केन्द्रीय विद्यालय संगठन
KENDRIYA VIDYALAYA SANGATHAN

केन्द्रीय विद्यालय संगठन, कोलकाता सम्भाग

KENDRIYA VIDYALAYA SANGATHAN, KOLKATA REGION

अध्ययनसामग्री/ STUDYMATERIAL

Academic Session–2023-24

Class –Eleven(XI)

Subject-MATHEMATICS

Subject Code – 041

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A note to the students….

It is a true that after class X there is a huge gap of syllabus in almost all the subjects, but systematic plan of study and labour in correct direction is going to make preparation easy.

This study material has been designed to help students of class XI understand the concepts of mathematics in a more effective way. The material covers all the topics that are part of the syllabus and provides a comprehensive understanding of each topic. The aim of this study material is to help students develop a strong foundation in mathematics, which will be useful for their future studies.

The study material has been prepared byteachers of mathematics of different KVs of Kolkata region, who have years of experience in teaching the subject.

Main features of the material

- Latest CBSE curriculum and design of question paper has been provided so that latest trend of the question paper can clearly be understood
- All the necessary synopsis and essential formula are included in the beginning of material for quick revision
- A good number of practice questions from each chapter are included according to the latest pattern

All care have been taken to include those information which are relevant and in support with the text book, but in no way it is the replacement for NCERT book and NCERT Exempler rather it is supplement to those books. Your teacher will help you to choose the key areas of scoring and in revision after completion of syllabus

Best of luck for your future

From All the teachers

COURSE STRUCTURE

CLASS XI (2023-24)

One Paper | Total Period–240 [35 Minutes each]

Three Hours | **Max Marks: 80**

No.	Units	No. of Periods	Marks
I.	Sets and Functions	60	23
II.	Algebra	50	25
III.	Coordinate Geometry	50	12
IV.	Calculus	40	08
V.	Statistics and Probability	40	12
	Total	240	80
	Internal Assessment		20

*No chapter/unit-wise weightage. Care to be taken to cover all the chapters.

Unit-I: Sets and Functions

1. **Sets** **(20) Periods**

Sets and their representations, Empty set, Finite and Infinite sets, Equal sets, Subsets, Subsets of a set of real numbers especially intervals (with notations). Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set. Properties of Complement.

2. **Relations & Functions** **(20) Periods**

Ordered pairs. Cartesian product of sets. Number of elements in the Cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto R x R x R).Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special type of relation. Pictorial representation of a function, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs. Sum, difference, product and quotients of functions.

3. **Trigonometric Functions** **(20) Periods**

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity sin2x + cos2x = 1, for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing *sin (x±y)* and *cos (x±y)* in terms of sin*x*, sin*y*, cos*x* & cos*y* and their simple applications. Deducing identities like the following:

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}, \cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$$

$$\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha + \beta)\sin\frac{1}{2}(\alpha - \beta)$$

Identities related to sin2*x*, cos2*x*, tan2 *x*, sin3*x*, cos3*x* and tan3*x*.

Unit-II: Algebra

1. **Complex Numbers and Quadratic Equations** **(10) Periods**

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve some of the quadratic equations. Algebraic properties of complex numbers. Argand plane

2. **Linear Inequalities** **(10) Periods**

Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line.

3. **Permutations and Combinations** **(10) Periods**

Fundamental principle of counting. Factorial *n*. (n!) Permutations and combinations, derivation of Formulae for ^nP_r and ^nC_r and their connections, simple applications.

4. **Binomial Theorem** **(10) Periods**

Historical perspective, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, simple applications.

5. **Sequence and Series** **(10) Periods**

Sequence and Series. Arithmetic Mean (A.M.) Geometric Progression (G.P.), general term of a G.P., sum of *n* terms of a G.P., infinite G.P. and its sum, geometric mean (G.M.), relation between A.M. and G.M.

Unit-III: Coordinate Geometry

1. Straight Lines (15) Periods

Brief recall of two dimensional geometry from earlier classes. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axis, point -slope form, slope-intercept form, two-point form, intercept form, Distance of a point from a line.

2. Conic Sections (25) Periods

Sections of a cone: circles, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.

3. Introduction to Three-dimensional Geometry (10) Periods

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points.

Unit-IV: Calculus

1. Limits and Derivatives (40) Periods

Derivative introduced as rate of change both as that of distance function and geometrically. Intuitive idea of limit. Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions. Definition of derivative relate it to scope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

Unit-V Statistics and Probability

1. Statistics (20) Periods

Measures of Dispersion: Range, Mean deviation, variance and standard deviation of ungrouped/grouped data.

2. Probability (20) Periods

Events; occurrence of events, 'not', 'and' and 'or' events, exhaustive events, mutually exclusive events, Axiomatic (set theoretic) probability, connections with other theories of earlier classes. Probability of an event, probability of 'not', 'and' and 'or' events.

MATHEMATICS

QUESTION PAPER DESIGN

CLASS – XI (2023-24)

Time: 3 Hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	44	55
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	20	25
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	16	20
	Total	80	100

1. *No chapter wise weightage. Care to be taken to cover all the chapters*
2. *Suitable internal variations may be made for generating various templates keeping the overall weightage to different form of questions and typology of questions same.*

Choice(s):

There will be no overall choice in the question paper.

However, 33% internal choices will be given in all the sections

INTERNAL ASSESSMENT	**20 MARKS**
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities	10 Marks

SETS

CONCEPTS AND RESULTS

**** Set :**a set is a well-defined collection of objects.

If a is an element of a set A, we say that " a belongs to A" the Greek symbol $\in$(epsilon) is used to denote the phrase 'belongs to'. Thus, we write a $\in$A. If 'b' is not an element of a set A, we write b $\notin$A and read "b does not belong to A".

There are two methods of representing a set :

(i) Roster or tabular form (ii) Set-builder form.

In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within brackets { }. For example, the set of all even positive integers less than 7 is described in roster form as {2, 4, 6}.

In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set. For example, in the set {a, e, i, o, u}, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property. Denoting this set by V, we write V = {x : x is a vowel in English alphabet}

**** Empty Set :**A set which does not contain any element is called the empty set or the null set or the void set. The empty set is denoted by the symbol φ or { }.

**** Finite and Infinite Sets :**A set which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

**** Equal Sets :**Two sets A and B are said to be equal if they have exactly the same elements and we write A = B. Otherwise, the sets are said to be unequal and we write $A \neq B$.

**** Subsets :**A set A is said to be a subset of a set B if every element of A is also an element of B.

In other words, $A\subset B$ if whenever a $\in$A, then a $\in$B. Thus $A \subset B$ if $a \in A \Rightarrow a \in B$

If A is not a subset of B, we write $A \not\subset B$.

** Every set A is a subset of itself, i.e., $A \subset A$.

** φ is a subset of every set**.**

** If $A\subset B$ and $A \neq B$, then A is called a proper subset of B and B is called superset of A.

** If a set A has only one element, we call it a singleton set. Thus, { a } is a singleton set.

** Closed Interval : $[a , b] = \{x : a \leq x \leq b\}$

** Open Interval : $(a , b) = \{ x : a < x < b\}$

** Closed open Interval : $[a , b) = \{x : a \leq x < b\}$

** Open closed Interval : $(a , b] = \{ x : a < x \leq b \}$

**** Power Set :**The collection of all subsets of a set A is called the power set of A. It is denoted by P(A)

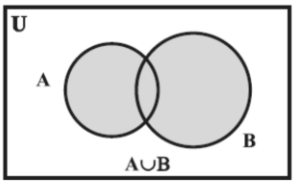
If A is a set with n(A) = m, then it can be shown that $n [P(A)] = 2^m$.

** **Universal Set :**The largest set under consideration is called Universal set.

** **Union of sets :**The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both). In symbols, we write.

$A \cup B = \{ x : x \in A \text{ or } x\in B \}$.

$x \in A \cup B \Rightarrow x \in A$ or $x \in B$



$x\notin A \cup B \Rightarrow x \notin A$ and $x\notin B$

** **Some Properties of the Operation of Union**

(i) $A \cup B = B \cup A$ (Commutative law)

(ii) $(A\cup B) \cup C = A \cup (B \cup C)$ (Associative law)

(iii) $A \cup \varphi = A$ (Law of identity element, φ is the identity of $\cup$)

(iv) $A \cup A = A$ (Idempotent law)

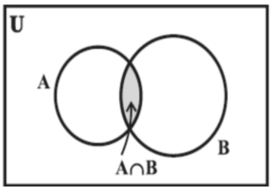
(v) $U \cup A = U$ (Lawof U)

** **Intersection of sets :**The intersection of two sets A and B is the set of all those elements which belong to bothA and B.

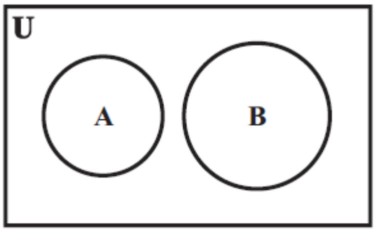
Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$

$x \in A \cap B \Rightarrow x \in A$ and $x \in B$

$x \notin A \cap B \Rightarrow x \notin A$ or $x \notin B$



** **Disjoint sets :**If A and B are two sets such that $A \cap B = \varphi$, then A and B are called disjoint sets.



** **Some Properties of Operation of Intersection**

(i) $A \cap B = B \cap A$ (Commutative law).

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law).

(iii) $\varphi \cap A = \varphi$, $U \cap A = A$ (Law of φ and U).

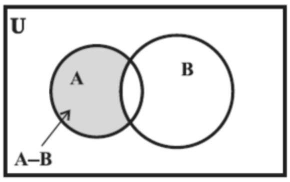
(iv) $A \cap A = A$ (Idempotent law)

(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law) i. e., $\cap$ distributes over $\cup$

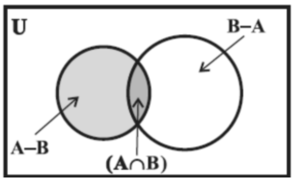
** **Difference of sets :**The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

Symbolically, we write A – B and read as " A minus B".

$A - B = \{ x : x \in A \text{ and } x \notin B \}$.

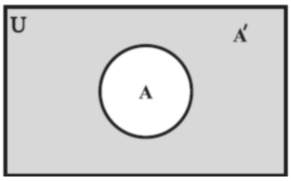


* The sets A – B, A ∩B and B – A are mutually disjoint sets, i.e., the intersection of any of these two sets is the null set.



** **Complement of a Set :**Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write A' to denote the complement of A with respect to U.

Thus, $A' = \{x : x \in U \text{ and } x \notin A \}$. Obviously $A' = U - A$



** **Some Properties of Complement Sets**

1. Complement laws: (i) $A \cup A' = U$ (ii) $A \cap A' = \varphi$

2. De Morgan's law: (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

3. Law of double complementation : $(A')' = A$

4. Laws of empty set and universal set $\varphi' = U$ and $U' = \varphi$.

** **Practical Problems on Union and Intersection of Two Sets :**

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B)$, if $A \cap B = \varphi$.

(iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.

RELATIONS & FUNCTIONS
CONCEPTS AND RESULTS

** **Cartesian Products of Sets :** Given two non-empty sets P and Q. The cartesian product $P \times Q$ is theset of all ordered pairs of elements from P and Q, i.e., $P \times Q = \{ (p , q) : p \in P, q \in Q \}$

** Two ordered pairs are equal, if and only if the corresponding first elements, are equal and the second elements are also equal.

** If there are p elements in A and q elements in B, then there will be pq elements in $A \times B$, i.e. if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

** If A and B are non-empty sets and either A or B is an infinite set, then so is $A \times B$.

** $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$. Here (a, b, c) is called an ordered triplet.

****Relation :** A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product $A \times B$. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in $A \times B$. The second element is called the image of the first element.

** The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R.

**The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the co-domain of the relation R. Range $\subseteq$ co-domain.

** A relation may be represented algebraically either by the Roster method or by the Set-builder method.

** An arrow diagram is a visual representation of a relation.

** The total number of relations that can be defined from a set A to a set B is the number of possible subsets of $A \times B$. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$ and the total number of relations is 2^{pq}.

** A relation R from A to A is also stated as a relation on A.

** **Function:** A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

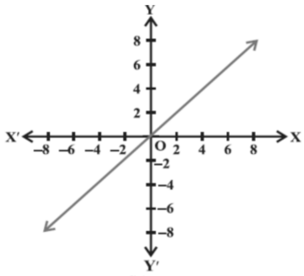
In other words, a function f is a relation from a non-empty set A to a non-empty set B such that the domain of f is A and no two distinct ordered pairs in f have the same first element.

If f is a function from A to B and $(a, b) \in f$, then $f(a) = b$, where b is called the image of a under f and a is called the pre-image of b under f.

** A function which has either R or one of its subsets as its range is called a real valued function. Further, if its domain is also either R or a subset of R, it is called a real function.

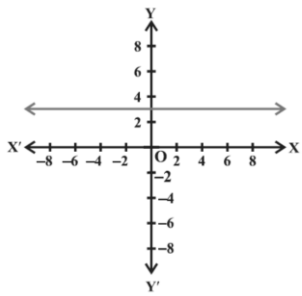
Some functions and their graphs

** **Identity function** Let **R** be the set of real numbers. Define the real valued function $f : \mathbf{R} \to \mathbf{R}$ by $y = f(x) = x$ for each $x \in \mathbf{R}$. Such a function is called the identity function. Here the domain and range of f are **R**.



$f(x) = x$

****Constant function :** Define the function $f: \mathbf{R} \to \mathbf{R}$ by $y = f(x) = c$, $x \in \mathbf{R}$ where c is a constant and each $x \in \mathbf{R}$. Here domain of f is **R** and its range is {c}.

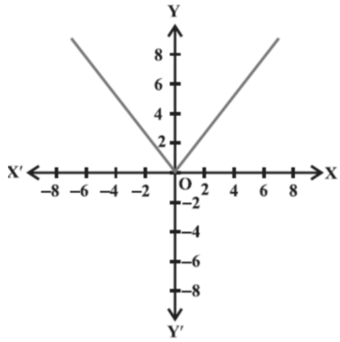


$f(x) = 3$

Polynomial function : A function f : **R**→**R** is said to be polynomial function if for each x in **R**, $y = f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$, where n is a non-negative integer and $a_0, a_1, a_2, ..., a_n \in$ **R**.

** **Rational functions :** are functions of the type $\frac{f(x)}{g(x)}$, where f(x) and g(x) are polynomial functions of x defined in a domain, where $g(x) \neq 0$.

** **The Modulus function :** The function f: **R**→**R** defined by $f(x) = |x|$ for each $x \in$ **R** is called modulus function. For each non-negative value of x, f(x) is equal to x.
But for negative values of x, the value of f(x) is the negative of the value of x, i.e., $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$.

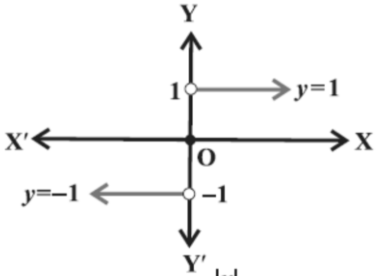


$f(x) = |x|$

** **Signum function :** The function f : **R**→**R** defined by

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

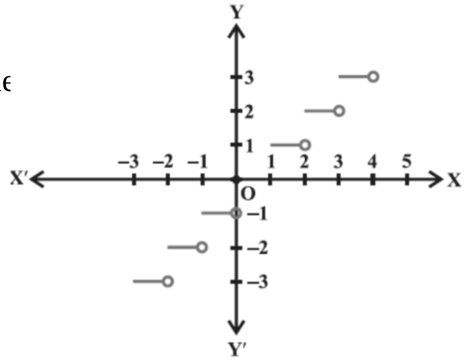
is called the signum function. The domain of the signum function is **R** and the range is the set {–1, 0, 1}.



$f(x) = \frac{|x|}{x}$

** **Greatest integer function :**
The function f :**R** → **R** defined by f(x) = [x], $x \in$ **R** assume the value of the greatest integer, less than or equal to x. Such a function is called the greatest integer function.

$[x] = -1$ for $-1 \leq x < 0$
$[x] = 0$ for $0 \leq x < 1$
$[x] = 1$ for $1 \leq x < 2$
$[x] = 2$ for $2 \leq x < 3$ and so on.



$f(x) = [x]$

Algebra of real functions

** **Addition of two real functions :** Let f : X → **R** and g : X → **R** be any two real functions, where $X \subset$ **R**. Then, we define (f + g): X → **R** by (f + g) (x) = f (x) + g (x), for all $x \in X$.

** **Subtraction of a real function from another :** Let f :X → **R** and g:X → **R** be any two real functions, where $X \subset$ **R**. Then, we define (f – g) : X→**R** by (f – g) (x) = f(x) – g(x), for all $x \in X$.

** **Multiplication by a scalar :** Let f : X→**R** be a real valued function and α be a scalar. Here by scalar, we mean a real number. Then the product α f is a function from X to **R** defined by
$(\alpha f)(x) = \alpha f(x), x \in X$.

** **Multiplication of two real functions :** The product (or multiplication) of two real functions f :X→**R** and g :X→**R** is a function fg :X→**R** defined by (fg) (x) = f(x) g(x), for all $x \in X$.

**** Quotient of two real functions** Let f and g be two real functions defined from X→**R** where $X \subset$ **R**. The quotient of f by g denoted by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$, $x \in X$

TRIGONOMETRIC FUNCTIONS
CONCEPTS AND RESULTS

Angles : Angle is a measure of rotation of a given ray about its initial point.

**** <u>Measurement of an angle</u>.**

****English System (Sexagesimal system)**

(i) 1 right angle = 90 degrees = 90^o. (ii) 1^o = 60 minutes = 60’. (iii) 1’ = 60 second = 60’’.

****French System (Centesimal system)**

(iv) 1 right angle = 100 grades = 100 g. (v) 1 g = 100 minutes = 100 ‘ (vi) 1’ = 100 seconds = 100 ‘’

****Circular System.**

(vii) $180^o = 200^g = \pi$ radians = 2 right angles, where a radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.

(viii) The circular measure θ of an angle subtended at the centre of a circle by an arc of length **l** is equal to the ratio of the length **l** to the radius r of the circle.

(ix) Each interior angle of a regular polygon of n sides is equal to $\frac{2n-4}{n}$ right angles.

T-ratios	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	–1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	n.d	$-\sqrt{3}$	–1	$-\frac{1}{\sqrt{3}}$	0

**** <u>Formulae for t-ratios of Allied Angles :</u>**

All T-ratio changes in $\frac{\pi}{2} \pm \theta$ and $\frac{3\pi}{2} \pm \theta$ while remains unchanged in $\pi \pm \theta$ and $2\pi \pm \theta$.

$\sin\left(\frac{\pi}{2} \pm \theta\right) = \cos\theta$ $\quad$ $\sin\left(\frac{3\pi}{2} \pm \theta\right) = = \cos\theta \ \frac{\pi}{2}$

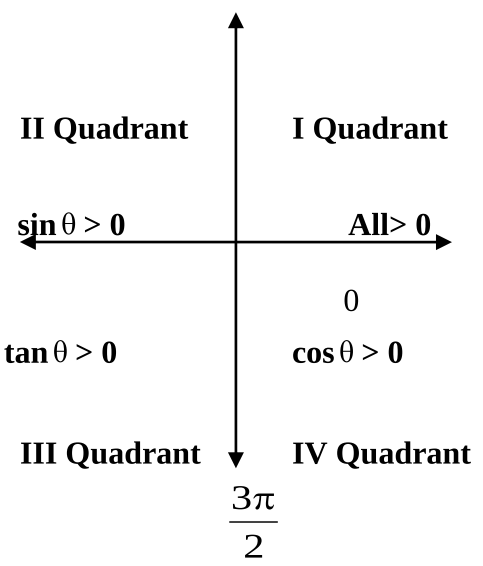
$\cos\left(\frac{\pi}{2} \pm \theta\right) = \mp\sin\theta$ $\quad$ $\cos\left(\frac{3\pi}{2} \pm \theta\right) = \pm\sin\theta$

$\tan\left(\frac{\pi}{2} \pm \theta\right) = \mp\cot\theta$ $\quad$ $\tan\left(\frac{3\pi}{2} \pm \theta\right) = \mp\cot\theta$

$\sin(\pi \pm \theta) = \mp\sin\theta$ $\quad$ $\sin(2\pi \pm \theta) = \pm\sin\theta \ \pi$

$\cos(\pi \pm \theta) = = \cos\theta$ $\quad$ $\cos(2\pi \pm \theta) = \cos\theta$

$\tan(\pi \pm \theta) = \pm\tan\theta$ $\quad$ $\tan(2\pi \pm \theta) = \pm\tan\theta$



** **Sum and Difference formulae :**

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$, $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$. $\tan\left(\dfrac{\pi}{4} + A\right) = \dfrac{1 + \tan A}{1 - \tan A}$,

$\tan\left(\dfrac{\pi}{4} - A\right) = \dfrac{1 - \tan A}{1 + \tan A}$ $\cot(A + B) = \dfrac{\cot A.\cot B - 1}{\cot B + \cot A}$ $\cot(A - B) = \dfrac{\cot A.\cot B + 1}{\cot B - \cot A}$

$\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

$\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

****Formulae for the transformation of a product of two circular functions into algebraic sum of two circular functions and vice-versa.**

$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$

$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

$\sin C + \sin D = 2 \sin \dfrac{C + D}{2} \cos \dfrac{C - D}{2}$, $\sin C - \sin D = 2 \cos \dfrac{C + D}{2} \sin \dfrac{C - D}{2}$.

$\cos C + \cos D = 2 \cos \dfrac{C + D}{2} \cos \dfrac{C - D}{2}$, $\cos C - \cos D = -2 \sin \dfrac{C + D}{2} \sin \dfrac{C - D}{2}$.

****Formulaefor t-ratios ofmultiple and sub-multipleangles** :

$\sin 2A = 2 \sin A \cos A = \dfrac{2 \tan A}{1 + \tan^2 A}$.

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

$1 + \cos 2A = 2\cos^2 A$ $1 - \cos 2A = 2\sin^2 A$ $1 + \cos A = 2\cos^2 \dfrac{A}{2}$ $1 - \cos A = 2\sin^2 \dfrac{A}{2}$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$, $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$.

$\sin 3A = 3 \sin A - 4 \sin^3 A$, $\cos 3A = 4 \cos^3 A - 3 \cos A$

$\sin 15^o = \cos 75^o = \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$. & $\cos 15^o = \sin 75^o = \dfrac{\sqrt{3} + 1}{2\sqrt{2}}$,

$\tan 15^o = \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3} = \cot 75^o$ & $\tan 75^o = \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1} = 2 + \sqrt{3} = \cot 15^o$.

$\sin 18^o = \dfrac{\sqrt{5} - 1}{4} = \cos 72^o$ and $\cos 36^o = \dfrac{\sqrt{5} + 1}{4} = \sin 54^o$.

$\sin 36^o = \dfrac{\sqrt{10 - 2\sqrt{5}}}{4} = \cos 54^o$ and $\cos 18^o = \dfrac{\sqrt{10 + 2\sqrt{5}}}{4} = \sin 72^o$.

$\tan\left(22\dfrac{1}{2}\right)^o = \sqrt{2} - 1 = \cot 67\dfrac{1}{2}^o$ and $\tan\left(67\dfrac{1}{2}\right)^o = \sqrt{2} + 1 = \cot\left(22\dfrac{1}{2}\right)^o$.

COMPLEX NUMBERS
CONCEPTS AND RESULTS

* A number of the form (a + ib) where a b $\in$ R, the set of real numbers, and i = $\sqrt{-1}$ (iota) is called a complex number. It is denoted by z, z = a + ib. "a" is called the real part of complex number z and "b" is the imaginary part i.e. Re(z) = a and Im(z) = b.
* Two complex numbers are said to be equal i.e. $z_1 = z_2$.

$\Leftrightarrow$ (a + ib) = (c + id)

$\Leftrightarrow$ a = c and b = d

$\Leftrightarrow$ Re (z_1) = Re (z_2) & $\text{Im}(z_1) = \text{Im}(z_2)$.

* A complex number z is said to be purely real if Im(z) = 0
and is said to be purely imaginary if Re(z) = 0.
* The set R of real numbers is a proper subset of the set of complex number C, because every real number can be considered as a complex number with imaginary part zero.
* $i^{4n} = (i^4)^n = (1)^n = 1$ $\qquad$ $i^{4n+1} = i^{4n}.i = (1).i = i$
  $i^{4n+2} = i^{4n}.\ i^2 = (1)(-1) = -1$ $\qquad$ $i^{4n+3} = i^{4n}.i^3 = (1)(-i) = -\ i.$

Algebra of Complex Numbers

** **Addition of two complex numbers :**Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers. Then, the sum $z_1 + z_2$ is defined as follows: $z_1 + z_2 = (a + c) + i\,(b + d)$, which is again a complex number.

The addition of complex numbers satisfy the following properties:

(i) The closure law The sum of two complex numbers is a complex number, i.e., $z_1 + z_2$ is a complex number for all complex numbers z_1 and z_2 .

(ii) The commutative law For any two complex numbers z_1 and z_2, $z_1 + z_2 = z_2 + z_1$

(iii) The associative law For any three complex numbers z_1, z_2, z_3, $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$.

(iv) The existence of additive identity There exists the complex number 0 + i 0 (denoted as 0), called the additive identity or the zero complex number, such that, for every complex number z, z + 0 = z.

(v) The existence of additive inverse To every complex number z = a + ib, we have the complex number – a + i(– b) (denoted as – z), called the additive inverse or negative of z. Thus z + (–z) = 0 (the additive identity).

** **Difference of two complex numbers :**Given any two complex numbers z_1 and z_2, the difference $z_1 - z_2$ is defined as follows: $z_1 - z_2 = z_1 + (-\ z_2)$.

** **Multiplication of two complex numbers :**Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers. Then, the product $z_1 z_2$ is defined as follows: $z_1 z_2 = (ac - bd) + i(ad + bc)$

**The multiplication of complex numbers possesses the following properties :

(i) **The closure law** The product of two complex numbers is a complex number, the product $z_1 z_2$ is a complex number for all complex numbers z_1 and z_2.

(ii) **The commutative law** For any two complex numbers z_1 and z_2, $z_1 z_2 = z_2 z_1$

(iii) **The associative law** For any three complex numbers z_1, z_2, z_3, $(z_1 z_2) z_3 = z_1 (z_2 z_3)$.

(iv) **The existence of multiplicative identity** There exists the complex number 1 + i 0 (denoted as 1), called the multiplicative identity such that z.1 = z, for every complex number z.

(v) **The existence of multiplicative inverse** For every non-zero complex number z = a + ib or a + bi ($a \neq 0$, $b \neq 0$), we have the complex number $\frac{a}{a^2+b^2}+i\frac{-b}{a^2+b^2}$ (denoted by $\frac{1}{z}$ or z^{-1}), called the multiplicative inverse of z such that $z.\frac{1}{z}=1$ (the multiplicative identity).

(vi) **The distributive law** For any three complex numbers z_1, z_2, z_3,

(a) $z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$ (b) $(z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3$

****Division of two complex numbers :**Given any two complex numbers z_1 and z_2, where $z_2 \neq 0$, the quotient $\frac{z_1}{z_2}$ is defined by $\frac{z_1}{z_2} = z_1.\frac{1}{z_2}$.

