Reading Material-Mathematics

Class IX (2025-26)

Subject Code – 041 & 241

Chapter 3: Coordinate Geometry

3.3 Plotting a Point in the Plane if its Coordinates are Given

Uptil now we have drawn the points for you, and asked you to give their coordinates. Now we will show you how we place these points in the plane if we know its coordinates. We call this process "plotting the point".

Let the coordinates of a point be (3, 5). We want to plot this point in the coordinate plane. We draw the coordinate axes, and choose our units such that one centimetre represents one unit on both the axes. The coordinates of the point (3, 5) tell us that the distance of this point from the y - axis along the positive x - axis is 3 units and the distance of the point from the x - axis along the positive y - axis is 5 units. Starting from the origin O, we count 3 units on the positive x - axis and mark the corresponding point as A. Now, starting from A, we move in the positive direction of the y - axis and count 5 units and mark the corresponding point as P (see Fig.3.15). You see that the distance of P from the y - axis is 3 units and from the x - axis is 5 units. Hence, P is the position of the point. Note that P lies in the 1st quadrant, since both the coordinates of P are positive. Similarly, you can plot the point Q (5, – 4) in the coordinate plane. The distance of Q from the x - axis is 4 units along the negative y - axis, so that its y - coordinate is – 4 (see Fig.3.15). The point Q lies in the 4th quadrant. Why?

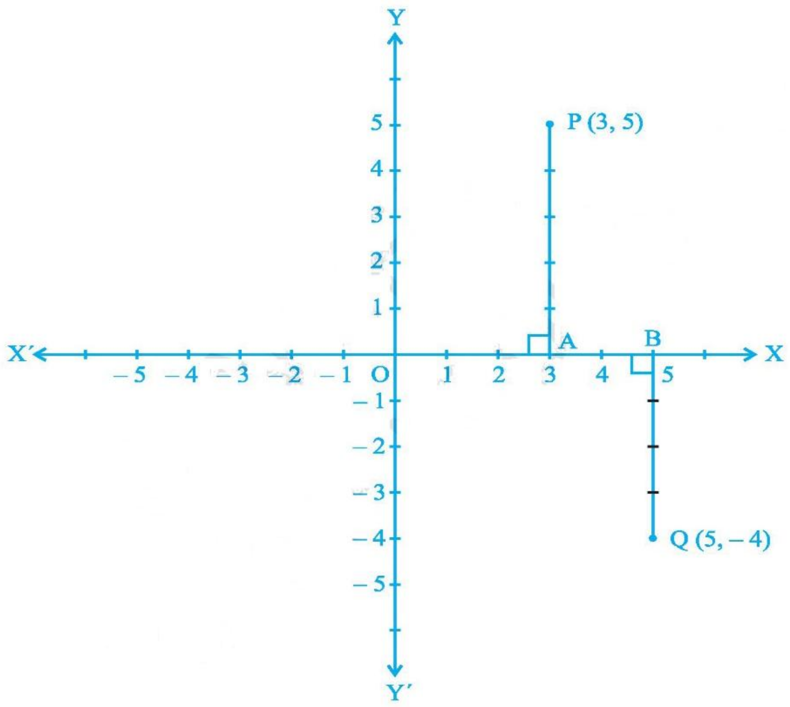

Fig. 3.15

Example 3 : Locate the points (5, 0), (0, 5), (2, 5), (5, 2), (–3, 5), (–3, –5), (5, –3) and (6, 1) in the Cartesian plane.

Solution : Taking 1cm = 1unit, we draw the x - axis and the y - axis. The positions of the points are shown by dots in Fig.3.16.

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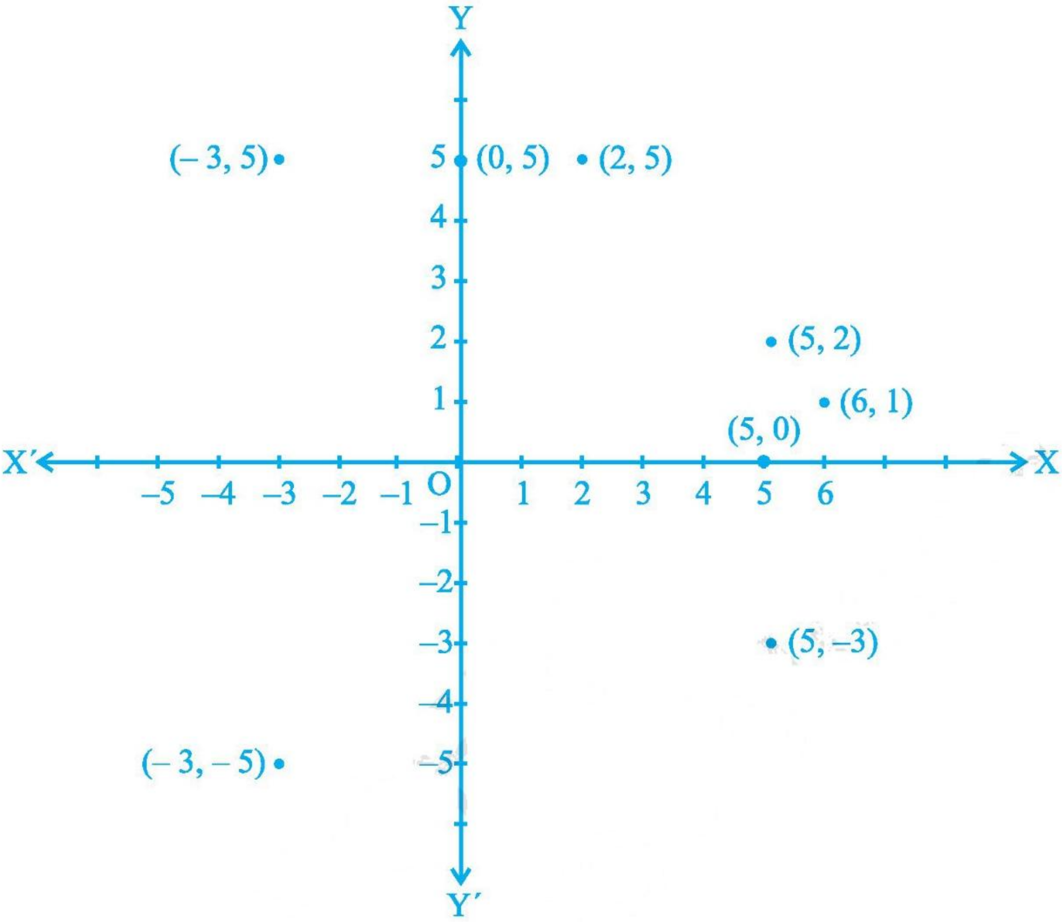


Fig. 3.16

Note : In the example above, you see that (5, 0) and (0, 5) are not at the same position. Similarly, (5, 2) and (2, 5) are at different positions. Also, (–3, 5) and (5, –3) are at different positions. By taking several such examples, you will find that, if $x \neq y$, **then the position of (x, y) in the Cartesian plane is different from the position of (y, x)**. So, if we interchange the coordinates x and y, the position of (y, x) will differ from the position of (x, y). This means that the order of x and y is important in (x, y). Therefore, (x, y) is called an ordered pair. The ordered pair $(x, y) \neq$ ordered pair (y, x), if $x \neq y$. Also $(x, y) = (y, x)$, if $x = y$.

Example 4 : Plot the following ordered pairs (x, y) of numbers as points in the Cartesian plane. Use the scale 1cm = 1 unit on the axes.

x	– 3	0	– 1	4	2
y	7	–3.5	– 3	4	– 3

Solution : The pairs of numbers given in the table can be represented by the points (– 3, 7), (0, –3.5), (– 1, – 3), (4, 4) and (2, – 3). The locations of the points are shown by dots in Fig.3.17.

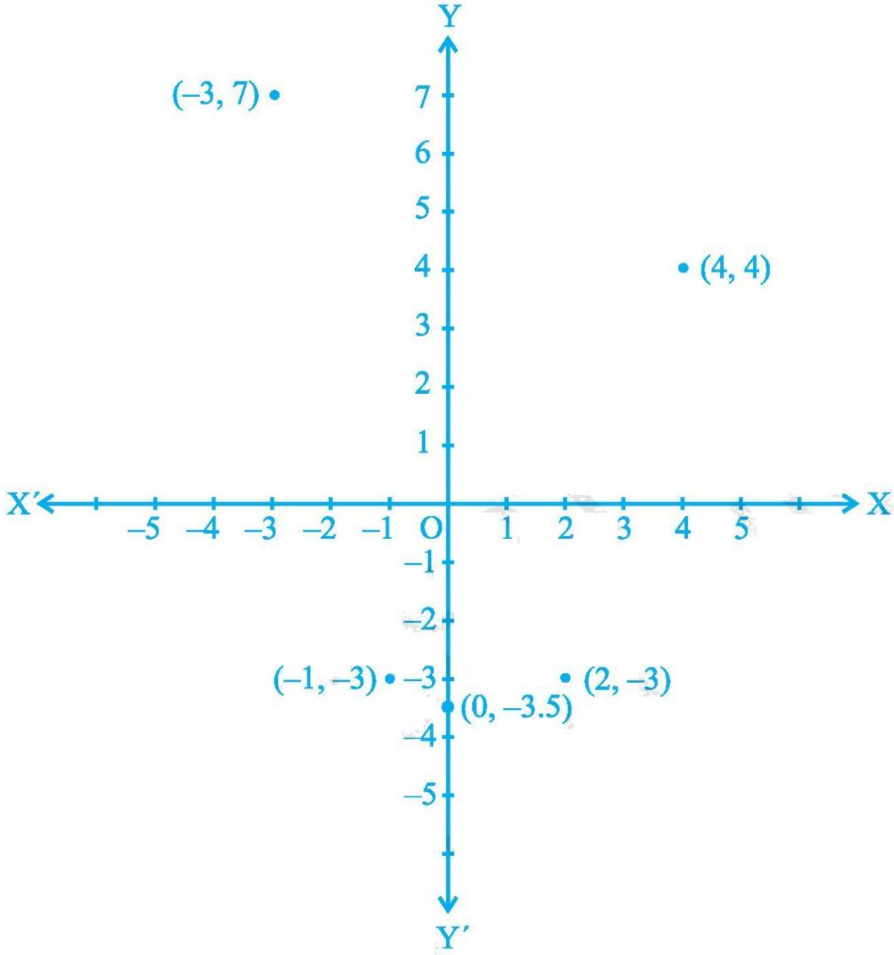


Fig. 3.17

Activity 2 : *A game for two persons* (Requirements: two counters or coins, graph paper, two dice of different colours, say red and green):

Place each counter at (0, 0). Each player throws two dice simultaneously. When the first player does so, suppose the red die shows 3 and the green one shows 1. So, she moves her counter to (3, 1). Similarly, if the second player throws 2 on the red and 4 on the green, she moves her counter to (2, 4). On the second throw, if the first player throws 1 on the red and 4 on the green, she moves her counter from (3, 1) to (3 + 1, 1 + 4), that is, adding 1 to the x - coordinate and 4 to the y - coordinate of (3, 1).

The purpose of the game is to arrive first at (10, 10) without overshooting, i.e., neither the abscissa nor the ordinate can be greater than 10. Also, a counter should not coincide with the position held by another counter. For example, if the first player's counter moves on to a point already occupied by the counter of the second player, then the second player's counter goes to (0, 0). If a move is not possible without overshooting, the player misses that turn. You can extend this game to play with more friends.

Remark : Plotting of points in the Cartesian plane can be compared to some extent with drawing of graphs in different situations such as Time-Distance Graph, Side-Perimeter Graph, etc which you have come across in earlier classes. In such situations, we may call the axes, t-axis, d-axis, s-axis or p-axis, etc. in place of the x and y axes.

EXERCISE 3.3

1. In which quadrant or on which axis do each of the points (– 2, 4), (3, – 1), (– 1, 0), (1, 2) and (– 3, – 5) lie? Verify your answer by locating them on the Cartesian plane.
2. Plot the points (x, y) given in the following table on the plane, choosing suitable units of distance on the axes.

x	– 2	–1	0	1	3
y	8	7	– 1.25	3	– 1

4.4 Graph of a Linear Equation in Two Variables

So far, you have obtained the solutions of a linear equation in two variables algebraically. Now, let us look at their geometric representation. You know that each such equation has infinitely many solutions. How can we show them in the coordinate plane? You may have got some indication in which we write the solution as pairs of values. The solutions of the linear equation in Example 3, namely,

$$x + 2y = 6 \qquad (1)$$

can be expressed in the form of a table as follows by writing the values of y below the corresponding values of x :

Table 1

x	0	2	4	6	. . .
y	3	2	1	0	. . .

In the previous chapter, you studied how to plot the points on a graph paper. Let us plot the points (0, 3), (2, 2), (4, 1) and (6, 0) on a graph paper. Now join any two of these points and obtain a line. Let us call this as line AB (see Fig. 4.2).

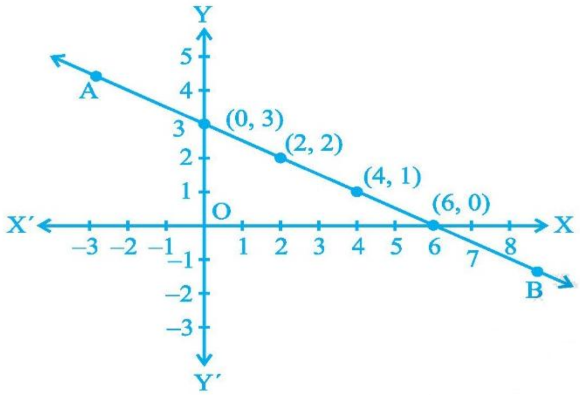


Fig. 4.2

Do you see that the other two points also lie on the line AB? Now, pick another point on this line, say (8, –1). Is this a solution? In fact, $8 + 2(-1) = 6$. So, (8, –1) is a solution. Pick any other point on this line AB and verify whether its coordinates satisfy the equation or not. Now, take any point not lying on the line AB, say (2, 0). Do its coordinates satisfy the equation? Check, and see that they do not.

Let us list our observations:

1. Every point whose coordinates satisfy Equation (1) lies on the line AB.
2. Every point (a, b) on the line AB gives a solution $x = a$, $y = b$ of Equation (1).
3. Any point, which does not lie on the line AB, is not a solution of Equation (1).

So, you can conclude that every point on the line satisfies the equation of the line and every solution of the equation is a point on the line. In fact, a linear equation in two variables is represented geometrically by a line whose points make up the collection of solutions of the equation. This is called the *graph* of the linear equation. So, to obtain the graph of a linear equation in two variables, it is enough to plot two points corresponding to two solutions and join them by a line. However, it is advisable to plot more than two such points so that you can immediately check the correctness of the graph.

Remark : The reason that a, degree one, polynomial equation $ax + by + c = 0$ is called a *linear* equation is that its geometrical representation is a straight line.

Example 5 : Given the point (1, 2), find the equation of a line on which it lies. How many such equations are there?

Solution : Here (1, 2) is a solution of a linear equation you are looking for. So, you are looking for any line passing through the point (1, 2). One example of such a linear equation is $x + y = 3$. Others are $y - x = 1$, $y = 2x$, since they are also satisfied by the coordinates of the point (1, 2). In fact, there are infinitely many linear equations which are satisfied by the coordinates of the point (1, 2). Can you see this pictorially?

Example 6 : Draw the graph of $x + y = 7$.

Solution : To draw the graph, we need at least two solutions of the equation. You can check that $x = 0$, $y = 7$, and $x = 7$, $y = 0$ are solutions of the given equation. So, you can use the following table to draw the graph:

Table 2

x	0	7
y	7	0

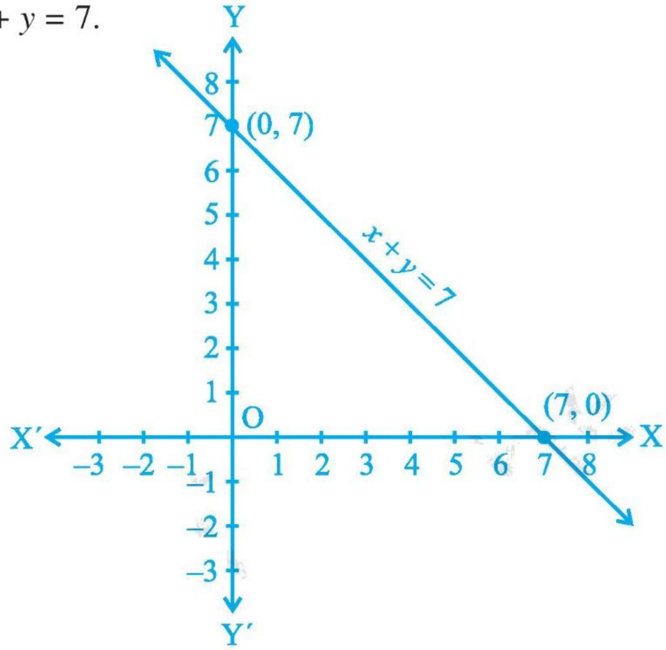


Fig. 4.3

Draw the graph by plotting the two points from Table 2 and then by joining the same by a line (see Fig. 4.3).

Example 7 : You know that the force applied on a body is directly proportional to the acceleration produced in the body. Write an equation to express this situation and plot the graph of the equation.

Solution : Here the variables involved are force and acceleration. Let the force applied be y units and the acceleration produced be x units. From ratio and proportion, you can express this fact as $y = kx$, where k is a constant. (From your study of science, you know that k is actually the mass of the body.)

Now, since we do not know what k is, we cannot draw the precise graph of $y = kx$. However, if we give a certain value to k, then we can draw the graph. Let us take $k = 3$, i.e., we draw the line representing $y = 3x$.

For this we find two of its solutions, say (0, 0) and (2, 6) (see Fig. 4.4).

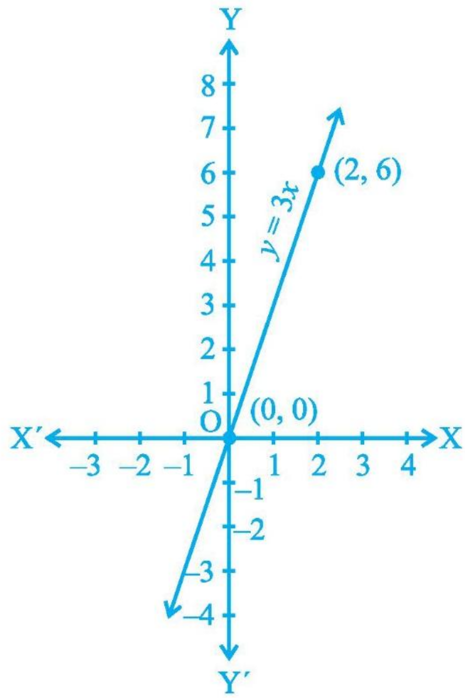


Fig. 4.4

From the graph, you can see that when the force applied is 3 units, the acceleration produced is 1 unit. Also, note that (0, 0) lies on the graph which means the acceleration produced is 0 units, when the force applied is 0 units.

Remark : The graph of the equation of the form $y = kx$ is a line which always passes through the origin.

