केंद्रीय विद्यालय संगठन

Kendriya Vidyalaya Sangathan

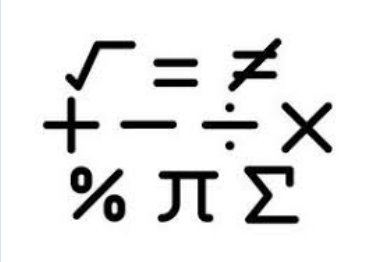
सत्र/ Session 2023-24

अध्ययन सामग्री गणित

Study Material Maths

कक्षा - ग्यारहवी

Class - XI



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Kendriya Vidyalaya Sangathan

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केन्द्रीय विद्यालय संगठन

KENDRIYA VIDYALAYA SANGATHAN

JABALPUR REGION

अध्ययन सामग्री/STUDY MATERIAL

सत्र/ SESSION: 2023-24

कक्षा/CLASS- ELEVENTH(XI)

गणित/MATHEMATICS

विषय कोड/Subject Code - 041

DIRECTED BY :-
MR. PANKAJ KUMAR JAIN
PRINCIPAL, KV BALAGHAT

DIRECTOR'S MESSAGE

It is with profound delight and utmost pride that I announce the publication of our study material for class XI (MATHEMATICS) for the session 2023-24. It's my firm belief that access to quality education should know no boundaries, transcending social and economic constraints. Our collective vision is to empower all students with the tools for success and intellectual growth.

With their steadfast dedication PGT-MATHEMATICS of Jabalpur Region of Kendriya Vidyalaya Sangathan have invested their knowledge, expertise, and passion into meticulously crafting these study materials to complement the classroom learning experience of the students. These materials serve as invaluable aids for self-study since they are comprehensive, well-structured, and presented in a manner that is easy to comprehend.

It is with pleasure that I place on record my commendation for the commitment and dedication of the team of all PGT Mathematics of Kendriya Vidyalaya Sangathan Jabalpur Region.

Wishing you all the very best in your academic journey!

PANKAJ KUMAR JAIN

PRINCIPAL
KENDRIYA VIDYALAY
BALAGHAT

CONTENT DEVELOPMENT TEAM OF PGT MATHEMATICS

BOOKLATE FOR CLASS - XI , SUBJECT :- MATHEMATICS (041)

S.NO.	NAME OF CHAPTER	PGT (MATHEMATICS)	KENDRIYA VIDYALAYA
1	1. Sets.	MR. DURAG SINGH DANGI	AMARKANTAK
2	2. Relations & Functions	MR. VIVEK VISHWAKARMA	CHANDAMETTA BARKUHI (WCL)
3	3.Trigonometric Functions	MRS. VANITA BAHEY	CHHINDWARA NO.II
4	4. Complex Numbers and Quadratic Equations	MR. DEVENDRA KUMAR SINGH	CHHINDWARA NO.II
5	5. Linear Inequalities	MR. DIGVIJAY SINGH	DINDORI
6	6. Permutations and Combinations	MR R K TRIPATHI	JAYANT COLLIERY
7	7.Binomial Theorem	MR. RAKESH KUMAR PATKAR	JABALPUR GCF NO.I
8	8. Sequence and Series	Mrs. SARITA SINGH	SAGAR NO.I (CANT.)
9	9. Straight Lines	Mrs. REKHA BARETHIYA	SAGAR NO.I (CANT.)
10	10. Conic Sections	Mrs. Rashmi Soni	KATNI NO.II (RLY)
11	11. Introduction to Three-dimensional Geometry	MR JITENDRA KUMAR TIWARI	KATNI NO.I (OF)
12	12. Limits and Derivatives	MR. RAJENDRA KUMAR BARMAN	NOWROZABAD SECL
13	13.Statistics	MR. Neeraj chaurasia	PANNA
14	14.Probability	MR. Anil Kumar Tamrakar	REWA NO.II
15	REVIEW AND COMPILATION	MR. RITESH AGRAWAL	BALAGHAT

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KENDRIYA VIDYALAYA SANGATHAN JABALPUR
STUDY MATERIAL FOR STUDENTS: 2023-24
CLASS XI

Subject: Mathematics

(Chapter -1: Sets)

Points to Remember:

- A set is a well-defined collection of objects.
- A set which does not contain any element is called *empty set*.
- A set which consists of a definite number of elements is called *finite set*, otherwise, the set is called *infinite set*.
- Two sets A and B are said to be equal if they have exactly the same elements.
- A set A is said to be subset of a set B, if every element of A is also an element of B. Intervals are subsets of **R**.
- The union of two sets A and B is the set of all those elements which are either in A or in B.
- The intersection of two sets A and B is the set of all elements which are common. The difference of two sets A and B in this order is the set of elements which belong to A but not to B.
- The complement of a subset A of universal set U is the set of all elements of U which are not the elements of A.
- For any two sets A and B, $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$
- The collection of all subsets of a set A is called the *power set* of A. It is denoted by P(A). In P(A), every element is a set.
- In general, if A is a set with $n(A) = m$, then it can be shown that $n[P(A)] = 2^n$
- In general, if A and B are finite sets, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- If $n(A \cap B) = \varphi$ then $n(A \cup B) = n(A) + n(B)$

Some Properties of Complement Sets

- Complement laws: (i) $A \cup A' = U$ (ii) $A \cap A' = \varphi$
- De Morgan's law: (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$
- Law of double complementation: $(A')' = A$
- Laws of empty set and universal set $\varphi' = U$ and $U' = \varphi$.

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MCQS

Q.1: If the sets A and B are given by A = {1, 2, 3, 4}, B = {2, 4, 6, 8, 10} and the universal set U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then:
(a) (A U B)' = {5, 7, 9} (b) (A ∩ B)'= {1, 3, 5, 6, 7}
(c) (A ∩ B)' = {1, 3, 5, 6, 7, 8} (d) None of these

Q.2: If A = {1, 2, 3, 4}, B = {2, 3, 5, 6} and C = {3, 4, 6, 7}, then
(a) A – (B ∩ C) = {1, 3, 4} (b) A – (B ∩ C) = {1, 2, 4}
(c) A – (B U C) = {2, 3} (d) A – (B U C) = {Ø}

Q.3: The number of the proper subset of {a, b, c} is:
(a) 3 (b) 8 (c) 6 (d) 7

Q.4: Which one is different from the others?
(a) empty set (b) void set (c) zero set (d) null set

Q.5: Which of the following collections is a set ?
(a) The collection of all the days of a week (b) A collection of 11 best hockey player of India.
(c) The collection of all rich person of Delhi (d) A collection of most dangerous animals of India.

Q.6: If A U B = Ø then n(A U B) is equal to:
(a) n(A) + n(B) - n(A∩B) (b) n(A) - n(B) + n(A∩B)

(c) n(A) + n(B) + n(A∩B) (d) n(A) - n(B) - n(A∩B)

Q.7: The number of subsets of a set containing n elements is:

(a) n (b) 2^{n-1} (c) n^2 (d) 2^n

Q.8: Set A = {0, 7, 26, 63} in set-builder form is:

(a) {x: x ∈ N, x = n3 – 1 and n ≤ 7} (b) {x: x ∈ N, x = n2 – 1 and n ≤ 5}

(c) {x: x ∈ N, x = n2 – 1 and n < 4} (d) {x: x ∈ N, x = n3 – 1 and n ≤ 4}

Q.9: If B = {Ø} then:

(a) B is an empty set (b) B is a finite set

(c) B is an infinite set (d) B is not a set

Q.10: If n(A) = 2 then n[P(P(A))] is:

(a) 8 (b) 4 (c) 12 (d) 16

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THREE LEVELS OF GRADED QUESTIONS

Level –I

(Very Short Answer Type Questions- 2 Marks)

Q.1: Write down the set {5, 25, 125, 625} in the set builder form

Q.2: If n(A) = 2 then find n[P(P(A))]

Q.3: Write down {x: x ∈ R, - 4 < x ≤ 6} as interval.

Q.4: Write the set A ={x: x ∈ R, 2x + 11= 15} in the roaster form.

Q.5: If A and B are two sets such that A ⊂ B, then what is A U B?

Q.6: If S = {x: x is a positive multiple of 3 less than 100} and
P = {x: x is a prime number less than 20}.Then find n(S) + n(P)

Q.7: Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n.

Q.8: Let S = the set of all triangles, P = the set of all isosceles triangles, Q = the set of all equilateral triangles, R = the set of all right-angled triangles. What do the sets P ∩ Q and R – P represents respectively ?

Q.9: Let V = {a, e, i, o, u}, V – B = {e, o} and B – V = {k}. Find the set B.

Q.10: If A = {a, {b}}, then find P(A)

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Level –II

(Short Answer Type Questions- 3 Marks)

Q.1: Write down all the subsets of the set {1, 2, 3}

Q.2: Draw appropriate venn diagram for each of the following:
(a) (A U B)' (b) A – B

Q.3: Prove that A= B if P(A) = P(B)

Q.4: A survey shows that 63%of Indians like cheese where as 76% like apples. If x % of Indians like cheese and apples. Find the value of x.

Q.5: Using properties of sets prove that $(A \cup B)' = A' \cap B'$.

Q.6: Using Venn Diagram prove that: $A \cap (B - C) = (A \cap B) - (A \cap C)$.

Q.7: For all sets A, B and C, if A ⊂B, then A U C ⊂ B U C

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Level -III

(Long Answer Type Questions- 5 Marks)

Q.1: If A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13} , C= {11, 13, 15} , D = {15, 17}, Find
(a) (A U B) ∩ C (b) C - D (c) (A ∩ B) ∩ (B U C) (d) B ∩ D

Q.2: In a survey of 100 students the number of students studying the various languages were found to be: English only18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48,

Sanskrit and Hindi 8, no language 24.Find
(i) How many students were studying Hindi?
(ii) How many students were studying English and Hindi?
(iii) How many students were studying Sanskrit only?
Q.3: In a survey of 25 students of a school it was found that 15 study Mathematics, 12 study Physics, 11 study Chemistry, 9 study both Mathematics & Physics, 4 study both Physics & Chemistry, 5 study both Chemistry and Mathematics and 3 students all of the above three subjects. Find the number of students who study:
(a) Only Mathematics
(b) At least one of the three subjects.
Q.4: In a group of students ,100 students know Hindi, 50 know English and 25 know both . Each of the students know either Hindi or English. How many students are there in the group .
Q.5: If $n(A - B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$.
Q.6: If $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6\}$ and $N = \{1, 3, 5\}$, then verify that
$L - (M \cup N) = (L - M) \cap (L - N)$.
Q.7: If A, B and C be sets. Then, show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

ASSERTION & REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

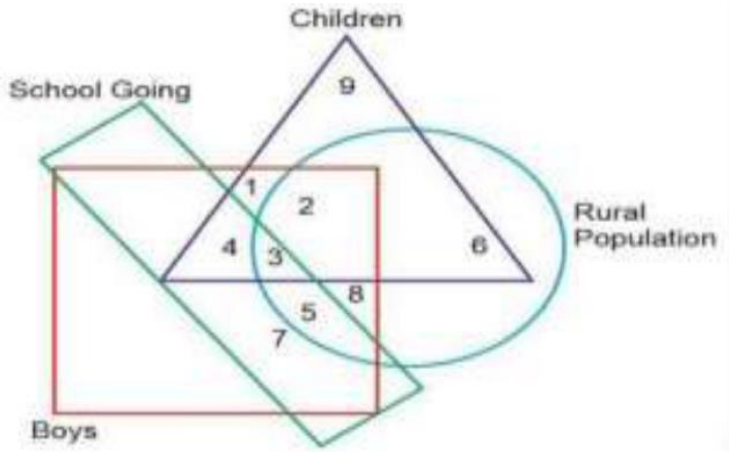
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

Q.1: Assertion : The number of non-empty subsets of the set {a, b, c, d} are 15.
Reason : Number of non-empty subsets of a set having n elements are $2^n - 1$.
Q.2: Suppose A, B and C are three arbitrary sets and U is a universal set.
Assertion : If $B = U - A$, then $n(B) = n(U) - n(A)$.
Reason : If $C = A - B$, then $n(C) = n(A) - n(B)$.
Q.3: Assertion : Let $A = \{1, \{2, 3\}\}$, then $P(A) = \{\{1\}, \{2, 3\}, \emptyset, \{1, \{2, 3\}\}\}$.
Reason : Power set is set of all subsets of A.
Q.4: Assertion : The subsets of the set $\{1, \{2\}\}$ are $\{\}$, $\{1\}$, $\{\{2\}\}$ and $\{1, \{2\}\}$.
Reason : The total number of proper subsets of a set containing n elements is $2^n - 1$.
Q.5: Assertion : For any two sets A and B, $A - B \subset B'$
Reason : If A be any set, then $A \cap A' = \emptyset$

CRITICALTHINKING TYPE QUESTIONS

CASE BASED QUESTION-1:

Venn diagrams were invented by a logician John Venn as a way of picturing relationships between different groups of things. These diagrams, also called Set diagrams or Logic diagrams, are widely used in mathematics, statistics, logic, teaching, linguistics, computer science and business
In the following diagram, triangle shows children, circle shows rural population, rectangle shows school going population & square shows boys.



Based on the information stated above answer the below given questions-:

(i) The rural boys not going to school are denoted by which number?

(a)1 (b) 2 (c)1,2 (d) 2,8

(ii) The children from rural population not going to school are denoted by which number?

(a)1 (b) 2 (c) 6 (d) 2,6

(iii) What is represented by number 4?

(a) Children who are not from rural population (b) Children who are boys

(c) School going boys (d) School going boys who are not from rural population

(iv) School going boys from village are denoted by which number?

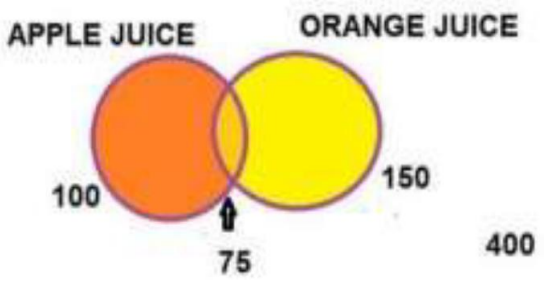
(a) 3 (b) 3,5 (c) 3,4 (d) 3, 4, 5,7

(v) Number of children who are not from rural population?

(a) 8 (b) 2 (c) 7 (d) 9

CASE BASED QUESTION-2:

In D.A.V School, Bahadurgarh, a survey was done on 400 students it was found that 100 like to take apple juice , 150 like to take orange juice and 75 like both apple as well as orange juice.



Based on above information, answer the following questions:

(i) Number of students who like either of the drink :

(a) 400 (b) 175 (c) 250 (d) 325

(ii) Number of students who likes neither apple juice nor orange juice :

(a) 225 (b) 325 (c) 75 (d) 25

(iii) Number of students who likes only apple juice:

(a) 125 (b) 75 (c) 100 (d) 25

(iv) Number of students who likes only orange juice :

(a) 75 (b) 25 (c) 100 (d) 125

(v) Which information we get from the given data :

(a) n (A U B) =n(A) + n (B) (b) n(A U B) < n (A ∩ B)

(c) n(A U B) < n (U) (d) n (A ∩ B) = n(A) + n (B)

(Chapter -2: RELATION AND FUNCTION)

1. A relation is a subset of cartesian products.
a) True
b) False

2. Let A={1,2,3,4,5} and R be a relation from A to A, R = {(x, y): y = x + 1}. Find the domain.
a) {1,2,3,4,5}
b) {2,3,4,5}
c) {1,2,3,4}
d) {1,2,3,4,5,6}

3. Let A={1,2,3,4,5} and R be a relation from A to A, R = {(x, y): y = x + 1}. Find the codomain.
a) {1,2,3,4,5}
b) {2,3,4,5}
c) {1,2,3,4}
d) {1,2,3,4,5,6}

4. Let A={1,2,3,4,5} and R be a relation from A to A, R = {(x, y): y = x + 1}. Find the range.
a) {1,2,3,4,5}
b) {2,3,4,5}
c) {1,2,3,4}
d) {1,2,3,4,5,6}

5. If set A has 2 elements and set B has 4 elements then how many relations are possible?
a) 32
b) 128
c) 256
d) 64

6. Is relation from set A to set B is always equal to relation from set B to set A.
a) True
b) False

7. If A={1,4,8,9} and B={1, 2, -1, -2, -3, 3,5} and R is a relation from set A to set B {(x, y): $x=y^2$}. Find domain of the relation.
a) {1,4,9}
b) {-1,1, -2,2, -3,3}
c) {1,4,8,9}
d) {-1,1, -2,2, -3,3,5}

8. If A={1,4,8,9} and B={1, 2, -1, -2, -3, 3,5} and R is a relation from set A to set B {(x, y): $x=y^2$}. Find codomain of the relation.
a) {1,4,9}
b) {-1,1, -2,2, -3,3}
c) {1,4,8,9}
d) {-1,1, -2,2, -3,3,5}

9. If A={1,4,8,9} and B={1, 2, -1, -2, -3, 3,5} and R is a relation from set A to set B {(x, y): $x=y^2$}. Find range of the relation.

a) {1,4,9}

b) {-1,1, -2,2, -3,3}

c) {1,4,8,9}

d) {-1,1, -2,2, -3,3,5}

10. Let A={1,2} and B={3,4}. Which of the following cannot be relation from set A to set B?

a) {(1,1), (1,2), (1,3), (1,4)}

b) {(1,3), (1,4)}

c) {(2,3), (2,4)}

d) {(1,3), (1,4), (2,3), (2,4)}

Each of these questions contains two statements Assertion (A) and Reason (R). Each of the questions has four alternative choices, any one of the which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation of A.

(b) A is true, Ris true; R is not a correct explanation of A.

(c) A is true; R is false.

(d) A is false; R is true.

Assertion (A): If $(x+1, y-2) = (3,1)$, then $x = 2$ and $y = 3$.

Reason (R): Two ordered pairs are equal, if their corresponding elements are equal.

Let $A = \{1,2,3,4,6\}$. If R is the relation on A defined by $\{(a,b): a, b \in A, b \text{ is exactly divisible by } a\}$.

Assertion (A): The relation R in Roster form is $\{(6,3), (6,2), (4,2)\}$.

Reason (R): The domain and range of R is $\{1,2,3,4,6\}$.

Assertion (A): If $(4x+3, y) = (3x+5, -2)$, then $x = 2$ and $y = -2$.

Reason (R): If $A = \{-1,3,4\}$, then $A \times A$ is $\{(-1,-1), (-1,3), (-1,4), (3,-1), (4,-1), (3,4)\}$.

Assertion (A): Let $A = \{1,2\}$ and $B = \{3,4\}$. Then, number of relations from A to B is 16.

Reason (R): If $n(A) = p$ and $n(B) = q$, then number of relations is 2^{pq}.

Assertion (A): The cartesian product of two non-empty sets P and Q is denoted as $P \times Q$ and $P \times Q = \{(p,q): p \in P, q \in Q\}$.

Reason (R): If $A = \{$ red, blue $\}$ and $B = \{b, c, s\}$, then $A \times B = \{($ red, $b)$, (red, c), (red, s), (blue, b), (blue, c), (blue, s)$\}$.

Assertion (A): The domain of the relation $R = \{(x+2, x+4): x \in N, x < 8\}$ is $\{3,4,5,6,7,8,9\}$.

Reason (R): The range of the relation $R = \{(x+2, x+4): x \in N, x < 8\}$ is $\{1,2,3,4,5,6,7\}$.

Let R be a relation defined by $R = \{(x, x+5): x \in \{0,1,2,3,4,5\}\}$ Then, consider the following

Assertion (A): The domain of R is $\{0,1,2,3,4,5\}$.

Reason (R): The range of R is $\{0,1,2,3,4,5\}$.

Assertion (A): The range of the function $f(x) = 2 - 3x, x \in R, x > 0$ is R.

Reason (R): The range of the function $f(x) = x^2 + 2$ is $[2, \infty)$.

2 MARKS

Short Answer Type Questions

Q1. If A = {-1, 2, 3 } and B = {1, 3}, then determine

(i) AxB (ii) BxC (c) BxB (iv) AxA

Q2. If P = {x : x < 3, x e N}, Q= {x : x≤2,x ∈ W}. Find (P∪ Q) x (P∩ Q), where W is the set of whole numbers.

Q3. IfA={x:x∈ W,x < 2}, 5 = {x : x∈N, 1 <.x < 5}, C= {3, 5}. Find
(i) Ax(B∩Q) (ii) Ax(B∪C)

Q4. In each of the following cases, find a and b. (2a + b, a − b) = (8, 3) (ii) {a/4, a − 2b) = (0, 6 + b)

Q5. Given A = {1,2,3,4, 5}, S= {(x,y) :x∈ A,y∈ A}.Find the ordered pairs which satisfy the conditions given below
x+y = 5 (ii) x+y<5 (iii) x+y>8

Q6. Given R = {(x,y) : x,y ∈ W, $x^2 + y^2 = 25$}. Find the domain and range of R

Q7. If R_1 = {(x, y)| y = 2x + 7, where x∈ R and -5 ≤ x ≤ 5} is a relation. Then find the domain and range of R_1.

Q8. If R_2 = {(x, y) | x and y are integers and $x^2 + y^2 = 64$} is a relation. Then find R_2

Q9. If R_3 = {(x, |x|) | x is a real number} is a relation. Then find domain and range

Q10.
If f and g are two real valued ftmctions defined as f(x) = 2x + 1, $g(x) = x^2 + 1$, then find.

(i) f + g (ii) f - g (iii) fg (iv)f/g

3 MARKS

Q.1 Redefine the function: f(x) = |x − 1| − |x + 6|. Write its domain also.

Q.2: The Cartesian product A × A has 9 elements among which are found (−1, 0) and (0,1). Find the set A and the remaining elements of A × A.

Q.3: Express the function f: A—R. $f(x) = x^2 - 1$. where A = { −4, 0, 1, 4) as a set of ordered pairs.

Q.4: Assume that A = {1, 2, 3,...,14}. Define a relation R from A to A by R = {(x, y) : 3x − y = 0, such that x, y ∈ A}. Determine and write down its range, domain, and codomain.

Q 5. The function f is defined by

$$f(x) = \begin{cases} 1 - x, & x < 0 \\ 1, & x = 0 \\ x + 1, & x > 0 \end{cases}$$

Draw the graph of f(x).

Q 6. A relation R is defined from a set A = {2, 3, 4, 7} to a set B = { 3, 6, 9, 0} as follows R = ((x,y) ∈ R : x is relatively prime to y; x ∈ A, y ∈ B). Express R as a set of ordered pairs and determine the domain and range.

Q 7. Draw the graph of the function f: R → R defined by f (x) = x^3, x ∈ R

Q.8: Find the domain and range of the real function f(x) = x/1+x^2.

9. In each of the following cases, find a and b.
(i)(2a + b, a − b) = (8, 3)
(ii) {a/4, a − 2b) = (0, 6 + b)

Q 10. Let f and g be real functions defined by f(x) = 2x+ 1 and g(x) = 4x − 7.
(i) For what real numbers x, f(x) = g(x)?
(ii) For what real numbers x, f (x) < g(x)?

Q 11. The ordered pair (5, 2) belongs to the relation R ={(x, y): y = x − 5, x, y∈Z}

Q 12. Find the domain for which the functions f (x) = $2x^2$ − 1 and g (x) = 1 − 3x and check whether they are equal.

Q 13. Let A = {1, 2, 3}, B = {4} and C = {5}
- (i) Verify that: A x (B − C) = (A x B) − (A x C)
- (ii) Find (A x B) ∩ (A x C).

5 MARKS

1. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as, R = {(x, y) : x ϵ A, y ϵ B and y = x^2}

(a) Depict this relation using arrow diagram.

(b) Find domain of R.

(c) Find range of R.

(d) Write co-domain of R.

2. Let R = { (x, y) : x, y ϵ N and y = 2x} be a relation on N. Find :

(i) Domain

(ii) Codomain

(iii) Range

Is this relation a function from N to N

Find the domain of the real function, f(×) = $\sqrt{x^2-4}$

Let $f=\{(1,1),(2,3),(0,-1),(-1,3),.....\}$ **be a function from z to z defined by** $f(x)=ax+b$, **for same integers a and b determine a and b.**

Let N **be the set of natural numbers and the relation** R **be define in** N **by** $R=$ $[(x,y): y=2x, x, y\in N]$. **what is the domain, co domain and range of** R **? Is this relation a function?**

6.Let R be a relation from Q to Q defined by $R=\{(a,b):a,b\in Q \text{ and } a-b\in z,\}$ **show that**

$(i)(a,a)\in R$ for all $a\in Q$ $(ii)(a,b)\in R$ implies that $(b,a)\in R$

$(iii)(a,b)\in R$ and $(b,c)\in R$ implies that $(a,c)\in R$

The function $'f'$ **which maps temperature in Celsius into temperature in Fahrenheit is defined by**

$f(c)=\frac{9}{5}c+32$ find $f(0)$, f (40) , f (-40) & f(35)

8.Find the domain and the range of the function $f(x)=3x^2-5$. **Also find** $f(-3)$ **and the numbers which are associated with the number 43 m its range.**

Find the domain of the function, □ f (×) = $\frac{x^2+2x+3}{x^2-5x+6}$ **Find the range of the following functions,**

.Let f **be the subset of** $Q\times Z$ **defined by**

$f=\left\{\left(\frac{m}{n},m\right):mn\in Z,n\neq 0\right\}$. **Is** f **a function from** Q to Z? **Justify your answer**

11.Let a relation $R=\{(0,0),(2,4),(-1,2),(3,6),(1,2)\}$ **then**

(i) write domain of R

(ii) write range of R

(iii) write R the set builder form

(iv) represent R by an arrow diagram

12.Let $A=\{1,2,3\}$, $B=\{1,2,3,4\}$ **and** $R=\{(x,y):(x,y)\in A\times B,y=x+1\}$

(i) find $A\times B$

(ii) write R in roster form

(iii) write domain & range of R

(iv) represent R by an arrow diagram

Find the domain and the range of the following functions $f(x)=\frac{1}{\sqrt{5-x}}$

.Let $f(x)=x+1$ **and** $g(x)=2x-3$ **be two real functions. Find the following functions**

$(i)\ f+g$ $(ii)f-g$ $(iii)fg$ $(iv)\frac{f}{g}$ $(v)f^2-3g$

Find the domain and the range of the following functions

$(i)f(x)=\frac{x-3}{2x+1}$ $(ii)f(x)=\frac{x^2}{1+x^2}$ $(iii)f(x)=\frac{1}{1-x^2}$

If $A=\{1,2,3\}$ $B=\{3,4\}$ **and** $c=\{4,5,6\}$

find $(i)\ A\times(B\cup C)$ $(ii)\ A\times(B\cap C)$ $(iii)\ (A\times B)\cap(B\times C)$

Let $A=\{1,2\}, B=\{1,2,3,4\}, C=\{5,6\}$ **and** $D=\{5,6,7,8\}$ **verify that** $(i)\ A\times(B\cap C)=(A\times B)\cap(A\times C)$

$(ii)\ A\times C$ is subset of $B\times D$

Let $A=\{1,2,3,4,5,6\}$ **define a relation** R **from** A **to** A **by**

$R=\{(x,y): y=x+1, x,y\in A\}$

(i) write R **in the roaster form**

(ii) write down the domain, co-domain and range of R

(iii) Represent R **by an arrow diagram**

Class 11 Maths Chapter 2

RELATIONS AND FUNCTIONS

Case Study Question 1:

Method to Find the Sets When Cartesian Product is Given

For finding these two sets, we write first element of each ordered pair in first set say A and corresponding second element in second set B (say).
Number of Elements in Cartesian Product of Two Sets

If there are p elements in set A and q elements in set B, then there will be pq elements in $A\times B$ i.e., if $n(A)=p$ and $n(B)=q$, then $n(A\times B)=pq$

Based on the above two topic, answer the following questions.

(i) If $A\times B=\{(a,1),(b,3),(a,3),(b,1),(a,2),(b,2)\}$. Then, A and B are (a) $\{1,3,2\},\{a,b\}$
(b) $\{a,b\},[1,3]$
(c) $\{a,b\},\{1,3,2\}$
(d) None of these

(ii) If the set A has 3 elements and set B has 4 elements, then the number of elements in $A\times B$ is
(a) 3
(b) 4
(c) 7
(d) 12

(iii) A and B are two sets given in such a way that $A\times B$ contains 6 elements. If three elements of $A\times B$ are $(1,3)$, $(2,5)$ and $(3,3)$, then A, B are
(a) $\{1,2,3\},\{3,5\}$
(b) $\{3,5\},,\{1,2,3\}$
(c) $\{1,2\},[3,5]$
(d) $\{1,2,3),[5\}$

(iv) The remaining elements of $A\times B$ in (iii) is
(a) $(5,1),(3,2),(3,5)$
(b) $(1,5),(2,3),(3,5)$
(c) $(1,5),(3,2),(5,3)$
(d) None of the above

(v) The cartesian product $P\times P$ has 16 elements among which are found $(a,1)$ and $(b,2)$. Then, the set P is
(a) $\{a,b\}$
(b) $\{1,2\}$
(c) $\{a,b,1,2\}$
(d) $\{a,b,1,2,4\}$

Case Study Question 2:

Ordered Pairs The ordered pair of two elements a and b is denoted by (a, b): a is first element (or first component) and b is second element (or second component).
Two ordered pairs are equal if their corresponding elements are equal. i.e., $(a, b) = (c, d) \Rightarrow a = c$ and $b = d$ Cartesian Product of Two Sets For two non-empty sets A and B, the cartesian product $A \times B$ is the set of all ordered pairs of elements from sets A and B.
In symbolic form, it can be written as

$$A \times B = \{(a, b): a \in A, b \in B\}$$

Based on the above topics, answer the following questions.

