + q)dx}{ax^2 + bx + c}$, $\int \frac{(px + q)dx}{\sqrt{ax^2 + bx + c}}$, $\int \sqrt{a^2 \pm x^2} dx$, $\int \sqrt{x^2 - a^2} dx$, $\int \sqrt{ax^2 + bx + c} dx$ to be evaluated.

Fundamental Theorem of Calculus (without proof).Basic properties of definite integrals and evaluation ofdefinite integrals.

4. Applications of the Integrals

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)

5. Differential Equations

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx}$ + py = q, where p and q are functions of x or constants.

$\frac{dx}{dy}$ + px = q, where p and q are functions of y or constants.

Unit-IV: Vectors and Three-Dimensional Geometry

1.Vectors

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three - dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

Unit-V: Linear Programming

1.Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1.Probability 30 Periods Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean of random variable

IMPORTANT TRIGONOMETRIC RESULTS & SUBSTITUTIONS

** **Formulae for t-ratios of Allied Angles :**

All T-ratio changes in $\frac{\pi}{2} \pm \theta$ and $\frac{3\pi}{2} \pm \theta$ while remains unchanged in $\pi \pm \theta$ and $2\pi \pm \theta$.

$\sin\left(\frac{\pi}{2} \pm \theta\right) = \cos\theta$ $\sin\left(\frac{3\pi}{2} \pm \theta\right) = = \cos\theta \ \frac{\pi}{2}$

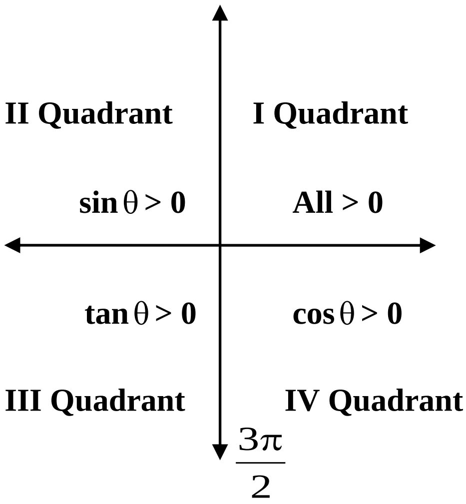
$\cos\left(\frac{\pi}{2} \pm \theta\right) = \mp\sin\theta$ $\cos\left(\frac{3\pi}{2} \pm \theta\right) = \pm\sin\theta$

$\tan\left(\frac{\pi}{2} \pm \theta\right) = \mp\cot\theta$ $\tan\left(\frac{3\pi}{2} \pm \theta\right) = \mp\cot\theta$

$\sin(\pi \pm \theta) = \mp\sin\theta$ $\sin(2\pi \pm \theta) = \pm\sin\theta \ \pi$

$\cos(\pi \pm \theta) = = \cos\theta$ $\cos(2\pi \pm \theta) = \cos\theta$

$\tan(\pi \pm \theta) = \pm\tan\theta$ $\tan(2\pi \pm \theta) = \pm\tan\theta$



** **Sum and Difference formulae :**

sin(A + B) = sin A cos B + cos A sin B
sin(A – B) = sin A cos B – cos A sin B
cos(A + B) = cos A cos B – sin A sin B
cos(A – B) = cos A cos B + sin A sin B

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$, $\tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A}$,

$\tan\left(\frac{\pi}{4} - A\right) = \frac{1 - \tan A}{1 + \tan A}$, $\cot(A + B) = \frac{\cot A.\cot B - 1}{\cot B + \cot A}$ $\cot(A - B) = \frac{\cot A.\cot B + 1}{\cot B - \cot A}$

$\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

$\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

****Formulae for the transformation of a product of two circular functions into algebraic sum of two circular functions and vice-versa.**

2 sinA cos B = sin (A + B) + sin(A – B)
2 cosA sin B = sin (A + B) – sin(A – B)
2 cosA cos B = cos (A + B) + cos(A – B)
2 sinA sin B = cos (A – B) – cos(A + B)

$\sin C + \sin D = 2\sin\frac{C + D}{2}\cos\frac{C - D}{2}$, $\sin C - \sin D = 2\cos\frac{C + D}{2}\sin\frac{C - D}{2}$.

$\cos C + \cos D = 2\cos\frac{C + D}{2}\cos\frac{C - D}{2}$, $\cos C - \cos D = -2\sin\frac{C + D}{2}\sin\frac{C - D}{2}$.

** **Formulae for t-ratios of múltiple and sub-múltiple angles** :

$\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1 + \tan^2 A}$.

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \frac{1 - \tan^2 A}{1 + \tan^2 A}$

$1 + \cos 2A = 2\cos^2 A$, $1 - \cos 2A = 2\sin^2 A$, $1 + \cos A = 2\cos^2\frac{A}{2}$ $1 - \cos A = 2\sin^2\frac{A}{2}$

$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$, $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$.

$\sin 3A = 3\sin A - 4\sin^3 A$, $\cos 3A = 4\cos^3 A - 3\cos A$

$\sin 15^o = \cos 75^o = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$. & $\cos 15^o = \sin 75^o = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$,

$\tan 15^o = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3} = \cot 75^o$ & $\tan 75^o = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} = 2 + \sqrt{3} = \cot 15^o$.

$\sin 18^o = \dfrac{\sqrt{5}-1}{4} = \cos 72^o$ & $\cos 36^o = \dfrac{\sqrt{5}+1}{4} = \sin 54^o$.

$\sin 36^o = \dfrac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54^o$ & $\cos 18^o = \dfrac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72^o$.

$\tan\left(22\frac{1}{2}\right)^o = \sqrt{2} - 1 = \cot\ 67\frac{1}{2}^o$ & $\tan\left(67\frac{1}{2}\right)^o = \sqrt{2} + 1 = \cot\left(22\frac{1}{2}\right)^o$.

** **Properties of Triangles** : In any Δ ABC,

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ [Sine Formula]

$\cos A = \dfrac{b^2+c^2-a^2}{2bc}$, $\cos B = \dfrac{c^2+a^2-b^2}{2ca}$, $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$.

** **Projection Formulae** : $a = b\cos C + c\cos B$, $b = c\cos A + a\cos C$, $c = a\cos B + b\cos A$

** **Some important trigonometric substitutions :**

$\sqrt{a^2+x^2}$	Put $x = a\tan\theta$ or $a\cot\theta$
$\sqrt{x^2-a^2}$	Put $x = a\sec\theta$ or $a\,\mathrm{cosec}\,\theta$
$\sqrt{a+x}$ or $\sqrt{a-x}$ or both	Put $x = a\cos 2\theta$
$\sqrt{a^n+x^n}$ or $\sqrt{a^n-x^n}$ or both	Put $x^n = a^n\cos 2\theta$
$\sqrt{1+\sin 2\theta}$	$=\sin\theta+\cos\theta$
$\sqrt{1-\sin 2\theta}$	$=\cos\theta-\sin\theta,\ 0<\theta<\frac{\pi}{4}$
	$=\sin\theta-\cos\theta,\ \frac{\pi}{4}<\theta<\frac{\pi}{2}$

****General solutions:**

$*\cos\theta = 0 \Rightarrow \theta = n\pi,\ n \in Z$

$*\sin\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2},\ n \in Z$

$*\tan\theta = 0 \Rightarrow \theta = n\pi,\ n \in Z$

$*\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha,\ n \in Z$

$*\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha,\ n \in Z$

$*\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha,\ n \in Z$

RELATIONS AND FUNCTIONS

SOME IMPORTANT RESULTS/CONCEPTS

** **Relation :** A relation R from a non-empty set A to a non-empty set B is a subset of $A \times B$.

A **relation R in a set A is called

(i) **Reflexive**, if $(a, a) \in R$, for every $a \in A$,

(ii) **Symmetric**, if $(a, b) \in R$ then $(b, a) \in R$,

(iii)**Transitive**, if $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$.

** Equivalence Relation :R is equivalence if it is reflexive, symmetric and transitive.

** **Function** :A relation **f** : $A \to B$ is said to be a function if every element of A is correlated to unique element in B.

* A is domain

* B is codomain

* For any x element $x \in A$, function **f** correlates it to an element in B, which is denoted by f(x) and is called image of x under **f** . Again if y= f(x), then x is called as pre- image of y.

* Range = $\{f(x) \mid x \in A\}$. Range $\subseteq$ Codomain

* The largest possible domain of a function is called domain of definition.

Composite function :Let two functions be defined as **f** : $A \to B$ and **g** : $B \to C$. Then we can define a function **gof**: $A \to C$ is called thecomposite function of**f** and **g.**

** **Different type of functions** : Let **f** : $A \to B$ be a function.

*__f__ is **one to one (injective) mapping**, if any two different elements in A is always correlated to different elements in B, i.e. $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ or $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

*__f__ is **many one mapping,** if $\exists$ at least two elements in Asuch that their images are same.

*__f__ is **onto mapping** (subjective), if each element in B is having at least one preimage.

*__f__ is**into mapping** if range $\subseteq$ codomain.

***f** is **bijective mapping** if it is both one to one and onto.

RESULTS :

1. A relation R in a set A, if n(A)= n then,

a) Number of relation from A to A = 2^{n^2}

b) Number of reflexive relation from A to A = 2^{n^2-n}

c) Number of symmetric relation from A to A = $2^{\frac{n(n+1)}{2}}$

d) Number of relation from A to A which are not symmetric = $2^{(n^2-n)/2}$

2. Let a function f : X → Y where n (X) = n & n (Y) = m.

a) Total number of functions = m^n= (no of elements in co-domain)$^{\text{no. of elements in domain}}$

b) Total number of one-one functions=$\begin{cases} {}^mc_n n!, & \text{if } m \geq n \\ 0, & \text{if } m < n \end{cases}$

c) Total number of many one functions=$\begin{cases} m^n - {}^mc_n n!, & \text{if } m \geq n \\ m^n, & \text{if } m < n \end{cases}$

d) Total number of onto

functions$\begin{cases} m^n - {}^mc_1(m-1)^n + {}^mc_2(m-2)^n - {}^mc_3(m-3)^n + \ldots, & \text{if } m < n \\ m!, & \text{if } m = n \\ 0, & \text{if } m > n \end{cases}$ =

$\begin{cases} m^n - {}^mc_1(m-1)^n + {}^mc_2(m-2)^n - {}^mc_3(m-3)^n + \ldots, & \text{if } m < n \\ m^n, & \text{if } m > n \end{cases}$

e) Total number of into functions=$\begin{cases} {}^mc_1(m-1)^n - {}^mc_2(m-2)^n + {}^mc_3(m-3)^n - \ldots, & \text{if } m \leq n \\ m^n, & \text{if } m > n \end{cases}$

f) Total number of one-one and onto functions= m!

g) Total number of constant functions = m

h) Total number of onto functions from the set { 1, 2, 3, ….,n} = n!

II) Some illustrations/examples:

(i) MCQ

1. Let A = { 1, 2, 3, ….,n}and B= {a, b}. then the number of surjections from A to B is

(a) ${}^nP_2$ (b) 2^n-2 (c)2^n-1 (d) none of these

2. Let f:R → R defined by f(x) = 1+ x^2. Choose the correct answer

(a) both one -one and onto (b)one-one but not onto

(c) onto but not one-one (d) Neither one- one nor onto

3. The maximum number of equivalence relation on the set A ={ 2, 4, 6} is/are

(a) 1 (b) 2 (c) 3 (d) 5

4. Let the relation R in the set A = { x∊ Z: 0 ≤ x ≤ 12}, given by R = { (a,b): |a-b| is multiple of 4 }. Then the equivalence class of 1is

(a) {1, 5, 9} (b) { 0, 1, 2, 5} (c)∅ (d) A

Ans . 1 (b), 2 (d) , 3 (c) , 4(a)

(ii) Case based study:

An organization conducted bike race under 2 different categories-boys and girls. Totally there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let B = {b_1,b_2,b_3} G={g_1,g_2} where B represents the set of boys selected and G the set of girls who were selected for the final race. Ravi decides to explore these sets for various types of relations and functions.

1. Ravi wishes to form all the relations possible from B to G. How many such relations are possible?

2. Let $R: B \rightarrow G$ be defined by R = { (b_1,g_1), (b_2,g_2),(b_3,g_1)}. Check R is/are injective/ surjective/bijective.

3. Ravi wants to find the number of injective functions from B to G. How many numbers of injective functions are possible?

Ans . 1. 2^6, 2. Surjective, 3. 0

(iii) Short answer type:

1. In each of the following cases, state whether the function is one-one , onto or bijective. Justify your answer:

(i) $f: R \rightarrow R$ defined by $f(x) = 3 - 4x$

(ii) $f: R \rightarrow R$ defined by $f(x) = 1 + x^2$

Sol. (i)$f: R \rightarrow R$ defined by $f(x) = 3 - 4x$

Let $x_1, x_2 \in R$ be such that $f(x_1) = f(x_2)$

⇨ $3 - 4x_1 = 3 - 4x_2 \Rightarrow -4x_1 = -4x_2 \Rightarrow x_1 = x_2$

$\therefore$ f is one − one

Let $y \in R$ be any real number.

$\therefore f(x) = y \Rightarrow 3 - 4x = y$

$\Rightarrow x = \frac{3-y}{4}$

$\therefore f\left(\frac{3-y}{4}\right) = y$

$\therefore$ Corresponding to every element $y \in R$, there exists $\left(\frac{3-y}{4}\right)$ such that $f\left(\frac{3-y}{4}\right) = y$

$\Rightarrow$ f is onto.

ii) $\mathbf{f: R \to R}$ defined by $\mathbf{f(x) = 1 + x^2}$

Let $x_1, x_2 \in R$ be such that $f(x_1) = f(x_2)$

$\Rightarrow 1 + x_1^2 = 1 + x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x = \pm x_2$

$\Rightarrow f(x_1) = f(-x_1)$

$\Rightarrow f is one - one$

Let $y \in R$ be any real number

$\therefore f(x) = y \Rightarrow 1 + x^2 = y$

As $x^2 \geq 0 \Rightarrow 1 + x^2 \geq 1 \Rightarrow y \geq 1$

$\therefore$ Range of f is greater than or equal to 1

$\therefore R_f \neq R$

$\Rightarrow$f is onto.

Thus, fis neither one-one nor onto.

(iv) Long answer type:

Show that the relation R in the set A={ x∈ Z: 0≤ x ≤ 12} given by R={$|a - b|$is a multiple of 4} is an equivalence relation. And the equivalence class of 2 i.e. [2]

Sol Reflexive. $|a - a| = 0 \in A$,

Symmetry.

let (a,b)∈R

$|a - b|$is multiple of 4

$= |-(b - a)|$is multiple of 4

$|(b - a)|$is multiple of 4

hence (b,a)∈R

Transitive:

let $(a, b), (b, c) \in R$

then , $|a - b| = 4m$ and $|b - c| = 4n$.

$a - b = \pm 4m$ and $b - c = \pm 4n$

$a - b + b - c = \pm 4m \pm 4n = \pm 4M$

$|a - c|$is multiple of 4

hence (a,c)∈R

[2]= {0, 4, 8, 12}

III. Questions for practices:

(i) MCQ :

Q.1 Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to b ∀ a, b ∈ T. Then R is

(A) reflexive but not transitive (B) transitive but not symmetric

(C) equivalence (D) none of these

Q.2. Let us define a relation R in R as aRb if a ≥ b. Then R is

(A) an equivalence relation (B) reflexive, transitive but not symmetric

(C) symmetric, transitive but not reflexive (D) neither transitive nor reflexive but symmetric.

Q.3. If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is

(A) 720 (B) 120 (C) 0 (D) none of these

Q.4 Let f : R → R be defined by f (x) = 1/x ∀ x ∈ R. Then f is

(A) one-one (B) onto (C) bijective (D) f is not defined

Q.5 . Which of the following functions from Z into Z are bijections?
(A) f (x) = x^3 (B) f (x) = x + 2 (C) f (x) = 2x + 1 (D) f (x) = x^2 + 1
Q.6. Let f : [2, ∞) → R be the function defined by f (x) = x^2 – 4x + 5, then the range of f is
(A) R (B) [1, ∞) (C) [4, ∞) (D) [5, ∞)
Q.7.. Let f : R → R be defined by f (x) = x^2 + 1. Then, pre-images of 17 and – 3, respectively, are
(A) φ, {4, – 4} (B) {3, – 3}, φ (C) {4, –4}, φ (D) {4, – 4, {2, – 2}
Q.8. . For real numbers x and y, define xRy if and only if x – y + $\sqrt{2}$ is an irrational number. Then the relation R is
(A) reflexive (B) symmetric (C) transitive (D) none of these
Q.9. The maximum number of equivalence relations on the set A = {1, 2, 3} are
(A) 1 (B) 2 (C) 3 (D) 5
Q.10. If the set A contains 3 elements and the set B contains 4 elements, then the number of one-one mappings from A to B is
(A) 144 (B) 81 (C) 24 (D) 64
ASSERTION - REASON TYPE QUESTIONS:
Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reasonis correct.
Q.11. Let A = {1, 2, 3, 4, 6}. If R is the relation on A defined by {(a, b) : a, b ∈A, b is exactly divisible by a}. **Assertion :** The relation R in Roster form is {(6, 3), (6, 2), (4, 2)}.
Reason : The domain and range of R is {1, 2, 3, 4, 6}..
Q.12.**Assertion :** Let f and g be two real functions givenby f = {(0, 1), (2, 0), (3, –4), (4, 2), (5, 1)} and g = {(1, 0), (2, 2), (3, –1), (4, 4), (5, 3)} Then, domain of f· g is given by {2, 3, 4, 5}.
Reason : Let f and g be two real functions. Then, (f·g)(x) = f {g(x)}.
Ans: 1.C, 2 B, 3. C 4.D, 5. B, 6. B, 7. C 8. A, 9.D, 10.C11. D, 12. C

(ii) Case based study.

1 Students of Grade 11, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = x - 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L.

Answer the following using the above information.
(i) Let $f: R \to R$ be defined by $(x) = x - 4$. Find the range of $f(x)$
(ii) Let R = {(L_1, L_2) : L_1 is parallel to L_2 and L_1 : y = x – 4} then write of the equation L_2 ?
(iii) Let relation R be defined by R = {(L_1, L_2): $L_1 \parallel L_2$ where L_1, L_2 € L} then show that R is equivalence relation.

2. Raji visited the Exhibition along with her family. The Exhibition had a huge swing, which attracted many children. Raji found that the swing traced the path of a Parabola as given by $y = x^2$.

Answer the following questions using the above information.

(i) Let f: {1,2,3,….}→{1,4,9,….} be defined by $f(x) = x2$. Prove it is bijective.

(ii) Check the function f: Z→Z defined by $(x) = x^2$ is injective , surjective.

(iii) Let : $N \to R$ be defined by $f(x) = x^2$. Write Range of the function.

(iv) Show that : $R \to R$ be defined by $f(x) = x^2$ is not one-one.

(v) Let f: $N \to N$ be defined by $(x) = x^2$. Write the domain of f.

3. Kriti and Kirat are two friends studying in class XII in a school at Chandigarh. While doing their mathematics project on Relations and Functions they have to collect the name of five metro cities and four cities other than metro cities of India; and present the name of cities in the form of sets. They have collected the name of cities and write in the form of sets given as follows:

A={ five metro cities of India}= { Delhi, Mumbai, Bangalore, Calcutta, Pune}

and B = {four non metro cities of India} = { Patiala, Agra, Jaipur, Ahmedabad}

Answer the following questions using the above information.

(i) How many functions exist from A to B.

(ii) Riya wants to know how many relation are possible from A to B.

(iii) Karan wants to know how many reflexive relation on set B.

(iv) How many symmetric relation on set A.

(v) Let R : A → A defined by R = { (x, y) : Total number of vehicles in Delhi(x) is greater than total number of vehicles in Mumbai(y)}. Show that R is neither reflexive nor symmetric.

Ans. 1. (i) R, (ii) y= x +c. , 2. (ii)Neither injective nor surjective, (iii) {1,4,9,16…}, (v) N
3.(i)1024, (ii)2^{20}, (iii) 2^{12}, (iv)2^{15}.

(iii) Short answer type:

1. Let R be the relation on the set {1, 2, 3, 4} given by $R = \{(1,2),(2,2),(1,1),(1,3),(3,2),(3,3),(4,4)\}$
Check whether the function is reflexive, symmetric or transitive?

2. Show that the relation R in R is defined by R = { (a, b) : a ≤ b } is reflexive and transitive but not symmetric

3. Determine whether given relation in N is reflexive, symmetric and transitive where
R = {(x, y) : y = x + 5 and x < 4}.

4. Show that the relation R defined in the set A of all polygons as R = {(P_1, P_2): P_1 and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

5. Show that the number of equivalence relation in the set {1, 2, 3} containing (1, 2) and (2, 1) is two.

6. Prove that the function $f : R \to R$ defined by $4x^2 + 12x + 15 = 0$ is one-one.

7. If f: R → R be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then find the value of $(fof)(x)$

8. Is g = {(1, 1), (2, 3), (3, 5), (4, 7)} a function? If g is described by g (x) = αx + β, then what value should be assigned to α and β.

9. Let f : R → R be the function defined by f (x) = $\frac{1}{2-\cos x}$ ∀x ∈ R. Then, find the range of f.

10. Give an example of mapping
(i) Which is one-one but not onto
(ii) Which is not one-one but onto
(iii) Which is neither one-one nor onto

Ans.1.Reflexive and transitive only. 4. Neither reflexive nor symmetric and nor transitive
1. Reflexive, symmetric and nor transitive, 7.x , 8. α=2, β= –1, 9. [1/3,1]

(IV) Long answer type:

1. Let A = {1, 2, 3, ... 9}and R be the relation in A×A defined by (a , b) R (c, d) = (a + c , b + d). Show that R is equivalence relation and also obtain the equivalent class [(2, 5)].

2. Prove that R is an equivalence relation, R:N ×N→N defined as
(a, b) R (c, d) if and only if ad(b + c) = bc(a + d).

3. Let f : W→W be defined as $f = \begin{cases} n-1 & \text{if n is odd} \\ n+1 & \text{if n is even} \end{cases}$ Show that f is bijective, W is whole number.

4. Give an example of a relation which is
(i) Symmetric but neither reflexive nor transitive.
(ii) Transitive but neither reflexive nor symmetric.
(iii) Reflexive and symmetric but not transitive.
(iv) Reflexive and transitive but not symmetric.
(v) Symmetric and transitive but not reflexive.

5. Show that the function f : R → {x ∈ R : – 1 < x < 1} defined by $f(x) = \frac{1}{1+|x|}$, x ∈ R is one-one and onto function.

6. Show that the function f : R → R given by $f(x) = x^3$ is injective.

7. Let $A = R - \{3\}$ and $= R - \{1\}$, consider the function f: A → B defined by $f(x) = \frac{x-2}{x-3}$. Is f one-one and onto? Justify your answer.

8. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

9. Show that the function f : R → R defined by $f(x) = \frac{x^2}{1+x^2}$, x ∈ R is neither one-one nor onto function

10. Let A = {– 1, 0, 1, 2}, B = {– 4, – 2, 0, 2} and f, g : A → B be functions definedby $f(x) = x^2 - x$, x ∈ A and $g(x) = 2\left|x - \frac{1}{2}\right| - 1$x ∈ A. Are f and g equal? Justify your answer.

Ans. 1. 7. Both one-one and onto. 10. Yes

PRACTICE PAPER (I) **M.M: 20**

Q. 1. Let f : R → R be defined by f (x) = 1/x ∀ x ∈ R. Then f is — 1
(A) one-one (B) onto (C) bijective (D) f is not defined

2. Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}. Then, — 1
(a) f is a relation from A to B (b) f is a function from A to B
(c) Both (a) and (b) (d) None of these

3. If A = {2, 3, 4, 5} and B = {3, 6, 7, 10}. R is a relation defined by R = {(a, b) : a is relatively prime to b, a ∈ A and b ∈ B}, then domain of R is — 1
(a) {2, 3, 5} (b) {3, 5} (c) {2, 3, 4} (d) {2, 3, 4, 5}

4 There are three relations R_1, R_2 and R_3 such that R_1 = {(2, 1), (3, 1), (4, 2)}, R_2 = {(2, 2), (2, 4), (3, 3), (4, 4)} and R_3 = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)} Then, — **1+1**
(a) R_1 and R_2 are functions (b) R_2 and R_3 are functions
(c) R_1 and R_3 are functions (d) Only R_1 is a function

5 If g = {(1, 1), (2, 3), (3, 5), (4, 7)} is a function described by the formula, g (x) = αx + β then what values should be assigned to D and E? — **1+1**
(a) α=2, β= 1 (b) α=2, β= -1 (c) α=1, β= -1 (d) α=3, β= -1

6. Show that the relation R defined in the set A of all polygons as R = {(P_1, P_2) : P_1 and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5? — **2**

7 Let f : R → R be the function defined by f (x) =$\frac{1}{2-\cos x}$∀x ∈R. Then, find the range of f. — **2**

8 Show that the relation R in the set A={ x∈ Z: 0≤ x ≤ 12} given by R={|a − b|is divisible by 4} is an equivalence relation. And the equivalence class of 3 i.e.[3] — **5**

9

The maths teacher of class XII dictates amaths problem as follows.

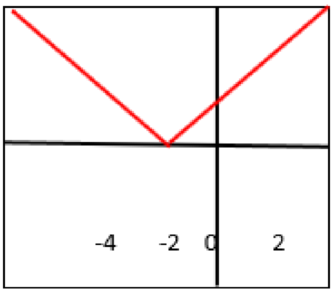
'' Draw the graph of the function, f of x is equal to modulus of x plus three minus one in the closed interval -3 to +3' '

Three students **R**akesh, **S**ravya and **N**avyahaveinterpreted the same dictation in three different ways and they have noted the function as f(x)= |x +3-1|, f(x)=|x|+3-1 and f(x)=|x+3|-1 respectively. All three have drawn the graphs correctly for their respectivefunctions

Based on the above information answer the following.

9(i) Sravya draws the graph in ' V shape ' . Write co-ordinate of vertex. — **1**

9(ii) — **1**



Observe the adjacent figure. who is draw it.

