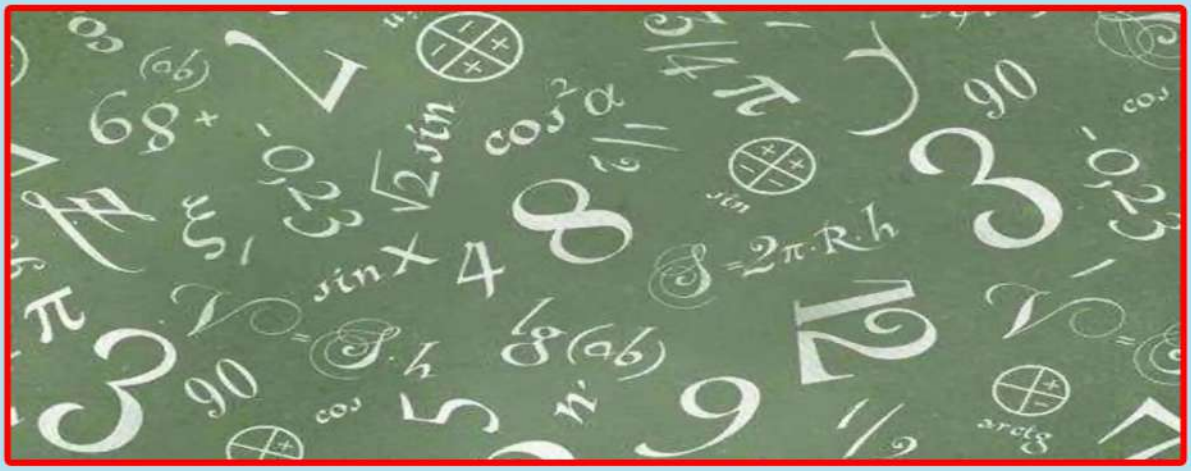


KENDRIYA VIDYALAYA SANGATHAN REGIONAL OFFICE, ERNAKULAM

CLASS XII

STUDY MATERIAL

MATHEMATICS - [041]

Based on Latest CBSE Exam Pattern
for Session 2023-24

OUR PATRON

HON. DEPUTY COMMISSIONER KVS RO ERNAKULAM REGION



SHRI . SANTHOSHKUMAR N

INSPIRATION



Shri. PATIL R A
Hon.Assistant Commissioner
KVS RO ERNAKULAM



Shri.GANESH S INDRALE
Hon.Assistant Commissioner
KVS RO ERNAKULAM

Co-ordinated by



Shri. K P Sudhakaran
Principal,
KV 1 CPCRI KASARAGOD

SANTHOSH KUMAR N
Deputy Commissioner



सन्तोष कुमार एन
उपायुक्त

केन्द्रीय विद्यालय संगठन, क्षेत्रीय कार्यालय, एरणाकुलम

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F.31350/KVS/RO/EKM/2023-24/Acad./CBSE

Date: 25.07.2023

MESSAGE FROM DEPUTY COMMISSIONER

I am delighted to announce the release of Support Material/Study material for the students of class XII. In our relentless pursuit of academic excellence, we have been constantly revising and upgrading our teaching methodologies and resources. And one of the important resources is the support material which is prepared by Kendriaya Vidyalayas for the students of class XII. It aims minimum level of learning for each student and also to foster deeper understanding of subject, which stimulate critical thinking and helps in achieving better score in CBSE exam.

I hope this support material will greatly benefit the academic journey of class XII not only in pursuit of good result in CBSE exams but also helpful for various entrance examinations.

Let us march ahead with dedicated minds and relentless endeavours for better future through better education.

With warm regards

भवदीय,

संतोष कुमार

25/7/23

(सन्तोष कुमार एन)
उपायुक्त

PART A

SAMPLE PAPERS

KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION

CLASS: XII SMAPLE QUESTION PAPER (2023-24) 1

SUB: MATHEMATICS

General Instructions : *Read the following instructions very carefully and strictly follow them:*

i. This question paper contains 38 questions. All questions are compulsory.

ii. This question paper is divided into five Sections - A, B, C, D and E.

iii. In Section A, Questions no. I to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.

iv. In Section B,Questions no.21to25 are very short answer type questions,carrying 2marks each.

v. In Section C, Questions no. 26 to 31 are short answer type questions, carrying 3 marks each.

vi. In Section D, Questions no. 32 to 35 are long answer type questions carrying 5 marks each.

vii. In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E. Use of calculators is not allowed.

SECTION-A

1. If matrix A = [1 2 3] then $A.A^T$ is matrix with following one element
   a. 7 b. 21 c. 14 d.28
2. If A is skew symmetric matrix of order 3, then the value of det A is
   a. 3 b. 0 c. 9 d. 27
3. If A is a 3 x 3 matrix such that det A = 8 then det (3A) is
   a. 8 b. 24 c. 72 d. 216
4. Which of the following is true for the given function .

$$f(x) = \begin{cases} \sin x + 2 \text{ if } x \neq 0 \\ 3 \text{ if } x = 0 \end{cases}$$

   a. Continuous at x = 0 c . not continuous at 0
   b. Differentiable at 0 d. none of the above
5. Find the magnitude of the vector $\vec{a} = 2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$
   a. 2 b. 7 c. 29 d. 49
6. Find the sum of the order and degree of the differential equation
   $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$
   a. 5 b. 3 c. 4 d. 2
7. The corner point of a feasible region is a point in the region which is the ___ of two boundary lines
   a. Intersection b. disjoint c. middle point d. none of these
8. If P (1, 5, 4) and Q(4, 1, -2) then the direction ratios pf vector PQ is
   a. < 3, -4, -6> b. < -3, 4, 6> c. <3, 4, 6> d. none of these
9. The value of $\int_2^3 \frac{x}{x2+1}$ dx is
   a. log 4 b. log(3/2) c. 1/2 log2 d. log(9 /4)
10. If A and B are non singular square matrices of the same order , then $(AB^{-1})^{-1}$ is
    a. $A^{-1}B$ b . $A^{-1}B^{-1}$ c. BA^{-1} d. AB

11. The solution set of the inequality $3x + 5y < 4$ is
    a. An open half plane not containing the orgin
    b. An open plane containing the orgin
    c. The whole XY plane not containing the line $3x + 5y = 4$
    d. A closed half plane containing the orgin

12. The scalar product of the vector $3\hat{\imath} - \hat{\jmath} - 2\hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ is
    a. -2 b. -1 c. -3 d. -6

13. If A is a square matrix of order 3 and det (A) = 5 then det (adj A) is
    a. 5 b. 25 c. 125 d. 1/5

14. Two cards are drawn at random from a pack of 52 cards one by one without replacement. What is the probability of getting first card red and second card jack?
    a.1/ 26 b. 1/13 c. 1/ 52 d. none of these

15. The general solution of the differential equation $y\,dx - x\,dy = 0$ is
    a. $xy=c$ b. $x=cy^2$ c. $y = cx$ d. $y = cx^2$

16. If two vectors a and b are such that $|\vec{a}| = 2$; $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$ then $|a-2b|$ is
    a. $\sqrt{2}$ b. $2\sqrt{6}$ c. 24 d. $2\sqrt{2}$

17. If $y = x^x$ then dy/dx is
    a. $x^x(1 + \log x)$ b. $\log x$ c. $\log x^x$ d. none of these

18. P is a point on the line joining the points A (0, 5, -2) B (3, -1, 2). If the x co-ordinate of P is 6, then its z co-ordinate is
    a. 10 b. 6 c. -6 d. -10

ASSERTION – REASON BASED QUESTION

In the following questions a statement of assertion (A) is followed by a statement of reason (R) . choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and r are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

19. Assertion : A relation $R = \{ |a- b| < 2 \}$ defined on the set $A = \{ 1,2,3,4,5 \}$ is reflexive
    Reason : A relation R on the set A is said to be reflexive if for (a, b) € R and (b, c) € R we have (a, c) € R

20. Assertion : The function defined by $f(x) = \cos x^2$ is a continuous function
    Reason : The sine function is continuous in its domain

SECTION-B

The following questions are of two marks each

21. Find the intervals in which the function $f(x) = x^3 - 4x^2 - 4x - 10$ is strictly decreasing

22. if $\vec{a}$ *and* $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$, then prove that $2\vec{a} + \vec{b}$ is perpendicular to $\vec{b}$.

Or

If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x-axis at angles of 30^0 and 120^0 respectively then find the value of $|\vec{a} + \vec{b}|^2$

23. find the value of k for which the function

f(x) = $\begin{cases} \frac{x^2+3x-10}{x-2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$ is continuous at x=2

24. if the position vector of A ,B, C are $-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$, $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $7\hat{\imath} - \hat{k}$ respectively then find the $|\overrightarrow{AB}|$ and $|\overrightarrow{BC}|$

25. check whether the modulus function f :R → R given by f(x)= $|x|$ is one to one and onto ?

Or

Evaluate : $sin^{-1}\left(sin\frac{3\pi}{5}\right)$

SECTION-C

The following questions are of three marks each

26. a) Find $\int x tan^{-1}x\, dx$

OR

b) Evaluate $\int_0^{\pi} \frac{x\, sinx}{1+cos^2x}\, dx$

27. a) Out of a group of 8 waiters in a hotel, 6 are very kind and cooperative with their customers so they are very popular , while the other two remain reserved. For a five star hotel three waiters are selected at random. Find the probability distribution of the number of very popular waiters

Or

b) Suppose that 5 boys out of 40 boys and 8 girls out of 50 girls are good speakers .assuming that there are equal number of boys and girls, find the probability of choosing a good speaker

28. a) find the particular solution of the differential equation $\log(\frac{dy}{dx}) = 3x + 4y$, if x = y = 0

Or

b) Solve the differential equation $(x + 2y^2)\frac{dy}{dx} = y$ given that y=1 when $x = 2$

29. $\int \sqrt{tanx} + \sqrt{cotx}\, dx$

30. Solve the following Linear Programming Problem graphically:

Maximize Z = 4500x + 5000y

Subject to constraints : 25000x + 40000y ≤ 7000000

x + y ≤ 250

x , y ≥ 0

31. evaluate $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x}\, dx$

SECTION-D

The following questions are of five marks each

32. Using integration, find the area of triangle ABC, where A (2,3) , B(4,7) and C (6,2) .

33. a) Find the shortest distance between the lines x + 1 = $\frac{y}{2}$ = -z and x = y+2 = 1- z

Or

b) find the coordinates of the foot of perpendicular and the length of perpendicular drawn from the point P(5,4,2) to the line $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + \hat{k} + \lambda((2\,\hat{\imath} + 3\,\hat{\jmath} - \hat{k})$

34. find $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$. Use this to solve the following system of equations

x-y+z=4 ; x – 2y -2z =9 , 2x + y +3z = 1

35. let N denote the set of all natural numbers and R be the relation on N×N defined as

(a,b) R (c,d) if ad(b+c) = bc(a+d), show that R is an equivalence relation

Or

Show that the relation R in the set A ={ 1,2,3,4,5} given by R ={(a,b): | a-b | is divisible by 2}

is an equivalence relation. Write all the equivalence classes of R.

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two case study questions have three sub parts (i),(ii),(iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. Case-study 1: Read the following passage and answer the questions given below.



Raja purchases 3 pens, 2 pencils and 1 mathematics instrument box and pays Rs. 41 to the shop keeper. His friends, Daya and Anil purchases 2 pens, 1 pencil, 2 instrument boxes and 2 pens, 2 pencils and 2 mathematical instrument boxes respectively. Daya and Anil pays Rs. 29 and Rs. 44 respectively.

Based on the above information answer the following

(i) Find the cost of one pen.

(ii) Find the cost of one pen and one mathematical instrumental box.

(iii) Find the cost of one pencil and one mathematical instrumental box.

OR

Find the cost of one pen, one pencil and one mathematical instrument.

37. Case-study 2: Read the following passage and answer the questions given below.

In the office three employees Mehul, Janya and Charvi process incoming matter related to a particular project. Mehul processes 40% of the matter and Janya and Charvi process rest of the matter equally. It is found that 6% of matter processed by Mehul has an error where as for Janya and Charvi error rate is 4% and 3% respectively.



(i) Find the conditional probability that an error is committed in processing by Janya while processing the matter.

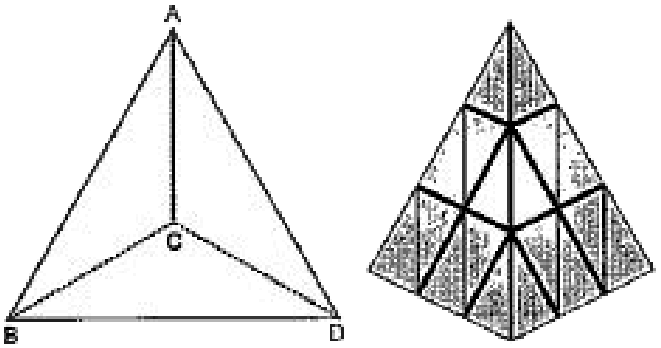
(ii) What is the probability that the matter processed by Janya has an error?

(iii) What is the probability of an error in processing the matter?

OR

(iv) The processed matter is checked and the selected matter has an error, what is the probability that it was processed by Mehul?

38. **Case-study 3:** Read the following passage and answer the questions given below.



A building is to be constructed in the form of a triangular pyramid, ABCD as shown in the figure. Let its angular points are A(0, 1, 2), B(3, 0, 1), C(4, 3, 6) and D(2, 3, 2) and G be the point of intersection of medians of Δ BCD.

(i) Find the coordinates of point G.

(ii) Find the length of vector $\overrightarrow{AG}$.

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KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION

SAMPLE PAPER -2

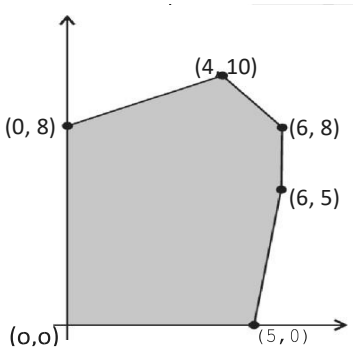
SUBJECT:-MATHEMATICS (041)

Time: - 3 Hours **CLASS: XII** **Max Marks: - 80**

General Instructions:

1. This Question paper contains - five sections **A, B, C, D** and **E**. Each section is compulsory. However, there are internal choices in some questions.
2. Section **A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section **B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section **C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section **D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section **E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

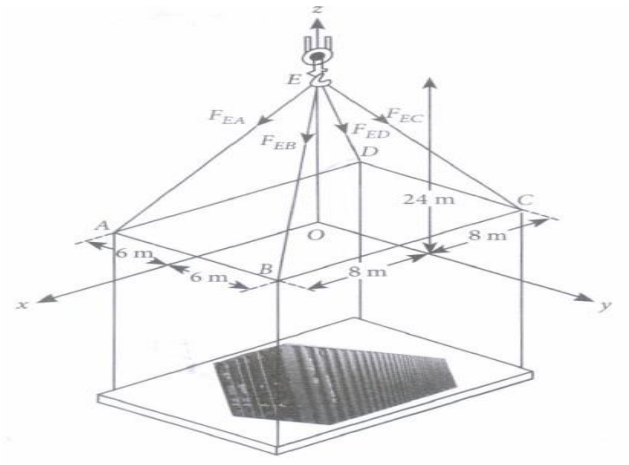
Q. No	**SECTION – A** **(Multiple Choice Questions)** Each question carries 1 mark**.**
1	If $A = [a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, \text{when } i \neq j \\ 0, \text{when } i = j \end{cases}$, then A^2 is (a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
2	If the matrix A= $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew-symmetric matrix , then the value of $a + b$ is (a) 2 (b) -2 (c) 0 (d) -5
3	If A is square matrix of order 3, such that $\|A\| = 8$,then $\|-A\|$ equals (b) 16 (b) -8 () 8 (d) -16
4	The area of a triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 sq. units. The value of k will be (a) 9 (b) 3 (c) – 9 (d) 6
5	If A and B are invertible matrices, then which of the following is not correct? (a) adj A = |A|. A^{-1} (b) $\det(A)^{-1} = [\det(A)]^{-1}$ (c) $(AB)^{-1} = B^{-1} A^{-1}$ (d) $(A + B)^{-1} = B^{-1} + A^{-1}$
6	The function f (x) = [x], where [x] denotes the greatest integer function, is continuous at (a) 4 (b) – 2 (c) 1 (d) 1.5
7	A man is watching an aero plane which is at the coordinate point A (4, -1, 3), assuming the man is at O (0, 0, 0). At the same time, he saw a bird at the coordinate point B (2, 0, 4). The unit vector along $\overrightarrow{AB}$ is (a) $\frac{2}{6}\hat{\imath} + \frac{1}{6}\hat{\jmath} + \frac{1}{6}\hat{k}$ (b) $\frac{-2}{\sqrt{6}}\hat{\imath} + \frac{1}{6}\hat{\jmath} + \frac{1}{6}\hat{k}$ (c) $\frac{-2}{\sqrt{6}}\hat{\imath} + \frac{1}{\sqrt{6}}\hat{\jmath} + \frac{1}{\sqrt{6}}\hat{k}$ (d) $\frac{4}{\sqrt{6}}\hat{\imath} + \frac{2}{\sqrt{6}}\hat{\jmath} + \frac{3}{\sqrt{6}}\hat{k}$

8	The value of the expression $\left\|\vec{a} \times \vec{b}\right\|^2 + \left(\vec{a}.\vec{b}\right)^2$ is $(a)\vec{a}.\vec{b}$ $(b)\|\vec{a}\|.\left\|\vec{b}\right\|$ (c) $\|\vec{a}\|^2 \left\|\vec{b}\right\|^2$ (d) $(\vec{a}.\overrightarrow{b})$
9	On which of the following intervals is the function f given by f (x) = x^{100} + sin x –1 decreasing (a) (0,1) (b) $\frac{\pi}{2}, \pi$ (c) $0, \frac{\pi}{2}$ (d) None of these
10	$\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to (a)tan x + cot x + c (b) sin x + cos x + c (c) tan x − cot x + c (d) sin x − cos x + c
11	P is a point on the line joining the points (0,5, −2) and B(3, −1,2). If the x-coordinate of P is 6, then its z-coordinate is (a) 10 (b) 6 (c) -6 (d) -10
12	The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is (a) 3 (b) 2 (c) 1 (d) not defined
13	The feasible solution for a LPP is shown in given figure. Let Z=3x-4y be the objective function. Minimum of Z occurs at a) (0,0) b) (0,8) c) (5,0) d) (4,10) 
14	If is $\vec{a}$ nonzero vector of magnitude 'a' and λ a nonzero scalar, then $\lambda\vec{a}$ is unit vector if (a) λ = 1 (b) λ = – 1 (c) a = | λ | (d) a = $\frac{1}{\|\lambda\|}$
15	The coordinates of the foot of the perpendicular drawn from the point (2, 5, 7) on the x-axis are given by (a) (2, 0, 0) (b) (0, 5, 0) (c) (0, 0, 7) (d) (0, 5, 7)
16	$If\ \left\|\vec{a} + \bar{b}\right\| = \left\|\vec{a} - \bar{b}\right\|$, then the angle between $\overrightarrow{a}$ and $\vec{b}$ is (a) 90° (b)60° (c)110° (d)180°
17	The corner points of the feasible region determined by the system of linear constraints are (0.10), (5,5), (15,15), (0,20). Let = px + qy , where p, q > 0. Conditions on p and q so that the maximum value of Z occurs at both the points (15,15) and (0,20) is (a) p = q (b) p = 2q (c) q = 2p (d) q = 3p

18	If A and B are two events such that P(A)+P(B)- P(A and B)=P(A), then (a)P(B/A) =1 (b)P(A/B) =1 (c) P(A/B) =0 (d) P(B/A) =0
	ASSERTION-REASON BASED QUESTIONS In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) **Both A and R are true and R is the correct explanation of A.** (b) **Both A and R are true but R is not the correct explanation of A.** (c) **A is true but R is false.** (d) **A is false but R is true.**
19	A: The Principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)+2\ \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is equal to $\frac{5\pi}{4}$ R: If domain of $\cos^{-1}x$ and $\sin^{-1}x$ are respectively (0, π) and $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
20	A: The following straight lines are perpendicular to each other. $$\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}\ and\ \frac{1-x}{-1}=\frac{y+2}{2}=\frac{3-z}{3}$$ R: Let line L-l passes through the point (x_1, y_1, z_1,) and parallel to the vector whose direction ratios are a_1, b_1, and c_1, and let line L- 2 passes through the point (x_2, y_2, z_2,) and parallel to the vector whose direction ratios are a_2, b_2, and c_2, · Then the lines L-1 and L-2 are perpendicular if $a_1.a_2+b_1.b_2+c_1.c_2=0$
	SECTION – B **(This section comprises of very short answer type-questions (VSA) of 2 marks each.)**
21	Find the domain of the function defined by $f(x)=\sin^{-1}\sqrt{x-1}$ **OR** Write in the simplest form: $\tan^{-1}\frac{\cos x-\sin x}{\cos x+\sin x}$, $-\frac{\pi}{4}<x<\frac{3\pi}{4}$.
22	Find $\frac{dy}{dx}$ of the function $y^x=x^y$ **OR** Find the values of k so that the function f is continuous at the indicated point $f(x)=\begin{cases}kx+1, & \text{if } x\le\pi\\ \cos x, & \text{if } x>\pi\end{cases}$ at $x=\pi$
23	Evaluate: $\int\frac{(x^4-x)^{\frac{1}{4}}}{x^5}dx$ **OR** Evaluate: $\int\frac{dx}{5-8x-x^2}$
24	Find the least value of 'a' such that function f given by $f(x)=x^2+ax+1$ is strictly increasing on $(1,2)$. **OR** It is given that at $x=1$ function x^4-62x^2+9x+9 attains maximum value on the interval $[0,2]$. Find the value of a.
25	The volume of a cube is increasing at the rate of 9 cubic cm per sec . How fast is the surface area increasing when the length of an edge is 10 cm.

SECTION C **(This section comprises of short answer type questions (SA) of 3 marks each)**	
26	If $y = (\tan^{-1}x)^2$, show that $(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$.
27	Evaluate: $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$
28	Evaluate $\int x \sin^{-1} x \, dx$ **OR** Evaluate $\int_0^{\frac{\pi}{2}} \log \sin x \; dx$
29	Show that the differential equation $(x-y)\frac{dy}{dx} = x + 2y$ is homogeneous and solve it. **OR** Find the general solution of the differential equation $x\frac{dy}{dx} + y - x + xy \cot x = 0 \; (x \neq 0)$
30	Solve the following Linear Programming Problem graphically: Maximize $Z = 5x + 2y$ subject to the constraints: $x - 2y \leq 2$, $3x + 2y \leq 12$, $-3x + 2y \leq 3$, $x \geq 0$, $y \geq 0$.
31	Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X). **OR** The random variable X has a probability distribution P(X) of the following form, where k is some number : $P(X)=\begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$ (a) Determine the value of k. (b) Find P ($X < 2$), (c) Find P ($X \geq 2$),
SECTION D **(This section comprises of long answer-type questions (LA) of 5 marks each)**	
32	Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2+1}$, $\forall x \in R$ is neither one-one nor onto. **OR** Let $A = \{1,2,3,\ldots,9\}$ and R be the relation in $A \times A$ defined by (a, b) R (c, d) if $a + d = b + c$, for $(a,b),(c,d)$ in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalence class [(2,5)].

33	If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$ find A^{-1}. Hence, solve the system of equations: $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$ and $x + y - 2z = -3$
34	Make a rough sketch of the region $\{(x, y): 0 \le y \le x^2, 0 \le y \le x, 0 \le x \le 2\}$ and find the area of the region using integration.
35	Prove that the lines $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular if $pp' + rr' + 1 = 0$. **OR** Find the equation of line passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ & $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.
SECTION E **(This section comprises of with two sub-parts. First two case study questions have three subparts of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each.)**	
36	In an elliptical sport field, the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  (i) If the length and the breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x. (ii) Find the critical point of the function. (iii) Use Second Derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.
37	Consider the following diagram, where the forces in the cable are given.



Based on the above information, answer the following questions.

(i) Find The vector of Direction Ratios of a Line $\overrightarrow{ED}$

(ii) The length of the cable $\overrightarrow{EB}$ is

(iii) Find The sum of all vectors of Direction Ratios along the cables.

OR

Find the cartesian equation of line along $\overrightarrow{EA}$

38 One day, a sangeet mahotsav is to be organised in an open area of Rajasthan. In recent years, it has rained only 6 days each year. Also, it is given that when it actually rains, the weatherman correctly forecasts rain 80% of the time. When it doesn't rain, he incorrectly forecasts rain 20% of the time. If leap year is considered, then answer the following questions.



(i) Find the probability that the weatherman predict rain.

(ii) Find the probability that it will rain on the chosen day, if weatherman predict rain for that day

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KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION

SAMPLE PAPER - 3

Class:-XII

Session 2023-24

Mathematics (Code-041)

Time: 3 hours Maximum marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section – A

[This section comprises of MCQ 's of 1 marks each

1) The matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is

a)a skew-symmetric matrix (b) a symmetric matrix

(c) a diagonal matrix (d) an upper triangular matrix

2) Find the value of the matrix A= $\begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$ is singular

a) 0, 1 b)1,3 (c) 0, 3 d) 3,2

3) The area of a triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 sq. units.

The value of k will be

(a) 9 (b) 3 (c) -9 (d) 6

4) A function f(x) = $\begin{cases} \frac{sinx}{x} + \cos x & x \neq 0 \\ 2k & x = 0 \end{cases}$ is continuous at x = 0 for

(a) k = 1 (b) k = 2 (c) k = $\frac{1}{2}$ d)) k = $\frac{3}{2}$

5) The value of λ for which the vectors $3\hat{\imath}-6\hat{\jmath}+\hat{k}$ and $2\hat{\imath}-4\hat{\jmath}+\lambda\hat{k}$ are parallel is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{2}{5}$

6) Find the degree of the differential equation $\left(1+\frac{dy}{dx}\right)^3 = \left(\frac{dy}{dx}\right)^2$

a) 0 b)1 c) 2 d)3

7) The corner points of the feasible region determined by the system of linear constraints are (0,10) ,(5,5), (15,5), (0,20). Let Z=px+qy , where p,q>0. Condition on p and q so that the maximum of Z occurs at both the points (5,15) and (0,20) is

a) p=3q b) p =2q c) p =q d) q =3p

8)The vectors from origin to the points A and B are $\vec{a}= 2\hat{\imath}-3\hat{\jmath}+2\hat{k}$ and $\vec{b}= 2\hat{\imath}+3\hat{\jmath}+\hat{k}$, respectively then the area of triangle OAB is

(a) 340 (b) √25 (c) √229 (d) $\frac{1}{2}$ √229

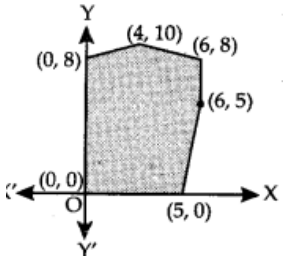
9) Evaluate $\int_0^{2\pi} \sin(\frac{\pi}{4}+\frac{x}{2})$ dx

(a) -2√2 (b) -2 (c) √2 (d) 2√2

10) Find $\begin{bmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{bmatrix}$

(a) $9x^2(x+y)$ (b) $9y^2(x+y)$ (c) $3y^2(x+y)$ (d) $7x^2(x+y)$

11) The feasible solution for a LPP is shown in the of following figure . Let Z=3x-4y be the objective function.



Minimum of Z occurs at:

a)(0,0) b) (0,8) c) (5,0) d) (4,10)

12. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be vectors with magnitudes 3, 4 and 5 respectively and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then the values of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

(a) 47 (b) 25 (c) 50 (d) -25

13) If A and B are invertible square matrices of size n x , then which of the following is not true?

(a) $|AB| = |A||B|$ (b) $|KA| = k^n |A|$ (c) $|A+B| = |A|+|B|$ (d) $|A^{-1}| = |A|^{-1}$

14) Let A and B be independent events with P(A) = 1/4 and

P(A ∪ B) = 2P(B) – P(A). Find P(B)

a) $\frac{1}{4}$ b) $\frac{3}{5}$ c) $\frac{2}{3}$ d) $\frac{2}{5}$

15) The solution of differential equation $(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$ is

(a) $(e^y + 1)\sin x = c$ (b) $e^x \sin x = c$ (c) $(e^x + 1)\cos x = c$ (d) none of these

16) The vectors $\overrightarrow{AB} = 3\hat{\imath}+4\hat{k}$ and $\overrightarrow{AC}=5\hat{\imath}-2\hat{\jmath}+4\hat{k}$ are the side of a ΔABC. The length of the median through A is

(a) √18 (b) √72 (c) √33 (d) √288

17) Write the number of points where f(x) = |x + 2| + |x – 3| is not differentiable.

(a) 2 (b) 3 (c) 0 (d) 1

18) Distance of the point (α, β, γ) from y-axis is

(a) β (b) |β | (c) |β| + |γ | (d) $\sqrt{\alpha^2+\gamma^2}$

ASSERTION REASONING BASED QUESTIONS

In the following questions , a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices

a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion

b.) Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.

c.) Assertion is true but the reason is false.

d.) Assertion is false and reason is true.

19) Assertion (A): The minimum value of the expression $\boldsymbol{x^2 + 2bx + cisc - b^2}$

Reason (R): The first order derivative of the expression at $\boldsymbol{x = -b}$ is zero.

20) Assertion (A): If f : R→R is a function defined by $f(x) = \frac{2x+1}{3}$, $f^{-1}(x) = \frac{3x-1}{2}$

Reason (R): f(x) is not a bijection.

Section –B

[This section comprises of very short answer type questions (VSA) of 2 marks each

21.Find the principal value of $\tan^{-1}\left\{sin\left(-\frac{\pi}{2}\right)\right\}$

OR

Find the domain of $\cos^{-1}(2x-3)$

22. Find the intervals in which the function f: R→R defined by $f(x) = \cot^{-1}x + x$ increases

23.Find the absolute maximum value of the function $f(x) = 4x - \frac{1}{2}x^2$ in the interval $[-2, \frac{9}{2}]$

OR

Find the maximum profit that a company can make if the profit function is given by

$P(x) = 41 - 72x - 18x^2$

24. Find $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}}\,dx$

25. The volume of a cube is increasing at a rate of 7 cm^3/sec. How fast is the surface area increasing when the length of an edge is 12 cm

Section- C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Evaluate $\int \frac{x^2+1}{(x+3)(x-1)^2}\,dx$

27 Let, X denote the number of colleges where you will apply after your results and P(X = *x*) denotes your probability of getting admission in *x* number of colleges. It is given that

$$P(X=x) = \begin{cases} kx, & if\ x = 0\ or\ 1 \\ 2kx, & if\ x = 2 \\ k(5-x), & if\ x = 3\ or\ x = 4 \\ 0 & , if\ x > 4 \end{cases}$$

where *k* is a positive constant. Find the value of *k*. Also find the probability that you will get admission in (i) at most 2 colleges (ii) at least 2 colleges.

28. Evaluate $\int_0^{\pi} \frac{x\,tanx}{secx+tanx}\,dx$

OR

$\int(\sqrt{cotx}+\sqrt{tanx}\quad dx$

29.Find the particular solution of the differential equation $\frac{dy}{dx}+ytanx=3x^2+x^3tanx\ \ ,y=0$ when $x=\frac{\pi}{3}$

OR

Solve $x\frac{dy}{dx}=y-x\tan\frac{y}{x}$

30. Solve the following Linear Programming Problem graphically:

Maximise $Z=5x+2y$ subject to the constraints

$x-2y\leq 2, 3x+2y\leq 12, -3x+2y\leq 3\ , x\geq 0, y\geq 0$

OR

Solve the following Linear Programming Problem graphically:

Minimise $Z=-50x+20y$ subject to the constraints

$$2x-y\geq\ -5\ \ , 3x+y\geq 3\ , 2x-3y\leq 12\ \ x\geq 0, y\geq 0$$

31. If $y=(x+\sqrt{1+x^2})^n$,then show that $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=n^2y$

Section- D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. Make a rough sketch of the region { (x,y): $x^2+y^2\leq 1\leq x+y$ } and find the area of the region using the method of integration.

33. Let N denote the set of all natural numbers R be the relation on N XN defined by

(a,b) R (c,d) ↔ ad(b+c) = bc(a+d),check whether R is an equivalence relation on N X N

OR

Show that the function f: R → R defined by $f(x) = 3x^3+5$ for all x∈ R is a bijection

34. If A = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ B = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, then find A B and use this to solve the system of equations y+2z=7 x− y = 3 2x+3y+4z = 17

35. Find the shortest distance between the lines $\vec{r}$= (4$\vec{\imath}$ -$\vec{\jmath}$)+ λ ($\vec{\imath} + 2\vec{\jmath} - 3\vec{k}$) and

$\vec{r}$= ($\vec{\imath}$ –$\vec{\jmath} + 2\vec{k}$)+ μ (2$\vec{\imath} + 4\vec{\jmath} - 5\vec{k}$)

OR

Show that the lines $\vec{r}$= ($\vec{\imath}$ +$\vec{\jmath} - \vec{k}$)+ λ (3$\vec{\imath} - \vec{\jmath}$) and $\vec{r}$= (4$\vec{\imath} - \vec{k}$)+ μ (2$\vec{\imath} + 3\vec{k}$)intersect .Also find their point of intersection .

Section –E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

36.A doctor is to visit a patient. From the past experience, it is known that the probabilites that he will come by cab, train, bike or by other means of transport are 0.3,0.2,0.1 and 0.4 respectively . The probabilites that he will be late are 0.25,0.3 ,0.35 and 0.1 , if he comes by cab, train, bike or other means of transport, then he will not be late.

i)When he arrives, he is late. What is the probability that he comes by train?

ii)When he arrives, he is late. What is the probability that he comes by cab?

iii)When he arrives, he is late. What is the probability that he comes by other means of transport?

OR

iv) What is the probability that doctor is late by any means ?

37. A potter made a mud vessel where the shape of the pot is based on $f(x) = |x - 3| + |x - 2|$ where $f(x)$ represent the height of the pot

i) When $x > 4$, what will be the height interms of height?

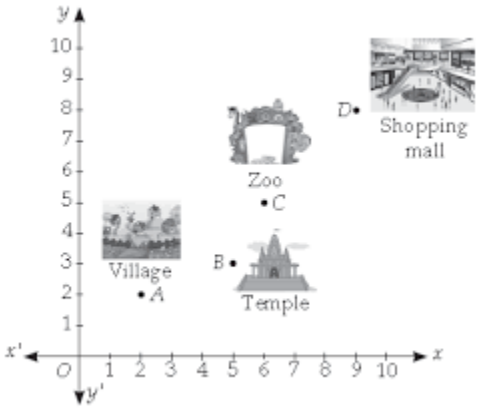
ii) When the x value lies between $(2,3)$ then the function is ……

iii) What is $\frac{dy}{dx}$ at $x = 3$?

OR

iv) If the potter is trying to make a pot using the function $f(x) = [x]$,will he get a pot or not ?Why ?

38. Ishaan left from his village on weekend. First, he travelled up to temple. After this, he left for the zoo. After this he left for shopping in a mall. The positions of Ishaan at different places is given in the following graph.



Based on the above information, answer the following questions.

i) What is the position vector of B

ii) Find the vector $\overrightarrow{BC}$ in terms of $\hat{\imath}$ and $\hat{\jmath}$?

iii) Find the length of vector $\overrightarrow{AD}$?

OR

iv) If $M = 4\hat{\jmath} + 3\hat{k}$, then write the unit vector in the direction of M?

…….

KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION

Sample paper 4

Class XII

Mathematics (Code-041)

Time Allowed: 3 Hours Maximum Marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Part A

1. If A and B are symmetric matrices of the same order, then (AB′ –BA′) is a
   (A) Skew symmetric matrix (B) Null matrix
   (C) Symmetric matrix (D) None of thes
2. If A and B are invertible matrices, then which of the following is not correct
   (A) adj A = |A|. A^{-1} (B) $\det(A^{-1}) = [\det(A)]^{-1}$
   (C) $(AB)^{-1} = B^{-1} A^{-1}$ (D) $(A + B)^{-1} = B^{-1} + A^{-1}$
3. If A is a matrix of order 3 × 3, and |3A| =27 then $|adj2A|$=
   (A) 18 (B) 8
   (C) 64 (D) None of these
4. If f (x)=$\begin{cases} \pi x sin(\frac{1}{x}) & if\ x \neq 0 \\ p & x = 0 \end{cases}$ is continuous at x = 0.then P
   (A) 1 (B) π
   (C) $\frac{\pi}{2}$ (D) None of these
5. The Value of q for which the lines $\vec{r} = \hat{\imath} + 2\hat{\jmath} + \hat{k} + s(\hat{\imath} + q\,\hat{\jmath} - \hat{k})$ and $\vec{r} = -\hat{\imath} - \hat{\jmath} + \hat{k} + t(-q\hat{\imath} + 2\,\hat{\jmath} + \hat{k})$ are perpendicular
   (A) 0 (B) 1
   (C) -1 (D) None of these
6. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$ is
   A) 1 (B) 2
   (C) 3 (D) None of these

7. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let Z = px + qy, where p, q > 0. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is

   (A) p = q (B) p = 2q

   (C) q = 2p (D) q = 3p

8. Which of the following points is a feasible solution to the linear constraints $2x+y\leq 10$, $x+2y\geq 8$

   (A) (0,0) (B) (3,2)

   (C) (4,2) (D) None of these

9. If G is the centroid of the triangle ABC then $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}$

   (A) $\vec{o}$ (B) 3 $\overrightarrow{GC}$

   (C) 2 $\overrightarrow{GC}$ (D) $\overrightarrow{GC}$

10. If $\sqrt{3}\left[\vec{a}.\vec{b}\right]=\left[\vec{a}\times\vec{b}\right]$ then angle between $\vec{a}$ and $\vec{b}$ is

    (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

    (C) $\frac{\pi}{4}$ (D) None of these

11. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqrt{\frac{1-cos2x}{2}}$ =

    (A) 2 (B) π

    (C) $\frac{\pi}{2}$ (D) None of these

12. If $A=\begin{pmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{pmatrix}$ then $|A|$=

    (A) 0 (B) pqr

    (C)$(p-q)$ (q -r) (r -p) (D) None of these

13. Projection of $2\hat{\imath}+\hat{\jmath}$ on $\hat{\imath}+\hat{k}$ =

    (A) 0 (B) 1

    (C) -1 (D) None of these

14. If $y=\sin^2(x^3)$then $\frac{dy}{dx}$ =

    (A) $2\sin x^3 \cos x^3$ (B) $3x^2\sin x^3 \cos x^3$

    (C) $6x^2\sin x^3 \cos x^3$ (D) $3x^2\cos^2 x^3$

15. Area of the parallelogram with diagonals $2\hat{\jmath}$ on $\hat{\imath}+2\hat{k}$

    (A) $\sqrt{5}$ sq.units (B) 5 sq.units

    (C) $2\sqrt{5}$ sq.units (D) None of these

16. Integrating factor of the differential equation $x\frac{dy}{dx}$ - y = sinx

    (A) x (B) -x

    (C) $\frac{1}{x}$ (D) e^{-x}

17. If a line make angle $\frac{\pi}{3}$ with x axis ,$\frac{\pi}{2}$ with y axis then the acute angle made by the line with z axis is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

18. A and B are two independent events events such that P(A'UB) =

(A) P(A') P(B) (B) 1-P(A ∪B)

(C) 1- P(A)PB') (D) P(A) P(B')

19. Assertion A: The difference between the greatest and smallest value of f(x) = $\frac{\pi}{4}$ -$\tan^{-1} x$ on [0,1]is $\frac{\pi}{4}$

Reason (R): If a function g decreases on [a, b], then greatest value of g=g(a) and least value of g = g(b)

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true and R is not the correct explanation of A

(C) A is true and R is false

(A) A is false and Rare true and R

20. Assertion (A) If X = {1,2,3} then number of onto functions that can be defined from X to X is 9

Reason (R): If X is a finite set and f:X→X is a onto function then f is one to one

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true and R is not the correct explanation of A

(C) A is true and R is false

(A) A is false and R are true and R

Part B

21. Find the principal value of $\tan^{-1}[\tan(\frac{67\pi}{8})]$

Or

The domain of $y = \cos^{-1}(x^2 - 4)$

22. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then how fast is the slope of curve changing when x = 3?

23. Find the interval in which $f(x)= x^2e^{-x}$ is increasing

Or

Find the maximum value f(x) = $\frac{1}{9x^2+6x+5}$

24. Evaluate: $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\, dx$

25. Find the point(s) of inflection of the function $f(x)= 4x^3 - 18x^2 + 27x - 7$

Part C

26. $\int \frac{1}{(x^2+1)(x^2+4)}\, dx$

27. Evaluate: $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$

Or

Evaluate: $\int [\frac{1}{logx} - \frac{1}{(logx)^2}]dx$

28. Find the particular solution to the differential equation: $2ye^{\frac{x}{y}}$ dx+(y-2x$e^{\frac{x}{y}}$)$dy = 0$ given that x=0 then y=1

Or

Find the particular solution to the differential equation: $(tan^{-1}y$ –x)dy = $(1+y^2)dx$ given that when x=0,then y=0

29. If $y\sqrt{x^2+1}$ = log $(\sqrt{x^2+1} - x)$ then prove that: $(x^2 + 1)\frac{dy}{dx} + xy + 1 = 0$

30. Two cards are drawn with out replacement from a pack of 52 cards. Find the mean of the number of red cards drawn

31. Solve the following linear programming problem graphically.

Maximise: Z= 5x + 2 y

Subject to the constraints: x -2y ≤ 2, 3x + 2y ≤ 12, -3x+2y≤3,x ≥ 0 and y ≥ 0

Or

Minimise: Z=3x+5y subject to the constraints: x+3y≥3, x+y≥2, x≥1, y≥0

Section D

32. Make a rough sketch and find the area of the region**: {(x, y):$0 \leq x \leq y^2, 0 \leq x \leq y, 0 \leq y \leq 2$},** using method of integration

33. Show that the relation on the set A = {x ∈ Z: 0 ≤ x ≤ 12}, given by R = {(a, b): $|a - b|$ is a multiple of 4 is an equivalence relation. Also find the set of elements related to 1

OR

Show that the function f : R → R defined by f (x) = $\frac{x}{x^2+1}$ ∀ x∈R , is neither one-one nor onto.

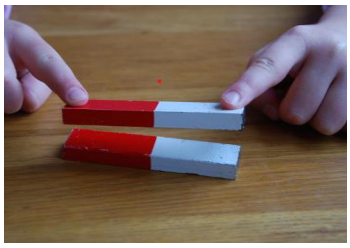
34. In a survey report for sanctioning a stoppage for trains, A rail way track is represented as the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ and a city center is represented as the point P (2,3, -8). Find the coordinate of the stop Q on the line so that distance PQ is shortest. also find the equation of PQ

OR

Two submarines are moving along the lines $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ and $\frac{x-1}{0} = \frac{y+1}{-2} = \frac{z}{1}$ at which point on their way they are nearest each other?

35. . If A = $\begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$; Find A^{-1}. Hence solve the system of equations. 4x+2y+3z=2, x+y +z=1; 3x+y-2z= 5

Part E



36. Mr Kumar kept a pin of negligible weight at a point A (1,2,3) and magnet M_1 at B (3,4,2) and Magnet M_2 at point C(7,5,5) of a rectangular coordinate system. magnet M_1 can exert a pulling force to the pin having magnitude 3 newton along the line joining of AB and magnet M_2 can also pull the pin towards it having magnitude of 7 newton along the line joining of AC

    (i) Express the force $\overline{F_1}$, the pulling force of magnet M_1 acts on the pin in terms of unit vectors $\hat{\imath}, \hat{\jmath}\ and\ \hat{k}$

    (ii) Express the force $\overline{F_2}$, the pulling force of magnet M_2 acts on the pin in terms of unit vectors $\hat{\imath}, \hat{\jmath}\ and\ \hat{k}$

    (iii) Find the magnitude of resultant of the forces $\overline{F_1}$ and $\overline{F_2}$ and direction cosines of the resultant

    Or

    Find direction cosines of $\overline{F_1} \times \overline{F_2}$

37. If f(x)=$\begin{cases} |x-1|+x & if\ x<1 \\ [x] & if\ x \geq 1 \end{cases}$

    (i) Find LHD of the function f(x) at x=1

    (ii) Find RHD of the function f (x) at x=1

    (iii) State whether f '(1) exists or not, if exist write the value of f '(1)

38. 

    A shopkeeper sells three types of flower seeds A, B and C. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%.

    (i) Calculate the probability of a randomly chosen seed to germinate

    (ii) If it is known that the seed did not germinate, Calculate the probability that it will be the seed of type C,

END

KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION

Sample paper -5

MATHEMATICS

TIME-3 Hrs **CLASS XII** **M.Marks-80**

General instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory.
2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has **5 Very Short Answer (VSA)-type** questions of 2 mark each.
4. **Section C** has **6 Short Answer (SA)-type** questions of 3 mark each.
5. **Section D** has **4 Long Answer (LA)-type** questions of 5 mark each.
6. **Section E** has **3 sources based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION A

MCQ (Each question carries 1 mark)

1. If AB-BA is Skew symmetric matrix and A is symmetric matrix, then B is
   a. Identity matrix b.Skew symmetric matrix
   c.symmetric matrix d.Scalar matrix

2. If A is a square matrix such that $A^2 = A$, then $(I+A)^3 - 7A$ is equal to
   a. A b. I-A c. I d. 3A
3. If A is an invertible matrix of order 3x3 and det A=0, then det A^{-1} is
   a. not defined b. 3 c. 1 d.0
4. If $A\begin{bmatrix}\cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}$, then $A+A^1=I$, the value of α is
   a. π/6 b. π/3 c.π d.3π/2
5. If $\begin{vmatrix}x+1 & x-1 \\ x-3 & x+2\end{vmatrix} = \begin{vmatrix}4 & -1 \\ 1 & 3\end{vmatrix}$, then write the value of x
   a. -2 b. 1/2 c. 2 d. 3
6. The function f(x) = [x], where [x] denotes the greatest integer function, is continuous at
   a. 4 b. -2 c. 1 d. 1.5
7. For the curve $\sqrt{x}+\sqrt{y}=1$, $\frac{dy}{dx}$ at (1/4,1/4)is
   a.1/2 b. 1 c. -1 d. 2
8. $\int_{-\pi/2}^{\pi/2} \sin^5 x\,dx$ equal to
   a. 0 b.-1 c.1 d. 5

9. $\int e^x(\cos x - \sin x)dx$ equal to

   a. $e^x \cos x + c$ b. $e^x \sin x + c$

   c. $-e^x \cos x + c$ d. $- e^x \cos x + c$

10. Which of the following is not a homogeneous function of x and y

    a. $2xy+x^2$ b.2x-y c.$\cos^2(y/x)+y/x$ d. sin x-cos y

11. The degree of the differential equation is $\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is

    a. 4 b. 3/2 c. not defined d. 2

12. The projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$ is

    a. 0 b. 1 c. -1 d. none of these

13. For what value of **a** the vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear

    a. 4 b. -4 c. 2 d. -2

14. If Θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a} - \vec{b}| = |\vec{a} + \vec{b}|$, where Θ is

    a. 0 b. π/4 c.π/2 d.π

15. The direction ratio of the line passing through two points (2,-4,5) and (0,1,-1) are

    a. (-2,5,-6) b.(-2,0,-6) c.(-2,5,0) d.(2,-3,4)

16. The corner points of the feasible region for a LPP are p(0,5), Q(1,5),R(4,2) and S(12,0). The minimum value of objective function Z= 2x+5y is at the point

    a. P b. Q c. R d. S

17. The point which does not lie in the half plane $2x+3y-12 \leq 0$ is

    a. (-1,2) b. (2,1) c. (2,3) d.(-3,2)

18. If p(A)= 3/10 and p(B)=2/5 and p(AUB)=3/5 ,then P(B/A) + P(A/B)

    a. 1/4 b. 1/3 c.5/12 d.7/12

ASSERTION- REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices

a. Both A and B are true and R is the correct explanation of A

b. . Both A and B are true and R is not the correct explanation of A

c. A is true but R is false

d. A is false but R is true

19. **Assertion(A):** Principal value of $\sin^{-1}(1/\sqrt{2})$ is π/4
    Reason (R): Principal value of $\cot^{-1}(-1/\sqrt{3})$ is π/3

20. **Assertion(A):**The acute angle between the line $\vec{r} = \hat{i} + \hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j})$ and the x-axis is π/4

    Reason (R): The acute angle between the lines $\vec{r} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + \lambda(a_1 i + b_1\hat{j} + c_1\hat{k})$ and $\vec{r} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} + \mu(a_2 i + b_2\hat{j} + c_2\hat{k})$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

SECTION B

21. Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\sqrt{3}$

OR

Evaluate $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

22. The length x of a rectangle is decreasing at the rate of 3cm/minutes and the write y is increasing at the rate of 2 cm/minutes. When x = 10 cm and y=6cm, find the rate of change of (a) the perimeter and (b) the area of the rectangle

23. If x = a(cosΘ + ΘsinΘ) , y = a(sinΘ - ΘcosΘ), find dy/dx

OR

If $y=(\sin x-\cos x)^{(\sin x-\cos x)}$, find dy/dx

24. Find λ and μ if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=0$

25. Find the Cartesian and vector equation of the line passes through the point (-2,4,-5) and parallel to the line given by $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$

SECTION C

26. Find $\int\frac{dx}{(x^2+1)(x-1)}$

27. Find $\int_{-5}^{2}|x+2|dx$ **OR** Find $\int_{0}^{\pi/4}\log(1+\tan x)dx$

28. Find $\int\frac{1}{9x^2+6x+5}dx$

29. Solve the differential equation $x\frac{dy}{dx}+2y=x^2,\quad x\neq 0$

OR

Solve the differential equation $x\,dy-y\,dx=(x^2+y^2)^{1/2}dx$

30. Solve the following LPP graphically:
Minimise z=3x+4y, subject to x+2y≤8, 3x+2y≤12, x,y≥0

31. Assume that each born child is equally likely to be a boy or girl. If a family has two children, what is the conditional probability that both are girls given that

a. Youngest is a girl b .at least one is a girl

OR

Probability of solving specific problem independently by A and B are 1/2 and 1/3 respectively. If both try to solve the problem independently. Find the probability that

a. The problem is solved b. exactly one of them solves the problem

SECTION D

32. Show that the relation R defined by (a,b)R(c,d) ⇒ a+d=b+c in the set N is an equivalence relation

OR

Let A ={ x Ɛ Z : 0 ≤ x ≤12 }. Show that R={(a,b):a,bƐA, |a-b| is divisible by 4} is an equivalence relation. Find the set of all elements related to 1

33. Find the area bounded by the circle $x^2 + y^2 = 4$ and the line y=x in the first quadrant

34. Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

x – y + 2z = 1 , 2y – 3z = 1 , 3x – 2y + 4z = 2

35. Find the vector equation of line passing through the point (1,2,-4) and perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

OR

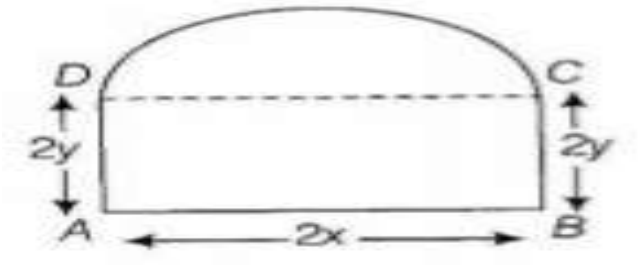
Find the shortest distance between the lines

$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$, $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

SECTION E

36. Case-Study 1: Read the following passage and answer the questions given below.

An architect designs a building for a small company . The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semi-circular opening. The window is having a perimeter of 10 meter as shown in the figure.



Based on the above information answer the following:

i.Find the relation between the variables ,If 2x and 2y represents the length and breadth of the rectangular portion of the window

ii. Find the combined area (A) of the rectangular region and semi-circular region of the window expressed as a function of x

iii. Find the length of the rectangular portion of the window should be, if the owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible.

OR

iii.Find the maximum area of the whole window

37.Case- study 2 :Read the following and answer

Scientist want to know the Oil- Reserves in sea so they travel over the sea along the curve $f(x) = (x+1)^3 (x-3)^3$ by an airplane. A student of class XII discusses the characteristic of the curve.



Answer the following questions on the basis of the information given above

I. Find the first order derivative of the given function

ii. Find the critical points of the given function

iii. Find the interval in which the given function is strictly increasing

OR

iii.The interval in which the given function is decreasing

38. Case-Study 3: Read the following passage and answer the questions given below.



The reliability of a HIV test is specified as follows:

Of people having HIV,90% of the test detect the disease but 10% go undetected. Of people free of HIV,99% of the test are judged HIV -ive but 1% are diagnosed as showing HIV +ive .From a large population of which only 0.1% have HIV, one person is selected at random ,given the HIV test ,and the pathologist reports him/her as HIV +ive.

(i) What is the probability that the person's HIV test is diagnosed as +ive

(ii) What is the probability that the person actually has HIV

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~~SAMPLE QUESTION PAPER~~ 6

Class:-XII Mathematics

Time: 3hours **Maximum marks:80**

General Instructions:

1. This Question paper contains-five sections **A,B,C,D** and **E**. Each section is compulsory. However ,there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1mark each.
3. **Section B** has 5 **Very Short Answer(VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer(LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** of 4 marks each with sub-parts.

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Section A (MCQ ,Each question carries 1mark)

1. If A and B are symmetric matrices of same order ,then AB – BA is a

(A) skew-symmetric matrix (B) symmetric matrix (C) zero matrix (D)identity matrix

2. Matrices A and B will be inverses of each other only if

(A) AB=BA (B) AB=BA=0 (C) AB=0, BA=I (D)AB=BA=I

3. If A = $\begin{bmatrix} 2 & P & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$ A^{-1} exists, then

(A) p = 2 (B) $p \neq 2$(C) $p \neq -2$ (D) none of these

4. The function f(x)= [x] ,where [x] denotes the greatest integer function , is continuous at

(A) 1 (B) 0 (C) -2 (D) 3/2

5. The angle between two vectors $\vec{a}$ and $\vec{b}$with magnitudes $\sqrt{3}$ and 4 respectively, and $\vec{a}.\vec{b}=2\sqrt{3}$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{5\pi}{2}$

6. The degree of the differential equation $\sqrt{1+(\frac{dy}{dx})^2} = \frac{d2y}{dx2}$

(A) 4 (B) 1/2 (C) $not\ defined$ (D) 2

7 .Objective function of LPP is

(A) a relation between the variables (B) a linear constraint (C)a function to be optimized (D) none of these

8.If $\vec{a}= \hat{\imath}+\hat{\jmath}+2\hat{k}$ and $\vec{b}= 2\hat{\imath}+\hat{\jmath}-2\hat{k}$, then the unit vector in the direction of $2\vec{a}-\vec{b}$ is

(A) $\frac{1}{\sqrt{37}}(-\hat{\jmath}+6\hat{k})$ (B) $\frac{1}{\sqrt{37}}(\hat{\jmath}-6\hat{k})$ (C) $\frac{1}{\sqrt{37}}(\hat{\jmath}+6\hat{k})$ (D) none of these

9.$\int_0^{\pi/2} cosxe^{sinx}dx$ is equal to

(A) e-1 (B) e+1 (C) e (D) 2e

10.If A is a square matrix of order 3 such that |A| = 3 , then the value of |adj(adjA)| is

(A) 9 (B) 81 (C) 6 (D) 27

11.The maximum value of Z=11x+7y subject to $x \leq 3$, $y \leq 2$, x ,y ≥ 0 is

(A) 37 (B) 30 (C) 47 (D) 28

12.If $|\vec{a}+\vec{b}| = 60$,$|\vec{a}-\vec{b}| = 40$ and $|\vec{b}| = 46$ then $|\vec{a}|$ is equal to

(A) 22 (B) 20 (C) 24 (D) 32

13.If A^2 - A + I = 0 then the inverse of A is

(A) A+I (B) I - A (C) $A - I$ (D) I

14. Two integers are selected at random from integers 1 to 11.If the sum is even , then find the probability that both the numbers are odd

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $\frac{4}{5}$

15. The general solution of the differential equation $\frac{ydx-xdy}{y} = 0$ is

(A) xy = c (B)$x = cy^2$ (C) $y = cx$ (D) $y = cx^2$

16.If |a| = 10, |b| = 2 and $\vec{a}.\vec{b} = 12$, then the value of $|\vec{a} x \vec{b}|$ is

A) 5 (B) 10 (C) 14 (D) 16

17.If y = $\tan^{-1}(\frac{sinx+cosx}{cosx-sinx})$, then $\frac{dy}{dx}$ is

A) 1 (B) sinx (C) $cosx$ (D) $\frac{\pi}{2}$+x

18. Find the value of k for which the lines $\frac{x+3}{-3} = \frac{y-1}{k} = \frac{z-5}{5}$ and $\frac{x+1}{-6} = \frac{y-2}{2} = \frac{z-5}{10}$ are parallel

A) k = 1 (B) k = 2 (C) $k = 3$ (D) k = 4

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A)is false but (R) is true.

19. ASSERTION(A): Let A = {2,4,6}, B= {3,5,7,9} and defined a function f = { (2,3),(4,4), (6,7) from A to B. Then f is not onto.

REASON (R):A function f :A → B is said to be onto, if every element of B is the image of some element of A under f

20. ASSERTION (A): f(x) = tanx – x always increases

REASON (R): Any function y = f(x) is increasing if $\frac{dy}{dx} > 0$

SECTION – B

This Section comprises of very short answer type of question (VSA) of 2 marks each

21) Find the value of $\cos^{-1}\cos\left(\frac{13\pi}{6}\right) + \tan^{-1}\tan\left(\frac{5\pi}{6}\right)$.

OR

Evaluate $\sin(\cot^{-1}(\cos(\tan^{-1} 1)))$.

22) Find the intervals in which the function $f(x) = (x+2)e^{-x}$, is increasing or decreasing.

23) Find the maximum and the minimum values of $f(x) = x + \sin 2x$ in the interval $[0,2\pi]$.

24) Evaluate $\int \frac{x^4}{1+x^5} dx$.

25) If $y = (\tan^{-1}x)^2$, show that $(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$.

SECTION – C

26) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$.

27) From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the prob. distrn of the number of defective bulbs.

28) Evaluate $\int e^x \left(\frac{\sin 4x - 4}{1 - \cos 4x}\right) dx$.

Or

Evaluate $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$.

29) Solve the differential equation.

$\cos^2 x \frac{dy}{dx} + y = \tan x$

Or

Solve the differential equation.

$(1+y^2)dx = (\tan^{-1}y - x)dy$

30) Solve the following L.P.P graphically

Maximize Z = 50x + 15y

Subject to $5x + y \leq 100$

$x + y \leq 60$

$x, y \geq 0$

Or

Minimize Z = 5x + 7y

Subject to $2x + y \geq 8$

$x + 2y \geq 10$

$x, y \geq 0$

31) If $x = a(\cos\Theta + \log\tan\frac{\Theta}{2})$ and $y = a\sin\Theta$; find $\left(\frac{d^2y}{dx^2}\right)$ at $\Theta = \frac{\pi}{4}$.

Section D(5 marks each)

32. Let N denote the set of all natural numbers and R be the relation on NxN defined by (a,b)R(c,d) if ad(b+c) = bc(a+d).Show that R is an equivalence relation.

33. If $A = \begin{pmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{pmatrix}$, then find A^{-1} .Using A^{-1}, solve the system of equations

$2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$

OR

Show that $A = \begin{pmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix}$ satisfies the equation $A^3 - A^2 - 3A - I = 0$ and hence find A^{-1}

34.Find the area of the region lying in the first quadrant and enclosed by the x- axis, the line y = x and the circle $x^2 + y^2 = 32$

35. Show that the lines $\vec{r} = \hat{3\imath}+2\hat{\jmath}- 4\hat{k} + \lambda (\hat{\imath}+2\hat{\jmath}+2\hat{k})$ and $\vec{r} = \hat{5\imath} - 2\hat{\jmath} + \mu (3\hat{\imath}+2\hat{\jmath}+6\hat{k})$ are intersecting. Hence , The line equations find their point of intersection.

OR

Find the coordinates of the foot of the perpendicular and length of the perpendicular drawn from the point P(5,4,2) to the line $\vec{r} = \hat{-\imath}+3\hat{\jmath}+\hat{k} + \lambda (2\hat{\imath}+3\hat{\jmath}-\hat{k})$. Also find the image of P in this line.

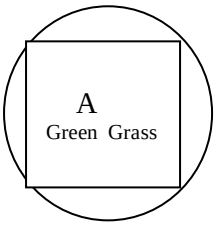
SECTION E (Case study questions)

36. In a school, teacher asks a question to three students, Mohan, Sruthi and Ritu. The probability of solving the question by Mohan, Sruthi and Ritu are 30%, 25% and 45% respectively. The probability of making error by Mohan, Sruthi and Ritu are 1%, 1.2% and 2% respectively. Based on the above information, answer the following questions.

i. Find the total probability of committing an error in solving the question.

ii. if the solution of question is checked by teacher and has some error, then find the probability that the question is not solved by Mohan.

37. An architect designs a garden in society. The garden is in the shape of a rectangle inscribed in a circle or radius 10m



On the basis of the above information, answer the following questions:

i. If 2x and 2y represents the length and breadth of the rectangular part, then find the relation between the variables. Find the area of the Green Grass A expressed as a function of x

ii. Show that the the area A is maximum when $x= 5\sqrt{2}$

38. A student drew 2 skew lines as shown below with their points through which they pass and their directions $\overrightarrow{b1}$ and $\overrightarrow{b2}$. The equations of these two lines l_1 and l_2 are given by $\frac{x+2}{2} = \frac{y}{3} = \frac{z-2}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

Based on the above information, answer the following:

i. Find the $\overrightarrow{b1}$ X $\overrightarrow{b2}$

ii. Find the shortest distance between the skew lines l_1 and l_2

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Sample Question Paper-7

Class XII Session 2023-24

Mathematics

Time Allowed: 3 Hours Maximum Marks: 80

General Instructions :

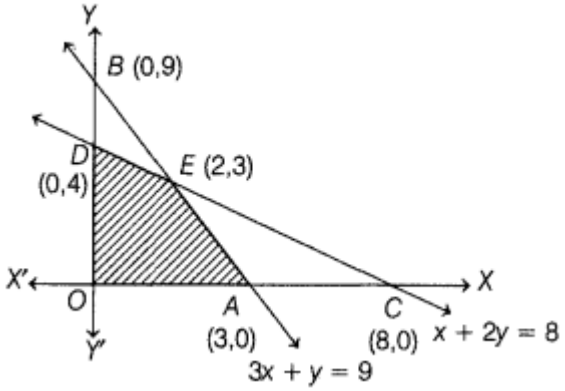
1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(Multiple Choice Questions) Each question carries 1 mark

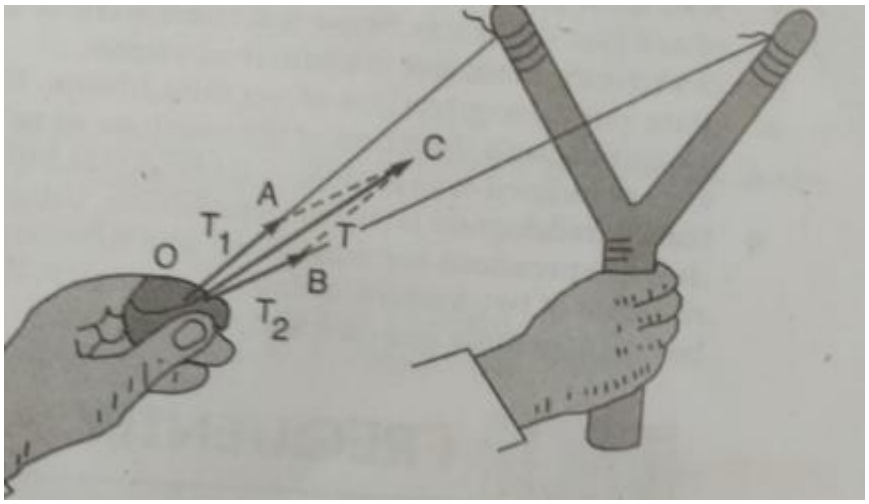
1	If A & B are two invertible matrices of order 3 and $\|A\|$ =2 and$\|(AB)^{-1}\|$ = -1/6 find$\|B\|$ a) -2 b) -3 c)$\frac{-1}{2}$ d)$\frac{-1}{3}$	
2	If A = $\begin{bmatrix} x & 2 \\ 2 & x \end{bmatrix}$ and $\|A^3\|$ = 27 find x a) $-\sqrt{7}$ b) $\pm\sqrt{7}$ c) 1 d) 7	
3	If A is square matrix such that |A| = 5 ,find the value of | AA^T | a) 5 b)25 c) -5 d)$\frac{1}{5}$	
4	Find derivative of sin(log x) $w.r.t.$ log(x^2) a)$\frac{\cos(logx)}{2}$ b) cos(logx) c) sin(logx) d)$\frac{sinl(ogx)}{2}$	
5	[X 1] $\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}$ =0 Find x	

	a) 0 b) -1 c) -2 d) 2	
6	P and Q are matrices of order 3 x 2 , n x p and n = p . Find the order of PQ a) 3x3 b) 3x2 c)2x3 d)2x2	
7	The function f(x) = $\frac{4-x^2}{4x-x^3}$ a) Discontinuous at exactly one point b) Discontinuous at exactly two points c) Discontinuous at exactly three points d) None of these	
8	$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$ a)$\frac{\pi}{6}$ b)$\frac{\pi}{12}$ c)$\frac{\pi}{24}$ d)$\frac{\pi}{4}$	
9	Determine the sum of order and degree of the differential equation $\sqrt{1+(\frac{dy}{dx})^2}$ = 3x - $\frac{dy}{dx}$ a) 2 b) 3 c) 1 d) 4	
10	Solve $\frac{dy}{dx} + y = e^{-x}$ a) ye^{x} = x +c b) ye^{-x} = x +c c) ye^{x} = - x +c d) ye^{x} = x	
11.	A vector in the direction of $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ that has a magnitude 15 is a) $\frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3}$ b) b) $15\hat{\imath} - 30\hat{\jmath} + 30\hat{k}$ c) c)$\hat{\imath} - 2\hat{\jmath} + 15\hat{k}$ d) $5\hat{\imath} - 10\hat{\jmath} + 10\hat{k}$	
12	Area of a parallelogram whose one diagonal is $2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and one side is $3\hat{\imath} + \hat{\jmath} - \hat{k}$ a) $\hat{\imath} - 4\hat{\jmath} + \hat{k}$	

	b) $3\sqrt{2}$ c) 6 unit d) $6\sqrt{2}$ units	
13	Write a unit vector in the directions sum of the vectors $\widehat{2\imath} + \widehat{2\jmath} - 5\,\hat{k}$ and $\widehat{2\imath} + \hat{\jmath} - 7\,\hat{k}$ a) $\frac{\widehat{2\imath} + \widehat{2\jmath} - 5\,\hat{k}}{13}$ b) $\frac{\widehat{2\imath} + \widehat{2\jmath} - 5\,\hat{k}}{169}$ c) $\frac{\widehat{2\imath} + \hat{\jmath} - 7\,\hat{k}}{13}$ d) $\frac{\widehat{4\imath} - 3\hat{\jmath} - 12\hat{k}}{13}$	
14	If a line makes angles 90^0,60^0 and θ with x- axis,y-axis and z-axis respectively . Find the measure of θ. a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$	
15	Find the angle between two lines, $\frac{x-2}{3} = \frac{y+5}{2} = \frac{z-1}{6}$ and $\frac{x-7}{1} = \frac{y}{2} = \frac{z+6}{2}$ a) $sin^{-1}(\frac{19}{21})$ b) $sin^{-1}(\frac{21}{19})$ c) $cos^{-1}(\frac{19}{21})$ d) $cos^{-1}(\frac{21}{19})$	
16	Corner points of the feasible region determined by the system of linear constraints are (0,3) (1,1) (3,0).let Z = px + qy where p,q> 0. Find the condition on p,q so that the minimum occurs at(3,0),(1,1) a) P=2q b) b) p=q/2 c) c)p=3q d) d)p=q	
17	The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below.  	

	Which of the following is not a constraint to the given Linear Programming Problem? a)$3x + y \leq 9$ b)$x + 2y \leq 8$ c) $x + 2y \geq 8$ d) $3x + y = 9$	
18	If P (A) = 0.4 P (B) = 0.8,P (B/A) 0.6 ,FIND P (AU B) a) 0.24 b) 0.3 c) 0.48 d) 0.96	
	ASSERTION-REASON BASED QUESTIONS In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
19	Assertion (A) : The greatest integer function f(x) = [x] is one-one Reason (R) : A function is one-one then f(x) = f(y) then x=y	
20	Assertion (A) : f(x) =$e^x$is an increasing function $\forall\, x \in R$ Reason (R) : If $f(x) \geq 0$, then f(x) is an increasing function.	
	SECTION B This section comprises of very short answer type-questions (VSA) of 2 marks each	
21	Find the value of sin^{-1} $(\cos \frac{3\pi}{5})$ OR Evaluate tan ($\cos^{-1}(8/17)$	
22	Differentiate $\tan^{-1}$ $(\frac{1+cosx}{sinx})$ with respect to x. OR Find the derivative of $\mathbf{sin^{-1}}$ $(\frac{2^{x+1}}{1+4^x})$w.r.to x	
23	Find the intervals in which the following function is a) increasing b) decreasing $F(x) = 10 - 6x - 2x^2$	
24	Find the value of k, so that the function $f(x) = \begin{cases} \left(\frac{1-\cos 4x}{8x^2}\right), & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$	
25	$\int_1^3 \frac{1}{16-x^2}$ dx	
	SECTION C	

	(This section comprises of short answer type questions (SA) of 3 marks each)	
26	$\int xtan^{-1}$x dx OR $\int \frac{xe^x}{(x+1)^2}$dx	
27	$\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}$ dx	
28	Solve X $\frac{dy}{dx}$ - y + x tan(y/x) = 0 OR $(1+y^2)$ + (x - $e^{tan^{-1}y}$)$\frac{dy}{dx}$ = 0	
29	Solve 2(y+3) - xy$\frac{dy}{dx}$ = 0 given that y(1) = -2	
30	Solve the following LPP graphically: Minimise Z = 5x + 10y subject to the constraints x + 2y ≤ 120 x + y ≥ 60, x – 2y > 0 and x, y ≥ 0 OR Maximise and minimise Z = x + 2y subject to the constraints x + 2 y ≥ 100 2x – y ≤ 0 2x+ y ≤ 200 x, y ≥ 0	
31	10% of the bulbs produced in a factory are of red colour and 2% are red and defective. If one bulb is picked up at random, determine the probability of its being defective if it is red.	
	SECTION D (This section comprises of long answer-type questions (LA) of 5 marks each)	
32	Relation R defined in a set of natural numbers N as R = { (a,b) : a- b is an even natural number and a,b∈ N} is neither reflexive nor symmetric but transitive OR Show that the function f: R → R defined by f(x) =$\frac{X}{X^2+1}$, ∀ x ∈ R is neither one-one nor onto.	
33	A total amount of rs 7000 is deposited in three savings bank accounts with annual interest 5%,8%, 8 $\frac{1}{2}$ % respectively. The total annual interest from these three account is 550. Equal amounts have been deposited in 5% and 8% saving account . find the amount deposited in each of the three accounts respectively using matrix method	
34	Find the area enclosed by the parabola y^2 = x and the line y = x +2 and x axis in the first quadrant using integration	
35	Show that the lines$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Also, find the point of intersection OR Find the vector and Cartesian equation of a line passing through P(2,-1,3) and	

	perpendicular to the lines $\bar{r} = \hat{\imath} + \hat{\jmath} - \hat{k} + \lambda\ (2\hat{\imath} - 2\hat{\jmath} + \hat{k})$ and $\bar{r} = 2\hat{\imath} - \hat{\jmath} - 3\hat{k} + \mu\ (\hat{\imath} + 2\ \hat{\jmath} + 2\hat{k})$	
	SECTION E (This section comprises of 3 case-study/passage-based questions of 4 marks each)	
36	An architect designs a building for a multi national company. The floor consists of rectangular regions with semicircular ends having a perimeter of 200m. 1) If x, y represent the length and breadth of rectangular region then find the relation between the variables 2) The area of the rectangular region 3) Maximum area	
37	Ishan and Varun decided to play with dice to keep themselves busy at home as their schools are closed due to heavy rain.varunthrw a die repeatedly until a six obtained.he denote the number throws by X Based on the above information answer the following 1)the probability when X = 2 2)T he probability that X = 4 3)The value of P(X> 6)	
38	When a stone held at the point O (0,0) on the rubber band is pulled, the tensions T1 and T2 are produced along OA and OB in the two segments of the rubber band. The coordinates of A and B are (2,4) and (5,2) respectively.  Based on the information answer the following questions. 1. Find$\overrightarrow{OA}$? 2. Find the magnitude of $\overrightarrow{OB}$. 3. Find the unit vector along $\overrightarrow{OC}$.	

KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION

SAMPLE QUESTION PAPER No: 8

Class:-XII

Session 2023-24 Mathematics (Code-041)

Time: 3 hours Maximum marks: 80

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General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each. 6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts

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Section –A (Multiple Choice Questions)

Each question carries 1 mark

1) Let X, Y, Z, W and P are matrices of order 2 × n, 3 × k, 2 × p, n × 3 and p × k, respectively. The restriction on n, k and p so that PY + WY will be defined are

a) k = 3, p = n b) k is arbitrary, p = 2

c) p is arbitrary, k = 3 d) k = 2, p = 3

2) Let A be a non-singular square matrix of order 3 × 3. Then |adj A| is equal to:

a) | A | b) | A |^2 c) | A |^3 d) 3|A|

3) If A, B and C are angles of a triangle, then the determinant $\begin{pmatrix} -1 & CosC & CosB \\ CosC & -1 & Cos\ A \\ CosB & CosA & -1 \end{pmatrix}$

a) 0 b) -1 c) 1 d) 2

4) If $f(x) = |x| + |x - 2|$, then

(a) f(x) is continuous at x=0 but not at x=2 (b) f(x) is continuous at x=0 and at x=2

(c) f(x) is continuous at x=2 but not at x=0 (d)None of these

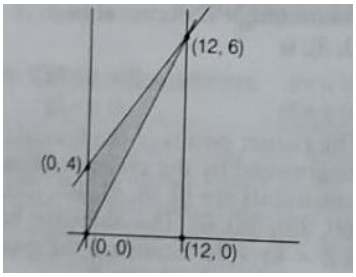
5) Unit vector perpendicular to each of the vectors $3i + j + 2k$ and $2i - 2j + 4k$ is

a) $\frac{i+j+k}{\sqrt{3}}$ b) $\frac{i-j+k}{\sqrt{3}}$ c) $\frac{i-j-k}{\sqrt{3}}$ d) $\frac{i+j-k}{\sqrt{3}}$

6) Integrating factor of the differential equation $Cosx\ \frac{dy}{dx} + y\ Sinx = 1$ is

$a)\ Cosx \quad b)\ Tanx \quad c)\ Sec\ x \quad d)\ Sinx$

7) The feasible region for an LPP is shown shaded in the figure. Let Z = 4x-3y be objective function. Maximum value of Z is:



a) 0 b) 8 c) 30 d) -18

8) If $|a| = |b| = |a + b| = 1$ then $|a - b|$ is equal to

a) 1 b) $\sqrt{3}$ c) 0 d) None

9) $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+Cos\ 2x}$ is equal to

a) 1 b) 2 c) 3 d) z

10) A = $\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ and B = $\begin{pmatrix} -2 & -2 \\ 0 & -1 \end{pmatrix}$ then $(A + B)^{-1}$

a) $\begin{pmatrix} -1 & 1 \\ 1 & -1/2 \end{pmatrix}$ b) doesn't exist c) is a skew symmetric d) None

11) Feasible region for an LPP shown shaded in the following figure. Minimum ofZ = 4x+3y occurs at the point:

a) (0,8) b) (2,5) c) (4,3) d) (9,0)

12) The area of parallelogram whose adjacent sides are

$i - 2j + 3k\ and\ 2i + j - 4k$ is

(a) $10\sqrt{6}$
(b) $5\sqrt{6}$
(c) $10\sqrt{3}$
(d) $5\sqrt{3}$

13) Given that A = $[a_{ij}]$ is a square matrix of order 3×3 and |A| = -7, then the value of $\sum_{i=1}^{3} a_{i2}\ A_{i2}$, where A_{ij} denotes the cofactor of element a_{ij} is

(a) 7

(b) -7

(c) 0

(d) 49

14) An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?

A. 3/7

B. 7/3

C. 1/7

D. 1/3

15) What is the degree of differential equation $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$?

A. 2

B. 3

C. 4

D. 5

16) If $|a \times b| = 4 \; and \; |a.b| = 2$, then $|a|^2 \, |b|^2$ is equal to:

A. 4

B. 6

C. 20

D. 2

17) If $x \; sin(a + y) = sin \; y, then \; dy/dx$ is equal to

a. $[\sin^2(a + y)]/\sin a$
b. $\sin a / [\sin^2(a + y)]$
c. $[\sin (a + y)] /\sin a$
d. $\sin a / [\sin(a + y)]$

18) The direction cosines of the y-axis are:

a) (9, 0, 0)

b) (1, 0, 0)

c) (0, 1, 0)

d) (0, 0, 1)

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

19) Assertion (A) : $f(x) = -|x + 1| + 3$ is defined for all real values of x except x = - 1.

Reason (R): Maximum value of f(x) is 3 and Minimum value does not exist

20)

Assertion (A): The function $f: R \to R, f(x) = |x|$ is not one-one

Reason(R): The function $f(x) = |x|$ is not onto .

Section –B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21) Write the principal value of $\tan^{-1}[\sin(-\pi/2)]$

OR

Write the value of $\tan^{-1}[2\sin(2\cos^{-1}\sqrt{3}/2)]$

22) Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing in R.

23) Find the absolute maximum value of $y = x^3 - 3x + 2$ in $0 \le x \le 2$.

24) Evaluate $\int_0^1 \frac{xe^x}{(x+1)^2}\,dx$

25) Check whether the function f: R → R defined by $f(x) = x^3 + x$ has any critical point/s or not ? If yes, then find the point/s.

SECTION C

(This section comprises of short answer type questions of 3 marks each)

Q26 Evaluate $\int \frac{x}{x^2+3x+2}$ dx

Q 27 A random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine a) k

b) P(0 < X <3)

c) P (X >6)

Or

Evaluate $\int_0^{\pi} \frac{xsinx}{1+cos^2x}$ dx

29 Solve the following differential equation

x $\frac{dy}{dx}$ = y- x tan $\frac{y}{x}$

or

Solve the following differential equation

x cos $(\frac{y}{x})$ $\frac{dy}{dx}$ = y cos $(\frac{y}{x})$ + x , x≠0

30 Solve the following Linear programming problem graphically

Minimise Z= 5x+10y subject to x+2y ≤120, x+y ≥ 60 , x-2y ≥ 0 , x,y ≥ 0

Or

Solve the following Linear programming problem graphically

Maximise Z= 3x+9y subject to the constraints

x+3y ≤ 60, x+y ≥ 10 , x ≤ y , x≥0, y≥0

31 If y= $e^{acos^{-1}x}$ where -1≤ x ≤1 Show that (1-x^2) $\frac{d^2y}{dx^2}$ - x$\frac{dy}{dx}$ - $a^2y = 0$

SECTION D

(This section comprises of long answer type questions of (LA) of 5 marks each)

32 Using integration find the area of the region { (x,y): 0 ≤ y ≤ x^2, 0≤ y ≤ x, 0≤ x ≤ 2 }

33 Let N denote the set of all natural numbers and R is the relation on NXN defined by (a,b) R (c,d) iff

ad (b+c) = bc(a+d) .Show that R is an equivalence relation

or

Show that f: N-→N defined by f (n) = $\begin{cases} n+1 \ if \ n \ is \ odd \\ n-1 \ if \ n \ is \ even \end{cases}$ is one one and onto

34 If A= $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ Find A^{-1} and hence solve the system of linear equations

x +2y +z =4

-x+y +z =0

x-3y +z=2

35 Find the shortest distance between the lines $\vec{r} = (\hat{\imath} + 2\,\hat{\jmath} + \widehat{k}) + \lambda\,(\hat{\imath} - \hat{\jmath} + \widehat{k})$ and $\hat{r} = (2\,\hat{\imath} - \hat{\jmath} - \widehat{k}) + \mu\,(2\,\hat{\imath} + \hat{\jmath} + 2\hat{k})$

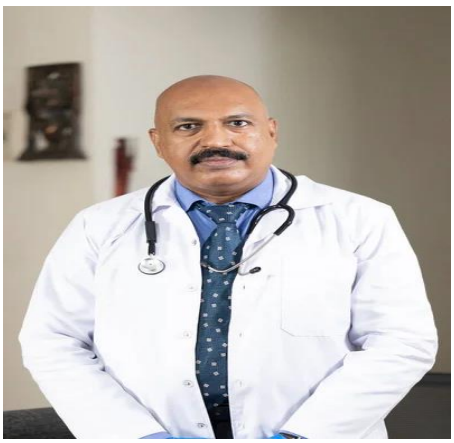
OR

Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

SECTION E

(This section comprises of 3 case study passage based questions of 4 marks each with sub parts

The first two case study questions have three sub parts i),ii),iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each)



36 Mr Rajesh is a doctor. He is to visit a visit a patient. From the past experience it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{2}{5}$.The probabilities that he will be late are $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{1}{12}$ *if* he comes by train,bus and scooter respectively, but if he comes by other means of trasport, then he will not be late. When he arrives, he is late.

i) What is the probability that he comes by train when it is known that he is late ? (1mark)

ii) What is the probability that he comes by bus when it is known that he is late (1 mark)

iii) What is the probability of the doctor getting late when he comes by other means of transport? (2 marks)

or

What is the probability of the doctor getting late?

37 Mr Naveen is appearing for a competitive examination. He was asked to attempt the following questions.

Let $\vec{a}$, $\vec{b}$ are three non zero vectors.

i) If $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ then what is the relation between $\vec{a}$ and $\vec{b}$? (1 mark)

ii) If $\vec{a}$ and $\vec{b}$ are unit vectors and Θ is the angle between them find $|\vec{a}-\vec{b}|$ (1 mark)

iii) Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, find $\vec{a}$ (2 marks)

or

Find area of the parallelogram if $\vec{a} = \hat{\imath}-2\hat{\jmath}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ are the diagonals of the parallelogram

38 The CEO of a multi-national company is interested in maximizing the profit of the company within a short .period. If $P(x) = -5x^2 + 125x + 37500$ is the total profit function of the company where x is the production of the company

i) What will be the production when the profit is maximum ? (2 marks)

ii) What will be production of the company when the profit is Rs 38250 ? (2 marks)

END

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KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION

SAMPLE QUESTION PAPER NO.9

CLASS XII MATHEMATICS-2023-24

Time:3 hours Maximum marks:80

General Instructions:

1.This Question paper contains-five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2.Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3.Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4.Section C has 6 Short answer (SA)-type questions of 3 marks each.

5.Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6.Section E has 3 Source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section- A

(Multiple Choice Questions)

Each question carries 1 mark.

Q1. If $[2 \quad 1 \quad 3]\begin{bmatrix}-1 & 0 & -1\\ -1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\ 0\\ -1\end{bmatrix} = A$,then write the order of the matrix A

(a) 1× 2 (b) 1× 1 (c) 3× 3 (d) 3× 1

Q2. If A = $\begin{bmatrix}1 & 0\\ 5 & 6\end{bmatrix}$ $then\ |A^{-1}| = ----$

(a) $\frac{-1}{6}$ (b) 6 (c) -6 (d) $\frac{1}{6}$

Q3. If A = $\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}, and\ B = \begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}, then$

(a) $A^{-1} = 6B$ (b) $A^{-1} = B$ (c) $B^{-1} = B$ (d) $B^{-1} = \frac{1}{6}A$

Q4. Determine the value of 'k' for which the following function is continuous at x =3

f(x) = $\begin{cases}\frac{(x+3)^2 - 36}{x-3}, x \neq 3\\ k\ ,\ \ x = 3\end{cases}$

(a) k = 12 (b) k = 3 (c) k=6 (d) k = -12

Q.5 The direction cosines of the line 5x-3 = 15y+7 = 3-10z are

(a) $\frac{1}{5}, \frac{1}{10}, \frac{1}{7}$ (b) $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}$ (c) $\frac{1}{5}, \frac{1}{15}, \frac{-1}{10}$ (d) $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$

Q.6 The sum of order and degree of the differential equation $\frac{d}{dx}\left\{(\frac{dy}{dx})^4\right\} = 0$ is

(a) 3 (b) 4 (c) 5 (d) 2

Q7. The minimum value of Z = 3x+8y subject to the constraints

$x \leq 20, y \geq 10$ *and* $x \geq 0, y \geq 0$ is :

(a) 80 (b) 140 (c) 0 (d) 60

Q8. Let $\vec{a} = \alpha\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} - \beta\hat{k}$, if $\vec{a}$ and $\vec{b}$ are collinear vectors, then the value of $\alpha + \beta$ is

(a) -6 (b) -2 (c) -8 (d) 12

Q9. The value of $\int_0^6 \sqrt{36 - X^2}$ dx is

(a) 9π (b) $\frac{9\pi}{2}$ (c) $\frac{\pi}{9}$ (d) $\frac{\pi}{2}$

Q10. The value of $\begin{vmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{vmatrix}$ is

(a) 0 (b) - x y z (c) x y z (c) 1

Q11. The corner points of the feasible region determined by the system of linear inequalities are (0,0),(4,0),(2.4) and (0,5).If the maximum value of Z = ax + by ,where a, b > 0 occurs at both (2,4) and (4,0),then

(a) a=2b (b) 2a =b (c) a= b (d) 3a=b

Q12. The cross product of the vectors $\hat{\imath} - \hat{\jmath} - 2\hat{k}$ and $\hat{\imath} + \hat{\jmath}$ is

(a) $2\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ (b) $2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ (c) $2\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ (d) $2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

Q13. If for any 2×2 square matrix A, A .(adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$,then the value of $|A|$ is

(a) 7 (b) 6 (c) 8 (d) 4

Q14. Two cards are drawn at random from a pack of 52 cards one by one without replacement.

The probability of getting first card Red and second card Jack is

(a) $\frac{1}{26}$ (b) $\frac{1}{52}$ (c) $\frac{26}{51}$ (d) $\frac{1}{51}$

Q15. The general solution of $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{\frac{1}{3}}$ is

(a) $y^{\frac{2}{3}} - x^{\frac{2}{3}} = C$ (b) $y^{\frac{2}{3}} + x^{\frac{2}{3}} = C$ (c) $y^{\frac{1}{3}} - x^{\frac{1}{3}} = C$ (d) $y^{\frac{1}{3}} + x^{\frac{1}{3}} = C$

Q16.If $\vec{a}$ and $\vec{b}$ denote the position vectors of points A and B respectively and P is a point on AB such that AP =2 PB ,Then the position vector of P is

(a) $\frac{\vec{a}+2\vec{b}}{3}$ (b) $\frac{2\vec{a}+\vec{b}}{3}$ (c) $\frac{\vec{a}+3\vec{b}}{3}$ (d) $\frac{2\vec{a}+2\vec{b}}{3}$

Q17. The set of all points where the function f(x) = x+ 2 + $|x-3|$ is differentiable is

(a) $(0,\infty)$ (b) $(3,\infty)$ (c) $(-\infty,3)$ (d) $(-\infty,3)\cup(3,\infty)$

Q18. The x -co ordinate of a point on the line $\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$ is 4,then its z - co ordinate is

(a) $\frac{4}{3}$ (b) 4 (c) -1 (d) 1

ASSERTION -REASON BASED QUESTIONS

In the following 2 questions, a statement of Assertion(A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a)Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Q19. ASSERTION (A): f(x) = - $|x+1|$ +3 is defined for all real values of x, except at x= -1

REASON (R): Maximum value of f(x) is 3 and minimum value of f(x) does not exist.

Q20. ASSERTION (A): The number of on to functions from a set P containing 5 elements to a set Q containing 2 elements is 30.

REASON (R): No of on to functions from a set containing m elements to a set Containing n elements is n^m.

SECTION B

(This section comprises of very short answer type questions (VSA) of 2 marks each)

Q21. Write $\tan^{-1}\sqrt{\frac{1-cosx}{1+cosx}}$, $x < \pi$, in simplest form

OR

Evaluate $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\cos^{-1}(0)$

Q22. Find the intervals in which the function f given by f(x) = 2 x^2 -3x is (a) increasing

(b) decreasing?

Q23. Find the absolute maximum and absolute minimum values of the function f given by

f(x) = $2x^3 - 15x^2 + 36x + 1$ on the interval [1,5].

OR

An edge of a variable cube is increasing at the rate of 3cm/s. How fast is the volume of the Cube increasing when the edge is 10 cm long.

Q24. Find $\int e^x(tan^{-1}\, x + \frac{1}{1+x^2})\, dx$

Q25.Find the maximum profit that a company can make, if the profit function is given by

P(x)= 41 +24 x-18x^2

Q26. Find $\int \frac{X^2}{(x^2+1)(x^2+4)}\, dx$

Q27. A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them is selected is 0.6. Find the probability that B is selected

Q28. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$

OR

Evaluate $\int \sqrt{\frac{1+x}{1-x}}\, dx$, x ≠ 1

Q29. Show that the differential equation $2y\, e^{\frac{x}{y}}\, dx + \left(y - 2xe^{\frac{x}{y}}\right) dy = 0$ is homogeneous and find its particular solution, given that, x = 0 when y = 1

OR

Find the particular solution of the differential equation $\frac{dy}{dx} + y\cot x = 2x + x^2$ cot x (x ≠ 0) given that y = 0 when x = $\frac{\pi}{2}$

Q30. Minimise Z = 3x + 5y subject to the constraints:

x + 2y ≥ 10

x + y ≥ 6

$3x + y \geq 8$

$x, y \geq 0$

OR

Solve the following linear programming problem graphically:

Minimise Z = 200 x + 500 y

subject to the constraints:

$x + 2y \geq 10$

$3x + 4y \leq 24$

$x \geq 0, y \geq 0$

Q31. If x= a sin 2t, y = a (cos 2t + log tan t), then find $\frac{dy}{dx}$

Q32. Find the area of the region included between the parabola $y = \frac{3x^2}{4}$ and the line

$3x - 2y + 12 = 0$

Q33. Let A= R - {3} and B = R - {1} .Consider the function f: A→ B defined by f (x) = $\frac{x-2}{x-3}$.

Prove that f is one-one and onto.

OR

Show that the relation R in the set A = $\{x: x \in Z, 0 \leq X \leq 12\}$ given by

R= $\{(a,b): a,b \in A, |a-b| \; is \; divisible \; by \; 3\}$ is an equivalence relation. Also find the equivalence class [4]

Q34.If A = $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$ find A^{-1}. Using A^{-1},solve the following system of equations.

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2$

$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5$

$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$, x, y, z $\neq 0$

Q35. An aeroplane is flying along the line $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda (2\hat{\imath} - \hat{\jmath} + \hat{k})$ where ' λ ' is a scalar and another aeroplane is flying along the line $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu (3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$ where ' μ ' is a scalar. Find the shortest possible distance between them.

OR

Find the co-ordinates of the foot of perpendicular drawn from the point A (1, 8, 4) to the line joining the points B (0, –1, 3) and C (2, –3, –1).

SECTION - E

(This section comprises of 3 case- study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii). (iii) of marks 1.1,2 respectively. The third case study question has two sub parts of 2 marks each.)

Q36. Read the following passage and answer the questions given below:

There are ten cards numbered 1 to 10 and they are placed in a box and then mixed up thoroughly. Then one card is drawn at random from the box. Based on the above, answer the following questions:

(i) What is the probability that the number on the drawn card is greater than 4? 2

(ii) If it is known that the number on the drawn card is greater than 4, then what is the probability that it is an even number? 2

Q37. Ginni purchased an air plant holder which is in the shape of a tetrahedron. Let A, B,

C and D are the coordinates of the air plant holder where A (1, 1, 1), B (2, 1, 3),

C= (3, 2, 2) and D = (3,3,4).

Based on the above information, answer the following questions

1. Find $\overrightarrow{AB}$.

a. $-\hat{i}-2\hat{k}$

b. $2\hat{i}+\hat{k}$

c. $\hat{i}+2\hat{k}$

d. $-2\hat{i}-\hat{k}$

2. Find $\overrightarrow{AC}$

a. $2\hat{i}-\hat{j}-\hat{k}$

b. $2\hat{i}+\hat{j}+\hat{k}$

c. $-2\hat{i}-\hat{j}+\hat{k}$

d. $\hat{i}+2\hat{j}+\hat{k}$

3. Find $\overrightarrow{AD}$

a. $2\hat{i}-2\hat{j}-3\hat{k}$

b. $\hat{i}+\hat{j}-3\hat{k}$

c. $3\hat{i}+2\hat{j}+2\hat{k}$

d. $2\hat{i}+2\hat{j}+3\hat{k}$

4. Area of ΔABC =

a = $\frac{\sqrt{11}}{2}$ sq. units

b = $\frac{\sqrt{14}}{2}$ sq. units

c = $\frac{\sqrt{13}}{2}$ sq. units

d = $\frac{\sqrt{17}}{2}$ sq. units

Q38.Read the following passage and answer the questions given below.



A person wants to plant some trees in the community park of his area. The local nursery charges the cost of planting trees by the following formula C(x) = $x^3 - 45x^2 + 600x$, where x is the number of trees and C(x) is the cost of planting x trees in rupees. The owner of local nursery has imposed a restriction that it can plant 10 to 20 trees in one community park for a fair distribution.

(1) What is the derivative of C(x) with respect to x? Find the possible number of trees, if $C'(X)$=0
(2) For how many trees should the person place order, so that his expenses are least? Also find the least amount (in rupees) that the person has to spend?

KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION

SAMPLE QUESTION PAPER-10

Class:-XII

Session 2023-24

Mathematics (Code-041)

Time: 3 hours **Maximum marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section –A

(Multiple Choice Questions)

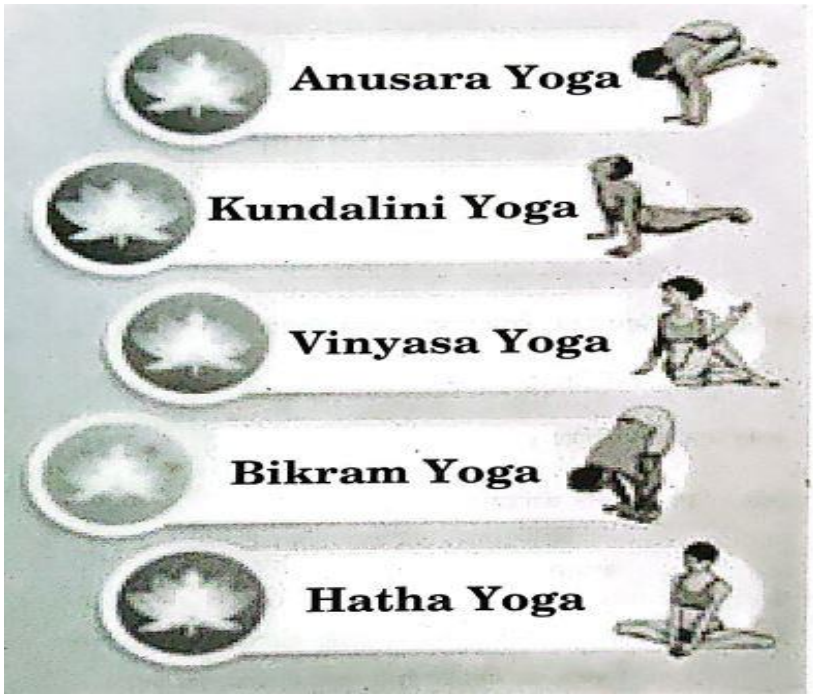
Each question carries 1 mark

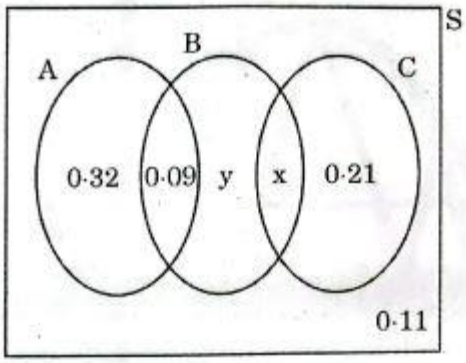
QN.NO	QUESTION
1	If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, then A+A' is a) skew –symmetric b) symmetric c) diagonal matrix d) zero matrix
2	If A is a square matrix of order 3 such that $\|A\| = -5$, then value of $\|-A\|$ is (a) 125 (b) - 125 (c) 5 (d) – 5
3	If A (3, 4), B(-7,2), C(x, y) are collinear, then which of the following is true? a) x+5y+17=0 (b)x+5y+13=0 (c)x-5y+17=0 (d) x-5y-17=0

4	If f(x) = $\frac{x^2-25}{x-5}$,when x≠5 K , when x=5 is continuous at x=5, find the value of k. a)5 b)10 c)25 d)0
5	The degree of the differential equation $[1+(\frac{dy}{dx})^2]^3=(\frac{d^2y}{dx^2})^2$ is (a) 4 (b)3 (c) 2 (d) Not defined
6	If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both X- axis and Y-axis, then the angle which it makes with positive Z –axis is: a) $\frac{\pi}{4}$ b)$\frac{3\pi}{4}$ c) $\frac{\pi}{2}$ d)0
7	In an LPP, if the objective function Z = ax + by has same maximum at two corner points of the feasible region, then the number of points at which maximum value of Z occurs is a) 0 b) 1 c) 2 d) infinite
8	For any integer n, the value of $\int_{-\pi}^{\pi} e^{cos^2x}\sin^3(2n+1)x\,dx$ is (a) -1 (b) 0 (c) 1 (d) 2
9	$\vec{a}$ and $\vec{b}$ are to non –zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0.The angle between $\vec{a}$ and $\vec{b}$ is a) $\frac{\pi}{4}$ b) π c) $\frac{\pi}{2}$ d) 0
10	The value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ is equal to a) 0 (b) 1 (c) x+y+z (d) 2(x+y+z)
11	The corner points of the feasible region determined by a system of linear inequalities are (0, 0), (4, 0), (2, 4) and (0, 5). If the maximum value of Z = ax + by where a,b > 0 occurs at both (2, 4) and (4, 0), then a) a = 2b b) 2a = b c) a = b d) 3a = b
12	If $\vec{a}$ and $\vec{b}$ are vectors such that $\lvert\vec{a}\rvert$ =3, $\lvert\vec{b}\rvert = \frac{\sqrt{2}}{3}$ and $\lvert\overrightarrow{a\times b}\rvert$=1, then the angle between $\vec{a}$ and $\vec{b}$ is a) π/2 b) π/3 c) π/6 d) π/4

13	If A=$\begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}$ then the value of $\lvert adj(A)\rvert$ is a) x^3 (b) x^6 (c) x^9 (d) x^{27}
14	A problem in Mathematics is given to three students whose chances of solving it are 1/2, 1/3 , 1/4 respectively. If the events of their solving the problem are independent then the probability that the problem will be solved, is a) 1/4 (b) 1/3 (c) 1/2 (d) 3/4
15	The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is : a) 1/x +1/y =C (b) log x-log y=C (c) xy=C (d) x + y=C
16	The vector $2\hat{\imath} + \hat{\jmath} - \hat{k}$ is perpendicular to $\hat{\imath} - 4\hat{\jmath} + \alpha\hat{k}$ if α is equal to a)10 b) -4 c) -2 d) -3
17	For the curve $\sqrt{x}+\sqrt{y} = 1$, $\frac{dy}{dx}$at (1/4,1/4) is a)1 b)1/2 c) -1 d)none of these
18	The distance of a point P (a,b,c) fom x-axis is (a) $\sqrt{a^2 + b^2}$ (b) a^2+b^2 (c) a^2+c^2 (d) $\sqrt{b^2 + c^2}$
ASSERTION-REASON BASED QUESTIONS In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
19	Assertion(A): If $y=\sin^{-1}\frac{2x}{1+x^2}$ then $\frac{dy}{dx} = \frac{2}{1+x^2}$ Reason(R): $\sin 2A = \frac{2tanA}{1+tan^2A}$
20	Assertion(A): A={1,2,3} B={4,5,6,7},f={(1,4),(2,5),(3,4), } is a function from A to B, and f is one-one Reason(R): A function f is one-one if distinct elements of A have distinct images in B.
SECTION B This section comprises of very short answer type-questions (VSA) of 2 marks each	
21	Find the value of $cos^{-1}\cos(\frac{-7\pi}{6})$ OR Prove that the function f is surjective where f:N→N such that f(n)=$\frac{n+1}{2}$, if n is odd =$\frac{n}{2}$,if n is even

<table>
<tr><td>22</td><td>Find the intervals in which the function f given by f(x)=tanx-4x ,x$\in (0,\frac{\pi}{2})$ is strictly increasing</td></tr>
<tr><td>23</td><td>Find the maximum profit that a company can make, if the profit function is given by P (x)=41-72x-18x^2
OR
Find the maximum and minimum value of the function f(x)=sin2x+5</td></tr>
<tr><td>24</td><td>Find$\int \frac{tan^3x}{cos^3x}$ dx</td></tr>
<tr><td>25</td><td>The volume of a cube is increasing at the rate of 9cm^3/s.How fast is its surface area increasing when the length of an edge is 10cm</td></tr>
<tr><td colspan="2">SECTION C
(This section comprises of short answer type questions (SA) of 3 marks each)</td></tr>
<tr><td>26</td><td>Evaluate$\int_0^{\pi} \frac{xtanx}{secxcosecx}$dx</td></tr>
<tr><td>27</td><td>The probability distribution of a random variable X is given below
<table>
<tr><td>X</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>P(X)</td><td>$\frac{k}{2}$</td><td>$\frac{k}{3}$</td><td>$\frac{k}{6}$</td></tr>
</table>
(i)Find the value of k
(ii)Find P(1≤ X<3)
(iii) Find the mean of X</td></tr>
<tr><td>28</td><td>Evaluate $\int \frac{(x^2-3x)}{(x-1)(x-2)}dx$
OR
Find $\int \frac{sin^6x+cos^6x}{sin^2xcos^2x}$dx</td></tr>
<tr><td>29</td><td>Find the particular solution of the differential equation
x$\frac{dy}{dx}$=+x $cos^2(\frac{y}{x})$ =y; given that x=1,y=$\frac{\pi}{4}$
OR
Solve the differential equation $(1+x^2)\frac{dy}{dx}$+2xy-4x^2=0 ,subject to the condition y(0)=0</td></tr>
<tr><td>30</td><td>Solve the LPP graphically Maximize Z=22x+18y
Subject to the constraints 3x+2y≤ 48, x+y≤ 20, x,y≥ 0
OR
Solve the following LPP graphically
Minimize Z=13x-15y
subject to the constraints x+y≤ 7,2x-3y+6≥ 0</td></tr>
<tr><td>31</td><td>If $y^x=e^{y-x}$, then prove that $\frac{dy}{dx}=\frac{(1+logx)^2}{logy}$</td></tr>
</table>

SECTION D **(This section comprises of long answer-type questions (LA) of 5 marks each)**	
32	**Using integration , find the area of region{(x,y):$x^2 + y^2 \leq 9, x + y \geq 3$}**
33	**Show that the relation R defined by (a,b)R(c,d)⇔ $a + d = b +$ c on the set AXA , where A={1,2,3,.....10} is an equivalence relation. Hence write the equivalence class of [(3,4)]** **OR** **Consider f:R-{$-\frac{4}{3}$}→R-{$\frac{4}{3}$} given by f(x)=$\frac{4x+3}{3x+4}$ .Show that f is bijective**
34	**34.If A=$\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$ then find A^{-1} and use it to solve the following questions** **x+2y-z=6,3x+2y-2z=3 ,2x-y+z=2**
35	**Find the shortest distance between the lines** **$\bar{r}$=(4$\hat{\imath}$-$\hat{\jmath}$) +λ ($\hat{\imath}$+2$\hat{\jmath}$-3$\hat{k}$) and$\bar{r}$=($\hat{\imath}$-$\hat{\jmath}$ + 2$\hat{k}$) +μ (2$\hat{\imath}$+4$\hat{\jmath}$-5$\hat{k}$)** **OR** **Find then length and the foot of the perpendicular drawn from the point (2,-1,5) on the line** $$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$$
SECTION E **(This section comprises of 3 case-study/passage-based questions of4 marks each with two sub-parts. First two case study questions have three sub -parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each**	
36	**There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below.** 

	The venn diagram below represents the probabilities of three different types of yoga, A,B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44  **On the basis of theabove information, answer the following questions** **(i) Find the value of x.** **(ii) Find the value of y** **(iii) (a) Find P$(\frac{C}{B})$.** **OR** **(iii) (b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.** **.**
37	**Solar Panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels. A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture , suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters P_1 (6,8,4) , P_2 (21,8,4), P_3 (21,16,10) and P_4 (6,16,10)**  **On the basis of the above information, answer the following questions**

	1. What are the components to the two edge vectors defined by $\vec{A}$ = PV of P_2 – PV of P_1 and $\vec{B}$ = PV of P_4– PV of P_1? (where PV stands for position vector) **2. Write the vector in standard notation with $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ (where $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are the unit vectors along the three axes).** **3. What are the magnitudes of the vectors $\vec{A}$ and $\vec{B}$ and in what units?** **OR** **What are the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof?**
38	**The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight, where $x \leq 3$**  **On the basis of the above information, answer the following questions** **1) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.** **2) Does the rate of growth of the plant increase or decrease in the first three days? What will be the height of the plant after 2 days?**

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SAMPLE QUESTION PAPER

CLASS-XII MATHEMATICS

General Instructions :

1. This question paper contains – five sections A,B,C,D and E.Each section is compulsory. However there are internal choices in some questions.
2. Section A has 18 MCQ'S and 2 Assertion Reason questions of 1 mark each.
3. Section B has 5 Very Short Answer-type questions of 2 marks each
4. Section C has 6 Short Answer-type questions of 3 marks each
5. Section D has 4 Long Answer-type questions of 5 marks each
6. Section E has 3 Source based/ case based/ passage based/ integrated units of assessment of 4 marks each with sub-parts

SECTION-A

(Each carries 1 mark)

1. $\begin{bmatrix} 3y-x & -2x \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}$, then the value of x +y is

   (a) 1 (b) 2 (c) 5 (d) 3
2. If A is matrix of order 3x3 such that |A| = 5, then |A(adjA)| is equal to

   (a) 25 (b) 125 (c) 5 (d) 1/125
3. For any square matrix A, A^2 -A+I = 0 then A^{-1} is equal to

   (a) A (b) A+I (c) I-A (d) A-I
4. The function f(x)= $\begin{cases} \frac{sin3x}{x}, x \neq 0 \\ \frac{k}{2}, x = 0 \end{cases}$ is continuous at x = 0,then the value of k is

   (a) 2 (b) 4 (c) 6 (d) 8
5. Let $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a}.\vec{b} = |\vec{b}|^2$ and $|\vec{a} - \vec{b}| = \sqrt{7}$ $|\vec{b}|$ equals

   (a) 7 (b) 14 (c) $\sqrt{7}$ (d) 21
6. The solution of $\frac{dy}{dx} - y = 1, y(0) = 1$ *is given by*

   (a) $xy = -e^x$ (b) $xy = -e^{-x}$ (c) $xy = -1$ (d) $y = 2e^x - 1$
7. The solution set of the in equation 3x+5y <7 is

   (a) Whole XY-plane except the points lying on the line 3x+5y=7
   (b) Whole XY-plane along with the points lying on the line 3x+5y=7
   (c) Open half plane containing the origin except the points of the line 3x+5y=7
   (d) Open half plane not containing the origin
8. If $\frac{d}{dx}$f(x)=logx, then f(x) is equal to

   (a) $\frac{1}{x}$ +C (b) x(logx-1)+C (c) x (log x +x) +C (d) e^{logx} +C

9. P is a point on the line segment joining the points (3,2,-1) and (6,2,-2). If x-coordinate of P is 5, then its y-coordinate is

(a) 2 (b) 1 (c) -1 (d) -2

10. If $A=\begin{bmatrix}2 & \lambda & -3\\ 0 & 2 & 5\\ 1 & 1 & 3\end{bmatrix}$, then A^{-1} exist, if

(a) $\lambda = 2$ (b) $\lambda \neq 2$ (c) $\lambda \neq -2$ (d) None of these

11. Solution of the following linear programming problem: Maximize z=5x+6y subject to $y \leq 2x+1$, $5x+2y \leq 20$ and $x \geq 0$, $y \geq 0$ is

(a) 6 (b) 20 (c) 40 (d) has no feasible region

12. If $\vec{a} = ((\hat{\imath} + 3\hat{\jmath} - 2\hat{k}) \times (-\hat{\imath} + 3\hat{k})$, then the value of $|\vec{a}|$

(a) 91 (b) $\sqrt{91}$ (c) 31 (d) $\sqrt{31}$

13. If $A=\begin{vmatrix}2 & 2\\ 9 & 4\end{vmatrix}$ and $I=\begin{vmatrix}1 & 0\\ 0 & 0\end{vmatrix}$, then $10A^{-1}$ is equal to

(a) A-4I (b) 6I-A (c) A-6I (d) 4I-A

14. The probability that A speaks truth is $\frac{4}{5}$ and that of B speaking truth is $\frac{3}{4}$. The probability that A and B contradict each other in stating the same fact is

(a) $\frac{7}{20}$ (b) $\frac{1}{5}$ (c) $\frac{3}{20}$ (d) $\frac{4}{5}$

15. The number of arbitrary constants in the particular solution of a differential equation of third order is:

(a) 3 (b) 2 (c) 1 (d) 0

16. If $|a \times b| = 4$ and $|a.b| = 2$, then $|a|^2 |b|^2$ is equal to:

(a) 4 (b) 6 (c) 20 (d) 2

17. If $y = (x + \sqrt{1+x^2})^n$, then $(1+x^2)\frac{d^2y}{d^2x} + \frac{dy}{dx}$ is

(a) n^2y (b) $-n^2y$ (c) -y (d) -y

18. If $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ are intersecting lines then k =

(a) $\frac{3}{2}$ (b) $\frac{9}{2}$ (c) $\frac{-2}{9}$ (d) $\frac{-3}{2}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) (A) is true but (R) is false.
(c) (A) is false but (R) is true.
(d) Both (A) and (R) are false

19. Assertion(A) If $x=at^2$ and $y=2at$ where t is the parameter and a is a constant , then $\frac{d^2y}{d^2x} = \frac{-1}{t^2}$

Reason (R) $\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} \div \frac{d^2y}{dt^2}$

20. Consider the function f: R→ R defined as f(x) =$\frac{x}{x^2+1}$

Assertion(A): f(x) is not bijective

Reason(R): f(x) is one- one but not onto

SECTION –B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. Find the principal value of $\tan^{-1}(\tan\frac{7\pi}{6})$

OR

Write the simplest form: $\sin^{-1}\left[\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right]$, $0<x<1$

22. The sides of an equilateral triangle are increasing at the rate of 2cm/s. Find the rate at which the area increases ,when the side is 10cm

23. Find the maximum and minimum values of the function f(x)=5+sin2x

OR

Find the domain of $\cos^{-1}(2x-1)$

24. Find $\int \frac{\sin(x-a)}{\sin(x+a)}\, dx$

25. Find the intervals in which the following function is increasing or decreasing

f(x) = $-2x^3-9x^2-12x+1$

SECTION-C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Find $\int e^{cot^{-1}x}\left(\frac{1-x+x^2}{1+x^2}\right)$ dx

27.Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also find the mean number of random variable.

OR

A and B throw a die alternately till one of them gets a six and wins the game. Find their respective probabilities of winning if A starts the game first.

28. Evaluate : $\int_{log\sqrt{2}}^{log\sqrt{3}} \frac{1}{(e^x+e^{-x})(e^x-e^{-x})}$ dx

29.Find the general solution of the differential equation :

$(xy-x^2)$dy = y^2dx

OR

Find the general solution of the differential equation :

$(x^2+1)\frac{dy}{dx}$ + 2xy = $\sqrt{x^2+4}$

30. Solve the following linear programming problem graphically :

Minimize : Z = 5x +7y

Subject to constraints : $2x + y \geq 8\,,\ x + 2y \geq 10,\ x, y \geq 0.$

31. If (a +bx) $e^{\frac{y}{x}}$ = x, Prove that x $\frac{d^2y}{dx^2}$ = $\left(\frac{a}{a+bx}\right)^2$

OR

Differentiate $sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r. to $sin^{-1}(2x\sqrt{1-x^2})$

SECTION-D

[This section comprises of long answer type questions (LA) of 5 marks each]

32.Using integration, find the area of the region bounded by y = $\sqrt{3}$ x and y = $\sqrt{4-x^2}$ and y- axis in the first Quadrant.

33. If N denote the set of natural numbers and R is the relation on N X N , defined by (a,b) R (c,d) , if ad(b+c) = bc (a+d) is an equivalence relation.

OR

A function f : $[-4,4] \rightarrow [0,4]$, given by f(x) = $\sqrt{16-x^2}$. Show that f is an onto function but not one-one. Further find all possible values of a , f(a) = $\sqrt{7}$.

34. A= $\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ B= = $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, Find AB, Use this solve the following system of equations, x-2y = 3, 2x-y-z = 2, -2y +z = 3

35. Find the equation of the diagonals of the parallelogram PQRS, whose vertices are P (4,2,-6) Q (5,-3,1) R (12,4,5) S(11,9,-2) .Use this equation find the point of intersection of diagonals.

OR

A line l passes through point (-13,2) and is perpendicular to both of the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$

Find the vector equation of the line. Hence obtain its distance from origin.

SECTION –E

[This section comprises of 3 Source based/ case based/ passage based/ integrated units of assessment of 4 marks each with sub-parts . The first two case study questions have 3 sub-parts (i),(ii), (iii) of marks 1,1,2 respectively. The third case study questions has 2 sub-parts of 2 marks each.]

36.Read the following passage and answer the questions given below.



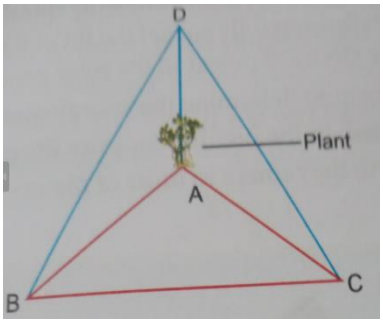
The temperature of a person during an intestinal illness is given by f(x) = $-0.1x^2$ +mx +98.6, $0 \leq x \leq 12, m\ being\ a\ constant$, f(x) is the temperature in degree Fahrenheit at x days.

(i) Is the function differentiable in the interval (0,12). Justify your answer
(ii) Is 6 is the critical point of the function , then find the value of the constant m
(iii) Find the interval in which the function is strictly increasing / strictly decreasing

OR

(iii) Find the points of absolute minima /absolute maxima in the interval [0,12]. Also find the corresponding absolute minimum / absolute maximum values of the function

37. Read the following passage and answer the questions given below.



Raghav purchased an air plant holder which is in the shape of tetrahedron. Let A,B,C,D be the co-ordinates of the air plant holder where A (1,2,3) B (3,2,1) C (2,1,2) D (3,4,3)

(i) Find the vector $\overrightarrow{AB}$
(ii) Find the vector $\overrightarrow{CD}$
(iii) Find the unit vector along $\overrightarrow{BC}$

OR

(iii)Find the area of $\Delta\ (BCD)$

38. Read the following passage and answer the questions given below



There are two antiaircraft guns named as A and B. The probabilities that the shells fixed from them hits an airplane are 0.3and 0.2 respectively. Both of them fired one shell at an airplane at the same time

(i) What is the probability that the shell fired from exactly one of them hit the plane?
(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B ?

KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION

SAMPLE QUESTION PAPER -12

SESSION – 2023 -24

Class: XII **Subject: Mathematics**

Max Mark: 80 **Hours: 3 hours.**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6 . Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

SECTION A

This section comprises of very short answer type questions (MCQ) of 2 marks each.

1. The vector equation of a straight line that passes through the point (3, 4,5) and parallel to the vector $2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ *is*

a. $\vec{r} = (2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) + \mu(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$

b. $\vec{r} = (3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}) + \mu(2\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$

c. $\vec{r} = (-2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + \mu(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$

d. $\vec{r} = (-3\hat{\imath} - 4\hat{\jmath} - 5\hat{k}) + \mu(2\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$

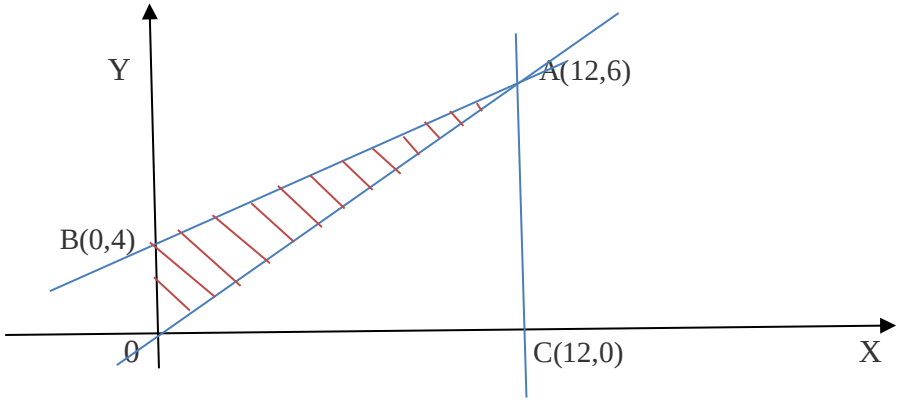
2. The integrating factor of the differential equation $\frac{dy}{dx} + ycotx = 2x + x^2 cot\, x$ is

a. Cot x b. sin x c. x d.-Cosec^2 x

3. The value of x if $\begin{vmatrix} 6 & 1 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} 3x & 1 \\ 2x & 2 \end{vmatrix}$ is

a. 3 b. 4 c. 1 d. 2

4. The maximum value of $F = 3x - 4y$, if the feasible region for LPP is shown in the figure is



a. 0 b. 8 c. 12 d.-16

5. The sum of degree and order of the given differential equation $\sqrt[3]{\frac{d^2y}{dx^2}}=\sqrt{\frac{dy}{dx}}$ is equal to

a. 3 b. 7/3 c. 4 d. 5/2

6. The probability distribution of the random variable x is given by

$$P(X=x)=\begin{cases} kx^2, & x=1,2,3 \\ 2kx, & x=4,5,6 \\ 0, & other\ wise \end{cases}$$

where k is a constant. Find k

a. ½ b. 1/44 c. 3/44 d. 1/3

7. The $\int \frac{e^{tan^{-1}x}}{1+x^2}dx$ is equal to

a. $e^{tan^{-1}}x+c$
b. $\tan^{-1}e^x+c$
c. $\tan^{-1}x+c$
d. $e^{1+x^2}+c$

8. $The\ \ value\ of \int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ is

a.$\frac{\pi}{2}$ b. $\frac{\pi}{6}$ c. $\frac{\pi}{4}$ d. $\frac{\pi}{12}$

9. If $\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}6\\ 3\\ 2\end{bmatrix}, then$ the value of 2x+y-z is:

a. 1 b. 2 c. 3 d.5

10. If A is a square matrix of order 3 and |A| = 5, then the value of |2A′| is

a. -10 b. 10 c. -40 d. 40

11. The value of k for which the function$(x)=\begin{cases}\frac{1-\cos 4x}{8x^2}, & x\neq 0\\ k, & x=0\end{cases}$ is continuous at x=0 is

a. 0 b. -1 c. 1 d.2

12. If A = $\begin{bmatrix}1 & 2 & -1\\ -1 & 1 & 2\\ 2 & -1 & 1\end{bmatrix}$, then |adj(adjA)|

a. 14^4 b. 14^3 c. 14^2 d. 14

13. If A = $\begin{bmatrix} 2 & \beta & -4 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$,the value of β for which A^{-1} exist is given by

a. $\beta \neq -2$ b. -2 c. $\beta \neq 2$ d. 2

14. If $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = \mathbf{3}$, $|\vec{b}| = \mathbf{5}$, $|\vec{c}| = \mathbf{7}$ then the angle between If $\vec{\boldsymbol{a}}$ ***and*** $\vec{\boldsymbol{b}}$ is

a. $\frac{\pi}{2}$ b. $\frac{\pi}{6}$ c. $\frac{2\pi}{3}$ d. $\frac{\pi}{3}$

15. If $|\vec{a} \times \vec{b}| = \mathbf{4}$, $|\vec{a} \cdot \vec{b}| = 2$, then $|\vec{a}|^2 |\vec{b}|^2$ is

a. 6 b. 2 c. 20 d. 8

16. If the lines $\frac{x-2}{3} = \frac{y-1}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are perpendicular, find the value of k

a. -2/5 b. -2/7 c. 4 d. 2/7 4

17. $\vec{\boldsymbol{a}}$ is a unit vector and $(\vec{\boldsymbol{x}} - \vec{\boldsymbol{a}}) \cdot (\vec{\boldsymbol{x}} + \vec{\boldsymbol{a}}) = 8$, then find $|\vec{\boldsymbol{x}}|$

a. 2 b. 5 c. 4 d. 3

18. The corner points of the feasible region determined by the system of linear constraints are (0,3) (1,1) and (3,0).Let $Z = px + qy$; $p, q > 0$.Condition on p and q so that minimum of Z occurs at (3,0) and (1,1) is

a. $p = 2q$ b. $p = \frac{q}{2}$ c. $p = 3q$ d. p =q

Questions numbers **19** and **20** are A**ssertion and Reason based questions** carrying 1 mark each. Two statements are given , one labelled Assertion (A) and the other labelled Reason (R).Select the correct answer from the codes (a),(b),(c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion (A)** : f: N→N given by f(x)=5x is injective but not surjective
Reason (R) : If co-domain ≠ range, then the function is not surjective

20. **Assertion (A) :** $|\sin x|$ is continuous for all $x \in R$
Reason (R) : Sin x and $|x|$ are continuous in R

SECTION-B

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. Find the value of $\sin^{-1}[\cos(\sin^{-1}\frac{\sqrt{3}}{2})]$

OR

Express in the simplest form $\tan^{-1}\frac{2\sqrt{x}}{1-x}$

22. Find the intervals in which the function $F: R \to R$ defined by f(x) =10 -6x-$2x^2$ are strictly increasing or decreasing

23. Find the maximum and minimum value of the function f(x) =$9x^2 + 12x + 2$

OR

The total revenue in Rupees received from the sale of x units of a product is given by R(x) =$13x^2$+26x+15.Find the marginal revenue when x=7.

24. Evaluate $\int_0^{\frac{\pi}{2}} \frac{sin^4 x}{sin^4 x+cos^4 x}$dx

25. Check whether the function, defined by f(x)=sinx+cosx , $0 \leq x \leq 2\pi$ has any critical point/s or not? If yes find the points

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Integrate:$\frac{e^x}{(1+e^x)(2+e^x)}$

27. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable and the expectation of X.

28. Find $\int \frac{1}{\cos(x-a)\cos(x-b)}\,dx$

OR

Evaluate $\int_0^{\frac{\pi}{2}}(2logsinx - logsin2x)dx$

29. Solve the differential equation :x logx $\frac{dy}{dx} + y = \frac{2}{x}logx$

OR

Solve the differential equation : xcos($\frac{y}{x}$) $\frac{dy}{dx}$=y cos($\frac{y}{x}$) + x

30. Solve the following Linear Programming Problem graphically:
Minimise Z=200x+500y
Subject to the constraints:
x+2y$\geq$ 10 ; 3x+4y$\leq$ 24 ; x, $y \geq 0$

OR

Solve the following Linear Programming Problem graphically
Maximise Z=4x+y
Subject to the constraints:
$x + y \leq 50$; $3x+y \leq 90$; $x, y \geq 0$

31. If $y=e^{a\cos^{-1}x}$, $-1 \leq x \leq 1$, show that $(1-x^2)y_2 - xy_1 - a^2y=0$.

SECTION -D

This section comprises of long answer type questions (LA) of 5 marks each.

32. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$

33. Let R be the relation on **NxN** defined by ***(a, b) R (c, d)*** if and only if

ad (b+ c) =bc(a+ d). Prove that R is an equivalence relation

OR

Show that the function f **:N →N** defined by

***f* (x) = x + 1 if x is odd**

x−1, if x is even

is both one-one and onto function.

34. Using the product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the following system of linear equations:

$$x - y + 2z = 1 \ , \ 2y - 3z = 1 \ , \ 3x - 2y + 4z = 2$$

35. Show that the lines c and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ intersect.

Also , find the co-ordinates of their point of intersection.

OR

Find the shortest distance between the lines

$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{5-Z}{5}$.

SECTION E

This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.

36. Read the following passage and answer the questions given below:

In an Office three employees Jayant, Sonia and Oliver process incoming copies of a certain form. Jayant processes 50%of the forms, Sonia processes 20% and Oliver the remaining 30% of the forms. Jayant has an error rate of 0.06, Sonia has an error rate of 0.04 and Oliver has an error rate of 0.03.

Based on the above information, answer the following questions.



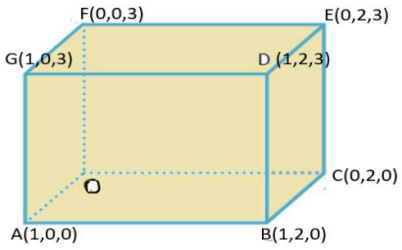
1) Find the total probability of committing an error in processing the form.
2) The manager of the Company wants to do a quality check. During inspection, he selects a form at random from the days output of processed form. If the form selected at random has an error, find the probability that the form is not processed by Jayant.

OR

Let E be the event of committing an error in processing the form and let $\boldsymbol{E_1}, \boldsymbol{E_2}, \boldsymbol{E_3}$ be the events that Jayant , Sonia and Oliver processed the form. Find the value of $\sum_{i=1}^{3} P(E_i | E)$.

37. Anu made a cuboidal fish tank having coordinates

O(0,0,0) , A(1,0,0) , B(1,2,0) , C(0,2,0) , D(1,2,3) , E(0,2,3) , F(0,0,3) and G(1,0,3)



Based on the above information ,answer the following questions.

1) Direction cosines of **AB** are

(a) < 1,1,0 > (b) < 0,2,0 > (c) <0,1,0> (d) none of these

2) Equation of diagonal **OD** is

(a) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ (b) $\frac{x}{0} = \frac{y}{1} = \frac{z}{2}$ (c) $\frac{x}{0} = \frac{y}{1} = \frac{z}{2}$ (d) none of these

3) The lines **BC** and **DE** are

(a) parallel (b) intersecting (c) skew lines (d) perpendicular

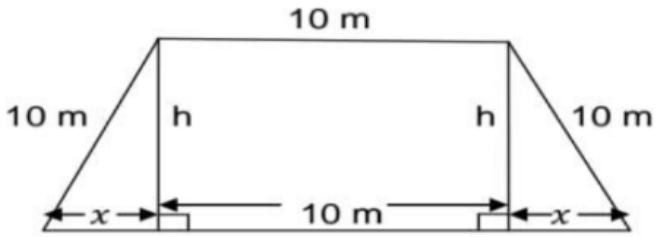
4) Equation of line **BF** is

(a) $\frac{x}{1} = \frac{y}{2} = \frac{z}{-3}$ (b) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ (c) $\frac{x}{0} = \frac{y}{1} = \frac{z}{2}$ (d) none of these.

38. Read the following passage and answer the questions given below:

The front gate of a building is in the shape of a trapezium as shown below. Its three sides other than base are 10m each. The height of the gate is h meter. On the basis of this information and figure given below answer the following questions:





1) Find area **A** of the gate expressed as a function of **x**.
2) Find value of $\frac{dA}{dx}$.
3) For which positive value of **x,** $\frac{dA}{dx}$ **= 0 .**
4) Find maximum area of trapezium.

KENDRIYA VIDYALAYA SANGATHAN ERNAKULAM REGION

SAMPLE PAPER(13)
CLASS XII

TIME: 3 HRS **MATHEMATICS** **MM: 80**

General Instructions:

1. This question paper contains-**five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has **18 MCQ's** and **02 Assertion-Reason** based questions of 1 mark each.
3. **Section B** has **5 Very Short Answer (VSA)-**type questions of 2 marks each.
4. **Section C** has **6 Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has **4 Long Answer(LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION-A

1. Let A={3,5} , then number of reflexive relation**s** on A is

   (a) 2 (b) 4 (c) 0 (d) 8

2. Sin $\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

   (a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

3. For a square matrix of A,A^2-A+I=0, then A^{-1}=

   (a) A (b) A+I c) A – I (d) I-A

4. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$,then x is equal to

   (a) 6 (b) ± 6 (c) -6 (d) 0

5. If A is a square matrix of order 3, such that A. adj(A) = 10 I , then $|adj\ A|$ is equal to

   (a) 20 (b) 10 (c) 100 (d) 101

6. The function f(x)=$[x]$ where$[x]$ denote greatest integer function, is continuous at

   (a) x=0 (b) x=1.5 (c) x=-2 (d) x=4

7. The angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ *is*

   $(a)\frac{\pi}{6}$ $(b)\ \cos^{-1}\left(\frac{1}{3}\right)$ $(c)\frac{\pi}{2}$ $(d)\pi$

8. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$,then value of x and y is

(a)x=3, y=1 (b) x=2, y=3 (c) x=2, y=4 (d) x=3,y=3

9. If for any 2×2 matrix A, A (adjA)=$\begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}$,$|A|$ is equal to

(a) 6 (b)36 (c)12 (d) 24

10. The projection of the vector $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ on the vectors $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$ *is*

$(a)\frac{5}{3}\sqrt{6}$ $(b)\ \frac{1}{3}$ $(c)\frac{1}{2}$ $(d)\ \sqrt{6}$

11. The sum of the order and degree of the following differential equation

$$\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\}=0 \quad is$$

(a) 4 (b) 2 (c) 1 (d) 3

12. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6,0), (6, 8) and (0, 5). Let F = 4x + 6y be the objective function. The minimum value of F occurs at

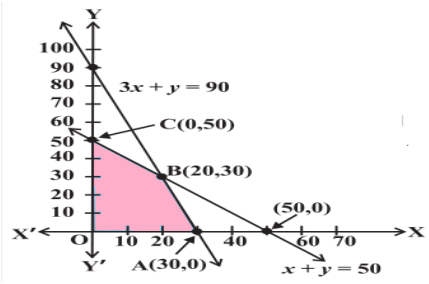
(a) (0,2) only

(b) (3,0) only

(c) The midpoint of the line segment joining the points (0, 2) and (3, 0) only

(d) Any point on the line segment joining the points (0, 2) and (3, 0)

13. The corner points of the shaded bounded feasible region of an LPP are (0, 0), (30, 0), (20, 30) and (0, 50) as shown in the figure.



The maximum value of the objective function Z = 4x+y is

(a) 120 (b) 130 (c) 140 (d) 150

14. Let A and B be two events. If P(A)=0.2, P(B)=0.4, P(AUB)=0.6 then $P\left(\frac{A}{B}\right)$ is equal to

(a) 1 (b) 0 (c) 0.2 (d) 0.4

15. If $f'(x)=x^2e^{x^3}$,then f(x) is

(a) $\frac{1}{3}e^{x^3}$+C (b) $\frac{1}{3}e^{x^4}$+C (c) $\frac{1}{2}e^{x^3}$+C (d) $\frac{1}{2}e^{x^2}$+C

16. What is the value of μ so that the vectors $\vec{a}= 2\hat{\imath} + \mu\hat{\jmath} + \hat{k}$ and $\vec{b}= \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ are perpendicular to each other.

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{5}{2}$ (d) $\frac{1}{5}$

17.The integrated factor of the differential equation: $(1+x^2)\frac{dy}{dx} + y = e^{tan^{-1}x}$ is

(a) $\frac{1}{e^{tan^{-1}x}}$ (b) $2\,e^{tan^{-1}x}$ (c) $3e^{tan^{-1}x}$ (d) $e^{tan^{-1}x}$

18.The value of $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ *is*

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

ASSERTION- REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct out of the following choices.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true and R is not the correct explanation of A

(c) A is true but R is false.

(d) A is false but R is true.

19.Assertion (A): $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

Reason (R): $\sin^{-1}(\sin(\theta)) = \theta$ if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

20.Assertion (A): The angle between the lines $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ & $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$ is 90°

Reason (R): Skew lines are lines in different planes which are parallel and intersecting.

SECTION B

21.Find the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$

OR

Check the injectivity and surjectivity of the function defined as $f : N \to N$ given by $f(x) = x^2$.

22.If $x\cos(a+y) = \cos y\,, then\ prove\ that\ \frac{dy}{dx} = \frac{\sin a}{cos^2(a+y)}$

23.The volume of a sphere is increasing at the rate of $8\frac{cm^3}{s}$.Find the rate at which its surface increasing when the radius of the sphere is 12cm.

24.Find the vector of magnitude 6, which is perpendicular to both the vectors
$2\hat{\imath} - \hat{\jmath} + \ \ 2\hat{k}\ and\ 4\,\hat{\imath} - \hat{\jmath} + 3\hat{k}$

OR

Find the equation of a line in vector and cartesian form which passes through the point (1,2,3) and is parallel to the vector$3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

25.If the equation of a line AB is $\frac{3-x}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$,Find the direction cosines of the line parallel to AB.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26.Evaluate $\int \frac{\text{x } \sin^{-1}(x^2)}{\sqrt{1-x^4}}$ dx

OR

Evaluate $\int_0^1 \ tan^{-1}\left(\frac{1-2x}{1+x-x^2}\right)$

27.Probability of solving specific problem independently by A and B are ½ and 1/3 respectively. If both try to solve the problem independently, find the probability that

(I) The problem is solved
(II) exactly one of them solves the problem.

OR

Let 'x' denote the number of colleges where you will apply after your results and P(X = x) denotes your probability of getting admission in x number of colleges. It is given that

$$P(X = x) = \begin{cases} kx, & if\ \ x = 0\ and\ \ 1 \\ 2kx, & if x = 2 \\ k(5-x), & if x = 3\ \ and\ 4 \end{cases}$$

Find the value of k

28.Evaluate $\int_0^{\pi/4} log\ \ (\ 1 + tan\,x\ )\,dx$

OR

$$\int_0^{\pi} \frac{xsinx}{1+cos^2x}dx$$

29.Find the general solution to the differential equation: $x\frac{dy}{dx} = y - xtan(\frac{y}{x})$

OR

Find the particular solution to the differential equation: $(tan^{-1}y\text{-}x)dy = (1+y^2)dx$ given that when x=0,then y=0

30.Solve the following linear programming (LPP) graphically. Maximize Z =6x + 3y subject to the constraints, 4x + y ≥ 80 , X + 5y ≥ 115 , 3x + 2y ≤ 150 , x, y ≥ 0

31.Evaluate: $\int \frac{1}{(x^2+1)(x^2+4)}\,dx$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32.Sketch the graph of y = | x + 3 |, and evaluate $\int_{-6}^{0} |\, x + 3 \,|\, dx$

33.Let A= {1,2, 3, . . ,9} and R be the relation in AXA defined by (a, b)R(c, d) if a+d=b+c for (a, b), (c, d) in AxA. Prove that R is an equivalence relation. Also obtain the equivalence class of {2,5}.

34.Find the vector equation of the line passing through the point (1,2,-4) and perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

OR

An insect is crawling along the line $\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and another insect is crawling along the line $\vec{r} = (-4\hat{i} - \hat{k}) + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$. At what points on the lines should they reach so that the distance between them is the shortest. Find the shortest possible distance between them.

35.The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

OR

Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

x – y + 2z = 1, 2y – 3z = 1, 3x – 2y + 4z = 2.

SECTION E

(This section comprises of two sub-parts. First two case study questions have three sub of marks 1, 1, 2 respectively. The third case study question has two sub marks each.)

36.Case-Study 1: Read the following passage and answer the questions given below. The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours.



(i) If ‘N’ is the number of bacteria, find the corresponding differential equation?

(ii) Find the general solution of the differential equation?

(iii) If N_0 is the initial count of bacteria, find the count of bacteria after 10 hours

OR

The value of $\int \frac{1}{k\,x}$ dx

37.A doctor is to visit a patient. From the past experience, it is known that the

probabilities that he will come by cab, metro, bike or by other means of transport are respectively 0.3, 0.2, 0.1 and 0.4. The probabilities that he will be late are 0.25, 0.3, 0.35 and 0.1 if he comes by cab, metro, bike and other means of transport respectively.



Based on the above information, answer the following questions.

(i) When the doctor arrives late, what is the probability that he comes by metro?

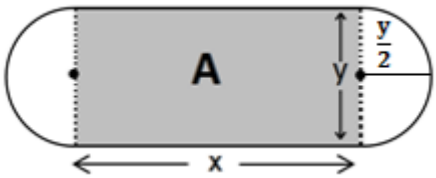
(ii) When the doctor arrives late, what is the probability that he comes by cab?

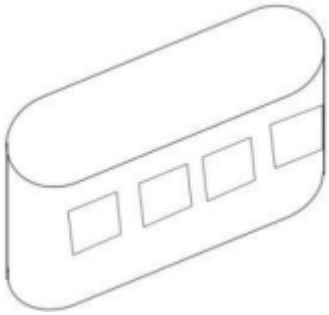
(iii)When the doctor arrives late, what is the probability that he comes by bike?

OR

When the doctor arrives late, what is the probability that he comes by other means of transport?

38.An architect designs a building for a multi-national company. The floor consists of a rectangular region with semicircular ends having a perimeter of 200m as shown below:





Design of Floor Building

Based on the above information answer the following:

(i) If x and y represent the length and breadth of the rectangular region, then find the relation between x and y

(ii) Find The maximum value of area A

KENDRIYA VIDYALAYA SANGTHAN ERNAKULAM REGION

Sample Question Paper-14

CLASS: XII 2023-24

Class XII

Mathematics (Code-041)

Time Allowed: 3 Hours Maximum Marks: 80

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A (Multiple Choice Questions) Each question carries 1 mark

1	If A and B are matrices of same order, then (AB' – BA') is a (a) skew- symmetric matrix (b) null matrix (c) symmetric matrix (d)unit matrix	1
2	If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, *then xis equal to* (a) 6 (b) ± 6 (c) - 6 (d) 0	1
3	Let Abe a square matrix of order 3, then |KA| is equal to (a) K |A| (b) K^2 |A| (c) K^3 |A| (d) 3K |A|	1
4	The function f(x) = [x], where [x] denotes the greatest integer function, is continuous at (a) 4 (b) -2 (c) 1 (d)1.5	1

5	The coordinates of the foot of perpendicular from the point (2,-3,4)on the y-axis is (a) (2,3,4) (b)(-2,-3,-4) (c)(0,-3,0) (d) (2,0,4)	1
6	The degree of differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\,sin\left(\frac{dy}{dx}\right)$ is (a) 1 (b) 2 (c) 3 (d) not defined	1
7	Find the value of λ such that the vectors $\vec{a} = 2\hat{\imath} + \lambda\,\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\,\hat{\jmath} + 3\,\hat{k}$ are orthogonal (a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{5}{2}$	1
8	The corner points of the feasible region of an LPP are (0,0), (0,8), (2,7), (5,4), (6,0) . The maximum profit P = 3x +2y occurs at (a) (5,4) (b)) (0,8) (c)) (6,0) (d)) (2,7)	1
9	$\int \frac{dx}{sin^2x cos^2x}$ is equal to (a) tan x + cot x +c (b) $(\tan x + \cot x)^2$ +c (c) tan x- cot x + c (d) $(\tan x - \cot x)^2$ + c	1
10	If A= $\begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} exist if (a) λ= 2 (b) $\lambda \neq 2$ (c) $\lambda \neq -2$ (d) none of these	1
11	In an LPP if the objective function Z = ax + by has the same maximum value on two corner points of the feasible region , then the number of points of which maxZ occurs is (a) 0 (b) 2 (c) finite (d) infinite	1
12	Find the value of a +b if the points (2,a,3) ,(3,-5,b) and (-1,11,9) are collinear (a) 0 (b) 1 (c) -1 (d) 2	1
13	For any 2x2 matrix , if A(adjA) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then |A| is equal to (a) 20 (b) 10 (c) 0 (d)100	1
14	Two dice are thrown , if it is known that the sum of the number on the die was less than 6, the probability of getting a sum 3, is (a) $\frac{1}{18}$ (b) $\frac{5}{18}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$	1

15	Integrating factor of $x\frac{dy}{dx} - y = x^4 - 3x$ is (a) x (b) log x (c) $\frac{1}{x}$ (d) -x	1
16	The area of triangle formed by vertices O,A,B where $\overrightarrow{OA} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\overrightarrow{OB} = -3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ is (a) $3\sqrt{5}$ sq. units (b) $5\sqrt{5}$ sq. units (c) $6\sqrt{5}$ sq. units (d) 4 sq. units	1
17	The function f(x) $= \frac{x-1}{x(x^2-1)}$ is discontinuous at (a) exactly one point (b)) exactly two points (c)) exactly three points (d)no point	1
18	P is a point on the line segment joining the points (3,2,-1) and (6,2,-2). If x co-ordinate of P is 5, then y co-ordinate is (a) 2 (b) 1 (c) -1 (d)-2	1
	ASSERTION-REASON BASED QUESTIONS In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
19	Assertion (A): A function f :N→ N defined as f(x) = x^2 is injective : Reason (R): A function f: A → B is injective if every element of B has a pre-image in A	1
20	Assertion (A): f(x)=log x is defined for all x Є (0, ∞) Reason (R): If f'(x) >0 , then f(x) is strictly increasing function	1
	SECTION B This section comprises of very short answer type-questions (VSA) of 2 marks each	
21	Find the value of $sin^{-1}\left[sin\left(-\frac{17\pi}{8}\right)\right]$ OR Express $tan^{-1}\left(\frac{cosx}{1-sinx}\right)$, $\frac{3\pi}{2} < x < \frac{\pi}{2}$ in simplest form	2
22	Find the intervals in which the function y = $[x(x-2)]^2$ is increasing function	2

23	Find the absolute maximum and absolute minimum values of the function f(x) = $sin^2 x - cosx$, x ∈ [0,π] OR Find the absolute maximum and absolute minimum values of the function f(x) = 4x - $\frac{x^2}{2}$, x ∈ [-2, $\frac{9}{2}$]	2
24	*Evaluate* $\int_0^1 x^2(1-x)^2 dx$	2
25	The amount of pollution content in air in a city due to x-diesel vehicles is given by p(x)= 0.005 x^3 +0.02 x^2 +30x . find the marginal increase in pollution content when 3 diesel vehicles are added	2
	SECTION C (This section comprises of short answer type questions (SA) of 3 marks each)	
26	Find $\int \frac{dx}{(x+1)(x+2)}$	3
27	Find the probability distribution of number of doublets in three throws of a pair of dice	3
28	Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ **OR** **Evaluate :** $\int_0^{\pi} \frac{x \sin x}{1+cos^2 x}\, dx$	3
29	**Find the general solution of the differential equation** x^2 dy + ($xy+y^2$) dx=0 OR Find the general solution of the differential equation: sec^2x. tany dx - sec^2y tanx dy = 0	3
30	Solve the following problem graphically: Minimise **Z = 200 x + 500 y** subject to the constraints: **x + 2y ≥ 10 , 3x + 4y ≤ 24 , x ≥ 0, y ≥ 0**	3

	OR Solve the following problem graphically: Minimise Z = 50x + 70y subject to the constraints: $2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$	
31	Differentiate with respect to x : $\sin^{-1}\left(\frac{2^{x+1}.}{1+(4)^x}\right)$	3
	SECTION D (This section comprises of long answer-type questions (LA) of 5 marks each)	
32	Find the product AB, where $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and use it to solve the equations: $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$	5
33	Find the identity element for * on A, if any. Show that the relation R in the set NxN defined by (a,b) R (c,d) $\Leftrightarrow$ $a^2 + d^2 = b^2 + c^2$ is an equivalence relation. Also find [(1,2)] OR Prove that the function, $f : N \rightarrow N$ is defined by $f(x) = x^2 + x + 1$ is one-one but not onto.	5
34	Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not. OR Find the coordinates of the foot of the perpendicular drawn from the point A $(-1, 8, 4)$ to the line joining the points B $(0, -1, 3)$ and C $(2, -3, -1)$. Also find the perpendicular distance.	5
35	Find the Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$	5
	SECTION E (This section comprises 3 case study -/ passage based questions of 4 marks each)	
36	Anitha walks 4km towards west from her home and reaches her friend Geetu's house. Then together they walk 3km in a direction 30^0 east of north and reaches school .	

	 Based on the above information, answer the following questions.	
	1. Find Anitha's displacement from her house to Geetu's house?	1
	2. Find Geetu's displacement from her house to school?	1
	3. Find Anitha's displacement from her house to school?	2
37	For three persons A , B and C the chances of being selected as a Manager for a firm are in the ratio 4:1:2 respectively. The respective probabilities for them to introduce new products are 0.3, 0.8 and 0.5  Based on above information answer the following	
	(i) What is the probability that A will be selected as Manager?	1
	(ii) What is the probability that new product is introduced ?	1

	(iii) What is the probability of B for introducing new product? OR What is the probability of A for introducing new product?	2
38	Case Study -3: Read the following passage and answer the questions given below  The temperature of a person during an intestinal illness is given by $f(x) = -0.2x^2 + kx + 47.5$, $0 \leq x \leq 12$ m being a constant ,where f(x) is the temperature in °C at x days. (i) If 10 is the critical point of the function then find the value of the constant k. (ii) Find the intervals in which the function is strictly increasing/strictly decreasing. (iii) Find absolute maximum and absolute minimum value OR Find point of local maximum / local minimum	 1 1 2

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Sample Question Paper -15

(Session 2023-24)

Class XII Sub: Mathematics

Time Allowed: 3 Hours Maximum Marks: 80

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A (Multiple Choice Questions) Each question carries 1 mark

1. If A=$\begin{pmatrix} 3 & -4 \\ 5 & -6 \end{pmatrix}$ and if AB=BA=$2I_2$, the identity matrix, then B is

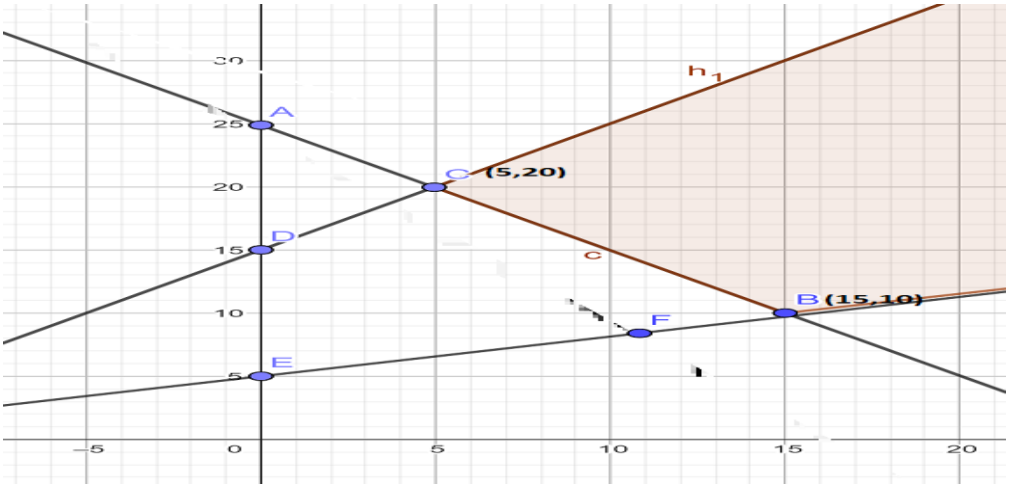
a) $\begin{bmatrix} -3 & 5 \\ -4 & 6 \end{bmatrix}$ b) $\begin{bmatrix} -6 & 4 \\ -5 & 3 \end{bmatrix}$ c) $\begin{bmatrix} -6 & -5 \\ 4 & 3 \end{bmatrix}$ d) $\begin{bmatrix} -3 & 2 \\ \frac{-5}{2} & \frac{3}{2} \end{bmatrix}$

2. If A is a square matrix of order 4 and det(adjA)=125, then value of det(3A) is
   a) 405 b) 15 c) 135 d) 125
3. If A is a skew symmetric matrix of order 3, then A is a
   a) Non Singular matrix b) Singular matrix
   c) Identity matrix d) Invertible matrix
4. The lines $\frac{5x-3}{10} = \frac{2-3y}{9} = \frac{7z}{21}$ and $\frac{x+13}{2} = \frac{y-1}{-3} = \frac{2z+5}{6}$ are
   a) Parallel b) Skew lines c) Intersecting lines d) Perpendicular lines
5. The value of 'a' for which the function f defined as

$(x) = \begin{cases} asin\frac{\pi}{2}(x+1), & x \le 0 \\ \frac{tanx-sinx}{x^3}, & x > 0 \end{cases}$ is continuous at x=0

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{\pi}{2}$ (d) $\frac{-\pi}{2}$

6. The feasible region of an LPP is shown in the figure. If z=-3x+5y, then the minimum value of z occurs



a) A single point b)at infinitely many points c) at (5, 20) d) at no point

7. The function y= acosx +bsinx is the solution of

a) $\frac{dy}{dx} = y$ b) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$ c) $\frac{dy}{dx} + y = 0$ d) $\frac{d^2y}{dx^2} + y = 0$

8. The value of τ *if the vector* $\vec{a} + \vec{b}$ *is perpendicular to* $\vec{c}$,

$\vec{a} = 9\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} - 5\hat{k}$ and $\vec{c} = \tau\hat{\imath} + 7\hat{\jmath} + 3\hat{k}$

a) -1 b) 1 c) 0 d) none of these

9. $\int_0^1 \frac{x\sin^{-1}(x^2)}{\sqrt{1-x^4}}$ dx equals to

a) $\frac{\pi^2}{4}$ b) $\frac{\pi}{8}$ c) $\frac{\pi}{2}$ d) $\frac{\pi^2}{8}$

10. If P+Q=$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ and P-2Q =$\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$, then P is

a) $\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & \frac{-1}{3} \\ \frac{1}{3} & 1 \end{bmatrix}$ c) $\begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$ d) $\begin{bmatrix} \frac{1}{3} & \frac{-2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

11. The corner points of the bounded feasible region determined by a system of linear constraints are (1,1),(0,3) and (3,0) Let Z =px+qy, where p,q > 0. The condition on p and q so that the minimum of Z occurs at (3,0) and (1,1) is

a) p=2q b)p=$\frac{q}{2}$ c) q=$\frac{p}{3}$ d) p=q

12. The triangle with vertices A,B and C whose position vectors are $\vec{a} = 3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$,

$\vec{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ *and* $\vec{c} = \hat{\imath} - 3\hat{\jmath}$-$5\hat{k}$ is

a) an iscosceles triangle b) a right angled triangle

c) equilateral triangle d) acute angled triangle

13. If A is of order 3xp, B is of order 4xm and C is of order nx2 and if $AB\mathbf{C}^{\mathbf{T}}$ is a square matrix, then value of p+m+n is

a) 8 b) 11 c) 9 d) 10

14. Two are cards are picked at random from a pack of 52 playing cards. If both the cards are spade cards, then probability that both are number cards is

a) $\frac{9}{13}$ b) $\frac{4}{13}$ c) $\frac{6}{13}$ d) none of these

15. The integrating factor of the differential equation $x\frac{dy}{dx} - xycotx + x^2 = 0$ is

a) sinx b) cosecx c) tanx d) secx

16. The line $\vec{r} = 2\hat{\imath} - 5\hat{k} + \tau(\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$ passing through the point

a) (3,3,3) b) (3,-3,3) c) (-3, 3, -3) d) (3,3,-3)

17. The derivative of $sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

a) $\frac{2}{\sqrt{1-x^4}}$ b) $\frac{-2}{\sqrt{1-x^4}}$ c) -1 d) none of these

18. The area of a triangle with two sides $\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $-4\hat{\jmath} + 2\hat{k}$ is

a) $\frac{\sqrt{21}}{2}$ b) $2\sqrt{21}$ c) $\sqrt{21}$ d) $\sqrt{42}$

ASSERTION- REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A): The function

f: N→ N $defined\ as\ f(x) = \begin{cases} \frac{n+1}{2}, & if\ n\ is\ odd \\ \frac{n}{2}, & if\ n\ is\ even \end{cases}$ $is\ a\ bijective\ function$ Reason (R) : A function is bijective if it is one one and onto

20. Assertion (A): The function $f(x)= 2x^3+3x^2-36x$ is increasing in the interval $(-\infty, -3)U(2, \infty)$

Reason (R) : First derivative of f(x) with respect to x is 0 at x=-3 and 2

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21. Find the domain of the function $\sec^{-1}(2x-3)$

OR

Find the principal value of $\text{cosec}^{-1}\left(\sec\frac{50\pi}{7}\right)$

22. A particle moves along the curve $6y=x^3+2$. Find the points on the curve, at which the y coordinate is changing 2 times as fast as the x coordinate.

23. If $y=\sqrt{sinx+\sqrt{sinx+\sqrt{sinx+\cdots+\infty}}}$, then prove that $\frac{dy}{dx}=\frac{cosx}{2y-1}$

OR

Show that the function $f(x)=\begin{cases}1+x, & if\ x\leq 2\\ 5-x, & if\ x>2\end{cases}$ is continuous but not differentiable at x=2.

24. Find $\int(\sin(logx)+\cos(logx))dx$

25. Evaluate $\int_1^3|2x-3|\,dx$

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Find the difference between the greatest and least values of the function f(x)=sin2x-x on the interval on the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

OR

If x=sint and y= sinpt, then prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0$

27. Find $\int sin^{-1}\left(\sqrt{\frac{x}{a+x}}\right)$ dx

28. An Urn contains 5 red and 2 black balls. Two balls are drawn at random without replacement. Let X denotes the number of black balls drawn. What are the possible values of X? Is X a random variable? If yes find the mean of X.

29. Solve the differential equation $e^x tanydx+(1-e^x)sec^2ydy=0$

OR

Find the particular solution of the differential equation $x^2dy+(xy+y^2)dx=0$, given that x=1,y=1.

30. Evaluate $\int_0^4\frac{dx}{\sqrt{x^2+2x+3}}$

OR

Find $\int\frac{2x^2-1}{x^2(x^2+2)}\,dx$

31. Solve the following Linear Programming Problem graphically :
Maximize z= 7x+6y subject to the constraints

$4x + 6y \leq 240,\ 6x + 3y \leq 240,\ x \geq 10,\ x \geq 0,\ y \geq 0$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each) (5x4=20)

32. Show that the function f: R→ R $defined\ by\ f(x) = \frac{x}{x^2+1}$, x∈ R is neither one one nor onto.

OR

Determine whether the relation defined on the set R of all real numbers as R={(a,b): a-b+$\sqrt{3}$ is an irrational number} is an equivalence relation.

33. A wire of length 25cm is to be cut into two pieces. One of the wire is to be made into a square and the other into a circle. What should be the lengths of the two pieces, so that the combine area of the square and circle is minimum?

34. Find the coordinates of the foot of the perpendicular and length of the perpendicular drawn from the point P(5,4,2) to the line $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + \hat{k} + \tau(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$

OR

Show that the following lines are intersecting lines and the point of intersection. $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{2-x}{-1} = \frac{y-4}{3} = \frac{z-6}{5}$

35. Find the product of two matrices A and B where A=$\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$,
B=$\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}$ and use the product to solve the following equations
x-y+z=4, x-2y-2z=9, 2x+y+3z=1

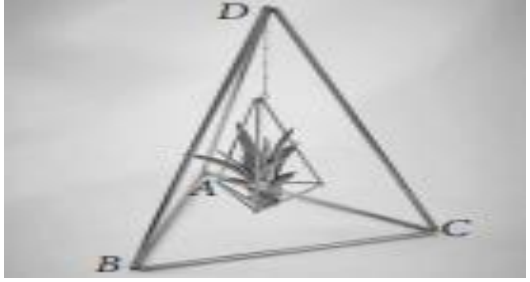
SECTION E

This section comprises of 3 case-study/passage-based questions of First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts (i) and (ii) of 2 marks each.

CASE-BASED/DATA-BASED

36. Read the following passage and answer the questions given below

Ginni purchased an air plant holder which is in the shape of a tetrahedron.
Let A, B, C and D are the coordinates of the air plant holder where A ≡ (1, 1, 1), B ≡ (2, 1, 3), C ≡ (3, 2, 2) and D ≡ (3, 3, 4).

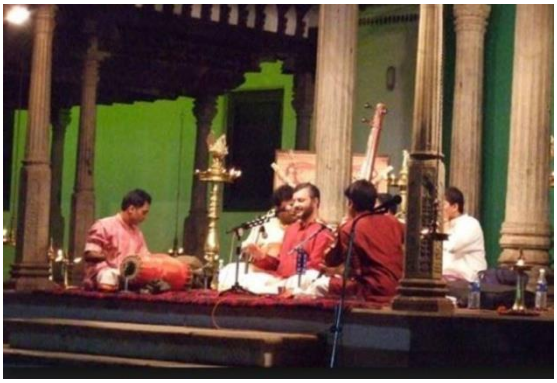


i) Find a unit the vector along the vector $\overrightarrow{AD}$ (1 mark)

ii) Find the projection of $\overrightarrow{CD}$ on $\overrightarrow{AC}$ (1 mark)

iii) Find the area of ΔDBC. (2 marks)

37. One day, a sangeet Mahotsav is to be organized in an open area of Rajasthan. In recent years, it has rained only 6 days each year. Also, it is given that when it actually rains, the weatherman correctly forecasts rain 80% of the time. When it doesn't rain, he incorrectly forecasts rain 20% of the time.
If leap year is considered, then answer the following questions.



.

i. Find the probability that it rains on chosen day (1 mark)

ii. Find the probability that the weatherman predicts correctly (1 mark)

iii. Find the probability that it will not rain on the chosen day, if weatherman predict rain for that day, (2 marks)

38. A student designs an open honeybee nest on the branch of a tree, whose plane face is parabolic shape with eqation $x^2=4y$ the branch of tree is given by a straight line y=4.



Now answer the questions given below

(i) Draw the figure and find the point of intersection of parabola $x^2=4y$ and straight line Y=4

(2 marks)

(ii) Find the area of the region bounded by the parabola and straight line (2 marks)

KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION
SAMPLE QUESTION PAPER-16

Class:-XII

Session 2023-24

Mathematics (Code-041)

Time: 3 hours **Maximum marks: 80**

General Instructions:

1. ***This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.***
2. ***Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.***
3. ***Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.***
4. ***Section C has 6 Short Answer (SA)-type questions of 3 marks each.***
5. ***Section D has 4 Long Answer (LA)-type questions of 5 marks each.***
6. ***Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.***

Section –A

(Multiple Choice Questions)

Each question carries 1 mark

1 If $[x+y+z \quad x+z \quad y+z] = [9 \quad 5 \quad 7]$ then value of x - y + z is _____

a) 1 (b) 2 (c) 3 (d) 4

2

For what value of x: $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix} = O?$

a) 1 (b) 2 (c) -1 (d) -2

3 Find the value of k if area of triangle is 4 sq. units and vertices are (k ,0) (4, 0) (0,2) is

(a) 0,8 (b) 2,7 (c) 0,7 (d)1,8

4 Find the value of k so that the function is continuous where $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & if \quad x \neq \frac{\pi}{2} \\ 3, & if \quad x = \frac{\pi}{2} \end{cases}$ $at \ x = \frac{\pi}{2}$

a) 8 (b) 7 (c) 6 (d) 3

5 If P(2,3,-6) and Q (3,-4,5) are two points, the direction cosines of the line PQ are

(a) $-\frac{1}{\sqrt{171}}, -\frac{7}{\sqrt{171}}, -\frac{11}{\sqrt{171}}$ **(b)** $\frac{1}{\sqrt{171}}, -\frac{7}{\sqrt{171}}, \frac{11}{\sqrt{171}}$

(c) $\frac{1}{\sqrt{171}}, \frac{7}{\sqrt{171}}, \frac{11}{\sqrt{171}}$ **(d)** $-\frac{7}{\sqrt{171}}, -\frac{1}{\sqrt{171}}, \frac{11}{\sqrt{171}}$

6 The solutions of the differential equation $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$ is:

(a) $y(1+x^2) = c + x$ (b) $\frac{-y}{1+x^2} = c + x$

(c) $y\, log\, log\,(1+x^2) = c + x$ (d) $y(1+x^2) = C + x$

7 The maximum value of Z = 3x +4y subjected to constraints x +y ≤ 40, x + 2y ≤ 60; x ≥ 0 and y ≥ 0 is

(a)120 (b)140 (c)100 (d)160

8 The unit vector in the direction of the sum of the vectors, $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ is

(a) $\frac{4}{\sqrt{29}}\hat{\imath} + \frac{5}{\sqrt{29}}\hat{\jmath} - \frac{2}{\sqrt{29}}\hat{\jmath}$ (b) $\frac{4}{\sqrt{29}}\hat{\imath} + \frac{3}{\sqrt{29}}\hat{\jmath} - \frac{2}{\sqrt{29}}\hat{\jmath}$

(c) $\frac{5}{\sqrt{29}}\hat{\imath} + \frac{3}{\sqrt{29}}\hat{\jmath} - \frac{2}{\sqrt{29}}\hat{\jmath}$ (d) $\frac{4}{\sqrt{29}}\hat{\imath} + \frac{3}{\sqrt{29}}\hat{\jmath} + \frac{2}{\sqrt{29}}\hat{\jmath}$

9 What is the value of $\int_0^{\pi/2} \frac{\sqrt{tanx}}{\sqrt{tanx}+\sqrt{cotx}}\, dx$?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{8}$ (d) None of these

10 Given that A is a square matrix of order 3 and |A| = -4, then |adj A| is equal to

(a) -4 (b) 4 (c) -16 (d) 16

11 Feasible region in the set of points which satisfy

a) the objective functions

(b) some the given constraints

(c) all of the given constraints

(d) none of these

12 if $\vec{a}$ and $\vec{b}$ are unit vectors , then what is the angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\vec{a} - \vec{b}$ to be a unit vector

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

13 Find the value of x for which the matrix A=$\begin{pmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{pmatrix}$ is singular

(a) 0, 1 (b) 1,3 (c) 0,3 (d)3,2

14 Let A and B be two given events such that P (A) = 0.6, P (B) =0.2 and P (A/ B) = 0.5. Then P(A' /B') is

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{8}$ (d) $\frac{16}{7}$

15 Find the general solution of $\frac{dy}{dx} = \sqrt{4 - y^2}$ $(-2 < y < 2)$

(a)$y = x + c$ (b) $y/2 = x + c$

(c) $y^2 = x + c$ (d) none of these

16

If $\vec{a} = 2\hat{i} - 3\hat{j} - \hat{k}, \vec{b} = -\hat{i} + \hat{k}, \vec{c} = 2\hat{j} - \hat{k}$, then the area of parallelogram, whose diagonals are $(\vec{a} + \vec{b})$ and $(\vec{b} + \vec{c})$ is _____

(a) 1 sq. unit (b) 2 sq. unit (c) ½ sq. unit (d) ¼ sq. unit

17 The function $f(x) = [x(x-3)]^2$ is increasing in :

(a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) (1, 3) (d) $[0, 1.5] \cup (3, \infty)$

18 If a line has direction ratios 2, – 1, – 2, determine its direction cosines:

(a) ⅓, ⅔, -⅓ (b) ⅔, -⅓, -⅔

(c) -⅔, ⅓, ⅔ (d) None of the above

ASSERTION-REASON BASED QUESTION

Read Assertion and reason carefully and write correct option for each question

(a) Both A and R are correct; R is the correct explanation of A.

(b) Both A and R are correct; R is not the correct explanation of A.

(c) A is correct; R is incorrect.

(d) R is correct; A is incorrect.

19. **Assertion (A):** x = 0 is the point of local maxima of the function f given by

$$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$$

Reason(R): $f'(x) = 0$ *at* $x = 0$ *and also* $f''(x) < 0$ *at* $x = 0$

20. **Assertion (A)** The relation R in the set A of all the books in a library of a college, given by R= {(x, y): x and y have same number of pages}is not equivalence relation.

Reason (R) Since R is reflexive, symmetric and transitive

Section –B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. Find the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

OR

Find the value of $tan^2 \sec^{-1} 2 + cot^2 \operatorname{cosec}^{-1} 3$

22. Find the interval in which f(x) = $-2x^3 - 9x^2 - 12x + 1$ is strictly increasing or strictly decreasing.

23. Find the absolute maximum and absolute minimum of

f(x) = $\sin x + \cos x, x \in [0, \pi]$

OR

At what points in the interval $[0, 2\pi]$ does the function $\sin 2x$ attains its maximum value?

24. Evaluate $\int_0^{\pi} sin^2 x/2 - cos^2 x/2$ dx.

25. A particle moves along the curve 6y=$x^3 + 2$ .Find the points on the curve at which the y coordinate is changing 8 times as fast as the x coordinate.

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Evaluate $\int \frac{1}{x(x^4-1)}\ dx$.

27. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

28. Evaluate $\int_{-5}^{5} |x + 2| dx$

OR

Evaluate $\int \sqrt{1 - 4x - x^2}$ dx

29. Solve the differential equation x dy – y dx=$\sqrt{x^2 + y^2}$ dx

OR

Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$ given that y=0 when x=1.

30. Minimise Z=10x + 4y subject to constrains 4x+y≥ 80, 2x+y≥ $60, x, y \geq 0$.

OR

Maximise Z= 10500x+9000y subject to the constraints x + y≤ $50, 2x + y \leq 80, x, y \geq \mathbf{0}$.

31. Find the derivative of $\tan^{-1} \frac{\sqrt{1-x^2}}{x}$ $w\ r\ to\ \cos^{-1} 2x\sqrt{1 - x^2}$

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. Find the area of the region {(x, y): $x^2 + y^2 \leq 4$, $x + y \geq 2$}

33. Show that the relation R on the set Z of integers, given by R= {(a, b):2 divides a−b} is an equivalence relation.

OR

A function f: [-4, 4] → [0, 4] is given by f(x) =$\sqrt{\mathbf{16 - x^2}}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which f (a) =√7.

34. If A=$\begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$ Find A^{-1} and hence solve $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=2, \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=3$

$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}$ = -4.

35. Find the coordinate of the foot of the perpendicular and the length of the perpendicular drawn from the point P(5,4,2) to the line $\vec{r}=-\hat{\imath}+3\hat{\jmath}+\hat{k}+\lambda(2\hat{\imath}+3\hat{\jmath}-\hat{k})$. Also find the image of the point P in this line.

OR

Find the vector and Cartesian equations of the line which is perpendicular to the lines with equations

$$\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4} \quad \text{and} \quad \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

and passes through the point (1, 1, 1). Also find the angle between the given lines

Section –E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts]

The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively.

The third case study question has two sub parts of 2 marks each.)

36. Solar Panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels. A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture, suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters P (6, 8, 4), Q (21, 8, 4), R (21, 16, 10) and S (6, 16, 10)



1. What are the components to the two edge vectors defined by $\overrightarrow{PQ}$ and $\overrightarrow{PS}$?
2. Find the magnitudes of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PS}$.
3. Find the unit vector along $\overrightarrow{PQ}$

OR

Find a vector perpendicular to both $\overrightarrow{PQ}$ and $\overrightarrow{PS}$.

37.



A shopkeeper sells three types of flower seeds A1,A2 and A3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%. Calculate the probability

(i) of a randomly chosen seed to germinate

(ii) that it will not germinate given that the seed is of type A3.

(iii) that it is of the type A2 given that a randomly chosen seed does not germinate.

38. Sand is pouring from a pipe at the rate of 12 cm^3/ second the falling sand forms a cone on the ground in such a way that the height of the cone is always $1/6^{th}$ of the radius of the base. Based on above information answer the following:

(i) Write the expression for volume in terms of height only.

(ii) What is the rate of Change of height, when height is 4 cm?

Prepared by

Mr Santhosh B

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KENDRIYA VIDYALAYA SANGATHAN , ERNAKULAM REGION

SAMPLE QUESTION PAPER-17

Class:-XII

Session 2023-24

Mathematics (Code-041)

Time: 3 hours **Maximummarks:80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. **Section A** has 18 MCQ's and 02Assertion-Reason based questions of 1 mark each.

3. **Section B** has 5 Very Short Answer (VSA)-typequestions of 2 marks each.

4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.

5. **Section D** has 4 Long Answer (LA)-typequestions of 5 marks each.

6. **Section E** has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section –A

(Multiple Choice Questions)

Each question carries 1 mark

1. If A is a square matrix such that A^2 = A then $(I + A)^3$ -7A is equal to

(a) A (b) I-A (c) I (d) 3A

2. If A and B are square matrices of order 3 such that |A| = -1, |B| = 3 then find the value of |3AB|?

(a) -81 (b) -27 (c) 27 (d) None of these

3. If the points (2, -3), (d, -1) and (0, 4) are collinear find the value of d?

(a) $\frac{7}{10}$ (b) $\frac{10}{7}$ (c) 1 (d) $\frac{4}{7}$

4. Determine the value of k so that the function f (x)=$\begin{cases} \frac{\sin 2x}{5x} & if\ x \neq 0 \\ k & if\ x = 0 \end{cases}$ is continuous at x = 0 :

(a) 1 (b) $\frac{2}{5}$ (c) $\frac{5}{2}$ (d) $\frac{-2}{5}$

5. Find the foot of the perpendicular drawn from the point (2, -3, 4) on the y-axis?

(a) (2, 0, 4) (b) (0, 3, 0) (c) (0, -3, 0) (d) (-2, 0, -4)

6. Integrating factor of the differential equation $\frac{dy}{dx}$ + y tan x – sec x = 0 is

(a) cos x (b) sec x (c) $e^{\cos x}$ (d) $e^{\sec x}$

7. The maximum value of z = 4x + 2y subject to the constraints 2x + 3y ≤ 18, x + y ≥ 10, x, y ≥ 0 is

(a) 36 (b) 40 (c) 20 (d) None of these

8. The value of $\hat{\imath}$. ($\hat{\jmath}$. $\hat{k}$) + $\hat{\jmath}$. ($\hat{\imath}$. $\hat{k}$) + $\hat{k}$. ($\hat{\imath}$. $\hat{\jmath}$) is

(a) 0 (b) -1 (c) 1 (d) 3

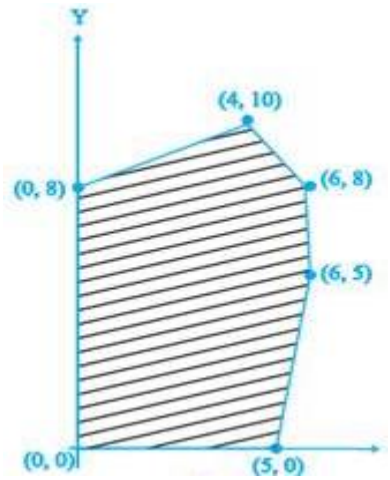
9. If $\int_0^a \frac{1}{1+4x^2}$ dx = $\frac{\pi}{8}$ then' a' equals

(a) $\frac{\pi}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\pi}{4}$ (d) 1

10. If A is a matrix of order 3 and |A| = 8 then |adj A| is

(a)1 (b) 2 (c) 2^3 (d) 2^6

11. The feasible solution for LPP is shown in given figure. Let z = 3x – 4y be the objective function minimum of z occurs at



(a) (0, 0) (b) (0, 8) (c) (5, 0) (d) (4, 10)

12. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}$ + $\vec{b}$ + $\vec{c}$ = 0. Then the value of $\vec{a}$. $\vec{b}$ + $\vec{b}$. $\vec{c}$ + $\vec{c}$. $\vec{a}$ is

(a) 1 (b) 3 (c) $\frac{-3}{2}$ (d) None of these

13. Given that A is a square matrix of order 3 and |A| = -2, then |adj(2A)| is equal to

(a) -2^6 (b) 4 (c) -2^8 (d) 2^8

14. The probability distribution of a random variable X is given below:

X	2	3	4	5
P(X)	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

Then the value of k is

(a) 8 (b) 16 (c) 32 (d) 48

15. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

(a) $e^x + e^{-y} = C$ (b) $e^x + e^y = C$

(c) $e^{-x} + e^y = C$ (d) $e^{-x} + e^{-y} = C$

16. The value of d for which two vectors $2\,\hat{\imath} - \hat{\jmath} + 2\,\hat{k}$ and $3\,\hat{\imath} + d\,\hat{\jmath} + \hat{k}$ are perpendicular is

(a) 2 (b) 4 (c) 6 (d) 8

17. Derivative of x^2 with respect to x^3 is

(a) $\frac{1}{x}$ (b) $\frac{2}{3x}$ (c) $\frac{2}{3}$ (d) $\frac{3x}{2}$

18. If the lines $\frac{x-2}{3} = \frac{y-1}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are perpendicular. Find the value of k

(a) $\frac{-2}{5}$ (b) $\frac{-2}{7}$ (c) 4 (d) $\frac{2}{7}$

Question number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. The principal value of cot^{-1} ($\sqrt{3}$) is $\frac{\pi}{6}$.

Reason (R) : Domain of cot^{-1}x is R – {-1, 1}.

20. Assertion: Let R be the relation on the set of integers z given by R = {(a,b): 2 divide a-b} is an equivalence relation.

Reason R: A relation R in a set A is said to be an equivalence relation of R is reflexive, symmetric and transitive.

Section –B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. Find the value of sin [$\frac{\pi}{3}$ - $sin^{-1}(\frac{-1}{2})$]

OR

Find the principal value of $cos^{-1}(\cos \frac{2\pi}{3})$ + $sin^{-1}(\sin \frac{2\pi}{3})$

22. Show that the function f(x) = tan x – x is always increasing.

23. Find the points of local maxima or local minima, if any. Also find the local maximum or local minimum values of the function as the case may be

f (x) = sin x + cos x, where$0 < x < \frac{\pi}{2}$

OR

Find the maximum profit that a company can make, if the profit function is given

P(x) = 41 + 24x - 18x^2

24. Find: $\int \frac{dx}{\sqrt{3-2x-x^2}}$dx

25. Check whether the function f (x) =x^5 + x, has any critical points or not? If yes, then find the points.

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Find : $\int \frac{2x}{(x^2+1)(x^2+2\ )}$dx

27. Find : : $\int \frac{(2x-5)e^{2x}}{(2x-3)^3}$ dx

OR

Evaluate $\int_0^{2\pi} \frac{1}{1+e^{sinx}}$ dx

28. Find the particular solution of the differential equation x$\frac{dy}{dx}$ = y- x tan $\frac{y}{x}$, given that y=$\frac{\pi}{4}$ at x=1

OR

Find the general solution of the differential equation $\log\left(\frac{dy}{dx}\right) = ax + by$

29.If x = sint , y= sin pt, then prove that $(1- x^2)\frac{d^2y}{dx^2}$ -x $\frac{dy}{dx}$ $+p^2$ y =0

30. There are two coins. One of them is a biased coin such that P(head) : P(tail) is 1:3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

31. Solve the following Linear Programming problem graphically.

Minimise : Z = 5x + 10 y subject to the constraints
x+2y≤ 120, x+y≥ 60 , x – 2y≥ 0, x≥ 0, y≥ 0

OR

Maximise : Z = 5x + 8 y subject to the constraints

x+y = 5, x≤4, y ≥2, x , y ≥0

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

32.Sketch the region bounded by the line 2x + y =8 , y=2, y=4 and the y-axis. Hence obtain the area using integration.

33 . Let A = {1,2,3,…9} and R be the relation on AxA defined as (a ,b) R (c,d), if and only if a +d = b + c. Prove that R is an equivalence relation. Also obtain the equivalence class[(2,5)].

OR

Consider f: **R**→[-9 ,∞) given by f(x) = $5x^2$ + 6x -9. Prove that f is bijective.

34.If A = $\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, B= $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of linear equations: x- 2y =3

2x – y- z =2

-2y + z =3

35.Find the value of b so that the lines : $\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4}$

$\frac{x-4}{5} = \frac{y-1}{2} = z$ are intersecting lines.Also find the point of intersection of these given lines.

OR

Find the vector and the cartesian equations of the line passing through the point (1,2,-4)and Parallel to the line joining the points A(3,3,-5) and B (1 ,0,-11). Hence, find the distance between the two lines.

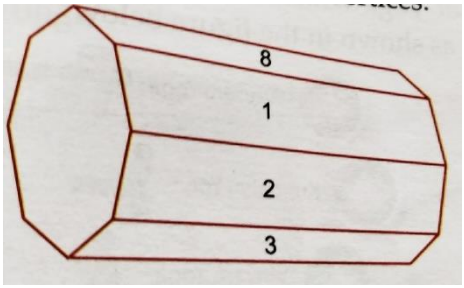
Section –E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts.

The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively.

The third case study question has two sub parts of 2 marks each.)

36. An octagonal prism is three- dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number obtained on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X

X	1	2	3	4	5	6	7	8
P(X)	P	2p	2p	p	2p	p^2	$2p^2$	$7p^2$+p

(i) Find the value of p

(ii) FindP(X>6)

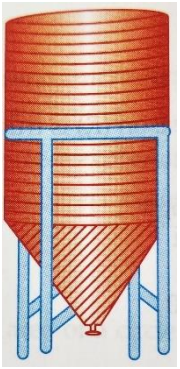
(iii) (a)Find P(X=3m) ,where m is a natural number.

OR

(iii)(b)Find the mean E(X)

37.**Read the following passage and answer the following questions.**

A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.



A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s. The semi-vertical angle of the conical tank is 45^0.

(i) Find the volume of water in the tank in terms of its radius r.

(ii) Find the rate of change of radius at an instant when r= $2\sqrt{2}$ cm.

(iii)(a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when r= $2\sqrt{2}$ cm.

OR

(iii)(b) Find the rate of change of height 'h' at an instant when slant height is 4 cm.

38.Read the following passage and answer the following questions.

Solar panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electric power in the solar panels.

A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted.Suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of metres P_1(6,8,4), P_2(21,8,4), P_3(21,16,10) and P_4(6,16,10) .

(i) Find the components to the two edge vectors defined by $\vec{A}$=PV of P_2- PV of P_1 and $\vec{B}$= PV of P_4- PV of P_1 where PV stands for position vector.

(ii)(a)Find the magnitudes of the vectors $\vec{A}$ and $\vec{B}$

(b)Find the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof.

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SAMPLE QUESTION PAPER-18

Class:-XII

Session 2023-24

Mathematics (Code-041)

Time : 3 hours Maximum Marks : 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of I mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with

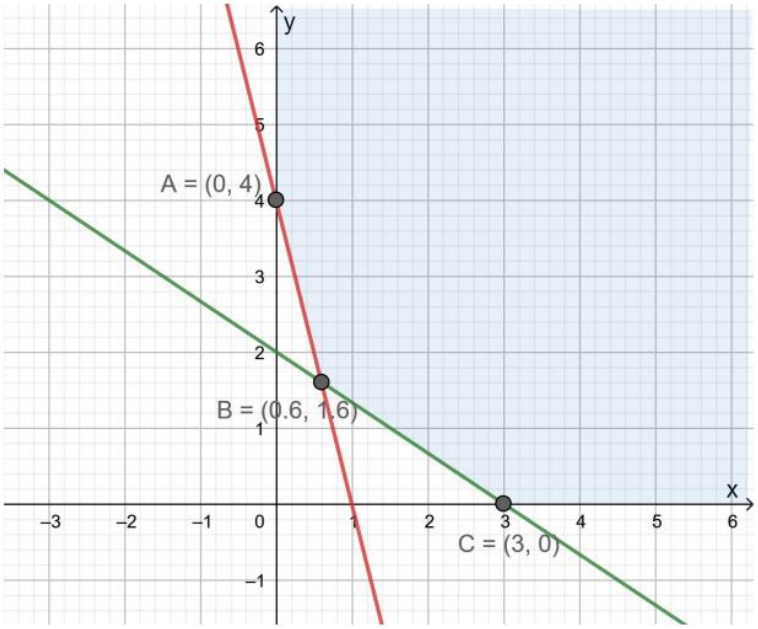
SECTION-A

(Multiple Choice Questions)

Each question carries 1 mark

Q1.	If A is a square matrix such that $A^2 = (I + A)^2 - 3A$ is (a) I (b) $2A$ (c) $3I$ (d) A
Q2.	If $A = \begin{bmatrix} a & 2 \\ 2 & a \end{bmatrix}$ and $(\|A\|)^3 = 125$. Then a is (a) ± 3 (b) 5 (c) ± 2 (d) 4
Q3.	If area of a triangle with vertices (3,2), (-1,4) and (6,k) is 7 sq units, then possible values of k is (a) 3 (b) -4 (c) -3,4 (d) 3,-4
Q4.	If the function $f(x) = \begin{cases} kx^2, if\ x \geq 1 \\ 4, if\ x < 1 \end{cases}$ is continuous at $x = 1$, then the value of k is (a) -4 (b) 4 (c) 0 (d)any real number
Q5.	If the direction cosine of a given line are $\frac{1}{k}, \frac{1}{k}, \frac{1}{k}$, then the value of k is (a) $\frac{1}{\sqrt{2}}$ (b) $\pm\frac{1}{\sqrt{3}}$ (c) 1 (d) $\pm\sqrt{3}$
Q6.	If p and q are the degree and order of the differential equation :

	$(\frac{d^2y}{dx^2})^2 + 3\frac{dy}{dx} + \frac{d^3y}{dx^3} = 4$, then the value of $2p - 3q$ is: (a) 7 (b) -7 (c) 3 (d) -3
Q7.	The objective function for a given linear programming problem is $Z = ax + by - 5$. If z attains the same value at $(1,2)$ and $(3,1)$ then : (a) $a + 2b = 0$ (b) $a + b = 0$ (c) $a = b$ (d)$2a - b = 0$
Q8	The value of $\int_0^2 \sqrt{4 - x^2}\ dx$ is : (a) $\frac{\pi}{2}$ (b) π (c) $-\pi$ (d) None of these
Q9	The matrix $\begin{bmatrix} 4+3k & 3 \\ 1+2k & 2 \end{bmatrix}$ is singular matrix for k equal to (a) 0 (b) -1 (c) 1 (d) No value of k
Q10	ABCD is a rhombus whose diagonals intersect at E. Then $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED}$ equals : (a) $\vec{0}$ (b) $\overrightarrow{AD}$ (c) 2 $\overrightarrow{BC}$ (d) 2 $\overrightarrow{AD}$
Q11.	For what value of p is $\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right) P$ a unit vector : (a) $\pm\frac{1}{\sqrt{3}}$ (b) $\pm\sqrt{3}$ (c) ± 1 (d) $\pm\frac{1}{3}$
Q12	If A is a square matrix of order 3×3 such that $A(adj\ A) = 10I$ then $\lvert adj\ A \rvert$ is
Q13	Three people A, B and C fire a target in turn. Their probabilities of hitting the target are 0.2, 0.3 and 0.5 respectively. The probability that target is hit is : (a) 0.993 (b) 0.94 (c) 0.72 (d) 0.90
Q14	The integrating factor for the differential equation $\frac{dy}{dx} + y\tan x - \sec x = 0$ is : (a) $\tan x$ (b) $\sec^2 x$ (c) $\sec x$ (d) $\tan^2 x$
Q15	The general point on the line $\vec{r} = \left(2\hat{\imath} + \hat{\jmath} - 4\hat{k}\right) + \lambda(3\hat{\imath} + 2\hat{\jmath} - \hat{k})$ is : (a) (2,1,4) (b) 3,2,1 (c) (-1,1,3) (d) $(2 + 3\lambda, 1 + 2\lambda, -4 - \lambda)$
Q16	Derivative of $\sqrt{e^{\sqrt{x}}}$ is (a) $\sqrt{e^{\sqrt{x}}}$ (b) $e^{\sqrt{x}}$ (c) $\frac{1}{2\sqrt{e^{\sqrt{x}}}}$ (d) $\frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}}$
Q17	The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is : (a) 0° (b) 30° (c) 45° (d) 90°
Q18	The corner points of the shaded unbounded feasible region of an LPP are (0, 4), (0.6, 1.6) and (3, 0) as shown in the figure. The minimum value of the objective

	function Z = 4x + 6y occurs at  (a)(0.6, 1.6) only (b) (3, 0) only (c) (0.6, 1.6) and (3, 0) only (d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)
	Assertion Reasoning Based Questions
	In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices: (a) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A). (b) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A). (c) Assertion (A) is true and Reason (R) is false. (d) Assertion (A) is false and Reason (R) is true.
Q19	Assertion (A) :The function $y = [x(x-2)]^2$ is increasing in $(0,1) \cup (2,\infty)$ Reason (R) : $\frac{dy}{dx} = 0$, When $x = 0,1,2$
Q20	Assertion (A) :A function $f(x) = \cos x, x \in [0, \frac{3\pi}{2}]$ is bijective Reason (R) : For one-one function, each element in domain has unique image in the co-domain

	SECTION B
	This section comprises of very short answer type questions (VSA) of 2 marks each.
Q21	If $\cos^{-1} x + \cos^{-1} y = 2\pi$ then find the value of $x^{100} + y^{200} + \frac{1}{x^{100} \cdot y^{100}}$ **OR** Find the range of principal value branch of the function $f(x) = 3\cos^{-1}(\frac{1}{2x-1}) - 2$
Q22	Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R
Q23	Find the local maxima and local minimum, if any of the function f given by $f(x) = \sin x + \cos x, 0 < x < \frac{\pi}{2}$ **OR** Without using derivatives, find the maximum and minimum values if any of function given by $f(x) = \sin 2x + 5$
Q24	Evaluate $\int_{-1}^{1} x \lvert x \rvert dx$
Q25	A particle moves along the curve $x^2 = 2y$. At what point the ordinate increases at the same rate as abscissa.
	SECTION C
	This section comprises of short answer type questions (SA) of 3 marks each
Q26	Find $\int \frac{(x^2+1)e^x}{(x+1)^2} dx$
Q27	The probability distribution of the discrete variable X is given as X : 2 3 4 5 P(X): $\frac{5}{K}$ $\frac{7}{K}$ $\frac{9}{K}$ $\frac{11}{K}$ Where K is a real number and K ≠ 0. Then i) Determine the value of K ii) Find P (X>4) iii) Find P (X≤ 3)
Q28	Evaluate $\int \frac{x^2+4}{x^4+16} dx$ **OR** Evaluate $\int_0^1 x(1-x)^n$ dx
Q29	Solve the differential equation $x(1+y^2)dx - y(1+x^2)dy = 0$, given that y=1 when x =0 **OR** Solve $(1+y^2)dx = (tan^{-1}y - x)dy$
Q30	Solve the following Linear programming problem graphically Maximise Z = 50x+15y subject to the constraints 5x+y ≤ 100,x+y ≤ 60,x.y,≥0 Solve the following Linear programming problem graphically,

	Minimize Z = x+2y, Subject to the constraints x+2y ≥ 100,2x – y ≤0,2x+y ≤ 200 x,y ≥0
Q31	If e^y(x+1) =1,Show that $\frac{d^2y}{dx^2} = (\frac{dy}{dx})^2$
	SECTION D
	This section comprises of long answer type questions (LA) of 5 Marks each.
Q32	Find the area of the region included between the parabola y = $\frac{3}{4}$ x^2 and the line 3x-2y +12 =0
Q33	Let N be the set of all natural numbers and R be a relation on NxN defined by (a,b) R (c,d) iff ad(b+c) = bc(a+d) .Then Show that R is an equivalence relation **OR** Let f : N → N be defined by f(n) = { $\frac{n+1}{2}$,if n is odd , $\frac{n}{2}$ if n is even for all n € N . State whether f is bijective or not
Q34	Solve using matrix method the system of equations 2x+y -3z =13 3x+2y +z =4 x+2y – z =8
Q35	Show that the lines $\vec{r}$ = (i^ +j^ – k^) + λ(3i – j^) and $\vec{r}$ = (4i^ - k^) + μ (2i^ +3k^) intersect . Also find their point of intersection OR Find the length and foot of the perpendicular drawn from the point (2, -1 , 5) on the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$
	SECTION E
	This section comprises 3 Case study / passage based questions of 4 marks each with subparts. The first two case study Questions have three subparts (i),(ii), (iii) of marks 1,1,2 respectively. The third case study Question has two subparts of 2 marks each.
Q36	A building contractor undertakes a job to construct 4 flats on a plot along with parking area. The probability of all construction workers not present for the job is 0.65. If they are not present and still works get completed is 0.35.The probability that work will be completed when all workers are present is 0.80.Based on the above information answer the following :if E_1 represent the event when all workers were not present for the job and E_2 represents event when workers were present . E represents completing the construction work at time. i)What is the probability that workers are present for the job? a)0.65 b) 1 c) 0.35 d) 0.80 ii)What is the probability that construction will be completed on time? a)0.51 b)0.60 c) 0.39 d) 0.45 iii) What is the probability that workers are not present given that construction work is completed on time? a)0.54 b)0.40 c) 0.49 d) 0.45

	OR What is the probability that workers were present given that construction job was completed on time ? a) 0.55 b) 0.50 c) 0.45 d) 0.73
Q37	A packaging company got orders to make open boxes of maximum volume from rectangular sheets of dimensions 45cm x 24cm.The execution department of company suggested to cut squares of equal side from all corners of rectangular sheet and folding up the flaps .If square of side x cm is cut from each corner ,then answer the following : i) Height of the box is a)(45- x) cm b) (24- x) cm c) 2x cm d)x cm ii)Volume of the box is a) $x(45-2x)(24-2x)\ cm^3$ b) $x(45-x)(24-x)\ cm^3$ c) $(45-2x)(24-2x)\ cm^3$ d) $(45-x)(24-x)\ cm^3$ iii)The side of the square for which volume is maximum is a)7cm b)5cm c)18 cm d)3cm **OR** Maximum volume of the box is a) $2450cm^3$ b) $1080\ cm^3$ c) $800\ cm^3$ d) $2400\ cm^3$
Q38	A cycle racer in the morning is cycling in a free cycling track and sometimes moving zig – zag .It is found that the path traced by the cyclist is given by the curve $f(x) =(x-1)\ (x-2)^2$. If at any part of time cyclist is at point P(x,y).Based on the above information and tracking path to be a mathematical statement ,answer the following i)The intervals for which f(x) is increasing a) $(-\infty.\frac{4}{3})$ b) $(\frac{4}{3},2)$ c) $(2, \infty)$, d) $(-\infty,\frac{4}{3}) \cup (2, \infty)$ ii) The intervals for which f(x) is decreasing a) $(-\infty,\frac{4}{3})$ b) b) $(\frac{4}{3},2)$ c) c)$(2, \infty)$ d) d) $(-\infty,\frac{4}{3}) \cup (2, \infty)$

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SAMPLE PAPER 19

Class:-XII Session 2023-24

Mathematics (Code-041)

Time: 3 hours Maximum marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts

Section –A

(Multiple Choice Questions) Each question carries 1 mark

Q1. If a matrix A is both symmetric and skew-symmetric, then

(a) A is a diagonal matrix (b) A is a zero matrix
(c) A is a scalar matrix (d) A is a square matrix

Q2. If $\begin{bmatrix} x+y & 2x+z \\ x-y & 2z+w \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$ then the values of x, y, z and w respectively are

(a) 2, 2, 3, 4 (b) 2, 3, 1, 2 (c) 3, 3, 0, 1 (d) 2, 3, 1, 4

Q3. Given that A is a square matrix of order 3 and |A| = -4, then |adj A| is equal to

(a) -4 (b) 4 (c) -16 (d) 16

Q4. A function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x & , x \neq 0 \\ 2k & , x = 0 \end{cases}$ is continuous at x = 0 for

(a) k = 1 (b) k = 2 (c) k = -1 (d) k = -2

Q5. Find the projection of $(2\vec{i} - \vec{j} + \vec{k})$ on $(\vec{2i} - 2\vec{j} + \vec{k})$.

(a) $\frac{7}{3}$ (b) $\frac{7}{\sqrt{6}}$ (c) $\frac{7}{6}$ (d) $\frac{7}{5}$

Q6. The number of arbitrary constants in the particular solution of a differential equation of third order is:

(a) 3 (b)2 (c) 1 (d)0

Q7. The optimal value of the objective function is attained at the points:

(a)on X-axis (b)on Y-axis

(c)corner points of the feasible region (d)at the origin

Q8. ABCD is a rhombus whose diagonals intersect at E. Then $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED}$ equals

(a) 0 (b) $\overrightarrow{AD}$ (c) $2\overrightarrow{BD}$ (d) $2\overrightarrow{AD}$

Q9. Compute $\int_2^3 \frac{cosx - \sin x}{4}$ dx.

a) $\frac{1}{4}$(sin 2 + cos 3 – sin 3 – cos 2) b) $\frac{1}{4}$ (sin 3 – cos 3 – sin 2 – cos 2)

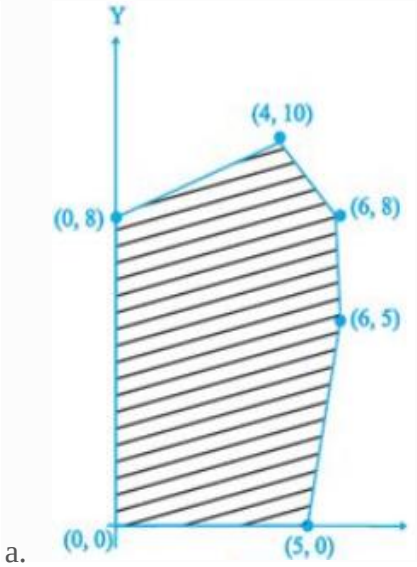
c) $\frac{1}{4}$ (sin 3 + cos 3 – sin 2 – cos 2) d)$\frac{1}{4}$ (sin 3 + cos 3 + sin 2 – cos 2)

Q10. If the points (2, -3), (k, -1) and (0, 4) are collinear, then find the value of 4k.

(a) 4 (b) 7/140 (c) 47 (d) 40/7

Q11. The feasible solution for an LPP is shown in Figure. Let Z = 3x – 4y be the objective function. Minimum of Z occurs at

a.



(a) (0, 8) (b) (0, 0) (c) (5 , 0) (d) (4, 10)

Q12. If C is the mid-point of $\overrightarrow{AB}$ and P is any point outside $\overrightarrow{AB}$, then

(a) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = 0$ (b) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = 0$ (c) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$ (d) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$

Q13. If A is invertible matrix of 3 X 3 and $|A| = 7$ then find $\left|A^{-1}\right|$

(a) 7 (b) 49 (c) 1/7 (d) 1/49

Q14. Find P(E|F), where E: no tail appears, F: no head appears, when two coins are

tossed in the air.

(a)0 (b). ½ (c)1 (d).75

Q15. Integrating factor of the differential equation $\frac{dy}{dx}$ + y tanx – sec x = 0 is

(a) cos x (b) sec x (c) $e^{\cos x}$ (d) $e^{\sec x}$

Q16. If the lines $\frac{x-1}{-3} = \frac{y-2}{-2k} = \frac{z-3}{2}$ *and* $\frac{x-1}{k} = \frac{y-2}{1} = \frac{z-3}{5}$ are perpendicular, then find k**.**

(a) 2 (b) 4 (c) 6 (d) 8

Q17. The set of points where f(x)=1+|x| x is differentiable, is

(a) $(-\infty,0) \cup (0,\infty)$ (b) $(-\infty,-1) \cup (-1,\infty)$ (c) $(-\infty, \infty)$ (d) $(0, \infty)$

Q18. The angle between the lines passing through the points (4, 7, 8), (2, 3, 4) and (-1, -2, 1),

(1, 2, 5) is

(a) 0 (b) π/2 (c) π/4 (d) π/6

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Q19. Assertion: The minimum value of the expression $x^2 + 2ax + b$ is $b-a^2$

Reason: The first order derivative of the expression at x = −a is equal to zero.
The minimum value of the expression $x^2 + 2ax + b$ is $b-a^2$

Q20. Assertion (A) : The relation R on the set N×N, defined by (a, b) R (c, d) ⇔ a+d = b+c for all (a, b), (c, d) ∈ N×N is an equivalence relation.
Reason (R) Any relation R is an equivalence relation, if it is reflexive, symmetric and transitive.

Section –B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

Q21. Find the value of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right)$

OR

Write the value of $\cos^{-1}(\cos 7\pi/6)$.

Q22. Show that $f(x) = e^{\frac{1}{x}}$, $x \geq 0$ is a decreasing function for all $x \geq 0$.

Q23. Find the maximum value and minimum value of $f(x)=\sin x + \cos x$, $x \in [0,\pi]$

OR

Calculate the adjacent sides of a rectangle with a given perimeter of 100 cm and enclose the maximum area.

Q24. Determine the antiderivative F of "f" , which is defined by $f(x) = 4x^3 - 6$, where $F(0) = 3$.

Q25. Let $f(x)=\sin(3x)$, for $0 \leq x \leq \pi$, Where does f(x) have critical points?

If yes, then find the point/s.

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

Q26. Evaluate $\int \tan^8 x \sec^4 x \, dx$

Q27. Find the probability distribution of the number of kings drawn when two cards are drawn one by one without replacement from a pack of 52 playing cards.

Q28.Evaluate $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}} dx$

OR

$$\int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx$$

Q29. Solve the differential equation:: $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$, y=0 when x=1.

OR

Solve the differential equation: $ye^y \, dx = (y^3 + 2xe^y) \, dy$.

Q30. Solve the following Linear Programming Problem graphically:

Maximise $Z = 5x + 3y$ subject to $3x+5y \leq 15,\ 5x+2y \leq 10,\ x \geq 0,\ y \geq 0$

OR

Solve the following Linear Programming Problem graphically:

Minimise $Z = 2x + 3y$ subject to contraints : $x \geq 0$, $y \geq 0$, $1 \leq x+2y \leq 10$

Q31. If $\sin y = x \sin(a + y)$, prove that $\frac{dy}{dx} = \frac{sin^2(a+y)}{\sin a}$

Section –D

(This section comprises of long answer type of questions (LA) of 5 marks each)

Q32. Using integration find the area of the region bounded by the lines y = 3x+2 , x = -1, x = 1 and . the X-axis

OR

Using integration, find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Q33. Let N be the set of all natural numbers and let R be a relation on NXN defined by

(a,b)R(c,d) <=> ad(b+c)= bc(a+d) for all (a,b),(c,d) $\in NXN$. Prove that R is an equivalence relation.

Q34. Find the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$.Using the product solve the system of equations

x-y+2z=1, 2y-3z =1 , 3x-2y+4z = 2.

OR

If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}.Using A^{-1}solve the system of equations 2x-3y+5z = 11, 3x+2y -4z=-5,

x+y-2z = -3.

Q35. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

Section –E

(This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

Q36. In answering a question on a multiple choice test for class XII, a student either knows the answer or guesses. Let 3/5 be the probability that he knows the answer and 2/5 be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability 1/3. Let E1, E2, E be the events that the student knows the answer, guesses the answer and answers correctly respectively.



(i) What is the value of P(E1)?

(ii) What is the value of P(E | E1) ?

(iii) Find

$$\sum_{k=1}^{k=2} P(E|E_k)\, P(E_k)$$

OR

What is the probability that the student knows the answer given that he answered it correctly?

Q37.

Solar Panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels.



A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture , suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters P_1 (6,8,4) , P_2 (21,8,4), P_3 (21,16,10) and P_4 (6,16,10)

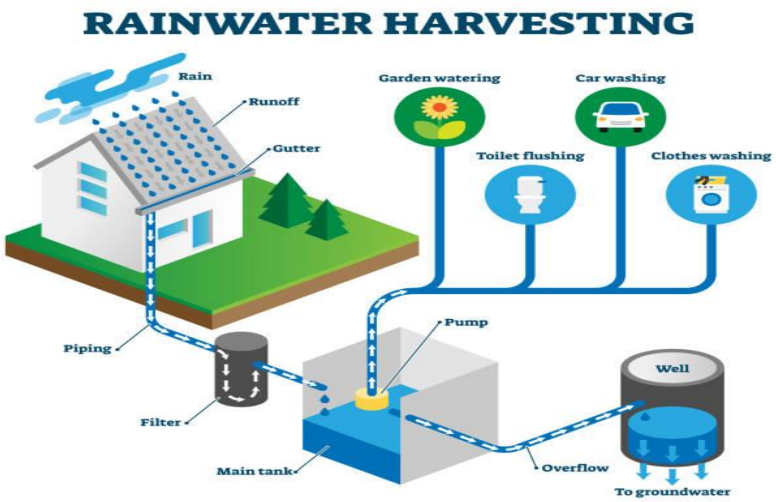
1. What are the components to the two edge vectors defined by $\vec{A}$ = PV of P_2 – PV of P_1 and $\vec{B}$ = PV of P_4 – PV of P_1? (where PV stands for position vector)
2. Write the vector in standard notation with $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ (where $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are the unit vectors along the three axes).
3. What are the magnitudes of the vectors $\vec{A}$ and $\vec{B}$ and in what units?

OR

What are the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof?

Q38. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and capacity of 250 cubic meter. The cost of land is Rs 5000 per sq.m.and the

cost of digging increases with depth and for the whole tank it is Rs 40000 h^2, where h is the depth of the tank in meters, x is the side of the square base of the tank in meters.



Based on the above information answer the following questions.

(i) Find the total cost C of digging the tank in terms of x.

(ii) Find the value of x for which the cost is minimum.

OR

Check whether the cost function C(x) expressed in terms of x is increasing or not, where x>0

……

KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION

SAMPLE QUESTION PAPER-20

Class:-XII

Session 2023-24

Mathematics (Code-041)

Time: 3 hours **Maximum marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts

Section –A
(Multiple Choice Questions)
Each question carries 1 mark

1. If A = $\begin{bmatrix} cos \propto & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$ then $A + A' = I$ if the value of $\propto$ is

   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{3\pi}{2}$

2. If A is a square matrix of order 3 such that $|adjA| = 64$, then, $|A|$ is

(a) 16 (b) ± 8 (c) ± 4 (d) ± 16

3. Let $f: R \to R$ be defined as $f(x) = x^4$ Then,

(a) f is one-one onto (b) f is onto but not one-one.

(b) (c) one- one but not onto f (d) f is neither one one nor onto.

4. The value of $\sin\left[\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 1

5. The function $f(x) = \begin{cases} \frac{kx}{|x|} & if\ x < 0 \\ 3 & if\ x \geq 0 \end{cases}$ is continuous at x = 0 then the value of k is

(a) 0 (b) – 1 (c) 1 (d) -3

6. If $y = log(cose^x)$, then $\frac{dy}{dx}$ is

(a) $-e^x tane^x$ (b) $e^x tane^x$ (c) $-tane^x$ (d) $-e^x cote^x$

7. If $x = t^2$, $y = t^3$, then $\frac{d^2y}{dx^2}$ is

(a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3}{4}$

8. The function $f(x) = x^3 + 3x$ is increasing in an interval;

(a) $(-\infty, 0)$ (b) $(0, \infty)$ (c) R (d) (0, 1) .

9. The point on the curve $y = x^2$, whare the rate of change of x-coordinate is equal to the rate of change of y-coordinate is

(a) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (b) $\left(\frac{1}{3}, \frac{1}{4}\right)$ (c) $\left(\frac{1}{4}, \frac{1}{6}\right)$ (d) $\left(\frac{1}{5}, \frac{1}{25}\right)$

10. $\int e^x\,(1 - cotx + cot^2x)$ dx = .

(a) $e^x cotx + C$ (b) $-e^x cotx + C$ (c) $e^x cosecx + C$ (d) - $e^x cosecx + C$

11. $\int_{-1}^{1} \frac{|x-2|}{x-2} dx, x \neq 2$ is equal to

(a) 1 (b) – 1 (c) 2 (d) – 2

12. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line y = 3 is

(a) 2 (b) $\frac{9}{4}$ (c) $\frac{9}{3}$ (d) $\frac{9}{2}$

13. The area of the region bounded by the curve *y = sinx* between the ordinates x = 0, x = $\frac{\pi}{2}$ and the x- axis is

(a) 2 sq. units (b) 4sq. units (c) 1 sq. units (d) 3 sq. unit.

14. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ respectively are

(a) 2, 2 (b) 2. 3 (c) 3, 2 (d) 3,3

15. The integrating factor of the differential equation $x\frac{dy}{dx} + 2y = x^2$ is

(a) x^2 (b) x (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$

16. The angle between the vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4 respectively and $\vec{a}.\vec{b} = 2\sqrt{3}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{5\pi}{2}$

17. The number of feasible solutions of the L.P.P given as Maximise Z = 15x + 30y, subject to the constraints: 3x + y ≤ 12, x + 2y ≤ 10, x ≥ 0 , y ≥ 0 is

(a) 1 (b) 2 (c) 3 (d) infinite.

18. X and Y are independent events such that $(X \cap Y') = \frac{2}{5}$, $P(X) = \frac{3}{5}$,then $P(Y)$ equal to

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{3}$ (d)) $\frac{1}{5}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

19. Assertion (A) :The relation R on the set A = { 1, 2, 3, 4, 5, 6 } defined as R = { (x, y) : y is divisible by x } is not an equivalence relation

Reason (R) : The relation R will be an equivalence relation, if it is reflexive, symmetric and transitive.

20. Assertion (A) : A balloon, which always remains spherical, has a variable radius. The rate at which its volume is increasing with the radius when the radius is 10 cm, is 400π cm^3/cm.

Reason (R): Rate of change of volume(v) of the balloon with respect to radius (r)

is $\frac{dv}{dr} = \left(\frac{2}{3}\pi\right).3r^2$

Section –B
[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. Let R be the relation in the set Z of integers given by R = {(a, b) : 2 divides a – b }.
Show that the relation is transitive. Write the equivalence class [0]

22. Prove that the function $f(x) = x^3 - 3x^2 + 4x - 100$ is strictly increasing in **R.**

23. Evaluate : $\int \frac{sin^2x - cos^2x}{sin^2x cos^2x} dx$.

24. Draw a rough sketch of $y = |x + 3|$ and evaluate $\int_{-6}^{0} |x + 3|\, dx$.

25. Solve the differential Equation: $\frac{dy}{dx} = \sqrt{4 - y^2}$, – 2 < y < 2

Section – C
[This section comprises of short answer type questions (SA) of 3 marks each]

26. Prove that $\sin^{-1}(\frac{8}{17}) + \sin^{-1}(\frac{3}{5}) = \cos^{-1}(\frac{36}{85})$

OR

Prove that $2\tan^{-1}(\frac{3}{4}) - \tan^{-1}(\frac{17}{31}) = \frac{\pi}{4}$

27.If A is a square matrix such that $A^2=I$, find the simplified value of $(A-I)^3 + (A+I)^3 - 7A$

28.Find the value of k, so that the function f(x)=$\begin{cases} \frac{1-cos4x}{8x^2} & \text{,if } x \neq 0 \\ k \ , if\ x = 0 \end{cases}$

29 Find the particular solution of the differential equation $\log(\frac{dy}{dx}) = 3x+4y$,given that y=0 when x=0.

OR

Evaluate: $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)\, dx$

30.Find the image of the point (1,6,3) on the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

31.Find the probability distribution of number of doublets in three tosses of a pair of dice.

Section – D
[This section comprises of long answer type questions (LA) of 5 marks each]

32. . Using the matrix method, solve the following system of linear equations

x-y+2z=7, 3x+4y-5z=-5, 2x-y+3z=12

33.Evaluate $\int \frac{1}{cos^4x + sin^2x cos^2 x + sin^4x} dx$

OR

Evaluate $\int(\sqrt{tanx} + \sqrt{cotx})$ dx

34. Find the value of α so that the four points A,B,C,D with position vectors $4\hat{\imath} + 5\hat{\jmath} + \hat{k}, -\hat{\jmath} - \hat{k}$,

$3\hat{\imath} + \alpha\hat{\jmath} + 4\hat{k}$, and $-4\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$, respectively are coplanar

35. Find the equation of the line passing through the point (-1,3,-2) and perpendicular to the lines

$\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$

OR

Find the shortest distance between the following lines

$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

SECTION E

[**This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)**

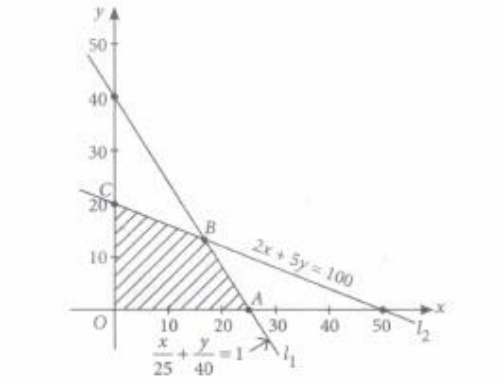
36. There are two rocket launching vehicles, named as A and B. The probabilities that the rocket launched from them reach the desired orbit of moon's surface are 0.3 and 0.2 respectively. Both of them launched one rocket at the same time.

(i) If it is known that the rocket launched from exactly one of them reach the orbit, then what is the probability that it was launched from A?

(ii) If it is known that the rocket launched from exactly one of them reach the orbit, then what is the probability that it was launched from B?

(iii) What is the probability that the rocket launched from exactly one of them reach the desired orbit?

37. Deepa rides her car at 25 km/hr, She has to spend Rs. 2 per km on diesel and if she rides it at a faster speed of 40 km/hr, the diesel cost increases to Rs. 5 per km. She has Rs. 100 to spend on diesel. Let she travels x kms with speed 25 km/hr and y kms with speed 40 km/hr. The feasible region for the LPP is shown below:



Based on the above information, answer the following questions.

(i) What is the point of intersection of line l_1 and l_2,

(ii) Find the corner points of the feasible region shown in above graph

(iii) If Z = x + y be the objective function. Find the point in which the maximum value of Z=30

38. A telephone company in a town has 500 subscribers on its list and collects fixed charges of 300 per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of Rs 1, one subscriber will discontinue the service.

i) If Rs x be the increase in annual subscription then what is the total revenue of the company after increment?

ii) Find the maximum profit that the company can make if the profit function is given by

$P(x) = 41 + 24x - 18x^2$.

KENDRIYA VIDYALAYA SANGATHAN ERNAKULAM REGION
SAMPLE QUESTION PAPER - 21 (SESSION:2023-2024)
CLASS XII
SUBJECT: MATHEMATICS (041)

Time Allowed:3Hours **Max.Marks:80**

General Instructions:

1.This question paper contains **-five sections A B, C, D and E.** Each section is compulsory. However, there are internal choices in some questions.
2.**Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.**
3.Section B has 5 Very Short Answer (VSA)- type questions of 2 marks each.
3.Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5.Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6.Section E has 3 source based/case based/passage based /integrated units of assessment (4 marks each) with sub parts.
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Q. No	SECTION A (Multiple Choice Questions) Each question carries 1 mark
1	If $\begin{vmatrix} sin\ \alpha & cos\ \beta \\ cos\ \alpha & sin\ \beta \end{vmatrix} = \frac{1}{2}$ where α and β are acute angles, then the value of $\alpha + \beta$ is (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c)$\frac{3\pi}{2}$ (d) π
2	If A and B are symmetric matrices of the same order, then AB' − BA' is a (a)Null matrix (b) Skew symmetric matrix (c) Symmetric matrix (d) Identity matrix
3	If $y = \log(\log(\log(x^5))$ then $\frac{dy}{dx}$ is equal to (a)$\frac{5}{x\ log\ (log\ (x^5)}$ (b) $\frac{5}{x\ log\ (x^5) log\ (log(x^5))}$ (c) $\frac{5x^4}{log\ (x^5) log\ (log(x^5))}$ (d)$\frac{5x^4}{log\ (log(x^5))}$
4	$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+cos2x}$ is equal to (a) 1 (b) 2 (c) 3 (d) 4
5	The integrating factor of the differential equation $(1 - y^2)\frac{dx}{dy} + xy = ay$ (-1<y<1) is (a) $\frac{1}{y^2-1}$ (b) $\frac{1}{\sqrt{y^2-1}}$ (c) $\frac{1}{1-y^2}$ (d) $\frac{1}{\sqrt{1-y^2}}$

6	Area of the parallelogram when diagonals are given by $\vec{a} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{b} = \hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ is (a) $10\sqrt{3}$ (b) $5\sqrt{3}$ (c) 8 (d) 4
7	If $\left\|\frac{A^{-1}}{2}\right\| = \frac{1}{k\|A\|}$ where A is a 3x3 matrix, then the value of k is (a) $\frac{1}{8}$ (b) 8 (c) 2 (d) $\frac{1}{2}$
8	A is a square matrix of order 3 such that A (adj A) = $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$,then the value of |adjA| is (a) 5 (b) 10 (c) 25 (d) 125
9	If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non singular matrix and a ϵ A then the set A is (a) R (b) [0} (c) {4} $(d)\ R - \{4\}$
10	Let f (x) $= \begin{cases} \frac{\sqrt{4+x}-2}{x} & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at x = 0 then k is equal to (a) $\frac{1}{2}$ (b) 4 (c) $\frac{1}{4}$ (d) 2
11	The degree of the differential equation $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is (a) 4 (b) $\frac{3}{2}$ (c) not defined (d) 2
12	Position vector of the midpoint of line segment AB is $3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$. If position vector of the point A is $2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$, then the position vector of the point B is (a) $\frac{5}{2}\hat{\imath} + \frac{5}{2}\hat{\jmath} - \frac{7}{2}\hat{k}$ (b) $4\hat{\imath} + \hat{\jmath} - 2\hat{k}$ (c) $5\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$ (d) $\frac{\hat{\imath}}{2} + \frac{\hat{\jmath}}{2} + \frac{\hat{k}}{2}$
13	If $\vec{a} + \vec{b} = \hat{\imath}$ and $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ then $\left\|\vec{b}\right\|$ equals (a) $\sqrt{14}$ (b)3 (c) $\sqrt{12}$ (d) 4
14	Direction cosines of the line $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$ are (a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ (b) $\frac{2}{\sqrt{157}}, \frac{-3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$ (c) $\frac{2}{\sqrt{157}}, \frac{3}{\sqrt{157}}, \frac{6}{\sqrt{157}}$ (d) $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$
15	If the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles then p is equal to (a) $\frac{65}{11}$ (b) $\frac{75}{11}$ (c) $\frac{70}{11}$ (d) $\frac{35}{11}$
16	The number of corner points of the feasible region determined by the constrains $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$. (a) 2 (b) 3 (c) 4 (d)5

17	The corner points of the feasible region for a linear programming problem are P (0,5) , Q (1,5) , R (4,2) and S (12,10). The Minium value of the objective function $z = 2x + 5y$ is at the point (a) P (b) Q (c) R (d) S
18	Let A and B be two events. If P(A) = 0.2, P(B) = 0.4 and P(AUB) = 0.6 then P(A/B) is (a) 0.8 (b) 0.5 (c) 0.3 (d) 0

ASSERTION -REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false.
(d) A is false but R is true

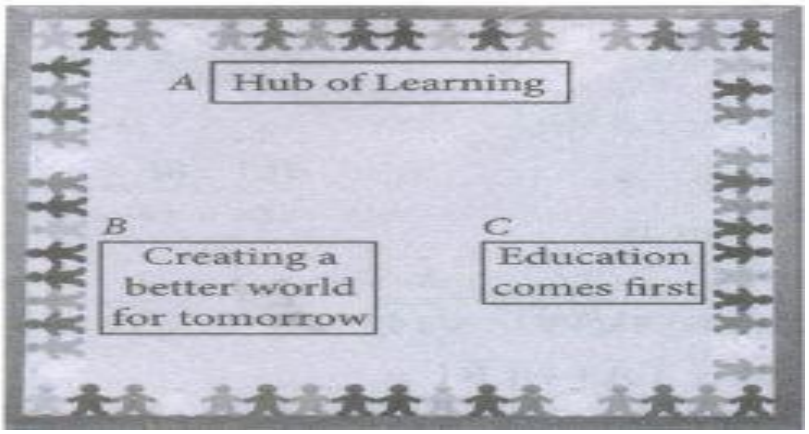
19	Assertion (A): f(x) = |x -1| + |x -2| is continuous but not differentiable at x= 1,2 Reason(R): Every differentiable function is continuous
20	Assertion(A): The function f: R → R defined by $f(x) = x^3$ is one-one Reason (R): Every polynomial function is one one

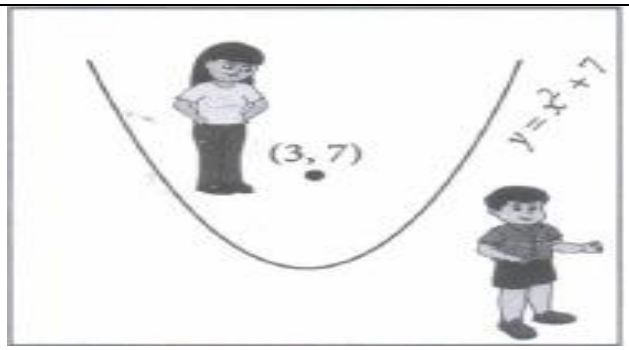
SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21	Prove that $cot^{-1}\left(\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right) = \frac{x}{2}$ $x\epsilon\left(0,\frac{\pi}{4}\right)$ OR Express $\tan^{-1}\left(\frac{cosx}{1-sinx}\right), \frac{-3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.
22	A balloon, which is always remains spherical is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when r = 15 cm.
23	Find the maximum and minimum value of the function $f(x) = 5 + sin2x$ OR The function $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value at $x = 1$in the interval [0,2]. Find the value of a.
24	Find the intervals in which the function $f(x) = log(2+x) - \frac{2}{2+x}$, x >-2 is increasing or decreasing.

25	Evaluate $\int_0^{2\pi} \frac{1}{1+e^{sinx}}\,dx$
	SECTION C **This section comprises of short answer type-questions (SA) of 3marks each**
26	If $x^y + y^x = a^b$,then find $\frac{dy}{dx}$?
27	Solve the differential equation $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$,subject to the initial condition y(0) =0. **OR** Solve the differential equation : $e^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0.$
28	Evaluate $\int \frac{x^2}{(x^2+1)(x^2+4)}\,dx$ **OR** Evaluate $\int \frac{cosx}{(1+sinx)(2+sinx)}\,dx$
29	Evaluate $\int_0^{\pi} \frac{x}{1+sin\,x}$ dx
30	Solve the following LPP Graphically: Minimise: $Z = -3x + 4y$ Subject to constraints:$x + 2y \leq 8, 3x + 2y \leq 12, x \geq 0, y \geq 0$ OR Solve the following LPP Graphically: Maximise:$Z = 3x + 2y$ Subject to constraints:$x + 2y \leq 10, 3x + y \leq 15, x \geq 0, y \geq 0$
31	Two numbers are selected at random (without replacement) from the first six positive integers. Let X denotes the larger of the two numbers obtained. Find the mean of X
	SECTION D **(This section comprises of long answer-type questions (LA) 0f 5 marks each)**
32	Let R be the relation on set A =$\{x: x \in Z, 0 \leq x \leq 20\}$ $given\ by$ $R = \{(a,b): (a-b) is\ multiple\ of\ 4\}$, Prove that R is an equivalence relation.Also write all the elements related to 1. **OR** Prove that the function $f: [0,\infty) \to R$ given by $f(x) = 9x^2 + 6x - 5$ is one-one but not onto.Modify the codomain of the function f to make it onto.

33	If $A=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$, B $=\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}$,then find AB and use it to solve the following system of equations :$x-2y=3, 2x-y-z=2$ and $-2y+z=3$
34	Two lines are given by $\vec{r}=(6\hat{\imath}+2\hat{\jmath}+2k)+\lambda(\hat{\imath}-2\hat{\jmath}+2\hat{k})$ & by $\vec{r}=(-4\hat{\imath}-k)+\mu(3\hat{\imath}-2\hat{\jmath}-2\hat{k})$. At what points on the lines the distance between them is shortest. Also find the shortest distance. **OR** Find the image of the point P(1,6,3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$
35	Find the area of the region bounded by the curve$x^2+y^2=9$,$x=\sqrt{3}$y and x-axis in the first quadrant
	SECTION E **(This section comprises of 3 case-study /passage based questions of 4 marks each with sub-parts.First two case study questions have three sub-parts(i),(ii),(iii) of marks 1,1,2 respectively.The third case study question has two sub-parts of 2 marks each.**
36	**Case-Study 1:** Three slogans on chart papers are to be placed on a school bulletin board at the point A(Hub of Learning),B(Creating a better world for tomorrow) and C(Education comes first).The coordinates of these points are (1,4,2),(3,-3,-2) and (-2,2,6) respectively.Based on the above information answer the following questions .  (i)Let $\vec{a}$,$\vec{b}$ and $\vec{c}$ be the position vector of the points A,B and C respectively,then find $\vec{a}+\vec{b}+\vec{c}$ (ii)Find the unitvector along $\vec{a}+\vec{b}+\vec{c}$ (iii)Suppose ,if the given slogans are to be placed on a straight line,thenfind the value of $\left\lvert\,\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right\rvert$ **OR** Find the area of Δ ABC
37	**Case-Study 2:**. A student Vivek is running on a playground along the curve given by y= x^2+7. Another student Devabhadra placed at (3,7) on playground wants to hit Vivek by paper ball when Vivek is nearest to her.



Based on the above information,answer the following questions.
(i)If (x_1, y_1) is the position of the Vivek on the curve $y = x^2+7$, then express D^2 , the function representing square of its distance from Devachandra placed at (3,7) in terms of x_1.
(ii) Find the critical point of the function D^2.
(iii) Find the position of Vivek when Devabhadra will hit the paper ball.

OR

(iii) Use Second derivative test to find the minimum value of the distance between Vivek and Devabhadra?

38 **Case-Study 3:**.

In pre-board examination of class XII,Science stream with Physics and Mathematics of a particular school,10 % of the students failed in Physics and 5% failed in Mathematics and 2% Failed in both Physics and Mathematics .A student is selected at random from the class.



Based on the above information,answer the following questions
(i)Find the probability that the selected student has passed in atleast one of the two subjects?
(ii)Find the Probability that the selected student has passed in Mathematics,if it is known that he has failed in Physics?

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SAMPLE QUESTION PAPER-22

Class XII

Session 2023 – 24

Mathematics

General Instructions:

i) This question paper contains 5 sections A, B, C, D and E.
ii) Section A has 18 MCQs and 2 Assertion – Reason based questions of 1 mark each.
iii) Section B has 5 very short answer type questions of 2 marks each.
iv) Section C has 6 short answer type questions of 3 marks each.
v) Section D has 4 long answer type questions of 5 marks each
vi) Section E has 3 case-based question of 4 marks and each with subparts.

SECTION A

(This section comprises of multiple-choice type questions (MCQ) of 1 mark each)

1. If A is a square matrix such that $A^2 = I$, then $(A + I)^3 + (A - I)^3 - 7A$ is equal to
   a) A b) I – A c) I + A d) 3A
2. If A = $[a_{ij}]$ is a square matrix of order 2, such that $a_{ij} = \begin{cases} 1, when\ i \neq j \\ 0, when\ i = 0 \end{cases}$ then A^2 is
   a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
3. If |A| = 2, where A is a 2×2 matrix, then $|4A^{-1}|$ equals
   a) 4 b) 2 c) 8 d) $\frac{1}{32}$
4. The values of a and b such that the function $f(x) = \begin{cases} 5, if\ x \leq 2 \\ ax + b, if\ 2 < x < 10 \\ 21, if\ x \geq 10 \end{cases}$
   a) 1 &2 b) 2 & 1 c) -1 & -2 d) -2 & -1
5. P is a point on the line joining the points (0,5, -2) and (3, -1,2), if x co-ordinate of P is 6 then the z co-ordinate is
   a) 10 b) 6 c) -6 d) -10
6. The sum of the order and degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = e^x$ is
   a) 2 b) 3 c) 5 d) 4
7. If the corner points of the feasible region for LPP where the objective function is z = 3x – 4y are (0,0), (6.12), (6,16) and (0,4), then the maximum value of z is
   a) 12 b) 8 c) 0 d) 18
8. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then the angle between $\vec{a}$ & $\vec{b}$
   a) 90° b) 60° c) 120° d) 180°
9. If $\int_0^1 (3x^2 + 2x + k)\, dx = 0$, then value of k is
   a) 8 b) -4 c) -2 d) 10

10. If A.(adj A) = $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then value of |A| + |adj A| is equal to

a) 12 b) 9 c)3 d) 27

11. The number of corner points of the feasible region determined by the constraints $x - y \leq 0, 2y \leq x + 2, x \geq 0, y \geq 0$ is

a) 2 b) 3 c) 4 d) 5

12. If $\vec{a} + \vec{b} = i$ and $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$, then $|\vec{b}|$ equals

a) $\sqrt{14}$ b) 3 c) $\sqrt{12}$ d) $\sqrt{17}$

13. Let A be a 3×3 matrix such that |adj A| = 64, then |A| is equal to

a) 8 only b) -8 only c) 64 d) 8 or -8

14. Given two independent events A and B such that P(A) = 0.3 and P(B) = 0.6, then the value of P(A and not B) is

a) 0.14 b) 0.12 c) 0.16 d) 0.18

15. The general solution of the differential equation $y^2dx - x^2dy = 0$ (given x, y > 0) is of the form

a) $x = cy^2$ b) $x - y = cxy$ c) $y - x = cx$ d) $y = cx^2$

16. The value of $\hat{\imath} \cdot (\hat{\jmath} \times \hat{k}) + \hat{\jmath} \cdot (\hat{\imath} \times \hat{\imath}) + \hat{k} \cdot (\hat{\imath} \times \hat{\jmath})$

a) 0 b) -1 c) 3 d) 1

17. The intervals in which the function f(x) = $x^2 - 4x + 6$ is strictly increasing are

a) $(-\infty, 2) \cup (2, \infty)$ b) $(-\infty, 2)$ c) $(2, \infty)$ d) $(-\infty, 2] \cup [2, \infty)$

18. Direction cosines of a line making equal angle with positive direction of co-ordinate axes are

a) 1, 1, 1 b) $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$ c) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ d) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

ASSERTION – REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer from the following.

a) Both A and R are true and R is the correct explanation of A
b) Both A and R are true and R is not the correct explanation of A
c) A is true but R is false
d) A is false but R is true

19. Assertion (A): All trigonometric functions have their inverse over their respective domains
Reason (R): The inverse of $\tan^{-1} x$ exists for some x ∈ R

20. Assertion (A): f(x) = $x^3 - 3x^2 + 4x$, x ∈ R is strictly increasing on R
Reason (R): Function f(x) is strictly increasing in (a,b) if f'(x) > 0 for each x ∈ (a,b)

SECTION B

(This section comprises of very short answer type questions (VSA) of 2 marks each)

21. Find the value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

OR

Evaluate $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$

22. Show that the function f(x) = $x^3 - 3x^2 + 6x - 100$ is strictly increasing on R.
23. Find the maximum value of the function f(x) = $\frac{1}{4x^2+2x+3}$

OR

Find the maximum profit that a company can make if the profit function is given by P(x)= $15 + 26x - 13x^2$, where 'x' is the number of units

24. Evaluate $\int_0^{\pi/2} \frac{(\cos x)^5}{(\sin x)^5+(\cos x)^5} dx$
25. An edge of a cube is increasing at the rate of 3cm/sec. Find the rate of change in its volume when the edge of cube is 10 cm.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Find $\int \frac{x^2+x+1}{(x+1)^2(x+2)} dx$
27. From a lot of 30 bulbs which includes 6 defective bulbs, a sample of 2 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs and hence find the mean number of defective bulbs.
28. Evaluate $\int_1^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}} dx$

Or

Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log \tan x \, dx$

29. Find the particular solution of the differential equation $e^x \tan y \, dx + (2e^x) \sec^2 y \, dy = 0$, given that $y = \frac{\pi}{4}$ when $x = 0$

Or

Find the particular solution of the differential equation $(3xy + y^2)dx + (x^2 + xy)dy = 0$ for $x = 1, y = 2$

30. Solve the following LPP graphically

Minimise $Z = 2x + 4y$, subject to the constraints $x + y \geq 8,\ x + 4y \geq 12,\ x \geq 3,\ y \geq 2$ and $x, y \geq 0$

Or

Maximise $Z = 105x + 90y$ subject to the constraints $x + y \leq 50,\ 2x + y \leq 80,\ x \geq 20$ and $x, y \geq 0$

31. If $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$ and $y = a \sin t$ then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$

SECTION D

(This section comprises of long answer type questions (LA) of 5 marks each)

32. Find the area of the region bounded by the line $3x - y = 3$ and the lines $x = \frac{1}{2}$ and $x = 2$ using the method of integration.
33. Let N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by $(a, b)R(c, d)$, if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

Or

Let $f: R - \left\{-\frac{4}{3}\right\} \to R$ be a function defined as f(x) $= \frac{4x}{3x+4}$. Show that f is one-one function. Also check whether f is an onto function or not.

34. Find the inverse of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse A^{-1}, solve the system of linear equations $x - y + 2z = 1;\ 2y - 3z = 1;\ 3x - 2y + 4z = 3$.
35. Find the image of the point $(2, -1, 5)$ in the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$

Or

Vertices B and C of ΔABC lie on the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z}{4}$. Find the area of ΔABC given that A has coordinates $(1, -1, 2)$ and the line segment BC has length of 5 units.

SECTION E

[This secton comprises of 3 case-study / passage-based questions of 4 marks each with sub parts. The first two case study question have the 4 sub parts (a),(b),(c),(d) of marks 1 each. The 3rd case study question has two sub parts of 2 marks each.]

36. If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition. Based on the above information, answer the following questions.

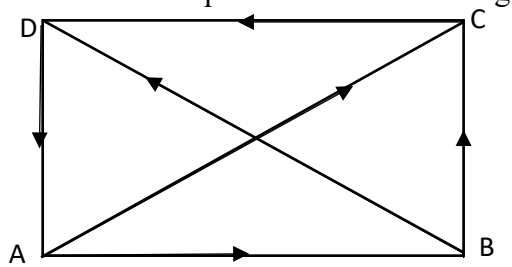
I. If $\vec{p}, \vec{q}, \vec{r}$ are the vectors represented by the sides of a triangle taken in order, then $\vec{q} + \vec{r} =$
   a) $\vec{p}$
   b) $\overrightarrow{2p}$
   c) $\overrightarrow{-p}$
   d) None of the above

II. If ABCD is a parallelogram and AC and BD are its diagonals, then $\overrightarrow{AC} + \overrightarrow{BD}$
   a) $2\overrightarrow{DA}$
   b) $2\overrightarrow{AB}$
   c) $2\overrightarrow{BC}$
   d) $2\overrightarrow{BD}$

III. If ABCD is a parallelogram, where $\overrightarrow{AB} = \overrightarrow{2a}$ and $\overrightarrow{BC} = \overrightarrow{2b}$, then $\overrightarrow{AC} - \overrightarrow{BD} =$
   a) $3\vec{a}$
   b) $4\vec{a}$
   c) $2\vec{b}$
   d) $4\vec{b}$

OR

If ABCD is a quadrilateral whose diagonals are $\overrightarrow{AC}$ and $\overrightarrow{BD}$, then $\overrightarrow{BA} + \overrightarrow{CD} =$



a) $\overrightarrow{AC} + \overrightarrow{DB}$
b) $\overrightarrow{AC} + \overrightarrow{BD}$
c) $\overrightarrow{BC} + \overrightarrow{AD}$
d) $\overrightarrow{BD} + \overrightarrow{CA}$

37. Aman has a rectangular painting canvas having a total area of 24 ft^2 which includes a border of 0.5 on the left, right and a border of 0.75 ft on the bottom, top inside it.



