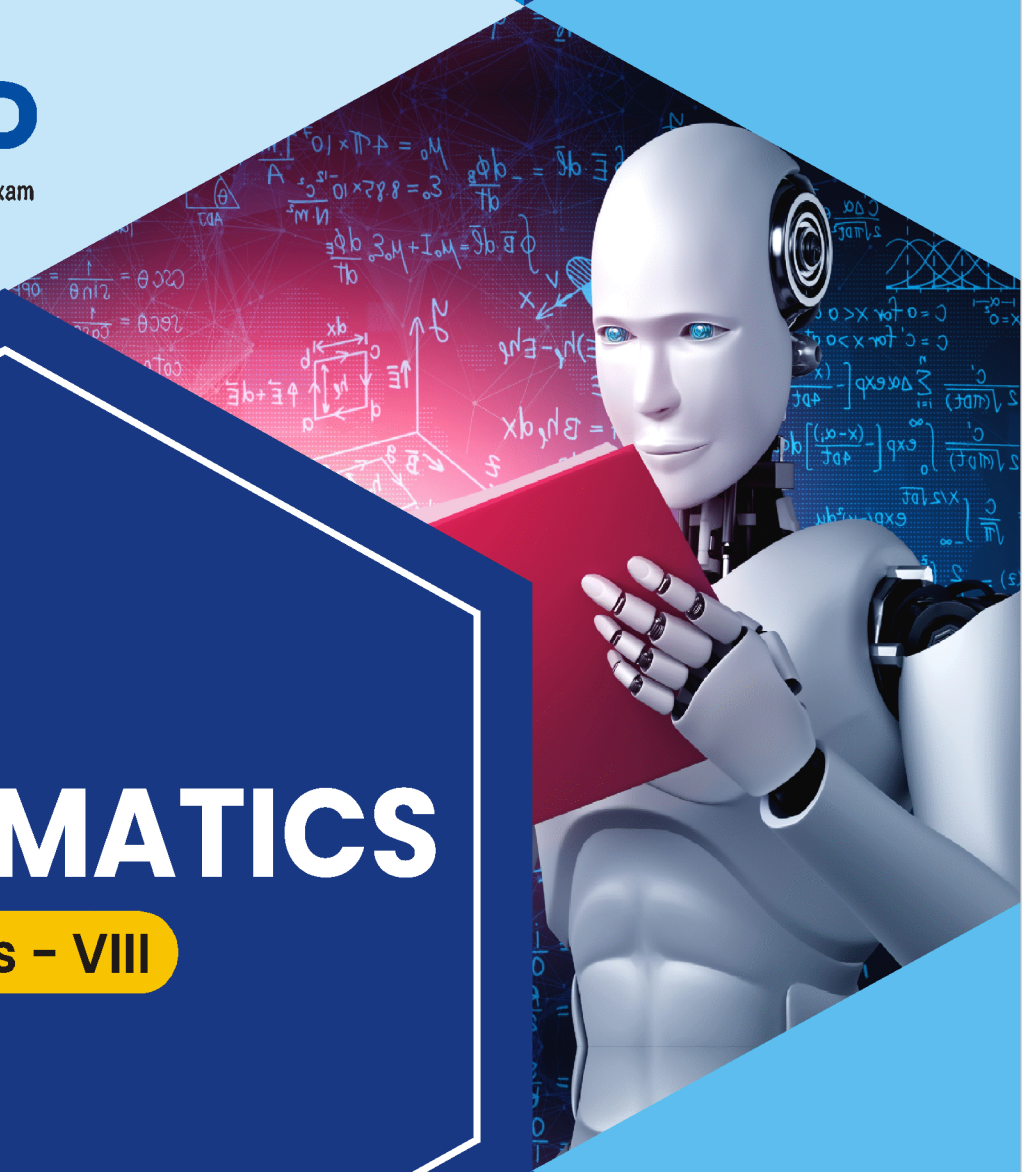
MATRIX OLYMPIAD

The Most Innovative Talent Recognition Exam



MATHEMATICS

Class - VIII

MATRIX

Campus : Piprali Road, Sikar, Rajasthan 332001

Phone : 01572-241911, 01572-243911

Website: www.matrixedu.in

Few words for the Readers

Dear Reader,

"*Matrix Olympiad is established to encourage school students to go a step further than their regular studies, and get a chance and exposure to competition on a wide scale. It also helps students enhance their learning of basic cognitive skills and deeper knowledge of subjects like Science, Mathematics, English, Mental Ability, Social Studies.“Matrix Olympiad helps students nurture their minds for higher targets of tomorrow and enables them to study School for JEE, NEET, CLAT, NDA, Olympiads , NSEJS, NTSE , STSE etc.*"

The above thought has been our guiding principle while designing and collating the study material for **Matrix Olympiad** . And hence, we hope that this particular material will be helpful towards your preparation for **Matrix Olympiad**.

Our team at **MATRIX** has put in their best efforts for making this particular module interesting and relevant for you. Additional efforts have been made to ensure that the content is easy to understand and error free to the extent possible. However, there might remain some inadvertent errors in answer keys and theoretical portion and we would welcome your valuable feedback regarding the same.
If there are any suggestions for corrections, please write to us at **smd@matrixacademy.co.in** and we would be highly grateful.

Finally, we would like to end this message by a famous quote by Ernest Hemingway - "*There is no friend as loyal as a book*." So, please give your study material the time and attention it deserves, and it will surely help you reach newer heights in your fight with competition examinations.

With love and best wishes !
Team MATRIX

CONTENTS

RATIONAL NUMBERS 1

Concepts

INTRODUCTION

In Mathematics, we frequently come across simple equations to be solved. For example,

the equation $x + 2 = 13$ (1)