(i) If $(a - 3, b + 7) = (3,7)$, then the value of a and b are
(a) 6,0
(b) 3,7
(c) 7,0
(d) $3, -7$

(ii) If $(x + 6, y - 2) = (0,6)$, then the value of x and y are
(a) 6,8
(b) $-6, -8$
(c) $-6,8$
(d) $6, -8$

(iii) If $(x + 2,4) = (5,2x + y)$, then the value of x and y are
(a) $-3,2$
(b) 3,2
(c) $-3, -2$
(d) $3, -2$

(iv) Let A and B be two sets such that $A \times B$ consists of 6 elements. If three elements of $A \times B$ are $(1,4)$, $(2,6)$ and $(3,6)$, then
(a) $(A \times B) = (B \times A)$
(b) $(A \times B) \neq (B \times A)$
(c) $A \times B = \{(1,4), (1,6), (2,4)\}$
(d) None of the above

(v) If $n(A \times B) = 45$, then $n(A)$ cannot be
(a) 15
(b) 17
(c) 5
(d) 9

(Chapter -3: TRIGONOMETRIC FUNCTIONS)

MCQ

1. The value of tan 1° tan 2° tan 3° ... tan 89° is:

(a) 0 (b) 1 (c) 1⁄2 (d) Not defined

2. If α + β = π/4, then the value of (1 + tan α) (1 + tan β) is :

(a) 1 (b) 2 (c) – 2 (d) Not defined

Q.NO.3 The greatest and least values of sinx , cosx are respectively

a. 1, -1 b.$\frac{1}{2}$,$\frac{-1}{2}$ c. $\frac{1}{4}$, $\frac{-1}{4}$ d.2,-2

Q.NO.4 Value of $2\sin^2\frac{2\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6} + \cos^2\frac{2\pi}{3}$ is

a. $\frac{1}{2}$ b.$\frac{-1}{2}$ c. 1 d.$\frac{3}{2}$

Q.NO.5 The conversion of 40°20' into radians is:

a. π radians b.$\frac{15\pi}{9}$ c.$\frac{121\pi}{540}$ d. None of these

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Q.NO.1 ASSERTION (A):The ratio of the radii of two circles at the centres of which two equal arcs substend angles of 30° and 70° is 21 : 10.

Reason(R):Number of radians in an angle subtended at the centre of a circle by an arc is equal to the ratio of the length of the arc to the radius of the circle.

Q.NO.2 ASSERTION (A):$\frac{\cos(\pi+x).\cos(-x)}{\sin(\pi-x)(\cos\frac{\pi}{2}+x)} = \cot^2 x$

Reason(R):$\cos(\pi+\theta) = -\cos\theta$, $\cos(-\theta) = \cos\theta$, $\sin(\pi-\theta) = \sin\theta$, $\sin(-\theta) = \sin\theta$

2 MARKS QUESTIONS

Q.NO.1 Find the radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm (use π = 22/7).

Q.NO.2 Show that cos 60° + cos 120° + cos 240° - sin 300° = 0

Q.NO.3 Show that $\frac{1+\sin 2x-\cos 2x}{1+\sin 2x+\cos 2x} = \tan x$

Q.NO.4 If x lies in second quadrant, then show that $\frac{\sqrt{1-sinx}}{\sqrt{1+sinx}} + \frac{\sqrt{1+sinx}}{\sqrt{1-sinx}} = 2\sec x$

Q.NO.5 Show that $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2(\frac{x-y}{2})$

3 MARKS QUESTIONS

Q.NO.1 Show that $\cos 2\theta.\cos\frac{\theta}{2} - \cos 3\theta . \cos\frac{9\theta}{2} = \sin 5\theta .\sin\frac{5\theta}{2}$

Q.NO.2 Prove that $\sin^2 A = \cos^2(A-B)+\cos^2 B - 2\cos(A-B)\cos A.\cos B$

Q.NO.3 If $\sin A = \frac{3}{5}$, $\cos B = \frac{-12}{13}$, where A and B both lie in second quadrant , find the value of sin(A+B)

Q.NO.4 Prove that $\tan 8x - \tan 6x - \tan 2x = \tan 9x.\tan 6x.\tan 2x$

Q.NO.5 Prove that $\frac{\sin A+\sin 3A+sin5A+sin7A}{\cos A+cos3A+cos5A+cos7A} = \tan 4A$

5 MARKS QUESTIONS

Q.NO.1 If $\tan x = \frac{-5}{12}$ and x lies in 2nd quadrant , find the value of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, $\tan\frac{x}{2}$

Q.NO.2 Prove that $\cos^2 x + \cos^2(x + \frac{\pi}{3})+\cos^2(x - \frac{\pi}{3}) = \frac{3}{2}$

Q.NO.3 Prove that $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

CASE BASED QUESTIONS

Q.NO.1 Rajiv constructs two right angled triangles in the fourth quadrant in such a way that the measure of triangle gives $\cos A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$ where $\frac{3\pi}{2} < A$ and $B < 2\pi$.

Based on the above information ,answer the following questions

1) Find the value of cos(A+B)

2) Find the value of sin(A-B)

3) Find the value of tan(A+B)

Q.NO.2 In a class test of class XI , a teacher asked to students to consider $A + B = \frac{\pi}{4}$, where A and B are acute angles.

Based on the above information ,answer the following questions

1) Find the value of (1+tan A).(1- tanB)

2) Find the value of (cot B - 1).(cot A - 1)

3) Find the value of sin(A+B)-cos(A+B)+tan(A+B)

MARKING SCHEME

Q.NO.1 Correct option: (b) 1

Solution: $\tan 1° \tan 2° \tan 3° \ldots \tan 89°$

$= [\tan 1° \tan 2° \ldots \tan 44°] \tan 45° [\tan(90° - 44°) \tan(90° - 43°) \ldots \tan(90° - 1°)]$

= [tan 1° tan 2° ... tan 44°] [cot 44° cot 43°..... cot 1°] × [tan 45°]

= [(tan 1° × cot 1°) (tan 2° × cot 2°) .....(tan 44° × cot 44°)] × [tan 45°]

We know that, tan A × cot A =1 and tan 45° = 1

Hence, the equation becomes as;

= 1 × 1 × 1 × 1 × ...× 1= 1 {As 1n = 1}

Q.NO.2 Correct option: (b) 2

Solution: Given, α + β = π/4

Taking "tan" on both sides,

tan(α + β) = tan π/4

We know that, tan(A + B) = (tan A + tan B)/(1 – tan A tan B)

and tan π/4 = 1.

So, (tan α + tan β)/(1 – tan α tan β) = 1

tan α + tan β = 1 – tan α tan β

tan α + tan β + tan α tan β = 1....(i)

(1 + tan α)(1 + tan β) = 1 + tan α + tan β + tan α tan β

= 1 + 1 [From (i)]=2

Q.NO.3 Correct option: a 1,-1

Q.NO.4 Correct option: d $\frac{3}{2}$

Q.NO.5 Correct option: c $\frac{121\pi}{540}$

ASSERTION-REASON BASED QUESTIONS

Q.NO.1 (d) If the radii of the two circles are r_1 and r_2 and l is the length of the arc in either case , then

l = r_1 (circular measure of 30°) = r_1($\frac{30\pi}{180}$ **) and l = r_1 (circular measure of 70°) = r_1(** $\frac{70\pi}{180}$ **)**

$\frac{r_{1\pi}}{6} = \frac{7r_{2\pi}}{18} \Rightarrow \frac{r_1}{r_2} = \frac{7}{3}$

Q.NO.2 (a) $\frac{cos(\pi+x).cos(-x)}{sin(\pi-x)(cos\frac{\pi}{2}+x)} = \frac{-cosx.cos\,x}{sinx(-sinx)}$ **= $\cot^2 x$**

∴ cos($\pi + \theta$) = -cosθ, cos(-θ) = cos θ, sin($\pi - \theta$) = sinθ , sin(-θ) = sin θ

2 MARKS QUESTIONS

Q.NO.1 Solution: Given, Length of the arc = l = 37.4 cm

Central angle = θ = 60° = 60π/180 radian = π/3 radians

We know that,r = l/θ **(1 mark)**

= (37.4) * (π / 3)= (37.4) / [22 / 7 * 3]= 35.7 cm (1 mark)

Q.NO.2 LHS $\cos 60° + \cos 120° + \cos 240° - \sin 300°$

$\cos 60° + \cos(180° - 60°) + \cos(270° - 30°) - \sin(360° - 60°)$ (1 mark)

$\cos 60° - \cos 60° - \cos 30° + \sin 60°$(1/2 mark)

$-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = 0$ =RHS (1/2 mark)

Q.NO.3 LHS $\frac{1+sin2x-\cos 2x}{1+sin2x+cos2x}$

$\frac{sin2x+1-cos2x}{sin2x+1+cos2x}$ (1/2 mark)

$\frac{2sinxcosx+2sin^2x}{2sinxcosx+2cos^2x}$ (1/2 mark)

$\frac{2sinx(cosx+sinx)}{2cosx(sinx+cosx)}$ (1/2 mark)

$\frac{sinx}{cosx}$ =tanx (1/2 mark)

Q.NO.4 LHS $\frac{\sqrt{1-sinx}}{\sqrt{1+sinx}} + \frac{\sqrt{1+sinx}}{\sqrt{1-sinx}}$

$\frac{1-sinx+1+sinx}{\sqrt{1-sin^2x}}$ (1 mark)

$\frac{2}{\sqrt{cos^2x}}$ (1/2 mark)

$\frac{2}{cosx}$ = secx(1/2 mark)

Q.NO.5 LHS $(cosx-cosy)^2 + (sinx-siny)^2$

$Cos^2x+cos^2y-2cosxcosy + sin^2x+sin^2y-2sinxsiny$ (1/2 mark)

$Sin^2x+cos^2x+sin^2y+cos^2y$ -2(sinx siny +cosx cosy)(1/2 mark)

1+1 -2 cos(x-y) (1/2 mark)

2 – 2 cos(x-y)

2 {1 – cos (x-y)}

2 . 2 $\sin^2(\frac{x-y}{2})$ (1/2 mark)

4 $\sin^2(\frac{x-y}{2})$

3 MARKS QUESTIONS

Q.NO.1 LHS $\cos 2\theta \cos\frac{\theta}{2} - \cos 3\theta \,.\, cos\frac{9\theta}{2}$

Using formula $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$ and

$\cos A - \cos B = -2 \sin\frac{A+B}{2} \sin\frac{A-B}{2}$

$\frac{1}{2}$ [cos $(2\theta + \frac{\theta}{2})$ + cos $(2\theta - \frac{\theta}{2})$] - $\frac{1}{2}$ [cos$(3\theta + \frac{9\theta}{2})$ + cos$(3\theta - \frac{9\theta}{2})$] (1 mark)

$\frac{1}{2}$ [cos $\frac{5\theta}{2}$ + cos $\frac{3\theta}{2}$ - cos $\frac{15\theta}{2}$ - cos$(\frac{-3\theta}{2})$]

$\frac{1}{2}$ [cos$\frac{5\theta}{2}$ + cos$\frac{3\theta}{2}$ - cos$\frac{15\theta}{2}$ - cos$\frac{3\theta}{2}$] (1/2 mark)

$\frac{1}{2}$ [cos$\frac{5\theta}{2}$ - cos$\frac{15\theta}{2}$]

$\frac{1}{2}$ [-2 sin$\frac{\frac{5\theta}{2}+\frac{15\theta}{2}}{2}$ sin$\frac{\frac{5\theta}{2}-\frac{15\theta}{2}}{2}$] (1 mark)

- sin $\frac{20\theta}{4}$ sin$(\frac{-10\theta}{4})$

Sin 5θ sin$\frac{5\theta}{2}$ = RHS proved(1/2 mark)

Q.NO.2 RHS $\cos^2$(A-B) +$\cos^2$B - 2 cos(A-B) cosA cosB

$\cos^2$B + $\cos^2$(A-B) – 2 cos (A-B) cosA cosB

$\cos^2$B + cos (A-B)[cos(A-B) – 2cosA cosB] (1/2 mark)

$\cos^2$B + cos (A-B)[cosA cosB +sinA sinB – 2cosA cosB] (1mark)

$\cos^2$B + cos (A-B)[sinA sinB - cosA cosB]

$\cos^2$B - cos (A-B) [cosA cosB - sinA sinB] (1/2 mark)

$\cos^2$B - cos (A-B)cos(A+B)

$\cos^2$B – ($\cos^2$A - $\sin^2$B) (1 mark)

$\cos^2$B +$\sin^2$B - $\cos^2$A

1 - $\cos^2$A= $\sin^2$A = LHS Proved(1/2 mark)

Q.NO.3 sin A = $\frac{3}{5}$ cos B = $\frac{-12}{13}$

where A and B both lie in second quadrant

cos A =$\pm\sqrt{1 - sin^2 A}$ = $\pm\sqrt{1 - \frac{9}{25}}$ =$\pm\frac{4}{5}$(1/2 mark)

A lies in second quadrant so that cosA = - $\frac{4}{5}$(1/2 mark)

sin B=$\pm\sqrt{1 - cos^2 B}$ =$\pm\sqrt{1 - \frac{144}{169}}$ =$\pm\frac{5}{13}$(1/2 mark)

B lies in second quadrant so that sin B=$\frac{5}{13}$(1/2 mark)

Sin(A+B) = sinA cosB + cosA sinB(1/2 mark)

= $\frac{3}{5}$ X $\frac{-12}{13}$ + - $\frac{4}{5}$ x $\frac{5}{13}$(1/2 mark)

= $\frac{-36}{65}$ - $\frac{20}{65}$ =$\frac{-56}{65}$ (1/2 mark) ans

Q.NO.4 LHS tan8x – tan 6x – tan 2x

tan 8x – (tan 6x+tan2x)

by using formula tan8x = tan(6x+2x)=$\frac{tan\,6x+tan2x}{1-tan6x\,tan2x}$ (1 mark)

tan 8x (1-tan6x tan2x)=tan6x+tan2x

tan8x – [tan8x(1-tan6xtan2x)] (1/2 mark)

tan8x – [tan8x –tan8x tan6x tan2x] (1/2 mark)

tan8x – tan8x +tan8x tan6x tan2x(1/2 mark)

tan8x tan6x tan2x=RHS proved(1/2 mark)

Q.NO.5 LHS $\frac{\sin A+\sin 3A+sin5A+sin7A}{\cos A+cos3A+cos5A+cos7A}$

$=\frac{2\,sin\left(\frac{A+3A}{2}\right)cos\left(\frac{A-3A}{2}\right)+2sin\left(\frac{5A+7A}{2}\right)cos\left(\frac{5A-7A}{2}\right)}{2\,cos\left(\frac{A+3A}{2}\right)cos\left(\frac{A-3A}{2}\right)+2cos\left(\frac{5A+7A}{2}\right)cos\left(\frac{5A-7A}{2}\right)}$ **(**1 mark)

$=\frac{2\,sin\left(\frac{4A}{2}\right)cos\left(\frac{-2A}{2}\right)+2sin\left(\frac{12A}{2}\right)cos\left(\frac{-2A}{2}\right)}{2\,cos\left(\frac{4A}{2}\right)cos\left(\frac{-2A}{2}\right)+2cos\left(\frac{12A}{2}\right)cos\left(\frac{-2A}{2}\right)}$

$=\frac{sin2A\,cos\,(-A)+sin6Acos(-A)}{cos2Acos(-A)+cos6Acos(-A)}$ **(**1/2 mark)

$=\frac{sin2A+sin6A}{cos2A+cos6A}$

$=\frac{2\,sin\left(\frac{2A+6A}{2}\right)cos\left(\frac{2A-6A}{2}\right)}{2\,cos\left(\frac{2A+6A}{2}\right)cos\left(\frac{2A-6A}{2}\right)}$ **(**1 mark)

$=\frac{sin4Acos(-2A)}{cos4Acos(-2A)}$

=tan4A = RHS proved(1/2 mark)

5 MARKS QUESTIONS

Q.NO.1 tanx = $\frac{-5}{12}$ **x lies in 2nd quadrant**

$\frac{\pi}{2} \leq$ **x** $\leq \pi$

$\frac{\pi}{4} \leq \frac{x}{2} \leq \frac{\pi}{2}$ **(1/2 mark)**

$1+\tan^2 x = \sec^2 x$ (1/2 mark)

$1+(\frac{-5}{12})^2 = \sec^2 x$

$1+\frac{25}{144} = \sec^2 x$

$\frac{169}{144} = \sec^2 x$

$\pm \frac{13}{12}$ **=secx(1/2 mark)**

x lies in second quadrant cos x =$\frac{-12}{13}$(1/2 mark)

cos$\frac{x}{2}$ = $\sqrt{\frac{1+cosx}{2}}$ =$\sqrt{\frac{1-\frac{12}{13}}{2}}$ = $\sqrt{\frac{1}{26}}$ =$\frac{1}{\sqrt{26}}$ (1 mark)

sin$\frac{x}{2}$ =$\sqrt{\frac{1-cosx}{2}}$ =$\sqrt{\frac{1+\frac{12}{13}}{2}}$ = $\sqrt{\frac{25}{26}}$ =$\frac{5}{\sqrt{26}}$ (1 mark)

tan$\frac{x}{2}$ = $\frac{sin\frac{x}{2}}{cos\frac{x}{2}}$ =5 (1 mark)

Q.NO.2 LHS cos^2x + cos^2(x +$\frac{\pi}{3}$)+cos^2(x - $\frac{\pi}{3}$)

$\frac{1}{2}$ [2cos^2x +2 cos^2x + cos^2(x +$\frac{\pi}{3}$)+cos^2(x - $\frac{\pi}{3}$)] (1 mark)

$\frac{1}{2}$ [(1+cos2x)+1+cos(2x+$\frac{2\pi}{3}$) +1+ cos(2x - $\frac{2\pi}{3}$)] (1 mark)

$\frac{1}{2}$ [3+cos2x +{ cos(2x+$\frac{2\pi}{3}$) + cos(2x - $\frac{2\pi}{3}$)] (1/2 mark)

$\frac{1}{2}$ [3+cos2x+2cos2x cos$\frac{2\pi}{3}$] (1 mark)

$\frac{1}{2}$ [3+cos2x+2cos2x ($\frac{-1}{2}$)] (1 mark)

$\frac{1}{2}$ [3+cos2x - cos2x] =$\frac{3}{2}$ (1/2 mark)

RHS

Q.NO.3 LHS cos 6x

=cos3(2x) (1/2 mark)

=4$cos^3$2x – 3cos2x (1 mark)

= 4[2cos^2x-1]^3 – 3 (2cos^2x-1) (1/2 mark)

= 4 [(2cos^2x)^3 -1 -3 . 2cos^2x (2cos^2x – 1)]-6cos^2x+3 (1 mark)

=4[8cos^6x – 1 -12 cos^4x +6cos^2x] – 6cos^2x +3(1 mark)

=32cos^6x – 4 – 48cos^4x +24cos^2x – 6cos^2x +3 (1 mark)

=32cos^6x – 48cos^4x +18cos^2x -1

CASE BASED QUESTIONS

Q.NO.1 Given , cosA= $\frac{4}{5}$, where $\frac{3\pi}{2} < A$, B $< 2\pi$

∴ sin A =$-\sqrt{1-cos^2A}$ = $-\frac{3}{5}$ (1/2 mark)

Cos B = $\frac{12}{13}$

∴ sin B =$-\sqrt{1-cos^2B}$ = $-\frac{5}{13}$ (1/2 mark)

i) cos(A+B) = cos A cos B – sin A sin B = $\frac{4}{5}$. $\frac{12}{13}$ -($-\frac{3}{5}$).($-\frac{5}{13}$) = $\frac{33}{65}$ (1 mark)

ii) sin (A - B) = sin A cos B - cos A .sin B = $-\frac{3}{5}$. $\frac{12}{13}$ - $\frac{4}{5}$. ($-\frac{5}{13}$) = $\frac{16}{65}$ (1 mark)

iii) sin (A + B) = sin A cos B + cos A .sin B = $-\frac{3}{5}$. $\frac{12}{13}$ + $\frac{4}{5}$. ($-\frac{5}{13}$) = $-\frac{56}{65}$ (1 mark)

tan (A + B) = $\frac{sin\,(A+B)}{cos\,(A+B)}$ = $\frac{\frac{-56}{65}}{\frac{33}{65}}$ = $-\frac{56}{33}$ (1 mark)

Q.NO.2 A + B =$\frac{\pi}{4}$,

tan(A + B) = $tan\ \frac{\pi}{4}$,

$i)\ \frac{tan\,A+tan\,B}{1-tan\,A\,tan\,B}$ = 1

tan A + tan B = 1 – tan A tan B

tan A + tan B + tan A tan B =1 (eq no 1)

1 + tan A + tan B + tan A tan B =1 + 1

(1 + tan A) + tan B (1 + tan A) = 2

(1 + tan A) (1 + tan B) = 2

ii) on dividing both sides of eq 1 bby tan A tan B ,

$$\frac{tan\,A+tan\,B+tan\,A\,tan\,B}{tan\,A\,tan\,B} = \frac{1}{tan\,A\,tan\,B}$$

cot B + cot A + 1= cot A cot B

cot A cot B - cot B - cot A = 1

cot A cot B - cot B - cot A + 1= 2

cot A (cot B – 1) – (cot B – 1) = 2

(cot B – 1) (cot A – 1) = 2

iii) A + B =$\frac{\pi}{4}$,
sin (A + B) = sin $\frac{\pi}{4}$
sin (A + B) = $\frac{1}{\sqrt{2}}$
cos (A + B) = cos $\frac{\pi}{4}$
cos (A + B) = $\frac{1}{\sqrt{2}}$
tan(A + B) = $tan\ \frac{\pi}{4}$
tan(A + B) = 1
sin(A+B)-cos(A+B)+tan(A+B) = $\frac{1}{\sqrt{2}}$ - $\frac{1}{\sqrt{2}}$ +1 = 1

(Chapter -4: COMPLEX NUMBERS)

MCQs

1. **If a + ib = c + id, then**

   (a) $a^2 + c^2 = 0$ (b) $b^2 + c^2 = 0$ (c) $b^2 + d^2 = 0$ (d) $a^2 + b^2 = c^2 + d^2$

2. **If 1 – i, is a root of the equation $x^2 + ax + b = 0$, where a, b ∈ R, then the value of a – b is**

   (a) -4 (b) 0 (c) 2 (d) 1

3. **If $[(1 + i)/(1 – i)]^x = 1$, then**
   (a) x = 2n + 1, where n ∈ N (b) x = 4n, where n ∈ N
   (c) x = 2n, where n ∈ N (d) x = 4n + 1, where n ∈ N

4. **The simplified value of $(1 – i)^3/(1 – i^3)$ is**

   (a) 1 (b) -2 (c) –I (d) 2i

5. **sin x + i cos 2x and cos x – i sin 2x are conjugate to each other for:**

   (a) x = nπ (b) x = [n + (1/2)] (π/2) (c) x = 0 (d) No value of x

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Answers with explanation

Ans . Q.1. (d) $a^2 + b^2 = c^2 + d^2$

$a + ib = c + id$

$\Rightarrow |a + ib| = |c + id|$

$\Rightarrow \sqrt{(a^2 + b^2)} = \sqrt{(c^2 + d^2)}$

Squaring on both sides, we get;

$a^2 + b^2 = c^2 + d^2$

Ans. Q.2. Correct option: (a) -4

Given that 1 – i is the root of $x^2 + ax + b = 0$.

Thus, 1 + i is also the root of the given equation since non-real complex roots occur in conjugate pairs.

Sum of roots = $-a/1 = (1 – i) + (1 + i)$

$\Rightarrow a = – 2$

Product of roots, $b/1 = (1 – i)(1 + i)$

$b = 1 – i^2$

$b = 1 + 1$ {since $i^2 = -1$}

$\Rightarrow b = 2$

Now, $a - b = -2 - 2 = -4$

Ans. Q.3. Correct option: (b) x = 4n, where n ∈ N

Solution: Given,
$[(1 + i)/(1 - i)]^x = 1$
By rationalizing the denominator,
$[(1 + i)(1 + i)/ (1 - i)(1 + i)]^x = 1$
$[(1 + i)^2/ (1 - i + i - i^2)]^x = 1$
$[(1 + i^2 + 2i)/(1 + 1)]^x = 1$
$[(1 - 1 + 2i)/ 2]^x = 1$
$i^x = 1$
Thus, $i^x = i^{4n}$, where n is any positive integer.

Ans Q.4. Correct option: (b) -2

Solution:

$(1 - i)^3/(1 - i^3)$

$= (1 - i)^3/(1^3 - i^3)$

$= (1 - i)^3/ [(1 - i)(1 + i + i^2)]$

$= (1 - i)^2/(1 + i - 1)$

$= (1 - i)^2/i$

$= (1 + i^2 - 2i)/i$

$= (1 - 1 - 2i)/i$

$= -2i/i$

$= -2$

Ans Q.5. Correct option: (d) No value of x

Solution:

Consider sin x + i cos 2x and cos x – i sin 2x are conjugate to each other.

So, sin x – i cos 2x = cos x – i sin 2x

On comparing real and imaginary parts of both sides, we get

$\Rightarrow$ sin x = cos x and cos 2x = sin 2x

$\Rightarrow$ sin x/cos x = 1 and (cos 2x/sin 2x) = 1

$\Rightarrow$ tan x = 1 and tan 2x = 1

Now, consider tan 2x = 1

Using the formula tan 2A = 2 tan A/(1 – $\tan^2$A),

(2 tan x)/(1 – $\tan^2$x) = 1

However, this is not possible for tan x = 1.

Therefore, for no value of x, sinx + i cos 2x and cos x – i sin 2x are conjugate to each other.

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ASSERTION REASONING QUESTIONS

Instructions : Each question consists of two statements, namely, Assertion (A) and Reason (R).For selecting the correct answer, use the following code:
(a) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A).
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is true.

Q.1. Assertion(A) : If z_1= 2+3i and z_2 = 3-2i, then $z_1 - z_2$ = -1+5i
Reason (R) : If z_1 = a + ib and z_2 = c + id, then $z_1 - z_2$ = (a-c) + i(b-d)

Q.2. Assertion(A) : If (1 + i)(x + iy) = 2-5i, then x = -3/2 and y = -7/2

Reason (R) : if a + ib = c + id, then a=c and b = d

Q.3. Assertion(A) : The multiplicative inverse of 2 – 3i is 2 +3i

Reason (R) : If z = 3+4i, then $\bar{z}$ = 3-4i

Answers

Q.1. (a) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).

Q.2. (a) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).

Q.3. (d) Assertion (A) is false and Reason (R) is true.

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VSA Type Questions (2 marks each)

Q.1. **Write the complex number $i^9 + i^{19}$ in the form of a + ib.**

Q.2. Express the given expression (1 + i) (1 + 2i) in the form a + ib and find the values of a and b.

Q.3. Determine the multiplicative inverse of 4 – 3i.

Q.4. Find the modulus of the complex number : (1 + 2i) / (1 – 3i).

Q.5. If $\left(\frac{1+i}{1-I}\right)^m$ = 1, find the least positive integral value of m.

Answers

Sol Q(1) : Given number: $i^9 + i^{19}$.

The expression $i^9 + i^{19}$ can be represented as follows:

$i^9 + i^{19} = (i^2)^4 . i + (i^2)^9 . i$ …(1)

We know that, $i^2 = 1$.

On substituting $i^2 = -1$, we get

$i^9 + i^{19} = (-1)^4.i + (-1)^9.i$

$i^9 + i^{19} = 1.i + (-1).i$

$i^9 + i^{19} = i – i$

$i^9 + i^{19} = 0$.

Therefore, $i^9 + i^{19}$ in the form of a + ib is 0 + i0.

Sol Q(2) : Given expression: (1 + i) (1 + 2i)

Hence, (1 + i) (1 + 2i) = 1(1) + 1(2i) + i + 2i(i)

(1 + i) (1 + 2i) = $1 + 2i + i + 2i^2$

(1 + i) (1 + 2i) = 1 + 2i + i + 2(-1) [As, $i^2 = -1$]

(1 + i) (1 + 2i) = 1 + 2i + i – 2

(1 + i) (1 + 2i) = -1 + 3i

Hence, the expression (1 + i) (1 + 2i) in the form of a + bi is -1 + 3i.

Thus, the value of a = -1 and b = 3.

Sol Q(3) : Let z = 4 – 3i.

As we know, the multiplicative inverse of z is 1/z.