I. If Aman wants to paint in the maximum area, then she needs to maximize
   a) Area of outer rectangle
   b) Area of inner rectangle
   c) Area of top border
   d) None of these

II. If x is the length of the outer rectangle, then area of inner rectangle in terms of x is
   a) $(x+3)\left(\frac{24}{x}-2\right)$
   b) $(x-1)\left(\frac{24}{x}+1.5\right)$
   c) $(x-1)\left(\frac{24}{x}-1.5\right)$
   d) $(x-1)\left(\frac{24}{x}\right)$

III. If the inner rectangle is maximum, then x is equal to
   a) 2ft
   b) 3 ft
   c) 4ft
   d) 5ft

OR

The maximum area of inner rectangle
   a) 12.5 ft^2
   b) 13 ft^2
   c) 13.5 ft^2
   d) 14 ft^2

38. Read the following passage and answer the following questions given below



There are two antiaircraft guns, named as A & B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

a) What is the probability that the shell fired from exactly one of them hit the plane?

b) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

SAMPLE PAPER 1

Answer/Solutions

SECTION-A

1	C
2	C
3	D
4	C
5	B
6	A
7	A
8	A
9	C
10	C
11	B
12	B
13	B
14	A
15	C
16	B
17	A
18	B
19	A
20	C

SECTION-B

21. given $f(x) = x^3 - 4x^2 - 4x + 15$

$F'(x) = 3x^2 - 4x - 4 = (x-2)(x + \frac{2}{3})$

Now $f'(x) = 0$

i.e $3x^2 - 4x - 4 = 0$ $\Rightarrow x = 2, \frac{-2}{3}$

so the intervals are $(-\infty, \frac{-2}{3}), (\frac{-2}{3}, 2), (2, \infty)$

when $x \in (-\infty, \frac{-2}{3})$ then $f'(x) > 0$ and when $x \in (2, \infty)$ then $f'(x) > 0$

but when $x \in (\frac{-2}{3}, 2)$ then $f'(x) < 0$ so f(x) is decreasing in the interval $(\frac{-2}{3}, 2)$

22. sol. Given $|\vec{a} + \vec{b}| = |\vec{a}|$

or $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}.\vec{b} = |\vec{a}|^2$

$\Rightarrow |\vec{a}|^2 - |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}.\vec{b} = 0$

$\Rightarrow |\vec{b}|^2 + 2\,\vec{a}.\vec{b} = 0$

$\Rightarrow .\vec{b}(.\vec{b} + 2\,\vec{a}) = 0$

Hence $2\vec{a} + \vec{b}$ is perpendicular to $\vec{b}$

Or

Sol. Given $|\vec{a}| = |\vec{b}| = 1$ and $\vec{a} \cdot \vec{b} = 0$ because angle between $\vec{a}$ and $\vec{b}$ is $120^0 - 30^0 = 90^0$

$\because \quad |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a} \cdot \vec{b}$

$= 1+1+0 \quad = 2$

$|\vec{a} + \vec{b}| \quad = \sqrt{2}$

23. sol. Given $f(x) = \begin{cases} \frac{x^2+3x-10}{x-2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$ is continuous at x=2

So LHL = f(2)

$$\lim_{x \to 2^-} f(x) = k$$

$\Rightarrow \lim_{x \to 2} \frac{x^2+3x-10}{x-2} = k$

$\Rightarrow \lim_{x \to 2} \frac{(x+5)(x-2)}{x-2} = k$

$\Rightarrow$ k=7 ans.

24. Given the points are $A(-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$, B$(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ and C$(7\hat{\imath} - \hat{k})$

$\overrightarrow{AB} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) - (-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$

$\overrightarrow{BC} = 7\hat{\imath} - \hat{k} - (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) = 6\hat{\imath} - 2\hat{\jmath} - 4\hat{k}$

$\overrightarrow{AC} = (7\hat{\imath} - \hat{k}) - (-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) = 9\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$

$|\overrightarrow{AB}| = \sqrt{3^2 + (-1)^2 + (-2)^2} = \sqrt{14}$

$|\overrightarrow{BC}| = \sqrt{6 + (-2)^2 + (-4)^2} = 2\sqrt{14}$

25. the modulus function f :R $\rightarrow$ R given by f(x) = $|x|$

* F(x) is not one to one because f(1) = f(-1) = 1 where 1 , -1 ϵ R

* f(x) is not onto because the negative values belongs to the codamain don't have the pre-image

Or

Sol. Given $sin^{-1}\left(sin\frac{3\pi}{5}\right)$

$= sin^{-1}\left(sin\left(\pi - \frac{2\pi}{5}\right)\right)$

$= sin^{-1}\left(sin\frac{2\pi}{5}\right) \qquad = \frac{2\pi}{5}$ ANS.

26. Solution : I = $\int x tan^{-1}x\, dx$

$= \int x tan^{-1}x\, dx$

$= tan^{-1}x \int x\, dx - \int [\frac{d}{dx} tan^{-1}x \int x\, dx]\, dx$

$= tan^{-1}x \frac{x^2}{2} - \frac{1}{2}\int \frac{x^2}{1+x^2} dx$

$= \frac{x^2}{2} tan^{-1}x - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2} dx$

$= \frac{x^2}{2} tan^{-1}x - \frac{1}{2} [\int [1 - \frac{1}{1+x^2}] dx$

$= \frac{x^2}{2} tan^{-1}x - \frac{1}{2} [x - tan^{-1}x\ ]$ + c Ans.

OR

Solution : let I = $\int_0^{\pi} \frac{x\, sinx}{1+cos^2x}\, dx$ ---------- i

I = $\int_0^{\pi} \frac{(\pi-x)\, sin(\pi-x)}{1+cos^2(\pi-x)}\, dx$ $\{ \because \int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx \}$

I = $\int_0^{\pi} \frac{(\pi-x)\, sinx}{1+cos^2x}\, dx$ ---------- ii

Adding i and ii we get

2I = $\int_0^{\pi} \frac{\pi\, sinx}{1+cos^2x}\, dx$ ---------- iii

Let cosx = t when x = 0 then then t = 1 and when x = π then then t = -1

∴ $-sinx$ dx = dt put these values in iii

∴ 2I = $\int_1^{-1} \frac{-\pi}{1+t^2}\, dt = \int_{-1}^{1} \frac{\pi}{1+t^2}\, dt = \pi\, [tan^{-1}]_{-1}^{1}$

2I = $2\frac{\pi^2}{4}$

So I = $\frac{\pi^2}{4}$ ans.

27. out of 6 waiters three have to be selected so n = 6 , X = {1,2,3}

P (x=1) = $\frac{{}^6c_1 \times {}^2c_2}{{}^8c_3} = \frac{3}{28}$

P (x=2) = $\frac{{}^6c_2 \times {}^2c_1}{{}^8c_3} = \frac{15}{28}$

P (x=3) = $\frac{{}^6c_3}{{}^8c_3} = \frac{10}{28}$

The probability distribution is

X	1	2	3
P(x)	$\frac{3}{28}$	$\frac{15}{28}$	$\frac{10}{28}$

Or

b) let the events, E_1 : choosing a boy

E_2 : choosing a girl

Let A be the event of choosing a good speaker

$\therefore P(E_1) = P(E_2) = \frac{1}{2}$, $P(\frac{A}{E_1}) = \frac{5}{40}$, $P(\frac{A}{E_2}) = \frac{8}{50}$

$\therefore$ P(choosing a good speaker) = $P(E_1) \times P(\frac{A}{E_1}) + P(E_2) \times P(\frac{A}{E_2})$

$= \frac{1}{2} \times \frac{5}{40} + \frac{1}{2} \times \frac{8}{50}$

$= \frac{57}{400}$ Ans.

28. Solution : given $\log(\frac{dy}{dx}) = 3x + 4y$

Or $\frac{dy}{dx} = e^{(3x+4y)}$ (applying exponents concepts on both sides)

$\frac{dy}{dx} = e^{3x}\ e^{4y}$

$\frac{dy}{e^{4y}} = e^{3x}\ dx$

$\int e^{-4y}\ dy = \int e^{3x}\ dx$ (integerating both sides)

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c$ ---------- (i)

Put x=y=0 so $c = \frac{-7}{12}$ put this in (i)

$\therefore \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$ is the particular solution

OR

Solution : given $(x + 2y^2)\frac{dy}{dx} = y$

$\frac{dx}{dy} - \frac{x}{y} = 2y$

Here $P = -\frac{1}{y}$, $Q = 2y$

$I.F = e^{\int -\frac{1}{y} dy}$

$I.F = \frac{1}{y}$

$\therefore \frac{x}{y} = 2y + c$ ----------(i)

Put y=1 and $x = 2$ in (i)

$\therefore$ C = 0 put this in (i)

$\therefore \quad \frac{x}{y} = 2y$ **Ans.**

29. Solution: $\int \sqrt{tanx} + \sqrt{cotx}\, dx =$

$= \int [\frac{\sqrt{sinx}}{\sqrt{cosx}} + \frac{\sqrt{cosx}}{\sqrt{sinx}}]\, dx$

$= \int [\frac{sinx+cosx}{\sqrt{sinx\ cosx}}]\, dx$

$= \sqrt{2} \int [\frac{sinx+cosx}{\sqrt{2sinx\ cosx}}]\, dx$

$= \sqrt{2} \int [\frac{sinx+cosx}{\sqrt{2sinx\ cosx}}]\, dx$

$= \sqrt{2} \int [\frac{sinx+cosx}{\sqrt{1-(sinx-cosx)^2}}]\, dx$ ---------- (i)

Let $sinx - cosx$ = t ----------(ii)

So $(sinx + cosx)$ dx = dt ----------(iii)

From (i),(ii) and (iii) we have

$\sqrt{2} \int \frac{dt}{\sqrt{1-t^2}}\, dt$

$= \sqrt{2} \sin^{-1}t + c$

$= \sqrt{2} \sin^{-1}(sinx - cosx) + c$

30. Given Maximize Z = 4500x + 5000y

Subject to constraints : 25000x + 40000y ≤ 7000000

Or 5x + 8y ≤ 1400

x + y ≤ 250

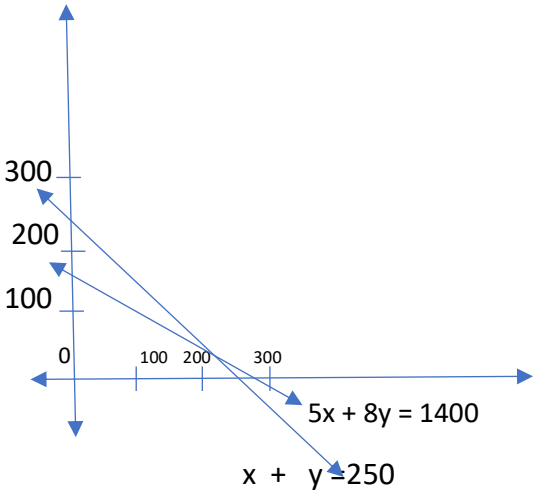
x , y ≥ 0

let 5x + 8y = 1400 so

x	0	200	280
y	175	50	0

And let x + y =250

x	0	250	200
y	250	0	50



Corner points	Z= 4500x + 5000y
A(200, 50)	4500x200 + 5000 x 50 = **1150000**
B(0,175)	0 + 5000 x 175 = 875000
O(0,0)	0+0 = 0
C(250,0)	4500x250 = 1125000

The maximum value is 1150000 at x = 200 and y = 50

31 . I=$\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x} dx$ ---------- (i)

I =$\int_0^{\pi} \frac{\pi - x}{a^2\cos^2 x + b^2 \sin^2 x} dx$ ----------(ii) { $\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ }

2I = 2 $\int_0^{\frac{\pi}{2}} \frac{\pi}{a^2\cos^2 x + b^2 \sin^2 x} dx$ { adding i and ii }

2I = 2π $\int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} dx$ -----(iii) {dividing the numerator and denominator by $\cos^2 x$}

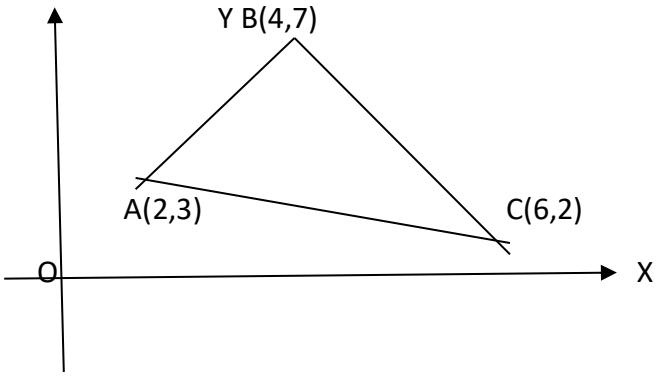
Let tanx = t

$Sec^2x\,dx = dt$, when x= 0 then t= 1, when x = $\frac{\pi}{2}$ then t= □ put these values in (iii)

2I= 2π $\int_0^{\propto} \frac{dt}{\left(\frac{a}{b}\right)^2 + t^2}$

I = $\frac{\pi^2}{2ab}$ ANS.

32.



Equation of AB is y =$2x - 1$

Equation of BC is y = $-\frac{5}{2}x + 17$

Equation of CA is y = $\frac{-x}{4} + \frac{7}{2}$

$$\left.\begin{matrix} Required \\ area \end{matrix}\right\} = \int_2^4 (2x-1)\,dx + \int_4^6 \left(-\frac{5}{2}x + 17\right) dx \; - \int_2^6 (-\frac{x}{4} + \frac{7}{2})\,dx$$

= 9 $sq.units$

33. Let line L_1 : x + 1 = $\frac{y}{2}$ = - z

Or $\frac{x-(-1)}{1} = \frac{y-0}{2} = \frac{z-0}{-1}$

Let $\vec{a}_1 = -\hat{\imath}$, $\vec{b}_1 = \hat{\imath} + 2\,\hat{\jmath} - \hat{k}$

And L_2 : x = y+2 = 1-z

Or $\frac{x-0}{1} = \frac{y-(-2)}{1} = \frac{z-1}{-1}$

Let $\vec{a}_2 = -2\,\hat{\jmath} + \hat{k}$, $\vec{b}_2 = \hat{\imath} + \hat{\jmath} - \hat{k}$

The shortest distance = $\left|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$ ----------(i)

Where $\vec{b}_1 \times \vec{b}_2 = (\hat{\imath} + 2\,\hat{\jmath} - \hat{k}) \times (\hat{\imath} + \hat{\jmath} - \hat{k}) = -\hat{\imath} - \hat{k}$

and $|\vec{b}_1 \times \vec{b}_2| = \sqrt{2}$

$\vec{a}_2 - \vec{a}_1 = \hat{\imath} - 2\,\hat{\jmath} + \hat{k}$

Put the values in (i)

So The shortest distance = $\left|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right| = \left|\frac{(-\hat{\imath} - \hat{k}).(\hat{\imath} - 2\,\hat{\jmath} + \hat{k})}{\sqrt{2}}\right|$

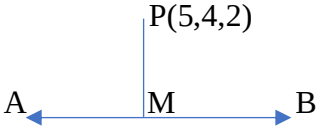
$= \left|\frac{-2}{\sqrt{2}}\right| = \sqrt{2}$ Ans.

Or

b) sol. Given $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + \hat{k} + \lambda((2\,\hat{\imath} + 3\,\hat{\jmath} - \hat{k})$

let $a_1 = 2$, $b_1 = 3$, $c_1 = -1$

the cartesian form of the line is $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ = t (let)



so the point on the line is A(2t-1, 3t+3 , -t+1)
let P(5,4,2) be the given point
the direction ratios of PA are : 2t-1-5 , 3t+3-4 , -t+1-2
= 2t-6 , 3t-1 , -t-1
Let a_2 = 2t-6 , b_2 = 3t-1 ,c_2 = -(t+1)
The Dr's of AB are : $a_1 = 2$, $b_1 = 3$, $c_1 = -1$
The Dr's of PM are : a_2 = 2t-6 , b_2 = 3t-1 , c_2 = -(t+1)
Here PM perpendicular to AB So $a_1a_2 + b_1b_2 + c_1c_2 = 0$
⇨ 2(2t-6)+3(3t-1)-1(-t-1) = 0
⇨ t = 1 put the value of t in a_2 , b_2 , c_2

∴ $a_2 = 1$, $b_2 = 6$, $c_2 = 0$ which are the coordinates of M

Now the length of PM = $\sqrt{(5-1)^2 + (4-6)^2 + (2-0)^2}$

$= 2\sqrt{6}$ Ans.

34. sol. Let A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$=8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

AB = 8I

$(AB)B^{-1} = 8\ I\ B^{-1}$

$A= 8\ B^{-1}$

Given the system of equations are : x-y+z=4 ; x – 2y -2z =9 ; 2x + y +3z = 1

$$\text{Let } D = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix} = B\ ,\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}\ ,\ C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

DX= C => BX = C OR $X = B^{-1}C$

$\Rightarrow X = \frac{1}{8} AC$ $\{ \because B^{-1} = \frac{1}{8} A \}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

x = 3 , y = -2 , z = -1 Ans.

35. sol. Given R be the relation on N×N defined as

(a,b) R (c,d) if ad(b+c) = bc(a+d),

To show : R is an equivalence relation

i. Reflexive : (a,b) R (a,b) because ab(b+a) = ba(a+b) for all (a,b) ϵ N x N

ii. Symmetric : if (a,b) R (c,d) then ad(b+c) = bc(a+d)
And if (c,d)R (a,b) then bc(a+d)= ad(b+c)
so R is symmetric for all (a,b) , (c,d) ϵ N x N

iii. Transitive : let (a,b) R (c,d) then ad(b+c) = bc(a+d)

Or $\frac{(b+c)}{bc} = \frac{(a+d)}{ad} \Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ ----------(i)

Let (c,d) R (e,f) then cf(d+e) = de(c+f) for a,b,c,d,e,f ϵ N

Or $\frac{(d+e)}{de} = \frac{(c+f)}{cf} \Rightarrow \frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$ ----------(i)

Adding (i) and (ii) we have $\frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$

$\Rightarrow \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f}$

$\Rightarrow \frac{e+b}{eb} = \frac{f+a}{af}$

$\Rightarrow af(e+b) = eb(f+a)$

$\Rightarrow$ (a,b) R (e,f) hence R is transitive

Hence R is an equivalence relation.

b) sol. Given : set A ={ 1,2,3,4,5} and

R ={(a,b): $|$a-b$|$ is divisible by 2}

$\therefore$ R = { (1,1),(1,3),(1,5) ,(2,2),(2,4),(3,1),(3,3),(3,5),(4,2),(4,4),(5,1),(5,3),(5,5)}

Reflexive : $|$a-a$|$ = 0 is divisible by 2 so (a,a) ϵ R hence R is reflexive

Symmetric : if $|$a-b$|$ is divisible by 2 then $|$b-a$|$ is also divisible by 2

Hence (a,b) ϵ R => (b,a) ϵ R

Transitive : from the given relation we see that if (a,b) ϵ R

then $|$a-b$|$ is divisible by 2

and if (b,c) ϵ R

then $|$b-c$|$ is divisible by 2

which implies $|$a-c$|$ is also divisible by 2 so (a,c) ϵ R

hence the relation is transitive

$\because$ the relation is reflexive symmetric and transitive so R is equivalence

The equivalence classes of R are [1] ={1,3,5} , [2] = {2,4}

36.	Let x, y and z be the cost of a pen, pencil, and mathematical instrument box. According to the question we have $3x + 2y + z = 41$ $2x + y + 2z = 29$ $2x + 2y + 2z = 44$ Corresponding matrix of the equation is, $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 41 \\ 29 \\ 44 \end{bmatrix}$ $\|A\| = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 2 \end{vmatrix} = 3(2-4) - 2(4-4) + 1(4-2) = -4 \neq 0$ adj $A = \begin{bmatrix} -2 & -2 & 3 \\ 0 & 4 & -4 \\ 2 & -2 & -1 \end{bmatrix}$ $A^{-1} = \frac{1}{-4}\begin{bmatrix} -2 & -2 & 3 \\ 0 & 4 & -4 \\ 2 & -2 & -1 \end{bmatrix}$ Then $X = A^{-1}B$ $X = \frac{1}{-4}\begin{bmatrix} -2 & -2 & 3 \\ 0 & 4 & -4 \\ 2 & -2 & -1 \end{bmatrix} \begin{bmatrix} 41 \\ 29 \\ 44 \end{bmatrix}$ $= \begin{bmatrix} 2 \\ 15 \\ 5 \end{bmatrix}$ Thus $x = 2, y = 15, z = 5$ (i) The cost of one pen = Rs. 2 (ii) The cost of one pen and one mathematical instrumental box = cost of one pen + cost of one mathematical instrumental = Rs.2 + Rs. 5 =Rs.7 (iii) The cost of one pencil and one mathematical instrumental box = cost of one pencil + cost of one mathematical instrumental box = Rs.2 + Rs.15 = Rs.20 **OR** Cost of one pen, one pencil and one mathematical instrument = = Cost of one pen+ cost of one pencil+ cost of one mathematical instrument = Rs.2 + Rs.15 + Rs.5 = Rs.22	1 1 2 2

37.	Considering E_1: matter processed by Mehul E_2: matter processed by Janya E_3: matter processed by Charvi E: matter has an error (i) $P(E/E_2) = 0.04$ (ii) Probability = $P(E_1).P(E/E_2)$ $= 0.3 \times 0.04$ $= 0.012$ (iii) $P(E) = P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2) + P(E_3)\,P(E/E_3)$ $= 0.4 \times 0.06 + 0.3 \times 0.04 + 0.3 \times 0.03$ $= 0.024 + 0.012 + 0.009$ $= 0.045$ **OR** (iv) $P(E_1/E) = \frac{P(E1)\,P(E/E1)}{0.045}$ $= \frac{0.024}{0.045}$ $= \frac{8}{15}$	1 1 2 2
38.	(i) Clearly, G be the centroid of Δ BCD, therefore coordinates of G are $\left(\frac{3+4+2}{3}, \frac{0+3+3}{3}, \frac{1+6+2}{3}\right)$ = (3, 2, 3) (ii) Since, $A = (0, 1, 2)$ and $G = (3,2,3)$ $\overrightarrow{AG} = (3-0)\hat{\imath} + (2-1)\hat{\jmath} + (3-2)\hat{k}$ $= 3\hat{\imath} + \hat{\jmath} + \hat{k}$ Thus, $\lvert\overrightarrow{AG}\rvert = \sqrt{3^2 + 1^2 + 1^2}$ $= \sqrt{9+1+1}$ $= \sqrt{11}$	2 2

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SAMPLE PAPER -2 (2023-24)

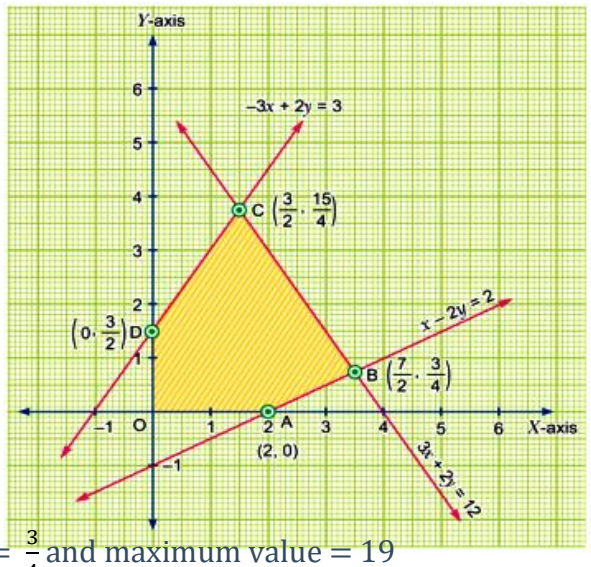
SUBJECT:-MATHEMATICS (041)

Time: - 3 Hours **CLASS: XII** **Max Marks: - 80**

Q. No	MARKING SCHEME
1	(d) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
2	(b) -2
3	(b) -8
4	(b) 3
5	(d) $(A + B)^{-1} = B^{-1} + A^{-1}$
6	(d) 1.5
7	(c) $\frac{-2}{\sqrt{6}}\hat{\imath} + \frac{1}{\sqrt{6}}\hat{\jmath} + \frac{1}{\sqrt{6}}\hat{k}$
8	(c) $\lvert\vec{a}\rvert^2 \lvert\vec{b}\rvert^2$
9	(d) None of these
10	(c) $\tan x - \cot x + c$
11	(b) 6
12	(d) not defined
13	(b) (0,8)
14	(d) $a = \frac{1}{\lvert\lambda\rvert}$
15	(a) (2, 0, 0)
16	(a) 90°
17	(d) $q = 3p$
18	(b) P(A/B) =1
19	(c) **A is true but R is false.**
20	(a) **Both A and R are true and R is the correct explanation of A.**
21	f (x) = $\sin^{-1}\sqrt{x-1}$ $-1 \le \sqrt{x-1} \le 1 \Rightarrow 0 \le \sqrt{x-1} \le 1$ $0 \le x-1 \le 1 \Rightarrow 1 \le x \le 2, x \in [1, 2]$ OR $\tan^{-1}\frac{1-\tan x}{1+\tan x} = \tan^{-1}\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x} = \tan^{-1}\tan\left(\frac{\pi}{4}-x\right) = \frac{\pi}{4}-x$
22	Taking logarithm on both sides x log y = y log x Diff.w.r.t.x $\frac{x}{y}\frac{dy}{dx} + \log y = \frac{y}{x} + \log x\frac{dy}{dx}$

	$\left(\frac{x}{y}-\log x\right)\frac{dy}{dx}=\left(\frac{y}{x}-\log y\right)$ So $\frac{dy}{dx}=\frac{y(y-x\log y)}{x(x-y\log x)}$ OR f(x) is continous at $x=\pi$ $\lim_{x\to\pi} f(x) = f(\pi)$ $\lim_{x\to\pi^+} f(x) = \lim_{x\to\pi^-} f(x) = k\pi+1$ $\lim_{x\to\pi} \cos x = \lim_{x\to\pi} (kx+1) = k\pi+1$ $-1 = k\pi+1$ $k = \frac{-2}{\pi}$
23	$\int \frac{(x^4-x)^{\frac{1}{4}}}{x^5}dx = \int \frac{\left(1-\frac{1}{x^3}\right)^{\frac{1}{4}}}{x^4}dx$ Put $1-\frac{1}{x^3}=t \quad i.e., \ \frac{1}{x^4}dx=\frac{dt}{3}$ $\int \frac{(x^4-x)^{\frac{1}{4}}}{x^5}dx = \frac{1}{3}\int t^{\frac{1}{4}}dt = \frac{4}{15}\left(1-\frac{1}{x^3}\right)^{\frac{5}{4}}+c$ **OR** $\int \frac{dx}{5-8x-x^2} = \int \frac{1}{(\sqrt{21})^2-(x+4)^2}dx = \frac{1}{2\sqrt{21}}log\left\lvert\frac{\sqrt{21}+(x+4)}{\sqrt{21}-(x+4)}\right\rvert+c$
24	We have, $f(x)=x^2+ax+1$ $f'(x)=2x+a>0 \Longrightarrow x>\frac{-a}{2}$ Therefore, we have to find the least value of a such that $x>\frac{-a}{2}$, when x ∈ (1,2). Thus, the least value of a for f to be increasing on (1,2) is given by, $\frac{-a}{2}=1 \Longrightarrow a=-2$ OR $f(x)=x^4-62x^2+9x+9$ $f'(x)=4x^3-124x+a$ As, function attains maximum value at x=1. So, f (1) = 0 $4(1)^3-124+a=0 \Longrightarrow a=120$
25	For writing $\frac{dV}{dt}=9$ cm³/sec V=x³ and for calculating $\frac{dx}{dt}=\frac{3}{2}$ cm/sec For S=6x² and $\frac{ds}{dt}$=3.6 cm²/sec
26	$y_1=\frac{2\tan^{-1}x}{1+x^2}$

	$\Rightarrow (1+x^2)y_1 = 2\tan^{-1}x$ $\Rightarrow (1+x^2)y_2 + 2xy_1 = \frac{2}{1+x^2}$ $\Rightarrow (x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$
27	$\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{c}{x+3}$ A =3/8 ,B =1/2 ,c = 5/8 On substituting and finding integration $= \frac{3}{8}\log\lvert x-1\rvert - \frac{1}{2(x-1)} + \frac{5}{8}\log\lvert x+3\rvert + c$ Or
28	$I = \sin^{-1}x \int x\,dx - \int \left\{\left(\frac{d}{dx}\sin^{-1}x\right)\int x\,dx\right\}dx$ $= \frac{x^2\sin^{-1}x}{2} - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}$ $\frac{x^2\sin^{-1}x}{2} + \frac{1}{2}\left\{\int \sqrt{1-x^2}\,dx - \int \frac{dx}{\sqrt{1-x^2}}\right\}$ $\frac{1}{4}(2x^2-1)\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} + C$ **OR** Let $I = \int_0^{\frac{\pi}{2}} \log\sin x\ dx$ ---(i) Then, using P-4 $I = \int_0^{\frac{\pi}{2}} \log\sin(\frac{\pi}{2}-x)\ dx = \int_0^{\frac{\pi}{2}} \log\cos x\ dx$ ---(ii) Adding the two values of I, we get $2I = \int_0^{\frac{\pi}{2}} (\log\sin x + \log\cos x)\,dx$ $2I = \int_0^{\frac{\pi}{2}} (\log\frac{\sin 2x}{2})\,dx$ $2I = \int_0^{\frac{\pi}{2}} \log\sin 2x\,dx - \int_0^{\frac{\pi}{2}} \log 2\,dx$ Put $2x = t$ in the first integral. Then $2\,dx = dt$, when $x = 0$, $t = 0$ and when $x = \frac{\pi}{2}$, $t = \pi$ Therefore $2I = \frac{1}{2}\int_0^{\pi} \log\sin t\,dt - \frac{\pi}{2}\log 2$ Now in the first integral we use P-6 $2I = \frac{2}{2}\int_0^{\frac{\pi}{2}} \log\sin t\,dt - \frac{\pi}{2}\log 2$ Now in the first integral we use P-0

	$2I = \int_0^{\frac{\pi}{2}} \log \sin x \, dx - \frac{\pi}{2}\log 2$ $2I = \mathrm{I} - \frac{\pi}{2}\log 2 \quad ; \quad \mathrm{I} = -\frac{\pi}{2}\log 2$
29	Showing homogeneous Substituting y= vx And getting $\frac{1-v}{v^2+v+1}\mathrm{dv} = \frac{dx}{x}$ Solving and getting solution $-\frac{1}{2}\log\frac{x^2+xy+y^2}{x^2} + \sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = \log x + \mathrm{c}$ OR Writing in the form $\frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)\mathrm{y}=1$ I.F= $e^{\int\left(\frac{1}{x}+\cot x\right)dx} = x\sin x$ $y(x \sin x) = \int x \sin x dx + \mathrm{c}$ Finding solution is $xy\sin x = -x\cos x + \sin x + \mathrm{c}$
30	 Hence, Z is maximum at x = $\frac{7}{2}$, y = $\frac{3}{4}$ and maximum value = 19
31	X can take values 2, 3, 4, 5, 6 (∵1 cannot be greater than the other selected number)

CORNER POINTS	Z =5x+ 2y
(0,0)	*0*
(2,0)	10
$\left(\frac{7}{2}, \frac{3}{4}\right)$	19 (MAXIMUM)
$\left(\frac{3}{2}, \frac{15}{4}\right)$	15
$(0, \frac{3}{2})$	3

X	2	3	4	5	6

P(X)	$\frac{2}{30}$	$\frac{4}{30}$	$\frac{6}{30}$	$\frac{8}{30}$	$\frac{10}{30}$
X.P(X)	$\frac{4}{30}$	$\frac{12}{30}$	$\frac{24}{30}$	$\frac{40}{30}$	$\frac{60}{30}$

Mean of X=E(X)=$\sum$X P(X) $= \frac{4+12+24+40+60}{30} = \frac{140}{30} = \frac{14}{3}$

OR

The random variable X has a probability distribution P(X) of the following form, where k is some number :

X	0	1	2	OTHERWISE	
P(X)	K	2K	3K	0	

(a) As Sum of all probabilities should be
$\Rightarrow k+2k+3k=1 \quad \Rightarrow k=1/6$

(b) $P(x<2)=p(x=0)+p(x=1)=k+2k=3(\frac{1}{6})=\frac{1}{2}$

(c) $P(x\geq 2)=3k+0=3(\frac{1}{6})+0=\frac{1}{2}$

32

ONE -ONE

For $x_1, x_2 \in R$, consider

$f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$

$\Rightarrow x_1x_2^2 + x_1 = x_2x_1^2 + x_2$

$\Rightarrow x_1x_2^2 + x_1 - x_2x_1^2 - x_2 = 0$

$\Rightarrow x_1x_2(x_2 - x_1) - 1(x_2 - x_1) = 0$

$\Rightarrow (x_1x_2 - 1)(x_2 - x_1) = 0$

$\Rightarrow (x_1x_2 = 1)$ or $(x_2 = x_1)$

We note that there are point, x_1 and x_2 with $x_1 \neq x_2$ and $f(x_1) = f(x_2)$, for instance, if we take $x_1 = 2$ and $x_2 = \frac{1}{2}$, then we have $f(x_1) = \frac{2}{5}$ and $f\,x_2) = \frac{2}{5}$

But $2 \neq \frac{1}{2}$. Hence f is not one-one.

Onto :-

Also, f is not onto for if so then for $1 \in R \; \exists \; x \in R$ such that $f(x) = 1$

which gives $\frac{x}{x^2+1} = 1$. But there is no such x in the domain R, since the equation

$x^2 - x + 1 = 0$ does not give any real value of x.

OR

Reflexive: Since $(a, b)R(a, b)$ as $a + b = b + a, \forall a, b \in A$ So, R is reflexive.

Symmetric: Let $(a, b)R(a, b)$ $a + d = b + c \Rightarrow c + b = d + a \Rightarrow (c, d)R(a, b)$

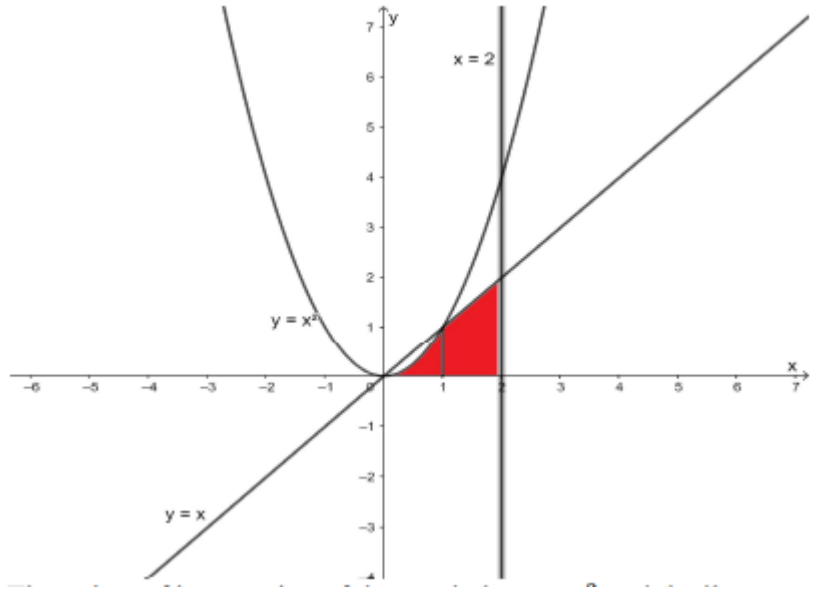
Transitive: Let $(a, b)R(c, d)$ and $(c, d)R(e, f)$

then $a + d = b + c$ and $c + f = d + e$,a+d=b+c and d+e=c+f

(a+d)-(d+e) =(b+c)-(c+f) $\Rightarrow$ a+f=b+e $\Rightarrow$ (a,b)R (e,f)

$\Rightarrow$ R is an equivalence Relation

Equivalence class of [(2,5)] is {(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)}

33	Here$\|A\| = -1$, A is a non singular matrix Adj $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ $AX = B$,$x = 1$, $y = 2$ and $z = 3$
34	The points of intersection of parabola $y = x^2$ and the line $y = x$ are $(0,0)$ and $(1,1)$ For correct sketching  Required area $= \int_0^1 x^2 dx + \int_1^2 x dx$ $\left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^2$ Finding correct answer $= \frac{11}{6}$ sq.unit
35	We have, $x = py+q \Rightarrow y = \frac{x-q}{p}$ ...(i) And $z = ry+s \quad \Rightarrow \quad y = \frac{z-s}{r}$ ...(ii) [Using Eqs. (i) and (ii)] $\Rightarrow \frac{x-q}{p} = \frac{y}{1} = \frac{z-s}{r}$ ....(iii) Similarly, $\Rightarrow \frac{x-q'}{p'} = \frac{y}{1} = \frac{z-s'}{r'}$ ....(iv) if these given lines are perpendicular to each other, then $\Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\Rightarrow pp' + 1 + rr' = 0$ $\Rightarrow pp' + rr' + 1 = 0$, Which is the required condition.

	OR Equation of line passing through the point (2, 1, 3) $\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c}$ ----------(1) As (1) is perpendicular to given lines $1.a + 2.b + 3.c = 0$ $-3.a + 2.b + 5.c = 0 \quad \Rightarrow \frac{a}{10-6} = \frac{b}{-9-5} = \frac{c}{2+6} \quad ; \quad \frac{a}{4} = \frac{b}{-14} = \frac{c}{8}$ Hence equation of line is $\frac{x-2}{4} = \frac{y-1}{-14} = \frac{z-3}{8}$
36	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \Rightarrow \frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \quad \Rightarrow y^2 = \frac{b^2}{a^2}(a^2 - x^2)$ $\Rightarrow y = \frac{b}{a}\sqrt{a^2 - x^2}$ (i) Area , $A = (2x)(2y) = \frac{4b}{a}x\sqrt{a^2 - x^2}$ $\frac{dA}{dx} = \frac{4b}{a}\left[x.\frac{1}{2\sqrt{a^2 - x^2}}.-2x + \sqrt{a^2 - x^2}.1\right] = \frac{4b}{a}\left[\frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}}\right]$ (ii) put $\frac{dA}{dx} = 0$, then $x = \frac{a}{\sqrt{2}}$ (iii)lenghth $= 2x = \sqrt{2}a$; width $= 2y = \sqrt{2}b$
37	(i) The coordinates of D are (-8, -6, 0) and that of E are (0, 0, 24) $\therefore$ Vector $\overrightarrow{ED}$ is $(-8-0)\hat{i}+(-6-0)\hat{j}+(0-24)\hat{k}$ i.e., $-8\hat{i}-6\hat{j}-24\hat{k}$ (ii) The coordinates of B are (8, 6, 0) and that of E are (0, 0, 24), therefore length of cable EB $= \sqrt{(8-0)^2 + (6-0)^2 + (0-24)^2} = \sqrt{676} = \sqrt{26}$ units (iii) The coordinates of A are (8, -6, 0) , B are (8, 6, 0) C are (-8,6,0) , D are (-8, -6, 0) and that of E are (0, 0, 24). SO $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED} = (8\hat{i}-6\hat{j}-24\hat{k}) + (8\hat{i}+6\hat{j}-24\hat{k}) + (-8\hat{i}+6\hat{j}-24\hat{k}) + (-8\hat{i}-6\hat{j}-24\hat{k})$ $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED} = -96\hat{k}$ OR The coordinates of A are (8, -6, 0) and that of E are (0, 0, 24). So cartesian equation of line along $\overrightarrow{EA}$= $\frac{x-0}{8-0} = \frac{y-0}{-6-0} = \frac{z-24}{0-24} \quad \Rightarrow \quad \frac{x}{-4} = \frac{y}{3} = \frac{z-24}{12}$
38	Let E be the event that it rains on chosen day, F be the event that it does not rain on chosen

day and A be the event the weatherman predict rain.

Then we have, $P(E) = \frac{6}{366}$, $P(F) = \frac{360}{366}$,

$P(A \mid E) = \frac{8}{10}$ and $P(A \mid F) = \frac{2}{10}$

(i) $P(A) = P(E)\,P(A \mid E) + P(F)\,P(A \mid F)$

$= \frac{6}{366} \times \frac{8}{10} + \frac{360}{366} \times \frac{2}{10} = \frac{768}{3660} = \frac{64}{305}$

(ii) $P\left(\frac{E}{A}\right) = \frac{P(E)\,P(A \mid E)}{P(E)\,P(A \mid E) + P(F)\,P(A \mid F)}$

$$= \frac{\frac{6}{366} \times \frac{8}{10}}{\frac{6}{366} \times \frac{8}{10} + \frac{360}{366} \times \frac{2}{10}} = \frac{1}{16}$$

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SAMPLE PAPER 3

Class:-XII

Session 2023-24

Mathematics (Code-041)

ANSWER KEY

SECTION A

1.(a) 2.(c) 3. (b) 4.(a) 5 (a) 6.(d) 7.(d) 8. (d) 9(d) 10 (b)

11(b) 12. (d) 13 (c) 14 (d) 15 (a) 16 (c) 17 (a) 18 (d) 19 (a) 20 (c)

SECTION B (2 Marks each)

21. $\tan^{-1}(\sin(\left(-\frac{\pi}{2}\right)))$

$.=\tan^{-1}(-\sin(\frac{\pi}{2}))$ (1)

$=\tan^{-1}(-1)$ (1/2)

$=-(\frac{\pi}{4})$ (1/2)

22. $0 \leq \cos^{-1}(2x-3) \leq \pi$ (1/2)

$\text{Cos}(\pi) \leq 2x-3 \leq \text{Cos}(0)$ (1/2)

$-1 \leq 2x-3 \leq 1$ (1/2)

$1 \leq x \leq 2$ or Domain = [1,2] (1/2)

23. $f(x)=4x-\frac{1}{2}x^2$, $x\epsilon[-2,9/2]$

$f'(x)=4-x$ (1/2)

For maxima orminima $f^1(x)=0$

$\Rightarrow 4-x=0$

x=4 is a critical point (1/2)

For absolute maximum and minimum we need to find f value at x=4,−2,9/2

$$\left.\begin{array}{l} f(4) = 16 - 8 = 8 \\ f(-2) = -8 - 2 = -10 \\ (9/2) = 18 - 81/8 = 7.875 \end{array}\right\}$$ (1)

Hence absolute maximum at x=4 and value is8 absolute minimum at x=−2 and value is −10

OR

. $P(x) = 41- 72\ x – 18\ x^2$

$P^{1}(x) = -72-36\ x$

$P'(x) = 0 \Longrightarrow x= -2$ which is the critical point

$P''(x) = -36 < 0$,hence P(x) is maximum at $x = -2$ and max value = $P(-2)= 149$

24. Let I = $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}}\,dx$ (1/2)

$I = \int_1^2 \frac{\sqrt{3-x}}{\sqrt{3-x}+\sqrt{x}}\,dx$ (1/2)

Adding above 2 equations $2\ I = \int_1^2 \frac{\sqrt{x}+\sqrt{3-x}}{\sqrt{3-x}+\sqrt{x}}\,dx$ (1/2)

$2\ I = \int_1^2 dx$

I= ½ (1/2)

25. Given $\frac{dv}{dt} = 7\ cm^3/sec$ (1/2)

For a cube Volume (V) = x^3 (1/2)

$\frac{dv}{dt} = 3x^2\frac{dx}{dt}$ (1/2)

$7= 3\ x12^2\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{7}{432}$ cm/sec (1/2)

SECTION C

26. $\frac{x^2+1}{(x+3)(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-3}$ $\frac{1}{2}$

$x^2 + 1 = A(x-1)(x-3) + B(x+3) + C(x-1)^2$ $\frac{1}{2}$

Compairing coefficients ,we get $A = \frac{3}{8}$, $B = \frac{1}{2}$, $C = \frac{5}{8}$ 1

$\int \frac{x^2+1}{(x+3)(x-1)^2}\,dx = \frac{3}{8}\log|x-1| - \frac{1}{2(x-1)} + \frac{5}{8}\log|x+3| + C$ 1

27.

x	0	1	2	3	4
$P(x)$	0	k	4k	2k	k

$0 + k + 4k + 2k + k + 0 = 1$ $\frac{1}{2}$

$8k = 1$

$k = \frac{1}{8}$ $\frac{1}{2}$

$i)\ P(x \leq 2) = k + 4k = 5k = \frac{5}{8}$ `1

$ii)\ P(x \geq 2) = 4k + 2k + k = 7k = \frac{7}{8}$ 1

28.

$I = \int_0^{\pi} \frac{x\ tanx}{secx + tanx}\ dx \ldots\ldots\ldots (1)$

$I = \int_0^{\pi} \frac{(\pi - x)\tan(\pi - x)}{sec(\pi - x) + tan(\pi - x)}\ dx$

$\int_0^{\pi} \frac{(\pi - x)\tan(\pi - x)}{secx + tanx}\ dx \ldots\ldots\ldots (2)$ 1

$(1) + (2)\quad 2I = \pi \int_0^{\pi} \frac{\tan x}{secx + tanx}\ dx$ $\frac{1}{2}$

$= \pi \int_0^{\pi} tanx(secx - tanx)\ dx\ = \pi[secx - tanx + 1]_0^{\pi} = \pi(\pi - 2)$ 1

$I = \frac{\pi(\pi - 2)}{2}$ $\frac{1}{2}$

OR

$I = \int(\sqrt{cotx} + \sqrt{tanx}\ dx\ = \int \frac{cosx + sinx}{\sqrt{sinx\ cosx}}\ dx$ $\frac{1}{2}$

Put $sinx - cosx = t \qquad (\cos x + sinx)dx = dt$ $\frac{1}{2}$

$(sinx - cosx)^2 = t^2$

$1 - 2sinxcosx = \frac{1 - t^2}{2}$ $\frac{1}{2}$

There fore $I = \int \frac{dt}{\sqrt{\frac{1 - t^2}{2}}}\ = \sqrt{2}\ \sin^{-1} t + C = \sqrt{2}\ \sin^{-1}(sinx - cosx) + C$ $1\frac{1}{2}$

29.

$$\frac{dy}{dx} + y\,tanx = 3x^2 + x^3 tanx$$

Compairing given equation with linear form $P = tanx\,, Q = 3x^2 + x^3 tanx$ $\frac{1}{2}$

Integrating factor $e^{\int tanx dx} = e^{\log secx} = secx$ $\frac{1}{2}$

General solution is $y\,secx = \int (3x^2 + x^3 tanx\,)\,secx\,dx + C$

$= \int (3x^2 secx\,dx) + \int x^3 tanx\;\; secx\,dx + C$

$= \int (3x^2 secx\,dx) + x^3 secx - \int 3x^2 secx\,dx + C$

$y\,secx = x^3 secx + C$ 1

$y\; = x^3 + C\;cosx$

Now $y = 0$ when $x = \frac{\pi}{3}$

$0 = (\frac{\pi}{3})^3 + \mathrm{C}\cos\frac{\pi}{3}$

$C = -\frac{2\pi^3}{27}$ $\frac{1}{2}$

Required particular solution is

$y\; = x^3 - \frac{2\pi^3}{27}\;cosx$ $\frac{1}{2}$

OR

$x\frac{dy}{dx} = y - x\tan\frac{y}{x}$ ……..(1)

$\frac{dy}{dx} = \frac{y}{x} - \tan\frac{y}{x}$ $\frac{1}{2}$

The given equation is homogeneous

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in(1) $\frac{1}{2}$

$cotv\ dv = -\frac{dx}{x}$ $\frac{1}{2}$

Integrating both sides $\int cotv\ dv = -\int \frac{dx}{x}$

$\log|sinv| = -\log|x| + logC$ 1

Hence $xsin\frac{y}{x} = C$ is the required solution $\frac{1}{2}$

30.

For correct graph $1\frac{1}{2}$

Corner points	$Z = 5x + 2y$
(0,0)	0
(2,0)	10
$\left(\frac{7}{2}, \frac{3}{4}\right)$	19 maximum
$\left(\frac{3}{2}, \frac{15}{4}\right)$	15
$(0, \frac{3}{2})$	3

$\frac{1}{2}$

Z is maximum at $\left(\frac{7}{2}, \frac{3}{4}\right)$ and maximum value is 19. 1

OR

For correct graph $1\frac{1}{2}$

Corner points	$Z = -50x + 20y$
(0,5)	100
(0,3)	60
(1,0)	-50
(6,0)	-300

The feasible region is unbounded .There fore -300 may or may no be the minimum value of Z

Consider the graph $-50x + 20y < -300$

The resulting open half plane has points common with the feasible region.Therefore Z has no minimum value subject to the given constraints. $1\frac{1}{2}$

Z is maximum at $\left(\frac{7}{2},\frac{3}{4}\right)$ and maximum value is 19. 1

31.

Differentiating w r t x

$$\frac{dy}{dx} = n(x+\sqrt{1+x^2})^{n-1}(1+\frac{2x}{2\sqrt{1+x^2}})$$

$$= n(x+\sqrt{1+x^2})^{n-1}(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}) \quad \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{n(x+\sqrt{1+x^2})^{n}}{\sqrt{1+x^2}} \quad \frac{1}{2}$$

$$\sqrt{1+x^2}\ \frac{dy}{dx} = n(x+\sqrt{1+x^2})^{n}$$

$$\sqrt{1+x^2}\ \frac{dy}{dx} = ny \quad 1$$

Differentiating again w r to x

$$\sqrt{1+x^2}\ \ \frac{d^2y}{dx^2}+\frac{2x}{2\sqrt{1+x^2}}\frac{dy}{dx} = n\ \frac{dy}{dx}$$

$$(1+x^2)\ \ \frac{d^2y}{dx^2}+x\ \frac{dy}{dx} = n\sqrt{1+x^2}\ \frac{dy}{dx} \quad 1$$

$$(1+x^2)\ \ \frac{d^2y}{dx^2}+x\ \frac{dy}{dx} = n\sqrt{1+x^2}\ \frac{ny}{\sqrt{1+x^2}}$$

$$(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx} = n^2y$$

SECTION D (5 MARKS EACH)

32. Required area = $\int_0^1 area\ under\ circle - area\ under\ line$ (1)

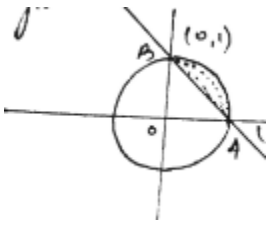


Figure (1 Mark)

$= \int_0^1 (\sqrt{1-x^2} - 1 + x)\, dx$ (1)

$= [\frac{X}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}(x) - x + \frac{x^2}{2}]^1$ (1)

$= \frac{\pi}{4} - \frac{1}{2}$ (1)

33. Let R be defined on N×N as

(a,b)R(c,d)⇔ad(b+c)=bc(a+d). ...

Reflexivity:

We can write ab(b+a)=ba(a+b) for all a,b∈N

Since, sum and product of natural numbers obeys commutative property

Hence, by def (1), we can write

(a,b)R(a,b) for all (a,b)∈N×N

Hence, R is reflexive. $(1\frac{1}{2})$

Symmmetry :

Let (a,b)R(c,d)

⇒ad(b+c)=bc(a+d)

⇒da(c+b)=cb(d+a) (Since, sum and product of natural numbers obeys commutative property)

or cb(d+a)=da(c+b)

⇒ (c,d) R (a,b)

Hence, R is symmetric $(1\frac{1}{2})$

Transitivity :

Let (a,b),(c,d),(e,f)∈N×N

Let (a,b)R(c,d) and (c,d)R(e,f)

ad(b+c)=bc(a+d) and cf(d+e)=de(c+f)

$\frac{b+c}{bc} = \frac{a+d}{ad}$ and $\frac{d+e}{de} = \frac{c+f}{cf}$

or $\frac{1}{b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{d}$ and $\frac{1}{d} + \frac{1}{e} = \frac{1}{c} + \frac{1}{f}$

adding above equations and cancelling common terms we have $\frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f}$

af (b+e)=be(a+f) (2)

Hence ,R is transitive

OR

Function f: R → R defined by $f(x) = 3x^3+5$

Let $f(x_1) = f(x_2)$ (1/2)

$3x_1^3+5 = 3x_2^3 +5$ (1/2)

$3(x_1- x_2)(x_1^2+x_1 x_2 + x_2^2) = 0$ (1)

$(x_1- x_2) = 0$ or $x_1 = x_2$,hence f is one-one (1)

For onto let $f(x) = y$ then $y = 3x^3+5$ (1/2)

$x = (\frac{y-5}{3})^{1/3}$ (1)

For every $y \in Y$, there exist $x \in X$ such that $f(x)= f((\frac{y-5}{3})^{1/3} = y$, hence f is onto (1)

Hence f is bijective

34. $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$B A = 6 I$ or $A^{-1} = \frac{1}{6} B$ (2)

Now given system can be written as $AX = C$ or $X= A^{-1} C$ (1)

$$X = \frac{1}{6} B C$$

$$X=\frac{1}{6}\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \quad (2)$$

x= 2, y= -1, z= 4

35. Given lines $\vec{r}= (4\vec{\imath} -\vec{\jmath})+ \lambda (\vec{\imath} + 2\vec{\jmath} - 3\vec{k})$ and

$\vec{r}= (\vec{\imath} -\vec{\jmath} + 2\vec{k})+ \mu (2\vec{\imath} + 4\vec{\jmath} - 5\vec{k})$

$\vec{a_1}= (4\vec{\imath} -\vec{\jmath})$ $\vec{b_1} = (\vec{\imath} + 2\vec{\jmath} - 3\vec{k})$

$\vec{a_2} = (\vec{\imath} -\vec{\jmath} + 2\vec{k})+ \vec{b_2} = (2\vec{\imath} + 4\vec{\jmath} - 5\vec{k})$

$\vec{a_2} - \vec{a_1} = (-3\vec{\imath} + 2\vec{k})$ $\vec{b_1} \times \vec{b_2} = (2\vec{\imath} - \vec{\jmath})$ (2)

$|\vec{b_1} \times \vec{b_2}| = \sqrt{5}$ (1)

Shortest distance $= \left|\frac{(\vec{a_2} - \vec{a_1}).\vec{b_1} \times \vec{b_2}}{|\vec{b_1} \times \vec{b_2}|}\right| = \frac{6}{\sqrt{5}}$ (2)

OR

$\vec{r}= (\vec{\imath}+\vec{\jmath}-\vec{k})+ \lambda(3\vec{\imath}-\vec{\jmath})$ **and** $\vec{r}= (4\vec{\imath}-\vec{k})+ \mu(2\vec{\imath}+3\vec{k})$

$\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}=\lambda$ and $\frac{x-4}{2}=\frac{y-0}{0}=\frac{z+1}{3}=\mu$ (2)

A (1+3λ,1- λ,-1) B (4+2μ,0,-1+3μ) (1)

If lines are intersecting 1- λ =0 ⇒ λ =1 , -1+3μ= -1⇒ $\mu = 0$

Also λ =1 &$\mu = 0$ satisfies 1+3λ=4+2μ .Hence lines are intersecting (1)

Point of intersection is given by (4,0,-1) (1)

SECTION E

36.let $A =$ the doctor visit the patient late

let E_1, E_2, E_3 and E_4 be the events that the doctor come by by cab, train, bike or other means of transport

$P(E_1) = 0.3\,, P(E_2) = 0.2\,\,, P(E_3) = 0.1\,, P(E_4) = 0.4$

$P\left(\frac{A}{E_1}\right) = 0.25\,, P\left(\frac{A}{E_2}\right) = 0.3,$

$P\left(\frac{A}{E_3}\right) = 0.35\,, P\left(\frac{A}{E_4}\right) = 0.1$ ½

$$i)P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\,P\left(\frac{A}{E_2}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right)+P(E_2)\,P\left(\frac{A}{E_2}\right)+P(E_3)\,P\left(\frac{A}{E_3}\right)+P(E_4)\,P\left(\frac{A}{E_4}\right)}$$

$$= \frac{0.2\times 0.3}{0.3\times 0.25+0.2\times 0.3+0.1\times 0.35+0.4\times 0.1} = \frac{2}{7}$$

½

$$ii)P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right)+P(E_2)\,P\left(\frac{A}{E_2}\right)+P(E_3)\,P\left(\frac{A}{E_3}\right)+P(E_4)\,P\left(\frac{A}{E_4}\right)}$$

$= \frac{0.3\times 0.25}{0.3\times 0.25+0.2\times 0.3+0.1\times 0.35+0.4\times 0.1} = \frac{5}{14}$ 1

$$iii)P\left(\frac{E_4}{A}\right) = \frac{P(E_4)\,P\left(\frac{A}{E_4}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right)+P(E_2)\,P\left(\frac{A}{E_2}\right)+P(E_3)\,P\left(\frac{A}{E_3}\right)+P(E_4)\,P\left(\frac{A}{E_4}\right)}$$

$= \frac{0.4\times0.1}{0.3\times0.25+0.2\times0.3+0.1\times0.35+0.4\times0.1} = \frac{4}{21}$ 2

OR

$$iii) P\left(\frac{E_3}{A}\right) = \frac{P(E_3)\,P\left(\frac{A}{E_3}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + P(E_3)\,P\left(\frac{A}{E_3}\right) + P(E_4)\,P\left(\frac{A}{E_4}\right)}$$

$= \frac{0.35\times0.1}{0.3\times0.25+0.2\times0.3+0.1\times0.35+0.4\times0.1} = \frac{1}{6}$ 1

P(doctor is late by any means) $= P\left(\frac{E_1}{A}\right) + P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right) + P\left(\frac{E_4}{A}\right)$

$= \frac{2}{7} + \frac{5}{14} + \frac{4}{21} + \frac{1}{6} = 1$ 1

37 . $f(x) = |x-3| + |x-2|$

$$f(x) = \begin{cases} (x-3)+(x-2)\,, & x \geq 3 \\ -(x-3)+(x-2\,), & 2 < x < 3 \\ -(x-3)-(x-2)\,, & x \leq 2 \end{cases}$$

$$f(x) = \begin{cases} 2x-5\,, & x \geq 3 \\ 1\,, & 2 < x < 3 \\ -2x+5\,, & x \leq 2 \end{cases}$$ **$\frac{1}{2}$**

when $x > 4$, $f(x) = 2x-5$ **$\frac{1}{2}$**

i) When $2 < x < 3$, $f(x) = 1$ **1**

ii)

$$f^{l}(x) = \begin{cases} 2\,, & x \geq 3 \\ 0\,, & 2 < x < 3 \\ -2\,, & x \leq 2 \end{cases}$$ **1**

LHD at $\boldsymbol{x = 3} = \mathbf{-2}$

RHD at $x = 3 = 2$

LHD ≠ RHD

The function is not differentiable 1

iii) No, because it is not continuous 1

$f(x) = [x]$ is not continuous **1**

38. Here (5, 3) are the coordinates of B. $\frac{1}{2}$

∴ P.V. of B $= 5\hat{\imath} + 3\hat{\jmath}$ $\frac{1}{2}$

i) P.V. of C $= 6\hat{\imath} + 5\hat{\jmath}$ $\frac{1}{2}$

$\overrightarrow{BC} = 6\hat{\imath} + 5\hat{\jmath} - (5\hat{\imath} + 3\hat{\jmath}) = \hat{\imath} + 2\hat{\jmath}$ $\frac{1}{2}$

ii) P.V of A $= 2\hat{\imath} + 2\hat{\jmath}$ P.V of D $= 9\hat{\imath} + 8\hat{\jmath}$ $\frac{1}{2}$

$\overrightarrow{AD} = 9\hat{\imath} + 8\hat{\jmath} - (2\hat{\imath} + 2\hat{\jmath}) = 7\hat{\imath} + 6\hat{\jmath}$ **1**

$|\overrightarrow{AD}| = \sqrt{7^2 + 6^2} = \sqrt{85}$ units $\frac{1}{2}$

OR

iii) $\widehat{M} = \frac{\vec{M}}{|\vec{M}|}$ $\frac{1}{2}$

$= \frac{4\hat{\jmath}+3\hat{k}}{\sqrt{4^2+3^2}}$ **1**

$= \frac{4\hat{\jmath}+3\hat{k}}{5}$ $\frac{1}{2}$

……

SAMPLE PAPER 4

Marking scheme

Part A

1. (A) Skew symmetric matrix
2. (D) $(A + B)^{-1} = B^{-1} + A^{-1}$
3. (C) 64
4. (D) None of these
5. (B) 1
6. (C) 3
7. (D) q = 3p
8. (C) (4,2)
9. (A) $\vec{o}$
10. (B) $\frac{\pi}{3}$
11. (A) 2
12. (A) 0
13. (D) None of these
14. (C) $6x^2 \sin x^3 \cos x^3$
15. (A) $\sqrt{5}$ sq.units
16. (C) $\frac{1}{x}$
17. (A) $\frac{\pi}{6}$
18. (C) 1- P(A)PB')
19. (A) Both A and R are true and R is the correct explanation of A
20. (A) A is false and R are true and R

Part B

21. $\tan^{-1}[\tan(8\pi + \frac{3\pi}{8})] = \tan^{-1}[\tan(\frac{3\pi}{8})] = \tan^{-1}[\tan\frac{3\pi}{6})]$ 1 mark

    - $\frac{3\pi}{8}$ 1 mark

    or

    Let $y = \cos^{-1}(x^2 - 4) \Rightarrow \cos y = x^2 - 4$ i.e. $-1 \le x^2 - 4 \le 1$ (since $-1 \le \cos y \le 1$) 1 mark

    $\Rightarrow 3 \le x^2 \le 5 \Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup \in [\sqrt{3}, \sqrt{5}]$ 1 mark

22. Slope of curve $\frac{dy}{dx} = 5 - 6x^2$ ½ mark

    Rate of change of slope $= \frac{d}{dt}(\frac{dy}{dx}) = -12x \cdot \frac{dx}{dt}$ 1 mark

    $= -12 \cdot (3) \cdot (2) = -72$ units/sec. Thus, slope of curve is decreasing at the rate of 72 units/sec ½ mark

23. $F'(x) = -x^2 e^{-x} + 2xe^{-x} = f(x) = -xe^{-x}(x-2)$ 1 mark

    Critical points x=0,2 ½ mark

    Increasing in [0,2] ½ mark

    Or

    $f(x) = \frac{1}{9x^2+6x+5}$ maximum when $9x^2 + 6x + 5$ is minimum ½ mark

    Minimum value of $9x^2 + 6x + 5 = (3x+1)^2 + 4$ is 4 1 mark

    Hence maximum value of $f(x) = \frac{1}{4}$ ½ mark

24. I= $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx$ ---(1) apply P3 I=$\int_2^8 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}$ --(2) ½ mark

(1) + (2) => 2I =$\int_2^8 1dx$ = $[x]_2^8$ 1 mark

I =3 ½ mark

25. f ′ (x) = $12x^2 - 36x + 27 = 3(4x^2 - 12x + 9) = 3(2x-3)^2$ 1 mark

f ′ (x) = 0 ⇒ x = $\frac{3}{2}$ (critical point) Since f ′ (x) ≥0 for all x ⇒ x = $\frac{3}{2}$ point of inflection 1 mark

Part C

26. $\int \frac{1}{(x^2+1)(x^2+4)}dx$

Put $x^2 = y$

$\int \frac{1}{(y+1)(y+4)}dx = \int \frac{1}{(y+1)(y+4)}dx$

$\frac{1}{(y+1)(y+4)} = \frac{A}{(y+1)} + \frac{B}{y+4}$

$1 = A(y+4) + B(y+1)$

Put y=-1 ,A=$\frac{1}{3}$put y=-4 get B=$\frac{-1}{3}$ 1mark

$\int \frac{1}{(y+1)(y+4)}\,dx = \int\{\frac{\frac{1}{3}}{(y+1)} + \frac{-\frac{1}{3}}{y+4}\}dx = \int\{\frac{\frac{1}{3}}{(x^2+1)} + \frac{-\frac{1}{3}}{x^2+4}\}dx$ 1 mark

$\frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\frac{x}{2} + C$ 1 mark

27. $\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx$ multiply numerator and Denominator by$\sqrt{a-x}$ $\frac{1}{2}$ mark

$I = \int_{-a}^{a}\sqrt{\frac{(a-x)^2}{a^2-x^2}}\,dx = \int_{-a}^{a}\frac{a-x}{\sqrt{a^2-x^2}} = \int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}} - \int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx$ 1 mark

by property 7, $\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx = 0$ because $\frac{x}{\sqrt{a^2-x^2}}$ $is\ odd$ 1 mark

$I = a\sin^{-1}\frac{x}{a}\,]_{-a}^{a} = a[\frac{\pi}{2} - -\frac{\pi}{2}] = a\pi$ $\frac{1}{2}$ mark

Or

$put\ logx = t\ then\ x = e^t, dx = e^t dt$ 1 mark

$\int[\frac{1}{logx} - \frac{1}{(logx)^2}]dx$

$= \int[\frac{1}{t} - \frac{1}{t^2}]e^t dt$ 1 mark

$(x) = \frac{1}{t}$ then f′(x)=$-\frac{1}{t^2}$ integral of the form $\int(f(x) + f'(x))\,e^x dx = e^x f(x) + c$

$= \frac{1}{t}e^t + C = \frac{1}{logx}e^{logx} + C$

$= \frac{x}{logx} + C$ 1 mark

28. Puy x=uy then$\frac{dx}{dy}$ =u +y$\frac{du}{dy}$ ½ mark

$\frac{dx}{dy} = \frac{2xe^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}}$ =>u $+ y\frac{du}{dy} = \frac{2ue^u - 1}{2e^u}$ => y$\frac{du}{dy} = \frac{2ue^u - 1}{2e^u}$ -u=> y$\frac{du}{dy} = \frac{2ue^u - 1 - 2u2e^u}{2e^u}$ 1 mark

$\int 2e^u du = -\int\frac{dy}{y}$ =>$2e^u = -log|y| + C$=>$2e^{\frac{x}{y}} = -log|y| + C$ 1mark

$2e^{\frac{x}{y}} + log|y| = 2$ ½ mark

Or

Linear in x , $\frac{(tan^{-1}y - x)}{(1+y^2)} = \frac{dx}{dy} \Rightarrow \frac{(tan^{-1}y}{(1+y^2)} - \frac{x}{(1+y^2)} = \frac{dx}{dy} \Rightarrow \frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{(tan^{-1}y}{(1+y^2)}$

1 mark

I.F.$= e^{\int \frac{1}{(1+y^2)}} = e^{\tan^{-1}y}$ ½ mark

Solution is $xe^{\tan^{-1}y} = \int e^{\tan^{-1}y} \frac{(tan^{-1}y}{(1+y^2)} dy$ ½ mark

Put $\tan^{-1}y = t \; then \frac{1}{(1+y^2)} dy = dt$, $\int e^{\tan^{-1}y} \frac{(tan^{-1}y}{(1+y^2)} dy = \int te^t dt = te^t - e^t$

Solution is

$xe^{\tan^{-1}y} = \tan^{-1}y \, e^{\tan^{-1}y} - e^{\tan^{-1}y} + C \Rightarrow x = \tan^{-1}y - 1 + c\, e^{-\tan^{-1}y}$ 1 mark

29. Differentiating $y\frac{x}{\sqrt{x^2+1}} + \sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}-x} \times (\frac{x}{\sqrt{x^2+1}} - 1)$ $1\frac{1}{2}$ marks

$y\frac{x}{\sqrt{x^2+1}} + \sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}-x} \frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}$ $\frac{1}{2}$mark

multiply by $\sqrt{x^2+1}$ and simplify

$xy + (x^2+1)\frac{dy}{dx} = -1$ 1 mark

30.

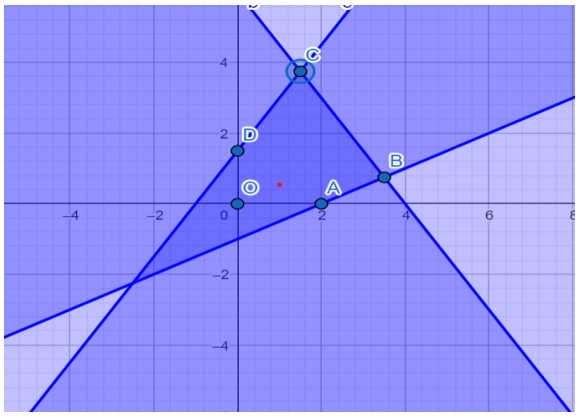
X denote the number of red cards drawn in 2 draws, X takes the values 0,1,2, $\frac{1}{2}$mark

X	0	1	2
P(X)	$\frac{25}{102}$	$\frac{52}{102}$	$\frac{25}{102}$

$1\frac{1}{2}$marks

Mean= 1 ½ mark

31.



$1\frac{1}{2}$marks

Feasible region pentagon OABCD

Z at O (0,0)=0

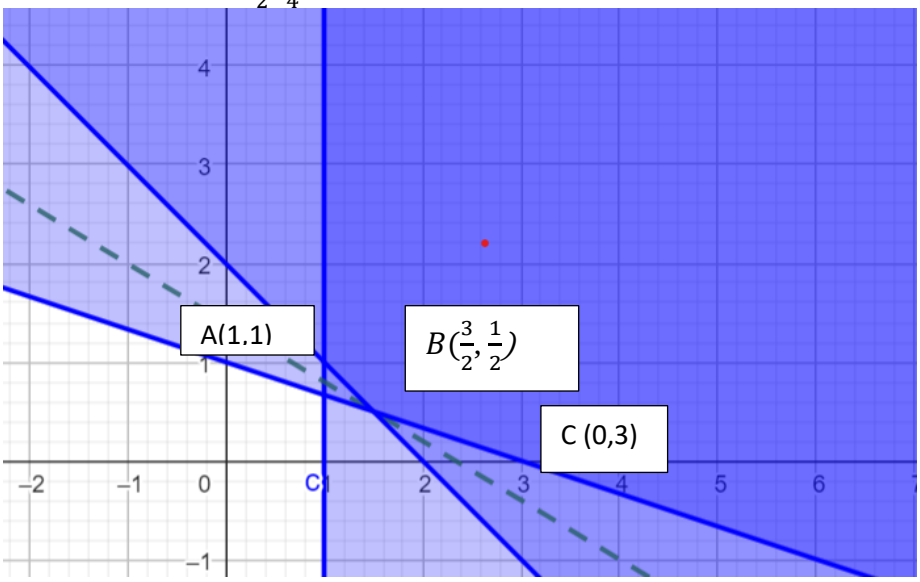
Z at A(2,0)=10

Z at B($\frac{7}{2}, \frac{3}{4}$) =19

Z at D(0, $\frac{3}{2}$)=3

Z at C($\frac{3}{2}, \frac{15}{4}$)= 15 $1\frac{1}{2}$marks

Maximum value B($\frac{7}{2}, \frac{3}{4}$) =19



Z at A(1,1)=8

Z at B($\frac{3}{2}, \frac{1}{2}$)=7

Zat C(3,0)=9 minimum value 7 at x=$\frac{3}{2}$ and y=$\frac{1}{2}$

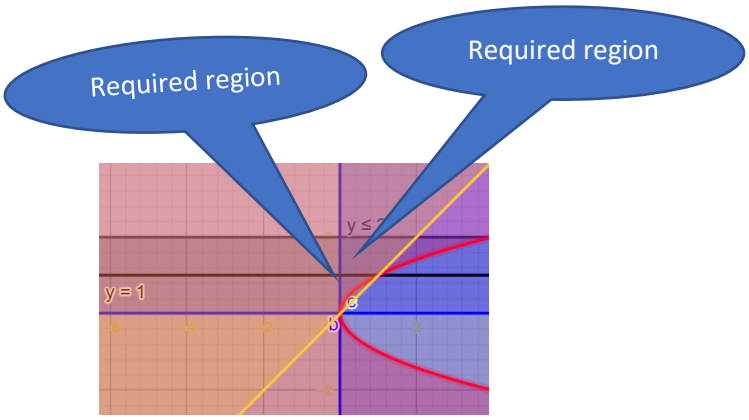
Part D

32. eliminating x from y^2=x and y=x we get y=0 0r y=1

Required area =$\int_0^1 y^2\, dy + \int_1^2 y\, dy$

$=\frac{1}{3}y^3]_0^1 + \frac{1}{2}y^2]_1^2$

$=\frac{1}{3} + \frac{3}{2} = \frac{11}{6}$ sq units



33. Reflexive because for every a∈A , $|a - a|$ =0 is a multiple of 4 1 mark

Symmetry because for every a,b ∈A such that $|a - b|$ is a multiple of 4 =>$|b - a|$ is a multiple of 4 1 mark

Transitive, for a,b and c ∈A, such that aRb and bR c

$=>|a-b|$ is a multiple of 4 =>a-b is a multiple of 4

And $|b-c|$ is a multiple of 4 =>b-c is a multiple of 4

a-b is a multiple of 4 and b-c is a multiple of 4=>a-b+b-c=a-c is a multiple of 4=> $|a-c|$ is a multiple of 2 marks

[1] ={1,5,9} 1 mark

[1] ={1,5,9}

or

For x,y ∈ R, consider f (x) = f (y) ⇒ $\frac{x}{x^2+1}=\frac{y}{y^2+1}$ ½ mark

- ⇨ $xy^2+x = yx^2+y$ ½ mark
- ⇨ $xy^2-yx^2 = y-x$ ½ mark
- ⇨ xy(y-x)-(y-x)=0 ½ mark
- ⇨ (y-x)(xy-1)=0 ½ mark
- ⇨ X=y or x=$\frac{1}{y}$ f(x) is not one to one ½ mark

Not onto 1 has no preimage when f(x)=1 =>$\frac{x}{x^2+1}=1$ 1 mark

=>$x^2-x+1=0$ which has no real root so 1 has no preimage 1 marks

1 Mark each

34. Find the foot of the perpendicular

D.R of line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}=>\frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3}=m$ *is* -2,6,-3 1 Mark

Any point on the line(-2m+4,6m,-3m+1) let it be Q ½ mark

D.R of PQ=-2m+2,6m-3,-3m+9 1 Mark

PQ Perpendicular to the line =>-2(-2m+2)+6(6m-3)+-3(-3m+9)=0=>m =1 1 mark

Foot of perpendicular became Q(2,6,-2) ½ mark

Equation of PQ: $\frac{x-1}{0}=\frac{y-3}{3}=\frac{z+8}{6}$ 1 mark

Or

Any point on $\frac{x}{1}=\frac{y}{-1}=\frac{z}{1}$ = p…(1) P is (p,-p,p) ½ mark

Any point on $\frac{x-1}{0}=\frac{y+1}{-2}=\frac{z}{1}=q$---(2) Q is(1,-2q-1,q) 1marl

D.r PQ p-1, -p+2q+1, p-q 1 mark

d.r of (1) 1,-1,1 and D.r of (2) 0,-2,1 1 mark

using PQ ⊥ (1) and (2) and getting q = 0, p=$\frac{2}{3}$ 1 mark

Required points P ($\frac{2}{3},\frac{2}{3},\frac{2}{3}$) and Q (1, -1,0) ½ mark

35. Writing in the form AX=B $\frac{1}{2}mark$

$|A =|=-8$ $1mark$

$A^{-1}=\frac{-1}{8}\begin{bmatrix}-3 & 7 & -1\\ 5 & -17 & -1\\ -2 & 2 & 2\end{bmatrix}$ $1\frac{1}{2}mark$

X=$A^{-1}B$ $\frac{1}{2}mark$

X= $\frac{1}{2}$,y = $\frac{3}{2}$, = -1 $1\frac{1}{2}mark$

36. (i) $\hat{2\imath} + 2\hat{\jmath} - \hat{k}$ 1 mark

(ii) $6\hat{\imath} + 3\hat{\jmath} + 2\,\hat{k}$ 1 mark

(iii) $8\hat{\imath} + 5\hat{\jmath} + \hat{k}$,D.Cs $\frac{8}{3\sqrt{10}}, \frac{5}{3\sqrt{10}}, \frac{1}{3\sqrt{10}}$ (1+1) marks

Or $7\hat{\imath} - 10\hat{\jmath} - 6\,\hat{k}$,D.Cs $\frac{7}{\sqrt{185}}, \frac{-10}{\sqrt{185}}, \frac{-6}{\sqrt{185}}$

37. (i)LHD=0 2 marks

(ii) RHD=0 1 mark

LHD=RHD=0=f'(1) 1 mark

38. A,B,C and G are the events seed A chosen, Seed B chosen ,Seed C chosen and G seed germinate then

(i) the probability of a randomly chosen seed to germinate

P(G)=P(A)P(G/A)+ P(B)P(G/B)+ P(C)P(G/C) 1 mark

$=\frac{4}{10}\times\frac{45}{100}+\frac{4}{10}\times\frac{60}{100}+\frac{2}{10}\times\frac{35}{100}=\frac{49}{100}$ (1marks)

P(C/G') $=\frac{P(C)\times P(G'/C)}{P(G')}=\frac{P(C)[1-P(\frac{G}{C})]}{1-P(G)}$

$=\frac{\frac{2}{10}\times[1-\frac{35}{100}]}{1-\frac{49}{100}}=\frac{13}{51}$

SAMPLE PAPER 5

MARKING SCHEME

QUESTION NO	ANSWERS	MARKS
	SECTION A	
1.	c.symmetric matrix	1
2.	c-I	1
3.	a.not defined	1
4.	b-π/3	1
5.	c-2	1
6.	d-1.5	1
7.	c- -1	1
8.	a- 0	1
9.	a- $e^x \cos x + c$	1
10.	d. sin x-cosy	1
11.	d- 2	1
12.	a-0	1
13.	b- -4	1
14.	c-π/2	1
15.	a- (-2,5,-6)	1
16.	c-R	1
17.	c-(2,3)	1
18.	d-7/12	1
19.	c-. A is true but R is false	1
20.	a- Both A and B are true and R is the correct explanation of A	1
	SECTION B	
21.	π/3+2π/6+π/3 π **OR** $\tan^{-1}\left[2\sin\left(2x\frac{\pi}{6}\right)\right]$, $\tan^{-1}\left[2\frac{\sqrt{3}}{2}\right]$ **π/3**	11/2 1/2 1/2 1 1/2
22.	dp/dx=2(-3+2)=-2cm/min dA/dx=-3x6+10x2=2cm^2/min	1 1
23.	dy/dΘ=aΘcosΘ, dx/dΘ=aΘsinΘ dy/dx=tanΘ OR Taking log on both side 1/y(dy/dx)= (sin x-cos x) $\frac{\cos x+\sin x}{\sin x-\cos x}$ +log(sin x-cos x) (sin x + cos x) dy/dx = (sin x-cos x)$^{(\sin x-\cos x)}$((sin x-cos x) $\frac{\cos x+\sin x}{\sin x-\cos x}$ +	1/2+1/2 1 1/2 1 1/2

	log(sin x-cos x) (sin x+ cos x))	
24.	λ=3 μ=27/2	1 1
25.	Dr(3,-5,6) $\frac{x+2}{3}=\frac{y-4}{-5}=\frac{z+5}{6}$ $\vec{r}=-2\hat{i}+4\hat{j}-5\hat{k}+\lambda(3\hat{i}-5\hat{j}+6k)$	1/2 1 1/2
	SECTION C	
26.	$\frac{1}{(x^2+1)(x-1)}=\frac{Ax+B}{(x^2+1)}+\frac{C}{(x-1)}$ A =-1/2 B=-1/2 C=1/2 $\int\frac{dx}{(x^2+1)(x-1)}=\frac{-1}{4}\log\lvert x^2+1\rvert+\frac{1}{2}\tan^{-1}x+\frac{1}{2}\log\lvert x-1\rvert+c=$	1/2 1 11/2
27.	$\int_{-5}^{-2}-(x+2)dx+\int_{-2}^{2}(x+2)d$ -{(2-4-(25/2-10)]+[(2+4)-(2-4)] 9/2+8=25/2 **OR** $2I=\int_0^{\pi/4}\log 2dx-I$ I=π/8 log2	1 1 1 2 1
28.	$\frac{1}{9}\int\frac{1}{\left(x+\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2}dx$ $\frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right)+c$	2 1
29.	P=2/x Q=x I.F=x^2 $Yx^2=x^4/4+c$ **OR** $\frac{dy}{dx}=\frac{y+\sqrt{x^2+y^2}}{x}$ $\frac{dv}{\sqrt{1+v^2}}=\frac{dx}{x}$ $\log\left\lvert v+\sqrt{1+v^2}\right\rvert=\log\lvert x\rvert+c$ $\log\left\lvert\frac{y}{x}+\sqrt{1+(\frac{y}{x})^2}\right\rvert=\log\lvert x\rvert+c$	1/2 1 11/2 1/2 1 1 1/2

30.	Each graph with solution region Corner points(0,0) (4,0)(2,3)(0,4) Minimum value=-12	1 1 1
31.	{BB,BG,GB,GG}=S A= both are girls B= youngest is a girl C= at least one is a girl P(A/B)=1/2 p(A/C)=1/3 **OR** $p(\overline{A}) = 1/2 \quad p(\overline{B}) = 2/3$ a.P(problem is solved)=1-(1/2x2/3)=2/3 b.(P exactly one of them solves the problem)=1/3+1/6=3/6=1/2	1 1+1 1 1 1
	SECTION D	
32.	1.(a,b)R(a,b)gives a+b=b+a which is true. R is reflexive 2.(a,b)R(c,d) ⇒ a+d=b+c (c,d)R(a,b) ⇒ a+d=b+c (a,b)R(c,d) ⇒ (c,d)R(a,b) R is symmetric 3. (a,b)R(c,d) ⇒ a+d=b+c --(1) (c,d)R(e,f) ⇒ e+d=f+c ---(2) Adding(a+d)+ (f+c)=(b+c)+(e+d) a+f=b+e gives (a,b)R(e,f) R is transitive R is an equivalence relation **OR** 1.|a-a|=0 which is divisible by 4. R is reflexive 2.(a,b)ЄR implies |a-b| is divisible by 4 implies |b-a| is divisible by 4 implies (b,a)ЄR R is symmetric 3. (a,b)(c,d)ЄR implies |a-b| is divisible by 4, implies |c-d|is divisible by 4 a-b=±4m,c-d=±4n a-c=±4m+±4n |a-c|ЄR R is transitive R is an equivalence relation Set of elements related to 1 is{1,5,9}	1 1 2 1 1 1 1 1 1
33.	Figure and point of intersection (√2,√2) $A = \int_0^{\sqrt{2}} x\,dx + \int_{\sqrt{2}}^{2} \sqrt{4-x^2}\,dx$ =1+2(π/2)-1-2(π/4)=π/2	2 1 2
34.	$A.C = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Since AC =I ,A^{-1} =C Then A. X = B	1 1 11/2

	$X= A^{-1}B = C .B=\begin{bmatrix}0\\5\\3\end{bmatrix}$ x=0, y=5, z=3	11/2
35.	$\frac{x-1}{a}=\frac{y-2}{b}=\frac{z+4}{c}$ 3a-16b+7c=0 3a+8b-5c=0 a/24=b/36=c/72 $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ **OR**	1/2 1 1 11/2 1
	$\vec{a}_2-\vec{a}_1=-6\hat{i}-15\hat{j}+3\hat{k}$ $\vec{b}_1 x\vec{b}_2=-6\hat{i}-15\hat{j}+3\hat{k}$ s.d=270/√270=√270units	1 1 3
	SECTION E	
36.	i. 2x+4y+πx=10 ii. A= 10x - (2+$\boldsymbol{\pi}$/ **2**)x^2 iii. **20**/(**4**+$\boldsymbol{\pi}$)m **OR** iii.A = **50**/(**4**+$\boldsymbol{\pi}$)m^2	1 1 2
37.	i. $6(x+1)^2(x-3)^2(x-1)$ ii. -1,1,3 iii. (1,3) U (3,∞) **OR** iii.(-∞, −1)U(-1,1)	1 1 2 2
38.	E : person selected is actually having HIV E' : person selected is actually not having HIV A : person's HIV test is diagnosed as +ive. P(E) = 0.001 P(E') =0.999 P(A|E) = 0.90 P(A|E') = 0.01 (i) $P(A) = P(E)P(A\|E) + P(E')P(A\|E') = 001X.90 + 0.999X0.01=0.01089$ (ii) $P(E\|A) = \frac{P(E)P(A\|E)}{P(E)P(A\|E)+P(E')P(A\|E')}$ $= \frac{90}{1089}$ = 0.083 approx.	2 2 2

SAMPLE PAPER 6

SECTION A

1.(A) 2. (D) 3. (D) 4. (D) 5.(B) 6. (D) 7. (C) 8. (C) 9. .(A) 10 .(B)

11. (C) 12.(A) 13. .(B) 14. .(B) 15. (C) 16. (D) 17 (A) 18(A) 19. (a) 20.(a)

SECTION B

21) $COS^{-1}\,COS\left(\frac{13\,\pi}{6}\right) = COS^{-1}\,COS\left(2\pi + \frac{\pi}{6}\right)$

$= COS^{-1}\,COS\left(\frac{\pi}{6}\right) = \frac{\pi}{6} \in (0,\pi)$ $(\frac{1}{2}\ M)$

$tan^{-1}\,tan\left(\frac{5\,\pi}{6}\right) = tan^{-1}\left(tan\left(\pi - \frac{\pi}{6}\right)\right)$

$= -\frac{\pi}{6} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (1 M)

$\therefore$ Sum = $\frac{\pi}{6} + -\frac{\pi}{6} = 0$ $(\frac{1}{2}\ M)$

Or

$sin\left(cot^{-1}\left(cos\,\frac{\pi}{4}\right)\right) = sin\left(cot^{-1}\,\frac{1}{\sqrt{2}}\right)$ $(\frac{1}{2}\ M)$

$= sin\left(sin^{-1}\left(\frac{\sqrt{2}}{\sqrt{3}}\right)\right)$ (1 M)

$= -\sqrt{\frac{2}{3}}$ $(\frac{1}{2}\ M)$

22) $f(x) = (x+2)e^{-x}$

$f'(x) = (x+2)(-e^{-x}) + e^{-x}(1)$

$= -e^{-x}(x+1)$ $(\frac{1}{2}\ M)$

$f'(x) = 0 \Rightarrow e^{-x}(x+1) = 0$

$\Rightarrow$ x+1 = 0 $\Rightarrow$ x=-1

$\therefore$ Intervals are (-∞, -1) (-1, ∞) $(\frac{1}{2}\ M)$

<u>In (-∞, -1)</u>

Put x = -2, $f'(x) =$ (-)(+)(-)= +ve

$\therefore$ f(x) is st. ↑ in (-∞, -1). $(\frac{1}{2}\ M)$

<u>In (-1, ∞)</u>

Put x = 0, $f'(x) =$ (-)(+)(+)= $-$ve

$\therefore$ f(x) is st. ↓ in (-1, ∞). $(\frac{1}{2}\ M)$

23) $f'(x) = 1 + 2Cos\,2x$ $(\frac{1}{2}\ M)$

$f'(x) = 0 \Rightarrow$ x = $\frac{\pi}{3}$ or x = $\frac{2\pi}{3}$ $(\frac{1}{2}\ M)$

$f(0) = 0, f\left(\frac{\pi}{3}\right) = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$

$f\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ $(\frac{1}{2}\ M)$

$f(2\pi) = 2\pi$

$\therefore$ Max Value = 2π $(\frac{1}{2}\ M)$

24) Put $x^5 = t$ $(\frac{1}{2}\ M)$

$5x^4 dx = dt$

$x^4 dx = \frac{dt}{5}$ $(\frac{1}{2}\ M)$

I $= \int \frac{\frac{dt}{5}}{1+t} = \frac{1}{5}\int \frac{dt}{1+t} = \frac{1}{5}\log(1+t) + C$ $(\frac{1}{2}\ M)$

$= \frac{1}{5}\log(1+x^5) + C$ $(\frac{1}{2}\ M)$

25) Difference w.r.t x →

$\frac{dy}{dx} = 2\ tan^{-1}\ x\ \frac{1}{1+x^2}$ $(\frac{1}{2}\ M)$

$\therefore (1+x^2)y_1 = 2\ tan^{-1}\ x$ $(\frac{1}{2}\ M)$

Difference w.r.t x →

$(1+x^2)y_2 + y_1(2x) = 2 \text{ x } \frac{1}{1+x^2}$ $(\frac{1}{2}\ M)$

$(1+x^2)y_2 + 2x\ (1+x^2)y_1 = 2$ $(\frac{1}{2}\ M)$

SECTION C

26) I $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{tan\ x}}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\frac{\sqrt{sin\ x}}{\sqrt{cos\ x}}}$ $(\frac{1}{2}\ M)$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{cos\ x}\ dx}{\sqrt{cos\ x}+\sqrt{sin\ x}}$ $(\frac{1}{2}\ M)$

I $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{cos\left(\frac{\pi}{2}-x\right)}\ dx}{\sqrt{cos\left(\frac{\pi}{2}-x\right)}+\sqrt{sin\left(\frac{\pi}{2}-x\right)}}$ $(\frac{1}{2}\ M)$

I $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{sin\ x}\ dx}{\sqrt{sin\ x}+\sqrt{cos\ x}}$ $(\frac{1}{2}\ M)$

Adding these,

2 I $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1\ dx = [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ $(\frac{1}{2}\ M)$

I $= \frac{\pi}{12}$ $(\frac{1}{2}\ M$

27) Let X denote the no. of defective bulbs

Then X = 0, 1, 2 $(\frac{1}{2}\ M)$

P(X=0) $= \frac{{}^3C_0 \text{ x } {}^7C_2}{{}^{10}C_2} = \frac{7}{15}$ $(\frac{1}{2}\ M)$

P(X=1) $= \frac{{}^3C_1 \text{ x } {}^7C_1}{{}^{10}C_2} = \frac{7}{15}$ $(\frac{1}{2}\ M)$

P(X=2) $= \frac{{}^3C_2 \text{ x } {}^7C_0}{{}^{10}C_2} = \frac{1}{15}$ $(\frac{1}{2}\ M)$

$\therefore$ Prob. Distrn of X is given by

X	0	1	2	Total
P(X)	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$	1

(1 M)

28) $\int e^x \left(\frac{2\sin 2x \cos 2x - 4}{2\sin^2 2x}\right) dx$ (1 M)

$= \int e^x (\cot 2x - 2\,\mathrm{cosec}^2 2x) dx$ ($\frac{1}{2}$ M)

$= \int e^x \big(f(x) + f'(x)\big) dx$ ($\frac{1}{2}$ M)

$= e^x . f(x)$ + C

$= e^x . \cot 2x$ + C (1 M)

Or

I $= \int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}$ dx

$= \int_0^\pi \frac{e^{\cos(\pi - x)}}{e^{\cos(\pi - x)} + e^{-\cos(\pi - x)}}$ dx (1 M)

I $= \int_0^\pi \frac{e^{-\cos x}}{e^{-\cos x} + e^{\cos x}}$ dx (Using $\int_0^a f(x)dx = \int_0^a f(a-x)dx$) ($\frac{1}{2}$ M)

Adding these →

$2I = \int_0^\pi 1$ dx $= [x]_0^\pi$ (1 M)

$= \pi$

$\therefore I = \frac{\pi}{2}$ ($\frac{1}{2}$ M)

29) Dividing by $\cos^2 x$ →

$\frac{dy}{dx} + sec^2 x . y = sec^2 x . \tan x$ ($\frac{1}{2}$ M)

This is of the form, $\frac{dy}{dx} + Py = Q$

Where P = $sec^2 x$, Q = $sec^2 x \tan x$

$\int P\,dx = \int sec^2 x\,dx = \tan x$

$e^{\int P\,dx} = e^{\tan x}$ ($\frac{1}{2}$ M)

Soln is $y . e^{\int P\,dx} = \int Q\, e^{\int P\,dx} dx$ ($\frac{1}{2}$ M)

$i.e.\ y . e^{\tan x} = \int sec^2 x \tan x . e^{\tan x} dx$ ($\frac{1}{2}$ M)

Put tan x = t $\therefore sec^2 x\,dx = dt$

$\therefore$ Soln is $y . e^{\tan x} = \int t\, e^t\, dt$ ($\frac{1}{2}$ M)

$= t\, e^t - \int e^t dt$

$=$ tan x . $e^{\tan x} - e^{\tan x}$ + C

$\therefore$ Soln is $y = (\tan x - 1) + C\, e^{-\tan x}$ ($\frac{1}{2}$ M)

Or

$\frac{dx}{dy} = \frac{\tan^{-1} y - x}{1 + y^2}$

$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$ ($\frac{1}{2}$ M)

This is of the form, $\frac{dy}{dx} + Px = Q$;

Where P = $\frac{1}{1+y^2}$, Q = $\frac{tan^{-1}y}{1+y^2}$

$\int P\,dy = \int \frac{1}{1+y^2}dy = tan^{-1}y$ $(\frac{1}{2}\,M)$

$e^{\int P\,dy} = e^{(tan^{-1}y)}$

∴ Soln is $x.e^{\int P\,dy} = \int Q\,e^{\int P\,dy}$dy + C $(\frac{1}{2}\,M)$

i.e. $x.e^{tan^{-1}y} = \int \frac{tan^{-1}y}{1+y^2}.e^{tan^{-1}y}$dy + C $(\frac{1}{2}\,M)$

Put $tan^{-1}y = t$

$\frac{1}{1+y^2}dy = dt$

∴ Soln is $x.e^{tan^{-1}y} = \int t\,e^t\,dt$

$= t\,e^t - \int e^t dt$

$= t\,e^t - e^t + C$

$= e^t$(t-1) + C

i.e. $x.e^{tan^{-1}y} = e^{tan^{-1}y}(tan^{-1}y - 1)$+ C $(1\,M)$

30) Drawing 1st line $(\frac{1}{2}\,M)$

Drawing 2nd line $(\frac{1}{2}\,M)$

Shading the feasible region $(\frac{1}{2}\,M)$

Writing the corner points O(0,0), A(20,0), B(10,50), C(0,60) $(\frac{1}{2}\,M)$

Corner Points	Value of Z = 50x + 15y
O(0,0)	15
A(20,0)	1000
B(10,50)	1250
C(0,60)	900

$(\frac{1}{2}\,M)$

∴ Max. Value = 1250 $(\frac{1}{2}\,M)$

Or

Drawing each lines $(1\,M)$

Shading the feasible region $(\frac{1}{2}\,M)$

Writing the corner points O(0,0), A(20,0), B(10,50), C(0,60) $(\frac{1}{2}\,M)$

Corner Points	Value of Z = 50x + 15y
A(10,0)	50
B(2,4)	38
C(0,8)	56

$(\frac{1}{2}\,M)$

∴ Min. Value = 38 $(\frac{1}{2}\,M)$

31) $\frac{dx}{d\theta}$ = a (-sin Θ + $\frac{1}{tan\frac{\theta}{2}} x\ sec^2\frac{\theta}{2}\ x\ \frac{1}{2}$)

= a (-sin Θ + $\frac{cos\frac{\theta}{2}}{sin\frac{\theta}{2}}$ x $\frac{1}{cos^2\frac{\theta}{2}}\ x\ \frac{1}{2}$)

= a (-sin Θ + $\frac{1}{sin\ \theta}$)

= a ($\frac{1-sin^2\theta}{sin\ \theta}$) = $\frac{a\ cos^2\theta}{sin\ \theta}$ (1 M)

$\frac{dy}{d\theta}$ = a cos Θ ($\frac{1}{2}$ M)

∴ $\frac{dy}{dx}$ = $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ = $\frac{a\ cos\theta.\ sin\theta}{a\ cos^2\theta}$ = tan Θ ($\frac{1}{2}$ M)

∴ $\frac{d^2y}{dx^2}$ = $\frac{d}{dx}(tan\ \theta)$ = $sec^2\theta$ x $\frac{d\theta}{dx}$

= $sec^2\theta$ x $\frac{sin\ \theta}{a\ cos^2\theta}$

= $\frac{sin\ \theta}{a\ cos^4\theta}$

∴ $\left(\frac{d^2y}{dx^2}\right)\Theta=\frac{\pi}{4}$ = $\frac{sin\frac{\pi}{4}}{a\ cos^4\frac{\pi}{4}}$ = $\frac{\frac{1}{\sqrt{2}}}{a\left(\frac{1}{\sqrt{2}}\right)^4}$ = $\frac{1}{a\left(\frac{1}{\sqrt{2}}\right)^3}$ = $\frac{2\sqrt{2}}{a}$ (1 M)

SECTION D

32. To prove reflexive (1m)

Symmetric: (a,b)R(c,d) → ad(b+c) = bc(a+d).

→ bc(a+d)= ad(b+c)

→ cb(d+a)= da(c+b)

→(c,d) R (a,b) (2m)

Transitive: (a,b)R(c,d) and (c,d) R (e,f) → ad(b+c) = bc(a+d).and cf(d+e) = de(c+f)

→ $\frac{b+c}{bc}$ = $\frac{a+d}{ad}$ and $\frac{d+e}{de}$ = $\frac{c+f}{cf}$

→ $\frac{1}{c}+\frac{1}{b}=\frac{1}{d}+\frac{1}{a}$ and $\frac{1}{e}+\frac{1}{d}=\frac{1}{f}+\frac{1}{c}$

Adding both ,we get → $\frac{1}{c}+\frac{1}{b}+\frac{1}{e}+\frac{1}{d}=\frac{1}{d}+\frac{1}{a}+\frac{1}{f}+\frac{1}{c}$

→ $\frac{1}{b}+\frac{1}{e}=\frac{1}{a}+\frac{1}{f}$

→(a,b) R (e,f) (2m)

33. $|A| = -1$ (1m)

$$\text{adj}(A) = \begin{pmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{pmatrix}$$ (2m)

$$A^{-1} = \text{adj}(A) / |A| = \begin{pmatrix} 0 & -2 & -1 \\ 1 & 9 & 5 \\ -2 & -23 & -13 \end{pmatrix}$$ (1/2m)

The system of equations can be written in matrix form as $A^T X = B$

$$\text{Where } A^T = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix}$$ (1/2 m)

$$\text{then } X = (A^{-1})^T B = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix} \begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

x= 1, y= 2, z= 3 (1 m)

OR

$$A^2 = \begin{pmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{pmatrix}$$ (1m) $$A^3 = \begin{pmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{pmatrix}$$ (1 m)

To prove $A^3 - A^2 - 3A - I = 0$ (1m)

$$A(A^2 - A - 3I) = I \quad \text{then } A^{-1} = A^2 - A - 3I = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix}$$ (2m)

34.Circle $x^2 + y^2 = 32$ having centre (0,0) and $4\sqrt{2}$, figure (1m)

Required area = $\int_0^4 x \, dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}$ (2m)

$= 8 + 4\pi - 8 = 4\pi$ sq.units (2m)

35.The line equations can be written as lines $\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2} = \lambda$ and lines $\frac{x-5}{3} = \frac{y+2}{2} = \frac{z}{6} = \mu$

Then $x = \lambda+3$ $y = 2\lambda+2$ $z = 2\lambda-4$ and $x = 3\mu+5$ $y = 2\mu-2$ $z = 6\mu$

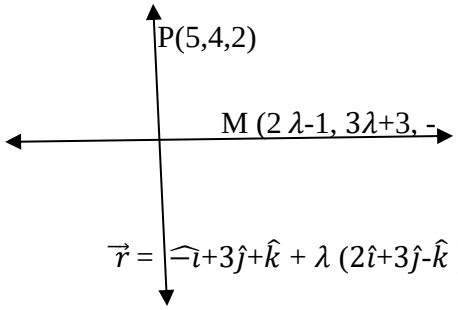
By solving $\lambda+3 = 3\mu+5$ and $2\lambda+2 = 2\mu-2$ we get $\lambda = -4 \; and \; -2$ (3m)

It satisfies the equation $2\lambda-4 = 6\mu$ .Therefore the lines are intersecting. (1m)

The point of intersection is $(\lambda+3, 2\lambda+2, 2\lambda-4)$ at $\lambda = -4$ = (-1,-6 , -12) (1m)

OR

The given line can be written as $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = \lambda$



Then x=2 λ-1, y= 3λ+3 z= - λ+1 (1m)

λ+1)

Let M (2 λ-1, 3λ+3, - λ+1) be the foot of the perpendicular $\vec{r} = \hat{\imath}+3\hat{\jmath}+\hat{k} + \lambda\,(2\hat{\imath}+3\hat{\jmath}-\hat{k})$.

Then PM perpendicular to the given line

Dr.s of PM is(2λ-6, 3λ-1, - λ-1) (1m)

2(2λ-6)+3(3λ-1) -1(- λ-1) = 0 then λ= 1 (1m)

M(1,6,0) then by distance formula PM=2$\sqrt{6}$units (1m)

Let R be the image of the point P .Then M is the midpoint of PR . By midpoint formula R(-3,8,-2) (1m)

36. Let $E_{1=}$ equation I solved by Mohan

$E_{2=}$ equation I solved by Sruthi

E_3 = equation solved by Ritu

A = Question has some error

i. P(A) = 0.015 (2 m)

ii. Required probability = 1 – $P(E_1/A)$ = 0.8 (2 m)

37. i. $x^2 + y^2 = 100$, A = (2x) (2y) = 4x $\sqrt{100 - x^2}$ (2m)

ii. Area= $4x\sqrt{100 - x^2}$ =200 (2m)

38.

i. $\overrightarrow{b1} \times \overrightarrow{b2} = 5\hat{\imath} - \hat{\jmath} - 7\hat{k}$ (2 m)

ii. Shortest distance = $\sqrt{3}$ units (2 m)

	ANSWERS OF SAMPLE PAPER 7	
	1) b 2)b 3)b 4)a 5)d 6)b 7)c 8)c 9)a 10)a 11)d 12)b 13)d 14)a 15)c 16)b 17)c 18)d19)d 20)a	
21	sin^{-1} (cos $\frac{3\pi}{5}$) = sin^{-1} (sin($\frac{\pi}{2} - \frac{3\pi}{5}$) = - $\frac{\pi}{10}$ OR 15/8	
22	Y = $\tan^{-1}(\frac{1+cosx}{sinx})$ = $\tan^{-1}$ (cotx/2) = $\frac{\pi}{2} - \frac{x}{2}$ $\frac{dy}{dx} = -1/2$ OR $\frac{dy}{dx} = \frac{2^{x+1}}{1+4^x}$ log2	
23	a) $(-\infty, -3/2)$ b) (-3/2 , ∞)	
24	K=1	
25	I = log(21/5)	
26	I = $\frac{x^2 tan^{-1} x}{2}$ - $\frac{x}{2}$ - $\frac{tan^{-1} x}{2}$ +c OR I = $\frac{e^x}{x+1}$ +	
27	Put $e^x = t$ I = $\text{Sin}^{-1}(\frac{e^x+2}{3})$	
28	| xsin(y/x) | = e^c	

	OR 2x $= e^{tan^{-1}y}$ +A $e^{tan^{-1}y}$	
29	X^2 (y +3) = e^{y+c}	
30	Minimum value of z = 300 at (60,0) OR The maximum value of Z is 400 at 0(0, 200) and the minimum value of Z is 100 at all the points on the line segment joining A(0, 50) and B(20, 40).	
31	1/5	
32	Proof OR Proof	
33	X+y+z =7000 10x+16y+17z = 110000 x-y =0 A-1 $= \frac{1}{8}\begin{bmatrix} 17 & -1 & 1 \\ 17 & -1 & -7 \\ -26 & 2 & 6 \end{bmatrix}$ X = 1125 Y= 1125 Z=4750	
34	$\frac{7}{6}$	
35	Proof Point of intersection (-1,-1,-1) OR r= 2i -j +3k +μ (-6 I – 3j +6k)	

36	1) $200= 2x +\pi y$ 2) $\frac{2}{\pi}$ $(100x – x^2)$ 3) $\frac{500}{\pi}$	
37	1)5/36 2) 125/1296 3) $(5/6)^5$	
38	1) $2i+4j$ 2) $\sqrt{29}$ 3) $\frac{7i+6j}{\sqrt{85}}$	

MARKING SCHEME OF SAMPLE QUESTION PAPER 8

Section-A(1 MARK EACH)

1) a
2) b
3) a
4) b
5) c
6) c
7) c
8) b
9) a
10) a
11) b
12) d
13) b
14) a
15) a
16) c
17) a
18) c
19) d
20) b

Section-B (2M each)

21) $\tan^{-1}[\sin(-\pi/2)]$

$\tan^{-1}[-\sin(\pi/2)]$ (1)

$\tan^{-1}[-1] = -\frac{\pi}{4}$ (1)

22) $f(x) = 4x^3 - 18x^2 + 27x - 7$

$f^1(x) = 12x^2-36x+27$ (1)

$= 3(4x^2-12x+9)$

$= 3(2x-3)^2$

≥ 0 and hence always increasing. (1)

23) $y = x^3 - 3x + 2$ **in** $0 \leq x \leq 2$.

$\frac{dy}{dx} = 3x^2-3$

$\frac{dy}{dx}$ = 0 implies $3x^2-3=0$

Implies x= ± 1 (1)

Now find f(0) ,f(1) and f(2)

f(0) = 2

f(1) = 0

f(2) = 4

Therefore Absolute maximum value =4 (1)

24) $\int_0^1 \frac{xe^x}{(x+1)^2} dx$

$\frac{xe^x}{(x+1)^2} = \frac{[(x+1)-1]\, e^x}{(x+1)^2}$

$= \left(\frac{1}{x+1} - \frac{1}{(x+1)^2} \right) e^x$ **(1)**

$\int_0^1 \frac{xe^x}{(x+1)^2} dx = \int_0^1 \left(\frac{1}{x+1} - \frac{1}{(x+1)^2} \right) e^x$ **dx**

$= \int_0^1 [f(x) + f^1(x)]\ e^x$ dx form

$= [\ e^x \frac{1}{x+1}\ ]$ apply limits 0 to 1 , we get $\frac{e}{2}$ -1 (1)

25) f(x) $= x^3 + x$, x is real

$f^1(x) = 3x^2+1$

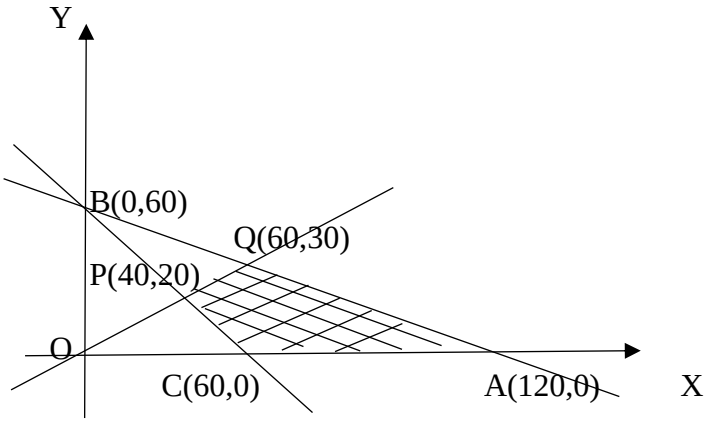
Put $f^1(x)$ =0 implies $3x^2+1 = 0$

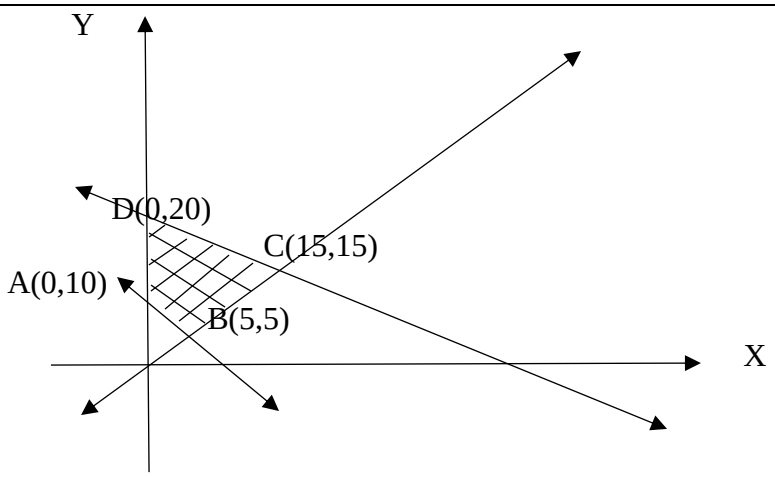
$x^2 = \frac{-1}{3}$ then x is not real .

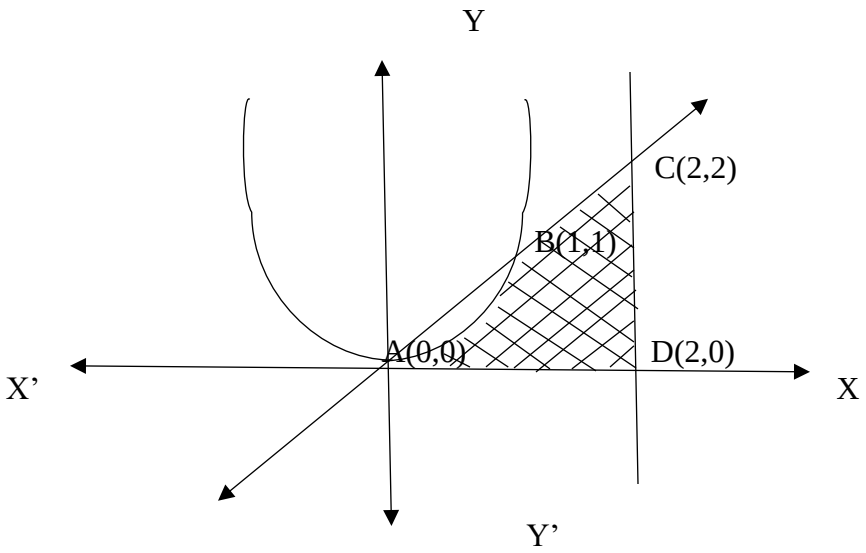
Therefore , no critical points .