Example 8 : For each of the graphs given in Fig. 4.5 select the equation whose graph it is from the choices given below:

(a) For Fig. 4.5 (i),

(i) $x + y = 0$ (ii) $y = 2x$ (iii) $y = x$ (iv) $y = 2x + 1$

(b) For Fig. 4.5 (ii),

(i) $x + y = 0$ (ii) $y = 2x$ (iii) $y = 2x + 4$ (iv) $y = x - 4$

(c) For Fig. 4.5 (iii),

(i) $x + y = 0$ (ii) $y = 2x$ (iii) $y = 2x + 1$ (iv) $y = 2x - 4$

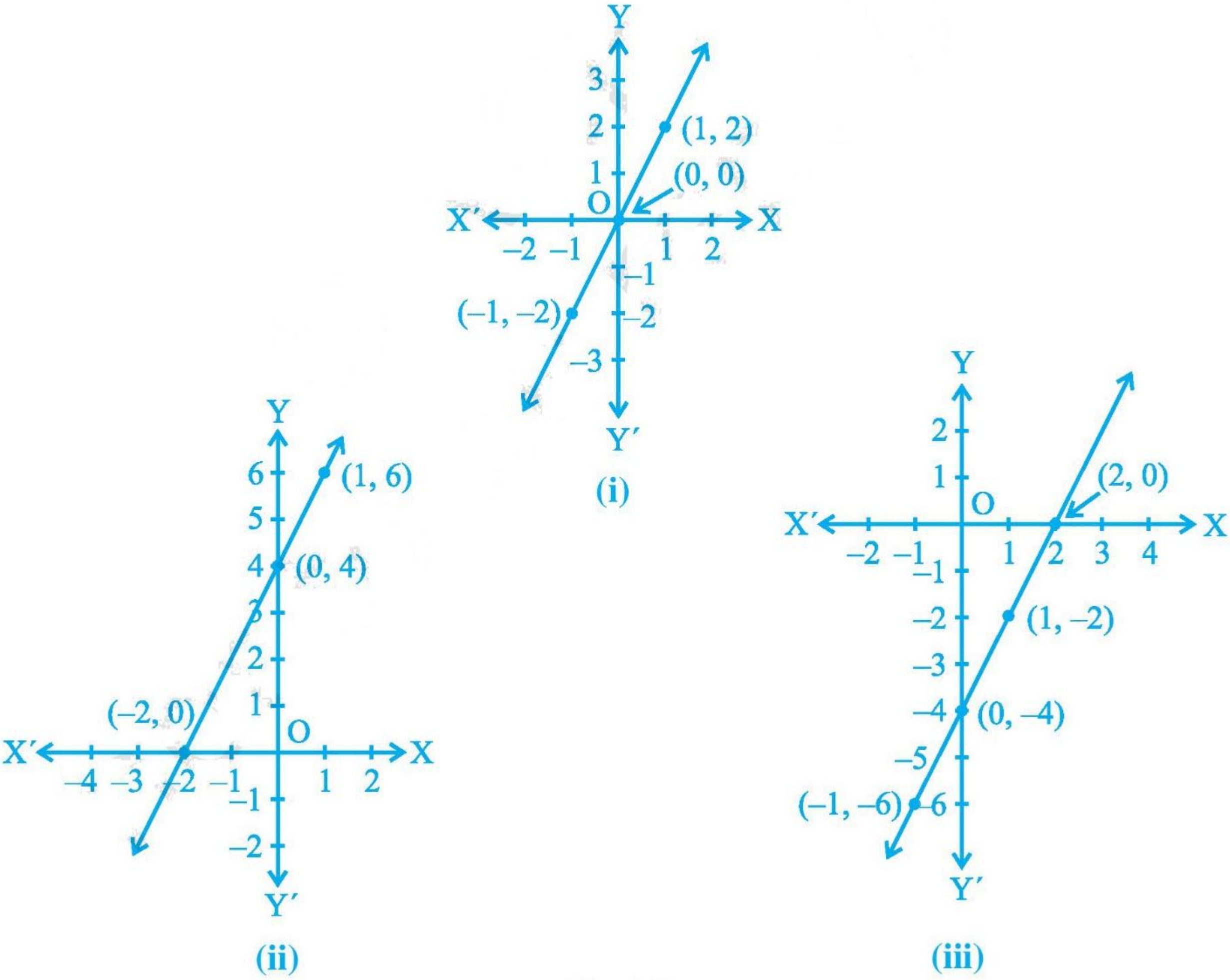


Fig. 4.5

Solution : (a) In Fig. 4.5 (i), the points on the line are (–1, –2), (0, 0), (1, 2). By inspection, $y = 2x$ is the equation corresponding to this graph. You can find that the y-coordinate in each case is double that of the x-coordinate.

(b) In Fig. 4.5 (ii), the points on the line are (–2, 0), (0, 4), (1, 6). You know that the coordinates of the points of the graph (line) satisfy the equation $y = 2x + 4$. So, $y = 2x + 4$ is the equation corresponding to the graph in Fig. 4.5 (ii).

(c) In Fig. 4.5 (iii), the points on the line are (–1, –6), (0, –4), (1, –2), (2, 0). By inspection, you can see that $y = 2x – 4$ is the equation corresponding to the given graph (line).

EXERCISE 4.3

1. Draw the graph of each of the following linear equations in two variables:

   (i) $x + y = 4$ (ii) $x - y = 2$ (iii) $y = 3x$ (iv) $3 = 2x + y$

2. Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?

3. If the point (3, 4) lies on the graph of the equation $3y = ax + 7$, find the value of a.

4. The taxi fare in a city is as follows: For the first kilometre, the fare is ₹ 8 and for the subsequent distance it is ₹ 5 per km. Taking the distance covered as x km and total fare as ₹ y, write a linear equation for this information, and draw its graph.

5. From the choices given below, choose the equation whose graphs are given in Fig. 4.6 and Fig. 4.7.

For Fig. 4. 6

(i) $y = x$

(ii) $x + y = 0$

(iii) $y = 2x$

(iv) $2 + 3y = 7x$

For Fig. 4.7

(i) $y = x + 2$

(ii) $y = x - 2$

(iii) $y = -x + 2$

(iv) $x + 2y = 6$

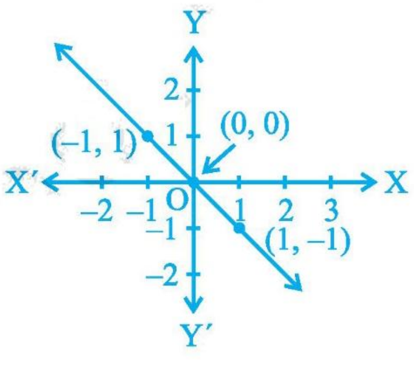


Fig. 4.6

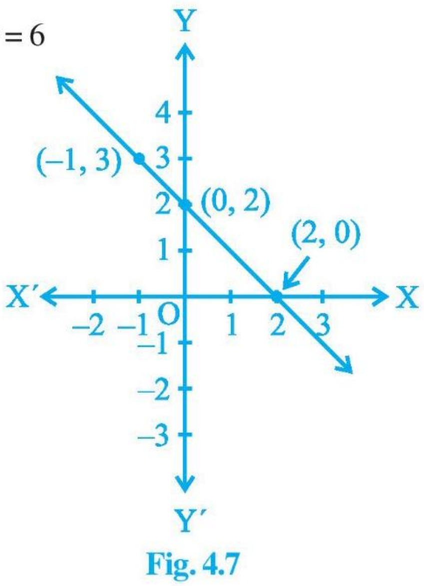


Fig. 4.7

6. If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also read from the graph the work done when the distance travelled by the body is

(i) 2 units (ii) 0 unit

7. Yamini and Fatima, two students of Class IX of a school, together contributed ₹ 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as ₹ x and ₹ y.) Draw the graph of the same.

8. In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:

$$F = \left(\frac{9}{5}\right)C + 32$$

(i) Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.

(ii) If the temperature is 30°C, what is the temperature in Fahrenheit?

(iii) If the temperature is 95°F, what is the temperature in Celsius?

(iv) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?

(v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.

6.5 Parallel Lines and a Transversal

Recall that a line which intersects two or more lines at distinct points is called a **transversal** (see Fig. 6.18). Line l intersects lines m and n at points P and Q respectively. Therefore, line l is a transversal for lines m and n. Observe that four angles are formed at each of the points P and Q.

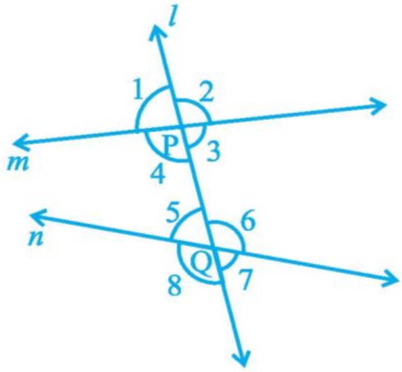


Fig. 6.18

Let us name these angles as ∠ 1, ∠ 2, . . ., ∠8 as shown in Fig. 6.18.

∠ 1, ∠ 2, ∠ 7 and ∠ 8 are called **exterior angles**, while ∠ 3, ∠ 4, ∠ 5 and ∠ 6 are called **interior angles**.

Recall that in the earlier classes, you have named some pairs of angles formed when a transversal intersects two lines. These are as follows:

(a) **Corresponding angles :**

(i) ∠ 1 and ∠ 5 (ii) ∠ 2 and ∠ 6

(iii) ∠ 4 and ∠ 8 (iv) ∠ 3 and ∠ 7

(b) **Alternate interior angles :**

(i) ∠ 4 and ∠ 6 (ii) ∠ 3 and ∠ 5

(c) **Alternate exterior angles:**

(i) ∠ 1 and ∠ 7 (ii) ∠ 2 and ∠ 8

(d) **Interior angles on the same side of the transversal:**

(i) ∠ 4 and ∠ 5 (ii) ∠ 3 and ∠ 6

Interior angles on the same side of the transversal are also referred to as **consecutive interior** angles or **allied** angles or **co-interior** angles. Further, many a times, we simply use the words alternate angles for alternate interior angles.

Now, let us find out the relation between the angles in these pairs when line m is parallel to line n. You know that the ruled lines of your notebook are parallel to each other. So, with ruler and pencil, draw two parallel lines along any two of these lines and a transversal to intersect them as shown in Fig. 6.19.

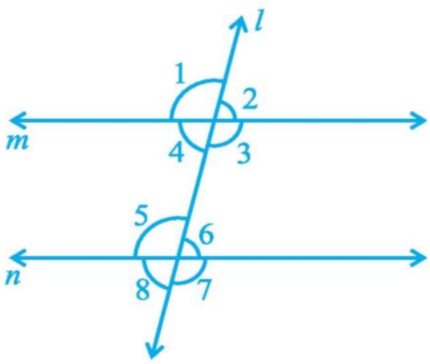


Fig. 6.19

Now, measure any pair of corresponding angles and find out the relation between them. You may find that : $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 4 = \angle 8$ and $\angle 3 = \angle 7$. From this, you may conclude the following axiom.

Axiom 6.3 : *If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.*

Axiom 6.3 is also referred to as the **corresponding angles axiom**. Now, let us discuss the converse of this axiom which is as follows:

If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel.

Does this statement hold true? It can be verified as follows: Draw a line AD and mark points B and C on it. At B and C, construct $\angle$ ABQ and $\angle$ BCS equal to each other as shown in Fig. 6.20 (i).

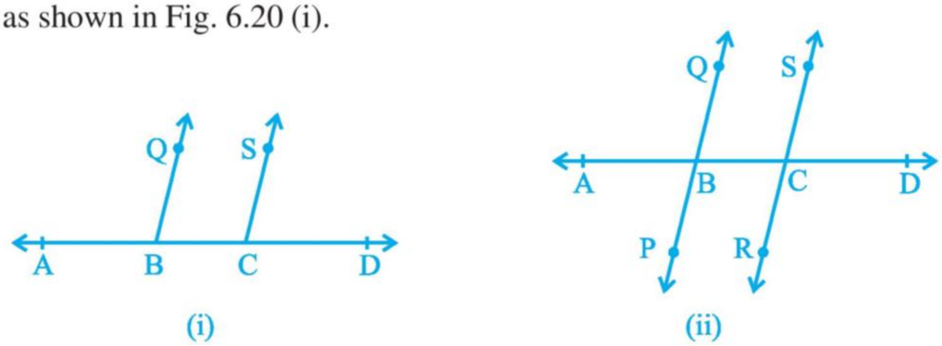


Fig. 6.20

Produce QB and SC on the other side of AD to form two lines PQ and RS [see Fig. 6.20 (ii)]. You may observe that the two lines do not intersect each other. You may also draw common perpendiculars to the two lines PQ and RS at different points and measure their lengths. You will find it the same everywhere. So, you may conclude that the lines are parallel. Therefore, the converse of corresponding angles axiom is also true. So, we have the following axiom:

Axiom 6.4 : *If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.*

Can we use corresponding angles axiom to find out the relation between the alternate interior angles when a transversal intersects two parallel lines? In Fig. 6.21, transveral PS intersects parallel lines AB and CD at points Q and R respectively.

Is $\angle$ BQR = $\angle$ QRC and $\angle$ AQR = $\angle$ QRD?

You know that $\angle$ PQA = $\angle$ QRC (1)

(Corresponding angles axiom)

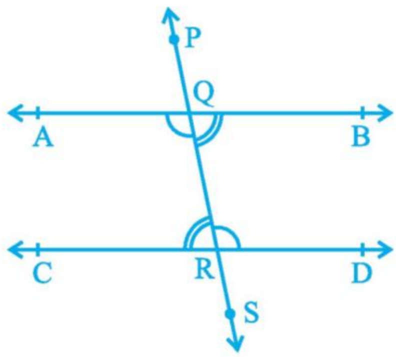


Fig. 6.21

Is $\angle$ PQA = $\angle$ BQR? Yes! (Why ?) (2)

So, from (1) and (2), you may conclude that

$\angle$ BQR = $\angle$ QRC.

Similarly, $\angle$ AQR = $\angle$ QRD.

This result can be stated as a theorem given below:

Theorem 6.2 : *If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.*

Now, using the converse of the corresponding angles axiom, can we show the two lines parallel if a pair of alternate interior angles is equal? In Fig. 6.22, the transversal PS intersects lines AB and CD at points Q and R respectively such that $\angle$ BQR = $\angle$ QRC.

Is AB || CD?

$\angle$ BQR = $\angle$ PQA (Why?) (1)

But, $\angle$ BQR = $\angle$ QRC (Given) (2)

So, from (1) and (2), you may conclude that

$\angle$ PQA = $\angle$ QRC

But they are corresponding angles.

So, AB || CD (Converse of corresponding angles axiom)

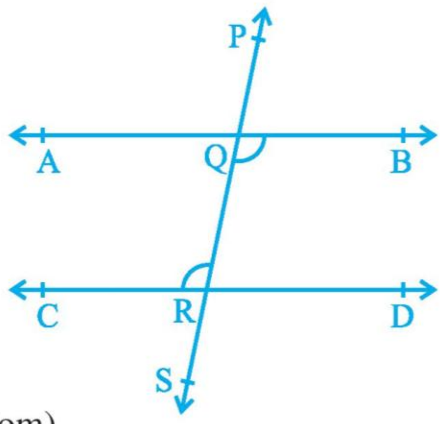


Fig. 6.22

This result can be stated as a theorem given below:

Theorem 6.3 : *If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.*

In a similar way, you can obtain the following two theorems related to interior angles on the same side of the transversal.

Theorem 6.4 : *If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.*

Theorem 6.5 : *If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.*

You may recall that you have verified all the above axioms and theorems in earlier classes through activities. You may repeat those activities here also.

6.7 Angle Sum Property of a Triangle

In the earlier classes, you have studied through activities that the sum of all the angles of a triangle is 180°. We can prove this statement using the axioms and theorems related to parallel lines.

Theorem 6.7 : *The sum of the angles of a triangle is 180º.*

Proof : Let us see what is given in the statement above, that is, the hypothesis and what we need to prove. We are given a triangle PQR and $\angle 1$, $\angle 2$ and $\angle 3$ are the angles of Δ PQR (see Fig. 6.34).

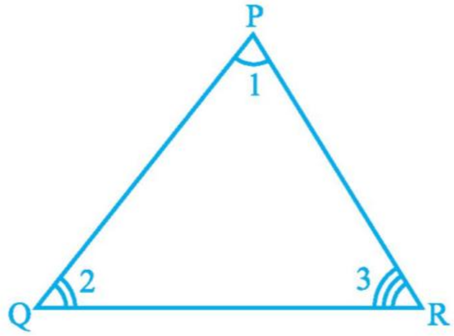


Fig. 6.34

We need to prove that $\angle 1 + \angle 2 + \angle 3 = 180°$. Let us draw a line XPY parallel to QR through the opposite vertex P, as shown in Fig. 6.35, so that we can use the properties related to parallel lines.

Now, XPY is a line.

Therefore, $\angle 4 + \angle 1 + \angle 5 = 180°$ (1)

But XPY || QR and PQ, PR are transversals.

So, $\angle 4 = \angle 2$ and $\angle 5 = \angle 3$

(Pairs of alternate angles)

Substituting $\angle 4$ and $\angle 5$ in (1), we get

$$\angle 2 + \angle 1 + \angle 3 = 180°$$

That is, $\angle 1 + \angle 2 + \angle 3 = 180°$

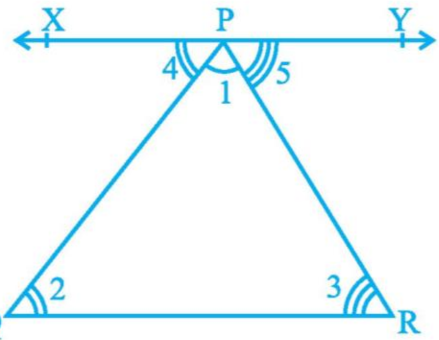


Fig. 6.35

Recall that you have studied about the formation of an exterior angle of a triangle in the earlier classes (see Fig. 6.36). Side QR is produced to point S, $\angle$ PRS is called an exterior angle of ΔPQR.

Is $\angle 3 + \angle 4 = 180°$? (Why?) (1)

Also, see that

$\angle 1 + \angle 2 + \angle 3 = 180°$ (Why?) (2)

From (1) and (2), you can see that

$$\angle 4 = \angle 1 + \angle 2.$$

This result can be stated in the form of a theorem as given below:

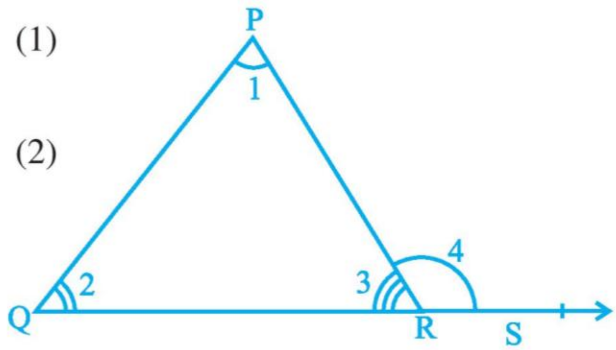


Fig. 6.36

Theorem 6.8 : *If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.*

It is obvious from the above theorem that an *exterior angle of a triangle is greater than either of its interior apposite angles.*

Now, let us take some examples based on the above theorems.

Example 7 : In Fig. 6.37, if QT $\perp$ PR, $\angle$ TQR = 40° and $\angle$ SPR = 30°, find x and y.

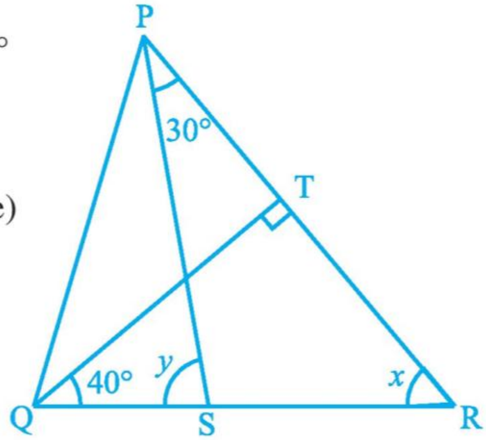


Fig. 6.37

Solution : In Δ TQR, $90° + 40° + x = 180°$

(Angle sum property of a triangle)

Therefore, $x = 50°$

Now, $y = \angle SPR + x$ (Theorem 6.8)

Therefore, $y = 30° + 50°$

$= 80°$

Example 8 : In Fig. 6.38, the sides AB and AC of ΔABC are produced to points E and D respectively. If bisectors BO and CO of $\angle$ CBE and $\angle$ BCD respectively meet at point O, then prove that

$$\angle BOC = 90° - \frac{1}{2} \angle BAC.$$

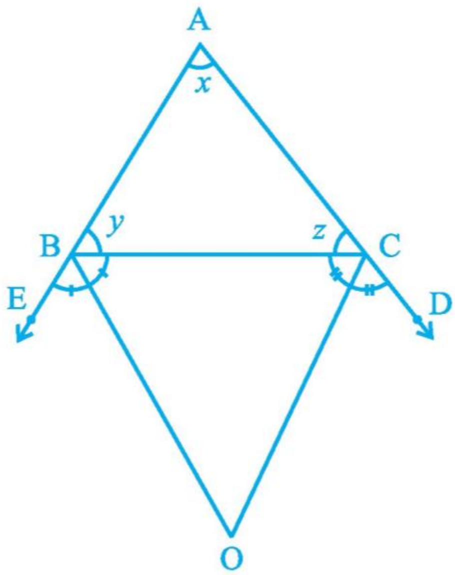


Fig. 6.38

Solution : Ray BO is the bisector of $\angle$ CBE.