Hence, 1/z = 1/ (4+3i)

Therefore, the multiplicative inverse of 4 – 3i is:

$= \frac{1}{4 - 3i} \times \frac{4+ 3i}{4+ 3i}$

$= \frac{4+ 3i}{16+9}$

$= \frac{4+ 3i}{25}$

Therefore, the multiplicative inverse of 4 – 3i is (4 + 3i)/25.

Sol Q(4) : $\left|\frac{1+2i}{1-3i}\right| = \frac{|1+2i|}{|1-3i|} = \frac{\sqrt{1^2+2^2}}{\sqrt{1^2+(-3)^2}} = \frac{\sqrt{5}}{\sqrt{10}} = \frac{1}{\sqrt{2}}$

Sol Q(5) $\left(\frac{1+i}{1-I}\right)^m = 1$

$$\Rightarrow \left(\frac{1+i}{1-I} \times \frac{1+i}{1+i}\right)^m = 1$$

$$\Rightarrow \left(\frac{1+2i-1}{1+1}\right)^m = 1$$

$$\Rightarrow (i^m = 1)$$

Therefore, the least positive integral value of m is 4.

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SA Type Questions (3 marks each)

Q.1. If z_1 and z_2 are the two complex numbers, then show that :

$Re\ (z_1z_2) = Re\ z_1\ Re\ z_2 - Im\ z_1\ Im\ z_2$.

Q.2. Compute the value of $(1-i)^n\ [1-(1/i)]^n$ for a positive integer "n".

Q.3. Express in the form of a + bi : $(1/3 + 3i)^3$

Q.4. Express in the form of a + ib: $(3i - 7) + (7 - 4i) - (6 + 3i) + i^{23}$

Q.5. The complex numbers sin x + i cos 2x is the conjugate to itself. Find the value of x.

Answers

Sol Q(1) : Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$

Now, take the product of these two complex numbers.

$z_1z_2 = (a_1 + ib_1)(a_2 + ib_2)$

$z_1z_2 = a_1(a_2 + ib_2) + ib_1(a_2 + ib_2)$

$z_1z_2 = a_1a_2 + ia_1b_2 + ia_2b_1 + i^2b_1b_2$

$z_1z_2 = a_1a_2 + ia_1b_2 + ia_2b_1 - b_1b_2$ [since $i^2 = -1$]

Now, the above equation can be rearranged as follows:

$z_1z_2 = a_1a_2 - b_1b_2 + ia_1b_2 + ia_2b_1$

$z_1z_2 = (a_1a_2 - b_1b_2) + i(a_1b_2 + a_2b_1)$

Here, the real part is:

$Re\ (z_1z_2) = a_1a_2 - b_1b_2$

Thus,

$Re\ (z_1z_2) = Re\ z_1\ Re\ z_2 - Im\ z_1\ Im\ z_2$.

Hence, $Re\ (z_1z_2) = Re\ z_1\ Re\ z_2 - Im\ z_1\ Im\ z_2$ is proved.

Sol Q(2) : Given: $(1 – i)^n [1- (1/i)]^n$

We know that $i^4 = 1$,

Hence, $(1 – i)^n [1- (1/i)]^n = (1 – i)^n [1- (i^4/i)]^n$

$(1 – i)^n [1- (1/i)]^n = (1 – i)^n [1 – i^3]^n$

Further, the above equation is written as follows:

$(1 – i)^n [1- (1/i)]^n = (1 – i)^n (1 + i)^n$ [Since $i^3 = -i$]

$(1 – i)^n [1- (1/i)]^n = [(1 – i) (1 + i)]^n$

The expression (1 – i) (1 + i) is of the form (a – b)(a + b), which is equal to $a^2 – b^2$.

Here, a = 1, b = 1

Thus, $(1 – i)^n [1- (1/i)]^n = (1 – i^2)^n$

$(1 – i)^n [1- (1/i)]^n = (1 – (-1))^n$ [As, $i^2 = -1$]

$(1 – i)^n [1- (1/i)]^n = (1 + 1)^n$

$(1 – i)^n [1- (1/i)]^n = 2^n$.

Therefore, the value of $(1 – i)^n [1- (1/i)]^n$ is 2^n.

Sol Q(3) : $(1/3 + 3i)^3 = (1/3)^3 + (3i)^3 + 3(1/3) (3i) (1/3 +3i)$

$= (1/27) + 27i^3 + 3i(1/3 +3i)$

$= 1/27 + 27(-i) + i + 9i^2$ [$i^3 = -i$]

$= 1/27 – 27i + i – 9$ [$i^2 = -1$]

$= (1/27 – 9) + i(-27 + 1)$

$= -242/27 – 26i$

Sol Q(4) : $(3i – 7) + (7 – 4i) – (6 + 3i) + i^{23}$

$= 3i – 7 + 7 – 4i – 6 – 3i + i^{23}$

$= -4i – 6 + i^{22+1}$

$= -4i – 6 + (i^2)^{11} . i$

$= -4i – 6 + (-1)^{11} . i$

$= -4i – 6 – i$

Thus, $(3i – 7) + (7 – 4i) – (6 + 3i) + i^{23} = -6 – 5i$

Sol Q(5) : Complex number = sin x + i cos 2x (given)

Conjugate number = sin x – i cos 2x

Now, sin x + i cos 2x = sin x – i cos 2x

⇒ sin x = cos x and sin 2x = cos 2x [comparing real and imaginary part]

⇒ tan x = 1

and tan 2x = 1

Now, both of them are not possible for the same value of x.

Therefore, x has no value.

==========**LA Type Questions (5 marks each)**

Q.1. If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$

Q.2. If $(x+iy)^3 = u+iv$, where $u,v,x,y \in R$, then show that $\frac{u}{x}+\frac{v}{y} = 4(x^2-y^2)$

Q.3. Find x and y , if $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i} = i, where\ i = \sqrt{-1}\ and\ x, y\ \in R$

Answers

Sol.Q.1. we know that $|z|^2 = z.\bar{z}$

$$\Rightarrow \quad \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2 = \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\overline{\frac{\beta-\alpha}{1-\bar{\alpha}\beta}}\right)$$

$$= \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\frac{\bar{\beta}-\bar{\alpha}}{1-\alpha\bar{\beta}}\right) \quad \textbf{using properties of conjugate}$$

$$= \frac{1-\bar{\alpha}\beta-\alpha\bar{\beta}+|\alpha|^2}{1-\bar{\alpha}\beta-\alpha\bar{\beta}+|\alpha|^2} \quad \text{simplifying and using } |z|^2 = z.\bar{z} \textbf{ as well as } |\beta| = 1$$

$$= 1$$

$$\Rightarrow \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1 \quad \textbf{proved .}$$

Sol.Q.2. **Given, $(x+iy)^3 = u+iv$**

$$\Rightarrow x^3 + (iy)^3 + 3.x.iy(x+iy) = u + iv$$

$$\Rightarrow (x^3-3xy^2) + i(3x^2y-y^3) = u + iv$$

on equating real and imaginary parts , we get

$$u = x^3-3xy^2, v = 3x^2y-y^3$$

$$\Rightarrow \frac{u}{x} = x^2 - 3y^2 \text{ and } \frac{v}{y} = 3x^2 - y^2$$

$$\Rightarrow \frac{u}{x}+\frac{v}{y} = 4(x^2-y^2)$$

Sol.Q.3. Given equation is $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i} = i$

simplifying and comparing the real and imaginary parts from both sides, we get

$$4x + 9y - 3 = 0 \qquad \text{----- (1)}$$

$2x-7y-13=0$ ----(2)

Solving (1) and (2) , we get

x = 3 and y = -1

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CASE Based Questions

Q.1. A complex number z is said to be purely real iff $\bar{z} = z$ and purely imaginary iff $\bar{z} = -z$, where $\bar{z}$ is the conjugate of z. If z = x + iy , then $\bar{z} = x - iy$.

Based on the above information, answer the following questions :

(i) Which of the followings is not true

(a) $\overline{z_1 + z_2} = \bar{z_1} + \bar{z_2}$ (b) $\overline{z_1 z_2} = \bar{z_1}\bar{z_2}$ (c) $\overline{z_1 - z_2} = \bar{z_2} - \bar{z_1}$ (d) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z_1}}{\bar{z_2}}$ 1 mark

(ii) If $z_1 = 1 + i \ and \ z_2 = 2 - i, then \ \overline{z_1 z_2}$ = ?

(a) 3-i (b) i-3 (c) 3 + i (d) none of these 1 mark

(iii) If $z = \frac{3+2iSin\theta}{1-2iSin\theta} (0 < \theta \leq \pi/2)$ is purely imaginary , then θ is equal to

(a) $\pi/4$ (b) $\pi/6$ (c) $\pi/3$ (d) $\pi/12$ 2 marks

Q.2. Two complex numbers z_1 = a + ib and z_2 = c+ id are said to be equal iff a = c and b = d.

On the basis of above information, answer the following questions :

(i) If (x+y) + i(x-y) = 4 + 6i, then xy = ?

(a) 5 (b) -5 (c) 4 (d) -4

(ii) If $\frac{(i+i)^2}{2-i} = x + iy$, then the value of x + y is

(a) 1/5 (b) 3/5 (c) 4/5 (d) 2/5

Answers

Q.1. (i) a (ii) c (iii) c

Q.2. (i) b (ii) d

(Chapter -5: LINEAR INEQUALITY)

LINEAR INEQUALITY

Q1) If $|x - 1| > 5$, then

(a) $x \in (-4, 6)$

(b) $x \in [-4, 6]$

(c) $x \in (-\infty, -4) \cup (6, \infty)$

(d) $x \in [-\infty, -4) \cup [6, \infty)$

Q2) If $x < 5$, then

(a) $-x < -5$

(b) $-x \leq -5$

(c) $-x > -5$

(d) $-x \geq -5$

Q3) If $-3x + 17 < -13$, then

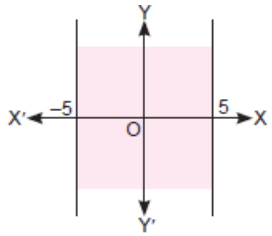
(a) $x \in (10, \square)$

(b) $x \in [10, \square)$

(c) $x \in (-\square, 10]$

(d) $x \in [-10, 10)$

Q4)The inequality representing following graph is



a) $|x| < 5$ (b) $|x| \leq 5$ (c) $|x| > 5$ (d) $|x| \geq 5$

Q5) If x, y and b are real numbers and $x < y$, $b < 0$, then

a) $\frac{x}{b} < \frac{y}{b}$
b) $\frac{x}{b} > \frac{y}{b}$
c) $\frac{x}{b} \geq \frac{y}{b}$
d) $\frac{x}{b} \leq \frac{y}{b}$

2 MARKS

Q6) Draw the graphical solution of the following inequation : $x > 3$

Q7) Solve for x : $3x - 4 \geq -2x + 6$ when (*i*) x is a natural number (*ii*) x is a real number.

Q8) Solve : $3x + 5 < x - 7$, when (*i*) x is an integer (*ii*) x is a real number.

Q9) Draw the graphical solution of the following system of inequation :

$$\frac{4-3X}{5} < \frac{2X-5}{4}$$

Q10) Solve $-12x > 30$, when (*i*) x is a natural number (*ii*) x is an integer.

3 MARKS

Q11) Solve :$-5 \leq \frac{2-3X}{4} \leq 9$

Q12) Solve : $5(2x - 7) - 3(2x + 3) \leq 0$ and $2x + 19 \leq 6x + 47$ and represent the solution on number line.

Q13) Draw the graphical solution of the following system of inequation : $-3 \leq \frac{4-7X}{2} \leq 18$

Q14) I.Q. of a person is given by formula I.Q. = $\frac{MENTAL\ AGE}{CHRONOLOGICAL\ AGE} \times 100$, where M.A. stands for mental age and C.A., stands for chronological age. If $75 \leq$ I.Q. ≤ 135 for a group of 9 year children. Find the range of their mental age.

Q15) Given set A = $\{x \in Z : 0 \leq x \leq 10\}$. Solve the inequation $-2x + 6 \leq 5x - 4$ in set A.

CASE BASE 4 MARKS

Q16

The marks of four students out of 100 in 4 test are given below and the grading scheme is also given. Read the given information carefully and answer the following.

Name	Test-1	Test-2	Test-3	Test-4
Prachi	85	93	94	89
Reshma	75	86	76	75
Ankit	92	83	44	60
Sunil	59	81	62	73

Grading system

Average Marks (x)	Grade
$x \geq 91$	A_1
$90 \geq x \geq 81$	A_2
$80 \geq x \geq 71$	B_1
$70 \geq x \geq 61$	B_2
$60 \geq x \geq 51$	C

(i) To get grade A_1 the minimum marks Prachi should score in test-5 is

(a) 94 (b) 95

(c) 90 (d) ≥ 94

(ii) The marks Reshma should score in Test-5 to get grade A_2 is

(a) ≥ 95 (b) 93

(c) ≥ 93 (d) 90

(iii) If Ankit scored 91 marks in his test-5, then he will get grade

(a) A_2 (b) B_1

(c) B_2 (d) C

(iv) To get average marks more than Ankit, the minimum marks Sunil should score is

(a) 96 (b) 95

(c) 98 (d) 97

Q1717 A manufacturer has 640 liters of 8% boric acid solution and 1300 liters of 2% boric acid solution. To dilute the 8% boric acid solution manufacturer added some amount of 2% boric acid solution into it.

(i) The maximum amount of 2% boric acid solution to be added so that resulting mixture is to be more than 4% boric acid is

(a) 1280 *l* (b) 1278 *l*

(c) 1300 *l* (d) 1200 *l*

(ii) The maximum amount of 2% boric acid solution left after adding to 8% boric acid solution so that resulting mixture to be less than or equal to 6% boric acid is

(a) 20 *l* (b) 800 *l*

(c) 980 *l* (d) 1000 *l*

(iii) If manufacturer used water in place of 2% boric acid solution for dilution of 8% boric acid solution, then amount of water to be added so that resulting mixture is more than 2% boric acid is

(a) ≤ 1920 *l* (b) 3840 *l*

(c) 1920 *l* (d) <1920 *l*

(iv) The minimum amount of water to be added to 8% boric acid solution so that resulting mixture is less than 4% boric acid solution is

(a) 641 *l* (b) 640 *l*

(c) 648 *l* (d) 650 *l*

18 A company produces certain items. The manager in the company used to make data record on daily basis about the cost and revenue of these items separately. The cost function is given by C(x) = 40x + 14000 and the revenue function is given by R(x) = 80x + 6000, where x is the number of items produced and sold considering that all items produced are sold.

(i) Number of items produced to get some profit is

(a) > 200 (b) 200

(c) ≥ 200 (d) > 102

(ii) If the cost function becomes C(x) = 40x + 12000 and revenue function remains same then number of items produced to get some profit is

(a) ≥ 150 (b) > 150

(c) > 200 (d) ≤ 150

(iii) To get at least ₹ 26,000 revenue per day the minimum number of items produced and sold per day is

(a) 255 (b) 257

(c) 250 (d) 300

(iv) The profit company will get after producing minimum number of items to get at least 26,000 revenue per day

(a) 2,400 (b) 20,000

(c) 24,000 (d) 2,000

(v) If the company has only ₹ 22,000 to invest in producing items per day. The maximum number of items that can be produced is

(a) 250 (b) ≥ 200

(c) 200 (d) ≤ 200

5 MARKS

Q19)A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18% ?

Q20)A company manufactures cassettes. Its cost and revenue functions are $C(x) = 26{,}000 + 30x$ and $R(x) = 43x$, respectively, where x is the number of cassettes produced and sold in a week. How many cassettes must be sold by the company to realise some profit ?

Q21)The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, find the range of pH value for the third reading that will result in the acidity level being normal.

Assertion and reasoning

Q1) Assertion : If a<b,c<0 then $\frac{a}{c} < \frac{b}{c}$,

Reasoning:if both sids are divided by same negative quantity, then inequality is reversed

Q2) Assertion: the line divide the cartisian plane in two parts

Reasoning:if a point P(a,b) lie on line dx+ey =f then da+eb=f .

(Chapter -6: PERMUTATION AND COMBINATION)

Q -1 In how many ways 5 girls and 3 boys be seated in a row, so that no two boys are together?

1. 1440
2. 14400
3. 3600
4. None of the above

Q-2 If ${}^{2n}C_3 : {}^{n}C_2 = 12 : 1$, then the value of n is ?

1. 6
2. 5
3. 4
4. None of the above

Q-3 In how many ways a committee, consisting of 5 men and 6 women can be formed from 8 men and 10 women?

1. 266
2. 5040
3. 11760
4. None of the above

Q-4 In how many different ways can the letters of the word 'GEOGRAPHY' be arranged such that the vowels must always come together?

1. 2520
2. 2530
3. 15130
4. 15120

Q-5 In how many ways can the word CHRISTMAS be arranged so that the letters C and M are never adjacent?

1. $8! \times (7/2)$
2. $9! \times (7/2)$
3. $8! \times (9/2)$
4. None of the above

Q-1	2
Q-2	2
Q-3	3

Q-4	4
Q-5	1

Assertion and Reason Questions
Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (C) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.
1. Assertion: If the letters W, I, F, E are arranged in a row in all possible ways and the words (with or without meaning) so formed are written as in a dictionary, then the word WIFE ocurs in the 24" position.
Reason: The number of ways of arranging four distinct objects taken all at a time is C(4,4).
2. Assertion: A number of four different digits is formed with the help of the digits 1,2,3,4,5,6,7 in all possible ways. Then, number of ways which are exactly divisible by 4 is 200.
Reason: A number divisible by 4, if unit place digit is divisible by 4
3. Assertion: Product of five consecutive natural numbers is divisible

2 MARKS QUESTION

Question 1:

Find the 3-digit numbers that can be formed from the given digits: 1, 2, 3, 4 and 5 assuming that

a) digits can be repeated.

b) digits are not allowed to be repeated.

Solution:

a) By the multiplication principle, the number of ways in which three-digit numbers can be formed from the given digits is 5 × 5 × 5 = 125

b) By the multiplication principle, the number of ways in which three-digit numbers can be formed without repeating the given digits is 5 × 4 × 3 = 60

Question 2:

A coin is tossed 6 times, and the outcomes are noted. How many possible outcomes can be there?

Solution:

When we toss a coin once, the number of outcomes we get is 2 (Either Head or tail)

So, in each throw, the no. of ways to get a different face will be 2.

Therefore, by the multiplication principle, the required no. of possible outcomes is

2 x 2 x 2 x 2 × 2 × 2 = 64

Question 3:

Evaluate the following

(i) 6 ! (ii) 5 ! – 2 !

Solution:

(i) 6! = 1 × 2 × 3 × 4 × 5 × 6 = 720

(ii) 5! = 1 × 2 × 3 × 4 x 5 = 120

As 2! = 1 × 2 = 2

Therefore, 5 ! – 2 ! = 120-2 = 118.

Question 4:

From a team of 6 students, in how many ways can we choose a captain and vice-captain assuming one person can not hold more than one position?

Solution:

From a team of 6 students, two students are to be chosen in such a way that one student will hold only one position.

Here, the no. of ways of choosing a captain and vice-captain is the permutation of 6 different things taken 2 at a time.

So, ${}^{6}P_{2}$ = 6! / (6 -2)! = 6! / 4! = 30

Question 5:

How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

Solution:

Number of letters in word EQUATION` = 8

n = 8

If all letters of the word used at a time

r = 8

Different numbers formed = nPr

= ${}^{8}P_{8}$

= 8!/(8 8)!

= 8!/0!

= 8!/1

= 8!

= 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1

= 40320

3 MARKS QUESTION

1. How many numbers are there between 99 and 1000, having at least one of their digits 7?

Solution:

Numbers between 99 and 1000 are all three-digit numbers.

Total number of 3 digit numbers having at least one of their digits as 7 = (Total numbers of three-digit numbers) – (Total number of 3 digit numbers in which 7 does not appear at all)

= (9 × 10 × 10) – (8 × 9 × 9)

= 900 – 648

= 252

2. How many 5-digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once?

Solution:

Let ABCDE be a five-digit number.

Given that the first two digits of each number are 6 and 7.

Therefore, the number is 67CDE.

As repetition is not allowed and 6 and 7 are already taken, the digits available for place C are 0, 1, 2, 3, 4, 5, 8, 9, i.e. eight possible digits.

Suppose one of them is taken at C, now the digits possible at place D is 7.

Similarly, at E, the possible digit is 6.

Therefore, the total five-digit numbers with given conditions = 8 × 7 × 6 = 336.

3. Find the number of permutations of the letters of the word ALLAHABAD.

Solution:

Given word – ALLAHABAD

Here, there are 9 objects (letters) of which there are 4As, 2 Ls and rest are all different.

Therefore, the required number of arrangements = 9!/(4! 2!)

= (1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9)/ (1 × 2 × 3 × 4 × 1 × 2)

= (5 × 6 × 7 × 8 × 9)/2

= 7560

4. **In how many of the distinct permutations of the letters in MISSISSIPPI do the four Is not come together?**

Solution:

Given word – MISSISSIPPI

M – 1

I – 4

S – 4

P – 2

Number of permutations = 11!/(4! 4! 2!) = (11 × 10 × 9 × 8 × 7 × 6 × 5 × 4!)/ (4! × 24 × 2)

= 34650

We take that 4 I's come together, and they are treated as 1 letter,

∴ Total number of letters=11 – 4 + 1 = 8

⇒ Number of permutations = 8!/(4! 2!)

= (8 × 7 × 6 × 5 × 4!)/ (4! × 2)

= 840

Therefore, the total number of permutations where four Is don't come together = 34650 – 840 = 33810

5. In a small village, there are 87 families, of which 52 families have at most 2 children. In a rural development programme, 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made?

Solution:

Given,

Total number of families = 87

Number of families with at most 2 children = 52

Remaining families = 87 – 52 = 35

Also, for the rural development programme, 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children.

Thus, the following are the number of possible choices:

52C18 × 35C2 (18 families having at most 2 children and 2 selected from other types of families)

52C19 × 35C1 (19 families having at most 2 children and 1 selected from other types of families)

52C20 (All selected 20 families having at most 2 children)

Hence, the total number of possible choices = 52C18 × 35C2 + 52C19 × 35C1 + 52C20

5 MARKS QUESTION

1. A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

Solution:

Given,

Men = 2

Women = 3

A committee of 3 persons to be constituted.

Here, the order does not matter.

Therefore, we need to count combinations.

There will be as many committees as combinations of 5 different persons taken 3 at a time.

Hence, the required number of ways = 5C3

= 5!/(3! 2!)

= (5 × 4 × 3!)/(3! × 2)

= 10

Committees with 1 man and 2 women:

1 man can be selected from 2 men in 2C1 ways.

2 women can be selected from 3 women in 3C2 ways.

Therefore, the required number of committees = 2C1 × 3C2

= 2 × 3C1

= 2 × 3

= 6

2. Determine the number of 5 card combinations out of a deck of 52 cards, if there is exactly one ace in each combination.

Solution:

Given a deck of 52 cards

There are 4 Ace cards in a deck of 52 cards.

According to the given, we need to select 1 Ace card out of the 4 Ace cards

∴ The number of ways to select 1 Ace from 4 Ace cards is 4C1

⇒ More 4 cards are to be selected now from 48 cards (52 cards – 4 Ace cards)

∴ The number of ways to select 4 cards from 48 cards is 48C4

Number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination = 4C1 × 48C4

= 4 × [48!/(44! 4!)]

= 4 × [(48 × 47 × 46 × 45 × 44!)/ (44! × 24)]

= 4 × 2 × 47 × 46 × 45

= 778320

3. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has

(i) no girls

(ii) at least one boy and one girl

(iii) at least three girls

Solution:

Given,

Number of girls = 7

Number of boys = 7

(i) No girls

Total number of ways the team can have no girls = 4C0 × 7C5

= 1 × 21

= 21

(ii) at least one boy and one girl

1 boy and 4 girls = 7C1 × 4C4 = 7 × 1 = 7

2 boys and 3 girls = 7C2 × 4C3 = 21 × 4 = 84

3 boys and 2 girls = 7C3 × 4C2 = 35 × 6 = 210

4 boys and 1 girl = 7C4 × 4C1 = 35 × 4 = 140

Total number of ways the team can have at least one boy and one girl = 7 + 84 + 210 + 140

= 441

(iii) At least three girls

Total number of ways the team can have at least three girls = 4C3 × 7C2 + 4C4 × 7C1

= 4 × 21 + 7

= 84 + 7

= 91

9. How many numbers greater than 1000000 can be formed using the digits 1, 2, 0, 2, 4, 2, 4?

Solution:

Given numbers – 1000000

Number of digits = 7

The numbers have to be greater than 1000000, so they can begin either with 1, 2 or 4.

When 1 is fixed at the extreme left position, the remaining digits to be rearranged will be 0, 2, 2, 2, 4, 4, in which there are 3, 2s and 2, 4s.

Thus, the number of numbers beginning with 1 = 6!/(3! 2!) = (6 × 5 × 4 × 3!)/(3! × 2)

= 60

The total numbers begin with 2 = 6!/(2! 2!) = 720/4 = 180

Similarly, the total numbers beginning with 4 = 6!/3! = 720/6 = 120

Therefore, the required number of numbers = 60 + 180 + 120 = 360.

10. 18 mice were placed in two experimental groups and one control group, with all groups equally large. In how many ways can the mice be placed into three groups?

Solution:

Number of mice = 18

Number of groups = 3

Since the groups are equally large, the possible number of mice in each group = 18/3 = 6

The number of ways of placement of mice =18!

For each group, the placement of mice = 6!

Hence, the required number of ways = 18!/(6!6!6!) = 18!/(6!)3

Case study

Read the Case study given below and attempt any 4 subparts:

One evening, four friends decided to play a card game Rummy. Rummy is a card game that is played with decks of cards. To win the rummy game a player must make a valid declaration by picking and discarding cards from the two piles given. One pile is a closed deck, where a player is unable to see the card that he is picking, while the other is an open deck that is formed by the cards discarded by the players. To win at a rummy card game, the players have to group cards invalid sequences and sets.

In rummy, the cards rank low to high starting with Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. Ace, Jack, Queen, and King each have 10 points. The remaining cards have a value equal to their face value. For example, 5 cards will have 5 points, and so on. Four cards are drawn from a pack of 52 playing cards, then:

i. How many different ways can this is done

ii. Exactly one card of each suit

iii. all cards of the same suit

While reading the instructions, he observed that the question paper consists of 12 questions divided into two parts - Part I and Part II, containing 5 and 7 questions, respectively.

Based on the given information, answer the following questions.

(i) If Anish is required to attempt 8 questions in all, selecting at least 3 from each part, then in how many ways can he select the questions?

(ii) If Anish is required to attempt 8 questions in all, selecting at most 3 from part I, then in how many ways can he select the questions?

Ques. Members of the BTS group, RM, Suga, V, Jin, Jungkook,Jimin, J Hope, are to be seated in a row for a photo session

i) Total number of ways of sitting arrangement of seven members is

(a) 28 (b) 5040 (c) 2520 (d) None of these

(ii) Total number of arrangements so that RM and Jin are at extreme positions

(a) 120 (b) 60 (c) 240 (d) 360

iii) Total number of arrangements if Jungkook is sitting in the middle is

(a) 720

(b) 360

(c) 120

(d) None of these

(Chapter -6: PERMUTATION AND COMBINATION)

MCQ

Q -1 In how many ways 5 girls and 3 boys be seated in a row, so that no two boys are together?

5. 1440
6. 14400
7. 3600
8. None of the above

Q-2 If ${}^{2n}C_3 : {}^{n}C_2 = 12 : 1$, then the value of n is ?

5. 6
6. 5
7. 4
8. None of the above

Q-3 In how many ways a committee, consisting of 5 men and 6 women can be formed from 8 men and 10 women?

5. 266
6. 5040
7. 11760
8. None of the above

Q-4 In how many different ways can the letters of the word 'GEOGRAPHY' be arranged such that the vowels must always come together?

5. 2520
6. 2530
7. 15130
8. 15120

Q-5 In how many ways can the word CHRISTMAS be arranged so that the letters C and M are never adjacent?

5. 8! × (7/2)
6. 9! × (7/2)
7. 8! × (9/2)
8. None of the above

Q-1	2
Q-2	2
Q-3	3
Q-4	4
Q-5	1

Assertion and Reason Questions
Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (C) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
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1. Assertion: If the letters W, I, F, E are arranged in a row in all possible ways and the words (with or without meaning) so formed are written as in a dictionary, then the word WIFE ocurs in the 24" position.
Reason: The number of ways of arranging four distinct objects taken all at a time is C(4,4).
2. Assertion: A number of four different digits is formed with the help of the digits 1,2,3,4,5,6,7 in all possible ways. Then, number of ways which are exactly divisible by 4 is 200.
Reason: A number divisible by 4, if unit place digit is divisible by 4
3. Assertion: Product of five consecutive natural numbers is divisible

2 MARKS QUESTION

Question 1:

Find the 3-digit numbers that can be formed from the given digits: 1, 2, 3, 4 and 5 assuming that

a) digits can be repeated.

b) digits are not allowed to be repeated.