QN NO	ANSWER SECTION C	MARKS
26	Let I=$\int \frac{x}{x^2+3x+2}$ dx Let x= A $\frac{d}{dx}$ (x^2+3x+2) +B x= (2x+3)A+B x=2Ax+3A+B 2A=1 and 3A+B=0 A=$\frac{1}{2}$ and B=$\frac{-3}{2}$ I=$\frac{1}{2}\int \frac{2x+3}{x^2+3x+2}$ dx - $\frac{3}{2}$ $\int \frac{dx}{x^2+3x+2}$ =>I =$\frac{1}{2} I_1$ - $\frac{3}{2} I_2$ Let I_1=$\int \frac{2x+3\ dx}{x^2+3x+2}$ and I_2 =$\int \frac{dx}{x^2+3x+2}$ To evaluate I_1 put t=x^2+3x+2 dt= (2x+3) dx I_1 =$\int \frac{dt}{t}$=log $\|t\|$ +c_1 = log $\|x^2+3x+2\|$ +c_1 $I_2 = \int \frac{dx}{x^2+3x+2}$ =$\int \frac{dx}{x^2+3x+\frac{9}{4}+2-\frac{9}{4}}$ =$\int \frac{dx}{(x+\frac{3}{2})^2-(\frac{1}{2})^2}$ =$\frac{1}{2\ x\ \frac{1}{2}}$ log $\left\|\frac{x+1}{x+2}\right\|$ +c_2 On substituting the values of I_1 and I_2 I=$\frac{1}{2}$log $\|x^2+3x+2\|$ -$\frac{3}{2}$ log $\left\|\frac{x+1}{x+2}\right\|$ +c	1 1 1/2 1/2
27	a) K+2k+2k+3k+k^2+$2k^2$+$7k^2$+k=1 $10k^2$+9k-1=0 Solving to get the value of k=1/10, -1 But k≠-1, Hence k=1/10 b) P(0<X<3)=P(1)+P(2)=1/10+2/10=3/10 c) P(X>6)=P(X=7)=17/100	1 1 1
28	$\int_0^1 x(1-x)^n dx$ =$\int_0^1 (1-x)\{1-(1-x)\}^n$dx=$\int_0^1 (1-x)x^n dx$ Using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ =$\int_0^1 (x^n - x^{n+1})dx$ = $\left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$ = $\left[\frac{1}{n+1} - \frac{1}{n+2}\right] - 0$ =$\frac{1}{(n+1)(n+2)}$	1 1/2 1 1/2

	Or Let I=$\int_0^{\pi} \frac{xsinx}{1+cos^2x}dx$ I=$\int_0^{\pi} \frac{(\pi-x)sin(\pi-x)dx}{1+cos^2(\pi-x)}$ I=$\int_0^{\pi} \frac{(\pi-x)sinxdx}{1+cos^2x} = \pi\int_0^{\pi} \frac{sinxdx}{1+cos^2x} - I$ 2I= $\pi\int_0^{\pi} \frac{sinxdx}{1+cos^2x}$ I=$\frac{\pi}{2}\int_0^{\pi} \frac{sinxdx}{1+cos^2x}$ Put cosx=t so that -sinxdx=dt. When x=0, t=1 and when x=π, t=-1 I=$\frac{-\pi}{2}\int_1^{-1} \frac{dt}{1+t^2}$ =π$\int_0^1 \frac{dt}{1+t^2}$ =π$[\tan^{-1}t]_0^1$= π$[\tan^{-1}1 - \tan^{-1}0]$=π$\left[\frac{\pi}{4}-0\right]$=$\frac{\pi^2}{4}$	1/2 1 1 ½
29	x$\frac{dy}{dx}$=y-xtan$\left(\frac{y}{x}\right)$ $\frac{dy}{dx}$=$\frac{y-x\tan\left(\frac{y}{x}\right)}{x}$ $\frac{dy}{dx}$=$\frac{y}{x} - \frac{\tan\left(\frac{y}{x}\right)}{x}$ which is a homogeneous differential equation On putting y=vx, $\frac{dy}{dx}=v+\frac{dv}{dx}$ v+x$\frac{dy}{dx}$=v-tanv x$\frac{dy}{dx}$=-tanv $\frac{dv}{tanv}$=-$\frac{dx}{x}$ cotvdv=-$\frac{dx}{x}$ On integrating both sides, $\int cotvdv$=-$\int \frac{dx}{x}$ log$\|sinv\|$=-log$\|x\|$+C log$\|sinv\|$+log$\|x\|$=C log$\|xsinv\|$=C log$\left\|xsin\frac{y}{x}\right\|$=C $xsin\frac{y}{x}=e^C$ $xsin\frac{y}{x}$=A $sin\frac{y}{x}=\frac{A}{x}$ y=xsin$^{-1}\left(\frac{A}{x}\right)$	½ ½ 1 1

	Or	
30	$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$ which is a homogeneous differential equation On putting $y=vx$, $\frac{dy}{dx}=v+x\frac{dv}{dx}$ $x\cos v\left[v+x\frac{dv}{dx}\right]=vx\cos v+x$ $vx\cos v+x^2\cos v\frac{dv}{dx}=vx\cos v+x$ $x^2\cos v\frac{dv}{dx}=x$ $\cos v dv=\frac{dx}{x}$ On integrating both sides, $\int \cos v dv=\int \frac{dx}{x}$ $\sin v=\log\lvert x\rvert+C$ $\sin\left(\frac{y}{x}\right)=\log\lvert x\rvert+C$	½ 1 1 ½
	 Draw the graph of $x+2y\leq120$, $x+y\geq60$, $x-2y\geq0$, $x\geq0$, $y\geq0$ For shading the feasible region Corner points of the feasible region are C(60,0), A(120,0), Q(60,30) & P(40,20) At C(60,0), Z=5x60 +10x0=300 At A(120,0), Z=5x120 + 10x0=600 At Q(60,30), Z= 5x60 +10x30=600 At P(40,20), Z= 5x40 + 10x20=400 Hence Z is minimum at C(60,0) and min Z=300 Or	1 ½ ½ 1

31	 For drawing each inequality Corner points of the feasible region are A(0,10), Z=90 B(5,5), Z=60 C(15,15), Z=180 D(0,20), Z=180 Maximum value of Z occurs at two points C(15,15) & D(0,20) & it is 180 in each case	1 ½ 1 ½
	$y=e^{a\cos^{-1}x}$ $\frac{dy}{dx} = e^{a\cos^{-1}x} \times \frac{-a}{\sqrt{1-x^2}}$ $\frac{dy}{dx} = \frac{-ay}{\sqrt{1-x^2}}$ $\sqrt{1-x^2} \times \frac{dy}{dx} = -ay$ Again differentiating with respect to x, $\sqrt{1-x^2} \times \frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{2x}{2\sqrt{1-x^2}} = -a\frac{dy}{dx}$ Multiplying throughout by $\sqrt{1-x^2}$, we get $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$	½ ½ 1 1

Q NO.	ANSWER SECTION D	MARKS
32	 Equation of curve and lines are $y=x^2$ $y=x$ $x=2$ The point of intersection of curve and line is $x^2 = x$ x=0,1 y=0,1 Thus, the points of intersection are A(0,0) and B(1,1) Required area=Area of shaded region ABCD $=\int_0^1 y(parabola)dx + \int_1^2 y(line)dx$ $=\int_0^1 x^2 dx + \int_1^2 xdx = \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^2$ $=\left[\frac{1}{3} - 0\right] + \left[\frac{4}{2} - \frac{1}{2}\right] = \frac{1}{3} + \frac{3}{2} = \frac{2+9}{6} = \frac{11}{6}$sq units	1 1 1 2
33	We are given that for (a,b), (c,d) ∈ NxN (a,b) R (c,d) ⇔ ad (b+c) = bc (a+d) ⇔ $\frac{b+c}{bc} = \frac{a+d}{ad} \Leftrightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a} \Leftrightarrow \frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c}$ This means that (a,b) R (c,d) if $\frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c}$ Reflexivity: Since if $\frac{1}{a} + \frac{1}{b} = \frac{1}{b} + \frac{1}{a}$ for all (a,b) ∈ N xN, therefore, R is reflexive. Symmetry: For (a,b), (c,d) ∈ NxN, (a,b) R (c,d)⇒ $\frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c} \Rightarrow \frac{1}{b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{d} \Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ ⇒ (c,d) R (a,b) Thus, (a,b) R (c,d) ⇒ (c,d) R (a,b) ⇒ R is symmetric Transitivity: For (a,b), (c,d) & (e,f) ∈ NxN, (a,b) R (c,d) and (c,d) R (e,f) $\frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c}$ & $\frac{1}{c} + \frac{1}{f} = \frac{1}{d} + \frac{1}{e} \Rightarrow \left(\frac{1}{a} + \frac{1}{d}\right) + \left(\frac{1}{c} + \frac{1}{f}\right) = \left(\frac{1}{b} + \frac{1}{c}\right) + \left(\frac{1}{d} + \frac{1}{e}\right)$	1 1 1 1 ½

	$\frac{1}{a}+\frac{1}{f}=\frac{1}{b}+\frac{1}{e} \Rightarrow$ (a,b) R (e,f) So (a,b) R (c,d) & (c,d) R (e,f) ⇒ (a,b) R (e,f) ⇒ R is transitive Hence R is an equivalence relation. Or Let m and n be two distinct elements of N. We shall show that *f(m)≠ f(n)* Case (i) When both m and n are odd, then *f(m)*=m+1, *f(n)*= n+1 *and* m≠n ⇒ m+1≠ n+1 ⇒ *f(m)≠ f(n)* Case (ii) When both m and n are even, then *f(m)*=m-1, *f(n)*= n-1 *and* m≠n ⇒ m-1≠ n-1 ⇒ *f(m)≠ f(n)* Case (iii) When m is odd and n is even, then *f(m)*=m+1 is even and *f(n)*= n-1 is odd so that *f(m)≠ f(n)* in this case also. Similarly, if m is even and n is odd, we shall find that *f(m)≠ f(n)* So, in all cases m≠n ⇒ *f(m)≠ f(n)* ⇒ *f is one-one.* Next, we shall prove that f is onto If n ∈ N is any element, then *f(n-1)*= (n-1)+1=n if n is even & *f(n+1)*= (n+1)-1=n if n is odd This means that every element n ∈ N is *f*-image of some element in N as n=*f(n-1)* if n is even and n=*f(n+1)* if n is odd Hence *f* is onto. Thus, *f* is both one-one and onto.	½ 1 1 1 2
34	We have, $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$ $\lvert A\rvert=\begin{vmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{vmatrix}$ =1(1+3)+1(2+3)+1(2-1)= 4+5+1 =10 ≠ 0 Therefore A^{-1} *exist* find the cofactors C_{ij} of elements of a_{ij} in A C_{11}=4, C_{12}=-5, C_{13}=1, C_{21}=2, C_{22}=0, C_{23}=-2, C_{31}=2, C_{32}=5, C_{33}=3 ⇒ adj(A)= $\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$ $A^{-1}=\frac{1}{\lvert A\rvert}adj(A)=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$ Now, let us solve the given system of linear equations which can be written in matrix form as $\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 1\\ 1 & -3 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$ $X=(A^T)^{-1}B$ $A^TX=B$, where $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$	1 1 ½ ½

	$\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{10}\begin{bmatrix}4&2&2\\-5&0&5\\1&-2&3\end{bmatrix}^T\begin{bmatrix}4\\0\\2\end{bmatrix}=\frac{1}{10}\begin{bmatrix}4&-5&1\\2&0&-2\\2&5&3\end{bmatrix}\begin{bmatrix}4\\0\\2\end{bmatrix}$ $=\frac{1}{10}\begin{bmatrix}18\\4\\14\end{bmatrix}=\begin{bmatrix}9/5\\2/5\\7/5\end{bmatrix}$ x=$\frac{9}{5}$, y=$\frac{2}{5}$ and z=$\frac{7}{5}$, which is the required solution.	2
35	Given lines are $\vec{r}=\overrightarrow{a_1}+\lambda\overrightarrow{b_1}$ $\vec{r}=\overrightarrow{a_2}+\mu\overrightarrow{b_2}$ $\overrightarrow{a_1}=\hat{\imath}+2\hat{\jmath}+\hat{k}$ $\overrightarrow{a_2}=2\hat{\imath}-\hat{\jmath}-\hat{k}$ $\overrightarrow{b_1}=\hat{\imath}-\hat{\jmath}+\hat{k}$ $\overrightarrow{b_2}=2\hat{\imath}+\hat{\jmath}+2\hat{k}$ Required shortest distance $=\left\lvert\frac{(\overrightarrow{b_1}\times\overrightarrow{b_2}).(\overrightarrow{a_2}-\overrightarrow{a_1})}{\lvert\overrightarrow{b_1}\times\overrightarrow{b_2}\rvert}\right\rvert$ $\overrightarrow{a_2}-\overrightarrow{a_1}=(2\hat{\imath}-\hat{\jmath}-\hat{k})-(\hat{\imath}+2\hat{\jmath}+\hat{k})$ $=\hat{\imath}-3\hat{\jmath}-2\hat{k}$ $\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\1&-1&1\\2&1&2\end{vmatrix}$ $=\hat{\imath}(-2-1)-\hat{\jmath}(2-2)+\hat{k}(1+2)$ $=-3\hat{\imath}+3\hat{k}$ Required S.D. $=\frac{\lvert(-3\hat{\imath}+3\hat{k}).(\hat{\imath}-3\hat{\jmath}-2\hat{k})\rvert}{\lvert-3\hat{\imath}+3\hat{k}\rvert}$ $=\frac{\lvert(-3)\times1+0\times(-3)+3\times(-2)\rvert}{\sqrt{(-3)^2+3^2}}$ $=\frac{9}{\sqrt{18}}=\frac{9}{3\sqrt{2}}=\frac{3}{\sqrt{2}}$ units. Or Equations of given lines are $\vec{r}=-\hat{\imath}-\hat{\jmath}-\hat{k}+\lambda(7\hat{\imath}-6\hat{\jmath}+\hat{k})$ $\vec{r}=3\hat{\imath}+5\hat{\jmath}+7\hat{k}+\mu(\hat{\imath}-2\hat{\jmath}+\hat{k})$ Which are of the form $\vec{r}=\overrightarrow{a_1}+\lambda\overrightarrow{b_1}$ $\vec{r}=\overrightarrow{a_2}+\mu\overrightarrow{b_2}$ $\overrightarrow{a_1}=-\hat{\imath}-\hat{\jmath}-\hat{k}$ $\overrightarrow{a_2}=3\hat{\imath}+5\hat{\jmath}+7\hat{k}$ $\overrightarrow{b_1}=7\hat{\imath}-6\hat{\jmath}+\hat{k}$ $\overrightarrow{b_2}=\hat{\imath}-2\hat{\jmath}+\hat{k}$ $\overrightarrow{a_2}-\overrightarrow{a_1}=(3\hat{\imath}+5\hat{\jmath}+7\hat{k})-(-\hat{\imath}-\hat{\jmath}-\hat{k})$	1 ½ ½ 1 2 1 ½

	$=4\hat{\imath}+6\hat{\jmath}+8\hat{k}$ $\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}$ $=\hat{\imath}(-6+2)-\hat{\jmath}(7-1)+\hat{k}(-14+6)$ $=-4\hat{\imath}-6\hat{\jmath}-8\hat{k}$ S.D between the given lines $=\left\lvert\frac{(\overrightarrow{b_1}\times\overrightarrow{b_2}).(\overrightarrow{a_2}-\overrightarrow{a_1})}{\lvert\overrightarrow{b_1}\times\overrightarrow{b_2}\rvert}\right\rvert$ $=\frac{\lvert(-4\hat{\imath}-6\hat{\jmath}-8\hat{k}).(4\hat{\imath}+6\hat{\jmath}+8\hat{k})\rvert}{\sqrt{(-4)^2+(-6)^2+(-8)^2}}$ $=\frac{\lvert(-4)\times4+(-6)\times6+(-8)\times8\rvert}{\sqrt{116}}$ $=\frac{116}{\sqrt{116}}=\sqrt{116}=2\sqrt{29}$ units.	1 ½ 2
Q NO.	ANSWER SECTION E	MARK
36	Let E1 is the event that the doctor comes by train E2 is the event he comes by bus E3 is the event that he comes by scooter E4 is the event he comes by other means of transport A is the event that he is late (i) $P(E_1/A)=1/4$ (ii) $P(E_2/A)=1/3$ (iii) $P(A/E_4)=0$ Or $P(A)=3/20$	 1 1 2
37	(i) $\vec{a}\perp\vec{b}$ $(ii)\ 2\sin\frac{\theta}{2}$ (iii) $\pm2(\vec{b}\times\vec{c})$ Or $\frac{\sqrt{70}}{2}$	1 1 2
38	(i) 12.5 (ii) 10	2 2

MARKING SCHEME OF SAMPLE PAPER, NO 9 – MATHEMATICS

SECTION: A (Solution of MCQ s of 1 mark each)

QN .NO	ANS	HINTS/SOLUTION	MARKS
1	(b)	1 x 1	1
2	(d)	$\frac{1}{6}$	1
3	(d)	$B^{-1} = \frac{1}{6} A$	1
4	(a)	k = 12	1
5	(d)	$\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$	1
6	(a)	3	1
7	(a)	80	1
8	(c)	-8	1
9	(a)	9π	1
10	(c)	x y z	1
11	(a)	a=2b	1
12	(d)	$2\hat{\imath}$ -2 $\hat{\jmath}$ +2 $\hat{k}$	1
13	(c)	8	1
14	(a)	$\frac{1}{26}$	1
15	(a)	$y^{\frac{2}{3}} - x^{\frac{2}{3}} = C$	1
16	(a)	$\frac{\vec{a} + 2\vec{b}}{3}$	1
17	(d)	$(-\infty, 3) \cup (3, \infty)$	1
18	(c)	-1	1
19	(d)	(A) is false but (R) is true	1
20	(c)	(A) is true but (R) is false	1

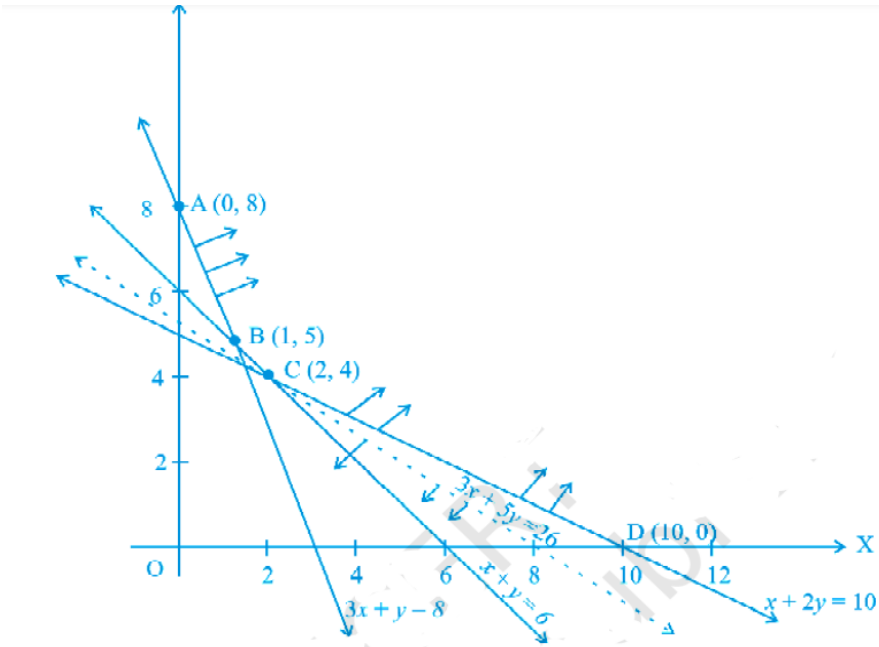
SECTION: B

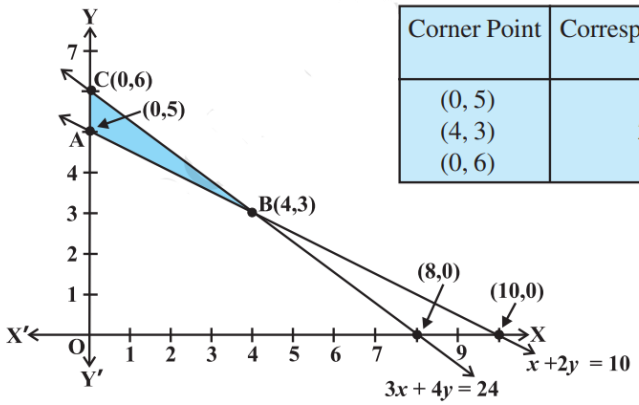
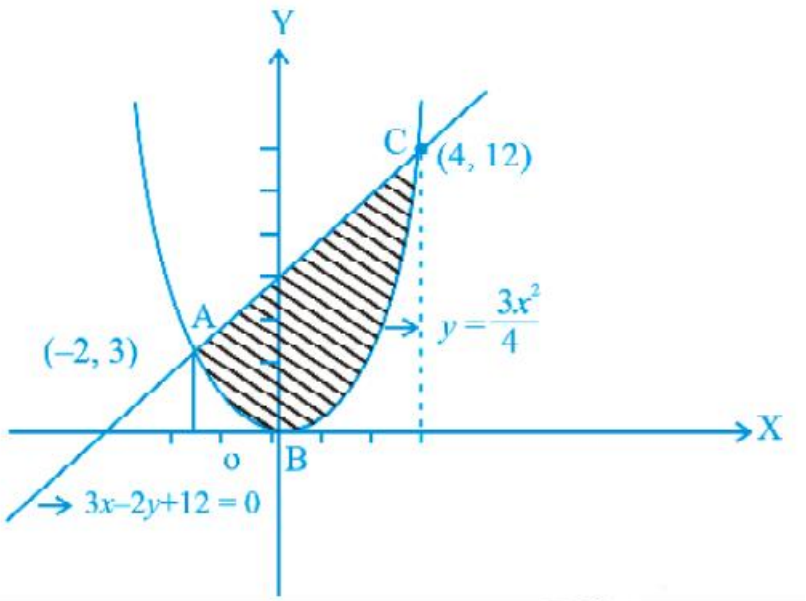
[This section comprises of solution of very short answer type questions(VSA) of 2 mark each]

21	$tan^{-1}\left(\sqrt{\frac{1-cosx}{1+cosx}}\right) = \tan^{-1}\sqrt{\frac{2sin^2\left(\frac{x}{2}\right)}{2cos^2\left(\frac{x}{2}\right)}}$ $= \tan^{-1} tan\frac{x}{2}$ $= \frac{x}{2}$ OR $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0) = \frac{3\pi}{4} + \frac{2\pi}{6} + \frac{\pi}{2}$	1 $\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$

	$= \frac{19\pi}{12}$	$\frac{1}{2}$
22	f(x) = $2x^2$-3x $f'(x) = 4x - 3,\ f'(x) = 0 \Rightarrow x=\frac{3}{4}$ $in\ \left(-\infty, \frac{3}{4}\right)\ f'(x) < 0$ f is decreasing in $\left(-\infty,\frac{3}{4}\right)$ $in\left(\frac{3}{4},\infty\right)\ f'(x) > 0$, f is increasing in $\left(\frac{3}{4},\infty\right)$	1 $\frac{1}{2}$ $\frac{1}{2}$
23	$f'(x)$ = $6x^2$-30x+36 $=6(x-3)(x-2)$ $f'(x)= 0 \Rightarrow$ x = 2,3 f(1) = 24, f(2) =29,f(3)=28,f(5)=56 Absolute maximum is 56 occurring at x=5 Absolute minimum is 24 at x=1 OR V=x^3,$\frac{dx}{dt} = 3cm/s$ $\frac{dv}{dt} = 3x^2\ \frac{dx}{dt}$ $\left(\frac{dv}{dt}\right)x = 10\ = 900cm^3/s$	1 1 $\frac{1}{2}$ 1 $\frac{1}{2}$
24	I = $\int e^x(tan^{-1}x + \frac{1}{1+x^2})\ dx$ Consider f(x) = tan^{-1} x, then f ′(x) = $\frac{1}{1+x^2}$ Thus, the given integrand is of the form e^x [f (x) + f ′(x)]. Therefore, $\int e^x(tan^{-1}x + \frac{1}{1+x^2})\ dx = e^x tan^{-1}x$ + C	 1 1
25	p'(x)=24-36x, p'(x)=0 $\Rightarrow x = \frac{2}{3}$ second derivative at $\frac{2}{3} = -36 < 0$ Maximum profit= p ($\frac{2}{3}$)=49	$1\frac{1}{2}$ $\frac{1}{2}$
	SECTION -C (This section comprises solutions of short answer type questions (SA) of 3 marks each.)	
26	$\int \frac{x^2}{(x^2+1)(x^2+4)}\ dx$ consider $\frac{x^2}{(x^2+1)(x^2+4)}$ and put x^2 = y.	

	Then $\frac{X^2}{(x^2+1)(x^2+4)} = \frac{y}{(y+1)(y+4)}$ Write $\frac{y}{(y+1)(y+4)} = \frac{A}{(y+1)} + \frac{B}{(y+4)}$ So that y = A (y + 4) + B (y + 1) Comparing coefficients of y and constant terms on both sides, we get A + B = 1 and 4A + B = 0, which give A = $\frac{-1}{3}$ and B = $\frac{4}{3}$ Thus, $\frac{X^2}{(x^2+1)(x^2+4)} = \frac{-1}{3(X^2+1)} + \frac{4}{3(X^2+4)}$ Therefore, $\int \frac{X^2}{(x^2+1)(x^2+4)} = \frac{-1}{3}\int \frac{dx}{(x^2+1)} + \frac{4}{3}\int \frac{dx}{(x^2+4)}$ $= \frac{-1}{3}\tan^{-1} x + \frac{4}{3}\tan^{-1}\frac{x}{2} + C$	1 1 1
27	Let p be the probability that B gets selected. P (Exactly one of A, B is selected) = 0.6 (given) ⇒ P (A is selected; B is not selected; B is selected, A is not selected) = 0.6 ⇒ P (A∩B′) + P (A′∩ B) = 0.6 ⇒ P (A) P (B′) + P (A′) P (B) = 0.6 ⇒ (0.7) (1 – p) + (0.3) p = 0.6 ⇒ p = 0.25 Thus the probability that B gets selected is 0.25.	1 1 1
28	$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}\, dx}{\sqrt{\cos x} + \sqrt{\sin x}}$ ---------- (1 $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$ 2I = $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx = [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ I = $\frac{\pi}{12}$ OR $\int \sqrt{\frac{1+x}{1-x}}\, dx = \int \frac{1}{\sqrt{1-x^2}}\, dx + \int \frac{x dx}{\sqrt{1-x^2}} = sin^{-1} x + I_1$ $I_1 = \int \frac{x dx}{\sqrt{1-x^2}} = \sqrt{1-x^2} + C$ I = $sin^{-1} x + \sqrt{1-x^2} + C$	$\frac{1}{2}$ 1 1 $\frac{1}{2}$ 1+ $\frac{1}{2}$ 1 $\frac{1}{2}$

29	$2e^{\frac{x}{y}} + \log\|y\| = 2$ (Textbook example qn) OR $y = x^2 - \frac{\pi^2}{4\,sins\,x}$ $(\sin x \neq 0)$ (Text book example qn)	3 3
30	For finding the value of Z at the vertices and minimum value of Z as 26  Draw the graph of 3x + 5y < 26 by dotted line. We see that the open half plane determined by 3x + 5y < 26 and R do not have a point in common. Thus, 26 is the minimum value of Z. OR	1 $1\frac{1}{2}$ $\frac{1}{2}$

	 Corner Point | Corresponding value of Z (0, 5) | 2500 (4, 3) | **2300** ← Minimum (0, 6) | 3000 A, B and C are (0,5), (4,3) and (0,6) respectively. Now we evaluate Z = 200x + 500y at these points. Hence, minimum value of Z is 2300 attained at the point (4, 3)	1 2
31	$\frac{dx}{dt} = 2acos2t$ $\frac{dy}{dt}$ =2a$\left(-sin2t + \frac{sec^2t}{2\,tant}\right)$ =2a $\frac{cos^2 2t}{sin2t}$ $\frac{dy}{dx}$= cot 2t.	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1
	SECTION -D (This section comprises of solutions of long answer type qns (LA) of 5 marks each)	
32		1

	For getting point of intersection as (-2,3) and (4,2) (1 mark) Required area= $\int_{-2}^{4}\left(\frac{3x+12}{2}-\frac{3x^2}{4}\right)dx$ = 27 sq units (2+1 marks)	
33	f (x_1)= f(x_2) $\Rightarrow \frac{x_1-2}{x_1-3}=\frac{x_2-2}{x_2-3}$ $(x_{1-2})(x_{2-3})=(x_{1-3})(x_{2-2}) \Rightarrow x_1=x_2$ ⇒f is one-one $y=\frac{x-2}{x-3} \Rightarrow y(x-3)=x-2 \Rightarrow x=\frac{3y-2}{y-1}$ for every y≠ 1, $y\ \epsilon\ co-domain B, there\ exists\ x=\frac{3y-2}{y-1}\epsilon\ A\ such\ that$ f(x)=y hence f is on to OR (a, a) $\epsilon R for\ all\ a\ \epsilon A, since\lvert a-a\rvert=0, which\ is\ divisible\ by\ 3$ Hence R is reflexive. For proving R is symmetric For proving transitivity,and writing R is an equivalence relation For getting [4]={1,4,7,10}	1 $1\frac{1}{2}$ $1\frac{1}{2}$ 1 1 1 2 1
34	$\lvert A\rvert=\begin{vmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{vmatrix}$=1200, $A\neq 0$ henceA^{-1} exists Adj A=$\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$ $A^{-1}=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$ System of equations can be written as AX=B Where A=$\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$,X=$\begin{bmatrix}\frac{1}{X}\\ \frac{1}{Y}\\ \frac{1}{Z}\end{bmatrix}$,B=$\begin{bmatrix}2\\ 5\\ -4\end{bmatrix}$, Solution is X= A^{-1}B Getting the answer as x=2,y= -3,z=5	1 1 $\frac{1}{2}$ 1 $1\frac{1}{2}$

35	For formula $\overrightarrow{b_1} \times \overrightarrow{b_2}$= $3\hat{\imath}$-$\hat{\jmath}$-$7\hat{k}$, $\left\|\overrightarrow{b_1} \times \overrightarrow{b_2}\right\|$=$\sqrt{59}$, $\overrightarrow{a_2}$ -$\overrightarrow{a_1}$ = $\hat{\imath}$-$\hat{k}$ $substituting\ \ and\ getting\ the answer\ \frac{10}{\sqrt{59}}$ OR Equation of the line through B and C is $\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4}$=$\lambda$ Any point on BC $(2\lambda, -2\lambda - 1, -4\lambda + 3), D.R\ of\ AB = (2, -2, -4)$ Getting the value of λ=$\frac{-5}{6}$ $Getting\ foot\ of\ perpendicular\ as\ \left(\frac{-5}{3}, \frac{2}{3}, \frac{19}{3}\right)$	1 1+1 2 1 1 2 5
	SECTION - E (This section comprises solution of 3 case- study/passage-based questions of 4 marks each with two sub parts. The solution of first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. Solution of 3rd case study question has two subparts of 2 marks each.)	
36	(i) $\frac{3}{5}$ (ii) $\frac{1}{2}$	2 2
37	1. (c) 2. (b) 3. (d) 4. (b)	1 1 1 1
38	(i)C(x)= $x^3 - 45x^2 + 600x$ C'(x)=$3x^2 - 90x + 600$=0 $\Rightarrow$x=10,20 (ii) C"(x)= 6x-90, C"(10)= -30<0 , C"(20)=30>0 C(x) is minimum at x=20 Therefore, the person must place the order for 20 trees in order to spend the least amount.	1 1 1+1

SAMPLE QUESTION PAPER-10

MARKING SCHEME

CLASS XII

MATHEMATICS (CODE-041)

SECTION: A (Solution of MCQs of 1 Mark each)

Q no.	Answer	Hints/Solution
1	b	Symmetric matrix
2	c	$\|-A\| = (-1)^3\|A\|$ $= -(-5)$ $= 5$
3	c	If A (3,4), B(-7,2), C(x, y) are collinear, then area of triangle ABC=0 i.e $\frac{1}{2}\begin{vmatrix} 3 & 4 & 1 \\ -7 & 2 & 1 \\ x & y & 1 \end{vmatrix}=0$ i.e $\begin{vmatrix} 3 & 4 & 1 \\ -7 & 2 & 1 \\ x & y & 1 \end{vmatrix}=0$ $i.e\ 3(2-y)-4(-7-x)+1(-7y-2x)=0$ i.e x-5y+17=0
4	b	LHL=RHL=k k=10
5	c	Degree=2
6	c	$\text{Cos}\gamma=0$ $\gamma=\frac{\pi}{2}$
7	d	Every point on the line joining these two corner points gives same maximum
8	b	0
9	c	$\pi/2$
10	a	$\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ $=(x+y)(x-y)-(y+z)(z-y)+(z+x)(z-x)$ $=(x^2-y^2)-(z^2-y^2)+(z^2-x^2)$ $=0$
11	a	Z(2, 4) = Z(4, 0) => 2a + 4b = 4a => a = 2b
12	d	$\pi/4$

13	**b**	$\|adj(A)\| = \|A\|^{n-1}$ = x^6	
14	**d**	**3/4**	
15	**c**	xy= C	
16	**c**	-2	
17	**c**	-1	
18	**d**	$\sqrt{b^2+c^2}$	
19	**a**	**Both A and R are true and R is the correct explanation of A**	
20	**d**	**A is false but R is true**	
Section –B **[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]**			
21	$cos^{-1}\cos(\frac{-9\pi}{4})$ $=cos^{-1}\cos(\frac{9\pi}{4})$ $=cos^{-1}\cos(2\pi+\frac{\pi}{4})$ $=cos^{-1}(\cos\frac{\pi}{4})$ $=\frac{1}{\sqrt{2}}$ OR f(1) =1 f(2)=1 since f(1)=f(2) ,f is surjective		$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1
22	f(x)=tanx-4x $f'(x)=sec^2$x-4 For f(x) to be strictly increasing f'(x)>0 $\Rightarrow sec^2$x-4>0 $\Rightarrow sec^2$x>4 $\Rightarrow cos^2$x<$\frac{1}{4}$ $\Rightarrow \frac{-1}{2} < cosx < \frac{1}{2}$ $\Rightarrow \frac{\pi}{3}$ <x<$\frac{\pi}{2}$ [x∈ $(0,\frac{\pi}{2})$		$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
23	For maxima and minimaP'(x)=0$\Rightarrow -72-36x=0$ $\Rightarrow x=-2$ $and P''$x)=-36 So P(x) is maximum at x=-2 The maximum value of P(x)=41-72× $-2-18(-2)^2$=113 OR -1≤ $sin2x \le 1$ -1+5 ≤ $sin2x+5 \le$ 1+5 4≤ $sin2x+5 \le 6$ Maximum value=6,Minimum value=4		$\frac{1}{2}$ $\frac{1}{2}$ 1 1 1
24	I=$\int \frac{tan^3x}{cos^3x}$dx =$\int \frac{sin^3x}{cos^6x}$ dx Let cos x=t$\Rightarrow sinxdx=-dt$ I=$\int(\frac{-1}{t^6}+\frac{1}{t^4})$dt =$\frac{t^{-5}}{5}+\frac{t^{-3}}{-3}$ +C =$\frac{1}{5cos^5x}-\frac{1}{3cos^3x}$ +C		$\frac{1}{2}$ $\frac{1}{2}$

	$=\frac{sec^5x}{5}-\frac{sec^3x}{3}$ +C	1
25	Let x be the side of the cube V=x^3 $\frac{dv}{dt}$=3$x^2\frac{dx}{dt}$ 9=3$x^2\frac{dx}{dt}$ $\frac{dx}{dt}=\frac{3}{x^2}$ S=6x^2 $\frac{dS}{dt}$=12x$\frac{dx}{dt}$=12x($\frac{3}{x^2}$)=$\frac{36}{x}$ $\frac{dS}{dt}$/x=10cm =$\frac{36}{10}cm^2$/s	1 1
	Section –C **[This section comprises of solution short answer type questions (SA) of 3 marks each]**	
26	I=$\int_0^\pi \frac{xtanx}{secxcosecx}$dx=$\int_0^\pi xsin^2$x dx .....(1) =$\int_0^\pi(\pi-x)sin^2x(\pi-$x)dx =$\int_0^\pi(\pi-x)sin^2$xdx .....(2) Adding (1) and (2) we get 2I=$\pi\int_0^\pi sin^2$xdx =$\frac{\pi}{2}\int_0^\pi(1-cos2x)\,dx$ =$\frac{\pi}{2}$ (x-$\frac{sin2x}{2}$) =[$[x-\frac{sin2x}{2}]_0^\pi=\frac{\pi^2}{4}$	1 $\frac{1}{2}$ $1\frac{1}{2}$
27	$\sum P(X_i)$=1⇒ $\frac{k}{2}+\frac{k}{3}+\frac{k}{6}$=1 $\frac{3k+2k+k}{6}$ =1 ⇒ $k=1$ P(1≤ $X\leq 3$) =P(1)+P(2) =$\frac{5k}{6}=\frac{5}{6}$ Mean=$\sum p_ix_i$=1×$\frac{k}{2}+2\times\frac{k}{3}+3\times\frac{k}{6}$ =$\frac{10k}{6}=\frac{5}{6}$	1 1 1
28	I=$\int\frac{(x^2-3x)}{(x-1)(x-2)}dx=\int\frac{(x^2-3x)}{(x^2-3x+2}dx$ =$\int\frac{(x^2-3x+2-2)}{(x^2-3x+2)}dx=\int dx-\int\frac{2dx}{x^2-3x+2}$ =x-2$\int\frac{dx}{(x-\frac{3}{2})^2-(\frac{1}{2})^2}dx$=x-2log$\left\lvert\frac{x-\frac{3}{2}-\frac{1}{2}}{x-\frac{3}{2}+\frac{1}{2}}\right\rvert$+C =x-2log$\left\lvert\frac{x-2}{x-1}\right\rvert$+C OR I=$\int\frac{sin^6x+cos^6x}{sin^2xcos^2x}=\int\frac{(sin^3x)^2+(cos^3x)^2}{sin^2xcos^2x}$dx =$\int\frac{(sin^2x+cos^2x)(sin^4x-sin^2xcos^2x+cos^4x)}{sin^2xcos^2x}$dx =$\int\frac{(sin^4x-sin^2xcos^2x+cos^4x)}{sin^2xcos^2x}$dx =$\int tan^2x\,dx-\int dx+\int cot^2dx$ =$\int(sec^2x-1)dx$-x+$\int(cosec^2x-1)dx$ =$\int sec^2xdx-\int dx$-x+$\int cosec^2$xdx-$\int dx$ =tanx-cotx+3x+C	 1 $1\frac{1}{2}$ $\frac{1}{2}$ 1 1 1
29	Given differential equation can be written as $\frac{dy}{dx}=\frac{y}{x}-cos^2(\frac{y}{x})$ .....(1)	1 $\frac{1}{2}$

Let y=vx $\therefore \frac{dy}{dx} = v + x\frac{dy}{dx}$ $\frac{1}{2}$

Equation (1) becomes v+x$\frac{dy}{dx}$ =v-cos^2v

$$sec^2 v dv = -\frac{dx}{x}$$

Integrating both sides we get tanv=-logIxI+C

tan$\frac{y}{x}$ =-logIxI +C 1

x=1,y=$\frac{\pi}{4}$ $\Rightarrow$ $C = 1$ $\frac{1}{2}$ $\frac{1}{2}$

Particular solution is tan$\frac{y}{x}$ =-1+logIxI+1

OR

$(1 + x^2)\frac{dy}{dx}$+2xy=$4x^2$ $\frac{1}{2}$

$\frac{dy}{dx}$ +$\frac{2x}{(1+x^2)}$y =$\frac{4x^2}{1+x^2}$

It is linear differential of the form $\frac{dy}{dx}$ +Py=Q

P=$\frac{2x}{(1+x^2)}$ Q=$\frac{4x^2}{1+x^2}$ 1

I F= $e^{\int P dx}$=$e^{log|1+x^2|}$ =1+x^2

Solution of d e is

y$(1 + x^2)$=$\int \frac{4x^2}{1+x^2}(1 + x^2)dx$+C 1

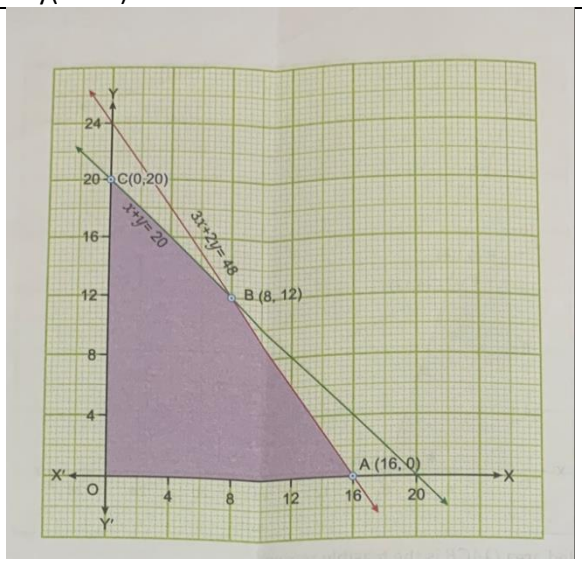
=$\int 4x^2 dx$+C $\frac{1}{2}$

y$(1 + x^2$ =$\frac{4x^3}{3}$ +C

x=0,y=0 $\therefore C = 0$

3y(1+x^2)=$4x^3$

30



1 ½

1

The region satisfying the inequalities is shown in the figure. Feasible region is shaded region with corner points O(0,0) A (16,0) B (8,12)

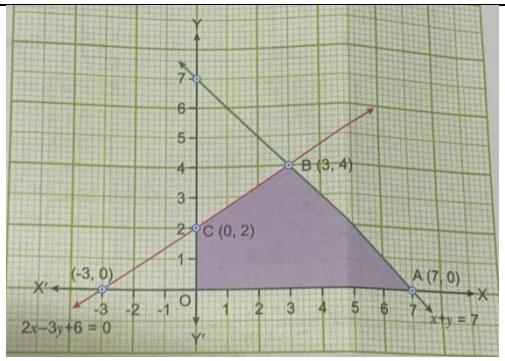
C (0,20) ½

Corner points	Z=22x+18y	
O(0,0)	0	
A (16,0)	352	
B (8,12)	392	Maximum
C (0,20)	360	

1 ½

The maximum value of Z is 390 at B (8,12)

OR



1

½

Feasible region is shaded region shown as OABC is bounded and co ordinate of its corner points are O(0,0) A (7,0) B(3,4) and D(0,2) respectively

Corner points	Z=13x-15y
O(0,0)	0
A (7,0)	91
B (3,4)	-21
C (0,2)	-30 Minimum

Hence the minimum value of Z is -30 at (0,2)

31 Given $y^x=e^{y-x}$ $\frac{1}{2}$

Taking logarithm on both sides we get xlogy= y-x

$\Rightarrow$x(1+logy)=y 1

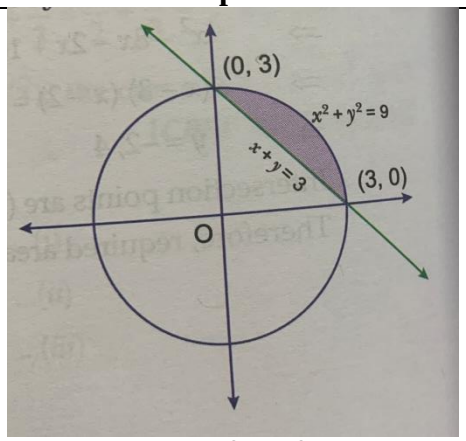
$\Rightarrow x=\frac{y}{1+logy}$

Differentiating both sides $\frac{dx}{dy} = \frac{(1+logy)1-y(0+\frac{1}{y})}{(1+logy)^2} = \frac{logy}{(1+logy)^2}$ $1\frac{1}{2}$

Section –D

[This section comprises of solution of long answer type questions (LA) of 5 marks each]

32 $1\frac{1}{2}$



Given region{(x,y):$x^2+y^2 \le 9, x+y \ge 3$}

On plotting we have the required region as shaded region

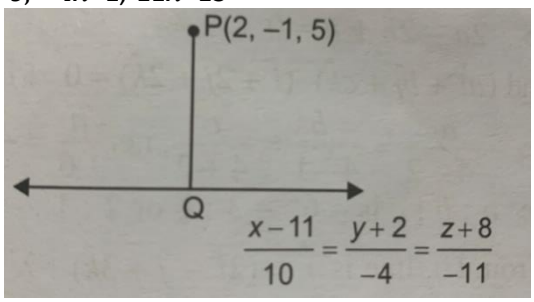
$\therefore Area\ of\ the$ shaded region=$\int_0^3 \sqrt{9-x^2}\,dx - \int_0^3 (3-x)dx$

$$[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}sin^{-1}(\frac{x}{3})]_0^3 - [3x - \frac{x^2}{2}]_0^3$$ 1

=(0+$\frac{9}{2}sin^{-1}(1)$-0)-(9-$\frac{9}{2}$)=$\frac{9\pi}{4}-\frac{9}{2}$ = $\frac{9}{4}(\pi-2)$ 1

$1\frac{1}{2}$

33	Let a,b∈ A $then$ a+b=b+a (a,b) R(a,b) SoR is reflexive Let (a,b) R(c,d) ⇒a+d=b+c ⇒c+b=d+a ⇒ (c,d)R(a,b) So R is symmetric	1
	Let (a,b) R(c,d) and (c,d) R (e,f) ⇒a+d=b+c and c+f=d+e $\Rightarrow a-b=c-d \; and \; c-d=e-f \Rightarrow a-b=e-f$ a+f=b+e	1
	so(a,b) R(e,f) so R transitive [(3,4)]={(1,2),(2,3),(3,4),(4,5),(5,6),(6,7),(7,8),(8,9),(9,10)}	2
	OR	
	f(x)=$\frac{4x+3}{3x+4}$	1
	f($x_{1)=f(x_2)} \Rightarrow \frac{4x_1+3}{3x_1+4}=\frac{4x_2+3}{3x_2+4} \Rightarrow$ (4x_1+3) (3x_2+4) = (4x_2+3)= (3x_1+4) $\Rightarrow 7x_1=7x_2 \Rightarrow 7x_1=x_2$	
	Hence f(x) is one-one	1
	Let y=$\frac{4x+3}{3x+4}$ y(3x+4)=4x+3 3xy+4y=4x+3⇒3xy-4x=3-4y x(3y-4)=3-4y⇒ $x=\frac{3-4y}{3y-4}$	2
	$$\frac{-4}{3}=\frac{3-4y}{3y-4}$$ -12y=16=9-12y 16=9 which is not possible Hence x cannot be $\frac{-4}{3}$	1
	Hence for every y∈ R-{$\frac{4}{3}$} there exist x∈ R-{-$\frac{4}{3}$} such that f(x)=y Hence f is on to function $\therefore f \; is \; bijective$	1
34	|A|=7	1
	Adj(A)=$\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$	$1\frac{1}{2}$
	A^{-1}=$\frac{1}{\|A\|}$adjA=$\frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$	$\frac{1}{2}$
	The given system of equations can be written as AX=B	1
	Where X=$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$,B=$\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$ X=A^{-1}B	

	$\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{7}\begin{bmatrix}0&1&2\\-7&7&-7\\-7&5&-4\end{bmatrix}\begin{bmatrix}6\\3\\2\end{bmatrix}=\frac{1}{7}\begin{bmatrix}7\\-35\\-35\end{bmatrix}=\begin{bmatrix}1\\-5\\-5\end{bmatrix}$ ∴x=1,y=-5,z=-5	1 1
35	$\bar{r}=(4\hat{\imath}-\hat{\jmath})+\lambda(\hat{\imath}+2\hat{\jmath}-3\hat{k})$ $\overrightarrow{a_1}$ = $4\hat{\imath}-\hat{\jmath}+0\hat{k}$ $\overrightarrow{b_1}$ = $\hat{\imath}+2\hat{\jmath}+-3\hat{k}$ $\bar{r}=(\hat{\imath}-\hat{\jmath}+2\hat{k})+\lambda(2\hat{\imath}+4\hat{\jmath}-5\hat{k})$ $\overrightarrow{a_2}$= = $\hat{\imath}-\hat{\jmath}+2\hat{k}$ $\overrightarrow{b_2}$ = $2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ $\overrightarrow{a_2}-\overrightarrow{a_1}$ = $-3\hat{\imath}+2\hat{k}$ $\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\1&2&3\\2&4&-5\end{vmatrix}=\hat{\imath}(-10+12)-\hat{\jmath}(-5+6)+\hat{k}(4-4)=2\hat{\imath}-\hat{\jmath}+10\hat{k}$ $\left\lvert\overrightarrow{b_1}\times\overrightarrow{b_2}\right\rvert=\sqrt{4+1+0}=\sqrt{5}$ d$=\left\lvert\frac{(a_2-a_1).(\overrightarrow{b_1}\times\overrightarrow{b_2})}{\lvert\overrightarrow{b_1}\times\overrightarrow{b_2}\rvert}\right\rvert=\left\lvert\frac{(-3\hat{\imath}+0\hat{\jmath}+2\hat{k}).(2\hat{\imath}+\hat{\jmath}+\hat{k})}{\sqrt{4+1+0}}\right\rvert$ $=\left\lvert\frac{-3\times2+0\times-1+2\times0}{\sqrt{5}}\right\rvert=\left\lvert\frac{-6+0+0}{\sqrt{5}}\right\rvert=\frac{6}{\sqrt{5}}$ OR General point on the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}=\lambda$ is Q$(10\lambda+11,-4\lambda-2,-11\lambda-8)$ .....(1) Direction ratio s of PQ are $10\lambda+11,-4\lambda-2+1,-11\lambda-8-13$ I,e $10\lambda+9,-4\lambda-1,-11\lambda-13$  If PQ is perpendicular to the given line ,then $10(10\lambda+9)-4(-4\lambda-1)-11(-11\lambda-13)=0$ $237\lambda=-237\Rightarrow\lambda=-1$ Substituting in (1) we get foot of the perpendicular Q (1,2,3) Length of the perpendicular PQ$=\sqrt{(2-1)^2+(-1-2)^2+(5-3)^2}=\sqrt{14}$	$\frac{1}{2}$ 1 1 1 $1\frac{1}{2}$ ½ ½ 1 1 1

		1

Section –E

[This section comprises solution of 3 case- study/passage based questions of 4 marks each with two sub parts. Solution of the first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. Solution of the third case study question has two sub parts of 2 marks each.)

36

(i) x + 0·21 = 0·44 which implies x = 0·23

(ii) 0·41 + y + 0·44 + 0.11= 1 which impliesy = 0·04

(iii) (a) $P(\frac{C}{B}) = \frac{P(C \cap B)}{P(B)}$

P(B)= 0.09+0.04+0.23=0.36

$P(\frac{C}{B}) = \frac{0.23}{0.36} = \frac{23}{36}$

OR

(iii) (b) P(A or B but not C)

= 0·32 + 0.09 + 0.04
= 0·45

37

1) Components of $\vec{A}$ **=15,0,0**
Components of $\vec{B}$ **=0,8,6**
2) $\vec{A} = 15i+0j+0k$
$\vec{B} = 0i+8j+6k$
3) Magnitude of $\vec{A}$ **= 15 unit**
Magnitude of $\vec{B} = \sqrt{64+36} = 10$ **unit**

OR

$\vec{N} = \vec{A} \times \vec{B}$

$$\vec{N} = \begin{vmatrix} i & j & k \\ 15 & 0 & 0 \\ 0 & 8 & 6 \end{vmatrix}$$

$= -90\hat{j} + 120\hat{k}$
Components are -90 , 120

38

$$y = 4x - \frac{1}{2}x^2$$

(1)The rate of growth of the plant with respect to the number of daysexposed to sunlightis given by $\frac{dy}{dx} = 4-x$

(2) Let rate of growth berepresented by the function g(x)=$\frac{dy}{dx}$

Now g'(x)= $\frac{d}{dx}\left(\frac{dy}{dx}\right) = -1 < 0$

Which implies g(x) decreases

So the rate of growth of the plant decreases for the first three days.

Height of the plant after 2 days is $y = 4\times2 - \frac{1}{2}2^2$

= 6 cm.

SAMPLE QUESTION PAPER -11

MARKING SCHEME

CLASS-XII MATHEMATICS

SECTION-A (Solution of MCQ's of 1 mark each)

1.	(d)	$\begin{bmatrix} 3y-x & -2x \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}$ -2x= -2, $\Rightarrow$ x=1 ,3y-x=5, $\Rightarrow$y=2, x+y =2
2.	(b)	|A(adjA| =|A|^n =$5^{3=}$ 125
3.	(c)	A^2 -A+I=0, A-1(A2 -A+I)=0, $A^{-1}A^2$-A^{-1}A+A^{-1} I=0 A- I +A^{-1}=0 , A^{-1} = I- A
4.	(c)	3 $\lim_{x\to 0} \frac{sin3x}{3x} = \frac{k}{2}$, 3=k/2, k =6
5.	(c)	$\hat{a}$-$\hat{b}$|^2 =($\hat{a}$-$\hat{b}$). ($\hat{a}$-$\hat{b}$) = |a⃗|2 -2(a.) ⃗ b ⃗ + |b ⃗| 2 7 = 14- 2$\|\vec{b}\|^2$ + $\|\vec{b}\|^2$ 7=14 - $\|\vec{b}\|^2$, $\overrightarrow{\|\vec{b}\|^2}$ =7$\Rightarrow$$|\vec{b}|$ =$\sqrt{7}$
6.	(d)	$\frac{dy}{dx} - y = 1$ $\frac{dy}{dx}$ = 1+y $\int \frac{dy}{1+y} = \int dx$, Log|1+y|=x ,y(0)=1, C=log2 Solution is log|1+y|=x+log2, log(1+y)/2=x $\Rightarrow$ (1+y)/2=e^x 1+y=2e^x $\Rightarrow$ $y = 2e^x$-1
7.	(c)	Open half plane containing the origin except the points of the line 3x+5y=7
8.	(b)	$\frac{d}{dx}$f(x)=logx, $\int \frac{d}{dx} f(x) = \int \log x$

		f (x)=$\int logx.1\ dx$ logx.x-x+c =x(logx-1)+c
9.	(a)	Let P be the point which divide the points(3,2,-1) and (6,2,-2) in the ratio k:1 x-coordinate of P is 5 $\Rightarrow \frac{6k+3}{k+1}$ =5 $\Longrightarrow$ k=2, then its y-coordinate is $\frac{2k+2}{k+1}=\frac{6}{3}=2$
10.	(d)	A=$\begin{bmatrix}2 & \lambda & -3\\ 0 & 2 & 5\\ 1 & 1 & 3\end{bmatrix}$, |A|≠0, λ=(-8)/5
11.	(c)	Corner points are (0,1) (4,0) and (2,5) . Maximum value of 5x+6y=40 at (2,5)
12.	(b)	$\vec{a}$ =(($\hat{\imath}$ +3$\hat{\jmath}$-2$\hat{k}$) X ($-\hat{\imath}$ +3$\hat{k}$), = (9$\hat{\imath}$ -j+3$\hat{k}$), $\|\vec{a}\|=\sqrt{9^2+1^2+3^2}=\sqrt{91}$
13.	(c)	A=$\begin{vmatrix}2 & 2\\ 9 & 4\end{vmatrix}$ and I=$\begin{vmatrix}1 & 0\\ 0 & 0\end{vmatrix}$, $10A^{-1}=10\ \frac{adjA}{\|A\|}$ $=10\ \frac{\begin{bmatrix}4 & -2\\ -9 & 2\end{bmatrix}}{8-18}$ $10\ \frac{\begin{bmatrix}4 & -2\\ -9 & 2\end{bmatrix}}{-10}=\frac{\begin{bmatrix}-4 & 2\\ 9 & -2\end{bmatrix}}{-1}$=A- 6I
14.	(a)	P(A) = 4/5 P(B) = 3/4. P(A and B contradict each other in stating the same fact)=P(A' B +AB')=1/5 x 3/4+(4)/(5) x 1/4 =7/20
15.	(d)	No arbitrary constants in the particular solution of a differential equation of any order
16.	(c)	If |a × b| = 4 ,----(1) |a.b| = 2,-----(2), Squaring and adding (1) and (2) . |a × b| ^{2} +|a.b| ^{2}= 16+4=20 $\|\vec{a}\ x\vec{b}\|=\|\vec{a}\|\ \|\vec{b}\|\ \sin\theta$ $\vec{a}\ .\vec{b}\|=\|\vec{a}\|\ \|\vec{b}\|\ \cos\theta$ |a|^2 |b|^2 =20
		If y =$(x+\sqrt{1+x^2})^n$,

17.	(a)	$\frac{dy}{dx}$ ==$n(x+\sqrt{1+x^2})^{n-1}$ (1+$\frac{1}{2\sqrt{1+x^2}}$2x) = $n(x+\sqrt{1+x^2})^{n-1}\frac{(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}$ =$n(x+\sqrt{1+x^2})^{n}\frac{1}{\sqrt{1+x^2}}$ =ny$\frac{1}{\sqrt{1+x^2}}$ $\sqrt{1+x^2}\frac{dy}{dx}$ =ny $\sqrt{1+x^2}\frac{d^2y}{d^2x}+\frac{dy}{dx}\frac{1}{2\sqrt{1+x^2}}$2x =n$\frac{dy}{dx}$ $\sqrt{1+x^2}\frac{d^2y}{d^2x}+\frac{dy}{dx}\frac{1}{\sqrt{1+x^2}}$x =n$\frac{dy}{dx}$ (1+x^2) $\frac{d^2y}{d^2x}+x\frac{dy}{dx}$ =n $\sqrt{1+x^2}\frac{dy}{dx}$ (1+x^2) $\frac{d^2y}{d^2x}+x\frac{dy}{dx}$ =n^{2}y
18.	(b)	If (x-1)/2=(y+1)/3=(z-1)/4 and (x-3)/1=(y-k)/2=z/1 For some λ andμ,2λ+1=3μ-1,3λ-1=2μ+k, and 4λ+1=μ Solving we get λ-(-3)/2 and μ=-5 Substituting in ,3λ-1=2μ+k,, we get k=9/2
19.	(d)	Assertion (A) is false as (d^2 y)/(dx^2)=(-1)/t^2 .dt/dx =(-1)/t^2 .1/2at=(-1)/(2at^3) Reason (R) is false as (d^2 y)/(dx^2)=d/dx(dy/dx) Both (A) and (R) are false
20.	(b)	Assertion (A) is true because f: R→ $\boldsymbol{R}$ defined by f(x):R =$\frac{x}{x^2+1}$ is not one one function Reason(R) is false (A) is true but (R) is false.

SECTION- B

[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

21.	tan^{-1}(tan$\frac{7\pi}{6}$)= tan^{-1}tan ($\pi+\frac{\pi}{6}$) = tan^{-1}(tan$\frac{\pi}{6}$) =$\frac{\pi}{6}$ OR sin$^{-1}\left[\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right]$ =: sin$^{-1}\left[\frac{\sqrt{1+COS2\theta}+\sqrt{1-COS2\theta}}{2}\right]$ = sin$^{-1}\left[\frac{COS\theta+sin\theta}{\sqrt{2}}\right]$ =$\frac{\pi}{4}+\frac{1}{2}cos^{-1}x$	1 1 1 1
22.	$\frac{da}{dt}$=2cm/s A=$\frac{\sqrt{3}}{4}$ a^2 , $\frac{dA}{dt}$ =$\frac{\sqrt{3}}{4}$(2a) $\frac{da}{dt}$ =10$\sqrt{3}$ cm^2/s	1 1

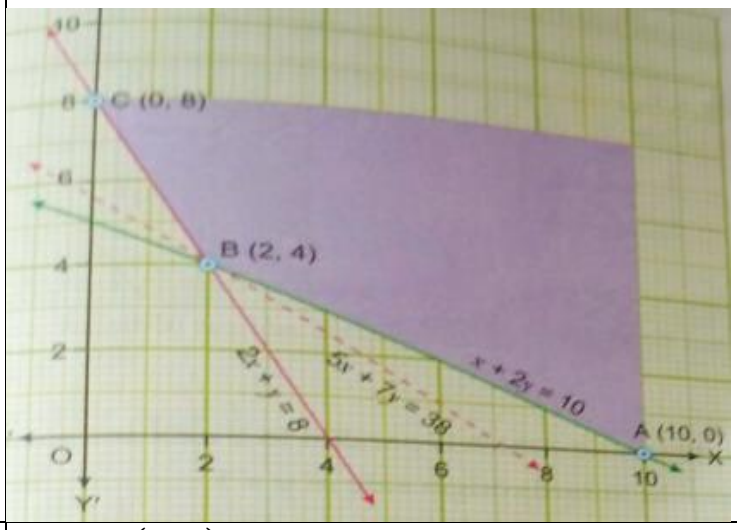
23.	Maximum value= 6 and minimum value =4 OR $-1<2x-1<1 \implies 0<2x<2 \implies 0<x<1$ Domain=[0,1]	1,1 1 1
24.	Let x+a=t $\int \frac{\sin(t-2a)}{sint}$ dt $= \int \frac{\sin t cos2a - costsin2a}{sint}$ dt$= \int cos2a - \cot t \; sin2a \; dt$ =xcos2a-sin2a logsin(x+a)	1/2 1/2 1
25.	$f(x)=-2x^3-9x^2-12x+1$ $f'(x) =-6x^2-18x-12 =-6(x^2+3x+2) =-6(x+2)(x+1)$ $f'(x) = 0 \Rightarrow x=-2$ and $x=-1$ intervals | Sign of f'(x) | Nature of f(x) (-∞,-2) | negative | decreasing (-2,-1) | positive | increasing (-1,∞) | negative | decreasing Hence f(x) is increasing in (-2,-1) and decreasing in (-∞,-2)U(-1,∞)	1 1 1

SECTION-C

[This section comprises of short answer type questions (SA) of 3 marks each]

26.	$\int e^{cot^{-1}x} \left(\frac{1-x+x^2}{1+x^2}\right)$ dx Let $cot^{-1}x = t$, $\frac{-1}{1+x^2}$ dx = dt $= \int -e^t \, (1 - \cot t + cot^2 t)dt$ $= \int -e^t \, (cosec^2 t - \cot t)dt = \int e^t \, (\cot t - cosec^2 t)dt$ $= e^t \cot t = e^{cot^{-1}} x \cot cot^{-1}x = x \, e^{cot^{-1}x} + c$	1 1 1
	Let no. of red balls = no. of green balls = x, Total balls =2x Probability of no red balls = $\frac{x}{2x} \times \frac{x}{2x}$ = ¼ Probability of 1 red balls = 2 $\times \frac{x}{2x} \times \frac{x}{2x}$ = ½	½ 1 ½

27.	Probability of 2 red balls = $\frac{x}{2x} \times \frac{x}{2x} = ¼$ Required mean = 0 X ¼ + 1 X ½ + 2 × ¼ = 1 **OR**, Let W and F be the probabilities of win and fail respectively when single die thrown alternatively. P(getting 6) = 1/6 , P(not getting 6) = 5/6 . Starting with A, P(A wins 1st throw) =W P(A wins 3rd throw) =$F_A F_B$W P(A wins 5th throw) =$F_A F_B F_A F_B$W P(A win) = 1/6 + 5/6.5/6.1/6 + 5/6.5/6.5/6.5/6.1/6+…… = 1/6$\left(1 + \left(\frac{5}{6}\right)^2\right) + \left(\frac{5}{6}\right)^4 + \cdots \ldots \ldots +)$ = 1/6 × $\frac{1}{1-\frac{25}{36}}$ = 6/11 P(B wins) = 1 - 6/11 = 5/11	1 1/2 1 1 1/2
28.	$\int_{\log\sqrt{2}}^{\log\sqrt{3}x} \frac{1}{(e^x+e^{-x})(e^x-e^{-x})}$ dx = $\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{\left(e^x+\frac{1}{e^x}\right)(e^x-\frac{1}{e^x})}$ = $\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{e^{2x}}{(e^{2x}+1)(e^{2x}-1)}\, dx$, e^{2x} = t, $2e^{2x}dx$ = dt , x = $\log\sqrt{3}$, t=3, x = $\log\sqrt{2}$, t=2, = ½ $\int_2^3 \frac{dt}{(t+1)(t-1)}$ = ½ .1/2 $\log\left\lvert\frac{t-1}{t+1}\right\rvert = \frac{1}{4}\left(log\frac{2}{4} - log\frac{1}{3}\right)$ = ¼ $\log\frac{3}{2}$	½ 1 ½ 1
29.	$(xy - x^2)$dy = y^2dx $\frac{dy}{dx} = \frac{y^2}{xy-x}$, y= vx , dy/dx= v + x .dv/dx V + x dv/dx = $\frac{v^2}{v-1}$ x.dv/dx = $\frac{v^2}{v-1}$ - v = $\frac{v}{v-1}$ $\frac{v-1}{v}$ dv = $\frac{dx}{x}$ $\int(1 - \frac{1}{v})$ dv = $\int \frac{dx}{x}$ v- $\log\lvert v\rvert = \log\lvert x\rvert + \log\lvert c\rvert$, v= log vcx $\frac{y}{x}$ = log cy , cy = $e^{y/x}$ **OR** $(x^2+1)\frac{dy}{dx}$ +2xy = $\sqrt{x^2+4}$ $\frac{dy}{dx} + \frac{2x}{x^2+1}y = \frac{\sqrt{x^2+4}}{x^2+1}$ IF = $e^{\int \frac{2x}{x^2+1}dx} = x^2+1$ y.IF = $\int Q.IF\, dx$, y.(x^2+1) = $\int \frac{\sqrt{x^2+4}}{x^2+1}$ (x^2+1) dx $(x^2+1)y = x/2\sqrt{x^2+4}$ + 2 log $\left\lvert x + \sqrt{x^2+4}\right\rvert$ +c	1 1 1 ½ ½ ½ 1 ½
30.	$2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$.	

	Corner points Z = 5x +7y (10,0) Z=50 (2,4) Z= 38…….Minimum (0,8) Z = 56 Since the feasible region is unbounded , draw the graph of the inequality 5x +7y < 38, , this graph does not have any point common with feasible region.  Hence minimum value of is 38 at (2,4).	1 1 1
31.	/x = log $\left(\frac{x}{a+bx}\right)$, logx – log (a + bx) $\frac{x\frac{dy}{dx}-y}{x^2}$ = $\frac{1}{x}-\frac{b}{a+bx}$, $x\frac{dy}{dx}-y=x^2\left(\frac{1}{x}-\frac{b}{a+bx}\right)$ = $\frac{ax}{a+bx}$= Differentiating again,, x $\frac{d^2y}{dx^2}+\frac{dy}{dx}-\frac{dy}{dx}=\frac{(a+bx)a-ax\,b}{(a+bx)^2}$ x $\frac{d^2y}{dx^2}=\left(\frac{a}{a+bx}\right)^2$ **OR** u =$sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$, v =$sin^{-1}(2x\sqrt{1-x^2})$ X= sinθ, $\theta = sin^{-1}$x U= $sec^{-1}\left(\frac{1}{\sqrt{1-sin^2\theta}}\right)$= sec^{-1}(1/cosθ) = sec^{-1} (secθ)= θ= sin^{-1}x V = $sin^{-1}(2sin\theta\sqrt{1-sin^2\theta})$ = $sin^{-1}(2sin\theta cos\theta)$= $sin^{-1}(sin2\theta)$ =2 θ= $2sin^{-1}$x du/dv = ½.	½ 1 1 ½ 1 1 1

SECTION-D

[This section comprises of long answer type questions (LA) of 5 marks each]

	Reflexive : (a,b) R (a,b) implies ab(b+a) = ba (a+b) which is true	1

32.	Hence R is reflexive Symmetric : If (a,b) R (c,d) , then ad(b+c) = bc (a+d) da(c +b) = cb (d+a) implies = cb (d+a) = da(c +b) so, (c,d) R(a,b) Hence R is symmetric. Transitive : If (a,b) R (c,d) , and (c,d) R (e,f) ad(b+c) = bc (a+d) and cf (d +e) = de (c+f) $\frac{b+c}{bc} = \frac{a+d}{ad}$, $\frac{d+c}{dc} = \frac{c+f}{cf}$, 1/c + 1/b = 1/d + 1/a 1/e + 1/d = 1/f + 1/c Addiing, 1/c + 1/b + 1/e + 1/d = 1/d + 1/a +1/f + 1/c 1/b +1/e = 1/a +1/f , $\frac{b+e}{be} = \frac{a+f}{af}$, af (b +e) = be (a +f) So (a,b) R (e,f) , Hence R is transitive. Hence R is an equivalence relation **OR** function f : $[-4,4] \to [0,4]$, given by f(x) = $\sqrt{16-x^2}$, x = 0 ∈ $[-4,4]$, f (0) = $\pm$ 4 , so it is not one-one function y = $\sqrt{16-x^2}$, x = $\sqrt{16-y^2}$ and each value of x∈ $[-4,4]$ $16-y^2 \geq 0$, $y^2-16 \leq 0$, -4≤ $y \leq 4$, but y = $\sqrt{16-x^2} \geq 0$ So, $0 \leq y \leq 4$, so for every y ∈ [0,4] , there exists some x∈ $[-4,4]$ so it is onto. f(a) = $\sqrt{7}.\sqrt{16-a^2}$ = $\sqrt{7}$, 16- a^2 = 7, a^2 = 9 , a= $\pm$3	1 21/2 ½ 1 1 1 2 1
33.	Intersection point of x^2+y^2= 4, y = $\sqrt{3}$ x $x^2+3x^2=4$, , x = $\pm$1, x =1, y = $\sqrt{3}$ Required area = $\int_0^{\sqrt{3}} x\ of\ line\ dy$ + $\int_{\sqrt{3}}^{2} x\ of\ circle\ dy$ $\int_0^{\sqrt{3}} \frac{y}{\sqrt{3}}\ dy$ + $\int_{\sqrt{3}}^{2} \sqrt{4-y^2}$ dy = $\left[\frac{y^2}{2\sqrt{3}}\right]_0^{\sqrt{3}}$ + $\left(\frac{y}{2}\sqrt{4-y^2}+\frac{1}{2}\times 4\ sin^{-1}y/2\right)_{\sqrt{3}}^{2}$ = $\frac{\sqrt{3}}{2}$ +(2 sin^{-1} 1 - $\frac{\sqrt{3}}{2}$ - 2 $sin^{-1}\frac{\sqrt{3}}{2}$ = $\frac{\sqrt{3}}{2}+2\times\frac{\pi}{2}-\frac{\sqrt{3}}{2}-2\times\frac{\pi}{3}$ = $\pi-\frac{2\pi}{3}=\frac{\pi}{3}$ sq.units	1 1 ½ 1/2 11/2 1/2

34.	AB = $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ = I , $A^{-1} = B$ or $B^{-1} = A$ Equation can be written as, $\begin{bmatrix}1 & -2 & 0\\ 2 & -1 & 1\\ 0 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}3\\ 2\\ 3\end{bmatrix}$ $B'X = C$, X= $(B')^{-1}$C, X = $(B^{-1})'$C= A' C = $\begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}\begin{bmatrix}3\\ 2\\ 3\end{bmatrix} = \begin{bmatrix}1\\ -1\\ 1\end{bmatrix}$, x =1, y = -1 , z = 1	2 ½ ½ 2
35.	Equation of diagonal PR is $\frac{x-X1}{X2-X1} = \frac{y-y1}{y2-y1} = \frac{z-z1}{z2-z1}$, $\frac{x-4}{8} = \frac{y-2}{2} = \frac{z+6}{11}$ Equation of diagonal QS is , $\frac{x-5}{6} = \frac{y+3}{12} = \frac{z-1}{-3}$ or , $\frac{x-5}{2} = \frac{y+3}{4} = \frac{z-1}{-1}$ General point on PR is $(8\alpha + 4, 2\alpha + 2, 11\alpha - 6)$ General point on QS is $(2\beta + 5, 4\beta - 3, -\beta + 1)$ Intersection of diagonals if , $8\alpha + 4 = 2\beta + 5$ and $2\alpha + 2 = 4\beta - 3$ So, $8\alpha - 2\beta = 1$ and $2\alpha - 4\beta = -5$, $solving\ \alpha = \frac{1}{2}\ and\ \beta = \frac{3}{2}$ Intersection point of diagonals = (4+4, 1+2, 11/2-6) = (8, 3, -1/2) **OR** Equation of line through point (-13,2) with direction ratio (a,b,c) is $\frac{x+1}{a} = \frac{y-3}{b} = \frac{z+2}{c}$ Line is perpendicular to given lines, a +2b +3c = 0, -3a +2b +5c =0 a/2 = b/-7 = c/4 , Required line is $\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$ vector form is, $\vec{r} = -1\,\hat{i} + 3\hat{j} \pm 2\,\hat{k} + \alpha(2\,\hat{i} - 7\,\hat{j} + 4\,\hat{k})$ Direction ratio of line passing through origin and $(2\alpha + 1, -7\alpha + 3, 4\alpha - 2)$ So, $2\alpha + 1 \pm 7 \times (-7\alpha + 3) + 4(4\alpha - 2) = 0$, α = 31/69 Distance from origin = $\sqrt{\frac{345}{69}}$	1 1 1 1 1 1 1 ½ 1 1 1/2

SECTION-E

[This section comprises of 3 Source based/ case based/ passage based/ integrated units of assessment of 4 marks each with sub-parts . The first two case study questions have 3 sub-parts (i),(ii), (iii) of marks 1,1,2 respectively. The third case study questions has 2 sub-parts of 2 marks each.]

	(i) YES (ii) m= 1.2 (iii) strictly increasing in [0,6] strictly decreasing	(1+1+2)

36.	[6,12] OR Point of local absolute maxima =6, and absolute maximum value = 102.2. Point of absolute minima = 0 , and absolute minimum value = 98.6	
37.	i) $(\overrightarrow{AB}) = (2\hat{\imath}\text{-}2\hat{k})$ (ii) $\overrightarrow{CD} = \hat{\imath} + 3\hat{j} + \hat{k}$ (iii) $\overrightarrow{BC} = -\hat{\imath} - \hat{j} + \hat{k}$ .Unit vector along $\overrightarrow{BC} = \frac{-\hat{\imath}-\hat{j}+\hat{k}}{\sqrt{3}}$ OR $\overrightarrow{BD} = 2\hat{j} + 2\hat{k}$ area of $\Delta\,(BCD)$ = ½ $\left\lvert\overrightarrow{BC} \times \overrightarrow{BD}\right\rvert$ = 1/2$\begin{vmatrix} i & j & k \\ -1 & -1 & 1 \\ 0 & 2 & 2 \end{vmatrix}$= $\sqrt{6}$ sq.units	(1+1+2)
38.	(i) 0.38 (ii) 7/19	(2+2)

Sample Question Paper 12- Answers

MARKING SCHEME

SECTION A

1. (b) *Equation of the straight line passing through one point* $\vec{a} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ *and parallel to* $\vec{b}$ $=2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ is given by $\vec{r} = (3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}) + \mu(2\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$ using the formula $\vec{r} = \vec{a} + \mu\vec{b}$ 1M

2. (b) Integrating factor $= e^{\int pdx} = e^{\int cotxdx} = e^{\log sinx} =$ sinx 1M

3. (b) 1M

$$\begin{vmatrix} 6 & 1 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} 3x & 1 \\ 2x & 2 \end{vmatrix}$$

➔ $18 - 2 = 6x - 2x$

➔ $4x = 16$

➔ $x = 4$

4. (c)

Corner points	Value of $F = 3x - 4y$
O(0,0)	0
A(12,6)	12 ➔*maximum*
B(0, 4)	-16

5. (c) $\sqrt[3]{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx}}$ Raising the power 6 we get $\left(\frac{d^2y}{dx^2}\right)^2 = \left(\frac{dy}{dx}\right)^3$ 1M

∴ Order=2 and degree=2 ➔ *sum=4*

6. (b) 1M

X	1	2	3	4	5	6	Total
$P(X = x)$	k	4k	9k	8k	10k	12k	1

$\sum p_i = 1$ ➔ *44k=1* ➔ $k = \frac{1}{44}$

7. (a) $\int \frac{e^{tan^{-1}x}}{1+x^2} dx$ Let t=$\tan^{-1} x$ 1M

$= \int e^t \, dt$ $\frac{dt}{dx} = \frac{1}{1+x^2}$

$= e^t$+C

$=e^{tan^{-1}}x + c$

8. (d) $\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = \tan^{-1} x]_1^{\sqrt{3}} = \tan^{-1}\sqrt{3} - \tan^{-1} 1 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$ 1M

9. (d) $\begin{bmatrix}1 & 1 & 1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\3\\2\end{bmatrix}$, ➔ $\begin{bmatrix}x+y+z\\y+z\\z\end{bmatrix} = \begin{bmatrix}6\\3\\2\end{bmatrix}$, 1M

➔ z=2,y=1,x=3 ➔ 2x+y-z = 5

10. (d) $|2A'|=2^3\, IA'I=8\times 5 = 40$ 1M

11. (c) $k = \lim_{x\to 0} \frac{1-\cos 4x}{8x^2} = \lim_{x\to 0} \frac{2sin^2 2x}{8x^2} = 1$ 1M

12. (a) $|adj(adjA)| = |(adjA)|^2 = (|A|^2)^2 = |A|^4 = 14^4$ 1M

13. (a) $|A| \neq 0$ ➔ $\beta \neq -2$ 1M

14. (d) $\vec{a} + \vec{b} + \vec{c} = 0$, 1M

➔ $a + b = -c$

➔ $(\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = -\vec{c}.-\vec{c}$

➔ $|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{c}|^2$

➔ $9+25+2|\vec{a}||\vec{b}|\cos\theta = 49$

➔ $\cos\theta = \frac{1}{2}$ ➔ $\theta = \frac{\pi}{3}$

15. (c) $|\vec{a} \times \vec{b}|^2 + (\vec{a}.\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 = 16+4 = 20$ 1M

16. (a) lines $\frac{x-2}{3} = \frac{y-1}{1} = \frac{4-z}{k} \ldots\ldots\ldots (1)$ 1M

$\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2} \ldots\ldots\ldots (2)$ are perpendicular

$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0$ ➔ $3k + 2 + 2k = 0$ ➔ k = -2/5

17. (d) $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 8$ 1M

➔ $|\vec{x}|^2 - |\vec{a}|^2 = 8$

➔ $|\vec{x}|^2 = 9$

➔ $|\vec{x}| = 3$

18. (a) $Z = px + qy$ is minimum occurs at (3,0) and (1,1) is 1M

$\therefore 3p = p + q \rightarrow p = 2q$

19. (a) Both A and R are true and R is the correct explanation of A. 1M

20. (a) Both A and R are true and R is the correct explanation of A. 1M

SECTION B

21. $\sin^{-1}[\cos(\sin^{-1}\frac{\sqrt{3}}{2})]$

$=\sin^{-1}(cos\frac{\pi}{3})$ 1

$=\sin^{-1}\frac{1}{2}$ ½

$=\frac{\pi}{6}$ ½

OR

$\sqrt{x}=\tan\theta\ , \theta = \tan^{-1}\sqrt{x}$ ½

$\tan^{-1}\frac{2tan\theta}{1-tan^2\theta}$ ½

$\tan^{-1}(tan2\theta)$ ½

$2\theta = 2\tan^{-1}\sqrt{x}$ ½

22. $F(x) = 10-6x-2x^2$

$\frac{dy}{dx}$ = -6−4x ½

$\frac{dy}{dx}$ = 0 implies x =$\frac{-3}{2}$ ½

Intervals are $(-\infty, \frac{-3}{2}), (\frac{-3}{2}, \infty)$ ½

Increasing in the interval $(-\infty, -\frac{3}{2})$

Decreasing in the interval $(\frac{-3}{2}, \infty)$ ½

23. f(x)= $9\left[(x+\frac{2}{3})^2\right]-2$ 1

Minimum value is -2 ½

No maximum value ½

OR

R(x)= $13x^2+26x+15$

$R^1(x)= 26x+26$ 1

$R^1(x)$ at x= 7 is 208 1

24. I=$\int_0^{\frac{\pi}{2}} \frac{sin^4x}{sin^4x+sin^4x}$ dx

I= $\int_0^{\frac{\pi}{2}} \frac{cos^4x}{sin^4x+cos^4x} dx$ ½

2I = $\int_0^{\frac{\pi}{2}} 1dx$ 1

I= $\frac{\pi}{4}$ ½

25. f(x) = sinx+cos x

$f^1(x)$= cos x-sinx 1

$f^1(x)$= 0 $implies$ x =$\frac{\pi}{4}, \frac{5\pi}{4}$ ½

Yes function has critical points, $\frac{\pi}{4}$ & $\frac{5\pi}{4}$ ½

26. Let,e^x= t, e^x dx= dt ½

$$\int \frac{dt}{(1+t)\,(2+t)}$$ ½

$$\frac{1}{(1+t)(2+t))} = \frac{1}{1+t} + \frac{-1}{2+t}$$ 1

$\int \frac{dt}{(1+t)\,(2+t)}$ = $\log|1+t| - log|2+t| + c$ ½

$= \log|1 + e^x| - log|2 + e^x| + c$ ½

27. X=2, 3,4,5,6. ½

X	2	3	4	5	6
P(X)	1/15	2/15	3/15	4/15	5/15
X.P(X)	2/15	6/15	12/15	20/15	30/15

2

½

Expectation of X= $\sum XP(X)$

$= \frac{70}{15} = \frac{14}{3}$

28. $\frac{1}{\sin(b-a)} \int \frac{\sin[(x-a)-(x-b)]}{\sin(x-a)\sin(x-b)} dx$ ½

$= \frac{1}{\sin(b-a)} \int \frac{\cos(x-b)}{\sin(x-b)} - \frac{\cos(x-a)}{\sin(x-a)} dx$ ½

$= \frac{1}{\sin(b-a)} \int [\cot(x-b) - \cot(x-a)] dx$ ½

$= \frac{1}{\sin(b-a)} [\log\sin(x-b) - \log\sin(x-a)] + c$ ½

$= \frac{1}{\sin(b-a)} \log(\frac{\sin(x-b)}{\sin(x-a)} + c$ ½

½

OR

$I = \int_0^{\frac{\pi}{2}} \log \frac{\sin^2 x}{2\sin x \cos x} dx$

$= \int_0^{\frac{\pi}{2}} \log\left(\frac{tanx}{2}\right) dx$

½

$I = \int_0^{\frac{\pi}{2}} logtanx - log2dx$……(1)

$I = \int_0^{\frac{\pi}{2}} logcotx - log2dx$……(2) ½

2I=:$\int_0^{\frac{\pi}{2}} log1 - 2log2dx$ adding (1) & (2) ½

2I=$-\int_0^{\frac{\pi}{2}} 2log2dx$ ½

I=$\int_0^{\frac{\pi}{2}} log2dx$

= $\frac{-\pi}{2}$log 2 ½

29. $\frac{dy}{dx} + \frac{y}{xlogx} = \frac{2}{x^2}$ ½

I.F=logx 1

y logx=$\int \frac{2}{x^2}$logx ½

=$\frac{-2logx}{x} - \frac{2}{x} + c$ ½ ½

OR

$\frac{dy}{dx} = \frac{ycos\left(\frac{y}{x}\right)+x}{xcos(\frac{y}{x})}$ Putting, y =vx

v+x$\frac{dv}{dx}$ =$\frac{vcosv+1}{cosv}$, ½

x$\frac{dv}{dx} = \frac{1}{cosv}$ 1

$\int cosv\, dv = \int \frac{dx}{x}$

Sin($\frac{y}{x}$) =logx+c 1

½

30. Drawing the graph of two inequalities and shading the feasible region $1\frac{1}{2}$

Corner pts	Value of Z
(0,5)	2500
(4,3)	2300
(0,6)	3000

1

½

Minimum value is 2300

OR

Drawing the graph of two inequalities and shading the feasible region $1\frac{1}{2}$

Corner pts	Value of Z
(0,0)	0
(30,0)	120
(20,30)	110
(0,50)	50

1

½

Maximum value is 120

31. $y_1=\frac{ae^{a\cos^{-1}x}}{\sqrt{1-x^2}}$

$y_1\sqrt{1-x^2} = ae^{a\cos^{-1}x}$ ½

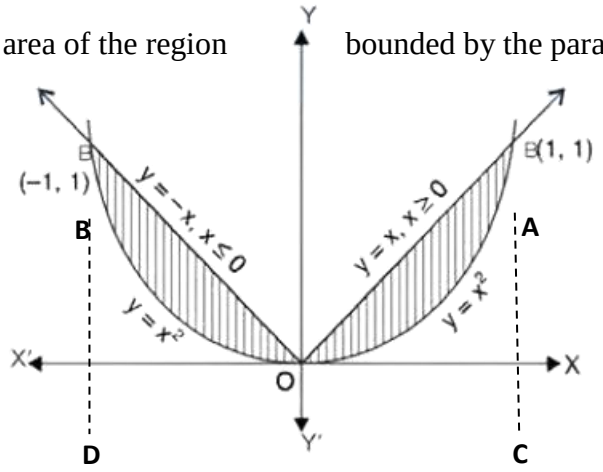
$y_1\frac{-2x}{2\sqrt{1-x^2}}+\sqrt{1-x^2}\ y_2=\frac{a^2e^{a\cos^{-1}x}}{\sqrt{1-x^2}}$ ½ 1

$-xy_1+(1-x^2)y_2=a^2y$ ½

$(1-x^2)y_2-xy_1-a^2y=0$ ½

SECTION D

32. The area of the region bounded by the parabola y = x^2 and y = $|x|$ can be represented as



1 Mark

The given area is symmetric about y axis

Therefore, area OACO = area OBDO

The **point of intersection of parabola** $x^2 = y$, and line, y = x, is A (1

1 Mark

Area of OACO = area of Δ OAB – area OBACO

1 Mark

i.e **Area of Δ OAB** = $\int_0^1 x dx = \frac{1}{2}$

1 Mark

Area of OBACO = $\int_0^1 y\,dx = \int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$

⇒ Area of OACO = Area of Δ OAB - Area of OBACO

$$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

Therefore , required area = $2\left[\frac{1}{6}\right] = \frac{1}{3}$ units.

1 Mark

33. Let R be defined on **N×N** as

(a,b) R (c,d) ⟺ ad(b+c)=bc(a+d)…….....(1)

Reflexivity:

We can write ab (b+a) = ba (a+b) for all a,b∈N

Since, sum and product of natural numbers obeys commutative property

Hence, by def (1), we can write

(a,b)R(a,b) for all (a,b) ∈ N×N

Hence, R is reflexive.

1 Mark

Symmmetry :

Let (a,b)R(c,d)

⇒ ad (b+c) = bc (a+d)

⇒ da (c+b) = cb (d+a) (Since, sum and product of natural numbers obeys commutative property)

or cb(d+a)=da(c+b)

⇒(c,d)R(a,b)

Hence, R is symmetric

Transitivity : 1 ½ Mark

Let (a,b),(c,d),(e,f) ∈ N×N

Let (a,b)R(c,d) and (c,d)R(e,f)

ad(b+c)=bc(a+d) and cf(d+e)=de(c+f)

$\Rightarrow \frac{ab}{a-b} = \frac{cd}{c-d}$ and $\frac{Cd}{C-d} = \frac{ef}{e-f}$ 2 Mark

$\Rightarrow \frac{ab}{a-b} = \frac{ef}{e-f}$

⇒ af (b+e) = be(a+f)

⇒(a,b)R(e,f)

Hence ,R is transitive

∴ R is Equivalence Relation. ½ Mark

OR

Suppose $f(x_1) = f(x_2)$. Note that if x_1 is odd and x_2 is even,

then we will have $x_1 + 1 = x_2 - 1$, i.e., $x_2 - x_1 = 2$ which is impossible. 1½ Mark

Similarly, the possibility of x_1 being even and x_2 being odd can also be ruled out,

using the similar argument.

Therefore, both x_1 and x_2 must be either odd or even.

Suppose both x_1 and x_2 are odd.

Then $f(x_1) = f(x_2) \Rightarrow x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2$. 1 Mark

Similarly, if both x_1 and x_2 are even,

then also $f(x_1) = f(x_1) \Rightarrow x_1 - 1 = x_2 - 1 \Rightarrow x_1 = x_2$. 1 Mark

Thus, f is one-one

Also, any odd number 2r + 1 in the co-domain N is the image of 2r + 2 in the domain N

and any even number 2r in the co-domain N is the image of 2r – 1 in the domain N.

Thus, $\boldsymbol{f}$ is onto. 1½ Mark

34. Product = $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ 1 Mark

$A^{-1} = \begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ 1 Mark

(1 mark)

$\mathbf{AX = B}$, $\mathbf{X} = \boldsymbol{A^{-1}B}$ ark

$X = \begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}\begin{bmatrix}1\\1\\2\end{bmatrix} = \begin{bmatrix}0\\5\\3\end{bmatrix}$

Therefore, x = 0, y = 5, z = 3. 2½ Mark

35. Let $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda$

Any general point on this line , say P = $(\lambda + 4, -4\lambda - 3, 7\lambda - 1)$

Again let $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \mu$

Any general point on this line, say Q = $(2\mu + 1, -3\mu - 1, 8\mu - 10)$ 1 Mark

If the both lines intersect then one potin will be common in two lines

Let **P** and **Q** coincide

$\lambda + 4 = 2\mu + 1,\ -4\lambda - 3 = -3\mu - 1,\ 7\lambda - 1 = 8\mu - 10$ 1 Mark

$\lambda - 2\mu = 3 \ldots..(i),\ -4\lambda + 3\mu = 2 \ldots..(ii),\ 7\lambda - 8\mu = -9$…..(iii)

Soliving (i) and (ii) we get $\boldsymbol{\lambda = 1}$ and $\boldsymbol{\mu = 2}$ 1 Mark

Substituting these values in (iii) we get

$7(1) - 8(2) = -9$

$-9 = -9$,which is true 1 Mark

Therefore, the given lines intersect each other

Point of intersection is P = $(1+4, -4 - 3, 7 - 1)$

$\mathbf{P = (5, -7, 6)}$ 1 Mark

OR

Given equation are

$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{5-Z}{5}$

Now $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{I} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$ = $\hat{\imath}(80-56)$- $\hat{\jmath}(-15-21) + \hat{k}(24+48)$ 1 Mark

1 Mark

$=24\hat{\imath} + 36\hat{\jmath} + 72\hat{k}$ 1 Mark

$$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{24^2 + 36^2 + 72^2} = 12 \times 7$$

Also, $\vec{a}_2 - \overrightarrow{a_1} =$ (15-8) $\hat{\imath} + (29+9)\hat{\jmath} + (5-10)\hat{k} = 7\hat{\imath} + 38\hat{\jmath} - 5\hat{k}$ 1 Mark

S.D = $\left|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}).\vec{b}_1 \times \vec{b}_2}{|\vec{b}_1 \times \vec{b}_2|}\right|$ 1 Mark

$= \frac{98}{7} =$ 14 units 1 Mark

SECTION E

36. Let $\mathbf{E_1, E_2, E_3}$ be the events that Jayant, Sonia and Oliver processed the form, which are clearly pairwise mutually exclusive and exhaustive set of events.

Then $P(E_1) = \frac{50}{100} = \mathbf{\frac{5}{10}}$, $P(E_2) = \frac{20}{100} = \mathbf{\frac{1}{5}}$ *and* $P(E_3) = \frac{30}{100} = \mathbf{\frac{3}{10}}$.

Also, let **E** be the event of committing an error

We have, $P(E \setminus E_1) = 0.06$, $P(E \setminus E_2) = 0.04$, $P(E/E_3) = 0.03$

1) The total probability of committing an error in processing the form is given by

$P(E) = P(E_1).P(E/E_1) + P(E_2).P(E/E_2) + P(E_3).P(E/E_3)$

$= \frac{30+8+9}{1000} = \frac{47}{1000} = 0.047$ 2M

2) The total probability of committing an error in processing the form is given by

$$P(E_1 \mid E) = \frac{P(E \mid E_1) \times P(E_1)}{P(E \mid E_1) \cdot P(E_1) + P(E \mid E_2) \cdot P(E_2) + P(E \mid E_3) \cdot P(E_3)}$$

$$= \frac{0.06 \times \frac{50}{100}}{0.06 \times \frac{50}{100} + 0.04 \times \frac{20}{100} + 0.03 \times \frac{30}{100}} = \frac{30}{47}$$

1M

Therefore, the required probability that the form is not processed by Jayant given that form has an error

$$= P\left(\bar{E}_1 \mid E\right) = 1 - P(E_1 \mid E) = 1 - \frac{30}{47} = \frac{17}{47}$$ 1M

OR

$\sum_{i=1}^{3} P(E_i \mid E) = P(E_1 \mid E) + P(E_2 \mid E) + P(E_3 \mid E) = 1$

Since, sum of the posterior probabilities is 1.

(we have $\sum_{i=1}^{3} P(E_1 \mid E) = P(E_1 \mid E) + P(E_2 \mid E) + P(E_3 \mid E)$

$= \frac{P(E \cap E_1)+P(E \cap E_2)+P(E \cap E_3)}{P(E)}$ $= \frac{P\left((E \cap E_1) \cup (E \cap E_2) \cup (E \cap E_3)\right)}{P(E)}$ 1M

as $E_i \; and \; E_j \; ; i \neq j$, are mutually exclusive events.

$= \frac{P(E \cap (E_1 \cup E_2 \cup E_3)}{P(E)} = \frac{P(E \cap S)}{P(E)} = \frac{P(E)}{P(E)} = 1; ' \; S' being \; sample \; space)$ 1M

37. $\frac{x_2-x_1}{r} = \frac{y_2-y_1}{r} = \frac{z_2-z_1}{r}$

1) Applying $\frac{a}{r} = \frac{b}{r} = \frac{c}{r}$ $< 0,1,0 >$ 1M

2) Applying

$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

$\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ 1M

3) Parallel 1M

4) $\frac{x}{1} = \frac{y}{2} = \frac{z-3}{-3}$ 1M

38. 1) Area = $\frac{1}{2}$ h (a+b)

$= \frac{1}{2}\sqrt{100 - x^2}\,[10 + 2x + 10]$

= (10+x) $\sqrt{100 - \; x^2}$ 1M

2) Differentiating Area with respect to x and simplifying 1M

$= \frac{-2x^2-10x+100}{\sqrt{100-x^2}}$

3) $-2x^2$- 10x + 100 = 0 by solving x=5 1M

4) Substituting x=5 in 1 Area = 75sq m 1M

KENDRIYA VIDYALAYA SANGATHAN ERNAKULAM REGION

SAMPLE PAPER 13

MARKING SCHEME TIME 3 HRS MM 80

QUESTION NO	ANSWER	MARK
1	(b)	1
2	(a)	1
3	(d)	1
4	(b)	1
5	(c)	1
6		1
7		1
8	(b)	1
9	(c)	1
10	(a)	1
11	(d)	1
12	(d)	1
13	(a)	1
14	(b)	1
15	(a)	1
16	(c)	1
17	(d)	1
18	(d)	1
19	(d)	1
20	(c)	1
21	$\tan^{-1}\left[2\sin\left(2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$ $\Rightarrow \tan^{-1}\left[2\sin\left(2X\frac{\pi}{6}\right)\right]$ $\Rightarrow \tan^{-1}\left[2\sin\left(\frac{\pi}{3}\right)\right]$ $\Rightarrow \tan^{-1}\left[2\frac{\sqrt{3}}{2}\right]$ $\Rightarrow \tan^{-1}\left[\sqrt{3}\right]=\frac{\pi}{3}$ OR To prove injectivity To prove surjectivity	½ ½ ½ ½ 1 1
22	$X=\frac{\cos y}{\cos(a+y)}$ $\Rightarrow \frac{dy}{dx}=\frac{\cos(a+y)(-\sin y)-\cos y\cos(a+y)}{\cos^2(a+y)}$	½ 1

	⇨ $\frac{\sin a}{cos^2\ (a+y)}$	1/2
23	To get $\frac{dr}{dt}=\frac{1}{72\pi}$ To get $\frac{dA}{dt}=\frac{4}{3}$	1 1
24	To get $\bar{a}\ X\bar{b} = -\hat{\imath} +2\hat{\jmath} +2\hat{k}$ To get unit vector along $\bar{a}\ X\bar{b}$ To get vector of magnitude 6 OR Cartesian form is $\frac{x-1}{3}\ \frac{y-1}{2}\ \frac{z-3}{-2}$ Vector form is $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\ +\mu(\ 3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}\ )$	1 ½ ½ 1 1
25	$Direction\ cosines\ are \frac{-3}{7},\ \frac{-2}{7},\ \frac{6}{7}$	2
	SECTION-C	
26	**Put** $\sin^{-1}(\ x^2\ ) = t$ ⇨ $\frac{x}{\sqrt{1\ -x^4}}$ **dx =** $\frac{t}{2}$**dt** ⇨ $\frac{1}{2}\int$ **tdt** ⇨ $\frac{t^2}{4}$**+ c** ⇨ $\frac{(\ \sin^{-1}(\ x^2\ ))^2}{4}$ **+ c** OR $\int_0^1 \tan^{-1}\frac{(\ (1-x\ ) - x))}{1+x\ (\ 1-x)}$**dx** ⇨ $\int_0^1 \tan^{-1}(1-x)\ dx$ - $\int_0^1 \tan^{-1} x\ dx$ ⇨ $\int_0^1 \tan^{-1} x\ dx$ - $\int_0^1 \tan^{-1} x\ dx$ = 0	½ 1 ½ ½ ½ 1 1 ½ ½
27	(1) **1- (½ x 2/3) = 2/3** (2) **½ x 2/3 + ½ x 1/3 = 1/2**	1 ½ 1 ½
28	**Writing P_4 Property** ⇨ **I=** $\int_0^{\frac{\pi}{4}} log\ \left(\ 1 + \tan(\frac{\pi}{4} - x\ \right)\ dx$ ⇨ **I =** $\int_0^{\pi/4} log\ \left(1 + \frac{1-tan\ x}{1+tan\ x}\right)\ dx$ ⇨ **2I =** $\int_0^{\pi/4} log\ \left(\frac{2}{1+tan\ x}\right)\ dx$	½ ½ 1 ½ ½

	⇨ $\mathbf{I = \frac{\pi}{8} \log 2}$	½
	OR	
	$\mathbf{I=\int_0^{\pi} \frac{xsinx}{1+\cos^2 x}\,dx}$**.....(1)**	
	$\mathbf{I=\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}\,dx}$	
	⇨ $\mathbf{\int_0^{\pi} \frac{(\pi-x)sinx}{1+\cos^2 x}\,dx}$**.....(2)**	½
		½
	(1)+ (2)	
	⇨ $\mathbf{2I=\int_0^{\pi} \frac{xsinx+(\pi-x)sinx}{1+\cos^2 x}\,dx}$	
	⇨ $\mathbf{\pi\int_0^{\pi} \frac{sinx}{1+\cos^2 x}\,dx}$	½
	⇨ $\mathbf{\pi\int_1^{-1} \frac{-dt}{1+t^2}}$	½
	⇨ $\mathbf{-\pi[\tan^{-1} t]_1^{-1}}$	½
	⇨ $\mathbf{-\pi[-\frac{\pi}{4}-\frac{\pi}{4}] = -\pi[-\frac{\pi}{2}] \Rightarrow I=\pi^2/2}$	
		½
29	$\mathbf{\frac{dy}{dx} = \frac{y}{x} - tan\left(\frac{y}{x}\right)}$**put y=vx then** $\mathbf{\frac{dy}{dx} = v + x\frac{dv}{dx}}$	½
	$\mathbf{\Rightarrow v + x\frac{dv}{dx}}$ **=v-tanv**	½
	$\mathbf{\Rightarrow -\int cotv\,dv = \int \frac{dx}{x}}$	1
	$\mathbf{\Rightarrow -\log\left\lvert\sin\left(\frac{y}{x}\right)\right\rvert = \log\lvert x\rvert + C}$	1
	OR	
	$\mathbf{\frac{(tan^{-1}y}{(1+y^2)} - \frac{x}{(1+y^2)} = \frac{dx}{dy}}$	
	⇨ $\mathbf{\frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{(tan^{-1}y}{(1+y^2)}}$	½
	⇨ **I.F.=** $\mathbf{e^{\int \frac{1}{(1+y^2)}} = e^{\tan^{-1} y}}$	½
	⇨ **Solution is** $\mathbf{xe^{\tan^{-1} y} = \int e^{\tan^{-1} y} \frac{(tan^{-1}y}{(1+y^2)}dy}$	½
	Put $\mathbf{\tan^{-1} y = t, then \frac{1}{(1+y^2)}dy = dt}$	
	⇨ $\mathbf{\int e^{\tan^{-1} y} \frac{(tan^{-1}y}{(1+y^2)}dy}$ $\mathbf{\int te^t dt = te^t - e^t + C}$	½
	= solution is $\mathbf{xe^{\tan^{-1} y} = \tan^{-1} y\, e^{\tan^{-1} y}\, e^{\tan^{-1} y} + C}$	½
	=> $\mathbf{x = \tan^{-1} y - 1 + ce^{-\tan^{-1} y}}$	

		½
30.	**To Draw the graph** **To get corner points** **Corner Point** | **Z = 6x + 3y** **(2,72)** | **228** **(15,20)** | **150** **(40,15)** | **285** **Max z = 285 at (40,15)**	1 ½ ½ 1
31.	$\int \frac{1}{(x^2+1)(x^2+4)}dx$ **Put** $x^2 = y$ ⇨ $\int \frac{1}{(y+1)(y+4)}dx = \int \frac{1}{(y+1)(y+4)}dx$ $Let \frac{1}{(y+1)(y+4)} = \frac{A}{(y+1)} + \frac{B}{y+4}$ ⇨ $I=A(y+4)+B(y+1)$ **Put y=-1 ,A=**$\frac{1}{3}$**put y=-4 get B=**$\frac{-1}{3}$ ⇨ $\int \frac{1}{(y+1)(y+4)}dx$ ⇨ $\int\{\frac{\frac{1}{3}}{(y+1)} + \frac{\frac{-1}{3}}{y+4}]dx$ ⇨ $\int\{\frac{\frac{1}{3}}{(x^2+1)} + \frac{\frac{-1}{3}}{x^2+4}]dx$ ⇨ $\frac{1}{3}\tan^{-1}x - \tan^{-1}\frac{x}{2}$**+C**	 ½ ½ 1 ½ ½
	SECTION-D	
32	**Sketching the graph.** **Area = 2** $\int_{-3}^{0}(x+3)dx$ **Correct Integration** **2 (0 - (**$\frac{9}{2}$ **- 9)) = 9**	2 1 2
33	**To prove relation is reflexive.** **To prove relation is symmetric.** **To prove relation is transitive.** **To get the equivalence class of (2,5)**	1 ½ 1 ½ 1 ½ ½
34	$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$ ⇨ **3a -16b + 7 c = 0** ⇨ **3a + 8 b – 5 c = 0**	1 1

	⇨ **a = 2 , b = 3 , c = 6**	1
	⇨ **Vector equation is** $\vec{r} = \vec{a} + \lambda \vec{b}$	1
	⇨ $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$	
	OR	
	Cartesian equation of the two given lines are	1
	$\frac{x-6}{1} = \frac{y-2}{-2} = \frac{z-2}{2} = \lambda$ .....(1) $\Rightarrow P = (\lambda+6, -2\lambda+2, 2\lambda+2)$	
	$\frac{x+4}{3} = \frac{y}{-2} = \frac{z+1}{-2}$.....(2) $\Rightarrow Q = (3\mu-4, -2\mu, -2\mu-1)$	½
	Direction ratios of PQ are	½
	$3\mu-\lambda-10, -2\mu+2\lambda-2, -2\mu-2\lambda-3$	
	Direction ratios of line (1) are 1 ,-2 ,2	
	Direction ratios of line (2) are 3 , -2 , -2	
	Distance between the two given lines will be shortest if PQ is	½
	perpendicular to line (1) as well as to line (2)	½
	$\therefore 1(3\mu-\lambda-10) - 2(-2\mu+2\lambda-2) + 2(-2\mu-2\lambda-3) = 0 \Rightarrow 3\mu-9\lambda-12 = 0$.....(3)	
	$3(3\mu-\lambda-10) - 2(-2\mu+2\lambda-2) - 2(-2\mu-2\lambda-3) = 0 \Rightarrow 17\mu-3\lambda-20 = 0$.....(4)	
	Solving (3) and (4)	
	λ **= -1 and** μ **=1**	
	The point on the lines they should reach so that the distance between them is the shortest is P(5, 4, 0) and Q(-1, -2, -3).	2
	Shortest distance = PQ = 9	
		1
35	**Forming equations as**	
	X + y +z = 6	½
	Y + 3z =11	
	X + z = 2y	
	⇨ **AX= B, X = A^{-1} B where A^{-1}=** $\frac{\text{adj A}}{\lvert A\rvert}$	½
	| A|= 9	½
	A^{-1} = $\frac{1}{\lvert 9\rvert}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$	2 ½
	X = 1 , Y = 2 , z = 3	1
	OR	
	$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	2

	$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ $\Rightarrow$ **x = 0, y = 5, z = 3**	2 1
	SECTION-E	
36.	**(i)** $\frac{dN}{dt} = k\,N$ **(ii)** $\log \lvert N \rvert = k\,t + C$ **(iii)** $9\,N_0$ **OR** $\frac{1}{k} \log \lvert x \rvert + C$	2 1 1
37	**(i) 2/7** **(ii) (ii) 5/14** **(iii) (iii) 1/6 OR (iii) 4/21**	1 1 2
38	**(i)** $2x + \pi y = 200$ **(ii) Area A =** $\frac{5000}{\pi}$	1 3

KENDRIYA VIDYALAYA SANGTHAN ERNAKULAM REGION

Sample Question Paper- 14 Class XII - 2023-24

Mathematics (Code-041)

Time Allowed: 3 Hours Maximum Marks: 80

SECTION – A

1	a	1
2	b	1
3	c	1
4	d	1
5	c	1
6	d	1
7	d	1
8	a	1
9	c	1
10	d	1
11	d	1
12	a	1
13	b	1
14	c	1
15	c	1
16	a	1
17	c	1
18	a	1
19	a	1
20	a	1
21	$sin^{-1}\left[-sin\left(\frac{17\pi}{8}\right)\right] = sin^{-1}\left[-sin\left(2\pi + \frac{\pi}{8}\right)\right]$	1 1

	$= sin^{-1}\left[-sin\left(\frac{\pi}{8}\right)\right] = -\frac{\pi}{8}$ OR $tan^{-1}\left[\frac{cos^2\frac{x}{2}-sin^2\frac{x}{2}}{\left(cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}\right]$ $= tan^{-1}\left[\frac{cos\frac{x}{2}-\sin\frac{x}{2}}{cos\frac{x}{2}-\sin\frac{x}{2}}\right]$ $= tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-tan\frac{x}{2}}\right]$ $= tan^{-1}\left[tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$ $=\frac{\pi}{4}+\frac{x}{2}$	1 1
22	y = $[x(x-2)]^2$ $\frac{dy}{dx}$ =4 x(x-1) (x-2) for increasing $\frac{dy}{dx}$> 0 Increasing x Є (0,1) ∪ (2,∞)	1 1
23	$f'(x)$= 2 sinx cosx + sinx $f'(x)$= 0 ⇒ Sinx (2cosx+1) =0 Sin x=0 or cos x = -1/2 x= 0 or x= $\frac{2\pi}{3}$ f(0) = -1 (absolute minimum f($\frac{2\pi}{3}$) = 5/4 (absolute maximum f(π) =1 OR f'(x) =4-x f'(x) =0 x=4 f(-2) =-10 (absolute minimum) f(4)= 8 (absolute maximum) f(9/2) = 63/8	1 1 1 1
24	$\int_0^1(1-x)^2\ [1-(1-x)]^n$ dx = $\int_0^1(1-2x+x^2)\ x^n$ dx =$\int_0^1(x^n-2x^{n+1}+x^{n+2})$ dx = $\frac{1}{n+1}-\frac{2}{n+2}+\frac{1}{n+3}$	1 1
25	P(x) = 0.005 x^3 +0.02 x^2 +30x . P' (x) = 0.015 x^2 +0.04 x +30 P' (x) at x=3 = 0.015 3^2 +0.04 +30 = 30.255	1 1
26	$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$ ……..(1) 1 = A(x+2) + B(x+1) If x= -2, then B = -1 If x = -1 then A = 1 Eq. 1 will become $\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} + \frac{-1}{x+2}$ Integrate both sides $\int\frac{dx}{(x+1)(x+2)} = \int\frac{dx}{(x+1)}$ -1 $\int\frac{dx}{(x+2)}$ = log $\lvert x+1\rvert$ - log $\lvert x+2\rvert$ +c	
27	Double lets (D) are (1,1) , (2,2) , (3,3) ,(4,4), (5,5) ,(6,6)	

	P(D) = $\frac{6}{36} = \frac{1}{6}$ N: Non double lets P(N) = $\frac{30}{36} = \frac{5}{6}$ P(X=0) = P(NNN) = P(N) P(N) P(N) = $\frac{5}{6}$ x $\frac{5}{6}$ x $\frac{5}{6}$ =$(\frac{5}{6})^3$ P(X=1) = P(DNN) +P(NDN) + P(NND) = $\frac{1}{6}$ x $\frac{5}{6}$ x $\frac{5}{6}$ + $\frac{5}{6}$ x $\frac{1}{6}$ x $\frac{5}{6}$ + $\frac{5}{6}$ x $\frac{5}{6}$ x $\frac{1}{6}$ = $\frac{75}{6^3}$ P(X=2) = P(DDN) +P(DND) + P(NDD) = $\frac{1}{6}$ x $\frac{1}{6}$ x $\frac{5}{6}$ + $\frac{1}{6}$ x $\frac{5}{6}$ x $\frac{1}{6}$ + $\frac{5}{6}$ x $\frac{1}{6}$ x $\frac{1}{6}$ = $\frac{15}{6^3}$ P(X=3) = P(DDD) = P(D) P(D) P(D) = $\frac{1}{6}$ x $\frac{1}{6}$ x $\frac{1}{6}$ = $\frac{1}{6^3}$ Distribution Table X | Number of double lets X | 0 | 1 | 2 | 3 P(X) | $(\frac{5}{6})^3$ | $\frac{75}{6^3}$ | $\frac{15}{6^3}$ | $\frac{1}{6^3}$	1 1 1
28	Put x = tanθ $\int_0^1 \frac{\log(1+x)}{1+x^2}$dx = $\int_0^{\frac{\pi}{4}} \log(1+\tan\theta)d\theta$ I = $\int_0^{\frac{\pi}{4}} \log(1+\tan\theta)d\theta$ = $\int_0^{\frac{\pi}{4}} \log(1+\tan(\frac{\pi}{4}-\theta)d\theta$ $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ = $\int_0^{\frac{\pi}{4}} \frac{2}{1+tan\theta}$ dθ = $\int_0^{\frac{\pi}{4}} log2$ dθ - I 2I = $\frac{\pi}{4}$ log2 I = $\frac{\pi}{8}$ log2 OR I = $\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+cos^2(\pi-x)}$ dx $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ I = $\int_0^{\pi} \frac{\pi \sin x}{1+cos^2 x}$ dx - I 2I = $\int \frac{-dt}{1+t^2}$ (t = cos$x$, dt = -sin$x$) 2I = - π $\tan^{-1} t$ = - π[$\tan^{-1} cos\, x]\pi_0$ [$\tan^{-1} cos\, x]_0^{\pi}$ I = - π [- $\frac{\pi}{4}$ - $\frac{\pi}{4}$] = $\pi^2/4$	1 1 1

29

$\frac{dy}{dx} = \frac{-xy-y^2}{x^2} = \frac{-y}{x} - (\frac{y}{x})^2$ (Let y = vx $\frac{dy}{dx}$ = v + x $\frac{dv}{dx}$)

$v + x\frac{dv}{dx} = -v - v^2$

$x\frac{dv}{dx} = -2v - v^2$

$\frac{dv}{v^2+2v} = -\frac{dx}{x}$ on integration

$\frac{1}{2}\log\left|\frac{v}{v+2}\right| - \log x + c$

OR

$\int \frac{\sec^2 x}{tanx}dx = \int \frac{\sec^2 y}{tany}dy$ ……variable separable

$\int \frac{1}{t}dt = \int \frac{1}{p}dp$ where t = tanx and p = tany

$\log|t| = \log|p| + c$

$\frac{tanx}{tany} = k$

30

Solution:

Given,

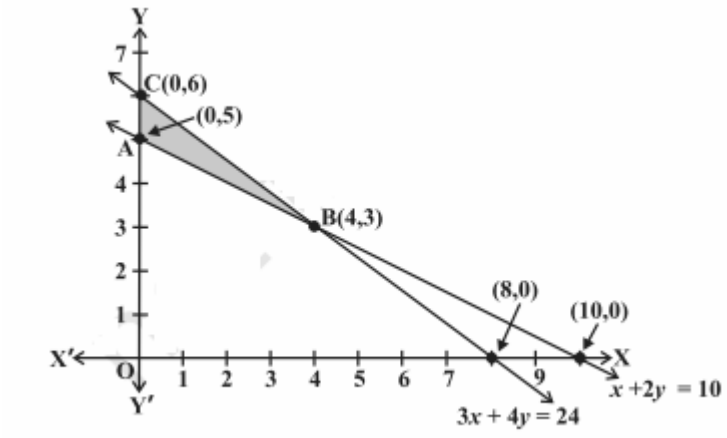
Minimise Z = 200 x + 500 y … (1) subject to the constraints:

x + 2y ≥ 10 … (2)

3x + 4y ≤ 24 … (3)

x ≥ 0, y ≥ 0 … (4)

Let us draw the graph of x + 2y = 10 and 3x + 4y = 24 as below.



Corner point	Value of Z
(0, 5)	2500

(4, 3)	2300← Minimum
(0, 6)	3000

Hence, the minimum value of Z is 2300 is at the point (4, 3).

OR

Minimise Z = 50x + 70y … (1)

subject to the constraints:

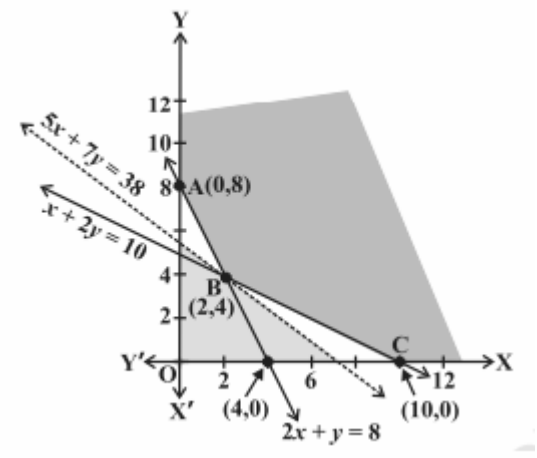
2x + y ≥ 8 … (2)

x + 2y ≥ 10 … (3)

x, y ≥ 0 … (4)

Let us draw the graph of 2x + y = 8 and x + 2y = 10 as given below.

Let us draw the graph of 2x + y = 8 and x + 2y = 10 as given below.

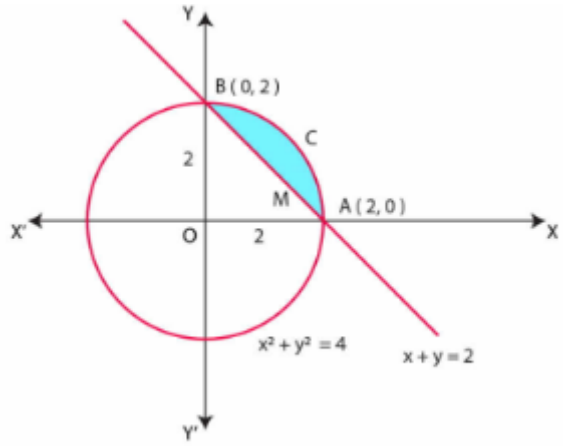


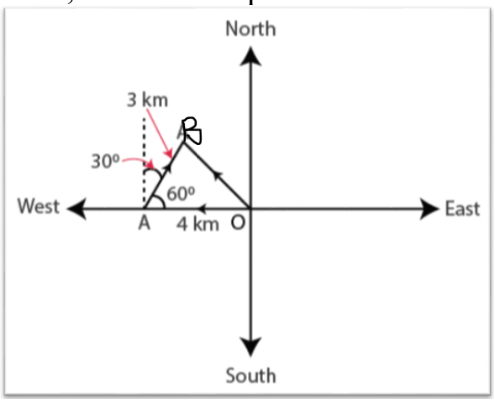
Here, observe that the feasible region is unbounded.