is solved when $x = 11$, because this value of x satisfies the given equation. The solution 11 is a natural number. On the other hand, for the equation

$x + 5 = 5$ (2)

the solution gives the whole number 0 (zero). If we consider only natural numbers, equation (2) cannot be solved. To solve equations like (2), we added the number zero to the collection of natural numbers and obtained the whole numbers. Even whole numbers will not be sufficient to solve equations of type

$x + 18 = 5$ (3)

Do you see 'why'? We require the number –13 which is not a whole number. This led us to think of integers, (positive and negative). Note that the positive integers correspond to natural numbers. One may think that we have enough numbers to solve all simple equations with the available list of integers. Now consider the equations

$2x = 3$ (4)

$5x + 7 = 0$ (5)

for which we cannot find a solution from the integers.(Check this)

We need the numbers $\frac{3}{2}$ to solve equation (4) and $\frac{-7}{5}$ to solve equation (5).

This leads us to the collection of **rational numbers.**

We have already seen basic operations on rational numbers.We now try to explore some properties of operations on the different types of numbers seen so far.

1. NATURAL NUMBERS

The numbers from 1 onwards i.e., 1, 2, 3 and so on are called natural numbers. The set of natural number is denoted by N.

1.1 PROPERTIES OF NATURAL NUMBERS

(i) For every natural number, there is a successor.

(ii) 1 is the smallest natural number.

(iii) There are finite number of natural numbers between two natural numbers.

(iv) Every natural number (except 1) can be obtained by adding 1 to the previous natural number.

(v) For the natural number 1, there is no 'Previous' natural number.

(vi) There are infinite number of natural numbers.

1.2 LAWS OF NATURAL NUMBERS

If a, b and c are three natural numbers :

1. Closure Law :

Addition	a + b is a natural number.
Subtraction	If a > b, then a – b is a natural number.
Multiplication	a × b is a natural number.
Division	a ÷ b need not be a natural number.

Example 1

4, 3 are natural numbers belongs to N, $4 + 3 = 7 \in N$, $4 \times 3 = 12 \in N$, $3 - 4 \notin N$, $3/4 \notin N$.