****Modulus a Complex Number :**Let z = a + ib be a complex number. Then, the modulus of z, denoted by | z |, is defined to be the non-negative real number $\sqrt{a^2+b^2}$, i.e., $|z| = \sqrt{a^2+b^2}$

** **Properties of Modulus** :

If z, z_1, z_2 are three complex numbers then

(i) $|z| = 0 \Leftrightarrow z = 0$ i.e., real part and imaginary part are zeroes.

(ii) $|z| = |\bar{z}| = |-z|$

(iii) $z.\bar{z} = |z|^2$

(iv) $|z_1.z_2| = |z_1|.|z_2|$

(v) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $z_2 \neq 0$

(vi) $|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z}_2)$

(vii) $|z_1-z_2|^2 = |z_1|^2 + |z_2|^2 - 2Re(z_1\bar{z}_2)$

(viii) $|z_1+z_2|^2 + |z_1-z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

****Conjugate of a Complex Number :**Let z = a + ib then its conjugate is denoted by $\bar{z} = (a - ib)$.

****Properties of conjugates** :

(i) $\overline{(\bar{z})} = z$

(ii) $z + \bar{z} = 2Re(z)$

(iii) $z - \bar{z} = 2iIm(z)$

(iv) $z + \bar{z} = 0 \Rightarrow$ z is purely real.

(v) $z.\bar{z} = [Re(z)]^2 + [Im(z)]^2$.

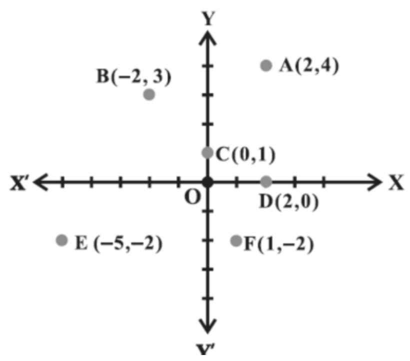
(vi) $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$

(vii) $\overline{z_1.z_2} = \bar{z}_1.\bar{z}_2$

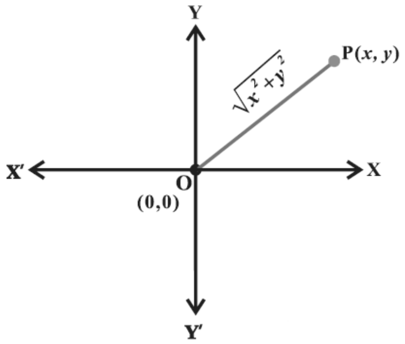
(viii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$, $z_2 \neq 0$

****Argand Plane and Polar Representation**

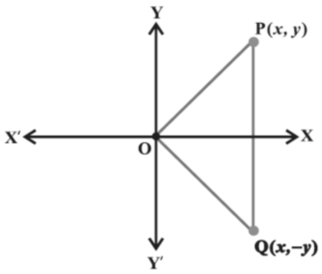
Some complex numbers such as 2 + 4i, – 2 + 3i, 0 + 1i, 2 + 0i, – 5 –2i and 1 – 2i which correspond to the ordered pairs (2, 4), (– 2, 3), (0, 1), (2, 0), (–5, –2), and (1, – 2), respectively, have been represented geometrically by the points A, B, C, D, E, and F, respectively.



The plane having a complex number assigned to each of its point is called the complex plane or the Argand plane.



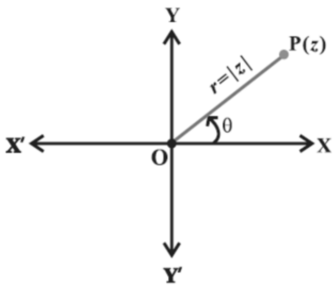
In the Argand plane, the modulus of the complex number $x + iy = \sqrt{x^2 + y^2}$ is the distance between the point P(x, y) to the origin O (0, 0). The points on the x-axis corresponds to the complex numbers of the form a + i 0 and the points on the y-axis corresponds to the complex numbers of the form 0 + i b.
The x-axis and y-axis in the Argand plane are called, respectively, the real axis and the imaginary axis.
The representation of a complex number z = x + iy and its conjugate z = x – iy in the Argand plane are, respectively, the points P (x, y) and Q (x, – y).



Geometrically, the point (x, – y) is the mirror image of the point (x, y) on the real axis.

** **Polar representation of a complex number :**

Let the point P represent the non zero complex number z = x + iy. Let the directed line segment OP be of length r and θ be the angle which OP makes with the positive direction of x-axis .



The point P is uniquely determined by the ordered pair of real numbers (r, θ), called the polar coordinates of the point P. We consider the origin as the pole and the positive direction of the x axis as the initial line.

We have, x = r cos θ, y = r sin θ and therefore, z = r (cos θ + i sin θ) is said to be the polar form of the complex number. Here $r = \sqrt{x^2 + y^2} = |z|$ is the modulus of z and θ is called the argument (or amplitude) of z which is denoted by arg z .

For any complex number z ≠ 0, there corresponds only one value of θ in $0 \le \theta < 2\pi$. However, any other interval of length 2π, for example $-\pi < \theta \le \pi$, can be such an interval. We shall take the value of θ such that $-\pi < \theta \le \pi$, called **principalargument** of z and is denoted by arg z, unless specified otherwise.

****Quadratic Equations :**Roots of the quadratic equation $ax^2 + bx + c = 0$ with real coefficients a, b, c , a ≠ 0 and $b^2 - 4ac < 0$ are $\dfrac{-b \pm \sqrt{4ac - b^2}\, i}{2a}$.

LINEAR INEQUALITIES
MAIN CONCEPTS AND RESULTS

* Two real numbers or two algebraic expressions related by the symbol '<', '>', '≤' or '≥' form an **inequality**.
* **Numerical inequalities :** $3 < 5$; $7 > 5$
* **Literal inequalities:** $x < 5$; $y > 2$; $x \ge 3$, $y \le 4$
* **Double inequalities :** $3 < 5 < 7$, $2 < y < 4$
***Strict inequalities :** $ax + b < 0$, $ax + b > 0$, $ax^2 + bx + c > 0$
* **Slack inequalities :** $ax + by \le c$, $ax + by \ge c$, $ax^2 + bx + c \le 0$
* **Linear inequalities :** $ax + b < 0$, $ax + b \ge 0$
* **Quadratic inequalities :** $ax^2 + bx + c > 0$, $ax^2 + bx + c \le 0$

Basic properties of inequality

1. **If $a \le b \Rightarrow a + c \le b + c$**
2. **If $a \le b \Rightarrow -b \le -a$**
3. **If $a \le b \Rightarrow \frac{1}{a} \ge \frac{1}{b}$**
4. **$|x| \le a \Rightarrow -a \le x \le a$**
5. **$|x - b| \le a \Rightarrow b - a \le x \le b + a$**
6. **$|x - b| \ge a \Rightarrow x \le b - a$ or $x \ge b + a$**

PERMUTATIONS AND COMBINATIONS
MAIN CONCEPTS AND RESULTS

** Fundamental principle of counting, or(the multiplication principle): "If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrence of the events in the given order is m × n."

****Factorial notation** The notation n! represents the product of first n natural numbers, i.e., the product $1 \times 2 \times 3 \times \ldots \times (n-1) \times n$ is denoted as n!. We read this symbol as 'n factorial'.

Thus, $1 \times 2 \times 3 \times 4 \ldots \times (n-1) \times n = n!$

$$n! = n(n-1)!$$
$$= n(n-1)(n-2)! \text{ [provided } (n \ge 2)]$$
$$= n(n-1)(n-2)(n-3)! \text{ [provided } (n \ge 3)]$$

****Permutations** A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

** The number of permutations of n different objects taken r at a time, where $0 < r \le n$ and the objects do not repeat is $n(n-1)(n-2)\ldots(n-r+1)$, which is denoted by

$$P(n, r) \quad \text{OR} \quad {}^{n}P_{r} = \frac{n!}{(n-r)!}, 0 \le r \le n$$

** ${}^{n}P_{0} = 1 = {}^{n}P_{n}$

** The number of permutations of n different objects taken r at a time, where repetition is allowed, is n^r.

** The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is $\frac{n!}{p_1!p_2!\ldots p_k!}$.

**The number of permutations of an dissimilar things taken all at a time along a circle is (n -1) !.

** The number of ways of arranging a distinct objects along a circle when clockwise and anticlockwise arrangements are considered alike is $\frac{1}{2}(n-1)!$.

** The number of ways in which (m + n) different things can be divided into two groups containing m and n things is $\frac{(m+n)!}{m!\,n!}$.

Combinationof n different objects taken r at a time, denoted by ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$.

** $^nP_r = {}^nC_r\, r!$, $0 \le r \le n$

** $^nC_0 = 1 = {}^nC_n$

$^nC_1 = n = {}^nC_{n-1}$

$^nC_2 = \dfrac{n(n-1)}{2!} = {}^nC_2$

$^nC_3 = \dfrac{n(n-1)(n-3)}{3!} = {}^nC_{n-3}$

** $^nC_r = {}^nC_s \Rightarrow r = s \text{ or } r + s = n$

**** $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

BINOMIAL THEOREM
MAIN CONCEPTS AND RESULTS

** Binomial theorem for any positive integer n

$$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}.b + {}^nC_2a^{n-2}.b^2 + \ldots\ {}^nC_{n-1}a.b^{n-1} + {}^nC_nb^n = \sum_{k=0}^{n} {}^nC_ka^{n-k}b^k$$

** The coefficients ^nC_r occurring in the binomial theorem are known as binomial coefficients.

** There are (n +1) terms in the expansion of $(a+b)^n$, i.e., one more than the index.

** $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_{n-1}x^{n-1} + {}^nC_nx^n$

** $(1-x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 - \ldots + (-1)^n\ {}^nC_nx^n$.

** $^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n$.

** $^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \ldots + (-1)^{n-1}\ {}^nC_n = 0$.

** General Term in the expansion of $(a+b)^n$ = $t_{r+1} = {}^nC_ra^{n-r}.b^r$

**Middle term in the expansion of $(a+b)^n$

(i) $\left(\dfrac{n}{2}+1\right)^{th}$ term if n is even (ii) $\left(\dfrac{n+1}{2}\right)^{th}$ and $\left(\dfrac{n+1}{2}+1\right)^{th}$ terms if n is odd.

SEQUENCE AND SERIES
CONCEPTS AND RESULTS

**** Sequence :** is an arrangement of numbers in a definite order according to some rule. A sequence can also be defined as a function whose domain is the set of natural numbers or some subsets of the type {1, 2, 3....k). ** A sequence containing a finite number of terms is called a finite sequence. A sequence is called infinite if it is not a finite sequence.

** **Series :** If a_1, a_2, a_3,…,an, be a given sequence. Then, the expression $a_1 + a_2 + a_3 +,\ldots + a_n + \ldots$

** **Arithmetic Progression (A.P.) :**is a sequence in which terms increase or decrease regularly by the same constant.

A sequence a_1, a_2, a_3,…, a_n,… is called arithmetic sequence or arithmetic progression if $a_n + 1 = a_n + d$, $n \in N$, where a_1 is called the first term and the constant term d is called the common difference of the A.P.

** The n^{th} term (general term) of the A.P. a, a + d, a + 2d, ... is **$a_n = a + (n – 1)\, d$.**

**If a, b, c are in A.P. and $k(\neq 0)$ is any constant, then

(i) a + k, b + k, c + k are also in A.P.

(ii) $a - k$, $b - k$, $c - k$ are also in A.P.

(iii) ak, bk, ck are also in A.P

(iv) $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in A.P.

** If a, a + d, a + 2d, …, a + (n – 1) d be an A.P. Then $l = a + (n - 1)d$.

$$\text{Sum to n terms } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[a + l]$$

** **Arithmetic mean (A.M.)** between two numbers a and b is $\frac{a+b}{2}$.

** **n arithmetic means** between two numbers a and b are $a + \frac{(b-a)}{n+1}, a + \frac{2(b-a)}{n+1}, a + \frac{3(b-a)}{n+1}, \ldots, a + \frac{n(b-a)}{n+1}$.

** Sum of n A.M.^{S} = n(single A.M.)

** Three consecutive terms in A.P. are a – d , a, a + d.

Four consecutive terms in A.P. are a – 3d , a – d , a + d, a + 3d.

Five consecutive terms in A.P. are a – 2d , a – d , a, a + d, a + 2d.

These results can be used if the sum of the terms is given.

** In an A.P. the sum of terms equidistant from the beginning and end is constant and equal to the sum of first and last terms.

** m^{th} term from end of an A.P. = $(n - m + 1)^{th}$ term from the beginning.

****Geometric Progression (G . P.) :A**sequence is said to be a geometric progression or G.P., if the ratio of any term to its preceding term is same throughout.

A sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called geometric progression, if each term is non-zero and

$\frac{a_{k+1}}{a_k} = r(\text{constant})$, for $k \geq 1$.

By taking $a_1 = a$, we obtain a geometric progression, a, ar, ar^2, ar^3,…., where a is called the first term and r is called the common ratio of the G.P.

** **General term of a G .P. =** $a_n = ar^{n-1}$**.**

** **Sum to n terms of a G .P. =** $\frac{a(r^n - 1)}{r - 1}$ if $r > 1$ **and** $\frac{a(1 - r^n)}{1 - r}$ if $r < 1$.

** **Sum of terms of an infinite G.P. =** $\frac{a}{1-r}$**.**

** **Geometric Mean (G .M.)**: of two positive numbers a and b is the number is $\sqrt{ab}$.

** **n geometric mean** between two numbers a and b are $a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}, a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}, a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}, \ldots a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$.

** Three consecutive terms in G.P. are $\frac{a}{r}$, a, ar .

Four consecutive terms in G.P. are $\frac{a}{r^3}, \frac{a}{r}$, ar, ar^3.

Five consecutive terms in G.P. are $\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2.

These results can be used if the product of the terms is given.

STRAIGHT LINES
CONCEPTS AND RESULTS

** Any point on the X-axis is (x, 0) and on the Y-axis is (0, y)

** Distance between two points $A(x_1, y_1)$ & $B(x_2, y_2)$ is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

**** Section formula**

(i) Coordinates of a point dividing the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ internally in the ratio m : n is $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$.

(ii) Coordinates of a point dividing the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ externally in the ratio m : n is $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$.

** Coordinates of the mid point of the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ is $\left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right)$

** Centroid of a ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

** In centre of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ is $\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$ where a = BC, b = AC, c = AB.

**AreaofΔABC withvertices $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ = $\frac{1}{2}$ $|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.

** Equation of any line parallel to X-axis is y = a, & equation of X-axis is y = 0.

** Equation of any line parallel to Y-axis is x = b & equation of Y axis is x = 0.

** Slope of line inclined at an angle θ with the + ve X- axis = tan θ.

** Slope of a line parallel to X-axis = 0 , slope of a line parallel to Y-axis = undefined.
Slope of a line equally inclined to the coordinate axes is –1 or 1.

** Slope of a line joining the points $A(x_1, y_1)$, $B(x_2, y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$.

** Slope of the line ax + by + c = 0, is $-\frac{a}{b}$.

** If two lines are parallel, then their slopes are equal.

** If two lines are perpendicular, then the product of their slopes is –1.

** Any equation of the form Ax + By + C = 0, with A and B are not zero, simultaneously, is called the general linear equation or general equation of a line.

(i) If A = 0, the line is parallel to the x-axis (ii) If B = 0, the line is parallel to the y-axis

(iii) If C = 0, the line passes through origin.

** Equation of a line having slope = m and cutting off an intercept ‘**c**’ and Y-axis is y = mx + c.

** Equation of a line through the point (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$.

** Equation of a line making intercepts of ‘a’ & ‘b’ on the respective axes is $\frac{x}{a} + \frac{y}{b} = 1$

** The equation of the line having normal distance from origin p and angle between normal and the positive x-axis ω is given by x cosω + y sinω = p .

** Distance of a point $P(x_1, y_1)$ from the line ax + by + c = 0 is $d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.

** Equation of the line parallel to ax + by + c = 0 is ax + by + λ = 0.

** Equation of the line perpendicular to ax + by +c= 0 is bx – ay + λ = 0.

****** If two lines are intersecting and θ is the angle between them, then $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$ where m_1 = slope of first line, m_2 = slope of second line and θ = acute angle.

If tan θ = negative ⇒θ = obtuse angle between the intersecting lines.

****** Distance between two parallel lines $ax + by + c_1 = 0$ &$ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

CONIC SECTIONS

CONCEPTS AND RESULTS

CIRCLES

A circle is a locus of a point which moves in a plane such that its distance from a fixed point in that plane in always constant. The fixed point is said to be the circle and the constant distance is said to be the radius.

** The equation of a circle with centre (h , k) and the radius r is $(x - h)^2 + (y - k)^2 = r^2$.

** The equation of a circle with centre (0 , 0) and the radius r is $x^2 + y^2 = r^2$.

** General equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ whose centre is (–g, –f) and radius is $\sqrt{g^2 + f^2 - c}$

** Equation of a circle when end points of diameter as $A(x_1, y_1)$, $B(x_2, y_2)$ is given by $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

** Length of intercepts made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on the X and Y-axes are $2\sqrt{g^2 - c}$ and $2\sqrt{f^2 - c}$.

CONICS

Conic Section or a conic is the locus of a point which moves so that its distance from affixed point bears a constant ratio to its distance from a fixed line.

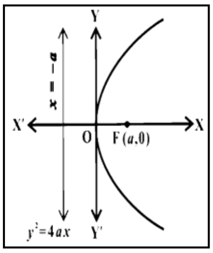
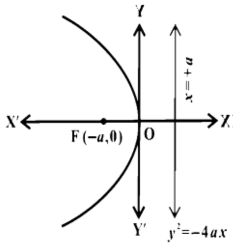
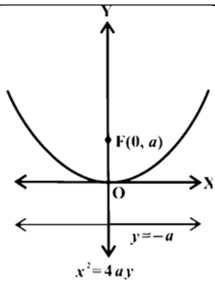
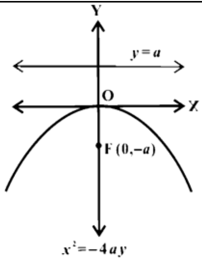
The fixed point is called the **focus**, the straight line the **directrix** and the constant ratio denoted by **e** is called the **eccentricity**.

Eccentricity (e) $\mathbf{e} = \frac{\text{distance between } P(x, y) \text{ \& Focus}}{\text{distance between } P(x, y) \text{ \& Directrix}}$.

If e = 1, then conic is a parabola.
If e < 1, then conic is a an ellipse.
If e > 1, then conic is a hyperbola.
If e = 0, then conic is a circle.

PARABOLA

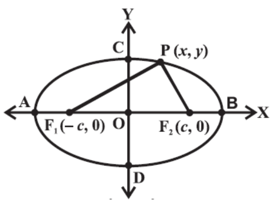
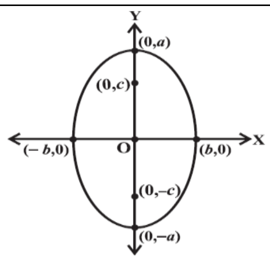
A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point in the plane.

S. No		$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
					
1.	Vertex	(0,0)	(0,0)	(0,0)	(0,0)
2.	Focus	(a,0)	(-a,0)	(0,a)	(0,-a)
3	Equation of directrix	x + a = 0	x – a = 0	y + a = 0	y – a = 0
4.	Equation of Axis	(x-axis), y = 0	(x-axis), y = 0	(y-axis), x = 0	(y-axis), x = 0
5.	Length of Latus Rectum	4a	4a	4a	4a
6.	Focal distance of a p(x, y)	x + a	x – a	y + a	y – a

** Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the hyperbola.

** **Position of a point with respect to a parabola** :The point (x_1, y_1) with respect to the parabola $y^2 = 4ax$ lies (i) outside , if $y_1^2 - 4ax_1 > 0$ (ii) on it if $y_1^2 - 4ax_1 = 0$ (iii)inside if $y_1^2 - 4ax_1 < 0$.

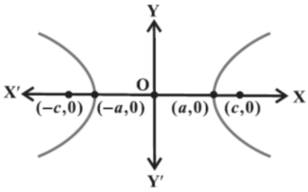
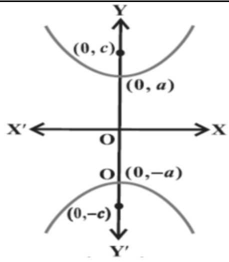
ELLIPSE: ** An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant. Ellipse is also defined as “The ratio of the distances between P(x, y) & Focus and P(x, y) & directrix is always a constant and this constant is said to eccentricity & always less than 1.

S. No.			
1.	Equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
2.	Centre	(0 , 0)	(0 , 0)
3.	Vertices	$(\pm a , 0)$	$(0 , \pm a)$
4.	Foci	$(\pm ae, 0)$ or $(\pm c , 0)$where $c^2 = a^2 - b^2$	$(0 \pm ae)$ or $(0 \pm c)$where $c^2 = a^2 - b^2$
5.	Equations of irectrices	$x = \pm\frac{a}{e}$.	$y = \pm\frac{a}{e}$
6.	Eccentricity	e = c/a	e = c/a
7.	Length of major axis	2a	2a

8.	Length of minor axis	2b	2b
9.	Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

**** HYPERBOLA**

A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.

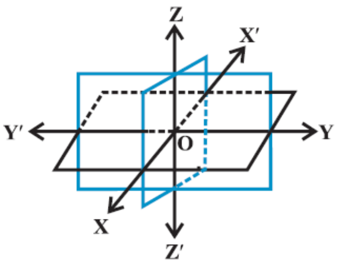
S. N			
1	Equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$
2.	Vertices	$(\pm a, 0)$	$(0, \pm a)$
3.	Foci	$(\pm ae, 0)$ or $(\pm c, 0)$ where $c^2 = a^2 + b^2$	$(0 \pm ae)$ or $(0 \pm ae)$ where $c^2 = a^2 + b^2$
4.	Eccentricity	e = c/a	e = c/a
5.	Equations of directrices	$x = \pm\frac{a}{e}$.	$y = \pm\frac{a}{e}$
6.	Centre	(0, 0)	(0, 0)
7.	Length of Transverse axis	2a	2a
8.	Length of conjugate axis	2b	2b
9.	Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
10.	Equation of Transverse axis	y = 0	x = 0
11.	Equation of conjugate axis	x = 0	y = 0

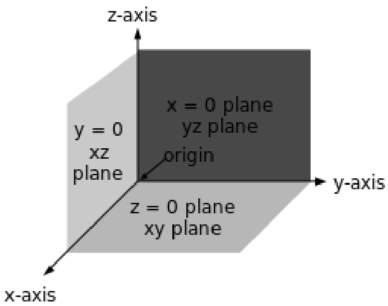
INTRODUCTION TO THREE-DIMENSIONAL GEOMETRY

MAIN CONCEPTS AND RESULTS

****Coordinate Axes and Coordinate Planes in Three Dimensional Space** :In three dimensions, the coordinate axes of a rectangular Cartesian coordinate system are three mutually perpendicular lines. The axes are called the x, y and z-axes.

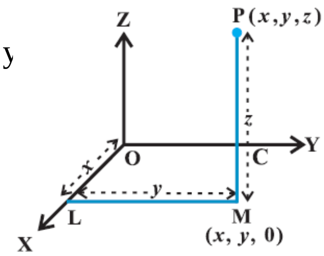
The three planes determined by the pair of axes are the coordinate planes, called XY, YZ and ZX-planes.





The three coordinate planes divide the space into eight parts known as octants.

** **Coordinates of a Point in Space :** The coordinates of a point P in three dimensional geometry is always written in the form of triplet like (x, y



Here x, y and z are the distances from the YZ**,** ZX and XY-planes.

Any point (i) on x-axis is of the form (x, 0, 0)

(ii) on y-axis is of the form (0, y, 0)

(iii) on z-axis is of the form (0, 0, z).

Any point (i) in XY-plane is of the form (x, y, 0)

(ii) in YZ-plane is of the form (0, y, z)

(iii) on ZX-plnane is of the form (x, 0, z).

** The three coordinate planes divide the space into eight parts known as octants.

Octants $\rightarrow$ Coordinates $\downarrow$	I XOYZ	II X′OYZ	III X′OY′Z	IV XOY′Z	V XOYZ′	VI X′OYZ′	VII X′OY′Z′	VIII XOY′Z′
x	+	–	–	+	+	–	–	+
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

** Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.

** **Section Formula** :The coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_1, y_1, z_1) internally and externally in the ratio m : n are given by

$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$, $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$ respectively.

** The coordinates of the mid point of the line joining P(x_1, y_1, z_1) and Q (x_2, y_2, z_2) is

$$\left(\frac{x_1+y_1+z_1}{2}, \frac{x_2+y_2+z_2}{2}\right)$$

** The coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1),(x_2, y_2, z_2) & (x_3, y_3, z_3) are $\left(\frac{x_1+y_1+z_1}{3}, \frac{x_2+y_2+z_2}{3}, \frac{x_3+y_3+z_3}{3}\right)$.

LIMITS AND DERIVATIVES
MAIN CONCEPTS AND RESULTS

Def : $\lim\limits_{x \to a} f(x) = l$*, if to a given* $\in > 0$*, there exists a +ve number S such that* $|f(x) - l| < \in$ *for* $|x - a| < \delta$.

*** Some Standard Results on Limits :*

** *If f(x) = K, a constant function, then* $\lim\limits_{x\to a} f(x) = K$.

** $\lim\limits_{x\to a} K.f(x) = K \lim\limits_{x\to a} f(x)$

** $\lim\limits_{x\to a} [f(x) \pm g(x)] = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} g(x)$

** $\lim\limits_{x\to a} f(x).\, g(x) = \lim\limits_{x\to a} f(x).\, \lim\limits_{x\to a} g(x)$

** $\lim\limits_{x\to a} \left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$.

** $\lim\limits_{x\to a} \log f(x) = \log\left(\lim\limits_{x\to a} f(x)\right)$.

** $\lim\limits_{x\to a} [f(x)]^{1/n}. = \left[\lim\limits_{x\to a} f(x)\right]^{1/n}$ *Provided* $\left[\lim\limits_{x\to a} f(x)\right]^{1/n}$ *is a real number.*

*****Sandwich Theorem (or squeeze principle).***

If f, g, h are functions such that $F(x) \le g(x) \le h(x)$ ***as*** $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} h(x) = l$, ***then*** $\lim\limits_{x \to a} g(x) = l$

** $\lim\limits_{x\to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$.

** $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$, *Also* $\lim\limits_{\theta\to 0} \dfrac{\theta}{\sin\theta} = 1$

** $\lim\limits_{\theta\to 0} \dfrac{\tan\theta}{\theta} = 1$, *Also* $\lim\limits_{\theta\to 0} \dfrac{\theta}{\tan\theta} = 1$

** $\lim\limits_{x\to 0} \dfrac{e^x - 1}{x} = \log e = 1$

** $\lim\limits_{x\to 0} \dfrac{a^x - 1}{x} = \log a$

** $\lim\limits_{x\to 0} (1 + x)^{1/x} = e$

** $\lim\limits_{n\to\infty} \left(1 + \dfrac{1}{n}\right)^n = C$

** $\lim\limits_{x\to 0} \dfrac{\sin^{-1} x}{x} = 1$

** $\lim\limits_{x\to 0} \dfrac{\tan^{-1} x}{x} = 1$

** **Some Standard Results of differentiation**

** $\dfrac{d}{dx}(x^n) = nx^{n-1} (x \in R, n \in R, x > 0)$

** $\dfrac{d}{dx}(x) = 1$

** $\dfrac{d}{dx}(c) = 0$ (where c is a constant)

** $\dfrac{d}{dx}(e^x) = e^x$

** $\dfrac{d}{dx}(a^x) = a^x \log_e a (a \in R, a > 0)$

** $\dfrac{d}{dx}(\log_e x) = \dfrac{1}{x} (x > 0)$

** $\dfrac{d}{dx}(\sin x) = \cos x$

** $\dfrac{d}{dx}(\cos x) = -\sin x$

** $\dfrac{d}{dx}(\tan x) = \sec^2 x$

** $\dfrac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$

** $\dfrac{d}{dx}(\sec x) = \sec x \tan x$

** $\dfrac{d}{dx}(\mathrm{cosec} x) = -\mathrm{cosec} x \cot x$

STATISTICS
CONCEPTS AND RESULTS

** Mean $\bar{x} = \dfrac{\sum x_i}{n}$

$= \dfrac{\sum f_i x_i}{\sum f_i}$

$= a + \left(\dfrac{\sum f_i u_i}{\sum f_i}\right)h$, where a is the assumed mean , h is the class size and $u_i = \dfrac{x_i - a}{h}$

** Median $= \left(\dfrac{n+1}{2}\right)^{th}$ observations (arranged in ascending or descending order) & the number of observations is odd.