Corner point	Value of Z = 50x + 70y	
A(0, 8)	560	
B(2, 4)	380 = Minimum	
C(10, 0)	500	

Therefore, the minimum value of Z is 380 obtained at the point (2,4)

31	$y = \sin^{-1}\frac{2^{x+1}}{1+4^x}$ $= \sin^{-1}\frac{2\,2^x}{1+(2^x)^2}$ $= \sin^{-1}(\sin 2\theta) = 2\tan^{-1}2^x$ $\frac{dy}{dx} = \frac{2^{x+2}\log 2}{1+4^x}$	
	SECTION D	
32	AB = 6I AX =C X = A^{-1} C X = $\frac{B}{6}$ C $\frac{1}{6}\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}\begin{bmatrix}3\\17\\7\end{bmatrix} = \frac{1}{6}\begin{bmatrix}12\\-6\\24\end{bmatrix} = \begin{bmatrix}2\\-1\\4\end{bmatrix}$	
33	R be the relation on NxN defined by (a,b) R (c,d) $\Leftrightarrow$ $a^2+d^2 = b^2+c^2$ $\forall$ (a,b) ϵ NxN, (a,b) R (a,b) $\Leftrightarrow$ $a^2+b^2 = b^2+a^2$ is true. $\therefore$ R is reflexive relation (a,b)R (c,d) $\Leftrightarrow$ $a^2+d^2 = b^2+c^2$ $\Leftrightarrow$ $b^2+c^2 = a^2+d^2$ $\Leftrightarrow$ $c^2+b^2 = d^2+a^2$ $\Leftrightarrow$ (c,d) R (a,b) $\forall$ (a,b),(c,d) ϵ NxN $\therefore$ R is symmeric relation. (a,b)R (c,d) $\Leftrightarrow$ $a^2+d^2 = b^2+c^2$ (c,d)R (e,f) $\Leftrightarrow$ $c^2+f^2 = d^2+e^2$ Adding $a^2+d^2+c^2+f^2 = b^2+c^2+d^2+e^2$ $\Leftrightarrow$ $a^2+f^2 = b^2+e^2$ $\Leftrightarrow$ (a,b)R (e,f) $\forall$ (a,b),(c,d),(e,f) ϵ NxN $\therefore$ R is transitive relation. $\therefore$ R is an equivalence relation . [(1,2)] = {(1,2)} OR $f(x) = (x+\frac{1}{6})^2+\frac{3}{4}$ Let $x_1, x_2 \epsilon$ N such that $f(x_1) = f(x_2) \Rightarrow (x_1+\frac{1}{6})^2+\frac{3}{4} = (x_2+\frac{1}{6})^2+\frac{3}{4}$ $\Rightarrow x_1 = x_2$	

	∴ f is one - one Let $(x+\frac{1}{6})^2+\frac{3}{4}$ = y ⇒ x = y - $\frac{5}{4}$ If y=1, x ∉ N. Co-domain ≠ Range ∴ f is not onto	
34	The lines in standard form are $\frac{x-1}{-3}=\frac{y-2}{\lambda/7}=\frac{z-3}{2}$ and $\frac{x-1}{-3\lambda/7}=\frac{y-5}{1}=\frac{z-6}{-5}$ [∵ two lines with DR's a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$] -3 x $\frac{-3\lambda}{7}+\frac{\lambda}{7}$ x 1 + 2x-5 = 0 ⇒ $\lambda = 7$ ∴ lines are $\frac{x-1}{-3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-1}{-3}=\frac{y-5}{1}=\frac{z-6}{-5}$ Any point from line1 is (-3λ +1, λ +2, 2 λ+3) Substitute in line2 $\frac{-3\lambda+1-1}{-3}=\frac{\lambda+2-5}{1}=\frac{2\lambda+3-6}{-5}$ ⇒ $\lambda=\frac{\lambda-3}{1}=\frac{2\lambda-3}{-5}$ No value of λ satisfying the above equation . ∴ lines are not intersecting.	
35	Equation of circle is $x^2+y^2=4$…….(i) $x^2+y^2=2^2$ Equation of a lines is x + y = 2 ……(iii)  Area of OACB, bounded by the circle and the coordinate axes is $\int_0^2 \sqrt{2^2-x^2}\ dx = \left[\frac{x\sqrt{2^2-x^2}}{2}+\frac{2^2}{2}\sin^{-1}\frac{x}{2}\right]_0^2$ $= 2\frac{\pi}{2} - 0 = \pi$ sq.units	

	Area of triangle OAB, bounded by the straight line and the coordinate axes is $\int_0^2 Y_L$ dx = $\int_0^2 (2 - x)$ dx = 2 sq.units Hence, the required area = Area of OACB – Area of triangle OAB = (π – 2) sq.units	
36	It is given that Let O be the position of Anitha's house and B be the position of i Geetu's house nd final positions of the girl, respectively. Then, the girl's position can be shown as **Solution:** It is given that Let O and B be the initial and final positions of the girl, respectively. Then, the Anitha's position can be shown as  	
(i)	$\overrightarrow{OA}$ = - 4$\hat{\imath}$	1
(ii)	$\overrightarrow{AB}$ = $\lvert AB \rvert \cos 60^0\, \hat{\imath} + \lvert AB \rvert \sin 60^0\, \hat{\jmath}$ $= \frac{3}{2}\hat{\imath} + \frac{3\sqrt{3}}{2}\hat{\jmath}$	1
(iii)	$\overrightarrow{OB}$ = $\overrightarrow{OA}$ + $\overrightarrow{AB}$	2

	$= \frac{-5}{2}\hat{\imath} + \frac{3\sqrt{3}}{2}\hat{\jmath}$	
37	i) P(A) $=\frac{4}{7}$ P(B) $= \frac{1}{7}$ P(C) $= \frac{4}{7}$ ii) P(E) = P(A) P(E/A) + P(B) P(E/B) + P(C) P(E/C) $= \frac{4}{7}$ x 0.3 + $\frac{1}{7}$ x 0.8 + $\frac{4}{7}$ x 0.5 = $\frac{3}{7}$ iii) P(B/E) $= \frac{8}{30}$ OR iv) P(A/E) $= \frac{2}{5}$	1 1 2
38	f^i(x)= -0.4x + k =0 (i). K=4 [∵ x=10 giv en] (ii). f^i(x) positive in in the interval [0,10) . ∴ function is increasing in [0,10). (iii). F(0) = 47. 5 Absolute minimum value f(12) = -0.2 x 12^2 + 4x12 + 47.5 = 66.7 f(10) = -0.2x100 + 4x10 + 47.5 = 67.5 Absolute maximum value	1 1 2

PREPARED BY

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2. MERCY MATHEW - KV PORT TRUST

MARKING SCHEME

SAMPLE PAPER 15

Qn. No	Answer	mark
	SECTION A	
1	Option **c**: {1,-1,I,-1}	1mark
2	Option **c**: B-A	1mark
3	Option **b**: Singular matrix	1mark
4	Option **a:** Parallel	1mark
5	Option **a:** 12	1mark
6	Option **b:** at infinitely many points	1mark
7	Option **d:** $\frac{d^2y}{dx^2}+y=0$	1mark
8	Option **a:** -1	1mark
9	Option **d:** $\frac{\pi^2}{8}$	1mark
10	Option **c:** $\begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$	1mark
11	Option **b:** $p=\frac{q}{2}$	1mark
12	Option**b:** a right angled triangle	1mark
13	Option **c:** 9	1mark
14	Option **c:** $\frac{6}{13}$	1mark
15	Option **b:** cosecx	1mark
16	Option **d:** (3,3,-3)	1mark
17	Option **d:** none of these	1mark
18	Option **c:** $\sqrt{21}$	1mark
19	Option **d:** A is false but R is true	1mark
20	Option **b:** Both A and R are true but R is not the correct explanation of A.	1 mark

	SECTION B	
21	(2x-3) $\in$ $(-\infty,-1) \cup (1,\infty)$ x $\in(-\infty,1) \cup (2,\infty)$ OR $\sec\frac{50\pi}{7} = -\sec\frac{\pi}{7}$ $\text{Cosec}^{-1}(-\sec\frac{\pi}{7}) = -\text{cosec}^{-1}(\text{cosec}(\frac{\pi}{2}-\frac{\pi}{7})$ $=\frac{\pi}{2}-\frac{\pi}{7}=5\frac{\pi}{14}$	1mark 1 mark 1/ 2mark 1 mark
22	$\frac{dy}{dx}=2$ $6\frac{dy}{dx}=3x^2$ The points are (-2.-1) and $(2,\frac{5}{3})$	1/ 2mark 1/ 2mark 1 mark
23	$Y=\sqrt{sinx+y}$ or $Y^2 = \text{sinx+y}$ $2y\frac{dy}{dx}=\cos x+\frac{dy}{dx}$ $\frac{dy}{dx}=\frac{\cos x}{2y-1}$ Or For showing LHL= RHL=F(2)=3 For showing LHD$\neq$RHD	1/ 2 mark 1 mark 1/2 mark 1 mark 1 mark
24	Take logx=t or $x=e^t$ $dx= e^t dt$ $\int e^t(sint+cost)dt = e^t sint = xsin(logx)+c$	12 mark 12 mark 1mark
25	$\int_0^2(2-x)dx+\int_2^3(x-2)dx$ For getting answer as $\frac{5}{2}$	1 mark 1 mark
	SECTION C	
26	F'(x)=0, $\cos2x=\frac{1}{2}$, $x=\frac{\pi}{6}, -\frac{\pi}{6}$ To identify maximum at $\frac{\pi}{6}$ and minimum at $-\frac{\pi}{6}$ Difference=$\frac{\pi}{2}--\frac{\pi}{2}=\pi$ OR $\frac{dx}{dt}=\cos t$ and $\frac{dy}{dt}=p\cos pt$ $\cos t\frac{dy}{dx}=p\cos pt$ $\cos t\frac{d^2y}{dx^2}-\frac{\sin t}{\cos t}\frac{dy}{dx}=\frac{p^2y}{\cos t}$, $\sqrt{1-x^2}\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0$	1mark 1 mark 1 mark 1 mark $\frac{1}{2}$ mark $1\frac{1}{2}$ marks
27	Let $x=a\tan^2\theta$, $\tan^2\theta=\frac{x}{a}$ $2a\int\theta(tan\theta sec^2\theta)\,d\theta = 2a\int utan^{-1}u du$ Integration by parts to get $(a+x)tan^{-1}\sqrt{\frac{a}{x}}-\sqrt{ax}+c$	1/2 mark 1mark $1\frac{1}{2}$ mark

<table>
<tr><td></td><td></td><td></td></tr>
<tr><td>28</td><td>X is a random variable
Probability distribution
<table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{10}{21}$</td><td>$\frac{10}{21}$</td><td>$\frac{1}{21}$</td></tr><tr><td>X_iP_i</td><td>0</td><td>$\frac{10}{21}$</td><td>$\frac{2}{21}$</td></tr></table>Mean = $\frac{4}{7}$</td><td>$\frac{1}{2}$ mark

2 marks

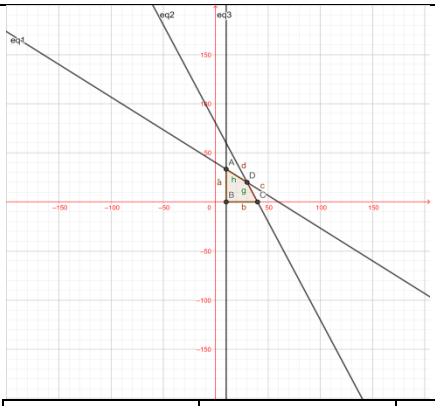
$\frac{1}{2}$ mark</td></tr>
<tr><td>29</td><td>$\int \frac{sec^2y}{tany}$ dy=$\int \frac{e^x}{1-e^x}dx$
tany= c(1-e^x)

OR
Put y=Vx
V+x$\frac{dv}{dx}$ = V+V^2
$\int \frac{dx}{x} = \int \frac{dv}{V^2}$
ylogx +x =y</td><td>$1\frac{1}{2}$ mark
$1\frac{1}{2}$ mark

$\frac{1}{2}$ mark
$\frac{1}{2}$ mark
$\frac{1}{2}$ mark
$1\frac{1}{2}$ mark</td></tr>
<tr><td>30</td><td>$\int_0^4 \frac{dx}{\sqrt{x^2+2x+3}} = \int_0^4 \frac{dx}{\sqrt{(x+1)^2+2}} = \log\left\|x+\sqrt{x^2+2x+3}\right\|$
$= \log\left\|\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right\|$
OR
$\int \frac{2x^2-1}{x^2(x^2+2)}dx = -\frac{1}{2}\int \frac{dx}{x^2}+\frac{5}{2}\int \frac{dx}{x^2+2}$
$= \frac{1}{2x}+\frac{5}{2\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+c$</td><td>2 mark

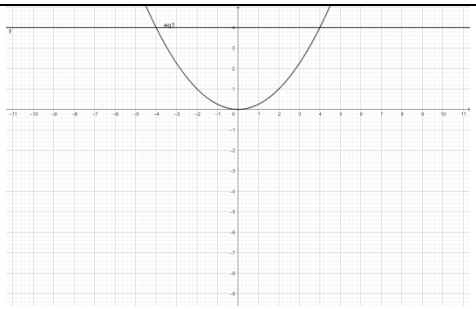
1 mark

2marks

1 mark</td></tr>
<tr><td>31</td><td> 
<table><tr><td>Corner Point</td><td>Z=7x+6y</td><td></td></tr><tr><td>D(30,40)</td><td>450</td><td>maximum</td></tr><tr><td>A(10,$\frac{100}{3}$)</td><td>270</td><td></td></tr><tr><td>B(10,0)</td><td>70</td><td></td></tr><tr><td>C(40,0)</td><td>280</td><td></td></tr></table></td><td>$1\frac{1}{2}$ mark for graph

$1\frac{1}{2}$ mark</td></tr>
</table>

	SECTION D	
32	For showing f(x) not one one . Reason f(x)=f($\frac{1}{x}$) Range of the function= $\left[-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right] \neq codomain$ Not onto OR Condition for Equivalence Relation Reflexive: aRa, $\forall a \in X$ as $a - a + \sqrt{3}$ is an irrational number Symmetric: If a-b+$\sqrt{3}$ is irrational, then b-a+$\sqrt{3}$ is also irrational. So aRb⇒bRa, $\forall (a, b) \in R$ Transitive: If a-b+$\sqrt{3}$ is irrational and if b-c+$\sqrt{3}$ is $irrational$, the $a - c + 2\sqrt{3}$ is $also$ $irrational$ I(a,b) and (b,c)∈ $R, then$ $(a, c) \in R$	2 marks 2mark 1 mark $\frac{1}{2}$ mark 1 mark $1\frac{1}{2}$ marks 2mark
33	Let x be the length of the first part. Length if other part =25−x Let r be the radius of circle.⇒r=$\frac{x}{2\pi}$ Let 'a' be the side of circle. a=$\frac{25-x}{4}$ A=$\pi(\frac{x}{2\pi})^2+(\frac{25-x}{4})^2 = \frac{\pi(x)^2}{4} + \frac{(25-x)^2}{16}$.....(1) Differentiating above equation w.r.t. x, we get $\frac{dA}{dx} = \frac{\pi x}{2} + \frac{(x-25)}{8}$.....(2) Putting $\frac{dA}{dx}=0$ ⇒x=$\frac{25\pi}{4+\pi}$ and $\frac{d^2A}{dx^2} < 0$ Length of pieces-$\frac{25\pi}{4+\pi}$ and $\frac{100}{4+\pi}$	$\frac{1}{2}$ mark 1 mark 1mark 2mark $\frac{1}{2}$ mark
34	For taking any point on the line as $(2\tau - 1, 3\tau + 3, -\tau + 1)$ For getting $\tau = 1$ The coordinates of the point is (1,6,0) The length of the perpendicular is $\sqrt{24}$ OR Any point on first line $(3\boldsymbol{\tau} - \mathbf{1}, \mathbf{5}\boldsymbol{\tau} - \mathbf{3}, \mathbf{7}\boldsymbol{\tau} - \mathbf{5})$ and on second line $(2+\boldsymbol{\mu}, \mathbf{3}\boldsymbol{\mu} + \mathbf{4}, \mathbf{5}\boldsymbol{\mu} + \mathbf{6})$ For getting $\boldsymbol{\tau} = \frac{1}{2}$ $\boldsymbol{\mu} = -\frac{3}{2}$ For showing intersecting lines and getting the coordinate as $(\frac{1}{2}, \frac{-1}{2}, \frac{-3}{2})$	1 mark 2 mark 1 mark 1 mark 1 mark 2 marks 2 marks
35	AB =8I_3 For identifying the coefficient matrix as B and B^{-1} as $\frac{A}{8}$	2marks 1 mark

	For getting x=3, y=-2 and z=-1	2 marks
	SECTION E	
36	i)$\frac{1}{\sqrt{29}}(2\vec{\imath}-4\vec{j}+3\vec{k})$ ii) $\frac{\sqrt{6}}{2}$ iii) $\frac{\sqrt{38}}{2}$ sq. unit	1 mark 1 mark 2 mark
37	I) $\frac{1}{6}$ ii) $\frac{4}{5}$ iii) $\frac{15}{16}$	1 mark 1 mark 2 mark
38	i.  ii.Area=$4\int_0^4 \sqrt{y}\,dy=\frac{64}{3}$ sq. unit	2mark 2mark

Q: NO	SAMPLE PAPER 16
1.	(a) 1 By definition of equality of matrices, we have $x+y+z=9$ ... (i) $x+z=5$ .... (ii) $y+z=7$ ... (iii) $(i)-(ii) \Rightarrow x+y+z-x-z=9-5$ $\Rightarrow y=4$ ... (iv) $(ii)-(iv) \Rightarrow x-y+z=5-4$ $\Rightarrow x-y+z=1$
2.	(C) $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$ Since Matrix multiplication is associative, therefore $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}0+4+0\\ 0+0+x\\ 0+0+2x\end{bmatrix}=O$ $\Rightarrow \begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}4\\ x\\ 2x\end{bmatrix}=O$ $\Rightarrow [4+2x+2x]=O \Rightarrow [4+4x]=O$ $\Rightarrow 4+4x=0 \Rightarrow 4x=-4 \Rightarrow x=-1$
3	(a) We have Area of triangle $=\frac{1}{2}\begin{vmatrix}k & 0 & 1\\ 4 & 0 & 1\\ 0 & 2 & 1\end{vmatrix}=4$ $\Rightarrow \lvert k(0-2)+1(8-0)\rvert=8$ $\Rightarrow k(0-2)+1(8-0)=\pm 8$ On taking positive sign $-2k+8=8$ $\Rightarrow -2k=0$ $\Rightarrow k=0$ On taking negative sign $-2k+8=-8$ $\Rightarrow -2k=-16$ $\Rightarrow k=8$ $\Rightarrow k=0, 8$
4.	(c) 6

	Here, $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & if \quad x\neq\frac{\pi}{2}\\ 3, & if \quad x=\frac{\pi}{2}\end{cases}$ $LHL=\lim_{x\to\frac{\pi}{2}^-} f(x)=\lim_{x\to\frac{\pi}{2}^-}\frac{k\cos x}{\pi-2x}$ Putting $x=\frac{\pi}{2}-h$ as $x\to\frac{\pi}{2}^-$ when $h\to 0$ $\therefore \lim_{x\to\frac{\pi}{2}^-} f(x)=\lim_{h\to 0}\frac{k\cos\left(\frac{\pi}{2}-h\right)}{\pi-2\left(\frac{\pi}{2}-h\right)}=\lim_{h\to 0}\frac{ksinh}{2h}=\frac{k}{2}\times\lim_{h\to 0}\frac{sinh}{h}=\frac{k}{2}\times 1=\frac{k}{2}$ Since f(x) is continuous at $x=\frac{\pi}{2}$, therefore LHL = $f\left(\frac{\pi}{2}\right)$ Also, $f\left(\frac{\pi}{2}\right)=3\Rightarrow\frac{k}{2}=3\Rightarrow k=6$
5	(b) $\frac{1}{\sqrt{171}}, -\frac{7}{\sqrt{171}}, \frac{11}{\sqrt{171}}$ P ≡ (2, 3, −6) and Q ≡ (3, −4, 5) Direction ratios = <1, −7, 11> Direction cosines $=\left(\frac{1}{\sqrt{1+49+121}}, \frac{-7}{\sqrt{1+49+121}}, \frac{11}{\sqrt{1+49+121}}\right)$ $=\left(\frac{1}{\sqrt{171}}, \frac{-7}{\sqrt{171}}, \frac{11}{\sqrt{171}}\right)$
6	(a) We have, $\frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{1}{(1+x^2)^2}$ Which is linear differential equation. Here, $P=\frac{2x}{1+x^2}$ and $Q=\frac{1}{(1+x^2)^2}$ $\therefore$ I.F. $=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$ $\therefore$ The general solution is $y(1+x^2)=\int(1+x^2)\frac{1}{(1+x^2)^2}+C$ $\Rightarrow \quad y(1+x^2)=\int\frac{1}{1+x^2}dx+C$ $\Rightarrow \quad y(1+x^2)=\tan^{-1}x+C$
7	(b) 140

	The feasible region lies on the origin side of x + y = 40 and x + 2y = 60, in first quadrant. The comer points of the feasible region are O(0, 0), A(0, 30), B(20, 20) and C(40, 0) ∴ At A(0, 30), P = 0 + 4 (30) = 120 At B(20, 20), P = 3(20) + 4(20) = 140 At C(40, 0), P = 3(40) + 0 = 120 ∴ Maximum value of P is 140.
8	(b) $\frac{4}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} - \frac{2}{\sqrt{29}}\hat{j}$ The sum of the given vectors is $\vec{a}+\vec{b}\ \ (=\vec{c}, say) = 4\hat{i}+3\hat{j}-2\hat{k}$ and $\lvert\vec{c}\rvert = \sqrt{4^2+3^2+(-2)^2} = \sqrt{29}$ Thus, the required unit vector is $\hat{c} = \frac{1}{\lvert\vec{c}\rvert}\vec{c} = \frac{1}{\sqrt{29}}(4\hat{i}+3\hat{j}-2\hat{k}) = \frac{4}{\sqrt{29}}\hat{i}+\frac{3}{\sqrt{29}}\hat{j}-\frac{2}{\sqrt{29}}\hat{k}$
9	(b) $\frac{\pi}{4}$ Let $I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}} dx$ ...(1) Using integration property $I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}} dx$ ...(2) Adding (1) and (2) $2I = \int_0^{\pi/2} dx = \frac{\pi}{2}$ $\Rightarrow I = \frac{\pi}{4}$
10	Correct option: (d) 16
11	(c) all of the given constraints
12	(a) 30^0

	as $\left\|\sqrt{3}\vec{a}-\vec{b}\right\|^2=\left(\sqrt{3}\vec{a}-\vec{b}\right)^2$ $=3\vec{a}^2+\vec{b}^2-2\sqrt{3}\vec{a}\cdot\vec{b}$ $1=3+1-2\sqrt{3}\ \vec{a}\cdot\vec{b}$ $\Rightarrow \vec{a}\cdot\vec{b}=\frac{\sqrt{3}}{2}$ $\therefore \cos\theta=\frac{\vec{a}\cdot\vec{b}}{\left\|\vec{a}\right\|\left\|\vec{b}\right\|}=\frac{\sqrt{3}}{2}\Rightarrow\theta=30°$
13	(c) 0,3 $A=\begin{bmatrix}3-x & 2 & 2\\ 2 & 4-x & 1\\ -2 & -4 & -1-x\end{bmatrix}$ If the matrix is singular, its determinant has to be zero. $\Rightarrow(3-x)[(4-x)(-1-x)+4]-2[2(-1-x)+2]+2[-8+2(4-x)]=0$ $\Rightarrow(3-x)[-4-4x+x+x^2+4]-2[-2-2x+2]+2[-8+8-2x]=0$ $\Rightarrow(3-x)[x^2-3x]+4x-4x=0$ $\Rightarrow(3-x)x(x-3)=0$ $\Rightarrow x=0,3$
14	(c) $\frac{3}{8}$ Given: $P(A)=0.6,\ P(B)=0.2,\ P\left(\frac{A}{B}\right)=0.5$ $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=0.5$ $\therefore P(A\cap B)=0.2\times0.5=0.1$ $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $=0.6+0.2-0.1=0.7$ $\therefore P\left(\frac{A'}{B'}\right)=\frac{P(A'\cap B')}{P(B')}=\frac{P(A\cup B)'}{1-P(B)}$ $=\frac{1-P(A\cup B)}{1-P(B)}$ $=\frac{1-0.7}{1-0.2}=\frac{0.3}{0.8}=\frac{3}{8}$ Ans
15	$y/2=x+c$

	$\frac{dy}{dx} = \sqrt{4 - y^2}$ Rearranging we get; $dy/\sqrt{(4 - y^2)} = dx$ Integrating both the sides, we get; $\int dy/\sqrt{(4 - y^2)} = \int dx$ We know that, by the formula; $\int 1/\sqrt{(a^2 - x^2)} = \sin^{-1}(x/a)$ Therefore, $\sin^{-1}y/2 = x+c$
16	(c) ½ sq. unit
17	Q. 7. The function $f(x) = [x(x - 3)]^2$ is increasing in : a) $(0, \infty)$ b) $(-\infty, 0)$ c) $(1, 3)$ d) $[0, 15] \cup (3, \infty)$
18	(b) ⅔, -⅓, -⅔ $= \frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}}$ $= \frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$
19	(a) Both A and R are correct; R is the correct explanation of A.
20	(c) A is correct; R is incorrect.
	SECTION B
21	$\tan^{-1}\left[2\sin\left(2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$ $= \tan^{-1}\left[2\sin\left(2\cos^{-1}\left(\cos\frac{\pi}{6}\right)\right)\right]$ $= \tan^{-1}\left[2\sin\left(2\left(\frac{\pi}{6}\right)\right)\right] = \tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]$ Or $\tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$ OR

	$\tan^2(\sec^{-1}2) + \cot^2(\operatorname{cosec}^{-1}3)$ $= \sec^2(\sec^{-1}2) - 1 + \operatorname{cosec}^2(\operatorname{cosec}^{-1}3) - 1$ $= 2^2 \times 1 + 3^2 - 2 = 11.$
22	f(x) = $-2x^3 - 9x^2 - 12x + 1$ $f'(x) = -6x^2$-18x-12 = $-6(x^2 + 3x + 2)$ = -6(x+1)(x+2) Turning points are -1 and -2

interval	x+1	x+2	-6(x+1)(x+2)
$x < -2$	−	−	− decreasing
$-2 < x < -1$	−	+	+ increasing
$x > -1$	+	+	−decreasing

Decreasing in $x < -2$ *and* $x > -1$ *and increasing in* $(-2, -1)$

23	f(x) = $\sin x + \cos x, x \in [0, \pi]$ $f'(x)$ =$\cos x - \sin x$ At the turning point $f'(x)$=0 and x = $\pi/4$ f(0) = 1 ,f(π) = -1 and f(= $\pi/4$) = $\sqrt{2}$ Absolute maximum = $\sqrt{2}$ and absolute minimum = -1 OR f(x) = $\sin 2x$ then $f'(x)$=$2\cos 2x$ and $f''(x) = -4\sin 2x$ At the turning point $f'(x)$=0 and 2x= $\frac{\pi}{2}$, $3\frac{\pi}{2}$,$5\frac{\pi}{2}$,$7\frac{\pi}{2}$ and x = $\frac{\pi}{4}$, $3\frac{\pi}{4}$,$5\frac{\pi}{4}$,$7\frac{\pi}{4}$ At x = = $\frac{\pi}{4}$, and $5\frac{\pi}{4}$ function has maximum value.
24	$\int_0^{\pi} sin^2x/2 - cos^2x/2$ dx = $\int -\cos x\, dx = -\sin x = -\sin\pi + \sin 0 = 0$
25	Given: Equation of the curve $6y = x^3 + 2$ .....(i) Let (x, y) be the required point on curve (i) According to the question, $\frac{dy}{dx} = 8$ .....(ii) ∴ From eq. (i), $6\frac{dy}{dx} = 3x^2$ $\Rightarrow 6\times 8 = 3x^2$ [From eq. (ii)] $\Rightarrow x^2 = \frac{6\times 8}{3}$ $\Rightarrow x = \pm 4$ Taking $x = 4,\ 6y = 64 + 2$ $\Rightarrow y = 11$

	∴ Required point is (4, 11). Taking $x=-4,\ 6y=-64+2 \Rightarrow y=\frac{-31}{3}$ ∴ Required point is $\left(-4,\frac{-31}{3}\right)$.
	Section – C
26	Let I = $\int\frac{1}{x\left(x^4-1\right)}dx = \int\frac{4x^3}{4x^3.x\left(x^4-1\right)}dx = \frac{1}{4}\int\frac{4x^3}{x^4\left(x^4-1\right)}dx$ ...(i) $\left[\because \frac{d}{dx}\left(x^4-1\right)=4x^3\right]$ Putting $x^4=t \Rightarrow 4x^3=\frac{dt}{dx} \Rightarrow 4x^3\,dx=dt$ Putting this value in eq. (i), I = $\frac{1}{4}\int\frac{dt}{t(t-1)} = \frac{1}{4}\int\frac{t-(t-1)}{t(t-1)}dt$ I = $\frac{1}{4}\int\left(\frac{t}{t(t-1)}-\frac{(t-1)}{t(t-1)}\right)dt = \frac{1}{4}\int\left(\frac{1}{(t-1)}-\frac{1}{t}\right)dt$ = $\frac{1}{4}\left[\log\lvert t-1\rvert-\log\lvert t\rvert\right]+c = \frac{1}{4}\log\left\lvert\frac{t-1}{t}\right\rvert+c = \frac{1}{4}\log\left\lvert\frac{x^4-1}{x^4}\right\rvert+c$
27	Let p represents the appearance of tail and q represents the appearance of head. Now $q=3p$ Since $p+q=1 \Rightarrow p+3p=1 \Rightarrow p=\frac{1}{4}$ and $q=1-\frac{1}{4}=\frac{3}{4}$ P (X = 0) = $^2C_0(q)^2=\left(\frac{3}{4}\right)^2=\frac{9}{16}$ P (X = 1) = $^2C_1q.p=2\times\frac{3}{4}\times\frac{1}{4}=\frac{6}{16}$ P (X = 2) = $^2C_2p^2=\left(\frac{1}{4}\right)^2=\frac{1}{16}$

Probability distribution:

x_i	0	1	2
p_i	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

28

$\int_{-5}^{5}|x+2|dx = \int_{-5}^{-2} -(x+2)\,dx + \int_{-2}^{5}(x+2)\,dx$

$= -\left(\frac{x^2}{2}+2x\right)_{-5}^{-2} + \left(\frac{x^2}{2}+2x\right)_{-2}^{5} = 29$

OR

$\int \sqrt{1-4x-x^2}\,dx = \int \sqrt{-x^2-4x+1}\,dx$

$= \int \sqrt{-\left(x^2+4x-1\right)}\,dx = \int \sqrt{-\left(x^2+4x+4-4-1\right)}\,dx = \int \sqrt{-\left[(x+2)^2+\left(\sqrt{5}\right)^2\right]}\,dx$

$= \int \sqrt{\left(\sqrt{5}\right)^2-(x+2)^2}\,dx = \frac{x+2}{2}\sqrt{\left(\sqrt{5}\right)^2-(x+2)^2} + \frac{\left(\sqrt{5}\right)^2}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+c$

$\left[\because \int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

$= \frac{x+2}{2}\sqrt{1-4x-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+c$

29

Given: Differential equation $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

$\Rightarrow \quad x\,dy = y\,dx + \sqrt{x^2+y^2}\,dx$

$\Rightarrow x\frac{dy}{dx}=y+\sqrt{x^2+y^2} \Rightarrow x\frac{dy}{dx}=y+x\sqrt{1+\left(\frac{y}{x}\right)^2}$

$\Rightarrow \frac{dy}{dx}=\frac{y}{x}+\sqrt{1+\left(\frac{y}{x}\right)^2}=f\left(\frac{y}{x}\right)$ [Dividing by x]

Therefore given differential equation is homogeneous.

Putting $\frac{y}{x}=v \Rightarrow y=vx \Rightarrow \frac{dy}{dx}=v.1+x\frac{dv}{dx}=v+x\frac{dv}{dx}$

Putting these values of $\frac{y}{x}$ and $\frac{dy}{dx}$ in eq. (i), we get

$v+x\frac{dv}{dx}=v+\sqrt{1+v^2} \Rightarrow x\frac{dv}{dx}=\sqrt{1+v^2} \Rightarrow \frac{dv}{\sqrt{1+v^2}}=\frac{dx}{x}$

Integrating both sides, $\int\frac{dv}{\sqrt{1+v^2}}=\int\frac{dx}{x} \Rightarrow \log\left(v+\sqrt{1+v^2}\right)=\log x+\log c$

Putting $\frac{y}{x}=v$, $\log\left(\frac{y}{x}+\sqrt{1+\left(\frac{y}{x}\right)^2}\right)=\log xc$

$\Rightarrow \log\left(\frac{y+\sqrt{x^2+y^2}}{x}\right)=\log cx \Rightarrow y+\sqrt{x^2+y^2}=cx^2$

OR

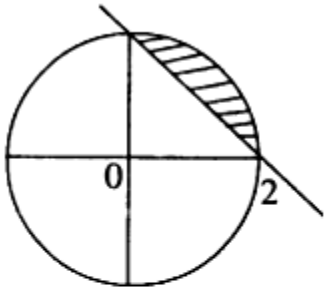
$$\frac{dy}{dx} = 1 + x + y + xy$$

$$= (1 + x)(1 + y)$$

or $$\int\frac{dy}{1+y} = \int(1+x)dx$$

$$\therefore \quad \log|1 + y| = x+\frac{x^2}{2}+C$$

When $x = 1, y = 0$ or $C = -\frac{3}{2}$

30	Plot the graph and shade the solution and feasible region is unbounded. Corner points are (30, 0),(10,40) and (0,80) Minimum value is 260 at (10, 40) OR Plot the graph and shade the solution and feasible region is bounded. Corner points are (0, 0),(40,0) (30,20)and (0,50) Maximum value is 495000 at (30,20)
31	Differentiate both functions separately by substituting x = $\cos\theta$ and obtain derivative as -1/2
	Section –D
32	**Required Area**  $\therefore \int_0^2 \sqrt{4-x^2} - \int_0^2 (2-x)dx$ $\left(\frac{x}{2}\sqrt{4-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right)_0^2 - \left(2x - \frac{x^2}{2}\right)_0^2$ $= (\pi - 2)$ sq. cm
33	(a, b)$\in R$ $\Rightarrow$ 2 divides a-b$\Rightarrow$ a-b=2m (a, a)$\in R$ $\Rightarrow$ a-a = 2m is true. Therefor R is reflexive. (a, b)$\in R$ $\Rightarrow$a-b=2m = -(b-a) = 2m$\Rightarrow$b-a=-2m = 2n is true , Therefor R is symmetric. (a, b)$\in R$ and (b, c)$\in R$ $\Rightarrow$ a-b=2m and b-c=2n then a- b + b -c = a-c= 2(m+n) is true. Therefor R is transitive and hence R is an equivalence Relation. OR

	Here we have to show that f(x) is both one – one and onto. For one – one, $f(x) = x^2 + 4,\ f(y) = y^2 + 4$ Now, for f(x) = f(y) $\Rightarrow x^2 + 4 = y^2 + 4$ $\Rightarrow x^2 = y^2$ $\Rightarrow x = y$ $\because$ for $f(x) = f(y) \Rightarrow x = y$, therefore f(x) is a one – one(injective) function For onto, $f(x) = y$ $\Rightarrow x^2 + 4 = y$ $\Rightarrow x^2 = y - 4$ $\Rightarrow x = \sqrt{y-4}$ Clearly $\sqrt{y-4}$ is a real number for all $y > 4$. Thus for all $y \in R$, there exists $x = \sqrt{y-4} \in R$, such that $f(x) = f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4 = y - 4 + 4 = y$ This shows that every element in co – domain has its preimage in domain. Hence f(x) is onto(surjective). Thus f(x) is both one – one and onto.
34	$A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix} \Rightarrow |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$ $= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$ $= 2(75) - 3(-110) + 10(72)$ $= 150 + 330 + 720 = 1200 \neq 0$ $C_{ij} = (-1)^{i+j} M_{ij}$ $C_{11} = 75 \quad C_{21} = 150 \quad C_{31} = 75$ $C_{12} = 110 \quad C_{22} = -100 \quad C_{32} = 30$ $C_{13} = 72 \quad C_{23} = 0 \quad C_{33} = -24$ $\text{adj } A = \begin{pmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{pmatrix}$ $A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{1200} \begin{pmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{pmatrix}$ $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5,\ \frac{6}{x} + \frac{9}{y} + \frac{20}{z} = -4$ Write matrix form and find x =2 y=-3 and z = 5.

35	Line is $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = \lambda$ then foot of the perpendicular is M (2$\boldsymbol{\lambda}$-1,3$\boldsymbol{\lambda}$+3,-$\boldsymbol{\lambda}$+1) And dr s of PM is 2$\boldsymbol{\lambda}$-6,3$\boldsymbol{\lambda}$-1,-$\boldsymbol{\lambda}$-1 and line is 2,3,-1 Line perpendicular to PM then 2(2$\boldsymbol{\lambda}$-6)+3(3$\boldsymbol{\lambda}$-1)-1(-$\boldsymbol{\lambda}$-1) =0 and then $\boldsymbol{\lambda}$ =1 M is (1,6,0) and PM =$\sqrt{24}$ Then using midpoint formula find image as (-3,8,,-2) OR Equation of the line through (1,1,1) is $\frac{x-1}{a} = \frac{y-1}{b} = \frac{z-1}{c}$ Then dr s of the line is a,b,c and given lines 1,2,4 and 2,3,4 Since lines perpendicular a+2b+4c=0 and 2a+3b+4c=0 and solve it to find drs are -4,4,-1 And equation as $\vec{r} = \hat{\imath} + \hat{\jmath} + \hat{k} + \boldsymbol{\lambda}(-\boldsymbol{4\hat{\imath}} + \boldsymbol{4\hat{\jmath}} - \boldsymbol{\hat{k}})$ **or** $\frac{x-1}{-4} = \frac{y-1}{4} = \frac{z-1}{-1}$ Angle between them as $\cos^{-1}\left(\frac{24}{\sqrt{609}}\right)$
	Section –E
36	i) Components are 15,0,0 and 0,8,6 ii) 15 unit and 10 unit iii) $\hat{\imath}$ or -90$\hat{\jmath}$ + 120 $\hat{k}$
37	i) 0.49 ii) 0.35 iii) 16 / 51
38	i) V = 12πh^3 ii) 1/48π

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SAMPLE QUESTION PAPER

SAMPLE PAPER 17

CLASS XII

MATHEMATICS (CODE-041)

SECTION: A (Solution of MCQs of 1 Mark each)

Qn.no	Answer	Hints / solutions
1.	c	
2.	a	
3.	b	
4.	b	
5	c	
6.	b	
7.	d	
8.	c	
9.	b	
10.	d	
11.	b	
12.	c	
13.	d	
14.	c	
15.	a	
16.	d	

17.	b	
18.	a	
19.	c	
20.	a	

Section –B

[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

21	$\sin[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)] = \sin[\frac{\pi}{3} + \frac{\pi}{6}]$ $= \sin\frac{\pi}{2} = 1$ OR $\cos^{-1}(\cos\frac{2\pi}{3}) + \sin^{-1}(\sin\frac{2\pi}{3}) = \frac{2\pi}{3} + \sin^{-1}(\sin\frac{\pi}{3})$ $= \frac{2\pi}{3} + \frac{\pi}{3} = \pi$.	1 1
22	f(x) = tan x – x f '(x) = sec^2 x – 1 f '(x) = $tan^2\ x$ f '(x) > 0 ∀ x ∈ R ∴ f (x) is always increases.	½ ½ 1

23	$f(x) = \sin x + \cos x$ $f'(x) = \cos x - \sin x = 0$ $x = \frac{\pi}{4}$ $f(\frac{\pi}{4}) = \sqrt{2}$ OR $\frac{dy}{dx}(P(x)) = 24 - 36x = 0$ $\therefore x = \frac{2}{3}$ $\{\{\frac{d^2}{dx^2}(P(x))\}_{x=\frac{2}{3}} = -36<0$ $\therefore$ Profit is maximum at $x = \frac{2}{3}$ Maximum Profit = 49.	1 1 1 1
24	$\int \frac{dx}{\sqrt{3-2x-x^2}} = \int \frac{dx}{\sqrt{2^2-(x+1)^2}}$ $= \sin^{-1}(\frac{x+1}{2}) + C$	1 1
25	$f(x) = x^5 + x$ $f'(x) = 5x^4 + 1 = 0$ for all $x \in R$, $x^4 \geq 0$ $\therefore f'(x) > 0$ Hence, no critical point exists.	1 ½ ½

Section –C

[This section comprises of solution short answer type questions (SA) of 3 marks each]

26.	Let I= $\int \frac{2x}{(x^2+1)(x^2+2)}dx$ $= \int \frac{2t}{(t+1)(t+2)}dt$ (taking $x^2=t$) $= \log\|t+1\| - \log\|t+2\| + C$ $= \log\|\frac{x^2+1}{x^2+2}\| + C$	1 1 ½ ½

27.	$\int \frac{(2x-5)e^{2x}}{(2x-3)^3}$ dx $= \int \frac{[(2x-3)-2)]e^{2x}}{(2x-3)^3}$ dx	½
	$= e^3 \int e^{2x-3}[\frac{1}{(2x-3)^2} - \frac{2}{(2x-3)^3}]$ dx	1
	$= \frac{e^3}{2} \int e^t[\frac{1}{t^2} - \frac{2}{t^3}]$ dt [By taking 2x-3=t]	½
	$= \frac{e^3}{2} e^t . \frac{1}{t^2}$ + C	½
	$= \frac{e^3}{2} e^{2x-3} . \frac{1}{(2x-3)^2}$ + C	
	$= \frac{e^{2x}}{2} . \frac{1}{(2x-3)^2}$ + C	½
	OR	
	$Let\ I = \int_0^{2\pi} \frac{1}{1+e^{sinx}}$ dx	½
	Also $I = \int_0^{2\pi} \frac{1}{1+e^{\sin(2\pi-x)}}$ dx $= \int_0^{2\pi} \frac{1}{1+e^{sinx}}$ dx	1
	2I $= \int_0^{2\pi} \frac{1+e^{sinx}}{1+e^{sinx}}$ dx $= \int_0^{2\pi} dx = 2\pi$ $\therefore$ I $= \pi$	1 ½
28.	x$\frac{dy}{dx}$ = y- x tan $\frac{y}{x}$, $\Rightarrow \frac{dy}{dx} = \frac{y}{x}$ - tan $\frac{y}{x}$ ----------(1) It is a homogeneous equation. Put y=vx Then equation(1) can be written as V+ x$\frac{dv}{dx} = v - \tan v \Rightarrow x\frac{dv}{dx} = -\tan v$	1
	$\Rightarrow \int -\cot v\ dv = \int \frac{dx}{x} \Rightarrow x \sin\left(\frac{y}{x}\right) = C$ At x=1,y=$\frac{\pi}{4}$, C=$\frac{1}{\sqrt{2}}$ $\Rightarrow x \sin\left(\frac{y}{x}\right) = \frac{1}{\sqrt{2}}$	1 ½ ½
	OR	

	$\log\left(\frac{dy}{dx}\right) = ax + by \Rightarrow \frac{dy}{dx} = e^{ax+by}$ $\Rightarrow \frac{dy}{e^{by}} = e^{ax}dx$ $\Rightarrow \int e^{-by}dy = \int e^{ax}dx$ $\Rightarrow -\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + C$ $\Rightarrow \frac{e^{ax}}{a} + \frac{e^{-by}}{b} = C'$	1 ½ 1 ½
29.	x = sint , y= sin pt $\frac{dx}{dt} = \cos t$, $\frac{dy}{dt} = \cos pt.\ p$ $\frac{dy}{dx} = \frac{p\cos pt}{cost}$ $\frac{d^2y}{dx^2} = -\frac{p^2 sinpt.cost + pcospt.sint}{cos^3 t}$ Substitute to get $(1-x^2)\frac{d^2y}{dx^2}$ -x $\frac{dy}{dx}$ $+p^2$ y =0	1 1 1
30	Let B be the event of biased coin and C be the event of fair coin. $\therefore$ P(B) = P(C) $= \frac{1}{2}$ P(H/B) $= \frac{1}{4}$, P(H/C) $= \frac{1}{2}$ P(B/H) $= \frac{P(B).(\frac{H}{B})}{P(B).P(\frac{H}{B}) + P(C).P(\frac{H}{C})}$ $= \frac{\frac{1}{2}x\frac{1}{4}}{\frac{1}{2}x\frac{1}{4} + \frac{1}{2}x\frac{1}{2}}$ $= \frac{1}{3}$	1 ½ 1 ½

31.	Graph of inequalities The corner points are(40,20) , (60,0) ,(120,0) and(60, 30) Minimum value of Z= 300 at (60, 0) **OR** Graph of inequalities The corner points are (3,2) and (0,5) Maximum value of Z is 40 at (0.5)	1 ½ 1 ½ 1 ½ 1 ½

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

32.	2x + y =8 ----------(i) y=2 ----------(ii) y=4 ----------(iii) Point of intersection of (i) and (ii) is (3 ,2) Point of intersection of (i) and (iii) is (2 ,4) For shaded region Required area = $\int_2^4 x \, dy = \int_2^4 \frac{(8-y)}{2} dy$ $=\frac{1}{2}[16 - 6]$ = 5 sq. units.	 2 1 2
33.	To prove reflexive, symmetric and transitive [(2,5)] = {(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)} **OR** To prove f is one-one To prove f is onto conclusion	3 ½ 1 ½ 2 2 1

34.	To find AB=I ----------(i) The matrix equation of the given system of equations is B^TX=C where X=$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, C=$\begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$ $\Rightarrow$ X =(B^T)^{-1}C =$(B^{-1})^T C$ = $(A)^T C$ (from (i) B^{-1}=A) = $\begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$ =$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ $\Rightarrow$ x=1, y=-1 and z=1	1 1 1 ½ 1 ½
35.	$\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4}$ ----------(i) $\frac{x-4}{5} = \frac{y-1}{2} = z$ ----------(ii) Let $\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4} = k_1$ and $\frac{x-4}{5} = \frac{y-1}{2} = z = k_2$ $\therefore$ Any point on line (i) is x= $2k_1$ +1 , y=$3k_1$+ b, z=$4k_1$ +3 Any point on line (ii) is x= $5k_2$ +4 , y=$2k_2$+ 1, z=k_2 For point of intersection of lines (i) and (ii), we have $2k_1$ +1 = $5k_2$ +4 and $3k_1$+ b =$2k_2$+ 1 On solving we get k_1=-1, k_2 =-1 Also, y = $3k_1$+ b and y= $2k_2$ +1 $\Rightarrow$ y= 3x(-1) +b and y = 2x(-1) +1=-1 $\Rightarrow$ y = -3 + b and y = -1 $\therefore$-3 + b = -1 $\Rightarrow$ b =-1 +3 =2 $\Rightarrow b = 2$ Point of intersection is (-1,-1,-1)	1 1 1 1 1

Section –E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts.

The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively.

The third case study question has two sub parts of 2 marks each.)

36. (i)We have $\sum_1^8 p_i = 1$ 1

$\Rightarrow p + 2p + 2p + p + 2p + p^2 + 2p^2 + 7p^2 + p = 1$

$\Rightarrow p = \frac{1}{10}$ 1

(ii)P(X> 6) = p(7)+p(8) $= 2p^2 + 7p^2 + p = 9p^2 + p$

$= \frac{19}{100}$

(iii) (a)P (X =3m) =P(3) +P(6)

$= 2p + p^2$ 2

$= 2x\frac{1}{10} + (\frac{1}{10})^2$

$= \frac{2}{10} + \frac{1}{100} = \frac{21}{100}$

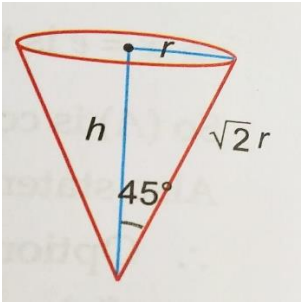
OR

(iii) (b)E(X) $= 1xp+2x2p+3x2p + 4p +5x2p +6xp^2 +7x2p^2$

$+8x (7p^2 + p)$

$=76\ p^2 + 33p$

$=4.06$

37.	 (i)We have . Tan $45^0 = \frac{r}{h} \Rightarrow 1 = \frac{r}{h}$ $\Rightarrow$r = h $\Rightarrow$ l =$\sqrt{2}$ r $\therefore$Volume of cone, V $= \frac{1}{3}\pi r^2$.h = V $= \frac{1}{3}\pi r^2$.r $= \frac{1}{3}\pi r^3$ (ii)We have, $\frac{dV}{dt}$ = -2cm^3/sec $\Rightarrow \frac{1}{3}\pi.3r^2 \frac{dr}{dt} = -2$ $\Rightarrow \frac{dr}{dt} = \frac{-2}{\pi r^2}$ — $\Rightarrow \frac{dr}{dt} = \frac{-1}{4\Pi}$cm/sec (iii)(a) We have , S=πrl =Πr x $\sqrt{2}$ r=$\sqrt{2}\,\pi r^2$ $\therefore$Rate of change of change of surface area, $\frac{dS}{dt} = \frac{d}{dt}(\sqrt{2}\,\pi r^2)$ =$\sqrt{2}\,\pi x 2r \frac{dr}{dt}$ =2$\sqrt{2}$ Πr x$\frac{-1}{4\Pi}$c = -2cm^2/sec (when r=2$\sqrt{2}$ cm) **OR** (iii) (b) We have volume of cone = $\frac{1}{3}\Pi r^2$h =$\frac{1}{3}\Pi h^3$ ($\because$ h = r) V = $\frac{1}{3}\Pi h^3$ $\frac{dV}{dt} = \frac{1}{3}\Pi$ x3$h^2 \frac{dh}{dt}$ $\Rightarrow$-2 = Π $h^2 \frac{dh}{dt}$	1 1 2

38.

$$\Rightarrow \frac{dh}{dt} = \frac{-2}{\Pi h^2} = \frac{-1}{4\Pi} \text{ cm / sec}$$

Given points are $P_1(6,8,4)$, $P_2(21,8,4)$, $P_3(21, 16,10)$ and P_4 (6, 16,10)

(i) We have $\vec{A}$ = PV of P_2 -PV of P_1= $15\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

Components of $\vec{A}$ are 15,0,0

$\vec{B}=0\hat{\imath} + 8\hat{\jmath} + 6\hat{k}$

Components of $\overrightarrow{B}$ are 0,8.6

(ii) (a) We have.

$|\vec{A}| = \sqrt{15^2 + 0^2 + 0^2}$ =15 units

$|\vec{B}| = \sqrt{0^2 + 8^2 + 6^2}$ =10 units

(b) We have ,

$$\vec{N} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 15 & 0 & 0 \\ 0 & 8 & 6 \end{vmatrix}$$

$= -90\hat{\jmath} + 120\hat{k}$

Its components are 0,-90, 120

----------x----------x----------

SAMPLE QUESTION PAPER

MARKING SCHEME 18

Class:-XII

Session 2023-24

Mathematics (Code-041)

SECTION-A

Q1.	(a) I
Q2.	(a) ± 3
Q3.	(d) 3,-4
Q4.	(c) 4
Q5.	(d) $\pm\sqrt{3}$
Q6.	(b) -7 $(p = 1, q = 3)$
Q7.	(d) $2a - b = 0$
Q8	(b) π
Q9	(d) No value of k
Q10	(a) $\vec{0}$
Q11.	(a) $\pm\frac{1}{\sqrt{3}}$
Q12	$A(adj\ A) = \lvert A\rvert I = 10I$ $\Longrightarrow \lvert A\rvert = 10$ $\lvert adj\ A\rvert = \lvert A\rvert^n = 10^2 = 100$
Q13	Probability of hitting the target = $1 - p(none) = 1 - p(\bar{A}) \cdot p(\bar{B}) \cdot p(\bar{C}) = 0.72$ (c) 0.72
Q14	(c) $\sec x$
Q15	(d) $(2 + 3\lambda, 1 + 2\lambda, -4 - \lambda)$
Q16	(d) $\frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}}$
Q17	(d) 90°
Q18	(d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)

	Assertion Reasoning Based Questions
Q19	(b) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct

	explanation of Assertion (A).	
Q20	(d) Assertion (A) is false and Reason (R) is true.	
	SECTION B	
Q21	$\cos^{-1} x + \cos^{-1} y = 2\pi = \pi + \pi$ $\cos^{-1} x = \pi, \cos^{-1} y = \pi$ $x = \cos \pi = -1$ $y = \cos \pi = -1$ According to the question the value is : $1 + 1 + 1 = 3$ **OR** For range - $0 \leq \cos^{-1}\left(\frac{1}{2x-1}\right) \leq \pi$ $0 \leq 3\cos^{-1}\left(\frac{1}{2x-1}\right) \leq 3\pi$ $-2 \leq 3\cos^{-1}\left(\frac{1}{2x-1}\right) - 2 \leq 3\pi - 2$ Therefore the range is : $[-2, 3\pi - 2]$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
Q22	$f(x) = x^3 - 3x^2 + 6x - 100$ $f'(x) = 3x^2 - 6x + 6$ $= 3(x^2 - 2x + 2)$ $=3((x-1)^2 + 1) > 0 \; \forall \; x \; \epsilon \; R$ Hence $f(x)$ is increasing on R	1 $\frac{1}{2}$ $\frac{1}{2}$
Q23	$f'(x) = \cos x + -\sin x$ For points of local maxima or minima, $f'(x) = 0$ $ie, \tan x = 1$ $\Longrightarrow x = \frac{\pi}{4}$ $f''(x) = -\sin x - \cos x, f''\left(\frac{\pi}{4}\right) < 0$ $\Longrightarrow x = \frac{\pi}{4}$ is a point of local maxima So local maximum value $= f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \sqrt{2}$ **OR** $-1 \leq \sin 2x \leq 1$ $4 \leq \sin 2x + 5 \leq 6$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$

	Therefore, Maximum value = 6 and Minimum value = 4	1
Q24	$f(x) = x\|x\|$ $f(-x) = (-x)\|-x\|$ $f(-x) = (-x)\|x\|$ $f(-x) = -f(x)$ So $f(x)$ is an odd function $\therefore \int_{-1}^{1} x\|x\|\,dx = 0$	1 1
Q25	$x^2 = 2y$ $2x\frac{dx}{dt} = 2\frac{dy}{dt}$ $2x\frac{dx}{dt} = 2\frac{dx}{dt}$ (Given $\frac{dx}{dt} = \frac{dy}{dt}$) $\Longrightarrow 2x = 2$ $\Longrightarrow x = 1$ $\therefore y = \frac{1}{2}$ The required point is $(1, \frac{1}{2})$	$1/2$ $1/2$ $1/2$ $1/2$
	SECTION C	
Q26	$\int e^x \frac{[(x^2-1)+2}{(x+1)^2})$ dx $= \int e^x[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}]dx$ $= (\frac{x-1}{x+1})e^x$ + c	1 1 1
Q27	i) $\frac{5}{K} + \frac{7}{K} + \frac{9}{K} + \frac{11}{K} = 1$ K= 32 ii) P(x>4) = P(x=5) = $\frac{11}{32}$ iii) P(x ≤ 3) = P(x =2) + P(x=3) $= \frac{5}{32} + \frac{7}{32}$ $= \frac{12}{32} = \frac{3}{8}$	1 1 1
Q28	Taking x^2 common factor from numerator and denominator and cancelling we get I = $\int \frac{1+\frac{4}{x^2}}{x^{2+\frac{16}{x^2}}}$dx	1

	$=\int \frac{1+\frac{4}{x^2}}{(x-\frac{4}{x})^2+8}dx$ Let $x-\frac{4}{x}=t$ then $(1+\frac{4}{x^2})$ dx = dt $=\int \frac{dt}{t^2+8}$ $=\frac{1}{\sqrt{8}}$ $\tan^{-1}$ $(\frac{t}{2\sqrt{2}})$ $=$ $\frac{1}{\sqrt{8}}$ $\tan^{-1}$ $(\frac{x^2-4}{2\sqrt{2}x})$ +c **OR** $I=\int_0^1(1-x)[1-(1-x)]^n dx$ $=\int_0^1(1-x)x^n dx$ $=[\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}]$ $=\frac{1}{n+1}-\frac{1}{n+2}$ $=\frac{1}{(n+1)(n+2)}$	1 1 1 1 1 1
Q29	Given equation is $\frac{y}{1+y^2}dy=\frac{x}{1+x^2}dx$ Integrating, $\frac{1}{2}\log(1+y^2)=\frac{1}{2}\log(1+x^2)+C$ **OR** $\frac{dx}{dy}=\frac{tan^{-1}y}{1+y^2}-\frac{x}{1+y^2}$ $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{tan^{-1}y}{1+y^2}$ $If = e^{tan^{-1}y}$ *solution is* $x.e^{tan^{-1}y}=\int \frac{tan^{-1}y}{1+y^2}e^{tan^{-1}y}dy$ $=e^{tan^{-1}y}tan^{-1}y - e^{tan^{-1}y}+c$ $=tan^{-1}y -1 +c\ e^{-tan^{-1}y}$	1 2 1 1 1
Q30	The vertices of the feasible region are 0(0,0),A (20,0),B(10.50),C(0,60) $Z_{(0,0)}=0$ $Z_{(20,0)}=1000$ $Z_{(10,50)}=1250$ $Z_{(0,60)}=900$ .Maximum value of Z is 1250 and occurs when x=10 and y=50.	1 1 1
Q31	$e^y=\frac{1}{x+1}$ $\log e^y=\log(\frac{1}{x+1})$ $\Rightarrow$y = -log (x+1) $\frac{dy}{dx}=-\frac{1}{x+1}$ ……(1)	1 1

	$\frac{d^2y}{dx^2} = \frac{1}{(x+1)^2}$ $= (\frac{dy}{dx})^2$	1
	SECTION D	
Q32	To get points of intersection ,solving the two equations, gives x= - 2 , 4 required area $= \int_{-2}^{4} [\frac{3x+12}{2} - \frac{3x^2}{4}]\, dx$ $= [\frac{3x^2}{4} + 6x - \frac{x^3}{4}]_{-2}^{4}$ = 27 square units	2 1 1 1
Q33	The given relation is (a,b)R (c,d) iff $\frac{a+d}{ad} = \frac{b+c}{bc}$ $\Rightarrow \frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c}$ Reflexive : Clearly $\frac{1}{a} + \frac{1}{b} = \frac{1}{b} + \frac{1}{a}$ $\Rightarrow$(a,b)R (a,b) R is reflexive Symmetric Let (a,b) R (c,d) $\Rightarrow \frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c}$ $\Rightarrow \frac{1}{b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{d}$ $\Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ $\Rightarrow$(a,d)R (a,b) R is Symmetric Tr. Let (a,b)R (c,d) and (c,d) R (e,f) $\Rightarrow \frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c}$ and $\Rightarrow \frac{1}{c} + \frac{1}{f} = \frac{1}{d} + \frac{1}{e}$ Adding and simplifying $\frac{1}{a} + \frac{1}{f} = \frac{1}{b} + \frac{1}{e}$ $\Rightarrow$ (a,b) R (e,f) R is transitive. Hence R is reflexive , symmetric and transitive It is an equivalence relation	1 1 1 1 1 1
Q34	IAI = -16 $\neq$0 Hence , A^{-1} exists Adj A $= \begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix}$ The solution of the system is given by $X = A^{-1}B$ $= \frac{-1}{16} \begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix} \begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix}$ $= \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ x=1, y = 2 , z= -3	1 2 1 1

Q35	$\bar{a}_2 - \bar{a}_1 = 3\bar{\imath} - \bar{\jmath}$ $\bar{b}_1 \times \bar{b}_2 = -3\bar{\imath} - 9\bar{\jmath} + 2\bar{k}$ $(\bar{a}_2 - \bar{a}_1).(\bar{b}_1 \times \bar{b}_2) = -9+9+0 = 0$ Shortest distance =0 Hence lines are intersecting When they intersect $1+3\lambda = 4 + 2\mu$ also $1-\lambda = 0 \Rightarrow \lambda = 1$ $1+3x1 = 4+2\mu$ $\Rightarrow \mu = 0$ The point of intersection is (4,0,-1) **OR** Any point on the given line is $(10\lambda+11,-4\lambda-2,-11\lambda-8)$…….(1) Dr's of the perpendicular $10\lambda+11-2,-4\lambda-2+1,-11\lambda-8-5$ ie, $10\lambda+9,-4\lambda-1,-11\lambda-13$. As the lines are perpendicular we have $10(10\lambda+9) - 4(-4\lambda-1) - 11(-11\lambda-13) = 0$ $\Rightarrow \lambda = -1$. Substituting in (1) ,the foot of the perpendicular is(1,2,3) Also the perpendicular distance is PQ = $\sqrt{(2-1)^2+(-1-2)^2+(5-3)^2} = \sqrt{14}$	2 2 1 1 1 1 1 1
Q36	(i) (c) ,$P(E_2) = 1-P(E_1) = 1-0.65 = 0.35$ (ii) (a) $P(E) = P(E_1).P(EE_1) + P(E_2)P(EE_2)$ $= 0.65 \times 0.35 + 0.35 \times 0.80$ $= 0.35 \times 1.45 = 0.51$ iii)(d) $P(E_1/E) = \frac{P(E_1)PE/E_1}{P(E)} = 0.65 \times 0.35/0.51$ $= 0.45$ **OR** (a)$P(E_2/E) = \frac{P(E_2).P(E/E_2)}{P(E)}$ $= \frac{0.35\ X\ 0.80}{0.51}$ $= 0.55$	1 1 2
Q37	i)d ii) a $V = x(45-2x)(24-2x)$ $= 1080x - 90x^2 - 48x^2 + 4x^3$ $= 4x^3 - 138x^2 + 1080x$ iii)b $\frac{dv}{dx} = 12x^2 - 276x + 1080$	1 1

	$\frac{dv}{dx}$ =0 ⇒ x=18 ,5 . here 18 rejected and hence x=5 **OR** a) Vol =5(45-10)(24-10) =2450 cm^3	2
Q38	i)d ii) b $f^I(x)$ =(x-2)(3x-4) $f^I(x)$ =0 gives x = 4/3,2 Case-1 Consider (-∞,4/3) Take 0 € (-∞,4/3) $f^I(x)$ at x=0 = -2 x -4 = 8 > 0 hence f(x) is increasing Case – 2 Consider (4/3,2)x 5 > 0 Take 3/2€(4/3,2) $f^I(x)$ at x=3/2 is-1/2 x ½ <0, hence f(x) is decreasing. Case - 3. Consider (2,∞) Take 3€ (2,∞) $f^I(x)$ at x=3 is 1 x 5 > 0,hence f(x) is increasing.	2 2

KENDRIYA VIDYALAYA SANGATHAN, ERNAKULAM REGION

Class:-XII , Session 2023-24

Mathematics (Code-041)

SAMPLE PAPER 19 - MARKING SCHEME

SECTION A

1. (b) A is a zero matrix
2. (a) 2, 2, 3, 4
3. (d) 16
4. (a) k = 1
5. (a) $\frac{5}{6}(\vec{\imath} - 2\ \vec{\jmath} + \vec{k})$
6. (d)0
7. (c)corner points of the feasible region
8. (a) 0
9. c)$\frac{1}{4}$ (sin 3 + cos 3 - sin 2 – cos 2)
10. (d) 40/7
11. (a) (0, 8)
12. (d) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$
13. (c) 1/7
14. (a)0
15. (b) sec x
16. (a) 2
17. (c) (-∞, ∞)
18. (a) 0
19. (b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
20. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

SECTION B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. $\sin(\frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})) \Rightarrow \sin(\frac{\pi}{3} + \frac{\pi}{6})$ 1

$\Rightarrow \sin(\frac{\pi}{2}) = 1$ 1

OR

$\cos^{-1}(\cos 7\pi/6) \Rightarrow \cos^{-1}(\pi + \frac{\pi}{6})$ ½

$\Rightarrow \cos^{-1}(-\cos(\frac{\pi}{6}))$ ½

$\Rightarrow \pi - \cos^{-1}(\frac{\sqrt{3}}{2})$ ½

$\Rightarrow \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ ½

22. $f(x) = e^{1/x}$ $x \geq 0$

$f'(x) = \frac{-e^{\frac{1}{x}}}{x^2} = 0$ 1

$\Rightarrow$for the function be decreasing f '(x) should be less than 0 .

As $e^{1/x} \geq 0$, $x^2 \geq 0$ f(x) is decreasing function for all $x \geq 0$. 1

23. $f(x)=\sin x + \cos x \Rightarrow f'(x) = \cos x - \sin x = 0 \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$ 1

$f''(x) < 0$, it has got a maximum value at $x = \frac{\pi}{4}$ ½

The maximum value is $\sqrt{2}$ ½

OR

We have $2x + 2y = 100 \Rightarrow x + y = 50$ ½

Let the area of rectangle be A. Therefore, $A = xy \Rightarrow A = x(50-x)$ ½

$\Rightarrow A = 50x - x^2$ ½

$\Rightarrow dA/dx = 50 - 2x$

For maximum area, $dA/dx = 0$.

Therefore, $50 - 2x = 0$ Therefore, $x = 25$ and $y = 25$. ½

Hence, the adjacent sides are 25 and 25 cm.

24. $f(x) = 4x^3 - 6$, $F(x) = x^4 - 6x + c$ 1

$F(0) = 3 \Rightarrow C = 3$ 1/2

$F(x) = x^4 - 6x + 3$ ½

25. $f(x)=\sin(3x)$, for $0 \leq x \leq \pi$

$\Rightarrow f'(x) = 3\cos 3x = 0$ 1

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ ½ + ½

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. $\int \tan^8 x \sec^4 x \, dx = \int \tan^8 x (\sec^2 x)^2 \, dx = \int \tan^8 x (1 + \tan^2 x) \sec^2 x \, dx$ ½ + ½

Put $\tan^2 x = t \quad \sec^2 x \, dx = dt$ ½

$= \int t^8 (1 + t^2) \, dt = \int t^8 + t^{10} \, dt$ ½

$= \frac{\tan^{11} x}{11} + \frac{\tan^9 x}{9} + c$ 1

27. $P(X=0) = {}^2C_0 (1/13)^0 (12/13)^2 = 1 \times 1 \times (12/13)^2 = 144/169$ 1

$P(X=1) = {}^2C_1 (1/13)^1 (12/13) = 24/169$ 1

$P(X=2) = 2C_2(1/13)^2 = 1\times(1/13)^2 = 1/169$ 1

X	0	1	2
P(X)	144/169	24/169	1/169

28. $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$ Rationalise

$= \int_0^1 \frac{\sqrt{1+x}+\sqrt{x}\,dx}{1}$ 1

$= \int_0^1 \frac{\sqrt{1+x}\;dx}{1} + \int_0^1 \frac{\sqrt{x}\;dx}{1}$ ½

Integrating ½

Getting the correct answer $\frac{4\sqrt{2}}{3}$ 1

OR

$$\int \frac{cos\,x}{(1-sin\,x)(2-sin\,x)}\,dx$$

$Put \sin x = t\,, \cos x\,dx = dt$ ½

$\int \frac{dt}{(1-t)(2-t)}$, $\frac{1}{(1-t)(2-t)} = \frac{A}{1-t} + \frac{B}{2-t}$ ½

Apply partial fraction method A= 1, B= -1 1

$= \log\left|\frac{2-sinx}{1-sinx}\right| + C$ 1

29. $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$

Identifying the DE as Homogenous y = vx , 1/2

$v + x\,\frac{dv}{dx} - v + cosec\,v = 0$ ½

$\int -sinv\,dv = \int dx/x$ ½

cos v = logx +c ½

simplifying and getting the answer

cos (y/x) = log cx 1

OR

$ye^y\,dx = (y^3 + 2xe^y)\,dy.$

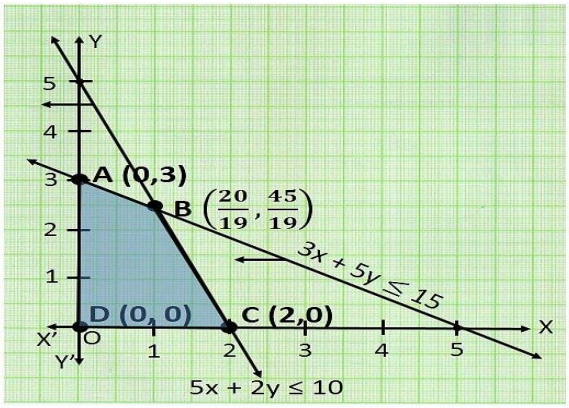
Converting into Linear Differential Equation

$\frac{dx}{dy} + (-\frac{2}{y})x = \frac{y^2}{e^y}$ 1

I F = $\frac{1}{y^2}$ and getting the solution ½ +1/2

Getting the solution $x = -y^2e^{-y} + cy^2$ 1

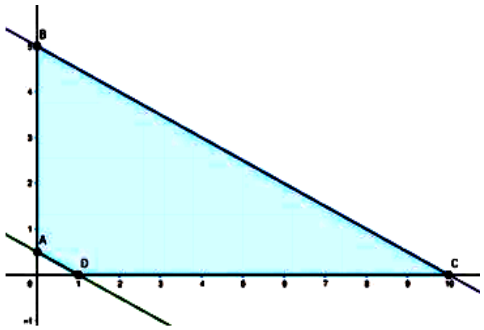
30.



Corner Points	Value of Z
(0, 3)	9
$\left(\frac{20}{19}, \frac{45}{19}\right)$	$\frac{235}{19}$ = 12.36 → Ma
(2, 0)	10

Hence, Z = $\frac{235}{19}$ is maximum at $\left(\frac{20}{19}, \frac{45}{19}\right)$ 2+1

OR



Corner Points	Z = 2x + 3y	
A(0,$\frac{1}{2}$)	$\frac{3}{2}$	Minimum
B(0,5)	15	
C(10,0)	20	
D(1,0)	2	

2+1

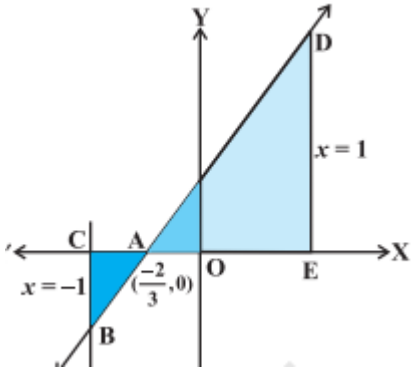
31. $\sin y = x \sin(a+y) \Rightarrow x = \frac{\sin y}{\sin(a+y)}$ ½

Apply quotient rule 1

$\frac{dx}{dy} = \frac{\sin y}{sin^2(a+y)}$ 1

taking reciprocal ½

32.



1

The required area = Area of the region ACBA + Area of the region ADEA

$$= \left| \int_{-1}^{\frac{-2}{3}} (3x+2)dx \right| + \int_{\frac{-2}{3}}^{1} (3x+2)dx$$

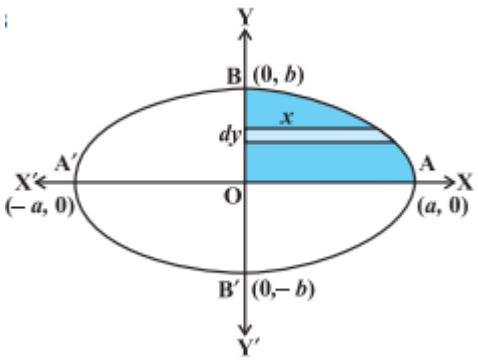
2

$$\left| \left[\frac{3x^2}{2} + 2x \right]_{-1}^{\frac{-2}{3}} \right| + \left[\frac{3x^2}{2} + 2x \right]_{\frac{-2}{3}}^{1}$$

2

$= \frac{1}{6} + \frac{25}{6} = \frac{26}{6} = \frac{13}{3}$ sq. units

OR



1

2

Required Area =

$$= 4\int_0^b x\,dy = 4\frac{a}{b}\int_0^b \sqrt{b^2-y^2}\,dy \;($$

$$= \frac{4a}{b}\left[\frac{y}{2}\sqrt{b^2-y^2}+\frac{b^2}{2}\sin^{-1}\frac{y}{b}\right]_0^b$$

$$= \frac{4a}{b}\left[\left(\frac{b}{2}\times 0+\frac{b^2}{2}\sin^{-1}1\right)-0\right]$$

$$= \frac{4a}{b}\frac{b^2}{2}\frac{\pi}{2}=\pi ab$$

2

33. Reflexive

Since ab(b+a) = ba(a+b)

=> (a,b)R(a,b)

So R is reflexive 1/2

Symmetric

(a,b)R(c,d) => ad(b+c)= bc(a+d)

=> bc(a+d)= ad(b+c)

=> cb(d+a)= da(c+b)

=> (c,d)R(a,b)

So R is Symmetric 2

Transitive

(a,b)R(c,d) => ad(b+c)= bc(a+d) => $\frac{b+c}{bc}=\frac{a+d}{ad}$ => $\frac{1}{b}+\frac{1}{c}=\frac{1}{a}+\frac{1}{d}$

(c,d)R(e,f) => cf(d+e)= de(c+f) => $\frac{d+e}{de}=\frac{c+f}{cf}$ => $\frac{1}{d}+\frac{1}{e}=\frac{1}{c}+\frac{1}{f}$

=> $\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}=\frac{1}{a}+\frac{1}{d}+\frac{1}{c}+\frac{1}{f}$

=> $\frac{1}{b}+\frac{1}{e}=\frac{1}{a}+\frac{1}{f}$

=> af(b+e) = be(a+f)

=> (a,b)R(e,f) 2

So R is transitive.

Since R is reflexive, symmetric and transitive it is an equivalence relation. 1/2

34..Product = $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ =I 2

Writing the matrix equation

$\begin{bmatrix}1 & -1 & 2\\ o & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ 1\\ 2\end{bmatrix}$ 1

$AX = B$ is the matrix equation

$X = A^{-1}B$

$= \begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}\begin{bmatrix}1\\ 1\\ 2\end{bmatrix} = \begin{bmatrix}0\\ 5\\ 3\end{bmatrix}$ 2

x=0,y=5 and z=3

OR

Writing the matrix equation

$\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}11\\ -5\\ -3\end{bmatrix}$

$AX = B$

$X = A^{-1}B = \left(\frac{1}{|A|}Adj\ A\right)B$

$|A| = -1 \neq 0$ 1

A^{-1} exists

$Adj\ A = \begin{bmatrix}0 & -1 & 2\\ 2 & -9 & 23\\ 1 & -5 & 13\end{bmatrix}$, $A^{-1} = -\frac{1}{1}\begin{bmatrix}0 & -1 & 2\\ 2 & -9 & 23\\ 1 & -5 & 13\end{bmatrix}$ 2

$X = A^{-1}B$

$= -\frac{1}{1}\begin{bmatrix}0 & -1 & 2\\ 2 & -9 & 23\\ 1 & -5 & 13\end{bmatrix}\begin{bmatrix}11\\ -5\\ -3\end{bmatrix}$ 1

$\Longrightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$

$x = 1, y = 2, z = 3$ 1

35. **Identifying** $\overrightarrow{a_1}, \overrightarrow{a_2}, \overrightarrow{b_1}, \overrightarrow{b_2}$ 1

$\overrightarrow{a_2}$-$\overrightarrow{a_1}$ = 4i+6j+8k 1

$\overrightarrow{b_1} X \overrightarrow{b_2}$= -4i-6j-8k 1

Formula and substitution 1

Shortest distance = $\sqrt{116}$ units. 1

36.

(i) 3/5 1

(ii) 1 1

(iii) 11/15 or 9/11 1+1

37.

1. 15,0,0 & 0 ,8,6 ½+1/2
2. 15i+0j+0k & 0i+8j+6k ½+1/2
3. 15 unit ,& :√(64 + 36) =√100 = 10 unit 1+1

OR

$\vec{N} = \vec{A}$ X $\vec{B}$ 1/2

$= \begin{vmatrix} i & j & k \\ 15 & 0 & 0 \\ 0 & 8 & 6 \end{vmatrix}$= -90j+120k , 1

Components are -90 &120 1/2

38.

(i) C= 5000x^2 + 4000 h^2 1

x^2h= 250

,h=250/ x^2

C= 5000x^2 + 4000x$\frac{62500}{x^4}$ 1

C = 5000x^2 + $\frac{2500000000}{x^4}$

(ii) $\frac{dC}{dx}$ = 10000x - $\frac{10000000000}{x^5}$ 1

For minimum $\frac{dC}{dx}$ =0

x^6=10^6 1/2

X=10 1/2

OR

$\frac{dC}{dx}$ = 10000x10 - $\frac{10000000000}{10^5}$ <0 1

So cost function is not increasing.

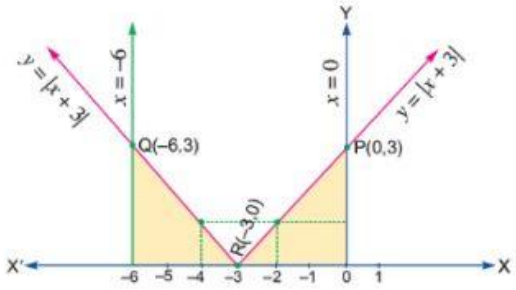
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SAMPLE QUESTION PAPER 20
Class:-XII
Session 2023-24
Mathematics (Code-041)

Time: 3 hours **Maximum marks: 80**

ANSWER KEY

	SECTION A
Qn.No	Value Points
1	(b) $\frac{\pi}{3}$
2	(b) ± 8
3	(d) f is neither one one nor onto.
4	(d) 1
5	(d) -3
6	$(a) \quad -e^x tane^x$
7	(b) $\frac{3}{4t}$
8	(c) R
9	(a) $\left(\frac{1}{2},\frac{1}{4}\right)$
10	(b) $-e^x cotx + C$
11	(d) – 2
12	(b) $\frac{9}{4}$
13	(c) 1 sq. units
14	(a) 2, 2
15	(a) x^2
16	(b) $\frac{\pi}{3}$
17	(d) infinite
18	(c) $\frac{1}{3}$
19	(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
20	(c) (A) is true but (R) is false.
	SECTION B
21	R = (a,b) : 2 divides (a-b) ⇒(a−b) is a multiple of 2. To find equivalence class 0, put b=0 **1** So, (a−0) is a multiple of 2 ⇒ a is a multiple of 2 [0} = { 0**,** ± 1, ± 2, ± 3, …} **1**
22	$f'(x)=3x^2-6x+4$ $=3(x^2-2x+1)+1$ **1** $f'(x) = 3(x-1)^2 + 1 > 0$ And so f(x) is increasing 1
23	

	$\int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$ $I = \int \frac{sec^2x - cosec^2x}{} dx$ solution is y = tanx + cotx + C	1 1
24	 $\text{Required area} = \int_{-6}^{0} \|x+3\| dx$ $= \int_{-6}^{-3} \|x+3\| dx + \int_{-3}^{0} \|x+3\| dx$ [By Property of definite integral] $= -\int_{-6}^{-3} (x+3) dx + \int_{-3}^{0} (x+3) dx$ $= -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0} = \frac{9}{2} + \frac{9}{2} = 9$ sq. units.	1 1
25.	$\frac{dy}{dx} = \sqrt{4 - y^2} (-2 < y < 2)$ or, $\frac{dy}{\sqrt{4-y^2}} = dx$ Now integrating we have, $\sin^{-1}\frac{y}{2} = x + c$ the solution as y = sin(x+c)	1 1

SECTION C

26. $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right)$

Let $\sin^{-1}\left(\frac{8}{17}\right) = A$

- ⇨ Sin A=8/17
- ⇨ $\sin^{-1}\left(\frac{3}{5}\right)$=B
- ⇨ Sin B = 3/5 , Cos A =$\frac{15}{17}$, Cos B= $\frac{4}{5}$ 2

Cos(A+B) =$\frac{15}{17}.\frac{4}{5} - \frac{8}{17}.\frac{3}{5}$

=36/85

A+B= $\cos^{-1}\left(\frac{36}{85}\right)$ 1

$\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$

(OR)

$2\,tan^{-1}\left(\frac{3}{4}\right)=\tan^{-1}\frac{\frac{6}{4}}{1-\frac{9}{10}}$ 1

$=\tan^{-1}\left(\frac{6}{4}\times\frac{16}{7}\right)$

$=\tan^{-1}\left(\frac{24}{7}\right)$

$2\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{17}{31}\right)$

$=\tan^{-1}\left[\frac{\frac{24}{7}+\frac{17}{31}}{1+\frac{24}{7}.\frac{17}{31}}\right]$

$=\tan^{-1}\left[\frac{625}{625}\right]=\tan^{-1}(1)=\frac{\pi}{4}$ 2

27. $(A-I)^3+(A+I)^3-7A$

$=A^3-3A^2I+3AI^2-I^3+A^3+3A^2I+3AI^2+I^3-7A$ 1 1/2

$=2A^3+6AI^2-7A$

$=2A+6A-7A=A$ 1 1/2

28. $\lim_{x\to 0} f(x)=f0)$

$\lim_{x\to 0}\frac{1-cos4x}{8x^2}==k$ 1 1/2

$\lim_{x\to 0}2\frac{sin^2 2x}{8x^2}=k$

⇨ K=1 1 1/2

29. $\frac{dy}{dx}=e^{3x}.e^{4y}$

$\frac{dy}{e^{4y}}=e^{3x}dx$ 1

$\int e^{-4y}\,dy=\int e^{3x}\,dx$

$\frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}+c$

$-\frac{1}{4}=\frac{1}{3}+c$

$c = -\frac{7}{12}$ 1

$\frac{e^{-4y}}{4} = \frac{e^{3x}}{3} - \frac{7}{12}$

$-3e^{-4y} = 4e^{3x} - 7$

$4e^{3x} + 3e^{-4y} = 7$ 1

OR

I = $\int_0^{\frac{\pi}{4}} \log(1 + tanx\ )dx$

= $\int_0^{\frac{\pi}{4}} \log\left(1 + \tan(\frac{\pi}{4} - x)\right) dx$ 1

I = $\int_0^{\frac{\pi}{4}} \log\ \left[1 + \frac{1-tanx}{1+tanx}\right] dx$

= $\int_0^{\frac{\pi}{4}} \log\ \left[\frac{2}{1+tanx}\right] dx$

I = $\int_0^{\frac{\pi}{4}} \log\ \ 2dx - I$

2I = $log2\ \ \int_0^{\frac{\pi}{4}} \log\ \ 1dx$

$log2\ [x]_0^{\frac{\pi}{4}}$

$2I = \frac{\pi}{4} log2$

$I = \frac{\pi}{8} log2$ 2

30. let R be the image of the point P (1,6,3) Q foot of the perpendicular PQ on the line AB is

$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$

$(x, y, z) = (\lambda, 2\lambda + 1, 3\lambda + 2)$

DR's of PQ is

$(\lambda - 1\ , 2\lambda - 5\ , 3\lambda - 1)$ 2

Since PQ perpendicular to AB

$\lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$

Therefore $\lambda = 1$

Therefore coordinate of Q is (1,3,5)

Q is the midpoint of P(1,6,3) and T (x, y, z)

$\frac{x+1}{2} = 1 \ \Rightarrow x = 1$

$\frac{y+6}{2} = 3 \Rightarrow y = 0$

$\frac{z+3}{2} = 5 \Rightarrow z = 7$

Therefore image of (1,6,3) is (1,0,7) 2

31. The total number of outcome = 36

Probability of getting a doublet in one doublet = $\frac{6}{36} = \frac{1}{6}$

Probability not getting doublet = 1- $\frac{1}{6} = \frac{5}{6}$

X denote the number of doublets in three tosses of pair of dies , X can have values 0,1,2,3 1

P(X=0) = $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$

P(X = 1) = ($\frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}$) 3 = 3× $\frac{25}{216}$ = $\frac{75}{216}$

P(X=2) = 3($\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}$) = $\frac{15}{216}$

P(X=3) = $\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$

Probability distribution is

X	0	1	2	3
P(X)	125/216	75/216	15/216	1/216

2

SECTION D

32. A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

$|A| = 4 \neq 0$ 1

A_{11} = 7, A_{12} = -19, A_{13} = -11

A_{21} = 1, $A_{22=}$ -1, A_{23} = -1

A_{31} = -3, A_{32} = 11, A_{33} = 7

$A^{-1} = \frac{adj\ A}{|A|} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$ 2

X = A^{-1}B

$= \frac{1}{4}\begin{bmatrix}8\\4\\12\end{bmatrix} = \begin{bmatrix}2\\1\\3\end{bmatrix}$

Therefore $x = 2, y = 1, z = 3$ 2

33. dividing by $cos^4\ x$

I = $\int \frac{sec^4\ x}{tan^4x + tan^2x+1}dx = \int \frac{sec^2\ x . sec^2\ x}{tan^4x + tan^2x+1}dx$ 1

Put $tanx = t \rightarrow sec^2\ x\ dx = dt$

I = $\int \frac{1+t^2}{t^4+t^2+1}dt = \int \frac{\left(1+\frac{1}{t^2}\right)}{t^2+\frac{1}{t^2}+1}dt = \int \frac{\left(1+\frac{1}{t^2}\right)}{\left(t-\frac{1}{t}\right)^2+3}dt$

Put $u = t - \frac{1}{t} \quad \rightarrow \quad 1 + \frac{1}{t^2}\ dt = du$

Therefore , I = $\int \frac{du}{x^2+(\sqrt{3})^2} \rightarrow$ I = $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right)$ + C 2

I = $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{3}}\right)$ + C

I = $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t^2-1}{\sqrt{3}\,t}\right)$ + C

I = $\frac{1}{\sqrt{3}}\tan^{-1}\left[\frac{tan^2x-1}{\sqrt{3}\,tanx}\right]$ + C 2

OR

I = $\int\left[\sqrt{tanx} + \sqrt{cotx}\right]\ dx$

= $\int \sqrt{tanx}(1 + cotx)\ dx$

Put $tanx = t^2$ $sec^2\ x\ dx$ = 2t dt 1

= $\int t\ \left(1 + \frac{1}{t^2}\right)\ \frac{2t}{1+t^4}\ dt$

= 2 $\int\ \frac{t^2+1}{t^4+1}\ dt$

I = 2 $\int\ \frac{\left(1+\frac{1}{t^2}\right)}{\left(t^2+\frac{1}{t^2}\right)}\ dt$

= 2 $\int\ \frac{\left(1+\frac{1}{t^2}\right)}{\left(t-\frac{1}{t}\right)^2+2}\ dt$ 2

Put $t - \frac{1}{t} = y \quad \rightarrow \quad (1 + \frac{1}{t^2})\ dt = dy$

I = 2 $\int\ \frac{dy}{y^2+(\sqrt{2})^2}$

I = $\frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{y}{\sqrt{2}}\right) + C$

$= \sqrt{2}\ \tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right) + C$

$= \sqrt{2}\ \tan^{-1}\left(\frac{t^2-1}{\sqrt{2}t}\right) + C$

$= \sqrt{2}\ \tan^{-1}\left(\frac{tanx-1}{\sqrt{2}tanx}\right) + C$ 2

34. $\underset{AB}{\rightarrow} = -4i - 6j - 2k$

$\underset{AC}{\rightarrow} = -i + (\alpha - 5)j + 3k,$

$\underset{AD}{\rightarrow} = -8i - j + 3k$ 1 1/2

Since $\underset{OA}{\rightarrow}, \underset{OB}{\rightarrow}, \underset{OC}{\rightarrow}, \underset{OD}{\rightarrow}$ are coplanar $[\underset{AB}{\rightarrow}, \underset{AC}{\rightarrow}, \underset{AD}{\rightarrow}] = 0$

ie, $\begin{vmatrix} -4 & -6 & -2 \\ -1 & (\alpha - 5) & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$ 1 1/2

expanding $\alpha = 9$ 2

35. equation of the line passing through (-1,3,-2) is $\frac{x+1}{a} = \frac{y-3}{b} = \frac{z+2}{c}$ ……(1) 1

The line (1) is $perpendicular$ to the given lines

$a + 2b + 3c = 0$

$-3a + 2b + 5c = 0$ 2

Therefore $\frac{a}{10-6} = \frac{b}{-9-5} = \frac{c}{2+6}$

$\frac{a}{4} = \frac{b}{-14} = \frac{c}{8}$

$\rightarrow \quad \frac{a}{2} = \frac{b}{-7} = \frac{c}{4}$

Therefore, the equation of the line is

$\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$ 2

OR

The given lines are

$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

$\underset{a_1}{\rightarrow} = 3\hat{\imath} + 5\hat{\jmath} + 7\,\hat{k}$ $\qquad$ $\underset{b_1}{\rightarrow} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\underset{a_2}{\rightarrow} = -\hat{\imath} - \hat{\jmath} - \hat{k}$ $\qquad\qquad$ $\underset{b_2}{\rightarrow} = 7\hat{\imath} - 6\hat{\jmath} + \hat{k}$

$\underset{a_2}{\rightarrow} - \underset{a_1}{\rightarrow} = -4\hat{\imath} - 6\hat{\jmath} - 8\hat{k}$ 1

$\underset{b_2}{\rightarrow} X \underset{b_1}{\rightarrow} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}$

$= \hat{\imath}(-2+6) - \hat{\jmath}(1-7) + \hat{k}(-6+14)$

$= 4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$, $\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{16+36+64} = \sqrt{116}$ 2

$(\underset{a_2}{\rightarrow} - \underset{a_1}{\rightarrow}) . (\overrightarrow{b_1} \times \overrightarrow{b_2}) = (-4\hat{\imath} - 6\hat{\jmath} - 8\hat{k}) . (4\hat{\imath} + 6\hat{\jmath} + 8\hat{k})$

$= -16 - 36 - 64 = -116$

Shortest distance $= \left|\frac{(\underset{a_2}{\rightarrow} - \underset{a_1}{\rightarrow}) . (\overrightarrow{b_1} \times \overrightarrow{b_2})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$

$= \left|\frac{-116}{\sqrt{116}}\right| = \sqrt{116}$ 2

SECTION E

36. (i)12/19, (ii)7/19, (iii)0.38 1+1+2

37. (i) (50/3,40/3) (ii), O(0,0),A(25,0) ,B (50/3,40/3), C(0,20) (iii) (50/3,40/3) 1+1+2

38. i) $R(x) = -x^2 + 200x + 150000$ 2+2

ii) 49

SAMPLE PAPER 21

KENDRIYA VIDYALAYA SANGATHAN ERNAKULAM REGION

CLASS XII

SUBJECT: MATHEMATICS (041)

SAMPLE QUESTION PAPER (SESSION:2023-2024)

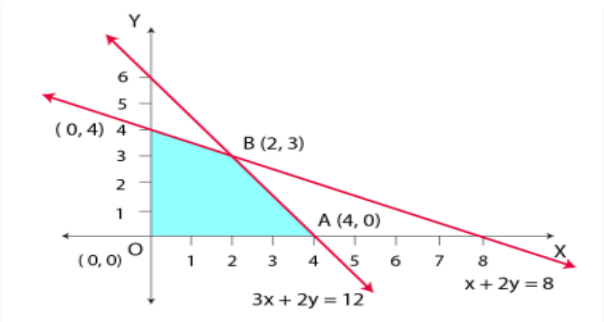
MARKING SCHEME

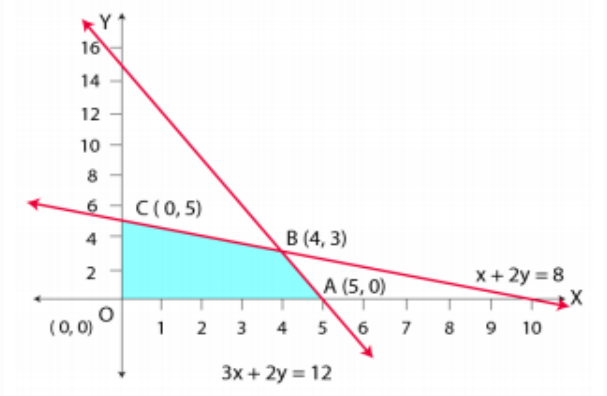
S.No	**PART A** **Section I**	**Marks**
1	(a) $\frac{2\pi}{3}$	1
2	(b) Skew symmetric matrix	1
3	(b) $\frac{5}{x\,log\,(x^5) log\,(log(x^5))}$	1
4	(a)1	1
5	(d) $\frac{1}{\sqrt{1-y^2}}$	1
6	(b) $5\sqrt{3}$	1
7	(b) 8	1
8	(c) 25	1
9	(d) $R - \{4\}$	1
10	(c) $\frac{1}{4}$	1
11	(d) 2	1
12	(b) $4\hat{i} + \hat{j} - 2\hat{k}$	1
13	(b) 3	1
14	(d) $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$	1
15	(c) $\frac{70}{11}$	1
16	(b) 3	1
17	(c) R	1
18	(d) 0	1
19	(b) Both A and R are true but R is not the correct explanation of A	1
20	(c) A is true but R is false.	1

	SECTION B (VSA questions of 2 Marks each)	
21	$\boldsymbol{cot^{-1}\left(\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right) = cot^{-1}\left(\frac{cos\frac{x}{2}+sin\frac{x}{2}+cos\frac{x}{2}+sin\frac{x}{2}}{cos\frac{x}{2}+sin\frac{x}{2}-(cos\frac{x}{2}+sin\frac{x}{2})}\right)}$ $= \boldsymbol{cot^{-1}\left(\frac{2cos\frac{x}{2}}{2sin\frac{x}{2}}\right)}$ $= \boldsymbol{cot^{-1}\left(cot\frac{x}{2}\right)}$ $= \boldsymbol{\frac{x}{2}}$	1
	OR	1
	$\tan^{-1}\left(\frac{cosx}{1-sinx}\right), \frac{-3\pi}{2} < x < \frac{\pi}{2}$ $=\tan^{-1}\left(\frac{cos^2\frac{x}{2}-sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}\right)$ $=\tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)$	1
	$= \tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$ $=\frac{\pi}{4}+\frac{x}{2}$	
22	$V = \frac{4}{3}\pi r^3$ $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $900 = 4\pi r^2 \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{900}{4\pi r^2}$ $\frac{dr}{dt}$ when $r = 15 = \frac{900}{4\pi(15^2)} = \frac{1}{\pi}$ cm/sec	1 1
23	Maximum value =6	1
	Minimum value =4	1
	OR	
	$f(x) = x^4 - 62x^2 + ax + 9$ $f'(x) = 4x^3 - 124x + a$ $f(x)$ has maximum value at $x = 1$ so $f'(1) = 0$ $4 - 124 + a = 0$ $a = 120$	1 1
24	$f(x) = log(2+x) - \frac{2}{2+x}$ $f'(x) = \frac{1}{2+x} - \frac{-2}{(2+x)^2}$ $= \frac{4+x}{(2+x)^2}$	1

	$4 + x > 0$ *when* $x > -2$ *so* $f(x)$ *is always an increasing function*	1
25	Let I = $\int_0^{2\pi} \frac{1}{1+e^{sinx}}\,dx$ (1) = $\int_0^{2\pi} \frac{1}{1+e^{sin(2\pi-x)}}\,dx$ (Using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$) I = $\int_0^{2\pi} \frac{1}{1+e^{-sinx}}\,dx$ $(sin(2\pi - x) = -sinx)$ I= $\int_0^{2\pi} \frac{e^{sinx}}{1+e^{sinx}}$dx (2) Adding (1) and (2) 2I = $\int_0^{2\pi} \frac{1+e^{sinx}}{1+e^{sinx}}\,dx = \int_0^{2\pi} 1dx = [x]_0^{2\pi} = 2\pi$ I = π	½ ½ ½ ½
	SECTION C **(Short Answer Questions of 3 Marks each**	
26	$x^y + y^x = a^b$ $u + v = a^b$ $\frac{du}{dx} + \frac{dv}{dx} = 0$ (1) $u = x^y$ $logu = ylogx$ Diff. wrp.x , $\frac{du}{dx} = x^y\left(\frac{y}{x} + logx\frac{dy}{dx}\right)$ Similarly, $\frac{dv}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + logy\right)$ $\frac{dy}{dx} = -\left(\frac{y^x logy + yx^{y-1}}{x^y logx + xy^{x-1}}\right)$	½ ½ 1 1

27	The given differential equation is $(1+x^2)\frac{dy}{dx}+2xy=4x^2$	1
	$\frac{dy}{dx}+\left(\frac{2x}{1+x^2}\right)y=\frac{4x^2}{1+x^2}$ it is a linear differential equation of the form $\frac{dy}{dx}$+Py =Q	
	where P =$\frac{2x}{1+x^2}$ and Q=$\frac{4x^2}{1+x^2}$	1/2
	Integrating Factor ,$e^{\int Pdx}=e^{log(1+x^2)}=1+x^2$	
	So solution of the differential equation will be ,$y(1+x^2)=\int 4x^2dx=\frac{4x^3}{3}$+c	1
	Using the initial condition get c =0. So the particular solution is $3y(1+x^2)=4x^3$	
	OR	½
	The given differential equation is $xe^{\frac{y}{x}}-y+x\frac{dy}{dx}=0$.	
	It can be expressed in the form $\frac{dy}{dx}=\frac{y}{x}-e^{\frac{y}{x}}$ It is a homogeneous differential equation	½
	Put $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$ and simplify, we will get $x\frac{dv}{dx}=-e^v$	1
	$\int\frac{dx}{x}=\int e^{-v}\,dv$ integrating and get the solution as $logx-e^{\frac{-y}{x}}=$ C	1 1/2
28	$$\int\frac{x^2}{(x^2+1)(x^2+4)}dx$$	
	Put $x^2=t$. $\frac{x^2}{(x^2+1)(x^2+4)}=\frac{t}{(t+1)(t+4)}=\frac{A}{t+1}+\frac{B}{t+4}$	1
	Using partial fraction method determine A $=-\frac{1}{3}$ and B =$\frac{4}{3}$	1
	$\int\frac{x^2}{(x^2+1)(x^2+4)}dx=\int\frac{-\frac{1}{3}}{x^2+1}dx+\int\frac{\frac{4}{3}}{x^2+4}dx$	
	$=-\frac{1}{3}\tan^{-1}(x)+\frac{4}{3}\left(\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right)$+C	1
	OR	
	$\int\frac{cosx}{(1+sinx)(2+sinx)}dx$	
	Put $sinx=t\Rightarrow cosxdx=dt$	1
	$\int\frac{cosxdx}{(1+sinx)(2+sinx)}=\int\frac{dt}{(1+t)(2+t)}$	
	Using partial fraction method determine A $=1$ and, B =-1	
	$\int\frac{cosx}{(1+sinx)(2+sinx)}dx$ =log$(1+t)-\log(t+2)$	1
	$=(log\big((1+sinx)-log(2+sinx)\big)$+C	1

29	Let I $= \int_0^{\pi} \frac{x}{1+\sin x} dx$ $= \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)} dx$ $I = \int_0^{\pi} \frac{\pi - x}{1+\sin x} dx$ $I + I = \int_0^{\pi} \frac{x}{1+\sin x} dx + \int_0^{\pi} \frac{\pi - x}{1+\sin x} dx$ $2I = \int_0^{\pi} \frac{x + \pi - x}{1+\sin x} dx$ $2I = \pi \int_0^{\pi} \frac{1}{1+\sin x} dx$ $= \pi \int_0^{\pi} \frac{1-\sin x}{1-\sin^2 x} dx$ $= \pi \int_0^{\pi} \frac{1-\sin x}{\cos^2 x} dx$ $= \pi \int_0^{\pi} (\sec^2 x - \tan x \sec x) dx$ $= \pi(\tan x - \sec x)_0^{\pi}$ $2I = 2\pi$, So $I = \pi$	1 1 1
30	The feasible region determined by the system of constraints, $x + 2y \leq 8, 3x + 2y \leq 12, x \geq 0, y \geq 0$ is given below.  O(0,0),A(4,0),B(2,3) and C(0,4) are the corner points of the feasible region . Hence the minimum value of the objective function is -12 at the point A (4,0) **OR** The feasible region determined by the system of constraints, $x + 2y \leq 10, 3x + y \leq 15, x \geq 0, y \geq 0$ is given below.	2



2

O(0,0),A(5,0),B(4,3) and C(0,5) are the corner points of the feasible region . 1

Corner Points	O (0,0)	A (5,0)	B (4,3)	C (0,5)
$Value\ of$ $Z = 3x + 2y$	0	15	18	10

Hence the maximum value of the objective function is 18 at the point B (4,3)

31 The random variable X can assume the values 2,3,4,5,6

$P(X = 2) = \frac{1}{15}$

$P(X = 3) = \frac{2}{15}$

$P(X = 4) = \frac{3}{15}$

$P(X = 5) = \frac{4}{15}$

$P(X = 6) = \frac{5}{15}$

$\frac{1}{2}$

1

X	2	3	4	5	6
P(X=x)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$
X P(x)	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{12}{15}$	$\frac{20}{15}$	$\frac{30}{15}$

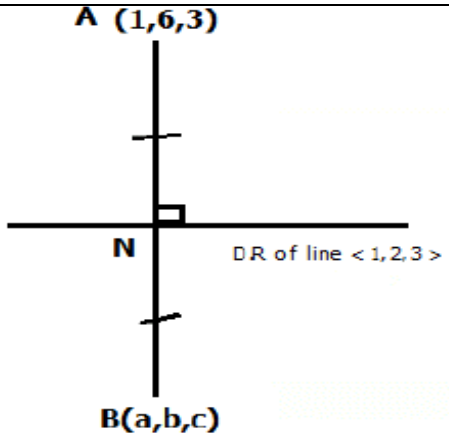
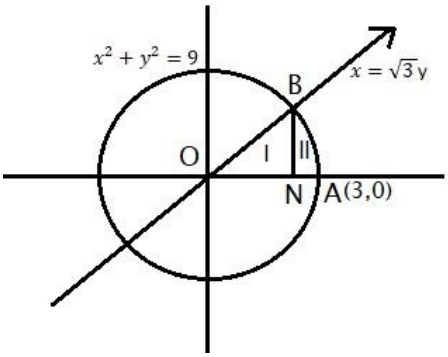
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Mean $= \sum XP(x) = \frac{70}{15} = \frac{14}{3}$

$\frac{1}{2}$

	SECTION D **(Long answer-type questions (LA) 0f 5 Marks each)**	
32	Let R be the relation on set A $=\{x: x \in Z, 0 \leq x \leq 20\}$ $given\ by$ $R = \{(a,b): (a-b) is\ multiple\ of\ 4\}$ For every a∈A $a - a = 0$, which is a multiple of 4. So $(a,a) \in R$ for every a∈A.So R is reflexive. If $(a,b) \in R$ then $(a-b) is\ multiple\ of\ 4$ $\Rightarrow (b-a) is\ multiple\ of\ 4$ $\Rightarrow (b,a) \in R.$ So R is symmetric. If $(a,b) \in R$ & If $(b,c) \in R$ $\Rightarrow (a-b) is\ multiple\ of\ 4$ $\&(b-c) is\ multiple\ of\ 4$ $\Rightarrow (a-b+b-c)$is a multiple of 4 $\Rightarrow (a-c)$is a multiple of 4 $\Rightarrow (a,c) \in R$ So R is Transitive. Hence R is an equivalence relation. The elements related to 1 in A is {1,5,9,13,17.} OR The function is $f: R_+ \to R$ given by, $f(x) = 9x^2 + 6x - 5$ To prove that the function is one − one. Let $f(x_1) = f(x_2) \Rightarrow$ $9x_1^2+6x_1-5=9x_2^2+6x_2-5 \Rightarrow 9x_1^2 - 9x_2^2 + 6x_1 - 6x_2 = 0$ $\Rightarrow (x_1 - x_2)(9x_1 + 9x_2 + 6)$ $\Rightarrow x_1 = x_2$ (since domain of f is positive real numbers $9x_1 + 9x_2 + 6 \neq 0$ $So\ f\ is$ one-one. To prove that $f(x)\ is$ not onto. Let $9x^2 + 6x - 5 = y \Rightarrow$ $(3x+1)^2 - 1 - 5 = y$ So $(3x+1)^2$ = y+6 $\Rightarrow x = \frac{\sqrt{y+6}-1}{3}$.But $x \geq 0$ $\frac{\sqrt{y+6}-1}{3} \geq 0$ .So $y \geq -5$.Hence Range =[-5,∞). Codomain =R Hence function is not onto. But if we modify Codomain =Range of the function. = [-5,∞). The give function become onto.	1 $1\frac{1}{2}$ $1\frac{1}{2}$ 1 1 1 1 1

33	Given, $A=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$, B $=\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}$ To prove that AB =I,Identity matrix of order 3 Hence A^{-1} =B Now the system of equations :$x-2y=3, 2x-y-z=2$ and $-2y+z=3$ can be written in the matrix form as A^TX = C,the constant matrix. So X=$(A^T)^{-1}$ C But $(A^T)^{-1}=(A^{-1})^T$ X =B^TC =$\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}^T\begin{bmatrix}3\\2\\3\end{bmatrix}$ $\begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}\begin{bmatrix}3\\2\\3\end{bmatrix}=\begin{bmatrix}1\\-1\\1\end{bmatrix}$ Hence the solution is $x=1, y=-1$ and $z=1$ =	1 1 2 1
34	Two lines are given by $\vec{r}=(6\hat{\imath}+2\hat{\jmath}+2k)+\lambda(\hat{\imath}-2\hat{\jmath}+2\hat{k})$ & $\vec{r}=(-4\hat{\imath}-k)+\mu(3\hat{\imath}-2\hat{\jmath}-2\hat{k})$. We have to find points on the lines so that the distance between them is shortest. Also find the shortest distance between the lines. Direction ratio of the first line is 1,-2,2 and d r's of the second line 3,-2,-2 Any point A on L1 is $(\lambda+6, 2-2\lambda, 2\lambda+2)$ & any point B on L2 is $(3\mu-4, -2\mu, -2\mu-1)$ Direction ratio of AB will be $\lambda-3\mu+10, -2\lambda+2\mu+2, \lambda+2\mu+3$ Using the concept that each line is perpendicular to the line of shortest distance, we will get two relations connecting λ and μ as $9\lambda-3\mu+12=0$ and $3\lambda-17\mu+20=0$. Solving the two equations and get $\lambda=-1$ and $\mu=1$. Hence A will be the point $(5,4,0)$ and B $(-1,-2,-3)$ Shortest distance =Distance between the points A and B = 9 units **OR** We have to find the image of the point P(1,6,3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$	½ ½ ½ 1 1 ½ 1

	 Any point on the line is $(\lambda, 2\lambda+1, 3\lambda+2)$ In particular for N,Direction ratio of AN are $\lambda-1, 2\lambda-5, 3\lambda-1$ and dr's of the line 1,2,3 AN is perpendicular to the line. So we can get $\lambda = 1$ Hence point N will be (1,3,5) N is the midpoint of A and B . So we will get the coordinates of the image B as (1,0,7)	½ ½ 2 1 1
35	To draw the correct figure  To get the points of intersection of $x^2+y^2=9$ and $x=\sqrt{3}y$ $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{-3\sqrt{3}}{2}, \frac{-3}{2}\right)$ The area of the pizza piece = $\int_0^{\frac{3\sqrt{3}}{2}} y_{line}\, dx + \int_{\frac{3\sqrt{3}}{2}}^{3} y_{circle}\, dx$ $= \int_0^{\frac{3\sqrt{3}}{2}} \frac{x}{\sqrt{3}}\, dx + \int_{\frac{3\sqrt{3}}{2}}^{3} \sqrt{9-x^2}\, dx$ $= \frac{3\pi}{4}$ sq.units.	1 1 1 2
	SECTION E **(Case-studies /passage based questions of 4 Marks each)**	
36	(i)$\vec{a} = \hat{\imath}+4\hat{\jmath}+2\hat{k}$, $\vec{b} = 3\hat{\imath}-3\hat{\jmath}-2\hat{k}$ and $\vec{c} = -2\hat{\imath}+2\hat{\jmath}+6\hat{k}$ $\vec{a}+\vec{b}+\vec{c} = -2\hat{\imath}+3\hat{\jmath}+6\hat{k}$	1

	(ii) The unitvector along $\vec{a}+\vec{b}+\vec{c}=\frac{2i+3j+6k}{7}$	1
	(iii) If the given points lie on the straight line, then the points will be collinear and so area of Δ ABC Will be zero. So $\left\| \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a} \right\|$ =0 **OR** Using Position Vectors of A,B and C find $\overrightarrow{AB}$and $\overrightarrow{AC}$ and determine area of Δ ABC $=\frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|=\frac{1}{2}\sqrt{1937}$ Square units.	2
37	(i)$D^2=f(x_1)=x_1^2$ -6x_1+9+x_1^4	1
	(b) Find the derivative of $f(x_1)$*and equate it to zero to get the critical point as* x_1= 1	1
	Find the position of Vivek when Devabhadra will hit the paper ball. = (1,8) **OR** Use Second derivative test to identify that (1,8) must be the position of Vivek to minimize the distance. The minimum distance between Vivek and Devabhadra =$\sqrt{5}$ units	2
38	(i)Let A be the event that selected student failed in Physics and B be the event that selected student failed in Maths.So $P(A)=\frac{10}{100}$, $P(B)=\frac{10}{100}$ and $P(A\cap B)=\frac{2}{100}$ (i) The probability that the selected student has passed in atleast one of the two subjects $=P(A'\cup B')$ $=P\langle(A\cap B)'\rangle=1-P(A\cap B)=1-\frac{2}{100}$ $=\frac{98}{100}$ **OR** (ii)The Probability that the selected student has passed in Mathematics,if it is known that he has failed in Physics = $P(B'/A)=\frac{P(B'\cap A)}{P(A)}$ $=\frac{P(A)-P(A\cap B)}{P(A)}$ $=\frac{\frac{10}{100}-\frac{2}{100}}{\frac{10}{100}}=\frac{8}{10}=\frac{4}{5}$	2

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SAMPLE QUESTION PAPER MARKING SCHEME - 22

Class XII

Session 2023 – 24

Mathematics

Answers:

1. a)
2. d)
3. c)
4. b)
5. b)
6. b)
7. c)
8. a)
9. c)
10. a)
11. b)
12. b)
13. d)
14. b)
15. b)
16. d)
17. c)
18. c)
19. d)
20. a)
21. $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right) = \cos^{-1}\left(\cos\left(\pi+\frac{\pi}{6}\right)\right) = \cos^{-1}\left(-\cos\frac{\pi}{6}\right) = \cos^{-1}\left(\cos\left(\pi-\frac{\pi}{6}\right)\right)$

$= \cos^{-1}\left(\cos\left(\frac{5\pi}{6}\right)\right) = \frac{5\pi}{6}$

$\cos^{-1}\left(\cos\left(\frac{13\pi}{6}\right)\right) = \cos^{-1}\left(\cos\left(2\pi+\frac{\pi}{6}\right)\right) = \cos^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$

$\therefore \cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right) + \cos^{-1}\left(\cos\left(\frac{13\pi}{6}\right)\right) = \frac{5\pi}{6}+\frac{\pi}{6} = \pi$

OR

$3\times\frac{\pi}{4}+2\times\frac{\pi}{6}+\frac{\pi}{2} = \frac{3\pi}{4}+\frac{\pi}{3}+\frac{\pi}{2} = \frac{9\pi+4\pi+6\pi}{12} = \frac{19\pi}{12}$

22. f'(x)$= 3x^2-6x+6 = 3(x^2-2x+2)$

$= 3[(x-1)^2+1] > 0$ for all x $\in R$

$\therefore$ f(x) is strictly increasing on R

23. $f(x) = \frac{1}{4x^2+2x+3}$

$4x^2 + 2x + 3 = \left(2x + \frac{1}{2}\right)^2 + \frac{11}{4} \geq \frac{11}{4}$

$\frac{1}{4x^2 + 2x + 3} \leq \frac{4}{11}$

f(x) $\leq \frac{4}{11}$

$\therefore$ max[f(x)] = $\frac{4}{11}$

OR

P'(x) = $26 - 26x$

P'(x) = 0 $\Rightarrow$ x = 1

P''(x) = $-26 < 0$

$\therefore$ P(x) is maximum when x = 1

Maximum profit = P(1) = 15 + 26 – 13 = 28

24. I = $\int_0^{\pi/2} \frac{(\cos x)^5}{(\sin x)^5 + (\cos x)^x} dx$ …eq(i)

I = $\int_0^{\pi/2} \frac{(\cos(\pi/2 - x))^5}{(\sin(\pi/2 - x))\ \ (\cos(\pi/2 - x))^5} dx$

Also I = $\int_0^{\pi/2} \frac{(\sin x)^5}{(\cos x)^5 + (\sin x)^x} dx$ …eq(ii)

(i) + (ii)

2I = $\int_0^{\pi/2} 1\, dx = [x]_0^{\pi/2} = \pi/2$

I = $\frac{\pi}{4}$

25. $\frac{da}{dt} = 3\ cm/sec$, a = 10 cm

$v = a^3$

$\frac{dv}{dt} = 3a^2 \frac{da}{dt}$

$= 3(10)^2 \cdot 3 = 900 cm^3/sec$

26. Let $\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$

$x^2 + x + 1 = A(x+1)(x+2) + B(x+2) + C(x+1)^2$

Put $= -1$, $(-1)^2 - 1 + 1 = A(-1+1)(-1+2) + B(-1+2) + C(-1+1)^2$

$1 = B$

$x = -2,\ (-2)^2 - 2 + 1 = A(-2+1)(-2+2) + B(-2+2) + C(-2+1)^2$

$3 = C$

$x = 0,\ (0)^2 - 0 + 1 = A(0+1)(0+2) + B(0+2) + C(0+1)^2$

$1 = 2A + 2B + C$

$1 = 2A + 2(1) + 3$

$A = -2$

$\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{3}{x+2}$

$\int \frac{x^2+x+1}{(x+1)^2(x+2)} dx = \int \frac{-2}{x+1} dx + \int \frac{1}{(x+1)^2} dx + \int \frac{3}{x+2} dx = -2\log|x+1| - \frac{1}{x+1} + 3\log|x+2| + C$

27. Let x denotes the number of defective bulbs in the sample

$\therefore x = 0,1,2$

$P(x=0) = P(1st\ bulb\ is\ non\ defective\ and\ 2nd\ is\ non\ defective) = \frac{24}{30} \times \frac{24}{30} = \frac{16}{25}$

$P(x=1)$

$= P(1st\ defective\ and\ 2nd\ non\ defective\ OR\ 1st\ non\ defective\ and\ 2nd\ defective)$

$= \frac{6}{30} \times \frac{24}{30} + \frac{24}{30} \times \frac{6}{30} = \frac{8}{25}$

$P(x=2) = P(1st\ and\ 2nd\ are\ defectives) = \frac{6}{30} \times \frac{6}{30} = \frac{1}{25}$

x	$P(x)$
0	$\frac{16}{25}$
1	$\frac{8}{25}$
2	$\frac{1}{25}$

Mean = $0 \times \frac{16}{25} + 1 \times \frac{8}{25} + 2 \times \frac{1}{25} = \frac{10}{25} = \frac{2}{5}$

28. $I = \int_1^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}} dx$ ..... (1)

$I = \int_1^2 \frac{\sqrt{1+2-x}}{\sqrt{1+2-x}+\sqrt{3-(1+2-x)}} dx = \int_1^2 \frac{\sqrt{3-x}}{\sqrt{3-x}+\sqrt{x}} dx$

$I = \int_1^2 \frac{\sqrt{3-x}}{\sqrt{3-x}+\sqrt{x}} dx$ ..... (2)

$(1) + (2) \Rightarrow 2I = \int_1^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}} dx + \int_1^2 \frac{\sqrt{3-x}}{\sqrt{3-x}+\sqrt{x}} dx$

$= \int_1^2 \frac{\sqrt{x}+\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}} dx = \int_1^2 1 dx = (x)_1^2 = 2 - 1 = 1$

$I = \frac{1}{2} \Rightarrow \int_1^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}} dx = \frac{1}{2}$

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log \tan x\, dx$ .....(1)

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log \tan\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right) dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log \tan\left(\frac{\pi}{2} - x\right) dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log \cot x\, dx$

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log \cot x\, dx$ ..... (2)

$(1) + (2) \Rightarrow 2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log \tan x\, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log \cot x\, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\log \tan x + \log \cot x)\, dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log (\tan x \cot x)\, dx$

$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log 1 dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 0\, dx = 0$

$I = 0 \Rightarrow \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log \tan x\, dx = 0$

29. $e^x \tan y\, dx + (2 - e^x)\sec^2 y\, dy = 0$

$(2 - e^x)\sec^2 y\, dy = -e^x \tan y\, dx$

$\frac{\sec^2 y}{\tan y} dy = -\frac{e^x}{2-e^x} dx$

$\int \frac{\sec^2 y}{\tan y} dy = \int -\frac{e^x}{2 - e^x} dx$

$\log \tan y = \log(2 - e^x) + C$

$y = \frac{\pi}{4}$ when $x = 0 \quad \Longrightarrow \quad \log \tan \frac{\pi}{4} = \log(2 - e^0) + C$

$\log 1 = \log 1 + C. \ \therefore C = 0$

Hence $\log \tan y = \log(2 - e^x) \Longrightarrow \tan y = 2 - e^x$

OR

$(3xy + y^2)dx + (x^2 + xy)dy = 0$

$(x^2 + xy)dy = -(3xy + y^2)dx$

$\frac{dy}{dx} = \frac{-(3xy+y^2)}{(x^2+xy)}$

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{-(3x(vx)+(vx)^2)}{(x^2+x(vx))} = -\frac{3v+v^2}{1+v}$

$x\frac{dv}{dx} = \frac{-3v-v^2}{1+v} - v = \frac{-3v-v^2-v-v^2}{1+v} = \frac{-4v-2v^2}{1+v} = \frac{-2v(2+v)}{1+v}$

$\frac{1+v}{v(2+v)} dv = -2\frac{dx}{x}$

$\int \frac{1+v}{v(2+v)} dv = -2\int \frac{dx}{x}$ ……(1)

$\frac{1+v}{v(2+v)} = \frac{A}{v} + \frac{B}{2+v}$

$1 + v = A(2 + v) + Bv$

Put $= 0$, $1 = 2A$

$A = \frac{1}{2}$

$v = -2, \ 1 - 2 = B(-2)$

$-1 = -2B$

$B = \frac{1}{2}$

$\frac{1+v}{v(2+v)} = \frac{\frac{1}{2}}{v} + \frac{\frac{1}{2}}{2+v}$

$\int \frac{1+v}{v(2+v)} dx = \int \left(\frac{\frac{1}{2}}{v} + \frac{\frac{1}{2}}{2+v}\right) dv = \frac{1}{2}\log v + \frac{1}{2}\log(2 + v)$

(1) $\Rightarrow \frac{1}{2}\log v + \frac{1}{2}\log(2+v) = -2\log x + C$

$\frac{1}{2}\log\left(\frac{y}{x}\right) + \frac{1}{2}\log\left(2+\frac{y}{x}\right) = -2\log x + C$

Given $x = 1, y = 2$

$\frac{1}{2}\log\left(\frac{2}{1}\right) + \frac{1}{2}\log\left(2+\frac{2}{1}\right) = -2\log 1 + C$

$\frac{1}{2}\log 2 + \frac{1}{2}\log 4 = 0 + C$

$\frac{1}{2}\log 8 = C$

$\therefore \frac{1}{2}\log\left(\frac{y}{x}\right) + \frac{1}{2}\log\left(2+\frac{y}{x}\right) = -2\log x + \frac{1}{2}\log 8$

$\log\left(\frac{y}{x}\right) + \log\left(2+\frac{y}{x}\right) = -4\log x + \log 8$

$\log\left(\frac{y}{x}(2+\frac{y}{x})\right) = \log\frac{8}{x^4}$

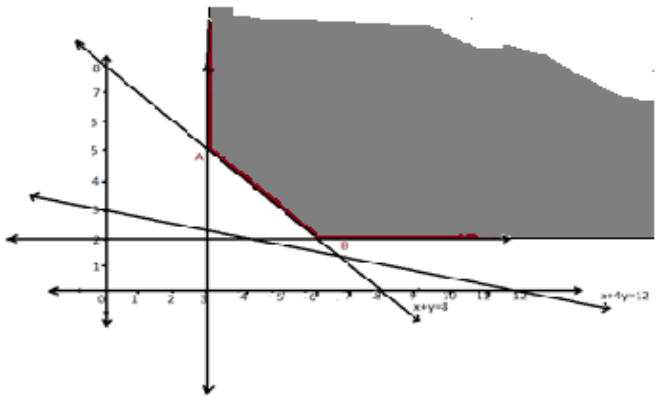
$\frac{2xy+y^2}{x^2} = \frac{8}{x^4}$. Hence $x^2(2xy+y^2) = 8$

30. $x + y = 8$

x	0	8
y	8	0

$x + 4y = 12$

x	0	4
y	3	2



The point A (3,5) and B((6,2)

Z at (3,5) = 2(3)+4(5)=26

Z at (6,2) = 2(6)+4(2)=20

Z is minimum when $x = 6$ *and* $y = 2$ and the minimum value is 20.