2. Commutative Law :

Addition	a + b = b + a	Commutative
Subtraction	a – b ≠ b – a	Not commutative
Multiplication	ab = ba	Commutative
Division	a ÷ b ≠ b ÷ a	Not commutative

Example 2

5 and 6 are natural numbers, $5 + 6 = 6 + 5 = 11$, $5 - 6 \neq 6 - 5$, $5 \times 6 = 6 \times 5 = 30$, $\frac{5}{6} \neq \frac{6}{5}$.

3. Associative Law :

Addition	a + (b + c) = (a + b) + c	Associative
Subtraction	a – (b – c) ≠ (a – b) – c	Not associative
Multiplication	a × (b × c) = (a × b) × c	Associative
Division	a ÷ (b ÷ c) ≠ (a ÷ b) ÷ c	Not associative

Example 3

7, 8 and 9 are natural numbers, $(7 + 8) + 9 = 7 + (8 + 9) = 24$, $(7 - 8) - 9 \neq 7 - (8 - 9)$, $(7 \times 8) \times 9 = 7 \times (8 \times 9) = 504$, $(7 \div 8) \div 9 \neq 7 \div (8 \div 9)$

4. Distributive Law :

a × (b + c) = ab + ac (over addition)

a × (b – c) = ab – ac (over subtraction)

5. Multiplicative Identity :

a × 1 = 1 × a = a; '1' is called the multiplicative identity of 'a'.

2. WHOLE NUMBERS

The natural numbers along with zero forms the collection of whole numbers.

2.1 PROPERTIES OF WHOLE NUMBERS

(i) The number 0 is the first and the smallest whole number.

(ii) There is no last or greatest whole number.

(iii) There are infinitely many or uncountable number of whole numbers.

(iv) All natural numbers are whole numbers.

(v) All whole numbers are not natural numbers.

2.2 LAWS OF WHOLE NUMBERS

If a, b and c are three whole numbers :

1. Closure Law :

Addition	$a + b$ is a whole number.
Subtraction	If $a > b$, then $a - b$ is a whole number.
Multiplication	$a \times b$ is a whole number.
Division	$a \div b$ need not be a whole number.

Example 1

(i) $0 + 5 = 5$, 5 is a whole number.

(ii) $0 - 8 = -8$, -8 is not a whole number but $8 - 0 = 8$, 8 is whole number.

(iii) $0 \times 9 = 0$, 0 is a whole number.

(iv) $8 \div 11 = \frac{8}{11}, \frac{8}{11}$ is not a whole number.

2. Commutative Law :

Addition	$a + b = b + a$	Commutative
Subtraction	$a - b \neq b - a$	Not commutative
Multiplication	$a \times b = b \times a$	Commutative
Division	$a \div b \neq b \div a$	Not commutative

Example 2

(i) $5 + 3 = 3 + 5 = 8$

(ii) $3 - 5 \neq 5 - 3$

(iii) $7 \times 3 = 3 \times 7 = 21$

(iv) $\frac{3}{7} \neq \frac{7}{3}$

3. Associative Law :

Addition	$a + (b + c) = (a + b) + c$	Associative
Subtraction	$a - (b - c) \neq (a - b) - c$	Not associative
Multiplication	$a \times (b \times c) = (a \times b) \times c$	Associative
Division	$a \div (b \div c) \neq (a \div b) \div c$	Not associative

Example 3

(i) $5 + (3 + 8) = (5 + 3) + 8 = 16$.

(ii) $5 - (3 - 8) \neq (5 - 3) - 8 \Rightarrow 5 - (-5) \neq (2 - 8) \Rightarrow 10 \neq -6$.

(iii) $5 \times (7 \times 3) = (5 \times 7) \times 3 \Rightarrow 5 \times 21 = 35 \times 3 \Rightarrow 105 = 105$

(iv) $5 \div (7 \div 3) \neq (5 \div 7) \div 3$

4. Distributive Law :

$a \times (b + c) = ab + ac$ (over addition)

$a \times (b - c) = ab - ac$ (over subtraction)

5. Existence of Additive Identity :

$a + 0 = 0 + a = a$; '0' is called additive identity of 'a'.

6. Multiplicative Identity :

$a \times 1 = 1 \times a = a$; '1' is called the multiplicative identity of 'a'.