$=$ mean of $\left(\dfrac{n}{2}\right)^{th}$ & $\left(\dfrac{n}{2}+1\right)^{th}$ observations (arranged in ascending or descending order) & the number of observations is odd.

$= l + \left(\dfrac{\frac{n}{2} - cf}{f}\right) \times h$ where, $l =$ lower limit of median class, $n =$ number of observations,

$cf =$ cumulative frequency of class preceding the median class, $f =$ frequency of median class,

$h =$ class size .

**Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$, where $l =$ lower limit of the modal class, $h =$ size of the class interval,

$f_1 =$ frequency of the modal class, $f_0 =$ frequency of the class preceding the modal class,

$f_2 =$ frequency of the class succeeding the modal class.

** **Measures of Dispersion:** The dispersion or scatter in a data is measured on the basis of the observations and thetypes of the measure of central tendency, used there. There are following measures ofdispersion: (i) Range, (ii) Quartile deviation, (iii) Mean deviation, (iv) Standard deviation.

** **Range:** Range of a series = Maximum value – Minimum value.

** **Mean Deviation :**

(i) For ungrouped data

$\text{M.D.}(\bar{x}) = \dfrac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$, where $\bar{x}$ = Mean $\text{M.D.}(M) = \dfrac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$, where M = Median

(ii) For grouped data

(a) Discrete frequency distribution

$$\text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{n} f_i|x_i - \bar{x}|}{\sum_{i=1}^{n} f_i} = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}| \qquad \text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{n}|x_i - M|$$

(b) Continuous frequency distribution

$$\text{M.D.}(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|, \text{ using } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right)h$$

$$\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{n} |x_i - M|, \text{ using } M = l + \left(\frac{\frac{n}{2} - cf}{f}\right)\times h$$

Standard deviation

For discrete data $\boldsymbol{\sigma} = \sqrt{\frac{\sum_1^n x_i^2}{n} - \left(\frac{\sum_1^n x_i}{n}\right)^2}$

For grouped frequency distribution $\boldsymbol{\sigma} = \sqrt{\frac{\sum_1^n f_i x_i^2}{n} - \left(\frac{\sum_1^n f_i x_i}{n}\right)^2}$

Shortcut method $\boldsymbol{\sigma} = \sqrt{\frac{\sum_1^n f_i d_i^2}{n} - \left(\frac{\sum_1^n f_i d_i}{n}\right)^2}$ where $d_i = x_i - A,\ A = assumed\ mean$

PROBABILITY

MAIN CONCEPTS AND RESULTS

** **Random Experiments :** An experiment is called random experiment if it satisfies the following two conditions:

(i) It has more than one possible outcome.

(ii) It is not possible to predict the outcome in advance.

** **Outcomes and sample space :** A possible result of a random experiment is called its outcome.

The set of all possible outcomes of a random experiment is called the sample space associated with the experiment. Each element of the sample space is called a sample point. Any subset E of a sample space S is called an event.

** **Impossible and Sure Events :** The empty set φ and the sample space S describe events. φ is called an impossible event and S, i.e., the whole sample space is called the sure event.

** **Compound Event :** If an event has more than one sample point, it is called a Compound event.

** **Complementary Event** : For every event A, there corresponds another event A' called the complementary event to A. It is also called the event 'not A'.

** **The Event 'A or B' :** When the sets A and B are two events associated with a sample space, then '$A \cup B$' is the event 'either A or B or both'. This event '$A \cup B$' is also called 'A or B'.

** **The Event 'A and B' :** If A and B are two events, then the set $A \cap B$ denotes the event 'A and B'.

** **The Event 'A but not B' :** the set A – B denotes the event 'A but not B'. $A - B = A \cap B'$

** **Mutually exclusive events :** two events A and B are called mutually exclusive events if the occurrence of any one of them excludes the occurrence of the other event, i.e., if they can not occur simultaneously. In this case the sets A and B are disjoint i.e. $A \cap B = \varphi$.

** **Exhaustive events :** if E1, E2, ..., En are n events of a sample space S and if

$E1 \cup E2 \cup E3 \cup \ldots \cup En = \bigcup_{i=1}^{n} E_i = S$, then E1, E2, ..., En are called exhaustive events.

if $E_i \cap E_j = \varphi$ for $i \neq j$ i.e., events E_i and E_j are pairwise disjoint and $\bigcup_{i=1}^{n} E_i = S$, then events $E_1, E_2, ..., E_n$ are

called **mutually exclusive and exhaustive events.**

** **Axiomatic Approach to Probability :** Let S be the sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S and range is the interval [0,1] satisfying the following axioms

(i) For any event E, $P(E) \geq 0$

(ii) $P(S) = 1$

(iii) If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$.

From the axiomatic definition of probability it follows that

(i) $0 \leq P(\omega_i) \leq 1$ for each $\omega_i \in S$

(ii) $P(\omega_1) + P(\omega_2) + ... + P(\omega_n) = 1$

(iii) For any event A, $P(A) = \Sigma P(\omega_i)$, $\omega_i \in A$.

** **Equally likely outcomes** : All outcomes with equal probability.

** **Probability of an event**: For a finite sample space with equally likely outcomes

Probability of an event $P(A) = \frac{n(A)}{n(S)}$ (A) □, where n(A) = number of elements in the set A,

n(S) = number of elements in the set S.

** **Probability of the event 'A or B' :** $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

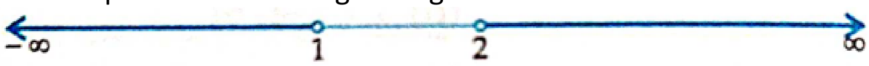
For mutually exclusive events A and B, we have $P(A \cup B)$ □□P(A) □□P(B)

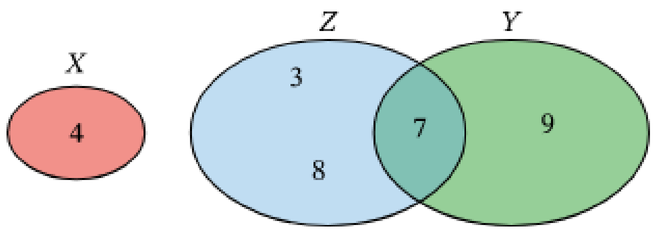
** **Probability of event 'not A' =** $P(A') = P(\text{not } A) = 1 - P(A)$.

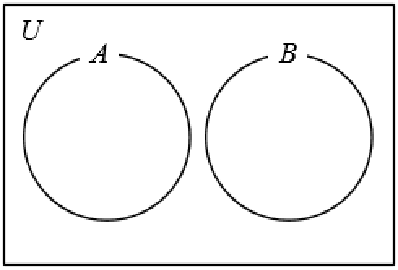
Practice materials

CHAPTER-1: SETS

1MARK QUESTIONS

Q.NO.	QUESTIONS	MARKS
1	The complement of the intersection of two sets is the union of their complements." This statement is called: (a) Complement Law (b) Associative Law (c) Idempotent Law (d) De Morgan's Law	**1**
2	If A={1,2,3,4,5}, then the number of proper subsets of A is (a) 120 (b) 30 (c) 31 (d)32	**1**
3	The set of circles passing through the origin (0,0) (a)Finite set (b) infinite set (c) Null set (d) none of these	**1**
4	The shaded part of a line is in given figure can also be described as  A. $(-\infty,1) \cup (2,\infty)$ B. $(-\infty,1] \cup [2,\infty)$ C.(1,2) D. [1,2]	**1**
5	**Roster for set C = {x : $x^2 + 7x - 8 = 0$, x ∈ R} is** (a) {– 8, 1} (b) [- 8, 1] (c) (- 8, 1) (d) {- 1, 8}	**1**

6	**The set builder form of interval [-4, 9) is:** (a) {x: x∈ R, $-4 \leq x \leq 9$} (b) {x: x∈ R, $-4 \leq x < 9$} (c) {x: x∈ R, $-4 < x < 9$} (d) {x: x∈ R, $-4 < x \leq 9$}	**1**
7	**Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}.Then which one is correct** **(a)** $(\boldsymbol{A} \cup \boldsymbol{B})' = \boldsymbol{A}' \cup \boldsymbol{B}'$**(b)** $(\boldsymbol{A} \cap \boldsymbol{B})' = \boldsymbol{A}' \cup \boldsymbol{B}'$ **(c)** $(\boldsymbol{A} \cap \boldsymbol{B})' = \boldsymbol{A}' \cap \boldsymbol{B}'$**(d)** $\boldsymbol{A}' \cap \boldsymbol{B}' = \boldsymbol{A}' \cup \boldsymbol{B}'$	**1**
8	For any two sets A and B, A∩ (A ∪ B) = (a)A (b) B (c) ∅ (d) none of these	**1**
9	Which of the following are examples of the singleton set? (a) $\{x: x \in Z, x^2 = 4\}$ (b) $\{x: x \in Z, x + 5 = 0\}$ (c) $\{x: x \in Z, x^2 = 16\}$ (d) $\{x: x \in Z, \lvert x \rvert = 1\}$	**1**
10	Let $A = \{1,2,\{3,4\},5\}$Which of the following are incorrect statement? (a) $\{3,4\} \subset A$ (b) $\{3,5\} \subset A$ (c) $\{\{3,4\}\} \subset A$ (d) 3,4∈ A	**1**
11	If $A = \phi$ then n[P(A)] = (a) 1 (b) 2 (c) 0 (d) 3	**1**
12	If $A = (2,4), B = [3,5)$ then $\boldsymbol{A} \cap \boldsymbol{B}$ = **(a) (3 , 4) (b) [3, 4) (c) [2, 5) (d) (3, 5)**	**1**
13	Let A and B be two sets such that n(A)= 16, n(B)= 14, n(A∪B)=25 then n(A∩B) is equal to (a) 30 (b)50 (c) 5 (d)none of these	**1**
14	The set A∪ A′ is (a) A (b) A′ (c)∅ (d) U	**1**
15	Set A and B have 3 and 6 elements respectively. What can be the minimum number of elements in A∪B? (a) 3 (b) 6 (c) 9 (d) 8	**1**
16	For the Venn - diagram given below, the set $(Z - Y) \times (X \cup Y)$ is:  (a) {(3,4), (3,7), (3,9), (8,4), (8,7), (8,9)} (b) {(4,8), (9,8), (7,8), (4,3), (9,3), (7,3)} (c) {(8,4), (8,9), (8,7), (4,3), (9,3), (7,3)} (d) {(4,8), (9,8), (7,8), (3,4), (3,9), (3,7)}	**1**
17	Which of the following sets is null set(s)?	**1**

	(a) {0} (b) {ϕ} (c) ϕ (d) sets of even prime number	
18	Which of the following is not a subset of **Q**? (a) Set of natural numbers (b) Set of integers (c) Set of rational numbers (d) Set of irrational numbers	**1**
19	If P={1, 2, 3, 4}, Q={2, 4, 6, 8} and R={3, 4, 5, 6} then P ∩ (Q U R) = (a) {1, 2, 3, 8} (b) {2, 3, 4} (c) {1, 5, 6, 8} (d) {1, 2, 3, 4, 5, 6, 8}	**1**
	In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices: *(a) Both A and R are true and R is the correct explanation of A.* *(b) Both A and R are true but R is not the correct explanation of A.* *(c) A is true but R is false.* *(d) A is false but R is true.*	**1**
20	Assertion (A): 'The collection of all natural numbers less than 100' is a set. Reason (R) : A set is a collection of the objects.	**1**
21	**Assertion (A):**  If $A \subset B$ for any two sets $A$and B, then adjacen diagram represents correct relationship between **Reason (R):** If $A \subset B$, then all elements of A are also in$B$.	**1**

2MARKS QUESTIONS

Q. NO	QUESTION	MARK
1	Write the set $A = \{x: x \in Z, x^2 < 20\}$ in roster form.	2M
2	Which of the following sets are empty sets? (i) $A = \{x: x^2 - 3 = 0 \; and \; x \; is \; rational\}$ (ii) $\boldsymbol{B} = \{x \in R: 0 < x < 1\}$	2M
3	Write down all possible subsets of each of the following sets: (i) $\{1, \{1\}\}$ (ii) {1,2,3}	2M
4	Write the following as intervals: (i) $\{x: x \in R, -12 < x < -10\}$ (ii) $\{x: x \in R, 3 \leq x \leq 4\}$	2M

5	What Universal Set would you propose for each of the following: (i) the set of isosceles triangle? (ii) the set of right triangle.	2M
	3 MARKS QUESTIONS	
1	Let $U = \{1,2,3,4,5,6,7,8,9\}, A = \{2,4,6,8\}\ and\ B = \{2,3,5,7\}. Verify\ that,$ $(i)(A \cup B)' = A' \cap B'\ (ii)(A \cap B)' = A' \cup B'.$	3M
2	Which of the following sets are finite and which are infinite: (i) A = $\{x: x\epsilon\ Z\ and\ x^2 - 5x + 6 = 0\}$ (ii) $B = \{x: x\ \epsilon\ Z\ \ and\ x^2\ is\ even\}$ (iii) C = $\{x : x\ \epsilon\ Z\ and\ x > -10\}$	3M
3	Let A and B be two sets. Prove that $(A - B) \cup B = A\ if\ and\ only\ if\ B \subset$ A.	3M
4	Let $U = \{1,2,3,4,5,6,7,8,9\}$ $A = \{1,2,3,4\}$, B = {2,4,6,8}, C = {3,4,5,6}. Find (i) $(A \cap C)'(ii)(A')'(iii)(B - C)'$	3M
5	Which of the following pairs of sets are equal? Justify your answer (i) $A = \{x: x\ is\ a\ letter\ of\ the\ word\ "LOYAL"\}$ B = $\{x: x\ is\ a\ letter\ of\ the\ word\ "ALLOY"\}$. (II) A = $\{x: x\ \epsilon\ Z\ and\ x^2 \leq 8\ \}$ B = $\{x: x\ \epsilon\ R, and\ x^2 - 4x + 3 = 0\}$	3M
	4 MARKS QUESTIONS	
1	Three friends were having get together. Suddenly they decided to play with their names using sets. Name of friends were AARTI, CHARVI and AYSHA. They asked each other the following questions. (i) How letters used for AARTI are written in roster form as a set? (a) $\{A, R, T, I\}$ (b) $\{x: x\ is\ a\ letter\ of\ the\ word\ AARTI\}$ (c) {A,T,I} (d) none of these (ii) What is the difference of set of letters of CHARVI and AYSHA? (a) $\{C, R, V, I\}$ (b) {C,S,V,I} (c) {C,T,V,I} (d) {C,V,I} (iii) Form a union of sets taking the letters of names of friends. $(a)\{A, R, T, I, C, H, V, Y, S\}$ (b) {A,R,T,I,C,H,V,} (c) {A,R,C,H,V,Y,S} (d) none of these (iv) Form a set of intersection of sets taking the letters of names of friends. (a) {A} (b) {A,R,T,I,C,H,V} (c) {A,R,C,H,V,Y,S} (d) none of these	4M

2 | 4M

After explaining operation on sets, Mathematics teacher in class wrote there sets as $A = \{2, 3, 4, 5\}$, $B = \{6, 7, 8\}$, $C = \{x: x \text{ is prime number less than } 10\}$. She asked the students that the following questions will judge how much you have understood. She asked the students to write down the answers and later they can check from the answers written by teacher and give marks.

(i) AU B =
(a) {2,3,4,5,6,7,8} (b) {2,3,4,5} (c) {6,7,8} (d) none of these

(ii) $(AUB) \cap C =$
(a) {2,3,5,7} (b) {2,3} (c) {5,7} (d) none of these

(iii) $(C - B) =$
(a) {2,3,5} (b) {2,3,5,7} (c) {3,5,7} (d) none of these

(iv) $(A \cap C) - B$
(a) {2,3,5} (b) {2,3} (c) {3,5} (d) none of these

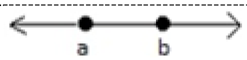
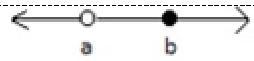
3

Case-Study: Passage-based question: Study the passage and table given below

and answer the questions (i) and (ii) given below:

The intervals are defined as the set of all real numbers lying between two given real numbers (end points / boundary points). It is a way of writing subsets of the set of all real numbers. Based on the inclusion / exclusion of end points the intervals are classified as – closed, open and semi closed / semi open intervals as shown in the following table.

Intervals

Intervals	Notations	inequalities	Number line representation
Closed	[a, b]	a $\leq x \leq b$	
Open	(a, b)	a < x < b	
Closed-Open	[a, b)	a $\leq x < b$	
Open - Closed	(a, b]	a < x $\leq b$	

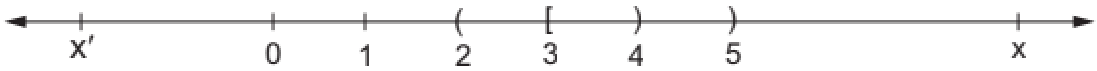
Intervals are sets so we can combine two or more intervals using

3(i)	To join the Indian Army under technical entry scheme the age of a candidate must be more than 16½ years and not above 19½ years. Represent the age limit using the interval.	2
3(ii)	According to weather report of Meteorological department the hottest month in Srinagar is July (minimum temperature 6°C, maximum temperature 32°C) and the coldest are December – January (temperature is between −15°C and 0°C). Represent the range of temperature in both the seasons as a single interval using the set	2
4	Sneha and Maria are best friends. Sneha likes Mathematics while Maria likes Statistics. They have created two non-empty sets A and B given by A = {x : x is a letter in 'I LOVE MATHEMATICS'} and B = {x : x is a letter in 'I LOVE STATISTICS'} Based on this information, answer the following questions.	
4(i)	Which of the following is True? (a) A = B (b) A ⊂ B (c) B ⊂ A (d) All of the above	1
4(ii)	A ∩ B is equal to (a) A (b) B (c) A ∪ B (d) ϕ	1
4(iii)	If number of proper subsets of A is n- more than number of proper subsets of B. Then find the value of n.	2

ANSWERS

1 MARK QUESTION

1	(d) De Morgan's Law
2	(c) 31 no. of proper subset = 2^n - 1
3	b infinite set
4	A. $(-\infty, 1) \cup (2, \infty)$
5	a {– 8, 1}
6	(b) {x: x∈ $\mathrm{R}, -4 \le \mathrm{x} < 9$}
7	(b) De Morgan's Law
8	(a) A use Venn diagram for this
9	(b) $\{x: x \in Z, x + 5 = 0\}$. Singleton set is a set having only one element.
10	(c) $\{\{3,4\}\} \subset A$
11	(a) 1 since no. of elements in ϕ = 0, then n[P(A)] =2^0 = 1

12	**(b) [3, 4)** 
13	(c) 5 use n (A ∪ B) = n (A) + n (B) – n (A ∩ B)
14	(d) U use Venn diagram
15	(b) 6 if $A \subset B$ then n(A∪B)= n(B) = 6
16	(a) {(3,4), (3,7), (3,9), (8,4), (8,7), (8,9)}
17	(c) φ
18	(d) Set of irrational numbers
19	(b) {2, 3, 4}
20	c A set is a well-defined collection of the distinct objects
21	d

2MARKS QUESTIONS

Q. NO	ANSWER
1	We observe that the integers whose squares are less than 20 are: $0, \pm 1, \pm 2, \pm 3, \pm 4$. Therefore, the set A in roster form is A =$\{-4, -3, -2, 0, 1, 2, 3, 4\}$
2	(i) Empty Set (ii) Non - Empty
3	(i) $\emptyset, \{1\}, \{\{1\}\}, \{1, \{1\}\}$ (ii) $\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$.
4	(i) $(-12, -10)$ (ii) $[3,4]$
5	(a) The set of all triangles in plane. (b) The set of all triangles in plane.

3 MARKS QUESTIONS

1	$(A \cup B)' = \{1 ,9\}$ $A' \cap B' = \{1 ,9\}$ $(A \cup B)' = A' \cap B'$

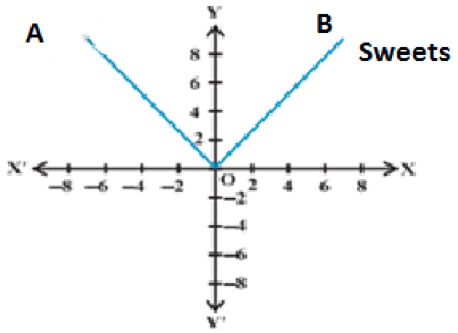
	$(A \cap B)' = \{1,3,4,5,6,7,8,9\}$ $A' \cup B' = \{1,3,4,5,6,7,8,9\}$ So, $(A \cap B)' = A' \cup B'$
2	$A = \{2, 3\}$ So, A is finite set. B = $\{\dots, -6, -4, -2, 0, 2, 4, 6, \dots\}$ So, B is infinite set. C = $\{-9, -8, -7, \dots\}$ So, C is infinite set.
3	Given, $(A - B) \cup B = A$ $(A \cap B') \cup B = A$ Or, $(A \cup B) \cap U = A$ Or, $(A \cup B) = A$ Or, $B \subset A$ Conversely, let $B \subset A$ $(A - B) \cup B = (A \cap B') \cup B = A \cup B = A$. (Proved)
4	(i) $(A \cap C)' = \{1,2,5,6,7,8,9\}$ (ii) $(A')' = \{1,2,3,4\}$ (iii) $(B - C)' = \{1,3,4,5,6,7,9\}$
5	(i) $A = B$ (ii) A $\neq B$

CASE BASED 4M

1	(i) (a) (ii) (a) (iii) (a) (iv) (a)
2	(i) (a) (ii) (a) (iii) (a) (iv) (a)

CHAPTER-2: RELATIONS AND FUNCTIONS

Q. NO	QUESTION	MARK
1	If AXA has 9 elements two of which are (-1, 0) and (0, 1), find the set A and the remaining elements of AXA.	2
2	**If A=$\{a, b\}$, find AXA.**	2
3	**If $A \times B = \{(p, q), (p, r), (m, q), (m, r)\}$, find A and B**	2
4	Write the relation R = $\{(x, x^3)$: x is a prime number less than 10$\}$ in roster form.	2
5	Find the values of a and b, when (a+3,b+2) =(5,1)	2
6	Find the domain and the range of the real function f(x) =$\sqrt{9 - x^2}$.	3
7	Let f, g: R→R be defined by f(x) = x + 1 and g(x) = 2x – 3. Find f – g, f. g and $\frac{f}{g}$.	3
8	Find the domain and the range of the real function f(x) =$\sqrt{(5 - x)}$.	3
9	Let f be the subset of Z x Z, defined by f = {(ab, a + b): a, b∈ Z}. Is f a function from Z to Z?Justify Your answer.	3
10	The function't' which maps temperature in degree Celsius in to temperature in degree Fahrenheit is defined by t(C) = $\frac{9C}{5}$+ 32. Find t (0), t (- 10) and the value of C, when t (C) = 212	3
11	In a school at Chandigarh, students of class XI were discussing about the relations and functions. Two StudentsAnkita and Babita form two sets A = {1, 2, 3, 4, 5} and B = {2, 4, 6}.  Based on the above information answer the following:	1+1+2 =4

	(1)Find n(A×B) (2)A correspondence of elements from A to B given as {(1, 2), (2, 2), (3, 4), (3, 6), (4, 4), (5, 6)}.Is it a function? Justify your answer. (3)If the function f: A⟶B such that (a, b)∈ f and a < b, defined by f = {(1, 2), (x, 4), (2, 4), (4, y), (5, 6)}, then find x and y.	
12	A is the anthills of an ant, at B some sweets are there and ant wants to reach at B. The path traced by an ant is shown in the following graph:  On the basis of the above graph find the following: (1)When ordinate is 6 then find abscissa (2)Which axis is line of symmetry for the graph? (3)Write the function for the graph along with domain and range.	1+1+2 =4

ANSWERS:

Q. NO	ANSWER	MARKS
1	Clearly -1, 0, 1 are elements of A. Therefore A= $\{-1,0,1\}$ Hence find AXA .Then remaining element of AXA are (-1,1),(-1,-1),(0,-1),(0,0),(1,-1),(1,0),(1,1).	2
2	AXA= $\{a,b\}$ x$\{a,b\}$ =$\{(a,a),(a,b),(b,a),(b,b)\}$	2
3	A=$\{p,m\}$, B=$\{q,r\}$	2
4	Prime numbers less than 10 are 2,3,5,7. R= $\{(2,8),(3,27),(5,125),(7,343)\}$	2
5	a+3 =5	2

	a= 2 and b-2 =1 b=3	
6	Domain = [-3, 3], Range = [0, 3]	3
7	(f – g) = - x + 4, f. g = $2x^2 - x - 3$, and $\frac{f}{g} = \frac{x+1}{2x-3}$	3
8	Domain = (- ∞, 5], Range = [0, ∞]	3
9	f is not a function because, if a and b both are positive or both are negative then ab is same but their images are not same.	3
10	t (0) =32^0F t(-10) =14^0F t(x) =212 therefore x=100 212^0F = 100^0C	3
11	(i) 15 (ii) No, Element 3 is having two images 4 and 6 (iii) x = 3 , y = 6	4
12	(i) ± 6 (ii) y-axis (iii) f(x) = |x|, the domain is R and Range is[0,∞)	4

CHAPTER-3: TRIGONOMETRIC FUNCTIONS

Q. NO	QUESTIONS	MARK
	2 marks each	
1	Evaluate $\tan 75^0$	2
2	Find the value of cos 1° cos 2° cos 3° … cos 179°.	2
3	Express 2 cos4x sin2x as an algebraic sum of sine or cosine.	2
4	If $\sin x = \frac{\sqrt{5}}{3}$, and $0 < x < \frac{\pi}{2}$, find the value of cos2x	2
5	Convert into radian measures: −47°30′	2
	3 marks each	
6	Find the radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm (use π = 22/7).	3
7	Prove that (√3 + 1) (3 – cot 30°) = $\tan^3 60°$ – 2 sin 60°.	3
8	A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?	3

9	Prove that $(\sin^4\theta - \cos^4\theta + 1)\,\mathrm{cosec}^2\theta = 2$	3
10	Prove that $\sin(40+\theta) \cdot \cos(10+\theta) - \cos(40+\theta) \cdot \sin(10+\theta) = 1/2$	3
	5 marks each	
11	Prove that: $\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$	5
12	Prove that: (cosec A – sin A)(sec A – cos A) = 1/(tan A + cot A)	5
13	Prove that $\cos 6x = 32\cos 2x - 48\cos 4x + 18\cos 2x - 1$	5
14	Find the values of other five trigonometric functions if sin x = 3/5, x lies in second quadrant.	5
15	Find the value of $\tan\frac{\pi}{8}$	5

ANSWERS

Q. NO	QUESTIONS	MARK
	2 marks each	
1	Evaluate $\tan 75^0$ Ans- Use the trigonometric addition formula for the tangent function $\tan 75^\circ = \tan(45^\circ + 30^\circ)$ $= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$ $= \frac{\sqrt{3}+1}{\sqrt{3}-1}$	1 1
2	Find the value of cos 1° cos 2° cos 3° … cos 179° . Since cos 90° = 0, we have cos 1° cos 2° cos 3° …cos 90°… cos 179° = 0	1 1

3	Express 2 cos4x sin2x as an algebraic sum of sines or cosine. Ans- $2\cos 4x \sin 2x = \sin(2x+4x) + \sin(2x-4x)$ $= \sin 6x + \sin(-2x)$ $= \sin 6x - \sin 2x$	 1 1
4	If $\sin x = \frac{\sqrt{5}}{3}$, and $0 < x < \frac{\pi}{2}$, find the value of cos2x Ans- We know that $\cos 2x = 1 - \sin^2 x$ $\cos 2x = 1 - 2\left(\frac{\sqrt{5}}{3}\right)^2$ $= 1 - 2 \times \frac{5}{9}$ $= -\frac{1}{9}$	 1 1
5	Convert into radian measures $-47^\circ 30'$ Ans- Convert into pure degree form and then convert to radian $-47^\circ 30' = -\left(47 + \frac{30}{60}\right)^\circ$ $= -\left(47 + \frac{1}{2}\right)^\circ$ $= -\left(\frac{95}{2} \times \frac{\pi}{180}\right)$ rad $= -\frac{19\pi}{72}$ rad	 1 1
	3 marks each	

6	Find the radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm (use $\pi = 22/7$). Solution: Given, Length of the arc = l = 37.4 cm Central angle = $\theta = 60° = 60\pi/180$ radian = $\pi/3$ radians We know that, $r = l/\theta$ $= (37.4) * (\pi / 3)$ $= (37.4) / [22 / 7 * 3]$ = 35.7 cm Hence, the radius of the circle is 35.7 cm.	3
7	Prove that $(\sqrt{3} + 1)(3 – \cot 30°) = \tan^3 60° – 2 \sin 60°$. Solution: LHS = $(\sqrt{3} + 1)(3 – \cot 30°)$ $= (\sqrt{3} + 1)(3 – \sqrt{3})$ $= 3\sqrt{3} – \sqrt{3}.\sqrt{3} + 3 – \sqrt{3}$ $= 2\sqrt{3} – 3 + 3$ $= 2\sqrt{3}$ RHS = $\tan^3 60° – 2 \sin 60°$ $= (\sqrt{3})^3 – 2(\sqrt{3}/2)$ $= 3\sqrt{3} – \sqrt{3}$ $= 2\sqrt{3}$ Therefore, $(\sqrt{3} + 1)(3 – \cot 30°) = \tan^3 60° – 2 \sin 60°$. Hence proved.	