9(iii) Find the distance between the vertices of the graphs of Rakesh and Navys graphs. — **2**

Ans 1.d, 2.a, 3d , 4c, 5.b, 7.[1/3,1], 9(i)(0,2), 9(ii) Rakesh, 9(iii)2$\sqrt{2}$

PRACTICE PAPER (II)

M.M: 30

1. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to b ∀ a, b ∈ T. Then R is **1**

(A) reflexive but not transitive (B) transitive but not symmetric

(C) equivalence (D) none of these

2 If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is **1**

(A) 720 (B) 120 (C) 0 (D) none of these

3 For real numbers x and y, define xRy if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation R is **1**

(A) reflexive (B) symmetric (C) transitive (D) equivalence

4 If A = {2, 3, 4, 5} and B = {3, 6, 7, 10}. R is a relation defined by R = {(a, b) : a is less than to b, a ∈ A and b ∈ B}, then domain of R is **1**

(a) {2, 3, 5} (b) {3, 5} (c) {2, 3, 4} (d) {2, 3, 4, 5}

5 Which of the following functions from Z into Z are bijections? **1+1**

(A) $f(x) = x^3$ (B) $f(x) = x + 2$ (C) $f(x) = 2x + 1$ (D) $f(x) = x^2 + 1$

6 The relation R defined on the set $A=\{1,2,3,4,5\}$ by $R=\{(a,b):|a^2-b^2|<16\}$ is given by: **1+1**

(a) $\{(1,1),(2,1),(3,1),(4,1),(2,3)\}$ (b) $\{(2,2),(3,2),(4,2),(2,4)\}$

(c) $\{(3,3),(4,3),(5,4),(3,4)\}$ (d) none of these

7 The function $f:R\to R$ defined by $f(x)=(x-1)(x-2)(x-3)$ is: **1+1**

(a) f is one-one but not onto (b) f is onto but not one-one

(c) f is both one-one and onto (d) neither one-one nor onto

8. Is g = {(1, 1), (2, 3), (3, 5), (4, 7)} a function? If g is described by g (x) = αx + β, then what value should be assigned to α and β. **4**

9 Show that the relation R in the set *A* of all the books in a library of a college, given by R = {(*x*, *y*): *x* and *y* have same number of pages} is an equivalence relation. **3**

10 Check whether the relation R in **R** defined as R = {(*a*, *b*): $a \le b^3$} is reflexive, symmetric or transitive. **3**

11 Show that the function *f*: **R*** → **R*** defined by $f(x)=\frac{1}{x}$ is one-one and onto, where **R*** is the set of all non-zero real numbers. Is the result true, if the domain **R*** is replaced by **N** with co-domain being same as **R***? **5**

12 Let A = {– 1, 0, 1, 2}, B = {– 4, – 2, 0, 2} and f, g : A → B be functions defined by $f(x) = x^2 - x$, x ∈ A and $g(x) = 2\left|x - \frac{1}{2}\right| - 1$ x ∈ A. Are f and g equal? Justify your answer. **5**

Ans 1c, 2c, 3d, 4d, 5c, 6a,b,c, 7b, 8 α=2, β= -1 12.yes

INVERSE TRIGONOMETRIC FUNCTIONS

* Domain & Range of the Inverse Trigonometric Function :

Functions	Domain	Range (Principal value Branch)
$\sin^{-1}$	$[-1,1]$	$[-\pi/2, \pi/2]$
$\cos^{-1}$:	$[-1,1]$	$[0,\pi]$
$\operatorname{cosec}^{-1}$:	$R-(-1\ ,\ 1)$	$[-\pi/2,\pi/2]-\{0\}$
$\sec^{-1}$:	$R-(-1\ ,\ 1)$	$[0,\pi]-\{\pi/2\}$
$\tan^{-1}$:	R	$(-\pi/2,\pi/2)$
$\cot^{-1}$:	R	$(0\ ,\ \pi)$

* Properties of Inverse Trigonometric Function

1. i $\sin^{-1}(\sin x)=x\ , x\in[-\pi/2,\pi/2]$ & $\sin(\sin^{-1}x)=x,\ \ x\in[-1\,,1]$

   ii. $\cos^{-1}(\cos x)=x\ \ , x\in[0\ ,\ \pi]$ & $\cos(\cos^{-1}x)=x,\ x\in[-1\,,1]$

   iii. $\tan^{-1}(\tan x)=x\ \ , x\in(-\pi/2,\pi/2)$ & $\tan(\tan^{-1}x)=x\ \ ,\ x\in \mathbf{R}$

   iv. $\cot^{-1}(\cot x)=x\ \ , x\in(0\ ,\ \pi)$ & $\cot(\cot^{-1}x)=x\ \ ,\ x\in\ \mathbf{R}$

   v. $\sec^{-1}(\sec x)=x\ , x\in[0\ ,\ \pi]-\pi/2$ & $\sec(\sec^{-1}x)=x\qquad x\in\mathbf{R}-(-1\,,1)$

   vi. $\operatorname{cosec}^{-1}(\operatorname{cosec}x)=x, [-\pi/2,\pi/2]-\{0\}$ & $\operatorname{cosec}(\operatorname{cosec}^{-1}x)=x\in\mathbf{R}-[-1\,,1]$

2. i. $\sin^{-1}x=\operatorname{cosec}^{-1}\frac{1}{x}$ & $\sin^{-1}x=\operatorname{cosec}^{-1}\frac{1}{x}$

   ii. $\cos^{-1}x=\sec^{-1}\frac{1}{x}$ & $\sec^{-1}x=\cos^{-1}\frac{1}{x}$

   iii. $\tan^{-1}x=\cot^{-1}\frac{1}{x}$ & $\cot^{-1}x=\tan^{-1}\frac{1}{x}$

3. i $\sin^{-1}(-x)=-\sin^{-1}x$ ii. $\tan^{-1}(-x)=-\tan^{-1}x$ iii. $\operatorname{cosec}^{-1}(-x)=-\operatorname{cosec}^{-1}x$

   iv $\cos^{-1}(-x)=\pi-\cos^{-1}x$ v $\sec^{-1}(-x)=\pi-\sec^{-1}x$ vi $\cot^{-1}(-x)=\pi-\cot^{-1}x$

Examples

Multiple Choice Questions :

1) The value of $\tan\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)$ is

a) **17/6** b) 1 c) 3/5 d) 3/4

Solution: $\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)=\tan\left(\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right)\right)=17/6$

2) The domain of $\sin^{-1}2x$ is

a) [-1, 1] b) (-1, 1) c) **[-1/2, 1/2]** d) (-1/2, 1/2)

Solution: $-1\le 2x\le 1 \Rightarrow -1/2\le x\le 1/2$

3) If $\cos^{-1}x=y$, then

a) $0\le y\le\pi$ b) $-\pi/2\le y\le\pi/2$ c) $-1\le y\le 1$ d) $x + y =1$

Solution: The range of $\cos^{-1}x$ is $[0, \pi]$

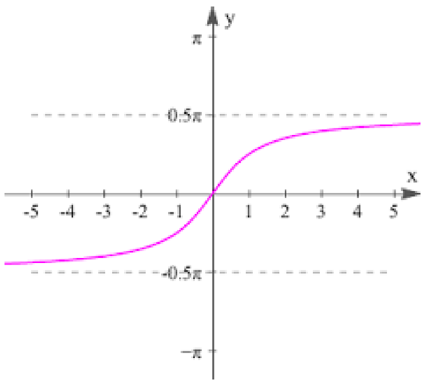
4) $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)=$

a) -1 b) **1** c) $\pi/2$ d) 0

Solution: $\sin(\pi/3 - (-\pi/6)) = \sin \pi/2 = 1$

Case Based Questions:

Ram and Mohan are students of class XII. One day their Mathematics teacher told them about Inverse trigonometric functions. Teacher sketch the graph of $y = \tan^{-1} x$ on the board as follows.



Based on the above information answer the following questions

i) The domain of $\tan^{-1}(3x + 2)$ is

a) **R** b) R^+ c) $(-\pi/2, \pi/2)$ d) $(0, \pi)$

Solution: x is real, implies $3x + 2$ is real. So domain is R

ii) Principal value of $\tan^{-1}(-1)$ is

a) $\pi/4$ b) **$-\pi/4$** c) -1 d) $3\pi/4$

Solution: $\tan(-\pi/4) = -1$ implies $\tan^{-1}(-1) = -\pi/4$

iii) The principal value of $\tan^{-1}(\tan(-6))$ is

a) -6 b) $2\pi - 6$ c) $6 - 2\pi$ d) None

Solution: $\tan^{-1}(\tan(-6)) = \tan^{-1}(-\tan 6) = \tan^{-1}(\tan(2\pi - 6)) = 2\pi - 6$

VSAQ/ SAQ:

1) If $\sin^{-1} x + \sin^{-1} y = 2\pi/3$, then find the value of $\cos^{-1} x + \cos^{-1} y$

Solution: $(\pi/2 - \cos^{-1} x) + (\pi/2 - \cos^{-1} x) = 2\pi/3$, implies $\cos^{-1} x + \cos^{-1} y = \pi/3$

2) Find the domain of $\sin^{-1}(x^2 - 4)$

Solution: **$-1 \le$** $x^2 - 4 \le 1$ => $3 \le x^2 \le 5$ => $x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

3) Find the value of $\sin^{-1}(\cos 33\pi/5)$

Solution: $\sin^{-1}(\cos 33\pi/5) = \sin^{-1}(\cos(6\pi + 3\pi/5)) = \sin^{-1}(\cos(3\pi/5)) = \pi/2 - \cos^{-1}(\cos(3\pi/5))$
$= \pi/2 - 3\pi/5 = -\pi/10$

Questions for Practice:

MCQs

1) If $\text{Sin}^{-1}(\cos x) = \pi/2 - x$, then

a) $-\pi \le x \le 0$ b) $0 \le x \le \pi$ c) $-\pi/2 \le x \le \pi/2$ d) $-\pi/4 \le x \le 3\pi/4$

2) $\text{Tan}^{-1} 1 =$

a) $\pi/2$ b) $\pi/3$ c) $\pi/4$ d) None

3) Principal value of $\cos^{-1}(-1/2)$ is

a) $\pi/4$ b) $2\pi/3$ c) $\pi/2$ d) $\pi/3$

4) The domain of $\sin 2x + \cos^{-1} 2x$ is

a) R b) $[-1, 1]$ c) $[-1/2, 1/2]$ d) $[-1, \pi + 1]$

5) Range of $\sec^{-1} x$ is

a) $[0, \pi]$ b) $[0, \pi/2) \cup (\pi/2, \pi]$ c) $[-\pi/2, \pi/2]$ d) $(0, \pi)$

6) If $\sin^{-1} x = y$ then

a) $0 \le x \le \pi$ b) $-\pi/2 \le y \le \pi/2$ c) $-1 < x < 1$ d) $-1 < y < 1$

7) The principal value of $\sin^{-1}(\sin 4\pi/5)$ is

a) $4\pi/5$ b) $\pi/5$ c) $-\pi/5$ d) $-4\pi/5$

8) The value of $\cot(\cos^{-1}(7/25))$ is

a) 25/24 b) 24/25 c) 7/24 d) 25/7

9) $\text{Sin}^{-1}\sqrt{(1 - x^2)}$ =

a) $\text{Sin}^{-1} x$ b) $\cos^{-1} x$ c) $\tan^{-1} x$ d) $\cot^{-1} x$

10) Assertion : The principal value of $\cos^{-1}(\cos 7\pi/6)$ is $5\pi/6$

Reason : $\cos^{-1}(\cos x) = x$, for all $x \in [0, \pi]$

a) A is true R is true and R is correct explanation for A

b) A is true R is true and R is not correct explanation for A

c) A is true but R is false d) A is false but R is true

d)

VSAQ / SAQ:

1) Find the value of $\cos^{-1}(\cos 13\pi/6)$
2) Write $\cot^{-1}(1/\sqrt{(x^2 - 1)})$ in the simplest form
3) Find the value of $\tan^{-1}(\tan 7\pi/6)$
4) If $\cos(\sin^{-1}\frac{1}{2} + \cos^{-1} x) = 0$, then find the value of x
5) Find the domain of $\sin^{-1}\sqrt{(x - 1)}$
6) Find the domain of $\cos^{-1}(-x^2)$
7) Find the range of $\sin^{-1} x + \cos^{-1} x + \tan^{-1} x$
8) Find the principal value of $\tan^{-1}(\cot 34\pi/5)$
9) Find the value of $\sin(\tan^{-1} x)$
10) Simplify : $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$

Answers:

MCQs:

1) b 2) c 3) b 4) c 5) b 6) b 7) b 8) b 9) c 10) a

VSAQ / SAQ:

1) $\pi/6$ 2) $\sec^{-1} x$ 3) $\pi/6$ 4) 1/2 5) [1, 2] 6) [-1, 1]
7) $[\pi/4, 3\pi/4]$ 8) $-3\pi/10$ 9) $x/\sqrt{(1 + x^2)}$ 10) $\pi/4 - x$

<u>Chapter test - 1</u>:

Each question carries 2 marks

1) Find the principal value of $\cos^{-1}(\cos(-680^0))$
2) Find the value of $\tan^2(\sec^{-1}2) + \cot^2(\text{cosec}^{-1}3)$
3) If $\cos^{-1}x > \sin^{-1}x$, then find x
4) Find the domain of $\cos x + \sin^{-1}x$
5) If $\sin^{-1}x + \sin^{-1}y = \pi$, then find $x^{2023} + y^{2023}$
6) Find the value of $\cot(\sin^{-1}x)$
7) Find the value of $\sin(\cos^{-1}(-3/5))$
8) Find the value of $\sin^{-1}(\cos(\sin^{-1} 1/2))$
9) Find the domain of $\sin^{-1}2x$
10) If $\tan^{-1}x = \pi/10$, then find the value of $\cot^{-1}x$

<u>Chapter test</u>: 2

Each question carries 2 marks

1) Write the value of $\tan^{-1}2\sin(2\cos^{-1}\frac{\sqrt{3}}{2})$
2) Prove that $\text{sce}^2(\tan^{-1} 2) + \text{cosec}^2(\cot^{-1}3) = 15$
3) Express $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ in the simplest form
4) Write the simplest form of $\tan^{-1}(\frac{3x-x^3}{1-3x^2})$
5) Write the simplest form of $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

6) Find the value of $\cot(\cos^{-1} x)$

7) Find the value of $\cos^{-1} (\cos 10)$

8) If $\cos^{-1} (-5/12) = \tan^{-1} x$, then find the value of x

9) Find the domain of $\tan^{-1} (3x + 4)$

10) Find the principal value of $2\cos^{-1} (1) + \sin^{-1} (1/\sqrt{2})$

11) The solution set of $\sin^{-1} x \le \cos^{-1} x$

12) Find the value of $\cos(\pi/3 + \cos^{-1} (-1))$

13) Find the value of $\sin^{-1}(\sin 5\pi/3)$

14) Find the value of $\cos(\tan^{-1} \{\cot(\sin^{-1} 1/2)\})$

15) Write the value of $\sin (\pi/3 - \sin^{-1}(-1/2))$

Answers

Chapter test:1

1) $2\pi/9$ 2) 11 3) $x \in (-1, 1/\sqrt{2})$ 4) [-1, 1] 5) 2

6) $\frac{\sqrt{1-x^2}}{x}$ 7) -24/25 8) $\pi/3$ 9) [-1/2, 1/2] 10) $2\pi/5$

Chapter test: 2

1) $\pi/3$ 3) $\frac{x}{\sqrt{1-x^2}}$ 4) $\frac{1}{2}\tan^{-1}x$ 5) $3 \tan^{-1} x$

6) $\pi/4 - x/2$ 7) $4\pi - 10$ 8) No solution 9) R 10) $5\pi/4$

11) $(-1/\sqrt{2}, 1/\sqrt{2})$ 12) $4\pi/3$ 13) $-\pi/3$ 14) 1/2 15) 1

MATRICES & DETERMINANTS

A matrix is a rectangular array of $m \times n$ numbers arranged in m rows and n columns.

$$A = \begin{bmatrix} a_{11} & a_{12} \ldots\ldots\ldots a_{1n} \\ a_{21} & a_{22} \ldots\ldots\ldots a_{2n} \\ \\ a_{m1} & a_{m2} \ldots\ldots\ldots a_{mn} \end{bmatrix}_{m\times n}$$

$\rightarrow$ **ROWS**

$\downarrow$**COLUMNS**

OR

$A = [a_{ij}]_{m\times n}$, where **i** = 1, 2,…., m ; **j** = 1, 2,….,n.

*** Row Matrix**: A matrix which has one row is called row matrix. $A = [a_{ij}]_{1\times n}$

*** Column Matrix** : A matrix which has one column is called column matrix. $A = [a_{ij}]_{m\times 1}$.

*** Square Matrix**: A matrix in which number of rows are equal to number of columns, is called a square matrix $A = [a_{ij}]_{m\times m}$

*** Diagonal Matrix** : A square matrix is called a Diagonal Matrix if all the elements, except the diagonal elements are zero. $A = [a_{ij}]_{n\times n}$, where $a_{ij} = 0$,$i \neq j$. $a_{ij} \neq 0$,$i = j$.

*** Scalar Matrix**: A square matrix is called scalar matrix it all the elements, except diagonal elements are zero and diagonal elements are some non-zero quantity. $A = [a_{ij}]_{n\times n}$, where $a_{ij} = 0$ if $i \neq j$. and $a_{ij} = k$, i = j.

*** Identity or Unit Matrix** : A square matrix in which all the non-diagonal elements are zero and diagonalelements are unity is called identity or unit matrix.

*** Null Matrices** : A matrices in which all element are zero.

*** Equal Matrices** : Two matrices are said to be equal if they have same order and all their corresponding elements are equal.

*** Sum of two Matrices :**If A = $[a_{ij}]$ and B = $[b_{ij}]$ are two matrices of the same order, say m × n, then, the sum of the two matrices A and B is defined as a matrix

C = $[c_{ij}]_{m\times n}$, where$c_{ij} = a_{ij} + b_{ij}$, for all possible values ofiand j.

* **Multiplication of a matrix :**

$kA = k\,[a_{ij}]_{m\times n} = [k\,(a_{ij})]_{m\times n}$

* **Negative of a matrix** is denoted by –A. We define–A = (– 1) A.

* **Difference of matrices :**If A = $[a_{ij}]$, B = $[b_{ij}]$ are two matrices of the same order, say m × n, then difference A – B is defined as a matrix D = $[d_{ij}]$, where $d_{ij} = a_{ij} - b_{ij}$,for all value of iand j.

***Properties of matrix addition :**

(i) **Commutative Law :**If A = and B are matrices of the same order, then A + B = B + A.

(ii)**Associative Law :**For any three matrices A, B &C of the same order, (A + B) + C = A + (B + C).

(iii)**Existence of identity:** Let A be an m × n matrix and O be an m × n zero matrix, then A + O = O + A = A. i.e. O is the additive identity for matrix addition.

(iv) **Existence of inverse :**Let A be any matrix, then we have another matrix as – A such that A + (– A) = (– A) + A= O. So – A is the additive inverse of A or negative of A.

* **Properties of scalar multiplication of a matrix :**If A and B be two matrices of the same order, and k and l arescalars, then

(i) k(A +B) = k A + kB, (ii) (k + l)A = k A + l A(iii) (k+ l) A = k A + l A

***Product of matrices:** If A& B are two matrices, then product AB is defined, if number of column of A = number of rows of B.

i.e. $A = [a_{ij}]_{m\times n}$, $B = [b_{jk}]_{n\times p}$ then $AB = [C_{ik}]_{m\times p}$,where $C_{ik} = \sum_{j=1}^{n} a_{ij}.b_{jk}$

***Properties of multiplication of matrices :**

(i) Product of matrices is not commutative. i.e. AB $\neq$ BA.

(ii) Product of matrices is associative. i.e. A(BC) = (AB)C

(iii) Product of matrices is distributive over addition i.e.(A+B) C = AC + BC

(iv) For every square matrix A, there exist an identitymatrix of same order such that IA = AI = A.
* **Transpose of matrix** : If A is the given matrix, then the matrix obtained by interchanging the rows and columns is called the transpose of a matrix.
* **Properties of Transpose** :
If A & B are matrices such that their sum & product are defined, then
(i). $(A^T)^T = A$ (ii). $(A+B)^T = A^T + B^T$ (iii). $(KA^T) = K.A^T$ where K is a scalar.
(iv). $(AB)^T = B^T A^T$ (v). $(ABC)^T = C^T B^T A^T$.
* **Symmetric Matrix** : A square matrix is said to be symmetric if A = A^T i.e. If $A = [a_{ij}]_{m\times m}$, then
$a_{ij} = a_{ji}$ for all i, j. Also elements of the symmetric matrix are symmetric about the main diagonal
* **Skew symmetric Matrix** : A square matrix is said to be skew symmetric if $A^T = -A$.
If $A = [a_{ij}]_{m\times m}$, then $a_{ij} = -a_{ji}$ for all i, j.
* **Elementary Operation (Transformation) of a Matrix:**
(i) Interchange of any two rows or two columns :$R_i \leftrightarrow R_j$,$C_i \leftrightarrow C_j$.
* Multiplication of the elements of any row or column by a non-zero number:
$R_i \rightarrow k R_i$, $Ci \rightarrow kC_i$, $k \neq 0$
*** Determinant** :To every square matrix we can assign a number called determinant
If A = $[a_{11}]$, det. A = | A | = a_{11}.
If A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $|A| = a_{11}a_{22} - a_{21}a_{12}$.
*** Properties** :
(i) The determinant of the square matrix A is unchanged when its rows and columns are interchanged.
(ii) The determinant of a square matrix obtained by interchanging two rows(or two columns) is negative of given determinant.
(iii) If two rows or two columns of a determinant are identical, value of the determinant is zero.
(iv) If allthe elements of a row or column of a square matrix A are multiplied by a non-zero number k, then determinant of the new matrix is k times the determinant of A.
(v) If elements of any one column(or row) are expressed as sum of two elements each, then determinant can be written as sum of two determinants.
(vi) Any two or more rows(or column) can be added or subtracted proportionally.
(vii) If A & B are square matrices of same order, then |AB| = |A| |B|
***Singular matrix**:A square matrix 'A' of order 'n' is said to be singular, if | A| = 0.
* **Non -Singular matrix** : A square matrix 'A' of order 'n' is said to be non-singular, if $|A| \neq 0$.
* If A and B are non-singular matrices of the same order, then AB and BA are also non-singular matrices of the same order.
* Let A be a square matrix of order n × n, then $|kA| = k^n|A|$.
* **Area of a Triangle:**area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$
* Equation of a line passing through (x_1, y_1)&$(x_2, y_2)$is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$
***Minor** of an element $a_{ij}$of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which elementa_{ij}lies. Minor of an element $a_{ij}$is denoted by M_{ij}.
* Minor of an element of a determinant of order n($n \geq 2$) is a determinant of order n – 1.
* **Cofactor** of an element a_{ij}, denoted by $A_{ij}$is defined by$A_{ij} = (-1)^{i+j}M_{ij}$, where $M_{ij}$is minor of a_{ij}.
* If elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero.
***Adjoint of matrix** :

If $A = [a_{ij}]$ be a square matrix then transpose of a matrix $[A_{ij}]$, where A_{ij} is the cofactor of A_{ij} element of matrix A, is called the adjoint of A.
Adjoint of A = Adj. A = $[A_{ij}]^T$.
A(Adj.A) = (Adj. A)A = | A| I.
* If A be any given square matrix of order n, then A(adjA) = (adjA) A = |A| I,
* If A is a square matrix of order n, then $|adj(A)| = |A|^{n-1}$.
***Inverse of a matrix** :Inverse of a square matrix A exists, if A is non-singular or square matrix A is said to be invertible and $A^{-1} = \frac{1}{|A|}$ Adj.A

***System of Linear Equations** :
$a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$.

$$\begin{bmatrix} a_1 & b_2 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \Rightarrow AX = B \Rightarrow X = A^{-1}B \quad ; \{ |A| \neq 0\}.$$

***Criteria of Consistency**.
(i) If $|A| \neq 0$, then the system of equations is said to be consistent & has a unique solution.
(ii) If |A| = 0 and (adj. A)B = 0, then the system of equations is consistent and has infinitely many solutions.
(iii) If |A| = 0 and (adj. A)B$\neq$ 0, then the system of equations is inconsistent and has no solution.

SOME ILLUSTRATIONS/EXAMPLES (WITH SOLUTION)

(i) Multiple Choice Questions:

Q.1. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is symmetric and Q is a skew symmetric matrix, then Q is equal to

(a) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -5/2 \\ 5/2 & 1 \end{bmatrix}$

Solution: Let $A = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$, $\therefore A' = \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}$

Now $A + A' = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 5 & 8 \end{bmatrix}$ $\quad \therefore \frac{1}{2}(A + A') = \begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix} = P$

$A - A' = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$ $\quad \therefore \frac{1}{2}(A - A') = \begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix} = Q$

So, Correct option is option (c).

Q.2. If A is 2×3 matrix such that AB and AB′ both are defined, then find the order of the matrix B
(a) 2×3 (b) 3×3 (c) 2×2 (d) Not defined
Solution: Let order of B be$m \times n$
$\because$ AB is defined, $\therefore$ No. of columns in A = No. of rows in B
$\Rightarrow 3 = m$
Order of B′ = $n \times m$
Again $\because$ AB′ is defined, $\therefore$ No. of columns in A = No. of rows in B′
$\Rightarrow 3 = n$
So, order of B = $m \times n = 3 \times 3$.
$\therefore$ **Correct option is (b).**

Q.3. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then the possible value(s) of x is/are
(a) 1 (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) $\pm\sqrt{3}$

Solution: Since $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2 - 20 = 2x^2 - 24 \quad \Rightarrow -18 + 24 = 2x^2 \quad \Rightarrow 2x^2 = 6 \quad \Rightarrow x = \pm\sqrt{3}$

$\therefore$ **Correct option is (d).**

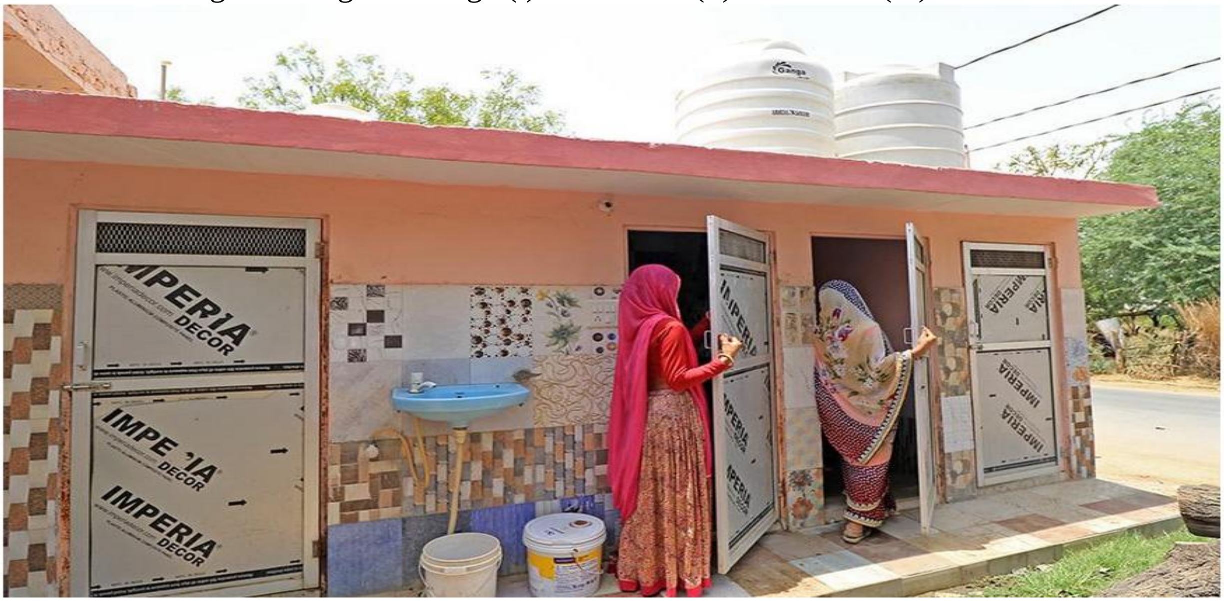
Q.4. If |A| = |kA|, where A is a square matrix of order 2, then sum of all possible values of k is
(a) 1 (b) -1 (c) 2 (d) 0
Solution: |A| = |kA| and n=2
$|A| = k^2 |A|$ $\quad (\because |kA| = k^n |A|)$

$\Rightarrow k^2 = 1$ $\quad\quad\quad\quad$ $\Rightarrow k = \pm 1$ $\quad\quad$ $\Rightarrow$ Sum of all values of k = + 1 – 1 = 0

∴ **Correct option is (d).**

(i) Case Based Study Question:

Q.5. To promote the usage of house toilets in villages especially for women, an organisation tried to generate awareness among the villagers through (i) house calls (ii) letters and (iii) announcements



The cost for each mode per attempt is (i) Rs 50 (ii) Rs 20 (iii)Rs40 respectively

The number of attempts made in the villages X, Y and Z are given below:

	(i)	**(ii)**	**(iii)**
X	400	300	100
Y	300	250	75
Z	500	400	150

Also the chance of making of toilets corresponding to one attempt of given modes is:

(i) 2% (ii) 4% (iii) 20%

Let A, B, C be the cost incurred by organisation in three villages respectively.