3. INTEGERS

The collection of whole numbers and negative of natural numbers together are called integers.

The collection 1, 2, 3, ... are said to be positive integers and – 1, – 2, – 3, ... are said to be negative integers.

3.1 PROPERTIES OF INTEGERS

(i) '0' is neither negative integer nor positive integer.

(ii) There is no smallest and greatest integer.

(iii) Negative integers are smaller than positive integers.

(iv) There are infinitely many or uncountable integers.

(v) All natural and whole numbers are integers.

(vi) All integers are neither natural nor whole numbers.

(vii) Multiplication of a positive and a negative integer is negative.

(viii) Multiplication of two positive integers is positive.

(ix) Multiplication of two negative integers is positive.

(x) Multiplication of odd number of negative integers is negative.

(xi) Multiplication of even number of negative integers is positive.

3.2 LAWS OF INTEGERS

If a, b and c are three integers :

1. Closure Law :

Addition	$a + b$ is an integer.
Subtraction	$a - b$ is an integer.
Multiplication	$a \times b$ is an integer.
Division	$a \div b$ need not be an integer.

Example 1

(i) $-7 + (-3) = -10$, an integer.

(ii) $4 + 3 = 7$, an integer.

(iii) $7 - 3 = 4$, an integer.

(iv) $5 - 10 = -5$, an integer.

(v) $-8 \times 5 = -40$, an integer.

(vi) $5 \div 8 = \frac{5}{8}$, which is not an integer.

2. Commutative Law :

Addition	$a + b = b + a$	Commutative
Subtraction	$a - b \neq b - a$	Not commutative
Multiplication	$a \times b = b \times a$	Commutative
Division	$a \div b \neq b \div a$	Not commutative

Example 2

(i) $5 + (-3) = (-3) + 5 = 2$

(ii) $5 - (-2) \neq (-2) - 5 \Rightarrow 7 \neq -7$.

(iii) $5 \times (-3) = (-3) \times 5 = -15$

(iv) $5 \div (-3) \neq (-3) \div 5$

3. Associative Law :

Addition	$a + (b + c) = (a + b) + c$	Associative
Subtraction	$a - (b - c) \neq (a - b) - c$	Not associative
Multiplication	$a \times (b \times c) = (a \times b) \times c$	Associative
Division	$a \div (b \div c) \neq (a \div b) \div c$	Not associative

Example 3

(i) $(-6) + [(-4) + (-3)] = [(-6) + (-4)] + (-3) \Rightarrow (-6) + (-7) = [(-10)] + (-3) \Rightarrow -13 = -13$.

(ii) $6 - (8 - 4) \neq (6 - 8) - 4 \Rightarrow 6 - 4 \neq (-2) - 4 \Rightarrow 2 \neq -6$.

(iii) $5 \times [(-2) \times (-3)] = [5 \times (-2)] \times (-3) \Rightarrow 5 \times 6 = (-10) \times (-3) \Rightarrow 30 = 30$.

(iv) $[(-10) \div 2] \div (-5) \neq (-10) \div [2 \div (-5)] \Rightarrow [(-5) \div (-5)] \neq (-10) \div (-0.4) \Rightarrow 1 \neq 25$.

4. Distributive Law :

$a \times (b + c) = ab + ac$ (over addition)

$a \times (b - c) = ab - ac$ (over subtraction)

5. Additive Identity :

$a + 0 = 0 + a = a$; '0' is called the additive identity of 'a'.

6. Additive Inverse :

$a + (-a) = (-a) + a = 0$; '$-a$' is called additive inverse of 'a'.

7. Multiplicative Identity :

$a \times 1 = 1 \times a = a$; '1' is called the multiplicative identity of 'a'.

4. RATIONAL NUMBERS

A number which can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a rational number.

Example 1

$\frac{-2}{3}, \frac{6}{7}$.

4.1 PROPERTIES OF RATIONAL NUMBERS

(i) All whole number, integers and natural numbers are rational numbers.

(ii) All rational numbers are not whole numbers, integers and natural numbers.