8	A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second? Solution: Given, Number of revolutions made by the wheel in 1 minute = 360 1 minute = 60 seconds Number of revolutions in 1 second = 360/60 = 6 Angle made in 1 revolution = 360° Angles made in 6 revolutions = 6 × 360° Radian measure of the angle in 6 revolutions = 6 × 360 × π/180 = 6 × 2 × π = 12π Hence, the wheel turns 12π radians in one second.	
9	Prove that $(\sin^4\theta - \cos^4\theta + 1) \operatorname{cosec}^2\theta = 2$ Solution: L.H.S. = $(\sin^4\theta - \cos^4\theta + 1) \operatorname{cosec}^2\theta$ $= [(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta) + 1] \operatorname{cosec}^2\theta$ Using the identity $\sin^2A + \cos^2A = 1$, $= (\sin^2\theta - \cos^2\theta + 1) \operatorname{cosec}^2\theta$ $= [\sin^2\theta - (1 - \sin^2\theta) + 1] \operatorname{cosec}^2\theta$ $= 2 \sin^2\theta \operatorname{cosec}^2\theta$ $= 2 \sin^2\theta (1/\sin^2\theta)$ = 2 = RHS	

10	Prove that $\sin(40+\theta)\cdot\cos(10+\theta)-\cos(40+\theta)\cdot\sin(10+\theta)=\frac{1}{2}$ Ans- We know, $\sin(a-b)=\sin a\cos b-\cos a\sin b$ L.H.S = $\sin(40+\theta)\cos(10+\theta)-\cos(40+\theta)\sin(10+\theta)$ $=\sin[40+\theta-10-\theta]=\sin 30$ $=\frac{1}{2}$	
	5 marks each	
11	Prove that $\cos^2 x+\cos^2\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right)=\frac{3}{2}$ LHS $=\cos^2 x+\cos^2\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right)$ $=\cos^2 x+[\cos(x+\frac{\pi}{3})]^2+[\cos(x-\frac{\pi}{3})]^2$ $=\cos^2 x+(\cos x\cos\frac{\pi}{3}-\sin x\sin\frac{\pi}{3})^2+(\cos x\cos\frac{\pi}{3}+\sin x\sin\frac{\pi}{3})^2$ $=\cos^2 x+[\cos x\left(\frac{1}{2}\right)-\sin x\left(\frac{\sqrt{3}}{2}\right)]^2+[\cos x\left(\frac{1}{2}\right)+\sin x\left(\frac{\sqrt{3}}{2}\right)]^2$ $=\cos^2 x+\frac{1}{4}(\cos x-\sqrt{3}\sin x)^2+\frac{1}{4}(\cos x+\sqrt{3}\sin x)^2$ $=\cos^2 x+\frac{1}{4}(\cos^2 x+3\sin^2 x-2\sqrt{3}\cos x\sin x)+\frac{1}{4}(\cos^2 x+3\sin^2 x+2\sqrt{3}\cos x\sin x)$ $=\cos^2 x+\frac{1}{4}(\cos^2 x+3\sin^2 x-2\sqrt{3}\cos x\sin x+\cos^2 x+3\sin^2 x+2\sqrt{3}\cos x\sin x)$ $=\cos^2 x+\frac{1}{4}(2\cos^2 x+6\sin^2 x)$ $=\cos^2 x+\frac{1}{2}\cos^2 x+\frac{3}{2}\sin^2 x$ $=\frac{3}{2}\cos^2 x+\frac{3}{2}\sin^2 x$ $=\frac{3}{2}(\cos^2 x+\sin^2 x)$ $=\frac{3}{2}(1)$ $=\frac{3}{2}$ = R H S	

12	Prove that: (cosec A – sin A)(sec A – cos A) = 1/(tan A + cot A) Solution: LHS = (cosec A – sin A)(sec A – cos A) = [(1/sin A) – sin A) [(1/cos A) – cos A] = [(1 – $\sin^2$A)/ sin A] [(1 – $\cos^2$A)/ cos A] Using the identity $\sin^2$A + $\cos^2$A = 1, = ($\cos^2$A/sin A) ($\sin^2$A/cos A) = cos A sin A….(i) RHS = 1/(tan A + cot A) = 1/[(sin A/cos A) + (cos A/sin A)] = (sin A cos A)/ ($\sin^2$A + $\cos^2$A) = (sin A cos A)/1 = sin A cos A….(ii) From (i) and (ii), LHS = RHS i.e. (cosec A – sin A)(sec A – cos A) = 1/(tan A + cot A) Hence proved.	

13	Prove that $\cos 6x = 32\cos 2x - 48\cos 4x + 18\cos 2x - 1$ L.H.S. $= \cos 6x$ $= \cos 2(3x)$ $= 2\mathrm{Cos}^2 3x - 1$ $= 2(4\cos 3x - 3\cos x)^2 - 1$ $= 2[16\cos 6x + 9\cos 2x - 24\cos 4x] - 1$ $= 32\cos 6x + 18\cos 2x - 48\cos 4x - 1$ $= 32\cos 6x - 48\cos 4x + 18\cos 2x 1$ $=$R.H.S.	

14	Find the values of other five trigonometric functions if sin x = 3/5, x lies in second quadrant Solution: Given, sin x = 3/5 It can be written as cosec x = 1 / sin x = 1 / (3/5) = 5/3. Using the trigonometry identity $\sin^2 x + \cos^2 x = 1$ $1 - (3/5)^2 = \cos^2 x$ $1 - (9/25) = \cos^2 x$ $\cos^2 x = 16/25$ cos x = ± 4/5 Since x lies in the second quadrant, the value of cos x is negative. So cos x = -4/5 sec x = 1/ cos x sec x = 1 / -4/5 sec x = -5/4 tan x = sin x/ cos x = (3/5)/(-4/5) = -3/4 cot x = 1/tan x = 1/(-3/4) = -4/3	

15	Find the value of $\tan\frac{\pi}{8}$ We know that $$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$ Therefore, we have $$\tan\left(2\frac{\pi}{8}\right) = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$$ $$\Rightarrow 1 = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$$ Put $\tan\frac{\pi}{8} = x$ $$1 = \frac{2x}{1-x^2}$$ $$\Rightarrow 2x = 1-x^2$$ $$\Rightarrow x = \frac{-1\pm\sqrt{2}}{1}$$ Since, $\frac{\pi}{8}$ lies in the first quadrant, the value must be positive, hence $$\tan\frac{\pi}{8} = \sqrt{2}-1$$	

CHAPTER-4: COMPLEX NUMBERS

Q. NO	QUESTION	MARK
1	Let $z = i^{99} + i^{118}$then z lies in (a) 1st quadrant (b) 2nd quadrant (c) 3rd quadrant (d) 4th quadrant	1
2	If $(1+i)(1+2i)(1+3i)\ldots\ldots(1+ni) = a - ib$, then value of $2\times 5\times 10\ldots.\times(1+n^2) =$ (a) a^2+b^2 (b) a^2-b^2 (c) a^2+4b^2 (d) a^2-4b^2	1
3	If z is a complex number such that $z^2 = (\bar{z})^2$ is a complex number such that	1

	a) zis purely real b) zis purely imaginary c) Either zis purely real or purely imaginary d) None of these	
4	Let The amplitude of $z = 1 + i$ is θ then $1 + tan\theta + tan^2\theta + tan^4\theta =$ (a) 1 (b) 2 (c) 3 (d) 4	1
5	If a real value of x satisfy the equation $\frac{3-4ix}{3+4ix} = a - ib\ (a, b \in R)$ then $a^2 + b^2 =$ (a) 1 (b) -1 (c) 2 (d) -2	1
6	The conjugate of the complex number $\frac{\mathbf{2+5}\boldsymbol{i}}{\mathbf{4-3}\boldsymbol{i}}$ is (a) $\frac{7-26i}{25}$ (b) $\frac{7+26i}{25}$ (c) $\frac{-7-26i}{25}$ (d) $\frac{-7+26i}{25}$	1
7	If $z = 3 + 5i$, then $z^3 + \bar{z} + 198$= (a) −3−5i (b)−3+5i (c) 3−5i (d) 3+5i	1
8	The inequality $\|z - 4\| < \|z - 2\|$ represents the region given by (a) $Re(z) > 0$ (b) $Re(z) < 0$ (c) $Re(z) > 2$ (d) None Of These	1
9	The least positive value of n, if $\left(\frac{1+i}{1-i}\right)^n = 1$ (a) 0 (b) 2 (c) 4 (d) 1	1
10	If $(x + iy)(2 - 3i) = 4 + i$ then value of $\frac{y+x}{y-x} =$ (a) $\frac{5}{13}$ (b) $\frac{14}{13}$ (c) $\frac{9}{19}$ (d) $\frac{19}{9}$	1
11	If $(x + iy)^{\frac{1}{3}} = a + ib$ then prove $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$	2
12	Prove $(x + 1 + i)(x + 1 - i)(x - 1 + i)(x - 1 - i) = x^4 + 4$	2

ANSWERS:

Q. NO	ANSWER	MARKS
1	(c)	
2	(a)	
3	(c)	
4	d	
5	a	
6	c	
7	d	
8	(d)	
9	c	
10	(d)	
11		
12	$(x + 1 + i)(x + 1 - i)(x - 1 + i)(x - 1 - i)$	

	$= ((x+1)^2 - i^2)((x-1)^2 - i^2)$ $=((x+1)^2 + 1)((x-1)^2 + 1)$ $=(x^2 + 2x + 2)(x^2 - 2x + 2)$ $= x^4 + 4$	

CHAPTER-5: LINEAR INEQUALITIES

Q. NO	QUESTION	MARK
	Directions for questions 1 to 10: Questions from 1 to 10 are multiple choice questions. There are 4 alternatives given for each questions from 1 to 10. Choose the best alternative out of these four.	
1	If $\|x+2\| \leq 9$, then (a) $x \in (-11, 7)$ (b) $x \in [-11, 7]$ (c) $x \in (-7, 11)$ (d) $x \in [-7, 11]$	1
2	If x is a real number and $\|x\| < 3$, then (a) $x \in (-3, 3)$ (b) $x \in [-3, 3]$ (c) $x \in \{(-\infty, -3) \cup (3, \infty)\}$ (d) $x \in R$.	1
3	If $-3x + 17 \leq -10$, then (a) $x \in [9, \infty)$ (b) $x \in (9, \infty)$ (c) $x \in [-9, \infty)$ (d) $x \in (-9, \infty)$	1
4	If $\frac{\|x-2\|}{x-2} \geq 0$, then (a) $x \in (-\infty, 2)$ (b) $x \in (-\infty, -2)$ (c) $x \in (2, \infty)$ (d) $x \in [2, \infty)$	1
5	If $x < 5$, then (a) $-x < -5$ (b) $-x > -5$ (c) $-x < 5$ (d) $-x > 5$	1
6	If $4x - 13 \leq x - 4$, then (a) $x \in [-3, \infty)$ (b) $x \in (-3, \infty)$ (c) $x \in (-\infty, 3]$ (d) $x \in [3, \infty)$	1
7	If $-3x < -12$, then (a) $x \in (-4, \infty)$ (b) $x \in [4, \infty)$ (c) $x \in (4, \infty)$ (d) $x \in [-4, \infty)$	1

8	Solve: $1 \leq \|x-2\|$, then (a) $x \in \{(-\infty, 1] \cup [3,4]$ — (b) $x \in \{(-3, 1] \cup [3,4]$ (c) $x \in \{(-\infty, 1] \cup [3, \infty)\}$ — (d) $x \in \{(-3, 1] \cup (3,4]$	1
9	Any linear inequality can have how many distinct solution? (a) Indefinitely many — (b) only one solution (c) only two solution — (d) only three solution	1
10	How many solutions does the inequality $30x \leq 200$ have when x is a natural number? (a) 7 — (b) 8 (c) 6 — (d) 10	1
	Directions for questions 11 to 15: In questions from 11 to 15 , there are two statements marked as Assertion (A) and Reason (R). Mark your answer as per the codes provided below: (a). Both A and R are true and R is the correct explanation of A. (b). Both A and R are true and R is not the correct explanation of A. (c). A is true and R is false. (d). A is false and R is true. (e). Both A and R are false.	
11	Assertion: If $a < b, c < 0$ then $\frac{a}{c} < \frac{b}{c}$. Reason: If both sides of an inequality are divided by the same negative quantity, then the inequality is reversed.	1
12	Assertion: The inequality $ax + by \leq c$, where *a, b* and *c* are real numbers, is a linear inequality. Reason: The solution of the inequality $4x - 7 \geq 9$, when x is a real number, is $(-\infty, 4]$.	1
13	Assertion: A line divides the Cartesian plane in two halves. Reason: If a point P(α, β) lies on the line $ax + by + c = 0$, then $a\alpha + b\beta + c = 0$.	1
14	Assertion: If $3x - 4 \leq -x + 8 \Longrightarrow x \in (-\infty, 3]$ Reason: Both sides of an inequality can be multiplied by positive quantity and same number can be added to both the sides of an inequality.	1
15	Assertion: The inequality $45x \leq 300$ has infinitely many solutions when x is an integer. Reason: There are infinitely many integers less than or equal to $\frac{20}{3}$.	1

	Directions for questions 16 to 20: Questions from 16 to 20 are true false type questions. State whether the given statement is true or false.	
16	If $x \geq -3$, then $5 - x \leq 8$.	1
17	If $p > 0$ & $q < 0$, then $p + q > p$.	1
18	If $x < y$ and $b < 0$, then $\frac{x}{b} > \frac{y}{b}$.	1
19	If $x > -2$ & $x < 9$, then $x \in (-2, 9)$	1
20	If $\|x\| \geq 3$, then $x \in [-3, 3]$.	1
21	A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?	4
22	Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.	4

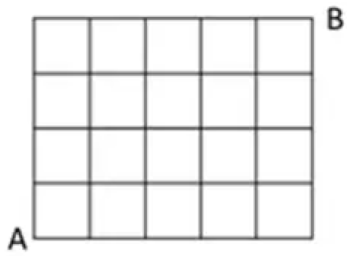
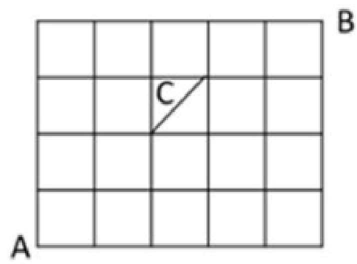
ANSWERS:

Q. NO	ANSWERS	MARKS
1	(b) $x \in [-11, 7]$	1
2	(a) $x \in (-3, 3)$	1
3	(a) $x \in [9, \infty)$	1
4	(c) $x \in (2, \infty)$	1
5	(b) $-x > -5$	1
6	(c) $x \in (-\infty, 3]$	1
7	(c) $x \in (4, \infty)$	1
8	(c) $x \in \{(-\infty, 1] \cup [3, \infty)\}$	1
9	(a) Indefinitely many	1
10	(c) 6	1
11	(d). A is false and R is true.	1
12	(c). A is true and R is false.	1
13	(b). Both A and R are true and R is not the correct explanation of A.	1
14	(a). Both A and R are true and R is the correct explanation of A.	1
15	(a). Both A and R are true and R is the correct explanation of A.	1

16	True	1
17	False	1
18	True	1
19	True	1
20	True	1
21	Let, x litres of 30% acid solution is required to be added. Then, Total mixture = $(x + 600)$ litres, Therefore, $30\%. x + 12\%$ of $600 > 15\%$ of $(x + 600)$ and $30\%. x + 12\%$ of $600 < 18\%$ of $(x + 600)$ Or, $\frac{30x}{100} + \frac{12}{100} \times 600 > \frac{15}{100} \times (x + 600)$ and $\frac{30x}{100} + \frac{12}{100} \times 600 < \frac{18}{100} \times (x + 600)$ Or, $30x + 7200 > 15x + 9000$ and $30x + 7200 < 18x + 10800$ Or, $15x > 9000 - 7200$ and $30x - 18x < 10800 - 7200$ Or, $15x > 1800$ and $12x < 3600$ Or, $x > \frac{1800}{15}$ and $x < \frac{3600}{12}$ Or, $x > 120$ and $x < 300$ Combining we get, $120 < x < 300$.	1 1 1 1
22	Let x be the smaller of the two consecutive odd natural number, so that the other one is x +2. Then, we should have x > 10 ... (1) and x + (x + 2) < 40 ... (2) Solving (2), we get 2x + 2 < 40 i.e., x < 19 ... (3) From (1) and (3), we get 10 < x < 19. Since x is an odd number, x can take the values 11, 13, 15, and 17. So, the required possible pairs will be (11, 13), (13, 15), (15, 17), (17, 19)	1 1 1 1

CHAPTER-6: PERMUTATIONS AND COMBINATIONS

Q. NO	QUESTION	MARK
1	Which one of the following is wrong (a) $n_{C_r} = n_{C_{r-1}}$ (b) $n_{P_r} = r!\, n_{C_r}$ (c) $n_{C_r} = n-1_{C_r} + n-1_{C_{n-r}}$ (d) $n+1_{C_r} = n_{C_r} + n_{C_{r-1}}$	1
2	The number of diagonals in a decagon is (a) 45 (b) 40 (c) 35 (d) 30	1
3	If $21_{C_{2r+15}} = 21_{C_{3r+6}}$ then r = (a) 1 (b) -1 (c) 9 (d) 0	1
4	The number of positive integral solution of equation $xy = 24$ is (a) 5 (b) 10 (c) 12 (d) 6	1
5	Number of words with or without meaning can be formed with the letters of the word BHARAT (a) 60 (b) 120 (c) 360 (d) 720	1
6	The number of arrangement of letters of the word HONEST in which H always precedes E and E precedes T (not necessarily adjacent) is (a) 360 (b) 240 (c) 720 (d) 120	1
7	$n_{P_4} = 20.n_{P_2}$, then n= (a) 7 (b) 6 (c) 8 (d) 5	1
8	Appu has 6 members in his family, grandpa, granny, parents, Appu and his sister, in how many ways a family photo graph can be taken in which grandpa and Granny will be always in the centre. (a) 24 (b) 48 (c) 120 (d) 720	1
9	Assertion (A) :$\sum_{47}^{51} n_{C_3} + 47_{C_4} = 52_{C_4}$ Reason (R) :$n+1_{C_r} = n_{C_r} + n_{C_{r-1}}$ (a) Both A and R are true and R is correct explanation of A (b) Both A and R are true but R is not correct explanation of A (c) A is true R is false (d)) R is true A is false	1
10	Assertion (A) :Total number of functions that can be defined from a set containing 3 elements to a set of 4 elements is 4^3 Reason (R) :Every image can have 3 pre image (a) Both A and R are true and R is correct explanation of A (b) Both A and R are true but R is not correct explanation of A (c) A is true R is false (d)) R is true A is false	1
11	How many numbers are there between 100 and 1000 which have exactly one of their digits as 6	2
12	Find total no of divisors of 180	2
13	Bishal is exploring with the number of five digits formed with 1,2,3,4,5 with all	

	digits used once. Try to answer the following questions (a) How many numbers Bishal can form. (b) Find Sum of unit digits of all numbers formed by Bishal (c) Find the sum of all the numbers he got	 1 1 2
14	There is a country called Mazeland in which all roads are either going north-south or east - west and all lanes are at a equal distance of 100 meters like shown in the figure. Now answer the following question (i) Gourav, a student of class XI has his home at A and school at B. the shortest distance from home to school is 1 KM. In how many ways he can go from his home to his school covering shortest distance.  (ii)Suppose there is a shortest alternative C in a square as shown in figure. He wants to use that path for commuting every time. In how many ways he can go from home to school.  	2 2

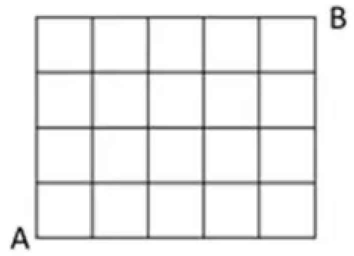
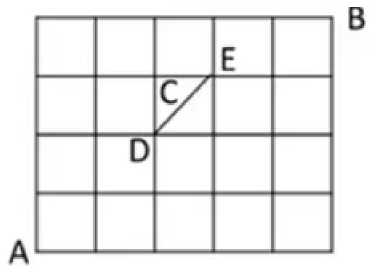
Q. NO	QUESTION	MARK
1	A coin is tossed 6 times, and the outcomes are noted. How many possible outcomes can be there?	2
2	How many words can be formed each of 2 vowels and 3 consonants from the letters of the given word – DAUGHTER?	2
3	It is needed to seat 5 boys and 4 girls in a row so that the girl gets the even places. How many are such arrangements possible?	2
4	Find the number of 5-card combinations out of a deck of 52 cards if each selection of 5 cards has exactly one king.	2
5	**In how many of the distinct permutations of the letters in MISSISSIPPI do the four Is not come together?**	3
6	**In a small village, there are 87 families, of which 52 families have at most 2 children. In a rural development programme, 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made?**	3

7	**Determine the number of 5 card combinations out of a deck of 52 cards, if there is exactly one ace in each combination.**	3
8	**How many numbers greater than 1000000 can be formed using the digits 1, 2, 0, 2, 4, 2, 4?**	3
9	**A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has** **(i) no girls** **(ii) at least one boy and one girl** **(iii) at least three girls**	4
10	Out of 7 boys and 5 girls a team of 7 students is to be formed. (i) Find the number of ways, if team contains at least 3 girls. (ii) Find the number of ways, if team contains at most 3 girls. (iii) Find the number of ways, if team contains exactly 3 girls. Or If exactly 3 girls ae selected and ae arranged in a row for photograph. Fid the number of ways, if all girls and all the boys stand together.	4
11	**How many permutations of the letters of the word 'MADHUBANI' do not begin with M but end with I?**	5
12	**How many different words can be formed from the letters of the word 'GANESHPURI'? In how many of these words:** **(i) the letter G always occupies the first place?** **(ii) the letters P and I, respectively, occupy the first and last place?** **(iii) Are the vowels always together?** **(iv) the vowels always occupy even places?**	5
13	**How many words can be formed with the letters of the word 'PARALLEL' so that all L's do not come together?**	5
14	Find the rank of the word SUCCESS, if all possible permutations of the word SUCCESS are arranged as in dictionary.	5
15	Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements. (i) Do the words start with P? (ii) Do all the vowels always occur together?	5

	(iii) Do all the vowels never occur together? (iv) Do the words begin with I and end in P?	