Solution:

a) By the multiplication principle, the number of ways in which three-digit numbers can be formed from the given digits is 5 × 5 × 5 = 125

b) By the multiplication principle, the number of ways in which three-digit numbers can be formed without repeating the given digits is 5 × 4 × 3 = 60

Question 2:

A coin is tossed 6 times, and the outcomes are noted. How many possible outcomes can be there?

Solution:

When we toss a coin once, the number of outcomes we get is 2 (Either Head or tail)

So, in each throw, the no. of ways to get a different face will be 2.

Therefore, by the multiplication principle, the required no. of possible outcomes is

2 x 2 x 2 x 2 × 2 × 2 = 64

Question 3:

Evaluate the following

(i) 6 ! (ii) 5 ! – 2 !

Solution:

(i) 6! = 1 × 2 × 3 × 4 × 5 × 6 = 720

(ii) 5! = 1 × 2 × 3 × 4 x 5 = 120

As 2! = 1 × 2 = 2

Therefore, 5 ! – 2 ! = 120-2 = 118.

Question 4:

From a team of 6 students, in how many ways can we choose a captain and vice-captain assuming one person can not hold more than one position?

Solution:

From a team of 6 students, two students are to be chosen in such a way that one student will hold only one position.

Here, the no. of ways of choosing a captain and vice-captain is the permutation of 6 different things taken 2 at a time.

So, ${}^{6}P_{2}$ = 6! / (6 -2)! = 6! / 4! = 30

Question 5:

How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

Solution:

Number of letters in word EQUATION` = 8

n = 8

If all letters of the word used at a time

r = 8

Different numbers formed = nPr

= ${}^{8}P_{8}$

= 8!/(8 8)!

= 8!/0!

= 8!/1

= 8!

= 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1

= 40320

3 MARKS QUESTION

1. How many numbers are there between 99 and 1000, having at least one of their digits 7?

Solution:

Numbers between 99 and 1000 are all three-digit numbers.

Total number of 3 digit numbers having at least one of their digits as 7 = (Total numbers of three-digit numbers) – (Total number of 3 digit numbers in which 7 does not appear at all)

= (9 × 10 × 10) – (8 × 9 × 9)

= 900 – 648

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Solution:

Let ABCDE be a five-digit number.

Given that the first two digits of each number are 6 and 7.

Therefore, the number is 67CDE.

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Suppose one of them is taken at C, now the digits possible at place D is 7.

Similarly, at E, the possible digit is 6.

Therefore, the total five-digit numbers with given conditions = 8 × 7 × 6 = 336.

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Solution:

Given word – ALLAHABAD

Here, there are 9 objects (letters) of which there are 4As, 2 Ls and rest are all different.

Therefore, the required number of arrangements = 9!/(4! 2!)

= (1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9)/ (1 × 2 × 3 × 4 × 1 × 2)

= (5 × 6 × 7 × 8 × 9)/2

= 7560

4. **In how many of the distinct permutations of the letters in MISSISSIPPI do the four Is not come together?**

Solution:

Given word – MISSISSIPPI

M – 1

I – 4

S – 4

P – 2

Number of permutations = 11!/(4! 4! 2!) = (11 × 10 × 9 × 8 × 7 × 6 × 5 × 4!)/ (4! × 24 × 2)

= 34650

We take that 4 I's come together, and they are treated as 1 letter,

∴ Total number of letters=11 – 4 + 1 = 8

⇒ Number of permutations = 8!/(4! 2!)

= (8 × 7 × 6 × 5 × 4!)/ (4! × 2)

= 840

Therefore, the total number of permutations where four Is don't come together = 34650 – 840 = 33810

5. In a small village, there are 87 families, of which 52 families have at most 2 children. In a rural development programme, 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made?

Solution:

Given,

Total number of families = 87

Number of families with at most 2 children = 52

Remaining families = 87 – 52 = 35

Also, for the rural development programme, 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children.

Thus, the following are the number of possible choices:

52C18 × 35C2 (18 families having at most 2 children and 2 selected from other types of families)

52C19 × 35C1 (19 families having at most 2 children and 1 selected from other types of families)

52C20 (All selected 20 families having at most 2 children)

Hence, the total number of possible choices = 52C18 × 35C2 + 52C19 × 35C1 + 52C20

5 MARKS QUESTION

1. A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

Solution:

Given,

Men = 2

Women = 3

A committee of 3 persons to be constituted.

Here, the order does not matter.

Therefore, we need to count combinations.

There will be as many committees as combinations of 5 different persons taken 3 at a time.

Hence, the required number of ways = 5C3

= 5!/(3! 2!)

= (5 × 4 × 3!)/(3! × 2)

= 10

Committees with 1 man and 2 women:

1 man can be selected from 2 men in 2C1 ways.

2 women can be selected from 3 women in 3C2 ways.

Therefore, the required number of committees = 2C1 × 3C2

= 2 × 3C1

= 2 × 3

= 6

2. Determine the number of 5 card combinations out of a deck of 52 cards, if there is exactly one ace in each combination.

Solution:

Given a deck of 52 cards

There are 4 Ace cards in a deck of 52 cards.

According to the given, we need to select 1 Ace card out of the 4 Ace cards

∴ The number of ways to select 1 Ace from 4 Ace cards is 4C1

⇒ More 4 cards are to be selected now from 48 cards (52 cards – 4 Ace cards)

∴ The number of ways to select 4 cards from 48 cards is 48C4

Number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination = 4C1 × 48C4

= 4 × [48!/(44! 4!)]

= 4 × [(48 × 47 × 46 × 45 × 44!)/ (44! × 24)]

= 4 × 2 × 47 × 46 × 45

= 778320

3. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has

(i) no girls

(ii) at least one boy and one girl

(iii) at least three girls

Solution:

Given,

Number of girls = 7

Number of boys = 7

(i) No girls

Total number of ways the team can have no girls = 4C0 × 7C5

= 1 × 21

= 21

(ii) at least one boy and one girl

1 boy and 4 girls = 7C1 × 4C4 = 7 × 1 = 7

2 boys and 3 girls = 7C2 × 4C3 = 21 × 4 = 84

3 boys and 2 girls = 7C3 × 4C2 = 35 × 6 = 210

4 boys and 1 girl = 7C4 × 4C1 = 35 × 4 = 140

Total number of ways the team can have at least one boy and one girl = 7 + 84 + 210 + 140

= 441

(iii) At least three girls

Total number of ways the team can have at least three girls = 4C3 × 7C2 + 4C4 × 7C1

= 4 × 21 + 7

= 84 + 7

= 91

9. How many numbers greater than 1000000 can be formed using the digits 1, 2, 0, 2, 4, 2, 4?

Solution:

Given numbers – 1000000

Number of digits = 7

The numbers have to be greater than 1000000, so they can begin either with 1, 2 or 4.

When 1 is fixed at the extreme left position, the remaining digits to be rearranged will be 0, 2, 2, 2, 4, 4, in which there are 3, 2s and 2, 4s.

Thus, the number of numbers beginning with 1 = 6!/(3! 2!) = (6 × 5 × 4 × 3!)/(3! × 2)

= 60

The total numbers begin with 2 = 6!/(2! 2!) = 720/4 = 180

Similarly, the total numbers beginning with 4 = 6!/3! = 720/6 = 120

Therefore, the required number of numbers = 60 + 180 + 120 = 360.

10. 18 mice were placed in two experimental groups and one control group, with all groups equally large. In how many ways can the mice be placed into three groups?

Solution:

Number of mice = 18

Number of groups = 3

Since the groups are equally large, the possible number of mice in each group = 18/3 = 6

The number of ways of placement of mice =18!

For each group, the placement of mice = 6!

Hence, the required number of ways = 18!/(6!6!6!) = 18!/(6!)3

Case study

Read the Case study given below and attempt any 4 subparts:

One evening, four friends decided to play a card game Rummy. Rummy is a card game that is played with decks of cards. To win the rummy game a player must make a valid declaration by picking and discarding cards from the two piles given. One pile is a closed deck, where a player is unable to see the card that he is picking, while the other is an open deck that is formed by the cards discarded by the players. To win at a rummy card game, the players have to group cards invalid sequences and sets.

In rummy, the cards rank low to high starting with Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. Ace, Jack, Queen, and King each have 10 points. The remaining cards have a value equal to their face value. For example, 5 cards will have 5 points, and so on. Four cards are drawn from a pack of 52 playing cards, then:

i. How many different ways can this is done

ii. Exactly one card of each suit

iii. all cards of the same suit

While reading the instructions, he observed that the question paper consists of 12 questions divided into two parts - Part I and Part II, containing 5 and 7 questions, respectively.

Based on the given information, answer the following questions.

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Ques. Members of the BTS group, RM, Suga, V, Jin, Jungkook,Jimin, J Hope, are to be seated in a row for a photo session

i) Total number of ways of sitting arrangement of seven members is

(a) 28

(b) 5040

(c) 2520

(d) None of these

(ii) Total number of arrangements so that RM and Jin are at extreme positions

(a) 120 (b) 60 (c) 240 (d) 360

iii) Total number of arrangements if Jungkook is sitting in the middle is

(a) 720 (b) 360 (c) 120 (d) None of these

(Chapter -7: BINOMIAL THEOPREM)

MCQ {5 QUESTIONS}

1	The number of terms in $((x-5)^2)^5$I A) 8 terms B) 9 terms C) 10 terms D)11 terms
2	Which one is True? A) $(1.2)^{4000} > 800$ B) $(1.2)^{4000} < 800$ C) $(1.2)^{4000} = 800$ D)$(1.2)^{4000} = 1600$
3	The term independent of x in the expansion of $\left(2x + \frac{1}{3x^2}\right)^9$ is A) 2nd B) 3rd C) 4th D) 5th
4	In the expansion of $\left(\frac{x}{3} - \frac{2}{x}\right)^6$, $x > 0$, the constant term is A) $\frac{160}{27}$ B)$\frac{16}{27}$ C) $\frac{-160}{27}$ D) $\frac{120}{27}$
5	If the coefficient of x^2 and x^3 in the expansion of $(3+mx)^9$ are equal , then the value of m is A) $-\frac{9}{7}$ B) $-\frac{7}{9}$ C) $\frac{9}{7}$ D) $\frac{7}{9}$

ASSERTION-REASON BASED QUESTIONS {2 QUESTIONS}

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Q1: Assertion (A) : The expansion of $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$
Reason (R) : If $x = -1$, then the above expansion is zero

Q2: Assertion (A) : $\sum_{k=1}^{n} k\left(n_{c_n}\right)^2 = n\,(2n - 1_{c_{n-1}})$
Reason (R) : : If 2^{2003} is divided by 15 then remainder is 8.

{2 marks questions}

Q1: Using binomial theorem, evaluate $(99)^5$.

Q2: Expand $\left(x^2 + \frac{3}{x}\right)^4$, x ≠ 0 b using Pascal triangle.

Q3: Expand $(1 + x + x^2)^4$

Q4: Prove that$(a+b)^4 - (a-b)^4$is an even number if *a* and b are integers:

Q5: Find 4th term of the expansion $\left(2x + \frac{3}{x}\right)^6$

{3 marks questions}

Q1: Prove that $\sum_{r=0}^{n} 3^r \ {}^nC_r = 4^n$

Q2: Find a, if the 4th and 5th term of the expansion $(2 + a)^7$ are equal.

Q3: Which of the following is larger $(99^{50} + 100^{50})$ or 101^{50}

Q4: Find the term independent of x in the expansion of$\left(2x + \frac{1}{3x^2}\right)^6$.

Q5:Find $(a + b)^6 - (a - b)^6$hence evaluate$\left(\sqrt{3} + \sqrt{2}\right)^6 - \left(\sqrt{3} - \sqrt{2}\right)^6$

{5 marks questions}

Q1: Find the value of $\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4$
Q2: Prove that $12^n-11n-1$ is divisible by 121 For all n∈N by using binomial theorem.
Q3: Show that $2^{4n+4}-15n-16$, where n ∈ N is divisible by 225.

CASE STUDY BASED QUESTIONS {3 QUESTIONS}

Q1: In class XI, teacher explained binomial theorem. Two students Shivani and Vishwani trying to solve the exercise. Shivani expanded $(1+x)^6$ by using Binomial theorem, Viswani expanded $(x+1)^6$.

Based on this above information answer the following questions.

1. According to Shivani find 4th term .
2. Find the value of ${}^6C_0+{}^6C_1+{}^6C_2+{}^6C_3+\ldots\ldots+{}^6C_6$
3. Find the positive value of x if 3rd terms of Shivani and Vishwani are equal.

Q2: In class XI, teacher explained binomial theorem. Two students Rohan and Mahek trying to solve the exercise. Mahek expanded $(x+y)^3$ by using Binomial theorem, Rohan expanded $(x+y)^3$.by using algebraic identity

Based on this above information answer the following questions.

1. According to Mahek find expansion
2. According to Rohan find expansion
3. Find the value of $(99)^3$ by using above result

(Chapter -8: SEQUENCE AND SERIES)

MCQS

Q1 First five terms of the sequence $a_n=(-1)^{n-1}5^{n+1}$ are

(a)25,125,625,-3125,15625

(b)25,-125,,625,-3125,15625

(c)25,125,625,-3125,-15625

(d)-25,125,-625,3125,-15625

Q2How many multiples of 4 lie between10 and 250

(a)126

(b) 23

(c)60

(d)130

Q3 If n Arithmetic means are introduced between 3 and 17 such that ratio of last mean to first mean is 3:1, then value of n is

(a)6

(b)8

(c)4

(d)None of these

Q4 If the sum of first n even natural numbers is equal to k times the sum of first n odd natural numbers ,then k is

a) $\frac{1}{n}$

(b) $\frac{n-1}{n}$

(c) $\frac{n+1}{2n}$

(d) $\frac{n+1}{n}$

Q5If a,b,c are in AP and x,y,z are in GP , then value of $x^{b-c}y^{c-a}z^{a-b}$ is

(a)0

(b)1

(c)xyz

(d) $x^a y^b z^c$

ASSERTION AND REASONING QUESTIONS

Q 1 ASSERTION: If the sum of the series $\frac{3}{\sqrt{5}}$ +$\frac{4}{\sqrt{5}}$ +$\sqrt{5}$ +…….. to 25 terms is $75\sqrt{5}$

REASON : If 27,x,3 are in GP, then x=±4

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) Both A and R false.

Q 2 ASSERTION: If the numbers $\frac{-2}{7}$, K,$\frac{-7}{2}$ are in GP ,then K=±1

REASON : If $a_1, a_2, a_3$are in GP, then $\frac{a_2}{a_1}$=,$\frac{a_3}{a_2}$

(a)Both A and R are true and R is the correct explanation of A.

(b)Both A and R are true and R is not the correct explanation of A.

©A is true but R is false.

(d)Both A and R false.

VERY SHORT ANSWERS (2 MARKS)

Q 1 Insert two arithmetic means between 2 and 10.

Q 2 Insert a geometric mean between 0.008 and 0.2 .

Q 3 The sum of n terms of a progression is $\frac{2^n-1}{3}$, show that it is a GP .

Q 4 Find a_8 of the sequence 0.3,0.06, 0.012,……..

Q 5 Find the sum of two arithmetic means between 6 and 27.

SHORT ANSWERS (3 MARKS)

Q 1 The ratio of the sum of first three terms is to that of first 6 terms of a GP is 125 : 152 .Find the common ratio

Q 2 If a, b ,c are in GP , and $a^{\frac{1}{x}}=b^{\frac{1}{y}}=c^{\frac{1}{z}}$,prove that x,y ,z are in AP.

Q 3 The sum of four numbers in GP is 60 and the arithmetic mean of the first and the last is 18 .Find the numbers.

Q 4 If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the AM between a and b ,then find value of n.

Q 5 Find the sum of the series up to n terms

0.5+0.55+0.555+……..

LONG ANSWERS (5 MARKS)

Q1. If the A.M. between pth and qth terms of an A.P. be equal to the A.M. between the rth and sth terms of A.P., then show that p+q = r+s.

Q2. If n arithmetic means are inserted between 20 and 80 such that the ratio of first mean to the last mean is 1:3, then find the value of n.

Q3. On the first day of new year i.e., on January Ramesh helped 3 persons. When those persons thanked him, he advised them not to thank but to help 3 more persons on second day and instructed them to do the same on third day. They move the chain similarly.

Assuming that the chain is not broken, answer the following:

(i) Find how many persons will be helped on 5^{th} day.
(ii) Find the total number of people helped in 5 days.
(iii) 6561 persons will be helped on which day?

OR

(iii)How many persons will be helped on 7^{th} day?

SOLUTIONS (MCQ)

ANS1 (b)25,-125,,625,-3125,15625

ANS 2 (c)60

ANS 3 (a)6

ANS 4 (c) $\frac{n+1}{2n}$

ANS 5 (b)1

(ASSERTION AND REASONING QUESTION)

ANS 1 © A is true but R is false.

ANS 2 (a)Both A and R are true and R is the correct explanation of A.

(VERY SHORT ANSWERS)

ANS -1

a=2 ,$a_4 = 10$,2+3d = 10 d=8/3 , $a_2 = \frac{14}{3}$, a_3=22/3

ANS-2

G= 0.04

ANS -3 ,

a_n=$\frac{2^n - 2^{n-1}}{3}$

$\frac{a_2}{a_1}$= $\frac{a_3}{a_2}$ =2

ANS 4

a=0.3 , r= 0.2

a_8=0.3$(0.2)^7$

ANS 5

a=6 ,a_4=27, 6+3d =27 , d=7 , $a_2 = 13$ $a_{3=20}$,sum=33

(SHORT ANSWERS)

ANS 1 $\frac{a(r^3-1)}{a(r^6-1)}$=$\frac{125}{152}$

$\frac{1}{r^3+1}$=$\frac{125}{152}$

r=3

ANS 2 Let $a^{\frac{1}{x}}$=$b^{\frac{1}{y}}$ =$c^{\frac{1}{z}} = k$, a=k^x,b=k^y,c=k^z

b^2=ac, k^{2y}=k^{x+z}

2y=x+z

X,y,z are in AP

ANS 3

a+ar+ar^2+ar^3=60

a(1+r)(1+r^2)=60 ……..(1)

a+ar^3=2x18

a(1+r^3)=36, a(1+r)(1+r^2-r) =36 …….(2)

dividing (2) by (1)

2r^2-5r+2=0

r =2,$\frac{1}{2}$, a=32

numbers are 32,16,8,4

ANS 4

$\frac{a^n+b^n}{a^{n-1}+b^{n-1}}=\frac{a+b}{2}$

By cross multiplication

a^{n-1}(a-b)=$b^{n-1}(a-b)$

a^{n-1}=b^{n-1}

$\frac{a^{n-1}}{b^{n-1}}$=1

$(\frac{a}{b})^{n-1}=(\frac{a}{b})^{0}$

n-1=0

n=1

ANS 5

S_n= 0.5 +0.55 +0.555+…….

$=\frac{5}{9}$(0.9+0.99+0.999+……)

=$\frac{5}{9}$((1-0.1)+(1-0.01)+(1-0.001)+……)

$=\frac{5}{9}$ [$(1+1+\cdots n\ times)-$ (0.1+0.01+0.001+……up to nth term)

$=\frac{5}{9}$ [n- $\frac{0.1(1-0.1^n)}{1-0.1}$]

$=\frac{5}{9}$ [n- $\frac{1(1-0.1^n)}{9}$]

$=\frac{5}{81}$ [9 n- 1+0.1^n]

LONG ANSWERS SOLUTION (5 MARKS)

ANS 1) Let a be the first term and d be the common difference of the given A.P. then

A_p = pth term = a + (p – 1) d ; a_q = qth term = a + (q - 1) d

A_r = rth term = a + (r - 1) d and, a_s = sth term = a + (s -1) d

It is given that

A.M. between a_p and a_q = A.M. between a_r and a_s

=> $\frac{1}{2}(a_p + a_q) = \frac{1}{2}(a_r + a_s)$

=> $a_p + a_q = a_r + a_s$

=> {a + (p - 1) d} + { a + (q - 1) d} = {a + (r -1) d} + {a + (s-1) d}

=> (p + q - 2) d = (r + s - 2) d

=> p + q = r + s

ANS 1 We have A,P_1, P_2, P_3,..... P_{14},B

(i) B is 14+2 = 16 th term.
(ii) a_{16} =3 ,a=1, 1+15d=3 ,d=2/15
(iii) D is the distance between two consecutive pillars
(iv) Distance of third pillar =a_4=1+3(2/15) =7/5km
OR
Position of 14th pillar =a_{15}=43/15 km

ANS 2

(i) Let side be x
x^2=50^2+50^2, x= 50$\sqrt{2}$ cm
(ii) Let side be y
y^2=$(25\sqrt{2})^2$+$(25\sqrt{2})^2$, y= 50 cm
$perimeter$= 4y =4x50 =200 cm

(iii) Length of each side Ist square, IIsquare ,……. Form a GP 100, 50$\sqrt{2}$,50….
Side of 6th square = a_6=100$(\frac{1}{\sqrt{2}})^5$=$\frac{25}{\sqrt{2}}$
OR
$(iii)a_7$=12.5

ANS 3

(i) No of persons helped by others form a GP
3,9,27,…….

a_5=3$(3)^4$=243
(ii)toal no of people got help on 5th day =3+9+27+81+243=363
(iii)8th day

OR

(iii) a_7=2187

(Chapter -9: STRAIGHT LINES)

MULTIPLE CHOICE QUESTIONS

1) The equation of straight line parallel to the $x-axis$ at a distance 5 units below it is
   (a) $y=-5$ (b) $y=5$ (c) $x=-5$ (d) $x=5$
2) The inclination of the line $\sqrt{3}\,x-y-1=0$ is
   (a) $\Theta = 30^0$ (b) $\Theta = 60^0$ (c) $\Theta = 45^0$ (d) $\Theta = 90^0$
3) The distance between the lines $3x+4y=9$ and $6x+8y=15$ is
   (a) 6 (b) 3 (c) 1/3 (d) 3/10
4) The point on y-axis which is equidistant from the points (3,2) and (- 5, - 2) is
   (a) (- 2,0) (b) (0, - 2) (c) (0, - 1) (d) (- 1,0)
5) Line through the points (- 2,6) and (4,8) is perpendicular to the line through the points (8,12) and (x,24) . The value of x is
   (a) 4 (b) 3 (c) 2 (d) 1

ASSERTION - REASONING

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

1) Assertion : The equation of the straight line which passes through the point (2, - 3) and the point of the intersection of the lines x + y + 4 = 0 and 3x – y – 8 = 0 is 2x – y – 7 = 0

Reason : Product of slopes of two perpendicular straight lines is - 1 .

2) Assertion : Equation of the horizontal line having distance a from the x-axis is either y = a or y = - a

Reason : Equation of the vertical line having distance b from the y-axis is either x = b or x = - b .

VSA type QUESTIONS (2 Marks)

1) Find the distance between the parallel lines $15x+8y-34=0$ and $15x+8y+31=0$.
2) Find the slope of the line which passes through the origin and the mid point of the line segment joining the points (0, - 4) and (8,0) .
3) Find the value of k for which the line $(k-3)x-(4-k^2)y+k^2-7k+6=0$ is parallel to x-axis .
4) Three consecutive vertices of a rhombus are (5,3) , (2,7) and (- 2,4) . Find the fourth vertex .
5) Find the equation of straight line through (2,3) and making an angle of 60^0 with the positive direction of x-axis .

SA type QUESTIONS (3 Marks)

1) Find the coordinates of the foot of the perpendicular drawn from the point (- 1,3) to the line $3x-4y-16=0$.
2) In what ratio the line joining the points A (- 1,1) and B (5,7) is divided by $x+y=4$.

3) If p is the length of the perpendicular from origin to the line $\frac{x}{a} + \frac{y}{b} = 1$, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

4) At what point the origin be shifted , if the coordinates of a point (4,5) become (- 3,9) ?

5) Find the equation of a line perpendicular to the line $x - 7y + 5 = 0$ and having x-intercept 3.Also find the distance of the point (1, - 1) from the required line .

LA type QUESTIONS (5 Marks)

1) Find equations of lines which pass through the point (3,4) and the sum of its intercepts on the axes is 14 .
2) (i) Find the equation of the line which intersects x-axis at a distance 2 units in the right of origin and y-axis at distance 3 units below to the origin.
   (ii) Find the distance of point (-2 , 3) from the line $5y = 12x - 2$.
   (iii) Find the angle between the x-axis and the line joining the points (3, -1) and (4, -2)

CASE STUDY -1

A girl standing at the junction (crossing) of two straight paths represented by the equations 2x – 3y + 4 = 0 and 3x + 4y – 5 = 0 wants to reach the path whose equation is 6x – 7y + 8 = 0 in the least time.

(i) Equation of path that she should follow is
a) 119 x+ 102 y = 125
b) 109 x+ 102 y = 125
c) 119 x+ 112 y = 125
d) 109 x+ 102 y = 105

(ii) The angle between the cross roads is
a) tan-1 (6/17)
b) tan-1 (1/17)
c) tan-1 (17/6)
d) tan-1 (6)

(iii) The y- intercept of the path used to reach in least time at 6x – 7y + 8 = 0 is
a) 125 / 119
b) 119 / 125
c) 102 / 125
d) 125/ 102

(iv) The x – intercept of the path used to reach in least time at 6x – 7y + 8 = 0 is
a) 125 / 119

b) 119 / 125
c) 102 / 125
d) 125/ 102

CASE STUDY -2

A triangular park has two of its vertices as B(-4, 1) and C(2, 11). The third vertex A is a point dividing the line joining the points (3, 1) and (6, 4) in the ratio 2 : 1.