Based on the above information answer the following questions

(A) Form a required matrix on the basis of the given information.

(B) Form a matrix, related to the number of toilets expected in villagers X, Y, Z after the promotion campaign.

(C) What is total amount spent by the organisation in all three villages X, Y and Z

OR

What are the total number of toilets expected after promotion campaign?

Solution:(A) Rs A, Rs B and Rs C are the cost incurred by the organisation for villages X, Y, Z respectively, therefore matrix equation will be

$$\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

(B) Let number of toilets expected in villagers X, Y, Z be x, y, z respectively

Therefore required matrix is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 2/100 \\ 4/100 \\ 20/100 \end{bmatrix}$$

(C). $\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} 20000 + 6000 + 4000 \\ 15000 + 5000 + 3000 \\ 25000 + 8000 + 6000 \end{bmatrix} = \begin{bmatrix} 30000 \\ 23000 \\ 39000 \end{bmatrix}$

Total money spent = 30000 + 23000 + 39000 = 92000 Rs

OR

From part (B) the required matrix for the expected number of toilets is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 2/100 \\ 4/100 \\ 20/100 \end{bmatrix} = \begin{bmatrix} 8+12+20 \\ 6+10+15 \\ 10+16+30 \end{bmatrix} = \begin{bmatrix} 40 \\ 31 \\ 56 \end{bmatrix}$$

So, total number of toilets expected in 3 villages are = 40 + 31 + 56 = 127

(iii) Short Answer Type Questions:

Q.6. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ then find the value of x and y.

Solution: Given $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ or $\begin{bmatrix} 2x - y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

So 2x – y = 10 and 3x + y = 5

On solving we get x = 3 and y = -4

Q.7. If A is a square matrix such that $A^2 = A$, show that $(I + A)^3 = 7A + I$.

Solution: L.H.S. = $(I + A)^3 = I^3 + A^3 + 3I^2A + 3IA^2$

$= I + A^2.A + 3IA + 3IA^2 = I + A.A + 3A + 3IA$

$= I + A^2 + 3A + 3A = I + A + 3A + 3A$

$= I + 7A$ = R.H.S. Proved.

(iv) Long Answer Type Questions:

Q.8. If $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, then find BA and use this to solve the system of equations: y + 2z = 7, x – y = 3 and 2x + 3y + 4z = 17.

Solution: $BA = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 6I$

$$B\left(\frac{1}{6}A\right) = I \quad \Rightarrow \quad B^{-1} = \frac{1}{6}A = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

The given equations can be re-written as, x – y = 3, 2x + 3y + 4z = 17, and y + 2z = 7

$$\therefore BX = C \text{ i.e. } \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow X = B^{-1}C \text{ i.e. } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

Hence, $x = 2, y = -1$ and $z = 4$

Q.9. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ prove that F(x) F(y) = F(x+y)

Solution: Given $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ so $F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence $F(x).F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos x\cos y - \sin x\sin y & -\cos x\sin y - \sin x\cos y & 0 \\ \sin x\cos y + \cos x\sin y & -\sin x\sin y + \cos x\cos y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence F(x).F(y) = F(x + y)

QUESTIONS FOR PRACTICE:

Multiple Choice Questions:

Q.1. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is equal to

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ **(c)** $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q.2. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of x and y is

(a) x = 3, y = 1 **(b)** x = 2, y = 3 **(c)** x = 2, y = 4 **(d)** x = 3, y = 3

Q.3. For any two matrices A and B **(a)** AB = BA **(b)** AB ≠ BA **(c)** AB = O **(d)** None of these

Q.4. which one is not correct **(a)** $(AB)' = B'A'$ **(b)** $A'B' = (BA)'$ **(c)** $(kA)' = kA'$ **(d)** $A' = A$

Q.5. If $\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}^2 = \begin{vmatrix} 3 & 2 \\ 1 & x \end{vmatrix} - \begin{vmatrix} x & 3 \\ -2 & 1 \end{vmatrix}$, then the value of x is: **(a)** 6 **(b)** 3 **(c)** 7 **(d)** 1

Q.6. If A is a square matrix of order 3 and |A| = 5, then |adj A| is **(a)** 5 **(b)** 25 **(c)** 125 **(d)** $\frac{1}{5}$

Q.7. The area of triangle with vertices (-3,0), (3,0) and (0,k) is 9 sq units. Then the value of k will be **(a)** 9 **(b)** 3 **(c)** -9 **(d)** 6

Q.8. Let $f(t) = \begin{bmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{bmatrix}$, then $\lim_{t \to 0} \frac{f(t)}{t^2}$ is equal to **(a)** 0 **(b)** – 1 **(c)** 2 **(d)** 3

Q.9. If A is a symmetric matrix, then A^3 is:

(a) Symmetric Matrix **(b)** Skew Symmetric Matrix **(c)** Identity matrix **(d)** Row Matrix

Assertion Reason Based Question:

Q.10. For A and B square matrices of same order, choose appropriate option

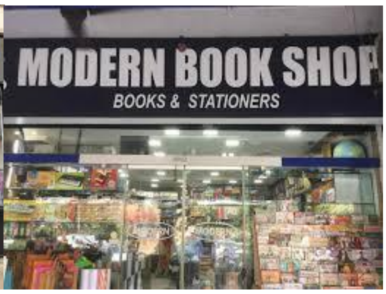
Assertion (A): $(A + B)^2 \neq A^2 + 2AB + B^2$

Reason (R): Generally, AB ≠ BA

(a) Both A & R are true & R is the correct explanation of A
(b) Both A & R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Case Study Based Questions:

Q.11. Read the following text and answer the following questions from (1 - 5) on the basis of the same:
A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below



Market	Products (in numbers)		
	Pencil	**Eraser**	**Sharpener**
A	10000	2000	18000
B	6000	20000	8000

If the unit Sale price of Pencil, Eraser and Sharpener are Rs 2.50, Rs 1.50 and Rs 1.00 respectively, and unit cost of the above three commodities are Rs 2.00, Rs 1.00 and Rs 0.50 respectively, then,
Answer the following questions using matrix method.

(i) Total revenue (in Rs) of market A is ….
(ii) Total revenue (in Rs) of market B …..
(iii) Cost (in Rs) incurred in market A ?
(iv) Find profits (in Rs) in market A and B respectively.
(v) Find gross profit (in Rs) in both market.

Q.**12.** A school wants to award its students for the values of honesty, regularity and hard work with a total cash award of Rs 6000. Three times the award money for hard work added to that given for honesty amounts to Rs 11000. The award money given for honesty and hard work together is double the one given for regularity.

Based on the above information, answer the following questions:

(i) If Rs x is awarded to honesty, Rs y to regularity and Rs z awarded to hard work, then what is the matrix equation representing the above situation ?

(ii) What is the value of |adj A| ?

(iii) What are the values of x, y, z respectively in this case ?

(iv) What is the value of $|A^{-1}|$?

(v) What is the value of A(Adj A) ?

Q.13. Two school KV-1 and KV-2 want to award their selected students on the values of sincerity, truthfulness and helpfulness. KV-1 wants to award Rs x each, Rs y each and Rs z each for the three respective values to 3, 2 and 1 students respectively with a total award money of Rs 1600. KV-2 wants to spend Rs 2300 to award its 4, 1, 3 students on the respective values (by giving the same amount of the three values as before). The total amount of the award for one prize on each is Rs 900.

Answer the following questions using matrix:

(i) Determine x + y + z

(ii) Calculate the value of 2x + y +3 z.

(iii) What is the value of y?

(iv) Find the value of 2x + 5y.

(v) Determine y – x.

Short Answer type Questions:

Q.14. If x = - 4 is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then find the sum of other two roots.

Q.15. If A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then find A^{2024}.

Q.16. If A is a 3×3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$.

Q.17. If A = $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then find (A – 2I) (A – 3I).

Q.18. If X = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, then find the value of $(X^2 - X)$.

Q.19. Matrix A = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ is a symmetric matrix, find values of a and b.

Q.20. Show that the elements on the main diagonal of a skew symmetric are all zero.

Q.21. Find the matrix X so that X $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.

Q.22. If A = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, then find $A^5 - A^4 - A^3 + A^2$.

Q.23. If A is a square matrix of order 3, |A′| = - 3, then find the value of |AA′|.

Long Answer type Questions:

Q.24. Solve the system of the following equations:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2, \qquad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5, \qquad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$$

Q.25. Express the following matrix as the sum of a symmetric and skew-symmetric matrix, and verify the result: $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$.

Q.26. If A = $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$, find AB and hence solve x – 2y = 10, 2x + y + 3z = 8, - 2y + z = 7

Q.27. If A = $\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, and B = $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations: x – 2y = 3, 2x – y – z = 2, – 2y + z = 3.

Q.28. If A = $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of linear equations: x – 2y = 10, 2x – y – z = 8, – 2y + z = 7.

ANSWERS:

Questions for Practice: 1. (d) 2.(b) 3. (d) 4. (d) 5. (a) 6. (b) 7. (b) 8.(a) 9. (a) 10. (a) 11(i). 46000 11(ii). 53000 11(iii). 31000 11(iv). (15000, 17000) 11(v). 32000

12(i). $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$ **12(ii). 36, 12(iii). (500, 2000, 3500), 12(iv).** $\frac{1}{6}$, **12(v). 6I, 13(i). 900 13(ii). 1900 13(iii). 300 13(iv). 1900 13(v). 100 14. 4, 15. O**

16. -1, 17. O, 18. 2I, 19. –2/3 & 3/2, 21. $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$, **22. O, 23. 9, 24. x = 2, y = –3, z = 5, 25.** $\begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$, **26. x = 4, y = – 3, z = 5, 27. AB = I;** $x = 1, y = -1, z = 1$, **28.** $A^{-1}\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$; $x = 0, y = -5, z = -3$.

CHAPTER TESTS

CLASS TEST –I

Max Marks: 20 **Time: 40 Min**

1.	If A is a 2×3 matrix such that AB and AB' both are defined, then find the order of the matrix B.	**1**
2.	If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew symmetric matrix, find the values a, b and c.	**1**
3.	Prove that AA' is always a symmetric matrix for any square matrix of A.	**1**
4.	If A and B are square matrices, each of order 2 such that |A|=3 and |B|= - 2, then write the value of |3AB|.	**1**
5.	If A is a square matrix of order 3 such that |adj A| = 225, find |A′|.	**1**
6.	If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then find the possible value(s) of x.	**1**
7.	Find the equation of the line joining A(1,3) and B(0,0) using determinants and find k if D(k,0) is a point such that area of triangle ABD is 3 sq units.	**2**
8.	Find A, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$	**2**
9.	Given $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find BA and use it to find the values of x, y, z from given equations: $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 17$	**5**
10.	If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that: $f(x)f(-y) = f(x - y)$	**5**

CLASS TEST –II

Max Marks: 30 **Time: 60 Min**

1.	If A is a square matrix of order 3 such that $A^2 = 2A$, then find the value of |A|.	**1**
2.	If A is a square matrix of order 3 and |A| = - 4, find |adj A|.	**1**
3.	If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then find the possible value(s) of x.	**1**
4.	If A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then find A^{2024}.	**1**
5.	If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$ = P + Q, where P is a symmetric and Q is a skew symmetric matrix, then find Q.	**1**

6. Let $A=\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Prove that $I+A=(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$. **2**

7. Find the equation of the line joining P(4,0) and Q(0,2) using determinants and find λ if R(λ,0) is a point such that area of triangle PQR is 4 sq units. **2**

8. Show that A = $\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $x^2 - 3x - 7 = 0$. Thus find A^{-1}. **2**

9. Express A = $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix. And verify it. **5**

10. If $A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$, Find A^{-1}. Hence, solve the system of equations : **5**

$3x + 3y + 2z = 1, \quad x + 2y = 4, \quad 2x - 3y - z = 5$

11. Find the product of two matrices A and B, where $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$, **5**

$B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it for solving the equations:

$x + y + 2z = 1, \quad 3x + 2y + z = 7, \quad 2x + y + 3z = 2$

Case Study Based Question

12. 1. Two farmers Ramakishan and Gurucharan Singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in Rs) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B

September sale (in Rs)$A = \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix}$ Ramakishan / Gurucharan

October sale (in Rs)$B = \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$ Ramakishan / Gurucharan

Answer the following questions using above information:

(i) Compute the total sales in September and October and write answer in terms of A and B. **1**

(ii) What is the value of A_{23} ? **1**

(iii) Determine the decrease in sales from September to October in terms of A and B. **1**

(iv) If Ramakishan receives 2% profit on gross sales, compute his profit (in Rs) for each variety sold in October using matrix method. **1**

ANSWERS

CLASS TEST –I

1. 3×3 2. (a = - 2, b = 0, c = -3) 4. – 4, 5. ±15, 6. ±6, 7. 3x – y = 0; k = ±2, 8. $[-1 \quad 2 \quad 1]$, 9. BA = 6 I; (x = - 14/3, y = - 23/3, z = 37/3).

CLASS TEST –II

1. 8, 2. 16, 3. $\pm\sqrt{3}$, 4. O, 5. $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$, 7. x + 2y = 4; λ=0,8,

8. $\begin{bmatrix} 2/7 & 3/7 \\ -1/7 & -5/7 \end{bmatrix}$, 9. A=$\begin{bmatrix} 3 & 1/2 & 1/2 \\ 1/2 & 0 & 1 \\ 1/2 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -3/2 & -1/2 \\ 3/2 & 0 & 2 \\ 1/2 & -2 & 0 \end{bmatrix}$,

10. $A^{-1} = -1/17\begin{bmatrix} -2 & 1 & -7 \\ -3 & -7 & 15 \\ -4 & 2 & 3 \end{bmatrix}$; (x = 2, y = 1, z = - 4) 11. AB = BA = 4I; (x = 2, y = 1, z = -1), 12. (i) A + B, (ii) A_{23} = 10000, (iii) A – B, (iv) Rs 100, 200, 120.

CONTINUITY& DIFFERENTIABILITY

* A function f is said to be continuous at x = a if
  Left hand limit = Right hand limit = value of the function at x = a

i.e. $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

i.e. $\lim_{h \to 0} f(a-h) = \lim_{h \to 0} f(a+h) = f(a)$.

* A function is said to be differentiable at x = a

if $Lf'(a) = Rf'(a)$ i.e $\lim_{h \to 0} \frac{f(a-h)-f(a)}{-h} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

(i) $\frac{d}{dx}(x^n) = n\,x^{n-1}$, $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$, $\frac{d}{dx}\left(\sqrt{x}\right) = -\frac{1}{2\sqrt{x}}$

(ii) $\frac{d}{dx}(x) = 1$

(iii) $\frac{d}{dx}(c) = 0, \forall\, c \in R$

(iv) $\frac{d}{dx}(a^x) = a^x \log a,\ a > 0,\ a \neq 1.$

(v) $\frac{d}{dx}(e^x) = e^x.$

(vi) $\frac{d}{dx}(\log_a x) =, \frac{1}{x \log a}$ a > 0, a ≠ 1, x

(vii) $\frac{d}{dx}(\log x) = \frac{1}{x}$, x > 0

(viii) $\frac{d}{dx}(\log_a |x|) = \frac{1}{x \log a}$, a > 0, a ≠1, x ≠ 0

(ix) $\frac{d}{dx}(\log |x|) = \frac{1}{x}$, x ≠ 0

(x) $\frac{d}{dx}(\sin x) = \cos x, \forall\, x \in R.$

(xi) $\frac{d}{dx}(\cos x) = -\sin x, \forall\, x \in R.$

(xii) $\frac{d}{dx}(\tan x) = \sec^2 x, \forall\, x \in R.$

(xiii) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x, \forall\, x \in R.$

(xiv) $\frac{d}{dx}(\sec x) = \sec x \tan x, \forall\, x \in R.$

(xv) $\frac{d}{dx}(\text{cosec}\, x) = -\text{cosec}\, x \cot x, \forall\, x \in R.$

(xvi) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$.

(xvii) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$.

(xviii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}, \forall\ x \in R$

(xix) $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}, \forall\, x \in R.$

(xx) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$.

(xxi) $\frac{d}{dx}(\text{cosec}^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}}$.

(xxii) $\frac{d}{dx}(|x|) = \frac{x}{|x|}$, x ≠ 0

(xxiii) $\frac{d}{dx}(ku) = k\frac{du}{dx}$

(xxiv) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

(xxv) $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$

(xxvi) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

SOME ILLUSTRATIONS:
MCQ

1. If y = log tan $\sqrt{x}$ then the value of $\frac{dy}{dx}$ is

(a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{\sec 2\sqrt{x}}{\sqrt{x}\tan x}$ (c) $2\sec^2\sqrt{x}$ (d) $\frac{\sec 2\sqrt{x}}{2\sqrt{x}\tan x}$

Ans: (d)

2. If y = ($\cos x^2$)^2 then $\frac{dy}{dx}$ is

(a) $-4x\sin 2x^2$ (b) $-x\sin x^2$ (c) $-2x\sin 2x^2$ (d) $-x\cos 2x^2$

Ans: (c)

3. If $y = \cot^{-1}(x^2)$ then the value of $\frac{dy}{dx}$ is

(a) $\frac{2x}{1+x4}$ (b) $\frac{2x}{\sqrt{1+4x}}$ (c) $\frac{-2x}{1+x4}$ (d) $\frac{-2x}{\sqrt{1+x2}}$

Ans: (c)

SHORT ANSWER TYPE QUESTIONS

****Q**. 1. If f(x) = $\begin{cases} 3ax+b, & \text{if } x>1 \\ 11 & \text{if } x=1 \\ 5ax-2b, & \text{if } x<1 \end{cases}$, continuous at x = 1, find the values of a and b.

Sol. $\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = f(1)$.........(i)

$\lim_{x\to1^-} f(x) = \lim_{h\to0} f(1-h) = \lim_{h\to0}[5a(1-h)-2b] = 5a-2b$

$\lim_{x\to1^+} f(x) = \lim_{h\to0} f(1+h) = \lim_{h\to0}[3a(1+h)+b] = 3a+b$

$f(1) = 11$

From (i) $3a+b = 5a-2b = 11$ and solution is $a = 3, b = 2$

******Q**. 2. If $y = \sin(m\sin^{-1}x)$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

Sol. $y = \sin(m\sin^{-1}x) \Rightarrow \frac{dy}{dx} = \cos(m\sin^{-1}x).\frac{m}{\sqrt{1-x^2}}$

$\Rightarrow \sqrt{1-x^2}\frac{dy}{dx} = m\cos(m\sin^{-1}x)$

Again diff.w.r.t.x, $\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{-2x}{\sqrt{1-x^2}}\right) = -m\sin(m\sin^{-1}x).\frac{m}{\sqrt{1-x^2}}$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -m^2\sin(m\sin^{-1}x) = -m^2y$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

LONG ANSWER TYPE QUESTIONS

**If $y = (\log_e x)^x + x^{\log_e x}$ find $\frac{dy}{dx}$.

Sol. $y = (\log_e x)^x + x^{\log_e x} = e^{\log\{(\log_e x)^x\}} + e^{\log\{x^{\log_e x}\}}$

$= e^{x\log\{(\log_e x)\}} + e^{\log_e x.\log_e x}$

$\frac{dy}{dx}. = e^{x\log\{(\log_e x)\}}\left[x.\frac{1}{\log x}.\frac{1}{x} + \log(\log x).1\right] + e^{\log_e x.\log_e x}\left[\frac{\log x}{x} + \frac{\log x}{x}\right]$

$= (\log_e x)^x\left[\frac{1}{\log x} + \log(\log x)\right] + x^{\log_e x}\left[2\frac{\log x}{x}\right]$

QUESTIONS FOR PRACTICE

Q 1. If f(x) = $\begin{cases} 3x-5 & x \le 5 \\ 2k & x > 5 \end{cases}$ is continuous at x=5 then k is (a)5 (b) 10 (c) 15 (d) $\frac{-2}{7}$

Q 2. The function f(x) = [x]is continuous at (a) 4 (b) -2 (c) 1 (d) 1.5

Q 3. If $x = t^2$ and $y = t^3$ then $\frac{d^2y}{dx^2}$ is equal to (a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3t}{2}$

Q 4. Derivative of x^2 w.r.t x^3 is (a) $\frac{1}{x}$ (b) $\frac{2}{3x}$ (c) $\frac{2}{3}$ (d) $\frac{3x}{2}$

Q 5. Assertion : f(x) = [x] not differentiable at x = 2

Reason : f(x) = [x] not continuous at x = 2

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true and R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

ANSWERS

1. (a) 2. (d) 3. (b) 4. (b) 5. (a)

SHORT ANSWER TYPE QUESTIONS

Q 1. . Find the number of points at which the function $f(x) = \frac{9-x^2}{9x-x^3}$ is discontinuous.

Q 2. . Find the value of k for which $f(x) = \begin{cases} \frac{\sin 2x}{5x} & , x \neq 0 \\ k & , x = 0 \end{cases}$ is continuous at x = 0.

Q 3. .Discuss the differentiability of the function f(x) = |x – 2| at x = 2.

Q 4. . Find : $\frac{d}{dx}\left[\log_e \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$

Q 5. Find : $\frac{d}{dx}\left[\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)\right]$, where $0 < x < \frac{\pi}{4}$

Q 6. . Find : $\frac{d}{dx}\left[x^{\sin x}\right]$

Q 7. Find the relationship between a and b so that the function defined by

$f(x) = \begin{cases} ax+1 & ,\text{if } x \leq 3 \\ bx+3 & ,\text{if } x > 3 \end{cases}$ is continuous at x = 3.

Q 8. If x=a(cost+tsint) and y = a (sint – t cost) find $\frac{d^2y}{dx^2}$

Q 9. If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$.

Q 10. Find : $\frac{d}{dx}\left[\log \sin\sqrt{x^2+1}\right]$

ANSWERS

1. Exactly three points (0,3 and -3) 2. $\frac{2}{5}$ **3. not differentiable at x = 2**

4. Sec x 5. $\frac{1}{2}$ **6.** $x^{\sin x}\left(\cos x.\log_e x + \frac{\sin x}{x}\right)$ **7.**

$\mathbf{3a - 3b = 2}$ is the relation between a and b

8. $\frac{\sec^3 t}{at}$ **10.** $\frac{x\cos\sqrt{x^2+1}}{\sqrt{x^2+1}.\sin\sqrt{x^2+1}}$

CLASS TEST -1 **M.M 30**

Each 3 marks

1. If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$ find $\frac{dy}{dx}$.

2. Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$.

3. If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$.

4. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$,

   find $\frac{d^2y}{dx^2}$.

5. If $y = (\tan^{-1} x)^2$, show that

   $(x^2+1)^2 y_2 + 2x(x^2+1) y_1 = 2$

6. If $y = \left(x + \sqrt{x^2+a^2}\right)^n$, prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2+a^2}}$

7. If $\log_e \sqrt{x^2+y^2} = \tan^{-1}\left(\frac{y}{x}\right)$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$

8. If $x^m . y^n = (x+y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$

9. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for, $-1 < x < 1$, prove that

   $\frac{dy}{dx} = -\frac{1}{1+x^2}$

10. If $y = (\log_e x)^x + x^{\log_e x}$ find $\frac{dy}{dx}$.

ANSWERS

1. $(\sin x)^x[x \cot x + \log \sin x] + \frac{1}{2} \times \frac{1}{\sqrt{x - x^2}}$

2. $\frac{y \tan x + \log \cos y}{x \tan y + \log \cos x}$

3. $\frac{\sec^3 x}{at}$

10. $(\log x)^{x-1}[1 + \log x \times \log(\log x)]$

CHAPTER TEST -2 **M.M –20**

.1. The function f(x) = [x] is continuous at 1M

(a) 4 (b) -2 (c) 1 (d) 1.5

2. If $x = t^2$ and $y = t^3$ then $\frac{d^2y}{dx^2}$ is equal to 1M

(a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3t}{2}$

. 3. Derivative of x^2 w.r.t x^3 is 1M

(a) $\frac{1}{x}$ (b) $\frac{2}{3x}$ (c) $\frac{2}{3}$ (d) $\frac{3x}{2}$

.4. If f(x) = $\begin{cases} 3x - 5 & x \le 3 \\ 2k & x > 3 \end{cases}$ is continuous at x=3 then k is 1M

(a)2 (b) 4 (c) 15 (d) $\frac{-2}{7}$

5. Assertion : f(x) = |x| not differentiable at x = 0 1M

Reason : f(x) = |x| not continuous at x = 0

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true and R is not the correct explanation of A

(c) A is true but R is false
(d) A is false but R is true

6. If $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$ 2M

7. If X = $a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = a\sin\theta$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ 2M

8. If f(x) = $\begin{cases} 3ax + b, & \text{if } x > 1 \\ 11 & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$, continuous at x = 1,find the values of a and b. 2M

9. $(\cos x)^y = (\sin y)^x$, then find $\frac{dy}{dx}$. 2M

10. Find $\frac{dy}{dx}$ at x =1, y = $\frac{\pi}{4}$ if $\sin^2 y + \cos xy = k$ 3M

11. Find $\frac{dy}{dx}$ if y = $e^{\tan^{-1}\sqrt{x}}$ 4M

ANSWERS

1. D 2. A 3. B 4. A 5. C 6. 1/a 7. 1 8. a=3, b=2 9. $\frac{dy}{dx} = \frac{\log(\sin y) + y\tan x}{\log(\cos y) - x\cot y}$ **10..** $\frac{\pi}{4(\sqrt{2}-1)}$ **11.** $\frac{e^{\tan^{-1}\sqrt{x}}}{2\sqrt{x}(1+x)}$

APPLICATION OF DERIVATIVE

** Whenever one quantity y varies with another quantity x, satisfying some rule y = f (x) , then $\frac{dy}{dx}$ (or f $'(x)$) represents the rate of change of y with respect to x and $\left[\frac{dy}{dx}\right]_{x=x_0}$ (or $f'(x_0)$) represents the rate of change of y with respect to x at $x = x_0$.