(iii) There are infinitely many rational numbers between two rational numbers.

Focus Point

Absolute Value of a Rational Number :

- Absolute value is denoted by the symbol '| |'.
- Absolute value of a positive number remains the same.
- Absolute value of a negative number is always positive.

Thus, for any rational number x, we write $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$.

4.2 LAWS OF RATIONAL NUMBERS

If a, b and c are three rational numbers :

1. Closure Law :

Addition	a + b is a rational number.
Subtraction	a – b is a rational number.
Multiplication	a × b is a rational number.
Division	a ÷ b need not be a rational number.

Example 1

(i) $\left(\frac{-5}{12}+\frac{-1}{4}\right)=\frac{\{-5+(-3)\}}{12}=\frac{-8}{12}=\frac{-2}{3}$, a rational number.

(ii) $\frac{-3}{4}+\frac{5}{6}=\frac{-9}{12}+\frac{10}{12}=\frac{-9+10}{12}=\frac{1}{12}$, a rational number.

(iii) $\frac{-27}{36}-\left(\frac{14}{36}\right)=\frac{-27-14}{36}=\frac{-41}{36}$, a rational number.

(iv) $\frac{-4}{5}\times\frac{-6}{11}=\frac{24}{55}$, a rational number.

(v) $\frac{2}{5}\div\frac{5}{3}=\frac{2}{5}\times\frac{3}{5}=\frac{6}{25}$, a rational number.

(vi) $\frac{2}{0}$ = not defined and hence, not a rational number.

2. Commutative Law :

Addition	$a + b = b + a$	Commutative
Subtraction	$a - b \neq b - a$	Not commutative
Multiplication	$a \times b = b \times a$	Commutative
Division	$a \div b \neq b \div a$	Not commutative

Example 2

(i) $\frac{5}{6}+\frac{-4}{9}=\frac{15}{18}+\frac{-8}{18}=\frac{15+(-8)}{18}=\frac{7}{18}$ and $\frac{-4}{9}+\frac{5}{6}=\frac{-8}{18}+\frac{15}{18}=\frac{-8+15}{18}=\frac{7}{18}$ $\therefore \frac{5}{6}+\frac{-4}{9}=\frac{-4}{9}+\frac{5}{6}$

(ii) $\frac{2}{3}-\frac{1}{6}=\frac{2}{3}+\frac{-1}{6}=\frac{2\times2+(-1)\times1}{6}=\frac{4-1}{6}=\frac{3}{6}=\frac{1}{2}$ and $\frac{1}{6}-\frac{2}{3}=\frac{1}{6}+\frac{-2}{3}=\frac{1+(-2)\times2}{6}=\frac{1-4}{6}=\frac{-3}{6}=\frac{-1}{2}$

$\therefore \frac{2}{3}-\frac{1}{6}\neq\frac{1}{6}-\frac{2}{3}$

(iii) $\frac{3}{4}\times\frac{5}{7}=\frac{3\times5}{4\times7}=\frac{15}{28}, \frac{5}{7}\times\frac{3}{4}=\frac{5\times3}{7\times4}=\frac{15}{28}$ $\therefore \frac{3}{4}\times\frac{5}{7}=\frac{5}{7}\times\frac{3}{4}$

(iv) $\frac{-5}{4}\div\frac{3}{7}=\frac{-5}{4}\times\frac{7}{3}=\frac{-35}{12}$ and $\frac{3}{7}\div\left(\frac{-5}{4}\right)=\frac{3}{7}\times\left(\frac{-4}{5}\right)=\frac{-12}{35}$ $\therefore \frac{-5}{4}\div\frac{3}{7}\neq\frac{3}{7}\div\left(\frac{-5}{4}\right)$

3. Associative Law :

Addition	$a + (b + c) = (a + b) + c$	Associative
Subtraction	$a - (b - c) \neq (a - b) - c$	Not associative
Multiplication	$a \times (b \times c) = (a \times b) \times c$	Associative
Division	$a \div (b \div c) \neq (a \div b) \div c$	Not associative