Based on the above information, answer the following questions:

a) The coordinates of third vertex A are

i) (5, 3)
ii) (3, 5)
iii) (-5, 3)
iv) (5, -3)

b) The equation of passing through B and C is

i) $5x – 3y – 23 = 0$
ii) $5x – 3y + 23 = 0$
iii) $3x + 5y – 23 = 0$
iv) $5x + 3y – 23 = 0$

c) The equation of line passing through A and parallel to BC is

i) $5x – 3y + 16 = 0$

ii) $5x – 3y + 34 = 0$

iii) $5x – 3y – 16 = 0$

iv) $5x + 3y – 16 = 0$

d) The equation of line passing through A and perpendicular to BC is

i) $3x + 5y – 30 = 0$

ii) $3x + 5y + 30 = 0$

iii) $3x – 5y + 30 = 0$

iv) $3x + 5y = 0$

e) The area of triangular field ABC is

i) 78 sq units

ii) 43 sq units

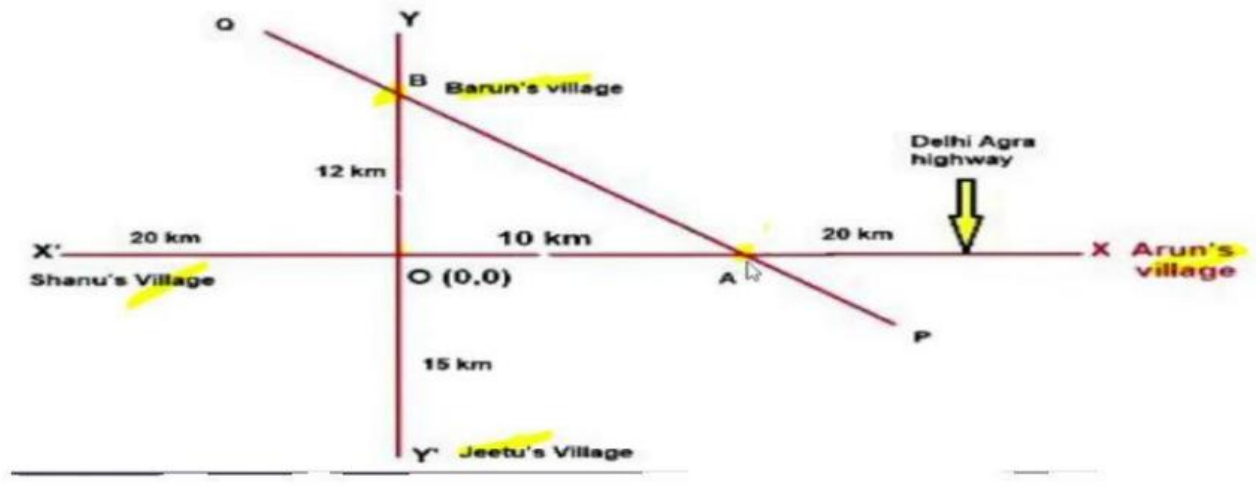
iii) 86 sq units

iv) 39 sq units

CASE STUDY -3

Read the paragraph given below and answer the following queations:

Villages of Shanu and Arun's are 50 km apart and are situated on Delhi Agra highway as shown in figure .
Another highway yy′ crosses Agra Delhi highway at O (0,0). A small local road PQ crosses both the highways at points A and B s.t. OA = 10 km and OB = 12 km. Also, the villages of Barun and Jeetu is 15km from O



Now answer the following questions:

a) What are the coordinates of A?

i) (10, 0)
ii) (10, 12)
iii) (0, 10)
iv) (0, 15)

b) What is the equation of line AB?

i) 5x + 6y = 60
ii) 6x + 5y = 60
iii) x = 10
iv) y = 12

c) What is the distance of AB from O (0,0)?

i) 60 km
ii) 60/√61 km
iii) √61 km
iv) 60 km

d) What is the slope of line AB?

i) 6/5
ii) 5/6
iii) -6/5
iv) 10/12

e) What is the length of line AB?

i) √61 km
ii) 12 km
iii) 10 km
iv) 2√61 km

ANSWERS

MCQ

1) (a) $y = -5$
2) (b) $\Theta = 60^0$
3) (d) 3/10
4) (b) (0, - 2)
5) (a) 4

ASSERTION AND REASON

1) (b)
2) (b)

VSA type QUESTIONS (2 Marks)

1) Distance between lines $ax + by + c = 0\ and\ ax + by + d = 0$ is $\frac{|c-d|}{\sqrt{a^2+b^2}}$
   Distance = $\frac{|-34-31|}{\sqrt{225+64}}$ …… 1
   = $\frac{|-65|}{\sqrt{289}} = \frac{65}{17}$ ……... 1

2) Mid point of line segment joining (0, - 4) and (8,0) = $\left(\frac{0+8}{2}, \frac{-4+0}{2}\right)$ = (4, - 2) ……..1
   Slope of required line = $\frac{-2-0}{4-0}$ = $-\frac{1}{2}$ …… 1

3) Since line is parallel to x-axis slope of the line = 0 ……..1/2
   $\frac{-(k-3)}{-(4-k^2)} = 0$ ……1/2
   $k - 3 = 0 \Longrightarrow k = 3$ …….. 1

4) Let (a,b) be the fourth vertex
   In rhombus diagonals bisect each other. ……. 1/2
   $\Longrightarrow \left(\frac{5+-2}{2}, \frac{3+4}{2}\right) = \left(\frac{2+a}{2}, \frac{7+b}{2}\right)$ ……. 1/2
   $\Longrightarrow$ a =1 , b = 0 …….. 1

5) Slope m = tan $60^0 = \sqrt{3}$ ……. 1
   Equation of line is $y - 3 = \sqrt{3}\,(x - 2)$
   $\Longrightarrow \sqrt{3}x - y + (3 - 2\sqrt{3}) = 0$ …… 1

SA type QUESTIONS (3 Marks)

1) Let P (- 1,3) be the given point

M(a,b) be the foot of the perperdicular on the line

Slope of the line joining (- 1,3) and (a,b) $= m_1 = \frac{b-3}{a+1}$

Slope of line $3x - 4y - 16 = 0 \Longrightarrow y = \frac{3}{4}x - 4 \Longrightarrow m_2 = \frac{3}{4}$ …… 1

The two lines are perpendicular , $m_1 m_2 = -1$

$\left(\frac{b-3}{a+1}\right) X \left(\frac{3}{4}\right) = -1 \Longrightarrow 3b - 9 = -4a - 4 \Longrightarrow 4a + 3b = 5$

Point (a,b) lies on $3x - 4y - 16 = 0 \Longrightarrow 3a - 4b = 16$ …… 1

Solving $a = \frac{68}{25}$ and $b = -\frac{49}{25}$

Foot of perpendicular = $\left(\frac{68}{25}, -\frac{49}{25}\right)$ …… 1

2) Let k :1 be the ratio in which the line joining A (- 1,1) and B (5,7) is divided by $x + y = 4$

The point of intersection = $\left(\frac{5k-1}{k+1}, \frac{7k+1}{k+1}\right)$ …… 1

The point also lie on the line $x + y - 4 = 0 \Longrightarrow \frac{5k-1}{k+1} + \frac{7k+1}{k+1} - 4 = 0$ …….. 1

$\Longrightarrow 8k - 4 = 0$

$\Longrightarrow k = \frac{1}{2} \Longrightarrow$ Ratio = 1 : 2 …….. 1

3) Length of perpendicular from (0,0) to $\frac{x}{a} + \frac{y}{b} = 1$ is p

$\Longrightarrow \left|\frac{0+0-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right| = p$ …….. 1

$\Longrightarrow \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} = p$ ……. 1

$\Longrightarrow \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = \frac{1}{p}$

$\Longrightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$ ……. 1

4) If (h,k) is the new origin ,then new coordinates (X,Y) of the point (x,y) are given by $x = X + h$ and $y = Y + k$ …… 1

(x,y) = (4,5) and (X , Y) = (-3,9)

$\Longrightarrow$ 4 = - 3 + h and 5 = 9 + k …… 1

$\Longrightarrow$ h = 7 and k = - 4

Origin = (7, - 4) ……. 1

5) Given equation $x - 7y + 5 = 0$

$\Longrightarrow y = \frac{1}{7}x + \frac{5}{7}$

Slope of given line = $\frac{1}{7}$ …… 1

Slope of line perpendicular to it is - 7

Required line has x-intercept = 3, so it passes through (3,0)

Equation of required line $y - 0 = -7(x - 3) \Longrightarrow 7x + y - 21 = 0$ …… 1

Distance of the point (1, - 1) from required line = $\frac{|7X1+(-1)-21|}{\sqrt{7^2+1^2}}$

$= \frac{|-15|}{\sqrt{50}} = \frac{15}{5\sqrt{2}} = \frac{3}{\sqrt{2}}$ ……. 1

LA type QUESTIONS (5 Marks)

1) x intercept a = a

y intercept b = 14 – a ……. 1

Equation of line ; $\frac{x}{a}+\frac{y}{b}=1 \Longrightarrow \frac{x}{a}+\frac{y}{14-a}=1$ …….. 1

Line $\frac{x}{a}+\frac{y}{14-a}=1$ is passing through (3,4) $\Longrightarrow \frac{3}{a}+\frac{4}{14-a}=1$

Simplifying as $a^2-13\,a+42=0$ …… 1

Solving above equation ; $a=6,7 \Longrightarrow a=6, b=8\ or\ a=7, b=7$ …… 1

Getting equation of lines ; $4x+3y=24$, $x+y=7$ …… 1

2) (i) x-intercept ; a = 2

y-intercept ; b = -3 …… 1

Equation of line ; $\frac{x}{a}+\frac{y}{b}=1 \Longrightarrow \frac{x}{2}+\frac{y}{-3}=1$

Simplifying $3x-2y-6=0$ …… 1

(ii) Distance = $\left|\frac{(12X-2)-(5\,X\,3)-2}{\sqrt{12^2}+(-5)^2}\right| = \frac{41}{13}$ …… 1

(iii) Slope of x-axis ; $m_1 = 0$ …….. 1/2

Slope of line joining the points (3, -1) and (4, -2) ; $m_2 = -1$ …….. 1/2

Calculation of angle = 45^0 …….. 1

CASE STUDY -1

(i) A (ii) C (iii) D (iv) A

CASE STUDY -2

a) i) (5, 3)

b) ii) 5x – 3y + 23 = 0

c) iii) 5x – 3y – 16 = 0

d) i) 3x + 5y – 30 = 0

e) iv) 39 sq units

CASE STUDY -3

a) i) (10, 0)

b) ii) 6x + 5y = 60

c) ii) 60/√61 km

d) iii) -6/5

e) iv) 2√61 *k*m

(Chapter -10 : CONIC SECTION)

1. The equation of the hyperbola with vertices at (0,±6) and e=5/3 is

(a) $\frac{x2}{36}-\frac{y2}{64}=1$ (b) $\frac{y2}{36}-\frac{x2}{64}=1$

(c) $\frac{x2}{64}-\frac{y2}{36}=1$ (d) $\frac{y2}{36}-\frac{x2}{64}=1$

Ans (b) Since the vertices are on the y-axis ,the equation is of the form (a) $\frac{y2}{a2}-\frac{x2}{b2}=1$ as vertices are (0,±6) , a=6 so the required equation is $\frac{y2}{36}-\frac{x2}{64}=1$

2. The conic represented by x=2(cos t + sin t) ,y=5(cos t - sin t) is

(a) a circle (b) a parabola (c) an Ellipse (d) a hyperbola

Ans From given equation x/2 =(cos t + sin t) ,y/5 =(cos t - sin t) Elliminating t from i & ii, we have

$\frac{x2}{4}+\frac{y2}{25}=2$ is an ellipse

3. Equation of the circle with centre on the y-axis and passing through the origin and the point (2, 3) is

(a) $x^2 + y^2 - 13y = 0$ (b) $3x^2 + 3y^2 + 13x + 3 = 0$

(c) $6x^2 + 6y^2 - 13x = 0$ (d) $x^2 + y^2 + 13x + 3 = 0$

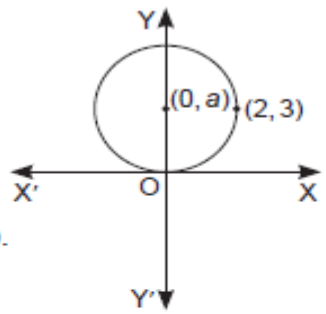
Ans (a), as the centre of the circle is on the y-axis.

So abscissa of the centre be 0.

Let coordinates of the centre be $(0, a)$

The circle passes through (0, 0).

$\therefore \quad r = a$



So equation of circle

$(x-0)^2 + (y-a)^2 = a^2$ ...(i)

$\Rightarrow x^2 + y^2 + a^2 - 2ay = a^2 \Rightarrow x^2 + y^2 - 2ay = 0$ ...(ii)

The circle passes through the point (2, 3)

So, $2^2 + 3^2 - 2a \times 3 = 0 \Rightarrow a = \frac{13}{6}$ {From (ii)}

Equation of the circle is

$\Rightarrow x^2 + \left(y - \frac{13}{6}\right)^2 = \left(\frac{13}{6}\right)^2$

$\Rightarrow x^2 + y^2 - \frac{13y}{3} + \frac{169}{36} = \frac{169}{36}$

$\Rightarrow 3x^2 + 3y^2 - 13y = 0$

4. The foci of an ellipse are (0, ± 6) and the length of its minor axis is 16. The equation of the ellipse is

(a) $\frac{x^2}{64}+\frac{y^2}{100}=1$ (b) $\frac{x^2}{100}+\frac{y^2}{64}=1$

(c) $\frac{x^2}{36}+\frac{y^2}{64}=1$ (d) $\frac{x^2}{16}+\frac{y^2}{36}=1$

Ans (a), as foci are $(0, \pm 6)$, i.e. foci lie on the y-axis.

So equation of the ellipse be $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1,\ a>b$

$$c^2 = a^2 - b^2$$

$$\Rightarrow \quad 6^2 = a^2 - 8^2$$

$\{b$ = length of minor axis $= \frac{1}{2} \times 16 = 8\}$

$$\Rightarrow \quad a^2 = 6^2 + 8^2 = 36 + 64 = 100$$

Hence equation of the ellipse be $\frac{x^2}{64}+\frac{y^2}{100}=1$

5. If the parabola $x^2 = 6y$, the focus and the equation of directrix are respectively.

(a) $\left(0, \frac{-3}{2}\right), y=\frac{3}{2}$ (b) $\left(0, \frac{3}{2}\right), y=\frac{3}{2}$

(c) $\left(0, \frac{3}{2}\right), y=\frac{-3}{2}$ (d) **none of these**

Ans: (c), given equation of the parabola is $x^2 = 4ay \Rightarrow a = 3/2$ Focus of the parabola is $\left(0, \frac{3}{2}\right)$ Its directrix is $y = -a \Rightarrow y = -3/2$

Assertion & Reasoning Question

Each of these questions contains two statements Assertion & Reason. Each of these questions also has four alternatives choices,

a) Assertion is correct. Reason is correct, Reason is correct explanation for assertion
b) Assertion is correct. Reason is correct, Reason is not a correct explanation for assertion
c) Assertion is correct. Reason is incorrect,.
d) Assertion is incorrect. Reason is correct,

1.Assertion: Latus rectum of a parabola ia a line segment perpendicular to the axis of the parabola,through the focus and whose end points lie on the parabola.

Reason : The equation of the hyperbola with foci on the y-axis is $\frac{x2}{a2}-\frac{y2}{b2}=1$

Ans: c) Assertion is correct but Reason is incorrect

2. Parabola is symmetric with respect to the axis of the parabola.

Assertion: If the equation has a term y^2,then the axis of symmetry is along the x-axis,

Reason: If the equation has a term x^2,then the axis of symmetry is along the x-axis

Ans (c)

3. Let the centre of an Ellipse is at (0,0)

Assertion:If major axis is on the y-axis and ellipse passes through the points (3,2) and (1,6), then the equation of ellipse is $x^2/10+y^2/40=1$

Reason: $x^2/a^2 + y^2/ b^2=1$ is an equation of ellipse if major axis is along y-axis

Ans **(a)** **Assertion**: Since, major axis is along y-axis. Hence, equation of ellipse will be of the form

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \qquad ...(i)$$

Given that (i) passes through the points (3, 2) and (1, 6) i.e, they will satisfy it

$$\therefore \quad \frac{3^2}{b^2}+\frac{2^2}{a^2}=1 \Rightarrow \frac{9}{b^2}+\frac{4}{a^2}=1 \qquad ...(ii)$$

$$\text{and} \quad \frac{1^2}{b^2}+\frac{6^2}{a^2}=1 \Rightarrow \frac{1}{b^2}+\frac{36}{a^2}=1 \qquad ...(iii)$$

Multiplying (ii) by 9 and then subtracting (iii) from it,

we get $\frac{80}{b^2}=8 \Rightarrow b^2=\frac{80}{8} \therefore b^2=10$

from eq (ii), we get $\frac{9}{10}+\frac{4}{a^2}=1 \Rightarrow \frac{4}{a^2}=1-\frac{9}{10} \Rightarrow a^2=40$

putting the value of $a^2=40$ and $b^2=10$ in (i), we get

$$\frac{x^2}{10}+\frac{y^2}{40}=1$$

Question carry 2 marks each:

1.Find the length of the axes, coordinates of foci, eccentricity of the conic section $16x^2 – 9y^2 – 144 = 0$.

ANS: Given conic section is

$16x^2 – 9y^2 – 144 = 0$

$\Rightarrow 16x^2 – 9y^2 = 144$

$\Rightarrow \frac{x^2}{9}-\frac{y^2}{16}=1$ [Hyperbola]

Here $a^2 = 9 \Rightarrow a = 3$ and $b^2 = 16 \Rightarrow b = 4$

Eccentricity (e) = $\sqrt{\frac{a^2+b^2}{a^2}}=\sqrt{\frac{9+16}{9}}=\sqrt{\frac{25}{9}}=\frac{5}{3}$

Length of transverse axis = $2a = 6$

Length of conjugate axis = $2b = 8$

Foci = $(\pm ae, 0) = \left(\pm 3\times\frac{5}{3}, 0\right) = (\pm 5, 0)$

2. Find the coordinates of the focus, the equation of directrix, vertex and length of latus rectum for the parabola $y^2 = -12x$.

ANS: Given parabola is $y^2 = -12x$
$\Rightarrow 4a = -12 \Rightarrow a = -3$
$\therefore$ Focus is $(-3, 0)$;
Directrix is $x + a = 0 \Rightarrow x - 3 = 0$
$\therefore$ Vertex is $(0, 0)$;
Length of latus rectum $= 4a = 12$

3. What is the eccentricity of hyperbola whose vertices and foci are $(\pm 2, 0)$ and $(\pm 3, 0)$ respectively ?

ANS: Vertices are $(\pm 2, 0)$, foci are $(\pm 3, 0)$
$\Rightarrow a = 2, c = 3$
$\therefore e = \frac{c}{a} = \frac{3}{2}$.

4. Find the centre and radius of the circle $2x^2 + 2y^2 - x = 0$.

Ans Centre = $\left(\frac{1}{4}, 0\right)$

And radius =1/4

5. For the following curve, find the coordinates of the focus, axis, equations of the directrix and length of the latusrectum

:$x^2 = -16y$.

ANS: Consider the equation $x^2 = -16y$
Comparing with $x^2 = 4ay$, we get
$4a = -16 \Rightarrow a = -4$.
$\therefore$ Coordinates of focus are (0, a), i.e. $(0, -4)$
Axis is y-axis.
Equation of directrix is $y + a = 0 \Rightarrow y - 4 = 0$
Length of latus rectum $= |4a| = 16$.

Question carry 3 marks each

1.Show that $4x^2 + y^2 - 8x + 2y + 1 = 0$ represents an ellipse. Find its eccentricity and length of latus rectum,

ANS: Consider the equation,
$4x^2 + y^2 - 8x + 2y + 1 = 0$
$\Rightarrow 4x^2 - 8x + y2 + 2y + 1 = 0$
$\Rightarrow 4[x^2 - 2x] + [y^2 + 2y + 1] = 0$
$\Rightarrow 4[(x-1)^2 - 1)] + (y+1)^2 = 0$
$\Rightarrow 4(x-1)^2 - 4 + (y+1)^2 = 0$
$\Rightarrow 4(x-1)^2 + (y+1)^2 = 4$
$\Rightarrow \frac{(x-1)^2}{1} + \frac{(y+1)^2}{4} = 1$ represents an ellipse.
Let $X = x - 1, Y = y + 1$ ... (*i*)
Also $a^2 = 4 \Rightarrow a = 2$; $b^2 = 1 \Rightarrow b = 1$
Eccentricity = $\sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$
Length of Latus rectum = $\frac{2b^2}{a} = \frac{2 \times 1}{2} = 1$

2. Find the equation of the hyperbola whose foci are (± 3√ 5 , 0) and the length of the latus rectum is 8.

Ans Let hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ ...(*i*)

Foci are (±3√ 5, 0) and latus rectum is 8.

$\therefore c = 3\sqrt{5}$ and $\frac{2b^2}{a} = 8$

$\Rightarrow \sqrt{a^2+b^2} = 3\sqrt{5}$ and $b^2 = 4a$

$\Rightarrow a^2 + b^2 = 45$ and $b^2 = 4a$ ...(*ii*)

$\Rightarrow a^2 + 4a - 45 = 0$

$\Rightarrow (a + 9)(a - 5) = 0$

$\Rightarrow a = -9, a = 5$

$a = -9$, not possible [From (*ii*)]

$\therefore a = 5$

Substituting in (*i*), we get

$b^2 = 4 \times 5 = 20$

Substituting for *a*, *b* in (*i*), we get hyperbola as $\frac{x^2}{25}-\frac{y^2}{20}=1$.

3. Write the equation of ellipse whose vertices are (± 5, 0) and foci (± 4, 0).

ANS: Vertices are (± 5, 0), foci (± 4, 0)

Ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ …(*i*)

We have $a = 5$, $c = 4 = \sqrt{a^2-b^2}$

$\Rightarrow 16 = 25 - b^2 \Rightarrow b^2 = 9$

Substituting for *a*, *b* in (*i*), we get

$\frac{x^2}{25}+\frac{y^2}{9} = 1$ as equation of the ellipse.

4. Find the equation of the circle passing through (0, 0) and making intercepts *a* and*b* on the coordinate axes.

ANS: Circle passes through origin. Let it meets axes at A, B. Then AB is

Coordinates of centre are

$P\left(\frac{a}{2}, \frac{b}{2}\right)$ and radius is $\frac{1}{2}\sqrt{a^2+b^2}$

$\therefore$ Equation of circle is $\left(x-\frac{a}{2}\right)^2+\left(y-\frac{b}{2}\right)^2 = \left(\frac{1}{2}\sqrt{a^2+b^2}\right)^2$

$\Rightarrow x^2 + y^2 - ax - by = 0$ is the required equation.

5. Find the equation of a circle whose centre is (3, –2) and which passes through the inter-section of the lines $5x + 7y = 3$ and $2x - 3y = 7$.

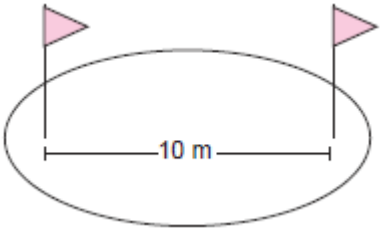
ANS: Point of intersection of $5x + 7y = 3$
and $2x - 3y = 7$ is (2, – 1)
Centre is (3, – 2).

Therefo $\frac{a^2}{4}r = \sqrt{\frac{(3-2)^2}{b^2}+\frac{(-2+1)^2}{4}} = \sqrt{\frac{1}{4}}$

$\therefore$ Circle is $(x-3)^2 + (y+2)^2 = 2$
$x^2 + y^2 - 6x + 4y + 11 = 0$

CBQ Question carry 5 marks each

1. A man is running on a race course such that the sum of distances of two flags posts from him is always 26 m and distance between two flag post is 10 m.



(i) The equation of the path is

(a) $\frac{x^2}{169}+\frac{y^2}{25}=1$ (b) $\frac{x^2}{25}+\frac{y^2}{169}=1$

(c) $\frac{x^2}{169}+\frac{y^2}{144}=1$ (d) $\frac{x^2}{144}+\frac{y^2}{169}=1$

(ii) The eccentricity of the path is

(a) $\frac{12}{13}$ (b) $\frac{3}{13}$

(c) $\frac{5}{13}$ (d) $\frac{13}{5}$

(iii) The length of latus rectum of the curve is

(a) $\frac{288}{13}$ m (b) $\frac{84}{13}$ m

(c) $\frac{144}{13}$ m (d) $\frac{5}{3}$ m

(iv) Coordinates of vertices of the path are

(a) $(0, \pm 12)$ (b) $(\pm 13, 0)$

(c) $(0, \pm 16)$ (d) $(\pm 5, 0)$

(v) Coordinates of the foci of the path are

(a) $(\pm 5, 0)$ (b) $(\pm 4, 0)$

(c) $(0, \pm 13)$ (d) $(0, \pm 12)$

ANS: (i) (c), The path is an ellipse

$2a = 26$

$\Rightarrow$ $a = 13, 2c = 10 \Rightarrow c = 5$

$$c^2 = a^2 - b^2$$

$$\Rightarrow \quad 25 = 169 - b^2$$

$$\Rightarrow \quad b^2 = 144$$

$$\Rightarrow \quad b = 12$$

Equation of path is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\Rightarrow \quad \frac{x^2}{169} + \frac{y^2}{144} = 1$$

(ii) (c), Eccentricity $= \frac{c}{a} = \frac{5}{13}$

(iii) (a), Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 144}{13}$

$$= \frac{288}{13} \text{ m}$$

(iv) (b), Coordinates of vertices = (± a, 0) = (± 13, 0)

(v) (a) Coordinates of foci = (± 5, 0)

2. A man visited India gate and moving on a path, he observed that his distance from India gate is always 500 metres. Taking India gate as origin. Answer the following:

(i) Find the equation of path of the man.

(ii) If he saw a balloon seller is at point (600, 800) then the man has to move towards India gate or away from it.

(iii) If man is considered to be at point (400, 300), then find the distance between balloon seller and man.

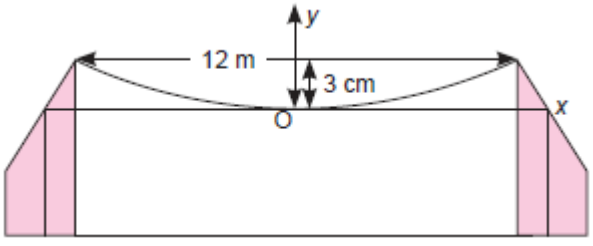
(iv) Find the locus of balloon seller if he also start moving around India gate on circular path from the point where he was standing.

(v) Find the area of triangle formed by joining the India gate, the position of man and the position of balloon seller.

ANS: (i) $x^2 + y^2 = 250000$ (ii) Man has to move away from India gate (iii) $100\sqrt{29}$ m

(iv) $x^2 + y^2 = 1000000$ (v) Area = 70000 m^2

3. A beam is supported at its ends by supports which are 12 m apart. Since the load is concentrated at its centre, there is a deflection of 3 cm at the centre and defected beam is in the shape of parabola. Now considering the centre of beam is at origin as shown in figure. Answer the following:



(i) The equation of parabola will be of the form

(a) $y^2 = 4ax$ (b) $x^2 = 4ay$

(c) $y^2 = -4ax$ (d) $x^2 = -4ay$

(ii) The focus of parabola is

(a) (0, 300) (b) (0, 3)

(c) (3, 0) (d) (12, 0)

(iii) The length of latus rectum of parabola is

(a) 1200 m (b) 1200 cm

(c) 12 m (d) 48 m

(iv) The points on the parabola where it is supported by two supports

(a) (± 6, 3) (b) $\left(\pm\frac{6}{100}, 3\right)$

(c) $\left(\pm 6, \frac{3}{100}\right)$ (d) (3, 6)

(v) How far from the centre is the deflection 1 cm?

(a) $2\sqrt{6}$ m (b) $2\sqrt{6}$ cm

(c) $2\sqrt{3}$ m (d) $2\sqrt{3}$ cm

ANS: (i) (b), Equation of parabola is $x^2 = 4ay$

(ii) (a), $x^2 = 4ay$

Point $\left(6, \frac{3}{100}\right)$ lies on parabola

$$36 = 4 \times a \times \frac{3}{100}$$

$$\Rightarrow \quad a = \frac{3600}{12} = 300$$

Focus = (0, 300)

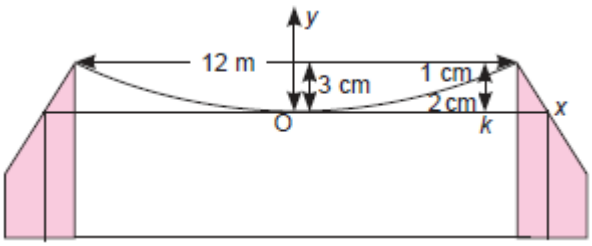
(iii) (a), Length of latus rectum is = 4a

= 4 × 300

= 1200 m

(iv) (c), The points are $\left(\pm 6, \frac{3}{100}\right)$

(v) (a), Where the deflection is 1 cm. Let the coordinates of point be $\left(k, \frac{2}{100}\right)$



$$x^2 = 4ay$$

$$k^2 = 4 \times 300 \times \frac{2}{100}$$

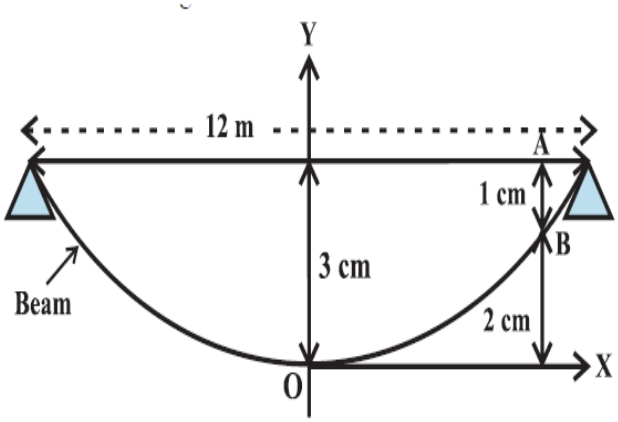
$$\Rightarrow \quad k^2 = 24$$

$$\Rightarrow \quad k = 2\sqrt{6}$$

At distance of $2\sqrt{6}$ m from centre deflection is 1 cm.

1.A beam is supported at its ends by supports which are 12 metres apart. Since the load is concentrated at its centre, there
is a deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola. How far from the centre is the deflection 1 cm?

Solution Let the vertex be at the lowest point and the axis vertical. Let the coordinate axis be chosen as



The equation of the parabola takes the form $x^2 = 4ay$.
Since it passes through (6, 3/100)
we have $(6)^2 = 4a(3/100)$ i.e. a=300 m
·let AB is the deflection of the beam which is 1/100 m
So,. Coordinates of B are $(x,2/100)$
Therefore $x^2 = 4 \times 300 \times 2/100=24$
i.e. $x = \sqrt{}$ 24 metres

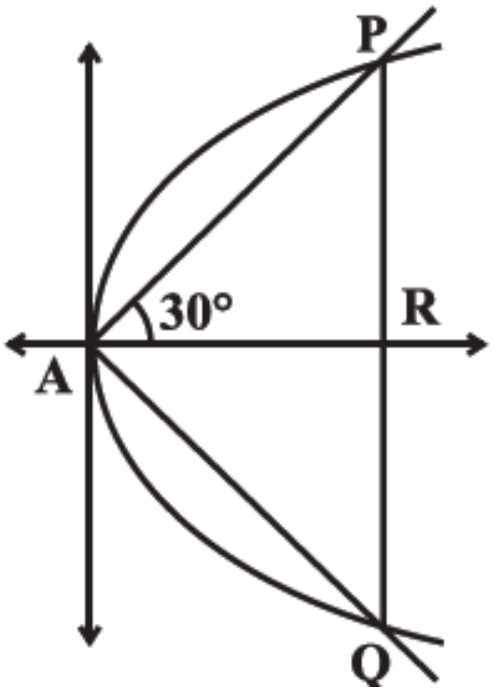
2. If the latus rectum of an ellipse with axis along *x*-axis and centre at origin is 10, distance between foci = length of minor axis, then the equation of the ellipse is
Ans Given that $2b^2/a = 10$
= and $2ae = 2b \Rightarrow b = ae$ *and*
Again, we know that
$b^2 = a^2(1 - e^2)$
or $2a^2e^2 = a^2 \Rightarrow \varepsilon = 1/\sqrt{2}$
Thus $a = b\sqrt{2}$
Again $2b^2/a = 10$
Or $b=5\sqrt{2}$

Therefore, the required equation of the ellipse is

$$\frac{x^2}{100} + \frac{y^2}{50} = 1$$

3.An equilateral triangle is inscribed in the parabola $y^2 = 4ax$ whose one vertex is at the vertex of the parabola. Find the length of the side of the triangle.