** Let I be an open interval contained in the domain of a real valued function f. Then f is said to be

(i) **increasing on I** if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) \le f(x_2)$ for all $x_1, x_2 \in$ I.

(ii) **strictly increasing on I** if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) < f(x_2)$ for all $x_1, x_2 \in$ I.

(iii) **decreasing** on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in$ I.

(iv) **strictly decreasing** on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) > f(x_2)$ for all $x_1, x_2 \in$ I.

** (i) f is **strictly increasing in (a, b)** if $f'(x) > 0$ for each $x \in (a, b)$

(ii) f is **strictly decreasing in (a, b)** if $f'(x) < 0$ for each $x \in (a, b)$

(iii) A function will be increasing (decreasing) in **R** if it is so in every interval of **R**.

** Slope of the tangent to the curve y = f (x) at the point (x_0, y_0) is given by $\left[\frac{dy}{dx}\right]_{(x_0, y_0)} \left(= f'(x_0)\right)$.

** The **equation of the tangent at (x_0, y_0)** to the curve y = f (x) is given by $y - y_0 = f'(x_0)(x - x_0)$.

** Slope of the normal to the curve y = f (x) at (x_0, y_0) is $-\frac{1}{f'(x_0)}$.

** The **equation of the normal at (x_0, y_0)** to the curve y = f (x) is given by $y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$.

** If slope of the tangent line is zero, then $\tan\theta = 0$ and so $\theta = 0$ which means the tangent line is parallel to the x-axis. In this case, the equation of the tangent at the point (x_0, y_0) is given by $y = y_0$.

** If $\theta \to \frac{\pi}{2}$, then $\tan\theta \to \infty$, which means the tangent line is perpendicular to the x-axis, i.e., parallel to the y-axis. In this case, the equation of the tangent at (x_0, y_0) is given by $x = x_0$.

** Let f be a function defined on an interval I. Then

(a) f is said to have a maximum value in I, if there exists a point c in I such that $f(c) \ge f(x)$, for all $x \in I$.

The number f (c) is called the maximum value of f in I and the point c is called a point of maximum value of f in I.

(b) f is said to have a minimum value in I, if there exists a point c in I such that $f(c) \le f(x)$, for all $x \in I$.

The number f (c), in this case, is called the minimum value of f in I and the point c, in this case, is called a point of minimum value of f in I.

(c) f is said to have an extreme value in I if there exists a point c in I such that f (c) is either a maximum value or a minimum value of f in I.

The number f (c), in this case, is called an extreme value of f in I and the point c is called an extreme point.

** **Absolute maxima and minima**

Let f be a function defined on the interval I and $c \in I$. Then

(a) f(c) is absolute minimum if $f(x) \ge f(c)$ for all $x \in I$.

(b) f(c) is absolute maximum if $f(x) \le f(c)$ for all $x \in I$.

(c) $c \in$ I is called the critical point off if $f'(c) = 0$

(d) Absolute maximum or minimum value of a continuous function f on [a, b] occurs at a or b or at critical points off (i.e. at the points where $f'$is zero)

If $c_1, c_2, \ldots, c_n$ are the critical points lying in [a , b],
then absolute maximum value of f = $\max\{f(a), f(c_1), f(c_2), \ldots, f(c_n), f(b)\}$

and absolute minimum value of f = min{f(a), f(c_1), f(c_2), … , f(c_n), f(b)}.

** **Local maxima and minima**

(a)A function f is said to have a local maxima or simply a maximum value at x= a if f(a ± h) ≤ f(a) for sufficiently small h

(b)A function f is said to have a local minima or simply a minimum value at x = a if f(a ± h) ≥ f(a).

** First derivative test :A function f has a maximum at a point x = a if

(i) f ′(a) = 0, and

(ii) f ′(x) changes sign from + ve to –ve in the neighbourhood of ‘a’ (points taken from left to right).

However, f has a minimum at x = a, if

(i) f ′(a) = 0, and

(ii) f ′(x) changes sign from –ve to +ve in the neighbourhood of ‘a’.

If f ′(a) = 0 and f ′ (x) does not change sign, then f(x) has neither maximum nor minimum and the point ‘a’ is called point of inflation.

The points where f ′(x) = 0 are called stationary or critical points. The stationary points at which the function attains either maximum or minimum values are called extreme points.

** **Second derivative test**

(i) a function has a maxima at x = a if f ′(x) 3. 4. b = 0 and f ″ (a) <0

(ii) a function has a minima at x = a if f ′(x) = 0 and f ″(a) > 0.

MULTIPLE CHOICE QUESTIONS

1.The rate of change of the area of a circle with respect to its radius r at r= 6cm is

(a) 10 π (b) 12 π (c) 8 π (d) 11 π

2. On which of the following is the function f given by f(x)= x^{100}+sinx+1 strictly decreasing

(a) (0,1) (b) $(\frac{\pi}{2},\pi)$ (c) $(0,\frac{\pi}{2})$ (d) None of these

3.Let the function f : R →R be defined by f(x)=2x + cos x. Then f(x)

(a) has a maximum at x = π (b) has a maximum at x=0

(c) is a decreasing function (d) is an increasing function

4. Which of the following functions is decreasing in$(0 , \frac{\pi}{2})$

(a) sin 2x (b) tan x (c) cos x (d) cos 3x

ASSERTION REASON BASED QUESTIONS

The following question contains STATEMENT-1(Assertion) and STATEMENT-2(Reason) and has the following four choices,only one of which is correct answer. Mark the correct choice.

(a) Statement-1 and Statement-2 are true.Statement-2 is a correct explanation for statement-1.

(b) Statement-1 and Statement-2 are true.Statement-2 is not a correct explanation for statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1is false Statement-2 is true.

Q. STATEMENT-1(Assertion) : The function f(x)= x^3- $3x^2$ + 6x - 10 is strictly increasing on R.

STATEMENT-2(Reason): A strictly increasing function i s an injective map.

SHORT ANSWER TYPE QUESTIONS

1. A stone is dropped into a quiet lake and waves move in circles at the speed of 5cm/sec. At the instant when the radius of circular wave is 8 cm,how fast is the enclosed area increasing ?

2. A balloon ,which always remains spherical on inflation is being inflated by pumping in 900 cubic centimeters of gas per second.Find the rate at which the radius of the balloon increases when the radius is 15 cm.

3. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2cm/sec.How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall ?

4. Sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base.How fast is the height of the sand of the cone increasing when the height is 4 cm ?

5. Find the intervals in which the following functions are strictly increasing or decreasing

(a) $-2x^3-9x^2-12x+1$ (b) $(x+1)^3(x-3)^3$

6. Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

7. Twenty meters of wire is available for fencing of a flower bed in the form of a circular sector. Find the maximum area of the flower bed.

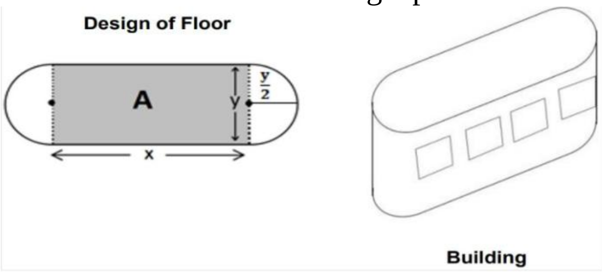
8. The sides of an equilateral triangle is increasing at the rate of 2cm/sec.Find the rate at which the area increases when the side is 10cm.

9. Find the rate of change of volume of a sphere with respect to it's surface area when the radius of the sphere is 12 cm.

10. Let x and y be the radius two circles such that $y=x^2+1$.Find the rate of change of circumference of the second circle with respect to the circumference of first circle.

CASE BASED QUESTIONS

Q.1) An architect designs a building for a multinational company the floor consist of rectangular region with semicircular ends having a perimeter of 200 m



(i) If x and y represent the length and breadth of rectangular region then find the relationship between the variables

(ii) Find the expression for the area of rectangular region.

(iii) Find the maximum area of the region.

Q.2) A telephone company in Gurgaon has 500 subscribers on its list and collect fixed charge of ₹300 per subscriber. The company proposes to increase the annual subscription and it is believed that every increase of ₹1 one subscriber will discontinue the service based on the above information answer the following

(i) if the annual subscription is increased by ₹x per subscriber then find the total revenue are of the company .

(ii) Find average revenue of the company when annual subscription is increased by ₹x per subscriber

(iii) Find the maximum annual revenue of the company.

Q.3) Sonam wants to repair sweet box for Diwali at home for making lower part of box she takes a square piece of cardboard of side 18 cm. Based on the above information answer the following questions

(i) If x cm be the length of each side of a square cardboard which is to be cut off from each corner of the square piece of side 18 cm then find the interval in which x lies.

(ii) Find the expression for volume of the open box.

(iii) Find the maximum volume of the box.

ANSWERS

MCQ's- 1. B 2. D 3. A 4.C 5.B

Short Answer Type Questions

1. 80 π 2.1/ π 3. 8/3cm/sec 4. 1/48 π 5. (a)-(-2,-1) increasing (b) x<-2,x>-1 Decreasing 6.Proof 7.25m^2 8.10$\sqrt{3}$$cm^2$/sec 9. 6$cm^3$/$cm^2$ 10.2x

CASE BASED ANSWERS

Q.1 (i) 2x+ $\pi y = 200$ (ii) 2 (100x-x^2) / π (iii) 5000/ π

Q.2 (i) (500-x)(300+x) (ii) [(500/x)-1][300+x] (iii) 160,000

Q.3 (i) (0,9) (ii) x(18-2x) (18-2x) (iii) 432 cm^3

TEST PAPER-1(20 Marks)

1. 1.The rate of change of the area of a circle with respect to its radius r at r=6cm is

(a) 10 π (b) 12 π (c) 8 π (d) 11 π 1Marks

2.A balloon ,which always remains spherical on inflationis being inflated by pumping in 900 cubic centimeters of gas per second.Find the rate at which the radius of the balloon increases when the radius is 15 cm. 3Marks

3.A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2cm/sec.How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall ? 3Marks

4.Sand is pouring from a pipe at the rate of 12cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base.How fast is the height of the sand of the cone increasing when the height is 4 cm ?

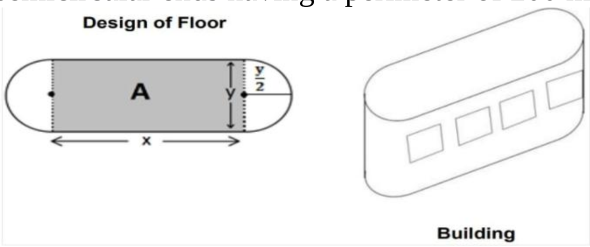
3Marks

5.Find the intervals in which the following functions are strictly increasing or decreasing

(a) $-2x^3-9x^2-12x+1$ (b)$(x+1)^3(x-3)^3$ 3Marks

6.Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θin $\left[0, \frac{\pi}{2}\right]$. 3Marks

Q.7. An architect designs a building for a multinational company the floor consist of rectangular region with semicircular ends having a perimeter of 200 m .



(i) If x and y represent the length and breadth of rectangular region then find the relationship between the variables

(ii) Find the expression for the area of rectangular region.

(iii) Find the maximum area of the region. 4 Marks

ANSWERS

1.B 2.1/ π 3. 8/3cm/sec 4. 1/48 π 5. (a)-(-2,-1) increasing (b) x<-2,x>-1 Decreasing

Q.7. (i) 2x+ $\pi y = 200$ (ii) 2 (100x-x^2) / π (iii) 5000/ π

TEST PAPER-1(30 Marks)

11. A stone is dropped into a quiet lake and waves move in circles at the speed of 5cm/sec. At the instant when the radius of circular wave is 8 cm, how fast is the enclosed area increasing ? 3Marks

12. A balloon ,which always remains spherical on inflation is being inflated by pumping in 900 cubic centimeters of gas per second.Find the rate at which the radius of the balloon increases when the radius is 15 cm. 3Marks

13. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall ? 3Marks

14. Sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base.How fast is the height of the sand of the cone increasing when the height is 4 cm ? 3Marks

15. Find the intervals in which the following functions are strictly increasing or decreasing

(b) $-2x^3-9x^2-12x+1$ (b)$(x+1)^3(x-3)^3$ 3Marks

16. Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$. 3Marks

17. Twenty meters of wire is available for fencing of a flower bed in the form of a circular sector. Find the maximum area of the flower bed. 3Marks

18. The sides of an equilateral triangle is increasing at the rate of 2cm/sec. Find the rate at which the area increases when the side is 10cm. 3Marks

19. Find the rate of change of volume of a sphere with respect to its surface area when the radius of the sphere is 12 cm. 3Marks

20. Let x and y be the radius two circles such that $y= x^2 +1$.Find the rate of change of circumference of the second circle with respect to the circumference of first circle. 3Marks

Answers

1.80 π 2.1/ π 3. 8/3cm/sec 4. 1/48 π 5. (a)-(-2,-1) increasing (b) x<-2,x>-1 Decreasing 6.Proof 7.$25m^2$ 8.$10\sqrt{3}cm^2/sec$ 9. $6cm^3/cm^2$ 10.2x

INDEFINITE INTEGRALS

SOME IMPORTANT RESULTS/CONCEPTS

* $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
* $\int 1.dx = x + C$
* $\int \frac{1}{x^n} dx = -\frac{1}{x^n} + C$
* $\int \frac{1}{\sqrt{x}} = 2\sqrt{x} + C$
* $\int \frac{1}{x} dx = \log_e x + C$
* $\int e^x dx = e^x + C$
* $\int a^x dx = \frac{a^x}{\log_e a} + C$
* $\int \sin x dx = -\cos x + C$
* $\int \cos x dx = \sin x + C$
* $\int \sec^2 x\, dx = \tan x + C$
* $\int \mathrm{cosec}^2 x\, dx = -\cot x + C$
* $\int \sec x . \tan x\, dx = \sec x + C$
* $\int \mathrm{cosec} x . \cot x\, dx = -\mathrm{cosec} x + C$
* $\int \tan x\, dx = -\log|\cos x| + C = \log|\sec x| + C$
* $\int \cot x\, dx = \log|\sin x| + C$
* $\int \sec x\, dx = \log|\sec x + \tan x| + C$

  $= \log\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C$
* $\int \mathrm{cosec} x\, dx = \log|\mathrm{cosec} x - \cot x| + C$

  $= -\log|\mathrm{cosec}\, x + \cot x| + C = \log\left|\tan\frac{x}{2}\right| + c$

* $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C,\ \text{if } x > a$
* $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C,\ \text{if } x > a$
* $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C, = -\frac{1}{a}\cot^{-1}\frac{x}{a} + C$
* $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a} + c = -\cos^{-1}\frac{x}{a} + C$
* $\int \frac{dx}{\sqrt{a^2 + x^2}} = \log|x + \sqrt{x^2 + a^2}| + C$
* $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log|x + \sqrt{x^2 - a^2}| + C$
* $\int \sqrt{x^2 + a^2} dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$
* $\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$
* $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$
* $\int \{f_1(x) \pm f_2(x) \pm .... f_n(x)\} dx$

  $= \int f_1(x)dx \pm \int f_2(x)dx \pm .... \pm \int f_n(x)dx$
* $\int \lambda f(x)dx = \lambda \int f(x)dx + C$
* $\int u.v\, dx = u.\int v.dx - \int \left[\int v.dx\right]\frac{du}{dx}.dx$

II Some Illustration / Examples

MCQ based questions with solutions

Q1. $\int \frac{1}{1+x^2} dx$ is equal to

(a) $\tan^{-1}x + c$ (b) $\sin^{-1}x + c$ (c) $\cos^{-1}x + c$ (d) $\sec^{-1}x + c$

Answer: (a) $\tan^{-1}x + c$

Q2. $\int \frac{e^x}{e^x+1} dx$ is equal to

(a) $x + \log(x^2 + 1)$ (b) e^x (c) $e^x + 1$ (d) $\log(e^x + 1)$

Answer: (d) $\log(e^x + 1)$

Q3. $\int \frac{1}{x \log x} dx$ is equal to

(a) logx +c (b) log(logx+c) (c) log(log(logx))+c (d) None of these

Answer: log(logx)+c

Q4. $\int e^x \sin x\, dx$ is equal to

(a) $\frac{1}{2}e^x(\sin x+\cos x)$ (b) $\frac{1}{2}e^x(\sin x-\cos x)$ (c)$e^x(\sin x-\cos x)$ (d) None of these

Answer: (b) $\frac{1}{2}e^x(\sin x-\cos x)$

ii) Case based study Question

Results:

(i) $\int \sec^2 x\,dx=\tan x+c$

(ii) $\int \sec x\tan x\,dx=\sec x+c$

(iii) $\int \frac{1}{ax+b}\,dx=\frac{\log(ax+b)}{a}+c$; ax+b>0

(iv) $\int \frac{f'(x)}{f(x)}dx=\log[f(x)]+c$

(v) $\int [f(x)]^n f'(x)dx=\frac{[f(x)]^{n+1}}{n+1}+c$; n≠-1

on the above information answer the following

(1) $\int \frac{1}{1+\sin x}dx$ is equal to

(a)tanx+secx+c (b) tanx-secx+c (c) $\tan^2 x-\sec x+c$ (d) tanx- $\sec^2 x+c$

(2) $\int \frac{\cos 2x+2\sin^2 x}{\cos^2 x}$ dx is equal to

(a)tanx+c (b) –tanx+c (c) –sinx+c (d) cosx+c

(3) $\int \frac{x+3}{x^2+4x+3}$dx is equal to

(a)logIx-1I+c (b) –logIx+1I+c (c) logIx+2I +c (d) logIx+1I+c

(4) $\int \frac{2x}{1+x^2}$dx is equal to

(a)log(1-x^2) + c (b) log(2+x^2) + c (c) log(1+x^2) + c (d) log(3+x^2) + c

Answer: Q1. (b)	**Q2. (a)**	**Q3. (d)**	**Q4. (c)**

(iii)Short Answer type questions with solutions:-

Q1. Evaluate: $\int \frac{1}{\sqrt{9+x^2}}$ dx

Sol. $\int \frac{1}{\sqrt{9+x^2}}dx=\int \frac{1}{\sqrt{3^2+x^2}}dx=\log I\frac{x+\sqrt{3^2+x^2}}{3}I+c$

Q2. Find $\int \sin^5\left(\frac{x}{2}\right).\cos\left(\frac{x}{2}\right)dx$

Sol. Put $\sin\frac{x}{2}=y$

⇨ $\frac{1}{2}\cos\left(\frac{x}{2}\right)dx=dy$

⇨ $\cos\left(\frac{x}{2}\right)dx=2\,dy$

so, $I=2\int y^5dy=2\frac{y^6}{6}=\frac{1}{3}y^6+c=\frac{1}{3}\sin^6\left(\frac{x}{2}\right)+c$

Q3. Integrate : $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$

Sol. Put $\tan^{-1}x=y$

⇨ $\frac{1}{1+x^2}dx=dy$

⇨ $I=e^y dy=e^y+c=e^{\tan^{-1}x}+c$

(iv)Long Answer type questions:

Q1. Find $\int \frac{2x}{(x^2+1)(x^2+2)}dx$

Sol. Put $x^2=y$ => 2x dx=dy

So, $I=\int \frac{dy}{(y+1)(y+2)}=\int\left[\frac{1}{(y+1)}-\frac{1}{(y+2)}\right]dy$

=logIy+1I-logIy+2I+c

$=\log I\frac{y+1}{y+2}I+c=\log I\frac{x^2+1}{x^2+2}I+c$

Q2. Evaluate: $\int \frac{dx}{\sqrt{3x^2+6x+12}}$

Sol. $I=\frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{x^2+2x+4}}=\frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{(x+1)^2+(\sqrt{3})^2}}=\frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{(t)^2+(\sqrt{3})^2}}$ (put x+1=t => dx=dt)

$= \frac{1}{\sqrt{3}} \log \left| t + \sqrt{t^2 + 3} \right| + c$ $= \frac{1}{\sqrt{3}} \log \left| (x + 1) + \sqrt{x^2 + 2x + 4} \right| + c$

III Practice Questons

(I) MCQ Questions for practices

1. Given $\int 2^x dx = f(x)+c$ then f(x)=

(a) 2^x (b) $2^x \log e^2$ (c) $\frac{2^x}{\log 2}$ (d) $\frac{2^{x+1}}{x+1}$

2. Given $\int \frac{1}{\sin^2 x \cos^2 x} dx$ is equal to

(a) $\sin^2 x - \cos^2 x + c$ (b) -1 (c) tan x +cot x+ c (d) tan x- cot x +c

3. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$ is equal to

(a) 2(sin x+ x cos θ) +c (b) 2(sin x- x cos θ) + c (c) 2(sin x +2x cos θ) +c (d) 2 (sin x- sin θ) +c

4. $\int \cot^2 x \, dx$ equals to

(a) Cot x – x +c (b) – cot x+ x +c (c) cot x +x +c (d) – cot x-x +c

ASSERTION REASON TYPE QUESTION

5.The following question consists of two statements – Assertion (A) and Reason (R). Answer the question selecting the appropriate option given below:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false
(d) A is false but R is true

Assertion: Geometrically, derivative of a function is the slope of the tangent to the corresponding curve at a point.

Reason: Geometrically, indefinite integral of a function represents a family of curves parallel go each other.

MCQ's 1-(c), 2-(d) 3-(a) 4-(d) 5 . Assertion Reason type Question: (b)

SHORT ANSWER TYPE QUESTIONS

1. Find $\int \frac{3+3\cos x}{x+\sin x} dx$
2. Find $\int \frac{dx}{\sqrt{5-4x-x^2}} dx$
3. Find $\int \frac{x^3-1}{x^2} dx$
4. Find $\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$
5. Find $\int \frac{dx}{x^2+16} dx$
6. Find $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$
7. Find $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$
8. Find $\int \sqrt{1 - \sin 2x} \, dx$
9. Find $\int \frac{(x^2+\sin^2 x)\sec^2 x}{1+x^2} dx$
10. Find $\int e^x \frac{x-3}{(x-1)^3} dx$
11. Find $\int \sin^{-1}(2x) dx$
12. Find $\int \frac{3-5\sin x}{\cos^2 x} dx$
13. Find $\int \frac{\tan^2 x . \sec^2 x}{1-\tan^6 x} dx$
14. Find $\int \sin x \log(\cos x) \, dx$
15. Find $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x . \cos^2 x} dx$

LONG ANSWER TYPE QUESTIONS

1. Find $\int \frac{6x+8}{3x^2+6x+2} dx$
2. Find $\int \frac{1-\tan^2 x}{1+\tan^2 x} dx$
3. Find $\int \frac{x^4}{1+x^{10}} dx$

4. Find $\int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx$

5. Find $\int \frac{\sin 8x}{\sqrt{1-\cos^4 4x}} dx$

Answer (Short answer type Questions):

1. $3 \log(x + \sin x) + c$
2. $\sin^{-1}(\frac{x+2}{3}) + c$
3. $\frac{x^2}{2} + \frac{1}{x} + c$
4. $-\log I \cos x \sin x I + c$
5. $\frac{1}{4} \tan^{-1}\left(\frac{x}{4}\right) + c$
6. $\tan x + c$
7. $\text{Log} \left|\tan x + \sqrt{\tan^2 x + 4}\right| + c$
8. $\sin x + \cos x + c$
9. $\tan x - \tan^{-1} x + c$
10. $\frac{e^x}{(x-1)^2} + c$
11. $x \sin^{-1} 2x + \frac{1}{2}\sqrt{1-4x^2} + c$
12. $3\tan x - 5 \sec x + c$
13. $\frac{1}{6} \log I \frac{1+\tan^3 x}{1-\tan^3 x} I + c$
14. $\cos x\,[1 - \log \cos x] + c$
15. $\tan x - \cot x - 3x + c$

Answer (Long answer type Questions):

1. $\log(3x^2 + 6x + 2) + \frac{2}{3} \log I \frac{x+1-\frac{1}{\sqrt{2}}}{x+1+\frac{1}{\sqrt{2}}} I + c$
2. $\frac{\sin 2x}{2} + c$
3. $\frac{1}{5} \tan^{-1} x^5 + c$
4. $\frac{1}{6} (\tan^{-1} x^3)^2 + c$
5. $-\frac{1}{8} \sin^{-1}(\cos^2 4x) + c$

V. Class Test

1. $\int x^2 \left(1 - \frac{1}{x^2}\right) dx$ is equal to (1)

   (a) $\frac{x^3}{3} - x + c$ (b) $\frac{x^3}{3} + x + c$ (c) x+1 (d) $-\frac{x^3}{3} + x + c$

2. $\int 3^x dx$ is equal to (1M)

(a) 3^x (b) $3^x + c$ (c) $\frac{3^x}{\log 3} + c$ (d) none of these

3. $\int \frac{x^3}{x+1} dx$ is equal to (1M)

(a) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log|1-x| + c$

(b) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log|1-x| + c$

(c) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log|1+x| + c$

(d) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log|1+x| + c$

4. $\int e^x \left(\frac{1-x}{1+x}\right)^2 dx$ is equal to (1M)

(a) $\frac{e^x}{1+x^2} + c$

(b) $\frac{-e^x}{1+x^2} + c$

(c) $\frac{e^x}{(1+x^2)^2}$ +c

(d) $\frac{-e^x}{(1+x^2)^2}$ +c

5.Evaluate $\int \frac{\sec^2(\log x)}{x} dx$ (2M)