ANSWERS:

Q. NO	ANSWER	MARKS
1	c	
2	(c)	
3	(d)	
4	(b)	
5	(c)	
6	(d)	
7	(a)	
8	(b)	
9	(a)	
10	(c)	
11	Case 1: 6 8 ways 9 ways =72 Case 2: 6 8 ways 9 ways =72 Case 3 6 9 ways 9 ways =81 Total =72+72+81=225	½ ½ ½ 1/2
12	$180 = 2^2 3^2 5$ Any number of the form $2^p 3^q 5^r$ will be divisor of 180 where p can be 0,1,2, q can be 0,1,2 and r can be 0,1. So total no of divisors is 3.3.2 = 18	
13	(a) 5! = 120 (b) 1 can be fixed in unit place in 4! Ways= 24 ways. Similarly each digits 2,3,4,5 can be fixed in in unit place in 24 ways. So the sum of the unit digits is 24(1+2+3+4+5) = 24. 15 =360 (c) Hence the sum of all numbers can be written as 360X10000 +360X1000+360X100+360X10 +360 = 3999960	

14	(i) he will cover distance in shortest path by moving 5 horizontal (H)units and 4 vertical(V) units. So the shortest path can be obtained by arranging the letter of word VVVVHHHHH. So the no of paths $=\frac{9!}{5!4!}$  (ii) In this case he has to reach D first ,then cover DE and then E to B. A toD is square of 2 by 2 and E to B is 2X1 so in this case no of paths is $\frac{4!}{2!\,2!} \times \frac{3!}{2!\,1!}$ 	

Q. NO	ANSWER	MARKS
1	When we toss a coin once, the number of outcomes we get is 2 (Either Head or tail) So, in each throw, the no. of ways to get a different face will be 2. Therefore, by the multiplication principle, the required no. of possible outcomes is $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$	2
2	No. of Vowels in the word – DAUGHTER is 3. No. of Consonants in the word Daughter is 5. No of ways to select a vowel = ${}^{3}c_{2} = 3!/2!(3-2)! = 3$ No. of ways to select a consonant = ${}^{5}c_{3} = 5!/3!(5-3)! = 10$ Now you know that the number of combinations of 3 consonants and 2 vowels = $10 \times 3 = 30$ Total number of words = $30 \times 5! = 3600$ ways.	2

3	5 boys and 4 girls are to be seated in a row so that the girl gets the even places. The 5 boys can be seated in 5! Ways. For each of the arrangements, 4 girls can be seated only at the places which are cross marked to make girls occupy the even places). B x B x B x B x B So, the girls can be seated in 4! Ways. Hence, the possible number of arrangements = 4! × 5! = 24 × 120 = 2880	2
4	Take a deck of 52 cards, To get exactly one king, 5-card combinations have to be made. It should be made in such a way that in each selection of 5 cards, or in a deck of 52 cards, there will be 4 kings. To select 1 king out of 4 kings = ${}^4c_1$ To select 4 cards out of the remaining 48 cards = ${}^{48}c_4$ To get the needed number of 5 card combination = ${}^4c_1$ x ${}^{48}c_4$ = 4x2x 47x 46×45 = 778320 ways.	2
5	Given word – MISSISSIPPI M – 1 I – 4 S – 4 P – 2 Number of permutations = 11!/(4! 4! 2!) = (11 × 10 × 9 × 8 × 7 × 6 × 5 × 4!)/ (4! × 24 × 2) = 34650 We take that 4 I’s come together, and they are treated as 1 letter,	3

	$\therefore$ Total number of letters=11 – 4 + 1 = 8 $\Rightarrow$ Number of permutations = 8!/(4! 2!) = (8 × 7 × 6 × 5 × 4!)/ (4! × 2) = 840 Therefore, the total number of permutations where four Is don't come together = 34650 – 840 = 33810	
6	Given, Total number of families = 87 Number of families with at most 2 children = 52 Remaining families = 87 – 52 = 35 Also, for the rural development programme, 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children. Thus, the following are the number of possible choices: ${}^{52}C_{18} \times {}^{35}C_2$ (18 families having at most 2 children and 2 selected from other types of families) ${}^{52}C_{19} \times {}^{35}C_1$ (19 families having at most 2 children and 1 selected from other types of families) ${}^{52}C_{20}$ (All selected 20 families having at most 2 children) Hence, the total number of possible choices = ${}^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20}$	3
7	Given a deck of 52 cards There are 4 Ace cards in a deck of 52 cards. According to the given, we need to select 1 Ace card out of the 4 Ace cards $\therefore$ The number of ways to select 1 Ace from 4 Ace cards is ${}^4C_1$ $\Rightarrow$ More 4 cards are to be selected now from 48 cards (52 cards – 4 Ace cards) $\therefore$ The number of ways to select 4 cards from 48 cards is ${}^{48}C_4$	3

	Number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination = ${}^{4}C_{1} \times {}^{48}C_{4}$ = 4 × [48!/(44! 4!)] = 4 × [(48 × 47 × 46 × 45 × 44!)/ (44! × 24)] = 4 × 2 × 47 × 46 × 45 = 778320	
8	Given numbers – 1000000 Number of digits = 7 The numbers have to be greater than 1000000, so they can begin either with 1, 2 or 4. When 1 is fixed at the extreme left position, the remaining digits to be rearranged will be 0, 2, 2, 2, 4, 4, in which there are 3, 2s and 2, 4s. Thus, the number of numbers beginning with 1 = 6!/(3! 2!) = (6 × 5 × 4 × 3!)/(3! × 2) = 60 The total numbers begin with 2 = 6!/(2! 2!) = 720/4 = 180 Similarly, the total numbers beginning with 4 = 6!/3! = 720/6 = 120 Therefore, the required number of numbers = 60 + 180 + 120 = 360.	3
9	Given, Number of girls = 7 Number of boys = 7 (i) No girls Total number of ways the team can have no girls = ${}^{4}C_{0} \times {}^{7}C_{5}$ = 1 × 21 = 21	4

	(ii) at least one boy and one girl 1 boy and 4 girls = ${}^7C_1 \times {}^4C_4 = 7 \times 1 = 7$ 2 boys and 3 girls = ${}^7C_2 \times {}^4C_3 = 21 \times 4 = 84$ 3 boys and 2 girls = ${}^7C_3 \times {}^4C_2 = 35 \times 6 = 210$ 4 boys and 1 girl = ${}^7C_4 \times {}^4C_1 = 35 \times 4 = 140$ Total number of ways the team can have at least one boy and one girl = 7 + 84 + 210 + 140 = 441 (iii) At least three girls Total number of ways the team can have at least three girls = ${}^4C_3 \times {}^7C_2 + {}^4C_4 \times 7C_1$ = 4 × 21 + 7 = 84 + 7 = 91	
10	(i) Ways to select at least 3 girls 3 girls 4 boys or 4 girls 3 boys or 5 girls 2 boys $= {}^5C_3 \times {}^7C_4 + {}^5C_4 \times {}^7C_3 + {}^5C_5 \times {}^7C_2$ $= 10 \times 35 + 5 \times 35 + 1 \times 21 = 350 + 175 + 21 = 546$ (ii) Ways to select at most 3 girls 3 girls 4 boys or 2 girls 5 boys or 1 girls 6 boys or 0 girl 7 boys $= {}^5C_3 \times {}^7C_4 + {}^5C_2 \times {}^7C_5 + {}^5C_1 \times {}^7C_6 + {}^5C_0 \times {}^7C_7$ $= 10 \times 35 + 10 \times 21 + 5 \times 7 + 1 \times 1 = 350 + 210 + 35 + 1 = 596$ (iii) Ways to select exactly 3 girls = ${}^5C_3 \times {}^7C_4 = 350$ Or Ways of arranging 3 girls and 4 boys if all girls and boys stand together = 2! X 3! X 4! = 2 X 6 X 24 = 288 Total ways of selecting and arranging = 288 X 350 = 100800	4
11	The word 'MADHUBANI'	5

	Total number of letters = 9 A total number of arrangements of word MADHUBANI excluding I: Total letters 8. Repeating letter A, repeating twice. The total number of arrangements that end with the letter I = 8! / 2! = [8×7×6×5×4×3×2!] / 2! = 8×7×6×5×4×3 = 20160 If the word starts with 'M' and ends with 'I', there are 7 places for 7 letters. The total number of arrangements that start with 'M' and end with the letter I = 7! / 2! = [7×6×5×4×3×2!] / 2! = 7×6×5×4×3 = 2520 The total number of arrangements that do not start with 'M' but end with the letter I = The total number of arrangements that end with the letter I – The total number of arrangements that start with 'M' and end with the letter I = 20160 – 2520 = 17640 *Hence, the total number of arrangements of the word MADHUBANI in such a way that the word is not starting with M but ends with I is 17640.*	
12	The word 'GANESHPURI' There are 10 letters in the word 'GANESHPURI'. The total number of words formed is ${}^{10}P_{10} = 10!$ **(i)** the letter G always occupies the first place? If we fix the first position with the letter G, and then the remaining number of letters is 9. The number of arrangements of 9 things, taken all at a time, is ${}^{9}P_{9} = 9!$ Ways.	5

Hence, the possible number of words using letters of 'GANESHPURI' starting with 'G' is 9!

(ii) the letters P and I, respectively, occupy the first and last place?

If we fix the first position with letters P and I in the end, then the remaining number of letters is 8.

The number of arrangements of 8 things, taken all at a time, is $^8P_8 = 8!$ Ways.

Hence, the possible number of words using letters of 'GANESHPURI' starting with 'P' and ending with 'I' is 8!

(iii) Are the vowels always together?

There are 4 vowels and 6 consonants in the word 'GANESHPURI'.

Consider 4 (A, E, I, U) vowels as one letter, then the total number of letters is 7 (A, E, I, U, G, N, S, H, P, R)

The number of arrangements of 7 things, taken all at a time, is $^7P_7 = 7!$ Ways.

(A, E, I, U) can be put together in 4! Ways.

Hence, a total number of arrangements in which vowels come together is 7! × 4!

(iv) the vowels always occupy even places?

Number of vowels in the word 'GANESHPURI' = 4(A, E, I, U)

Number of consonants = 6(G, N, S, H, R, I)

Even positions are 2, 4, 6, 8 or 10

Now, we have to arrange 10 letters in a row such that vowels occupy even places. There are 5 even places (2, 4, 6, 8 or 10). 4 vowels can be arranged in these 5 even places in $^5P_4$ ways.

The remaining 5 odd places (1, 3, 5, 7, 9) are to be occupied by the 6 consonants in $^6P_5$ ways.

So, by using the formula,

$P(n, r) = n!/(n - r)!$

$P(5, 4) \times P(6, 5) = 5!/(5 - 4)! \times 6!/(6 - 5)!$

	$= 5! \times 6!$ *Hence, a number of arrangements so that the vowels occupy only even positions is $5! \times 6!$*	
13	The word 'PARALLEL' There are 8 letters in the word 'PARALLEL', out of which 2 are As, 3 are Ls and the rest all are distinct. So by using the formula, $n!/ (p! \times q! \times r!)$ The total number of arrangements $= 8! / (2!\ 3!)$ $= [8\times7\times6\times5\times4\times3\times2\times1] / (2\times1\times3\times2\times1)$ $= 8\times7\times5\times4\times3\times1$ $= 3360$ Now, let us consider all L's together as one letter, so we have 6 letters, out of which A repeats 2 times and others are distinct. These 6 letters can be arranged in $6! / 2!$ ways. The number of words in which all L's come together $= 6! / 2!$ $= [6\times5\times4\times3\times2\times1] / (2\times1)$ $= 6\times5\times4\times3$ $= 360$ So, now the number of words in which all L's do not come together = total number of arrangements – The number of words in which all L's come together *$= 3360 - 360 = 3000$*	5
14	Alphabets present in the word **SUCCESS** are **S,U,C,E** Dictionary order of the alphabet's is **C,E,S,U** Number of words starting with **C (C _ _ _ _ _ _)** (no two **C** will be repeated but here we have three **S**) is **6!3!** Number of words starting with **E (E _ _ _ _ _ _)** (here we have two **C** and three **S**)	5

	is **6!2!3!** Now we want the word starting with **S** So, Number of words starting with **SC (SC _ _ _ _ _)** (here we have single **C** and two **S**) is **5!2!** Number of words starting with **SE (SE _ _ _ _ _)** (here we have two **C** and two **S**) is **5!2!2!** Number of words starting with **SS (SS _ _ _ _ _)** (here we have two **C** and single **S**) is **5!2!** Now next word will be **SUCCESS** Now rank of the word **SUCCESS** is **6!3!+6!2!3!+5!2!+5!2!2!+5!2!+1** **= 120+60+60+30+60+1=331**	
15	No. of letters =12 No. of Ns = 3 No. of Es = 4 No. of Ds = 2 Required number of arrangements = $\frac{12!}{3!2!4!} = 1663200$ (i) If starting with P, then total arrangements = 1 $\times \frac{11!}{3!2!4!}$ (ii) There are 5 vowels in the given word, which are 4 Ex and 1 I. since, they to always occur together, we treat them as a single object andtogether with remaining 7 objects can be arranged = $\frac{8!}{3!2!}$ and 5 vowels van be arranged in $\frac{5!}{4!}$ ways so total number of arrangements = $\frac{8!}{3!2!} \times \frac{5!}{4!} = 16800$ (iii) The required number of arrangements = 1663200 – 16800 = 1646400 (iv) Let us find I and P at the extreme ends (I at the left end and P at the right end) we are left with 10 letters. Hence, the required number of arrangements = $\frac{10!}{3!2!4!} = 12600$	5

CHAPTER-7: BINOMIAL THEOREM

Q. No.	Question	Marks
1	The number of ways 7 boys and 6 girls can be seated in a row so that they are alternate is a)3620800 b)3062800c)3628800 d)3645280	1
2	The value of P(n, n – 1) is	1

	a) n! b) n c) 2n d) 2n!	
3	If repetition of the digits is allowed, then the number of even natural numbers having two digits is a) 20 b) 25 c) 45 d) 90	1
4	If $\frac{1}{8!}+\frac{1}{9!}=\frac{x}{10!}$ $find\ x$ a) 64 b) 50 c) 90 d) 100	1
5	The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet? a) 50400 b) 50300 c) 50200 d) 50100	1
6	If ${}^{n}C_3 = {}^{n}C_5$ then find n: a) 4 b) 6 c) 8 d) 10	1
7	The number of odd numbers lying between 40000 and 70000 that can be made from the digits 0, 1, 2, 4, 5, 7 if digits can be repeated any number of times is a) 766 b) 1296 c) 1125 d) 655	1
8	If ${}^{8}C_{x} = {}^{8}C_1$ then x will be: a) 6 b)5 c) 7 d)4	1
9	Find the number of ways of choosing 4 cards from a pack of 52 playing cards when four cards belong to four different suits. a) 4^{13} b) 13^3 c) 13^5 d) 13^4	1
10	Find the value of ${}^{5}P_2$. a) 5 b) 10 c) 15 d)20	1
11	Given 4 flags of different colors, how many different signals can be generated, if a signal required the use of 2 flags one below the other?	3
12	7 men and 5 women are to be seated in a row so that no two women sit	3

	together. Find the number of ways they can be seated.	
13	Determine n if ${}^{2n}C_3 : {}^{n}C_3 = 16:1$	3
14	In how many ways a debate team of 3 boys and 3 girls are selected from 6 boys and 5 girls?	3
15	In how many ways can a cricket team of eleven be chosen out of a batch of 17 players for the following cases, if  1. There is no restriction on the selection 2. A particular player is always chosen 3. A particular player is never chosen	4
16	 Four friends are playing with cards. They are choosing 4 cards from a pack of 52 playing cards. Using these information answer the following questions. (i) How many of these four cards are of the same suit? (ii) How many of these four cards belong to four different suits? (iii) How many of these four cards are face cards? (iv) How many of these two are red cards and two are black cards?	4

ANSWERS

Q NO	ANSWERS	MARK
1	c	1
2	a	1
3	c	1
4	d	1
5	a	1
6	c	1
7	b	1
8	c	1
9	d	1
10	d	1
11	Here, the upper place of the flag can be filled in 4 ways by using the 4 flags	3

	of different colors. Now, the lower place of the flag can be filled in 3 ways by using the remaining 3 flags of different colors. $\therefore$ total number of signals can be generated $= 4 \times 3$ $= 12$	
12	Given, 7 men and 5 women are to be seated in a row so that no two women sit together. The number of ways they can be seated is 7 men can be sit asxMxMxMxMxMxMxMx Here x denote the space for women to sit. So there are 8 space and 5 women can be sit as ${}^{8}P_{5} = \frac{8!}{(8-5)!}$ $= \frac{8\times7\times6\times5\times4\times3!}{3!}$ $= 6720$ Now total number of arrangement = 7! × 6720 = 5040 × 6720 =33868800	3
13	$\frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{16}{1}$ $\Rightarrow \frac{(2n)!}{3!(2n-3)!} * \frac{3!(n-3)!}{n!} = \frac{16}{1}$ $\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} * \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 16$ $\Rightarrow \frac{(2)(2n-1)(2n-2)}{(n-1)(n-2)} = 16$ $\Rightarrow \frac{(2n-1)(n-1)}{(n-1)(n-2)} = 4$ $\Rightarrow \frac{(2n-1)}{(n-2)} = 4$ $\Rightarrow 2n-1 = 4(n-2)$ $\Rightarrow 2n-1 = 4n-8$ $\Rightarrow 4n-2n = 8-1$ $Hence,\ n = \frac{7}{2}$	3
14	A debate team of 3 boys and 3 girls is to be selected from 6 boys and 5 girls. 3 boys can be selected from 6 boys in ${}^{6}C_{3}$ ways. 3 girls can be selected from 5 girls in ${}^{5}C_{3}$ ways. Therefore, according to multiplication principle, number of ways in which a	3

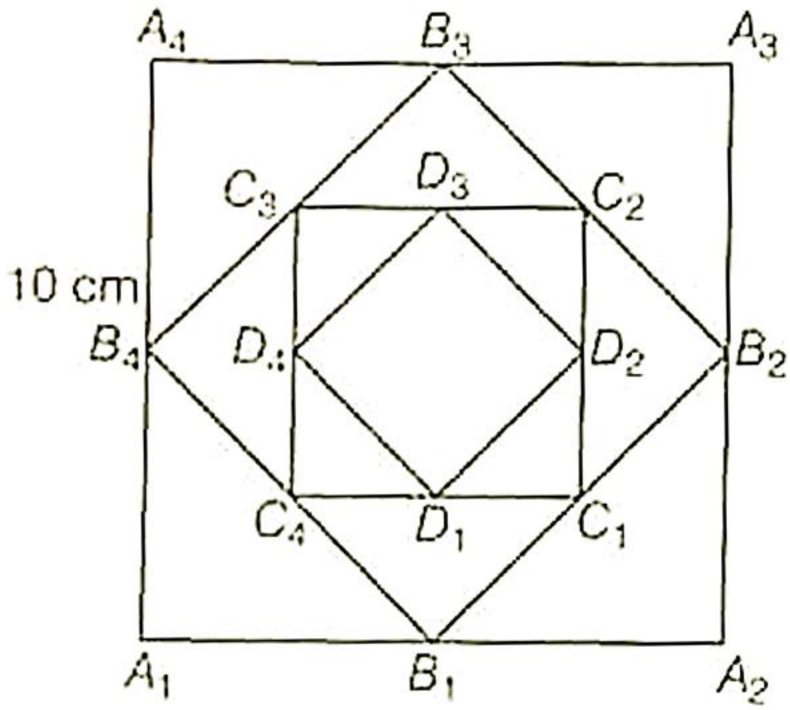
	team of 3 boys and 3 girls can be selected is given by $^6C_3 * {}^5C_3 = \left(\frac{6!}{3!3!}\right)*\left(\frac{5!}{3!2!}\right) = \left(\frac{6*5*4}{3*2}\right)*\left(\frac{5*4}{2}\right) = 20*10 = 200$	
15	Given: Total number of players = 17 1. There is no restriction on the selection The number of ways for team selection when there is no restriction is expressed by: $^{17}C_{11} = \frac{17!}{11!6!} = \frac{17*16*15*14*13*12*11!}{11!6!}$ $\Rightarrow \frac{17*16*15*14*13*12}{6*5*4*3*2} = 12376$ 2. A particular player is always chosen The number of ways a team selected when a particular player is always chosen is expressed by: $^{16}C_{10} = \frac{16!}{10!6!} = \frac{16*15*14*13*12*11*10!}{10!(6*5*4*3*2)} = 8008$ 3. A particular player is never chosen The number of ways a team selection is done when a particular player is never chosen can be expressed by: $^{16}C_{11} = \frac{16!}{11!5!} = \frac{16*15*14*13*12*11!}{11!(5*4*3*2*1)} = 4368$	4
16	(i) Required number of ways=$^{13}C_4+{}^{13}C_4+{}^{13}C_4+{}^{13}C_4=4\times{}^{13}C_4=2860$ (ii) Required number of ways=$^{13}C_1\times{}^{13}C_1\times{}^{13}C_1\times{}^{13}C_1=13^4$ (iii) Required number of ways=$^{12}C_4=495$ • Required number of ways=$^{26}C_2\times{}^{26}C_2=105625$	4

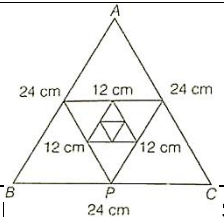
	SEQUENCE AND SERIES	
Q. NO	QUESTION	MARK
1	The 4th and 7th terms of a GP are $\frac{1}{18}$ and $\frac{-1}{486}$ respectively. Its first term is a) $\frac{2}{3}$ b) $-\frac{2}{3}$ c) $-\frac{3}{2}$ d) $\frac{3}{2}$	1
2	In a GP, the ratio between the sum of first 3 terms and the sum of first 6 terms is 125 : 152. The common ratio is a) $\frac{1}{2}$ b) $\frac{5}{6}$ c) $\frac{2}{3}$ d) $\frac{3}{5}$	1
3	If second term of a G.P. is 2 and the sum of its infinite terms is 8, then its first term is a) $\frac{1}{4}$ b) 2 c) $\frac{1}{2}$ d) 4	1
4	For any two positive numbers, we have a) None of these b) AM$\leq$ GM c) $AM = \frac{3}{4}GM$ d) AM$\geq$ GM	1
5	If a, b, c are in G.P., then a) $a(b^2 + c^2) = c(a^2 + b^2)$ b) $a(b^2 + a^2) = c(b^2 + c^2)$	1

	c) a^2 (b + c) = c^2 (a + b) d) none of these	
6	The two geometric means between the numbers 1 and 64 are a) 4 and 16 b) 8 and 16 c) 2 and 16 d) 1 and 64	1
7	If the sum of n terms of a GP is (2^n- 1) then its common ratio is a) $\frac{-1}{2}$ b) $\frac{1}{2}$ c) 2 d) 3	1
8	The third term of G.P. is 4. The product of its first 5 terms is a) 4^4 b) 4^3 c) 4^5 d) None of these	1
9	The sum of first three terms of a G.P. is to the sum of next three terms is 125 : 27. The common ratio of the G.P. is a) $\frac{1}{2}$ b) $\frac{5}{3}$ c) $\frac{3}{5}$ d) none of these	1
10	The sum of first eight terms of a G.P. is 82 times the sum of first four terms. The common ratio of the G.P. is a) 3 b) 2 c) 5 d) 4	1

11	The next term of the sequence 1, 1, 2, 4, 7, 13,.... is a) 21 b) 24 c) none of these d) 19	1
12	The sum of an infinite G.P. is 4 and the sum of the cubes of its terms is 92. The common ratio of the original G.P. is a) $-\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{1}{2}$	1
13	$1+\sqrt{3}+3+3\sqrt{3}$ + ... upto 10 terms = ? a) None of these b) $81(\sqrt{3}+1)$ c) $121(\sqrt{3}+1)$ d) $100(\sqrt{3}+1)$	1
14	f the nth term of the GP 3,$\sqrt{3}$, 1,... is $\frac{1}{243}$ then n = ? a) 14 b) 13 c) 12 d) 15	1
15	The next term of the sequence $\frac{1}{4},\frac{1}{36},\frac{1}{144},$ a) $\frac{1}{169}$ b) $\frac{1}{400}$ c) $\frac{1}{576}$ d) $\frac{1}{1296}$	1
16	The sum of the infinite GP $\left(1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots\infty\right)$ is:	1

	a) $\frac{3}{2}$ b) $\frac{4}{9}$ c) $\frac{5}{9}$ d) $\frac{2}{3}$	
17	The sum of an infinite GP is $\frac{80}{9}$ and its common ratio is $-\frac{4}{5}$. The first term of the GP is a) 16 b) 8 c) 20 d) 12	1
18	If (k - 1), (2k +1), (6k + 3) are in GP then k = ? a) - 2 b) 7 c) 0 d) 4	1
19	**Assertion (A):** If the numbers $\frac{-2}{7}$, K, $\frac{-7}{2}$ are in GP, then k = ± 1 . **Reason (R):** If a_1 , a_2 , a_3 are in GP, then $\frac{a_2}{a_1} = \frac{a_3}{a_2}$. a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A. c) A is true but R is false. d) A is false but R is true	1
20	**Assertion (A):** The sum of first 6 terms of the GP 4, 16, 64, ... is equal to 5460. **Reason (R):** Sum of first n terms of the G.P is given by $S_n = \frac{a(r^n-1)}{r-1}$, where a = first term r = common ratio . a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A. c) A is true but R is false. d) A is false but R is true.	1
21	**Read the text carefully and answer the questions:** A student of class XI draws a square of side	4

	10 cm. Another student joins the mid - point of this square to form a new square. Again, the mid - points of the sides of this new square are joined to form another square by another student. This process is continued indefinitely.  (i) Write the given information in progression and name progression. (ii) Find the sum of areas of all the square formed is (in sq cm)? (iii)Find the sum of the perimeter of all the square formed is (in cm)? (iv)Find the sum area of 5 squares?	
22	**Read the text carefully and answer the questions:** Each side of an equilateral triangle is 24 cm. The mid - point of its sides are joined to form another triangle. This process is going	4

23	 A sequence of no s said to be a geometric progression, if the ratio of each term, except the first one, by its preceding term is always constant. Rahul being a plant lover decides to continuously infinite. (i)Find the sum of perimeter of all triangles (in cm)? (ii) Find the sum of area of all the triangle (in sq cm)? (iii) Find the sum of perimeter of first 6 triangle is (in cm)? (iv) Find the sum of areas of first 4 triangles in sq cm?	4

open a nursery and he bought fewplants with pots. He wants to place potsin such a way that number of pots infirst row is 2, in second row is 4 and inthird row is 8 and so on....

Based on the above information, answer

the following questions.

(i) Find the constant multiple by which thenumber of pots is increasing inevery row.
(ii) Find the number of pots in 8th row
(iii) Find the difference in number of pots placed in 7th row and 5th row
(iv) If Rahul wants to place 510 pots in total, then find the total number of rows

formed in this arrangement.