Solution As shown in the figure APQ denotes the equilateral triangle with its equal sides of length l (say).



Here AP = l so AR = $l \cos 30° = l\sqrt{3}/2$

Also, PR = $l \sin 30° = l/2$

.

Thus $(\sqrt{3}/2, 1/2)$ are the coordinates of the point P lying on the parabola $y^2 = 4ax$.

Therefore,

$l^2/4 = 4a(l\sqrt{3}/2) \Rightarrow l = 8a\sqrt{3}$

Thus, $8a\sqrt{3}$ is the required length of the side of the equilateral triangle inscribed in the parabola $y_2 = 4ax$.

(Chapter -11 : INTRODUCTION TO THREE DIMENSIONAL GEOMETRY)

MCQ QUESTIONS

Q.1: Which octant do the point (−5,4,3) lie?

A. Octant I

B. Octant II

C. Octant III

D. Octant IV

Q.2: A point is on the x-axis. Which of the following represent the point?

A. (0, x, 0)

B. (0, 0, x)

C. (x, 0, 0)

D. None of the above

Q.3: Coordinate planes divide the space into _____ octants.

A. 4

B. 6

C. 8

D. 10

Q.4: What is the distance between the points (2, –1, 3) and (–2, 1, 3)?

A. $2\sqrt{5}$ units

B. 25 units

C. $4\sqrt{5}$ units

D. $\sqrt{5}$ units

Q.5: The maximum distance between points (3sin θ, 0, 0) and (4cos θ, 0, 0) is:

(a) 3 units

(b) 4 units

(c) 5 units

(d) Cannot be determined

Assertion and Reason Questions

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a) (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct, reason is a correct explanation for assertion

(b) Assertion is correct, reason is correct, reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason in correct

1 Assertion: The coordinates of the point which divides the join of A(2,-1,4) and B(4,3,2) in the ratio 2:3 externally is C(-2,-9,8)

Reason: If $P(x_1,y_1,z_1)$ and $Q(x_2,y_2.z_2)$ be two points, and let R be a point on PQ produced dividing it externally in the ratio m_1: m_2. Then the coordinates of R are

$$\left(\frac{m_1x_2-m_2x_1}{m_1-m_2},\frac{m_1y_2-m_2y_1}{m_1-m_2},\frac{m_1z_2-m_2z_1}{m_1-m_2}\right)$$

2. Assertion: If three vertices of a parallelogram ABCD are A(3,-1,2), 8(1,2,-4) and C(-1.1.2). then the fourth vertex is (1.-2,8).

Reason: Diagonals of a parallelogram bisect each other and mid-point of AC and BD coincide

TWO MARKS QUESTIONS

1. Find the distance between the points P(-2,4,1) and Q(1, 2, – 5).
2. Write the distance of point P(2,3,5) from the xy-plane.
3. Name the octants in which the following points lie: (-5,-4,7), (-7,2,-5)
4. How far apart are the points (2, 0, 0) and (-3, 0, 0)
5. Write the distance of point P(5,12,13) from the x-axis.

THREE MARKS QUESTIONS

1. Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).

2. Find the coordinates of a point equidistant from the four points O (0,0,0), A(a, 0, 0), B(0, b, 0) and C(0,0, c).

3. Prove that the points (a,b,c), (b,c,a) and (c,a,b) are the vertices of an equilateral triangle.

4. Show that the points (-2, 3, 5), (1, 2, 3) and (7, 0, -1) are collinear.

5. What are the coordinates of the vertices of a cube whose edge is 2 units, one of whose vertices coincides with the origin and the three edges passing through the origin, coincide with the positive direction of the axes through the origin?

FIVE MARKS QUESTIONS

1. Three consecutive vertices of a parallelogram ABCD are .4(6, -2,4), 6(2,4, -8), C(-2, 2, 4). Find the coordinates of the fourth vertex.

2. Show that the triangle ABC with vertices .4(0,4,1), 6(2,3, -1) and C(4, 5,0) is right angled.

3. The mid-point of the sides of a triangle are (1, 5, -1), (0,4, -2) and (2, 3,4). Find its vertices. Also, find the centroid of the triangle.

CASE BASED QUESTIONS

1. **A boy is standing at point O and observe three kites A, B and C in space. Taking O as origin if the coordinates of three kites A, B and C are (3, 4, 5), (–1, 3, 4) and (2, – 1, 4) respectively, then**
(i) the distance between kites A and B will be —-
(ii) The coordinates of a point on the y-axis which is at a distance of units from kite A are
(a) (0, 0, 5) (b) (0, 7, 0)
(c) (3, 0, 0). (d) (0, 5, 0)
(iii) The coordinates of point D so that ABCD is a parallelogram are
(a) (6, 0, 5) (b) (4, 9, 5)
(c) (–1, 3, 2) (d) (5, 6, 0)
(iv) If the points (0, –1, –7), (2, 1, –9) and (6, 5, –13) represent kites A, B and C then the kites
(a) are collinear
(b) form right angled triangle
(c) form an isosceles triangles
(d) form a rhombus

(Chapter -12 : LIMITS AND DERIVATIVES)

MCQs		
QN	**SOLUTION**	**MARKS**
1	**The value of** $\lim_{X\to\frac{1}{2}}\left(\frac{4X^2-1}{2X-1}\right)$ **=** **(A) 1 (B) 2 (C) 3 (D) 4**	**1**
2	**The value of** $\lim_{X\to 0}\left(\frac{tan^2 3x}{X^2}\right)$ **=** **(A) 0 (B) 3 (C)** ∞ **(D) 9**	**1**
3	**If f(x) =** $\frac{Ix-9I}{x-9}$**, then find** $\lim_{x\to 3^3} f(x)$ **(A) 1 (B) -1 (C) 2 (D) 0**	**1**
4	**The value of** $\lim_{x\to 3}(4x^3-2x^2-x+1)$ **=** **(A) 40 (B) 20 (C) 38 (D) 88**	**1**
5	**If f(x) =** x^2-5x+7**, then f' (3) is :** **(A) 11 (B) 1 (C) 4 (D) 0**	**1**
	ASSERTION - REASON QUESTIONS	
Q N	**QUESTION**	**MARKS**
	Directions (Q Nos. - 6&7) **In the questions given below are two statements labelled as Assertion (A) and Reason (R). In the context of the two statements, which one of the following is correct ?** **(a) Both A and B are correct, R is the correct explanation of A** **(a) Both A and B are correct, R is not the correct explanation of A** **(c) A is correct, R is incorrect** **(d) A is incorrect, R is correct**	
6	Assertion : **The value of** $\lim_{X\to 2^+}\left(\frac{X^2-1}{2X+4}\right) = \frac{3}{8}$ Reason : $\lim_{X\to a^+} f(x) = \lim_{h\to 0} f(a+h)$	**1**
7	**Assertion : The derivative of f(x) = cos x at x = 0 is 0** **Reason : For a function** $f(x)$**,** **f'(a) =** $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$	**1**
	VSA	
8	**Evaluate LHL and RHL of the** $\lim_{x\to 0}\frac{IxI}{x}$	**2**
9	**Evaluate :** $\lim_{x\to 1}\frac{X^{21}-1}{X^3-1}$	**2**
10	**Evaluate :** $\lim_{x\to 0}\frac{tan\,x}{x}$	**2**
11	**Find the derivative of** x^2 **- 3 at x = 5**	**2**
12	**Compute the derivative of f(x) =** $\sin^2 x$	**2**

Q N	SA QUESTION	MARKS
13	**Evaluate :** $\lim_{x\to 0}\left[\frac{\tan x - 2\tan 3x + \tan 5x}{x}\right]$	3
14	**Evaluate :** $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$	3
15	**Differentiate $2x^3 - 4x^2 + 6x + 8$ w.r.t. x**	3
16	**Differentiate** $(x + \frac{1}{x})^3$ **w.r.t. x**	3
17	**Compute the derivative of sin x.**	3

QN	LA QUESTION	MARKS
18	**Evaluate :** $\lim_{x\to 3}\frac{x^3-7x^2+15x-9}{x^4-5x^3+27x-27}$	5
19	**If f(x) =** $\begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \le x \le 1, \\ nx^3 + m, & x > 1 \end{cases}$ ***for what integers m and n does both*** $\lim_{x\to 0} f(x)$ **and** $\lim_{x\to 1} f(x)$ **exist ?**	5
20	**Find the derivative of : f(x) =** $\frac{4x + 5\sin x}{3x + 7\cos x}$	5

Q N	CASE STUDY QUESTION	MARKS
21	 **Mr. Shami, a maths teacher of class XI wants to check the knowledge of his students, so he writes a function f(x) = sin x + cos x, where f be a real valued function defined by** $f'(x) = = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ **For a function f(x) = sin x + cos x, answer the following questions ;** **(a) Find** $\frac{d}{dx}\{f'(x)\}$ **at x = 90°** **(b) If f(x) =** $\cos^2 x - \sin^2 x$**, then find the value of f'(45°)**	4
22	**A function f is said to be a rational function, if f(x) =** $\frac{g(x)}{h(x)}$**, where g(x) and h(x) are polynomial functions such that h (x) ≠ 0.** **Then** $\lim_{x\to a} f(x) = \lim_{x\to a}\frac{g(x)}{h(x)} = \frac{\lim_{x\to a} g(x)}{\lim_{x\to a} h(x)} = \frac{g(a)}{h(a)}$	

	However, if $h(a)$ = 0, then there are two cases arise : **(i) g(a) ≠ 0 (ii) $g(a) = 0$** **In the first case, we say that the limit does not exist.** **In the second case, we can find limit.** **Based on the above information, answer the following questions :** **(a) Find the value of** $$\lim_{x \to 2} \frac{x^2 - 4}{x^3 - 4x^2 + 4x}$$ **(b) Find the value of** $$\lim_{x \to 2} \frac{x^3 - 4x^2 + 4x}{x^2 - 4}$$	**2** **2**
23	 **One day Virat, a friend of Rohit, explain him a direct application mehod to find the derivative of polynomials. He explained that :** **Let f(x) = $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ be a polynomial function, where a_1s are all real numbers and $a_a \neq 0$. Then, the derivative function is given by** $\frac{df(x)}{dx} = n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \ldots + 2a_2 x + a_1$. **After explaining the method, he ask these two questions to Rohit :** **(a) Compute the derivative of : $5x^{100} - x^{48} + 1$.** **(b) Find the derivative of : $f(x) = 1 + x + x^2 + x^3 + \ldots + x^{50}$ at x = 1.** **Help Rohit to solve these questions.**	**4**

QN	MCQs SOLUTIONS	MARKS
1	(B) 2	1
2	(D) 9	1
3	(B) -1	1
4	(D) 88	1
5	(B) 1	1
	SOLUTION (ASSERTION - REASON QUESTIONS)	
6	(a) Both A and B are correct, R is the correct explanation of A	1
7	(a) Both A and B are correct, R is the correct explanation of A	1

	SOLUTION (VSA)	
8	LHL = $\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0} \frac{-x}{x} = -1$ RHL = $\lim_{x\to 0^+} \frac{|x|}{x} = \lim_{x\to 0} \frac{x}{x} = 1$	1 1
9	$\lim_{x\to 1} \frac{x^{21}-1}{x^3-1} = \lim_{x\to 1}\left[\frac{x^{21}-1}{x-1} \div \frac{x^3-1}{x-1}\right] = \lim_{x\to 1}\left[\frac{x^{21}-1}{x-1}\right] \div \lim_{x\to 1}\left[\frac{x^3-1}{x-1}\right]$ $= 21\,(1)^{20} \div 3\,(1)^2$ using $\lim_{x\to a} \frac{x^n-a^n}{x-a} = na^{n-1}$ $= 21 \div 3 = 7$	1/2 1/2 1/2 1/2
10	We have $\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{\sin x}{x \cos x}$ $= \lim_{x\to 0} \frac{\sin x}{x} . \lim_{x\to 0} \frac{1}{\cos x}$ using $\lim_{x\to 0} \frac{\sin x}{x} = 1$, using $\lim_{x\to 0} \cos x = 1$ $= 1.1 = 1$	1/2 1 1/2
11	Let $f(x) = x^2 - 3$, Accordingly, $F'(5) = \lim_{h\to 0} \frac{f(5+h)-f(5)}{h}$ $= \lim_{h\to 0} \frac{[(5+h)^2-3]-[(5)^2-3]}{h}$ $= \lim_{h\to 0} \frac{(5)^2+2.5.h+(h)^2-3-(5)^2+3}{h}$ $= \lim_{h\to 0} \frac{10h+(h)^2}{h}$ $= \lim_{h\to 0} (10 + h) = 10$	1/2 1 1/2
12	$\frac{df(x)}{dx} = \frac{d(\sin x.\sin x)}{dx}$ useing Leibnitz product rule = (sin x)' sin x + sin x (sin x)' = (cos x) sin x + sin x (cos x) = 2 sin x cos x = sin 2x	1/2 1/2 1/2 1/2

Q N	SA SOLUTIONS	MARKS
13	$\lim_{x\to 0}\left[\frac{\tan x - 2\tan 3x + \tan 5x}{x}\right]$ $= \lim_{x\to 0}\left[\frac{\tan x}{x} - \frac{2\tan 3x}{x} + \frac{\tan 5x}{x}\right]$ $= \lim_{x\to 0}\left[\frac{\tan x}{x} - 6\frac{\tan 3x}{3x} + 5\frac{\tan 5x}{5x}\right]$ $= \lim_{x\to 0}\left[\frac{\tan x}{x}\right]$ - 6 $\lim_{x\to 0}\left[\frac{\tan 3x}{3x}\right]$ + 5 $\lim_{x\to 0}\left[\frac{\tan 5x}{5x}\right]$ $= \lim_{x\to 0}\left[\frac{\tan x}{x}\right]$ -6 $\lim_{3x\to 0}\left[\frac{\tan 3x}{3x}\right]$ + 5 $\lim_{5x\to 0}\left[\frac{\tan 5x}{5x}\right]$ = 1 - 6(1) + 5(1) = 0	1 1 1
14	Putting x = 3 $\lim_{x\to 3} \frac{x^4-81}{2x^2-5x-3} = \frac{3^4-81}{2(3^2)-5(3)-3} = \frac{81-81}{18-15-3} = \frac{0}{0}$ since, it is of the form $\frac{0}{0}$	

	We simplify as $\lim_{x\to 3} \frac{x^4-81}{2x^2-5x-3} = \lim_{x\to 3} \frac{(x^2)^2-9^2}{2x^2-6x+x-3} = \lim_{x\to 3} \frac{(x-3)(x+3)(x^2+9)}{(x-3)(2x+1)} = \lim_{x\to 3} \frac{(x+3)(x^2+9)}{(2x+1)}$ **Putting x = 3** $= \frac{(3+3)(3^2+9)}{2(3)+1} = \frac{6(9+9)}{6+1} = \frac{6(18)}{7} = \frac{108}{7}$	1 1 1
15	**On differentiating both sides w.r.t. x. We get** $\frac{dy}{dx} = \frac{d(2x3-4x2+6x+8)}{dx} = 2\frac{d}{dx}x^3 - 4\frac{d}{dx}x^2 + 6\frac{d}{dx}(x) + \frac{d}{dx}(8)$ $= 2(3x^2) - 4(2x) + 6(1) + 0 = 6x^2 - 8x + 6$	$1\frac{1}{2}$ $1\frac{1}{2}$
16	**Let** $y = (x + \frac{1}{x})^3 = x^3 + 3x(\frac{1}{x^2}) + 3x^2(\frac{1}{x}) + \frac{1}{x^3} = x^3 + \frac{3}{x} + 3x + \frac{1}{x^3}$ **On differentiating both sides w.r.t. x. We get** $\frac{dy}{dx} = \frac{d}{dx}x^3 + \frac{d}{dx}(\frac{3}{x}) + \frac{d}{dx}(3x) + \frac{d}{dx}(\frac{1}{x^3})$ $= 3x^2 + 3\frac{d}{dx}x^{-1} + 3\frac{d}{dx}x + \frac{d}{dx}x^{-3}$ $= 3x^2 - \frac{3}{x^2} + 3 - \frac{3}{x^4}$	1 $1\frac{1}{2}$ 1/2
17	**Let f(x) = sin x, then** $\frac{df(x)}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h}$ $= \lim_{h\to 0} \frac{2\cos\left(\frac{2x+h}{2}\right)-\sin(\frac{h}{2})}{h}$ **using formula for sin A - sin B** $= \lim_{h\to 0} \cos(x+\frac{h}{2}) . \lim_{h\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} = \cos x . 1 = \cos x$	1 1 1

QN	LA SOLUTIONS	MARKS
18	**When x = 3, the expression** $\lim_{x\to 3} \frac{x^3-7x^2+15x-9}{x^4-5x^3+27x-27}$ **assumes the form** $\frac{0}{0}$ **So, (x - 3) is a factor of numerator and denominator** **Now, factorizing the numerator and denominator we get** $\lim_{x\to 3} \frac{x^3-7x^2+15x-9}{x^4-5x^3+27x-27}$ $= \lim_{x\to 3} \frac{(x-3)(x^2-4x+3)}{(x-3)(x^3-2x^2-6x+9)}$ $= \lim_{x\to 3} \frac{(x^2-4x+3)}{(x^3-2x^2-6x+9)}$ **form** $\frac{0}{0}$ $= \lim_{x\to 3} \frac{(x-3)(x-1)}{(x-3)(x^2+x-3)}$ $= \lim_{x\to 3} \frac{(x-1)}{(x^2+x-3)} = \frac{3-1}{9+3-3} = \frac{2}{9}$	1 2 2
19	**The given function is** $f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \le x \le 1, \\ nx^3 + m, & x > 1 \end{cases}$ $\lim_{x\to 0^-} f(x) = \lim_{x\to 0} (mx^2 + n) = m(0)^2 + n = n$ $\lim_{x\to +} f(x) = \lim_{x\to 0}(nx + m) = n(0) + m = m$	1

	Thus $\lim_{x\to 0} f(x)$ exists if m = n **$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} (nx + m)$ = n(1) + m = m + n** **$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} (nx^3 + m)$ = n $(1)^3$ + m = m + n** **So, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1} f(x)$.** **Thus $\lim_{x\to 1} f(x)$ exists for any integral value of m and n.**	**$1\frac{1}{2}$** **1** **$1\frac{1}{2}$**
20	**Let f(x) = $\frac{4x + 5\sin x}{3x + 7\cos x}$,** **By quotient rule,** **f'(x) =** $\frac{(3x + 7\cos x)\frac{d}{dx}(4x + 5\sin x) - (4x + 5\sin x)\frac{d}{dx}(3x + 7\cos x)}{(3x + 7\cos x)^2}$ $= \frac{(3x + 7\cos x)\left[4\frac{d}{dx}(x) + 5\frac{d}{dx}(\sin x)\right] - (4x + 5\sin x)\left[3\frac{d}{dx}(x) + 7\frac{d}{dx}(\cos x)\right]}{(3x + 7\cos x)^2}$ $= \frac{(3x + 7\cos x)(4 + 5\cos x) - (4x + 5\sin x)(3 - 7\sin x)}{(3x + 7\cos x)^2}$ $= \frac{12x + 15x\cos x + 28\cos x + 35\cos^2 x - 12x + 28x\sin x - 15\sin x + 35\sin^2 x}{(3x + 7\cos x)^2}$ $= \frac{15x\cos x + 28\cos x + 28x\sin x - 15\sin x + 35}{(3x + 7\cos x)^2}$ $= \frac{15x\cos x + 28\cos x + 28x\sin x - 15\sin x + 35(\sin^2 x + \cos^2 x)}{(3x + 7\cos x)^2}$ $= \frac{15x\cos x + 28\cos x + 28x\sin x - 15\sin x + 35}{(3x + 7\cos x)^2}$	**2** **2** **1**

Q N	**CASE STUDY QUESTIONS**	**MARKS**
21	**(a) We have, f'(x) = cos x - sin x** **So, $\frac{d}{dx}${f'(x)} = $\frac{d}{dx}$ (cos x - sin x)** **= $\frac{d}{dx}$ (cos x)$\frac{d}{dx}$ - (sin x)** **= - sin x - cos x** **So, $\frac{d}{dx}${f'(x)} at x = 90°** **= - sin 90° - cos 90°** **= - 1 - 0 = - 1** **(b) We have f(x) = $\cos^2$ x - $\sin^2$ x** **Then, f'(x) = - 2 cos x.sin x - 2 sin x.cos x** **= - 4 cos x.sin x** **So, f'(45°) = -4 x $\frac{1}{\sqrt{2}}$ x $\frac{1}{\sqrt{2}}$ = $\frac{-4}{2}$ = -2**	**2** **2**
22	**(a) Consider f(x) = $\frac{x^2 - 4}{x^3 - 4x^2 + 4x}$** **On putting x = 2, we get** **f(2) = $\frac{2^2 - 4}{2^3 - 4(2^2) + 4(2)}$ = $\frac{4 - 4}{8 - 16 + 8} = \frac{0}{0}$ i.e., it is of the form $\frac{0}{0}$** **so, let us first factories it** **consider, $\lim_{x\to 2} \frac{x^2 - 4}{x^3 - 4x^2 + 4x} = \lim_{x\to 2} \frac{(x+2)(x-2)}{x(x-2)^2} = \lim_{x\to 2} \frac{(x+2)}{x(x-2)} = \frac{2+2}{2(2-2)} = \frac{4}{0}$** **which is not defined.** **So, $\lim_{x\to 2} \frac{x^2 - 4}{x^3 - 4x^2 + 4x}$ does not exist.** **(b) Evaluating the function at 2, it is of the form $\frac{0}{0}$.**	**2** **2**

	Hence $\lim_{x\to 2} \frac{x^3-4x^2+4x}{x^2-4} = \lim_{x\to 2} \frac{x(x-2)^2}{(x+2)(x-2)} = \lim_{x\to 2} \frac{x(x-2)}{(x+2)} = \frac{2(2-2)}{(2+2)} = \frac{0}{4} = 0$	
23	**(a) Consider the polynomial : $5x^{100} - x^{48} + 1$** **A direct application of the above method gives the derivative of given polynomial $500\,x^{99} - 48\,x^{47}$** **(b) Consider the function : $f(x) = 1 + x + x^2 + x^3 + \ldots + x^{50}$ at $x = 1$** **A direct application of the above method gives the derivative of given function** **$= 1 + 2x + 3x^2 + 4x^3 + \ldots + 50x^{49}$** **At $x = 1$ the value of this function equals :** **$1 + 2 + 3 + \ldots + 50 = \frac{(50)(51)}{2} = 1275$**	**2** **2**

(Chapter -13 : STATISTICS)

CONCEPTS AND RESULTS

** Mean $\bar{x} = \dfrac{\sum x_i}{n}$

$= \dfrac{\sum f_i x_i}{\sum f_i}$

$= a + \left(\dfrac{\sum f_i u_i}{\sum f_i}\right) h$, where a is the assumed mean , h is the class size and $u_i = \dfrac{x_i - a}{h}$

** Median $= \left(\dfrac{n+1}{2}\right)^{th}$ observations (arranged in ascending or descending order) & the number of observations is odd.

$=$ mean of $\left(\dfrac{n}{2}\right)^{th}$ & $\left(\dfrac{n}{2}+1\right)^{th}$ observations (arranged in ascending or descending order) & the number of observations is odd.

$= l + \left(\dfrac{\frac{n}{2} - cf}{f}\right) \times h$ where, $l =$ lower limit of median class, $n =$ number of observations,

$cf =$ cumulative frequency of class preceding the median class, $f =$ frequency of median class,

$h =$ class size .

**Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$, where $l =$ lower limit of the modal class, $h =$ size of the class interval,

$f_1 =$ frequency of the modal class, $f_0 =$ frequency of the class preceding the modal class,

$f_2 =$ frequency of the class succeeding the modal class.

** **Measures of Dispersion:** The dispersion or scatter in a data is measured on the basis of the observations and the types of the measure of central tendency, used there. There are following measures of dispersion: (i) Range, (ii) Quartile deviation, (iii) Mean deviation, (iv) Standard deviation.

** **Range:** Range of a series = Maximum value – Minimum value.

** **Mean Deviation :** The mean deviation about a central value 'a' is the mean of the absolute values of the deviations of the observations from 'a'. The mean deviation from 'a' is denoted as M.D. (a).

(i) For ungrouped data

$$\text{M.D.}(\bar{x}) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|, \text{ where } \bar{x} = \text{Mean} \qquad \text{M.D.}(M) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|, \text{ where } M = \text{Median}$$

(ii) For grouped data

(a) Discrete frequency distribution

$$\text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{n} f_i|x_i - \bar{x}|}{\sum_{i=1}^{n} f_i} = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}| \qquad \text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{n}|x_i - M|$$

(b) Continuous frequency distribution

$$\text{M.D.}(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|, \text{ using } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right)h$$

$$\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{n}|x_i - M|, \text{ using } M = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Section A (MCQ)

Q.1 If the variance of the data is 121, the standard deviation of the data is:

(a) 121 (b) 11 (c) 12 (d) 21

Answer: (b) 11

Q.2**Find the median of 36, 72, 46, 42, 60, 45, 53, 46, 51, 49.**

A. 42 B. 45.5 C. 47.5 D. 45

Answer: C. 47.5

Q.3 Range of a data is equal to:

(a) Range = Max Value – Min Value (b) Range = Max Value + Min Value

(c) Range = (Max Value – Min Value)/2 (d) Range = (Max Value + Min Value)/2

Answer: (a) Range = Max Value – Min Value

Q.4 Mean deviation from mean of the following data ; 4,7,8,9,10,12,13,17 is

(a) 2 (b) 3 (c) 4 (d) 5

Answer : (b)

Q.5 If the mean of 1,3,4,5,7 is m ,the numbers 3,2,2,4,3,3,p have mean m-1 and median q then p + q is

(a) 4 (b) 5 (c) 6 (d) 7

Answer : (d) 7

Assertion Reason Questions

DIRECTION : In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true.

Q.1 Assertion (A) The mean deviation about mean for the data 4,7,8,9,10,12,13,17,3.

Reason (R) The mean deviation about the mean for 38,70,48,40,42,55,63,46,54 is 8.5.

Answer :- (c)

Q.2 Consider the following data

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Then, the mean deviation about the mean is 6.32.

Reason (R) Consider the following data

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Then, the mean deviation about the mean is 15 .

Answer :- (c)

Q.3 Assertion (A) The mean deviation about median calculated for series, where variability is very high, cannot be fully relied.

Reason (R) – The median is not a representative of central tendency for the series where degree of variability is very high.

Answer :- (a)

Section B Short Answer Type Question(2 Marks Questions)
Question 1. Find the mean deviation about the mean.

x_i	5	10	15	20	25
f_i	7	4	6	3	5

x_i	f_i	$f_i x_i$	$\lvert x_i - 14 \rvert$	$f_i \lvert x_i - 14 \rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	350		158

$$\text{Mean}(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{1}{25} \times 350 = 14$$

$$\therefore \quad \text{Mean deviation about mean} = \frac{1}{N}\sum_{i=1}^{n} f_i \lvert x_i - \bar{x} \rvert$$

$$= \frac{1}{25} \times 158 = 6.32$$

Find the mean deviation about the median for the data in Exercises 7 and 8.
Question 2.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Solution :-

x_i	f_i	$c.f.$	$\lvert x_i - 7 \rvert$	$f_i \lvert x_i - 7 \rvert$
5	8	8	2	16
7	6	14	0	0
9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48
	26			84

Here, $\frac{N}{2} = \frac{26}{2} = 13$

The *c.f.* just greater than 13 is 14 and the corresponding value of x is 7.

So, Median $(M) = 7$

$$\therefore \text{ M.D. about median } = \frac{1}{N}\sum_{i=1}^{n} f_i \lvert x_i - M \rvert$$

$$= \frac{1}{26} \times 84 = 3.23$$

Question 3.

x_i	15	21	27	30	35	
f_i	3	5	6	7	8	

Solution :-

x_i	f_i	$c.f.$	$\lvert x_i - 30 \rvert$	$f_i \lvert x_i - 30 \rvert$
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40
	29			148

Here, $\frac{N}{2} = \frac{29}{2} = 14.5$

The *c.f.* just greater than 14.5 is 21 and the corresponding value of x is 30.

$\therefore$ Median $(M) = 30$

$$\text{M.D. about median } = \frac{1}{N}\sum_{i=1}^{n} f_i \lvert x_i - M \rvert$$

$$= \frac{1}{29} \times 148 = 5.1$$

Find the mean deviation about the mean for the data in Exercises 9 and 10.

Question 4.