6. Evaluate $\int \frac{x^2+1}{(x-1)^2(x+3)} dx$ (2M)

7. Evaluate $\int \cos^2 3x \, dx$ (2M)

8. Evaluate $\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx$ (5M)

9 Evaluate $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$ (5M)

Class Test II

1.Given $\int 2^x dx$= f(x)+c then f(x) is (1M)

(b) 2^x (b) $2^x \log e^2$ (c) $\frac{2^x}{\log e^2}$ (d) $\frac{2^{x+1}}{x+1}$

2 Given $\int \frac{1}{\sin^2 x \cos^2 x} dx$ is equal to (1M)

(b) $\sin^2 x - \cos^2 x$ +c (b) -1 (c) tan x +cot x+ c (d) tan x- cot x +c

3 .$\int \tan^{-1}\sqrt{x}\, dx$ is equal to (1M)

(a)$(1+x)\tan^{-1}\sqrt{x} - \sqrt{x}$+c

(b) $x\tan^{-1}\sqrt{x} - \sqrt{x}$+c

(c)$\sqrt{x} - x\tan^{-1}\sqrt{x}$+c

(d)$\sqrt{x} - (x+1)\tan^{-1}\sqrt{x} + c$

4. $\int \sqrt{\frac{1-\cos 2x}{1+\cos 2x}} dx$ is equal to (1M)

(a) $-\log|\sec x|$ +c

(b) $\log|\sec x|$ +c

(b) $\log|\tan x|$ +c

(c) $\log|\cos x|$ +c

5. Evaluate $\int \sin^3(2x+1) dx$ (2M)

6. Evaluate $\int \sin^{-1} x \, dx$ (2M)

7. Evaluate $\int \sin 5x \sin 3x \, dx$ (2M)

8. Evaluate $\int \frac{1}{x\sqrt{x^4-1}} dx$ (2M)

11. Evaluate $\int \frac{2+\sin 2x}{1+\cos 2x} e^x \, dx$, (2M)

9. Evaluate $\int \frac{\cos 5x+\cos 4x}{1-2\cos 3x} dx$ (3M)

10. Evaluate $\int \frac{\sin x - x\cos x}{x(x+\sin x)} dx$ (3M)

12. Evaluate $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$, (5M)

13. Evaluate $\int \frac{1}{\sqrt{\sin^3 x \sin(x+a)}}$ dx(5M)

DEFINITE INTEGRALS
SOME IMPORTANT RESULTS/CONCEPTS

* $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F(x) = \int f(x)\,dx$

* $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

* $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

* $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

* $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

* $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

* $\int_{-a}^a f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function of } x. \\ 0 & \text{if } f(x) \text{ is an odd function of } x \end{cases}$

* $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x). \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$

MCQ's

QNo	Question	Mark	Correct Response
1	$\int_{-\pi/4}^{\pi/4} Sec^2x dx$ (a) -1 (b) 0 (c) 1 (d) 2	1	d
2	$\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4} dx$ is (a) 6 (b)0 (c) 1 (d) 4	1	a
3	$\int_0^{2/3} \frac{dx}{4+9x^2}$ is (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\pi/24$ (d) $\pi/4$	1	c
4	$\int_0^1 \frac{dx}{1+x^2}$ is (a) 0 (b) $\pi/4$ (c) $\pi/12$ (d) $\pi/6$	1	b
5	$\int_{-1}^1 x^{17} + x^{71}\, dx$ is (a) 1 (b)0 (c)2 (d) 4	1	b
6	$\int_0^{\pi/4} \tan^2x\, dx$ is (a)1-π/4 (b)1+π/4(c)1-π/2 (d) 1+π/2	1	a

Problems for Practice

All the questions carry 3 marks

1 Evaluate $\int_0^1 \frac{\sin x}{1+\sin x} dx$

2 Evaluate $\int_0^1 \cot^{-1}(1-x-x^2)dx$

3 Evaluate $\int_0^{\pi/2}(\sqrt{\tan x}+\sqrt{\cot x})\, dx$

4 Evaluate $\int_{-1}^{3/2}|x\sin\pi x|dx$

5 Evaluate $\int_0^1 \frac{xdx}{1+x^2}$

6 Evaluate $\int_1^3 |2x-1|\, dx$

7 Evaluate $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\, dx$

8 Evaluate $\int_{-2}^{2}\frac{x^2dx}{1+5^x}$

9 Evaluate $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{tanx}}$

10 Evaluate $\int_0^{\pi/2}(2logcosx - logsin2x)dx$

All the questions carry 5 marks

1 Evaluate $\int_{-1}^{2}|x^3 - x|dx$

2 Evaluate $\int_{-6}^{6}|x+3|\ dx$

3 Evaluate $\int_0^{\pi/2}\frac{xsinxcosx}{sin^4x + cos^{4x}}dx$

4 Evaluate $\int_0^{\pi}\frac{xdx}{a^2cos^2x+b^2sin^2x}dx$

5 Evaluate $\int_0^{\pi}\frac{xtanx}{tanx + secx}dx$

MCQ

1 $\int_0^2(x^2+3)dx$ is	(a)8	(b) 25/3	(c)26/3	(d) 9
2 $\int_0^{\pi}\sin^2 xdx$ is	(a)$\frac{\pi}{2}$	(b) $\frac{\pi}{3}$	(c) $\frac{\pi}{4}$	(d)π
3 $\int_0^{\pi}\frac{dx}{1+sinx}$ is	(a) $\frac{\pi}{4}$	(b) $\frac{\pi}{3}$	(c)$e^{\pi/2}$	(d) 2
4 $\int_0^1\frac{1-x}{1+x}dx$	(a) $\frac{\log 2}{2}$	(b) $\frac{\log 2}{2}$-1	(c) 2log2-1	(d) 2log2 +1
5$\int_0^{\pi/6}$cosx cos2x dx	(a) ¼	(b) 5/12	(c) 1/3	(d)-1/12
6 $\int_0^1\frac{dx}{e^x+e^{-x}}$	(a) 1-$\pi/4$	(b) $\tan^{-1}e$	(c)) $\tan^{-1}e + \pi/4$	(d) $\tan^{-1}e - \pi/4$
7 $\int_0^1\sqrt{\frac{1-x}{1+x}}dx$	(a) $\frac{\pi}{2}$	(b)$\frac{\pi}{2} - 1$	(c)$\pi/2 + 1$	(d)0
8 $\int_0^{\pi/2}\log(\frac{4+3sinx}{4+3cosx})dx$	(a) 2	(b) 3/4	(c) 0	(d) -2
9 $\int_0^1\tan^{-1}\frac{2x-1}{1+x-x^2}dx$	(a) 1	(b) 0	(c) -1	(d)$\pi/4$
10 $\int_0^{\pi/2}\frac{\sqrt{tanx}}{\sqrt{tanx}+\sqrt{cotx}}dx$ is	(a)$\frac{\pi}{2}$	(b) $\pi/3$	(c) $\pi/4$	(d) π
11 $\int_0^{\pi/2}\frac{dx}{1+tanx}$=	(a) $\frac{\pi}{2}$	(b) $\pi/3$	(c) $\pi/4$	(d) π
12 $\int_{-1}^{1}\sin^3x\cos^2x\ dx$	(a) 0	(b)1	(c)2	(d)3

ASSERTION AND REASONING BASED PROBLEMS

In the following questions a statement of assertion Statement 1 is followed by statement of reason Statement 2 Mark the correct choice as

(a) If statement 1 and statement 2 is true and statement 2 is the correct explanation of 1

(b) If statement 1 and statement 2 is true and statement 2 is not the correct explanation of 1

(c) If statement 1 is true and statement 2 is false

(d) If statement 1 is false and statement 2 is true

Now answer the following

1 Statement I$\int_0^{\pi/2}\sin^2xdx = \pi/4$ Statement II $\int_0^a f(x)dx = \int_0^a f(a-x)\ dx$

2Statement I $\int_2^3\frac{\sqrt{x}dx}{\sqrt{x}+\sqrt{5-x}}$ =1/2 Statement II $\int_0^{2a}f(x)dx$=2$\int_0^a f(x)dx$ if f(x)=f(2a-x)

ANSWERS

SHORT ANSWER 3 MARKS			
1	$\boldsymbol{\pi - 2}$	**6**	**6**
2	$\boldsymbol{\frac{\pi}{2} - \log 2}$	**7**	$\boldsymbol{\pi}$
3	$\boldsymbol{\pi/2\sqrt{2}}$	**8**	**8/3**
4	**3/π +1/π^2**	**9**	$\boldsymbol{\pi/12}$
5	**Log2/2**	**10**	$\boldsymbol{\frac{\pi}{2} - \log(\frac{1}{2})}$

LONG ANSWER 5MARKS	
1	11/4
2	45
3	$\frac{\pi^2}{16}$
4	$\frac{\pi^2}{2ab}$
5	$\frac{\pi(\pi-2)}{2}$

S NO	ANSWER	S No	Correct answer
1	c 26/3	7	(b) $\frac{\pi}{2} - 1$
2	A $\frac{\pi}{2}$	8	(c) 0
3	D 2	9	(b) 0
4	C 2log2 -1	10	(a)$\frac{\pi}{2}$
5	(d)-1/12	11	(a) $\frac{\pi}{2}$
6	(d) $\tan^{-1} e - \pi/4$	12	(a) 0

ASSERTION AND REASONING

1 Choice (a) is correct

2 Choice (c) is correct

QUESTION PAPER OF DEFINITE INTEGRAL

Marks 20

Q No	SECTION A	Marks
1	$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$ value is (a)$\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{`12}$	1
2	$\int_2^6 \frac{\sqrt{8-x}}{\sqrt{x}+\sqrt{8-x}} dx$ is (a) 0(b) 1(c)2 (d) 3	1
3	$\int_1^3 (x-1)(x-2)(x-3)dx$ is (a)3 (b) 1(c)2 (d)0	1
	SECTION B The question is response based 1 mark is awarded for writing correct answer and 2 marks is awarded for writing the correct explanation	
4	$\int_1^2 e^x(\frac{1}{x}-\frac{1}{x^2})dx$ (a)e(e-2)/2 (b)e(e-1) (c) 0 (d)1	2
	SECTION CThe question is case based	
5	For a function f(x) if f(-x)=f(x) it is called even function and f(-x)=-f(x) is called an odd function Again we have $\int_{-a}^{a} f(x)dx = \begin{cases} \int_0^a 2f(x)dx \text{ if } f(x) \text{ is even} \\ 0 \text{ if } f(x) \text{ is odd} \end{cases}$ (i)What is the nature of $f(x)=x^7 \sin x$ (ii)$\int_{-\pi}^{\pi} x^7 \sin x dx$ is (iii)$\int_{-\pi}^{\pi} x \sin x dx$ is	1 1 2
	SECTION DBoth questions are 3 marks each Evaluate $\int_0^{2\pi} \frac{\cos x}{\sqrt{4+3\sin x}} dx$	3
7	Evaluate $\int_{-1}^{1} 5x^4(\sqrt{x^5}+1)dx$	3
	SECTION EThe question is of 5 marks	
8	$\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x dx}{\cos^2 x + \sin^4 x}$	5

MARKING SCHEME

QNo	Value point	Mark
1	**Correct answer b**	**1**
2	**Correct answer c**	**1**
3	**Correct answer d**	**1**
4	**Correct answer is (a) The correct explanation is the student should apply the property that $\int e^x (f(x) + f'(x))dx = e^x f(x)$ c nad substitute the lower and upper limit**	**2**
5	**(i) Odd** **(ii) 0** **(iii) 2π**	**1** **1** **2**
6	**Correct substitution 4+3Sinx=t** **Correct change of limit** **Correct answer 0**	**1** **1** **1**
7	**Correct substitution** **Correct change of limit** **Correct answer 4√2/3**	**1** **1** **1**
8	**Correctly changing numerator to Sin2x and denominator to Cos 2x** **Correct substitution** **Solving and getting correct answer $\frac{\pi}{6\sqrt{3}}$**	**2** **1** **2**

QUESTION PAPER OF DEFINITE INTEGRAL

Marks 30

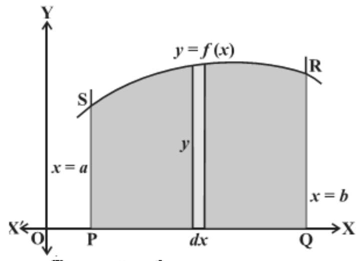
Q No	SECTION A	Marks
1	$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\operatorname{Cos} x}+\sqrt{\operatorname{Sin} x}} dx$ value is (a)$\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{`12}$	1
2	Evaluate $\int_{-1}^{1} \sin^3 x \cos^2 x dx$ (a) 0 (b)1 (c)2(d) 3	1
3	$\int_{-1}^{1} \frac{\sin x - x^2}{3-\|x\|} dx$ is (a) 0 (b) 1(c) 3 (d)5	1
	SECTION BThe question is response based 1 mark is awarded for writing correct answer and 2 marks is awarded for writing the correct explanation	
4	$\int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)} dx$ (a) a (b) –a (c) 1(d) 0	2
5	$\int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}} dx$ (a) 0 (b) 1 (c) 2 (d) 3	2
6	$\int_0^{2/3} \frac{dx}{4+9x^2}$ (a) $\frac{\pi}{12}$ (b) $\frac{\pi}{24}$ (c) $\pi/8$ (d)$\pi/2$	2
7	$\int_0^{\pi/2} \cos x e^{\sin x} dx$ (a) e+1 (b0 e (c) e-1 (d) 1	2
8	$\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$ (a) aπ (b) $\pi/2$ (c) a $\pi/4$ (d)	
	SECTION CAll the questions carry 3 marks each	
9	Evaluate the integral $\int_0^1 x e^{x^2} dx$	3
10	Evaluate $\int_0^4 \|x-1\| dx$	3
11	Evaluate $\int_0^2 x\sqrt{2-x}\, dx$	3
	SECTION DThe question are of 5 marks	
12	Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx$	5
13	$\int_0^{\frac{\pi}{2}} \frac{x dx}{\sin x + \cos x}$	5

MARKING SCHEME

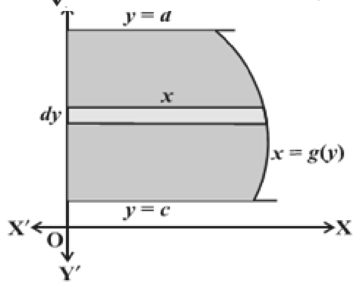
QNo	Value point	Mark
1	Correct answer a	1
2	Correct answer c	1
3	Correct answer a	1
4	Correct answer is (a) The correct explanation is the student should apply the property that $\int_0^a f(x)\,dx = \int_0^a f(a-x)dx$and substitute the lower and upper limit	2
5	Correct answer c Student should use the formula sin2x=2sinxcosx and change the denominator into sinx +cosx square then integrate	1 1
6	Correct answer is $\frac{\pi}{24}$Student should apply correct formula and apply limit	1+1
7	Correct answer is c (e-1) The student should correctly change the variable and integrate	1+1
8	Correct answer is b Correct application of the property and applying limit	1+1+1
9	Putting x^2 =t and 2xdx=dtChanging limit $\int_0^1 e^x/2$ dxGetting answer ½(e-1)	1+1+1
10	Applying property $\int_0^4 \lvert x-1\rvert\, dx = -\int_0^1 x-1\,dx + \int_1^4 x-1\,dx = 5$	1+1+1
11	Applying the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ Getting $\int_0^2 2x^{3/2} - x^{5/2}dx$ and integratingGetting answer $16\sqrt{2/15}$	1 1+1
12	Applying the property$\int_0^a f(x)dx = \int_0^a f(a-x)dx$Getting I =$\int_0^{\uparrow\pi} \frac{(\pi-x)\tan x}{\sec x+\cos x}$ Adding and getting 2I=$\int_0^{\pi} \frac{\pi \sin x}{1+\cos 2x}$dxSubstituting sinx as t and changing limit Getting answer as $\frac{\pi^2}{4}$	1+1 1+1 1
13	Applying the property$\int_0^a f(x)dx = \int_0^a f(a-x)dx$ Applying and getting $\int_0^{\pi/2} \frac{(\frac{\pi}{2}-x)}{\cos x+\cos x}$ dx Adding and getting $\int_0^{\pi/2} \frac{\frac{\pi}{2}dx}{\cos x+\cos x}$ Dividing the denominator by $\sqrt{2}$ Integrating and getting $\frac{\pi}{2\sqrt{2}}$log ($\sqrt{2}$ -1)	1 1 1 1 1

APPLICATIONS OF THE INTEGRALS
SOME IMPORTANT RESULTS/CONCEPTS

** Area of the region PQRSP = $\int_a^b dA = \int_a^b y\,dx = \int_a^b f(x)\,dx$.



** The area A of the region bounded by the curve x = g (y), y-axis and the lines y = c, y = d is given by A= $\int_c^d x\,dy = \int_c^d g(y)\,dy$



MCQ

Q1. Find the area enclosed by curve $4x^2 + 9y^2 = 36$

(a) 6л sq units (b) 4л sq units (c) 9л sq units (d) 36л sq units

Sol.

$4x^2 + 9y^2 = 36$

$\frac{4X^2}{36} + \frac{9y^2}{36} = 1$

$\frac{X^2}{9} + \frac{y^2}{4} = 1$

a = 3 , b = 2

Area of ellipse = л ab = л.3.2 = 6 л sq. units

Q2. The area of the region bounded by the curve x² = 4y and the straight line x = 4y – 2 is

a. ⅜ sq. units (b) ⅝ sq. units (c)⅞ sq. units (d) 9/8 sq. units

Answer: (d) 9/8 sq. units

Explanation:

For the curves $x^2 = y$ and x = 4y-2, the points of intersection are x = -1 and x = 2.

Hence, the required area, A = $_{-1}\int^{2}$ {[(x+2)/4]- [x^2/4] } dx

Now, integrate the function and apply the limits, we get

A = (¼)[(10/3)-(-7/6)]

A = (¼)(9/2) = 9/8 sq. units

Hence, the correct answer is option (d) 9/8 sq. units

Q3. The area enclosed between the graph of y = x^3 and the lines x = 0, y = 1, y = 8 is

(a) 7 (b) 14 (c) 45/4 (d)None of these

Answer: (c) 45/4

Explanation:

Given curve, $y=x^3$ or $x = y^{1/3}$.

Hence, the required area, A = $_1\int^8 y^{1/3}\,dy$

A = $[(y^{4/3})/(4/3)]_1^8$

Now, apply the limits, we get

A = (¾)(16-1)

A = (¾)(15) = 45/4.

Hence, option (c) 45/4 is the correct answer.

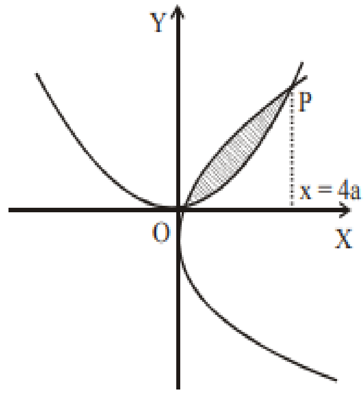
Q4. The area of the region bounded by the curve $y^2 = x$, the y-axis and between y = 2 and y = 4 is
(a) 52/3 sq. units (b)54/3 sq. units (c)56/3 sq. units (d)None of these
Answer: (c) 56/3
Explanation:
Given: $y^2 = x$
Hence, the required area, $A = {}_2\int^4 y^2\, dy$
$A = [y^3/3]_2^4$
$A = (4^3/3) – (2^3/3)$
A = (64/3) – (8/3)
A = 56/3 sq. units.
Q5. Area of region bounded by the curve $y^2 = 4x$,and its latus rectum above x axis
(a) 0 sq units (b) 4/3 sq units (c) 3/3 sq units(d) 2/3 sq units
Ans (b) 4/3 sq units
Q6. Area of region bounded by $y= x^3$, x axis , x=1 and x=-2
(b) -9 sq units (b) -15/4 sq units (c) 15/4 sq units (d) 17/4 sq units
Ans (b) -15 /4
Q7. Area of region bounded by curve y=x and $y = x^3$ is
(a) 1/2 sq units (b) 1 /4 sq units (c) 9/2 sq units (d) 9/4 sq units
Ans. (a) 1/2 sq units
Q8. The area enclosed by the circle $x^2+y^2 = 2$ is equal to:
(a) 4л sq units (b) 2√2л sq units (c) $4л^2$ sq units (d) 2л sq units
Ans. (d) 2л sq units
Q9. The area of the region bounded by the parabola $y = x^2$ and y = |x| is
(a) 3 (b) 1 / 2 (c) 1/3 (d) 2
Ans. (c) 1/3
Q10. The area of the region enclosed by the parabola $x^2 = y$, the line y = x + 2 and the x-axis, is
(a) 5 / 9 (b) 9 / 5 (c) 5 /6 (d) 2 /3
Ans (c) 5 /6

ASSERTION - REASON TYPE QUESTIONS :
Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct
Q.11 Assertion : The area bounded by the curve y = cos x in I quadrant with the coordinate axes is 1 sq. unit.
Reason : $\int_0^{\frac{л}{2}} \cos x\, dx = 1$
Ans (a) $\int_0^{\frac{л}{2}} \cos x\, dx = [\sin x]_0^{\pi/2} = \sin\frac{л}{2} - 0 = 1$
Q.12 Assertion : The area bounded by the curves $y^2 = 4a^2 (x – 1)$ and lines x = 1 and y = 4a is 16a 3 sq. units.
Reason : The area enclosed between the parabola $y^2 = x^2 – x + 2$ and the line y = x + 2 is 8 3 sq. units
Ans. (c)
Q.13Assertion : The area bounded by the circle $x^2 + y^2 = a^2$ in the first quadrant is given by
$\int_0^a \sqrt{a^2 - x^2}\, dx$
Reason : The same area can also be found by $\int_0^a \sqrt{a^2 - y^2}\, dy$
Ans. (b)
Q.14Assertion : The area bounded by the circle y = sin x and y = –sin x from 0 to л is 3 sq. unit.
Reason :The area bounded by the curves is symmetric about x-axis.
Ans. (d)

Short Answer Type Question

Q. Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$

Sol. Solving given curves for x , we get $x = 0$ and $x = 4a$



So the Required area = $\int_0^{4a}\left(\sqrt{4ax} - \frac{x^2}{4a}\right) dx$

$= \left[2\frac{2}{3}\sqrt{a}x^{\frac{3}{2}} - \frac{x^3}{12a}\right]_0^{4a}$

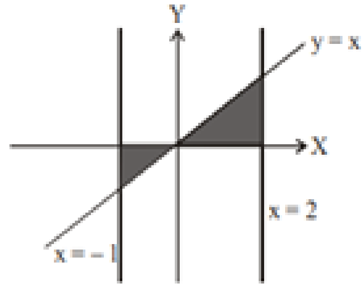
$= \frac{32}{3}a^2 - \frac{16}{3}a^2$

$= \frac{16}{3}a^2$

Q. Find the area bounded by the line y = x, x-axis and lines x = – 1 to x = 2.

Sol. We have, y = x, a line

Required Area = Area of shaded region



$= \left|\int_{-1}^{0} x\,dx\right| + \left|\int_0^2 x\,dx\right| = \left|\frac{x^2}{2}\right|_{-1}^{0} + \left|\frac{x^2}{2}\right|_0^2$

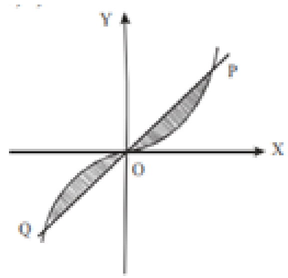
$= \left|-\frac{1}{2}\right| + \left|\frac{2}{1}\right| = 2 + \frac{1}{2} = \frac{5}{2}$ sq. units

Q. Find the area between the curves $y = x$ and $y = x^3$.

From the given function $y = x$ and $y = x^2$

$x^3 = x \Rightarrow x^3 - x = 0 \Rightarrow x(x-1)(x+1) = 0$

$x = 0, 1, -1$



The required area is symmetrical about the origin as shown in the diagram, So

Required Area $= 2\int_0^1 (x - x^3)\,dx$

$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]$

$= 2\left[\frac{1}{2} - \frac{1}{4}\right] = \frac{1}{2}$

Long Answer Type Questions :

Q. Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and the x-axis,

Sol. From the given equation

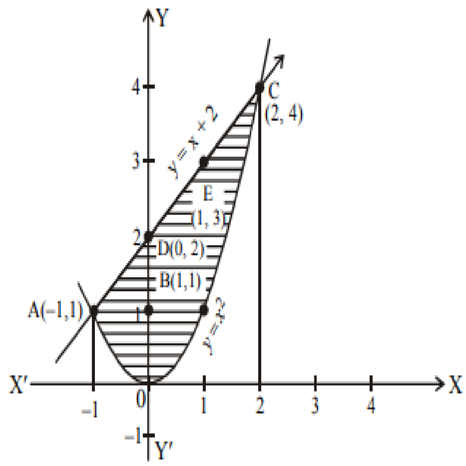
$x^2 = y$ and $y = x + 2$

$\Rightarrow x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = 2,\ x = -1$



For the parabola with vertex (0,0) and the axis of parabola is y-axis

	A	O	B	C
X	-1	0	1	2
Y	1	0	1	4

For the line y = x+2

	A	D	E	C
X	-1	0	1	2
Y	1	2	3	4

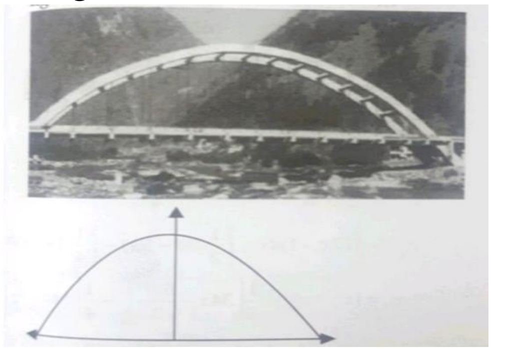
So the Required area = $\int_{-1}^{2}(x+2)dx$ - $\int_{-1}^{2} x^2\ dx$

$= \left[\frac{(x+2)^2}{2}\right]_{-1}^{2} - \left[\frac{x^3}{2}\right]_{-1}^{2}$

$= \frac{1}{2}[16 - 1]\left[\frac{8}{3} + \frac{1}{3}\right] = \frac{15}{3} - 3 = \frac{9}{2}$

CASE STUDY QUESTION : 1

The bridge connects two hills 100 feet apart. The arch of the bridge is in a parabolic form. The highest point on the bridge is 10 feet above the road at the middle of the bridge as seen in the figure



Based on the information given above, answer the following questions:

(i) the equation of the parabola designed on the bridge is

(a) $x^2 = 250\ y$

(b) $x^2 = -\ 250\ y$

(c) $y^2 = 250\ x$

(d) $y^2 = -\ 250\ x$

(ii) the value of the integral

(a) 1000 / 3 (b) 2500 / 3 (c) 1200 (d) 0

(iii) the integrand of the integral function.