ANSWERS

Q. NO.	ANSWER	MARK
1	c) $-\frac{3}{2}$	1
2	d) $\frac{3}{5}$	1
3	d) 4	1
4	d) AM$\geq$ GM	1
5	a) $a(b^2 + c^2) = c(a^2 + b^2)$	1
6	a) 4 and 16	1
7	c) 2	1

8	c) 4^5	1
9	c) $\frac{3}{5}$	1
10	a) 3	1
11	b) 24	1
12	d) $\frac{1}{2}$	1
13	c) $121(\sqrt{3}+1)$	1
14	b) 13	1
15	b) $\frac{1}{400}$	1
16	a) $\frac{3}{2}$	1
17	a) 16	1
18	d) 4	1
19	a) Both A and R are true and R is the correct explanation of A.	1
20	a) Both A and R are true and R is the correct explanation of A.	1
21	(I) Here side of first square is 10 cm. Side of second square is $5\sqrt{2}$ cm. Side of third square is 5 cm. Hence the given sequence is GP First term = 10 and common difference = $1/\sqrt{2}$ (ii) Area of squares are 100, 50, 25, ... Which is in GP A = 100 r = ½ Sum of all squares = 100+50+25+......... +∞ S = $\frac{a}{1-r}$ = $\frac{100}{1-\frac{1}{2}}$ = 200 (iii) Perimeters of squares are 40, $20\sqrt{2}$, 20, ... Which is in GP A = 40 r = $1/\sqrt{2}$ Sum of perimeters of all squares = 40 + $20\sqrt{2}+20+$ ....+ ∞ S = $\frac{a}{1-r}$ = $\frac{40}{1-\frac{1}{\sqrt{2}}}$ = $80+40\sqrt{2}$ (iv) Area of squares are 100, 50, 25, ...	1 1 1

	Which is in GP A = 100 r = ½ $S_5 = \frac{100(1-(\frac{1}{2})^5}{1-\frac{1}{2}} = \quad 96.85$	1
22	(i) Perimeter of first triangle = 24x3 = 72 Perimeter of second triangle = 12x3 = 36 Perimeter of third triangle = 6x3 = 18 Which is in GP, a = 72 and r = ½ Sum of all perimeter = 72 + 36 + 18 + ……. + ∞ $S = \frac{a}{1-r} = \frac{72}{1-\frac{1}{2}} = 144$	1
	(ii) Area of first triangle = $\frac{\sqrt{3}}{4}$x 576 Area of second triangle = $\frac{\sqrt{3}}{4}$ x 144 Area of third triangle = $\frac{\sqrt{3}}{4}$ x 36 Which is in GP Sum of all area of triangle = $\frac{\sqrt{3}}{4}$ x 576+$\frac{\sqrt{3}}{4}$ x 144+$\frac{\sqrt{3}}{4}$ x 36+……… +∞ $S = \frac{a}{1-r} = \frac{\frac{\sqrt{3}}{4}576}{1-\frac{1}{4}} = 192\sqrt{3}$	1
	(III) a = 72 and r = ½, n = 6 $S_6 = \frac{72(1-(\frac{1}{2})^6}{1-\frac{1}{2}} = \frac{567}{4}$	1
	(iv) Area of first triangle = $\frac{\sqrt{3}}{4}$ x 576 Area of second triangle = $\frac{\sqrt{3}}{4}$ x 144 Area of third triangle = $\frac{\sqrt{3}}{4}$ x 36 Which is in GP, a = $\frac{\sqrt{3}}{4}$ x 576, r = ¼ Using the formula, $S_n = \frac{a(1-r^n)}{1-r}$ We have $S_4 = \frac{765\sqrt{3}}{4}$	1

23	(i)The constant multiple by which thenumber of pots is increasing inevery row that isthe common ratio = 2 (Ans. a) (ii) The number of pots in 8th row is = 8^{th} term in G.P = $2x2^{8-1}$ =$2x2^7$ = 256 (Ans. b) (iii) The difference in number of potsplaced in 7th row and 5th row is = $2x2^6 - 2x2^4 = 2^5 (2^2 - 1) = 96$ (Ans. d) (iv) Let, to place 510 pots in total, then the total number of rows formed in this arrangement is n Then we can write, $\frac{2(2^n-1)}{2-1}$ = 510 $\Rightarrow (2^n - 1)$ =255 $\Rightarrow 2^n = 256$ $\Rightarrow 2^n = 2^8$ $\Rightarrow n = 8$ (Ans. b)	**1** **1** **1** **1**

LIMITS AND DERIVATIVES

Q. No.	QUESTION	MARK
1	1. What is the value of $\lim_{y\to 2} \frac{y^2-4}{y-2}$? a) 2 b) 4 c) 1 d) 0	1
2	What is the value of $\lim_{y\to\infty} \frac{2}{y}$? a) 0 b) 1 c) 2 d) ∞	1
3	What is the value of $\lim_{x\to\infty} \frac{x^2-9}{x^2-3x+2}$? a) 1 b) 2 c) 0	1

	d) Limit does not exist.	
4	What is the value of $\lim\limits_{x\to 4} \frac{x^2-2x-8}{x-4}$? a) 0 b) 2 c) 8 d) 6	1
5	$$\text{Evaluate} : \lim_{x\to 0} \frac{ax + \sin x}{\tan x + bx^2}$$ A) a + 1 B) a C) $\frac{a}{b}$ D) $\frac{a+1}{b+1}$	1
6	What is the value of $\lim y\to 4$ f(y)? It is given that f(y) = $y^2 + 6y$ (y ≥ 2) and f(y) = 0 (y < 2). a) 0 b) 16 c) 40 d) 30	1
7	What is the value of the limit f(x) = $\frac{2x^2+\frac{2}{x}}{2x^2-\frac{4}{x}}$ if x approaches infinity? a) 0 b) 1 c) 2 d) 4	1
8	What is the value of $\lim\limits_{x\to 4} \frac{x^2-4-3x}{x-3}$? a) 0 b) 4 c) 1 d) Limit does not exist	1
9	What is the value of $\lim\limits_{x\to 0} \frac{\sin 3x}{3x}$? a) 0 b) 1 c) 3 d) 1/3	1
10	What is the value of $\lim\limits_{x\to 0} \frac{x^2 \sec x}{\sin x}$? a) 3 b) 2 c) 1 d) 0	1
11	What is the value of $\lim\limits_{x\to\infty} \frac{x \sin\frac{2}{x}}{2}$? a) 1	1

	b) 2 c) 1/2 d) Limit does not exist	
12	Which of the following limits does not yield 1? a) limx→0 (sin x / x) b) limx→0(tan x / cot x) c) limx→0 [(1/e^x) +cosx] d) limx→0 x cosec x	1
13	Find the derivative of e^{x^2}. a) e^{x^2} b) 2x c) $2e^{x^2}$ d) $2xe^{x^2}$	1
14	What is the derivative of sin x tan x ? a) sin x + tan x sec x b) cos x + tan x sec x c) sin x + tan x d) sin x + tan x sec^{2}x	1
15	What is the derivative e^x sinx + e^x cos x? a) 0 b) 2 cosx c) 2e^x.sinx d) 2e^x.cos x	1
16	$\lim\limits_{x \to 0}[x - 1]$, where [.] is the greatest integer function, is equal to A) 1 B) 2 C) 0 D) does not exist	1
17	If y = $\frac{\sin(x+9)}{\cos x}$, then $\frac{dy}{dx}$ $at\ x = 0$ is (a)$\cos 9$(b) $\sin 9$ (c) 0 (d) 1	1
18	If f(x) = x sin x, then f'($\frac{\pi}{2}$) is equal to A) 0 B) 1 C) – 1 D) $\frac{1}{2}$	1
19	Derivative of f(x) = 1 + x + x^2 + x^3 + x^4 + …………….+x^{50} at x = 1 is A) 1725 B) 1275 C) 50 D) 0	1
20	Derivatives of the function $\sin x \cos x$ is equal to	1

	(a) - $\cos 2x$ (b) $\sin 2x$ (c) $\cos 2x$ (d) - $\sin 2x$	
21	**ASSERTION-REASON BASED QUESTIONS(Questions 9 to 10)** In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true. 9) **Assertion(A) :** $\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$ = 2 **Reason(R) :** $\lim_{x\to a}\frac{x^n-a^n}{x-a}$ = na^{n-1}	1
22	**Assertion(A) :** $\frac{d}{dx}(x^5-2x^4-2)$ = $5x^4-8x^3-2$ **Reason(R) :** Differentiation of x^n with respect to x is nx^{n-1}	1
23	Let y = $x^{x^{x^{\cdots\infty}}}$ then find $\frac{dy}{dx}$	2
24	If y = $x^{2023}+\log_{2023}x$ then find $\frac{dy}{dx}$	2
25	If y = $\frac{x-1}{x+1}$, then find $\frac{dy}{dx}$	2
26	Indeterminate forms of limits:On direct evaluation, if a limit takes the forms $\frac{0}{0}$, $\frac{\infty}{\infty}$, 0 x ∞ … … …, we use standard results for evaluating limits. There are a few indeterminate forms given below. $\frac{0}{0}$, $\frac{\infty}{\infty}$, 0 x ∞, ∞^0, 0^0, 1^∞, ∞ – ∞ etc. Find the following limits. (i) $\lim_{x\to 2}\frac{x^6-64}{x-2}$= ……… (a) 0 (b) 80 (c) 192 (d) 129 (ii) $\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}$ = ………	4

	(a) 0 (b) 3/2 (c) 2/3 (d) 15 (iii) $\lim_{x\to 0}\frac{\sqrt{1+3x}-\sqrt{1-3x}}{x}$ = ……… (a) 0 (b) 1 (c) 3 (d) 6 (iv) $\lim_{x\to 0}\frac{8^x-2^x}{x}$ = ……… (a) 0 (b) log 2 (c) log 4 (d) log 8	
27	Suppose f is a real valued function, the function defined by $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, wherever the limit exists is defined to be the derivative of f at x and is denoted byf′(x). This definition of derivative is also called the first principle of derivative. Let *f* and *g* be two functions such that their derivatives are defined in a common domain. Then (i) Derivative of sum of two functions is sum of the derivatives of the Functions. $\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}[f(x)]+\frac{d}{dx}[g(x)]$. (ii) Derivative of difference of two functions is difference of the derivatives of the functions. $\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}[f(x)]-\frac{d}{dx}[g(x)]$. (iii) Derivative of product of two functions is given by the following *product rule*.$\frac{d}{dx}[f(x).g(x)]=f(x).\frac{d}{dx}[g(x)]+g(x).\frac{d}{dx}[f(x)]$. (iv) Derivative of quotient of two functions is given by the following *quotient rule* (whenever the denominator is non–zero). $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x).\frac{d}{dx}[f(x)]-f(x).\frac{d}{dx}[g(x)]}{[g(x)]^2}$.	4

	Using the above concepts, answer the following questions: (i) What is derivative of sin x? (ii) What is derivative of cosx ? (iii) Find the derivative of tan x w.r.t. x. **OR** If y = $\frac{x}{\tan x}$, find $\frac{dy}{dx}$.	

ANSWERS:

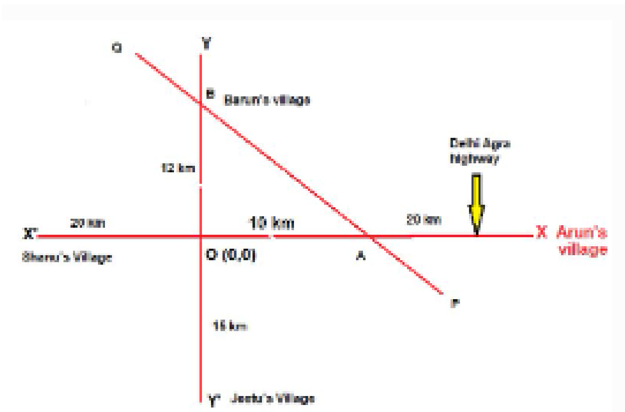
Q. No.	ANSWER	MARKS
1	b)	1
2	a)	1
3	a)	1
4	d)	1
5	d)	1
6	c)	1
7	b)	1
8	a)	1
9	b)	1
10	d)	1
11	a)	1
12	c)	1
13	d)	1
14	a)	1
15	d)	1
16	d)	1
17	a)	1
18	b)	1
19	b)	1

20	c)	1
21	b)	
22	d)	
23	Clearly y = x^y Taking logarithm both side we get logy = y logx Now differentiate both side w.r.t x we get $\frac{1}{y}\frac{dy}{dx} = \frac{y}{x} + logx\ \frac{dy}{dx}$ Simplify above we will get our required answer	
24	y = $x^{2023} + \log_{2023} x$ therefore y = $x^{2023} + \frac{\log_e x}{\log_e 2023}$ Therefore $\frac{dy}{dx}$ = 2023 $x^{2022} + \frac{1}{x \log_e 2023}$	1 1
25	$-\frac{2}{(1+\mathrm{x})^2}$	
26	(i) (c) (ii) (b) (iii) (c) (iv) (c)	
27	(i) $\frac{d(\sin x)}{dx} = \cos x$ (ii) $\frac{d(\cos x)}{dx} = -\sin x$ (iii) Let $f(x) = \tan x = \frac{\sin x}{\cos x}$. $\frac{dy}{dx} = \frac{\cos x . \frac{d}{dx}(\sin x) - \sin x . \frac{d}{dx}(\cos x)}{(\cos x)^2} = \frac{cos^2 x + sin^2 x}{cos^2 x}$. $= \frac{1}{cos^2 x} = sec^2 x.$ **OR** Let $y = \frac{x}{\tan x}$.	1 1 2 2

	Then, $\frac{dy}{dx} = \frac{\tan x.\frac{d}{dx}(\mathrm{x}) - \mathrm{x}.\frac{d}{dx}(\tan x)}{(\tan x)^2}$ $= \frac{\tan x - x.sec^2 x}{tan^2 x}$	

Straight line

Question no.	Answers	Marks
1.	The slope of line, whose inclination with the x-axis is 60°is (a) -√3 (b)√3 (c)1/√3 (d)-1/√3	1
2.	In which quadrant, the points (-2 ,-2) Lie? (a)1st (b)2nd (c)3rd (d)4th	
3.	A (1,1) and B (2,-3) are two points and D is a point on AB such that AD = 3AB, B lies Between A and D then coordinates of D are (a) (4,11) (b)(-4,11) (c)(-4,-11) (d)(4,-11)	1
4.	Find the slope of the lines passing through the points (6,3) and (-2,1).	1
5.	Find the value of k , such that slope of the lines passing through the points (2,4) and (-1,k) is 3.	1
6.	Find the slope of a line which makes -120° angles with the x-axis .	1
7.	Find the equation of a line which cuts off intercept 2 units on the negative side of y-axis and makes an angle of 45° with the positive direction of x-axis.	1
8.	Find the equation of a line which cuts of intercepts of 3 unit on the x-axis and 4 unit on the negative side of the y-axis.	1
9.	Find the equation of a line parallel to y-axis ,at a distance of 5/2 units to the left of y-axis.	1
10.	The slope of the line, whose equation is 5x +6y=7 is (a)5/6 (b)6/5 (c)-5/6 (d)-5	1
11.	The inclination of line √3x+3y=5 with the x-axis is (a) 180° (b)60° (c)30° (d)150°	1
12.	The distance between line x/a-y/b-1 and point (b,a) is (a)|b^2-a^2-ab|/$\sqrt{a^2+b^2}$ (b) |-b^2-a^2+ab|/$\sqrt{a^2+b^2}$ (c) |b^2+a^2-ab|/$\sqrt{a^2+b^2}$ (d)$\sqrt{b^2+a^2}$	1
13.	Distance between parallel lines x+3y-9=0 and x+3y+1=0 is (a)3 units (b)1 unit (c) √10 units (d)10 units	1

14.	The value of S for which the following lines are parallel: 2x+3y=7 and Sx+6y=8 (a)-2/3 (b)2/3 (c)4 (d)-4	1
15.	Reduce the equation of the line 2x-3y-4 =0 in the intercept form and hence find the intercept made by the line on the x-axis and the y-axis.	1
16.	Find the point(s) on the x-axis whose distance from the line x/3+y/4=4 unit.	1
17.	Find the value of k for which the line $(k-3)x-(4-k^2)y+k^2-7k+6=0$ is parallel to the x-axis .	1
18.	Find the value of k ,if the straight line 2x+3y+4+k(6x-y+12)=0 is perpendicular to the line 7x+5y-4=0.	1
19.	Find the Value of P for which the following lines are perpendicular :Px+3y=4 and 3x-4y=7.	1
20.	If the points (a,0),(b,0) and (3,4) are collinear, show that 3/a+4/b=1.	1
21.	If A and B are two persons sitting at the positions (2, -3) and (6,-5). If C is a third person who is sitting between A and B such that it divides the line AB in 1:3 ratio.  Based on the above information, answer the following questions. (i) Find the distance between A and B (ii) Find the equation of AB (iii)Find the coordinate of point C (iv) Find distance between A and C is	4
22.	Villages of Shanu and Arun's are 50km apart and are situated on Delhi Agra highway as shown in the following picture. Another highway YY' crosses Agra Delhi highway at O(0,0). A small local road PQ crosses both the highways at pints A and B such that OA=10 km and OB =12 km. Also, the villages of Barun and Jeetu are on the smaller high way YY'. Barun's village B is 12km from O and that of Jeetu is 15 km from O. 	4

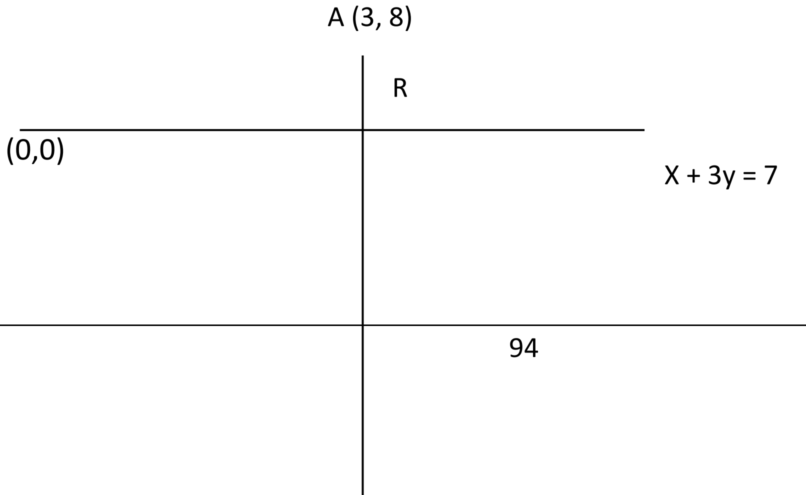
	(i) Find the coordinates of A (ii) Find the equation of line AB (iii) Find the distance of AB from O(0, 0) (iv) Find the slope of line AB	
23	A parking lot in an IT company is triangular shaped with two of its vertices at B(–2, 0) and C(1, 12). The third vertex A is at the midpoint of the line joining the points (1, 1) and (3, 11).  Based on the above information, answer the following questions. (i) Find the coordinates of A (ii) Find the equation of line parallel to BC and passing through the vertex A. (iii) Find the equation of line that passes through the points B(-2, 0) and C(1,12). (iv) Find the equation of line perpendicular to BC and passing through the vertex A.	4
24	Prove that the line through the point(x_1,y_1) are parallel to the line Ax+By+C=0 is A(x-x_1)+B(y-y_1)=0.	5
25	Find the image of the point(3,8) with respect to the line x+3y=7 assuming the line to be a plane mirror .	5

Question no.	Answers	Marks
1.	(C), (-2, -2) lies in the 3rd quadrant.	1
2.	(b), here θ = 60° Or, slope of the line = tan θ = tan60° = $\sqrt{3}$	1
3.	(d), we have AD = AB + BD Or, AD = AD/3 + BD Or, $\frac{2}{3}$AD= BD Or, AD : BD = 3: 2 D divides AB externally in the ratio 3 : 2 D(4 , -11)	1

	Coordinates of D are $(4, -11)$	
4.	Slope of the line passing through the points (6 , 3) and (-2 , 1) is $m = \frac{1-3}{-2-6} = \frac{-2}{-8} = \frac{1}{4}$	1
5.	Slope = 3 Therefore, $\frac{k-4}{-1-4} = 3$ or, $k - 4 = 9$ or, $k = -5$	1
6.	$m = \tan(-120°) = -\tan 120°$ $= -\tan(180° - 60°) = -[-\tan 60°] = \sqrt{3}$	1
7.	$c = -2$, $m = \tan 45° = 1$, equation is $y = 1x - 2$ Or, $x - y - 2 = 0$	1
8.	. x- intercept = 3, y-intercept = -4 Therefore, equation is $\frac{x}{3} + \frac{y}{(-4)} = 1$ or, $4x - 3y = 12$ $4x - 3y - 12 = 0$	1
9.	Line || to the y-axis is $x = k$ Here $k = \frac{-5}{2}$(left of the y-axis) Therefore, equation is $x = \frac{-5}{2}$	1
10.	(c) , equation is $5x + 6y = 7$ Slope = $-\frac{coefficient\ of\ x}{coefficient\ of\ y} = -5/6$	1
11.	(d), equation of the line is $\sqrt{3}x + 3y = 5$ Slope = $-\frac{coefficient\ of\ x}{coefficient\ of\ y} = -1/\sqrt{3}$ Let θ be the inclination of line with x-axis $m = \tan\theta = -1/\sqrt{3} = \tan\theta$ $\theta = 150°$	1
12.	.(a), equation of line is $\frac{x}{a} - \frac{y}{b} = 1$ Or, $bx - ay = ab$ Or, $bx - ay - ab = 0$ Therefore, distance of point (b, a) from line $bx - ay - ab = 0$ is $= \frac{\|b^2 + a^2 - ab\|}{\sqrt{a^2 + b^2}}$	1
13.	(c), lines are $x + 3y - 9 = 0$ ……….(i) $X + 3y + 1 = 0$ ……….(ii) We notice that the coefficients of x and y in (i) and (ii) are the same	1

	Therefore, Distance = $\left\| \frac{1-(-9)}{\sqrt{1+9}} \right\|$ = $\left\| \frac{10}{\sqrt{10}} \right\|$ = $\sqrt{10}$ units	
14.	(c), if lines are ||, slopes are equal Therefore, $\frac{-2}{3} = \frac{-S}{6}$ Or, S = 4.	1
15.	. Given equation is x + 2y – 4 = 0 Or, x + 2y = 4 Dividing by 4 we get $\frac{x}{4} + \frac{y}{2} = 1$ is the intercept form. Therefore, x-intercept = 4, y- intercept = 2	1
16.	let the required point be (α, 0), then the length of the perpendicular from(α, 0)on $\frac{x}{3} + \frac{y}{4} = 1$ or, 4x – 3y – 12 = 0 is 4 Therefore, $\left\| \frac{4\alpha - 3\times 0 - 12}{\sqrt{4^2+3^2}} \right\| = 4$ or, |4α – 12| = 201 Or, |α – 3| = 51 Therefore, α – 3 = +-5 or, α = 8, -2 Hence, the required points are (8,0) and (-2,0)	1
17.	Slope = $\frac{(k-3)}{4-k^2}$ if parallel to the x- axis Slope = 0 or, k – 3 = 0 or, k = 3.	1
18.	Slope of the line 2x + 3y + 4 + k(6x – y + 12) = 0 is $\left\{-\frac{(2+6k)}{3-k}\right\}$ and slope of the line 7x +5y – 4 = 0 is (-7/5). If the lines are perpendicular then $\left\{-\frac{(2+6k)}{3-k}\right\} \times \frac{(-7)}{5} = -1$ Or, 14 + 42k = -15 + 15k or k = $\frac{-29}{37}$	1
19.	the slope of Px + 3y = 4 is $m_1 = \frac{-P}{3}$ The slope of 3x – 4y = is $m_2 = \frac{3}{4}$ As the lines are perpendicular, $m_1 \times m_2 = -1$ $\frac{-P}{3} \times \frac{3}{4} = -1$ or P = 4	1
20.	If points A(a, 0), B(0, b) and C(3, 4) are collinear, then the slope of AB = slope of BC $\frac{b-0}{0-a} = \frac{4-b}{3-0}$	1

	Or, 3b = -4a + ab Or, 3b + 4a = ab or $\frac{3}{a}+\frac{4}{b}=1$	
21.	(i) $2\sqrt{5}$ (ii) X+2Y+4=0 (iii) $(3,-\frac{7}{2})$(iv) $\frac{\sqrt{5}}{2}$	4
22	(i)(a)(10, 0) (ii)(b)6x+5y=60 (iii)(b)$\frac{60}{\sqrt{61}}km$ (iv)(c) $-\frac{6}{5}$	4
23	(i)A is the mid-point of the points (1,1) and (3,11), By mid point formula A=$(\frac{1+3}{2},\frac{1+11}{2})$ Therefore, coordinates of A= (2, 6) (ii)Equation of line BC is $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$ y-0=$\frac{12-0}{1+2}(x+2) => 4x-y+8=0$ Equation of line parallel to BC and passes through (2,6) is y-6=4(x-2)=> 4x - y =2. (iii)Equation of line passes through two points B(-2, 0) and C(1, 12) is $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$ 4x-y-8=0. (iv) Equation of line perpendicular to BC and passes through (2,6) is y-6=- $\frac{1}{4}(x-2)$ x+4y=26.	
24	Slope of line Ax + By + C = 0 is $\frac{-A}{B}$ Therefore, slope of the parallel line = $\frac{-A}{B}$ Therefore, line through(x_1 , y_1) and parallel to given line is $y - y_1 = \frac{-A}{B}(x - x_1)$ Or, $A(x - x_1) + B(y - y_1) = 0$	5
25		



	B (α , β) Let B(α, β) be the image of A(3,8) in line x=3y=7 ∴ R is mid-point of AB R ([3+ α]/2, [8+ β]/2) R lies on x+3y=7 ∴ (3+α)/2+3× (8+ β)/2=7 Or, 3+α+24 +3β = 14 Or, α+3β=-13 …….. (i) Also,AB perpendicular line x+3y=7 ∴(β-8)/(α-3)×-1/3=-1 Or,β-8=3α-9 Or, 3α-β=1………….(ii) Solving (i) and (ii),we get α=-1,β=-4 ∴image is (-1,-4).	5

CONIC SECTION

Q. NO	QUESTION	MARK
1	The centre of the circle $x^2+y^2-2ax=0, a\neq 0$ (i) (0,0) (ii) (a,0) (iii) (-a,0) (iv) (0,a)	1
2	Radius of the circle $x^2+y^2+2x+2y-3=0$ is (i) $\sqrt{5}$ (ii) 1 (iii) $\sqrt{3}$ (iv) none	1
3	The radius of the circle $x^2+y^2-4x=0$, is (i) 1 (ii) 2 (iii) 4 (iv) $\sqrt{2}$	1
4	Vertex of the parabola $x^2=4ay$ is (i) $(a,0)$ (ii) $(-a,0)$ (iii) $(0,a)$ (iv) $(0,0)$	1
5	Focus of the parabola $y^2=4ax$	1

	(i) $(0,0)$ (ii) $(0,a)$ (iii) $(a,0)$ (iv) $(-a,0)$	
6	Length of latus rectum of a parabola $x^2 = 4ay$ is (i) a (ii) $2a$ (iii) $3a$ (iv) $4a$	1
7	Eccentricity of a parabola is (i) Less than 1 (ii) Equal to 1 (iii) Greater than 1 (iv) none	1
8	Eccentricity of ellipse is (i) Less than 1 (ii) Equal to 1 (iii) Greater than 1 (iv) none	1
9	Length of latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$ is (i) $4a$ (ii) $\frac{b^2}{a}$ (iii) $\frac{2b^2}{a}$ (iv) none	1
10	The equation of parabola with vertex (0,0) and focus $(-2,0)$ is (i) $y^2 = 8x$ (ii) $y^2 = -8x$ (iii) $y^2 = 4x$ (iv) $y^2 = -4x$	1
11	The length of major axis of ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$is (i) 6 (ii) 8 (iii) 12 (iv) 36	1
12	The length of conjugate axis of hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$, is (i) 8 (ii) 6 (iii) 4 (iv) 3	1
13	The length of latus rectum of the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$,is (i) $\frac{9}{4}$ (ii) $\frac{32}{3}$ (iii) $\frac{16}{3}$ (iv) $\frac{9}{2}$	1
14	The equation $4x^2+9y^2=36$ represents a/an	1

(i)	circle	(ii)	parabola
(iii)	ellipse	(iv)	hyperbola

15	The vertices of the hyperbola $\frac{y^2}{9} - \frac{x^2}{27} = 1$, is	1

(i)	$(0, \pm 3)$	(ii)	$(\pm 3, 0)$
(iii)	$(0, \pm 3\sqrt{3})$	(iv)	$(\pm 3\sqrt{3}, 0)$

16	The equation of directrix in the parabola $y^2 = -8x$ is	1

(i)	$x = 2$	(ii)	$x = -2$
(iii)	$y = 2$	(iv)	$y = -2$

17	The equation of circle passing through (2,3) with centre (h,k) and radius r is given by	1

(i)	$(x-h)^2 + (y-k)^2 = r^2$	(ii)	$(x-2)^2 + (y-3)^2 = r^2$
(iii)	$(2-h)^2 + (3-k)^2 = r^2$	(iv)	none

18	The focus of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$,is	1

(i)	$(0, \pm 2)$	(ii)	$(0, \pm 3)$
(iii)	$(0, \pm\sqrt{5})$	(iv)	$(0, \pm\sqrt{13})$

19	The focus of hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$, is	1

(i)	$(\pm\sqrt{7}, 0)$	(ii)	$(\pm 5, 0)$
(iii)	$(\pm 4, 0)$	(iv)	$(\pm 3, 0)$

20	He equation of directrix of the parabola of the form $x^2 = 4ay$ is	1

(i)	$x = a$	(ii)	$x = -a$
(iii)	$y = a$	(iv)	$y = -a$

21.	An arch is in the form of semi ellipse. It is 8 m wide and 2 m high at the centre. The equation of semi ellipse is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.	4

Answer the following questions:

(i) What is the length of semi major axis?