Income per day	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800
Number of persons	4	8	9	10	7	5	4	3

Solution:-

Income per day	Mid values x_i	f_i	$f_i x_i$	$\lvert x_i - 358 \rvert$	$f_i \lvert x_i - 358 \rvert$
0 – 100	50	4	200	308	1232
100 – 200	150	8	1200	208	1664
200 – 300	250	9	2250	108	972
300 – 400	350	10	3500	8	80
400 – 500	450	7	3150	92	644
500 – 600	550	5	2750	192	960
600 – 700	650	4	2600	292	1168
700 – 800	750	3	2250	392	1176
		50	17900		7896

$$\text{Mean } (\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{1}{50} \times 17900 = 358$$

Mean deviation about mean

$$= \frac{1}{N}\sum_{i=1}^{n} f_i \lvert x_i - \bar{x} \rvert = \frac{1}{50} \times 7896 = 157.92$$

Question 5.

Heights (in cm)	95-105	105-115	115-125	125-135	135-145	145-155
Number of boys	9	13	26	30	12	10

Solution :-

Height (in cms)	Mid values x_i	f_i	$f_i x_i$	$\lvert x_i - 125.3 \rvert$	$f_i \lvert x_i - 125.3 \rvert$
95 – 105	100	9	900	25.3	227.7
105 – 115	110	13	1430	15.3	198.9
115 – 125	120	26	3120	5.3	137.8
125 – 135	130	30	3900	4.7	141
135 – 145	140	12	1680	14.7	176.4
145 – 155	150	10	1500	24.7	247
		100	12530		1128.8

$$\text{Mean } (\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{1}{100} \times 12530 = 125.3$$

Mean deviation about mean

$$= \frac{1}{N}\sum_{i=1}^{n} f_i \lvert x_i - \bar{x} \rvert = \frac{1}{100} \times 1128.8 = 11.28$$

(3 Marks Questions)

Question 1.

From the prices of shares X and Y below, find out which is more stable in value:

X	Y
35	108
54	107
52	105
53	105
56	106
58	107
52	104
50	103
51	104
49	101

X	Y	$(X-\bar{X})$	$(Y-\bar{Y})$	$(X-\bar{X})^2$	$(Y-\bar{Y})^2$
35	108	−16	3	256	9
54	107	3	2	9	4
52	105	1	0	1	0
53	105	2	0	4	0
56	106	5	1	25	1
58	107	7	2	49	4
52	104	1	−1	1	1
50	103	−1	−2	1	4
51	104	0	−1	0	1
49	101	−2	−4	4	16
510	1050			350	40

$$\bar{X}=\frac{510}{10}=51, \bar{Y}=\frac{1050}{10}=105$$

$$\sigma_X=\sqrt{\frac{\Sigma(X-\bar{X})^2}{n}}=\sqrt{\frac{350}{10}}=5.92$$

$$\sigma_Y=\sqrt{\frac{\Sigma(Y-\bar{Y})^2}{n}}=\sqrt{\frac{40}{10}}=2$$

$$\text{C.V. of } X=\frac{\sigma_X}{\bar{X}}\times 100=\frac{5.92}{51}\times 100=11.60$$

$$\text{C.V. of } Y=\frac{\sigma_Y}{\bar{Y}}\times 100=\frac{2}{105}\times 100=1.9$$

$\because$ C.V. of Y < C.V. of X

Thus prices of share Y are more stable.

Question 2

Find the mean and standard deviation using short-cut method

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Solution :-

x_i	f_i	$u_i = x_i - 64$	$f_i u_i$	$f_i u_i^2$
60	2	–4	–8	32
61	1	–3	–3	9
62	12	–2	–24	48
63	29	–1	–29	29
64	25	0	0	0
65	12	1	12	12
66	10	2	20	40
67	4	3	12	36
68	5	4	20	80
	100		0	286

Let assumed mean $(A) = 64$

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} = 64 + \frac{0}{100} = 64$$

$$\text{S.D. } (\sigma) = \frac{1}{N}\sqrt{N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2}$$

$$= \frac{1}{100}\sqrt{100 \times 286 - (0)^2}$$

$$= \frac{1}{100}\sqrt{28600} = \frac{1}{100} \times 169.1 = 1.69$$

Find the mean and variance for the following frequency distributions in Exercises 13and 14.

Question 3

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

Solution :-

Classes	Mid values x_i	f_i	$u_i = \frac{x_i - 105}{30}$	$f_i u_i$	$f_i u_i^2$
0 – 30	15	2	–3	–6	18
30 – 60	45	3	–2	–6	12
60 – 90	75	5	–1	–5	5
90 – 120	105	10	0	0	0
120 – 150	135	3	1	3	3
150 – 180	165	5	2	10	20
180 – 210	195	2	3	6	18
		30		2	76

Let assumed mean $(A) = 105$

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} \times h = 105 + \frac{2}{30} \times 30 = 107$$

$$\text{Variance } (\sigma^2) = \frac{h^2}{N^2}[N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2]$$

$$= \frac{900}{900}[30 \times 76 - 4] = [2280 - 4] = 2276$$

Question 4.

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Solution:-

Classes	Mid values x_i	f_i	$u_i = \frac{x_i - 25}{10}$	$f_i u_i$	$f_i {u_i}^2$
0 – 10	5	5	–2	–10	20
10 – 20	15	8	–1	–8	8
20 – 30	25	15	0	0	0
30 – 40	35	16	1	16	16
40 – 50	45	6	2	12	24
		50		10	68

Let assumed mean $(A) = 25$

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} \times h = 25 + \frac{10}{50} \times 10$$

$$= 25 + 2 = 27$$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}[N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2]$$

$$= \frac{(10)^2}{(50)^2}[50 \times 68 - (10)^2] = \frac{100}{2500}[3400 - 100]$$

$$= \frac{1}{25} \times 3300 = 132$$

Question 5.

Find the mean, variance and standard deviation using short-cut method.

Heights in cms	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of children	3	4	7	7	15	9	6	6	3

Solution :-

Height in cms	Mid values x_i	f_i	$u_i = \frac{x_i - 92.5}{5}$	$f_i u_i$	$f_i u_i^2$
70 – 75	72.5	3	–4	–12	48
75 – 80	77.5	4	–3	–12	36
80 – 85	82.5	7	–2	–14	28
85 – 90	87.5	7	–1	–7	7
90 – 95	92.5	15	0	0	0
95 – 100	97.5	9	1	9	9
100 – 105	102.5	6	2	12	24
105 – 110	107.5	6	3	18	54
110 – 115	112.5	3	4	12	48
		60		6	254

Let assumed mean $(A) = 92.5$

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} \times h = 92.5 + \frac{6}{60} \times 5$$

$$= 92.5 + 0.5 = 93$$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}[N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2]$$

$$= \frac{(5)^2}{(60)^2}[60 \times 254 - (6)^2]$$

$$= \frac{25}{3600} \times 15204 = 105.58$$

$$\text{Standard deviation } (\sigma) = \sqrt{105.58} = 10.27$$

(5 marks Questions Question)

Question 1.

The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of Circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

[Hint: First make the data continuous by making the classes as 32.5 – 36.5, 36.5 – 40.5, 40.5 – 44.5, 44.5 – 48.5, 48.5 – 52.5 and then proceed.]

Diameters	Modified Classes	Mid values x_i	f_i	$u_i = \frac{x_i - 42.5}{4}$	$f_i u_i$	$f_i u_i^2$
33 – 36	32.5 – 36.5	34.5	15	–2	–30	60
37 – 40	36.5 – 40.5	38.5	17	–1	–17	17
41 – 44	40.5 – 44.5	42.5	21	0	0	0
45 – 48	44.5 – 48.5	46.5	22	1	22	22
49 – 52	48.5 – 52.5	50.5	25	2	50	100
			100		25	199

Let assumed mean $(A) = 42.5$

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} \times h = 42.5 + \frac{25}{100} \times 4$$

$$= 42.5 + 1 = 43.5$$

$$\text{Standard deviation}(\sigma) = \frac{h}{N}\sqrt{N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2}$$

$$= \frac{4}{100}\sqrt{100 \times 199 - (25)^2} = \frac{1}{25}\sqrt{19275}$$

$$= \frac{1}{25} \times 138.83 = 5.55$$

Question 2: The mean and variance of 8 observations are 9 and 9.25 respectively. If six of the observations are 6 , 7 , 10, 12 , 12 and 13 . Find the remaining two observations.
Solution :-

Let the remaining two observations be x and y.

Therefore, the observations are 6, 7, 10, 12, 12, 13, x, y.

$$\text{Mean, } \bar{x} = \frac{6+7+10+12+12+13+x+y}{8} = 9$$

$$\Rightarrow 60 + x + y = 72$$

$$\Rightarrow x + y = 12 \qquad \ldots(1)$$

$$\text{Variance} = 9.25 = \frac{1}{n}\sum_{i=1}^{8}\left(x_i - \bar{x}\right)^2$$

$$9.25 = \frac{1}{8}\left[(-3)^2 + (-2)^2 + (1)^2 + (3)^2 + (3)^2 + (4)^2 + x^2 + y^2 - 2\times 9(x+y) + 2\times(9)^2\right]$$

$$9.25 = \frac{1}{8}\left[9 + 4 + 1 + 9 + 9 + 16 + x^2 + y^2 - 18(12) + 162\right]$$

...[Using (1)]

$$9.25 = \frac{1}{8}\left[48 + x^2 + y^2 - 216 + 162\right]$$

$$9.25 = \frac{1}{8}\left[x^2 + y^2 - 6\right]$$

$$\Rightarrow x^2 + y^2 = 80 \qquad \ldots(2)$$

From (1), we obtain

$x^2 + y^2 + 2xy = 144$ ...(3)

From (2) and (3), we obtain

$2xy = 64$ ... (4)

Subtracting (4) from (2), we obtain

$x^2 + y^2 - 2xy = 80 - 64 = 16$

$x - y = \pm 4$ ... (5)

Therefore, from (1) and (5), we obtain

$x = 8$ and $y = 4$, when $x - y = 4$

$x = 4$ and $y = 8$, when $x - y = -4$

Thus, the remaining observations are 4 and 8.

Question 3: The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are 2 , 4 , 10 , 12 , and 14 . Find the remaining two observations .

Solution :-

Let the remaining two observations be x and y.

The observations are 2, 4, 10, 12, 14, x, y.

$$\text{Mean, } \bar{x} = \frac{2+4+10+12+14+x+y}{7} = 8$$

$$\Rightarrow 56 = 42 + x + y$$

$$\Rightarrow x + y = 14 \qquad \ldots(1)$$

$$\text{Variance} = 16 = \frac{1}{n}\sum_{i=1}^{7}\left(x_i - \bar{x}\right)^2$$

$$16 = \frac{1}{7}\left[(-6)^2 + (-4)^2 + (2)^2 + (4)^2 + (6)^2 + x^2 + y^2 - 2\times 8(x+y) + 2\times(8)^2\right]$$

$$16 = \frac{1}{7}\left[36 + 16 + 4 + 16 + 36 + x^2 + y^2 - 16(14) + 2(64)\right]$$

...[Using (1)]

$$16 = \frac{1}{7}\left[108 + x^2 + y^2 - 224 + 128\right]$$

$$16 = \frac{1}{7}\left[12 + x^2 + y^2\right]$$

$$\Rightarrow x^2 + y^2 = 112 - 12 = 100$$

$$x^2 + y^2 = 100 \qquad \ldots(2)$$

From (1), we obtain

$x^2 + y^2 + 2xy = 196$ ... (3)

From (2) and (3), we obtain

$2xy = 196 - 100$

$2xy = 96$ ... (4)

Subtracting (4) from (2), we obtain

$x^2 + y^2 - 2xy = 100 - 96$

$(x - y)^2 = 4$

$x - y = \pm 2$ ... (5)

Therefore, from (1) and (5), we obtain

$x = 8$ and $y = 6$ when $x - y = 2$

$x = 6$ and $y = 8$ when $x - y = -2$

Thus, the remaining observations are 6 and 8.

Section D (Case Study)

Q.1 You are gfiven the following data

Xi	2	5	6	8	10	12
fi	2	8	10	7	8	5

Based on the following data answer the following questions

I) Mean of the grouped data is (a) 7 (b) 7.5 (c) 8 (d) 8.5

II)Mean deviation about the mean is (a) 2.1 (b) 2.2 (c) 2.3 (d) 2.4

III) The value of median is (A) 5 (b) 6 (c) 7 (d) 8

IV) The mean deviation about mean is (a) 1.9 (2.0 (c) 2.2 (d)2.3

V) The difference between mean and median is (a)0 .9 (b) 0.7 (c) 0.5 (d) 0.3

Answer – (I) b , (II) c , (III) c ,(IV) d, (V) c

Q.2. You are given some observations as 34, 66, 30, 38, 44, 50, 40, 60,42, 51.

Based on the these observations, answer the following questions.

(i) The mean of the given data is (a) 40.5 (b) 45.0 (c) 45.5 (d) 50.50

(ii) The mean deviation about mean is (a) 10 (b) 9.5 (c) 9.1 (d) 9

(iii) The mean of the data is (a) 41 (b) 42 (c) 43 (d) 44

(iv) The mean deviation about the median is (a) 8.0 (b) 8.3 (c) 8.5 (d) 8.7

(v) The difference between mean deviation about mean and mean deviation about median is

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

Answer :- (i) c, (ii) d, (iii) c , (iv) d , (v) (c)

Q.3 Consider the data

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Based on above information answer the following questions.

(i) Mean is calculated by using the formula

$(a)\quad \bar{x} = \frac{\sum \text{fixi}}{N}$ (b) $\bar{x} = \sum \text{fi xi}$ (c) $\bar{x} = \frac{\sum fx^2}{N}$ (d) None of These

(ii) Variance is calculated by using the formula

(a) $\sigma^2 = \frac{1}{N}\sum \text{fi}(\text{xi} - \overline{x})^2$ (b) $\sigma^2 = \frac{1}{N}\sum \text{fi}(\text{xi} + \overline{x})^2$ (c) $\sigma^2 = \frac{1}{N}\sum \text{fi}(\text{xi} - \overline{x})$ (d) None of these .

(iii) Mean of the given data is

(a) 10 (b) 12 (c) 14 (d) 15

(iv) Variance of the given data is

(a) 40 (b) 45.8 (c) 41.5 (d) 39.8

(iv) Standard deviation of the given data

(a) 6.77 (b) 5 (c) 4.8 (d) 3.19

Answer :- (I) a (ii) a (iii) c (iv) b (v) a

(Chapter -14 : PROBABILITY)

MCQs (5 Questions)

1-. If A and B are two events such that $A \neq \emptyset$, $B \neq \emptyset$, then

(a). P (A/B) = P(A) P(B) (b) $P(A/B) = \frac{PA(\cap B)}{P(B)}$

(c). P(A/B) P(B/A) = 1 (d) P (A/B) = P(A) / P(B)

Ans: (b)

2-. -. If A and B are two events such that $P(A) \neq 0$, and P(B/A) = 1, then

(a). $A \subset B$ (b) $B \subset A$ (c) $B = \emptyset$, (d) $A = \emptyset$,

Ans: (a)

3-. If P (A/B) > P(A), then which of the following is correct ?

(a). P(B/A) < P(B) (b) $P(A \cap B) <$ P(A) . P(B)

(c) P(B/A) > P(B) (d) P(B/A) = P(B)

Ans: (c)

4-. If A and B are two events such that P(A) + P(B) - P(A and B) = P(A), then

(a). P(B/A) = 1 (b) P(A/B) = 1 (c) P(B/A) = 0 (d) P(A/B) = 0

Ans: (b)

5-. If A and B are two events such that P(A) > 0 and $P(B) \neq 1$, the P(A' / B') equals

(a). 1 - P(A/B) (b) 1 - P(A' / B) (c) $\frac{1-P(A \cup B)}{P(B')}$ (d) P(A') / P(B')

Ans: (c)

ASSERTION-REASON BASED QUESTIONS (2 Questions)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R).

Choose the correct answer out of the following choices.

(A) If Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) If Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) If (A) is true but (R) is false. (D) If (A) is false but (R) is true.

1-. ASSERTION (A): 20 persons are sitting in a row, two of these persons are selected at random. The probability that the two selected person are not together is 0.9.

REASON (R) : If $\bar{A}$ denotes the negation of event A, then $P(\bar{A})$ 1 – P(A)

Ans: (a)

2-. ASSERTION (A): the probability of drawing either a king or an ace from a pack of 52 playing card is 2/13.

REASON (R) : For any two events A and B $P(A \cup B) = P(A) + P(B) - P(A - B)$

Ans: (c)

2- MARKS QUESTIONS (5 questions)

1-. 10% of the bulbs produced in a factory are red colour and 2% are red colour and defective. If one bulb is picked at random, determine the probability of its being defective if I red.

Ans : 1/5

2-. Three dice are thrown at the same time. Find the probability of getting three two's if it is known that the sum of numbers on the dice was a six.

Answer: 1/10

3-. A class consists of 80 students; 25 of them are girls and 55 boys; 10 of them has joined coaching and remaining has not joined the coaching; 20 of them are healthy. What is the probability of selecting a healthy coaching going girl ?

Answer : 5/512

4-. The probability that a teacher will give an un-announced test during any class meeting is 1/5.If a student is absent twice, what is the probability that he will miss at least one test ?

Answer: 9/25

5-. A coin is tossed and a die is thrown. Find the probability that the outcome will be head or a number greater than 4, or both.

Answer: 2/3

3- MARKS QUESTIONS (5 questions)

1-. Suppose you have two coins which appear identical in your pocket. You known that one is fair and one is two headed. If you take one out, toss it and get a head, what is the probability that it was a fair coin ?

Answer : 1/3

2-. Three bags contains a number of red and white balls as follow:

Bag I : 3 red balls; Bag II : 2 red balls; bag III : 3 white balls

The probability that bag I will be chosen and a ball is selected from it is i/6, I = 1,2,3. If a white ball is selected, what is the probability that it come from Bag III ?

Answer : 9/11

3-. A shopkeeper sells three types of seeds A-1, A-2 and A-3. They are sold as a mixer where the proportions are 4 : 4 : 2 respectively. The germination rates of three types of seeds are 45%, 60% and 35% respectively. Calculate the probability that it will nor germinate given that the seed is of type A-3.

Answer: 35/100

4-. There are three coins. One is two headed coin, another is based coin that comes up heads 75% of the times and third is an unbased coin. One of the three coins is choosen at random and tossed, If shows head, what is the probability that it was the two headed coin ?

Answer: 4/9

5-. Bag A contain 3 red and 5 black balls, white bag B contains 4 red balls and 4 black balls. Two balls are transferred at random from bag A to bag B and then a ball is drawn from bag B at random. If the ball drawn from bag B is found to be red, find the probability that two red balls where transferred from bag A to bag B.

5- MARKS QUESTIONS (2 questions)

1-. Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail'. (Answer: 2/9)

2-. In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B? (Answer : 28/69)

CASE STUDY BASED QUESTIONS (PROBABILITIES)

CASE STUDY - 1

Attempt any four sub-parts from the questions given. Each sub-part carries 1 mark.

I Read the following text and answer the following questions basis of the same:

A coach is trying 3 players. He observed that the player A can hit the target 4 times in 5 shots, the player B can hit the target 3 times in 4 shots and the player C can hit the target 2 times in 3 shots.

1. Let the target is hit by A, B: the target is hit by B and C: the target is hit by A and C. Then, the probability that A, B and, C all will hit, is:

(A) 4/5 (B) 3/5 (C) 2/5 (D) 1/5 [Correct answer is C]

2. What is the probability that B, C will hit and A will lose?

(A) 1/10 (B) 3/10 (C) 7/10 (D) 4/10 [Correct answer is A]

3. What is the probability that 'any two of A, B and C will hit?

(A) 1/30 (B) 11/30 (C) 17/30 (D) 13/30 [Correct answer is D]

4. What is the probability that 'none of them will hit the target'?

(A) 1/30 (B) 1/60 (C) 1/15 (D) 2/15 [Correct answer is B]

5. What is the probability that at least one of A, B or C will hit the target ?

6. (A) 59/60 (B) 2/5 (C) 3/5 (D) 1/60 [Correct answer is A]

CASE STUDY - 2

II. Read the following text and answer the following questions on the basis of the same:

The reliability of a COVID PCR test is specified as follows: Of people having COVID, 90% of the test detects the disease but 10% goes undetecte. Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test, and the pathologist reports him/ her as COVID positive.

Q. 1. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually having COVID'?
(A) 0.001 (B) 0.1 (C) 0.8 (D) 0.9 [Ans. Option (D) is correct.]

Q. 2. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually not having COVID'?
(A) 0.01 (B) 0.99 (C) 0.1 D) 0.001 [Ans. Option (A) is correct.]

Q. 3. What is the probability that the 'person is actually not having COVID'?
(A) 0.998 (B) 0.999 (C) 0.001 (D) 0.111 [Ans. Option (B) is correct.]

Q. 4. What is the probability that the 'person is actually having COVID given that 'he is tested as COVID positive'?
(A) 0.83 (B) 0.0803 (C) 0.083 (D) 0.089 [Ans. Option (C) is correct].

Q.5. What is the probability that the 'person selected will be diagnosed as COVID positive'?
(A) 0.1089 (B) 0.01089 (C) 0.0189 (D) 0.189 [Ans. Option (B) is correct.]

CASE STUDY - 3

III. Read the following text and answer the following questions on the basis of the same:

In answering a question on a multiple choice test for class XII, a student either knows the answer or guesses. Let 3/5 be the probability that he knows the answer and 2/5 be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability 1/3. Let E_1, E_2, E be the events that the student knows the answer, guesses the answer and answers correctly respectively.

Q. 1. What is the value of $P(E_1)$?
(A) 2 /5 (B) 1/ 3 (C) 1 (D) 3/5 [Ans. Option (D) is correct].

Q. 2. Value of $P(E/ E_1,)$ is:
A) 1/3 B) 1 (C) 2/3 (D) 4/5 [Ans. Option (B) is correct.]

Q.3. $\sum_{k=1}^{k=2} P(E/Ek)\, P(Ek)$ Equals :

(A) 11/5 (B) 4/15 (C) 1/5 (D) 1 [Ans. Option (A) is correct.]

Q.4. $\sum_{k=1}^{k=2} P(Ek)$ Equals :

(A) 1/3 (B) 1/5 (C) 1 (D) 3/5 [Ans. Option (C) is correct.]

Q.5 What is the probability that the student knows the answer given that he answered it correctly?

(A) 2/11 (B) 5/3 (C) 9/11 (D) 13/3 [Ans. Option (C) is correct.]

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BLUE PRINT

CLASS XI: SAMPLE PAPER 2023-24

SUBJECT: MATHEMATICS (041)

S.NO	UNITS	NAME OF CHAPTERS TOPIC	SECTION –A (01 MARK EACH)		SECTION – B (VSA) (02 MARKS EACH)	SECTION – C (SA) (03 MARKS EACH)	SECTION – D (LA) (05MARKS EACH)	SECTION – E (CBQ) (04 MARKS EACH)	TOTAL
			MCQ	ARQ					
1.	SETS AND FUNCTIONS (23)	SETS	1(1)	1(1)				4(1)	6(3)
2.		RELATIONS AND FUNCTIONS	2(2)			6(2)			8(4)
3.		TRIGONOMETRIC FUNCTIONS	1(1)			3(1)	5(1)		9(3)
4.	ALGEBRA (25)	COMPLEX NUMBERS AND QUADRATIC EQUATIONS	1(1)			3(1)			4(2)
5.		LINEAR INEQUALITIES					5(1)		5(1)
6.		PERMUTAIONS AND COMBINATIONS	1(1)	1(1)				4(1)	6(3)
7.		BINOMIAL THEOREM	2(2)		2(1)				4(3)
8.		SEQUENCE AND SERIES	1(1)		2(1)	3(1)			6(3)
9.	CO ORDINATE GEOMETRY (12)	STRAIGHT LINES	1(1)				5(1)		6(2)
10.		CONIC SECTION	1(1)		2(1)				3(2)
11.		3D GEOMETRY	1(1)		2(1)				3(2)
12	CALCULAS (08)	LIMITS AND DERIVATIVES	3(3)		2(1)	3(1)			8(5)
13	STATISTICS AND PROBABILITY (12)	STATISTICS	1(1)				5(1)		6(2)
14		PROBABILITY	2(2)					4(1)	6(3)
		TOTAL	18(18)	2(2)	10(5)	18(6)	20 (4)	12(3)	80(38)

KENDRIYA VIDYALAYA SANGATHAN

JABALPUR REGION

SAMPLE PAPER 2023-24

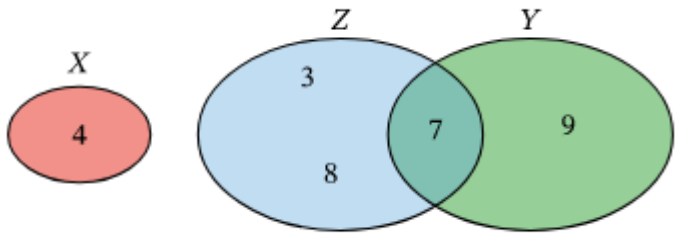
CLASS - XI

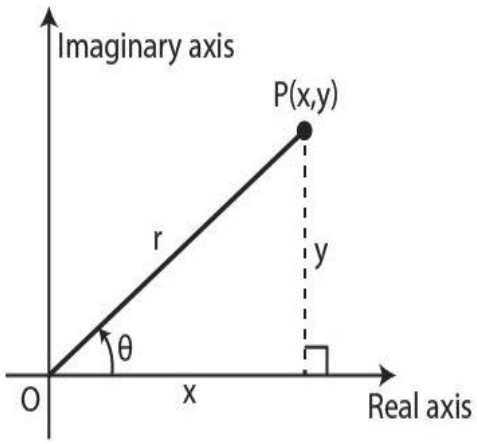
MATHEMATICS (041)

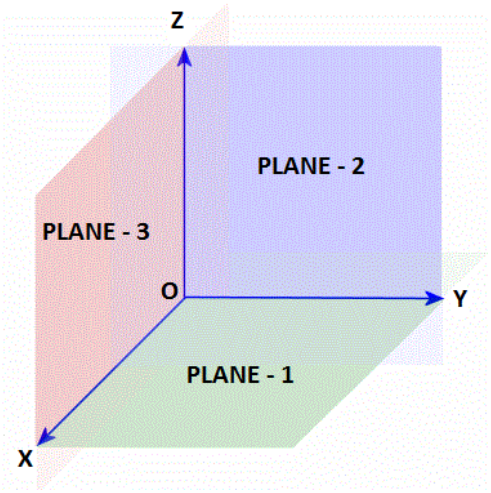
Time allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

i	This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions. You have to attempt only one of the alternatives in all such questions.
ii	Section A has 18 MCQ's and 02 Assertion - Reason based questions of 1 mark each.
iii	Section B has 5 Very Short Answer (VSA) - type questions of 2 marks each.
iv	Section C has 6 Short Answer (SA) - type questions of 3 marks each.
v	Section D has 4 Long Answer (LA) - type questions of 5 marks each.
vi	Section E has 3 source based / case based / passage based / integrated units of assessment (4 marks each) with sub parts.
vii	Use of calculator is not permitted. You may ask for logarithmic tables, if required.