Example 3

(i) $\frac{-3}{4}+\left(\frac{5}{6}+\frac{-4}{9}\right)=\frac{-3}{4}+\left(\frac{15}{18}+\frac{-8}{18}\right)=\frac{-3}{4}+\left(\frac{15-8}{18}\right)=\frac{-3}{4}+\frac{7}{18}=\frac{-27}{36}+\frac{14}{36}=\frac{-27+14}{36}=\frac{-13}{36}$ and

$\left(\frac{-3}{4}+\frac{5}{6}\right)+\left(\frac{-4}{9}\right)=\left(\frac{-9}{12}+\frac{10}{12}\right)+\frac{-4}{9}=\frac{-9+10}{12}+\frac{-4}{9}=\frac{1}{12}+\frac{-4}{9}=\frac{3}{36}+\frac{-16}{36}=\frac{3+(-16)}{36}=\frac{-13}{36}$

$\therefore \frac{-3}{4}+\left(\frac{5}{6}+\frac{-4}{9}\right)=\left(\frac{-3}{4}+\frac{5}{6}\right)+\frac{-4}{9}$

(ii) $\frac{-2}{3}-\left(\frac{-4}{5}-\frac{1}{2}\right)=\frac{-2}{3}-\left(\frac{-8-5}{10}\right)\Rightarrow\frac{-2}{3}-\left(\frac{-13}{10}\right)=\frac{-2}{3}+\frac{13}{10}=\frac{-20+39}{30}=\frac{19}{30}$ and $\left[-\frac{2}{3}-\left(\frac{-4}{5}\right)\right]-\frac{1}{2}$

$=\left[\frac{-2}{3}+\frac{4}{5}\right]-\frac{1}{2}=\left(\frac{-10+12}{15}\right)-\frac{1}{2}=\left(\frac{2}{15}\right)-\frac{1}{2}=\frac{4-15}{30}=\frac{-11}{30}$ so, $\frac{-2}{3}-\left[\frac{-4}{5}-\frac{1}{2}\right]\neq\left[\frac{-2}{3}-\left(\frac{-4}{5}\right)\right]-\frac{1}{2}$

(iii) $\left(\frac{-5}{4}\right)\times\left(\frac{-6}{11}\times\frac{2}{7}\right)=\frac{-5}{4}\times\frac{-12}{77}=\frac{(-5)\times(-12)}{4\times77}=\frac{(-5)\times(-3)}{1\times77}=\frac{15}{77}$ and $\left(\frac{-5}{4}\times\frac{-6}{11}\right)\times\frac{2}{7}=\left\{\frac{(-5)\times(-6)}{4\times11}\right\}\times\frac{2}{7}$

$=\left\{\frac{(-5)\times(-3)}{2\times11}\right\}\times\frac{2}{7}=\frac{15}{22}\times\frac{2}{7}=\frac{15\times2}{22\times7}=\frac{15}{77}\therefore\frac{-5}{4}\times\left(\frac{-6}{11}\times\frac{2}{7}\right)=\left(\frac{-5}{4}\times\frac{-6}{11}\right)\times\frac{2}{7}$

(iv) $\left[\frac{1}{2}\div\left(\frac{-1}{3}\right)\right]\div\frac{2}{5}=\left[\frac{1}{2}\times(-3)\right]\div\frac{2}{5}=\frac{-3}{2}\div\frac{2}{5}=\frac{-3}{2}\times\frac{5}{2}=\frac{-15}{4}$ and $\frac{1}{2}\div\left[\frac{-1}{3}\div\frac{2}{5}\right]=\frac{1}{2}\div\left[\frac{-1}{3}\times\frac{5}{2}\right]$

$=\frac{1}{2}\div\left[\frac{-5}{6}\right]=\frac{1}{2}\times\left(\frac{6}{-5}\right)=\frac{-3}{5}\quad\therefore\left[\frac{1}{2}\div\left(\frac{-1}{3}\right)\right]\div\frac{2}{5}\neq\frac{1}{2}\div\left[\frac{-1}{3}\div\frac{2}{5}\right]$