(ii) What is the length of semi minor axis?

(iii) Write the equation of ellipse.

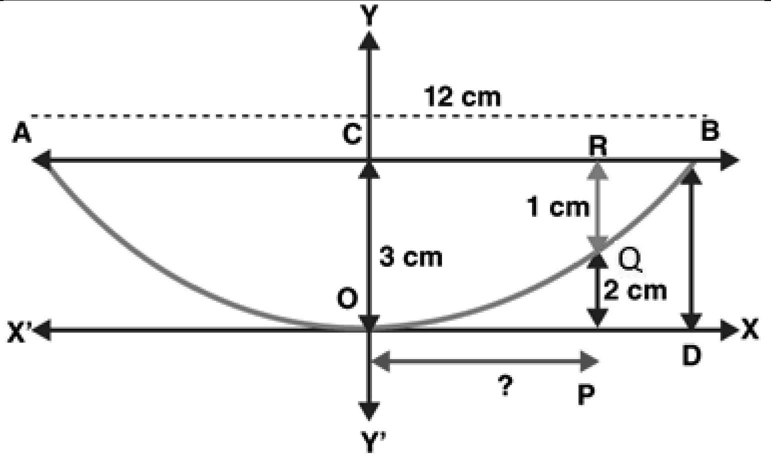
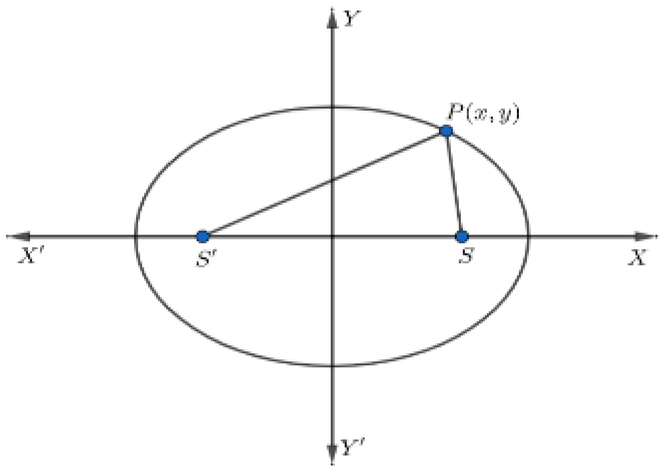
OR

Find the height of the arch at a point 1.5 m from the end.

Q. NO	ANSWER	MARKS
1	(ii) (a,0) because the equation of circle is written as $(x-a)^2+(y-0)^2 = a^2$	1
2	(i)$\sqrt{5}$ because the equation of circle is $(x+1)^2+(y+1)^2 = 5 = (\sqrt{5})^2$	1
3	(ii)2 because the equation of circle is $(x-2)^2+(y-0)^2 = 4 = 2^2$	1
4	(iv) $(0,0)$	1
5	(iii) $(a, 0)$	1
6	(iv)$4a$	1
7	(ii)Equal to 1	1
8	(i) Less than 1	1
9	(iii)$\frac{2b^2}{a}$	1
10	(ii)$y^2 = -8x$, since, $a = -2$, and equation is given by$y^2 = 4ax$	1
11	(iii)12 , The length of major axis = $2a$, here $a = 6$	1
12	(i)6 , length of conjugate axis = 2b, b = 3	1
13	(iv)length = $\frac{2b^2}{a} = \frac{2\times 9}{4}$, $a = 4, b = 3$	1
14	(iii)ellipse The equation dividing by 36 reduces to $\frac{x^2}{9} + \frac{y^2}{4} = 1$	1
15	(i)$(0, \pm 3)$, The vertices are $(0, \pm a)$, a = 3	1
16	(i) $x = 2$, here, $focus\ is(-2,0)$	1
17	(iii)$(2-h)^2+(3-k)^2 = r^2$,since,$(x-h)^2+(y-k)^2 = r^2, here\ x = 2, y = 3$	1
18	(iii)$(0, \pm\sqrt{5})$, The focus of given ellipse is $(0, \pm c)$, where $b^2 + c^2 = a^2$	1
19	(ii)$(\pm 5,0)$, The focus of given hyperbola $(\pm c, 0)$, where $a^2 + b^2 = c^2$	1
20	(iv) $y = -a$, As,focus is $(0, a)$	1
21	(i)Length of major axis = 2a = 8, a = 4, (ii)Length of minor axis = 2b = 4, b = 2, (iii)equation of ellipse = $\frac{x^2}{16} + \frac{y^2}{4} = 1$	1 1 2

	OR $\frac{x^2}{16}+\frac{y^2}{4}=1 \Rightarrow \frac{(4-1.5)^2}{16}+\frac{y^2}{4}=1 \Rightarrow \frac{6.25}{16}+\frac{y^2}{4}=1 \Rightarrow y^2=2.4375, y=$ $1.56\ (approx.)$ So, required height = 1.56 m.	2
22	(i) The relation between a , b and c in ellipse is given by $b^2+c^2=a^2$ (ii) The length of major axis = 2a = 10 Therefore , a = 5 (iii) $b^2+c^2=a^2 \Rightarrow b^2=a^2-c^2$, Given 2c = 8 , c = 4 $Hence, b^2=9 \Rightarrow b=3$ OR Required equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$	1 1 2 2

Q. NO	QUESTION	Mark
1	Find the centre and radius of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$	2
2	Given the ellipse with equation $9x^2 + 25y^2 = 225$, find the eccentricity and foci.	2
3	Find the equation of the parabola which is symmetric about the y-axis, and passes through the point (2, -3).	2
4	Find the eccentricity of the conic $9x^2 – 16y^2 = 144$.	2
5	Find the focus, vertex and equation of directrix of the parabola $y^2 = x$	3
6	Find the equation of the ellipse , whose length of the major axis is 20 and foci are $(0 , \pm 5)$	3
7	Find the equation of the hyperbola where foci are $(0 ,\pm12)$ and the length of the latus rectum is 36.	3
8	Find the equation of the circle whose radius is 5 and which touches the circle $x^2 +$ 4y -20 = 0 exexteexternally at the point (5,5).	3
9	A beam is supported at its ends by supports which are 12 metres apart. Since the load is concentrated at its centre, there is a deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola. How far from the centre is the deflection 1 cm ?	4

<table>
<tr><td></td><td>
</td><td></td></tr>
<tr><td>10</td><td>A man running a racecourse notes that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m. Find the equation of the posts traced by the man.

</td><td>4</td></tr>
<tr><td>11</td><td>Find the equation of the circle which passes through the points (3,7) ,(5,5) and has its centre on the line $x – 4y = 1$.</td><td>5</td></tr>
<tr><td>12</td><td>Find the equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13.</td><td>5</td></tr>
<tr><td>13</td><td>An arc is in the form of a semi-ellipse. It is 8 m wide and 2 m high at the centre. Find the height of the arc at a point 1.5 m from one end.</td><td>5</td></tr>
<tr><td>14</td><td>An equilateral triangle is inscribed in the parabola $y^2 = 4ax$ whose vertex is at the vertex of the parabola .Find the length of its side.</td><td>5</td></tr>
<tr><td>15</td><td>Find the equation of the circle which passes through the points (5 , - 8) , (2 , - 9) and (2 ,1). Find also the coordinates of its centre and radius.</td><td>5</td></tr>
</table>

ANSWERS:

Q. NO	ANSWER	MARKS
1	Centre (-4 , -5) and radius 7 units	2
2	Eccentricity = 4/5 , foci ($\pm$ 4, 0)	2
3	$3x^2 = -4y$	2
4	5/4	2
5	Vertex (0,0) , focus ($\frac{1}{4}$, 0) and equation of directrix , x = - $\frac{1}{4}$	3
6	$\frac{x^2}{75} + \frac{y^2}{100} = 1$	3
7	$3y^2 - x^2 = 108$	3
8	$(x-9)^2 + (y-8)^2 = 5^2$, centre (9,8) , radius = 5	3
9	$2\sqrt{6}$ metres	4
10	$\frac{x^2}{25} + \frac{y^2}{9} = 1$	4
11	$x^2 + y^2 + 6x + 2y = 90$	5
12	$25x^2 - 144y^2 = 900$	5
13	$\frac{\sqrt{39}}{4}$ m	5
14	$8a\sqrt{3}$	5
15	$x^2 + y^2 - 4x + 8y - 5 = 0$, centre (2 , - 4) and radius = 5	5

3-DIMENSIONAL GEOMETRY

SL NO	QUESTION	Marks
1	Which octant do the point (−5,4,3) lie? A. Octant I B. Octant II C. Octant III D. Octant IV	1
2	What is the distance between the points (2, –1, 3) and (–2, 1, 3)?	1

	A. $2\sqrt{5}$ units B. 25 units C. $4\sqrt{5}$ units D. $\sqrt{5}$ units	
3	The locus represented by $xy + yz = 0$ is: (a) A pair of perpendicular lines (b) A pair of parallel lines (c) A pair of parallel planes (d) A pair of perpendicular planes	1
4	Find the image of $(-2,3,4)$ in the y z plane. A. (-2, 3, 4) B. (2, 3, 4) C. (-2, -3, 4) D. (-2, -3, -4)	1
5	The distance of the point P(a, b, c) from the x-axis is: (a) $\sqrt{(a^2 + c^2)}$ (b) $\sqrt{(a^2 + b^2)}$ (c) $\sqrt{(b^2 + c^2)}$ (D) none of these	1
6	The maximum distance between points $(3\sin\theta, 0, 0)$ and $(4\cos\theta, 0, 0)$ is: (a) 3 units (b) 4 units (c) 5 units	1

	(d) Cannot be determined	
7	The plane 2x - (1+a)y + 3az = 0 passes through the intersection of the planes (A) 2xy = 0 and y + 3z = 0 (B) 2x - y = 0 and y-3z = 0 (C) 2x + 3z = 0 and y = 0 (D) 2x - 3z = 0 and y = 0	1
8	The locus of a point which moves so that the difference of the squares of its distances from two given points is constant, is a (a) Straight line (b) Plane (c) Sphere (d) None of these	1
9	Three planes x + y = 0, y + z = 0, and x + z = 0 (a) none of these (b) meet in a line (c) meet in a unique point (d) meet taken two at a time in parallel lines	1
10	The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinatesof A and B are (3, –5, 7) and (–1, 7, – 6), respectively, Then the coordinates of thepoint C. (a) (1 , 1 , 2) (b) (1 , 0 , 1) (c) (1 ,2 ,3) (d) (0 ,0 , 2)	1
11	YOZ-plane divides the line segment joining the points $(3,-2,-4)$ and $(2,4,-3)$ in the ratio- a) $1:2$ b) $-4:3$ c) $-2:3$	1

	d) $-3:2$	

ANSWER

1	B	1
2	A	1
3	D	1
4	B	1
5	C	1
6	C	1
7	B	1
8	C	1
9	C	1
10	A	1
11	d	1

SL NO	QUESTION	Marks
1	Find the distance of the point (3,4,5) from the origin .	2
2	Find the coordinate of the image of the point (1,3,-6) in YOZ plane	2
3	Find the distance between the points (-1,4.0) and (3,6,1)	2
4	Find the value of p such that the distance between the points (4,5,p) and (7,1,-3) is 13	2
5	.Find the coordinate of the foot of the perpendicular from the point (4,-3,5) on the Xaxis.	2
6	Find a point on x axis whose distance from the point (-1,4,2) is 3 $\sqrt{5}$	3
7	A is a point (1.3.4) and B is a point (1,-2,-1). A point P moves so that 3PA = 2PB . Find the locus of the point P	3
8	Show that (0,7,-10) ,(1,6,-6) and (4,9,-6 are the vertices of an isosceles triangle.)	3
9	Find the centroid of the triangle mid point of whose sides are (1,2,-3) (3,0,1) and (-1,1,-4)	3
10	Show that if $x^2 + y^2$ =1 .then the point (x. y.$\sqrt{1 - x^2 - y^2}$) is at a distance of 1 unit from the origin	3
11	Show that the points (-2,6,-2) (0,4,-1) (-2,3,1) and (-4,5,0)form the vertices of a square	3
12	Find the coordinate of a point equidistant from the four points O (0,0,0) ,A(l,0,0) B(0,m,0) and C(0,0,n)	3
13	A boy is standing at point O and observe three kites A, B and C in space. Taking O as origin if the coordinates of three kites A, B and C are (3,4,5), (1, 3, 4) and (2,-1,4) respectively, then (i) Find the distance between kites A and B	4

	(ii) Find the coordinates of a point on the y-axis which is at a distance of √35 units from kite A (iii) Find the coordinates of point D so that ABCD is a parallelogram (iv) If the points (0,-1,-7), (2, 1-9) and (6,5.-13) represent kites A, B and C then check whether the kites are collinear or not.	
14	The mid-points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2, 3, -1). Find the coordinates of the vertices of the triangle.	5

ANSWERS

1	distance of the point (3,4,5) from the origin $\sqrt{3^2+4^2+5^2}$ = $\sqrt{50}$ units	2
2	For image in YOZ plane x coordinate will change its sign. So. image= (-1,3,-6)	2
3	Let the points beA (-1,4.0) and B (3,6,1) So, AB =$\sqrt{(3+1)^2+(6-4)^2+(1-0)^2}$ =$\sqrt{21}$ units	2
4	Here $\sqrt{(4-7)^2+(5-1)^2+(p+3)^2}$ =13 On solving we get p=-15 ,9	2
5	foot of the perpendicular from the point (4,-3,5) on the Zaxis is lie on Z axis omly. So. Required foot of the perpendicular is (0,0,5)	2
6.	Let the point on the xaxis be (x.0.0) distance between the points (-1,4,2) from (x,0,0) = 3 $\sqrt{5}$ => $\sqrt{(x+1)^2+(0-4)^2+(0-2)^2}$ = 3 $\sqrt{5}$ On solving we get x-1 = ± 5 $=> x = 4 \, or - 6$ Therefore the points are (4,0,0)and (-6,0,0)	3
7	Let P (x.y.z) be any point on the locus and it is given that 3PA=2PB 3$\sqrt{(X-1)^2+(Y-3)^2+(Z-4)^2}$ =2$\sqrt{(X-1)^2+(Y+2)^2+(Z+1)^2}$ On squaring and solving we get $5x^2+5y^2+5z^2-10x-70y-80z+210=0$,which is the required locus	3
8	Let A(0,7,-10) ,B(1,6,-6) andC (4,9,-6) be the given points. By using distance formula we get AB = $\sqrt{1+1+16}$ =$\sqrt{18}$ BC=$\sqrt{9+9+0}$ =$\sqrt{18}$ Thus we get AB=BC HenceΔ ABC is an isosceles triangle	3
9	We know that centroid of the triangle formed by joining the mid points of given triangle coincide with the centroid of the original triangle. Hence the coordinate of the centroid is ($\frac{1+3-1}{3}$,$\frac{2+0+1}{3}$,$\frac{-3+1-4}{3}$) =(1,1,-2)	3
10	Distance of the point ((x.y.$\sqrt{1-x^2-y^2}$) from the origin	3

	$= \sqrt{(x-0)^2 + (y-0)^2 + \left(\sqrt{1-x^2-y^2} - 0\right)^2}$ $= \sqrt{x^2 + y^2 1 - x^2 - y^2}$ =1	
11	Let the points be A(-2,6,-2), B (0,4,-1), C (-2,3,1) andD (-4,5,0) To prove that ABCD IS A Square. Using distance formula we get AB = $\sqrt{(0+2)^2 + (4-6)^2 + (-1+2)^2} = \sqrt{9} = 3$ BC=$\sqrt{(-2-0)^2 + (3-4)^2 + (1+1)^2}$ =$\sqrt{9}$ =3 CD=$\sqrt{(-4+2)^2 + (5-3)^2 + (0-1)^2}$ =$\sqrt{9}$ =3 DA=$\sqrt{(-4+2)^2 + (5-6)2 + (0+2)^2}$ =$\sqrt{9}$ =3 Thus we get AB=BC=CD=DA Also, AC= =$\sqrt{18}$ and BD= $\sqrt{18}$ Thus we get AB=BC=CD=DA AND AC=BD Hence ABCD is a square	3
12	Let P (x,y,z) be the required point. Then OP=PA=PB=PC Now OP=PA $=> x^2 + y^2 + z^2 = (X-l)^2 + (Y-0)^2 + (Z-0)^2$ $=> X = \frac{l}{2}$ Similarly OP=PB $=> Y = \frac{m}{2}$ and OP=PC $=> Z = \frac{N}{2}$ Hence three coordinate of the required point is (l/2, m/2, n/2)	3
13	(i) 3√2 units (ii) (0,5,0) (iii)(6, 0,5) (iv) Collinear	4
14	Let A(x^1, y^1, z^1) , B(x^2 , y^2 , z^2) and C(x^3 , y^3 , z^3) be the vertices of ΔABC , such that D(5, 7, 11) E(0, 8, 5) and F(2, 3, - 1) be the mid-points of the sides BC, CA and AB respectively. The vertex A is (-3, 4, -7). The vertex B is (7, 2, 5). The vertex C is (3, 12, 17).	5

STATISTICS

Q. NO	QUESTION	MARK
1	Range of a data is equal to: a) Range = Max value – Min value b) Range = Max value + Min value c) Range = (Max value - Min value)/2 d) Range = (Max value + Min value)/2	1

2	If the variance of a data is 121, then the standard deviation of the data is: a) 121 b) 11 c) 12 d) 21	1
3	Relation between mean, median and mode is given by: a) Mode = 2 Median – 3 Mean b) Mode = 2 Median + 3 Mean c) Mode = 3 Median – 2 Mean d) Mode = 3 Median +2 Mean e)	1
4	The geometric mean of a series having arithmetic mean = 25 and harmonic mean = 16 is: a) 16 b) 20 c) 25 d) 30	1
5	The coefficient of variation is computed by: a) $\frac{S.D}{MEAN} \times 100$ b) $\frac{S.D}{MEAN}$ c) $\frac{MEAN}{S.D} \times 100$ d) $\frac{MEAN}{S.D}$	1
6	If the mean of first n natural numbers is $\frac{5n}{9}$, then $n =$ a) 5 b) 4 c) 9 d) 10	1
7	The sum of 10 items is 12 and the sum of their squares is 18. Then the standard deviation is: a) 1/5 b) 2/5 c) 3/5 d) 4/5	1
8	The algebraic sum of the deviation of 20 observations measured from 30 is 2. So, the mean of observations is: a) 30.0 b) 30.1 c) 30.2 d) 30.3	1
9	The median and SD of a distribution are 20 and 4 respectively. If each item is increased by 2, the new median and SD are: a) 20, 4 b) 22, 6 c) 22, 4 d) 20, 6	1

10	If Mean = Median = Mode, then it is a) Symmetric distribution b) Asymmetric distribution c) Both Symmetric distribution and Asymmetric distribution d) None of these	1
11	If one of the observations is zero then the geometric mean is: a) (Sum of observations)/2 b) (Multiplying of all observations)^{n} c) (Multiplying of all observations)$^{1/n}$ d) 0	1
12	Which one is the measure of dispersion method: a) Range b) Quartile deviation c) Mean deviation d) All of above	1
13	if the variation is V and standard deviation σ then a) $\sigma^2 = V$ b) $V^2 = \sigma$ c) $V = 1/\sigma$ d) $V\sigma = \pm 1$	1
14	The mean deviation with respect to median of the observations 5, 5, 5, 5, 5, 5, 5, 5 is a) 5 b) 0 c) 1 d) Equal to mean	1
15	If the two variables x and y are connected by $y = a + bx$, then a) $MD(\bar{y}) = \|a\|MD(\bar{x})$ b) $MD(\bar{y}) = \|b\|MD(\bar{x})$ c) $MD(\bar{y}) = MD(\bar{x})$ d) $MD(\bar{y}) = a + b.MD(\bar{x})$	1
16	If σ is the standard deviation of a variable X, then the standard deviation of $\frac{1}{c}(a + bX)$ is a) $\left\|\frac{a}{c}\right\|\sigma$ b) $\frac{a}{b}\sigma$ c) $a\sigma + b$ d) $\frac{1}{c}\sigma$	1
17	The standard deviation of a data will be minimum if it is determined respect to a) Mean b) Median c) Least observation d) None of above	1
18	Let x and y are two variables connected by 3y+4x = 8and range of x is 6, then the range of y is a) 4 b) 8 c) 10	1

	d) 12	
19	The standard deviation of the observations of a variable is 4. If 24 is subtracted from each observation, then the standard deviation of the obtained will be a) 4 b) -20 c) 20 d) 6	1
20	Let x and y are two variables connected by 7x + 8y = 56 and $MD(\bar{x}) = 4$, then $MD(\bar{y}) =$ a) 4.5 b) 7 c) 3.5 d) 32.5	1
21	The standard deviation of the observations of a variable is 2.7. If 1.5 is added to each observation, then the standard deviation of the obtained will be a) 1.5 b) 3 c) 4.2 d) 2.7	1
	CASE STUDY BASED- (4 Marks)	
1	Arya is doing one of his projects. For this he asked shoe size of 10 of his class-mates which are as follows: 6, 5, 5, 6, 8, 6, 7, 7, 8, 8  Based on the above information answer the following: a) What would be the mean shoe size for the data? i)6 ii) 6.25 iii) 6.6 iv) 7 b) What would be the median for the data? i)5 ii) 6 iii) 6.5 iv) 7 c) What would be the mean deviation respect to mean for the data?	4

	i)0.5 ii) 0.6 iii) 1 iv) 1.5 d) What would be the mean deviation respect to median for the data? i)0.75 ii) 1 iii) 1.25 iv) 6.5 e) Different measure of dispersion we studied are .... i)Range ii) Mean deviation iii) standard deviation iv) all of above	
2	For a group of 200 candidates, the mean and standard deviation of scores were found to be 40 and 15 respectively. Later on, it was discovered that the scores of 43 and 35 were misread as 34 and 53, respectively.	4

Student	Eng	Hind	Social sc.	Science	Maths
Aditya	39	59	84	80	41
Pramit	79	92	68	38	75
Sumit	41	60	38	71	82
Arya	71	77	87	75	42
Shivam	72	65	69	83	67
Anupam	46	96	53	71	39

Based on the above information answer the following:

a) Find the sum of correct scores.
i)7991 ii) 8000 iii) 8550 iv) 6572

b) Find the correct mean.

i)42.924 ii) 39.995 iii) 38.423 iv) 41.621

c) The formula of variance is

i) $\sum_{i=1}^{n}\frac{(x_i-\bar{x})^2}{n}$ ii) $\sum_{i=1}^{n}(x_i-\bar{x})^2$ iii) $\sum_{i=1}^{n}\frac{(x_i-\bar{x})^2}{f_i}$

iv) $\sum_{i=1}^{n} f_i(x_i-\bar{x})^2$

d)Find the correct variance

i) 280.3 ii) 235.6 iii) 224.143 iv) 226.521

e) Find the correct standard deviation

i) 14.971 ii) 11.321 iii) 16.441 iv) 12.824

Q. NO	QUESTION	MARK
1	If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.	2
2	Suppose that mean and standard deviation of 6 observations are 8 and 4 respectively . Let each observation be multiplied by 3. Find the new standard deviation of the resulting observations.	2
3	The scores of batsman A are 40 ,70,52,34,42,55,63,46,54,44. Find their variance.	2
4	Suppose we have 10 students in a class and the marks scored by them in a Mathematics test are 12, 14, 18, 9, 11, 7, 9, 16, 19, and 20 out of 20. Then find the mean deviation w.r.t the average value scored by the student in the class.	2
5	The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are 2, 4, 10, 12,14, find the remaining two observations.	3
6	Find the mean deviation about the mean for the data .	3

xi	5	10	15	20	25
fi	7	4	6	3	5

Q. NO	QUESTION	MARK
7	Find the mean and variance for the following frequency distribution.	3

Classes	Frequencies
0-30	2
30-60	3
60-90	5
90-120	10
120-150	3
150-180	5
180-210	2

Q. NO	QUESTION	MARK
8	A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only 32, 35, 37, 30, 33, 36, 35 and 37 pages. Find the standard deviation of the pages yet to be completed by them.	3

9

Descriptive Statistics for Salary Data of Assistant Professors across Gender and Division

Division	Female M	Female SD	Female N	Male M	Male SD	Male N	Overall M	Overall SD	Overall N
Humanities	49,838.52	9,224.04	24	45,796.33	8,719.61	24	48,736.10	9,138.01	48
Natural Sciences	56,365.24	6,898.76	21	54,213.14	5,871.54	28	55,135.47	6,354.56	49
Business	61,250.00	9,148.08	3	59,424.77	7,754.28	15	59,728.97	7,736.50	18
Overall	53,407.17	8,932.45	48	54,259.74	8,195.85	67	53,850.51	8,824.46	115

A]Which stream/division has most varied salary for female professors?

B]Which stream/division has most varied salary for female professors than overall salary variation?

C]Find the minimum amount of difference of SD of male and female professors and the division ?

4=1+1 +2

10

200 candidates marks in Chemistry with the mean and standard deviation 10 and 15 respectively. After that if was found that the scale 43 was misread as 34.

I] What is the correct total marks of 200 candidates ?

Ii] Find the correct mean?

III] Calculate the wrong sum of squares of marks?