(a) Even (b) Odd (c) Neither odd nor even (d) None

(iv) The area formed by the curve x 2 =250 y , x axis , y=0 and y=10 is

(a) $1000\sqrt{2}$ / 3 (b) 4 / 3 (c) 2000 / 3 (d) 0

(v) The area formed by the curve x 2 =250y, y axis , y=2 and y=4 is

(a) $1000\sqrt{2}$ / 3 (b) 0 (c) 1000 / 3 (d) none of these

Answers (i)b (ii) a (iii) a (iv) c (v) d

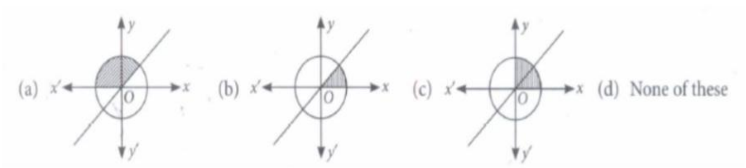
CASE STUDY QUESTION : 2

Consider the curve and line y = x in the first quadrant based on the given information, answer the following questions .

(i) Paint the intersection of both the given curve is

(a) (0,4) (b) $(0, 2\sqrt{2})$ (c) $(2\sqrt{2}, 2\sqrt{2})$ (d) $(2\sqrt{2},4)$

(ii) Which of the following shaded portion represent the area bounded by the given two curves ?



(iii) The value of the integral $\int_0^{2\sqrt{2}} x\,dx$ is

(a) 0 (b) 1 (c) 2 (d) 4

(iv) The value of the integral $\int_0^{2\sqrt{2}} \sqrt{16 - x^2}\,dx$ is

(i) 2(л-2) (b) 2(л-8) (c) 4(л-2) (d) 4(л+2)

(v) The area bounded the given curves is

(i) 3л sq. units (ii) л/2 sq. units (iii) л sq. units (d) 2л sq. units

CASE STUDY QUESTION : 3

Consider the following equations of curves y = cosx, y = x+1 and y=0. On the basis ofabove information, answer the following questions.

(i) The curves y = cosx and y =x+1 meet at(a) (1, 0) (b) (0 ,1) (c) (1 ,1) (d) (0, 0)

(ii) y = cosx meet x-axis at(a) (-л/2, 0) (b) (л/2, 0) (c) both (a) and (b) (d) None of these.

(iii) Value of the integral $\int_{-1}^{0}(x+1)dx$ is(a) 1 /2 (b) 2 /3 (c) 3 /4 (d) 1 /3

(iv) Value of the integral $\int_0^{\pi/2} \cos x\,dx$ is (a) 0 (b) -1 (c) 2 (d) 1

(v) Area bounded by the given curves is

(i) 1 /2 sq. units (ii) 3 /2 sq. units (iii) 3 /4 sq. units (d) 1 /4 sq. units

Short Answer type questions (Unsolved)

Q1. Find the area enclosed between curves $y = x^2 + 2$, $y = x$, $x = 0$, $x = 3$

Ans. 21 /2 sq. units

Q2. Find the area of the region bounded by the curve y=sinx between the lines x=0 , x=π/2 and the x-axis.

Ans 4 sq units

Q4. Find the area enclosed between curves $y = 4x – x^2$, $0 \leq x \leq 4$, x-axis

Ans. 32 /3 sq. units

Q5. Find the area enclosed between curves $y = x^2 + 2$, $y = x$, $x = 0$, $x = 3$

Ans. 21 /2 sq. units

Q6. Find the area $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$ { (x , y) : $x^2 + y^2 < 1 < x+y$}

Ans. π/4 - 1/ 2 sq. units.

Q7. Find the area enclosed between $y^2 = 4ax$ and its latus rectum.

Ans. 8 a^2/3 sq. units.

Q8. Find the area enclosed between curves $y = x^3$, $x = - 2$, $x = 1$, $y = 0$

Ans. 15 /4 sq. units

Q9. Find the area bounded by the curve y = cos x between x = 0 and x = 2π

Ans. 4 sq. units

Long Answer Type Questions : (Unsolved)

Q1. Find the area of the region bounded by the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Ans. *π*ab.

Q2. Using the method of integration find the area bounded by the curve $|x| + |y| = 1$

Ans. 4 sq. units

Q3. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

Ans. 4π + 32/3 sq. units

Q4. Draw the rough sketch and find the area of the region bounded by two parabolas $4y^2 = 9x$ and $3x^2 = 16y$ by using method of integration.

Ans. 4 sq. units

Q5. Find the area of the region bounded by the line y = 3x + 2, the x-axis and the ordinates x = –1 and x = 1.

Ans. 13 /3 sq. units.

DIFFERENTIAL EQUATIONS

SOME IMPORTANT RESULTS/CONCEPTS

** Order of Differential Equation : Order of the heighest order derivative of the given differential equation is called the order of the differential equation.

** Degree of the Differential Equation : Heighest power of the heighest order derivative when powers of all the derivatives are of the given differential equation is called the degree of the differential equatin

** Homogeneous Differential Equation : $\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}$, where $f_1(x,y)$ & $f_2(x,y)$ be the homogeneous function of same degree.

** Linear Differential Equation :

i. $\frac{dy}{dx} + py = q$, where p & q be the function of x or constant.

Solution of the equation is : $y.e^{\int p\,dx} = \int e^{\int p\,dx}.q\,dx$, where $e^{\int p\,dx}$ is Integrating Factor (I.F.)

ii. $\frac{dx}{dy} + px = q$, where p & q be the function of y or constant.

Solution of the equation is: $x.e^{\int p\,dy} = \int e^{\int p\,dy}.q\,dy$, where $e^{\int p\,dy}$ is Integrating Factor (I.F.)

Example1. The order and degree of the differential equation

$(\frac{dy}{dx})^2 + \frac{d^2y}{dx^2} + 5 = 0$ is given by

(a) order 1 and degree 2 (b) order 2 and degree 2

(c) order 2 and degree 1 (d) order 1 and degree 1

Solution : option (c)

Highest order derivative is $\frac{d^2y}{dx^2}$ So Order is 2.

Also power of $\frac{d^2y}{dx^2}$ is one So degree is 1.

Example 2 . The degree and order of the differential equation

$\frac{dy}{dx} + \sin(\frac{dy}{dx}) = 0$. is given by

(a) 1 and 1 (b) 1 and not defined

(c) not defines and 1 (d) None of these

Solution: option (b)

Here highest order derivative is $\frac{dy}{dx}$

So Order of differential equation is **1**.

Differential equation cannot be written as a polynomial in derivatives. Hence degree is **not defined**.

Example 3 . The number of arbitrary constants in General solution of a differential equation of order 2 is

(a) 2 (b) 1 (c) 3 (d) 0

Solution : option (a)

Since the number of arbitrary constants in general solution of a differential equation is equal to order of the differential equation . So here number of arbitrary constants in the general solution will be 2 as order of differential equation is given to be 2

Example 4 . The Integrating Factor of the given differential equation

$\frac{dy}{dx} - \frac{y}{x} = 2x^2$ is

(a) x^2 (b) x (c) $-\frac{1}{x}$ (d) $\frac{1}{x}$

Solution : option (d)

Comparing the given differential equation with

$\frac{dy}{dx} + Py = Q$ we get that here P = $-\frac{1}{x}$ and Q = $2x^2$.

Now Integrating factor (I.F.) = $e^{\int p\,dx} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = e^{\log\frac{1}{x}} = \frac{1}{x}$

Example 5. Two statements A and R are given below . Chose correct option from given options

Assertion (A) : The Integrating Factor of the differential equation $\frac{dy}{dx} - y = \text{Cos}x$ is e^{-x}

Reasoning (R) : A function of the form f (x , y) = x^n g ($\frac{y}{x}$) is called homogeneous function .

(a) Both A and R are true and R is the correct explanation of A
(b) both A and R are true but r is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true .

Solution : Option (b)
In the assertion statement A , P = - x
Integrating factor (I.F.) = $e^{\int p\,dx} = e^{\int -1\,dx} = e^{-x}$
Therefore Statement A is True
Statement R is also true (By definition of homogeneous Functions) but clearly R is not the correct explanation of A .

Example 6. Verify that the function y = acosx + bsinx is solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$

Solution : y = acosx + bsinx
Differentiating both sides w.r.t x

$\frac{dy}{dx}$ = - asinx + bcosx

Again differentiating both sides w.r.t x

$\frac{d^2y}{dx^2}$ = - acosx + (-bsinx) = - (acosx + bsinx)

Now L.H.S. = - (acosx + bsinx) + acosx + bsinx = 0 = R.H.S.
Hence it is verified that the given function is solution of given differential equation .

Example 7. Solve $e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$

Solution : The equation is $e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$

$$\frac{dy}{dx} = -\frac{e^x \tan y}{(1 - e^x)\sec^2 y}$$

$$\frac{dy}{dx} = -\left(\frac{e^x}{1 - e^x}\right)\left(\frac{\tan y}{\sec^2 y}\right)$$

By using variable separating method

$$\frac{\sec^2 y}{\tan y}dy = \frac{e^x}{e^x - 1}dx$$

Integrating both sides $\int \frac{\sec^2 y}{\tan y}dy = \int \frac{e^x}{e^x - 1}dx$

Log (tan y) = $\log(e^x - 1)$ + log c
Log (tan y) = $\log(e^x - 1)c$ by using (log a + log b = log ab)
tan y = $(e^x - 1)c$.

Example 8. Solve the following Homogeneous differential equation ;
2xy dy = ($x^2 + y^2$) dx

Solution : write the given equation as $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$

Since it is Homogeneous differential equation

Put y = vx $\Rightarrow \frac{dy}{dx}$ = v + x $\frac{dv}{dx}$ in the given equation

So , v + x $\frac{dv}{dx} = \frac{1 + v^2}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1 + v^2}{2v} - v \quad \Rightarrow x\frac{dv}{dx} = \frac{-(v^2 - 1)}{2v}$

Separating the variables and writing the equation as

$\frac{2v}{v^2 - 1}dv = -\frac{dx}{x}$

Integrating both sides and getting

$\int \frac{2v}{v^2-1}\,dv = -\int \frac{dx}{x} \Rightarrow \log|(v^2-1)| = -\log|x| + \log|c|$

$\Rightarrow \log|x(v^2-1)| = \log c \Rightarrow x(v^2-1) = \pm c = c_1$

Now replace v by $\frac{y}{x}$ to get $x^2 - y^2 = cx$

Example 9. Solve $(1 + x + y) = \frac{dx}{dy}$

Solution : $x+y+1 = \frac{dx}{dy}$

$\frac{dx}{dy} - x = y + 1$

This is like Type 2 linear differential equation as $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y.

Here $P = -1$ and $Q = 1 + y$

Integrating factor is $e^{\int -1\,dy} = e^{-y}$

Solution is

$x.e^{-y} = \int (1+y)\,e^{-y}\,dy + c$

$$= \int e^{-y}\,dy + \int y.e^{-y}\,dy + c = -e^{-y} - y.e^{-y} - e^{-y} + c$$

$x = -y - 2 + c\,e^{y}$

Example 10 . Find the particular solution of the given differential equation :

$\frac{dy}{dx} + y\text{Cot}x = 2x + x^2\text{Cot}x$ given that y = 0 when $x = \frac{\pi}{2}$

Solution : The given differential equation is the Linear differential equation of the type $\frac{dy}{dx} + Py = Q$.

Here $P = \cot x$ and $Q = 2x + x^2\cot x$

Integrating factor (I.F.) = $e^{\int P\,dx} = e^{\int \cot x\,dx} = e^{\log \sin x} = \sin x$

So the general solution of the given equation is

$y\sin x = \int (2x + x^2\cot x)\sin x\,dx$

$\Rightarrow y\sin x = \int 2x\sin x\,dx + \int (x^2\cos x)\,dx$

$\Rightarrow y\sin x = \sin x\,(\frac{2x^2}{2}) - \int (\cos x)\frac{2x^2}{2}\,dx + \int (x^2\cos x)\,dx$

$\Rightarrow y\sin x = x^2\sin x + c$

Now for the particular solution put y = 0 and $x = \frac{\pi}{2}$

So $0 = (\frac{\pi}{2})^2\sin\frac{\pi}{2} + c \Rightarrow c = -\frac{\pi^2}{4}$

Therefore the particular solution of given differential equation is

$y\sin x = x^2\sin x - \frac{\pi^2}{4}$

III Questions for Practice:

Que1. The sum of degree and order of the differential equation

$5x\,(\frac{dy}{dx})^2 - \frac{d^2y}{dx^2} - 6y = \log x$ is

(a) 2 (b) 3 (c) 1 (d) 4

Que2. Write the degree and order of the differential equations

$[1 + (\frac{dy}{dx})^2]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$

(a) order 1 and degree 2 (b) order 2 and degree 2

(c) order 2 and degree 1 (d) order 1 and degree 1

Que 3 . The number of arbitrary constants in a particular solution is

(a) 1 (b) 3 (c) 2 (d) 0

Que 4. Which one of the following is the general solution of the differential equation ydx – x dy = 0

(a) xy = c (b) $x = cy^2$ (c) $y = cx^2$ (d) y = cx

Que 5. Two statements A and R are given below . Chose correct option from given options

Assertion (A) : The general solution of the differential equation $e^{\frac{dy}{dx}} = x^2$ is y = 2x (log x – 1) + c

Reasoning (R): The integrating factor of linear differential equation $\frac{dy}{dx} + Py = Q$ is $e^{\frac{dP}{dx}}$

(a) Both A and R are true and R is the correct explanation of A
(b) both A and R are true but r is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true .

Short Answer Questions

Que 6. Verify that y = 3 Cos(logx) + 4Sin(logx) is a solution of the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Que 7. Verify that the given function xy= logy + c is solution of differential equation $\frac{dy}{dx} = \frac{y^2}{1-xy}$, $xy \neq 1$

Que 8. Solve $\sec^2x.\tan y\,dx + \sec^2y.\tan x\,dy = 0$.

Que 9. Solve $(1+ y^2)dx - y(1+x^2)dy = 0$.

Que 10. Solve $\cos^2x\frac{dy}{dx} + y = \tan x$.

Que 11. Solve $\frac{dy}{dx} + 2(\frac{y}{x}) = \frac{1}{x^2}$

Que 12. Solve $ydx + (x - y^2)dy = 0$.

Que 13. Solve $\frac{dy}{dx} - \frac{y}{x} = 2x^2$

Que 14.Solve ydx + x log ($\frac{y}{x}$) dy -2x dx = 0

Que 15. Find Particular solution of $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$ given that y=0 when x = 1

Long answer questions

Que 16. Find the general solution of the differential equation $\frac{dy}{dx} - y = \cos x$

Que 17. Show that the differential equation (x – y)$\frac{dy}{dx}$ = x + 2y is homogeneous equation and solve it .

Que 18. Solve : $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

Que 19. Solve the initial value problem : (x^3+ x^2 + x + 1)$\frac{dy}{dx}$ = $2x^2$ + x when y (0) = 1

Que 20. Find the particular solution of $\frac{dy}{dx} + y\cot x = 2x + x^2\cot x$ given that y=0 when x = $\frac{\pi}{2}$

Answers of practice questions

Ans 1 (b) Ans 2 (b) Ans 3 (d) Ans 4 (d) Ans 5 (c)

Ans 8 tanx tany = c

Ans 9. $\tan^{-1}y = 2\tan^{-1}x + c$ **Ans 10 $y = \tan x - 1 + c\,e^{-\tan x}$**

Ans 11 $y = \frac{1}{x} + \frac{c}{x^2}$ **Ans 12 $x = \frac{y^3}{3} + \frac{c}{y}$**

Ans 13 $y = x^3 + cx$ **Ans 14. $cy = \log\left|\frac{y}{x}\right| - 1$**

Ans 15.$\cos\left(\frac{y}{x}\right) = \log|ex|$ **Ans 16. $x^2(y^2 + 2xy) = c$**

Ans 17. $\log|x^2 + xy + y^2| = 2\sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3}x}\right) + c$

Ans 18 . $x^2 - y^2 = cx$

Ans 19. $y = \frac{1}{4}\log|(x+1)^2(x^2+1)^3| - \frac{1}{2}\tan^{-1}x + 1$

Ans 20. $y = x^2 - \frac{\pi^2}{4\sin x}$

TEST -1

M.M. 20 Topic: Differential Equations

Section A (MCQs and A & R) (01 mark Each)

Que 1. The number of arbitrary constants in Particular solution of a differential equation of order 4 is

(a) 2 (b) 4 (c) 1 (d) 0

Que 2 . The degree of differential equation $log\left(\frac{dy}{dx}\right)+\left(\frac{d^2y}{dx^2}\right)=3y$ is

(a) 1 (b) 2 (c) 0 (d) None of these

Que 3 .The Integrating Factor of differential equation $\frac{dy}{dx}+2y=e^x$ is

(a) e^{2x} (b) e^x (c) e^{-x} (d) e^{-2x}

Que 4 . A homogeneous differential equation can be solved by making the substitution

(a) v = xy (b) y = vx (c) x = v (d) y =v

Que 5. Two statements A and R are given below . Chose correct option from given options

Assertion (A) : The general solution of the differential equation $e^{\frac{dy}{dx}}=x^2$ is y = 2x(log x – 1) + c

Reasoning (R): The integrating factor of linear differential equation $\frac{dy}{dx}+$ Py = Q is $e^{\frac{dP}{dx}}$

(a) Both A and R are true and R is the correct explanation of A

(b) both A and R are true but r is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true .

SectionB (2 marks Each)

Que 6. Verify that the given function y = ax + 2a^2 is a solution of differential equation

$$2(\frac{dy}{dx})^2+x\left(\frac{dy}{dx}\right)-y=0$$

Que 7. Solve the differential equation : $\frac{dy}{dx}+\frac{y}{y^2+1}=0$

Section C (3 marks Each)

Que8. find the particular solution of log $(\frac{dy}{dx})=2x+3y$ given that x = 0 , y = 0

Que 9. Solve the differential equation $\frac{dy}{dx}+y=cosx-sinx$

Section D (5 marks)

Que 10. Find the particular solution of the given differential equation $\frac{dy}{dx}=1+x^2+y^2+x^2y^2$ given that y= 1 when x = 0

TEST 2

M.M. 30 **Topic : Differential Equations**

Section A (MCQs and A & R) (01 mark Each)

Que 1. The order and degree of the differential equation $(\frac{dy}{dx})^2+\frac{d^2y}{dx^2}+5=0$ is given by

(a) order 1 and degree 2 (b) order 2 and degree 2

(c) order 2 and degree 1 (d) order 1 and degree 1

Que 2. The degree and order of the differential equation$\frac{dy}{dx}+sin(\frac{dy}{dx})=0$. is given by

(a) 1 and 1 (b) 1 and not defined

(c) not defined and 1 (d) None of these

Que 3 Which one of the following is the general solution of the differential equation ydx – x dy = 0

(a) xy = c (b) x = cy^2 (c) y = cx^2 (d) y = cx

Que 4. Which of the following is a not a Homogeneous differentia equation :

(a) 2xy dy = (x^2+y^2) dx (b) xy dx – (x^2+y^2)dy = 0

(c) $(1+y^2)dx-y(1+x^2)dy=0.$ (d) None of these

Que 5. Two statements A and R are given below . Chose correct option from given options

Assertion (A) : The general solution of the differential equation x $\frac{dy}{dx}$ + 2y =x^2 is y = $\frac{x^2}{4}+cx^{-2}$

Reasoning (R): The solution of linear differential equation y (I.F)=$\int(Q\times I.F.)dx$+ c

(a) Both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true .

Section B (2 marks Each)

Que 6 Verify that y = 3 cos(logx) + 4sin(logx) is a solution of differential equation

$$x^2\left(\frac{d^2y}{dx^2}\right)+x\left(\frac{dy}{dx}\right)+y=0$$

Que 7 (i) Write the degree and order of the differential equations $\sqrt{1-y^2}dy+\sqrt{1-x^2}dx=0$

(ii) how many constants will be there in its general solution ?

Que 8. Solve the initial value problem Cos $(\frac{dy}{dx})$ = a given that y = 1 when x = 0

Que 9. Solve the differential equation $\frac{dy}{dx}= sin^{-1}$x

Section C (3 marks Each)

Que10 Solve the differential equation (y – $\sin^2$x) dx + tanx dy = 0

Que 11 Find the general solution of the differential equation (x^3 + y^3) dy = x^2y dx

Que 12. Find the particular solution of the differential equation

x$\left(\frac{dy}{dx}\right)-y+xsin\left(\frac{y}{x}\right)=0\ giventhatwhenx=2\,,y=\ \pi$

Que13. Verify that xy = logy + c is solution of differential equation $\frac{dy}{dx}=\ \frac{y^2}{1-xy}$

Section D (5 marks Each)

Que 14. Find the general solution of the differential equation x (y^3 + x^3)dy = (2y^4 + 5x^3)dx

Que15. Find the general solution of the differential equation $\frac{dy}{dx}-\ y\ = cosx$

ANSWERS (TEST 1)

Ans 1(d) Ans 2(d) Ans 3(a) Ans 4(b) Ans5(c)

Ans 7. $\frac{1}{2}log(\ 1+\ y^2\ )=\ -x+c$

Ans 8. $\frac{-1}{3}e^{-3y}=\frac{1}{2}e^{2x}-\ \frac{5}{6}$

Ans 9. $y=cosx+ce^{-x}$

Ans 10. $tan^{-1}y\ =x+\ \frac{x^3}{3}+\ \frac{\pi}{4}$

ANSWERS (TEST 2)

Ans 1(c) Ans 2(b) Ans 3(d) Ans 4(c) Ans5(a)

Ans 7 (i) Degree 1 , order 1 Ans 7 (ii) Number of constants is 1

Ans 8. $cos(\frac{y-2}{x})=a$

Ans 9. $xsin^1x+\ \sqrt{1-x^2}\ +c$

Ans 10. $ysinx=\frac{sin^3x}{3}\ +c$

Ans 11 $\frac{-x^3}{3y^3}+\ logy\ =c$

Ans 12 $x\,tan(\frac{y}{2x})=2$

Ans 14 . y^4 + 4x^3y = c x^3

Ans 15. $\frac{(\,sinx-cosx\,)}{2}+\ ce^x$

VECTORS
SOME IMPORTANT RESULTS/CONCEPTS

* Position vector of point $A(x, y, z) = \vec{OA} = x\hat{i} + y\hat{j} + z\hat{k}$
* If $A(x_1, y_1, z_1)$ and point $B(x_2, y_2, z_2)$ then $\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$
* If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$; $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$
* Unit vector parallel to $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|}$
* Scalar Product (dot product) between two vectors: $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$; θ is angle between the vectors
* $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$
* If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ then $\vec{a}.\vec{b} = a_1a_2 + b_1b_2 + c_1c_2$
* If $\vec{a}$ is perpendicular to $\vec{b}$ then $\vec{a}.\vec{b} = 0$
* $\vec{a}.\vec{a} = |\vec{a}|^2$
* Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$
* Vector product between two vectors:

  $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\ \hat{n}$; $\hat{n}$ is the normal unit vector

  which is perpendicular to both $\vec{a}$ & $\vec{b}$
* $\hat{n} = \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$
* If $\vec{a}$ is parallel to $\vec{b}$ then $\vec{a} \times \vec{b} = 0$
* Area of triangle (whose sides are given by $\vec{a}$ and $\vec{b}$) $= \dfrac{1}{2}|\vec{a} \times \vec{b}|$
* Area of parallelogram (whose adjacent sides are given by $\vec{a}$ and $\vec{b}$) $= |\vec{a} \times \vec{b}|$
* Area of parallelogram (whose diagonals are given by $\vec{a}$ and $\vec{b}$) $= \dfrac{1}{2}|\vec{a} \times \vec{b}|$

SOLVED EXAMPLES

1.The position vector of a point which divides the join of points with position vectors $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$. in the ratio 1 : 2 internally is (a)$\frac{3\bar{a}+2\bar{b}}{3}$ (b)$\bar{a}$ (c)$\frac{5\bar{a}-\bar{b}}{3}$ (d) $\frac{4\bar{a}+\bar{b}}{3}$

ANS: (d), as position vector = $\dfrac{2(\bar{a}+\bar{b})+1(2\bar{a}-\bar{b})}{1+2} = \dfrac{4\bar{a}+\bar{b}}{3}$

2. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$ find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$

answer. Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{a} \times \vec{c} = \vec{b} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{j} - \hat{k} \Rightarrow \hat{i}(z-y) + \hat{j}(x-z) + \hat{k}(y-x) = \hat{j} - \hat{k}$$

$\Rightarrow z - y = 0,\ x - z = 1,\ y - x = -1$

Also $\vec{a}.\vec{c} = 3 \Rightarrow x + y + z = 3$

$\Rightarrow x + z + z = 3 \Rightarrow x + 2z = 3$

and $x - z = 1 \Rightarrow z = \frac{2}{3},\ x = \frac{5}{3},\ y = \frac{2}{3}$

$\therefore \vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$ or $\frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$

3. . If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$, show that angle between $\vec{a}$ and $\vec{b}$ is 60^o.

answer. $\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow \vec{a} + \vec{b} = -\vec{c} \Rightarrow (\vec{a} + \vec{b}).(\vec{a} + \vec{b}). = (-\vec{c}).(-\vec{c})$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}.\vec{b} = |\vec{c}|^2 \Rightarrow 9 + 25 + 2\,\vec{a}.\vec{b} = 49$

$\Rightarrow 2\,\vec{a}.\vec{b} = 15 \Rightarrow 2|\vec{a}||\vec{b}|\cos\theta = 15 \Rightarrow 30\cos\theta = 15$

$\Rightarrow \cos\theta = 1/2 \Rightarrow \theta = 60^0$

III .Questions for Practice: Number of questions should be as mentioned in the table:

1. If $\vec{a}$ and $\vec{b}$ .are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\,\vec{a} - \vec{b}$ to be a unit vector?

(a) 30° (b) 45° (c) 60° (d) none of these

2. The value of λ for which vectors $2\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{i} - \lambda\hat{j} + 4\hat{k}$ are orthogonal is

(A)12 (B) 14 (C) 16 (D) none of these

3.If $|\bar{a} + \bar{b}| = |\bar{a} - \bar{b}| =$, then angle between $\bar{a}$ and $\bar{b}$ is

(a) 30° (b) 45° (c) 60° (d) 90°

4. If $|\vec{a}| = 5, |\bar{b}| = 13$ and $|\bar{a} \chi \bar{b}| = 25$, then $\bar{a} \cdot \bar{b}$ is equal to

(a) 12 (b) 5 (c) 13 (d) 60

5. Assertion- Reason question

Assertion: (a)Two vectors are said to be like vectors if they have the same direction but different magnitude.