4. Distributive Law :

a × (b + c) = ab + ac (over addition)

a × (b – c) = ab – ac (over subtraction)

Example 4

Consider any three rational number, say, $\frac{2}{3},\frac{-3}{5}$ and $\frac{7}{10}$. We have, $\frac{2}{3}\times\left(\frac{-3}{5}+\frac{7}{10}\right)=\frac{2\times(-3)}{3\times5}+$

$\frac{2\times7}{3\times10}=\frac{2\times(-1)}{1\times5}+\frac{1\times7}{3\times5}=\frac{-2}{5}+\frac{7}{15}=\frac{-2\times3+7}{15}=\frac{-6+7}{15}=\frac{1}{15}\quad\therefore\frac{2}{3}\times\left(\frac{-3}{5}+\frac{7}{10}\right)=\frac{2}{3}\times\left(\frac{-3}{5}\right)+\frac{2}{3}\times\frac{7}{10}$

5. Existence of Additive Identity (Role of 0) :

a + 0 = 0 + a = a; '0' is called the additive identity of 'a'.

Example 5

$\frac{2}{3}+0=\frac{2}{3}=0+\frac{2}{3}$

6. Existence of Additive Inverse (Negative of a Number) :

a + (– a) = (– a) + a = 0; '– a' is called additive inverse of 'a'.

Example 6

The additive inverse (negative) of $\frac{3}{5}$ is $-\frac{3}{5}$ and $-\frac{3}{5}$ is written as $\left(-\frac{3}{5}\right)$.

Therefore, $-\frac{3}{5}$ and $-\frac{3}{5}$ represent the same rational number, that is, $-\frac{3}{5}=-\frac{3}{5}$. or $-\left(\frac{-3}{5}\right)=\frac{3}{5}\Rightarrow-\left(-\frac{3}{5}\right)=\frac{3}{5}$

Similarly, we have $-\left(-\frac{5}{7}\right)=\frac{5}{7}, -\left(-\frac{15}{13}\right)=\frac{15}{13}, -\left(-\frac{7}{12}\right)=\frac{7}{12}$ and so on.

Note : The rational number which is equal to its negative is zero.

7. Multiplicative Identity (Role of 1) :

$a \times 1 = 1 \times a = a$; '1' is called multiplicative identity of 'a'.

8. Multiplicative Inverse (Reciprocal) :

$a \times \frac{1}{a} = \frac{1}{a} \times a = 1; \left(\frac{1}{a}\right)$ is called multiplicative inverse of 'a' or reciprocal of 'a'.

Example 7

$\frac{8}{21}$ should be multiplied by $\frac{21}{8}$ to get the product 1.

Similarly, $\frac{-5}{7}$ must be multiplied by $\frac{7}{-5}$ so as to get the product 1.

So, we say that $\frac{21}{8}$ is the reciprocal of $\frac{8}{21}$ and $\frac{7}{-5}$ is the reciprocal of $\frac{-5}{7}$.

A rational number $\frac{c}{d}$ is called the reciprocal or multiplicative inverse of another rational number $\frac{a}{b}$ if $\frac{a}{b} \times \frac{c}{d} = 1$.

Focus Point

- The reciprocal of 1 is 1 and the reciprocal of – 1 is – 1.

Also 1 and – 1 are the only rational numbers which are their own reciprocals.

- There is no rational number which when multiplied with 0, gives 1.

Therefore rational number 0 has no reciprocal or multiplicative inverse.

Example 1

Express each of the following as a rational number :

(i) $\frac{-2}{5}+\left(\frac{11}{5}+\frac{-3}{5}\right)$ (ii) $\left(\frac{-2}{5}+\frac{11}{5}\right)+\frac{-3}{5}$

What do you see ?

Solution :

(i) $\frac{-2}{5}+\left(\frac{11}{5}+\frac{-3}{5}\right)=\frac{-2}{5}+\left(\frac{11+(-3)}{5}\right)=\frac{-2}{5}+\frac{8}{5}=\frac{-2+8}{5}=\frac{6}{5}$

(ii) $\left(\frac{-2}{5}+\frac{11}{5}\right)+\frac{-3}{5}=\left(\frac{-2+11}{5}\right)+\frac{-3}{5}=\frac{9}{5}+\frac{-3}{5}=\frac{9+(-3)}{5}=\frac{6}{5}$

The answer obtained is same in both the cases. This verifies additive associativity of rational numbers.