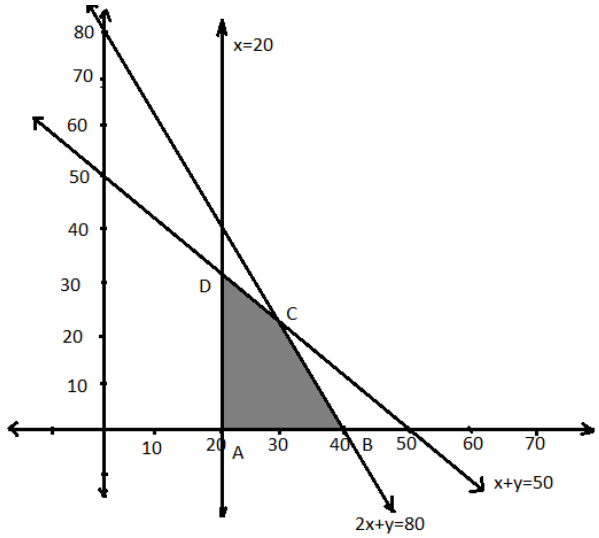
OR

$x + y = 50$

x	0	50
y	50	0

$2x + y = 80$

x	0	40
y	80	0



A(20,0) ,B(40,0),C(30,20) and D(20,30)
Z at A(20,0) = 105(20)+90(0) = 2100
Z at B(40,0) = 105(40)+90(0) =4200
Z at C(30,20) = 105(30)+90(20) = 4950
Z at D(0,20) = 105(0)+90(20) = 1800
Z is maximum when $x = 30\ and\ y = 20$ and the maximum value of Z is 4950

31. $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$ and $y = a\sin t$

$$\frac{dx}{dt} = a(-\sin t + \frac{1}{\tan\left(\frac{2}{2}\right)} \times \sec^2\frac{t}{2} \times \frac{1}{2})$$

$$= a(-\sin t + \frac{\cos\frac{t}{2}}{\sin\frac{t}{2}} \times \frac{1}{\cos^2\frac{t}{2}} \times \frac{1}{2}) = a(-\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}})$$

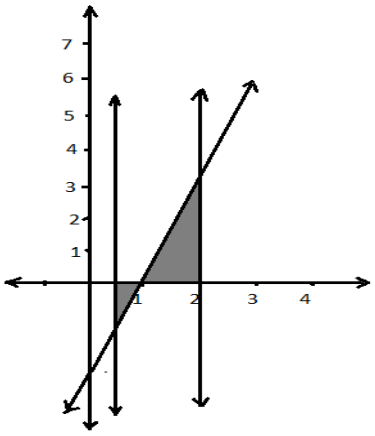
$$= a\left(-\sin t + \frac{1}{\sin t}\right) = a\left(\frac{-\sin^2 t + 1}{\sin t}\right) = a\left(\frac{\cos^2 t}{\sin t}\right)$$

$$\frac{dy}{dt} = a\cos t$$

$$\therefore \frac{dy}{dx} = \frac{a\cos t}{a\left(\frac{\cos^2 t}{\sin t}\right)} = \frac{\sin t}{\cos t} = \tan t$$

At $t = \frac{\pi}{4}$, $\frac{dy}{dx} = \tan\frac{\pi}{4} = 1$

32. Required area $= \left|\int_{\frac{1}{2}}^{1} y \text{ of the line } dx\right| + \int_{1}^{2} y \text{ of the line } dx = \left|\int_{\frac{1}{2}}^{1}(3x - 3)dx\right| + \int_{1}^{2}(3x-3)dx$



$$= \left|\left(3\frac{x^2}{2} - 3x\right)_{\frac{1}{2}}^{1}\right| + \left(3\frac{x^2}{2} - 3x\right)_{1}^{2}$$

$$= \left|\left(3\times\frac{1}{2} - 3\times 1\right) - \left(3\times\frac{1}{8} - 3\times\frac{1}{2}\right)\right| + \left(3\times\frac{2^2}{2} - 3\times 2\right) - \left(3\times\frac{1^2}{2} - 3\times 1\right)$$

$$= 3\left[\left|\frac{1}{2} - 1 - \frac{1}{8} + \frac{1}{2}\right| + 0 - \frac{1}{2} + 1\right] = 3\left[\frac{1}{8} + \frac{1}{2}\right] = \frac{15}{8} \text{ square units}$$

33. $ad(b+c) = bc(a+d) \Rightarrow adb + adc = bca + bcd$

$$\Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a} \text{ (dividing by } abcd)$$

$(a,b)R(c,d) \Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$

$\forall (a,b) \in N \times N, \frac{1}{a} + \frac{1}{b} = \frac{1}{b} + \frac{1}{a} \Rightarrow (a,b)R(a,b)$

Hence R is reflexive

$(a,b)R(c,d) \Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$

$$\Rightarrow \frac{1}{d} + \frac{1}{a} = \frac{1}{c} + \frac{1}{b} \Rightarrow \frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c} \Rightarrow (c,d)R(a,b)$$

Hence R is symmetric

$(a,b)R(c,d)\ and\ (c,d)R(e,f) \Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ and $\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$

$$\frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a} \ldots\ldots\ldots\ldots (1)$$

$$\frac{1}{f} + \frac{1}{c} = \frac{1}{e} + \frac{1}{d} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

$(1) - (2) \Rightarrow \frac{1}{b} - \frac{1}{f} = \frac{1}{a} - \frac{1}{e} \Rightarrow \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f} \Rightarrow (a,b)R(e,f)$

Hence R is transitive.

$\therefore R$ is an equivalence relation

Or

For $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\} \to R$ let $f(x_1) = f(x_2) \Rightarrow \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$

$$\Rightarrow 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$$

$$\Rightarrow 16x_1 = 16x_2 \Rightarrow x_1 = x_2$$

Hence f is one-one.

Let $y \in R \; such \; that \; y = \frac{4x}{3x+4} \Longrightarrow 3xy + 4y = 4x$

$x(3y - 4) = -4y$

$\therefore x = \frac{-4y}{3y-4} = \frac{-4y}{3\left(y-\frac{4}{3}\right)}$. Then $y = \frac{4}{3} \in R$ has no pre image in $R - \left\{-\frac{4}{3}\right\}$

Hence f is not an onto function.

34. $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

$A_{11} = +(8-6) = 2 \quad A_{12} = -(0+9) = -9 \quad A_{13} = +(0-6) = -6$

$A_{21} = -(-4+4) = 0 \quad A_{22} = +(4-6) = -2 \quad A_{23} = -(-2+3) = -1$

$A_{31} = +(3-4) = -1 \quad A_{32} = -(-3-0) = 3 \quad A_{33} = +(2-0) = 2$

$adjA = \begin{bmatrix} 2 & -9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$

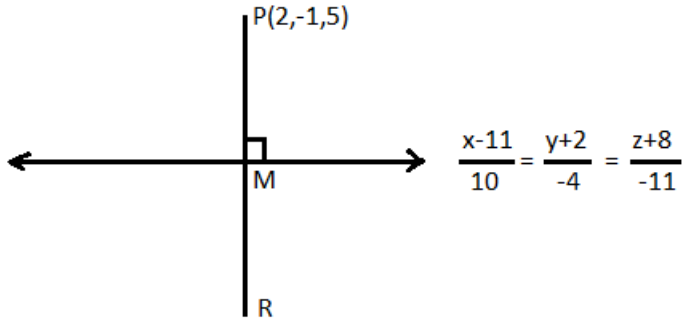
$|A| = 1(8-6) + 1(0+9) + 2(0-6) = 2 + 9 - 12 = -1$

$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{-1}\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}B = \frac{1}{-1}\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 2+0-3 \\ -9-2+9 \\ -6-1+6 \end{bmatrix} = -1\begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$x = 1, \quad y = 2, \quad z = 1$

35. Let R(a, b, c) be the image of P(2,-1,5). Then PR$\perp$ to the line and let M be the midpoint of PR.



Let $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda$, then $x = 10\lambda + 11, \quad y = -4\lambda - 2, \quad z = -11\lambda - 8$

Let $M(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$

Direction ratios of PM are $10\lambda + 11 - 2, -4\lambda - 2 - (-1), -11\lambda - 8 - 5 = (10\lambda + 9, -4\lambda - 1, -11\lambda - 13)$

PM$\perp$ to the line $\Longrightarrow 10(10\lambda + 9) - 4(-4\lambda - 1) - 11(-11\lambda - 13) = 0$

$100\lambda + 90 + 16\lambda + 4 + 121\lambda + 143 = 0$

$237\lambda + 237 = 0$

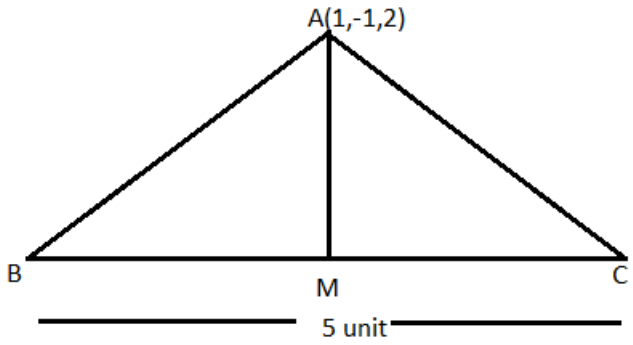
$\therefore \lambda = -1$

Hence the midpoint of PR is $M(1,2,3)$

Then $\frac{a+2}{2} = 1, \quad \frac{b+(-1)}{2} = 2, \quad \frac{c+5}{2} = 3 \Longrightarrow a = 0, b = 5, \; c = 1$

Hence the image of P(2,-1,5) is R(0,5,1)

OR



Equation of BC is $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z-0}{4} = \lambda$

Let M be the foot of the perpendicular drawn from M on BC

Then M$(2\lambda - 2, \lambda + 1, 4\lambda)$

Direction ratios of AM are $2\lambda - 3, \lambda + 2, 4\lambda - 2$

$AM \perp BC \Longrightarrow 2(2\lambda - 3) + 1(\lambda + 2) + 4(4\lambda - 2) = 0$

$\Longrightarrow 21\lambda - 12 = 0$

$\lambda = \frac{12}{21} = \frac{4}{7}$

Thus M $(\frac{-6}{7}, \frac{11}{7}, \frac{16}{7})$

AM = $\sqrt{(\frac{-6}{7} - 1)^2 + (\frac{11}{7} + 1)^2 + (\frac{16}{7} - 2)^2} = \sqrt{\frac{71}{7}}$

Hence are of $\Delta ABC = \frac{1}{2} \times BC \times AM = \frac{1}{2} \times 5 \times \sqrt{\frac{71}{7}} = \sqrt{\frac{1775}{28}}$ square units

36. I. c)

II. c) $\overrightarrow{AC} + \overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BC} + \overrightarrow{CD}$

$= \overrightarrow{AB} + 2\overrightarrow{BC} - \overrightarrow{AB} = 2\overrightarrow{BC}$

III.b) $\overrightarrow{AC} - \overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{BC} - \overrightarrow{CD}$

$= \overrightarrow{AB} - \overrightarrow{CD} = \overrightarrow{AB} + \overrightarrow{AB} = 4\overrightarrow{a}$

OR

d)In ΔABC $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC}$

In ΔBCD $\overrightarrow{BC} = \overrightarrow{BD} - \overrightarrow{CD}$

$\therefore \overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BD} - \overrightarrow{CD}$

$\Rightarrow \overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{BD} + \overrightarrow{CA}$

37. I. b)

II. c)

III. c) A(x) = $(x - 1)\left(\frac{24}{x} - 1.5\right)$

A'(x) = 0 ⇒ x = 4

A''(x) = $-\frac{48}{x^3}$ is negative when x = 4

OR

c) Maximum Area = $(4-1)\left(\frac{24}{4}-1.5\right)$ = 13.5 ft^2

38. I. $P(A\bar{B}) + P(\bar{A}B) = 0.3 \times 0.8 + 0.7 \times 0.2$

$= 0.38$

II. $\frac{P(\bar{A}B)}{P(A\bar{B})+P(\bar{A}B)} = \frac{0.14}{0.38} = \frac{7}{19}$

PART B

STUDY MATERIAL

CHAPTER 1

RELATIONS AND FUNCTIONS

GIST OF THE LESSON

1. If A≠∅ and B≠∅ then A×B= {(a, b): a∈ A and b∈ B} is called Cartesian Product of sets A and B. the element (a, b) is called ordered pair
2. A=∅ or B=∅ then A×B=∅
3. n(A×B) = n(A)×n(B)
4. If A≠∅ and B≠∅ then a set R is said to be a relation from A to B if R⊂ A×B
5. Number of relations that can be defined from A to B is $2^{n(A)\times n(B)}$
6. Let A≠∅ then a set R is a relation on A if R⊂ A×A
7. Notation aRb⇔(a,b)∈R
8. ∅⊂A×A is a relation on A known as empty relation or void relation or null relation
9. A×A⊂A×A is a relation on A known as universal relation on A
10. If R= {(a, a): a ∈A} known as identity relation on A
11. R is a reflexive relation on A if (a, a) ∈R for every a ∈A
    or R is a reflexive relation on A if aRa for every a ∈A
12. R is a symmetric relation on A if (a, b) ∈R => (b, a) ∈R for every a, b∈ A
    Or R is a symmetric relation on A if aRb =>bRa for every a, b∈ A
13. R is transitive on A if (a,b)∈R and (b, c)∈R=>(a, c)∈R for every a, b, c∈ A
    or R is transitive on A if aRb and bRc =>aRc for every a, b, c ∈A
14. R is an equivalence relation on A if it is reflexive ,symmetric and transitive
15. If R is an equivalence relation on A and a∈ A then equivalence class of a, [a]= {b∈ A:(b, a) ∈R}
16. Sets $A_1, A_2, A_3 \ldots A_n$ is a partition of set A if $A_i \cap A_j = \emptyset$ if i≠j and $A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n = A$
17. Equivalence relation defined on a set gives a partition of the set as equivalence classes and every partition of set gives an equivalence relation
18. If A≠∅ and B≠∅ ,a function f: A->B is a relation which associate each element of A to a Unique element of B, A is known as domain of f, B is known as co domain of f
19. If f(a)=b then b is known as image of a and a is known as pre-image of b
20. Set of all images of elements of A is known as range of f, Range of f ⊂B
21. A function f: A->B is one to one or injective if a≠ b =>f(a)≠f(b) ∀ a, b∈ A
    or f: A->B is one to one or injective if f(a)=f(b)=>a=b ∀ a, b∈ A
22. A function which is not one to one is known as many to one
23. A function f: A->B is onto or surjective if for each element b∈ B, there exists a∈ A such that f(a)=b
24. A function f: A->B is onto or surjective if Range of f=B
25. A function f: A->B which is not onto is known as into function
26. A function f: A->B which is both one to one and onto is known as a bijection
27. A function f: A->B which is both injective and surjective is known as a bijection

28. If n(A)=m and n(B)=n then

(i)Number of functions that can be defined from A to B =n^m

(ii)Number of one-to-one functions from A to B =$\frac{n!}{[n-m]!}$ If n≥m otherwise it is zero

(iii)Number of onto functions from A to B=$\sum_{r=1}^{n}(-1)^{n-r}\ {}^{n}C_r\ r^m$ if n≤m otherwise it is zero

(iv)Number of bijections from A to B = n! if n=m otherwise it is zero

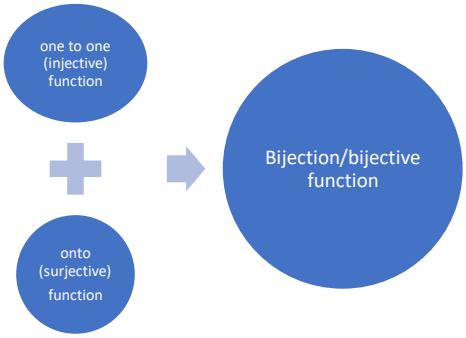
29. For a finite set A, if a function f: A->A is one to one then f is onto
30. For a finite set A, if a function f: A->A is onto then f is one to one
31. For a finite set A the number of bijection from A to A=number of onto functions from A to A=number of one to one function from A to A=n!
32. Graphical test for a function: if any straight line parallel to y axis does not cut the graph at more than one point then the graph represents a function
33. Graphical test for one-to-one function: if any straight line parallel to x axis does not cut the graph at more than one point then the graph represents a one-to-one function
34. A function f: A->B is invertible if and only if it is a bijecti

CONCEPT MAPPING

Relations and functions		
Equivalence relation Equivalence relation on a nonempty set A is relation which is reflexive, symmetric and transitive		**Bijections** A function F: A->B is a bijection if it is both one to one and onto
R is reflexive on A if (a,a)∈R ∀a∈ A Or aRa ∀a∈ A		**One to one or injective functions** A function F: A->B is one to one or injective If a≠ b=> F(a)≠F(b) ∀a, b ∈A Or if F(a)=F(b) =>a=b ∀a, b ∈A
R is symmetric on A if (a, b) ∈R => (b, a) ∀a, b ∈A or aRb=>bRa ∀a, b ∈A		**Onto or surjective functions** A function F: A->B is a onto or surjective If for every b∈ B there exists a∈ A such that f(a)=b Or Range of f = B
R is transitive on A if (a, b) ∈R and (b, c) ∈ R=>aRc ∀a, b, c ∈ A or aRb and bRc =>aRc ∀a, b, c ∈A		**Many one function** A function F: A-> B is many one if it is not one to one
Equivalence class of a=[a]		**Into functions** A function F: A-> B is into function if it is not onto

If R is an equivalence relation on A and a∈ A then Equivalence class of a=[a]=set of all elements of A related a = {b ∈Aa, b) ∈R}		





Multiple choice questions		
1.	Let X= {1,2,3} and R is relation defined in the set X as R = {(1,3), (2,2), (3,2)} then the minimum ordered pairs should include in relation R to make it reflexive and symmetric is	
	a) (1,1), (2,3) and (1,2)	b) (3,3), (3,1), and (1,2)
	c) (1,1), (3,3), (3,1), and (2,3)	d) (1,1), (3,3), (3,1), and (1,2)
2	A relation R is defined as follows on N. Which of the following is reflexive relation	

	(a) R= {(x, y):x>y, x, y ϵN}	(b) R= {(x, y):x+ y=10, x, yϵ N}
	© R= {(x, y):x+4y=10, x, y ϵN}	(d) R= {(x, y): xy is a square number x, yϵ N}
3	The number of equivalence relations that can be defined in the set A= {1,2,3} which containing the elements (1,2) is	
	(a) 0	(b) 1
	(c) 2	(d) 3
4	A relation R is defined on Z as aRb if and only if a^2-7ab + $6b^2$=0 then R is	
	a) Reflexive and symmetric	b) symmetric and not Reflexive
	c) transitive but not Reflexive	d) Reflexive but not symmetric
5	A relation R is defined on Z as aRb if and only if a-b+ √2 is an irrational number then R is	
	a) Reflexive	b) symmetric and Reflexive
	c) transitive and Reflexive	d) none of these
6	Let X= $\{x^2 : x \in N\}$ and the function f: N->X is defined as f(x)=x^2, x ∈ N then the function f is	
	a) Bijective	b) Not bijective
	c) Surjective only	d) Injective only
7	Which of the following function f: Z->Z is a bijection	
	a) f(x)=x^3	b) f(x)=x+2
	c) f(x)=2x+1	d) f(x)=$x^2 + 1$
8	Let f: R->R defined as f(x)=4+3cosx then f(x) is	
	a) Bijective	b) One to one but not onto
	c) Onto but not one to one	d) Neither one to one nor onto
9	The number of one-to-one functions that can be defined from the set {1,2,3,4,5} to {a, b}	
	a) 5	b) 0
	c) 2	d) 3
10	If F: N->N f(x)=$\begin{cases} \frac{n+1}{2} & if \quad n\ is\ odd \\ \frac{n}{2} & if \quad n\ is\ even \end{cases}$ then F(x) is	

	a) Bijective	b) One to one but not onto
	c) Onto but not one to one	d) Neither one to one nor onto
Short Answer questions		
11	Check whether the relation R on the set N of natural numbers given by R= {(a, b): b is a multiple of a } is reflexive, symmetric and transitive	
12	Let W denote the set of words in English dictionary. Define the relation R by R= {(x, y):x, y ∈W such that x and y have at least one letter in common}. Show that this relation R is reflexive and symmetric but not transitive	
13	An equivalence relation R in the set A divides it into equivalence classes A_1, A_2, A_3 Find (i) $A_1 \cup A_2 \cup A_3$ (ii) $A_1 \cap A_2 \cap A_3$	
14	Check whether the relation R on set of all real numbers **R** as R= {(a, b): a $\leq b^3$} is reflexive, symmetric and transitive	
15	Let R be a relation defined on the set of natural numbers N as R= {(x, y):x, y∈N,2x+y=11}. Verify whether R is reflexive, symmetric and transitive	
16	If F= {(1,2), (2,4), (3,1), (4, k)} is a one -to-one function from set A to A, where A= {1,2,3,4} then find the value of k, also find the number of bijections can defined from A to A	
17	A relation f defined in the set of real numbers R as f = {(a, b):$\sqrt{a}$ =b} Verify whether f is a function from R to R.	
18	Show that the function f: R->R given by f(x)=4x^3+7 is a bijection	
19	Let F: [2, ∞) ->B be a function defined as F(x)=5-4x+x^2 is a bijection then find B	
20	Let f: R- {$\frac{-4}{3}$} -> R- {$\frac{4}{3}$} given by f(x) =$\frac{4x+3}{3x+4}$ Show that f is a bijective function	
	LONG ANSWER QUESTIONS	
21	Show that the relation on the set A={x∈Z:0≤x≤12} given by R= {(a, b):$\|a-b\|$ is divisible by 4} is an equivalence relation Find all elements related to 1, equivalence class [1]	
22	Prove that the relation R in the set Z of integers defined as R= {(a, b): a +b is divisible by 2} is an equivalence relation. Write the equivalence class[0]	
23	Let N be the set of natural numbers and R be the relation on NXN defined by (a, b) R (c, d) if only if ad=bc for all a, b, c, d∈ N. Show that R is an equivalence relation	

24	Let A= {1,2, 3, …,9} and R be the relation on A×A defined as (a, b) R (c, d) if and only if a+ d=b+ c. Prove that R is an equivalence relation also obtain the equivalence class [(2,5)]
25	Let R be the relation on N×N defined by (a, b) R (c, d) if and only if ad (b+ c) =bc (a+ d), Prove that R is an equivalence relation
26	Show that the relation R defined on the set N×N defined as (a, b) R (c, d) if and only if $a^2+d^2=b^2+c^2$ is an equivalence relation
27	Show that the function f: R ->R given by f(x)= $\frac{x}{x^2+1}$ is neither one to one nor onto
28	Show that f: N->N, given by f(x)=$\begin{cases} x+1 & if\ x\ is\ odd \\ x-1 & if\ xis\ even \end{cases}$ is a bijection
29	Show that the function f: N->N defined ad f(x)=x^2+x+1 is one to one but not onto
30	Let A = [–1, 1]. Then, discuss whether the following functions defined on A are one-one, onto or bijective (i) f (x) = $\frac{x}{2}$(ii) g(x) = $\lvert x \rvert$ (iii) h(x)=x$\lvert x \rvert$ (iv) k(x) = x^2.
CASE STUDY QUESTIONS	
31	 During a Swachh Bharat Abhiyan organizing committee wanted collect and segregate Metal, Paper, glass, batteries, organic and plastic waste. In the set of all participants a relation R defined as R= {(x, y) ∈R: both the participants x and y collect the same type of waste} Based on the information given above answer the following questions (a) Check whether R is an Equivalence relation in the set of all participants (b) In how many groups the participants are divided on the basis of their waste collection assume that there are participants to collect all type of waste (c) State whether the waste collected from different groups are segregated or not?

32	 A Physical education teacher asked the class teacher to form four teams with 12 members each out of the 48 students for a Kabaddi match.The class teacher asked to the students to form teams in such way that "two students are in the same team if difference of their roll numbers is divisible by 4" Based on the above information anser the following questions (a)Is it possible to form the teams by the method adopted by class teacher? (b) Which roll numbers are members of the team in which roll number 5 belongs? (c) If R is a relation defined in the set of roll numbers as R={(x,y):difference of x and y is divisible by 4 } show that R is an equivalence relation?
33	 Farmers plant sapling along straight lines parallel to each other as in figure .Let us assume that saplings are planted along the line y=x +1 and paralell to it.Let L be the set of all lines on the field Answer the following using the above information (i) R_1 be a relation defined on L as R_1={(l_1, l_2): $l_1 \parallel l_2$,where $l_1, l_2 \in$L} then R_1 is …….. (a) Equivalence relation (b)only Reflexive (c)Not reflexive

	(c) Symmetric but not transitive (ii) Which of the following line is related the line y=x+1 as per definition of the relation R_1 (a) 2x-y+5=0 (b)2x+y=5 (c) 2x-2y=10 (d) x+y=1 (iii) R_2 be a relation defined on L as R_2={(l_1, l_2): l_1 ⊥ l_2,where l_1, l_2∈L} then R_2 is …….. (a) symmetric but neither reflexive nor transitive (b) reflexive and symmetric but not transitive © reflexive but neither symmetric not transitive (d) R is an equivalnce relation (iv) The function f:R->R defined by f(x)=x+1 is ……. (a) Injective but not surjective (b)Surjective but not injective (c) Bijective (d)Neither Injective Not Surjectie (v) Let function f:R->R defined by f(x)=x+1 then range of is (a) Q (b)Z ©W (d) R
34	

	Sherlin and Danju are playing Ludo by rolling the dice alternatly,it was observed that the possible outcomes of the die belongs to the set B={1,2,3,4,5,6}.Let A ={S,D},be the set of all players Answer the following questions (i)Let R: B->B defined as R={(x,y): y is divisible by x} then R is (a) reflexive and transitive but not symmetric (b) reflexive and symmetric not transitive © Not reflexive but symmetric and transitive (d) Equivalence relation (ii) How many relations can be defined from A to B (a) 2^4 (b) 2^{36} © 2^8 (d)2^{12} (iii)How many functions can be defined from A to B (a) 36 (b)64 © 720 (d)1024 (iv)Let R_1 be a relation on B defined as R_1={(1,2),(2,2),(1,3),(3,4),(3,1),(4,3),(5,5)} then R_1 is (a) Symmetric (b)Reflexive (c)transitive (d)None of these (v) How many surjections can be defined from A to B (a) 30 (b) 0 © 32 (d)64

35	 In a Master chef competition final round 3 chef were selected and Judges assigned three dishes D= $\{D_1, D_2, D_3,\}$ to the participants P= $\{P_1, P_2, P_3,\}$ and asked them to prepare dishes as per the following rules. Rule A: everybody has to prepare exactly one dish Rule B: No two participant is allowed to prepare same dish Rule C: All the dish must be prepared in the competition Answer the following questions in the context of functions
	(a) In how many ways all participants can choose a Dish as per rule A? Justify your answer (b) In how many ways everybody can choose a dish to prepare as per Rule B? Justify your answer ©In How many ways all participants can prepare exactly one dish as per rule C, Justify your answer

ANSWERS

Q.No	Answer
1	c
2	d
3	c
4	d
5	a
6	a
7	b
8	d
9	b
10	c
11	Reflexive and transitive but not symmetric
12	Prove reflexive, symmetry and give example for not transitive
13	(i) $A_1 \cup A_2 \cup A_3 = A$ (ii) $A_1 \cap A_2 \cap A_3 = \emptyset$
14	Not reflexive, not symmetric and not transitive, give examples in each case
15	Not reflexive, not symmetric and not transitive, give examples in each case
16	K=3 and number of bijections =24
17	Not a function because no negative numbers have image in R

18	Prove 1to1 and onto
19	B=[1,∞)
20	Prove 1to1 and onto
21	Prove reflexive, symmetry and transitive [1]={1,5,9}
22	Prove reflexive, symmetry and transitive, [0]=set of all even integers
23	Prove reflexive, symmetry and transitive
24	Prove reflexive, symmetry and transitive, [(2,5)]={(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)}
25	Prove reflexive, symmetry and transitive
26	Prove reflexive, symmetry and transitive
27	Prove or give examples for not 1to 1 and not onto
28	Prove by taking different cases
29	Prove 1 to 1 and not onto
30	(i) one to one but not onto (ii) not one to one and not onto (iii) one to one and onto (iv) not one to one and not onto
31	It is an equivalence relation,6 groups, yes
32	(i)yes (ii){1,5,9,13,17,21,25,29,33,37,41,45} (iii) Prove reflexive, symmetry and transitive
33	(i)a (ii)c (iii)a (iv)c (v)d
34	(i)a (ii)d (iii)a (iv)d (v)b
35	(i)number of functions from D to P=27 (ii) Number of one to one function from D to P=3! =6 (iii) Number of onto functions from D to P=3! =6

CHAPTER 2
INVERSE TRIGONOMETRIC FUNCTIONS

BASIC CONCEPTS AND FORMULAE:
TRIGONOMETRIC FUNCTIONS

FUNCTIONS	DOMAIN	RANGE
Y=sin x	R	[-1,1]
Y = cos x	R	[-1,1]
Y = tan x	R$-(2n+1)\frac{\pi}{2}, n \in Z$	R
Y = cosec x	R$-n\pi, n \in Z$	R-(-1,1)
Y = sec x	R$-(2n+1)\frac{\pi}{2}$	R-(-1,1)
Y = cot x	R$-n\pi, n \in Z$	R

IMPORTANT TRIGONOMETRIC RESULTS & SUBSTITUTIONS

** **Formulae for t-ratios of Allied Angles :**

All T-ratio changes in $\frac{\pi}{2} \pm \theta$ and $\frac{3\pi}{2} \pm \theta$ while remains unchanged in $\pi \pm \theta$ and $2\pi \pm \theta$.

$\sin\left(\frac{\pi}{2} \pm \theta\right) = \cos\theta$ $\quad$ $\sin\left(\frac{3\pi}{2} \pm \theta\right) = = \cos\theta$

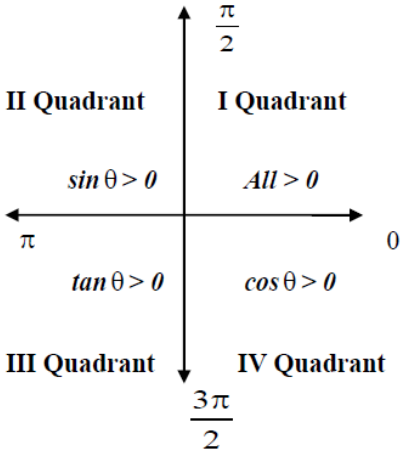
$\cos\left(\frac{\pi}{2} \pm \theta\right) = \mp\sin\theta$ $\quad$ $\cos\left(\frac{3\pi}{2} \pm \theta\right) = \pm\sin\theta$

$\tan\left(\frac{\pi}{2} \pm \theta\right) = \mp\cot\theta$ $\quad$ $\tan\left(\frac{3\pi}{2} \pm \theta\right) = \mp\cot\theta$

$\sin(\pi \pm \theta) = \mp\sin\theta$ $\quad$ $\sin(2\pi \pm \theta) = \pm\sin\theta$

$\cos(\pi \pm \theta) = = \cos\theta$ $\quad$ $\cos(2\pi \pm \theta) = \cos\theta$

$\tan(\pi \pm \theta) = \pm\tan\theta$ $\quad$ $\tan(2\pi \pm \theta) = \pm\tan\theta$



** **Sum and Difference formulae :**

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$, $\tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A}$,

$\tan\left(\frac{\pi}{4} - A\right) = \frac{1 - \tan A}{1 + \tan A}$, $\cot(A + B) = \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A}$ $\quad$ $\cot(A - B) = \frac{\cot A \cdot \cot B + 1}{\cot B - \cot A}$

$\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

$\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

** Formulae for t-ratios of múltiple and sub-múltiple angles :

$$\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}.$$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$1 + \cos 2A = 2\cos^2 A$ $\quad 1 - \cos 2A = 2\sin^2 A$ $\quad 1 + \cos A = 2\cos^2\frac{A}{2}$ $\quad 1 - \cos A = 2\sin^2\frac{A}{2}$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}, \qquad \tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}.$$

$\sin 3A = 3\sin A - 4\sin^3 A$, $\quad \cos 3A = 4\cos^3 A - 3\cos A$

$$\sin 15^\circ = \cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}. \quad \& \quad \cos 15^\circ = \sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}},$$

$$\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3} = \cot 75^\circ \quad \& \quad \tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2 + \sqrt{3} = \cot 15^\circ.$$

$$\sin 18^\circ = \frac{\sqrt{5}-1}{4} = \cos 72^\circ \quad \text{and } \cos 36^\circ = \frac{\sqrt{5}+1}{4} = \sin 54^\circ.$$

$$\sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54^\circ \quad \text{and } \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72^\circ.$$

$$\tan\left(22\frac{1}{2}\right)^\circ = \sqrt{2} - 1 = \cot\ 67\frac{1}{2}^\circ \quad \text{and } \tan\left(67\frac{1}{2}\right)^\circ = \sqrt{2} + 1 = \cot\left(22\frac{1}{2}\right)^\circ.$$

PRINCIPAL VALUE BRANCHES OF INVERSE TRIGONOMETRIC FUNCTIONS

FUNCTIONS	DOMAIN	PRINCIPAL VALUE BRANCH
$Y=\sin^{-1}x$	[-1,1]	$[-\frac{\pi}{2},\frac{\pi}{2}]$
$Y=\cos^{-1}x$	[-1,1]	$[0,\pi]$
$Y=\tan^{-1}x$	R	$(-\frac{\pi}{2},\frac{\pi}{2})$
$Y=\operatorname{cosec}^{-1}x$	R− (-1,1)	$[-\frac{\pi}{2},\frac{\pi}{2}]-\{0\}$
$Y=\sec^{-1}x$	R − (-1,1)	$[0,\pi]-\{\frac{\pi}{2}\}$
$Y=\cot^{-1}x$	R	$(0,\pi)$

Values of trigonometric functions

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
Sin*x*	**0**	**1**	**0**	**-1**	**0**	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
Cosx	**1**	**0**	**-1**	**0**	**1**	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
Tan*x*	**0**	**n.d**	**0**	**n.d**	**0**	$\frac{1}{\sqrt{3}}$	**1**	$\frac{\sqrt{3}}{1}$
Cosec*x*	**n.d**	**1**	**n.d**	**-1**	**n.d**	**2**	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$
Sec*x*	**1**	**n.d**	**-1**	**n.d**	**1**	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	**2**

$\mathbf{Cot}x$	**n.d**	**0**	**n.d**	**0**	**n.d**	$\sqrt{3}$	**1**	$\frac{1}{\sqrt{3}}$

Properties of Inverse trigonometric functions:

1. $\sin(\sin^{-1}x) = x, x \in [-1,1]$ *and* $\sin^{-1}(\sin x) = x, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
2. $(i)\ \sin^{-1}(\frac{1}{x}) = \operatorname{cosec}^{-1}x\ , x \geq 1$ *or* $x \leq -1$
   (ii)$\cos^{-1}(\frac{1}{x}) = \sec^{-1}x\ , x \geq 1$ *or* $x \leq -1$
   (iii)$\tan^{-1}(\frac{1}{x}) = \cot^{-1}x\ ,\ x > 0$
3. (i) $\sin^{-1}(-x) = -\sin^{-1}x, x \in [-1,1]$
   (ii) $\tan^{-1}(-x) = -\tan^{-1}x\ , x \in R$
   (iii)$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x\ |x| \geq 1$
4. (i) $\cos^{-1}(-x) = \pi - \cos^{-1}x, x \in [-1,1]$
   (ii)$\sec^{-1}(-x) = \pi - \sec^{-1}x\ |x| \geq 1$
   (iii) $\cot^{-1}(-x) = \pi - \cot^{-1}x\ , x \in R$
5. (i)$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\ , x \in [-1, 1]$
   (ii)$\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}\ , x \in R$
   (iii)$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\ , |x| \geq 1$

MULTIPLE CHOICE QUESTIONS

1. Simplified form of $\cos^{-1}(4x^3 - 3x)$
   (a) $3\sin^{-1}x$ (b) $3\cos^{-1}x$ (c) $\pi - 3\sin^{-1}x$ (d) $\pi - 3\cos^{-1}x$
2. If $y = \sec^{-1}x$ then
   (a) $0 \leq y \leq \pi$ (b) $0 \leq y \leq \frac{\pi}{2}$ (c) $-\frac{\pi}{2} < y < \frac{\pi}{2}$
   (d) none of these
3. The value of $\cos^{-1}(\frac{1}{2}) + 2\sin^{-1}(\frac{1}{2})$ is equal to
   (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d)$\frac{7\pi}{6}$
4. Principal value of $\cos^{-1}(-\frac{\sqrt{3}}{2})$
   (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{7\pi}{6}$
5. The principal value of $\sin^{-1}(\sin\frac{7\pi}{6})$
   (a) $\frac{\pi}{6}$ (b) $\frac{7\pi}{6}$ (c) $\frac{-\pi}{6}$ (d) $\frac{\pi}{3}$
6. *Choose correct option* : $\sin(\tan^{-1}x)$ =…… when $|x| < 1$
   $(a)\ \frac{x}{\sqrt{1-x^2}}$ $(b)\ \frac{1}{\sqrt{1-x^2}}$ $(c)\ \frac{1}{\sqrt{1+x^2}}$ $(d)\ \frac{x}{\sqrt{1+x^2}}$

SHORT ANSWER TYPE QUESTIONS

7. Find the value of expression : $2\sec^{-1} 2 + \sin^{-1}(\frac{1}{2})$
8. Express in simplest form : $\tan^{-1}(\frac{\sin x}{1+\cos x})$, $-\pi < x < \pi$
9. Express in simplest form : $\tan^{-1}(\frac{\cos x}{1-\sin x})$ where $-\frac{3\pi}{2} < x < \frac{\pi}{2}$
10. Express in simplest form : $\tan^{-1}(\frac{\cos x}{1+sinx})$ where $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
11. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, *then find the value of* $\cos^{-1} x + \cos^{-1} y$
12. Express in simplest form $\sin^{-1}(\frac{\sin x+\cos x}{\sqrt{2}})$, where $\frac{-\pi}{4} < x < \frac{\pi}{4}$
13. Find the value of $\sin[\frac{\pi}{3} - \sin^{-1}(-\frac{1}{2})]$
14. Write the principal value of $\tan^{-1}(1) + \cos^{-1}(-\frac{1}{2})$
15. Write the value of $\tan^{-1}[2\sin(2\cos^{-1}\frac{\sqrt{3}}{2})]$
16. Write $\cot^{-1}\frac{1}{\sqrt{x^2-1}}$, $|x| > 1$ *in simplest form*
17. Express in simplest form : $\tan^{-1}(\frac{\cos x-\sin x}{\cos x+\sin x})$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$
18. Write in simplest form $\tan^{-1}\{\frac{x}{\sqrt{a^2-x^2}}\}$, $-a < x < a$
19. Write in simplest form : $\tan^{-1}\sqrt{\frac{a-x}{a+x}}$, $-a < x < a$
20. Write in simplest form: $\sin^{-1}\frac{x}{\sqrt{x^2+a^2}}$

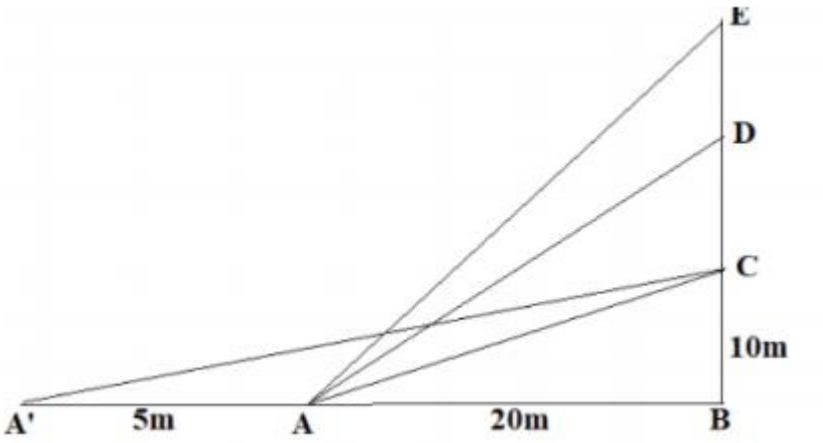
LONG ANSWER QUESTIONS

21. Write the simplest form of $\tan^{-1}(\frac{\sqrt{1-cosx}}{\sqrt{1+cosx}})$, where $-\pi < x < \pi$
22. Write in simplest form $\cot^{-1}(\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}})$ where $x\epsilon(0,\frac{\pi}{4})$
23. Prove that $\tan^{-1}(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$, $\frac{-1}{\sqrt{2}} \leq x \leq 1$
24. Prove that $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$
25. Find the value of $\tan^{-1}[2\cos(2\sin^{-1}\frac{1}{2})]$
26. Prove that $\tan^{-1}[\frac{\sqrt{1+\cos x}+\sqrt{1-cosx}}{\sqrt{1+cosx}-\sqrt{1-cosx}}] = \frac{\pi}{4}+\frac{x}{2}$, $0< x < \frac{\pi}{2}$
27. Prove that $\tan^{-1}[\frac{\sqrt{1+cosx}+\sqrt{1-cosx}}{\sqrt{1+cosx}-\sqrt{1-cosx}}] = \frac{\pi}{4} - \frac{x}{2}$ if $\pi < x < \frac{3\pi}{2}$
28. Prove that $\cot^{-1}[\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}] = \frac{\pi}{2} - \frac{x}{2}$, if $\frac{\pi}{2} < x < \pi$
29. Prove that $\tan^{-1}[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$, $-1 < x < 1$

CASE STUDY QUESTIONS

30. The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID -19 protocol. Ram , Robert and Rahim are the three engineers who are working on this project . If " A " is considered to be a person viewing the hoarding board 20 metres away from the building , standing at the edge of a pathway nearby . Ram ,Robert and Rahim suggested to the film to place the

hoarding board at three different locations namely C,D and E . "C " is at the height of 10 metres from the ground level. For the viewer A , the angle of elevation of " D "is double the angle of elevation of "C" .The angle of elevation of "E" is triple the angle of elevation of "C" for the same viewer .Look at the figure given and based on the above information answer the following .



Type equation here.

1. Measure of $< CAB =$
   (a) $\tan^{-1}(2)$(b)$\tan^{-1}(\frac{1}{2})$ (c) $\tan^{-1}(1)$(d) $\tan^{-1}(3)$
2. Measure of <DAB =
   (a) $\tan^{-1}(\frac{3}{4})$(b)$\tan^{-1}(3)$ (c) $\tan^{-1}(\frac{4}{3})$(d)$\tan^{-1} 4$
3. Measure of $< EAB =$
   (a) $\tan^{-1}(11)$(b) $\tan^{-1}(3)$(c) $\tan^{-1}(\frac{2}{11})$(d) $\tan^{-1}(\frac{11}{2})$
4. If A' is another viewer standing on the same line of observation across the road . If the width of the road is 5 metres , then the < CA' B is
   (a)$\tan^{-1}(\frac{1}{2})$ b) $\tan^{-1}(\frac{1}{8})$ (c)$\tan^{-1}(\frac{2}{5})$ (d)$\tan^{-1}(\frac{11}{21})$
5. Also find the difference between $< CA\,B\ and$ <C A'B
   (a) $\tan^{-1}\left(\frac{1}{12}\right)(b)\tan^{-1}\left(\frac{1}{8}\right)(c)\tan^{-1}\left(\frac{2}{5}\right)(d)\tan^{-1}(\frac{11}{21})$

31. .

A group of students of class XII visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Raj path (formerly called the Kingsway), is about 138 feet (42 metrs) in height.



1.What is the angle of elevation if they are standing at a distance of 42 m away from the monument ?

(a) $\tan^{-1} 1$

(b) $\sin^{-1} 1$

(c)$\cos^{-1} 1$

(d)$\sec^{-1} 1$

2.They want to see the tower at an angle of $\sec^{-1} 2$. So , they want to know the distance where they want to stand and hence find the distance .

(a)42 m (b) 20.12 m (c) 24.24 m (d) 24.64 m

3.If the altitude of the sun is at $\cos^{-1}(\frac{1}{2})$, then the height of the vertical tower that will cast a shadow of length 20 m is

(a) $20\sqrt{3}$ (b) $\frac{20}{\sqrt{3}}$ (c) $16\sqrt{3}$ (d)$\frac{16}{\sqrt{3}}$

4. The ratio of the length of a rod and its shadow is 1:2 . The angle of elevation of the sun is

(a) $\sin^{-1}\frac{1}{2}$(b) $\cos^{-1}\frac{1}{2}$(c)$\tan^{-1}\frac{1}{2}$ (d)$\cot^{-1}\frac{1}{2}$

5. Domain of $\sin^{-1} x$ is …….

.(a)) (-1,1) (b) {-1, 1} (c) [-1, 1] (d) R

32.

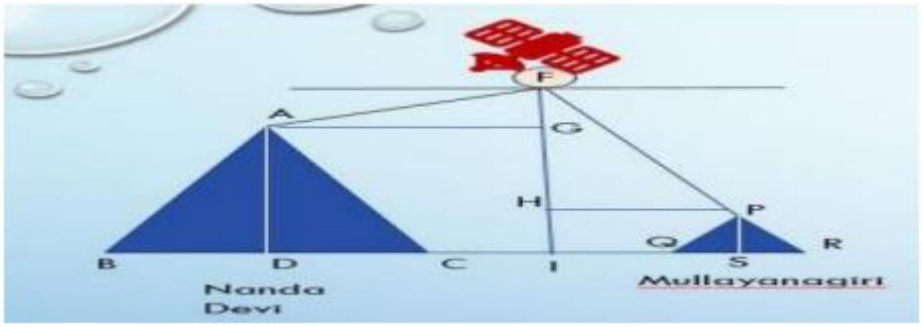


In the figure the angles of depression of the top and bottom of an 8 m tall building from the top of a multi storeyed building is $\tan^{-1}\frac{1}{\sqrt{3}}$ *and* $\sec^{-1}\sqrt{2}$ respectively

1. The height of the multi-storeyed building is
   (a) $4(1+\sqrt{3})$ m (b) 3(3+$\sqrt{3}$) m (c) 4(4+$\sqrt{3}$)m (d) 4(3+3$\sqrt{3}$)m
2. The distance between two building
   (a) 4(13+ $\sqrt{3}$) m (b)4(31+ $\sqrt{3}$) m (c) 2(3+ $\sqrt{3}$) m (d) 4(3+ $\sqrt{3}$)m
3. The value of $\tan^{-1}(1/\sqrt{3})$ is
   (a) $\sin^{-1}\left(\frac{1}{2}\right)$ $(b)\cos^{-1}\left(\frac{1}{2}\right)(c)\cos^{-1}(1/\sqrt{2})$ (d) $\sin^{-1}(\frac{\sqrt{3}}{2})$
4. The value of $\sec^{-1}\sqrt{2}$ is
   (a) $\sin^{-1}\left(\frac{1}{2}\right)$ $(b)\cos^{-1}\left(\frac{1}{2}\right)$ $(c)\cos^{-1}1$ (d) $\tan^{-1}1$
5. The range of $\cos^{-1}x$
   (a) $(0,\pi)$ (b)$[0,\pi]$ (c) $\{0,\pi\}$(d) $(0,\pi]$

33.

A satellite flying at height h is watching the top of the two tallest mountains in Uttarakhand and Karnataka , as
Nanda Devi (height 7816 metres) and Mullayanagiri (height1937 metres). The angles of depression from the satellite , to the top of Nanda Devi and Mullayanagiri are $\cot^{-1}\sqrt{3}$ and $\tan^{-1}\sqrt{3}$ respectively . If the distance between the peaks of the two mountains is 1937 km and the satellite is vertically above the midpoint of the distance between the two mountains . Look at the Figure given below and answer the Questions .



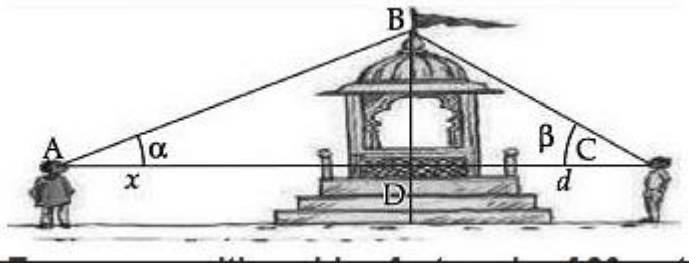
1.The distance of the satellite from the top of Nanda Devi hill is
(a)1139.4 km (b) 577.52km (c) 1937 km (d) 1025. 36 km

2. The distance of the satellite from the top of Mullayangiri is
(a) 1139.4 km (b) 577.52 km (c) 1937 km (d) 1025.36 km

3. The distance of the satellite from the ground is
(a) 1139.4 km (b) 577.52 km (c) 1937 km (d) 1025. 36 km

4 . What is the angle of elevation of the top of Nanda Devi if a man is standing at a distance of 7816 metre from Nanda Devi
(a) $\sec^{-1}(2)$ $(b)\cot^{-1}(1)(c)\sin^{-1}(\sqrt{3}/2)$ (d) $\cos^{-1}(\frac{1}{2})$

34.



Two men on either side of a temple of 30 m high observe its top at an angle of elevation α and β respectively . (as in figure) . The distance between the two men is $40\sqrt{3}$ metres and distance between the first person A and the temple is 30 $\sqrt{3}$ metre .

1. Find α

   (a)$\sin^{-1}(\frac{\sqrt{3}}{2}$)(b) $\sin^{-1}(\frac{1}{2})$ (c) $\sin^{-1}(1/\sqrt{2}$) (d)$\sin^{-1}(1)$

2. Find β

   (a) $\sin^{-1}(\frac{\sqrt{3}}{2})$ (b)$\sin^{-1}(\frac{1}{2})$ (c) $\sin^{-1}(1/\sqrt{2}$) (d)$\sin^{-1}(1)$

3. If A is moving towards the temple and the distance between A and temple is 10 $\sqrt{3}$ m find the angle of elevation which A makes

   (a) $\sin^{-1}(\frac{\sqrt{3}}{2})$ (b)$\sin^{-1}(\frac{1}{2})$ (c)$\sin^{-1}\frac{1}{\sqrt{2}}$ $(d)\sin^{-1}(1)$

4 . Then what is the change in angle α

(a) $\sin^{-1}(\frac{\sqrt{3}}{2})$ (b) $\sin^{-1}\left(\frac{1}{2}\right)$ (c) $\sin^{-1}(1/\sqrt{2}$)$\sin^{-1}(1)$

ANSWERS

1. (b) $3\cos^{-1}x$
2. (d) none of these
3. (c) $\frac{2\pi}{3}$
4. (c) $\frac{5\pi}{6}$
5. (c)$-\frac{\pi}{6}$
6. (d)$\frac{x}{\sqrt{1+x^2}}$
7. $\frac{5\pi}{6}$
8. $\frac{x}{2}$
9. $\frac{\pi}{4}+\frac{x}{2}$, $-\frac{\pi}{2}<\frac{\pi}{4}+\frac{x}{2}<\frac{\pi}{2}$
10. $\frac{\pi}{4}-\frac{x}{2}$, $where$ $-\frac{\pi}{2}\leq x\leq\frac{\pi}{2}$, so $0\leq\frac{\pi}{4}-\frac{x}{2}\leq\frac{\pi}{2}$
11. $\frac{\pi}{3}$
12. $x+\frac{\pi}{4}$
13. 1
14. $\frac{11\,\pi}{12}$
15. $\frac{\pi}{3}$

16. $\sec^{-1} x$
17. $\frac{\pi}{4} - x$
18. $\sin^{-1}(\frac{x}{a})$
19. $\frac{1}{2}\cos^{-1}(\frac{x}{a})$
20. $\tan^{-1}(\frac{x}{a})$
21. $\frac{x}{2}$
22. $\frac{x}{2}$

30 Case study Questions answers

1. (b) $\tan^{-1}(\frac{1}{2})$

2. (c) $\tan^{-1}(\frac{4}{3})$

3. $\tan^{-1}(\frac{11}{2})$

4. $\tan^{-1}(\frac{2}{5})$.

5.$\tan^{-1}(\frac{1}{12})$

31 Case study

1. (a) $\tan^{-1}(1)$, 2. (c) 24.24 m , 3. (a) $20\sqrt{3}$, 4. (c) $\tan^{-1}(\frac{1}{2})$ 5(c)[-1,1]

32 Case study

1. (a) $4(1+\sqrt{3})$ m , 2. (d) $4(3+\sqrt{3})$m , 3. (a) $\sin^{-1}(\frac{1}{2})$, 4. (d) $\tan^{-1}(1)$,5. (b) $[0, \pi]$

33 Case study

1.(a)1139.4 km , 2. (c) 1937 km 3 . (b) 577. 52 km 4 . (b)$\cot^{-1}(1)$

34 Case study

1. (b)$\sin^{-1}(\frac{1}{2})$ 2. (a) $\sin^{-1}(\frac{\sqrt{3}}{2})$ 3. (a)$\sin^{-1}(\frac{\sqrt{3}}{2})$ 4. (b)$\sin^{-1}(\frac{1}{2})$

CHAPTER 3

MATRICES

CONCEPT MAPPING

- Definition of a matrix
- Order of a Matrix
- Construction of a Matrix of given order
- TYPES OF MATRICES

  i) column matrix ii) row matrix iii) square matrix iv) diagonal matrix v) scalar matrix vi) identity matrix vii) zero or null matrix

- MATRIX OPERATIONS

  i) Addition

  ii)Subtraction

  iii)Scalar multiplication

  iv)Multiplication of matrices

- TRANSPOSE OF A MATRIX
- SYMMETRIC &SKEW –SYMMETRIC MATRICES
- INVERTIBLE MATRIX

GIST OF THE LESSON

MATRIX DEFINITION-

A matrix is an ordered rectangular array of numbers or functions

ROW MATRIX-

A matrix having only one row is called a row matrix

COLUMN MATRIX-

A matrix having only one column is called column matrix

SQUARE MATRIX

A matrix in which the number of rows is equal to the number of columns is called a square matrix. An $m \times n$ matrix is a square matrix if $m=n$

DIAGONAL MATRIX

A square matrix $B = [bij]_{mxm}$ is a diagonal matrix if all its non- diagonal elements are zero, that is if $bij = 0$, when $i \neq j$.

SCALAR MATRIX

A diagonal matrix $B = [bij]_{mxm}$ is a scalar matrix if $bij = 0$, when $i \neq j$ and $bij = k$, when $i = j$, for some constant k

IDENTITY MATRIX

A square matrix $A = [aij]$ $n \times n$ is an identity matrix if $aij= 1$,when $i = j$ $aij =0$,when $i \neq j$

ZERO MATRIX

A matrix is said to be zero matrix or null matrix if all its elements are zer

EQUALITY OF TWO MATRICES

Two matrices $A = [aij]$ and $B = [bij]$ are said to be equal if (i) they are of the same order (ii) each element of A is equal to the corresponding element of B, that is $aij = bij$ for all i and j

ADDITION OF MATRICES

The sum of two matrices is a matrix obtained by adding the corresponding elements of the given matrices. If $A = [aij]$ and $B = [bij]$ are two matrices of the same order, say $m \times n$. Then, the sum of the two matrices A and B is defined as a matrix $C = [cij]$ $m \times n$, where $cij = aij + bij$, for all possible values of i and j.

DIFFERENCE OF TWO MATRICES-

If $A = [aij]$, $B = [bij]$ are two matrices of the same order, say $m \times n$, then difference $A – B$ is defined as a matrix $D = [dij]$, where $dij = aij – bij$, for all values of i and j

MULTIPLICATION OF A MATRIX BY A SCALAR

$A= [aij]$ $m \times n$ is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by the scalar k.

PROPERTIES OF MATRIX ADDITION

1) Commutative Law - If A = [aij], B = [bij] are matrices of the same order, say m × n, then A + B = B + A.
2)Associative law- For any three matrices A = [aij], B = [bij], C = [cij] of the same order, say m × n, (A + B) + C = A + (B + C)
3)Existence of additive identity- Let A = [aij] be an m × n matrix and O be an m × n zero matrix, then A + O = O + A = A. In other words, O is the additive identity for matrix addition.
4)The existence of additive inverse -Let A = [aij] m × n be any matrix, then we have another matrix as – A =- [aij] m × n such that A + (– A) = (– A) + A= O. Then -Ais the additive inverse of A or negative of A.

PROPERTIES OF SCALAR MULTIPLICATION OF A MATRX

If A = [aij] and B = [bij] be two matrices of the same order, say m × n, and k and l are scalars ,then (i)k(A +B) = k A + kB, (ii) (k + l)A = k A + l A

MULTIPLICATION OF TWO MATRICES

The product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B. Let A = [aij] be an m × n matrix and B = [bjk] be an n × p matrix. Then the product of the matrices A and B is the matrix C of order m × p. To get the $(i, k)^{th}$ element c_{ik} of the matrix C, we take the i th row of A and kth column of B, multiply them elementwise and take the sum of all these products.

Non-commutativity of multiplication of matrices

Even if AB and BA are both defined, it is not necessary that AB = BA

PROPERTIES OF MULTIPLICATION OF MATRICES

1) The associative law : For any three matrices A, B and C. We have (AB) C = A (BC), whenever both sides of the equality are defined.
2) The existence of multiplicative identity: For every square matrix A, there exist an identity matrix of same order such that IA = AI = A.
3) The distributive law : For three matrices A, B and C. A (B+C) = AB + AC (ii) (A+B) C = AC + BC

TRANSPOSE OF A MATRIX

If A = [aij] be an m × n matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. Transpose of the matrix A is denoted by A' or A^T.

KO

For any matrices A and B of suitable orders,

(i) (A')' = A

(ii) (A + B)' = A' + B'

iii) (A - B)' = A' - B'

(iv) (A B)' = B' A'

SYMMETRIC AND SKEW SYMMETRIC MATRICES

A square matrix A = [aij] is said to be symmetric if A' = A, that is, [aij] = [aji] for all possible values of i and j.

A square matrix A = [aij] is said to be skew symmetric matrix if A' = – A, that is aji = – aij for all possible values of i and j. Now, if we put i = j, we have aij = – aji. Therefore 2aii = 0 or aii = 0 for all i's. This means that all the diagonal elements of a skew symmetric matrix are zero.

For any square matrix A with real number entries, A + A' is a symmetric matrix and A – A' is a skew symmetric matrix

Any square matrix can be expressed as the sum of a symmetric and skew symmetric matrices

INVERTIBLE MATRICES

If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the inverse matrix of A and it is denoted by A^{-1}. In that case A is said to be invertible

MULTIPLE CHOICE QUESTIONS

Question NO	Question	
1	If order of matrix A is 2x3 and order of matrix B is 3x4 , find the order of AB. 1) 2x4 2) 2x2 3) 4x2 4) 3x3	
2	If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then A+A' is 1) skew –symmetric 2) symmetric 3) diagonal matrix 4) zero matrix	
3	The number of all possible matrices of order2x3 with entry 1 or 2 1)16 2) 64 3) 6 4) 24	

4	If a matrix A is both symmetric and skew –symmetric , then A is necessarily a 1) diagonal matrix 2) zero square matrix 3) square matrix 4) Identity matrix	
5	If A = [aij] is a square matrix of order 2 such that aij = $\begin{cases}1 \; when \; i \neq j \\ 0 when \; i = j\end{cases}$, then A^2 is: 1) $\begin{pmatrix}1 & 0\\1 & 0\end{pmatrix}$, 2) $\begin{pmatrix}1 & 1\\0 & 0\end{pmatrix}$, 3) $\begin{pmatrix}1 & 1\\1 & 0\end{pmatrix}$, 4) $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ **SHORT ANSWER TYPE QUESTIONS**	
6	Show that A A^I and A^I A are both symmetric matrices for any matrix A.	
7	Give an example of two non-zero 2×2 matrices A and B such that $AB=0$	
8	If the matrix $\begin{bmatrix}0 & a & 3\\2 & b & -1\\c & 1 & 0\end{bmatrix}$ is skew symmetric, find a + b + c	
9	If 2 A + 3 B = $\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}$ and 3 A + 2 B = $\begin{bmatrix}-2 & 2\\1 & -5\end{bmatrix}$, find A and B.	
10	Find the matrix A such that A $\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}$ = $\begin{bmatrix}0 & -3\\10 & 3\end{bmatrix}$.	
11	Let A = $\begin{bmatrix}3 & 2 & 5\\4 & 1 & 3\\0 & 6 & 7\end{bmatrix}$. Express A as a sum of symmetric and skew symmetric matrices.	
12	If A = $\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$, f(x) = x^2 -2x – 3 , show that f(A) = 0.	

13	If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find the matrix K so that $A^2 = 8A + K$	
14	If A and B are matrices of same order, then prove that $AB^{I} - BA^{I}$ is skew symmetric matrix.	
15	If A is a square matrix such that $A^2 = I$. Then prove that $(A - I)^3 + (A + I)^3 - 7A = A$	
16	**CASE STUDY1** The following matrix depicts a number of students of a school who were awarded for discipline, attendance and obedience. discipline attendance obedience $\begin{pmatrix} 18 & 12 & 20 \\ 10 & 18 & 12 \end{pmatrix} \begin{matrix} \leftarrow girls \\ \leftarrow boys \end{matrix}$ If the prize money for three values were respectively Rs 500, Rs200 & Rs300 . Based on the above data, answer the following questions	

1	Total award money received by boys a)10000 b)12000 c)12200 d)17400	
2	Total award money received by girls a)12000 b)12200 c)10000 d)17400	
3	Total award money received by the students only for discipline a)14000 b)24000 c)17400 d)12200	
4	How much amount received by the girls for attendance? a)2400 b) 2600 c)2800 d)2000	
5	Who received more amount, boys or girls and how much? a)Girls: 1200 b)Boys:5200 c)Girls:5200 d)Boys: 1200	

17	A manufacturer produces three types of bolts, x, y and z which he sells in two markets. Annual sales (in Rs) are indicated below:		

Markets	Products		
	x	y	z
I	10000	2000	18000
II	6000	20000	8000

If unit sales prices of x, y and z are Rs.2.50, Rs.1.50 and Rs.1.00 respectively, then answer the following questions using the concept of matrices.

1	Find the total revenue collected from the Market-I. (a) Rs.44000 (b) Rs. 48000 (c) Rs.46000 (d) Rs.53000	
2	Find the total revenue collected from the Market-II. (a) Rs.5100 (b) Rs.53000 (c) Rs.46000 (d) Rs 49000	
3	If the unit costs of the above three commodities are Rs.2.00, Rs.1.00 and 50 paise respectively, then find the gross profit from both the markets. (a) Rs.53000 (b) Rs.46000 (c) Rs.34000 (d) Rs.32000	
4	If matrix A=[aij]2×2 , where aij=1, if i≠j , and aij= 0 if i=j , then A^2 is equal to (a) I (b) A (c) 0 (d) none of these	
5	If A and B are matrices of same order, then (AB' - BA') is a (a) skew-symmetric matrix (b) null matrix (c) symmetric matrix (d) unit matrix	

18 Three car dealers, say A, Band C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan,5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan,5 SUV cars in 2020.

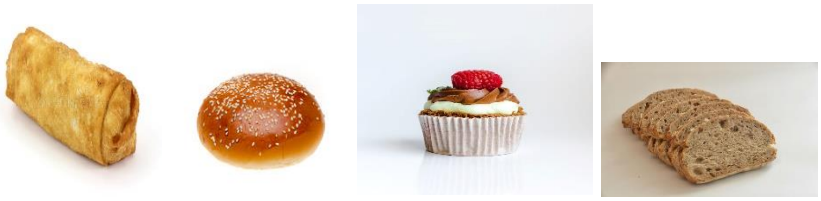


Based on the above information, answer the following questions.

	1	The matrix summarising sales data of 2019 is a) $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 30 & 5 \end{bmatrix}$ b) $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix}$ **c)** $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 100 & 30 & 5 \\ 120 & 50 & 10 \\ 90 & 40 & 2 \end{bmatrix}$ *d)* $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 200 & 50 & 6 \\ 100 & 30 & 5 \\ 300 & 150 & 20 \end{bmatrix}$	
	2	The matrix summarizing sales data of 2020 is a) $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}$ b) $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 120 & 50 & 10 \\ 100 & 60 & 5 \\ 90 & 40 & 2 \end{bmatrix}$ **c)** $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 100 & 60 & 5 \\ 120 & 50 & 10 \\ 90 & 40 & 2 \end{bmatrix}$ *d)* $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 200 & 50 & 6 \\ 100 & 60 & 5 \\ 300 & 150 & 20 \end{bmatrix}$	
	3	The total number of cars sold in two given years, by each dealer, is given by the matrix a) $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 190 & 100 & 7 \\ 300 & 80 & 11 \\ 420 & 200 & 30 \end{bmatrix}$ b) $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 300 & 80 & 11 \\ 190 & 100 & 7 \\ 420 & 200 & 30 \end{bmatrix}$ **c)** $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 420 & 200 & 30 \\ 300 & 80 & 11 \\ 190 & 100 & 7 \end{bmatrix}$ *d) None of these*	
	4	The increase in sales from 2019 to 2020 is given by the matrix a) $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 180 & 100 & 10 \\ 10 & 20 & 1 \\ 100 & 20 & 3 \end{bmatrix}$ b) $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 10 & 20 & 3 \\ 100 & 20 & 1 \\ 180 & 100 & 10 \end{bmatrix}$ **c)** $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 180 & 100 & 10 \\ 100 & 20 & 1 \\ 10 & 20 & 3 \end{bmatrix}$ *d)* $\begin{matrix} & H & S & SUV \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 100 & 20 & 3 \\ 180 & 100 & 10 \\ 10 & 20 & 3 \end{bmatrix}$	**1**

	5	If each dealer receive profit of Rs.50000 on sale of a Hatchback, Rs.100000 on sale of a Sedan and Rs. 200000 on sale of a SUV (then amount of profit received in the year 2020 by each dealer is given by the matrix a) $\begin{matrix}A\\B\\C\end{matrix}\begin{bmatrix}30000000\\15000000\\12000000\end{bmatrix}$ b) $\begin{matrix}A\\B\\C\end{matrix}\begin{bmatrix}12000000\\16200000\\34000000\end{bmatrix}$ *c)* $\begin{matrix}A\\B\\C\end{matrix}\begin{bmatrix}34000000\\16200000\\12000000\end{bmatrix}$ *d)* $\begin{matrix}A\\B\\C\end{matrix}\begin{bmatrix}15000000\\30000000\\12000000\end{bmatrix}$	**1**

19 Three people denoted by P1, P2, P3 intend to buy some rolls, buns, cakes and bread. Each of them needs these commodities in differing amounts and can buy them in two shops S1, S2. The individual prices and desired quantities of the commodities are given in the following tables:



Demanded quantity of foodstuff: shops $S1$ and $S2$:

	roll	bun	cake	bread
P_1	6	5	3	1
P_2	3	6	2	2
P_3	3	4	3	1

Prices in

	S_1	**S_2**
roll	**1.50**	**1.00**
bun	**2.00**	**2.50**
cake	**5.00**	**4.50**
bread	**16.00**	**17.00**

1	The amount spent by the person P1 in the shop S1 is **a) 55** **b) 50** **c) 60** **d) 62.5**	
2	The amount spent by person P1 in the shop S2 is **a) 49** **b) 45** **c) 48** **d) 51**	
3	P2 and P3 are friends and decide to buy all the items from the shop S2. How much will it cost them?	

		a) 53.5 b) 82 c) 104.5 d) None of the above	
	4	If the cost of production for S2 to give desired quantities of foodstuff is 113.5. Find the profit earned by S2 **a) 38.5 b) 41.5 c) 40 d) None of the above**	

20 Three schools A, B and C organized an event for collecting funds for helping the COVID effected victims. They sold toys, handmade boxes and notebook packets from the recycled material at a cost of 50, 30 and 40 each. The number of items sold is given below

ITEMS	A	B	C
TOYS	20	30	40
BOXES	50	40	50
NOTEBOOKS	40	20	30





Based on the above information answer the following

Q1. Amount collected by school A is
a) 3500 b) 4100 c) 4500 d) 5100
Q2. Amount collected by school C is
a) 5700 b) 6000 c) 3500 d) 4700
Q3. The total amount raised by all the schools is

a) 12100 b) 12200 c) 12300 d) 12400
Q4. Total amount raised by toys is
a) 4000 b) 4500 c) 5000 d) 5500

21 For what value of x: $\begin{pmatrix}1 & 2 & 1\end{pmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix} = 0$

22 If A = $\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$, Prove that A+ A^T is symmetric and A – A^T is skew symmetric matrices

23	If f(x) = $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then show that f(x) f(y) = f (x +y).	
24	If A= $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2+7A+2I =0$	
25	Find x, y, z if A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A^T = A^{-1}$	
26	If A = $\begin{bmatrix} 0 & -\tan\frac{\propto}{2} \\ \tan\frac{\propto}{2} & 0 \end{bmatrix}$, and I is the identity matrix of order 2, show that I + A = (I – A) $\begin{bmatrix} cos \propto & -sin \propto \\ sin \propto & cos \propto \end{bmatrix}$	
27	Find the value of a,b,c and d from the equation $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$	
28	If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ A $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, Find A	
29	Let A = $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$,C= $\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find a matrix D such that CD – AB = 0	
30	If A = $\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find A^2 -5A – 14 I . Hence, obtain A^3	

ANSWERS

QUESTION NO.	CORRECT OPTION
1	1
2	2
3	2
4	2
5	4

6	Prove that (A A^I)^I = A A^I and (A^I A)^I = A^I A	

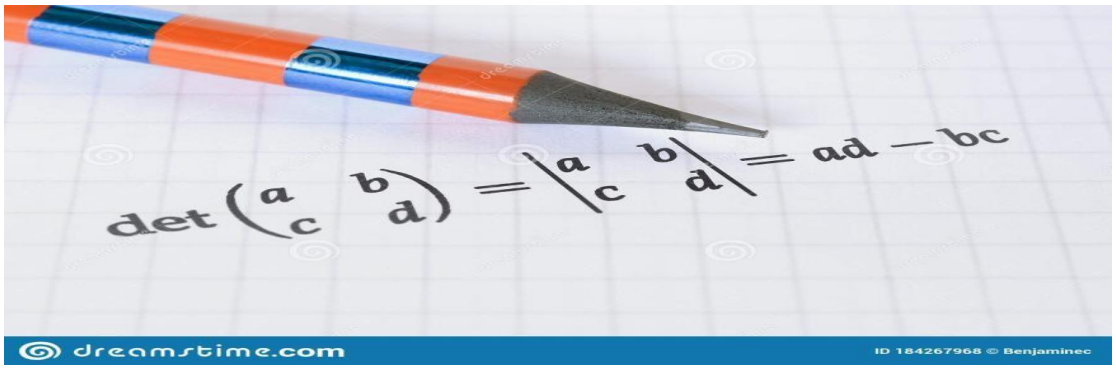
7	$\begin{bmatrix}1 & 0\\2 & 0\end{bmatrix}\begin{bmatrix}0 & 0\\-1 & 3\end{bmatrix}$	
8	A = -2, b = 0 , c = -3 a+b+c = -5	
9	A = $\begin{bmatrix}-2 & 0\\-1 & -3\end{bmatrix}$, B = $\begin{bmatrix}2 & 1\\2 & 2\end{bmatrix}$	
10	Let A = $\begin{bmatrix}a & b\\c & d\end{bmatrix}$ 2a+4b=0, 3a+5b=-3, 2c+4d=10, 3c+5d=3 A = $\begin{bmatrix}-6 & 3\\-19 & 12\end{bmatrix}$	
11	A = $\begin{bmatrix}3 & 3 & \frac{5}{2}\\3 & 1 & \frac{9}{2}\\\frac{5}{2} & \frac{9}{2} & 7\end{bmatrix}$ + $\begin{bmatrix}0 & -1 & \frac{5}{2}\\1 & 0 & -\frac{3}{2}\\-\frac{5}{2} & \frac{3}{2} & 0\end{bmatrix}$	
12	f(A) =A^2 -2A-3I = $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$	
13	K= $\begin{bmatrix}-7 & 0\\0 & -7\end{bmatrix}$	
14	Prove that $(AB^I - BA^I)^I = -(AB^I - BA^I)$, Using Properties	
15	$(A-I)^3 = A^3 -3A^2 I +3AI^2 -I^3$ $(A+I)^3 = A^3 +3A^2 I +3AI^2 +I^3$	

	Q1	Q2	Q3	Q4	Q5
16	c	d	a	a	c
17	c	b	d	a	a
18	b	a	c	c	c
19	b	a	c	c	
20	b	d	c	b	

21	**X = -1**	

22	Let P = A + A^T = $\begin{bmatrix}4 & 7\\ 1 & 10\end{bmatrix}$, and Q = A - A^T = $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$, prove that P^T =P and Q^T = -Q	
23	F (x + y) = $\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$	
24	**A^2=**$\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}$, A^3==$\begin{bmatrix}21 & 0 & 34\\ 12 & 8 & 23\\ 34 & 0 & 55\end{bmatrix}$	
25	AA^T =I $4y^2+z^2$ =1, $2y^2- z^2$ =0, $x^2+y^2+z^2$ =1 **X=**$\pm\frac{1}{\sqrt{2}}$**, y = =**$\pm\frac{1}{\sqrt{6}}$**, z = =**$\pm\frac{1}{\sqrt{3}}$**,**	
26	**I + A =** $\begin{bmatrix}1 & -\tan\frac{\propto}{2}\\ \tan\frac{\propto}{2} & 1\end{bmatrix}$**, I - A =** $\begin{bmatrix}1 & \tan\frac{\propto}{2}\\ -\tan\frac{\propto}{2} & 1\end{bmatrix}$,	
27	a=1 , b = 2, c = 3, d =4	
28	**A =** $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$	
29	**D =** $\begin{bmatrix}-191 & -110\\ 77 & 44\end{bmatrix}$	
30	**A^2 -5A -14I =** $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ A^3 = $\begin{bmatrix}187 & -195\\ -156 & 148\end{bmatrix}$	

CHAPTER 4
DETERMINANTS



CONCEPT MAPPING

DETERMINANT OF A SQUARE MATRIX

MINOR AND CO-FACTOR

AREA OF TRIANGLE

ADJOINT OF A MATRIX

INVERSE OF A MATRIX

SOLUTION OF EQUATION BY MATRIX METHOD

A non-singular square matrix of order n is invertible, if there exists a square matrix B of the same order such that $AB = I_n = BA$. Inverse of A is B and we write $A^{-1} = B$

Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} is denoted by M_{ij}

Minors

Inverse of a matrix

Adjoint's relation with Inverse

$$A^{-1} = \frac{1}{|A|} .\text{adj } A$$

Finding inverse of a matrix

cofactor

Use the minors and apply the below to get Cofactor :
$C_{ij} = (-1)^{(i+j)} * M_{ij}$
M_{ij} is the minor in the i^{th} row and j^{th} column.

Properties of inverse

1. $(A^{-1})^{-1} = A$
2. $(A^T)^{-1} = (A^{-1})^T$
3. $(AB)^{-1} = B^{-1}A^{-1}$
4. $(A^k)^{-1} = (A^{-1})k$, $k \in N$ (in particular $(A^2)^{-1} = (A^{-1})^2$)
5. $\text{adj}(A^{-1}) = (\text{adj } A)^{-1}$

$$|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$$

6. $A = \text{diagonal}(a_1 a_2 \ldots a_n) \Rightarrow A^{-1} = \text{diagonal}(a_1^{-1}\, a_2^{-1} \ldots a_n^{-1})$
7. A is symmetric $\Rightarrow A^{-1}$ is also symmetric.
8. A is diagonal, $|A| \neq 0 \Rightarrow A^{-1}$ is also diagonal.
9. A is a scalar matrix $\Rightarrow A^{-1}$ is also a scalar matrix.
10. A is triangular, $|A| \neq 0 \Rightarrow A^{-1}$ is also triangular.
11. Every invertible matrix possesses a unique inverse.

Adjoint's relation with cofactor

Adj A is the transpose of the cofactor matrix

GIST OF THE LESSON

To every square matrix we can assign a number called determinant

Value of determinant of a matrix of order 1

If A= [aij]. Then Det .A= IAI= $|aij|$ = aij

Value of determinant of a matrix of order 2

If A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then IAI = $a_{11}a_{22}$ - $a_{21}a_{12}$

Value of determinant of a matrix of order 3

. If A= $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then IAI = $a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$

If A= [aij]. Is a square matrix of order n ,then IAI =$|A^T|$

If A and B are square matrix of order n, then$|AB|=|A||B|$

Let A be a square matrix of order n × n, then | kA| is equal to k^n IAI

The area of a triangle whose vertices are (x_1 , y_1), (x_2 , y_2) and (x_3 , y_3), is

$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

The area of the triangle formed by three collinear points is zero.

Equation of line joining the points (x_1 , y_1) and (x_2 , y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=0$

Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij}.

Cofactor of an element a_{ij} ,denoted by A_{ij} is defined by $A_{ij} = (-1)^{i+j} M_{ij}$,where M_{ij} is minor of a_{ij}

Δ = a11A11+a12A12+a13A13.where Aij are cofactors of aij.

If elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero

$\Delta = a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$

The adjoint of a square matrix A = $[a_{ij}]$.n × n is defined as the transpose of the matrix $[A_{ij}]_{n \times n}$, where A_{ij} is the cofactor of the element a_{ij} . Adjoint of the matrix A is denoted by adj A.

If A be a square matrix of order n, then $|adjA|=|A|^{n-1}$.

If A be a square matrix of order n, then $|A\ adjA|=|A|^n$.

For any square matrix A, A(Adj.A)=(Adj.A)A=$|A|$I

If A be a square matrix of order n, then $|adj(adjA)|=|A|^{(n-1)^2}$

If A and B are square matrices of the same order, then adj(AB)=adjB.adj A

If A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then adj.A = $\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$ Note :to find adjA interchange diagonal elements and change the sign of non – diagonal elements.

If A is a singular matrix, then $|A|=0$

A square matrix A is said to be non-singular if $|A|\neq 0$

If A and B are nonsingular matrices of the same order, then AB and BA are also nonsingular matrices of the same order

A square matrix A is invertible if and only if A is nonsingular matrix ($|A|\neq 0$)

$A^{-1} = \frac{1}{|A|}(adjA)$

If A is an invertible matrix, then $|A|\neq 0$ and $(A^{-1})^T=(A^T)^{-1}$

If A is a non-singular matrix $|(kA)^{-1}| = \frac{1}{k\ |A|}$

$(AB)^{-1} = B^{-1}A^{-1}$

<u>Solution of system of linear equations using inverse of a matrix</u>

Consider the system of equations

$a_1 x + b_1 y + c_1 z = d_1$

$a_2 x + b_2 y + c_2 z = d_2$

$a_3 x + b_3 y + c_3 z = d_3$

$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$ AX = B, X = A^{-1} B

Where $A^{-1} = \frac{1}{|A|}(adjA)$

MULTIPLE CHOICE QUESTIONS

1	The area of a triangle with vertices (2 , - 6),(5 , 4)and(K , 4) is 35 sq. units then k is a)12 b) – 2 c) -12 , -2 d)12 , - 2

2	If A= $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is a singular matrix then value of b is A)-3 b) 3 c) 0 d) arbitrary
3	**Themaximum valueof** $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\sin\theta \end{vmatrix}$ A) -2 b) 1 c) -1 d) 2
4	Let A is a non – singular matrix of order 3 x 3 then $\lvert A\ (adj\ A)\rvert$ is equal to A)$\lvert A \rvert$ b) $\lvert A \rvert^2$ c) $\lvert A \rvert^3$ d) 3$\lvert A \rvert$
5	If A and B are invertible matrices of order 3,$\lvert A \rvert$=2 and $\lvert(AB)^{-1}\rvert$= - 1/ 6 , Find $\lvert B\rvert$ A) – 1/3 b) 3 c) -1/12 d) -3
SHORT ANSWER QUESTIONS	
6.	If A = $\begin{bmatrix} 2 & \beta & -4 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$ find value of β for which A^{-1} exist ?
7	If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, find the value of a_{32} A_{32}
8.	If A is a square matrix of order 3 x3 with $\lvert A\rvert = 9$, then write the value of $\lvert 2\,.\,adjA\rvert$
9.	For the matrix A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$,find the numbers a and b such that A^2 + a A + b I = **O**
10	There are two values of a which makes determinant, $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$ Then find sum of these numbers.
11	If A = $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, find the value of $\lvert A^2 - 2A \rvert$
12	Find k if the matrix $\begin{bmatrix} 1 & k & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3x 3 matrix A and $\lvert A\rvert = 4$
13	$\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix}$, then find x.

14	Show that if the determinant $\begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \cos 2\theta \\ -11 & 14 & 2 \end{vmatrix} = 0$ then $\sin\theta = 0$ or $\sin\theta = 1/2$.
15	Evaluate $\begin{vmatrix} \cos 15^0 & \sin 15^0 \\ \sin 75^0 & \cos 75^0 \end{vmatrix}$
16	If A = [a_{ij}] is a matrix of order 2 x2 , such that $\lvert A\rvert = -15$ and A_{ij} represents the cofactor of a_{ij} then find $a_{21}A_{21} + a_{22}A_{22}$
17	If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$ then find the value of x
18	If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ then find the value of k if $\lvert 2A\rvert = k \lvert A\rvert$
19	Show that the points (a, b+c) , (b , c+a) , and (c, a+b) are collinear
20	Find the interval in which Det (A) lies if $A = = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$
	LONG ANSWER QUESTIONS
21	If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ find x and y such that $A^2 + x I = y A$. Hence find A^{-1}
22	If $A = \begin{bmatrix} 12 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A^T)^{-1}$
23	Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = \mathbf{O}$ where I is 2 x 2 identity matrix and **O** is 2 x 2 zero matrix. Using this equation , find A^{-1}.
24	Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ compute A^{-1} and show that $2A^{-1} = 9I - A$
25	If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$
26	If x = – 4 is a root of $\Delta = \begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$ then find the other two roots
27	Let f (t) = $\begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$ then find $\lim_{t \to 0} \frac{f(t)}{t^2}$
28	Find the equation of the line joining A (1 , 3) and B (0, 0) using determinants and find k if D (k , 0) is a point such that area of triangle ABD is 3 sq. units.
29	If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, verify A (adj A) = $\lvert A\rvert$ I and find A^{-1}

30	Use the product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations $X - y + 2z = 1$, $2y - 3z = 1$, $3x - 2y + 4z = 2$
31	Find x , y and z if $A = \begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ satisfies $A^T = A^{-1}$
32	A typist charges Rs 145 for typing 10 English and 3 Hindi pages , while charges for typing 3 English and 10 Hindi pages are Rs 180 .Using matrices , find the charges of typing one English and one Hindi page separately.
33	If $A^{-1} = \begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$ and $B = \begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$ find $(AB)^{-1}$
34	Find A^{-1} , if $A = \begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$. Also show that $A^{-1} = \frac{A^2 - 3I}{2}$
35	The monthly income of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5:7 . if each saves rs 15,000 per month , find their monthly incomes using matrix method
	CASE STUDY QUESTIONS
36	Two schools A and B decided to award prizes to their students for three values honesty (x) ,punctuality (y) and obedience (z) .School A decided to award a total of Rs 11000 for the three values to 5 , 4 and 3 students respectively while school B decided to award Rs 10700 for the three values to 4 , 3 and 5 students respectively . If all the three prizes together amount to Rs 2700 , based on the information given answer the following questions 1. Form the equations in terms of x , y and z 2. Is it possible to solve the system of equations using matrix method 3. Find award prize for each of the three values
37	A mixture is to be made of three foods A , B , C . The three foods A ,B, C contain nutrients P , Q , R shown below .

Food	Grams per kg of nutrient		
	P	Q	R
A	1	2	5
B	3	1	1
C	4	2	1

1. Form linear equation representing the data if the mixture will have 8 kg of P, 5 kg of Q and 7 kg of R
2. How to form a mixture which will have 8 kg of P, 5 kg of Q and 7 kg of R.

38	Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50m, then its area will remain same, but if length is decreased by 10m and breadth is decreased by 20m, then its area will decrease by 5300 m^2 1. Based on the information given above, form equations in terms of x and y 2. Write down matrix equation represented by the given information 3. Find the value of x (length of rectangular field) 4. Find the value of y (breadth of rectangular field) 5. How much is the area of rectangular field?
39	Three shopkeepers Salim, Vijay and Venket are using polythene bags, handmade bags (prepared by prisoners) and newspaper's envelope as carry bags. It is found that the shopkeepers Salim, Vijay and Venket are using (20,30,40), (30,40,20) and (40, 20, 30) polythene bags, handmade bags and newspaper's envelopes respectively. The shopkeepers Salim, Vijay and Venket spent Rs.250, Rs.270 and Rs.200 on these carry bags respectively.    Using the concept of matrices and determinants, answer the following questions. (i) What is the cost of one polythene bag? **(a) Rs.1 (b) Rs.2 (c) Rs.3 (d) Rs.5** (ii) What is the cost of one handmade bag? **(a) Rs.1 (b) Rs.2 (c) Rs.3 (d) Rs. 5** (iii) What is the cost of one newspaper envelope **(a) Rs.1 (b) Rs.2 (c) Rs.3 (d) Rs. 5** (iv) Keeping in mind the social conditions, which shopkeeper is better? **(a) Salim (b) Vijay (c) Venket (d) None of these** (v) Keeping in mind the environmental conditions, which shopkeeper is better?

	(a) Salim (b) Vijay (c) Venket (d) None of these
40	**A trust invested some money in two types of bonds . The first bond pays 10 % interest and second bond pays 12 % interest .The trust received Rs 2800as interest . However , if trust had interchanged money in bonds ,they would have got Rs 100 less as interest .** **1. Write down linear equations based on information given above** **2. Write down matrix equation represented by the given information** **3.Find the amount invested in first bond** **4. Find the amount invested in first bond** **5. Find the total amount invested by the trust**

ANSWERS

MULTIPLE CHOICE QUESTIONS

1) d)12 , - 2	2) d) arbitrary	3) b) 1	4) c) $\lvert A \rvert^3$	5) d) -3

SHORT ANSWER QUESTIONS

6) $\beta \neq -2$	7) 110	8) 8 x 9^2 = 648	9) a = -4 , b = 1

10) $\Delta = 86$

$\rightarrow a^2 + 4a - 21 = 0$

$a = -7, 3$

Sum = -4

11) 25

12) $\lvert adj\ A \rvert = \lvert A \rvert^{n-1}$

$2k - 6 = 4^2$

K = 11

13) x = ± 3

14) $2 - 2\cos 2\theta - \sin 3\theta = 0$

$2 - 2(1 - 2\sin^2\theta) - (3\sin\theta - 4\sin^3\theta) = 0$

$\sin\theta\,(4\sin^2\theta + 4\sin\theta - 3) = 0$

$\sin\theta(2\sin\theta - 1)(2\sin\theta + 3) = 0$

$\sin\theta = 0$ or $\sin\theta = 1/2$

15) $\cos 90^0 = 0$

16) $\lvert A \rvert = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

Expanding along R_2

$a_{21}\,A_{21} + a_{22}\,A_{22} = -15$

17) x = 2

18) k = 4

19) Given points are collinear as $\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = 0$

20) Det (A) ∈ [2 ,4]

LONG ANSWER QUESTIONS

21) x = 8 , y = 8 ,

$A^{-1} = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

22) $|A^T| = 1$

$(A^T)^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

23) $A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

24) $|A| = 2$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$9I - A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$= 2A^{-1}$

25) $(AB)^{-1} = B^{-1}A^{-1} = \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$

26) $\Delta = 0$

$\rightarrow x^3 - 13x + 12 = 0$

$x = 1$ and $x = 3$ are other two roots.

27) $f(t) = -t^2 \cos t + t \sin t$

$\lim_{t \to 0} \frac{f(t)}{t^2} = 0$

28) Equation of line AB is $y = 3x$

Area of triangle = 3 sq units

$\rightarrow k = \pm 2$

29) $|A| = 1$

$$\text{Adj } A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \quad A(\text{adj } A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = |A| I$$

$$A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

30) Product $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

$X = 0, y = 5, z = 3$

31) $A^T = A^{-1}$

$AA^T = AA^{-1}$

$$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing corresponding elements

$2y^2 - z^2 = 0$

$4y^2 + z^2 = 1$

$x^2 + y^2 + z^2 = 1$

$x = \pm\frac{1}{\sqrt{2}}, \ y = \pm\frac{1}{\sqrt{6}}, \ z = \pm\frac{1}{\sqrt{3}}$

32) Let charges of typing 1 English page be Rs x and of 1 Hindi page be Rs y

$10x + 3y = 145$

$3x + 10y = 180$

$|A| = 91$ and $A^{-1} = \frac{1}{91}\begin{bmatrix} 10 & -3 \\ -3 & 10 \end{bmatrix}$

X = 10 , y = 15

33) $|B| = 1$

$B^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$, $(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

,

34) $|A| = 2$

$A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

$\frac{A^2 - 3I}{2} = \frac{1}{2}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \frac{3}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = A^{-1}$

35) 3x- 5y = 15000

4x – 7y = 15000

$|A| = -1$

$A^{-1} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}$

X = 30000

Y = 15000

Monthly income of Aryan = 3 x 30000= Rs 90000

Monthly income of Babban = 4 x 30000= Rs 1,20,000

CASE STUDY QUESTIONS.

36) 1 . 5x+4y+3z = 11000 , 4x + 3y +5z = 10700 , x + y + z = 2700

2. yes as $|A| = -3$

3. x = 1000 , y = 900 , z = 800

37) Let the food needed be x kg of A , y kg of B and z kg of C

x+3y+4z =8 , 2x +y+2z = 5 , 5x +y+z = 7

$|A| = 11$

x = 1 , y =1 , z = 1

The mixture is formed by mixing 1 kg of each of food A, B , C

38) 1. (x-50)(y+50)=xy

→ x – y = 50

(x- 10)(y – 20) = xy – 5300

→ 2x+y=550

2. $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

3. x= 200

4. y = 150

		5 Area =200 x 150 = 30, 000 sq .m
		39) 20x + 30y + 40z = 250, 30x + 40y + 20z = 270 40x + 20y + 30z .=. 200 $\|A\| =$ -27000 By matrix method cost of a polythene bag, a handmade bag and a newspaper envelope is Rs.1, Rs.5 and Rs.2 respectively. **(i) (a)** **(ii) (d)** **(iii) (b**) **(iv) (b) :** Vijay investing most of the money on hand-rnade bags. **(v) (a) :** Salim investing less amount of money on polythene bags.
		40) 1. 10x +12y = 280000 12x+10y =270000 2. $\begin{bmatrix}10 & 12\\12 & 10\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}280000\\270000\end{bmatrix}$ 3 . x= 10000 4. y = 15000

ANSWERS

MULTIPLE CHOICE QUESTIONS

1) d)12 , - 2	2) d) arbitrary	**3)** b) 1	4) c) $\lvert A \rvert^3$	**5)** d) -3

SHORT ANSWER TYPE QUESTIONS

6) $\beta \neq -2$	**7) 110**	**8) 8 x 9 ^{2} = 648**	**9) a = -4 , b = 1**

10) $\Delta = 86$

$\rightarrow a^2 + 4a - 21 = 0$

$a = -7, 3$

Sum = -4

11) 25

12) $\lvert adj\ A \rvert = \lvert A \rvert^{n-1}$

2k – 6 = 4^2

K = 11

13) x = ± 3

14) 2 – 2 $\cos 2\theta$ - $\sin 3\theta = 0$

2 – 2 (1- $2\sin^2\theta$**) – (** $3\sin\theta - 4\sin^3\theta$ **) = 0**

sin $\theta\ (4\sin^2\theta + 4\sin\theta - 3) = 0$

sin $\theta(2\ \textbf{sin}\ \theta - 1)(2\ \textbf{sin}\ \theta + 3) = 0$

$\sin\theta = 0$ or $\sin\theta = 1/2$

15) $\cos 90^0 = 0$

16) $\lvert A \rvert = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

Expanding along R_2

$a_{21}\ A_{21} + a_{22}\ A_{22}$ **= - 15**

17) x = 2

18) k = 4

19) Given points are collinear as $\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = 0$

20) Det (A) $\in$ **[2 ,4]**

LONG ANSWER TYPE QUESTIONS

21) x = 8 , y = 8 ,

A^{-1} = $\frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

22) $|A^T| = 1$

$(A^T)^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

23) **A^{-1} =** $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

24) $|A| = 2$

A^{-1} = $\frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$9\,I - A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$= 2\,A^{-1}$

25) $(AB)^{-1} = B^{-1}A^{-1} = \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$

26) $\Delta = \mathbf{0}$

$\rightarrow x^3 - 13x + 12 = 0$

$x = 1$ and $x = 3$ are other two roots.

27) $f(t) = -t^2 \cos t + t \sin t$

$\lim_{t \to 0} \frac{f(t)}{t^2} = 0$

28) Equation of line AB is $y = 3x$

Area of triangle = 3 sq units

$\rightarrow k = \pm 2$

29) $|A| = 1$

$$\text{Adj } A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \quad A\,(\text{adj } A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = |A|\,I$$

$$A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

30) Product = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

$X = 0,\ y = 5,\ z = 3$

31) $A^T = A^{-1}$

$A\,A^T = A\,A^{-1}$

$$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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40x + 20y + 30z .=. 200

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(i) (a)

(ii) (d)

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4 . x= 10000

5. y = 15000

CONTINUITY AND DIFFERENTIABILITY.

GIST OF THE LESSON FOR QUICK REVISION

Continuous Function

A real valued function f is said to be continuous, if it is continuous at every point in the domain of f.

Continuity of a function at a point

Suppose f is a real valued function on a subset of real numbers and let c be a point in the domain of f, then f is continuous at x=c, if $\lim_{x\to c} f(x) = f(c)$

Ie., $\lim_{x\to c+} f(x) = \lim_{x\to c-} f(x) = f(c)$

Then f(x) is continuous at x=c . Otherwise f(x) is discontinuous at x=c

Graphically, a function f(x) is said to be continuous at a point if the graph of the function has no break either on the left or the right in the neighbourhood of the point.

Some basic continuous functions:

1. Every constant function is continuous.
2. Every identity function is continuous.
3. Rational functions are always continuous
4. Every polynomial function is continuous.
5. Modulus function f(x)= |x| is continuous.
6. All trigonometric functions are continuous in their domain.
7. e^x, logx continuous in their domain.

Algebra of continuous function

Theorem 1 :

Let f and g be two real functions, continuous at a real number c, then

1. (f + g) is continuous at x=c
2. (f-g) is continuous at x=c
3. fg is continuous at x=c
4. f/g is continuous at x=c provided g(c) ≠0

Differentiability

A real valued function f, is said to be differentiable at x=c in its domain, if its left hand and right hand derivatives at x=c exists and are equal.

At x=a, right hand derivative,

Rf'(a)=$\lim_{x\to 0}\left(\frac{f(a+h)-f(a)}{h}\right)$ and left hand derivative, Lf'(a) = $\lim_{x\to 0}\left(\frac{f(a-h)-f(a)}{-h}\right)$

Thus f(x) is differentiable at x=a , if Rf'(a) = Lf'(a)

Otherwise, f(x) is not differentiable at x=a

Derivatives of some standard functions:

1. $\frac{d}{dx}(\text{constant}) = 0$
2. $\frac{d}{dx}(x^n) = nx^{n-1}$
3. $\frac{d}{dx}(\sin x) = \cos x$
4. $\frac{d}{dx}(\cos x) = -\sin x$
5. $\frac{d}{dx}(\tan x) = sec^2 x$
6. $\frac{d}{dx}(\text{cosec } x) = -cosec\ x \cot x$
7. $\frac{d}{dx}(\sec x) = \sec x \tan x$
8. $\frac{d}{dx}(\cot x) = -cosec^2 x$
9. $\frac{d}{dx}(e^x) = e^x$
10. $\frac{d}{dx}(a^x) = a^x \log a,\ \ a > 0$
11. $\frac{d}{dx}(\log_e x) = \frac{1}{x}, x > 0$
12. $\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}$, $a > 0$, $a \neq 1$

Algebra of derivatives

1. $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$
2. $\frac{d}{dx}(uv) = u.\frac{dv}{dx} + v.\frac{du}{dx}$ (product rule)
3. $\frac{d}{dx}(\frac{u}{v}) = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$ (quotient rule) where u and v are functions of x
4. $\frac{d(ku)}{dx} = k.\frac{du}{dx}$ where k is a constant

Chain Rule

Example 1:

If $y = \sin(x^2)$, then $\frac{dy}{dx} = \cos(x^2).\frac{d}{dx}(x^2) = \cos(x^2).2x = 2x\cos(x^2).$

Example 2:

If $y = \tan(2x+3)$, then $\frac{dy}{dx} = sec^2(2x+3).\frac{d}{dx}(2x+3)$

$= sec^2(2x+3).2 = 2sec^2(2x+3).$

Example 3 :

If $y = \sin(\cos(x^2))$, then $\frac{dy}{dx} = \cos(\cos(x^2)).\frac{d(\cos(x^2))}{dx}$

$= \cos(\cos(x^2)).-\sin(x^2).\frac{d}{dx}(x^2)$

$= \cos(\cos(x^2)).-\sin(x^2).2x$

$= -2x.\sin(x^2)\cos(\cos(x^2))$

Derivative of implicit function

Let f(x,y) = 0 be an implicit function of x, then, to find $\frac{dy}{dx}$, first differentiate both sides of equation w.r.t x and then take all terms involving $\frac{dy}{dx}$ to LHS and remaining terms to RHS , then find $\frac{dy}{dx}$.

Derivatives of inverse trigonometric functions

1. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$, -1 < x< 1
2. $\frac{d}{dx}(cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$, -1 < x< 1
3. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$
4. $\frac{d}{dx}(cot^{-1}x) = \frac{-1}{1+x^2}$
5. $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x.\sqrt{x^2-1}}$, |x| > 1
6. $\frac{d}{dx}(cosec^{-1}x) = \frac{-1}{x.\sqrt{x^2-1}}$, |x| > 1

Derivative of a function w.r.t another function

Let u = f(x) and v= g(x) be two given functions. Differentiate both functions w.r.t x separately and substitute in the following formula.

$\frac{du}{dv} = \frac{du}{dx} \div \frac{dv}{dx}$

Derivative of logarithmic functions

Suppose, given function is of the form $u(x)^{v(x)}$

In such cases, take logarithm on both sides and use properties of logarithm to simplify it. Then differentiate it.

Derivatives of parametric functions

If x= f(t) and y = g(t), then

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

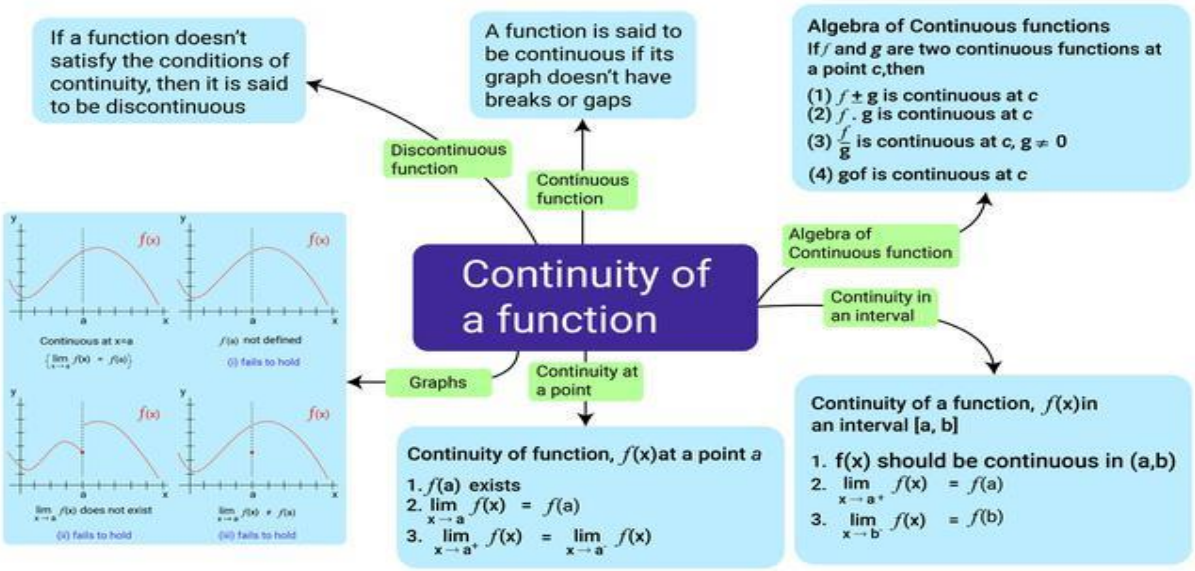
Second order derivative

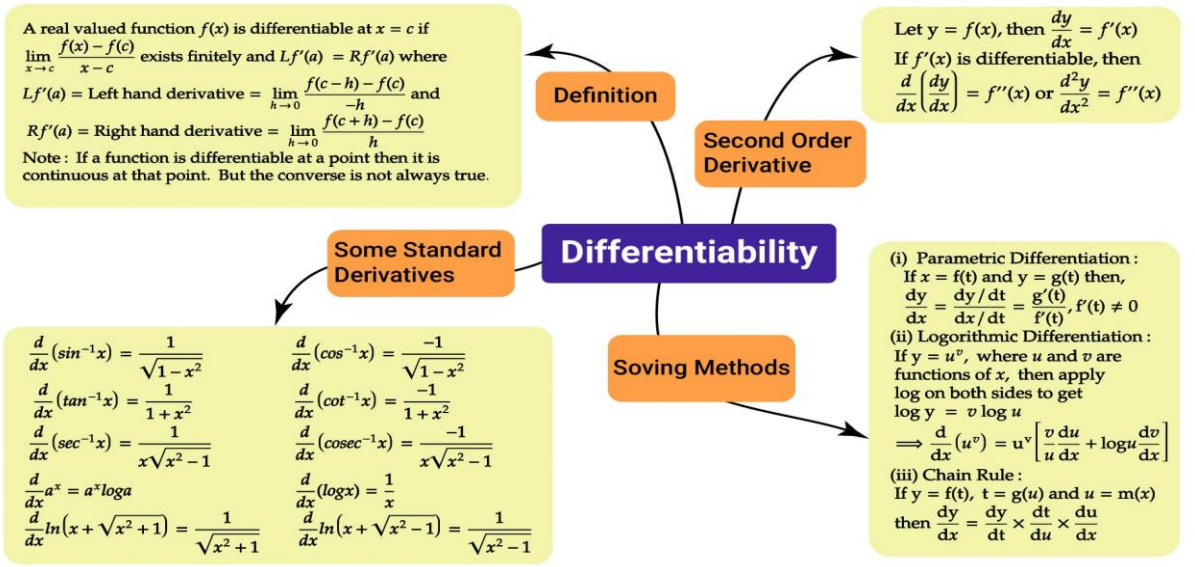
Let y=f(x) be a given function, then $\frac{dy}{dx}$ is called first derivative of y.

$\frac{d}{dx}(\frac{dy}{dx})$ is called the second order derivative of y w.r.t x and is denoted by

$\frac{d^2y}{dx^2}$ or y'' or y_2

MIND MAPPING





TYPES OF QUESTIONS

I.MCQ

1 If $f(x)=\begin{cases}3x-5, & x\le 5\\ 2k, & x>5\end{cases}$ is continuous at x=5, then k is

a.7/2 b.2/7 c.-7/2 d.-2/7

2 The function f(x) =[x] is continuous at

a.4 b.-2 c.1 d.1.5

3 If $x=t^2$ and $y=t^3$ then $\frac{d^2y}{dx^2}$ is equal to

a. $\frac{3}{2}$ b. $\frac{3}{4t}$ c. $\frac{3}{2t}$ d. $\frac{3t}{2}$

4 Derivative of x^2 with respect to x^3 is

a. $\frac{1}{x}$ b. $\frac{2}{3x}$ c. $\frac{2}{3}$ d. $\frac{3x}{2}$

5 For the curve √x+√y = 1, $\frac{dy}{dx}$ at (1/4,1/4) is

a.1 b.1/2 c.-1 d. none of these

II.SHORT ANSWER QUESTIONS

6 Check whether the function $f(x)=\begin{cases}3x+5, & x\geq 2\\ x^2, & x<2\end{cases}$ is continuous at x=2

7 Show that the function $f(x)=\begin{cases}\frac{x^2-x-6}{x-3}, & x\neq 3\\ 5, & x=3\end{cases}$ is continuous at x=3

8 If the function $f(x)=\begin{cases}3ax+b, & x>1\\ 11 & x=1\\ 5ax-2b & x<1\end{cases}$ is continuous at x=1, then find a and b

9 Prove that $f(x)=\begin{cases}1+x, & x\leq 2\\ 5-x, & x>2\end{cases}$ is not differentiable at x=2

10 Find the value of k for which

$$f(x)=\begin{cases}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & -1\leq x<0\\ \frac{2x+1}{x-1}, & 0\leq x\leq 1\end{cases}$$ is continuous at x=0

11 Discuss the continuity of the function at x=1/2 where the function

$$f(x)=\begin{cases}\frac{1}{2}+x, & 0\leq x<1/2\\ 1 & x=1/2\\ \frac{3}{2}+x & 1/2<x\leq 1\end{cases}$$

12 If $x^2 + 2xy + y^3 = 42$, find $\frac{dy}{dx}$

13 If siny =x sin(a+y). prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

14 If $y=e^{3x}\sin 4x 2^x$, find $\frac{dy}{dx}$

15 If $x^y = y^x$, find $\frac{dy}{dx}$

16 Differentiate $\tan^{-1}\left(\frac{2^{x+1}}{1-4^x}\right)$

17 If $y = \tan^{-1}x$, then find $\frac{d^2y}{dx^2}$ in terms of y alone

18 Differentiate $\log(1+x^2)$ with respect to $\tan^{-1}x$

19 If $y=ax^{n+1} + bx^{-n}$ and $x^2\frac{d^2y}{dx^2}=\lambda y$, then find λ

20 Find $\frac{dy}{dx}$, when $x = e^{\theta}\left(\theta + \frac{1}{\theta}\right)$ and $x = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$

III.CASE STUDY QUESTIONS

21 If a relation between x and y is such that y cannot be expressed in terms of x, then y is called an implicit function of x. When a given relation expresses y as an implicit function of x and we want to find $\frac{dy}{dx}$, then we differentiate every term of the given relation with respect x, remembering that a term in y is first differentiated w.r.t y and then multiplied by $\frac{dy}{dx}$

Based on the above information, find the value of $\frac{dy}{dx}$ in each of the following questions

(i) $x^3 + x^2y + xy^2 + y^3 = 81$

(a) $\frac{(3x^2 + 2xy + y^2)}{x^2 + 2xy + 3y^2}$ (b) $\frac{-(3x^2 + 2xy + y^2)}{x^2 + 2xy + 3y^2}$ (c) $\frac{(3x^2 + 2xy - y^2)}{x^2 - 2xy + 3y^2}$ (d) $\frac{3x^2 + xy + y^2}{x^2 + xy + 3y^2}$

(ii) $x^y = e^{x-y}$

(a) $\frac{x - y}{(1 + \log x)}$ (b) $\frac{x + y}{(1 + \log x)}$ (c) $\frac{x - y}{x(1 + \log x)}$ (d) $\frac{x + y}{x(1 + \log x)}$

(iii) $e^{\sin y} = xy$

(a) $\frac{-y}{x(y\cos y - 1)}$ (b) $\frac{y}{y\cos y - 1}$ (c) $\frac{y}{y\cos y + 1}$ (d) $\frac{y}{x(y\cos y - 1)}$

(iv) $\sin^2 x + \cos^2 y = 1$

(a) $\frac{\sin 2y}{\sin 2x}$ (b) $-\frac{\sin 2x}{\sin 2y}$ (c) $-\frac{\sin 2y}{\sin 2x}$ (d) $\frac{\sin 2x}{\sin 2y}$

(v) $y = (\sqrt{x})^{\sqrt{x}^{\sqrt{x}\cdots\infty}}$

(a) $\frac{-y^2}{x(2 - y\log x)}$ (b) $\frac{y^2}{2 + y\log x}$ (c) $\frac{y^2}{x(2 + y\log x)}$ (d) $\frac{y^2}{x(2 - y\log x)}$

22 If y = f(u)is a differentiable function of u and u = g(x is a differentiable function of x, then y = f(g(x)) is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$. This rule is known as CHAIN RULE.

Based on the above information, find the value of $\frac{dy}{dx}$ in each of the following questions

1.cos√x

a. $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$ b. $\frac{\sin\sqrt{x}}{2\sqrt{x}}$ c.sin√x d. -sin√x

2. $7^{x+\frac{1}{x}}$

a. $\left(\frac{x^2 - 1}{x^2}\right)7^{x+\frac{1}{x}}\log 7$ b. $\left(\frac{x^2 + 1}{x^2}\right)7^{x+\frac{1}{x}}\log 7$ c. $\left(\frac{x^2 - 1}{x^2}\right)7^{x-\frac{1}{x}}\log 7$

d. $\left(\frac{x^2 + 1}{x^2}\right)7^{x-\frac{1}{x}}\log 7$

3. $\sqrt{\frac{1-\cos x}{1+\cos x}}$

a. $\frac{1}{2}\sec^2\frac{x}{2}$ b. $-\frac{1}{2}\sec^2\frac{x}{2}$ c. $\sec^2\frac{x}{2}$ d. $-\sec^2\frac{x}{2}$

4. $\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)+\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

a. $-\frac{1}{x^2+b^2}+\frac{1}{x^2+a^2}$ b. $\frac{1}{x^2+b^2}+\frac{1}{x^2+a^2}$ c. $\frac{1}{x^2+b^2}-\frac{1}{x^2+a^2}$

d. $-\frac{1}{x^2+b^2}-\frac{1}{x^2+a^2}$

5. $\sec^{-1}x+\cos ec^{-1}\frac{x}{\sqrt{x^2-1}}$

a. $\frac{2}{\sqrt{x^2-1}}$ b. $\frac{-2}{\sqrt{x^2-1}}$ c. $\frac{1}{x\sqrt{x^2-1}}$ d. $\frac{2}{x\sqrt{x^2-1}}$

23 Let x = f(t) and y = g(t) be the parametric form with t as a parameter, then $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ = $\frac{g^1(t)}{f^1(t)}$ where $f^1(t)\neq 0$

Based on the above information, answer the following questions

1.The derivative of f(tanx)w.r.t g(secx) at x=π/4, where $f^1(1)=2$ and $g^1(\sqrt{2})=4$ is

a.1/√2 b.√2 c.0 d.1

2. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

a.1 b.-1 c.2 d.4

3. The derivative of e^{x^3} w.r.t logx is

a. e^{x^2} b. $e^{x^3}.3x^2.2$ c. $e^{x^3}.3x^3$ d. $e^{x^2}.3x^2+3x$

4. The derivative of $\cos^{-1}(2x^2-1)$ w.r.t $\cos^{-1}x$ is

a.2 b. $\frac{-1}{2\sqrt{1-x^2}}$ c.2/x d.$1-x^2$

5.If y=$\frac{u^4}{4}$ and $u=\frac{2}{3}x^3$,then find dy/dx

a. $\frac{2}{27}x^9$ b. $\frac{16}{27}x^{11}$ c. $\frac{8}{27}x^9$ d. $\frac{2}{27}x^{11}$

24 Logarithmic differentiationis a powerful technique to differentiate functions of the form $f(x)=u(x)^{v(x)}$ where both u and v are differentiable functions of x.Let the function $y=f(x)=u(x)^{v(x)}$, then $y^1=u(x)^{v(x)}\left[v(x)\frac{u^1(x)}{u(x)}+\log u(x)v^1(x)\right]$

Based on the above information, answer the following questions

1. Differentiate x^x

   a.$x^x(1+\log x)$ b. $x^x(1-\log x)$ c. $-x^x(1+\log x)$ d. $x^x \log x$

2. Differentiate $x^x+a^x+x^a+a^a$

   a.$1+\log x+a^x \log a+ax^{a-1}$ b. $x^x(1+\log x)+\log a+ax^{a-1}$ c. $x^x(1+\log x)+x^a \log x+ax^{a-1}$

   d. . $x^x(1+\log x)+a^x \log a+ax^{a-1}$

3.If $x= e^{x/y}$, then find dy/dx

a. $-\frac{(x+y)}{x\log x}$ b. $-\frac{(x-y)}{x\log x}$ c. $\frac{(x+y)}{x\log x}$ d. $\frac{(x-y)}{x\log x}$

4.If $y=(2-x)^3(3+2x)^5$, then find dy/dx

a. $(2-x)^3(3+2x)^5\left[\frac{15}{3+2x}-\frac{8}{2-x}\right]$ b. $(2-x)^3(3+2x)^5\left[\frac{15}{3+2x}+\frac{3}{2-x}\right]$

c. $(2-x)^3(3+2x)^5\left[\frac{10}{3+2x}-\frac{3}{2-x}\right]$ d. $(2-x)^3(3+2x)^5\left[\frac{10}{3+2x}+\frac{3}{2-x}\right]$

5.If y= $x^x e^{2x+5}$, then find dy/dx

a. $x^x e^{2x+5}$ b. $x^x e^{2x+5}(3-\log x)$ c. $x^x e^{2x+5}(1-\log x)$

d. $x^x e^{2x+5}(3+\log x)$

25 Let f(x) be a real valued function, then its left hand derivative(L.H.D) at the point a is $f^1(a-)=\lim_{x\to 0}\frac{f(a-h)-f(a)}{-h}$ and its Right hand derivative(R.H.D) at the point a is $f^1(a+)=\lim_{x\to 0}\frac{f(a+h)-f(a)}{h}$,also a function f(x)is said to be differentiable at x=a and if its L.H.D and R.H.D at x=a exist and are equal. For the function

$$f(x)=\begin{cases}|x-3| & x\geq 1\\ \frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4} & x<1\end{cases}$$

1. L.H.D of f(x)at x=1

   a.1 b.-1 c.0 d.2

2. f(x) is not differentiable at

   a.x=1 b.x=2 c.x=3 d.x=4

3.Find the value of $f^1(2)$

   a.1 b.2 c.3 d.4

4.Find the value of $f^1(-1)$

   a.x=1 b.x=2 c.x=-2 d.x=-1

5.R.H.D of f(x) at x=1 is

   a.1 b.-1 c.0 d.2

IV.LONG ANSWER QUESTIONS

26. If $x^m y^n=(x+y)^{m+n}$, prove that $\frac{dy}{dx}=\frac{y}{x}$
27. If y= $e^{m\sin^{-1}x}$, then show that $(1-x^2)y_2-xy_1-m^2y=0$ (hint $(1-x^2)^{1/2}y_1=my$)
28. If x= sint, y=sinpt, prove that $(1-x^2)y_2-xy_1+p^2y=0$
    (hint y_1=pcospt/cost $y_2=\frac{-p^2sinptcost+pcosptsint}{(cost)^3}$)
29. If y=log$(x+\sqrt{x^2+a^2})$, show that $(x^2+a^2)y_2+xy_1=0$
30. If x= a(cost+logtan$\frac{t}{2}$), y= asint evaluate $\frac{d^2y}{dx^2}$ at t=π/3
    (hint $\frac{dy}{dx}=tant, \frac{d^2y}{dx^2}=\frac{sint}{acos^4t}$)
31. If y = $x^{sinx-cosx}+\frac{x^2-1}{x^2+1}$, find $\frac{dy}{dx}$
32. If y= $(\tan^{-1}x)^2$, show that $(x^2+1)^2y_2+2x(x^2+1)y_1=2$ (hint $(1+x^2)y_1=2\tan^{-1}x$)
33. Show that f(x)= |x-5| is continuous but not differentiable at x=5
    (for continuous LHL=RHL=f(5), for differentiability LHD=RHD)
34. If y= $(x+\sqrt{x^2+1})^m$, then show that $(x^2+1)y_2+xy_1-m^2y=0$
35. If x=acosθ+bsinθ, y = asinθ-bcosθ, then prove that $y^2y_2-xy_1+y=0$
    (Hint$\frac{dy}{dx}=-y/x$)

ANSWERS

MCQ

1.a 2.d 3.b 4.b 5.c

SHORT ANSWERS

6.discontinuous at x=2

8. a=3 b=2

10. k= -1

12. $\frac{-2(x+y)}{2x+3y^2}$

14. e^{3x}sin4x2^x(3+4cot4x+log2)

15. $\frac{y(xlogy-y)}{x(ylogx-x)}$

16. $\frac{2^{x+1}log2}{1+4^x}$

17. -sin2y$\cos^2$y

18. 2x

19. λ =n(n+1)

20. $\frac{e^{-\theta}(\theta^2-\theta^3+\theta+1)}{(\theta^2+\theta^3+\theta-1)}$

CASE STUDY

QN.NO	1	2	3	4	5
21	b	c	d	d	d
22	a	a	a	b	d
23	a	b	c	a	a

24	a	d	d	c	d
25	b	b	c	d	c

LONG ANSWERS

30. 8√3/a

31. $\frac{dy}{dx} = x^{sinx-cosx}$[(sinx-cosx)/x+logx(cosx+sinx)]+$\frac{4x}{(x^2+1)^2}$

APPLICATION OF DERIVATIVES

Key Points:

Derivative as Rate of Change

- Let y=f(x) be a function. Then $\frac{dy}{dx}$ denotes the rate of change of y w.r.t x.
- The value of $\frac{dy}{dx}$ at x = x_0 i.e. $(dy/dx)_{x=x_0}$ represents the rate of change of y w.r.t x at x = x_0
- If two variables x and y are varying with respect to another variable t, i.e., if x = f(t) and y= g(t), then by Chain Rule, $\frac{dy}{dx} = \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)$ provided $\frac{dx}{dt} \neq 0$
- $\frac{dy}{dx}$ is positive if y increases as x increases and is negative if y decreases as x increases.