SECTION – A		
Direction (Q.1 - Q.18) - There are multiple choice type questions. Choose the correct answer:		
Q. NO.	**QUESTIONS**	**MARKS**
1	"The complement of the intersection of two sets is the union of their complements." This statement is called: (a) Complement Law (b) Associative Law (c) Idempotent Law (d) De Morgan's Law	1
2	For the Venn - diagram given below, the set $(Z - Y) \times (X \cup Y)$ is:  (a) {(3,4), (3,7), (3,9), (8,4), (8,7), (8,9)} (b) {(4,8), (9,8), (7,8), (4,3), (9,3), (7,3)} (c) {(8,4), (8,9), (8,7), (4,3), (9,3), (7,3)} (d) {(4,8), (9,8), (7,8), (3,4), (3,9), (3,7)}	1
3	If f is the identity function and g is the modulus function then which of the following is true? (a) $(f+g)(x) = \begin{cases} 0, & x \geq 0 \\ 2x, & x < 0 \end{cases}$ (b) $(f+g)(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$ (c) $(f+g)(x) = \begin{cases} 2x, & x \geq 0 \\ -2x, & x < 0 \end{cases}$ (d) $(f+g)(x) = \begin{cases} -2x, & x \geq 0' \\ 2x, & x < 0 \end{cases}$	1
4	The angle made by an arc of unit length of a unit circle is (a) 1 degree (b) 360 degree (c) 1 radian (d) π radians	1
5	How many chords can be drawn through 15 points on a circle? (a) 30 (b) 90 (c) 105 (d) 210	1

Q. NO.	QUESTIONS	MARKS
6	In the figure given below, the point $P(x, y)$ represents the complex number $z = x + iy$. The length r of line segment OP is called:  (a) Conjugate of z. (b) Modulus of z. (c) Real part of z. (d) Imaginary part of z.	1
7	Consider the following statements about Pascal's triangle: (i) Every number in a row is the sum of the two numbers diagonally above it (ii) Sum of the numbers in a row is equal to 2^n (iii) It is a triangular arrangement of numbers that gives the coefficients in the expansion of any binomial expression, such as $(x + y)^n$ Which of the (i), (ii) and (iii) are always true: (a) Only (i) and (ii) (b) Only (i) and (iii) (c) Only (ii) and (iii) (d) All are true	1
8	Total number of terms in the expansion of $(x + a)^{51} + (x - a)^{51}$ after simplification are: (a) 26 (b) 52 (c) 25 (d) 104	1
9	If the Arithmetic Mean and Geometric Mean of two different positive real numbers a and b are A.M. and G.M. respectively, then which of the following is always true? (a) A.M. $\leq$ G.M. (b) A.M. $<$ G.M. (c) A.M. $\geq$ G.M. (d) A.M. $>$ G.M.	1

Q. NO.	QUESTIONS	MARKS
10	The equation of straight line parallel to the $x-axis$ at a distance 5 units below it is: (a) $y=-5$ (b) $y=5$ (c) $x==-5$ (d) $x=5$	1
11	The coordinates of the centre and radius of the circle whose equation in standard form is $(x+3)^2+(y-5)^2=16$, are: (a) Centre: $(-3,5)$, Radius: 4 (b) Centre: $(3,-5)$, Radius: 4 (c) Centre: $(-3,5)$, Radius: 16 (d) Centre: $(3,-5)$, Radius: 16	1
12	In the figure given below, the names of the planes 1, 2 and 3 are respectively:  Plane - 1 Plane - 2 Plane - 3 (a) YZ ZX XY (b) XY YZ ZX (c) XZ YZ XY (d) XY ZX YZ	1
13	The value of $\lim_{x\to 0}\frac{x}{x}$ is: (a) 0 (b) 1 (c) ∞ (d) Does not exist	1
14	Derivative of a function $y=f(x)$ with respect to x at $x=a$ is: (a) Slope of line parallel to $x-axis$ at the point $(a,f(a))$ (b) Slope of line parallel to $y-axis$ at the point $(a,f(a))$ (c) Slope of tangent to the curve $y=f(x)$ at the point $(a,f(a))$ (d) Slope of the line passing through origin and the point $(a,f(a))$	1

Q. NO.	QUESTIONS	MARKS
15	The value of $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 3x}\right)$ is: (a) $\frac{3}{4}$ (b) 4 (c) 3 (d) $\frac{4}{3}$	1
16	Which of the following is not a measure of dispersion: (a) Median (b) Range (c) Mean deviation (d) Standard deviation	1
17	If A and B are two mutually exclusive events then which of the following are true: (i) $A \cap B = \phi$ (ii) $P(A) = P(B)$ (iii) $P(A) + P(B) = 1$ (iv) $P(A \cup B) = P(A) + P(B)$ (a) Only (i) and (ii) (b) Only (ii) and (iii) (c) Only (i) and (iv) (d) Only (iii) and (iv)	1
18	Three integers are chosen at random from 1 to 10. The probability that they are non-consecutive is: (a) $\frac{1}{15}$ (b) $\frac{14}{15}$ (c) $\frac{13}{15}$ (d) $\frac{7}{15}$	1

ASSE RTION - REASON BASED QUESTIONS

Direction (Q.19 - Q.20) -

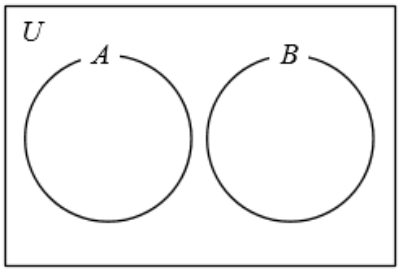
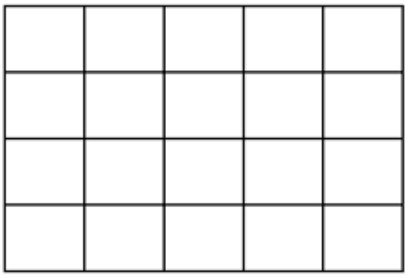
In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Q. NO.	QUESTIONS	MARKS
19	**Assertion (A):**  If $A \subseteq B$ for any two sets A and B, then adjacent Venn diagram represents correct relationship between A and B. **Reason (R):** If $A \subseteq B$, then all elements of A are also in B.	1
20	**Assertion (A):**  In the given figure total number of rectangles of any size is 150. **Reason (R):** In a $m \times n$ grid (m horizontal and n vertical straight lines) total numbers of rectangles formed is $m_{C_2} \times n_{C_2}$ (Selecting two horizontal and two vertical straight lines simultaneously)	1

SECTION – B

Direction (Q.21 - Q.25) - This section comprises of very short answer type-questions (VSA) of 2 marks each.

Q. NO.	QUESTIONS	MARKS
21	Using binomial theorem, expand $\left(x^2 + \frac{3}{x}\right)^4, x \neq 0$ **OR** Using the binomial theorem, show that $6^n - 5n$ always leaves remainder 1 when divided by 25.	2
22	Insert two arithmetic means between 49 and 28.	2

Q. NO.	QUESTIONS	MARKS
23	Find the equation of parabola with vertex $(0, 0)$ and focus $(0, -6)$. **OR** Find the eccentricity and the coordinates of foci of the ellipse $9x^2 + 4y^2 = 36$.	2
24	Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point P (3, –2, 5).	2
25	Evaluate: If $\lim_{x\to 2}\left(\frac{x^n - 2^n}{x-2}\right) = \lim_{x\to 4}(5x^2 + 2x - 8)$ and $n \in N$, then find n.	2

SECTION – C		
Direction (Q.26 - Q.31) - This section comprises of short answer type-questions (SA) of 3 marks each.		
Q. NO.	**QUESTIONS**	**MARKS**
26	Given a relation $R = \{(x, y) : x, y \in W,\ x^2 + y^2 = 25\}$, where W is the set of all whole numbers. Find: (i) R in roster form (ii) Domain of R (iii) Range of R	3
27	Find the domain and range of the function: $f(x) = \frac{x-3}{2x+1}$ **OR** The function f is defined by $f(x) = \begin{cases} 1 - x, & x < 0 \\ 1, & x = 0 \\ x + 1, & x > 0 \end{cases}$. Draw the graph of $f(x)$.	3
28	Prove that: $\sin 105^0 + \cos 105^0 = \frac{1}{\sqrt{2}}$	3
29	Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form.	3
30	Find the sum of the sequence: $6, 66, 666, 6666, \ldots\ldots\ldots$ to n terms. **OR** In a G.P, the 3rd term is 24 and 6th term is 192. Find the 10th term.	3

Q. NO.	QUESTIONS	MARKS
31	Find the derivative with respect to x: $\frac{x \tan x}{\sec x+\tan x}$ **OR** $\frac{x^5-\cos x}{\sin x}$	3

SECTION – D

Direction (Q.32 - Q.35) - This section comprises of long answer type-questions (LA) of 5 marks each.

Q. NO.	QUESTIONS	MARKS
32	If $\cos x = -\frac{1}{3}$, x lies in 3rd Quadrant, then find the values of (i) $\sin\frac{x}{2}$ (ii) $\cos\frac{x}{2}$ **OR** Prove that: $sin10^0\ sin50^0\ sin60^0\ sin70^0 = \frac{\sqrt{3}}{16}$.	5
33	A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?	5
34	Solve the questions (i), (ii) and (iii) given below: (i) Find the equation of the line which intersects $x-axis$ at a distance 2 units in the right of origin and $y-axis$ at distance 3 units below to the origin. (ii) Find the distance of point $(-2,3)$ from the line $5y = 12x - 2$. (iii) Find the angle between the $x-axis$ and the line joining the points $(3,-1)$ and $(4,-2)$. **OR** Find equations of lines which pass through the point $(3,4)$ and the sum of its intercepts on the axes is 14.	2 1 2 5
35	Calculate mean, variance and standard deviation for the following distribution.	5

Classes	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
Frequency	3	7	12	15	8	3	2

SECTION – E		
Direction (Q.36 - Q.38) - This section comprises of 3 case-study / passage-based questions of 4 marks each.		
Q. NO.	**QUESTIONS**	**MARKS**
36	**Case-Study: Read the Case study given below and attempt (i), (ii) and any one subparts of (iii):** Ravi, a student of class XI had been anxiously waiting for having a get together with his classmates as he could not be with them for a stretch of two years in Corona Period. He invited four of his best friends Amar, David, sheetal and Fatima to his birthday party. After cutting cake they gathered to take a group photograph of all of them sitting in a row.  **(i)** How many different photographs can be clicked? **(ii)** In how many of these photographs would Fatima be not sitting at middle? **(iii)**In how many of these photographs would Ravi and Amar be sitting side by side? **OR (The option is only for part (iii))** In how many of these photographs would David and Sheet be not sitting together?	4 1 1 2
37	**Case-Study: Read the Case study given below and attempt (i), (ii) and any one subparts of (iii):** Four persons Abdul , Kavita, Monika and Pramod are appearing in an interview to get a job in a software company. Chances of Abdul's selection are same as that of Pramod. Chances of Kavita's selection are double that of Abdul. Chances of Monika's selection are four times that of Abdul. One and only one among these are sure to be selected.	4

Q. NO.	QUESTIONS	MARKS
	 **(i)** What is the probability that Abdul gets selection? **(ii)** What is the probability that Kavita gets selection? **(iii)** What is the probability that Abdul or Monika get selection? **OR (The option is only for part (iii))** What is the probability that Pramod does not get selection?	1 1 2
38	**Passage-based question: Study the passage and table given below and answer the questions (i) and (ii) given below:** The intervals are defined as the set of all real numbers lying between two given real numbers (end points / boundary points). It is a way of writing subsets of the set of all real numbers. Based on the inclusion / exclusion of end points the intervals are classified as – closed, open and semi closed / semi open intervals as shown in the following table. Intervals Name of interval / Notation / Inequality description / Number line representation Closed — [a, b] — $a \leq x \leq b$ Open — (a, b) — $a < x < b$ Closed - Open and Open - Closed — [a, b) — $a \leq x < b$; (a, b] — $a < x \leq b$	4

Q. NO.	QUESTIONS	MARKS
	Intervals are sets so we can combine two or more intervals using set operations.	
	(i) To join the Indian Army under technical entry scheme the age of a candidate must be more than 16½ years and not above 19½ years. Represent the age limit using the interval.	2
	(ii) According to weather report of Meteorological department the hottest month in Srinagar is July (minimum temperature 6°C, maximum temperature 32°C) and the coldest are December – January (temperature is between −15°C and 0°C). Represent the range of temperature in both the seasons as a single interval using the set operations.	2

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SAMPLE PAPER 2023-24

MARKING SCHEME

MATHEMATICS (041) **CLASS: XI**

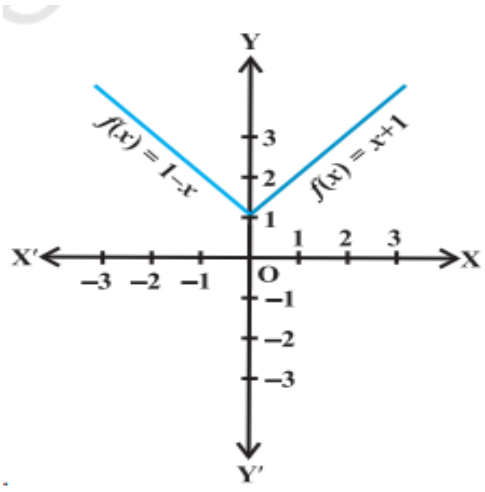
SECTION - A			
Q.NO.	**ANSWER**	**SOLUTION**	**MARKS**
1	**d**	De Morgan's Law	**1**
2	**a**	{(3,4), (3,7), (3,9), (8,4), (8,7), (8,9)}	**1**
3	**b**	$(f+g)(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$ $(f+g)(x) = f(x) + g(x) = x + \lvert x \rvert$ $(f+g)(x) = f(x) + g(x) = x + x = 2x \text{ , } when\ x \geq 0$ $(f+g)(x) = f(x) + g(x) = x - x = 0 \text{ , } when\ x < 0$	**1**
4	**c**	1 radian	**1**
5	**c**	$15_{C_2} = 105$	**1**
6	**b**	Modulus of z	**1**
7	**d**	All are true	**1**
8	**a**	26 ● $(x+a)^{51}$ has 52 terms all are positive ● $(x-a)^{51}$ has 52 terms - 26 positive and 26 negative ● 26 negative terms of $(x-a)^{51}$ cancel with the 26 corresponding positive terms of $(x+a)^{51}$ ● 26 positive terms of $(x-a)^{51}$ combine with 26 corresponding positive terms of $(x+a)^{51}$	**1**
9	**d**	A.M. > G.M.	**1**
10	**a**	$y = -5$	**1**
11	**a**	Centre: $(-3, 5)$, Radius: 4	**1**
12	**b**	Plane - 1 - XY Plane - 2 - YZ Plane - 3 - ZX	**1**
13	**b**	1 $(As\ x \to 0\ so\ x \neq 0)$	**1**
14	**c**	Slope of tangent to the curve $y = f(x)$ at the point $\big(a, f(a)\big)$	**1**
15	**d**	$\frac{4}{3}$ $\left(\because \lim_{x\to 0}\left(\frac{\sin mx}{\sin nx}\right) = \frac{m}{n}\right)$	**1**
16	**a**	Median	**1**
17	**c**	Only (i) and (iv)	**1**
18	**b**	$\frac{14}{15}$ ● Total number of selections $= 10_{C_3} = 120$ ● Cases for consecutive integers are $(1,2,3), (2,3,4), (3,4,5)(4,5,6)(5,6,7), (6,7,8), (7,8,9), (8,9,10)$ ● Total number of Cases for consecutive integers = 8 ● Total number of favourable cases = 120 - 8 = 112 ● Required probability $= \frac{112}{120} = \frac{14}{15}$	**1**

Q.NO.	ANSWER	SOLUTION	MARKS
19	**d**	A is false but R is true	**1**
20	**a**	Both A and R are true and R is the correct explanation of A	**1**

SECTION - B

Q.NO.	SOLUTION	MARKS
21	• $4_{C_0}\,(x^2)^4\left(\frac{3}{x}\right)^0 + 4_{C_1}\,(x^2)^3\left(\frac{3}{x}\right)^1 + 4_{C_2}\,(x^2)^2\left(\frac{3}{x}\right)^2 + 4_{C_3}\,(x^2)^1\left(\frac{3}{x}\right)^3$ $+4_{C_4}\,(x^2)^0\left(\frac{3}{x}\right)^4$ (Applying binomial theorem)	**1**
	• $x^8 + 12\,x^5 + 54\,x^2 + \frac{108}{x} + \frac{81}{x^4}$ (Simplifying)	**1**
	OR	
	• $6^n - 5n = (1+5)^n - 5n = (1 + 5n + n_{C_2}\,5^2 + n_{C_3}\,5^3 + \cdots + n_{C_n}\,5^n) - 5n$	**1**
	• $6^n - 5n = 1 + 5^2\left(n_{C_2} + n_{C_3}\,5^1 + \cdots + n_{C_n}\,5^{n-2}\right) = 1 + 25\,k$	**1**
22	• Let the two A.M. are A_1 and $A_2 \implies 49,\ A_1,\ A_2, 28$ are in A.P.	**½**
	• $a_4 = 28 \implies 49 + 3d = 28$	**½**
	• $d = -7$	**½**
	• $A_1 = 42\,,\quad A_2 = 35$	**½**
23	• It is downward parabola of the form $x^2 = -4ay$ with $a = 6$	**1**
	• Hence equation of parabola is: $x^2 = -4 \times 6y \implies x^2 = -24\,y$	**1**
	OR	
	• Equation of ellipse in standard form: $\frac{x^2}{2^2} + \frac{y^2}{3^2} = 1$	
	(It is vertical ellipse in standard form: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a < b$)	**½**
	• $a = 2, b = 3\,,\quad c = \sqrt{3^2 - 2^2} = \sqrt{5}$	**½**
	• $e = \frac{c}{b} = \frac{\sqrt{5}}{3}$	**½**
	• Foci: $(0, \pm c)$ i.e. $\left(0, \pm\sqrt{5}\right)$	**½**
24	• A point on y-axis is $(0, k, 0)$	**½**
	• Writing and applying distance formula	**1**
	• Getting $k = 2, -6$ and writing answer $(0, 2, 0)$ and $(0, -6, 0)$	**½**

Q.NO.	SOLUTION	MARKS
25	• $\lim_{x\to 2}\left(\frac{x^n-2^n}{x-2}\right) = n.2^{n-1}$ • $\lim_{x\to 4}(5x^2+2x-8) = 80$ • $n.2^{n-1} = 80 \Rightarrow n = 5$	1 ½ ½

SECTION - C		
Q.NO.	**SOLUTION**	**MARKS**
26	• $R = \{(0,5),(3,4),(4,3),(5,0)\}$ • Domain: $A = \{0,3,4,5\}$ • Range: $B = \{0,3,4,5\}$	1 1 1
27	• Finding domain: $D_f = R - \left\{-\frac{1}{2}\right\}$ • Separating $x = \frac{y+3}{1-2y}$ • Finding range: $R_f = R - \left\{\frac{1}{2}\right\}$ **OR** • Drawing the graph of: $f(x) = 1-x, \quad x < 0$ • Drawing the graph of: $f(x) = 1, \quad x = 0$ • Drawing the graph of: $f(x) = x+1, \quad x > 0$ • 	1 1 1 1 1 1

Q.NO.	SOLUTION	MARKS
28	• $\sin 105^0 + \cos 105^0 = \sin(90^0 + 15^0) + \cos 105^0 = \cos 15^0 + \cos 105^0$	**1**
	• $\cos 105^0 + \cos 15^0 = 2\cos\frac{105^0+15^0}{2}.\cos\frac{105^0-15^0}{2} = 2\cos 60^0 \cos 45^0$	**1**
	• $2.\frac{1}{2}.\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$	**1**
29	• $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \frac{-1+9i}{(1-4i)(1+i)}\left(\frac{3-4i}{5+i}\right)$	**½**
	• $= \frac{33+31i}{28-10i}$	**1**
	• $= \frac{33+31i}{28-10i} \times \frac{28+10i}{28+10i} = \frac{614+1198i}{(28)^2-(10i)^2} = \frac{614+1198i}{784+100}$	**1**
	• $= \frac{2(307+599i)}{884} = \mathbf{\frac{307}{442} + \frac{599}{442}i}$	**½**
30	• $S_n = 6 + 66 + 666 + 6666 +$ …… to n terms	
	• $S_n = 6\,(1 + 11 + 111 + 1111 +$ …… to n terms)	**½**
	• $S_n = \frac{6}{9}(9 + 99 + 999 + 9999 +$ …… to n terms)	**½**
	• $S_n = \frac{6}{9}[(10-1) + (100-1) + (1000-1) + (10000-1) + \ldots$ to n terms]	**½**
	• $S_n = \frac{6}{9}[(10 + 100 + 1000 + 10000 + \cdots$ to n terms$) - n]$	**½**
	• $S_n = \frac{6}{9}\left[\frac{10\,(10^n-1)}{10-1} - n\right] = \mathbf{\frac{6}{9}\left[\frac{10\,(10^n-1)}{9} - n\right]}$	**1**
	OR	
	• $a_3 = 24 \Longrightarrow ar^2 = 24$	**½**
	• $a_6 = 192 \Longrightarrow ar^5 = 192$	**½**
	• $\frac{ar^2}{ar^5} = \frac{24}{192} \Longrightarrow r = 2$	**½**
	• Getting $a = 6$	**½**
	• Getting $a_{10} = ar^9 = \mathbf{3072}$	**1**

Q.NO.	SOLUTION	MARKS
31	• $y = \frac{x\ \tan x}{\sec x+\tan x}$ • $\frac{dy}{dx} = \frac{(\sec x+\tan x)\frac{d}{dx}(x\ \tan x) - x\ \tan x\frac{d}{dx}(\sec x+\tan x)}{(\sec x+\tan x)^2}$ …….. (1) (Applying quotient rule) • Calculation of $\frac{d}{dx}(x\ \tan x) = xsec^2x + \tan x$ (Applying product rule) • Calculation of $\frac{d}{dx}(\sec x + \tan x) = \sec x\,.\tan x + sec^2x$ • Substituting in (1) and simplifying to get $\boldsymbol{\frac{dy}{dx} = \frac{x\sec x\,(\sec x-\tan x)+\tan x}{\sec x+\tan x}}$ **REMARK: It can also be solved as follow:** • $y = \frac{x\ \tan x}{\sec x+\tan x} = \frac{x\ \sin x}{1+\sin x}$ and proceed as above to get • $\boldsymbol{\frac{dy}{dx} = \frac{x\ \cos x + \sin x + \sin^2 x}{(1+\sin x)^2}}$ **OR** • $y = \frac{x^5-\cos x}{\sin x}$ • $\frac{dy}{dx} = \frac{\sin x\frac{d}{dx}(x^5-\cos x)-(x^5-\cos x)\frac{d}{dx}\sin x}{sin^2x}$ ……. (i) (Applying quotient rule) • Calculation of $\frac{d}{dx}(x^5 - \cos x) = 5x^4 + \sin x$ • Calculation of $\frac{d}{dx}\sin x = \cos x$ • Substituting in (1) and simplifying to get • $\boldsymbol{\frac{dy}{dx} = \frac{5x^4\ \sin x - x^5\cos x + 1}{sin^2x}}$	1 ½ ½ 1 **1** ½ ½ **1**

SECTION - D		
Q.NO.	**SOLUTION**	**MARKS**
32	• $\pi \leq x \leq \frac{3\pi}{2} \implies \frac{\pi}{2} \leq \frac{x}{2} \leq \frac{3\pi}{4}$ (As given)	1
	(i)	
	• $\cos x = 1 - 2\sin^2\frac{x}{2} \implies 2\sin^2\frac{x}{2} = 1 - \cos x$	1
	• $2\sin^2\frac{x}{2} = 1 - \left(-\frac{1}{3}\right) \implies \sin\frac{x}{2} = \pm\sqrt{\frac{2}{3}} = \pm\frac{\sqrt{6}}{3}$	1
	• $\mathbf{\sin\frac{x}{2} = \frac{\sqrt{6}}{3}}$ $\left(\because \frac{\pi}{2} \leq \frac{x}{2} \leq \frac{3\pi}{4}, i.e.\, x\ is\ in\ II\ quadrant\right)$	
	(ii)	
	• $\cos x = 2\cos^2\frac{x}{2} - 1 \implies 2\cos^2\frac{x}{2} = 1 + \cos x$	1
	• $2\cos^2\frac{x}{2} = 1 - \frac{1}{3} \implies \cos\frac{x}{2} = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$	1
	• $\mathbf{\cos\frac{x}{2} = -\frac{\sqrt{3}}{3}}$ $\left(\because \frac{\pi}{2} \leq \frac{x}{2} \leq \frac{3\pi}{4}, i.e.\, x\ is\ in\ II\ quadrant\right)$	
	OR	
	• $sin10^0\ sin50^0\ sin60^0\ sin70^0 = \frac{\sqrt{3}}{2} sin10^0\ sin50^0\ sin70^0$	½
	• $= \frac{\sqrt{3}}{4}(2\ sin50^0\ sin10^0)\ sin70^0$	½
	• $= \frac{\sqrt{3}}{4}(cos40^0 - cos60^0)\ sin70^0$	1
	• $= \frac{\sqrt{3}}{4}\left(cos40^0 - \frac{1}{2}\right)\ sin70^0$	½
	• $= \frac{\sqrt{3}}{8}(2\ cos40^0 - 1)\ sin70^0$	½
	• $= \frac{\sqrt{3}}{8}(2\ cos40^0\ sin70^0 - sin70^0)$	½
	• $= \frac{\sqrt{3}}{8}(sin110^0 + sin30^0 - sin70^0)$	½
	• $= \frac{\sqrt{3}}{8}\left(sin110^0 + \frac{1}{2} - sin110^0\right)$	½
	• $= \mathbf{\sqrt{3}/16}$	½
33	• Let x litres of 2% solution be added then total mixture = $x + 640$ litres	½
	• Formation of inequality: 2% of x + 8% 640 > 4% of $(x + 640)$	1
	$\implies x + 2560 > 2x + 1280$	1
	$\implies x < 1280$	
	• Formation of inequality: 2% of x + 8% 640 < 6% of $(x + 640)$	1
	$\implies x + 2560 < 3x + 1920$	1
	$\implies x > 320$	
	Hence the amount of 2% solution to should be more than 320 litres but less than 1280 litres.	½

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(i)

- $x-$ intercept:$a=2$
  $y-$ intercept: $b=-3$ — ½
- Equation of line: $\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{x}{2}+\frac{y}{-3}=1$ — 1
- $\boldsymbol{3x-2y-6=0}$ (Simplifying) — ½

(ii)

- Distance $=\left|\frac{(12\times-2)-(5\times3)-2}{\sqrt{12^2+(-5)^2}}\right|=\boldsymbol{\frac{41}{13}}$ **units** — 1

(iii)

- Slope of $x-axis$: $m_1=0$ — ½
- Slope of line joining the points $(3,-1)$ and $(4,-2)$: $m_2=-1$ — ½
- Calculation of Angle $=\boldsymbol{45^0}$ — 1

OR

- $x-$ intercept: $a=a$
  $y-$ intercept: $b=14-a$ — 1
- Equation of line: $\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{x}{a}+\frac{y}{14-a}=1$ — 1
- Line $\frac{x}{a}+\frac{y}{14-a}=1$ is passing through $(3,4) \Rightarrow \frac{3}{a}+\frac{4}{14-a}=1$ — 1
- Simplifying as $a^2-13a+42=0$
- Solving above equation: $a=6,7 \Rightarrow \boldsymbol{a=6, b=8}$ $\boldsymbol{OR}$ $\boldsymbol{a=7, b=7}$ — 1
- **Getting equations of lines: $\boldsymbol{4x+3y=24}$, $\boldsymbol{x+y=7}$** — 1

35

Classes	**Frequency (f_i)**	**Mid - Point (x_i)**	$\boldsymbol{f_i x_i}$	$\boldsymbol{d_i=x_i-\bar{x}}$ $\boldsymbol{d_i=x_i-62}$	$\boldsymbol{d_i{}^2}$	$\boldsymbol{f_i d_i{}^2}$
30 - 40	3	35	105	-27	729	2187
40 - 50	7	45	315	-17	289	2023
50 - 60	12	55	660	-7	49	588
60 - 70	15	65	975	3	9	135
70 - 80	8	75	600	13	169	1352
80 - 90	3	85	255	23	529	1587
90 - 100	2	95	190	33	1089	2178
TOTAL	**N = 50**		**3100**			**10050**

For correct mean - 2 Marks

For correct variance - 2 Marks

- Calculation of Mean: $\bar{x}=\frac{1}{N}\sum_{i=1}^{7} f_i x_i=\frac{3100}{50}=62$
- Calculation of Variance: $\sigma^2=\frac{1}{N}\sum_{i=1}^{7} f_i\,\boldsymbol{d_i{}^2}=\frac{10050}{50}=201$
- Calculation of S.D.: $\sigma=\sqrt{201}=14.18$

For correct S.D. - 1 Marks

SECTION - E		
Q.NO.	**SOLUTION**	**MARKS**
36	**(i)** $5 \times 4 \times 3 \times 2 \times 1 = \mathbf{120}$ **(ii)** • When Fatima is sitting at middle, the numbers of photographs are: $1_{P_1} \times 4_{P_4} = 1 \times 24 = 24$ • When Fatima is not sitting at middle, the numbers of photographs are: $\mathbf{120 - 24 = 96}$ **(iii)** • When Ravi and Amar be sitting side by side, the numbers of photographs are: $4_{P_4} \times 2 = \mathbf{48}$ **OR** • When David and sheetal be sitting together, the numbers of photographs are: $4_{P_4} \times 2 = 48$ • When David and sheetal be not sitting together, the numbers of photographs are: $\mathbf{120 - 48 = 72}$	1 1 2 1 1
37	**(i)** Consider the events $A \equiv$ Abdul gets selection $B \equiv$ Kavita gets selection $C \equiv$ Monika gets selection $D \equiv$ Pramod gets selection $P(A) + P(B) + P(C) + P(D) = 1$ $P(A) + 2\,P(A) + 4\,P(A) + P(A) = 1$ $\boldsymbol{P(A) = \frac{1}{8}}$ **(ii)** $\boldsymbol{P(B) = 2\,P(A) = \frac{1}{4}}$ **(iii)** $\boldsymbol{P(A\ or\ C) = P(A) + P(C) = \frac{5}{8}}$ (A and C are Mutually exclusive) **OR** $\boldsymbol{P(D') = 1 - P(D) = 1 - P(A) = \frac{7}{8}}$	1 1 2

Q.NO.	SOLUTION	MARKS
38	**(i)** $16.5 < Age \leq 19.5$ **The open - closed interval:** $(\mathbf{16.5}, \mathbf{19.5}]$ **(ii)** $[\mathbf{6}, \mathbf{32}] \cup (-\mathbf{15}, \mathbf{0})$	2 2