Therefore, $\angle CBO = \frac{1}{2} \angle CBE$

$= \frac{1}{2} (180° - y)$

$= 90° - \frac{y}{2}$ (1)

Similarly, ray CO is the bisector of $\angle$ BCD.

Therefore, $\angle BCO = \frac{1}{2} \angle BCD$

$= \frac{1}{2} (180° - z)$

$= 90° - \frac{z}{2}$ (2)

In Δ BOC, $\angle$ BOC + $\angle$ BCO + $\angle$ CBO = 180° (3)

Substituting (1) and (2) in (3), you get

$$\angle \text{BOC} + 90° - \frac{z}{2} + 90° - \frac{y}{2} = 180°$$

So, $$\angle \text{BOC} = \frac{z}{2} + \frac{y}{2}$$

or, $$\angle \text{BOC} = \frac{1}{2}(y + z) \quad (4)$$

But, $x + y + z = 180°$ (Angle sum property of a triangle)

Therefore, $y + z = 180° - x$

Therefore, (4) becomes

$$\angle \text{BOC} = \frac{1}{2}(180° - x)$$

$$= 90° - \frac{x}{2}$$

$$= 90° - \frac{1}{2}\angle \text{BAC}$$

EXERCISE 6.3

1. In Fig. 6.39, sides QP and RQ of Δ PQR are produced to points S and T respectively. If $\angle$ SPR = 135° and $\angle$ PQT = 110°, find $\angle$ PRQ.

2. In Fig. 6.40, $\angle$ X = 62°, $\angle$ XYZ = 54°. If YO and ZO are the bisectors of $\angle$ XYZ and $\angle$ XZY respectively of Δ XYZ, find $\angle$ OZY and $\angle$ YOZ.

3. In Fig. 6.41, if AB || DE, $\angle$ BAC = 35° and $\angle$ CDE = 53°, find $\angle$ DCE.

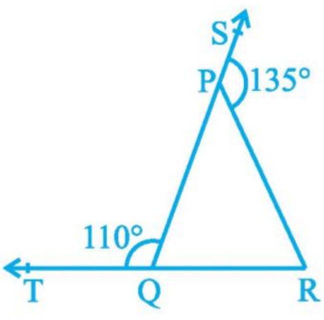


Fig. 6.39

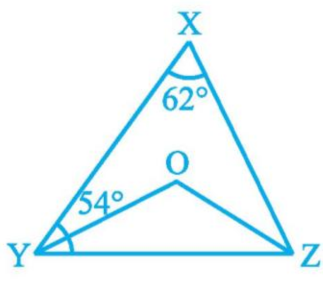


Fig. 6.40

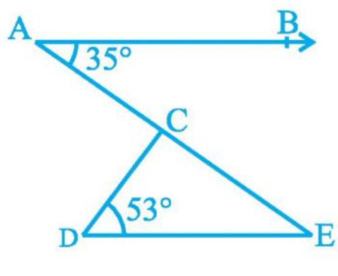


Fig. 6.41

4. In Fig. 6.42, if lines PQ and RS intersect at point T, such that $\angle$ PRT = 40°, $\angle$ RPT = 95° and $\angle$ TSQ = 75°, find $\angle$ SQT.

5. In Fig. 6.43, if PQ $\perp$ PS, PQ $\parallel$ SR, $\angle$ SQR = 28° and $\angle$ QRT = 65°, then find the values of x and y.

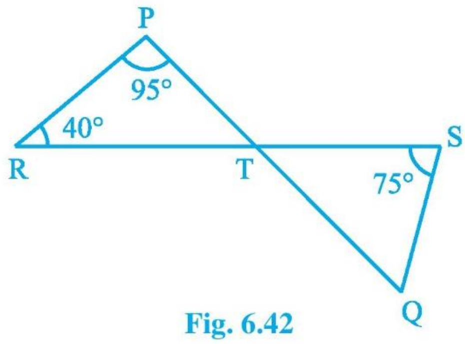


Fig. 6.42

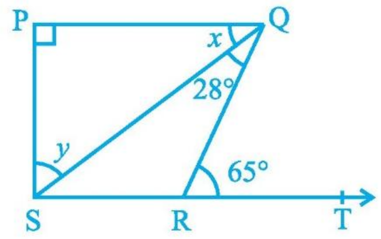


Fig. 6.43

6. In Fig. 6.44, the side QR of Δ PQR is produced to a point S. If the bisectors of $\angle$ PQR and $\angle$ PRS meet at point T, then prove that $\angle QTR = \frac{1}{2} \angle QPR$.

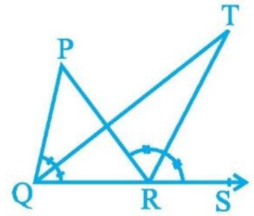


Fig. 6.44

Chapter 7: Triangles

7.6 Inequalities in a Triangle

So far, you have been mainly studying the equality of sides and angles of a triangle or triangles. Sometimes, we do come across unequal objects, we need to compare them. For example, line-segment AB is greater in length as compared to line segment CD in Fig. 7.41 (i) and ∠ A is greater than ∠ B in Fig 7.41 (ii).

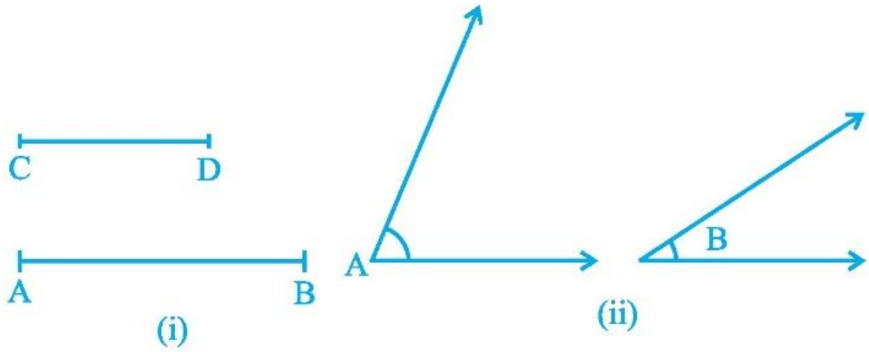


Fig. 7.41

Let us now examine whether there is any relation between unequal sides and unequal angles of a triangle. For this, let us perform the following activity:

Activity : Fix two pins on a drawing board say at B and C and tie a thread to mark a side BC of a triangle.

Fix one end of another thread at C and tie a pencil at the other (free) end . Mark a point A with the pencil and draw Δ ABC (see Fig 7.42). Now, shift the pencil and mark another point A′ on CA beyond A (new position of it)

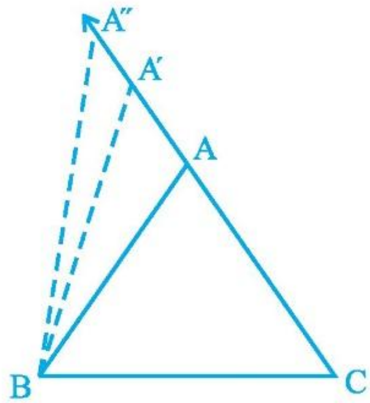


Fig. 7.42

So, A′C > AC (Comparing the lengths)

Join A′ to B and complete the triangle A′BC. What can you say about ∠ A′BC and ∠ ABC?

Compare them. What do you observe?

Clearly, ∠ A′BC > ∠ ABC

Continue to mark more points on CA (extended) and draw the triangles with the side BC and the points marked.

You will observe that as the length of the side AC is increased (by taking different positions of A), the angle opposite to it, that is, ∠ B also increases.

Let us now perform another activity :

Activity : Construct a scalene triangle (that is a triangle in which all sides are of different lengths). Measure the lengths of the sides.

Now, measure the angles. What do you observe?

In Δ ABC of Fig 7.43, BC is the longest side and AC is the shortest side.

Also, ∠ A is the largest and ∠ B is the smallest.

Repeat this activity with some other triangles.

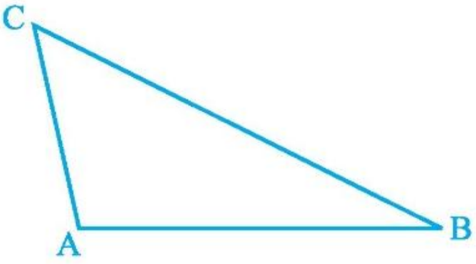


Fig. 7.43

We arrive at a very important result of inequalities in a triangle. It is stated in the form of a theorem as shown below:

Theorem 7.6 : *If two sides of a triangle are unequal, the angle opposite to the longer side is larger (or greater).*

You may prove this theorem by taking a point P on BC such that CA = CP in Fig. 7.43.

Now, let us perform another activity :

Activity : Draw a line-segment AB. With A as centre and some radius, draw an arc and mark different points say P, Q, R, S, T on it.

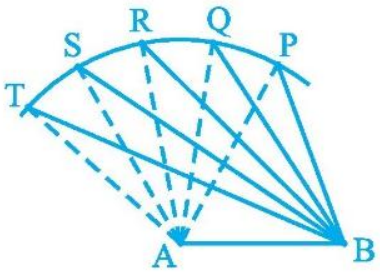


Fig. 7.44

Join each of these points with A as well as with B (see Fig. 7.44). Observe that as we move from P to T, ∠ A is becoming larger and larger. What is happening to the length of the side opposite to it? Observe that the length of the side is also increasing; that is ∠ TAB > ∠ SAB > ∠ RAB > ∠ QAB > ∠ PAB and TB > SB > RB > QB > PB.

Now, draw any triangle with all angles unequal to each other. Measure the lengths of the sides (see Fig. 7.45).

Observe that the side opposite to the largest angle is the longest. In Fig. 7.45, ∠ B is the largest angle and AC is the longest side.

Repeat this activity for some more triangles and we see that the converse of Theorem 7.6 is also true. In this way, we arrive at the following theorem:

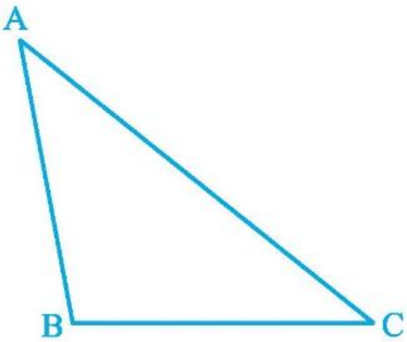


Fig. 7.45

Theorem 7.7 : *In any triangle, the side opposite to the larger (greater) angle is longer.*

This theorem can be proved by the method of contradiction.

Now take a triangle ABC and in it, find AB + BC, BC + AC and AC + AB. What do you observe?

You will observe that $AB + BC > AC$,

$BC + AC > AB$ and $AC + AB > BC$.

Repeat this activity with other triangles and with this you can arrive at the following theorem :

Theorem 7.8 : *The sum of any two sides of a triangle is greater than the third side.*

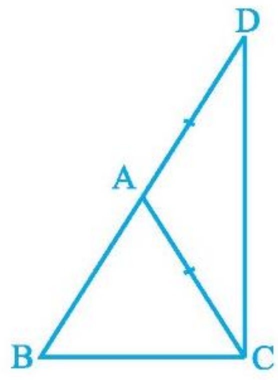


Fig. 7.46

In Fig. 7.46, observe that the side BA of Δ ABC has been produced to a point D such that AD = AC. Can you show that ∠ BCD > ∠ BDC and BA + AC > BC? Have you arrived at the proof of the above theorem.

Let us now take some examples based on these results.

Example 9 : D is a point on side BC of Δ ABC such that AD = AC (see Fig. 7.47). Show that AB > AD.

Solution : In Δ DAC,

AD = AC (Given)

So, ∠ ADC = ∠ ACD

(Angles opposite to equal sides)

Now, ∠ ADC is an exterior angle for ΔABD.

So, ∠ ADC > ∠ ABD

or, ∠ ACD > ∠ ABD

or, ∠ ACB > ∠ ABC

So, AB > AC (Side opposite to larger angle in Δ ABC)

or, AB > AD (AD = AC)

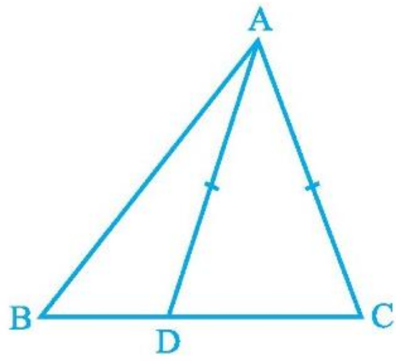


Fig. 7.47

EXERCISE 7.4

1. Show that in a right angled triangle, the hypotenuse is the longest side.
2. In Fig. 7.48, sides AB and AC of Δ ABC are extended to points P and Q respectively. Also, ∠ PBC < ∠ QCB. Show that AC > AB.

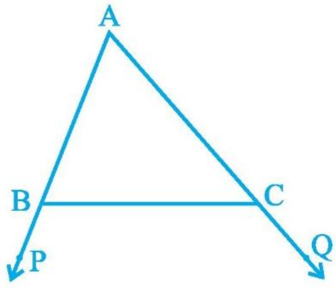


Fig. 7.48

3. In Fig. 7.49, ∠ B < ∠ A and ∠ C < ∠ D. Show that AD < BC.

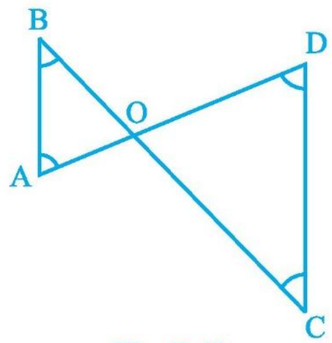


Fig. 7.49

4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see Fig. 7.50). Show that ∠ A > ∠ C and ∠ B > ∠ D.

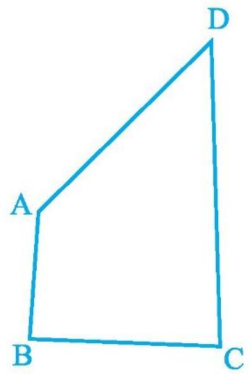


Fig. 7.50

5. In Fig 7.51, PR > PQ and PS bisects ∠ QPR. Prove that ∠ PSR > ∠ PSQ.

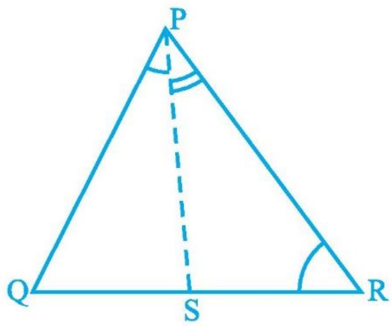


Fig. 7.51

6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

CHAPTER 9

AREAS OF PARALLELOGRAMS AND TRIANGLES

9.1 Introduction

In Chapter 5, you have seen that the study of Geometry, originated with the measurement of earth (lands) in the process of recasting boundaries of the fields and dividing them into appropriate parts. For example, a farmer *Budhia* had a triangular field and she wanted to divide it equally among her two daughters and one son. Without actually calculating the area of the field, she just divided one side of the triangular field into three equal parts and joined the two points of division to the opposite vertex. In this way, the field was divided into three parts and she gave one part to each of her children. Do you think that all the three parts so obtained by her were, in fact, equal in area? To get answers to this type of questions and other related problems, there is a need to have a relook at areas of plane figures, which you have already studied in earlier classes.

You may recall that the part of the plane enclosed by a simple closed figure is called a *planar region* corresponding to that figure. The magnitude or measure of this planar region is called its *area*. This magnitude or measure is always expressed with the help of a number (in some unit) such as 5 cm^2, 8 m^2, 3 hectares etc. So, we can say that area of a figure is a number (in some unit) associated with the part of the plane enclosed by the figure.

We are also familiar with the concept of congruent figures from earlier classes and from Chapter 7. *Two figures are called congruent, if they have the same shape and the same size*. In other words, if two figures A and B are congruent (see Fig. 9.1) , then using a tracing paper,

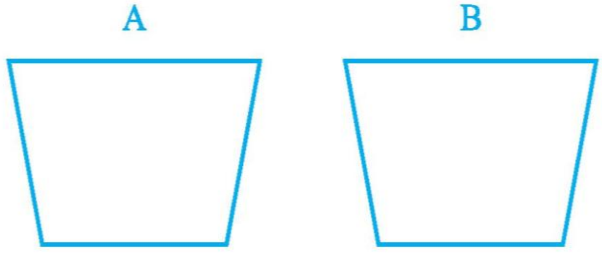


Fig. 9.1

you can superpose one figure over the other such that it will cover the other completely. So *if two figures A and B are congruent, they must have equal areas*. However, the converse of this statement is *not true*. In other words, *two figures having equal areas need not be congruent*. For example, in Fig. 9.2, rectangles ABCD and EFGH have equal areas (9×4 cm^2 and 6×6 cm^2) but clearly they are not congruent. (Why?)

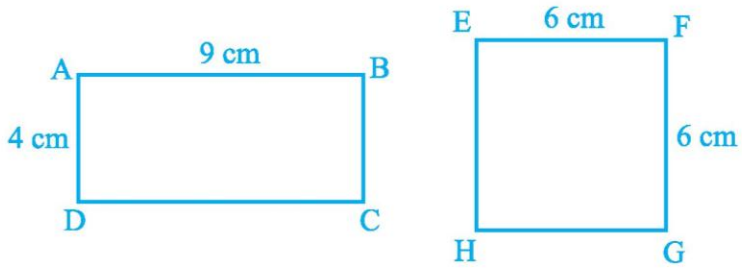


Fig. 9.2

Now let us look at Fig. 9.3 given below:

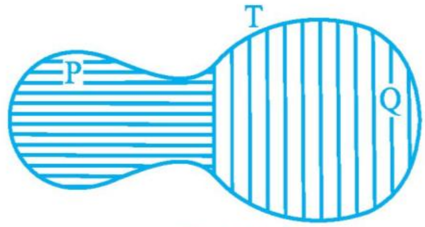


Fig. 9.3

You may observe that planar region formed by figure T is made up of two planar regions formed by figures P and Q. You can easily see that

Area of figure T = Area of figure P + Area of figure Q.

You may denote the area of figure A as ar(A), area of figure B as ar(B), area of figure T as ar(T), and so on. Now you can say that *area of a figure is a number (in some unit) associated with the part of the plane enclosed by the figure with the following two properties:*

(1) *If* A *and* B *are two congruent figures, then ar*(A) = ar(B);

and (2) *if a planar region formed by a figure* T *is made up of two non-overlapping planar regions formed by figures* P *and* Q*, then* ar(T) = ar(P) + ar(Q).

You are also aware of some formulae for finding the areas of different figures such as rectangle, square, parallelogram, triangle etc., from your earlier classes. In this chapter, attempt shall be made to consolidate the knowledge about these formulae by studying some relationship between the areas of these geometric figures under the condition when they lie on the same base and between the same parallels. This study will also be useful in the understanding of some results on 'similarity of triangles'.