IV]Calculate the correct standard deviation

4=1+1 +1+1

11	Find the mean deviation about median for the following data : Heights (in cm) 95-105 105-115 115-125 125-135 135-145 145-155 Number of Girls. 9 15 23 30 13 10	5
12	The yield of wheat and rice per acre for 10 districts of a state is as under: District 1 2 3 4 5 6 7 8 9 10 Wheat 12 10 15 19 21 16 18 9 25 10 Rice 22 29 12 23 18 15 12 34 18 12 Calculate for each crop, (i) Range (ii) Mean Deviation about Mean (iii)Which crop has greater Standard Deviation?	5
13	Find the mean deviation (M.D) from the mean and the standard deviation (S.D) of the A.P. a, a + d, a + 2 d,......,a + 2n.d	5
14	Suppose that mean and standard deviation of 6 observations are 8 and 4 respectively. On checking it is found that observation 3 was as wrong insert. Find the new standard deviation of the observations if I]the wrong value is removed II] the wrong value is changed by 11.	5
15	Calculate the mean deviation from the mean of the following data:	5

Class	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	4	6	10	20	10	6	4

Q. NO	ANSWER	MARKS
1	Range = Max value – Min value (a)	1

2	Variation=$V = \sigma^2 = 121$ Req. S.D = $\sigma = \sqrt{121} = 11$ (b)	1
3	Mode = 3 Median – 2 Mean (c)	1
4	Relation among A.M, G.M, H.M: $GM^2 = AM \times HM$ $GM^2 = 16 \times 25$, $GM = \sqrt{16 \times 25} = 20$ (b)	1
5	$\frac{S.D}{MEAN} \times 100$,$MEAN \neq 0$ (a)	1
6	Mean=$\frac{1+2+3.....+n}{n}$ $$\frac{n(n+1)}{2n} = \frac{5}{9}$$ $$9n + 9 = 10n$$ $n = 9$ (c)	1
7	$$\sum x = 12, \qquad \sum x^2 = 18$$ So, $\sigma^2 = V = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{18}{10} - \left(\frac{12}{10}\right)^2 = 9/25$ $\sigma = \sqrt{9/25} = 3/5$ (c)	1
8	$$\sum (x_i - 30) = 2$$ $$\sum x_i - 30 \times 20 = 2$$ $$\frac{\sum x_i}{20} - 30 \times \frac{20}{20} = \frac{2}{20}$$ $Mean - 30 = 0.1$ $$Mean = 30.1$$	1
9	If each observation is increased by 2, new median = 20+2 =22 But σ will remain same as 4 as variance is independent respect to change of origin. 22, 4 (c)	1
10	Mean = Median = Mode, in a symmetric distribution (a)	1
11	The G.M of 0, 1, 2, 3, 4, ……., n = $(0 \times 1 \times 2 \times 3 \ldots \ldots . \times n)^{1/n} = 0$(d)	1
12	All of above(d)	1
13	$V = \sigma^2$ (a)	1
14	Median = 5	1

	Mean deviation w.r to median for each observation = $\|5-5\|=0$ Req. M.D =0 (b)	
15	$y=a+bx$ implies $\bar{y}=a+b\bar{x}$ implies $MD(\bar{y})=\|b\|MD(\bar{x})$ (b)	1
16	Let, $u=\frac{1}{c}(a+bX)$ $u=\frac{a}{c}+\frac{b}{c}X$ $\sigma_u=\left\|\frac{b}{c}\right\|X$ (a)	1
17	standard deviation of a data will be minimum respect to Median (b)	1
18	$3y+4x=8\rightarrow y=\frac{8}{3}-\frac{4}{3}x \rightarrow range(y)=\left\|-\frac{4}{3}\right\|range(x)=\frac{4}{3}\times 8=6$ (b)	1
19	New observations u_i can be written as $u_i=x_i-4$ $\sigma_u=\sigma_x=4$ i.e standard deviation will be remained same (a)	1
20	$7x+8y=56\rightarrow 8y=56-7x$ y =56/8-7/8x $MD(\bar{y})=\|-7/8\|MD(\bar{x})$ $MD(\bar{y})=7/8\times 4=3.5$ (c)	1
21	$u_i=x_i+5$ $\sigma_u=\sigma_x=2.7$ i.e standard deviation will be remained same (d)	
CASE STUDY BASED (4)-Answer		
1	Given data: 6, 5, 5, 6, 8, 6, 7, 7, 8, 8 a) Req. mean =m =$\frac{6+5+5+6+8+7+7+8+8}{10}=\frac{6.6}{10}=6.6$ (iii) b) To find median arranging the numbers in increasing order: 5, 5, 6, 6, **6, 7**,7, 8,8, 8 Req. median = M=$\frac{6+7}{2}=\frac{13}{2}=6.5$ (iii) c) Obtained mean = m = 6.6 Req mean deviation about mean $=\frac{\sum\|x_i-m\|}{n}=\frac{(0.6\times3)+(1.6\times2)+(0.4\times2)+(1.4\times3)}{10}=\frac{10}{10}=1$ (iii) d) Obtained median = M = 6.5 Req mean deviation about median $=\frac{\sum\|x_i-M\|}{n}=\frac{(0.5\times3)+(1.5\times2)+(0.5\times3)+(1.5\times3)}{10}=$	4

	$\frac{10}{10} = 1$ (ii) e) All of above	
2	We have, n = 200, incorrect mean = 40 and incorrect standard deviation = 15. Now, incorrect mean = 40 $\Rightarrow \frac{incorrect\ \sum x_i}{200} = 20$ $\Rightarrow incorrect\ \sum x_i = 800$ $\boldsymbol{\Rightarrow correct\ \sum x_i = 8000 - (43+35) + (34+53) = 7991}$ **(i)** **So, correct mean =** $\boldsymbol{\frac{7991}{200} = 39.955}$ **(ii)** And incorrect SD = 15 $\Rightarrow$ incorrect variance = $15^2 = 225$ $\Rightarrow \frac{incorrect\ \sum x_i^2}{200} - (incorrect\ mean)^2 = 225$ $\Rightarrow \frac{incorrect\ \sum x_i^2}{200} - (40)^2 = 225$ $\Rightarrow incorrect\ \sum x_i^2 = 200(225 + 1600) = 365000$ $\Rightarrow$ correct $\sum x_i^2 = incorrect \sum x_i^2 - (34^2 + 53^2) + (43^2 + 35^2) = 36500 - 3965 + 3074 = 364109$ So, **correct variance** = $\frac{1}{200}$correct $\sum x_i^2 - (correct\ mean)^2 = \frac{1}{200}(364109) - \left(\frac{799}{200}\right)^2 = 1820.545 - 1596.402$ $= \mathbf{224.143}$ **(iv)** So, c**orrect SD**= $\boldsymbol{\sqrt{224.143} = 14.971}$ **(i)** **The correct formula of the variance =** $\boldsymbol{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n}}$ **(i)**	4

Q. NO	ANSWER	MARKS
1	Range = Greatest value-Lowest value	1

	So 36.8=Greatest value-13.4 So Greatest value=36.8+13.4=50.2	½ ½
2	Here no of value n=6, mean M =8, Standard deviation (S.D)=4 If each value is multiplied by 3 , New Standard deviation is 3 * 4=12.	2
3	Scores xi = 40 ,70,52,34,42,55,63,46,54,44. Here n=10. Mean M= (40+70+52+34+42+55+63+46+54+44)/10=500/10=50 So standard deviation $=\sqrt{[(40^2+70^2+52^2+34^2+42^2+55^2+63^2+46^2+54^2+44^2)/10 - 50^2]}$ $=\sqrt{[1600+4900+2704+1156+1764+3025+3969+2116+2916+1946)/10-2500]}$ $=\sqrt{[2609.6-2500]}=\sqrt{109.6}=10.5$ approximately.	½ ½ ½ ½
4	The average value scored by the student in the class is, Mean (Average) = (12 + 14 + 18 + 9 + 11 + 7 + 9 + 16 + 19 + 20)/10 = 135/10 = 13.5 Then, the average value of the marks is 13.5 Mean Deviation = {|12-13.5| + |14-13.5| + |18-13.5| + |9-13.5| + |11-13.5| + |7-13.5| + |9-13.5| + |16-13.5| + |19-13.5| + |20-13.5|}/10 = 34.5/10 = 3.45	1 1
5	The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are 2, 4, 10, 12,14. Let remaining two observations are x, y. So 2+4+10+12+14+x+y=56 Thus x+y=14…..(1) Again $2^2+4^2+10^2+12^2+14^2+x^2+y^2=(16+8^2)7=560$ then $x^2+y^2= 100$….(2) Solving (1) and (2) x=8 ,y=6 .	½ 1 ½ 1 ½ 1 ½
6	(table below) Mean=350/25=14 Mean deviation about the mean=158/25=6.32	1 1 ½ ½

xi	5	10	15	20	25	total
fi	7	4	6	3	5	25
Xi fi	35	40	90	60	125	350
Ixi-14Ifi	63	16	6	18	55	158

7 (Marks: 2)

Classes	Frequencies fi	xi	xi fi	Xi²fi
0-30	2	15	30	450
30-60	3	45	135	6075
60-90	5	75	375	28125
90-120	10	105	1050	110250
120-150	3	135	405	54675
150-180	5	165	825	136125
180-210	2	195	390	76050
TOTAL	30		3210	514125

Mean=3210/ 30=107 — ½

Variance=514125/30-107^2=5688.5 — ½

8

Xi [no of remaining page]	28	25	23	30	27	24	25	23	Total
fi	1	1	1	1	1	1	1	1	8
Xi fi	28	25	23	30	27	24	25	23	205
xi^2 fi	784	625	529	900	729	576	625	529	5297

Marks: 1, 1

standard deviation of the pages yet to be completed by them is $\sqrt{[5297/8-\{205/8\}^2]}=\sqrt{5.725}=2.39$ — ½, ½

9

A]Humanities — 1

B]All — 1

C]Humanities — 1

9224.04-8719.61=504.43 — 1

10

200 candidates marks in Chemistry with the mean and standard deviation 10 and 15 respectively. After that if was found that the scale 43 was misread as 34.

I] Correct total marks of 200 candidates =2000-34+43=2009 — 1

Ii] The correct mean=2009/200=10.045 — 1

III] The wrong sum of squares of marks=$[15^2+10^2]200=65000$. — 1

IV]The correct standard deviation=$\sqrt{[\{65000-34^2+43^2\}/200-10.045^2]}=15.1$ — 1

11 (Marks: 2)

Class	Frequency fi	Cmf	Class mark xi	xi -Me	[xi-Me]fi	Ixi-MeI fi
95-105	9	9	100	-26	-234	234

105-115	15	24	110	67.14	1007.1	1007.1
115-125	23	47	120	77.14	1774.22	1774.22
125-135	30	77	130	87.14	2614.2	2614.2
135-145	13	90	140	97.14	1262.82	1262.82
145-155	10	100	150	107.14	1071.4	1071.4
Total	100					7963.74

1

N=100
so l=125
F=47
f=30
d=10 1
Me=l+[N/2-F]d/f =126 1

Thus Mean deviation w.r.t Me =7963.74/100=79.6374

12 (i) Range

a. Wheat

Range = H – L

= 25 – 9 ½

=16

b. Rice

Range = H – L

=34 – 12 ½

= 22

(ii) Mean Deviation about Mean ½

Mean : Wheat has Mean (12+10+15+19+21+16+18+9+25+10)/10=16.5 ½

Rice has mean (22+29+12+23+18+15+12+34+18+12)/10=19.5

So mean deviation about mean are:

Wheat

M.D(M)={|12-16.5| + |10-16.5| + |15-16.5| + |19-16.5| + |21-16.5| + |16-16.5| + |18-16.5| + |9-16.5| +|25-16.5|+ |10-16.5|}/10=44/10=4.4 ½

Rice ½

M.D(M)={|22-19.5| + |29-19.5| + |12-19.5| + |23-19.5| + |18-19.5| + |15-19.5| + |12-19.5| + |34-19.5| +|18-19.5|+ |12-19.5|}/10=54/10=5.4

(iii)Which crop has greater Standard deviation

Variance[Rice]=$\sum$[Xi − M]2 /N=458.5/10=45.85 1

Variance[Wheat]=$\sum$[Xi − M]2 /N=322.5/10=32.25 1

	So rice has greater Standard deviation.	
13	To find mean deviation (M.D) from the mean and the standard deviation	
	(S.D) of the A.P.	
	a, a + d, a + 2 d,......,a + 2n.d	1/2
	Here no of terms is 2n+1.	½
	Mean $=[\ \{a+2nd\}(2n+1)/2]/(2n+1)=(a+2nd)/2$	1
	S.D.$=\sqrt{[\{(2n+1)a^2+2n\ d^2 + (2ad+4ad+....4nad)\}/(2n+1)-(a+2nd)^2/4]}$	1
	$=\sqrt{[\{(2n+1)a^2+2n\ d^2 + 2adn(2n+1)-(2n+1)a^2-(2n+1)4adn-4n^2d^2(2n+1)\}/(2n+1)]}$	1
	$=\sqrt{[2n\ d^2/(2n+1)\ -2adn-4n^2d^2}$	1
14	I]the wrong value is removed:	
	The sum of 6 values=8*6=48,if 3 is removed sum of 5 values =48-3=45	½
	So new mean=45/5=9	½
	Sum of squares of 6 values=[16+64]6=480	½
	Sum of squares of 5 values=480-9=471	
	Thus standard deviation of 5 values$=\sqrt{[471/5 -81]}=\sqrt{[94.2-81]}=\sqrt{13.2}$	½
	II] the wrong value is changed by 11:	
	The sum of new 6 values=48-3+11=56	½
	So new mean=56/6=28/3	½
	Sum of squares of new 6 values=480-9+121=592	1
	Thus standard deviation$=\sqrt{[592/6-(28/3)^2]}=\sqrt{[\{1776-1568\}/18]}=\sqrt{11.55}$	1
15		3

Class	Frequency fi	Class mark xi	xi fi	xi -M	[xi-M]fi	Ixi-M I fi
10-20	4	15	60	-27.86	-111.44	111.44
20-30	6	25	150	-17.86	-107.16	107.16
30-40	10	35	350	-7.86	-78.6	78.6
40-50	20	45	900	2.14	42.8	42.8
50-60	10	55	550	12.14	121.4	121.4
60-70	6	65	390	22.14	132.84	132.84
Total	56		2400			594.24

Mean M = 2400/56 — 1

= 42.86 — 1

Mean deviation w.r.t mean= 594.24 /56 i.e. 10.61

PROBABILITY

Q. NO	QUESTION	MARK
1	Consider the experiment in which a coin is tossed repeatedly until a head comes up. Describe the sample space.	2
2	A die is rolled. Let, E bethe event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?	2
3	Given$\mathrm{P(A)} = \frac{3}{5}$ and $\mathrm{P(B)} = \frac{1}{5}$. Find $\mathrm{P(A\ or\ B)}$, if A and B are mutually exclusive events.	2
4	Events A and B are such that $\mathrm{P(not\ A\ or\ not\ B)} = 0.25$, state whether A and Bare mutually exclusive events or not.	2
5	In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that: I. The student opted for NCC orNSS. II. The student has opted neither NCC nor NSS. III. The student has opted NSS but not NCC.	3
6	Find the probability that win a hand of 7 cards is drawn from a well shuffled deck of 52 cards, it contains I. All kings. II. 3 kings. III. At least 3 kings.	3
7	In a relay race, there are 5 teams A, B, C, D and E. I. What is the probability that A,B& C finish first, second and third respectively. II. What is the probability that A, B & C are first three to finish (in any order).	3
8	Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that I. You both enter the same section? II. You both enter the different section?	3
9	Rahul went to a fair. There he saw in a shop a lottery seller was selling lotteries. He asked the shopkeeper about this lottery game and he got the information that among these 10000 tickets, there are 10 prizes will be awarded. He is willing to know  I. What is the probability of not getting a prize, if he buys one ticket? II. What is the probability of not getting a prize if he buys 2 tickets? III. What is the probability of not getting a prize if he buys 10 tickets?	4

10	Shivnath went to a fair. In the fare he saw a shopkeeper was mixing tickets numbered 1 to 20 thoroughly and asking customers to take out a ticket randomly. He is willing to know  I. What is the probability that drawn ticket number is a multiple of 3? II. What is the probability that drawn ticket number is a multiple of 7? III. If drawn ticket number is a multiple of 3 or 7 then he wins. What is the probability of his winning?	4
11	A fair coin is tossed four times, and a person win ₹ 1 for each head and lose ₹ 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.	5
12	Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.	5
13	If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5 and 7, what is the probability of forming a number divisible by 5 when the digits are repeated?	5
14	If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5 and 7, what is the probability of forming a number divisible by 5 when the repetitions of digits are not allowed?	5
15	A and B are two events such that $P(A) = 0.54, P(B) = 0.69$ and $P(A \cap B) = 0.35.$ Find I. $P(A \cup B)$ II. $P(A')$ III. $P(A' \cap B')$ IV. $P(A \cap B')$ V. $P(A' \cap B)$	5

Q. NO	ANSWERS	MARK
1	In the experiment head may come up on the first toss, or on the second toss, or on the third toss and so on till headis obtained. Hence, the desired sample space is$S = \{H, TH, TTH, TTTH, TTTTH, \dots\}$.	2
2	1,2,3,4,5and6arethepossibleoutcomeswhen adieisthrown.So, $S = \{1, 2, 3, 4, 5, 6\}$ Asperttheconditionsgiventhequestion Ebetheevent“dieshows4” $E = \{4\}$ Fbetheevent“dieshowsevennumber” $F = \{2, 4, 6\}$ $E \cap F = \{4\}$ $\therefore E \cap F \neq \phi$ …[becausethereisacommonelementinEandF] Therefore, EandFarenotmutuallyexclusiveevent.	2
3	If A and B are mutually exclusive events, then	2

	$P(A \text{ or } B) = P(A) + P(B) = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$	
4	$P(\text{not } A \text{ or not } B) = 0.25$ $\Rightarrow P(A' \cup B') = 0.25$ $\Rightarrow P[(A \cap B)'] = 0.25$ $\Rightarrow 1 - P(A \cap B) = 0.25$ $\Rightarrow P(A \cap B) = 0.75$ Hence, A and Barenot mutually exclusive events.	2
5	Let, A and B be the sets of students who have opted for NCC and NSS respectively. $\therefore n(A) = 30, n(B) = 32, n(A \cap B) = 24, n(A \cup B) = 60$ I. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\Rightarrow P(A \cup B) = \frac{30}{60} + \frac{32}{60} - \frac{24}{60}$ $\Rightarrow P(A \cup B) = \frac{38}{60} = \frac{19}{30}$ II. $P(\text{neither } A \text{ nor } B)$ $= P(A' \cap B')$ $= P[(A \cup B)']$ $= 1 - P(A \cup B)$ $= 1 - \frac{19}{30}$ $= \frac{11}{30}$ III. $P(B \text{ but not } A)$ $= P(A' \cap B)$ $= P(B) - P(A \cap B)$ $= \frac{32}{60} - \frac{24}{60}$ $= \frac{8}{60}$ $= \frac{2}{15}$	3
6	The total number of possible hands= $C(52, 7)$ I. Number of hands with 4 kings $= C(4, 4) \times C(48, 3)$(other 3 cards must be chosen from remaining 48 cards) Hence,$P(\text{a hand wil have 4 kings}) = \frac{C(4,4)\times C(48,3)}{C(52,7)} = \frac{1}{7735}$ II. Number of hands with 3 kings and 4 non-king cards $= C(4, 3) \times C(48, 4)$ Hence, $P(\text{a hand will have 3 kings}) = \frac{C(4,3)\times C(48,4)}{C(52,7)} = \frac{9}{1547}$ III. $P(\text{atleast 3 kings}) = P(\text{3 kings or 4 kings}) = P(\text{3 kings}) + P(\text{4 kings}) = \frac{9}{1547} + \frac{1}{7735} = \frac{46}{7735}$	3
7	If we consider the sample space consisting of all finishing orders in the first three places, we will have$P(5, 3) = 60$ sample points, each with a probability of $\frac{1}{60}$. I. A, B and C finish first, second, and third respectively. There is only one finishing order for this, i.e. ABC. $\therefore P(\text{A, B and C finish first, second and third respetively}) = \frac{1}{60}$ II. A, B and C are the first three finishers. There will be $3! = 6$arrangements for A, B and C. $\therefore P(\text{A, B and C are the first three to finish}) = \frac{6}{60} = \frac{1}{10}$	3

8	The total number of students is 100. I and my friend are among the 100 students. Two sections of 40 are 60 students are formed. Total number of ways of selecting two students out of 100students is C(100, 2) I. Both of us will enter the same section if both of us are among either 40 students or 60 students. Number of ways in which both of us enter the same section is $C(40, 2) + C(60, 2)$ $\therefore \text{P(Both of us will enter the same section)} = \frac{C(40, 2) + C(60, 2)}{C(100, 2)} = \frac{17}{33}$ II. $\text{P(Both of us will enter different section)} = 1 - \frac{17}{33} = \frac{16}{33}$	3
9	I. Out of 10000 tickets, one ticket can be chosen in $C(10000, 1) = 10000$ ways. There are 9990 tickets not containing a prize.Out of these 9990 tickets one can be chosen in $C(9990, 1) = 9990$ $\therefore \text{P(not getting a prize)} = \frac{9990}{10000} = \frac{999}{1000}$ II. Out of 10000 tickets, twotickets can be chosen in $C(10000, 2)$ways. As there are 9990 tickets not containing a prize. Out of these 9990 tickets two can be chosen in $C(9990, 2)$ ways. $\therefore \text{P(not getting a prize)} = \frac{C(9990,2)}{C(10000, 2)}$ III. Out of 10000 tickets, ten tickets can be chosen in $C(10000, 10)$ways. As there are 9990 tickets not containing a prize. Out of these 9990 tickets ten can be chosen in $C(9990, 10)$ ways. $\therefore \text{P(not getting a prize)} = \frac{C(9990,10)}{C(10000, 10)}$	4
10	Let, S be the sample space associated with the given random experiment. A and B denote the events getting a ticket bearing a number which is a multiple of 3 and 7 respectively, then $S = \{1, 2, 3, \ldots, 20\}, A = \{3,6,9, \ldots, 18\}, B = \{7,14\}$ I. $\text{P(ticket numbe is a multiple of 3)} = \frac{6}{20} = \frac{3}{10}$ II. $\text{P(ticket numbe is a multiple of 7)} = \frac{2}{20} = \frac{1}{10}$ III. $\text{P(winning)} = \frac{3}{10} + \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$	4
11	A coin is tossed 4 times then the respective sample space will contain $2^4 = 16$ elements. In 4 tosses of a coin, the possible number of heads are 0, 1, 2, 3 and 4 . Now we shall discuss all the 5 cases. **Case 1(0 Heads):** There is only one possibility, i.e.$\{(T, T, T, T)\}$ For 0 head, he will get $= -4 \times 1.5 = -₹6$ So, in this case a loss will incur of ₹6 $\therefore \text{P(a loss of ₹6)} = \frac{1}{16}$ **Case 2(1 Heads):** There are 4 possibilities, i.e.$\{(H, T, T, T), (T, H, T, T), (T, T, H, T), (T, T, T, H)\}$ For 1 head, he will get $= 1 - 3 \times 1.5 = -₹3.5$ So, in this case a loss will incur of ₹3.5 $\therefore \text{P(a loss of ₹3.5)} = \frac{4}{16} = \frac{1}{4}$	5

	Case 3(2 Heads): There are 4 possibilities, i.e. $\{(H,H,T,T),(T,H,H,T),(T,T,H,H),(H,T,T,H),(H,T,H,T),(T,H,T,H)\}$ For 2 heads, he will get $= 1+1-2\times 1.5 = -₹1$ So, in this case a loss will incur of ₹1 $$\therefore \mathrm{P(a\ loss\ of\ ₹1)} = \frac{6}{16} = \frac{3}{8}$$ **Case 4(3 Heads):** There are 4 possibilities, i.e. $\{(H,H,H,T),(T,H,H,H),(H,T,H,H),(H,H,T,H)\}$ For 3heads, he will get $= 1+1+1-1.5 = ₹1.5$ So, in this case a profit will incur of ₹1.5 $$\therefore \mathrm{P(a\ profit\ of\ ₹1.5)} = \frac{4}{16} = \frac{1}{4}$$ **Case 5(4 Heads):** There is only one possibility, i.e. $\{(H,H,H,H)\}$ For 4 head, he will get $= 4\times 1 = ₹4$ So, in this case a profit will incur of ₹4 $$\therefore \mathrm{P(a\ loss\ of\ ₹4)} = \frac{1}{16}$$	
12	Total number of ways by which three letters can be put into three envelopes$= 3! = 6$ $\text{Derrangement of n objects} = n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots+(-1)^n\frac{1}{n!}\right]$ $\therefore \text{Derrangement of 3 objects} = 3!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right] = 3\left(1-1+\frac{1}{2}-\frac{1}{6}\right) = 2$ $\text{P(no letter is in the correct envelope)} = \frac{2}{6} = \frac{1}{3}$ $\therefore \text{P(atleast one letter is in the correct envelope)} = 1-\frac{1}{3} = \frac{2}{3}$	5
13	Since 4 digit numbers greater than 5000 are formed. The thousand's place digit is either 7 or 5. The total number of 4 digits number greater than 5000 $= 2\times 5\times 5\times 5-1 = 249$ A number is divisible by 5, if the digit at it's unit place is either 0 or 5. $\therefore$ The total number of 4 − digits numbers greater than 5000 and divsible by $5 = 2\times 5\times 5\times 2-1 = 99$ $\therefore$ P(forming a number which is greater than 5000 and divisible by 5) $= \frac{99}{249} = \frac{33}{83}$	5
14	Since 4 digit numbers greater than 5000 are formed. The thousand's place digit is either 7 or 5. The total number of 4 digits number greater than 5000 $= 2\times 4\times 3\times 2 = 48$ A number is divisible by 5, if the digit at it's unit place is either 0 or 5. $\therefore$ The total number of 4 − digits numbers starting with 5 and divsible by $5 = 1\times 3\times 2\times 1 = 6$ $\therefore$ The total number of 4 − digits numbers starting with 7 and divsible by $5 = 1\times 3\times 2\times 2 = 12$ $\therefore$ The total number of 4 − digits numbers greater than 5000 and divsible by $5 = 6+12 = 18$ $\therefore$ P(forming a number which is greater than 5000 and divisible by 5) $= \frac{18}{48} = \frac{3}{8}$	5
15	I. $P(A\cup B) = P(A)+P(B)-P(A\cap B)$ $\Rightarrow P(A\cup B) = 0.54+0.69-0.35$	5

	$\Rightarrow P(A \cup B) = 0.88 = \frac{88}{100} = \frac{22}{25}$ II. $P(A')$ $= 1 - P(A)$ $= 1 - 0.54$ $= 0.46$ III. $P(A' \cap B')$ $= P[(A \cup B)']$ $= 1 - P(A \cup B)$ $= 1 - \frac{22}{25}$ $= \frac{3}{25}$ IV. $P(A \cap B')$ $= P(A) - P(A \cap B)$ $= 0.54 - 0.35$ $= 0.19$ $= \frac{19}{100}$ V. $P(A' \cap B)$ $= P(B) - P(A \cap B)$ $= 0.69 - 0.35$ $= 0.34$ $= \frac{34}{100}$ $= \frac{17}{50}$	