Reason: (b)Vector quantities do not have a specific direction.

(A) Both (a) and (b) are correct and (b)is the correct explanation of (a)

(B) Both (a) and (b) are correct and (b)is not the correct explanation of (a)

(C) (a)is correct but (b)is false

(D) (a)is false but (b)is correct

6. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$

7. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$.

8. For given vectors, $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -2\hat{i} + 3\hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

9. Find a vector in the direction of vector $3\hat{i} - 4\hat{j} + 5\hat{k}$ which has magnitude7 units.

10. Find the value of x for which $x(\hat{i} + 2\hat{j} + 3\hat{k})$ is a unit vector.

11. Find the value of λ for which the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - \lambda\hat{k}$ are collinear.

12. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$

13. Write the projection of vector $\hat{i} + \hat{j} + \hat{k}$ along the vector $\hat{j}$.

14. Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors.

15.Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a} \cdot \vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$.

ANSWERS: 1. (a) 30° 2. (B) 14 3.(d) 90° 4. (d) 60 5. : (C) (a)is correct but (b)is false

6. $\cos^{-1}\left(-\frac{1}{3}\right)$ **;7.** $\frac{1}{3}\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ **; 8.** $\frac{1}{\sqrt{6}}\left(\hat{i} + \hat{j} + \hat{k}\right)$ **; 9.** $\frac{7}{5\sqrt{2}}\left(3\hat{i} - 4\hat{j} + 5\hat{k}\right)$

10. $\pm\frac{1}{\sqrt{14}}$ **; 11.** $\lambda = 8$ **; 12. 5; 13. 1 ;14.** $p = \frac{2}{3}$ **15.** $\frac{8}{7}$

CLASS TEST 20 MARKS (two marks each)

1 Write two different vectors having same magnitude.

2. Write two different vectors having same direction.

3.Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis.

4. Find the scalar and vector components of the vector with initial point (2, 1, 3) and terminal point (– 5, 7, 7).

5. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$.

6. Find the unit vector in the direction of vector $\vec{PQ}$, where P and Q are the points(2, 3, 4) and (5, 6, 7), respectively.

7. For given vectors, $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -2\hat{i} + 3\hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

8. Find a vector of magnitude 4 units, and parallel to the resultant of the vectors $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

9. Find a vector in the direction of vector $3\hat{i} - 4\hat{j} + 5\hat{k}$ which has magnitude7 units.

10. Find the value of x for which $x(\hat{i} + 2\hat{j} + 3\hat{k})$ is a unit vector

ANSWERS

1. $2\hat{i} + 3\hat{j}$ **and** $3\hat{i} + 2\hat{j}$ **2.** $\hat{i} + \hat{j} + \hat{k}$ **and** $2\hat{i} + 2\hat{j} + 2\hat{k}$ **3.** $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

4. Scalar components : –7, 6 and 4, Vector components : –7 $\hat{i}$ **, 6** $\hat{j}$ **, and 4** $\hat{k}$ **.**

5. $\frac{1}{3}\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ **6.** $\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$ **7.** $\frac{1}{\sqrt{6}}\left(\hat{i} + \hat{j} + \hat{k}\right)$

8. $\pm\frac{2}{3}(4\hat{i} + 4\hat{j} + 2\hat{k})$ **9.** $\frac{7}{5\sqrt{2}}\left(3\hat{i} - 4\hat{j} + 5\hat{k}\right)$ **10.** $\pm\frac{1}{\sqrt{14}}$

CLASS TEST 30 MARKS (two marks each)

1. Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors.

2. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then write the values of θ for which $\vec{a} \cdot \vec{b} \geq 0$.

3. Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a} \cdot \vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$.

4. If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, find the angle between $\vec{a}$ and $\vec{b}$.

5. If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°, find $\vec{a} \cdot \vec{b}$.

6.For what value of λ are the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ perpendicular to each other?

7.If $\vec{a} \cdot \vec{a} = 0$and $\vec{a} \cdot \vec{b} = 0$, then what can be concluded about the vector $\vec{b}$?

8. If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 80$, then find $|\vec{x}|$.

9. If $\vec{a}$ is a unit vector and $(2\vec{x} - 3\vec{a}) \cdot (2\vec{x} + 3\vec{a}) = 91$, then write the value of $|\vec{x}|$.

10. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a} + \vec{b}| = 13$ and $|\vec{a}| = 5$, find the value of $|\vec{b}|$.

11. Write the value of $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j}$.

12. Write the value of $(\hat{k} \times \hat{j}) \cdot \hat{i} + \hat{j} \cdot \hat{k}$.

13. Write the value of $(\hat{i} \times \hat{j}) \cdot \hat{k} + (\hat{j} \times \hat{k}) \cdot \hat{i}$.

14. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, then what is the angle between $\vec{a}$ and $\vec{b}$?

15. Write a unit vector perpendicular to both the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$

ANSWERS

1. $p = \frac{2}{3}$

2. $0 \leq \theta \leq \frac{\pi}{2}$

3. $\frac{8}{7}$

4. $\frac{\pi}{3}$

5. $\sqrt{3}$

6. $\lambda = \frac{5}{2}$

7. $\vec{b}$ **may be any vector.**

8. 9

9. 5

10. 12

11. 1

12. – 1

13. 2

14. $\frac{\pi}{4}$

15. $-\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}$

THREE DIMENSIONAL GEOMETRY
SOME IMPORTANT RESULTS/CONCEPTS

** Direction cosines and direction ratios:

If a line makes angles α, β and γ with x, y and z axes respectively the $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the direction cosines denoted by l, m, n respectively and $l^2+m^2+n^2=1$

Any three numbers proportional to direction cosines are direction ratios denoted by a, b, c

$$\frac{l}{a}=\frac{m}{b}=\frac{n}{c} \qquad l=\pm\frac{a}{\sqrt{a^2+b^2+c^2}},\quad m=\pm\frac{b}{\sqrt{a^2+b^2+c^2}},\quad n=\pm\frac{c}{\sqrt{a^2+b^2+c^2}},$$

* Direction ratios of a line segment joining $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ may be taken as $x_2-x_1,\ y_2-y_1,\ z_2-z_1$
* Angle between two lines whose direction cosines are l_1,m_1,n_1 and l_2,m_2,n_2 is given by

$$\cos\theta=l_1l_2+m_1m_2+n_1n_2=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{\left(a_1^2+b_1^2+c_1^2\right)\left(a_2^2+b_2^2+c_2^2\right)}}$$

* For parallel lines $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ and

for perpendicular lines $a_1a_2+b_1b_2+c_1c_2=0$ or $l_1l_2+m_1m_2+n_1n_2=0$

** **STRAIGHT LINE :**

* Equation of line passing through a point (x_1,y_1,z_1) with direction cosines a, b, c: $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$
* Equation of line passing through a point (x_1,y_1,z_1) and parallel to the line: $\frac{x-\alpha}{a}=\frac{y-\beta}{b}=\frac{z-\gamma}{c}$ is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

* Equation of line passing through two point (x_1,y_1,z_1) and (x_2,y_2,z_2) is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$
* Equation of line (Vector form)

Equation of line passing through a point $\vec{a}$ and in the direction of $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\vec{b}$

* Equation of line passing through two points $\vec{a}$ & $\vec{b}$ and in the direction of $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\left(\vec{b}-\vec{a}\right)$
* Shortest distance between two skew lines: if lines are $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ $\vec{r}=\vec{a_2}+\lambda\vec{b_2}$

then Shortest distance $=\frac{\left(\vec{a_2}-\vec{a_1}\right).\left(\vec{b_1}\times\vec{b_2}\right)}{\left|\vec{b_1}\times\vec{b_2}\right|}$; $\vec{b_1}\times\vec{b_2}\neq 0$

$$\frac{\left|\left(\vec{a_2}-\vec{a_1}\right)\times\vec{b_1}\right|}{\left|\vec{b_1}\right|} \quad ;\vec{b_1}\times\vec{b_2}=0$$

SOME ILUSTRATIONS :

Q 1. The vector equation of the symmetrical form of equation of straight line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ is

(a) $\vec{r}=(3i+7j+2k)+\mu(5i+4j-6k)$ (b) $\vec{r}=(5i+4j-6k)+\mu(3i+7j+2k)$

(c) $\vec{r}=(5i-4j-6k)+\mu(3i-7j-2k)$ (d) $\vec{r}=(5i-4j+6k)+\mu(3i+7j+2k)$

Q 2. The angle between a line whose direction ratios are in the ratio 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12) is

(a) $cos^{-1}(2/3)$ (b) $cos^{-1}(-2/3)$ (c) $tan^{-1}(2/3)$ (d) None of these

Q3. If a line makes 60^0 and 45^0 angles with the positive direction of x-axis and z-axis respectively ,then find the angle that it makes with positive direction of y-axis .Hence ,write the direction cosines of the line.

Sol : Let $,\beta,\gamma$ be the angles which line makes with axes.

Hence the direction cosines of the lines are

$\therefore$ $\cos 60^0$, $\cos\beta$, $\cos 45^0$ by taking $\alpha = 60^0$ and $\gamma = 45^0$

$\therefore \frac{1}{4} + cos^2\beta + \frac{1}{2} = 1$

$\therefore \cos\beta = \pm\frac{1}{2}$

$\therefore \beta = 60^0$ or 120^0

So the direction cosines of the lines are ($\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$)or ($\frac{1}{2}, -\frac{1}{2}, \frac{1}{\sqrt{2}}$)

Q4. Find the direction cosines of a line which makes equal angles with the axes. How many such lines are there?

Sol: :Let α be the angle which the line makes with all axes.

$\therefore$Its direction cosines are $\cos\alpha$,$\cos\alpha$, $\cos\alpha$

$\therefore \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$ $\qquad\qquad$ $\therefore \cos^2\alpha = \frac{1}{3}$

$\therefore \cos\alpha = \pm\frac{1}{\sqrt{3}}$

$\therefore$The required direction cosines are $(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}})$ There are four distinct lines.

Q5. Show that the points (2,3,4) ,(-!,-2,1) and (5,8,7) are collinear.

Sol: Let P(2,3,4), Q (-!,-2,1) and R(5,8,7) be the given points.

He direction ratios of PQ are(-1-2,-2-3,1-4) direction ratios of QR are

(5+1,8+2,7-1) ie(-3,-5,-3) and (6,10,6) so$\frac{-3}{6} = \frac{-5}{10} = \frac{-3}{6}$

the lines are collinear.

Q6. Find the shortest distance between the following lines :

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

Sol. Given lines are $\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \lambda(\hat{i} - 2\hat{j} + \hat{k})$, and

$$\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k})\ \vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k})$$

$$\vec{a}_1 = 3\hat{i} + 5\hat{j} + 7\hat{k},\quad \vec{b}_1 = \hat{i} - 2\hat{j} + \hat{k};$$

$$\vec{a}_2 = -\hat{i} - \hat{j} - \hat{k},\quad \vec{b}_2 = 7\hat{i} - 6\hat{j} + \hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = -4\hat{i} - 6\hat{j} - 8\hat{k},\quad \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix} = 4\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\text{S.D.} = \frac{\left|(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|} = \frac{\left|(-4\hat{i} - 6\hat{j} - 8\hat{k}).(4\hat{i} + 6\hat{j} + 8\hat{k})\right|}{\left|4\hat{i} + 6\hat{j} + 8\hat{k}\right|}$$

$$= \frac{|-16-36-64|}{\sqrt{16+36+64}} = \frac{116}{\sqrt{116}} = \sqrt{116} = 2\sqrt{29}$$

Q7. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Find their point of intersection.

Sol. Any point on $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda$ is $(3\lambda - 1,\ 5\lambda - 3,\ 7\lambda - 5)$

Any point on $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu$ is$(\mu + 2,\ 3\mu + 4,\ 5\mu + 6)$

If the lines intersect than for some λ & μ

$3\lambda - 1 = \mu + 2 \quad \Rightarrow 3\lambda - \mu = 3 ....(i)$

$5\lambda - 3 = 3\mu + 4 \quad \Rightarrow 5\lambda - 3\mu = 7 ....(ii)$

$7\lambda - 5 = 5\mu + 6 \quad \Rightarrow 7\lambda - 5\mu = 11 .....(iii)$

From (i) & (ii) $\lambda = \frac{1}{2}, \ \mu = -\frac{3}{2}$ which satisfies (iii)

$\Rightarrow$ given lines intersect and point of intersection is $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$

Case study

The equation of motion of a Missile are x = 3t, y = -4t, z = t, where the time 't' is given in seconds, and the distance is measured in kilometres.

Based on the above answer the following:

Q 1.What is the path of the Missile?

Q 2.At what distance will the rocket be from the starting point (0, 0, 0) in 5 seconds?

1. Straight line **2 $\sqrt{650}$ kms**

QUESTIONS FOR PRACTICE

SHORT ANSWER TYPE QUESTIONS

1. Find the direction cosines of the line passing through the two points (1,– 2, 4) and (– 1, 1, – 2).

2. Find the direction cosines of x, y and z-axis.

3. If a line makes angles 90^o, 135^o, 45^o with the x, y and z axes respectively, find its direction cosines.

4. Find the acute angle which the line with direction-cosines $\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}, n \right\rangle$ makes with positive direction of z-axis.

5. Find the length of the perpendicular drawn from the point (4, –7, 3) on the y-axis.

6. Find the coordinates of the foot of the perpendicular drawn from the point (2, –3, 4) on the y-axis.

7. Find the coordinates of the foot of the perpendicular drawn from the point (–2, 8, 7) on the XZ-plane.

8. Find the image of the point (2, –1, 4) in the YZ-plane.

9. Find the vector and Cartesian equations for the line passing through the points (1, 2, –1) and (2, 1, 1).

10. Find the vector equation of a line passing through the point (-2, 3, 2) and parallel to the line $\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$.

11. Find the angle between the lines $\vec{r} = (2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$.

12. The two lines x =ay +b, z =cy +d ; and x =a' y +b' , z =c' y +d ' are perpendicular to each other, find the relation involving a, a', c and c'.

13. If the two lines L_1 : $x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$, L_2 : $x = 2, \frac{y}{-1} = \frac{z}{2-\alpha}$ are perpendicular, then find value of α.

14. Find the vector equation of the line passing through the point (–1, 5, 4) and perpendicular to the plane z = 0.

ANSWERS

1. $\left(-\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right)$

2. (1, 0, 0); (0, 1, 0) and (0, 0, 1)

3. $(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

4. $\frac{\pi}{4}$

5. 5 units

6. (0, –3, 0)

7. (–2, 0, 7)

8. (–2, –1, 4)

9. $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + 2\hat{k})$; $\frac{x-1}{1} = \frac{x-2}{-1} = \frac{z+1}{2}$ **10.** $\vec{r} = (-2\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$

11 $\cos^{-1}\frac{4}{21}$ **12. aa' +cc' = -1** **13.** $\frac{7}{3}$ **14.** $\vec{r} = -\hat{i} + 5\hat{j} + (4+\lambda)\hat{k}$

LONG ANSWER TYPE QUESTIONS

1. Find the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

2. Find the shortest distance between the following lines : $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

3. Find the equation of a line parallel to $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and passing through $2\hat{i} + 4\hat{j} + 5\hat{k}$. Also find the S.D. between these lines.

4. Find the equation of the line passing through (1, –1, 1) and perpendicular to the lines joining the points(4, 3, 2), (1, –1, 0) and (1, 2, –1), (2, 2, 1).

5. Find the value of λ so that the lines $\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7}$ are perpendicular to each other.

6. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Find their point of intersection.

7. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

8. Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance 5 units from the point P(1, 3, 3).

9. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not. $\frac{x-1}{2} = \frac{y-1}{3} = z$, $\frac{x+1}{5} = \frac{y-2}{1}$, $z = 2$.

ANSWERS

1. $\frac{3\sqrt{2}}{2}$ **units** **2.** $2\sqrt{29}$ **units**

3. $\vec{r}$**=** $(2\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 4\hat{j} + 5\hat{k})$ **and** $\frac{\sqrt{5}}{\sqrt{29}}$ or $\frac{\sqrt{145}}{29}$ **units**

4. $\frac{x-1}{-2} = \frac{y+1}{1} = \frac{z-1}{-1}$ **5.** $\lambda = -2$

6. $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$ **7.** $(1, 0, 7)$

8. $(-2, -1, 3)$ or $(4, 3, 7)$ **9.** $\frac{19}{\sqrt{195}}$ **, not intersecting**

TEST-1

20 MARKS 30 MINUTES

SECTIONA

Q 1 The line which passes through the origin and intersect the two lines $\frac{x-1}{2} = \frac{y+3}{4} = \frac{z-5}{3}$, $\frac{x-4}{2} = \frac{y+3}{3} = \frac{z-14}{4}$ is,

(a) $\frac{x}{1} = \frac{y}{-3} = \frac{z}{5}$ (b) $\frac{x}{-1} = \frac{y}{3} = \frac{z}{5}$ (c) $\frac{x}{1} = \frac{y}{3} = \frac{z}{-5}$ (d) $\frac{x}{1} = \frac{y}{4} = \frac{z}{-5}$

SECTION B

Q2. Study the two statements labelled as assertion (A) reason (R).
Point out if : (A) Both Assertion and reason are true and reason is correct explanation of assertion.
(B) Assertion and reason both are true but reason is not the correct explanation of assertion.
(C) Assertion is true, reason is false.
(D) Assertion is false, reason is true.
Assertion: Equation of line Passing through the point (1,2,3) and (2,-1,5) is (x – 1) /1 = (y − 2 /(−3) = (z – 3)/ 2
Reason : Equation of line passing through the point (x_1,y_1,z_1) and (x_2,y_2,z_2) is
$(x - x_1)/(x_2 - x_1) = (y - y_1)/(y_2 - y_1) = (z - z_1)/(z_2 - z_1)$

Q3. If the two lines L_1 : $x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$, L_2 : $x = 2, \frac{y}{-1} = \frac{z}{2-\alpha}$ are perpendicular, then find value of α.

SECTION C

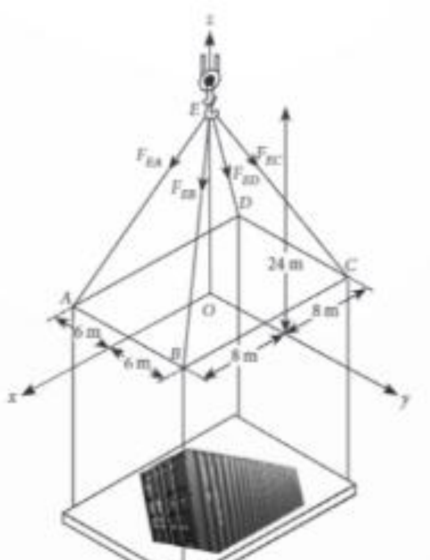
4. Find the equation of the line passing through (1, –1, 1) and perpendicular to the lines joining the points (4, 3, 2), (1, –1, 0) and (1, 2, –1), (2, 2, 1).

5. Find the value of λ so that the lines $\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7}$ are perpendicular to each other.

SECTION D

7. Find the shortest distance between the following lines :
$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

SECTION-E (CASE BASED)



Based on the above information ,answer the following questions
(i) What is the Cartesian equation of line along EA ?
(ii) *Findthevectorequationof* The vector $\overrightarrow{ED}$

ANSWERS

1.(a) 2.(a) 3.7/3 4. 5. $\lambda = -2$ **6.** $2\sqrt{29}$ units

7 (i) $\frac{x}{-4} = \frac{y}{3} = \frac{z-24}{12}$ (ii) $-8\hat{\imath}-6\hat{\jmath}-24\hat{k}$

TEST -2
MARKS-30 50 MINUTES

SECTION A

Q 1. The vector equation of the symmetrical form of equation of straight line
$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ is
(a) $\vec{r} = (3i + 7j + 2k) + \mu(5i + 4j - 6k)$ (b) $\vec{r} = (5i + 4j - 6k) + \mu(3i + 7j + 2k)$
(c) $\vec{r} = (5i - 4j - 6k) + \mu(3i - 7j - 2k)$ (d) $\vec{r} = (5i - 4j + 6k) + \mu(3i + 7j + 2k)$

SECTION B

Q2. Find the angle between the lines $\vec{r} = (2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$.

Q3. Find the vector equation of the line passing through the point (–1, 5, 4) and perpendicular to the plane z = 0.

Q4. Find the direction cosines of the line passing through the following points:
(-2, 4, -5), (1, 2,3).

SECTION-C

Q5 Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point P (1, 3, 3)

Q6 Find the coordinates of the foot of the perpendicular drawn from the point A(1, 8,4) to the line joining the points B(0, -1,3) and C(2, -3,-1).

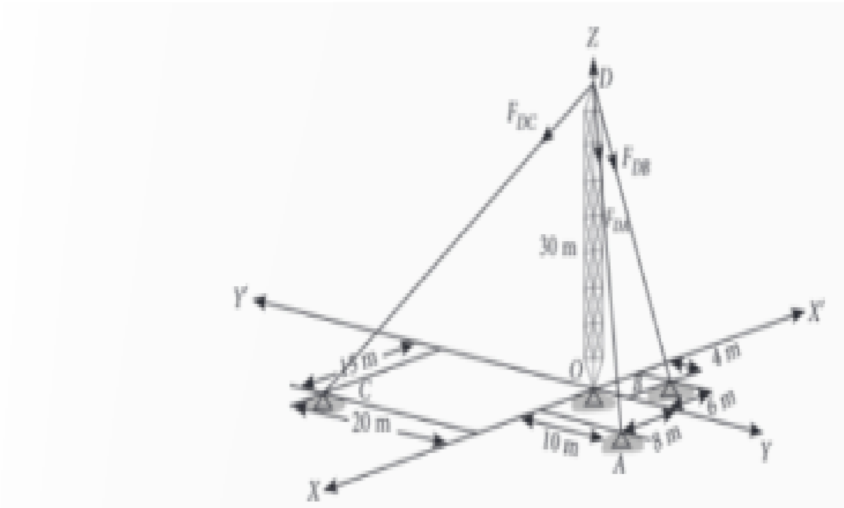
Q7 The Cartesian equation of a line AB is $\frac{2x-1}{\sqrt{3}}=\frac{y+2}{2}=\frac{z-3}{3}$. Find the direction cosines of a line parallel to AB.

SECTION-D

Q.8 Find the image of the point P (5, 9, 3) in the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$.

Q9 Find the equation of the line passing through the points P(-1,3,-2) and perpendicular to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$

SECTION-E



(i) Find the equation of line along AD.

(ii) Find the length of DC.

ANSWERS

1(d) **2.** $\cos^{-1}\left(\frac{4}{\sqrt{21}}\right)$ **3** $\vec{r}=-\hat{i}+5\hat{j}+(4+\lambda)\hat{k}$ **4** $\frac{3}{\sqrt{77}}$ **5(-2,-1,3) or (4,3,7)** **6** $\left(\frac{5}{3},\frac{-8}{3},\frac{-1}{3}\right)$

7 $(\sqrt{3}/\sqrt{55}, 4/\sqrt{55}, 6/\sqrt{55})$ **8 (1, 1, 11)** **9** $\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$ **10 (i)** $\frac{x}{4}=\frac{y}{5}=\frac{z-30}{-15}$ **(ii)** $5\sqrt{61}$

LINEAR PROGRAMMING

**** An Optimisation Problem** A problem which seeks to maximise or minimise a function is called an optimisation problem. An optimisation problem may involve maximisation of profit, production etc or minimisation of cost, from available resources etc.

**** Linear Programming Problem (LPP)**

A linear programming problem deals with the optimisation (maximisation/minimisation) of a **linear function** of two variables (say x and y) known as **objectivefunction** subject to the conditions that the variables are non-negative and satisfy a set of linear inequalities (called **linear constraints)**. A linear programming problem is a special type of optimisation problem.

**** Objective Function** Linear function $Z = ax + by$, where a and b are constants, which has to be maximised or minimised is called a linear objective function.

**** Decision Variables** In the objective function $Z = ax + by$, x and y are called decision variables.

**** Constraints** The linear inequalities or restrictions on the variables of an LPP are called **constraints**. The conditions $x \geq 0$, $y \geq 0$ are called non-negative constraints.

**** Feasible Region** The common region determined by all the constraints including non-negative constraints $x \geq 0$, $y \geq 0$ of an LPP is called the feasible region for the problem.

**** Feasible Solutions** Points within and on the boundary of the feasible region for an LPP represent feasible solutions.

**** Infeasible Solutions** Any Point outside feasible region is called an infeasible solution.

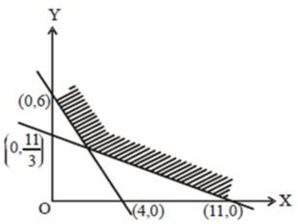
**** Optimal (feasible) Solution** Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.

****** Let R be the feasible region (convex polygon) for an LPP and let $Z = ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

****** Let R be the feasible region for a LPP and let $Z = ax + by$ be the objective function. If R is **bounded**, then the objective function Z has both a maximum and a minimum value on R and each of these occur at a corner point of R. If the feasible region R is **unbounded**, then a maximum or a minimum value of the objective function may or may not exist. However, if it exits, it must occur at a
corner point of R.

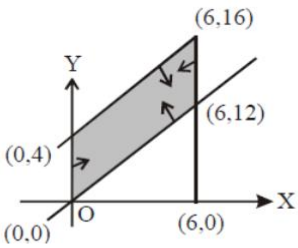
ILLUSTRATIONS

1. Find the maximum value of the objective function $Z = 5x + 10y$ subject to the constraints
   $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$.
2. Find the maximum value of $Z = 3x + 4y$ subjected to constraints $x + y \leq 40$, $x + 2y \leq 60$, $x \geq 0$ and $y \geq 0$
3. Find the points where the minimum value of Z occurs:
   $Z = 6x + 21y$, subject to $x + 2y \geq 3$, $x + 4y \geq 4$, $3x + y \geq 3$, $x \geq 0$, $y \geq 0$.
4. For the following feasible region, write the linear constraints.

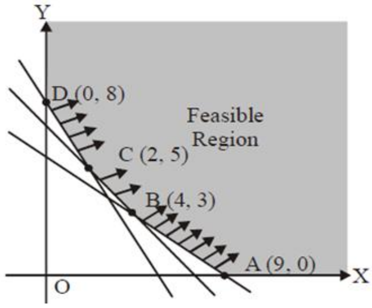


5. The feasible region for LPP is shown shaded in the figure.

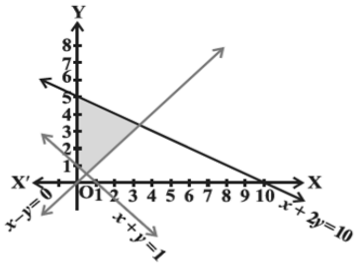
Let $Z = 3x - 4y$ be the objective function, then write the maximumvalue of Z .