9.3 Parallelograms on the same Base and Between the same Parallels

Now let us try to find a relation, if any, between the areas of two parallelograms on the same base and between the same parallels. For this, let us perform the following activities:

Activity 1 : Let us take a graph sheet and draw two parallelograms ABCD and PQCD on it as shown in Fig. 9.9.

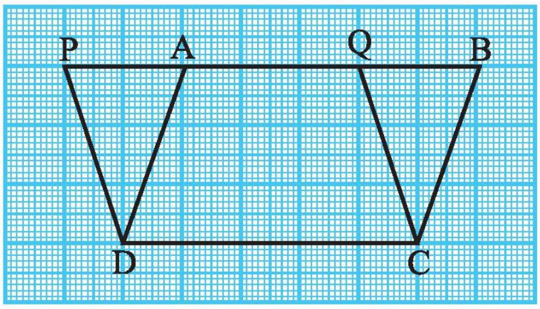


Fig. 9.9

The above two parallelograms are on the same base DC and between the same parallels PB and DC. You may recall the method of finding the areas of these two parallelograms by counting the squares.

In this method, the area is found by counting the number of complete squares enclosed by the figure, the number of squares a having more than half their parts enclosed by the figure and the number of squares having half their parts enclosed by the figure. The squares whose less than half parts are enclosed by the figure are ignored. You will find that areas of both the parallelograms are (approximately) $15 cm^2$. Repeat this activity* by drawing some more pairs of parallelograms on the graph sheet. What do you observe? Are the areas of the two parallelograms different or equal? If fact, they are equal. So, this may lead you to conclude that *parallelograms on the same base and between the same parallels are equal in area*. However, remember that this is just a verification.

Activity 2 : Draw a parallelogram ABCD on a thick sheet of paper or on a cardboard sheet. Now, draw a line-segment DE as shown in Fig. 9.10.

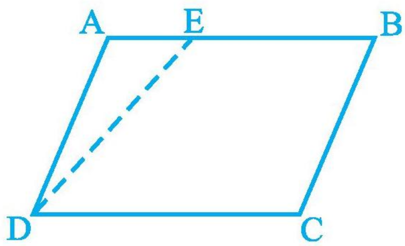


Fig. 9.10

*This activity can also be performed by using a Geoboard.

Next, cut a triangle A′ D′ E′ congruent to triangle ADE on a separate sheet with the help of a tracing paper and place Δ A′D′E′ in such a way that A′D′ coincides with BC as shown in Fig 9.11.Note that there are two parallelograms ABCD and EE′CD on the same base DC and between the same parallels AE′ and DC. What can you say about their areas?

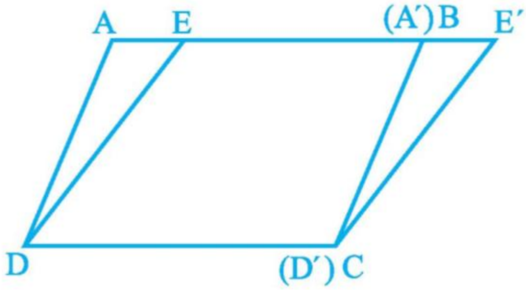


Fig. 9.11

As $\Delta ADE \cong \Delta A'D'E'$

Therefore ar (ADE) = ar (A′D′E′)

Also ar (ABCD) = ar (ADE) + ar (EBCD)

= ar (A′D′E′) + ar (EBCD)

= ar (EE′CD)

So, the two parallelograms are equal in area.

Let us now try to prove this relation between the two such parallelograms.

Theorem 9.1 : *Parallelograms on the same base and between the same parallels are equal in area.*

Proof : Two parallelograms ABCD and EFCD, on the same base DC and between the same parallels AF and DC are given (see Fig.9.12).

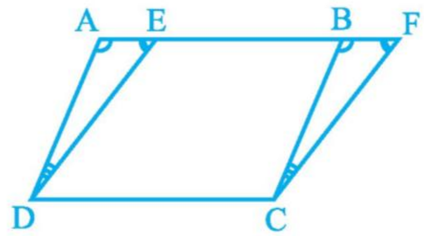


Fig. 9.12

We need to prove that ar (ABCD) = ar (EFCD).

In Δ ADE and Δ BCF,

∠DAE = ∠CBF (Corresponding angles from AD || BC and transversal AF) (1)

∠AED = ∠BFC (Corresponding angles from ED || FC and transversal AF) (2)

Therefore, ∠ADE = ∠BCF (Angle sum property of a triangle) (3)

Also, AD = BC (Opposite sides of the parallelogram ABCD) (4)

So, $\Delta ADE \cong \Delta BCF$ [By ASA rule, using (1), (3), and (4)]

Therefore, ar (ADE) = ar (BCF) (Congruent figures have equal areas) (5)

Now, ar (ABCD) = ar (ADE) + ar (EDCB)

= ar (BCF) + ar (EDCB) [From(5)]

= ar (EFCD)

So, parallelograms ABCD and EFCD are equal in area.

9.4 Triangles on the same Base and between the same Parallels

Let us look at Fig. 9.18. In it, you have two triangles ABC and PBC on the same base BC and between the same parallels BC and AP. What can you say about the areas of such triangles? To answer this question, you may perform the activity of drawing several pairs of triangles on the same base and between the same parallels on the graph sheet and find their areas by the method of counting the squares. Each time, you will find that the areas of the two triangles are (approximately) equal. This activity can be performed using a geoboard also. You will again find that the two areas are (approximately) equal.

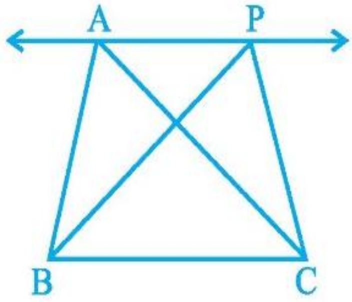


Fig. 9.18

To obtain a logical answer to the above question, you may proceed as follows:

In Fig. 9.18, draw CD || BA and CR || BP such that D and R lie on line AP(see Fig.9.19).

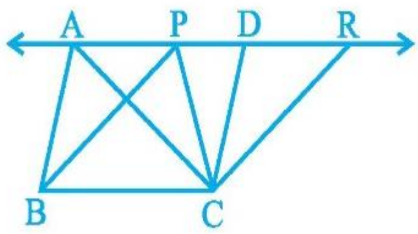


Fig. 9.19

From this, you obtain two parallelograms PBCR and ABCD on the same base BC and between the same parallels BC and AR.

Therefore, ar (ABCD) = ar (PBCR) (Why?)

Now Δ ABC $\cong$ Δ CDA and Δ PBC $\cong$ Δ CRP (Why?)

So, ar (ABC) = $\frac{1}{2}$ ar (ABCD) and ar (PBC) = $\frac{1}{2}$ ar (PBCR) (Why?)

Therefore, ar (ABC) = ar (PBC)

In this way, you have arrived at the following theorem:

Theorem 9.2 : *Two triangles on the same base (or equal bases) and between the same parallels are equal in area.*

Now, suppose ABCD is a parallelogram whose one of the diagonals is AC (see Fig. 9.20). Let AN $\perp$ DC. Note that

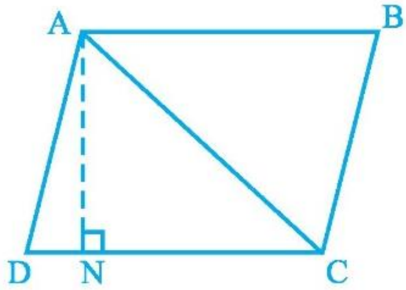


Fig. 9.20

$$\Delta\, ADC \cong \Delta\, CBA \qquad \text{(Why?)}$$

So, $$\text{ar (ADC)} = \text{ar (CBA)} \qquad \text{(Why?)}$$

Therefore, $$\text{ar (ADC)} = \frac{1}{2}\,\text{ar (ABCD)}$$

$$= \frac{1}{2}\,(\text{DC} \times \text{AN}) \qquad \text{(Why?)}$$

So, area of Δ ADC $= \frac{1}{2} \times$ base DC $\times$ corresponding altitude AN

In other words, *area of a triangle is half the product of its base (or any side) and the corresponding altitude (or height)*. Do you remember that you have learnt this formula for area of a triangle in Class VII ? From this formula, you can see that *two triangles with same base (or equal bases) and equal areas will have equal corresponding altitudes.*

For having equal corresponding altitudes, the triangles must lie between the same parallels. From this, you arrive at the following converse of Theorem 9.2 .

Chapter : Constructions

11.2 Basic Constructions

In Class VI, you have learnt how to construct a circle, the perpendicular bisector of a line segment, angles of 30°, 45°, 60°, 90° and 120°, and the bisector of a given angle, without giving any justification for these constructions. In this section, you will construct some of these, with reasoning behind, why these constructions are valid.

Construction 11.1 : *To construct the bisector of a given angle.*

Given an angle ABC, we want to construct its bisector.

Steps of Construction :

1. Taking B as centre and any radius, draw an arc to intersect the rays BA and BC, say at E and D respectively [see Fig.11.1(i)].
2. Next, taking D and E as centres and with the radius more than $\frac{1}{2}$ DE, draw arcs to intersect each other, say at F.
3. Draw the ray BF [see Fig.11.1(ii)]. This ray BF is the required bisector of the angle ABC.

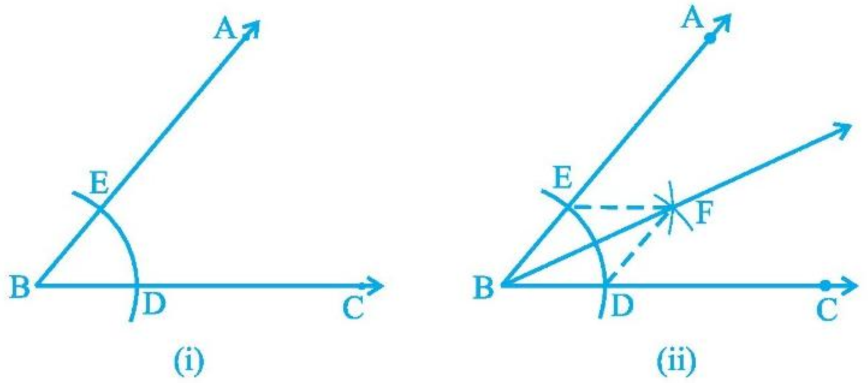


Fig. 11.1

Let us see how this method gives us the required angle bisector.

Join DF and EF.

In triangles BEF and BDF,

BE = BD (Radii of the same arc)

EF = DF (Arcs of equal radii)

BF = BF (Common)

Therefore, $\Delta BEF \cong \Delta BDF$ (SSS rule)

This gives $\angle EBF = \angle DBF$ (CPCT)

Construction 11.3 : *To construct an angle of 60^0 at the initial point of a given ray.*

Let us take a ray AB with initial point A [see Fig. 11.3(i)]. We want to construct a ray AC such that $\angle$ CAB = 60°. One way of doing so is given below.

Steps of Construction :

1. Taking A as centre and some radius, draw an arc of a circle, which intersects AB, say at a point D.
2. Taking D as centre and with the same radius as before, draw an arc intersecting the previously drawn arc, say at a point E.
3. Draw the ray AC passing through E [see Fig 11.3 (ii)].

   Then $\angle$ CAB is the required angle of 60°. Now, let us see how this method gives us the required angle of 60°.

   Join DE.

   Then, AE = AD = DE (By construction)

   Therefore, Δ EAD is an equilateral triangle and the $\angle$ EAD, which is the same as $\angle$ CAB is equal to 60°.

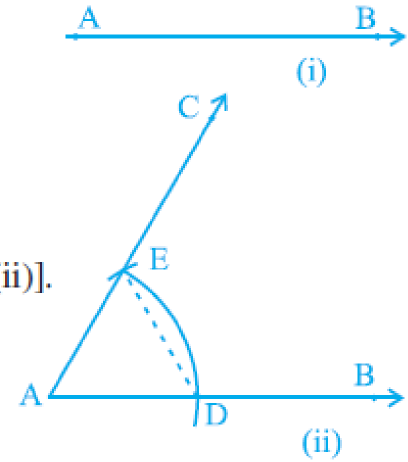


Fig. 11.3

EXERCISE 11.1

1. Construct an angle of 90^0 at the initial point of a given ray and justify the construction.

2. Construct an angle of 45^0 at the initial point of a given ray and justify the construction.

3. Construct the angles of the following measurements:

(i) 30° (ii) $22\frac{1}{2}^\circ$ (iii) 15°

4. Construct the following angles and verify by measuring them by a protractor:

(i) 75° (ii) 105° (iii) 135°

5. Construct an equilateral triangle, given its side and justify the construction.

11.3 Some Constructions of Triangles

So far, some basic constructions have been considered. Next, some constructions of triangles will be done by using the constructions given in earlier classes and given above. Recall from the Chapter 7 that SAS, SSS, ASA and RHS rules give the congruency of two triangles. Therefore, a triangle is unique if : (i) two sides and the included angle is given, (ii) three sides are given, (iii) two angles and the included side is given and, (iv) in a right triangle, hypotenuse and one side is given. You have already learnt how to construct such triangles in Class VII. Now, let us consider some more constructions of triangles. You may have noted that at least three parts of a triangle have to be given for constructing it but not all combinations of three parts are sufficient for the purpose. For example, if two sides and an angle (not the included angle) are given, then it is not always possible to construct such a triangle uniquely.

Construction 11.4 : *To construct a triangle, given its base, a base angle and sum of other two sides.*

Given the base BC, a base angle, say ∠B and the sum AB + AC of the other two sides of a triangle ABC, you are required to construct it.

Steps of Construction :

1. Draw the base BC and at the point B make an angle, say XBC equal to the given angle.
2. Cut a line segment BD equal to AB + AC from the ray BX.
3. Join DC and make an angle DCY equal to ∠BDC.
4. Let CY intersect BX at A (see Fig. 11.4).

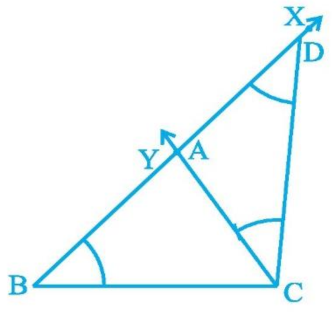


Fig. 11.4

Then, ABC is the required triangle.

Let us see how you get the required triangle.

Base BC and ∠B are drawn as given. Next in triangle ACD,

$$\angle ACD = \angle ADC \qquad \text{(By construction)}$$

Therefore, AC = AD and then

$$AB = BD - AD = BD - AC$$

$$AB + AC = BD$$

Alternative method :

Follow the first two steps as above. Then draw perpendicular bisector PQ of CD to intersect BD at a point A (see Fig 11.5). Join AC. Then ABC is the required triangle. Note that A lies on the perpendicular bisector of CD, therefore AD = AC.

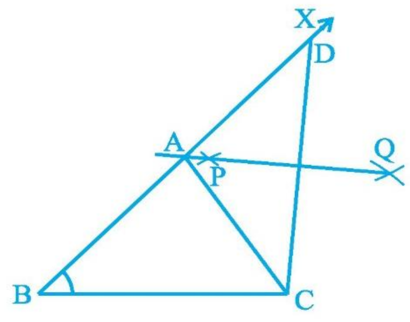


Fig. 11.5

Remark : The construction of the triangle is not possible if the sum AB + AC ≤ BC.

Construction 11.5 : *To construct a triangle given its base, a base angle and the difference of the other two sides.*

Given the base BC, a base angle, say ∠B and the difference of other two sides AB – AC or AC – AB, you have to construct the triangle ABC. Clearly there are following two cases:

Case (i) : Let AB > AC that is AB – AC is given.

Steps of Construction :

1. Draw the base BC and at point B make an angle say XBC equal to the given angle.
2. Cut the line segment BD equal to AB – AC from ray BX.
3. Join DC and draw the perpendicular bisector, say PQ of DC.
4. Let it intersect BX at a point A. Join AC (see Fig. 11.6).

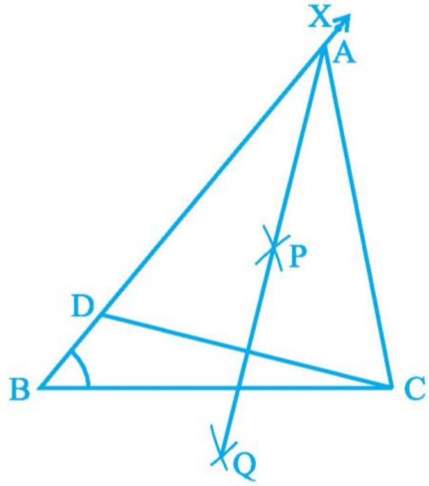


Fig. 11.6

Then ABC is the required triangle.

Let us now see how you have obtained the required triangle ABC.

Base BC and ∠B are drawn as given. The point A lies on the perpendicular bisector of DC. Therefore,

$$AD = AC$$

So, $$BD = AB - AD = AB - AC.$$

Case (ii) : Let AB < AC that is AC – AB is given.

Steps of Construction :

1. Same as in case (i).
2. Cut line segment BD equal to AC – AB from the line BX extended on opposite side of line segment BC.
3. Join DC and draw the perpendicular bisector, say PQ of DC.
4. Let PQ intersect BX at A. Join AC (see Fig. 11.7).

Then, ABC is the required triangle.

You can justify the construction as in case (i).

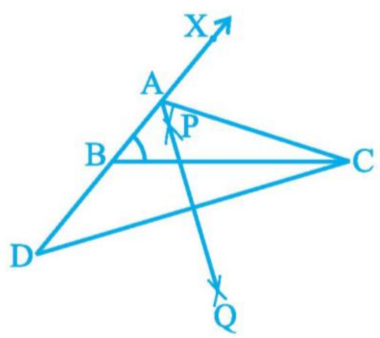


Fig. 11.7

Chapter 10: Heron's Formula (Areas)

12.3 Application of Heron's Formula in Finding Areas of Quadrilaterals

Suppose that a farmer has a land to be cultivated and she employs some labourers for this purpose on the terms of wages calculated by area cultivated per square metre. How will she do this? Many a time, the fields are in the shape of quadrilaterals. We need to divide the quadrilateral in triangular parts and then use the formula for area of the triangle. Let us look at this problem:

Example 4 : Kamla has a triangular field with sides 240 m, 200 m, 360 m, where she grew wheat. In another triangular field with sides 240 m, 320 m, 400 m adjacent to the previous field, she wanted to grow potatoes and onions (see Fig. 12.11). She divided the field in two parts by joining the mid-point of the longest side to the opposite vertex and grew patatoes in one part and onions in the other part. How much area (in hectares) has been used for wheat, potatoes and onions? (1 hectare = 10000 m^2)

Solution : Let ABC be the field where wheat is grown. Also let ACD be the field which has been divided in two parts by joining C to the mid-point E of AD. For the area of triangle ABC, we have

$$a = 200 \text{ m}, b = 240 \text{ m}, c = 360 \text{ m}$$

Therefore, $s = \dfrac{200 + 240 + 360}{2} \text{ m} = 400 \text{ m}.$

`

So, area for growing wheat

$$= \sqrt{400(400-200)(400-240)(400-360)}\ \text{m}^2$$

$$= \sqrt{400 \times 200 \times 160 \times 40}\ \text{m}^2$$

$$= 16000\sqrt{2}\ \text{m}^2 = 1.6 \times \sqrt{2}\ \text{hectares}$$

$$= 2.26 \text{ hectares (nearly)}$$

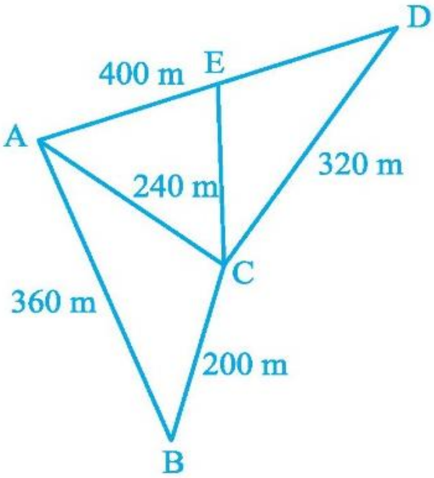


Fig. 12.11

Let us now calculate the area of triangle ACD.