Increasing and Decreasing Functions

- A function y = f (x) is said to be **increasing** on an interval (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in$ (a, b)

  Alternatively, a function y = f (x) is said to be increasing if $f'(x) \geq 0$ for each x in (a, b)

  (a) **strictly increasing** on an interval (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in$ (a, b).

  Alternatively, a function y = f (x) is said to be strictly increasing if $f'(x) > 0$ for each x in (a, b)

  (b) **decreasing** on (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in$ (a, b). Alternatively, a function y = f (x)) is said to be decreasing if $f'(x) \leq 0$ for each x in (a, b)

  (c) **strictly decreasing** on (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in$ (a, b).

  Alternatively, a function y = f (x) is said to be strictly decreasing if $f'(x)<0$ for each x in (a, b)

  (d) **constant** function in (a, b), if f (x) = c for all x ∈ (a, b), where c is a constant.

  Alternatively , f(x) is a constant function if $f'(x) = 0$.

- A point c in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable is called a critical point of f.

Maxima And Minima

Definition: Let f be a function defined on an interval I. Then

1) f is said to have a maximum value in I, if there exists a point c in I such that f(c) > f(x), for all x ∈ I. The number f(c) is called the maximum value of f in I and the point c is called a point of maximum value of f in I.
2) f is said to have a minimum value in I, if there exists a point c in I such that f (c) < f (x), for all x ∈ I. The number f (c), in this case, is called the minimum value of f in I and the point c, in this case, is called a point of minimum value of f in I.
3) f is said to have an extreme value in I if there exists a point c in I such that f (c) is either a maximum value or a minimum value of f in I. The number f (c), in this case, is called an extreme value of f in I and the point c is called an extreme point.

Local Maxima and Local Minima

Definition: Let f be a real valued function and let c be an interior point in the domain of f. Then

(a) c is called a point of local maxima if there is an h > 0 such that f (c) ≥ f (x), for all x in (c – h, c + h), x ≠ c. The value f(c) is called the local maximum value of f.

(b) c is called a point of local minima if there is an h > 0 such that f (c) ≤ f (x), for all x in (c – h, c + h). The value f(c) is called the local minimum value of f.

Geometrically, the above definition states that if x = c is a point of local maxima of f, then the graph of f around c will be as shown in Fig. below. Note that the function f is increasing (i.e., f ′(x) > 0) in the interval (c – h, c) and decreasing (i.e., f ′(x) < 0) in the interval (c, c + h). This suggests that f ′(c) must be zero,

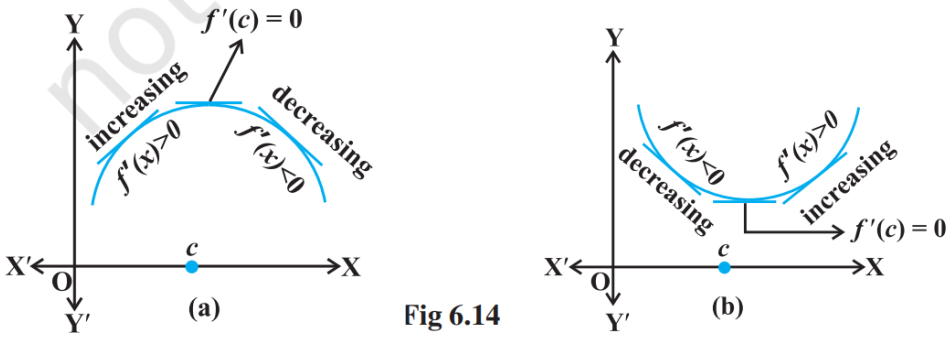


Fig 6.14

Theorem: Let f be a function defined on an open interval I. Suppose $c \in I$ be any point. If f has a local maxima or a local minima at $x = c$, then either $f'(c) = 0$ or f is not differentiable at c.

Definition: A point c in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable is called a critical point of f.

Theorem: (First Derivative Test) Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then

1) If $f'(x)$ changes sign from positive to negative as x increases through c, i.e., if $f'(x) > 0$ at every point sufficiently close to and to the left of c, and $f'(x) < 0$ at every point sufficiently close to and to the right of c, then c is a point of local maxima.
2) If $f'(x)$ changes sign from negative to positive as x increases through c, i.e., if $f'(x) < 0$ at every point sufficiently close to and to the left of c, and $f'(x) > 0$ at every point sufficiently close to and to the right of c, then c is a point of local minima.
3) If $f'(x)$ does not change sign as x increases through c, then c is neither a point of local maxima no a point of local minima. In fact, such a point is called point of inflection.

Theorem: (Second Derivative Test) Let f be a function defined on an interval I and $c \in I$. Let f be twice differentiable at c. Then

1) $x = c$ is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$ The value $f(c)$ is local maximum value of f.
2) $x = c$ is a point of local minima if $f'(c)\ 0 =$ and $f''(c) > 0$ In this case, $f(c)$ is local minimum value of f.
3) The test fails if $f'(c) = 0$ and $f''(c) = 0$. In this case, we go back to the first derivative test and find whether c is a point of local maxima, local minima or a point of inflexion.

Working rule for finding absolute maxima and/or absolute minima

Step 1: Find all critical points of f in the interval, i.e., find points x where either $f'(x) = 0$ or f is not differentiable.
Step 2: Take the end points of the interval.
Step 3: At all these points (listed in Step 1 and 2), calculate the values of f.
Step 4: Identify the maximum and minimum values of f out of the values calculated in Step 3. This maximum value will be the absolute maximum value of f and the minimum value will be the absolute minimum value of f.

Multiple Choice Questions

1. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of an edge is 12 cm?

A) 5/3 Sq.cm/sec
B) 7/3 Sq.cm/sec
C) 8/3 Sq.cm/sec
D) None of these

2. A particle moves along the curve $y = x^2 + 2x$. At what point(s) on the curve are the x and y coordinates of the particle changing at the same rate?

A) (1/3, 2/3)
B) (-1/3, 2/3)
C) (-1/3, -2/3)
D) (-1/2, -4/3)

3. The bottom of a rectangular swimming tank is 25 m by 40 m water is pumped into the tank at the rate of 500 cubic meters per minute. Find the rate at which the level of water in the tank is rising.?

A) ¼ m/min
B) 2/3 m/min
C) 1/3 m/min
D) ½ m/min

4. The total revenue received from the sale of x units of a product is given by $R(x) = 10x^2 + 13x + 24$. Find the marginal revenue when x = 5

A) 113 Rupees
B) 223 Rupees
C) 93 Rupees
D) 339 Rupees

5. The interval in which the function $y = x^2e^{-x}$ is increasing is

A) $(-\infty, 0)$
B) (0,2)
C) $(2, \infty)$
D) None of these

Answers:

1) C 2) D 3) D 4) A 5) B

SHORT ANSWER QUESTIONS

6. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of an edge is 12 cm?

7. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?

8. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle?

9. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

10. Show that the function f given by $f(x)=x^3-3x^2+7x$, $x \in R$, is strictly increasing on R.

11. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R

12. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

13. The total cost C (x) in Rupees associated with the production of x units of an item is given by

$$C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$$

Find the marginal cost when 17 units are produced.

14. Show that the function $f(x) = \log(1+x) - \frac{2x}{2+x}$, $x > -1$ is an increasing function of x throughout its domain.

15. Prove that $y = \frac{4\,Sin\theta}{2+Cos\theta} - \theta$ *is an increasing function of* θ *in* $\left[0, \frac{\pi}{2}\right]$

16. Find the rate of change of the volume of a sphere with respect to its surface area when the radius is 2 cm.?

17.Find the maximum and the minimum values of the function $f(x) = x + 2$, $x \in (0,1)$?

18. Find the local maximum and the local minimum values of the function $f(x) = \frac{-3}{4}x^4\ 8x^3 - \frac{45}{2}x^2 + 105$?

19. Find the maximum value of $f(x) = (x-1)^{\frac{1}{3}}(x-2)$ in [1,9]

20. Find two numbers whose sum is 24 and whose product is as large as possible?

6. Let x be the length of a side, V be the volume, and s be the surface area of the cube.Then, $V = x^3$ and $S = 6x^2$ where x is a function of time t.

It is given that $\frac{dV}{dt} = 8 \text{ cm}^3/\text{s}$.

Then, by using the chain rule, we have:

$$8 = \frac{dV}{dt} = \frac{d}{dt}\left(x^3\right) = \frac{d}{dx}\left(x^3\right)\cdot\frac{dx}{dt} = 3x^2.\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{8}{3x^2} \qquad (1)$$

$$\text{Now, } \frac{dS}{dt} = \frac{d}{dt}\left(6x^2\right) = \frac{d}{dx}\left(6x^2\right)\cdot\frac{dx}{dt} \qquad [\text{By chain rule}]$$

$$= 12x.\frac{dx}{dt} = 12x\cdot\left(\frac{8}{3x^2}\right) = \frac{32}{x}$$

Thus, when x = 12 cm, $\frac{dS}{dt} = \frac{32}{12} \text{ cm}^2/\text{s} = \frac{8}{3} \text{ cm}^2/\text{s}.$

Hence, if the length of the edge of the cube is 12 cm, then the surface area is increasingat the rate of 8/3 cm^2/s.

Qn. 7) Let x be the length of a side and V be the volume of the cube. Then,$V = x^3$.

$\therefore \frac{dV}{dt} = 3x^2\cdot\frac{dx}{dt}$ (By chain rule)

It is given that,

$$\frac{dx}{dt} = 3 \text{ cm/s}$$

$$\therefore \frac{dV}{dt} = 3x^2(3) = 9x^2$$

Thus, when x = 10 cm,

$$\frac{dV}{dt} = 9(10)^2 = 900 \text{ cm}^3/\text{s}$$

Hence, the volume of the cube is increasing at the rate of 900 cm^3/s when the edge is10 cm long.

Qn.8) Since the length (x) is decreasing at the rate of 5 cm/minute and the width (y) isincreasing at the rate of 4 cm/minute, we have:

$$\frac{dx}{dt} = -5 \text{ cm/min} \quad \text{and} \quad \frac{dy}{dt} = 4 \text{ cm/min}$$

a) The perimeter (P) of a rectangle is given by,

(a) P = 2(x + y)

$$\therefore \frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = 2(-5+4) = -2 \text{ cm/min}$$

Hence, the perimeter is decreasing at the rate of 2cm/min

(b) The area (A) of a rectangle is given by,

A = x × y

$$\frac{dA}{dt} = \frac{dx}{dt} \cdot y + x \cdot \frac{dy}{dt} = -5y + 4x$$

When x = 8 cm and y = 6 cm,

$$\frac{dA}{dt} = (-5 \times 6 + 4 \times 8) \text{ cm}^2/\text{min} = 2 \text{ cm}^2/\text{min}$$

Hence, the area of the rectangle is increasing at the rate of 2 cm^2/min.

Qn9) Let y m be the height of the wall at which the ladder touches. Also, let the foot of the ladder be x m away from the wall. Then, by Pythagoras theorem, we have: $x^2 + y^2 = 25$[Length of the ladder = 5 m]

$$\Rightarrow y = \sqrt{25 - x^2}$$

Then, the rate of change of height (y) with respect to time (t) is given by,

$$\frac{dy}{dt} = \frac{-x}{\sqrt{25 - x^2}} \cdot \frac{dx}{dt}$$

It is given that $\frac{dx}{dt} = 2 \text{ cm/s}$

.

$$\therefore \frac{dy}{dt} = \frac{-2x}{\sqrt{25-x^2}}$$

Now, when x = 4 m, we have:

$$\frac{dy}{dt} = \frac{-2\times 4}{\sqrt{25-4^2}} = -\frac{8}{3}$$

Hence, the height of the ladder on the wall is decreasing at the rate of $\frac{8}{3}$ cm/s .

Qn. 10)

Sol.

$f(x)=x^3-3x^2+7x$

$f'(x) = 3x^2 - 6x +7$

in every interval of R.

$= 3(x^2 - 2x + 1) + 4$

$= 3(x-1)^2 +4 > 0,$

.. the function fis strictly increasing on R. 1

Qn.11)

Sol.

$f(x) = e^{2x}$

$f'(x) = 2e^{2x} > 0$, for all x = R.

:. f(x) is strictly increasing on R.

Qn. 12)

Soln

$$V = \frac{1}{3}\pi r^2 h$$

It is given that,

$$h = \frac{1}{6}r \Rightarrow r = 6h$$

$$\therefore V = \frac{1}{3}\pi(6h)^2 h = 12\pi h^3$$

The rate of change of volume with respect to time (t) is given by,

$$\frac{dV}{dt} = 12\pi \frac{d}{dh}\left(h^3\right)\cdot\frac{dh}{dt}$$

$$= 12\pi\left(3h^2\right)\frac{dh}{dt}$$

$$= 36\pi h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = 12 \text{ cm}^3/\text{s}$$

,

when h = 4 cm, we have:

$$12 = 36\pi(4)^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{12}{36\pi(16)} = \frac{1}{48\pi}$$

Hence, when the height of the sand cone is 4 cm, its height is increasing at the rateof $\frac{1}{48\pi}$ cm/s .

Qn. 13) Soln :-

Marginal cost (MC) $= \frac{dC}{dx} = 0.007\left(3x^2\right) - 0.003(2x) + 15$ When $= 0.021x^2 - 0.006x + 15$

$\therefore$

x = 17, MC = 0.021 (17^2) − 0.006 (17) + 15
= 0.021(289) − 0.006(17) + 15

= 6.069 − 0.102 + 15

= 20.967

Hence, when 17 units are produced, the marginal cost is ₹ 20.967.

14) Soln :-

$$f'(x) = \frac{1}{1+x} - \frac{4}{(2+x)^2}$$

$$= \frac{x^2}{(2+x)^2(1+x)} = \frac{(+)}{(+)(+)} = +ve$$

Since x>-1 , 1+x >0

Therefore the fn . f is st. increasing .

15) Soln:-

$$f'(\theta) = 4\left(\frac{(2+cos\theta)cos\theta - sin\theta(-sin\theta)}{(2+cos\theta)^2}\right) - 1$$

$$= 4\left(\frac{2cos\theta + cos^2\theta + sin^2\theta}{(2+cos\theta)^2}\right) - 1$$

$$= 4\frac{(2cos\theta+1)-(2+cos\theta)^2}{(2+cos\theta)^2}$$

$=\frac{4cos\theta-cos^2\theta}{(2+cos\theta)^2}=\frac{cos\theta(4-cos\theta)}{(2+cos\theta)^2}=\frac{(+)(+)}{(+)}=+ve$

Therefore ,the function f is an increasing function of x

16) Soln :-

Radius of sphere(r) = 2cm

$V = 4/3\ \pi r^3$

$\frac{dV}{dr} = 4\pi r2$

$A = 4\pi r2$

$\frac{dA}{dr}=8\pi r$

$\frac{dV}{dA}=\frac{\frac{dV}{dr}}{\frac{dV}{dr}}$

$=\frac{4\pi r^2}{8\pi r}=\frac{r}{2}$

$\frac{dV}{dA_{r=2}}=\frac{2}{2}$ =1 cm

17) $f(x) = x + 2$

$f'(x) = 1$

so for any value of x $f'(x)$ cannot be 0.

So $f(x)$ has no critical points.

Hence $f(x)$ has neither local maximum not local minimum

18) $f(x)=\frac{-3}{4}\ x^4\ 8x^3-\frac{45}{2}\ x^2+105$

$f'(x) = -3x^3 - 24x^2 - 45x$

$= -3x(x^2 + 8x + 15)$

$= -3x(x + 3)(x + 5)$

$f''(x) = -9x^2 - 48x - 45$

$f'(x) = 0 \Longrightarrow -3x(x + 3)(x + 5) = 0$

$\Longrightarrow x = 0,\ x = -3,\ x = -5$

$f''(0)=-45< 0$

So x=0 is a local max. point.

f''(-3) =+18 >0

So x=-3 is a local min. point

f''(-5) =-30 < 0

So x=-5 is a local max. point.

19) $f(x) = (x - 1)^{\frac{1}{3}}(x - 2)$

$f'(x) = (x - 1)^{\frac{1}{3}}(1) + (x - 2) \times 1/3\ (x - 1) -2/3$

$= (x - 1) -2/3\ (x - 1 + \frac{x-2}{3})=\frac{x-1)^{\frac{-2}{3}}}{3}\ (3x - 3 + x - 2) =(4x-5)/3(x-1)\ 2/3$

$f'(x) = 0 \Longrightarrow x = 5/4$

Find the value of f at x= 5/4, x= 1 , &at x=9

$f(1) = 0$,
$f(9) = = 14$,
$f(5/4) < 0$
Maximum value of $f(x)$ is 14

20) Let x & y be the required nos.

X+y = 24 (given)

P = xy = x(24-x)

$=24x - x^2$

$\frac{dP}{dx}=24-2x$

$\frac{dP}{dx} = 0 \rightarrow 2x=24$ or x=12

Second derivative of P is -2 which is -ve
So P is a max. when x=12

When x=12 ,y = 12
So the required numbers are x=12 & y= 12

LONG ANSWER QUESTIONS

21. Show that the volume of the greatest cylinder which can be inscribed in a cone of height 'h' and semi vertical angle α is $\frac{4}{27}\prod h^3 tan^2\alpha$

22. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half that of the cone

23. An open box with a square base is to be made out of a given quantity of cardboard of area c^2 sq units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units

24. Show that the right circular cylinder of given volume, open at the top, has minimum total surface area if its height is equal to the radius of the base

25. Find the volume of the largest right circular cylinder that can be inscribed in sphere of radius r centimeter

26. Show that the semi vertical angle of a right circular cone of maximum volume and given slant height is $tan^{-1}\sqrt{2}$

27. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area

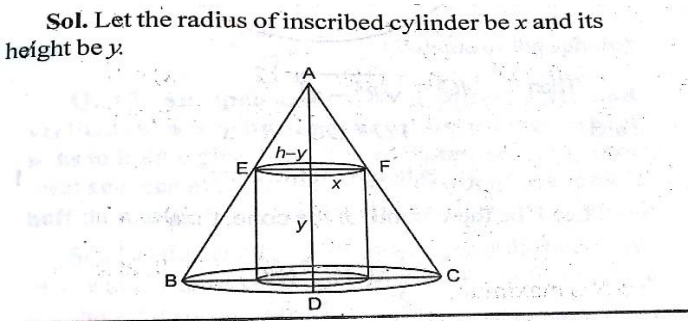
28. Show that the right circular cone of the least curved surface area and given volume has an altitude is equal to $\sqrt{2}$ times the radius of the base

29. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m. Find the dimensions of the rectangle that will produce the largest area of the window?

30. Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere?

LONG ANSWER QUESTIONS – Solutions

21.



V= $\pi x^2 y$

$=\pi(h-y)^2 \tan^2\alpha \cdot y$

$\frac{dv}{dx} = 0 \rightarrow$ y =h or h=h/3

$\frac{d^2y}{dx^2} < 0$ at y =h/3

Then V=$\frac{4}{27}\prod h^3 tan^2\alpha$

Qn.22)

Let r and h be the radius and height of the cone .Let x and y be the radius and height of the cylinder.

Curved surface area of the cylinder S= $\frac{2\pi h}{r}$ (rx-x^2)

$\frac{ds}{dx} = 0 \rightarrow$ x= r/2

$\frac{d^2s}{dx^2} < 0$ at x =r/3 , there S is maximum when x= r/2

Sol. Let x units be the side of the square base and y units be the height of the box.

$\therefore$ c^2 = Area of the base + Area of four walls

$$= x^2 + 4xy$$

$$\Rightarrow \quad y = \frac{c^2 - x^2}{4x} \qquad \text{...(i)} \quad \mathbf{1}$$

Now, volume of the box,

$$V = x^2 y$$

$$= \frac{x^2(c^2 - x^2)}{4x}$$

$$= \frac{1}{4}(c^2 x - x^3) \qquad \mathbf{1}$$

$$\therefore \quad \frac{dV}{dx} = \frac{1}{4}(c^2 - 3x^2)$$

and $$\frac{d^2V}{dx^2} = \frac{1}{4}(-6x) = -\frac{3}{2}x \qquad \mathbf{1}$$

Now, for maximum or minimum volume,

$$\frac{dV}{dx} = 0$$

$$\Rightarrow \quad \frac{1}{4}(c^2 - 3x^2) = 0$$

$$\Rightarrow \quad x = \frac{c}{\sqrt{3}} \qquad \mathbf{1}$$

Also, at $x = \frac{c}{\sqrt{3}}$, $\frac{d^2V}{dx^2} = -\frac{3}{2} \cdot \frac{c}{\sqrt{3}} < 0$

$\therefore$ V is maximum at $x = \frac{c}{\sqrt{3}}$ $\mathbf{1}$

and from (i), $y = \frac{c^2 - \frac{c^2}{3}}{4 \times \frac{c}{\sqrt{3}}} = \frac{c}{2\sqrt{3}}$

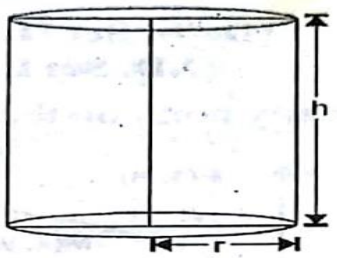
$\therefore$ Maximum value of $V = \frac{c^2}{3} \times \frac{c}{2\sqrt{3}}$

$$= \frac{c^3}{6\sqrt{3}} \qquad \mathbf{1}$$

Sol. Volume of a right circular cylinder,

$$V = \pi r^2 h$$

$$\Rightarrow \quad h = \frac{V}{\pi r^2}$$



1

Total surface area of a right circular cylinder, open at the top,

$$S = \pi r^2 + 2\pi r h$$

$$\Rightarrow \quad S = \pi r^2 + 2\pi r \frac{V}{\pi r^2}$$

$$\Rightarrow \quad S = \pi r^2 + \frac{2V}{r} \qquad \mathbf{1}$$

For minimum S,

$$\frac{dS}{dr} = 0$$

and $$\frac{dS}{dr} = 2\pi r - \frac{2V}{r^2} \qquad \mathbf{1}$$

$$\Rightarrow \quad 2\pi r - \frac{2V}{r^2} = 0$$

$$\Rightarrow \quad r^3 = \frac{V}{\pi}$$

or $$r = \left(\frac{V}{\pi}\right)^{1/3} \qquad \mathbf{1}$$

Now, $$\frac{d^2S}{dr^2} = 2\pi + \frac{4V}{r^3}$$

At $r = \left(\frac{V}{\pi}\right)^{1/3}$,

$$\frac{d^2S}{dr^2} = 2\pi + 4\pi = 6\pi > 0$$

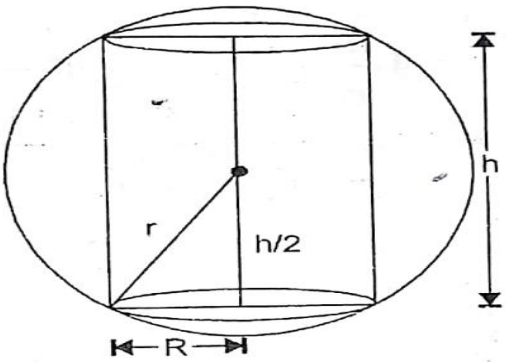
$\therefore$ At $r = \left(\frac{V}{\pi}\right)^{1/3}$, S is minimum **1**

25.

Sol. Since a right circular cylinder of radius 'R' and height 'h' is inscribed in a sphere of radius 'r'. Therefore we have

$$r^2 = \left(\frac{h}{2}\right)^2 + R^2$$

$$\Rightarrow \quad R^2 = \left(r^2 - \frac{h^2}{4}\right) \qquad \textbf{1}$$



1

Volume of the cylinder,

$$V = \pi R^2 h$$

$$\Rightarrow \quad V = \pi\left(r^2 - \frac{h^2}{4}\right)h$$

$$\Rightarrow \quad V = \pi r^2 h - \frac{\pi}{4}h^3$$

For maximum V,

$$\frac{dV}{dh} = 0$$

$$\therefore \quad \frac{dV}{dh} = \pi r^2 - \frac{3}{4}\pi h^2 = 0$$

$$\Rightarrow \quad h^2 = \frac{4r^2}{3}$$

$\Rightarrow \quad h = \frac{2r}{\sqrt{3}}$ 1

Also, $\frac{d^2V}{dh^2} = -\frac{3}{2}\pi h = -\sqrt{3}\pi r < 0$

At $h = \frac{2r}{\sqrt{3}}, \frac{d^2V}{dh^2} < 0$ 1

$\therefore$ Volume is maximum at

$$h = \frac{2r}{\sqrt{3}}$$

Maximum (largest) volume

$$= \pi\left(r^2 - \frac{h^2}{4}\right)h$$

$$= \pi\left(r^2 - \frac{r^2}{3}\right)\left(\frac{2r}{\sqrt{3}}\right) = \frac{4}{3\sqrt{3}}\pi r^3$$ 1

26.

V=$\frac{1}{3}\pi r^2\theta$

$l^2=r^2+h^2$

$r^{2=}l^2-h^2$

So , V=1/3 $\pi(l^2-h^{2)}$ h

$=\frac{1}{3}\pi\ (\ l^2h-h^3$

$\frac{dV}{dh}=\frac{1}{3}\pi(\ l^2-3h^2$

$\frac{dV}{dh}$ =0 implies $l^2=3h^2$

Implies h = $\frac{l}{\sqrt{3}}$

$\frac{d2y}{dx2}=\frac{1}{3}\pi$(-6h) = -ve

So V is max. when h = $\frac{l}{\sqrt{3}}$

When V is max. , tan θ =$\frac{r}{h}$ = $\frac{l^2-\frac{l^2}{3}}{\frac{l}{\sqrt{3}}}$ = $\sqrt{2}$

So θ = $\tan^{-1}\sqrt{2}$

Qn. 27) Let ABCD be a Rectangle inscribed in a given circle with centre at O and radius 'a'

Let AB= 2x and BC =2y

Then y= $\sqrt{a^2 - x^2}$

A= 4xy = 4x$\sqrt{a^2 - x^2}$

$\frac{dA}{dx}$ = 4 ($\frac{a^2-2x^2}{\sqrt{a^2-x^2}}$)

$\frac{dA}{dx}$ =0 implies x=a/2 then y= a/2

$\frac{d2A}{dx2} < 0$ at x=y

Hence area is maximum when the rectangle is a square

28.Let r be the radius,h be the height and l be the slant height of the cone.

V = 1/3 πr^2h

h =$\frac{3V}{\pi r^2}$

curved surface area S= πrl

$S^2 = \pi^2 r^2 l^2$

$= \pi^2 r^2 (h^2 + l^2)$

$Z = \pi^2 r^2 (\frac{9v^2}{\pi^2 r^4} + r^2)$

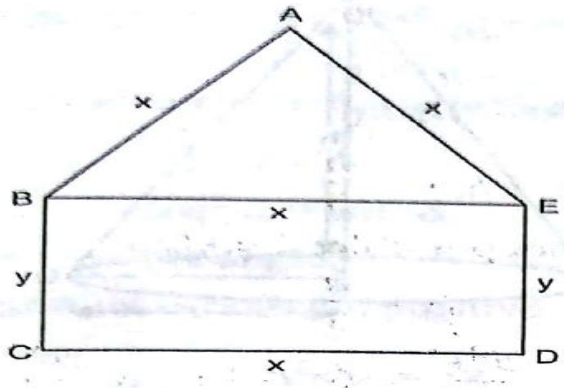
$\frac{dz}{dr}$ =0 implies $V^2 = \frac{4\pi^2 r^6}{18}$

Then h=$\sqrt{2}$r when $\frac{d2z}{dr2} > 0$

29.

OR

Let the side of the equilateral triangle = x and the breadth of the rectangle = y.



Therefore the perimeter of the complete window

$$3x + 2y = 12 \quad \ldots(1)$$ 1

Its area, $$A = \frac{\sqrt{3}}{4}x^2 + xy$$

$$\Rightarrow \quad A = \frac{\sqrt{3}}{4}x^2 + \frac{x}{2}[12 - 3x]$$

$$\Rightarrow \quad A = \frac{\sqrt{3}}{4}x^2 + 6x - \frac{3}{2}x^2$$

$$\Rightarrow \quad A = (\sqrt{3} - 6)\frac{x^2}{4} + 6x$$ 1

Differentiating w.r.t. x, we get

$$\frac{dA}{dx} = \frac{x}{2}(\sqrt{3} - 6) + 6 \quad \ldots(2)$$ 1

For maxima or minima of A,

$$\frac{dA}{dx} = 0$$

$$\Rightarrow \quad \frac{x}{2}(\sqrt{3} - 6) + 6 = 0$$ 1

$$\Rightarrow \quad x = \left(\frac{12}{6 - \sqrt{3}}\right)\left(\frac{6 + \sqrt{3}}{6 + \sqrt{3}}\right)$$

$$x = \frac{12(6 + \sqrt{3})}{33} = \frac{4}{11}(6 + \sqrt{3})$$

Differentiating equation (2) again w.r.t. x, we get

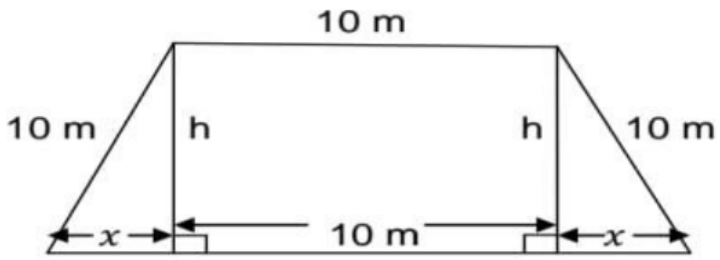
$$\frac{d^2A}{dx^2} = \frac{1}{2}(\sqrt{3} - 6) < 0$$ 1

Hence the area is maximum when $y=\frac{30-6\sqrt{3}}{11}$, $x=\frac{24+4\sqrt{3}}{11}$

CASE STUDY QUESTIONS

31) The front gate of a building is in the shape of a trapezium as shown below. Its three sides other than base are 10m each. The height of the gate is h meter. On the basis of this information and figure given below answer the following questions:





(i) . The area A of the gate expressed as a function of x is

a. (10+x)$\sqrt{100+x^2}$

b. (10-x)$\sqrt{100+x^2}$

c. (10+x)$\sqrt{100-x^2}$

d. (10-x)$\sqrt{100-x^2}$

(ii) . The value of $\frac{dA}{dx}$ is

a. $\frac{2x^2+10x-100}{\sqrt{100-x^2}}$

b. $\frac{2x^2-10x-100}{\sqrt{100-x^2}}$

c. $\frac{2x^2+10x+100}{\sqrt{100-x^2}}$

d. $\frac{-2x^2-10x+100}{\sqrt{100-x^2}}$

(iii). For which positive value of x , $\frac{dA}{dx}$ =0

a. 10

b. 5

c. 20

d. 15

(iv). If at the value of x where $\frac{dA}{dx}$ =0 area of trapezium is maximum then what is maximum area of trapezium ?

a. $25\sqrt{3}$ sqm

b. $100\sqrt{3}$ sqm

c. $75\sqrt{3}$ sqm

d. $50\sqrt{3}$ sqm

(v). If the area of trapezium is maximum then the value of $\frac{d^2A}{dx^2}$ is

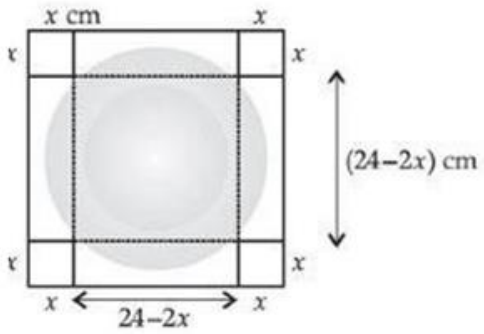
a. positive

b. negative

c. 0

d. none of these

32)An open box is to be made out of a piece of cardboard measuring 24 cm x 24 cm



by cutting of equal squares from the corners and turning up the sides.

Based on this information answer all the following Questions.

(i) The volume V(x) of the open box is

a) $4x^3 - 96x^2 + 576x$

b) $4x^3 + 96x^2 + 576x$

c) $2x^3 - 48x^2 + 288x$

d) $2x^3 + 48x^2 + 288x$

(ii) The value of dV/dx is

a) $12(x^2+16x-48)$
b) $12(x^2-16x+48)$
c) $12(x^2-16x-48)$
d) $12(x^2+16x+48)$

(iii) The value of d^2V/dx^2 is

a) 24(x+8)
b) 12(x-4)
c) 24(x-8)
d) 12(x+4)

(iv) For what value of the height, the volume of the open box is maximum

a) 3 cm

b) 9 cm

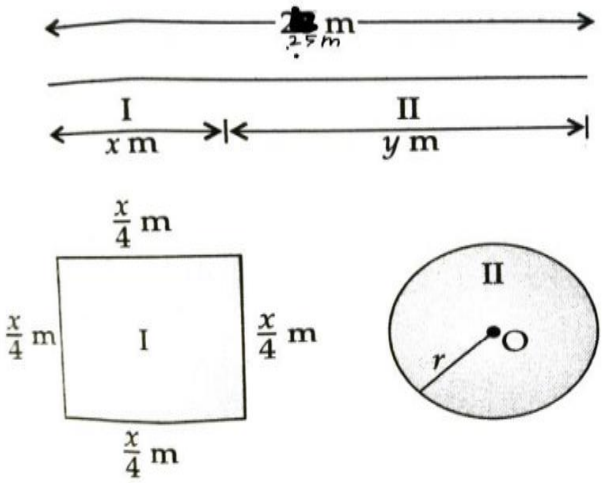
c) 1 cm

d) 4cm

(v) The volume is minimum if

a) $dV/dx=0$ and $d^2V/dx^2=0$
b) $dV/dx=0$ and $d^2V/dx^2<0$
c) $dV/dx=0$ and $d^2V/dx^2>0$
d) None of these

33) While constructing a house, a piece of wire of length 25cm is to be cut into pieces one of which is to bent into the form of a square and other into the form of a circle for the construction of two windows.





Based on the above information, answer the following question:

(i) What is the total area of the square and circle?

(a) $(x/4)^2 + \pi r^2$

(b) $(x/2)^2 + \pi r^2$

(c) $(x/4)^2 + \pi r$

(d) $(x/2)^2 + \pi r$

(ii) What is the relation of r with y?

(a) r = $y\ \pi$

(b) r = $y^2\pi$

(c) r = $xy\ \pi$

(d) r = $xy^2\pi$

(iii) If we talk about total length of wires then what is the relation between x and y?

(a) x+y =25

(b) x+y=28

(c) x+y =26

(d) x+y =27

(iv) When $dA\ dy$ = 0, then find the value of y

(a) 50π π+4 (ii) 75 π+8 (c) 25π π+4 (d) 100 π+8

(v) Again, when $\frac{dA}{dy}$ = 0, then the value of x

. (a) 50 π+4

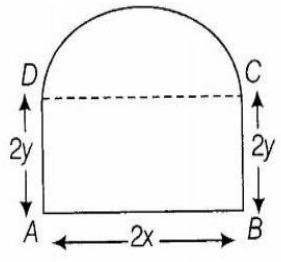
(b) 100 π+4

(c) 25 π+4

(d)50π π+4

5.The sum of the length hypotenuse and a side of a right-angled triangle is given by AC+BC = 10

34)

	An architect designs a building for a small company . The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semi circular opening. The window is having a perimeter of 10 meter as shown in the figure.  **Based on the above information answer the following:**	

(i)	**If 2x and 2y represents the length and breadth of the rectangular portion of the window, then the relation between the variables is** **(a) 4y – 2x = 10- π (b) 4y = 10- (2-π)x** **(c) 4y = 10- (2+π)x (d) 4y – 2x = 10 + π**	1
(ii)	**The combined area (A) of the rectangular region and semi-circular region of the window expressed as a function of x is** **(a) A= 10x + (2 + $\frac{\pi}{2}$) x^2 (b) A= 10x - (2+$\frac{\pi}{2}$) x^2** **(c) A= 10x - (2-$\frac{\pi}{2}$) x^2 (d) A = 4xy + $\frac{\pi}{2}x^2$, where y= $\frac{5}{2}$ + $\frac{1}{2}$ (2+π)x**	1
(iii)	**The maximum value of area of the whole window, A is** **(a) A = $\frac{50}{4+\pi}cm^2$ (b) A = $\frac{50}{4+\pi}m^2$ (c) A = $\frac{100}{4+\pi}m^2$ (d) A = $\frac{50}{4-\pi}m^2$**	1
(iv)	**The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible. For this to happen, the length of the rectangular portion of the window should be** **(a) $\frac{20}{4+\pi}$ m (b) $\frac{10}{4+\pi}$ m (c) $\frac{4}{10+\pi}$ m (d) $\frac{100}{4+\pi}$ m**	1

35) Scientist want to know the Oil- Reserves in sea so they travel over the sea along the curve f(x) = $(x+1)^3$ $(x-3)^3$ by an airplane. A student of class XII discuss the characteristic of the curve. Answer the following questions on the basis of the information given above

(i) The first order derivative of the given function is

(a) $3(x+1)^2(x-3)^2$

(b) $6(x+1)^2(x-3)^2(x-1)$

(c) 2(x-1)

(d) None of these

(ii) The critical point of the given function are

(a) -1,1,3

(b) 1,3,-2

(c) 1,2

(d) None of these

(iii) The interval in which the given function is strictly increasing is

(a) $(1,3) \cup (3,\infty)$

(b) $(-\infty, -1) \cup (-1,1)$

(c) $(1,3) \cup (-1,\infty)$

(d) None of these

(iv) The interval in which the given function is decreasing is

(a) $(1,3) \cup (3,\infty)$

(b) $(-\infty, -1) \cup (-1,1)$

(c) $(1,3) \cup (-1,\infty)$

(d) None of these

ANSWERS TO CASE STUDY QUESTIONS

Qn.31 (i) c (ii) d (iii) b (iv) c (v) b

Qn.32 (i) a (ii) b (iii) c (iv) d (v) c

Qn.33 (i) a (ii)b (iii) a (iv) c (v) b

Qn34 (i) (ii) (iii) (iv) (v)

Qn.35 (i) b (ii) a (iii)a (iv) b

CHAPTER 7 – INTEGRALS

DEFINITION OF INTEGRALS

Let $\frac{d}{dx}\mathrm{F}(x) = f(x)$. Then we write $\int f(x)\,dx = \mathrm{F}(x) + \mathrm{C}$. These integrals are called indefinite integrals or general integrals, C is called constant of integration. All these integrals differ by a constant.

PROPERTIES OF INTEGRALS

(I) The process of differentiation and integration are inverses of each other in the sense of the following results :

$$\frac{d}{dx}\int f(x)\,dx = f(x)$$

and $$\int f'(x)\,dx = f(x) + \mathrm{C}, \text{ where C is any arbitrary constant.}$$

(II) Two indefinite integrals with the same derivative lead to the same family of curves and so they are equivalent.

(III) $\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

(IV) For any real number k, $\int k\, f(x)\,dx = k\int f(x)\,dx$

(V) Properties (III) and (IV) can be generalised to a finite number of functions $f_1, f_2, \ldots, f_n$ and the real numbers, $k_1, k_2, \ldots, k_n$ giving

$$\int [k_1 f_1(x) + k_2 f_2(x) + \ldots + k_n f_n(x)]\,dx$$

$$= k_1\int f_1(x)\,dx + k_2\int f_2(x)\,dx + \ldots + k_n\int f_n(x)\,dx.$$

PRE-REQUISITE FORMULAS :

1. $sin^3x = \frac{3\,sinx - sin3x}{4}$

2. $cos^3x = \frac{3\,cosx + cos3x}{4}$

3. $cos^2 x = \frac{1+\cos 2x}{2}$

4. $sin^2 x = \frac{1-\cos 2x}{2}$

5. 2 cosx cosy= cos (x+y) +cos (x-y)

6. 2 sinx sin y = cos (x-y) – cos (x+y)

7. 2sinx cosy= sin(x+y) + sin (x-y)

8. 2cosx siny= sin (x+y) – sin (x-y)

3 .STANDARD INTEGRALS

(i) $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$. Particularly, $\int dx = x + C$

(ii) $\int \cos x \, dx = \sin x + C$ (iii) $\int \sin x \, dx = -\cos x + C$

(iv) $\int \sec^2 x \, dx = \tan x + C$ (v) $\int \operatorname{cosec}^2 x \, dx = -\cot x + C$

(vi) $\int \sec x \tan x \, dx = \sec x + C$

(vii) $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$ (viii) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$

(ix) $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$ (x) $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

(xi) $\int \frac{dx}{1+x^2} = -\cot^{-1} x + C$ (xii) $\int e^x dx = e^x + C$

(xiii) $\int a^x dx = \frac{a^x}{\log a} + C$ (xiv) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$

(xv) $\int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1} x + C$ (xvi) $\int \frac{1}{x} dx = \log|x| + C$

4.PARTIAL FRACTIONS

1. $\dfrac{px+q}{(x-a)(x-b)} = \dfrac{A}{x-a}+\dfrac{B}{x-b},\ a \neq b$

2. $\dfrac{px+q}{(x-a)^2} = \dfrac{A}{x-a}+\dfrac{B}{(x-a)^2}$

3. $\dfrac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \dfrac{A}{x-a}+\dfrac{B}{x-b}+\dfrac{C}{x-c}$

4. $\dfrac{px^2+qx+r}{(x-a)^2(x-b)} = \dfrac{A}{x-a}+\dfrac{B}{(x-a)^2}+\dfrac{C}{x-b}$

5. $\dfrac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \dfrac{A}{x-a}+\dfrac{Bx+C}{x^2+bx+c}$

where $x^2 + bx + c$ can not be factorised further.

5. INTEGRATION BY SUBSTITUTION

(i) $\int \tan x\,dx = \log|\sec x| + C$ (ii) $\int \cot x\,dx = \log|\sin x| + C$

(iii) $\int \sec x\,dx = \log|\sec x + \tan x| + C$

(iv) $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + C$

6. INTEGRATION OF SPECIAL FUNCTIONS

(i) $\int \dfrac{dx}{x^2-a^2} = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right| + C$

(ii) $\int \dfrac{dx}{a^2-x^2} = \dfrac{1}{2a}\log\left|\dfrac{a+x}{a-x}\right| + C$ (iii) $\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + C$

7. INTEGRATION BY PARTS

For given functions f_1 and f_2, we have

$$\int f_1(x)\cdot f_2(x)\,dx = f_1(x)\int f_2(x)\,dx - \int\left[\frac{d}{dx}f_1(x)\cdot\int f_2(x)\,dx\right]dx$$

$$\int e^x[f(x)+f'(x)]\,dx = \int e^x f(x)\,dx + C$$

8. INTEGRATION OF SPECIAL FUNCTIONS

(i) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

(ii) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

(iii) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

(iv) Integrals of the types $\int \frac{dx}{ax^2+bx+c}$ or $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ can be transformed into standard form by expressing

$$ax^2+bx+c = a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right] = a\left[\left(x+\frac{b}{2a}\right)^2+\left(\frac{c}{a}-\frac{b^2}{4a^2}\right)\right]$$

(v) Integrals of the types $\int \frac{px+q\,dx}{ax^2+bx+c}$ or $\int \frac{px+q\,dx}{\sqrt{ax^2+bx+c}}$ can be transformed into standard form by expressing

$px+q = A\frac{d}{dx}(ax^2+bx+c)+B = A\,(2ax+b)+B$, where A and B are determined by comparing coefficients on both sides.

DEFINITE INTEGRALS

9.PROPERTIES OF DEFINITE INTEGRALS

P_0 : $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

P_1 : $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$. In particular, $\int_a^a f(x)\,dx = 0$

P_2 : $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

P_3 : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

P_4 : $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

(Note that P_4 is a particular case of P_3)

P_5 : $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

P_6 : $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if $f(2a-x) = f(x)$ and

0 if $f(2a-x) = -f(x)$

P_7 : (i) $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is an even function, i.e., if $f(-x) = f(x)$.

(ii) $\int_{-a}^a f(x)\,dx = 0$, if f is an odd function, i.e., if $f(-x) = -f(x)$.

MIND MAPPING :



MULTIPLE CHOICE QUESTIONS:

1. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

a.2π *b.* $\frac{\pi}{12}$ c. 4 d. $\frac{\pi}{3}$

2. $\int_0^{\frac{\pi}{2}} log\,\frac{4+3sinx}{4+3cosx}$ dx equals

a.2 b. ¾ *c. 0* d. -2

3. $\int_e^{e^2} \frac{dx}{xlogx}$ equal to

a.2 b. *log2* *c. 0* d. 1

4. $\int_{-1}^{1} |x|\,dx$ is

2 b. -1 c. 1 d. 0

5. $\int_0^{\frac{\pi}{4}} \frac{cosx}{sin^2x}$dx is

a.2 b. $\sqrt{2}$ c. 1 d. -$\sqrt{2}$

SHORT ANSWER QUESTIONS :

1) Find $\int \frac{\log(sinx)}{tanx}$ dx
2) Find $\int \frac{\sqrt{tanx}}{sin\,x\cos x}$ dx
3) $\int tan^8 x sec^4$dx
4) $\int x^2\,logx\,dx$
5) $\int tan^4 x\ dx$= 1/3tan^3x-tanx +x +C
6) Evaluate $\int \frac{Cos2x-Cos2\propto}{Cosx-Cos\propto}\ dx$
7) Evaluate $\int_2^8 |x-5|$dx
8) Find $\int \sqrt{10-4x+4x^2}$ dx
9) Find$\int x\,sin^{-1}x\,dx$
10) Find $\int_0^{\frac{\pi}{4}} \frac{1+tanx}{1-tanx}\ dx$
11) Evaluate $\int e^x \left(\frac{1+sinx}{1+cosx}\right) dx$
12) Find $\int \frac{x^3}{x^4+3x^2+2}$ dx
13) Evaluate $\int \frac{dx}{2sin^2x+5\,cos^2x}$
14) $\int \frac{e^x}{(1+e^x)(2+e^x)}$ dx
15) $\int \frac{(x^2+1)e^x}{(x+1)^2}\ dx$
16) $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$
17) $\int \frac{e^x}{e^{2x}+6e^x+8}$ dx
18) $\int \frac{1}{sinxcos^3x}\ dx$

19) $\int \frac{\sin(x-a)}{\sin(x+a)}$ dx

20) $\int \frac{dx}{x(x^4-1)}$

LONG ANSWER QUESTIONS:

1.$\int \frac{sin^8x-cos^8x}{1-2sin^2xcos^2x}$ dx

2. Evaluate$\int \frac{x+2}{2x^2+6x+5}\ dx.$

3.Evaluate$\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$ dx

4. Find $\int \frac{x^2}{x^4+x^2-2}\ dx$

5. $\int \frac{2}{(1-x)(1+x^2)} dx$

6. $\int \frac{3x-1}{(x+2)^2}$ dx

7. Find $\int tan^{-1}\sqrt{\frac{1-x}{1+x}}$ dx

8. Evaluate $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{10-x}+\sqrt{x}}$ dx

9.Evaluate $\int_{-1}^{2}|x^3-x|\ dx.$

10.$\int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{9+16sin2x}$ dx

11. $\int_0^{\pi} \frac{x\ tanx}{secx+tanx}$dx

12.$\int_0^{\pi/2} \frac{sin^2x}{sinx+cosx}$ dx

13.$\int \frac{x^4}{(x+1)(x^2+1)}$dx

14. $\int_0^1 \frac{\log(1+x)}{1+x^2}$ dx

15. $\int_0^{\frac{\pi}{2}} \frac{xsinxcosx}{sin^4x+\ cos^4x}$ dx

CASE STUDY QUESTIONS :

CASE STUDY 1:

For any function f(x) , we have

$\int_a^b f(x)\ dx = \int_a^{c_1} f(x)\ dx + \int_{c_1}^{c_2} f(x)\ dx + \cdots + \int_{c_n}^{b} f(x)\ dx$; where $a < c_1 < c_2 < \cdots . < c_n < b$

On the basis of the above information , answer the following questions

$i)\ \int_{-1}^{1} |x|\ dx$

(A)1 (B)2 (C) − 1 (D)0

$ii)\ \int_0^2 |x - 1|\ dx$

(A)2 (B)1 (C) − 1 (D)0

$iii)\ \int_0^{\pi} |\cos x|\ dx$

(A) $\frac{\pi}{2}$ (B)2 (C) $\frac{\pi}{4}$ (D)0

$iv)\ \int_0^2 [x]\ dx$; where $[x]$ is the greatest integer function

(A)2 (B)1 (C) − 1 (D)0

CASE STUDY 2:

The given integral $\int f(x)dx$ can be transferred into another form by changing the independent variable *'x '* to 't' by substituting $x = g(t)$

Consider $I = \int f(x)dx$

Put $x = g(t)\ and \frac{dx}{dt} = g'(t)dt$

Thus $I = \int f(x)dx = \int f\big(g(t)\big)g'^{(t)}dt$

This change of variable formula is known as Integration by substitution.

$i)\ \int \frac{1}{x + x\log x} dx$

(A)$\log|1 + \log x| + C$ (B) $\log x + C$ (C)$\log|x + x\log x| + C$(D)$\log|x + \log x|$

$ii)\ \int \frac{(\sin^{-1} x)^2}{\sqrt{1 - x^2}} dx$

(A) $\frac{(\sin^{-1} x)^2}{2} + C$ (B) $\frac{(\sin^{-1} x)^3}{3} +$ (C) $\frac{\sin^{-1} x}{2} + C$ (D) $\sin^{-1} x + C$

$iii)\ \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

(A) $-2 \sin \sqrt{x} + C$ (B)$2 \cos \sqrt{x} + C$ (C) $-2 \cos \sqrt{x} + C$ (D)$\sqrt{x} + C$

$iv)\ \int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

(A) e (B) $2(e - 1)$ (C) $e - 1$ (D) $e + 1$

CASE STUDY 3:

The given integral $\int f(x)dx$ can be transformed into another form by changing the independent variable x to t by substituting x = g(t)

Consider $I = \int f(x)dx$

Put x = g(t) so that $\frac{dx}{dt}$ = g'(t)

d(x) = g'(t) dt

Thus $I = \int f(x)dx = \int f\big(g(t)\big)g'(t)dt$

This change of variable formula is one of the important tools available in the name of integration by substitution

(i) $\int 2x \sin(x^2 + 1)dx$ is equal to

A) $\cos(x^2 + 1) + C$ B) $-\cos(x^2 + 1) + C$ C) $\sin(x^2 + 1) + C$

D) $-\sin(x^2 + 1) + C$

(ii) $\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx$ is equal to

A) $-sin(\tan^{-1} x) + C$ B) $-cos(\tan^{-1} x) + C$ C) $cos(\tan^{-1} x) + C$ D) $sin(\tan^{-1} x) + C$

(iii) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ is equal to

A) $\frac{(\sin^{-1} x)^2}{2}$ + C B) $\frac{(\cos^{-1} x)^2}{2}$ + C C) $\frac{(\tan^{-1} x)^2}{2}$ + C D)None of these

(iv) $\int \frac{sinx}{(1+cosx)^2}$ dx is equal to

A) sinx + C B) $\frac{1}{1+cosx}$ + C C) (1 + cosx) +C D) None of these

CASE STUDY 4:

"The integration of the product of two functions = (first function) x (integral of the second function) - Integral of [(differential coefficient of the first function) x (Integral of the second function)]". This is found quite useful in integrating product of functions .

Based on the above information answer the following.

(i) $\int x tan^{-1} x \, dx$ is equal to:

A) $\frac{x^2}{2}\tan^{-1} x + \frac{x}{2} + \frac{\tan^{-1} x}{2} + C$ B) $\frac{x^2}{2}\tan^{-1} x - \frac{x}{2} - \frac{\tan^{-1} x}{2} + C$

C) $\frac{x^2}{2}\tan^{-1} x + \frac{\tan^{-1} x}{2} + C$, D) $\frac{x^2}{2}\tan^{-1} x - \frac{x}{2} + \frac{\tan^{-1} x}{2} + C$

(ii) $\int logx \, dx$ is equal to:

A) x log x - x + C , B) x log x + x + C

C) log x - x + C, D) None of these

(iii) $\int x \, sec^2 x \, dx$ is equal to:

A) x tan x + log |sec x| + C B) x tan x - log |sec x| + C

C) tan x - log |sec x| + C D) None of these

(iv) $\int x \, log 2x \, dx$ is equal to:

A) $\frac{x^2}{2}\log 2x - \frac{x^2}{4} + C$ B) $\frac{x^2}{2}\log 2x + \frac{x^2}{4} + C$

C) $\log 2x - \frac{x^2}{4} + C$ D)None of these

CASE STUDY 5:

$\int e^x(f'(x) + f(x))dx = \int e^x f'(x)dx + \int f(x)e^x dx$

$= \int e^x f'(x)dx + f(x)e^x - \int f'(x)e^x dx + C$

$= f(x)e^x + C$

Based on the above information answer the following.

(i) $\int e^x(sinx + \cos x)\, dx$ is equal to:

A)$-e^x sinx + C$, B) $e^x cosx + C$, C)$e^x sinx + C$, D)None of these

(ii) $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right)$ is equal to:

A) $\frac{e^x}{x} + C$, B) $\frac{e^{2x}}{x} + C$, C) $\frac{e^x}{2x} + C$, D) None of these

(iii)$\int e^x \left(\frac{1-sinx}{1-cosx}\right) dx$ is equal to:

A) $e^x cot\frac{x}{2} + C$, B) $e^{2x} cot\frac{x}{2} + C$, C)$-e^x cot\frac{x}{2} + C$, D)None of these

(iv)$\int e^x \left(\frac{x^2+1}{(x+1)^2}\right) dx$ is equal to:

A) $\frac{x-1}{x+1}e^x + C$, B) $\frac{x+1}{x-1}e^x + C$, C) $2x\, e^x + C$, D)None of these

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ANSWERS:

ANSWERS OF MCQ:

1.b 2. c 3. b 4.c 5. d

ANSWERS OF SHORT ANSWER QUESTIONS:

1.$\frac{(\log sinx)^2}{2} + c$ (Hint : put log (sinx) = t

2.$\int \frac{\sqrt{\tan x}}{\sin x\, cosx}\, dx = \int \frac{\sqrt{tanx}}{tanx}.sec^2 x\, dx$

Let tanx = t, $\sec^2 x\, dx = dt$

$\int \frac{1}{\sqrt{t}} dt = 2\sqrt{t} = 2\sqrt{tanx} + C$

3. $\int tan^8 x sec^4\, dx = \int tan^8 x(tan^2 x + 1)\, sec^2 x\, dx =$

$$= \int tan^{10} x sec^2 x\, dx + \int tan^8 x sec^2 x\, dx \quad (\tan x = t, sec^2\, x\, dx = dt)$$

$$= \frac{tan^{11} x}{11} + \frac{tan^9 x}{9} + C$$

4. $\int x^2 logx\, dx = \int logx.x^2\, dx = logx.\frac{x^3}{3} - \int \frac{1}{x}.\frac{x^3}{3}\, dx = logx.\frac{x^3}{3} - \frac{1}{3}\int x^2 dx$

$$= logx.\frac{x^3}{3} - \frac{x^3}{9} + c$$

5. $\int tan^4 x\, dx = 1/3 tan^3 x - tanx + x + C$

6 $\int \frac{Cos2x - Cos2\propto}{Cosx - Cos\propto} dx = 2(\sin x + x\cos\alpha) + C$

(Hint : $\cos 2x = 2cos^2 x - 1$ & $\cos 2\alpha = 2cos^2\alpha - 1$

7. $\int_2^8 |x-5| dx = 9$, (Hint : $\int_{-2}^{5} -(x-5)dx + \int_5^8 (x-5)dx$

$$I = \int \sqrt{10 - 4x + 4x^2}\, dx = \int \sqrt{(2x-1)^2 + (3)^2}\, dx$$

Put $t = 2x - 1$, then $dt = 2dx$.

Therefore, $I = \frac{1}{2}\int \sqrt{t^2 + (3)^2}\, dt$

$$= \frac{1}{2} t \frac{\sqrt{t^2+9}}{2} + \frac{9}{4} \log\left|t + \sqrt{t^2+9}\right| + C$$

$$= \frac{1}{4}(2x-1)\sqrt{(2x-1)^2 + 9} + \frac{9}{4} \log\left|(2x-1) + \sqrt{(2x-1)^2 + 9}\right| + C.$$

8.

9.

$$\int x\sin^{-1}x\,dx = \sin^{-1}x\cdot\frac{x^2}{2} - \frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2\cdot\sin^{-1}x}{2} + \frac{1}{2}\left\{\int\sqrt{1-x^2}\,dx - \int\frac{1}{\sqrt{1-x^2}}dx\right\}$$

$$= \frac{x^2\cdot\sin^{-1}x}{2} + \frac{1}{2}\left\{\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x - \sin^{-1}x\right\}$$

$$\text{or } \frac{x^2\cdot\sin^{-1}x}{2} + \frac{x\sqrt{1-x^2}}{4} - \frac{1}{4}\cdot\sin^{-1}x + C$$

10. $\int_0^{-\frac{\pi}{4}} \frac{1+tanx}{1-tanx}\,dx = \int_0^{-\frac{\pi}{4}} tan\left(\frac{\pi}{4}+x\right)dx = \log\ sec\left(\frac{\pi}{4}+x\right)_0^{-\frac{\pi}{4}}$

$= logsec0 - logsec\,\pi/4 \quad = log1 - log\sqrt{2}$

$=0 - -\frac{1}{2}\ log2 = -1/2 \log 2$

11. $\int e^x\left(\frac{1+sinx}{1+cosx}\right)dx = \int e^x\left(\frac{1}{1+cosx} + \frac{sinx}{1+cosx}\right)dx = \int e^x\left(\frac{1}{2cos^2\frac{x}{2}} + \frac{2sin\frac{x}{2}cos\frac{x}{2}}{2cos^2\frac{x}{2}}\right)dx =$

$\int e^x\left(\frac{1}{2}sec^2\frac{x}{2} + \tan\frac{x}{2}\right)dx = e^x tan\frac{x}{2} + c$

12. $\int \frac{x^3}{x^4+3x^2+2}$ dx (Hint : put x^2=t, and partial fraction, A= -1, B = 2)

= $\log\left|\frac{x^2+2}{\sqrt{x^2+1}}\right|$ +C

13. $\int \frac{dx}{2sin^2x+5\,cos^2x}$ (Hint: divide by cos^2x)

Put $\tan x = t$ so that $\sec^2 x\,dx = dt$. Then

$$\mathrm{I} = \int\frac{dt}{2t^2+5} = \frac{1}{2}\int\frac{dt}{t^2+\left(\sqrt{\frac{5}{2}}\right)^2}$$

$$= \frac{1}{2}\frac{\sqrt{2}}{\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{2}t}{\sqrt{5}}\right) + \mathrm{C}$$

$$= \frac{1}{\sqrt{10}}\tan^{-1}\left(\frac{\sqrt{2}\tan x}{\sqrt{5}}\right) + \mathrm{C}.$$

14. $\int \frac{e^x}{(1+e^x)(2+e^x)}$ dx (Hint: e^x=t) Ans: $\log\left|\frac{1+e^x}{2+e^x}\right|$ +C

15. $\int \frac{(x^2+1)e^x}{(x+1)^2}\,dx = e^x\left(\frac{x-1}{x+1}\right)$ +C (Hint : $\int e^x\left(\frac{x^2-1+1+1}{(x+1)^2}\right)$ dx= $\int e^x\left(\frac{x^2-1}{(x+1)^2}+\frac{2}{(x+1)^2}\right)$dx

$= \int e^x\left(\frac{x-1}{x+1}+\frac{2}{(x+1)^2}\right)$dx , f(x)= $\frac{x-1}{x+1}$, f'(x) =$\frac{2}{(x+1)^2}$)

16. $\int \frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(a-b)}\log\left|\frac{\cos(x-a)}{\cos(x+a)}\right|$ +C (Hint : 1 = $\frac{\sin[(x-b)-(x-a))]}{\sin(a-b)}$

17. $\int \frac{e^x}{e^{2x}+6e^x+8}$ dx = ½ $\log\left|\frac{e^x+2}{e^x+4}\right|$ +C (Hint :e^x= t)

18. $\int \frac{1}{sinxcos^3x}\,dx$= log|$tanx$| +1/2$tan^2$x + C (Hint : In the Denominator divide and multiply by cosx)

19.

$$\int \frac{\sin(x-a)}{\sin(x+a)}dx$$

Let , $(x+a)=t$

$$\int \frac{\sin(t-2a)}{\sin t}dt = \int \frac{\sin t\cos 2a-\cos t\sin 2a}{\sin t}dt = \int(\cos 2a-\cot t\sin 2a)dt$$

$$\therefore \int \frac{\sin(t-2a)}{\sin t}dt = (\cos 2a)t-(\sin 2a)\log|\sin t|+C$$

20. $\int \frac{dx}{x(x^4-1)}$ = 1/4 $\log\left|\frac{x^4-1}{x^4}\right|$ +C (Hint : Multiply and divide by x^3, and partial fraction)

ANSWERS OF LONG ANSWER QUESTIONS:

1. $\int \frac{sin^8x-cos^8x}{1-2sin^2xcos^2x}$ dx = $\int \frac{(sin^4x+cos^4x)(sin^4x-cos^4x)}{(sin^2x+cos^2x)^2-2sin^2xcos^2x}$ =

$\int \frac{(sin^4x+cos^4x)(sin^2x+cos^2x)(sin^2x-cos^2x)}{(sin^4x+cos^4x)}$ = $\int -cos\,2x\,dx = -\frac{1}{2}sin2x+C$

2. $\int \frac{x+2}{2x^2+6x+5}\,dx$= 1/4log|$2x^2+6x+5$| +1/2$tan^{-1}$(2x+3) +C

(Hint: x +2 = A (4x +6) + B

3. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$ dx= .$-2\sqrt{1-x}+cos^{-1}\sqrt{x}+\sqrt{x-x^2}$

4. Find $\int \frac{x^2}{x^4+x^2-2}\ dx$

Consider $\frac{x^2}{x^4+x^2-2}$ and put $x^2 = y$

$$\therefore \quad \frac{x^2}{x^4+x^2-2} = \frac{y}{y^2+y-2} = \frac{y}{(y+2)(y-1)}$$

Let $\quad \frac{y}{(y+2)(y-1)} = \frac{A}{y+2} + \frac{B}{y-1}$

$$\therefore \quad y = A(y-1) + B(y+2)$$

Put $y = 1$, $\quad 1 = A(0) + B(3)$

or $\quad B = \frac{1}{3}$

Put $y = -2$, $\quad -2 = A(-3) + B(0)$

or $\quad A = \frac{2}{3}$

$$\therefore \quad \frac{x^2}{x^4+x^2-2} = \frac{\frac{2}{3}}{(x^2+2)} + \frac{\frac{1}{3}}{(x^2-1)}$$

$$= \frac{2}{3(x^2+2)} + \frac{1}{3(x^2-1)}$$

$$\therefore \int \frac{x^2}{(x^2+2)(x^2-1)}dx = \frac{2}{3}\int \frac{1}{x^2+2}dx + \frac{1}{3}\int \frac{1}{x^2-1}dx$$

$$= \frac{2}{3}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right] + \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|\right] + C$$

$$= \frac{\sqrt{2}}{3}\tan^{-1}\frac{x}{\sqrt{2}} + \frac{1}{6}\log\left|\frac{x-1}{x+1}\right| + C$$

5. $\int \frac{2}{(1-x)(1+x^2)}dx$ = -log$|x-1|$ +1/2 log$|1+x^2|$ +tan^{-1}x +C

(Hint : $\frac{2}{(1-x)(1+x^2)} = \frac{A}{(1-x)} + \frac{Bx+C}{(1+x^2)}$

6. $\int \frac{3x-1}{(x+2)^2}$ dx= 3 log$|x+2|$ +$\frac{7}{x+2}$ +C (Hint : $\frac{3x-1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$)

7. $\int tan^{-1}\sqrt{\frac{1-x}{1+x}}$ dx $= \frac{1}{2}\left(xcos^{-1}x - \sqrt{1-x^2}\right)$ +C

(Hint : x= cos $2\theta, then\ by\ parts$)

8. $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{10-x}+\sqrt{x}}$ dx =3 (Hint: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

9. $\int_{-1}^2 |x^3 - x|\ dx$= 11/4 (Hint: $\int_{-1}^0 (x^3-x)\ dx + \int_0^1 -(x^3-x)dx + \int_1^2 (x^3-x)\ dx$)

10. $\int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{9+16sin2x}$ dx= $\frac{1}{40}$ log 9

(Hint : sin x-cosx = t, (sinx +cosx)dx = dt

$(-cosx + sinx)^2 = t^2$, 1- sin2x =t^2, sin2x = 1-t^2

I = $\int_{-1}^{0} \frac{dt}{25-16t^2}$

= $\frac{1}{40}$ log 9 (After applying limit in 1/40 log $\left|\frac{5+4t}{5-4t}\right|$

11. $\int_0^{\pi} \frac{x\,tanx}{secx+tanx}$dx = $\frac{\pi}{2}(\pi-2)$

(Hint : $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

2 I= $\pi \int_0^{\pi} \frac{tanx}{secx+tanx}$,

I = $\pi/2 \int_0^{\pi} \left(\frac{tanx\,(secx-tanx)}{(secx+tanx)(secx-tanx)}\right) dx$

= $\pi/2 \int_0^{\pi} (secx\,tanx - tan^2 x)dx$ integrate & applying the limits for getting the answer.

12.$LetI = \int_0^{\pi/2} \frac{sin^2 x}{sinx+cosx}$ dx & applying $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

2I = $\int_0^{\pi/2} \frac{1}{sinx+cosx}$ dx= $\int_0^{\pi/2} \frac{1}{\sqrt{2}} \left(\frac{dx}{\frac{1}{\sqrt{2}}sinx+\frac{1}{\sqrt{2}}cosx}\right)$

= $\int_0^{\pi/2} \frac{1}{\sqrt{2}}\left(\frac{dx}{\sin(x+\frac{\pi}{4})}\right)$dx

= $\int_0^{\pi/2} cosec\left(x+\frac{\pi}{4}\right)$dx

= $\frac{1}{\sqrt{2}}\left[log\left|cosec\left(x+\frac{\pi}{4}\right) - \cot(x+\pi/4\right|\right]_0^{\frac{\pi}{2}}$

= $\frac{1}{\sqrt{2}}$ log ($\sqrt{2}$ +1) After applying limit

13.$\int \frac{x^4}{(x-1)(x^2+1)}$dx = $\frac{x^2}{2}$ +x +1/2 log $|x-1|$ -1/4 log$|x^2+1|$ -1/2 $tan^{-1}x$ +C

(Hint: degree of numerator is greater than denominator , so divide.

$\frac{x^4}{(x-1)(x^2+1)}$ = (x+1) + $\frac{1}{(x-1)(x^2+1)}$, use partial fraction)

14.$\int_0^1 \frac{\log(1+x)}{1+x^2}$ dx = $\frac{\pi}{8}$ log 2

(Hint: Put x= tanθ, $to\ get\ \int_0^{\pi/4} \log(1+tan\theta)d\theta$ and then applying $\int f(x)dx = \int f(a-x)dx$)

15. $\int_0^{\frac{\pi}{2}} \frac{xsinxcosx}{sin^4x+cos^4x}$ dx= .$\frac{\pi^2}{16}$ (Hint: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and then divide numerator and denominator by cos^4x and put tan^2x = t)

ANSWERS OF CASE STUDY:

CASE STUDY 1:

(i)(A)1 (ii) (B)1 (iii) (B)2 (iv) (B)1

CASE STUDY 2:

(i)$(A)\log|1+\log x|+C$ (ii) (B) $\frac{(\sin^{-1}x)^3}{3}+C$ (iii) (C) $-2\cos\sqrt{x}+C$

(iv) (B)$2(e-1)$

CASE STUDY 3:

(i) B) $-\cos(x^2+1)+C$ (ii) B) $-cos(\tan^{-1}x)+C$, (iii) A) $\frac{(\sin^{-1}x)^2}{2}+C$

(iv) B) $\frac{1}{1+cosx}+C$

CASE STUDY 4:

(i) D) $\frac{x^2}{2}\tan^{-1}x-\frac{x}{2}+\frac{\tan^{-1}x}{2}+C$, (ii) A) x log x - x + C , (iii) B)x tan x - log |sec x| + C

(iv) A) $\frac{x^2}{2}\log 2x-\frac{x^2}{4}+C$

CASE STUDY 5:

(i) C)$e^x sinx+C$ (ii) A) $\frac{e^x}{x}+C$ (iii) C) $-e^x cot\frac{x}{2}+C$ (iv)A) $\frac{x-1}{x+1}e^x+C$

****

CHAPTER 8

APPLICATIONS OF INTEGRALS

LEARNING OUTCOMES

- Area under the simple curves
- Area of the region bounded by a curve and a line

Standard form of curves

- **Parabolas**

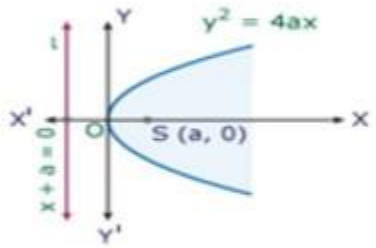


Fig.i

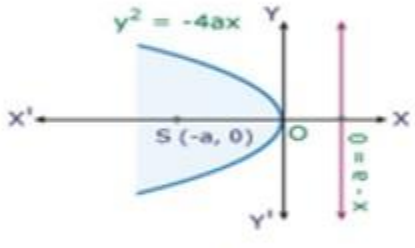


Fig.ii

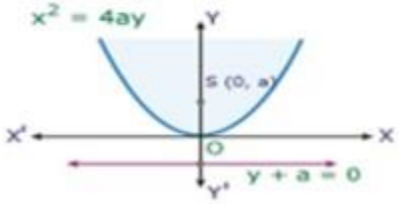


Fig.iii

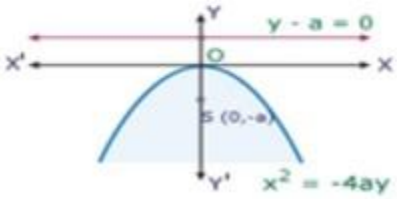
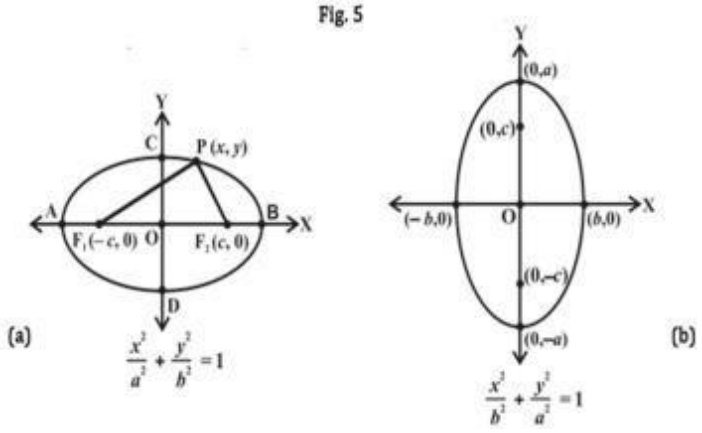


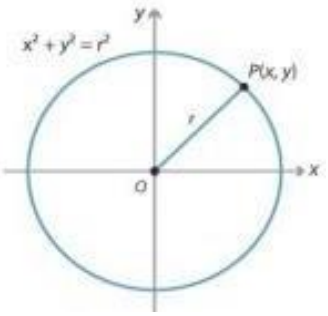
Fig.iv

ELLIPSE

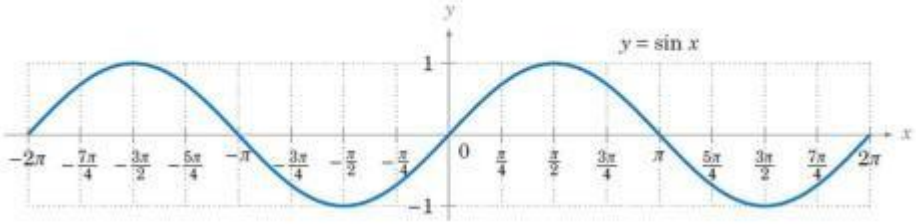
Fig. 5



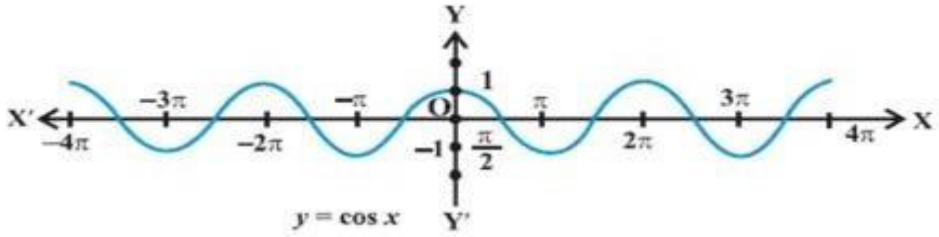
CIRCLE



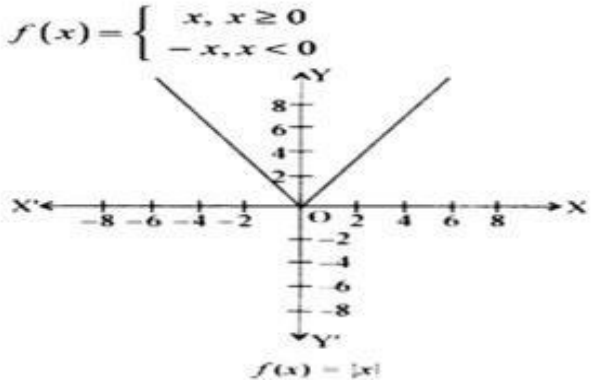
- **Sine curve**



COSINE CURVE

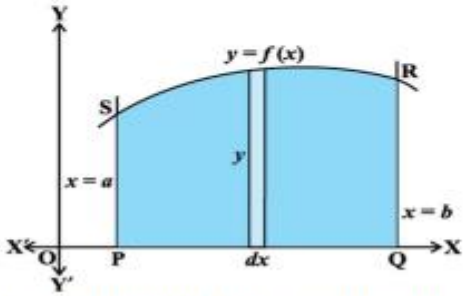


- **Modulus function**

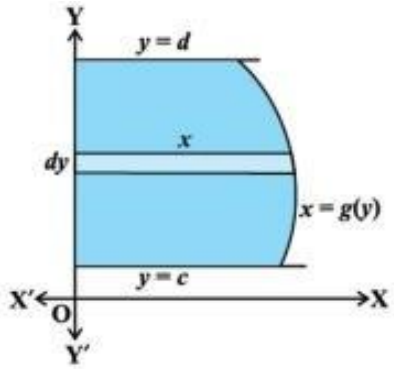


Area under simple curves

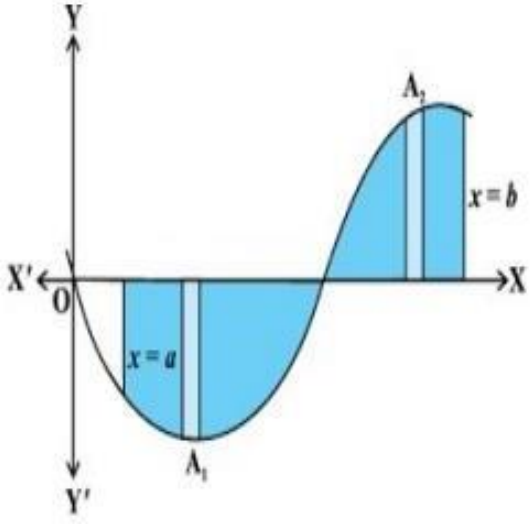
- The area A of the region bounded by the curve y=f(x), x-axis and the line x=a and x=b is given by A $= \int_a^b y\,dx = \int_a^b f(x)\,dx$



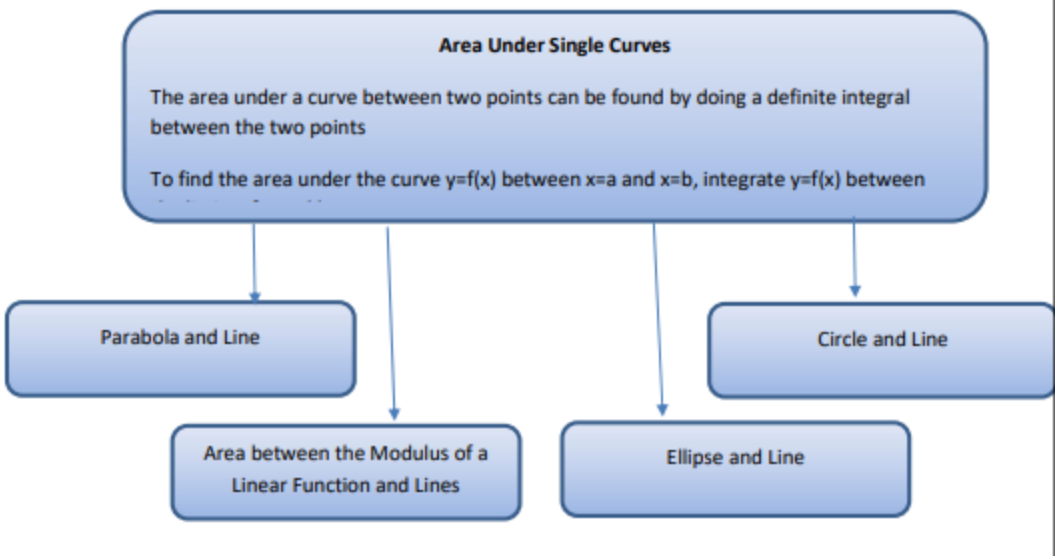
- The area A of the region bounded by the curve x=f(y), y-axis and the line y=c and y=d is given By $A = \int_c^d x\,dy = \int_c^d f(y)\,dy$



- The area A of the region bounded by the curve y=f(x), x-axis and the line x=a and x=b is given by A = $|A_1| + A_2$

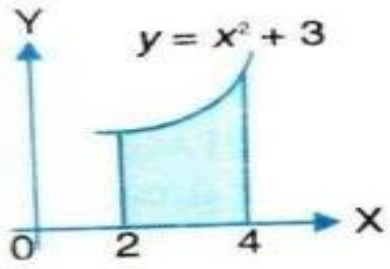


MIND MAP



MULTIPLE CHOICE QUESTIONS :

Q.1. The expression for finding the area of the shaded region is ----------



A) $\int_0^4 (x^2 + 3)\,dx$ (B) $\int_2^4 (x^2 + 3)dx$ (C) $\int_2^4 (x^2 + 4)\,dx$ (D) $\int_0^4 (x^2 - 3)dx$

Q.2.

The area of the region bounded by the circle $x^2 + y^2 = 1$ is

(A) 2π sq. units **(B)** π sq. units

(C) 3π sq. units **(D)** 4π sq. units

Q.3

. **Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is**

(A) 2 sq. units **(B)** 4 sq. units

(C) 3 sq. units **(D)** 1 sq. unit

Q.4

. **Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is**

(A) 2 **(B)** $\frac{9}{4}$

(C) $\frac{9}{3}$ **(D)** $\frac{9}{2}$

Q.5

. **The area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$ and $x = 3$ is**

(A) $\frac{7}{2}$ sq. units **(B)** $\frac{9}{2}$ sq. units

(C) $\frac{11}{2}$ sq. units **(D)** $\frac{13}{2}$ sq. units

SHORT ANSWER QUESTIONS:

Q.6 Find the area of the region bounded by the ellipse $\frac{x^2}{25}+\frac{y^2}{16} = 1$

Q.7 Find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line x + y = 2

Q.8 Find the area lying between the curves $y^2 = 4ax$ and y=2x

Q.9.Sketch the graph of y=$|x + 3|$ and evaluate $\int_{-6}^{0}|x + 3|\, dx$

Q.10 Find the area bounded by the curve y= x^3 ,the x-axis and the ordinates x=-2 and x=1

Q.11.Find the area of the region bounded by the line y=3x+2 , the x-axis and the ordinates $x = -1\ and\ x = 1$.

Q.12 Find the area under the given curve y=x^2 , x=-1, x=1 and x-axis

Q.13 Find the area bounded by the curve y= sin x between x=0 and x=2π

Q.14 Compute the area of the region bounded by the curve y= 2^x and the lines x=1 to x=3

Q.15Find the area bounded by the curve y= Cot x, X-axis and the lines $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{4}$

LONG ANSWER QUESTIONS:

Q. 16. Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

Q.17 Find the area of the region

$$\{(x, y) : x^2 + y^2 \leq 1 \leq (x + y)\}.$$

Q. 18. Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area using integration

Q.19

Using integration find the area of the region:
$\{(x, y) : 0 \leq y \leq x^2,\ 0 \leq y \leq x, 0 \leq x \leq 2\}$

Q .20

. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the straight line $3x + 4y = 12$.

Q.21 Find the area bounded by the curves $y=\sqrt{x}$, 2y+3=x and x-axis

Q.22 Find the area of the region bounded by the curves $x^2 + y^2 = 4$, y= $\sqrt{3}\,x$ and x -axis in the first quadrant

Q.23 Find the area of the region bounded by the parabola y^2 =x and the line 2y= x.

Q.24 Find the area of the region in the first quadrant enclosed by the Y-axis, the line y=x and the circle $x^2 + y^2 = 32$, using integration

Q.25 Using integration, find the area of the region bounded by the line x-y+2 = 0, the curve x=$\sqrt{y}$ and the Y -axis

CASE STUDY QUESTIONS :

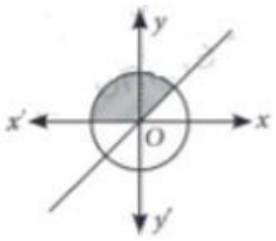
1. Consider the curve $x^2 + y^2 = 16$, and the line $y = x$ in the first quadrant.

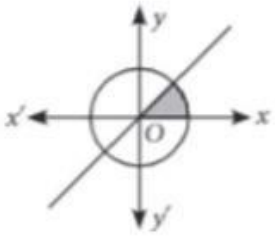
Based on the above information, answer the following questions.

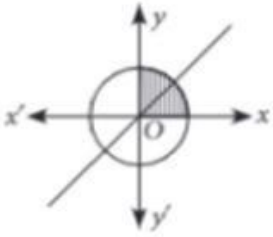
(i) Point of intersection of both the given curves is

(a) (0,4) (b) $(0, 2\sqrt{2})$ (c) $(2\sqrt{2}, 2\sqrt{2})$ (d) $(2\sqrt{2}, 4)$

(ii) Which of the following shaded portion represent the area bounded by the giventwo curves

a) 

(b) 

(c) 

d) none of these

(iii) The value of the integral $\int_0^{2\sqrt{2}} x\, dx$ is

(a) 0 (b) 1 (c) 2 (d) 4

(iv) The value of the integral $\int_{2\sqrt{2}}^{4} \sqrt{16 - x^2}$ is

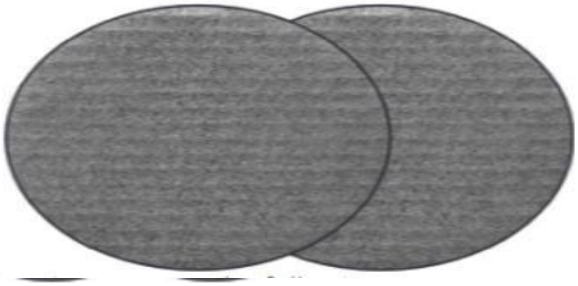
(a) $2(\pi - 2)$ (b) $2((\pi - 8)$ (c) $4(\pi - 2)$ (d) $4(\pi + 2)$

(v) Area bounded by the two given curves is

(a) 3π sq.units (b) $\frac{\pi}{2}$ sq units (c) π sq units (d) $2\pi\ sq\ units$

Q.2 Ajay cut two circular pieces of card boards and placed one upon other shown in the figure.Thefigure

represents the equation $(x-1)^2+y^2=1$,while other represents equation $x^2+y^2=1$,



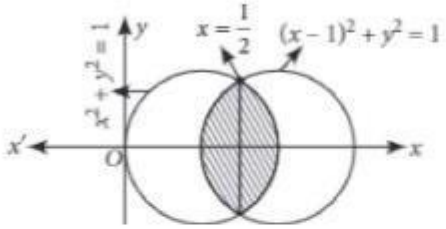
Based on the above information , answer the following

(i) Both the circular pieces of card board meet each other at

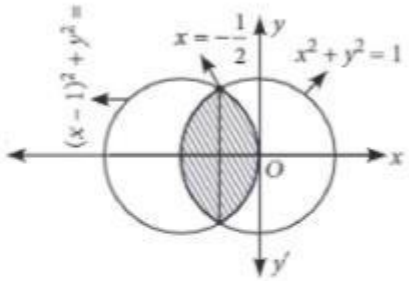
(a) x = 1 (b) $x=\frac{1}{2}$ (c) $x=\frac{1}{3}$ (d) $x=\frac{1}{4}$

(ii) Graph of given two curves can be drawn as

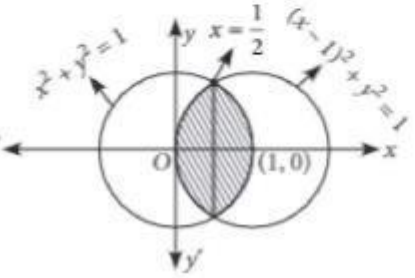
(a)



(b)



(c)



(d) None of these

(iii) Value of $\int_0^{\frac{1}{2}}\sqrt{1-(x-1)^2}\,dx$ is

(a) $\frac{\pi}{6}-\frac{\sqrt{3}}{8}$ (b) $\frac{\pi}{6}+\frac{\sqrt{3}}{8}$ (c) $\frac{\pi}{2}+\frac{\sqrt{3}}{4}$ (d) $\frac{\pi}{2}-\frac{\sqrt{3}}{4}$

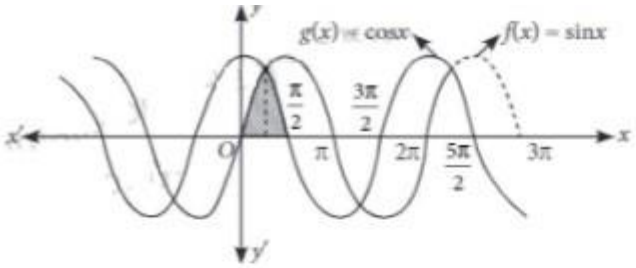
(iv) Value of $\int_{1/2}^{1} \sqrt{1-x^2}$ dx is

(a) $\frac{\pi}{2}+\frac{\sqrt{3}}{4}$ (b) $\frac{\pi}{6}+\frac{\sqrt{3}}{8}$ (c) $\frac{\pi}{6}-\frac{\sqrt{3}}{8}$ (d) $\frac{\pi}{2}-\frac{\sqrt{3}}{4}$

(v) Area of hidden portion of lower circle is

(a) $(\frac{2\pi}{3}+\frac{\sqrt{3}}{2})$ sq units (b) $(\frac{\pi}{3}-\frac{\sqrt{3}}{8})$ sq units (c) $(\frac{\pi}{3}+\frac{\sqrt{3}}{8})$ sq units (d) $(\frac{2\pi}{3}-\frac{\sqrt{3}}{2})$ sq units

Q.3 **Graphs of two functions f(x) = sin x and g(x) = cos x given below**



Based on the above informtion,answer the following questions

(i) In $[0,\pi]$ the curves f(x) = sin x and g(x) = cos x intersect at x =

(a)$\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) π

(ii) Value of $\int_0^{\pi/4} sinx$ dx is

(a) $1-\frac{1}{\sqrt{2}}$ (b) $1+\frac{1}{\sqrt{2}}$ (c) $2-\frac{1}{\sqrt{2}}$ (d) $2+\frac{1}{\sqrt{2}}$

(iii) Value of $\int_{\pi/4}^{\pi/2} \cos x$ dx is

(a) $1-\frac{1}{\sqrt{2}}$ (b) $1+\frac{1}{\sqrt{2}}$ (c) $2-\sqrt{2}$ (d) $2+\sqrt{2}$

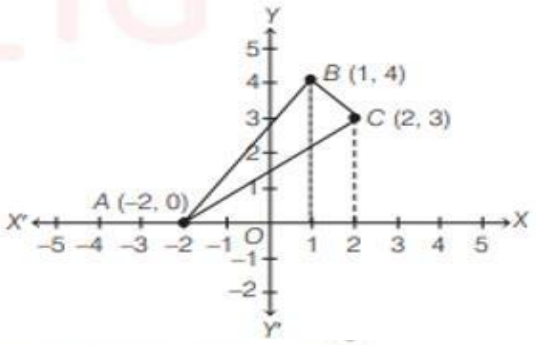
(iv) Value of $\int_0^{\pi} \sin x$ dx is

(a) 0 (b) 1 (c) 2 (d) -2

(v) Value of $\int_0^{\pi/2} \sin x$ dx isDIFFERE

(a) 0 (b) 1 (c) 3 (d) 4

Q.4 Location of three branches of a bank is represented by the three points A(-2,0) ,B(1,4) and C(2,3) as shown in figure .Point D is (2,0) .



(i) Equation of line AB is

(a) $y = \frac{4}{3}(x + 2)$ (b) $y = \frac{4}{3}(x + 1)$

(c) $y = \frac{4}{5}(x + 2)$ (d) $y = \frac{4}{5}(x + 1)$

(ii) Equation of line BC is

(a) $y = x + 5$ (b) $y = -x + 5$

(c) $y = x + 4$ (d) $y = -x + 4$

(iii) Area of region $ABCD$ is

(a) 19 sq units (b) $\frac{19}{2}$ sq units

(c) 17 sq units (d) 6 sq units

(iv) Area of ΔADC is

(a) 3 sq units (b) 4 sq units

(c) 6 sq units (d) 5 sq units

(v) Area of ΔABC is

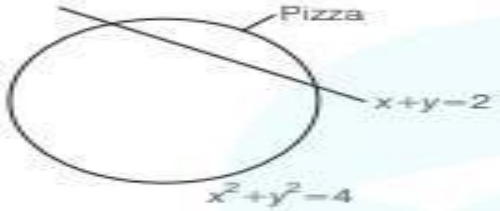
(a) 7 sq units (b) $\frac{3}{2}$ sq units

(c) 5 sq units (d) $\frac{7}{2}$ sq units

Q.5

Aman was celebrating his birthday with his friends. He ordered a pizza. He cut the pizza with a knife. Pizza was circular in shape which is represented by $x^2 + y^2 = 4$ and sharp edge of knife represents a straight line given by $x + y = 2$.





Based on the above information, answer the followwing questions.

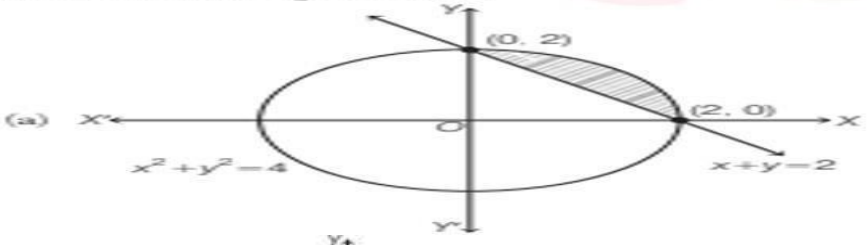
(i) The points of intersection of the edge of knife (line) and pizza shown in figure is (are)

(a) (0, 2) and (2, 0) (b) (0, 1) and (1, 0)

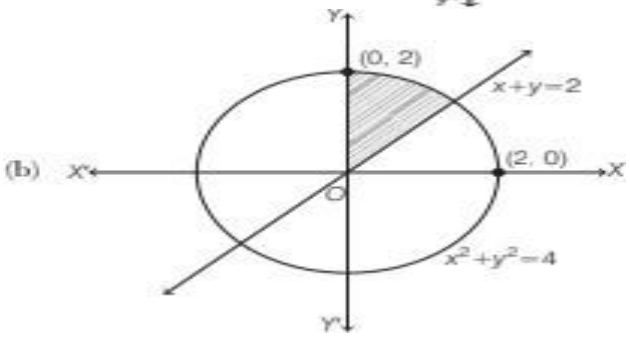
(c) (1, 2) and (2, 1) (d) (3, 1) and (1, 3)

(ii) Which of the following shaded portion represent the smaller area bounded by pizza and edge of knife in first quadrant?

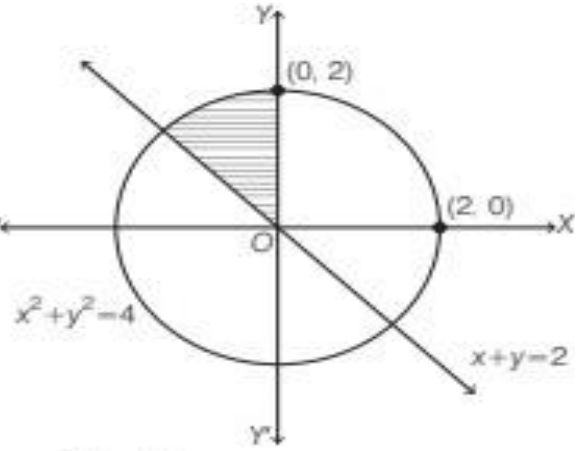
(a)



(b)



(c)



(d) None of the above

(iii) The area bounded by the sharp edge of knife with the coordinate axes is

(a) $\frac{1}{4}$ sq unit (b) $\frac{1}{2}$ sq unit

(c) 2 sq units (d) 3 sq units

(iv) Area of each slice of pizza, when Aman cut the pizza into 4 equal pieces is

(a) $\frac{\pi}{2}$ sq units (b) π sq units

(c) $\frac{\pi}{3}$ sq units (d) 2π sq units

(v) Area of whole pizza is

(a) 4π sq units (b) 3π sq units

(c) π sq units (d) $\frac{\pi}{2}$ sq units

ANSWERS:

MCQ ANSWERS:

Q.1 (B),

Q.2 (B)

Q.3 (A)

Q.4 (B)

Q.5(A)

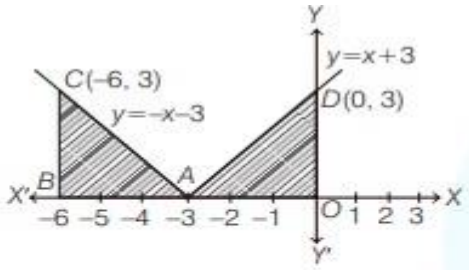
SHORT ANSWER QUESTIONS :

Q.6 20 π sq units

Q.7 ($\pi - 2$) sq units

Q.8 $\frac{1}{3} sq\ units$

Q.9



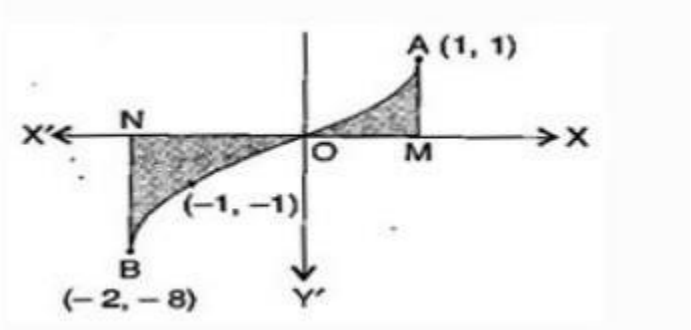
$\therefore$ Required area

= Area of region ABC + Area of region OAD

$= \int_{-6}^{-3} |x+3|\, dx + \int_{-3}^{0} |x+3|\, dx$

Integrating and applying limits to get the area shaded is equal to 9 sq.units

Q.10.



$$= \left|\int_{-2}^{0} x^3\,dx\right| + \left|\int_{0}^{1} x^3\,dx\right|$$

$$= \left|\left(\frac{x^4}{4}\right)_{-2}^{0}\right| + \left|\left(\frac{x^4}{4}\right)_{0}^{1}\right|$$

$$= \left|0 - \frac{16}{4}\right| + \left|\frac{1}{4} - 0\right|$$

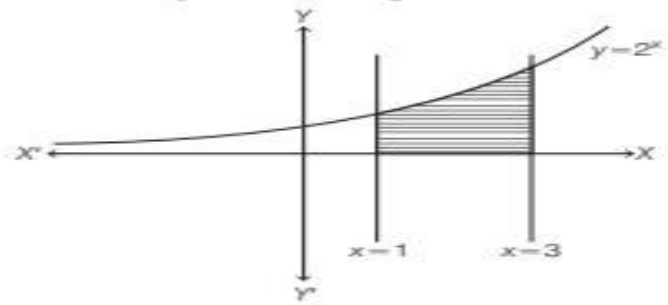
$$= 4 + \frac{1}{4} = \frac{17}{4} \text{ sq. units.}$$

Q.11 $\frac{13}{3} sq.units$

Q.12 $\frac{2}{3} sq.units$

Q.13 $4\ sq.units$

Q. 14



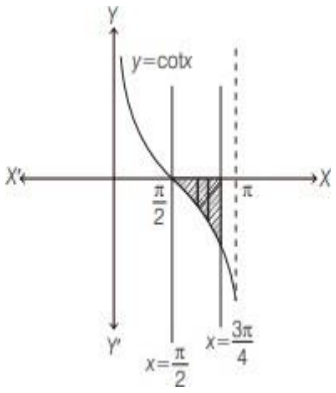
$$\therefore \text{Area of bounded region} = \int_1^3 2^x\,dx = \left[\frac{2^x}{\log 2}\right]_1^3$$

$$= \frac{1}{\log 2}[2^3 - 2^1] = \frac{1}{\log 2}(8 - 2)$$

$$= \frac{6}{\log 2} \text{ sq units}$$

Q.15

$$\therefore \text{Area of bounded region} = \left|\int_{\pi/2}^{3\pi/4} y\,dx\right|$$
$$= \left|\int_{\pi/2}^{3\pi/4} \cot x\,dx\right|$$
$$= \left|[\log|\sin x|]_{\pi/2}^{3\pi/4}\right|$$
$$= \left|\log\left|\sin\frac{3\pi}{4}\right| - \log\left|\sin\frac{\pi}{2}\right|\right|$$
$$= \left|\log\frac{1}{\sqrt{2}} - \log 1\right|$$
$$= |\log 1 - \log\sqrt{2} - \log 1|$$
$$= |-\log\sqrt{2}| = \log\sqrt{2} \text{ sq units}$$



ANSWERS OF LONG ANSWER QUESTIONS:

16. 27 sq.units

17. $\frac{\pi}{4} - \frac{1}{2} su.units$

18. $\frac{5\pi}{4} - \frac{1}{2} sq.units$

19 .$\frac{11}{6} sq.units$

20. $(3\pi - 6) sq.units$

21. 9 sq.units

22. $\frac{2\pi}{3} sq.units$

23.$\frac{4}{3} sq.units$

24. $4\pi sq.units$

25. .$\frac{10}{3}$sq.units

ANSWERS OF CASESTUDY QUESTIONS:

Q.1 (i) c (ii) b (iii) d (iv) a (v) d

Q.2 (i) b (ii) c (iii) a (iv) c (v) d

Q.3 (i) c (ii)a (iii) a (iv)c (v) b

Q.4 (i) a (ii) b (iii)b (iv) c (v)d

Q.5 (i) a (ii)a (iii) c (iv) b (v) a

CHAPTER 9

DIFFERENTIAL EQUATIONS

SYLLABUS

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx}$+ py = q, where p and q are functions of x or constants.

$\frac{dx}{dy}$+ px = q, where p and q are functions of y or constants.

Points To Remember

An equation involving derivatives of the dependent variable with respect to independent variable (variables) is known as a differential equation. If there is only one independent variable, then we call it an ordinary differential equation.

Order of a Differential Equation Order of a differential equation is defined as the order of the highest order derivative of the dependent variable with respect to the independent variable involved in the given differential equation.

Degree of a Differential Equation Degree of a differential equation, when it is a polynomial equation in derivatives, is defined as the highest power (exponent) of the highest order derivatives involved in the given differential equation.

Solution of a Differential Equation

A function which satisfies the given differential equation is called its solution.

General and Particular Solution of a Differential Equation

The solution which contains as many arbitrary constants as the order of the differential equation is called a general solution and the solution free from arbitrary constants is called a particular solution.

Methods of solving first order ,first degree Differential Equations

1. Differential equations with variable separable Variable separable method is used to solve an equation in which variables can be separated completely i.e. terms containing y should remain with dy and terms containing x should remain with dx

eg: y dx =x dy can be solved as $\frac{dx}{x} = \frac{dy}{y}$

Integrating both sides logx = logy + log c

$\frac{x}{y}$ = c

x = cy is the solution

2. Homogeneous differential equations

a) A differential equation which can be expressed in the form $\frac{dy}{dx}$ = f(x,y) where f(x,y) is a homogeneous function of degree zero is called homogeneous differential equation .

Example : $(x^2 + xy)\,dy = (x^2 + y^2)\,dx$

To solve this, we substitute y=vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

b) A differential equation which can be expressed in the form $\frac{dx}{dy}$ = f(x,y) where f(x,y) is a homogeneous function of degree zero is called homogeneous differential equation

Example

$\left(e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}}\left(\frac{x}{y}\right) dy = 0$

To solve this, we substitute x = vy and $\frac{dx}{dy} = v + y\frac{dv}{dy}$

3. Linear differential equations

a) A differential equation of the form $\frac{dy}{dx}$ + Py = Q where P and Q are constants or functions of x only is called first order linear differential equation

Its solution is given as $y\, e^{\int P dx} = \int Q e^{\int P dx}\, dx + c$

Example : $\frac{dy}{dx} + 3y = 2x$ has solution

$y\, e^{\int 3dx} = \int 2x e^{\int 3dx}\, dx + c$

b) A differential equation of the form $\frac{dx}{dy}$ + Px = Q where P and Q are constants or functions of y only is called first order linear differential equation

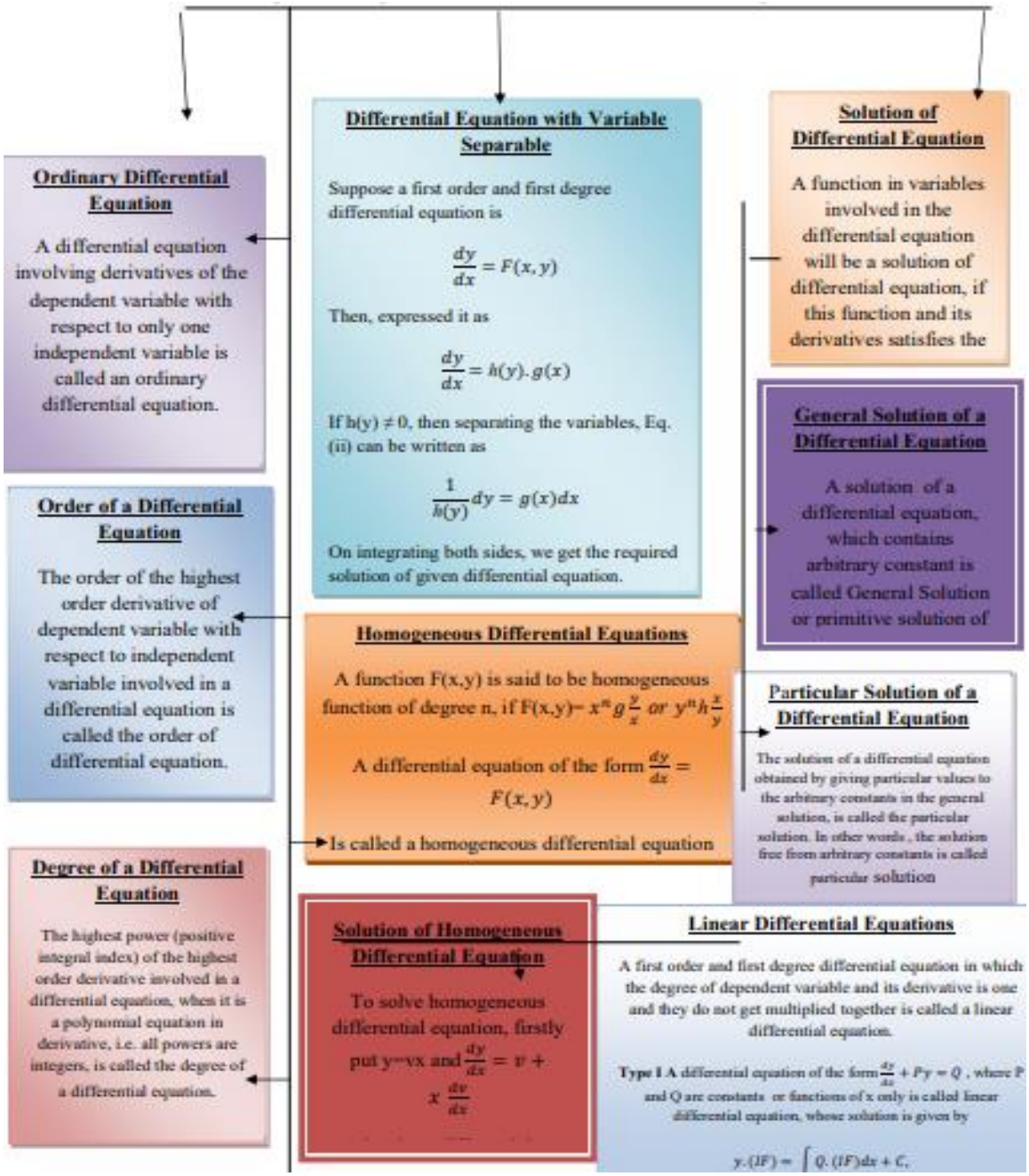
Its solution is given as $x\, e^{\int P dy} = \int Q e^{\int P dy}\, dy + c$

Example : $\frac{dx}{dy} - \frac{x}{y} = 2y$ has solution

$x\, e^{\int \frac{-1}{y} dy} = \int 2y\, e^{\int \frac{-1}{y} dy}\, dy + c$

MIND MAPPING

An equation containing an independent variable, dependent variable and derivative of dependent variable with respect to independent variable is called a differential equation.



MULTIPLE CHOICE QUESTIONS

1) The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is

(A) 4 (B) $\frac{3}{2}$ (C) not defined (D) 2

2) Solution of the differential equation tany $sec^2x\ dx$ + tanx $sec^2y\ dy$ = 0 is

(A) tanx + tany = k (B) tanx – tany = k

(C) $\frac{tan\ x}{tan\ y} = k$ (D) tanx . tany = k

3) The solution of $\frac{dy}{dx} + y = e^{-x}, y\ (0) = 0$ is :

(A) y = e^x (x – 1) (B) y = xe^{-x}

(C) y = xe^{-x} + 1 (D) y = (x + 1)e^{-x}

4)Integrating factor of the differential equation $\frac{dy}{dx}$ + y tan x – sec x = 0 is:

(A) $cosx$ (B) $secx$ (C) e^{cosx} (D) e^{secx}

5) The solution of the differential equation $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$ is :

(A) y $(1 + x^2)$ = c + $tan^{-1}\ x$ (B) $-\frac{y}{1+x^2} = c + tan^{-1}\ x$

(C) y log $(1 + x^2)$ = c + $tan^{-1}\ x$ (D) y $(1 + x^2)$ = c + $sin^{-1}\ x$

SHORT ANSWER QUESTIONS

1)Find the solution of $\frac{dy}{dx} = 2^{y-x}$.

2) Given that $\frac{dy}{dx} = e^{-2y}$ and y = 0 when x = 5

Find the value of x when y = 3

3) Solve the differential equation $(x^2 – 1)\ \frac{dy}{dx}$ + 2xy = $\frac{1}{(x^2-1)}$

4) Solve the differential equation $\frac{dy}{dx}$ + 2xy = y

5) Find the general solution of $\frac{dy}{dx} + ay = e^{mx}$

6) Solve the differential equation $\frac{dy}{dx} + 1 = e^{x+y}$

7) Solve: ydx – xdy = x^2 ydx

8) Solve the differential equation $\frac{dy}{dx} = 1 + x + y^2 + xy^2$, when y = 0, x = 0.

9)Find the general solution of (x + $2y^3$) $\frac{dy}{dx}$ = y

10) If y(x) is a solution of $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x \text{ and } y\,(0) =$ 1, *then find the value of* $y\left(\frac{\pi}{2}\right)$

11) Find the equation of a curve passing through origin and satisfying the differential equation (1 + x^2) $\frac{dy}{dx} + 2xy = 4x^2$

12) Solve : $x^2 \frac{dy}{dx} = x^2 + xy + y^2$

13) Find the general solution of $y^2\,dx + (x^2 - xy + y^2)\,dy = 0$

14) Solve : (x + y) (dx – dy) = dx + dy

15) Solve : 2 (y + 3) – xy $\frac{dy}{dx}$ = 0, given that y (1) = – 2

CASE STUDY QUESTIONS

1) A Veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the hospital, it was already dead. The pet lover wanted to find its time of death. He took the temperature of the cat at 11.30 pm which was 94.6°F. He took the temperature again after one hour; the temperature was lower than the first observation. It was 93.4°F. The room in which the cat was put is always at 70°F. The normal temperature of the cat was 98.6°F when it was alive. The doctor estimated the time of death using Newton law of cooling which is governed by the differential equation: $\frac{dT}{dt} \propto$ (T − 70), where 70°F is the room temperature and T is the temperature of the object at time t. Substituting the two different observations of T and t made, in the solution of the differential equation $\frac{dT}{dt} = k$(T − 70) where k is a constant of proportion, time of death is calculated.

1) State the degree of the above given differential equation.

2) Which method of solving a differential equation helped in calculation of the time of death?

a) Variable separable method

b) Solving Homogeneous differential equation

c) Solving Linear differential equation

d) all of the above

3) If the temperature was measured 2 hours after 11.30pm, will the time of death change? (Yes/No)

4) The solution of the differential equation $\frac{dT}{dt} = k(T - 70)$ is given by,

a) log | T – 70| = kt + C b) log | T – 70| = log |kt |+ C

c) T – 70 = kt + C d) T – 70 = kt C

5) If t = 0 when T is 72, then the value of c is

a) -2 b) 0 c) 2 d) Log 2

ANSWERS:

1) Degree is 1

2) (a) Variable separable method

3) No

4) (a) log | T – 70| = kt + C

5) (d)log 2

2) Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx}$ = K (50 − y) where x denotes the number of weeks and y the number of children who have been given the drops.

1. State the order of the above given differential equation.

2. Which method of solving a differential equation can be used to solve $\frac{dy}{dx}$ = k (50 − y).?

a) Variable separable method

b) Solving Homogeneous differential equation

c) Solving Linear differential equation

d) All of the above

3. The solution of the differential equation $\frac{dy}{dx}$= k (50 − y) is given by,

a) log | 50 – y| = kx + C b) - log | 50 – y| = kx + C

c) log | 50 – y| = log| kx |+ C d) 50 – y = kx + C

4. The value of c in the particular solution given that y(0)=0 and k = 0.049 is :

a) log 50 b) log $\frac{1}{50}$ c) 50 d) -50

5. Which of the following solutions may be used to find the number of children who have been given the polio drops?

a) y = 50 – e^{kx} b) y = 50 − e^{-kx} c) y = 50 (1 – e^{-k}) d) y = 50 (e^{kx}– 1)

ANSWERS:

1. Order is 1

2. (a) Variable separable method

3. (b) - log | 50 – y| = kx + C

4. (b) log $\frac{1}{50}$

5. (c) y = 50 ((1 – e^{-k})

3.

A thermometer reading 80°F is taken outside. Five minutes later the thermometer reads 60°F. After another 5 minutes the thermometer reads 50°F. At any time t the thermometer reading be T°F and the outside temperature be S°F.

Based on the above information, answer the following questions.



1. If λ is positive constant of proportionality, then $\frac{dT}{dt}$ is
   a) λ (T− S)
   b) λ (T+ S)
   c) λTS
   d) −λ (T− S)

2. The value of T(5) is
   a) 30°F
   b) 40°F
   c) 50°F
   d) 60°F

3. The value of T(10) is
   a) 50°F
   b) 60°F
   c) 80°F
   d) 90°F

4. Find the general solution of differential equation formed in given situation.
   a) $log\ T = St + c$
   b) $log\ (T - S) = -\lambda t + c$
   c) $log\ S = tT + c$
   d) $log\ (T + S) = \lambda t + c$

5. Find the value of constant of integration c in the solution of differential equation formed in given situation.

   a) log (60 - S)
   b) log (80 + S)
   c) log (80 - S)
   d) log (60 + 5)

ANSWERS:

1. (d) $-\lambda\,(T-S)$
2. (d) 60°F
3. (a)50°F
4. (b)$log\ S = tT + c$
5. (c) log (80 - S)

4) It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal. Let P denotes the principal at any time t and rate of interest be r % per annum.

Based on the above information, answer the following questions.

1. Find the value of $\frac{dP}{dt}$
   a) $\frac{Pr}{1000}$
   b) $\frac{Pr}{100}$
   c) $\frac{Pr}{10}$
   d) Pr



2. If P_0 be the initial principal, then find the solution of differential equation formed in given situation.
   a) $\log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$
   b) $\log\left(\frac{P}{P_0}\right) = \frac{rt}{10}$
   c) $\log\left(\frac{P}{P_0}\right) = rt$
   d) $\log\left(\frac{P}{P_0}\right) = 100rt$

3. If the interest is compounded continuously at 5% per annum, in how many years will ₹100 double itself?
   a) 12.728 years
   b) 14.789 years
   c) 13.862 years
   d) 15.872 years

4. At what interest rate will ₹100 double itself in 10 years? ($log_e 2 = 0.6931$).
   a) 9.66%
   b) 8.239%
   c) 7.341%
   d) 6.931%

5. How much will ₹1000 be worth at 5% interest after 10 years? ($e^{0.5} = 1.648$)
   a) ₹1648
   b) ₹1500
   c) ₹1664
   d) ₹1572

ANSWERS

1. (b) $\frac{Pr}{100}$
2. (a) $\log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$
3. (c) 13.862 years
4. (d) 6.931%
5. (a) ₹1648

5) If the equation is of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ or $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$, where $f(x,y), g(x,y)$ are homogeneous functions of same degree in x and y, then put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ so that the dependent variable y is changed toanother variable v and then apply variable separable method.

Based on the above information, answer the following questions.

1. The general solution of $x^2\frac{dy}{dx} = x^2 + xy + y^2$ is:
   a) $\tan^{-1}\frac{x}{y} = \log|x| + c$
   b) $\tan^{-1}\frac{y}{x} = \log|x| + c$
   c) $y = x\log|x| + c$
   d) $x = y\log|y| + c$

2. Solution of the differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ is :
   a) $x^3 + y^2 = cx^2$
   b) $\frac{x^2}{2} + \frac{y^3}{3} = y^2 + c$
   c) $x^2 + y^3 = cx^2$
   d) $x^2 + y^2 = cx^3$

3. Solution of the differential equation $(x^2 + 3xy + y^2)dx - x^2dy = 0$ is
   a) $\frac{x+y}{x} - \log x = c$
   b) $\frac{x+y}{x} + \log x = c$
   c) $\frac{x}{x+y} - \log x = c$
   d) $\frac{x}{x+y} + \log x = c$

4. General solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}\left\{\log\left(\frac{y}{x}\right) + 1\right\}$ is:
   a) $\log(xy) = c$
   b) $\log y = cx$
   c) $\log\left(\frac{y}{x}\right) = cx$
   d) $\log x = cy$

5. Solution of the differential equation $\left(x\frac{dy}{dx} - y\right)e^{\frac{y}{x}} = x^2\cos x$ is :
   a) $e^{\frac{y}{x}} - \sin x = c$
   b) $e^{\frac{y}{x}} + \sin x = c$
   c) $e^{\frac{-y}{x}} - \sin x = c$
   d) $e^{\frac{-y}{x}} + \sin x = c$

ANSWERS

1. (b) $\tan^{-1}\frac{y}{x} = \log|x| + c$
2. (d) $x^2 + y^2 = cx^3$
3. (d) $\frac{x}{x+y} + \log x = c$
4. (c) $\log\left(\frac{y}{x}\right) = cx$
5. (a) $e^{\frac{y}{x}} - \sin x = c$

LONG ANWER QUESTIONS

1. Show that the differential equation $2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ is homogeneous. Find the particular solution given that x=0 when y=1.