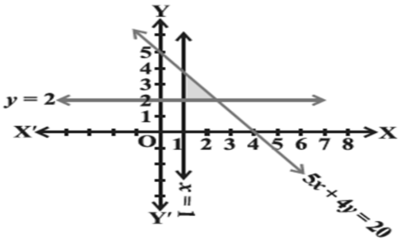
6. Feasible region for an LPP is shown shaded in the following figure. Find the point where minimum of $Z = 4x + 3y$ occurs.



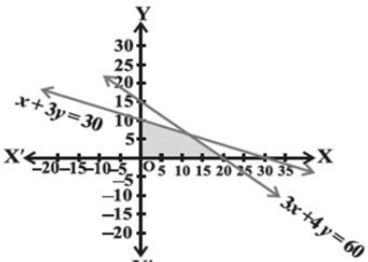
7. Write the linear inequations for which the shaded area in the following figure is the solution set.



8. Write the linear inequations for which the shaded area in the following figure is the solution set.



9. Write the linear inequations for which the shaded area in the following figure is the solution set.



10. Solve the following Linear Programming Problems graphically: Maximise $Z = 5x + 3y$ subject to $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$, $y \ge 0$.

ANSWERS

1. 600

2. 140

3. $\left(2, \frac{1}{2}\right)$

4. $x \ge 0, y \ge 0, 3x + 2y \ge 12, x + 3y \ge 11$

5. 0

6. (2,5)

7. $x + 2y \le 10, x + y \ge 1, x - y \le 0, x, y \ge 0$

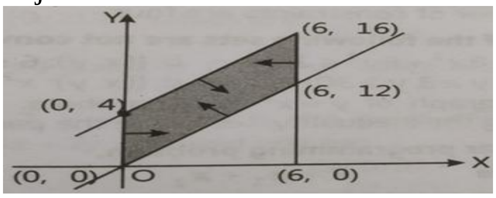
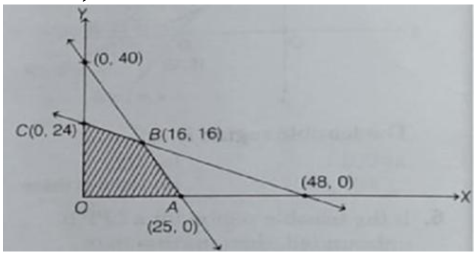
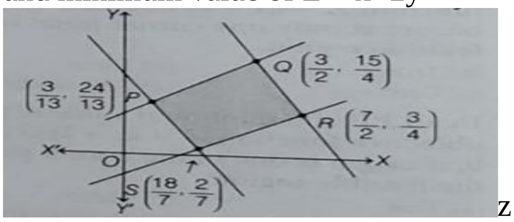
8. $5x + 4y \le 20, x \ge 1, y \ge 2$

9. $3x + 4y \le 60, x + 3y \le 30, x \ge 0, y \ge 0$

10. Maximum Z = $\frac{235}{19}$ **at** $\left(\frac{20}{19}, \frac{45}{19}\right)$

Test – 01 M.M. : 20

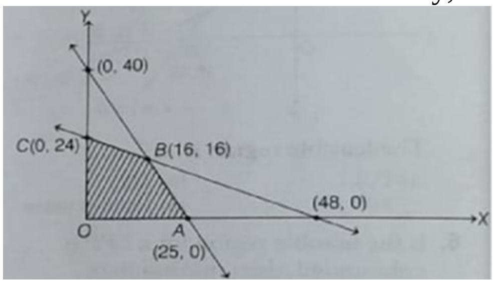
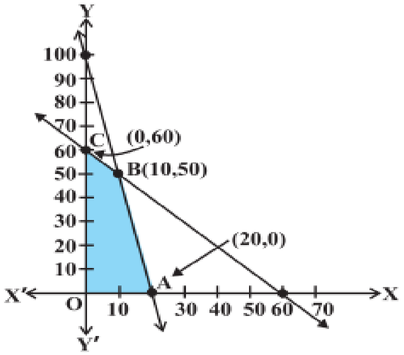
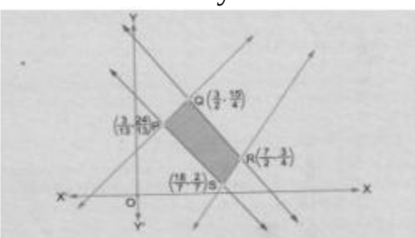
1	The set of all feasible solutions of a LPP is a ____ set. (a) Concave (b) Convex (c) Feasible (d) None of these	1
2	In a LPP, if the objective function Z = ax + by has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same……….value.	1

	(a) minimum (b) maximum (c) zero (d) none of these	
3	In the feasible region for a LPP is ………, then the optimal value of the objective function Z = ax+by may or may not exist. (a) bounded (b) unbounded (c) in circled form (d) in squared form	1
4	Region represented by $x \geq 0$, $y \geq 0$ is: (a) First quadrant (b) Second quadrant (c) Third quadrant (d) Fourth quadrant	1
5	The feasible region for an LPP is shown shaded in the figure. Let Z = 3x- 4y be objective function. Maximum value of Z is  (a) 0 (b) 8 (c) 12 (d) -18	1
6	The maximum value of Z = 4x+3y, if the feasible region for an LPP is as shown below, is  (a) 112 (b) 100 (c) 72 (d) 110	1
7	In the given figure, the feasible region for a LPP is shown. Find the maximum and minimum value of Z = x+2y  z (a) 8 , 3.2 (b) 9 , 3.14 (c) 9 , 4 (d) None	2
8	The linear programming problem minimize Z= 3x+2y, subject to constraints $x + y \leq 8$, $3x+5y \leq 15$, $x, y \geq 0$, has (a) One solution (b) No feasible solution (c) Two solutions (d) Infinitely many solutions	2
9	Corner points of the feasible region determined by the system of linear constraints are (0,3),(1,1) and (3,0). Let Z= px + qy, where p, q>0. Condition on p and q so that the minimum of Z occurs at (3,0) and (1,1) is (a) p=2q (b) $p=\frac{q}{2}$ (c) p=3q (d) p=q	2
10	Solve the following Linear Programming Problem : Max. Z = x + 2y Subject to the constraints : $x + y \leq 50$, $3x + y \leq 90$, $x \geq 0$, $y \geq 0$	3
11	Solve the following Linear Programming Problem : Min Z = x + 2y Subject to the constraints : $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, $y \geq 0$	5

ANSWER :

1 (b) 2 (b) 3 (b) 4 (a) 5 (a) 6 (a) 7 (b) 8.(a) 9. (b) 10. Max. z =100 11.min z= 100

Test : 02 M. M. – 30

1	The common region determined by all the constraints including non-negative constraints $x, y \geq 0$ of a linear programming problem is called (a) Feasible region (b) Feasible solution (c) Optimal solution (d) Constraints	1
2	In the feasible region for a LPP is ………, then the optimal value of the objective function $Z = ax + by$ may or may not exist. (a) bounded (b) unbounded (c) in circled form (d) in squared form	1
3	$Z = 250x + 75y$ is a linear objective function. Variables x and y are called ….. (a) Decision variables (b) Constraints (c) Constant (d) Objective function	1
4	Points within and on the boundary of the feasible region represent …… (a) Infeasible solution (b) Feasible solution (c) Objective solution (d) None	1
5	The maximum value of $Z = 4x+3y$, if the feasible region for an LPP is as shown below, is  (a) 112 (b) 72 (c) 110 (d) 100	1
6	Corner points of the feasible region determined by the system of linear constraints are (0,3),(1,1) and (3,0). Let Z= px + qy, where p, q>0. Condition on p and q so that the minimum of Z occurs at (3,0) and (1,1) is (a) p=2q (b) p= q/2 (c) p=3q (d) p=q	2
7	Write the linear inequations for which the shaded area in the following figure is the solution set  (a) $5x + y \leq 100$, $x + y \geq 60$ (c) $5x + y \geq 100$, $x + y \leq 60$ (b) $5x + y \geq 60$, $x + y \leq 100$ (d) $5x + y \leq 100$, $x + y \leq 60$	2
8	In figure, the feasible region (shaded) for a LPP is shown. Determine the maximum and minimum value of $Z = x+2y$.  (a) 8 , 3.2 (b) 9 , 3.14 (c) 9 , 4 (d) None	2
9	Solve the following Linear Programming Problem : Max. $Z = x + 2y$ Subject to the constraints : $x + y \leq 50$, $3x + y \leq 90$, $x \geq 0$, $y \geq 0$	3

10	Solve the following linear programming problem graphically: Maximise $Z = 3x + 4y$ subject to the constraints : $x + y \leq 4$, $x \geq 0$, $y \geq 0$.	3
11	Solve the following linear programming problem graphically: Maximise $Z = 4x + y$ subject to the constraints:$x + y \leq 50$, $3x + y \leq 90$, $x \geq 0$, $y \geq 0$	3
12	Find the minimum value of $Z = 11x + 7y$ Subject to $x + 3y \leq 9$, $x + y \leq 5$, $x \geq 0$, $y \geq 0$	5
13	Solve the Linear Programming graphically: Maximize $Z= 9x+3y$ subject to$2x + 3y \leq 13$, $3x + y \leq 5$, $x \geq 0$, $y \geq 0$	5

ANSWER :

1 (a) 2 (b) 3 (a) 4 (b) 5 (a) 6 (b) 7 (d) 8.(b) 9. Max. z =100 10. Max. z =16
11 Max Z = 110 12. Min Z = 21 13. Max Z = 15

PROBABILITY

SOME IMPORTANT RESULTS/CONCEPTS

** **Sample Space and Events** :
The set of all possible outcomes of an experiment is called the sample space of that experiment. It is usually denoted by S. The elements of S are called events and a subset of S is called an event.
$\phi\ (\subset S)$ is called an impossible event and
$S(\subset S)$ is called a sure event.
** **Probability of an Event.**
(i) If E be the event associated with an experiment, then probability of E, denoted by P(E) is defined as $P(E) = \dfrac{\text{number of outcomes in E}}{\text{number of total outcomes in sample space S}}$
it being assumed that the outcomes of the experiment in reference are equally likely.
(ii) P(sure event or sample space) = P(S) = 1 and P(impossible event) = $P(\phi) = 0$.
(iii) If $E_1, E_2, E_3, \dots, E_k$ are mutually exclusive and exhaustive events associated with an experiment (i.e. if $E_1 \cup E_2 \cup E_3 \cup \dots \cup E_k) = S$ and $E_i \cap E_j = \phi$ for $i, j \in \{1, 2, 3, \dots, k\}$ $i \neq j$), then
$P(E_1) + P(E_2) + P(E_3) + \dots + P(E_k) = 1$.
(iv) $P(E) + P(E^C) = 1$
** If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e. P (E|F) is given by
$P(E|F) = \dfrac{P(E \cap F)}{P(F)}$ provided $P(F) \neq 0$
** Multiplication rule of probability : $P(E \cap F) = P(E)\, P(F|E)$
$= P(F)\, P(E|F)$ provided $P(E) \neq 0$ and $P(F) \neq 0$.
** Independent Events :E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other.
Let E and F be two events associated with the same random experiment, then E and F are said to be independent if $P(E \cap F) = P(E) \,.\, P(F)$.
** Bayes' Theorem :If $E_1, E_2, \dots, E_n$ are n non empty events which constitute a partition of sample space S, i.e. $E_1, E_2, \dots, E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup \dots \cup E_n = S$ and A is any event of nonzero probability, then

$$P(E_i|A) = \frac{P(E_i).P(A|E_i)}{\sum_{j=1}^{n} P(E_j).P(A|E_j)} \quad \text{for any } i = 1, 2, 3, \dots, n$$

** The probability distribution of a random variable X is the system of numbers

X :	x_1	x_2	...	x_n
P(X) :	p_1	p_2	...	p_n

where, $p_i > 0$, $\sum_{i=1}^{n} p_i = 1$, i = 1, 1, 2,...,

** **Binomial distribution:** The probability of x successes P (X = x) is also denoted by P (x) and is given by $P(x) = {}^nC_x q^{n-x} p^x$, x = 0, 1,..., n. (q = 1 – p)

Illustrations

1.If P (A) = 0.8, P (B) = 0.5 and P (B|A) = 0.4, what is the value of P (A ∩ B)?

A. 0.32 B. 0.25 C. 0.1 D. 0.5

Answer: A. 0.32

Explanation: Given, P (A) = 0.8, P (B) = 0.5 and P (B|A) = 0.4
By conditional probability, we have;
$P(B|A) = P(A \cap B)/P(A) \Rightarrow P(A \cap B) = P(B|A).\, P(A) = 0.4 \times 0.8 = 0.32$

2.If P (A) = 6/11, P (B) = 5/11 and P (A ∪ B) = 7/11, what is the value of P(B|A)?

A. ⅓ B. ⅔ C. 1 D. None of the above

Answer: B. ⅔
Explanation: By definition of conditional probability we know;
P(B|A) = P(A ∩ B)/P(A) …(i)
Also,
P(A ∩ B) = P(A) + P(B) – P(A U B)
= 6/11 + 5/11 – 7/11= 4/11
Now putting the value of P(A ∩ B) in eq.(i), we get;
P(B|A) = (4/11)/(6/11) = 4/6 = ⅔
3.**Find P(E|F), where E: no tail appears, F: no head appears, when two coins are tossed in the air.**
A. 0 B. ½ C. 1 D. None of the above
Answer: A. 0
Explanation: Given,
E: no tail appears
And F: no head appears
⇒ E = {HH} and F = {TT}
⇒ E ∩ F = ϕ
As we know, two coins were tossed;
P(E) = ¼
P(F) = ¼
P(E ∩ F) = 0/4 = 0
Thus, by conditional probability, we know that;
P(E|F) = P(E ∩ F)/P(F)
= 0/(¼)
= 0
4.**If P(A ∩ B) = 70% and P(B) = 85%, then P(A/B) is equal to:**
A. 17/14 B. 14/17 C. ⅞ D. ⅛
Answer: B.14/17
Explanation: By conditional probability, we know;
P(A|B) = P(A ∩ B)/P(B)
= (70/100) x (100/85)
= 14/17
5. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, then what is the conditional probability that both are girls? Given that
(i) the youngest is a girl?
(ii) atleast one is a girl?
Answer:
Let B and b represent elder and younger boy child. Also, G and g represent elder and younger girl child. If a family has two children, then all possible cases are
S = {Bb, Bg, Gg, Gb}
∴ n(S) = 4
Let us define event A : Both children are girls, then A = {Gg} ⇒ n(A) = 1
(i) Let E_1 : The event that youngest child is a girl.
Then, E_1 = {Bg, Gg} and $n(E_1)$ = 2

so $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

and $A \cap E_1 = \{Gg\} \Rightarrow n(A \cap E_1) = 1$

so $P(A \cap E_1) = \frac{n(A \cap E_1)}{n(S)} = \frac{1}{4}$

Now, $P\left(\frac{A}{E_1}\right) = \frac{P(A \cap E_1)}{P(E_1)} = \frac{1/4}{1/2} = \frac{1}{2}$

∴ Required probability $= \frac{1}{2}$

(ii) Let E_2: The event that atleast one is girl.
Then, E_2 = {Eg, Gg, Gb} ⇒ $n(E_2) = 3$,
6.If P(not A) = 0.7, P(B) = 0.7 and P(B/A) = 0.5, then find P(A/B).
Answer:

Given, $P(A') = 0.7$, $P(B) = 0.7$ and $P\left(\frac{B}{A}\right) = 0.5$

Clearly, $P(A) = 1 - P(A') = 1 - 0.7 = 0.3$

Now, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow \quad 0.5 = \frac{P(A \cap B)}{0.3}$

$\Rightarrow \quad P(A \cap B) = 0.15$

$\therefore \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} \Rightarrow P\left(\frac{A}{B}\right) = \frac{3}{14}$

7. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event 'number is even' and B be the event 'number is marked red'. Find whether the events A and B are independent or not.
Or
A die, whose faces are marked 1,2, 3 in red and 4, 5, 6 in green , is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.
Answer:
When a die is thrown, the sample space is
S = {1, 2, 3, 4, 5, 6}
⇒ n(S) = 6
Also, A: number is even and B: number is red.
∴ A = {2, 4, 6} and B = {1, 2, 3} and A ∩ B = {2}
⇒ n(A) = 3, n(B) = 3 and n(A ∩ B) = 1

Now, $P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

$P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{6}$

Now, $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq \frac{1}{6} = P(A \cap B)$

$\therefore \quad P(A \cap B) \neq P(A) \times P(B)$

Thus, A and B are not independent events.
8. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.
Answer:
Let us denote the numbers on black die by B_1, B_2, ……., B_6 and the numbers on red die by R_1, R_2, ….., R_6.
Then, we get the following sample space.
s = {(B_1, R_1) ,(B_1, R_2), ……., (B_1, R_6), (B_2, R_2), ………, (B_6, B_1), (B_6, B_2), ……., (B_6,R_6)
Clearly, n(S) = 36
Now, let A be the event that sum of number obtained on the die is 8 and B be the event that red die shows a number less than 4.
Then, A = {(B_2, R_6), (B_6,R_2), (B_3,R_5), (B_5,R_3), (B_4,R_4)}
and B = {(B_1, R_1), (B_1,R_2), (B_1,R_3), (B_2,R_1), (B_2,R_2), (B_2,R_3) ,…….., (B_6,R_1), (B_6,R_2), (B_6,R_3)}
⇒ A ∩ B = {(B_6, R_2), (B_5, R_3)}
Now, required probability,
p(AB) = P(A∩B)P(B)=2361836=218=19

9. Evaluate P(A ∪ B), if 2P (A) = P(B) = 513 and P(A/ B) = 25.
Answer:

We have, $2P(A) = P(B) = \frac{5}{13}$

$\Rightarrow P(A) = \frac{5}{26}$ and $P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$

$\because P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\therefore \quad \frac{2}{5} = \frac{P(A \cap B)}{5/13}$

$\Rightarrow \quad P(A \cap B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13}$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$

$= \frac{5+10-4}{26} = \frac{11}{26}$

10. Prove that if E and F are independent events, then the events E and F' are also independent. (Delhi 2017)
Answer:
Given, E and F are independent events, therefore
⇒ PE(∩ F) = P(E) P(F) …….. (i)
Now, we have,
P(E ∩ F') + P(E ∩ F) = P(E)
P(E ∩ F') = P(E) – P(E ∩ F)
P(E ∩ F') = P(E) – P(E) P(F) [using Eq. (i))
P(E ∩ F') = P(E) [1 – P(F)]
P (E ∩ F') = P(E) P(F')
∴ E and F 'are also independent events.
Hence proved.

III : Problems for Practice:

1.A bag contains 5 red and 3 blue balls.If 3 balls are drawn at random without replacement the probability of getting exactly one red ball is (a)45/196 (b)135/392 (c)15/56 (d)15/29

2.A flashlight has 8 batteries out of which 3 are dead.If two batteries are selected without replacement and tested, the probability that both are dead is
(a)33/56 (b)9/64 (c)1/14 (d) 3/28

3. Probability that A speaks truth is 4/5.A coin is tossed .A reports that a head appears.The probability that actually there was a head is
(a)4/5 (b)1/2 (c)1/5 (d) 2/5

4. A and B are two students. Their chances of solving a problem correctly are 1/3 and ¼ respectively. If the probability of their making a common error is 1/20 and they obtain the same number , then the probability of their answer to be correct is
(a)1/12 (b)1/40 (c)13/120 (d) 10/13

5. mark the correct choice
(a) Statement-1 and statement-2 are true ; statement -2 is a correct explanation for statement -1
(b) Statement-1 and statement-2 are true ; statement -2 is not a correct explanation for statement -1
(c) Statement-1 is true , statement-2 is false
(d) Statement-1 is false , statement-2 is true
Statement-1 (assertion) 20 persons are sitting in a row. Two of these persons are selected at random. The probability that the two selected persons are not together is 0.9.
Statement-2 (Reason) If $\overline{A}$ denotes the negation of an event A, then $P(\overline{A}) = 1 - P(A)$.

6. In shop A, 30 tin pure ghee and 40 tin adulterated ghee are kept for sale while in shop B, 50 tin pure ghee and 60 tin adulterated ghee are there. One tin of ghee is purchased from one of the shops randomly and it is found

to be adulterated. Find the probability (i) Getting adulterated ghee (ii) it is getting from shop B.

7.Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 of 5 times. He throws a die and reports that it is actually a six. Find the probability that it is actually a six. Do you also agree that the value of truthfulness leads to more respect in the society ?

8. In a game of Archery, each ring of the Archery target is valued. The centre most ring is worth 10 point and rest of the rings are allotted points 9 to 1 in sequential order moving outwards. Archer A is likely to earn 10 points with a probability of 0.8 and Archer B is likely to earn 10 points with a probability of 0.9. Based on the above information answer the following questions. If both of them hit the Archery target , then find the probability that

(i) exactly one of them earns 10 points (ii) both of them earn 10 points.

9. If P(A) = 4/7, P(B) = 0, then find P (A/B).

10. Write the probability of an even prime number on each die, when a pair of dice is rolled.

11. Two independent events A and B are given such that P(A) = 0.3 and P(B) = 0.6 find P(A and not B).

12. A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event 3 on the die. Check whether A and B are independent events or not.

13. Let A and B be two events. If P(A)=0.2, P(B)=0.4,P(A∩B) = 0.6, then find P(A∪B).

14. How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%?

15. A problem in mathematics is given to 3 students whose chances of solving it are 1/2,1/3 and 1/4,. What is the probability that The (i) problem is solved (ii) exactly one of them will solve it?

16. X is taking up subjects, Mathematics, Physics and Chemistry in the examination. His probabilities of getting Grade A in these subjects are 0.2,0.3 and 0.5 respectively.

Find the probability that he gets (i) Grade A in all subjects. (ii) Grade A in no subject

(iii) Grade A in two subjects.

17. A speak truth in 60% of the case, while B in 90% of the cases. In what per cent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A?

18. There are three urns A, B and C. Urn A contains 4 white balls and 5 blue balls. Urn B contains 4 white balls and 3 blue balls.Urn C contains 2 white balls and 4 blue balls. One ball is drawn from each of these urns. What is the probability that out of these three balls drawn, two are white balls and one is a blue ball?

19.(a) 12 cards numbered 1 to 12,are placed in a box,mixed up thoroughly and then a card is drawn at random from the box.If it is known that the number on the drawn card is more than 3,find the probability that it is an even number.

(b) 12 cards, numbered 1 to 12 ,are placed in a box,mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 5, find the probability that it is an odd number.

20. 10%of the bulbs produced in a factory are of red colour and 2%are red and defective.If one bulb is picked up at random ,determine the probability of its being defective if it is red.

21. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively 3/10,1/5 ,1/10 and 2/5 . The probabilities that he will be late are 1/4, 1/3,and 1/12, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late.What is the probability that he comes by train?

22. Suppose that the reliability of a HIV test is specified as follows: of people having HIV, 90% of the test detects the disease but 10% go undetected. Of the people free of HIV, 99% of the tests are judged HIV-ve but 1% are diagnosed as showing HIV +ve. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ve. What is the probability that the person actually has HIV?

23. A letter is known to have come either from TATA NAGAR or from CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letters came from TATA NAGAR?

IV: Answers: 1.(c) 2.(d) 3.(a) 4.(d) 5. (a) 6. (i)43/77 (ii) 21/43 7. 4/9 8.(i)0.26 (ii) 0.72 , 9.does not exist. 10. 5/18 11.0.12 12. Yes 13. 0 14. 3 15.(i) ¾ (ii) 11/24 16. (i) 0.03 (ii) 0.28 (iii) 0.22 17. 42%, yes 18. 64/189 19. (a) 5/9 (b) 3/7 20. 1/5 21.1/2 22. 0.083 approx. 23. 7/11 .

TEST-1 (20 Marks)

SECTION A (1MARK)

1. A bag contains 5 white and 4 red balls. 2 balls are drawn from the bag. Find the probability that both balls are white.

2 find the probability of a red king in a pack of 52 cards.

SECTION B(4MARKS)

3 The probability that a bulb produced by a factory will fuse after 100 days of use is 0.05. Find the probability that out of such five such bulbs

(i) Not more than one

(ii) More than one

Will fuse after 100 days of use. 4 In a bolt factory, 3 machines A, B and C manufacture 25, 35 and 40 per cent of the total bolts manufactured.

Of these output, 5, 4 and 2 per cent are defective respectively. A bolt is drawn at random and is found to be defective. Find the probability that it was manufactured by either machine A. 4

5 An urn contains 4 white and 3 red balls. Let X be the number of red balls in a random draw of 3 balls. Find the mean and variance of X

SECTION C(6MARKS)

6 A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively 3/10, 1/5, 1/10and 2/5. The probabilities

that he will be late are 1/4,1/3 and 1/12 , if he comes by train, bus and scooter respectively, but if he comes by other

means of transport, then he will not be late. When he arrives, he is late.What is the probability that he comes by train?

TEST-2 (30 Marks)

SECTION A (1MARK)

1. Two coins are tossed. What is the probability of coming up two heads if it is known that at least one head comes up.

2. Four cards are drawn from 52 cards with replacement. Find the probability of getting at least 3 aces.

SECTION B(4MARKS)

3. A pair of dice is thrown 7 times. If getting a total of 7 is considered a success, what is the probability of getting

(i) Exactly 6 successes

(ii) At most 6 successes. 4

4. An insurance company insured 3,000 scooters, 4,000 cars and 5,000 trucks. The probabilities of the accident involving a scooter, a car and a truck are 0.02, 0.03 and 0.04 respectively. One of the insured vehicles meet with an accident. Find the probability that it is a scooter.

5. By examining the chest X-ray, the probability that T.B. is detected when a person is actually suffering from it is 0.99. The probability that the doctor diagnosis correctly that a person has T.B. on the basis of X-ray is 0.001. In a certain city, 1 in 1,000 persons suffers from T.B. A person selected at random is diagnosed to have T.B. What is the chance that the person has actually T.B.?

6.A bag contains 5 red marbles and 3 black marbles. Three marbles are drawn one by one without replacement.What is the probability that at least one of the three Marbles drawn be black if the first marble is red ?

SECTION C(6MARKS)

7. Suppose that the reliability of a HIV test is specified as follows: of people having HIV, 90% of the test detects the disease but 10% go undetected. Of the people free of HIV, 99% of the tests are judged HIV-ve but 1% are diagnosed as showing HIV +ve. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ve. What is the probability that the person actually has HIV ?

8. In an examination, an examinee either guesses or copies or knows the answer of multiple choice questions with four choices. The probability that he makes a guess is 1/3, and probability that he copies the answer is 1/6 . The probability that his answer is correct, given that he copied it, is 1/8 . Find the probability that he knew the answer to the question, given that he correctly answered it.