Here, we have $s = \frac{240 + 320 + 400}{2}$ m = 480 m.

So, area of Δ ACD $= \sqrt{480(480-240)(480-320)(480-400)}\ \text{m}^2$

$$= \sqrt{480 \times 240 \times 160 \times 80}\ \text{m}^2 = 38400\ \text{m}^2 = 3.84 \text{ hectares}$$

We notice that the line segment joining the mid-point E of AD to C divides the triangle ACD in two parts equal in area. Can you give the reason for this? In fact, they have the bases AE and ED equal and, of course, they have the same height.

Therefore, area for growing potatoes = area for growing onions

$$= (3.84 \div 2) \text{ hectares} = 1.92 \text{ hectares.}$$

Example 5 : Students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups. One group walked through the lanes AB, BC and CA; while the other through AC, CD and DA (see Fig. 12.12). Then they cleaned the area enclosed within their lanes. If AB = 9 m, BC = 40 m, CD = 15 m, DA = 28 m and $\angle$ B = 90°, which group cleaned more area and by how much? Find the total area cleaned by the students (neglecting the width of the lanes).

Solution : Since AB = 9 m and BC = 40 m, $\angle$ B = 90°, we have:

$$\text{AC} = \sqrt{9^2 + 40^2}\ \text{m}$$

$$= \sqrt{81 + 1600}\ \text{m}$$

$$= \sqrt{1681}\ \text{m} = 41\text{m}$$

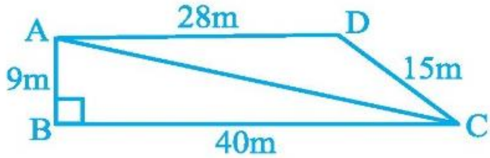


Fig. 12.12

Therefore, the first group has to clean the area of triangle ABC, which is right angled.

Area of Δ ABC $= \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 40 \times 9\ \text{m}^2 = 180\ \text{m}^2$$

The second group has to clean the area of triangle ACD, which is scalene having sides 41 m, 15 m and 28 m.

Here, $s = \frac{41 + 15 + 28}{2}$ m = 42 m

Therefore, area of Δ ACD = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-41)(42-15)(42-28)}\ \text{m}^2$

$= \sqrt{42 \times 1 \times 27 \times 14}\ \text{m}^2 = 126\ \text{m}^2$

So first group cleaned 180 m^2 which is (180 – 126) m^2, i.e., 54 m^2 more than the area cleaned by the second group.

Total area cleaned by all the students = (180 + 126) m^2 = 306 m^2.

Example 6 : Sanya has a piece of land which is in the shape of a rhombus (see Fig. 12.13). She wants her one daughter and one son to work on the land and produce different crops. She divided the land in two equal parts. If the perimeter of the land is 400 m and one of the diagonals is 160 m, how much area each of them will get for their crops?

Solution : Let ABCD be the field.

Perimeter = 400 m

So, each side = 400 m ÷ 4 = 100 m.

i.e. AB = AD = 100 m.

Let diagonal BD = 160 m.

Then semi-perimeter s of Δ ABD is given by

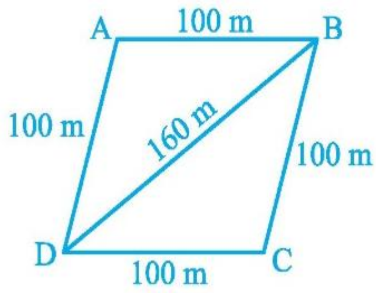


Fig. 12.13

$s = \frac{100 + 100 + 160}{2}$ m = 180 m

Therefore, area of Δ ABD = $\sqrt{180(180-100)(180-100)(180-160)}$

$= \sqrt{180 \times 80 \times 80 \times 20}\ \text{m}^2 = 4800\ \text{m}^2$

Therefore, each of them will get an area of 4800 m^2.

Chapter 11: Surface Areas and Volumes

CHAPTER 13

SURFACE AREAS AND VOLUMES

13.1 Introduction

Wherever we look, usually we see solids. So far, in all our study, we have been dealing with figures that can be easily drawn on our notebooks or blackboards. These are called *plane figures*. We have understood what rectangles, squares and circles are, what we mean by their perimeters and areas, and how we can find them. We have learnt these in earlier classes. It would be interesting to see what happens if we cut out many of these plane figures of the same shape and size from cardboard sheet and stack them up in a vertical pile. By this process, we shall obtain some *solid figures* (briefly called *solids*) such as a cuboid, a cylinder, etc. In the earlier classes, you have also learnt to find the surface areas and volumes of cuboids, cubes and cylinders. We shall now learn to find the surface areas and volumes of cuboids and cylinders in details and extend this study to some other solids such as cones and spheres.

13.2 Surface Area of a Cuboid and a Cube

Have you looked at a bundle of many sheets of paper? How does it look? Does it look like what you see in Fig. 13.1?

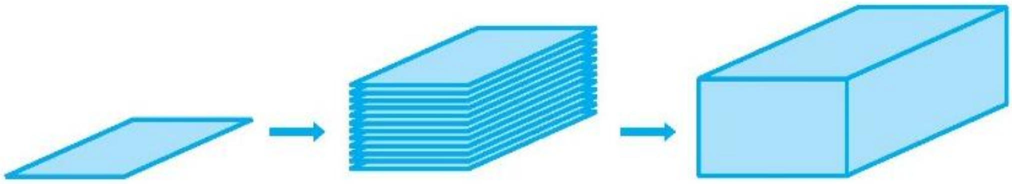
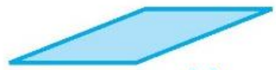


Fig. 13.1

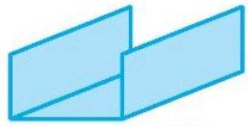
That makes up a cuboid. How much of brown paper would you need, if you want to cover this cuboid? Let us see:

First we would need a rectangular piece to cover the bottom of the bundle. That would be as shown in Fig. 13.2 (a)



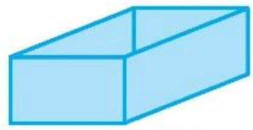
(a)

Then we would need two long rectangular pieces to cover the two side ends. Now, it would look like Fig. 13.2 (b).



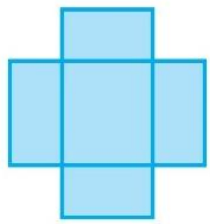
(b)

Now to cover the front and back ends, we would need two more rectangular pieces of a different size. With them, we would now have a figure as shown in Fig. 13.2(c).



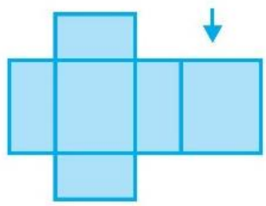
(c)

This figure, when opened out, would look like Fig. 13.2 (d).



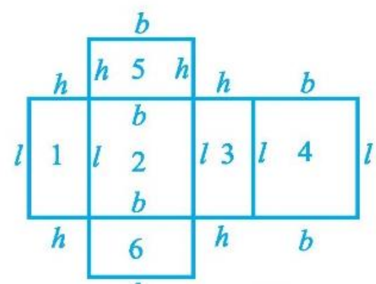
(d)

Finally, to cover the top of the bundle, we would require another rectangular piece exactly like the one at the bottom, which if we attach on the right side, it would look like Fig. 13.2(e).



(e)

So we have used six rectangular pieces to cover the complete outer surface of the cuboid.



(f)

Fig. 13.2

This shows us that the outer surface of a cuboid is made up of six rectangles (in fact, rectangular regions, called the faces of the cuboid), whose areas can be found by multiplying length by breadth for each of them separately and then adding the six areas together.

Now, if we take the length of the cuboid as l, breadth as b and the height as h, then the figure with these dimensions would be like the shape you see in Fig. 13.2(f).

So, the sum of the areas of the six rectangles is:

Area of rectangle 1 $(= l \times h)$

$+$

Area of rectangle 2 $(= l \times b)$

$+$

Area of rectangle 3 $(= l \times h)$

$+$

Area of rectangle 4 $(= l \times b)$

$+$

Area of rectangle 5 $(= b \times h)$

$+$

Area of rectangle 6 $(= b \times h)$

$= 2(l \times b) + 2(b \times h) + 2(l \times h)$

$= 2(lb + bh + hl)$

This gives us:

Surface Area of a Cuboid = $2(lb + bh + hl)$

where l, b and h are respectively the three edges of the cuboid.

Note : The unit of area is taken as the square unit, because we measure the magnitude of a region by filling it with squares of side of unit length.

For example, if we have a cuboid whose length, breadth and height are 15 cm, 10 cm and 20 cm respectively, then its surface area would be:

$$2[(15 \times 10) + (10 \times 20) + (20 \times 15)] \text{ cm}^2$$
$$= 2(150 + 200 + 300) \text{ cm}^2$$
$$= 2 \times 650 \text{ cm}^2$$
$$= 1300 \text{ cm}^2$$

Recall that a cuboid, whose length, breadth and height are all equal, is called a *cube*. If each edge of the cube is a, then the surface area of this cube would be

$$2(a \times a + a \times a + a \times a), \text{ i.e., } 6a^2 \text{ (see Fig. 13.3), giving us}$$

Surface Area of a Cube = $6a^2$

where a is the edge of the cube.

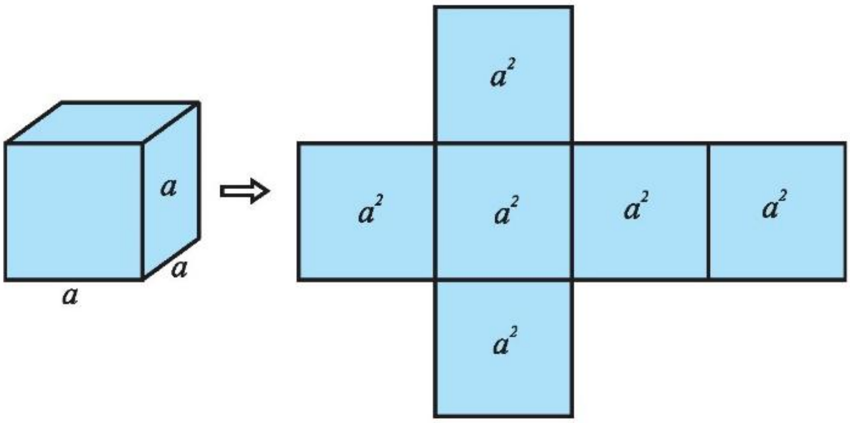


Fig. 13.3

Suppose, out of the six faces of a cuboid, we only find the area of the four faces, leaving the bottom and top faces. In such a case, the area of these four faces is called the **lateral surface area** of the cuboid. So, *lateral surface area of a cuboid of length l, breadth b and height h is equal to $2lh + 2bh$ or $2(l + b)h$*. Similarly, *lateral surface area of a cube of side a is equal to $4a^2$*.

Keeping in view of the above, the surface area of a cuboid (or a cube) is sometimes also referred to as the **total surface area**. Let us now solve some examples.

Example 1 : Mary wants to decorate her Christmas tree. She wants to place the tree on a wooden box covered with coloured paper with picture of Santa Claus on it (see Fig. 13.4). She must know the exact quantity of paper to buy for this purpose. If the box has length, breadth and height as 80 cm, 40 cm and 20 cm respectively how many square sheets of paper of side 40 cm would she require?

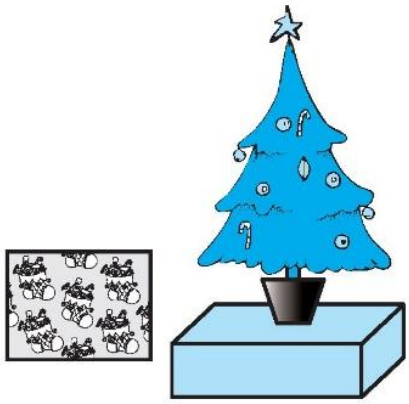


Fig. 13.4

Solution : Since Mary wants to paste the paper on the outer surface of the box; the quantity of paper required would be equal to the surface area of the box which is of the shape of a cuboid. The dimensions of the box are:

Length =80 cm, Breadth = 40 cm, Height = 20 cm.

The surface area of the box = $2(lb + bh + hl)$

$$= 2[(80 \times 40) + (40 \times 20) + (20 \times 80)] \text{ cm}^2$$

$$= 2[3200 + 800 + 1600] \text{ cm}^2$$

$$= 2 \times 5600 \text{ cm}^2 = 11200 \text{ cm}^2$$

The area of each sheet of the paper = $40 \times 40 \text{ cm}^2$

$$= 1600 \text{ cm}^2$$

Therefore, number of sheets required $= \dfrac{\text{surface area of box}}{\text{area of one sheet of paper}}$

$$= \frac{11200}{1600} = 7$$

So, she would require 7 sheets.

Example 2 : Hameed has built a cubical water tank with lid for his house, with each outer edge 1.5 m long. He gets the outer surface of the tank excluding the base, covered with square tiles of side 25 cm (see Fig. 13.5). Find how much he would spend for the tiles, if the cost of the tiles is ₹ 360 per dozen.

Solution : Since Hameed is getting the five outer faces of the tank covered with tiles, he would need to know the surface area of the tank, to decide on the number of tiles required.

Edge of the cubical tank = 1.5 m = 150 cm (= a)

So, surface area of the tank = $5 \times 150 \times 150 \text{ cm}^2$

Area of each square tile = side × side = $25 \times 25 \text{ cm}^2$

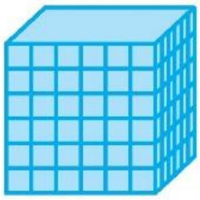


Fig. 13.5

So, the number of tiles required $= \dfrac{\text{surface area of the tank}}{\text{area of each tile}}$

$$= \frac{5 \times 150 \times 150}{25 \times 25} = 180$$

Cost of 1 dozen tiles, i.e., cost of 12 tiles = ₹ 360

Therefore, cost of one tile = ₹ $\dfrac{360}{12}$ = ₹ 30

So, the cost of 180 tiles = 180 × ₹ 30 = ₹ 5400

`

EXERCISE 13.1

1. A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is opened at the top. Ignoring the thickness of the plastic sheet, determine:

   (i) The area of the sheet required for making the box.

   (ii) The cost of sheet for it, if a sheet measuring 1m^2 costs ₹ 20.

2. The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of ₹ 7.50 per m^2.

3. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of ₹ 10 per m^2 is ₹ 15000, find the height of the hall.
   [**Hint :** Area of the four walls = Lateral surface area.]

4. The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?

5. A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.

   (i) Which box has the greater lateral surface area and by how much?

   (ii) Which box has the smaller total surface area and by how much?

6. A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.

   (i) What is the area of the glass?

   (ii) How much of tape is needed for all the 12 edges?

7. Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is ₹ 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.

8. Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m × 3 m?

13.3 Surface Area of a Right Circular Cylinder

If we take a number of circular sheets of paper and stack them up as we stacked up rectangular sheets earlier, what would we get (see Fig. 13.6)?

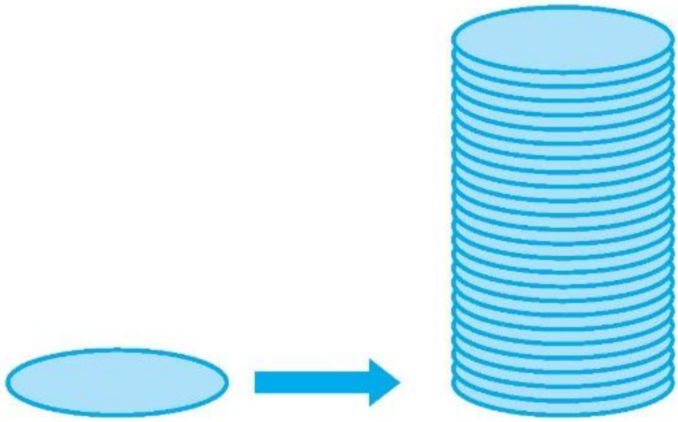


Fig. 13.6

Here, if the stack is kept vertically up, we get what is called a *right circular cylinder*, since it has been kept at right angles to the base, and the base is circular. Let us see what kind of cylinder is *not* a right circular cylinder.

In Fig 13.7 (a), you see a cylinder, which is certainly circular, but it is not at right angles to the base. So, we can *not* say this a *right* circular cylinder.

Of course, if we have a cylinder with a non circular base, as you see in Fig. 13.7 (b), then we also cannot call it a right circular cylinder.

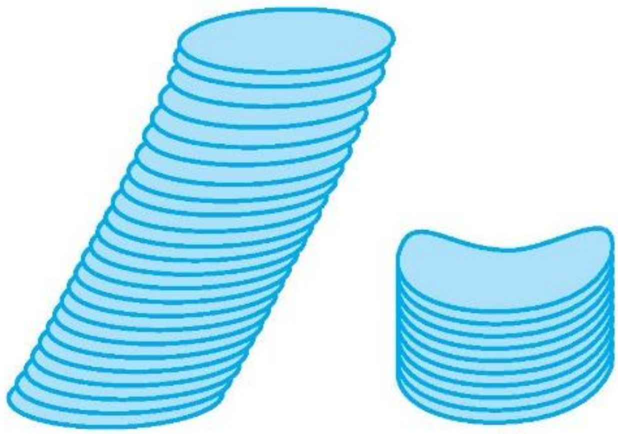


Fig. 13.7

Remark : Here, we will be dealing with only right circular cylinders. So, unless stated otherwise, the word cylinder would mean a right circular cylinder.

Now, if a cylinder is to be covered with coloured paper, how will we do it with the minimum amount of paper? First take a rectangular sheet of paper, whose length is just enough to go round the cylinder and whose breadth is equal to the height of the cylinder as shown in Fig. 13.8.

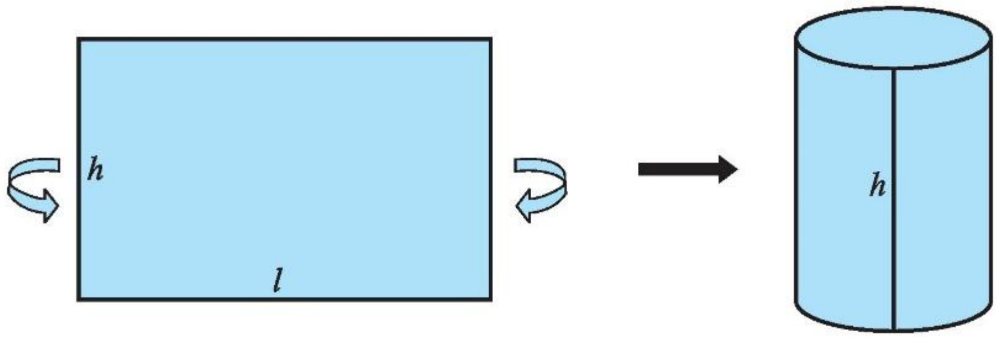


Fig. 13.8

The area of the sheet gives us the curved surface area of the cylinder. Note that the length of the sheet is equal to the circumference of the circular base which is equal to $2\pi r$.

So, curved surface area of the cylinder

$$= \text{area of the rectangular sheet} = \text{length} \times \text{breadth}$$

$$= \text{perimeter of the base of the cylinder} \times h$$

$$= 2\pi r \times h$$

Therefore, **Curved Surface Area of a Cylinder = $2\pi rh$**

where r is the radius of the base of the cylinder and h is the height of the cylinder.

Remark : In the case of a cylinder, unless stated otherwise, 'radius of a cylinder' shall mean' base radius of the cylinder'.

If the top and the bottom of the cylinder are also to be covered, then we need two circles (infact, circular regions) to do that, each of radius r, and thus having an area of πr^2 each (see Fig. 13.9), giving us the total surface area as $2\pi rh + 2\pi r^2 = 2\pi r(r + h)$.