2. Solve: $\frac{dy}{dx} - 3y\cot x = \sin 2x$;.Find the particular solution when $y = 2$ *when* $x = \frac{\pi}{2}$.

3. Find the particular solution of the differential equation $(1 + e^{2x})dy + (1 + y^2)e^x\,dx = 0$, given that $y = 1$, when $x = 0$.

4. Solve the Differential Equation $(1 + x^2)\frac{dy}{dx} + y = \tan^{-1}x$.

5. In a bank Principal increases at the rate of r % per year. Find the value of r if Rs. 100 double itself in 10 years (given log 2 =0.6931).

6. Solve the differential equation $xdy - y\,dx = \sqrt{x^2 + y^2}\,dx$.

7. Find the particular solution of the Differential Equation $(\tan^{-1} y - x)\,dy = (1 + y^2)dx$, given that x =0, y =0.

8. Find the general solution of the differential equation
$$\frac{dy}{dx} - y = \cos x$$

9. Solve the differential equation $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1,\ x \neq 0$

10. Solve the differential equation $x\frac{dy}{dx} + y - x + xy\cot x = 0,\ x \neq 0$

11. Find a particular solution of the differential equation

$\frac{dy}{dx} - 3y\,Cotx = \text{Sin}\,2x$, given y = 2 when x = $\frac{\pi}{2}$

12. Find a particular solution of the differential equation

$\frac{dy}{dx} + y\,Cotx = 2x + x^2 Cotx, x \neq 0$, given y = 0 when $x = \frac{\pi}{2}$

13. Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ given that y = 0 when x = 0

14. Find a particular solution of the differential equation

$\left[x\text{Sin}^2\left(y/x\right) - y\right]dx + xdy = 0$, given $y = \frac{\pi}{4}$ when x = 1

15. Find a particular solution of the differential equation

$2xy + y^2 - 2x^2\frac{dy}{dx} = 0$, given y = 2 when x = 1

ANSWERS:

ANSWERS OF MULTIPLE CHOICE QUESTIONS :

1) D 2) D 3) B 4) B 5) A

ANSWERS OF SHORT ANSWER QUESTIONS

1) $2^{-x} - 2^{-y} = k$

2) $\frac{e^6 + 9}{2}$

3) y(x^2- 1) = $\frac{1}{2}log\left(\left|\frac{x-1}{x+1}\right|\right) + k$

4) y = c.$e^{x - x^2}$

5) (a + m) y = $e^{mx} + ce^{-ax}$

6) (x - c)$e^{x+y} + 1 = 0$

7) $y = kxe^{\frac{-x^2}{2}}$

8) y = tan $\left(x + \frac{x^2}{2}\right)$

9) x = y $(y^2 + c)$

10) $\frac{1}{3}$

11) $y = \frac{4x^3}{3(1+x^2)}$

12) $tan^{-1}\left(\frac{y}{x}\right) = log|x| + c$

13) $tan^{-1}\left(\frac{x}{y}\right) + log\ y = c$

14) x + y = ke^{x-y}

15) $x^2\ (y+3)^3 = e^{y+2}$

ANSWERS OF LONG ANSWER QUESTIONS:

1. $2e^{\frac{x}{y}} + \log y = 2$

2. $y(\text{cosec}^3 x) = -2\ \text{cosec}x + 4$

3. $\tan^{-1} y = -\tan^{-1} e^x + \frac{\pi}{2}$

4. $e^{tan^{-1}x}(y + 1) = \tan^{-1}x\ e^{tan^{-1}x} + c$

5. r =10 log 2 = 6.931%

6. $y + \sqrt{x^2 + y^2} = c\,x^2$

7.

8. $x\,e^{tan^{-1}y} = (\tan^{-1}y - 1)e^{tan^{-1}y} + 1$

$y = \frac{1}{2}(\sin x - \cos x) + ce^x + c$

9. $y e^{2\sqrt{x}} = 2\sqrt{x} + c$

10. $y = cotx + \frac{1}{x} + \frac{c}{x\ sinx}$

11. $y = 4sin^3x - 2sin^2x$

12. $y\,sinx = x^2sinx - \frac{\pi^2}{4}$

13. $-3e^{-4y} = 4e^{3x} - 7$

14. $log|x| = cot\left(\frac{y}{x}\right) - 1$

15. $log|x| = \frac{-2x}{y} + 2$

CHAPTER -10- VECTOR ALGEBRA

SUMMARY

- **DEFINITION OF Vectors**
- **Definition of Scalars**
- **Position Vector**
- **Dc's and Dr's**
- **Types of vectors**
  - **Zero Vector**
  - **Unit Vector**
  - **Unit vector in the direction of vectors**
  - **Collinear Vectors**
  - **Coinitial vectors**
  - **Equal Vectors**
  - **Negative of a Vector**
  - **Unit Vectors along the coordinate axes.**
- **Addition of Vectors- Triangle law of addition / parallelogram law of addition**
- **Multiplication of a Vector by a Scalar**
- **Vector joining two points**
- **Section formula and mid point formula**
- **Dot product of vectors**
- **Properties of dot product of vectors**
- **Projection of vectors on a line**
- **Perpendicular vectors**
- **Finding the angle between the two vectors**
- **Expressing dot product in rectangular coordinates.**
- **Cross product of vectors**
- **Properties of Cross product of vectors**
- **Expressing cross product in rectangular coordinates.**
- **Unit vector perpendicular to two given vectors.**
- **Angle between two vectors**
- **Area of parallelogram when adjacent sides are given.**
- **Area of parallelogram when diagonals are given.**
- **Area of a triangle**
- **Area of a rectangle ABCD, when position vectors of A,B,C,D are given**

MIND MAPPING

- Position vector of a point P(x, y, z) is given as $\overrightarrow{OP} (= \vec{r})$ = x $\hat{i}$ +y $\hat{j}$ +z $\hat{k}$ and its magnitude by $\sqrt{x^2 + y^2 + z^2}$.
- The scalar components of a vector are its direction ratios, and represent its projections along the respective axes.
- The magnitude (r), direction ratios (a, b, c) and direction cosines (l, m, n) of any vector are related as: $l = \frac{a}{r}$, $m = \frac{b}{r}$, $n = \frac{c}{r}$
- $l^2 + m^2 + n^2 = 1$ but $a^2 + b^2 + c^2 \neq 1$ in general
- The vector sum of the three sides of a triangle taken in order is $\vec{0}$.
- The vector sum of two coinitial vectors is given by the diagonal of the parallelogram whose adjacent sides are the given vectors.
- The multiplication of a given vector by a scalar λ, changes the magnitude of the vector by the multiple |λ|, and keeps the direction same (or makes it opposite) according as the value of λ is positive (or negative).
- For a given vector $\vec{a}$, $\hat{a} = \frac{\vec{a}}{|a|}$ gives the unit vector in the direction of $\vec{a}$.
- The position vector of a point R dividing a line segment joining the points P and Q whose position vectors are respectively, in the ratio m : n

(i) internally, is given by $\frac{m\vec{b}+n\vec{a}}{m+n}$

(ii) externally, is given by $\frac{m\vec{b}-n\vec{a}}{m-n}$.

- $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are parallel or collinear iff $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

Dot product	Cross product
$\vec{a} \cdot \vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \cos\theta$	$\vec{a}\times\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \sin\theta\, \hat{n}$ where $\hat{n}$ is perpendicular to both $\vec{a}, b$.
$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ (commutative property)	$\vec{a}\times\vec{b} \neq \vec{b} \times \vec{a}$ (not commutative)
If $\vec{a} \perp \vec{b}$ *then* $\vec{a}.\vec{b} = 0$	If $\vec{a} \parallel \vec{b}$, *then* $\vec{a}\times\vec{b} = 0$
$\vec{a}.\vec{a} = \lvert\vec{a}\rvert^2$ $(\vec{a}.\vec{b})^2 = (\vec{a})^2 + (\vec{b})^2 + 2\vec{a}.\vec{b}$	$\vec{a}\times\vec{a} = 0$
If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ Then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$	If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ Then $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
Geometrical meaning projection of $\vec{a}$ *on* $\vec{b} = \frac{\vec{a}.\vec{b}}{\lvert\vec{b}\rvert}$	Geometrical meaning $\vec{a}\times\vec{b}$ = *vector area of a* parallelogram with $\vec{a}, \vec{b}$ represent the adjacent sides.
$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$	$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$
$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{i} \cdot \hat{k} = 0$	$\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$ $\hat{j} \times \hat{i} = -\hat{k}$, $\hat{k} \times \hat{j} = -\hat{i}$, $\hat{i} \times \hat{k} = -\hat{j}$

MCQ Questions

1. If points A (60 $\hat{\imath}$ + 3 $\hat{\jmath}$), (40 $\hat{\imath}$ – 8 $\hat{\jmath}$) and C (a$\hat{\imath}$- 52$\hat{\jmath}$) are collinear, then 'a' is equal to
   a) 40 b) -40 c) 20 d) -20
2. If $\vec{a}+\vec{b}+\vec{c}$ = 0, $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$then the angle between If $\vec{a}$ and $\vec{b}$ is
   a) $\frac{\pi}{6}$ b) $\frac{2\pi}{3}$ c) $\frac{5\pi}{3}$ d) $\frac{\pi}{3}$
3. If $\vec{a}$ = $\hat{\imath}$ + $\hat{\jmath}$ - $\hat{k}$, $\vec{b}$ = - $\hat{\imath}$ + 2 $\hat{\jmath}$ +2 $\hat{k}$ and $\vec{c}$ = - $\hat{\imath}$ + 2 $\hat{\jmath}$ - $\hat{k}$ then a unit vector normal to the vectors ($\vec{a}+\vec{b}$) and ($\vec{b}-\vec{c}$) is
   a) $\hat{\imath}$ b) $\hat{\jmath}$ c) $\hat{k}$ d) none of these
4. If $|\vec{a}\ x\ \vec{b}| = 4$, | $\vec{a}\ .\ \vec{b}$ |=2, then $|\vec{a}|^2$ $|\vec{b}|^2$ is
   a) 6 b) 2 c) 20 d) 8
5. The value of $\hat{\imath}$. ($\hat{\jmath}$ x $\hat{k}$) + $\hat{\jmath}$. ($\hat{\imath}$ x $\hat{k}$) + $\hat{k}$. ($\hat{\imath}$ x $\hat{\jmath}$) is
   a) 0 b) -1 c) 1 d) 3

Short Answer Questions

1. Find the unit vector in the direction of the sum of the vectors $\vec{a}$ = 2$\hat{\imath}$ +2 $\hat{\jmath}$ -5 $\hat{k}$, $\vec{b}$ = 2$\hat{\imath}$ + $\hat{\jmath}$ +3 $\hat{k}$.
2. Show that the points A(-2$\hat{\imath}$ +3 $\hat{\jmath}$ +5 $\hat{k}$), B ($\hat{\imath}$ +2 $\hat{\jmath}$ +3 $\hat{k}$), C (7$\hat{\imath}$ - $\hat{k}$) are collinear.
3. If $\vec{a}$ = 2 $\hat{\imath}$ + 2 $\hat{\jmath}$ +3 $\hat{k}$, $\vec{b}$ = - $\hat{\imath}$ + 2 $\hat{\jmath}$ + $\hat{k}$, $\vec{c}$ = 3 $\hat{\imath}$ + $\hat{\jmath}$ are such that $\vec{a}$ +λ $\vec{b}$ is perpendicular to $\vec{c}$,then find the value of λ.
4. Find the area of the ||gm whose adjacent sides are represented by the vectors $\vec{a}$ = 3 $\hat{\imath}$ + $\hat{\jmath}$ -2 $\hat{k}$, $\vec{b}$ = $\hat{\imath}$ -3 $\hat{\jmath}$ +4 $\hat{k}$
5. Find a vector of magnitude 3√2 units which makes an angle of π/4, π/2 with y and z-axes, respectively.
6. If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a}\ x\ \vec{b}| = 8$, find $\vec{a}\ .\ \vec{b}$
7. If $\vec{a}$ = 2 $\hat{\imath}$ -3 $\hat{\jmath}$ +$\hat{k}$, $\vec{b}$ = - $\hat{\imath}$ + $\hat{k}$, $\vec{c}$ = 2 $\hat{\jmath}$ -$\hat{k}$ are three vectors, find the area of the parallelogram having diagonals ($\vec{a}+\vec{b}$) and ($\vec{b}+\vec{c}$)
8. If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a}\ x\ \vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$
9. If $\vec{p}$ and $\vec{q}$ are the unit vectors forming an angle of 30^0 , find the area of the parallelogram having $\vec{a} = \vec{p} + 2\vec{q}$ and $\vec{b} = 2\vec{p} + \vec{q}$ as its diagonals.
10. $\vec{a}$ is a unit vector and ($\vec{x}-\vec{a}$) ($\vec{x}+\vec{a}$) = 8, then find $|\vec{x}|$

Long Answer Questions

1. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}$ = 0,then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$
2. The scalar product of the vector $\hat{\imath}$ + $\hat{\jmath}$ + $\hat{k}$ with a unit vector along the sum of vectors 2 $\hat{\imath}$ + 4 $\hat{\jmath}$ -5 $\hat{k}$ and λ$\hat{\imath}$ + 2 $\hat{\jmath}$ +3 $\hat{k}$ is equal to one. Find the value of λ.
3. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two find | $\vec{a}+\vec{b}+\vec{c}$ |

4. . If with reference to the righthanded system of mutually perpendicular unit vectors $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$, $\vec{\alpha}$ = $3\hat{\imath} - \hat{\jmath}$ and $\vec{\beta}$ = $2\,\hat{\imath} + \hat{\jmath} - 3\,\hat{k}$ then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$ where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$
5. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$ find the angle between $\vec{a}$ and $\vec{b}$
6. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then show that ($\vec{a} - \vec{d}$) is parallel to ($\vec{b} - \vec{c}$) , given that $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.
7. Find the area of the parallelogram whose diagonals are $\vec{a}$ and $\vec{b}$. Also find the area of the parallelogram whose diagonals are $2\,\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + 3\,\hat{\jmath} - \hat{k}$.
8. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ determine the vertices of a triangle, show that $\frac{1}{2}\left[\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}\right]$ gives the vector area of triangle. Hence deduce the condition that the three points $\vec{a}$, $\vec{b}$ and $\vec{c}$ are collinear. Also find the unit vector normal to the plane of the triangle.
9. Given that $\vec{a}$, $\vec{b}$ and $\vec{c}$ form a triangle such that $\vec{a} = \vec{b} + \vec{c}$. Find p,q,r,s such that area of triangle is $5\sqrt{6}$ where $\vec{a} = p\hat{\imath} + q\,\hat{\jmath} + r\,\hat{k}$, $\vec{b} = s\,\hat{\imath} + 3\,\hat{\jmath} + 4\hat{k}$ and $\vec{c} = 3\,\hat{\imath} + \hat{\jmath} - 2\hat{k}$.
10. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\jmath} - \hat{k}$, then find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

CASE STUDY QUESTIONS

CASE STUDY - I

Ginni purchased an air plant holder which is in the shape of a tetrahedron. Let A, B, C and D are the coordinates of the air plant holder where A (1, 1, 1), B (2, 1, 3), C=(3, 2, 2) and D = (3,3,4).

Based on the above information, answer the following questions

1. **Find the position vector of AB.**
   a. $-\hat{\imath} - 2\hat{k}$
   b. $2\hat{\imath} + \hat{k}$
   c. $\hat{\imath} + 2\hat{k}$
   d. $-2\hat{\imath} - \hat{k}$

2. **Find the position vector of AC.**
   a. $2\hat{\imath} - \hat{\jmath} - \hat{k}$
   b. $2\hat{\imath} + \hat{\jmath} + \hat{k}$
   c. $-2\hat{\imath} - \hat{\jmath} + \hat{k}$
   d. $\hat{\imath} + 2\hat{\jmath} + \hat{k}$

3. **Find the position vector of AD**
   a. $2\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$
   b. $\hat{\imath} + \hat{\jmath} - 3\hat{k}$
   c. $3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$
   d. $2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

4. **Area of ΔABC=**

a. $\frac{\sqrt{11}}{2}$ sq. units

b. $\frac{\sqrt{14}}{2}$ sq. units

c. $\frac{\sqrt{13}}{2}$ sq. units

d. $\frac{\sqrt{17}}{2}$ sq. units

CASE STUDY - II

Team A,B,C went for playing a tug of war game. Teams A, B, C, have attached a rope to a mental ring and its trying to pull the ring into their own area(learn areas shown below).

Team A pulls with force F1=$4\hat{\imath}+0\hat{\jmath}$ KN

Team B →F2= $2\hat{\imath}+4\hat{\jmath}$ KN

Team C →F3=$-3\hat{\imath}-3\hat{\jmath}$ KN

Based on the above information, answer the following.

1. **Which team will win the game?**
   a. **Team B**
   b. **Team A**
   c. **Team C**
   d. **No one**
2. **What is the magnitude of the teams combined force?**
   a. **7 KN**
   b. **1.4 KN**
   c. **1.5 KN**
   d. **2 KN**
3. **In what direction is the ring getting pulled?**
   a. **2.0 radian**
   b. **2.5 radian**
   c. **2.4 radian**
   d. **3 radian**
4. **What is the magnitude of the forces of Team B?**
   a. **2√5 KN**
   b. **6 KN**
   c. **2 KN**
   d. **√6KN**
5. **How many KN force is applied by Team A?**
   a. **5 KN**

b. 4 KN
c. 2 KN
d. 16 KN

CASE STUDY – III

A class XII student appearing for a competitive examination was asked to attempt the following questions.

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ *be three* non zero vectors.

1. **If $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ then**
   a. $\vec{a} \perp \vec{b}$
   b. $\vec{a} \| \vec{b}$
   c. $\vec{a}=\vec{b}$
   d. None of these
2. **If $\vec{a}=\hat{\imath}-2\hat{\jmath}$, $\vec{b}=2\hat{\imath}+\hat{\jmath}+3\hat{k}$ then evaluate $(2\vec{a}+\vec{b})\cdot[(\vec{a}+\vec{b})\times(\vec{a}-2\vec{b})]$**
   a. 0
   b. 4
   c. 3
   d. 2
3. **If $\vec{a}$ and $\vec{b}$ are unit vectors and θ be the angle between them then**

   $|\vec{a}-\vec{b}|$

   a. $\sin\frac{\theta}{2}$
   b. $2\sin\frac{\theta}{2}$
   c. $2\cos\frac{\theta}{2}$
   d. $\cos\frac{\theta}{2}$

4. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$ then $\vec{a} =$

   a. $2\left(\vec{b} \times \vec{c}\right)$

   b. $-2\left(\vec{b} \times \vec{c}\right)$

   c. $\pm 2\left(\vec{b} \times \vec{c}\right)$

   d. $2\left(\vec{b} \pm \vec{c}\right)$

5. The area of the parallelogram If $\vec{a} = \hat{i} - 2\hat{j}$, $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ as diagonals is

   a. 70

   b. 35

   c. √70/2

   d. √70

ANSWERS

MCQ QUESTIONS

1. b 2. d 3. a 4. c 5. c

SHORT ANSWER QUESTIONS

1) $\frac{4}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} - \frac{2}{\sqrt{29}}\hat{k}$ 3) 8 4) $10\sqrt{3}$ 5) $\pm 3\hat{i} + 3\hat{j}$ 6) 6

7) $\frac{\sqrt{21}}{2}$ 8) $\frac{\pi}{6}$ 9) $\frac{3}{4}$ 10) 3

LONG ANSWER QUESTIONS

1) $\frac{-3}{2}$ 2) 1 3) $5\sqrt{2}$ 4) λ=1/2 , $\overrightarrow{\beta_1} = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$ $\overrightarrow{\beta_2} = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

5) $\frac{\pi}{3}$ 7) $\frac{1}{2}\sqrt{62}$ 9) p = -8 , q=4, r=2 , s= -11,5 10) $\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

CASE STUDY – I ANSWERS

1. (c) **2. (b)** **3. (d)** **4. (b)** **5. (a)**

CASE STUDY – II ANSWERS

1. a. Team B **2. b. 1.4 KN** **3. c. 2.4 KN** **4. a. 2√5 KN** **5. b. 4 KN**

CASE STUDY – III ANSWERS

1. a **2. A** **3. B** **4. C** **5. C**

MCQ Questions - solutions

1.

-40

Given: Three points $A\left(60\hat{i}+3\hat{j}\right), B\left(40\hat{i}-8\hat{j}\right)$ and $C\left(a\hat{i}-52\hat{j}\right)$ are collinear.

Then, $\overrightarrow{AB}=\lambda\overrightarrow{BC}$.

We have,

$$\overrightarrow{AB}=\left(40\hat{i}-8\hat{j}\right)-\left(60\hat{i}+3\hat{j}\right)=-20\hat{i}-11\hat{j}$$

$$\overrightarrow{BC}=\left(a\hat{i}-52\hat{j}\right)-\left(40\hat{i}-8\hat{j}\right)=(a-40)\hat{i}-44\hat{j}$$

$$\overrightarrow{AB}=\lambda\overrightarrow{BC}$$

$$\Rightarrow -20\hat{i}-11\hat{j}=\lambda(a-40)\hat{i}-\lambda 44\hat{j}$$

$$\Rightarrow \lambda(a-40)=-20, -44\lambda=-11 \Rightarrow \lambda=\frac{1}{4}$$

$$\Rightarrow a-40=-80$$

$$\Rightarrow a=-40$$

2.

Given , $|\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7 \ldots (i)$

Let θ be the angle between $\vec{a}$ and $\vec{b}$.

Given that

$\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow |\vec{a} + \vec{b}| = |-\vec{c}|^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{c}|^2$

$\Rightarrow 2\vec{a}.\vec{b} = |\vec{c}|^2 - |\vec{a}|^2 - |\vec{b}|^2$

$\Rightarrow 2\vec{a}.\vec{b} = 7^2 - 3^2 - 5^2 \ldots\ldots\ldots\ldots\ldots\ldots$ [Using (i)]

$\Rightarrow 2\vec{a}.\vec{b} = 15$

$\Rightarrow 2|\vec{a}||\vec{b}| \cos\theta = 15$

$\Rightarrow 2(3)(5)\cos\theta = 15 \ldots\ldots\ldots\ldots\ldots\ldots$ [Using (i)]

$\Rightarrow \cos\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{3}$

3.

$$\vec{a}+\vec{b}=0\hat{i}+3\hat{j}+\hat{k}$$

$$\vec{b}-\vec{c}=0\hat{i}-0\hat{j}+3\hat{k}$$

$$\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}-\vec{c}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 3 & 1\\ 0 & 0 & 3\end{vmatrix}$$

$$=9\hat{i}$$

$$\left|\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}-\vec{c}\right)\right|=9\left|\hat{i}\right|$$

$$=9\left(1\right)$$

$$=9$$

Unit vector perpendicular to both $\vec{a}+\vec{b}$ and $\vec{b}-\vec{c}=\dfrac{\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}-\vec{c}\right)}{\left|\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}-\vec{c}\right)\right|}$

$$=\frac{9\hat{i}}{9}$$

$$=\hat{i}$$

4.

We know

$$\left(\vec{a}.\vec{b}\right)^2+\left|\vec{a}\times\vec{b}\right|^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2\ldots(1)$$

$$\left|\vec{a}.\vec{b}\right|=2(\text{ Given })$$

$$\Rightarrow\left|\vec{a}.\vec{b}\right|^2=\left(\vec{a}.\vec{b}\right)^2$$

From (1), we get

$$(2)^2+(4)^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2$$

$$\Rightarrow\left|\vec{a}\right|^2\left|\vec{b}\right|^2=20$$

5.

$$\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)$$
$$=\hat{i}.\hat{i}+\hat{j}.\left(-\hat{j}\right)+\hat{k}.\hat{k}$$
$$=\left|\hat{i}\right|^2-\left|\hat{j}\right|^2+\left|\hat{k}\right|^2$$
$$=1-1+1$$
$$=1$$

Short Answer Type – Solutions

1. **Ans: Let** $\vec{c}=\vec{a}+\vec{b}$

$$=\left(2\hat{i}+2\hat{j}-5\hat{k}\right)+\left(2\hat{i}+\hat{j}+3\hat{k}\right)$$
$$=4\hat{i}+3\hat{j}-2\hat{k}$$
$$\left|\vec{c}\right|=\sqrt{16+9+4}$$
$$=\sqrt{29}$$

The required unit vector is

$$\hat{c}=\frac{\vec{c}}{\left|\vec{c}\right|}$$

$$= \frac{4\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{29}}$$

$$= \frac{4}{\sqrt{29}}\hat{i}+\frac{3}{\sqrt{29}}\hat{j}-\frac{2}{\sqrt{29}}\hat{k}$$

2. Ans: $\overrightarrow{AB} = 3\hat{i}-\hat{j}-2\hat{k}$

$\overrightarrow{BC} = 6\hat{i}-2\hat{j}-4\hat{k}$

$\overrightarrow{CA} = 9\hat{i}-3\hat{j}-6\hat{k}$

$\left|\overrightarrow{AB}\right| = \sqrt{14}, \overrightarrow{BC} = 2\sqrt{14}$

$and \left|\overrightarrow{AC}\right| = 3\sqrt{14}$

$\left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$

Hence points A, B, C are collinear.

3. Ans: $\vec{a}+\lambda\vec{b} = \left(2\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(-\hat{i}+2\hat{j}+\hat{k}\right)$

$= (2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}$

$\left(\vec{a}+\lambda\vec{b}\right).\vec{c} = 0\left[\because \vec{a}+\lambda\vec{b} \perp \vec{c}\right.$

$\left[(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}\right].\left(3\hat{i}+\hat{j}\right) = 0$

$3(2-\lambda)+(2+2\lambda) = 0$

$-\lambda = -8$

$\lambda = 8$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

4. Ans:

$= -2\hat{i}-14\hat{j}-10\hat{k}$

$req.area = \left|\vec{a}\times\vec{b}\right|$

$= \sqrt{(-2)^2+(-14)^2+(-10)^2} = 10\sqrt{3}$

5. Solution:

From the give,

m = cos π/4 = 1/√2

n = cos π/2 = 0

Therefore, $l^2 + m^2 + n^2 = 1$

l^2 + (½) + 0 = 1

l^2 = 1 – ½

l = ±1/√2

Hence, the required vector is:

$\vec{r} = 3\sqrt{2}\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$

$\vec{r} = 3\sqrt{2}\left(\pm\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + 0\hat{k}\right)$

$\vec{r} = \pm 3\hat{i} + 3\hat{j}$

6.

We know

$$\left(\vec{a}.\vec{b}\right)^2 + \left|\vec{a}\times\vec{b}\right|^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2$$

$$\Rightarrow \left(\vec{a}.\vec{b}\right)^2 + 8^2 = 2^2\times 5^2 (\because \left|\vec{a}\times\vec{b}\right| = 8, \left|\vec{a}\right| = 2 \text{ and } \left|\vec{b}\right| = 5)$$

$$\Rightarrow \left(\vec{a}.\vec{b}\right)^2 + 64 = 100$$

$$\Rightarrow \left(\vec{a}.\vec{b}\right)^2 = 36$$

$$\Rightarrow \left(\vec{a}.\vec{b}\right) = 6$$

7.

It is given that $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}, \vec{b} = -\hat{i} + \hat{k}, \vec{c} = 2\hat{j} - \hat{k}$

$\therefore \vec{a} + \vec{b} = \left(2\hat{i} - 3\hat{j} + \hat{k}\right) + \left(-\hat{i} + \hat{k}\right) = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{b} + \vec{c} = \left(-\hat{i} + \hat{k}\right) + \left(2\hat{j} - \hat{k}\right) = -\hat{i} + 2\hat{j}$

We know that the area of parallelogram is $\frac{1}{2}\left|\vec{d}_1\times\vec{d}_2\right|$, where $\vec{d}_1$ and $\vec{d}_2$ are the diagonal vectors.

Now,

$$\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}+\vec{c}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ -1 & 2 & 0 \end{vmatrix} = -4\hat{i} - 2\hat{j} - \hat{k}$$

$\therefore$ Area of the parallelogram having diagonals $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{b}+\vec{c}\right)$

$$= \frac{1}{2}\left|\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}+\vec{c}\right)\right|$$

$$= \frac{1}{2}\left|-4\hat{i}-2\hat{j}-\hat{k}\right|$$

$$= \frac{1}{2}\sqrt{(-4)^2+(-2)^2+(-1)^2}$$

$$= \frac{\sqrt{21}}{2} \text{ square units}$$

Thus, the required area of the parallelogram is $\frac{\sqrt{21}}{2}$ square units.

8.

Let θ be the angle between $\vec{a}$ and $\vec{b}$.

$\vec{a}\times\vec{b} = 3\hat{i}+2\hat{j}+6\hat{k}$(Given)

$\Rightarrow \left|\vec{a}\times\vec{b}\right| = \sqrt{9+4+36}$

$= 7$

We know

$\left|\vec{a}\times\vec{b}\right| = |\vec{a}|\,\left|\vec{b}\right|\sin\theta$

$\Rightarrow 7 = (2)\,(7)\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

9.

$$\vec{a} = \hat{p} + 2\,\widehat{q}$$
$$\vec{b} = 2\,\widehat{p} + \hat{q}$$
$$\vec{a} \times \vec{b} = (\vec{p} + 2\vec{q}) \times (2\vec{p} + \vec{q})$$
$$= 2\vec{p} \times \vec{p} + \vec{p} \times \vec{q} + 4\vec{q} \times \vec{p} + 2\vec{q} \times \vec{q}$$
$$= 2\,(0) + \vec{p} \times \vec{q} - 4\vec{p} \times \vec{q} + 2\,(0)$$
$$= -3\vec{p} \times \vec{q}$$
$$\text{Area of the parallelogram} = \frac{1}{2}\left|\vec{a} \times \vec{b}\right|$$
$$= \frac{1}{2}\left|-3\,(\vec{p} \times \vec{q})\right|$$
$$= \frac{3}{2}\left|\vec{p}\right|\left|\vec{q}\right| \sin 30^o$$
$$= \frac{3}{2}(1)\,(1)\left(\frac{1}{2}\right)(\because \vec{p} \text{ and } \vec{q} \text{ are unit vectors})$$
$$= \frac{3}{4} \text{ sq. units}$$

10

Ans: $\left|\vec{a}\right| = 1$

$$\left(\vec{x} - \vec{a}\right).\left(\vec{x} + \vec{a}\right) = 8$$
$$\left|\vec{x}\right|^2 - 1 = 8$$
$$\left|\vec{x}\right|^2 = 9$$
$$\left|\vec{x}\right| = 3$$

Long Answer Questions – solutions

1. **Ans:** $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 1, \left|\vec{c}\right| = 1,$

$$\vec{a} + \vec{b} + \vec{c} = 0 \quad (Given)$$
$$\vec{a}.\left(\vec{a} + \vec{b} + \vec{c}\right)$$
$$\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$$
$$\left(\vec{a}\right)^2 + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$$
$$1 + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$$

$\vec{a}.\vec{b}+\vec{a}.\vec{c}=-1-------(i)$

similiorly

$\vec{b}.\vec{a}+\vec{b}.\vec{c}=-1-------(ii)$

again

$\vec{c}.\vec{a}+\vec{c}.\vec{b}=-1-------(iii)$

adding$(i),(ii)$*and*(iii)

$2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=-3 \qquad \left[\vec{a}.\vec{b}=\vec{b}.\vec{a}\right.$

$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-3/2$

2. **Ans:** $\vec{a}=2\hat{i}+4\hat{j}-5\hat{k}$

$\vec{b}=\lambda\hat{i}+\hat{j}+3\hat{k}$

$\vec{a}+\vec{b}=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

Unit vector along

$\vec{a}+\vec{b}=\dfrac{\vec{a}+\vec{b}}{\left|\vec{a}+\vec{b}\right|}$

$=\dfrac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+(6)^2+(-2)^2}}$

$=\dfrac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+40}}$

$ATQ\ \vec{c}.\left(\vec{a}+\vec{b}\right)=1$

$\left(\hat{i}+\hat{j}+\hat{k}\right).\left(\dfrac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{(2+\lambda)^2+40}\right)=1$

$\dfrac{(2+\lambda)+6-2}{\sqrt{(2+\lambda)^2+40}}=1$

$2+\lambda+4=\sqrt{(2+\lambda)^2+40}$

sq.both site

$\lambda^2+36+12\lambda=(2+\lambda)^2+40$

$\lambda=1$

3. **Ans:** $\vec{a}.\left(\vec{b}+\vec{c}\right)=0,\ \vec{b}.\left(\vec{c}+\vec{a}\right)=0\ \vec{c}.\left(\vec{a}+\vec{b}\right)=0,(Given)$

$\left|\vec{a}+\vec{b}+\vec{c}\right|^2=\left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right)$

$=\vec{a}.\vec{a}+\vec{a}.\left(\vec{b}+\vec{c}\right)+\vec{b}.\vec{b}+\vec{b}.\left(\vec{a}+\vec{c}\right)+\vec{c}.\vec{c}+\left(\vec{a}+\vec{b}\right)$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$$
$$= 9 + 16 + 25$$
$$= 50$$
$$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{50}$$
$$= 5\sqrt{2}$$

4. **Ans:** Let $\vec{\beta}_1 = \lambda\vec{\alpha}$ $\quad [\because \vec{\beta}_1 \| \text{to} \vec{\alpha}$

$$\vec{\beta}_1 = \lambda\left(3\hat{i} - \hat{j}\right)$$
$$= 3\lambda\hat{i} - \lambda\hat{j}$$
$$\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$$
$$= \left(2\hat{i} + \hat{j} - 3\hat{k}\right) - \left(3\lambda\hat{i} - \lambda\hat{j}\right)$$
$$= (2 - 3\lambda)\hat{i} + (1 + \lambda)\hat{j} - 3\hat{k}$$
$$\vec{\alpha}.\vec{\beta}_2 = 0 \qquad [\because \vec{\beta}_2 \perp \vec{\alpha}$$
$$3(2 - 3\lambda) - (1 + \lambda) = 0$$
$$\lambda = \frac{1}{2}$$
$$\vec{\beta}_1 = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$$
$$\vec{\beta}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$$

5. **Ans:** $\vec{a} + \vec{b} + \vec{c} = 0$

$$\vec{a} + \vec{b} = -\vec{c}$$
$$\left(\vec{a} + \vec{b}\right).\left(-\vec{c}\right) = -\vec{c}.\left(-\vec{c}\right)$$
$$\left(\vec{a} + \vec{b}\right).\left(\vec{a} + \vec{b}\right) = \vec{c}.\vec{c}$$
$$|\vec{a}|^2 + 2\vec{a}\vec{b} + |\vec{b}|^2 = |\vec{c}|^2$$
$$\vec{a}.\vec{b} = \frac{49 - 9 - 25}{2} = \frac{15}{2}$$
$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$
$$= \frac{1}{2}$$
$$\theta = 60$$

6.

Sol. Given, $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$

$\Rightarrow \quad \vec{a}\times\vec{b}-\vec{a}\times\vec{c}=\vec{c}\times\vec{d}-\vec{b}\times\vec{d}$

$\Rightarrow \quad \vec{a}\times\vec{b}-\vec{a}\times\vec{c}+\vec{b}\times\vec{d}-\vec{c}\times\vec{d}=\vec{0}$

$\Rightarrow \quad \vec{a}\times(\vec{b}-\vec{c})+(\vec{b}-\vec{c})\times\vec{d}=\vec{0}$ [By left and right distributive law]

$\Rightarrow \quad \vec{a}\times(\vec{b}-\vec{c})-\vec{d}\times(\vec{b}-\vec{c})=\vec{0}$ [$\because \ \vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$]

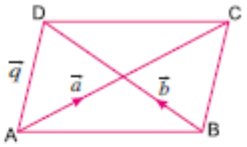
$\Rightarrow \quad (\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=\vec{0}$ [By right distributive law]

$\Rightarrow \quad (\vec{a}-\vec{d}) \ || \ (\vec{b}-\vec{c})$

7

Sol. Let $ABCD$ be a parallelogram such that

$$\overrightarrow{AB}=\vec{p}, \overrightarrow{AD}=\vec{q} \ \Rightarrow \ \overrightarrow{BC}=\vec{q}$$



By triangle law of addition, we get

$$\overrightarrow{AC}=\vec{p}+\vec{q}=\vec{a} \quad \text{[say]} \quad ...(i)$$

Similarly, $\overrightarrow{BD}=-\vec{p}+\vec{q}=\vec{b}$ [say] ...(ii)

On adding equation (*i*) and (*ii*), we get

$$\vec{a}+\vec{b}=2\vec{q} \ \Rightarrow \ \vec{q}=\frac{1}{2}(\vec{a}+\vec{b})$$

On subtracting equation (*ii*) from equation (*i*), we get

$$\vec{a}-\vec{b}=2\vec{p} \ \Rightarrow \ \vec{p}=\frac{1}{2}(\vec{a}+\vec{b})$$

Now, $\vec{p}\times\vec{q}=\frac{1}{4}(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})=\frac{1}{4}(\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b})$

$$=\frac{1}{4}[\vec{a}\times\vec{b}+\vec{a}\times\vec{b}] \ =\frac{1}{2}(\vec{a}\times\vec{b})$$

So, area of a parallelogram $ABCD = |\vec{p}\times\vec{q}|=\frac{1}{2}|\vec{a}\times\vec{b}|$

Now, area of a parallelogram, whose diagonals are $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$.

$$=\frac{1}{2}|(2\hat{i}-\hat{j}+\hat{k})\times(\hat{i}+3\hat{j}-\hat{k})|$$

$$=\frac{1}{2}|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 3 & -1\end{vmatrix}|$$

$$=\frac{1}{2}|[\hat{i}(1-3)-\hat{j}(-2-1)+\hat{k}(6+1)]|$$

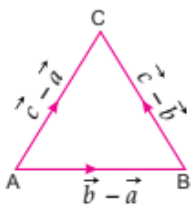
$$=\frac{1}{2}|-2\hat{i}+3\hat{j}+7\hat{k}|$$

$$=\frac{1}{2}\sqrt{4+9+49}$$

$$=\frac{1}{2}\sqrt{62} \text{ sq. units}$$

8

Sol. Since, $\vec{a}$ $\vec{b}$ and $\vec{c}$ are the vertices of a ΔABC as shown.



$\therefore$ Area of $\Delta ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

Now, $\overrightarrow{AB}=\vec{b}-\vec{a}$ and $\overrightarrow{AC}=\vec{c}-\vec{a}$

$\therefore$ Area of $\Delta ABC=\frac{1}{2}[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]$

$$= \frac{1}{2}\left|(\vec{b}\times\vec{c}) - (\vec{b}\times\vec{a}) - (\vec{a}\times\vec{c}) + (\vec{a}\times\vec{a})\right|$$

$$= \frac{1}{2}\left|(\vec{b}\times\vec{c}) + (\vec{a}\times\vec{b}) + (\vec{c}\times\vec{a}) + \vec{0}\right|$$

$$= \frac{1}{2}\left|(\vec{b}\times\vec{c}) + (\vec{a}\times\vec{b}) + (\vec{c}\times\vec{a})\right| \quad ...(i)$$

For three points to be collinear, area of the ΔABC should be equal to zero.

$$\Rightarrow \quad \frac{1}{2}[\vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{a}\times\vec{b}] = 0$$

$$\Rightarrow \quad \vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{a}\times\vec{b} = 0 \quad ...(ii)$$

This is the required condition for collinearity of three points $\vec{a}, \vec{b}$ and $\vec{c}$.

Let $\hat{n}$ be the unit vector normal to the plane of the ΔABC.

$$\therefore \quad \hat{n} = \frac{\overrightarrow{AB}\times\overrightarrow{AC}}{|\overrightarrow{AB}\times\overrightarrow{AC}|} = \frac{\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}}{|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}|}$$

9

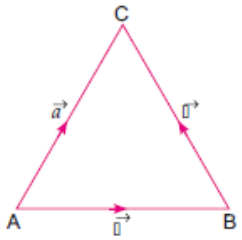
Sol. Given, $\vec{a} = \vec{b} + \vec{c}$

$$\Rightarrow \quad p\hat{i} + q\hat{j} + r\hat{k} = (s\hat{i} + 3\hat{j} + 4\hat{k}) + (3\hat{i} + \hat{j} - 2\hat{k})$$

$$\Rightarrow \quad p\hat{i} + q\hat{j} + r\hat{k} = (s+3)\hat{i} + 4\hat{j} + 2\hat{k}$$

Equating the co-efficient of $\hat{i}, \hat{j}, \hat{k}$ from both sides, we get

$$\Rightarrow \quad s + 3 = p;\ q = 4 \text{ and } r = 2 \quad ...(i)$$



Now, area of triangle $= \frac{1}{2}|\vec{b}\times\vec{c}|$

$$\Rightarrow \quad 5\sqrt{6} = \frac{1}{2}\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ s & 3 & 4 \\ 3 & 1 & -2 \end{vmatrix}\right| = \frac{1}{2}\left|(-6-4)\hat{i} - (-2s-12)\hat{j} + (s-9)\hat{k}\right|$$

$$\Rightarrow \quad 5\sqrt{6} = \frac{1}{2}\sqrt{10^2 + (2s+12)^2 + (s-9)^2} = \frac{1}{2}\sqrt{100 + 4s^2 + 144 + 48s + s^2 + 81 - 18s}$$

$$\Rightarrow \quad 5\sqrt{6} = \frac{1}{2}\sqrt{325 + 5s^2 + 30s}$$

Squaring both sides

$$\Rightarrow \quad 150 = \frac{1}{4}(325 + 5s^2 + 30s)$$

$$\Rightarrow \quad 600 - 325 = 5s^2 + 30s \qquad \Rightarrow \quad 5s^2 + 30s - 275 = 0$$

$$\Rightarrow \quad s = \frac{-30 \pm \sqrt{900 + 4\times5\times275}}{10} = \frac{-30 \pm \sqrt{6400}}{10} = \frac{-30 \pm 80}{10}$$

$$\Rightarrow \quad s = -11, 5 \quad ...(ii)$$

From (i) and (ii)

$$s = -11, 5;\ p = -8, 8;\ q = 4 \text{ and } r = 2$$

10

Sol. Let $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then,

$$(\vec{a} \times \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ c_1 & c_2 & c_3 \end{vmatrix} = (c_3 - c_2)\hat{i} + (c_1 - c_3)\hat{j} + (c_2 - c_1)\hat{k}$$

$\because$ $(\vec{a} \times \vec{c}) = \vec{b}$

$\Rightarrow$ $(c_3 - c_2)\hat{i} + (c_1 - c_3)\hat{j} + (c_2 - c_1)\hat{k} = \hat{j} - \hat{k}$

$\Rightarrow$ $c_3 - c_2 = 0, c_1 - c_3 = 1$ and $c_2 - c_1 = -1$ ...(*i*)

Also, $\vec{a} . \vec{c} = (\hat{i} + \hat{j} + \hat{k}) . (c_1\hat{i} + c_2\hat{j} + c_3\hat{k})$

$\Rightarrow$ $\vec{a} . \vec{c} = c_1 + c_2 + c_3$

$\Rightarrow$ $c_1 + c_2 + c_3 = 3$ $[\because \vec{a} . \vec{c} = 3]$ ...(*ii*)

$\Rightarrow$ $c_1 + c_2 + c_1 - 1 = 3$ $[\because c_1 - c_3 = 1]$ ... (*iii*)

$\Rightarrow$ $2c_1 + c_2 = 4$

On solving $c_1 - c_2 = 1$ and $2c_1 + c_2 = 4$, we get

$$3c_1 = 5 \Rightarrow c_1 = \frac{5}{3}$$

$\therefore$ $c_2 = (c_1 - 1) = \left(\frac{5}{3} - 1\right) = \frac{2}{3}$ and $c_3 = c_2 = \frac{2}{3}$

Hence, $\vec{c} = \left(\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$.

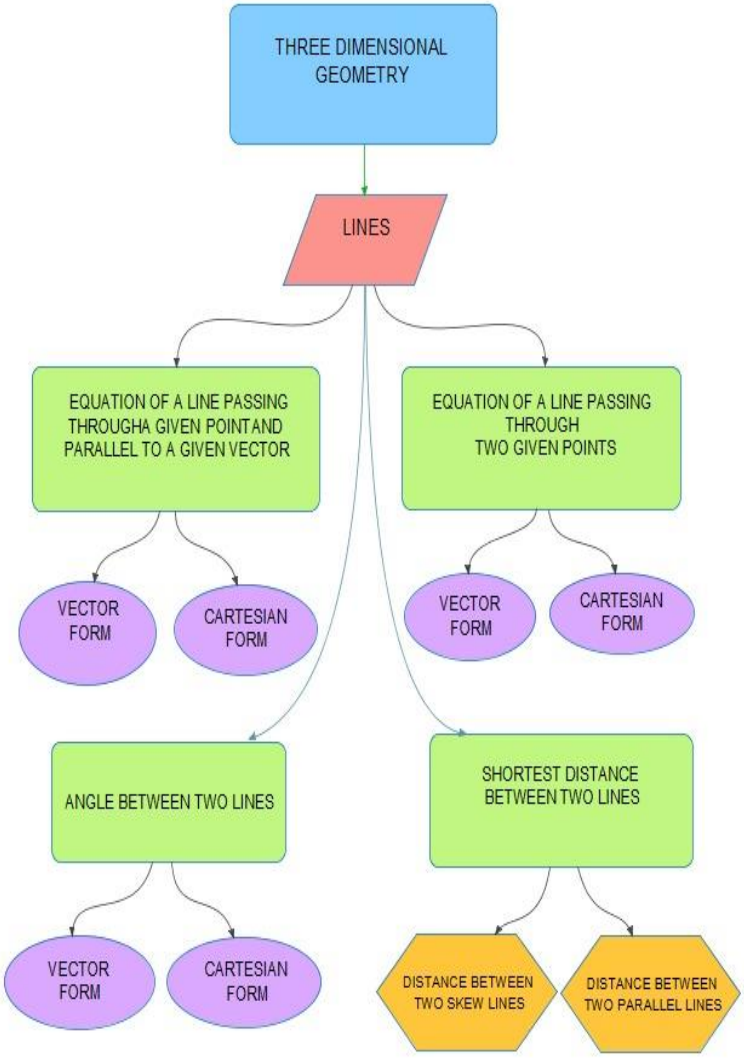
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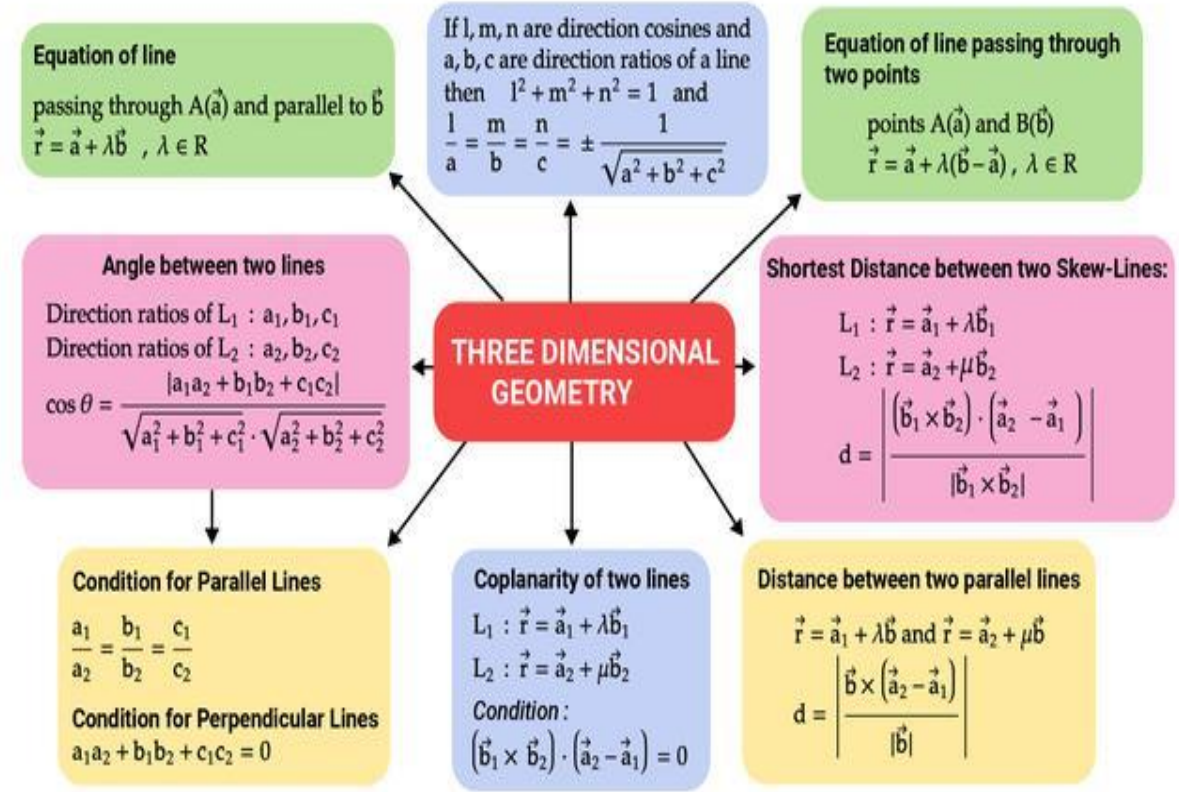
STUDENT SUPPORT MATERIAL 2022-2023

MATHEMATICS

THREE DIMENSIONAL GEOMETRY

CONCEPT MAPPING





THINGS TO REMEMBER

Direction Cosines of a line

A directed line *l* passing through origin making angles α, β, γ with x, y and z axes respectively are called direction angles. Cosine of these angles namely cos α, cos β, cos γ are called direction cosines of the directed line *l*.

Direction cosines of a line are denoted by *l,m,n*

***l* = cos α ,*m* = cos β , *n* = cos γ**

If *l, m* and *n* are the direction cosine of a line then $l^2 + m^2 + n^2 = 1$.

Direction ratios of a line

Any three numbers which are proportional to the direction cosines of a line are called the direction ratios of the line. Direction ratios of a line are denoted as *a,b,c* .

***l = ak, m = bk, n = ck, k* being a constant.**

$$l = \pm \frac{a}{\sqrt{a^2 + b^2 + c^2}} \quad m = \pm \frac{b}{\sqrt{a^2 + b^2 + c^2}} \quad n = \pm \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

The sign to be taken for *l, m* and *n* depend on the desired sign of *k*, either a positive or negative.

The direction ratios of the line segment joining the points P(x_1, y_1, z_1) and Q(x_2,y_2,z_2) may be taken as $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$.

Equation of line in space :

1) **(a) Equation of a line passing through a point with position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda \vec{b}$.**

(b) Equation of a line passing through the point (x_1, y_1, z_1) and having direction ratios a, b, c is $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

(c) Equation of a line passing through the point (x_1, y_1, z_1) and having direction cosines l, m, n is $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$

2) **(a) Equation of a line passing through two points with position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$.**

(b) Equation of a line passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$

Distance Formula :

1) **Distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by**
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

2) **(a) The Shortest Distance between the Skew Lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is $d = \left| \frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} X \vec{b_2})}{|(\vec{b_1} X \vec{b_2})|} \right|$**

(b) The Shortest Distance between the Skew Lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ **and** $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ **is** $d = \left| \frac{\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2}} \right|$

3) The distance between the parallel lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}$ **and** $\vec{r} = \vec{a}_2 + \mu \vec{b}$ **is** $d = \left| \frac{\vec{b} \times (\vec{a_2}-\vec{a_1})}{|\vec{b}|} \right|$

MCQ QUESTIONS

1	If the cartesian equation of a line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, write its vector equation. (a) $\vec{r} = (\hat{\imath} - 4\hat{\jmath} + 6\hat{k}) + \mu(5\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$ (b) $\vec{r} = (3\hat{\imath} - 4\hat{\jmath} + 3\hat{k}) + \mu(-5\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$ (c) $\vec{r} = (3\hat{\imath} + 4\hat{\jmath} + 3\hat{k}) + \mu(-5\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$ (d) $\vec{r} = (3\hat{\imath} - 4\hat{\jmath} + 3\hat{k}) + \mu(5\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$
2	Find the foot of the perpendicular drawn from the point (2,-3,4) on the y-axis. (a) (2,0,4) (b) (0.3.0) (c) (0,-3,0) (d) (-2,0,-4)
3	If the lines $\frac{x-2}{3} = \frac{y-1}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are perpendicular, find the value of **k .** (a) -2/5 (b) -2/7 (c) 4 (d) 2/7
4	The equation of a line is $\frac{x-1}{-2} = \frac{y+3}{3} = \frac{z+2}{6}$, find the direction cosines of a line parallel to the given line. (a) -2/7, 3/7, 6/7 (b) 2/7, -3/7,- 6/7 (c) -2 , 3 , 6 (d) 2 , -3 , -6

5	If a line makes angles 90 ^{0} and 60^{0}with the positive direction of x and y axes ,find the angle which it makes with positive direction of z -axis. (a) π/3 (b) π/4 (c) π/6 (d) 0
6	Write direction cosines of a line parallel to z-axis. (a) 1,0,0 (b) 0,0,1 (c) 1,1,0 (d) -1,-1,-1
7	The distance of a point P (a,b,c) fom x-axis is (a) $\sqrt{a^2+b^2}$ (b) a^2+b^2 (c) a^2+c^2 (d) $\sqrt{b^2+c^2}$
8	If α,β, ϒ are the angles that a line makes with the positive direction of x,y,z axis respectively then the direction cosines of the line are (a) cosα,sinβ, cosϒ (b) cosα,cosβ, cosϒ (c) sinα,sinβ, sinϒ (d) 1, 1, 1

MCQ ANSWERS

1	(b) $\vec{r} = (3\hat{\imath} - 4\hat{\jmath} + 3\hat{k}) + \mu\,(-5\hat{\imath} + 7\hat{\jmath} + 2\hat{k}\,)$
2	(c) (0,-3,0)
3	(a) -2/5
4	(a) -2/7, 3/7, 6/7
5	(c) π/6
6	(b) 0,0,1
7	(d) $\sqrt{b^2+c^2}$
8	(b) cosα,cosβ, cosϒ

2 MARKS QUESTIONS

1	If α,β, ϒ are the direction angles of a line, find the value of sin^{2}α + sin^{2}β+sin^{2}ϒ

2	Find the direction cosines of the following line: $\frac{3-x}{-1} = \frac{2y-1}{2} = \frac{z}{4}$
3	Find the value of k so that the lines x = -y = kz and x – 2 = 2y + 1 = -z + 1 are perpendicular to each other.
4	The x –coordinate of a point on the line joining the points P (2, 2, 1) and Q (5, 1, -2) is 4 . Find its z- co ordinate
5	Check whether the lines passing through (1, 1, 2) and (3,5,1) is parallel to the line through (4, 2,-1) and (2, -2 , 0)
6	Find the vector and Cartesian equation of the line passing through points (3, -2, -5) and ,(5, -4, 6)
7	Find the distance of the point (– 2, 4, – 5) from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$
8	If the equation of a line is x = ay + b, z = cy + d, then find the direction ratios of the line and a point on the line.
9	Find the equation of a line passing through the points P(-1, 3,2) and Q(-4,2,-2).Also ,if point R(5, 5, α) is collinear with P and Q then find the value of α
10	The points A(1,2,3), B(-1,-2,-3)and C(2,3,2) are the vertices of a parallelogram,then find the equation of CD.

ANSWERS OF 2 MARKS

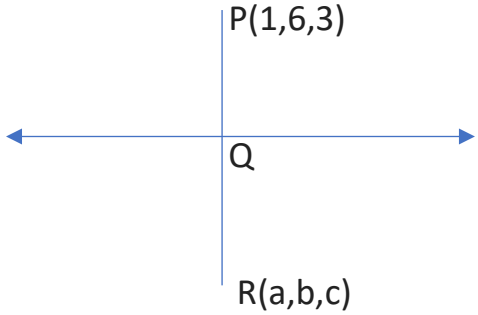
1	2
2	Direction ratios are 1 ,1 ,4 Direction cosines are $\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}$
3	The lines are : $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1/k}$ *and* $\frac{x-2}{1} = \frac{y+1}{1/2} = \frac{z-1}{-1}$ Since these lines are perpendicular $1x1 + -1x\frac{1}{2} + \frac{1}{k}x - 1 = 0$ k = 2

4	The equation of lines are $\frac{x-2}{3}=\frac{y-2}{-1}=\frac{z-1}{-3}$ Any point on this line is (3k+2, -k+2, -3k+1) 3k+2=4 $\Rightarrow$ k=2/3 z co ordinate = -3x2/3 + 1 =-2 + 1 = -1
5	The direction ratios of line joining (1, 1, 2) and (3, 5, 1) are 2,4,-1 The direction ratios of line joining (4, 2,-1)and (2, -2, 0) are -2,-4,1 Since the direction ratios are proportional they are parallel
6	i) Cartesian equation : $\frac{x-3}{2}=\frac{y+2}{-2}=\frac{z+5}{11}$ ii) Vector Equation : $\vec{a}+\lambda(\vec{b}-\vec{a})$ $(3\vec{i}-2\vec{j}-5\vec{k})+\lambda(2\hat{i}-2\hat{j}+11\vec{k})$
7	P (–2, 4, – 5) is the given point. Any point Q on the line is given by (3λ-3, 5λ + 4, 6λ-8) $\overrightarrow{PQ}=(3\lambda-1)\hat{\imath}+5\lambda\hat{\jmath}+(6\lambda-3)\hat{k}$ PQ and the given line are perpendicular $\therefore\ (3\lambda-1)3+5\lambda.5+(6\lambda-3)6=0 \Rightarrow \lambda=\frac{3}{10}$ $\overrightarrow{PQ}=\frac{1}{10}\hat{\imath}+\frac{1}{10}\hat{\jmath}-\frac{12}{10}\hat{k}$ Magnitude of PQ = $\sqrt{\frac{37}{10}}$
8	Using both the conditions $\frac{x-b}{a}=y \quad \frac{z-d}{c}=y$ $\frac{x-b}{a}=\frac{y-0}{1}=\frac{z-d}{c}$ Direction ratios of the given line are a, 1, c and a point on the given line and the point on the given line is (b,0,d)
9	The equation of the line passing through P and Q is $\frac{x+1}{3}=\frac{y-3}{1}=\frac{z-2}{4}$ Since the given point is collinear with the points P and Q , R(5, 5, α) satisfies the equation Substituting in the given equation , α = 10
10	Let D(a,b,c) A (1,2,3) B(-1,-2,-3) C(2,3,2) Midpoint of AC = Midpoint of BD (3/2,5/2,5/2) = (-1+a /2, -2+b /2,-3+c /2) a=4, b=7 , c=8 D(4,7,8) Equation of CD is $\frac{x-2}{2}=\frac{y-3}{4}=\frac{z-2}{6}$

LONG ANSWER TYPE QUESTIONS

1	Find the equation of the line passing through the point P (-1,3,-2) and perpendicular to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$
2	Find the co-ordinates of the foot of the perpendicular drawn from the point A (1,8,4) to the line joining B (0, -1,3) and C (2,-3,-1).
3	(a) Find the image of the point (1,6,3) in the line: $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ (b) Also, find the length of the segment joining the given point and its image
4	Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect. Also, find the point of intersection
5	Find the value of ,λ so that the lines $\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other
6	Find the equation of line passing through (1,2,3) and midpoint of the line joining (2,-1,3) and (1,2,5)
7	Show that the lines $\vec{r}=\left(\hat{\imath}+\hat{\jmath}-\hat{k}\right)+\lambda\,(3\hat{\imath}-\hat{\jmath})$ and $\vec{r}=\left(4\hat{\imath}-\hat{k}\right)+\mu\left(2\hat{\imath}+3\hat{k}\right)$ intersect each other . Find their point of intersection.
8	Find the shortest distance between the lines whose vector equations are $\vec{r}=(1-t)\,\hat{\imath}+(t-2)\,\hat{\jmath}+(3-2t)\,\hat{k}$ $\vec{r}=(s+1)\hat{\imath}+(2s-1)\hat{\jmath}-(2s+1)\hat{k}$
9	Find the shortest distance between the lines $\vec{r}=3\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(\hat{\imath}+2\hat{\jmath}+2\hat{k})$ and $\vec{r}=5\hat{\imath}-2\hat{\jmath}+\mu(3\hat{\imath}+2\hat{\jmath}+6\hat{k})$ If the lines intersect find their point of intersection
10	Find the equation of the line passing through the point P (2, -1,3) and perpendicular to the lines $\vec{r}=\left(\hat{\imath}+\hat{\jmath}-\hat{k}\right)+\lambda\left(2\hat{\imath}-2\hat{\jmath}+\hat{k}\right)$ and $\vec{r}=\left(2\hat{\imath}-\hat{\jmath}-3\hat{k}\right)+\mu\left(\hat{\imath}+2\hat{\jmath}+2\hat{k}\right)$

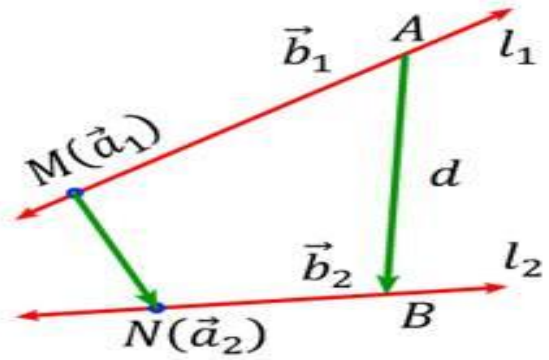
ANSWERS OF LONG ANSWER TYPE QUESTIONS

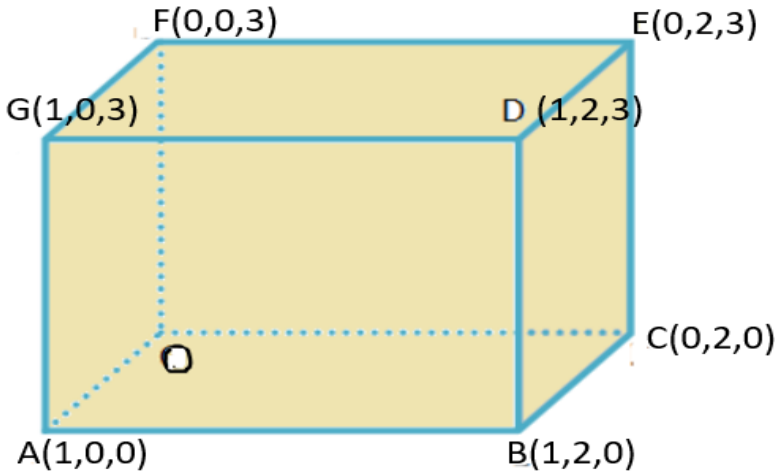
1	Equation of the line passing through P(-1,3,-2) is $\frac{x+1}{a}=\frac{y-3}{b}=\frac{z+2}{c}$ Since it is perpendicular to the two given lines a+2b+3c =0 and -3a+2b+5c =0 Solving a =2 , b=-7 , c=4 Equation of required line is $\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$
2	Let D be the foot of the perpendicular Equation of line BC is $\bar{r}$= -j+3k + λ (2i-2j -4k) Therefore any point D on the line is (2 λ, -1-2 λ, 3- 4 λ) Since AD is perpendicular to BC, (2 λ -1)x2 + (-1-2 λ -8)x(-2) + (3-4 λ -4)x(-4)=0 Solving we get λ = -5/6 So the required point D is (-5/3,2/3, 19/3)
3	 Q is the foot of the perpendicular from P Therefore Q is (λ ,2 λ +1,3 λ +2) PQ is perpendicular to the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$, so we have ($\lambda$ -1)x1 + (2 λ +1-6)x2 +(3 λ +2-3)x3=0 $\Rightarrow$ λ = 1 Q = (1,3,5) Using midpoint formula image is (1,0,7) Required distance PR = $2\sqrt{13}$ using distance formula
4	$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$ $\Rightarrow$ x= $2\lambda+1, y=3\lambda+2, z=4\lambda+3$ $\frac{x-4}{5}=\frac{y-1}{2}=z$ = k $\Rightarrow$ x=5k+4 ,y =2k+1, z=k Lines intersect means $2\lambda+1=5k+4$ and $3\lambda+2=2k+1$ $\lambda=-1\ and\ k=-1$ $4\lambda+3=k$ this equation is true for $\lambda=-1\ and\ k=-1$ The lines are intersecting and the point of intersection is (-1,-1,-1)

5	$\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{5z-10}{11} \Rightarrow \frac{x-1}{-3}=\frac{y-2}{2\lambda/7}=\frac{z-2}{11/5} \rightarrow (i)$ $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5} \Rightarrow \frac{x-1}{-\frac{3\lambda}{7}}=\frac{y-5}{1}=\frac{z-6}{-5} \rightarrow (ii)$ (i) and (ii) are perpendicular -3$(-\frac{3\lambda}{7})$ +$\frac{2\lambda}{7}$ (1) + $\frac{11}{5}$(-5) =0 $\Rightarrow \lambda = 7$
6	Midpoint of (2,-1,5) and (1,2,3) is $(\frac{3}{2},\frac{1}{2},4)$ Dr of line joining midpoint and (1,2,3) is ½,$\frac{-3}{2}$,1 Equation of the line is $\bar{r}$ = $\hat{\imath}+2\hat{\jmath}+3\hat{k}$ + λ$(\frac{1}{2}\hat{\imath}-\frac{\hat{3}}{2}\hat{\jmath}+\hat{k})$ Cartesian equation is $\frac{x-1}{1/2}=\frac{y-2}{-3/2}=\frac{z-3}{1}$
7	The two given lines will intersect if $(\hat{\imath}+\hat{\jmath}-\hat{k})+\lambda(3\hat{\imath}-\hat{\jmath}) = (4\hat{\imath}-\hat{k})+\mu(2\hat{\imath}+3\hat{k})$ for some particular values of λ and μ Equating coefficients of $\hat{\imath}\ and\ \hat{\jmath}$ and solving λ = 1 and $\mu = 0$ Substituting in the coefficient of $\hat{k}$, the equation is satisfied. ∴ the two lines intersect Putting λ = 1 in the first line , the point of intersection is (4,0,-1)
8	Shortest distance , d = $\frac{8}{\sqrt{29}}$
9	The shortest distance between the lines is 0 ∴The lines are intersecting Point of intersection is (−1,−6,−12)
10	Required equation of line is $\vec{r} = (2\hat{\imath}-\hat{\jmath}+3\hat{k})+\mu(2\hat{\imath}+\hat{\jmath}-2\hat{k})$

CASE STUDY QUESTIONS

1	A student drew 2 skew lines as shown below with their points through which they pass and their directions $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$. The equations of these two lines $l_1\ and l_2$ are given by$\frac{x+2}{2}=\frac{y}{3}=\frac{z-2}{1}$ and$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$

	 Based on the information ,answer the following 1.The vector $\overrightarrow{b_1}$ x $\overrightarrow{b_2}$ equals (a) $5\hat{\imath} - \hat{\jmath} - 7\hat{k}$ (b) $4\hat{\imath} - 5\hat{\jmath} - 6\hat{k}$ (c)$\hat{\imath} - \hat{\jmath} + \hat{k}$(d) none of these 2.The vector $\overrightarrow{MN}$ equals (a) $4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$ (b) $5\hat{\imath} - 6\hat{\jmath} - 5\hat{k}$ (c)$5\hat{\imath} + 5\hat{\jmath} + 5\hat{k}$ (d) none of these 3.The shortest distance between these two lines l_1 $and l_2$ is (a) √2 units (b) √3 units (c) √5 units (d) none of these 4.The lines $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{1}$ and $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{1}$are (a)parallel (b)intersecting (c) skew lines (d) perpendicular 5. The shortest distance between the lines $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z+1}{1}$ and $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z}{1}$is (a) √2 units (b) √3 units (c) √5 units (d) none of these
2	Anu made a cuboidal fish tank having coordinates O (0,0,0) ,A(1,0,0), B(1,2,0) C(0,2,0) ,D(1,2,3), E0,2,3),F(0,0,3) and G(1,0,3)



Based on the above information ,answer the following questions.

1.Direction cosines of AB are

(a) < 1,1,0 > (b) < 0,2,0 > (c) <0,1,0> (d) none of these

2.Equation of diagonal OD is

(a) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ (b) $\frac{x}{0}=\frac{y}{1}=\frac{z}{2}$ (c) $\frac{x}{0}=\frac{y}{1}=\frac{z}{2}$ (d) none of these

3. The lines BC and DE are

(a)parallel (b)intersecting (c) skew lines (d) perpendicular

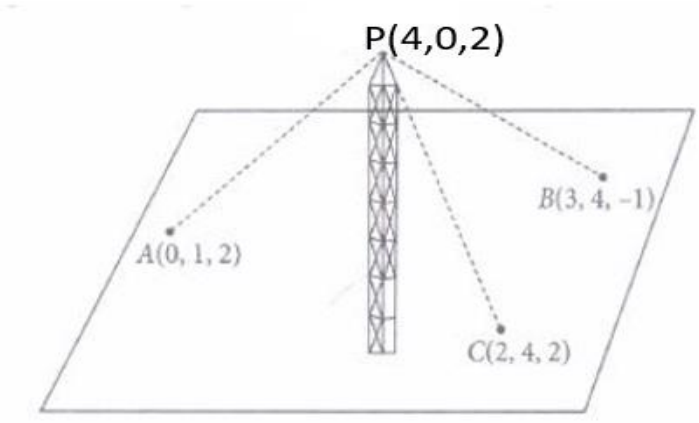
4.Equation of line BF is

(a) $\frac{x}{1}=\frac{y}{2}=\frac{z}{-3}$ (b) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ (c) $\frac{x}{0}=\frac{y}{1}=\frac{z}{2}$ (d) none of these

5.The lines AB and OD are

(a)parallel (b)intersecting (c) skew lines (d) perpendicular

3 A mobile tower stands at the top of a hill. Consider the surface on which tower stands as a plane having points A(0, 1,2), B(3, 4, -1) and C(2, 4, 2) on it. The mobile tower is tied with 3 cables from the point A, Band C such that it stands vertically on the ground. The peak of the tower is at the point (4,0,2), as shown in the figure.Let N(2,-3,1) be the foot of the perpendicular from the point P.



	Based on the above information, answer the following questions (i) The equation of line BC is (a) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ (b) $\frac{x-2}{1}=\frac{y-4}{0}=\frac{z-2}{-3}$ (c) $\frac{x-4}{3}=\frac{y}{-2}=\frac{z-2}{1}$ (d) none of these (ii)The equation of the perpendicular line drawn from the peak of tower to the foot of the perpendicular where N(2,-3,1) is the foot of the perpendicular is (a) $\frac{x}{3}=\frac{y}{-2}=\frac{z}{1}$ (b) $\frac{x-4}{-2}=\frac{y}{-3}=\frac{z-2}{-1}$ (c) $\frac{x-4}{3}=\frac{y}{2}=\frac{z-2}{1}$ (d) none of these (iii) The height of the tower from the ground is (a) 6 units (b) $\sqrt{14}$ units (c)$\sqrt{13}$ units (d) 14 units (iv)) The direction ratios of the line joining the points A and B are (a)1,2,3 (b) 1, 2,0 (c) 3, 3, -3 (d) 3,4 -1 (v) If Q is the reflection of the point P (4,0,2) in the line joining the points N(2,-3,1) and B(3,4,-1) ,then the coordinates of Q is (a)(1,2,3) (b) (1, 2,0) (c) (1,2,1) (d) (0,-6,0)

ANSWERS OF CASE STUDY QUESTIONS

1	i(a) ii(c) iii(b) iv (a) v(d)
2	i(c) ii (a) iii(a) iv (a) v (c)
3	i (b) ii(b) iii (b) iv(c) v(d)

KENDRIYA VIDYALAYA SANGATHAN –ERNAKULAM REGION

STUDY MATERIAL- LINEAR PROGRAMMING

ACADEMIC YEAR 2022-23

- **A linear programming problem is one that is concerned with finding optimal value (maximum or minimum) of a linear function of several variables (called objective function) subject to the condition that the variable are non negative and satisfy a set of linear in equalities (called linear constraints). Variables are sometimes called decision variables and are non negative.**
- **The common region determined by all the constraints including the non negative constraints of a linear programming problem is called the feasible region or solution region for the problem .**
- **Points within and on the boundary of the feasible region represent feasible solution of the constraints . Any point outside the region is an infeasible solution**
- **Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called the optimal solution .**

The following Theorems are fundamental in solving linear programming problems:

Theorem 1 Let R be the feasible region (convex polygon) for a linear programming problem and let Z = ax + by be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

Theorem 2 Let R be the feasible region for a linear programming problem, and let Z = ax + by be the objective function. If R is bounded, then the objective function Z has both a maximum and a minimum value on R and each of these occurs at a corner point (vertex) of R. If the feasible region is unbounded, then a maximum or a minimum may not exist. However, if it exists, it must occur at a corner point of R. Corner point method for solving a linear programming problem.

The method comprises of the following steps: (i) Find the feasible region of the linear programming problem and determine its corner points (vertices). (ii) Evaluate the objective function Z = ax + by at each corner point. Let M and m respectively be the largest and smallest values at these points. (iii) If the feasible region is bounded, M and m respectively are the maximum and minimum values of the objective function.

If the feasible region is unbounded, then (i) M is the maximum value of the objective function, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, the objective function has no maximum value. (ii) m is the minimum value of the objective function, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, the objective function has no minimum value.

If two corner points of the feasible region are both optimal solutions of the same type, i.e., both produce the same maximum or minimum, then any point on the line segment joining these two points is also an optimal solution of the same type

S

Corner points of the feasible region for anLPPare

MCQ QUESTIONS

1 Let F = 4x+ 6y be the objective function. The Minimum value of F occurs at .....

(a) only (0, 2) (b) only (3, 0) (c) the mid-point of the line segment joining the points (0, 2) and (3, 0) only (d) any point on the line segment joining the points (0, 2) and (3, 0).

2. Solution set of the inequality 2x+ y> 5 is .....

(a) The half plane containing origin (b) The open half plane not containing origin (c) xy- plane excepts the points on the line 2x+ y= 5 (d) None of these

3. T he point at which the maximum value of Z = 3x+ 2y subject to the constraints x+ 2y≤2, x ≥0, y ≥0 is .....

(a) (0, 0) (b) (1.5, – 1.5) (c) (2, 0) (d) (0, 2)

4. The feasible region of the inequality x+ y≤1 and x– y≤1 lies in ..... quadrants.

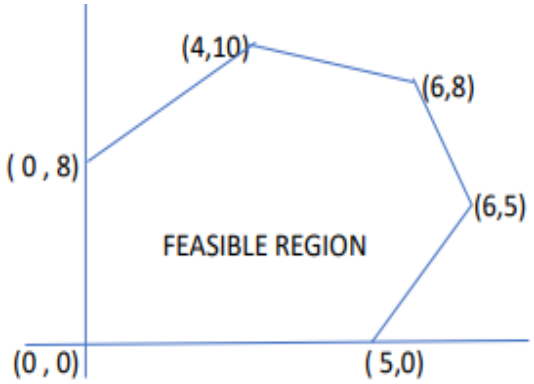
(a) Only I and II (b) Only I and III (c) Only II and III (d) All the four

5 The region represented by the in equation x – y≤ –1, x – y ≥ 0, x≥0, y≥0 is .....

(a) bounded (b) unbounded (c) do not exist (d) triangular region

SHORT ANSWER QUESTIONS

1. The feasible region of an L.P.P is shown here.If z=3x-4y is the objective function then Find the Min(Z)



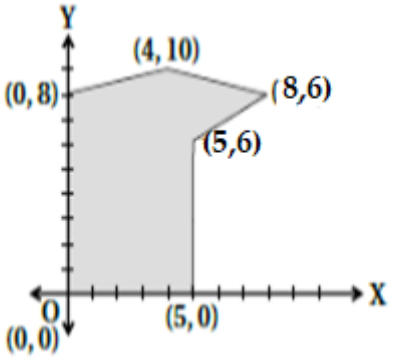
2. Find the Maximum value of Z=6X+8Y subject to the constraints 2x+y≤30,x+2y≤24,x≥0,y≥0

3. Find the point at which the maximum value of (3x + 2y) subject to the constraints x + y ≤2, x ≥ 0, y ≥ 0

4. The vertices of the feasible region determined by some linear constraints are (0, 2), (1, 1),(3, 3), (1, 5). Let Z = px+ qy where p, q> 0.Find the condition on p and q so that the maximum of Z occurs at both the points

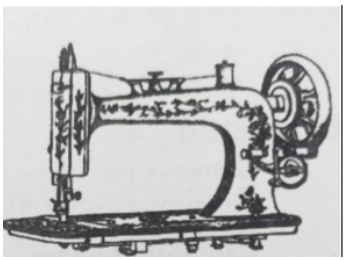
(3, 3) and (1, 5)

5. The feasible solution for a LPP is shown in Figure Let z = 3x – 4y be the objective function.Find the points at which maximum of Z occurs



CASE STUDY QUESTIONS

1. Suppose a dealer in rural area wishes to purchase a number of sewing machines. He has only Rs. 5760 to invest and has space for at most 20 items for storage. An electronic sewing machine costs him Rs. 360 and a manually operated sewing machine Rs. 240. He can sell an electronic sewing machine at a profit of Rs. 22 and a manually operated sewing machine at a profit of Rs.18. Based on the above information, answer the following question



1 Let x and y denote the number of electronic sewing machines and manually operated sewing machines purchased by the dealer. If it is assumed that the dealer purchased atleast one of the given machines then:

(a) $x+y \geq 0$ (b) $x+y < 0$ (c) $x+y > 0$ (d) $x+y \leq 0$

Q 2 Let the constraints in the given problem is represented by the following inequalities: $x+y \leq 20$; $360x+240y \leq 5760$ and $x,y \geq 0$. Then which of the following point lie in its feasible region

a) (0,24) (b) (8,12) (c) (20,2) (d) None of these

Q 3 If the objective function of the given problem is maximize Z = 22x+18y, then its optimal value occurs at (a) (40, 0) (b) (16, 0) (16, 8) (c) (8, 12) (d) (0, 2) (d) (0, 2

Q 4 In an LPP if the objective function Z=ax+by has the same maximum value on two corner points of the feasible region ,then the number of points at which Z_{MAX} is

a)0 b) 2 c) finite d) infinite

Q 5 If an LPP admits optimal solution at two consecutive vertices of a feasible region, then

(a) The required optimal solution is at a mid pointof the line joining two points.

(b) The optimal solution occurs at every point on the line joining these two points.

(c) The LPP under consideration is not solvable.

(d) The LPP under consideration must be reconstructed

2. A manufacturing company makes two models X and Y of a product. Each piece of model X requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model Y requires 12 labour hours of fabricating and 3 labour hours for finishing, the maximum labour hours available for fabricating and finishing are 180 and 30 respectively. The company makes a profit of Rs. 8000 on each piece of model X and Rs. 12000 on each piece of model Y. Assume x is the number of pieces of model X and y is the number of pieces of model Y. Based on the above information, answer the following questions



Q1.Which among these is not a constraint for this LPP?

(a) 9x+12y ≥ 180 (b) 3x+4y ≤ 60 (c) x+3y ≤ 30 (d) None of these

Q2.The shape formed by the common feasible region is:

(a) Triangle (b) Quadrilateral (c) Pentagon (d) Hexagon Q

Q3.Which among these is a corner point for this LPP?

(a) (0,20) (b) (6,12) (c) (12,6) (d) (10,0)

Q4. Maximum of Z occurs at

(a) (0,20) (b) (0,10) (c) (20,10)

Q5. The sum of maximum value of Z is:

(a) 168000 (b) 16000 (c) 120000 (d) 180000

3. An aeroplane can carry a Maximum of 200 passengers.A profit of ₹1000 is made on each executive class ticket and a profit of ₹600 is made on each Economy class ticket. The airline reserves at least 20 seats for the Executive class. However at least 4 times as many passengers prefer to travel by economy class than by executive class .It is given that the number of executive class ticket is 'x' and that of economy class ticket isy



'.

Q1.The maximum value of x+y is (A) 100 (B) 200 © 80 (D) 2

Q2.The relation between 'x' and 'y' is (A) x < y (B) x≥4y © y > 80 (D) y≥4x

Q3.Which among the following is not a constraint of this L.P.P (A) x≥80 (B) x+Y≤20 © x≥0 (D) y≥4x

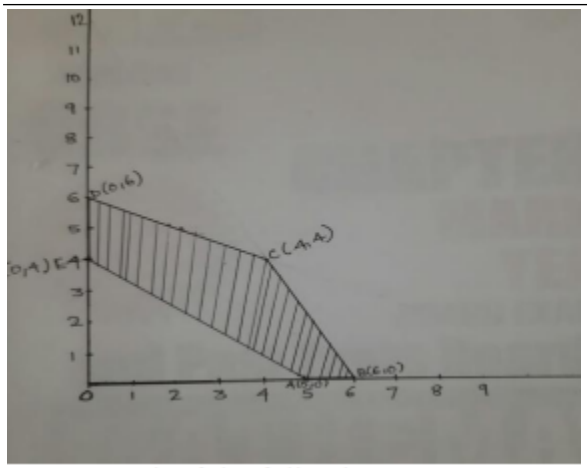
Q4.The profit when x=20 and y=80 is ₹ (A) 60000 (B) 68000 © 64000 (D) 13600

Q5.The maximum profit is

(A)128000. B)68000 C)120000 D)136000

LONG ANSWER QUESTIONS

1. Solve the following linear programming problem (L P P) graphically. Maximize Z=2x+5y, Subject to the constraints ; 2x+4y≤ 8,8,y≤ 66,x+y≤ 4 x,y≥ 0
2. The corner points of the feasible region determined by the system of linear constraints are as shown below



Answer each of the following (i)Let Z =600x+400y be the objective function.Find the maximum and minimum value of Z and also find the corresponding points at which the maximum and minimum value occurs. (ii)Let Z =ax + by ,where a, b >0 be the objective function.Find the condition on a and b so that the maximum value of Z occurs at A(2,8) and B(4,6).Also mention the number of optimal

solutions in this case.

3. Minimize and maximize Z=5x+2y subject to the following constraints x
   -2y≤ 2, 3x + 2y < 12<12 -3x+2y≤3, x≥ 0, y≥ 0

4. Minimise (Z) = 5x + 7y
   Subject to constraints 2x + y ≥ 8, x + 2y ≥ 10 and x ≥ 0, y ≥ 0

5. Maximize Z= 3x+5y such that x+3y≥ 3 , x≥ 2 , y≥ 0

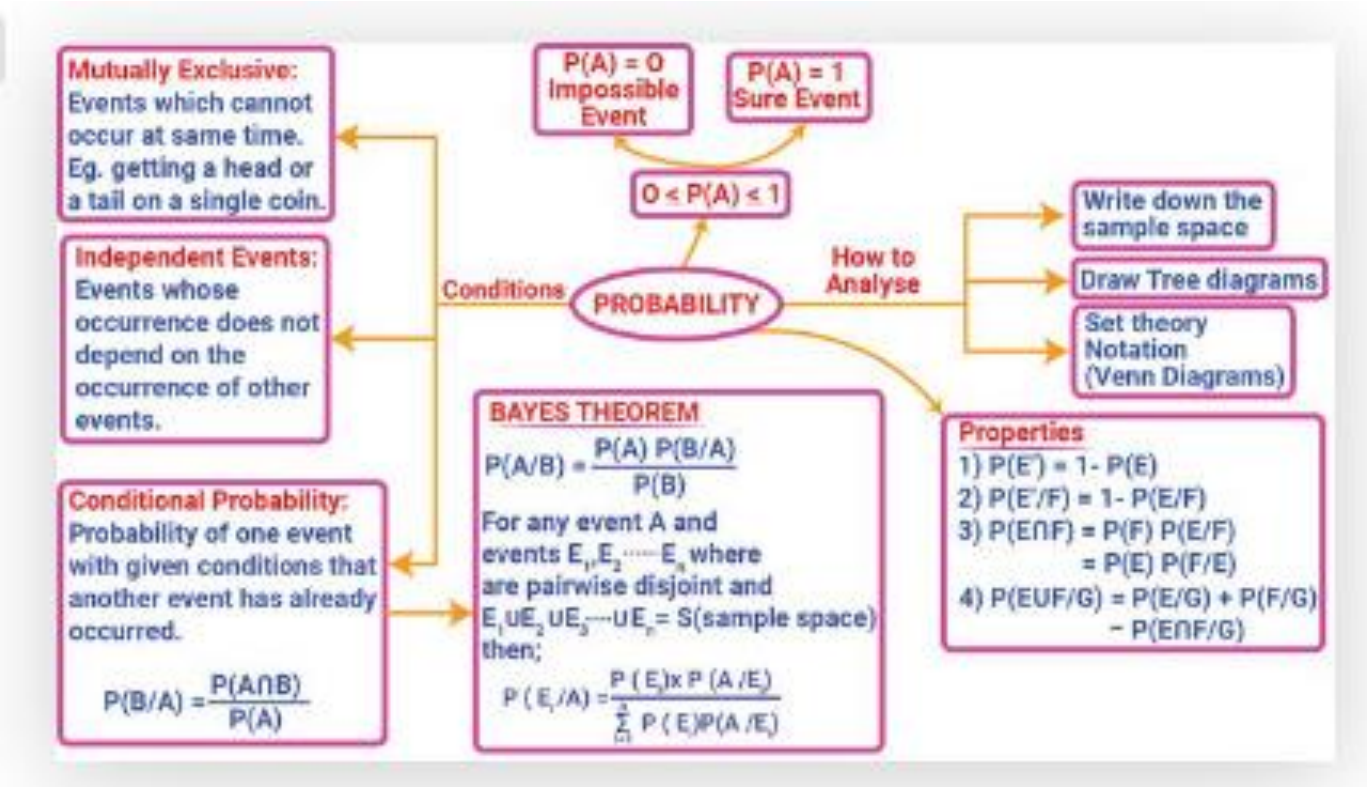
ANSWERS

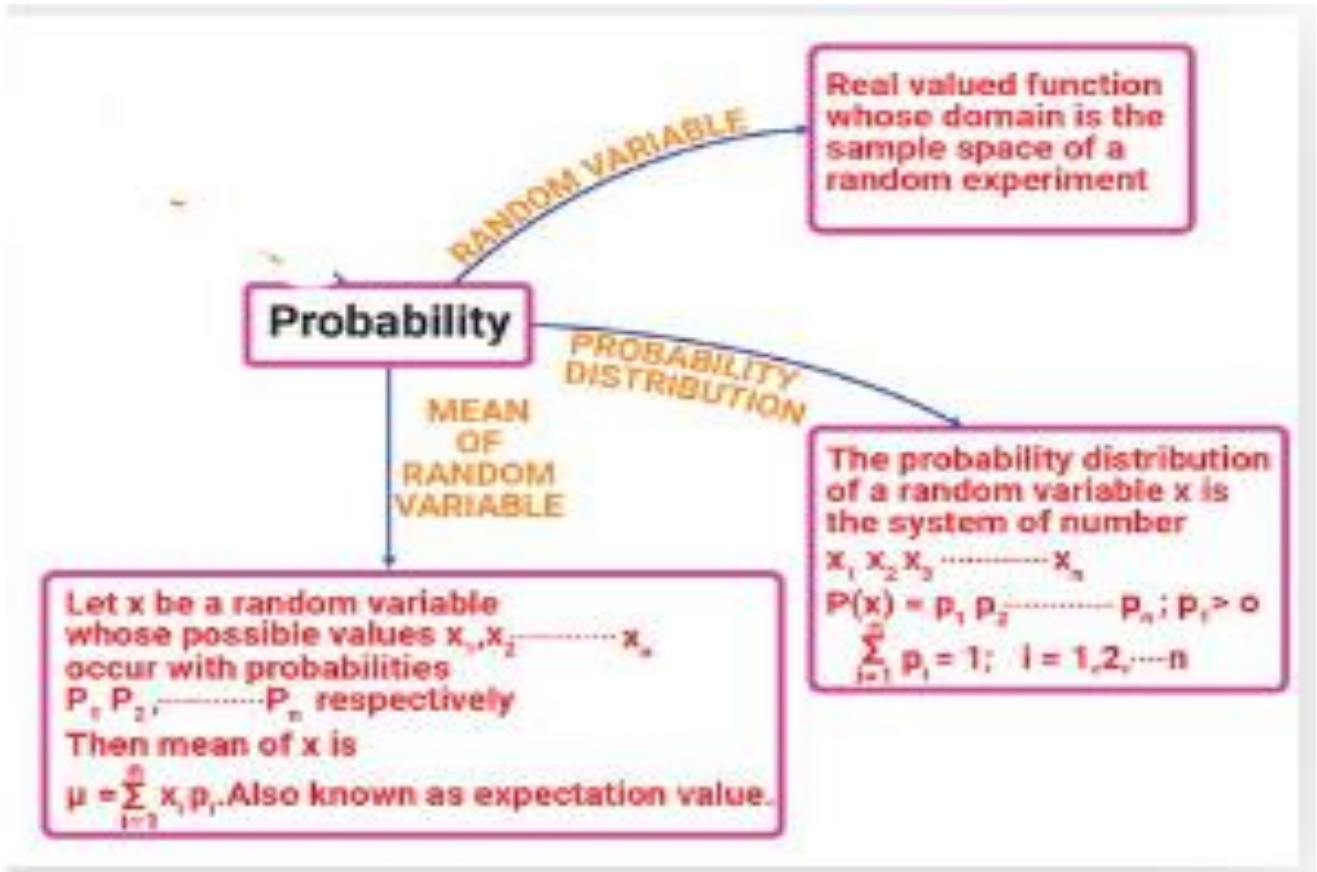
MCQ	1.d	2b	3c	4c	5c
SHORT ANSWER	1)-32	2)120	3.(2,0)	4.p=q	5.(5,0)
CASE SYUDY 1	1c	2b	3c	4c	5b
CASE STUDY 2	1a	2b	3c	4d	5a
CASE STUDY 3	1b	2d	3b	4b	5d
LONG ANSWER QUESTION	1)maxZ=10	2 (i)max 3600 (ii)a=b	3) z_{max} =19 Z_{mini}=0	4)38	5) 7

CHAPTER 13

PROBABILITY

CONCEPT AND MIND MAPPING





POINTS TO REMEMBER

Event: A subset of the sample space associated with a random experiment is called an event or a case.

e.g. In tossing a coin, getting either head or tail is an event.

Equally Likely Events: The given events are said to be equally likely if none of them is expected to occur in preference to the other.

e.g. In throwing an unbiased die, all the six faces are equally likely to come.

Mutually Exclusive Events: A set of events is said to be mutually exclusive, if the happening of one excludes the happening of the other, i.e. if A and B are mutually exclusive, then $(A \cap B) = \Phi$

e.g. In throwing a die, all the 6 faces numbered 1 to 6 are mutually exclusive, since if any one of these faces comes, then the possibility of others in the same trial is ruled out.

Exhaustive Events: A set of events is said to be exhaustive if the performance of the experiment always results in the occurrence of at least one of them. If E1, E2, …, En are exhaustive events, then $E1 \cup E2 \cup \ldots\ldots \cup En = S$.

e.g. In throwing of two dice, the exhaustive number of cases is $6^2 = 36$.

Since any of the numbers 1 to 6 on the first die can be associated with any of the 6 numbers on the other die.

Complement of an Event:

Let A be an event in a sample space S, then the complement of A is the set of all sample points of the space other than the sample point in A and it is denoted by A' or $\bar{A}$.

i.e. $A' = \{n : n \in S, n \notin A]$

Note:

(i) An operation which results in some well-defined outcomes is called an experiment.

(ii) An experiment in which the outcomes may not be the same even if the experiment is performed in an identical condition is called a random experiment.

Probability of an Event:

If a trial result is n exhaustive, mutually exclusive and equally likely cases and m of them are favourable to the happening of an event A, then the probability of happening of A is given by

$$P(A) = \frac{\text{Number of favourable cases to } A}{\text{Number of exhaustive cases}} = \frac{n(A)}{n(S)} = \frac{m}{n}$$

Note: (i) $0 \leq P(A) \leq 1$

(ii) Probability of an impossible event is zero.

(iii) Probability of certain event (possible event) is 1.

(iv) $P(A \cup A') = P(S)$

(v) $P(A \cap A') = P(\Phi)$

(vi) $P(A')' = P(A)$

(vii) $P(A \cup B) = P(A) + P(B) - P(A \cap S)$

Conditional Probability:

Let E and F be two events associated with the same sample space of a random experiment. Then, probability of occurrence of event E, when the event F has already occurred, is called a conditional probability of event E over F and is denoted by P(E/F).

$$P(E/F) = \frac{P(E \cap F)}{P(F)}, \text{ where } P(F) \neq 0$$

Similarly, conditional probability of event F over E is given as

$$P(F/E) = \frac{P(F \cap E)}{P(E)}, \text{ where } P(E) \neq 0$$

Properties of Conditional Probability:

If E and E are two events of sample space S and G is an event of S which has already occurred such that $P(G) \neq 0$, then

(i) $P[(E \cup F)/G] = P(F/G) + P(F/G) - P[(F \cap F)/G]$, $P(G) \neq 0$

(ii) $P[(E \cup F)/G] = P(F/G) + P(F/G)$, if E and F are disjoint events.

(iii) $P(F'/G) = 1 - P(F/G)$

(iv) $P(S/E) = P(E/E) = 1$

Multiplication Theorem:

If E and F are two events associated with a sample space S, then the probability of simultaneous occurrence of the events E and F is

$P(E \cap F) = P(E) \cdot P(F/E)$, where $P(F) \neq 0$ Or $P(E \cap F) = P(F) \cdot P(F/F)$, where $P(F) \neq 0$

This result is known as multiplication rule of probability.

Multiplication Theorem for More than Two Events:

If F, F and G are three events of sample space, then

$$P(E \cap F \cap G) = P(E) \cdot P\left(\frac{F}{E}\right) \cdot P\left(\frac{G}{E \cap F}\right)$$

Independent Events: Two events E and F are said to be independent, if probability of occurrence or non-occurrence of one of the events is not affected by that of the other. For any two independent events E and F, we have the relation

(i) $P(E \cap F) = P(F) \cdot P(F)$

(ii) $P(F/F) = P(F), P(F) \neq 0$

(iii) $P(F/F) = P(F), P(F) \neq 0$

Also, their complements are independent events,

i.e. $P(E' \cap F') = P(E') \cdot P(F')$

Note: If E and F are dependent events, then $P(E \cap F) \neq P(F) \cdot P(F)$.

Three events E, F and G are said to be mutually independent, if

(i) $P(E \cap F) = P(E) \cdot P(F)$
(ii) $P(F \cap G) = P(F) \cdot P(G)$
(iii) $P(E \cap G) = P(E) \cdot P(G)$
(iv) $P(E \cap F \cap G) = P(E) \cdot P(F) \cdot P(G)$

If at least one of the above is not true for three given events, then we say that the events are not independent.

Note: Independent and mutually exclusive events do not have the same meaning.

Baye's Theorem and Probability Distributions

Partition of Sample Space: A set of events E1, E2,…,En is said to represent a partition of the sample space S, if it satisfies the following conditions:

(i) $Ei \cap Ej = \Phi; i \neq j; i, j = 1, 2, \ldots\ldots n$
(ii) $E1 \cup E2 \cup \ldots\ldots \cup En = S$
(iii) $P(Ei) > 0, \forall i = 1, 2,\ldots, n$

Theorem of Total Probability:

Let events E1, E2, …, En form a partition of the sample space S of an experiment.

If A is any event associated with sample space S, then

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + \ldots + P(E_n) \cdot P(A/E_n) = \sum_{j=1}^{n} P(E_j) \cdot P(A/E_j)$$

Baye's Theorem:

If E1, E2,…,En are n non-empty events which constitute a partition of sample space S, i.e. E1, E2,…, En are pairwise disjoint $E1 \cup E2 \cup \ldots\ldots \cup En = S$ and $P(Ei) > 0$, for all $i = 1, 2, \ldots.. n$ Also, let A be any non-zero event, the probability

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)}, \forall\ i = 1, 2, 3, \ldots, n$$

Random Variable:

A random variable is a real-valued function, whose domain is the sample space of a random experiment. Generally, it is denoted by capital letter X.

Note: More than one random variables can be defined in the same sample space.

Probability Distributions:

The system in which the values of a random variable are given along with their corresponding probabilities is called probability distribution.

Let X be a random variable which can take n values $x_1, x_2, \ldots, x_n$.

Let $p_1, p_2, \ldots,$ pn be the respective probabilities.

Then, a probability distribution table is given as follows:

X	x_1	x_2	x_3	.....	x_n
P	p_1	p_2	p_3	.....	p_n

such that $p_1 + p_2 + p_3 + \ldots +$ pn = 1

Note: If xi is one of the possible values of a random variable X, then statement X = xi is true only at some point(s) of the sample space. Hence ,the probability that X takes value x, is always non-zero, i.e. $P(X = xi) \neq 0$.

Mean of random variable

Let X be a random variable whose possible values are $x_1, x_2\ x_3\ x_4$..... ….x_n occur with probabilities are $p_1, p_2\ p_3\ p_4$..... ….p_n respectively . then mean of X, denoted by μ , is the number $\sum_{i=1}^{n} X_i\ p_I$ ie. The mean of X is the weighted average of the possible value of X_i , each value being weighted by its probability with which it occurs.

The mean of a random variable X is also called the expectation of X, denoted by E(X)

Mean(μ) = E(X) = $\mu = \sum_{i=1}^{n}(x_i p_i) = x_1 p_1 + x_2 p_2 + \cdots . x_1 p_1$ **;**

μ is called the expected value of X, ie, **E(X).**

MULTIPLE CHOICE QUESTIONS

1. If $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ ia equal to:
   [(a) $\frac{1}{10}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{17}{20}$]
2. If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{4}$ then $P(A' \cap B')$ is
   [(a) $\frac{1}{12}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{3}{16}$]
3. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then the probability of getting exactly one red ball is
   [(a) $\frac{45}{196}$ (b) $\frac{135}{192}$ (c) $\frac{15}{56}$ (d) $\frac{15}{29}$]

4. The probability distribution of a discrete random variable X is given below

X	2	3	4	5
P(X)	5/k	7/k	9/k	11/k

   Then the value of k is
   [(a) 8 (b) 16 (c) 32 (d) 48]
5. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum 3 is
   [(a) $\frac{1}{18}$ (b) $\frac{5}{18}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$]

SHORT ANSWER QUESTIONS

6. Out of 8 outstanding students of school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.
7. A fair die is rolled consider the following events A= {2,4,6}, B={4,5} and C= {3,4,5,6}.
   Find . $P[(A \cup B)/C]$
8. For the following probability distribution

X	-4	-3	-2	-1	0
P(X)	0.1	0.2	0.3	0.2	0.2

   Find E(X)

9. The probabilities of A, B, C, solving a problem, are 1/3 , 2/7 and 3 / 8 respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them can solve the problem.
10. A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game, then find the probability that B wins.
11. A coin is biased so that the head is 3 times as likely to occur as tail. If coin is tossed twice, find the probability distribution for the number of tails
12. There are two bags, bag I and bag II. Bag I contains 4 white and 3 red balls while another bag II contains 3 white and 7 red balls. One ball is drawn at random from one of the bags and it is found to be white. Find the probability that it was drawn from bag I.

13. A four-digit number is formed using the digits 1,2,3,5 with no repetition. Find the probability that the number is divisible by 5.
14. Ten cards numbered from 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?
15. An urn contains 5 red balls, 6 green balls and 4 black balls. A ball is drawn at random from the urn. What is the probability that the ball drawn is either red or black?
16. A die is thrown twice and the sum of the numbers rising is noted to be 6. Calculate the is the conditional probability that the number 4 has arrived at least once?
17. If A and B are two independent events such that P (**A** ∩ **B**) = $\frac{2}{15}$ and P(**A** ∩ **B**) = $\frac{1}{6}$, then find P(**A**) and P(**B**)
18. A and B throw a die alternatively till one of them gets a number greater than four and wins the game. If A starts the game, what is the probability of B winning?
19. From a pack of 52 playing cards, a card is lost. From the remaining 51 cards, two cards are drawn at random (without replacement) and are found to be both diamonds. What is the probability that the lost card was a card of heart?
20. A and B throw a pair of dice alternatively. A win the game if he gets a total of '7' and B wins the game if he gets a total of '10'. If A starts the game, then find the probability that B wins.

CASE STUDY QUESTIONS

21. In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let $\frac{7}{9}$ be the probability that he knows the answer and $\frac{2}{9}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{9}$. Let E_1, E_2 , E be the events that the student knows the answer, guesses the answer and answers correctly respectively.



Based on the above information, answer the following:

(i) Find the value of $\sum_{k=1}^{k=2} P(E_k)$

(ii) What is the probability that the student knows the answer given that he answered it corrwectly ?

22. When a person has TB disease in lungs, the chest X-ray usually appears abnormal. By examining the chest X-ray, the probability that a person is diagnosed with TB when he is actually suffering from it is 0.99. The probability that the doctor incorrectly diagnoses a person to be having TB, on the basis of X-

ray reports is 0.001. In a certain city, 1 in 1000 persons suffers from TB. A person is selected at random and diagnosed to have TB.



Based on the above information, answer the following:

(i) What is the probability that the person actually having TB?

(ii) What is the probability that the person has no TB?

23. A company produing electric bulbs has factories at three location A, B, and C, and company got a bulk order of producing electric bulbs. The capacities at locations A and C are same and at loccation B is doule that of C,. Also it is known that 4% of bulbs produced at A and B are defective and 5 % produced at C are defective.



Based on the above information, answer the following questions:

(i) Find the probability of production capacity of factory at place C.

(ii) Calculate the probability of producing defective bulb.

24. A glass jar contains twenty white balls of plastic numbered from 1 to 20, ten red balls of plastic numbered from 1 to 10, forty yellow balls of plastic numbered from 1 to 40 and ten blue balls of plastic numbered from 1 to 10. If these 80 balls of plastic are thoroughly shuffled so that each ball has the same probability of being drawn.



Based on the above information answer the following:

(i) Determine the probabilities of drawing a ball of plastic that is red or yellow and numbered 1, 2, 3 or 4.

(ii) Discuss the probabilities of drawing a plastic ball which is numbered 5, 15, 25 or 35.

25. In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 6%, Sonia has an error rate of 4% and Iqbal has an error rate of 3%.



Based on the above information answer the following:

(i) The total probability of committing an error in processing the form.

(ii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay

LONG ANSWER QUESTIONS

26. There are three coins. One is a two headed coin(having heads on both faces), another is a biased coin that comes up heads 75% of the times and the third is an unbiased coin. One of the three coin is chosen at random and tossed , it shows heads what Is the probability , that it was the two headed coin.

27. Suppose that 5% of men and 0.25% of women have of gray hair. A gray haired person is selected at random. What is the probability of this person this being male? Assume that there are equal number of males and females.
28. A committee of 4 students is selected at random from a group consisting of 8boys and 4 girls. Given that there is at least 1 girl in the committee calculate the probability that there is exactly 2 girls in the committee.
29. Find mean (μ) for the following probability distribution.

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

30. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time. B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random. (a) What is the probability of getting a defective item? (b) If the item so chosen is found to be defective. What is the probability that it was produced by A?
31. Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. (a) What is the probability that person suffers heart attach even if he has followed any of the given two options? (b) It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.
32. Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces
33. Two cards are drawn from a pack of 52 cards. Find the probability distribution of number of aces
34. An Urn contains 4 white and 3 red balls. Find the probability distribution and mean of number of red balls in a random draw of three balls
35. Two cards are drawn with replacement from a pack of 52 cards. Find the probability distribution of number of kings

ANSWERS

	VERY SHORT ANSWER (MARK 1)
1	**(c)** $\because \quad P(A)=\frac{4}{5}, P(A\cap B)=\frac{7}{10}$ $\therefore \quad P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{7/10}{4/5}=\frac{7}{8}$
2	**(c)** Here, $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$ and $P(A/B)=\frac{1}{4}$ $\because \quad P(A/B)=\frac{P(A\cap B)}{P(B)}$ $\Rightarrow \quad P(A\cap B)=P(A/B)\cdot P(B)=\frac{1}{4}\cdot\frac{1}{3}=\frac{1}{12}$ Now, $P(A'\cap B')=1-P(A\cup B)$ $=1-[P(A)+P(B)-P(A\cap B)]$ $=1-\left[\frac{1}{2}+\frac{1}{3}-\frac{1}{12}\right]=1-\left[\frac{6+4-1}{12}\right]$ $=1-\frac{9}{12}=\frac{3}{12}=\frac{1}{4}$
3	**(c)** Probability of getting exactly one red (R) ball $=P_R\cdot P_{\overline{R}}\cdot P_{\overline{R}}+P_{\overline{R}}\cdot P_R\cdot P_{\overline{R}}+P_{\overline{R}}\cdot P_{\overline{R}}\cdot P_R$ $=\frac{5}{8}\cdot\frac{3}{7}\cdot\frac{2}{6}+\frac{3}{8}\cdot\frac{5}{7}\cdot\frac{2}{6}+\frac{3}{8}\cdot\frac{2}{7}\cdot\frac{5}{6}$ $=\frac{15}{4\cdot7\cdot6}+\frac{15}{4\cdot7\cdot6}+\frac{15}{4\cdot7\cdot6}$ $=\frac{5}{56}+\frac{5}{56}+\frac{5}{56}=\frac{15}{56}$
4	**(c)** We know that, $\Sigma P(X)=1$ $\Rightarrow \quad \frac{5}{k}+\frac{7}{k}+\frac{9}{k}+\frac{11}{k}=1$ $\Rightarrow \quad \frac{32}{k}=1$ $\therefore \quad k=32$
5	**(c)** Let E_1 = Event that the sum of numbers on the dice was less than 6 and E_2 = Event that the sum of numbers on the dice is 3 $\therefore \quad E_1=\{(1,4),(4,1),(2,3),(3,2),(2,2),(1,3),(3,1),(1,2),(2,1),(1,1)\}$ $\Rightarrow \quad n(E_1)=10$ and $E_2=\{(1,2),(2,1)\}\Rightarrow n(E_2)=2$ $\therefore$ Required probability $=\frac{2}{10}=\frac{1}{5}$
	SHORT ANSWERS (MARK 2)
6	**Ans:** Total number of students = 8 The number of ways to select 4 students out of 8 students $={}^8C_4=70$ The number of ways to select 2 boys and 2 girls $=={}^3C_2\times{}^5C_2=3\times10=30$ $\therefore$ Required probability $=\frac{30}{70}=\frac{3}{7}$

7	Given that, $A = \{2, 4, 6\}, B = \{4, 5\}, C = \{3, 4, 5, 6\}$ Now, $A \cup B = \{2, 4, 6\} \cup \{4, 5\} = \{2, 4, 5. 6\}$ So, $P(A \cup B) = \frac{4}{6} = \frac{2}{3}$ Now, $(A \cup B) \cap C = \{2, 4, 5, 6\} \cap \{3, 4, 5, 6\} = \{4, 5, 6\}$ So, $P[(A \cup B) \cap C] = \frac{3}{6} = \frac{1}{2}$ Also $P(C) = \frac{4}{6} = \frac{2}{3}$ Required probability $= P[(A \cup B)/C] = \frac{P[(A \cup B) \cap C]}{P(C)}$ $= \frac{1/2}{2/3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$
8	$E(X) = \Sigma X\, P(X)$ $= -4 \times (0.1) + (-3 \times 0.2) + (-2 \times 0.3) + (-1 \times 0.2) + (0 \times 0.2)$ $= -0.4 - 0.6 - 0.6 - 0.2 = -1.8$
9	25/56
10	5/17
11	<table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{9}{16}$</td><td>$\frac{3}{8}$</td><td>$\frac{1}{16}$</td></tr></table>
12	40/61
13	1/4
14	4/7
15	3/5
16	F: Addition of numbers is 6 E: 4 has appeared at least once So, that, we need to find P(E|F) **Finding P (E):** The probability of getting 4 atleast once is: E = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)} Thus , P(E) = 11/ 36 **Finding P (F):** The probability to get the addition of numbers is 6 is: F = {(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)} Thus, P(F) = 5/ 36 Also, E ∩ F = {(2,4), (4,2)} P(E ∩ F) = 2/36 Thus, P(E|F) = (P(E ∩ F)) / (P (F)) = (2/36)/ (5/36) Hence, Required probability is 2/5.
17	P(A) = 5 /6 *and* P(B) = 4 /5 OR :P(A) = 1 /5 *and* P(B) = 1 /6
18	B=**2** /**5** , A=**3**/5
19	**13**/50

20	$B = 5/17$
	CASE STUDY
21	**(i)** Let, E_1 = Knows the answer E_2 = guess the answer $\therefore\ P(E_1)=\frac{7}{9},\ P(E_2)=\frac{2}{9}$ $\sum_{k=1}^{k=2} P(E_k) = P(E_1)+P(E_2) = \frac{7}{9}+\frac{2}{9} = \frac{9}{9} = 1$ **(ii)** The required probability $P\left(\frac{E}{E_1}\right)=\frac{P(E_1).P(E_1\mid E)}{P(E_1).P(E_1\mid E)+P(E_2)P(E\mid E_2)} = \frac{\frac{7}{9}\times 1}{\frac{7}{9}\times 1+\frac{2}{9}\times\frac{1}{9}} = \frac{\frac{7}{9}}{\frac{65}{81}} = \frac{63}{65}$
22	Ans: Let E = event that the doctor diagnoses TB, E_1 = event that the person selected is suffering from TB, and E_2 = event that the person selected is not suffering from TB. Then, $P(E_1) = \frac{1}{1000}$ and $P(E_2) = 1-\frac{1}{1000} = \frac{999}{1000}$ $P(E/E_1)$ = probability that TB is diagnosed, when the person actually has TB = $\frac{99}{100}$ $P(E/E_2)$ = probability that TB is diagnosed, when the person has no TB = $\frac{1}{1000}$ (i) Using Bayes's theorem, we have $P(E_1/E)$ = probability of a person actually having TB, if it is known that he is diagnosed to have TB = $\frac{P(E/E_1)P(E_1)}{P(E/E_1)P(E_1)+P(E/E_2)P(E_2)} = \frac{\left(\frac{99}{100}\times\frac{1}{1000}\right)}{\left(\frac{99}{100}\times\frac{1}{1000}\right)+\left(\frac{1}{1000}\times\frac{999}{1000}\right)} = \frac{110}{221}$ (ii) $P(E_2/E)$ = probability of a person actually having no TB, if it is known that he is diagnosed to have TB = $\frac{P(E/E_2)P(E_2)}{P(E/E_1)P(E_1)+P(E/E_2)P(E_2)} = \frac{\left(\frac{1}{1000}\times\frac{999}{1000}\right)}{\left(\frac{99}{100}\times\frac{1}{1000}\right)+\left(\frac{1}{1000}\times\frac{999}{1000}\right)} = \frac{111}{221}$
23	**Ans:** (i) Let x be the production capacity at A & C. ⇒ 2x be the production capacity at B. ∵ A : B : C = 1 : 2 : 1, Sum of production capacity = 1 + 2 + 1 = 4 If P(A), P(B) and P(C) denote the probabilities of production at places A, B and C respective Hence, $P(C) = \frac{1}{4}$ (ii) Let E be the event that bulb produced is defective Then given that $P(E/A) = 4\% = \frac{4}{100}$, $P(E/B) = 4\% = \frac{4}{100}$, $P(E/C) = 5\% = \frac{5}{100}$

	Also $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{4} = \frac{1}{2}$, $P(C) = \frac{1}{4}$ $\therefore$ Probability that the bulb produced is defective = P(E) $\therefore$ $P(E) = P(A) \cdot P(E/A) + P(B) \cdot P(E/B) + P(C) \cdot P(E/C)$ $= \frac{1}{4} \times \frac{4}{100} + \frac{1}{2} \times \frac{4}{100} + \frac{1}{4} \times \frac{5}{100}$ $= \frac{4+8+5}{400} = \frac{17}{400}$
24	**Ans: (i)** Let A → event that Anand solves B → event that Sanjay solves C → event that Aditya solves $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$ $\therefore P(A') = \frac{1}{2}, P(B') = \frac{2}{3}, P(C') = \frac{3}{4}$ $P(A \cap B' \cap C') = P(A)\,P(B')\,P(C') = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$ **(ii)** $P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$ $= P(A)\,P(B')\,P(C') + P(A')\,P(B)\,P(C') + P(A')\,P(B')\,P(C)$ $= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} = \frac{11}{24}$
25	Ans: $P(V) = 50\% = \frac{50}{100}$, $P(S) = 20\% = \frac{20}{100}$, $P(I) = 30\% = \frac{30}{100}$ Let E be the event that error occurred. $P(E/V) = \frac{6}{100}$, $P(E/S) = \frac{4}{100}$, $P(E/I) = \frac{3}{100}$ (i) $P(E) = P(V)\,P(E/V) + P(S)\,P(E/S) + P(I)\,P(E/I) = \frac{50}{100} \times \frac{6}{100} + \frac{20}{100} \times \frac{4}{100} + \frac{30}{100} \times \frac{3}{100}$ $= \frac{300}{10000} + \frac{80}{10000} + \frac{90}{10000} = \frac{470}{10000} = 0.047$ (ii) $P\left(\frac{V}{E}\right) = \frac{P(V)P\left(\frac{E}{V}\right)}{P(E)} = \frac{50/100 \times 6/100}{47/1000} = \frac{30}{47}$ $\therefore$ P(Not processed by Vinya) $= 1 - \frac{30}{47} = \frac{17}{47}$.
	LONG ANSWER
26	4/9
27	20/21
28	168/425
29	Mean(μ)= 3/2
30	(a) 34 /1000 (b) 5/ 34
31	(a) 29/ 100 (b) 14 /29
32	<table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{188}{221}$</td><td>$\frac{32}{221}$</td><td>$\frac{1}{221}$</td></tr></table>

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$$P(X=0)=\frac{{}^{48}C_2}{{}^{52}C_2}=\frac{48\times47}{52\times51}=\frac{188}{221}$$

$$P(X=1)=\frac{{}^{4}C_1\times{}^{48}C_1}{{}^{52}C_2}=\frac{48\times4\times2}{52\times51}=\frac{32}{221}$$

$$P(X=2)=\frac{{}^{4}C_2}{{}^{52}C_2}=\frac{4\times3}{52\times51}=\frac{1}{221}$$

X	**0**	**1**	**2**
P(X0)	**188/221**	**32/221**	**1/221**

34

Let X be the no. of red balls in a random draw of three bal

As there are 3 red balls, possible values of X are 0, 1, 2, 3.

$$P(0)=\frac{{}^{3}C_0\times{}^{4}C_3}{{}^{7}C_3}=\frac{4\times3\times2}{7\times6\times5}=\frac{4}{35}$$

$$P(1)=\frac{{}^{3}C_1\times{}^{4}C_2}{{}^{7}C_3}=\frac{3\times6\times6}{7\times6\times5}=\frac{18}{35}$$

$$P(2)=\frac{{}^{3}C_2\times{}^{4}C_1}{{}^{7}C_3}=\frac{3\times4\times6}{7\times6\times5}=\frac{12}{35}$$

$$P(3)=\frac{{}^{3}C_3\times{}^{4}C_0}{{}^{7}C_3}=\frac{1\times1\times6}{7\times6\times5}=\frac{1}{35}$$

X	$P(X)$	$XP(X)$	$X^2P(X)$
0	4/35	0	0
1	18/35	18/35	18/35
2	12/35	24/35	48/35
3	1/35	3/35	9/35
Total	1	9/7	15/7

$$\text{Mean}=\Sigma XP(X)=\frac{9}{7}$$

35

X	P(X)	XP(X)
0	$\frac{^{48}C_2}{^{52}C_2}=\frac{1128}{1326}$	0
1	$\frac{^{4}C_1\,^{48}C_1}{^{52}C_2}=\frac{192}{1326}$	$\frac{192}{1326}$
2	$\frac{^{4}C_2}{^{52}C_2}=\frac{6}{1326}$	$\frac{12}{1326}$
		$E(X)=\frac{204}{1326}$