So, **Total Surface Area of a Cylinder = $2\pi r(r + h)$**

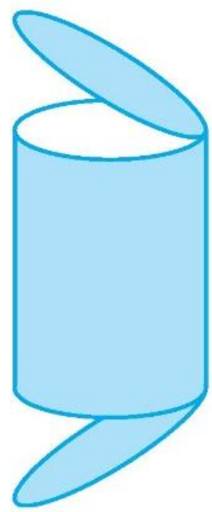


Fig. 13.9

where h is the height of the cylinder and r its radius.

Remark : You may recall from Chapter 1 that π is an irrational number. So, the value of π is a non-terminating, non-repeating decimal. But when we use its value in our calculations, we usually take its value as approximately equal to $\frac{22}{7}$ or 3.14.

Example 3 : Savitri had to make a model of a cylindrical kaleidoscope for her science project. She wanted to use chart paper to make the curved surface of the kaleidoscope. (see Fig 13.10). What would be the area of chart paper required by her, if she wanted to make a kaleidoscope of length 25 cm with a 3.5 cm radius? You may take $\pi = \frac{22}{7}$.

Solution : Radius of the base of the cylindrical kaleidoscope (r) = 3.5 cm.

Height (length) of kaleidoscope (h) = 25 cm.

Area of chart paper required = curved surface area of the kaleidoscope

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 3.5 \times 25 \text{ cm}^2$$

$$= 550 \text{ cm}^2$$

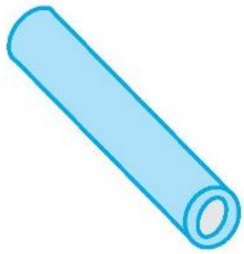


Fig. 13.10

EXERCISE 13.2

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder.
2. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?
3. A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4 cm (see Fig. 13.11). Find its
   (i) inner curved surface area,
   (ii) outer curved surface area,
   (iii) total surface area.

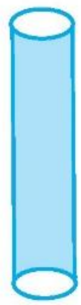


Fig. 13.11

4. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2.

5. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of ₹ 12.50 per m^2.

6. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height.

7. The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find

   (i) its inner curved surface area,

   (ii) the cost of plastering this curved surface at the rate of ₹ 40 per m^2.

8. In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.

9. Find

   (i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.

   (ii) how much steel was actually used, if $\frac{1}{12}$ of the steel actually used was wasted in making the tank.

10. In Fig. 13.12, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.

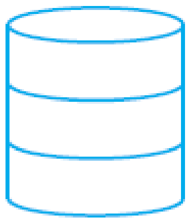


Fig. 13.12

11. The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition?

13.6 Volume of a Cuboid

You have already learnt about volumes of certain figures (objects) in earlier classes. Recall that solid objects occupy space. The measure of this occupied space is called the **Volume** of the object.

Note : If an object is solid, then the space occupied by such an object is measured, and is termed the **Volume** of the object. On the other hand, if the object is hollow, then interior is empty, and can be filled with air, or some liquid that will take the shape of its container. In this case, the volume of the substance that can fill the interior is called the **capacity of the container**. In short, the volume of an object is the measure of the space it occupies, and the capacity of an object is the volume of substance its interior can accommodate. Hence, the unit of measurement of either of the two is cubic unit.

So, if we were to talk of the volume of a cuboid, we would be considering the measure of the space occupied by the cuboid.

Further, the area or the volume is measured as the magnitude of a region. So, correctly speaking, we should be finding the area of a circular region, or volume of a cuboidal region, or volume of a spherical region, etc. But for the sake of simplicity, we say, find the area of a circle, volume of a cuboid or a sphere even though these mean only their boundaries.

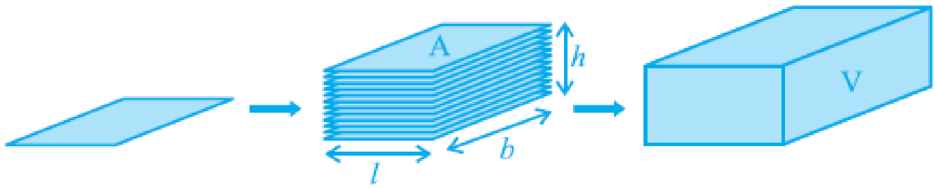


Fig. 13.23

Observe Fig. 13.23. Suppose we say that the area of each rectangle is A, the height up to which the rectangles are stacked is h and the volume of the cuboid is V. Can you tell what would be the relationship between V, A and h?

The area of the plane region occupied by each rectangle × height

= Measure of the space occupied by the cuboid

So, we get $A \times h = V$

That is, **Volume of a Cuboid = base area × height = length × breadth × height**

or $l \times b \times h$, where l, b and h are respectively the length, breadth and height of the cuboid.

Note : When we measure the magnitude of the region of a space, that is, the space occupied by a solid, we do so by counting the number of cubes of edge of unit length that can fit into it exactly. Therefore, the unit of measurement of volume is cubic unit.

Again **Volume of a Cube = edge × edge × edge = a^3**

where a is the edge of the cube (see Fig. 13.24).

So, if a cube has edge of 12 cm,

then volume of the cube = $12 \times 12 \times 12$ cm^3

= 1728 cm^3.

Recall that you have learnt these formulae in earlier classes. Now let us take some examples to illustrate the use of these formulae:

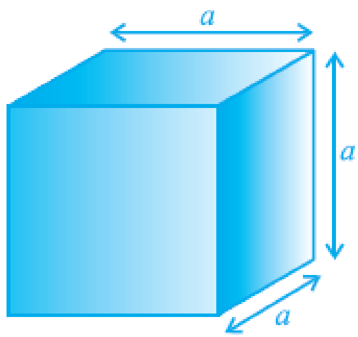


Fig. 13.24

Example11 : A wall of length 10 m was to be built across an open ground. The height of the wall is 4 m and thickness of the wall is 24 cm. If this wall is to be built up with bricks whose dimensions are 24 cm × 12 cm × 8 cm, how many bricks would be required?

Solution : Since the wall with all its bricks makes up the space occupied by it, we need to find the volume of the wall, which is nothing but a cuboid.

Here, Length = 10 m = 1000 cm

Thickness = 24 cm

Height = 4 m = 400 cm

Therefore, Volume of the wall = length × thickness × height

= $1000 \times 24 \times 400$ cm^3

Now, each brick is a cuboid with length = 24 cm, breadth = 12 cm and height = 8 cm

So, volume of each brick = length × breadth × height

= $24 \times 12 \times 8$ cm^3

$$\text{So, number of bricks required} = \frac{\text{volume of the wall}}{\text{volume of each brick}}$$

$$= \frac{1000 \times 24 \times 400}{24 \times 12 \times 8}$$

$$= 4166.6$$

So, the wall requires 4167 bricks.

Example 12 : A child playing with building blocks, which are of the shape of cubes, has built a structure as shown in Fig. 13.25. If the edge of each cube is 3 cm, find the volume of the structure built by the child.

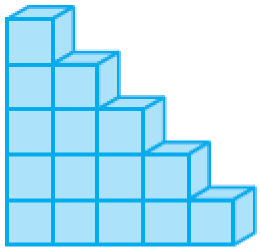


Fig. 13.25

Solution : Volume of each cube = edge × edge × edge

$= 3 \times 3 \times 3 \text{ cm}^3 = 27 \text{ cm}^3$

Number of cubes in the structure = 15

Therefore, volume of the structure = $27 \times 15 \text{ cm}^3$

$= 405 \text{ cm}^3$

EXERCISE 13.5

1. A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?
2. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? (1 m^3 = 1000 *l*)
3. A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?
4. Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of ₹ 30 per m^3.
5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.
6. A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?
7. A godown measures 40 m × 25 m × 15 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown.
8. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.
9. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

13.7 Volume of a Cylinder

Just as a cuboid is built up with rectangles of the same size, we have seen that a right circular cylinder can be built up using circles of the same size. So, using the same argument as for a cuboid, we can see that the volume of a cylinder can be obtained as : base area × height

= area of circular base × height = $\pi r^2 h$

So, **Volume of a Cylinder = $\pi r^2 h$**

where r is the base radius and h is the height of the cylinder.

Example 13 : The pillars of a temple are cylindrically shaped (see Fig. 13.26). If each pillar has a circular base of radius 20 cm and height 10 m, how much concrete mixture would be required to build 14 such pillars?

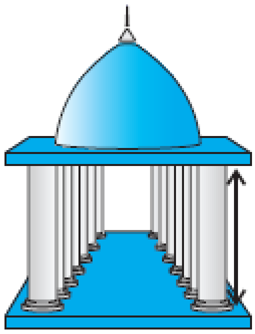


Fig. 13.26

Solution : Since the concrete mixture that is to be used to build up the pillars is going to occupy the entire space of the pillar, what we need to find here is the volume of the cylinders.

Radius of base of a cylinder = 20 cm

Height of the cylindrical pillar = 10 m = 1000 cm

So, volume of each cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 20 \times 20 \times 1000 \text{ cm}^3$$

$$= \frac{8800000}{7} \text{ cm}^3$$

$$= \frac{8.8}{7} \text{ m}^3 \text{ (Since } 1000000 \text{ cm}^3 = 1 \text{m}^3\text{)}$$

Therefore, volume of 14 pillars = volume of each cylinder × 14

$$= \frac{8.8}{7} \times 14 \text{ m}^3$$

$$= 17.6 \text{ m}^3$$

So, 14 pillars would need 17.6 m^3 of concrete mixture.

Example 14 : At a Ramzan Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small cylindrical glasses (see Fig. 13.27) of radius 3 cm up to a height of 8 cm, and sold for ₹ 15 each. How much money does the stall keeper receive by selling the juice completely?

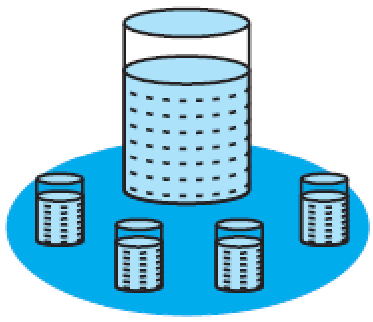


Fig. 13.27

Solution : The volume of juice in the vessel

$$= \text{volume of the cylindrical vessel}$$

$$= \pi R^2 H$$

(where R and H are taken as the radius and height respectively of the vessel)

$$= \pi \times 15 \times 15 \times 32 \text{ cm}^3$$

Similarly, the volume of juice each glass can hold = $\pi r^2 h$

(where r and h are taken as the radius and height respectively of each glass)

$$= \pi \times 3 \times 3 \times 8 \text{ cm}^3$$

So, number of glasses of juice that are sold

$$= \frac{\text{volume of the vessel}}{\text{volume of each glass}}$$

$$= \frac{\pi \times 15 \times 15 \times 32}{\pi \times 3 \times 3 \times 8}$$

$$= 100$$

Therefore, amount received by the stall keeper = ₹ 15 × 100

= ₹ 1500

EXERCISE 13.6

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? ($1000 \text{ cm}^3 = 1l$)
2. The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has a mass of 0.6 g.
3. A soft drink is available in two packs – (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?
4. If the lateral surface of a cylinder is 94.2 cm^2 and its height is 5 cm, then find (i) radius of its base (ii) its volume. (Use $\pi = 3.14$)

5. It costs ₹ 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of ₹ 20 per m^2, find

(i) inner curved surface area of the vessel,

(ii) radius of the base,

(iii) capacity of the vessel.

6. The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it?

7. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

8. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?

CHAPTER 14

STATISTICS

14.1 Introduction

Everyday we come across a lot of information in the form of facts, numerical figures, tables, graphs, etc. These are provided by newspapers, televisions, magazines and other means of communication. These may relate to cricket batting or bowling averages, profits of a company, temperatures of cities, expenditures in various sectors of a five year plan, polling results, and so on. These facts or figures, which are numerical or otherwise, collected with a definite purpose are called *data*. Data is the plural form of the Latin word *datum*. Of course, the word 'data' is not new for you. You have studied about data and data handling in earlier classes.

Our world is becoming more and more information oriented. Every part of our lives utilises data in one form or the other. So, it becomes essential for us to know how to extract meaningful information from such data. This extraction of meaningful information is studied in a branch of mathematics called *Statistics*.

The word 'statistics' appears to have been derived from the Latin word 'status' meaning 'a (political) state'. In its origin, statistics was simply the collection of data on different aspects of the life of people, useful to the State. Over the period of time, however, its scope broadened and statistics began to concern itself not only with the collection and presentation of data but also with the interpretation and drawing of inferences from the data. Statistics deals with collection, organisation, analysis and interpretation of data. The word 'statistics' has different meanings in different contexts. Let us observe the following sentences:

1. May I have the latest copy of 'Educational Statistics of India'.
2. I like to study 'Statistics' because it is used in day-to-day life.

In the first sentence, statistics is used in a plural sense, meaning numerical data. These may include a number of educational institutions of India, literacy rates of various states, etc. In the second sentence, the word 'statistics' is used as a singular noun, meaning the subject which deals with the collection, presentation, analysis of data as well as drawing of meaningful conclusions from the data.

In this chapter, we shall briefly discuss all these aspects regarding data.

14.2 Collection of Data

Let us begin with an exercise on gathering data by performing the following activity.

Activity 1 : Divide the students of your class into four groups. Allot each group the work of collecting one of the following kinds of data:

(i) Heights of 20 students of your class.

(ii) Number of absentees in each day in your class for a month.

(iii) Number of members in the families of your classmates.

(iv) Heights of 15 plants in or around your school.

Let us move to the results students have gathered. How did they collect their data in each group?

(i) Did they collect the information from each and every student, house or person concerned for obtaining the information?

(ii) Did they get the information from some source like available school records?

In the first case, when the information was collected by the investigator herself or himself with a definite objective in her or his mind, the data obtained is called *primary data*.

In the second case, when the information was gathered from a source which already had the information stored, the data obtained is called *secondary data*. Such data, which has been collected by someone else in another context, needs to be used with great care ensuring that the source is reliable.

By now, you must have understood how to collect data and distinguish between primary and secondary data.

EXERCISE 14.1

1. Give five examples of data that you can collect from your day-to-day life.

2. Classify the data in Q.1 above as primary or secondary data.

14.3 Presentation of Data

As soon as the work related to collection of data is over, the investigator has to find out ways to present them in a form which is meaningful, easily understood and gives its main features at a glance. Let us now recall the various ways of presenting the data through some examples.

Example 1 : Consider the marks obtained by 10 students in a mathematics test as given below:

55 36 95 73 60 42 25 78 75 62

The data in this form is called *raw data*.

By looking at it in this form, can you find the highest and the lowest marks?

Did it take you some time to search for the maximum and minimum scores? Wouldn't it be less time consuming if these scores were arranged in ascending or descending order? So let us arrange the marks in ascending order as

25 36 42 55 60 62 73 75 78 95

Now, we can clearly see that the lowest marks are 25 and the highest marks are 95.

The difference of the highest and the lowest values in the data is called the *range* of the data. So, the range in this case is 95 – 25 = 70.

Presentation of data in ascending or descending order can be quite time consuming, particularly when the number of observations in an experiment is large, as in the case of the next example.

Example 2 : Consider the marks obtained (out of 100 marks) by 30 students of Class IX of a school:

10	20	36	92	95	40	50	56	60	70
92	88	80	70	72	70	36	40	36	40
92	40	50	50	56	60	70	60	60	88

Recall that the number of students who have obtained a certain number of marks is called the *frequency* of those marks. For instance, 4 students got 70 marks. So the frequency of 70 marks is 4. To make the data more easily understandable, we write it in a table, as given below:

Table 14.1

Marks	Number of students (i.e., the frequency)
10	1
20	1
36	3
40	4
50	3
56	2
60	4
70	4
72	1
80	1
88	2
92	3
95	1
Total	30

Table 14.1 is called *an ungrouped frequency distribution table,* or simply a *frequency distribution table*. Note that you can use also *tally marks* in preparing these tables, as in the next example.

Example 3 : 100 plants each were planted in 100 schools during Van Mahotsava. After one month, the number of plants that survived were recorded as :

95	67	28	32	65	65	69	33	98	96
76	42	32	38	42	40	40	69	95	92
75	83	76	83	85	62	37	65	63	42
89	65	73	81	49	52	64	76	83	92
93	68	52	79	81	83	59	82	75	82
86	90	44	62	31	36	38	42	39	83
87	56	58	23	35	76	83	85	30	68
69	83	86	43	45	39	83	75	66	83
92	75	89	66	91	27	88	89	93	42
53	69	90	55	66	49	52	83	34	36

To present such a large amount of data so that a reader can make sense of it easily, we condense it into groups like 20-29, 30-39, . . ., 90-99 (since our data is from 23 to 98). These groupings are called 'classes' or 'class-intervals', and their size is called the *class-size* or *class width*, which is 10 in this case. In each of these classes, the least number is called the *lower class limit* and the greatest number is called the *upper class limit,* e.g., in 20-29, 20 is the 'lower class limit' and 29 is the 'upper class limit'.

Also, recall that using tally marks, the data above can be condensed in tabular form as follows:

Table 14.2

Number of plants survived	Tally Marks	Number of schools (frequency)
20 - 29	|||	3
30 - 39	~~||||~~ ~~||||~~ ||||	14
40 - 49	~~||||~~ ~~||||~~ ||	12
50 - 59	~~||||~~ |||	8
60 - 69	~~||||~~ ~~||||~~ ~~||||~~ |||	18
70 - 79	~~||||~~ ~~||||~~	10
80 - 89	~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ |||	23
90 - 99	~~||||~~ ~~||||~~ ||	12
Total		100

Presenting data in this form simplifies and condenses data and enables us to observe certain important features at a glance. This is called *a grouped frequency distribution table*. Here we can easily observe that 50% or more plants survived in 8 + 18 + 10 + 23 + 12 = 71 schools.

We observe that the classes in the table above are non-overlapping. Note that we could have made more classes of shorter size, or fewer classes of larger size also. For instance, the intervals could have been 22-26, 27-31, and so on. So, there is no hard and fast rule about this except that the classes should not overlap.

Example 4 : Let us now consider the following frequency distribution table which gives the weights of 38 students of a class:

Table 14.3

Weights (in kg)	Number of students
31 - 35	9
36 - 40	5
41 - 45	14
46 - 50	3
51 - 55	1
56 - 60	2
61 - 65	2
66 - 70	1
71 - 75	1
Total	38

Now, if two new students of weights 35.5 kg and 40.5 kg are admitted in this class, then in which interval will we include them? We cannot add them in the ones ending with 35 or 40, nor to the following ones. This is because there are gaps in between the upper and lower limits of two consecutive classes. So, we need to divide the intervals so that the upper and lower limits of consecutive intervals are the same. For this, we find the difference between the upper limit of a class and the lower limit of its succeeding class. We then add half of this difference to each of the upper limits and subtract the same from each of the lower limits.

For example, consider the classes 31 - 35 and 36 - 40.

The lower limit of 36 - 40 = 36

The upper limit of 31 - 35 = 35

The difference = 36 – 35 = 1

So, half the difference = $\frac{1}{2} = 0.5$

So the new class interval formed from 31 - 35 is (31 – 0.5) - (35 + 0.5), i.e., 30.5 - 35.5.

Similarly, the new class formed from the class 36 - 40 is (36 – 0.5) - (40 + 0.5), i.e., 35.5 - 40.5.

Continuing in the same manner, the continuous classes formed are:

30.5-35.5, 35.5-40.5, 40.5-45.5, 45.5-50.5, 50.5-55.5, 55.5-60.5, 60.5 - 65.5, 65.5 - 70.5, 70.5 - 75.5.

Now it is possible for us to include the weights of the new students in these classes. But, another problem crops up because 35.5 appears in both the classes 30.5 - 35.5 and 35.5 - 40.5. In which class do you think this weight should be considered?

If it is considered in both classes, it will be counted twice.

By convention, we consider 35.5 in the class 35.5 - 40.5 and not in 30.5 - 35.5. Similarly, 40.5 is considered in 40.5 - 45.5 and not in 35.5 - 40.5.

So, the new weights 35.5 kg and 40.5 kg would be included in 35.5 - 40.5 and 40.5 - 45.5, respectively. Now, with these assumptions, the new frequency distribution table will be as shown below:

Table 14.4

Weights (in kg)	Number of students
30.5-35.5	9
35.5-40.5	6
40.5-45.5	15
45.5-50.5	3
50.5-55.5	1
55.5-60.5	2
60.5-65.5	2
65.5-70.5	1
70.5-75.5	1
Total	40

Now, let us move to the data collected by you in Activity 1. This time we ask you to present these as frequency distribution tables.

Activity 2 : Continuing with the same four groups, change your data to frequency distribution tables.Choose convenient classes with suitable class-sizes, keeping in mind the range of the data and the type of data.

EXERCISE 14.2

1. The blood groups of 30 students of Class VIII are recorded as follows:

   A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O,

   A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O.

   Represent this data in the form of a frequency distribution table. Which is the most common, and which is the rarest, blood group among these students?

2. The distance (in km) of 40 engineers from their residence to their place of work were found as follows:

5	3	10	20	25	11	13	7	12	31
19	10	12	17	18	11	32	17	16	2
7	9	7	8	3	5	12	15	18	3
12	14	2	9	6	15	15	7	6	12

   Construct a grouped frequency distribution table with class size 5 for the data given above taking the first interval as 0-5 (5 not included). What main features do you observe from this tabular representation?

3. The relative humidity (in %) of a certain city for a month of 30 days was as follows:

98.1	98.6	99.2	90.3	86.5	95.3	92.9	96.3	94.2	95.1
89.2	92.3	97.1	93.5	92.7	95.1	97.2	93.3	95.2	97.3
96.2	92.1	84.9	90.2	95.7	98.3	97.3	96.1	92.1	89

   (i) Construct a grouped frequency distribution table with classes 84 - 86, 86 - 88, etc.

   (ii) Which month or season do you think this data is about?

   (iii) What is the range of this data?

4. The heights of 50 students, measured to the nearest centimetres, have been found to be as follows:

161	150	154	165	168	161	154	162	150	151
162	164	171	165	158	154	156	172	160	170
153	159	161	170	162	165	166	168	165	164
154	152	153	156	158	162	160	161	173	166
161	159	162	167	168	159	158	153	154	159

   (i) Represent the data given above by a grouped frequency distribution table, taking the class intervals as 160 - 165, 165 - 170, etc.

   (ii) What can you conclude about their heights from the table?

5. A study was conducted to find out the concentration of sulphur dioxide in the air in

parts per million (ppm) of a certain city. The data obtained for 30 days is as follows:

0.03	0.08	0.08	0.09	0.04	0.17
0.16	0.05	0.02	0.06	0.18	0.20
0.11	0.08	0.12	0.13	0.22	0.07
0.08	0.01	0.10	0.06	0.09	0.18
0.11	0.07	0.05	0.07	0.01	0.04

(i) Make a grouped frequency distribution table for this data with class intervals as 0.00 - 0.04, 0.04 - 0.08, and so on.

(ii) For how many days, was the concentration of sulphur dioxide more than 0.11 parts per million?

6. Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted down as follows:

0	1	2	2	1	2	3	1	3	0
1	3	1	1	2	2	0	1	2	1
3	0	0	1	1	2	3	2	2	0

Prepare a frequency distribution table for the data given above.

7. The value of π upto 50 decimal places is given below:

3.14159265358979323846264338327950288419716939937510

(i) Make a frequency distribution of the digits from 0 to 9 after the decimal point.

(ii) What are the most and the least frequently occurring digits?

8. Thirty children were asked about the number of hours they watched TV programmes in the previous week. The results were found as follows:

1	6	2	3	5	12	5	8	4	8
10	3	4	12	2	8	15	1	17	6
3	2	8	5	9	6	8	7	14	12

(i) Make a grouped frequency distribution table for this data, taking class width 5 and one of the class intervals as 5 - 10.

(ii) How many children watched television for 15 or more hours a week?

9. A company manufactures car batteries of a particular type. The lives (in years) of 40 such batteries were recorded as follows:

2.6	3.0	3.7	3.2	2.2	4.1	3.5	4.5
3.5	2.3	3.2	3.4	3.8	3.2	4.6	3.7
2.5	4.4	3.4	3.3	2.9	3.0	4.3	2.8
3.5	3.2	3.9	3.2	3.2	3.1	3.7	3.4
4.6	3.8	3.2	2.6	3.5	4.2	2.9	3.6

Construct a grouped frequency distribution table for this data, using class intervals of size 0.5 starting from the interval 2 - 2.5.

14.5 Measures of Central Tendency

Earlier in this chapter, we represented the data in various forms through frequency distribution tables, bar graphs, histograms and frequency polygons. Now, the question arises if we always need to study all the data to 'make sense' of it, or if we can make out some important features of it by considering only certain representatives of the data. This is possible, by using measures of central tendency or averages.

Consider a situation when two students Mary and Hari received their test copies. The test had five questions, each carrying ten marks. Their scores were as follows:

Question Numbers	1	2	3	4	5
Mary's score	10	8	9	8	7
Hari's score	4	7	10	10	10

Upon getting the test copies, both of them found their average scores as follows:

$$\text{Mary's average score} = \frac{42}{5} = 8.4$$

$$\text{Hari's average score} = \frac{41}{5} = 8.2$$

Since Mary's average score was more than Hari's, Mary claimed to have performed better than Hari, but Hari did not agree. He arranged both their scores in ascending order and found out the middle score as given below:

Mary's Score	7	8	(8)	9	10
Hari's Score	4	7	(10)	10	10

Hari said that since his middle-most score was 10, which was higher than Mary's middle-most score, that is 8, his performance should be rated better.

But Mary was not convinced. To convince Mary, Hari tried out another strategy. He said he had scored 10 marks more often (3 times) as compared to Mary who scored 10 marks only once. So, his performance was better.

Now, to settle the dispute between Hari and Mary, let us see the three measures they adopted to make their point.

The average score that Mary found in the first case is the *mean*. The 'middle' score that Hari was using for his argument is the *median*. The most often scored mark that Hari used in his second strategy is the *mode*.

Now, let us first look at the mean in detail.

The **mean** (or **average**) of a number of observations is the sum of the values of all the observations divided by the total number of observations.

It is denoted by the symbol $\overline{x}$, read as 'x bar'.

Let us consider an example.

Example 10 : 5 people were asked about the time in a week they spend in doing social work in their community. They said 10, 7, 13, 20 and 15 hours, respectively.

Find the mean (or average) time in a week devoted by them for social work.

Solution : We have already studied in our earlier classes that the mean of a certain number of observations is equal to $\dfrac{\text{Sum of all the observations}}{\text{Total number of observations}}$. To simplify our working of finding the mean, let us use a variable x_i to denote the ith observation. In this case, i can take the values from 1 to 5. So our first observation is x_1, second observation is x_2, and so on till x_5.

Also $x_1 = 10$ means that the value of the first observation, denoted by x_1, is 10. Similarly, $x_2 = 7$, $x_3 = 13$, $x_4 = 20$ and $x_5 = 15$.

Therefore, the mean $\overline{x} = \dfrac{\text{Sum of all the observations}}{\text{Total number of observations}}$

$$= \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$= \frac{10 + 7 + 13 + 20 + 15}{5} = \frac{65}{5} = 13$$

So, the mean time spent by these 5 people in doing social work is 13 hours in a week.

Now, in case we are finding the mean time spent by 30 people in doing social work, writing $x_1 + x_2 + x_3 + \ldots + x_{30}$ would be a tedious job. We use the Greek symbol Σ (for the letter Sigma) for *summation*. Instead of writing $x_1 + x_2 + x_3 + \ldots + x_{30}$, we write $\sum_{i=1}^{30} x_i$, which is read as 'the sum of x_i *as* i varies from 1 to 30'.

So, $$\overline{x} = \frac{\sum_{i=1}^{30} x_i}{30}$$

Similarly, for n observations $$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Example 11 : Find the mean of the marks obtained by 30 students of Class IX of a school, given in Example 2.

Solution : Now, $$\overline{x} = \frac{x_1 + x_2 + \cdots + x_{30}}{30}$$

$$\sum_{i=1}^{30} x_i = 10 + 20 + 36 + 92 + 95 + 40 + 50 + 56 + 60 + 70 + 92 + 88$$
$$80 + 70 + 72 + 70 + 36 + 40 + 36 + 40 + 92 + 40 + 50 + 50$$
$$56 + 60 + 70 + 60 + 60 + 88 = 1779$$

So, $$\overline{x} = \frac{1779}{30} = 59.3$$

Is the process not time consuming? Can we simplify it? Note that we have formed a frequency table for this data (see Table 14.1).

The table shows that 1 student obtained 10 marks, 1 student obtained 20 marks, 3 students obtained 36 marks, 4 students obtained 40 marks, 3 students obtained 50 marks, 2 students obtained 56 marks, 4 students obtained 60 marks, 4 students obtained 70 marks, 1 student obtained 72 marks, 1 student obtained 80 marks, 2 students obtained 88 marks, 3 students obtained 92 marks and 1 student obtained 95 marks.

So, the total marks obtained = $(1 \times 10) + (1 \times 20) + (3 \times 36) + (4 \times 40) + (3 \times 50) + (2 \times 56) + (4 \times 60) + (4 \times 70) + (1 \times 72) + (1 \times 80) + (2 \times 88) + (3 \times 92) + (1 \times 95)$

$= f_1x_1 + \ldots + f_{13}x_{13}$, where f_i is the frequency of the ith entry inTable 14.1.

In brief, we write this as $\sum_{i=1}^{13} f_i x_i$.

So, the total marks obtained $= \sum_{i=1}^{13} f_i x_i$

$= 10 + 20 + 108 + 160 + 150 + 112 + 240 + 280 + 72 + 80 + 176 + 276 + 95$

$= 1779$

Now, the total number of observations

$= \sum_{i=1}^{13} f_i$

$= f_1 + f_2 + \ldots + f_{13}$

$= 1 + 1 + 3 + 4 + 3 + 2 + 4 + 4 + 1 + 1 + 2 + 3 + 1$

$= 30$

So, the mean $\bar{x} = \dfrac{\text{Sum of all the observations}}{\text{Total number of observations}} = \left(\dfrac{\sum_{i=1}^{13} f_i x_i}{\sum_{i=1}^{13} f_i} \right)$

$= \dfrac{1779}{30} = 59.3$

This process can be displayed in the following table, which is a modified form of Table 14.1.

Table 14.12

Marks (x_i)	Number of students (f_i)	$f_i x_i$
10	1	10
20	1	20
36	3	108
40	4	160
50	3	150
56	2	112
60	4	240
70	4	280
72	1	72
80	1	80
88	2	176
92	3	276
95	1	95
	$\sum_{i=1}^{13} f_i = 30$	$\sum_{i=1}^{13} f_i x_i = 1779$

Thus, in the case of an ungrouped frequency distribution, you can use the formula

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

for calculating the mean.

Let us now move back to the situation of the argument between Hari and Mary, and consider the second case where Hari found his performance better by finding the middle-most score. As already stated, this measure of central tendency is called the *median*.

The **median** is that value of the given number of observations, which divides it into exactly two parts. So, when the data is arranged in ascending (or descending) order the median of ungrouped data is calculated as follows:

(i) When the number of observations (n) is odd, the median is the value of the $\left(\frac{n+1}{2}\right)^{\text{th}}$ observation. For example, if $n = 13$, the value of the $\left(\frac{13+1}{2}\right)^{\text{th}}$, i.e., the 7th observation will be the median [see Fig. 14.9 (i)].

(ii) When the number of observations (n) is even, the median is the mean of the $\left(\frac{n}{2}\right)^{\text{th}}$ and the $\left(\frac{n}{2}+1\right)^{\text{th}}$ observations. For example, if $n = 16$, the mean of the values of the $\left(\frac{16}{2}\right)^{\text{th}}$ and the $\left(\frac{16}{2}+1\right)^{\text{th}}$ observations, i.e., the mean of the values of the 8th and 9th observations will be the median [see Fig. 14.9 (ii)].

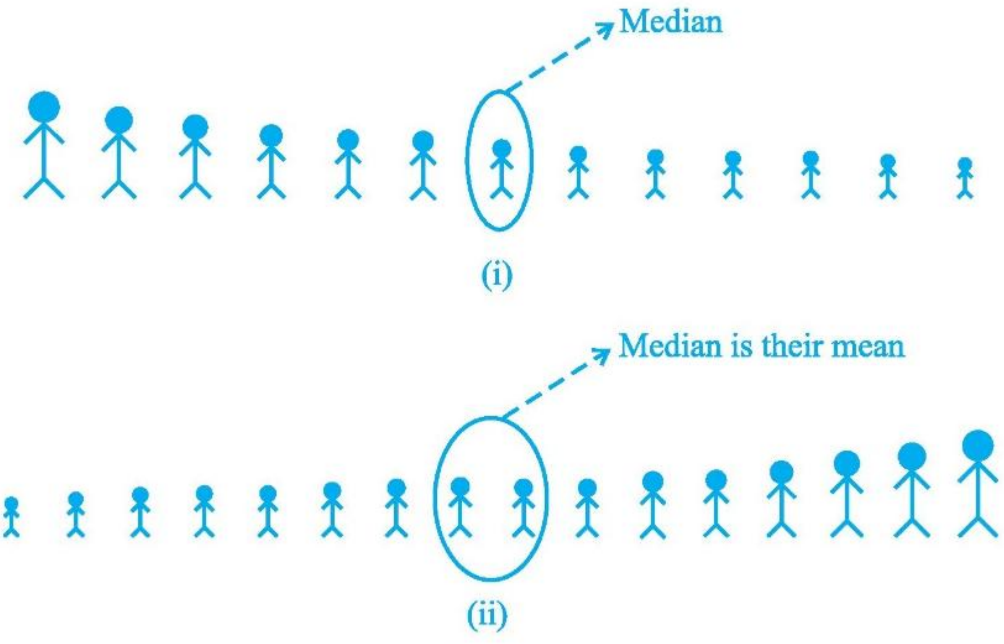


Fig. 14.9

Let us illustrate this with the help of some examples.

Example 12 : The heights (in cm) of 9 students of a class are as follows:

155 160 145 149 150 147 152 144 148

Find the median of this data.

Solution : First of all we arrange the data in ascending order, as follows:

144 145 147 148 149 150 152 155 160

Since the number of students is 9, an odd number, we find out the median by finding the height of the $\left(\frac{n+1}{2}\right)$th $= \left(\frac{9+1}{2}\right)$th = the 5th student, which is 149 cm.

So, the median, i.e., the medial height is 149 cm.

Example 13 : The points scored by a Kabaddi team in a series of matches are as follows:

17, 2, 7, 27, 15, 5, 14, 8, 10, 24, 48, 10, 8, 7, 18, 28

Find the median of the points scored by the team.

Solution : Arranging the points scored by the team in ascending order, we get

2, 5, 7, 7, 8, 8, 10, 10, 14, 15, 17, 18, 24, 27, 28, 48.

There are 16 terms. So there are two middle terms, i.e. the $\frac{16}{2}$th and $\left(\frac{16}{2}+1\right)$th, i.e., the 8th and 9th terms.

So, the median is the mean of the values of the 8th and 9th terms.

i.e, the median $= \frac{10+14}{2} = 12$

So, the medial point scored by the Kabaddi team is 12.

Let us again go back to the unsorted dispute of Hari and Mary.

The third measure used by Hari to find the average was the *mode*.

The **mode** is that value of the observation which occurs most frequently, i.e., an observation with the maximum frequency is called the mode.

The readymade garment and shoe industries make great use of this measure of central tendency. Using the knowledge of mode, these industries decide which size of the product should be produced in large numbers.

Let us illustrate this with the help of an example.

Example 14 : Find the mode of the following marks (out of 10) obtained by 20 students:

4, 6, 5, 9, 3, 2, 7, 7, 6, 5, 4, 9, 10, 10, 3, 4, 7, 6, 9, 9

Solution : We arrange this data in the following form :

2, 3, 3, 4, 4, 4, 5, 5, 6, 6, 6, 7, 7, 7, 9, 9, 9, 9, 10, 10

Here 9 occurs most frequently, i.e., four times. So, the mode is 9.

Example 15 : Consider a small unit of a factory where there are 5 employees : a supervisor and four labourers. The labourers draw a salary of ₹ 5,000 per month each while the supervisor gets ₹ 15,000 per month. Calculate the mean, median and mode of the salaries of this unit of the factory.

Solution : Mean = $\frac{5000 + 5000 + 5000 + 5000 + 15000}{5} = \frac{35000}{5} = 7000$

So, the mean salary is ₹ 7000 per month.

To obtain the median, we arrange the salaries in ascending order:

5000, 5000, 5000, 5000, 15000

Since the number of employees in the factory is 5, the median is given by the $\left(\frac{5+1}{2}\right)$th $= \frac{6}{2}$th = 3rd observation. Therefore, the median is ₹ 5000 per month.

To find the mode of the salaries, i.e., the modal salary, we see that 5000 occurs the maximum number of times in the data 5000, 5000, 5000, 5000, 15000. So, the modal salary is ₹ 5000 per month.

Now compare the three measures of central tendency for the given data in the example above. You can see that the mean salary of ₹ 7000 does not give even an approximate estimate of any one of their wages, while the medial and modal salaries of ₹ 5000 represents the data more effectively.

Extreme values in the data affect the mean. This is one of the weaknesses of the mean. So, if the data has a few points which are very far from most of the other points, (like 1,7,8,9,9) then the mean is not a good representative of this data. Since the median and mode are not affected by extreme values present in the data, they give a better estimate of the average in such a situation.

Again let us go back to the situation of Hari and Mary, and compare the three measures of central tendency.

Measures of central tendency	Hari	Mary
Mean	8.2	8.4
Median	10	8
Mode	10	8

This comparison helps us in stating that these measures of central tendency are not sufficient for concluding which student is better. We require some more information to conclude this, which you will study about in the higher classes.

EXERCISE 14.4

1. The following number of goals were scored by a team in a series of 10 matches:

   2, 3, 4, 5, 0, 1, 3, 3, 4, 3

   Find the mean, median and mode of these scores.

2. In a mathematics test given to 15 students, the following marks (out of 100) are recorded:

   41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60

   Find the mean, median and mode of this data.

3. The following observations have been arranged in ascending order. If the median of the data is 63, find the value of x.

   29, 32, 48, 50, x, $x+2$, 72, 78, 84, 95

4. Find the mode of 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18.

5. Find the mean salary of 60 workers of a factory from the following table:

Salary (in ₹)	Number of workers
3000	16
4000	12
5000	10
6000	8
7000	6
8000	4
9000	3
10000	1
Total	60

6. Give one example of a situation in which

   (i) the mean is an appropriate measure of central tendency.

   (ii) the mean is not an appropriate measure of central tendency but the median is an appropriate measure of central tendency.

CHAPTER 15

PROBABILITY

It is remarkable that a science, which began with the consideration of games of chance, should be elevated to the rank of the most important subject of human knowledge. ***—Pierre Simon Laplace***

15.1 Introduction

In everyday life, we come across statements such as

(1) It will **probably** rain today.

(2) I **doubt** that he will pass the test.

(3) **Most probably**, Kavita will stand first in the annual examination.

(4) **Chances** are high that the prices of diesel will go up.

(5) There is a 50-50 **chance** of India winning a toss in today's match.

The words 'probably', 'doubt', 'most probably', 'chances', etc., used in the statements above involve an element of uncertainty. For example, in (1), 'probably rain' will mean it may rain or may not rain today. We are predicting rain today based on our past experience when it rained under similar conditions. Similar predictions are also made in other cases listed in (2) to (5).

The uncertainty of 'probably' etc can be measured numerically by means of 'probability' in many cases.

Though probability started with gambling, it has been used extensively in the fields of Physical Sciences, Commerce, Biological Sciences, Medical Sciences, Weather Forecasting, etc.

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15.2 Probability – an Experimental Approach

In earlier classes, you have had a glimpse of probability when you performed experiments like tossing of coins, throwing of dice, etc., and observed their *outcomes*. You will now learn to measure the chance of occurrence of a particular *outcome* in an experiment.

Blaise Pascal (1623–1662)
Fig. 15.1

The concept of probability developed in a very strange manner. In 1654, a gambler Chevalier de Mere, approached the well-known 17th century French philosopher and mathematician Blaise Pascal regarding certain dice problems. Pascal became interested in these problems, studied them and discussed them with another French mathematician, Pierre de Fermat. Both Pascal and Fermat solved the problems independently. This work was the beginning of Probability Theory.

Pierre de Fermat (1601–1665)
Fig. 15.2

The first book on the subject was written by the Italian mathematician, J.Cardan (1501–1576). The title of the book was 'Book on Games of Chance' (Liber de Ludo Aleae), published in 1663. Notable contributions were also made by mathematicians J. Bernoulli (1654–1705), P. Laplace (1749–1827), A.A. Markov (1856–1922) and A.N. Kolmogorov (born 1903).

Activity 1 : (i) Take any coin, toss it ten times and note down the number of times a head and a tail come up. Record your observations in the form of the following table

Table 15.1

Number of times the coin is tossed	Number of times head comes up	Number of times tail comes up
10	—	—

Write down the values of the following fractions:

$$\frac{\text{Number of times a head comes up}}{\text{Total number of times the coin is tossed}}$$

and

$$\frac{\text{Number of times a tail comes up}}{\text{Total number of times the coin is tossed}}$$

(ii) Toss the coin twenty times and in the same way record your observations as above. Again find the values of the fractions given above for this collection of observations.

(iii) Repeat the same experiment by increasing the number of tosses and record the number of heads and tails. Then find the values of the corresponding fractions.

You will find that as the number of tosses gets larger, the values of the fractions come closer to 0.5. To record what happens in more and more tosses, the following group activity can also be performed:

Acitivity 2 : Divide the class into groups of 2 or 3 students. Let a student in each group toss a coin 15 times. Another student in each group should record the observations regarding heads and tails. [Note that coins of the same denomination should be used in all the groups. It will be treated as if only one coin has been tossed by all the groups.]

Now, on the blackboard, make a table like Table 15.2. First, Group 1 can write down its observations and calculate the resulting fractions. Then Group 2 can write down its observations, but will calculate the fractions for the combined data of Groups 1 and 2, and so on. (We may call these fractions as *cumulative fractions*.) We have noted the first three rows based on the observations given by one class of students.

Table 15.2

Group (1)	Number of heads (2)	Number of tails (3)	$\frac{\text{Cumulative number of heads}}{\text{Total number of times the coin is tossed}}$ (4)	$\frac{\text{Cumulative number of tails}}{\text{Total number of times the coin is tossed}}$ (5)
1	3	12	$\frac{3}{15}$	$\frac{12}{15}$
2	7	8	$\frac{7+3}{15+15}=\frac{10}{30}$	$\frac{8+12}{15+15}=\frac{20}{30}$
3	7	8	$\frac{7+10}{15+30}=\frac{17}{45}$	$\frac{8+20}{15+30}=\frac{28}{45}$
4	⋮	⋮	⋮	⋮

What do you observe in the table? You will find that as the total number of tosses of the coin increases, the values of the fractions in Columns (4) and (5) come nearer and nearer to 0.5.

Activity 3 : (i) Throw a die* 20 times and note down the number of times the numbers

*A die is a well balanced cube with its six faces marked with numbers from 1 to 6, one number on one face. Sometimes dots appear in place of numbers.

`

1, 2, 3, 4, 5, 6 come up. Record your observations in the form of a table, as in Table 15.3:

Table 15.3

Number of times a die is thrown	Number of times these scores turn up					
	1	2	3	4	5	6
20						

Find the values of the following fractions:

$$\frac{\text{Number of times 1 turned up}}{\text{Total number of times the die is thrown}}$$

$$\frac{\text{Number of times 2 turned up}}{\text{Total number of times the die is thrown}}$$

⋮

⋮

$$\frac{\text{Number of times 6 turned up}}{\text{Total number of times the die is thrown}}$$

(ii) Now throw the die 40 times, record the observations and calculate the fractions as done in (i).

As the number of throws of the die increases, you will find that the value of each fraction calculated in (i) and (ii) comes closer and closer to $\frac{1}{6}$.

To see this, you could perform a group activity, as done in Activity 2. Divide the students in your class, into small groups. One student in each group should throw a die ten times. Observations should be noted and cumulative fractions should be calculated.

The values of the fractions for the number 1 can be recorded in Table 15.4. This table can be extended to write down fractions for the other numbers also or other tables of the same kind can be created for the other numbers.

Table 15.4

Group (1)	Total number of times a die is thrown in a group (2)	$\frac{\text{Cumulative number of times 1 turned up}}{\text{Total number of times the die is thrown}}$ (3)
1	—	—
2	—	—
3	—	—
4	—	—

The dice used in all the groups should be almost the same in size and appearence. Then all the throws will be treated as throws of the same die.

What do you observe in these tables?

You will find that as the total number of throws gets larger, the fractions in Column (3) move closer and closer to $\frac{1}{6}$.

Activity 4 : (i) Toss two coins simultaneously ten times and record your observations in the form of a table as given below:

Table 15.5

Number of times the two coins are tossed	Number of times no head comes up	Number of times one head comes up	Number of times two heads come up
10	—	—	—

Write down the fractions:

$$A = \frac{\text{Number of times no head comes up}}{\text{Total number of times two coins are tossed}}$$

$$B = \frac{\text{Number of times one head comes up}}{\text{Total number of times two coins are tossed}}$$

$$C = \frac{\text{Number of times two heads come up}}{\text{Total number of times two coins are tossed}}$$

Calculate the values of these fractions.

Now increase the number of tosses (as in Activitiy 2). You will find that the more the number of tosses, the closer are the values of A, B and C to 0.25, 0.5 and 0.25, respectively.

In Activity 1, each toss of a coin is called a *trial*. Similarly in Activity 3, each throw of a die is a *trial*, and each simultaneous toss of two coins in Activity 4 is also a *trial*.

So, *a trial is an action which results in one or several outcomes*. The possible outcomes in Activity 1 were Head and Tail; whereas in Activity 3, the possible outcomes were 1, 2, 3, 4, 5 and 6.

In Activity 1, the getting of a head in a particular throw is an *event with outcome 'head'*. Similarly, *getting a tail is an event with outcome 'tail'*. In Activity 2, the getting of a particular number, say 1, is an *event* with outcome 1.

If our experiment was to throw the die for getting an even number, then the event would consist of three outcomes, namely, 2, 4 and 6.

So, an *event* for an experiment is the collection of some outcomes of the experiment. In Class X, you will study a more formal definition of an event.

So, can you now tell what the events are in Activity 4?

With this background, let us now see what probability is. Based on what we directly observe as the outcomes of our trials, we find the *experimental* or *empirical* probability.

Let n be the total number of trials. The *empirical probability* P(E) of an event E happening, is given by

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{The total number of trials}}$$

In this chapter, we shall be finding the empirical probability, though we will write 'probability' for convenience.

Let us consider some examples.

To start with let us go back to Activity 2, and Table 15.2. In Column (4) of this table, what is the fraction that you calculated? Nothing, but it is the empirical probability of getting a head. Note that this probability kept changing depending on the number of trials and the number of heads obtained in these trials. Similarly, the empirical probability of getting a tail is obtained in Column (5) of Table 15.2. This is $\frac{12}{15}$ to start with, then it is $\frac{2}{3}$, then $\frac{28}{45}$, and so on.

So, the empirical probability depends on the number of trials undertaken, and the number of times the outcomes you are looking for coming up in these trials.

Activity 5 : Before going further, look at the tables you drew up while doing Activity 3. Find the probabilities of getting a 3 when throwing a die a certain number of times. Also, show how it changes as the number of trials increases.

Now let us consider some other examples.

Example 1 : A coin is tossed 1000 times with the following frequencies:

Head : 455, Tail : 545

Compute the probability for each event.

Solution : Since the coin is tossed 1000 times, the total number of trials is 1000. Let us call the events of getting a head and of getting a tail as E and F, respectively. Then, the number of times E happens, i.e., the number of times a head come up, is 455.

So, the probability of E = $\frac{\text{Number of heads}}{\text{Total number of trials}}$

i.e., $$P(E) = \frac{455}{1000} = 0.455$$

Similarly, the probability of the event of getting a tail = $\frac{\text{Number of tails}}{\text{Total number of trials}}$

i.e., $$P(F) = \frac{545}{1000} = 0.545$$

Note that in the example above, P(E) + P(F) = 0.455 + 0.545 = 1, and E and F are the only two possible outcomes of each trial.

Example 2 : Two coins are tossed simultaneously 500 times, and we get

Two heads : 105 times

One head : 275 times

No head : 120 times

Find the probability of occurrence of each of these events.

Solution : Let us denote the events of getting two heads, one head and no head by E_1, E_2 and E_3, respectively. So,

$$P(E_1) = \frac{105}{500} = 0.21$$

$$P(E_2) = \frac{275}{500} = 0.55$$

$$P(E_3) = \frac{120}{500} = 0.24$$

Observe that $P(E_1) + P(E_2) + P(E_3) = 1$. Also E_1, E_2 and E_3 cover all the outcomes of a trial.

Example 3 : A die is thrown 1000 times with the frequencies for the outcomes 1, 2, 3, 4, 5 and 6 as given in the following table :

Table 15.6

Outcome	1	2	3	4	5	6
Frequency	179	150	157	149	175	190

Find the probability of getting each outcome.

Solution : Let E_i denote the event of getting the outcome i, where $i = 1, 2, 3, 4, 5, 6$. Then

Probability of the outcome 1 = $P(E_1) = \dfrac{\text{Frequency of 1}}{\text{Total number of times the die is thrown}}$

$$= \frac{179}{1000} = 0.179$$

Similarly, $P(E_2) = \dfrac{150}{1000} = 0.15$, $P(E_3) = \dfrac{157}{1000} = 0.157$,

$P(E_4) = \dfrac{149}{1000} = 0.149$, $P(E_5) = \dfrac{175}{1000} = 0.175$

and $P(E_6) = \dfrac{190}{1000} = 0.19$.

Note that $P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) = 1$

Also note that:

(i) The probability of each event lies between 0 and 1.

(ii) The sum of all the probabilities is 1.

(iii) $E_1, E_2, \ldots, E_6$ cover all the possible outcomes of a trial.

Example 4 : On one page of a telephone directory, there were 200 telephone numbers. The frequency distribution of their unit place digit (for example, in the number 25828573, the unit place digit is 3) is given in Table 15.7 :

Table 15.7

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	22	26	22	22	20	10	14	28	16	20

Without looking at the page, the pencil is placed on one of these numbers, i.e., the number is chosen at *random*. What is the probability that the digit in its unit place is 6?

Solution : The probability of digit 6 being in the unit place

$$= \frac{\text{Frequency of } 6}{\text{Total number of selected telephone numbers}}$$

$$= \frac{14}{200} = 0.07$$

You can similarly obtain the empirical probabilities of the occurrence of the numbers having the other digits in the unit place.

Example 5 : The record of a weather station shows that out of the past 250 consecutive days, its weather forecasts were correct 175 times.

(i) What is the probability that on a given day it was correct?

(ii) What is the probability that it was not correct on a given day?

Solution : The total number of days for which the record is available = 250

(i) P(the forecast was correct on a given day)

$$= \frac{\text{Number of days when the forecast was correct}}{\text{Total number of days for which the record is available}}$$

$$= \frac{175}{250} = 0.7$$

(ii) The number of days when the forecast was not correct = 250 – 175 = 75

So, P(the forecast was not correct on a given day) = $\frac{75}{250} = 0.3$

Notice that:

P(forecast was correct on a given day) + P(forecast was not correct on a given day)

$$= 0.7 + 0.3 = 1$$

Example 6 : A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of 1000 cases.

Table 15.8

Distance (in km)	less than 4000	4000 to 9000	9001 to 14000	more than 14000
Frequency	20	210	325	445

If you buy a tyre of this company, what is the probability that :

(i) it will need to be replaced before it has covered 4000 km?

(ii) it will last more than 9000 km?

(iii) it will need to be replaced after it has covered somewhere between 4000 km and 14000 km?

Solution : (i) The total number of trials = 1000.

The frequency of a tyre that needs to be replaced before it covers 4000 km is 20.

So, P(tyre to be replaced before it covers 4000 km) = $\dfrac{20}{1000} = 0.02$

(ii) The frequency of a tyre that will last more than 9000 km is $325 + 445 = 770$

So, P(tyre will last more than 9000 km) = $\dfrac{770}{1000} = 0.77$

(iii) The frequency of a tyre that requires replacement between 4000 km and 14000 km is $210 + 325 = 535$.

So, P(tyre requiring replacement between 4000 km and 14000 km) = $\dfrac{535}{1000} = 0.535$

Example 7 : The percentage of marks obtained by a student in the monthly unit tests are given below:

Table 15.9

Unit test	I	II	III	IV	V
Percentage of marks obtained	69	71	73	68	74

Based on this data, find the probability that the student gets more than 70% marks in a unit test.

Solution : The total number of unit tests held is 5.

The number of unit tests in which the student obtained more than 70% marks is 3.

So, P(scoring more than 70% marks) = $\frac{3}{5}$ = 0.6

Example 8 : An insurance company selected 2000 drivers at random (i.e., without any preference of one driver over another) in a particular city to find a relationship between age and accidents. The data obtained are given in the following table:

Table 15.10

Age of drivers (in years)	Accidents in one year				
	0	**1**	**2**	**3**	**over 3**
18 - 29	440	160	110	61	35
30 - 50	505	125	60	22	18
Above 50	360	45	35	15	9

Find the probabilities of the following events for a driver chosen at random from the city:

(i) being 18-29 years of age *and* having exactly 3 accidents in one year.

(ii) being 30-50 years of age *and* having one or more accidents in a year.

(iii) having no accidents in one year.

Solution : Total number of drivers = 2000.

(i) The number of drivers who are 18-29 years old and have exactly 3 accidents in one year is 61.

So, P (driver is 18-29 years old with exactly 3 accidents) = $\frac{61}{2000}$

$= 0.0305 \approx 0.031$

(ii) The number of drivers 30-50 years of age and having one or more accidents in one year = 125 + 60 + 22 + 18 = 225

So, P(driver is 30-50 years of age and having one or more accidents)

$$= \frac{225}{2000} = 0.1125 \approx 0.113$$

(iii) The number of drivers having no accidents in one year = 440 + 505 + 360

= 1305

Therefore, P(drivers with no accident) = $\frac{1305}{2000}$ = 0.653

Example 9 : Consider the frequency distribution table (Table 14.3, Example 4, Chapter 14), which gives the weights of 38 students of a class.

(i) Find the probability that the weight of a student in the class lies in the interval 46-50 kg.

(ii) Give two events in this context, one having probability 0 and the other having probability 1.

Solution : (i) The total number of students is 38, and the number of students with weight in the interval 46 - 50 kg is 3.

So, P(weight of a student is in the interval 46 - 50 kg) = $\frac{3}{38}$ = 0.079

(ii) For instance, consider the event that a student weighs 30 kg. Since no student has this weight, the probability of occurrence of this event is 0. Similarly, the probability of a student weighing more than 30 kg is $\frac{38}{38}$ = 1.

Example 10 : Fifty seeds were selected at random from each of 5 bags of seeds, and were kept under standardised conditions favourable to germination. After 20 days, the number of seeds which had germinated in each collection were counted and recorded as follows:

Table 15.11

Bag	1	2	3	4	5
Number of seeds germinated	40	48	42	39	41

What is the probability of germination of

(i) more than 40 seeds in a bag?

(ii) 49 seeds in a bag?

(iii) more that 35 seeds in a bag?

Solution : Total number of bags is 5.

(i) Number of bags in which more than 40 seeds germinated out of 50 seeds is 3.

P(germination of more than 40 seeds in a bag) = $\frac{3}{5}$ = 0.6

(ii) Number of bags in which 49 seeds germinated = 0.

P(germination of 49 seeds in a bag) = $\frac{0}{5} = 0.$

(iii) Number of bags in which more than 35 seeds germinated = 5.

So, the required probability = $\frac{5}{5} = 1.$

Remark : In all the examples above, you would have noted that the probability of an event can be any fraction from 0 to 1.

EXERCISE 15.1

1. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.

2. 1500 families with 2 children were selected randomly, and the following data were recorded:

Number of girls in a family	2	1	0
Number of families	475	814	211

Compute the probability of a family, chosen at random, having

(i) 2 girls (ii) 1 girl (iii) No girl

Also check whether the sum of these probabilities is 1.

3. Refer to Example 5, Section 14.4, Chapter 14. Find the probability that a student of the class was born in August.

4. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcome	3 heads	2 heads	1 head	No head
Frequency	23	72	77	28

If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.

5. An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The

information gathered is listed in the table below:

Monthly income (in ₹)	Vehicles per family			
	0	1	2	Above 2
Less than 7000	10	160	25	0
7000 – 10000	0	305	27	2
10000 – 13000	1	535	29	1
13000 – 16000	2	469	59	25
16000 or more	1	579	82	88

Suppose a family is chosen. Find the probability that the family chosen is

(i) earning ₹ 10000 – 13000 per month and owning exactly 2 vehicles.

(ii) earning ₹ 16000 or more per month and owning exactly 1 vehicle.

(iii) earning less than ₹ 7000 per month and does not own any vehicle.

(iv) earning ₹ 13000 – 16000 per month and owning more than 2 vehicles.

(v) owning not more than 1 vehicle.

6. Refer to Table 14.7, Chapter 14.

(i) Find the probability that a student obtained less than 20% in the mathematics test.

(ii) Find the probability that a student obtained marks 60 or above.

7. To know the opinion of the students about the subject *statistics*, a survey of 200 students was conducted. The data is recorded in the following table.

Opinion	Number of students
like	135
dislike	65

Find the probability that a student chosen at random

(i) likes statistics, (ii) does not like it.

8. Refer to Q.2, Exercise 14.2. What is the empirical probability that an engineer lives:

(i) less than 7 km from her place of work?

(ii) more than or equal to 7 km from her place of work?

(iii) within $\frac{1}{2}$ km from her place of work?

9. **Activity :** Note the frequency of two-wheelers, three-wheelers and four-wheelers going past during a time interval, in front of your school gate. Find the probability that any one vehicle out of the total vehicles you have observed is a two-wheeler.

10. **Activity :** Ask all the students in your class to write a 3-digit number. Choose any student from the room at random. What is the probability that the number written by her/him is divisible by 3? Remember that a number is divisible by 3, if the sum of its digits is divisible by 3.

11. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg):

    4.97 5.05 5.08 5.03 5.00 5.06 5.08 4.98 5.04 5.07 5.00

    Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

12. In Q.5, Exercise 14.2, you were asked to prepare a frequency distribution table, regarding the concentration of sulphur dioxide in the air in parts per million of a certain city for 30 days. Using this table, find the probability of the concentration of sulphur dioxide in the interval 0.12 - 0.16 on any of these days.

13. In Q.1, Exercise 14.2, you were asked to prepare a frequency distribution table regarding the blood groups of 30 students of a class. Use this table to determine the probability that a student of this class, selected at random, has blood group AB.

15.3 Summary

In this chapter, you have studied the following points:

1. An event for an experiment is the collection of some outcomes of the experiment.
2. The empirical (or experimental) probability P(E) of an event E is given by

$$P(E) = \frac{\text{Number of trials in which E has happened}}{\text{Total number of trials}}$$

3. The Probability of an event lies between 0 and 1 (0 and 1 inclusive).