Example 2

Simplify : $\frac{3}{8}+\frac{7}{2}+\frac{-3}{5}+\frac{9}{8}+\frac{-3}{2}+\frac{6}{5}$

Solution :

Re-arranging and grouping the number in pairs in such a way that each group contains a pair of rational numbers with a same denominator, we have.

$$\frac{3}{8}+\frac{7}{2}+\frac{-3}{5}+\frac{9}{8}+\frac{-3}{2}+\frac{6}{5}=\left(\frac{3}{8}+\frac{9}{8}\right)+\left(\frac{7}{2}+\frac{-3}{2}\right)+\left(\frac{-3}{5}+\frac{6}{5}\right)=\frac{3+9}{8}+\frac{7+(-3)}{2}+\frac{(-3)+6}{5}=\frac{12}{8}+\frac{4}{2}+\frac{3}{5}$$

$$=\frac{3}{2}+2+\frac{3}{5}=\frac{3\times5}{2\times5}+\frac{2\times10}{1\times10}+\frac{3\times2}{5\times2}=\frac{15}{10}+\frac{20}{10}+\frac{6}{10}=\frac{15+20+6}{10}=\frac{41}{10}$$

Example 3

Verify : $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$ for $\frac{a}{b}=\frac{-2}{3},\frac{c}{d}=\frac{5}{7}$ and $\frac{e}{f}=\frac{-1}{6}$

Solution :

We have, $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\left(\frac{-2}{3}+\frac{5}{7}\right)+\frac{-1}{6}=\frac{(-2)\times7+3\times5}{21}+\frac{-1}{6}=\frac{(-14)+15}{21}+\frac{-1}{6}=\frac{1}{21}+\frac{(-1)}{6}$

$=\frac{1\times2+(-1)\times7}{42}=\frac{2+(-7)}{42}=\frac{(-5)}{42}=\frac{-5}{42}$ and $\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)=\frac{-2}{3}+\left(\frac{5}{7}+\frac{-1}{6}\right)=\frac{-2}{3}+\left(\frac{5\times6+7\times(-1)}{42}\right)$

$=\frac{-2}{3}+\frac{30+(-7)}{42}=\frac{(-2)}{3}+\frac{23}{42}=\frac{(-2)\times14+23\times1}{42}=\frac{(-28)+(23)}{42}=\frac{-5}{42}$ $\therefore\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$

Example 4

The product of two rational numbers is $\frac{-28}{81}$. If one of the number is $\frac{14}{27}$, find the other.

Solution :

We have, product of two numbers $=\frac{-28}{81}$. One number $=\frac{14}{27}$ so, the other number is obtained by dividing the product by the given number.

$\therefore$ Other number $=\frac{-28}{81}\div\frac{14}{27}=\frac{-28\times27}{81\times14}=\frac{-(28\times27)}{81\times14}=\frac{-(2\times1)}{3\times1}=\frac{-2}{3}$

Example 5

Express each of the following expression in its lowest terms.

(i) $\left(\frac{12}{5}\times\frac{3}{4}\right)+\left(\frac{12}{5}\times\frac{7}{2}\right)$ (ii) $\left(\frac{2}{3}\times\frac{-5}{7}\right)-\left(\frac{2}{3}\times\frac{4}{5}\right)$

Solution :

(i) $\left(\frac{12}{5}\times\frac{3}{4}\right)+\left(\frac{12}{5}\times\frac{7}{2}\right)=\left(\frac{12\times3}{5\times4}\right)+\left(\frac{12\times7}{5\times2}\right)=\frac{9}{5}+\frac{42}{5}=\frac{9+42}{5}=\frac{51}{5}$

(ii) $\left(\frac{2}{3}\times\frac{-5}{7}\right)-\left(\frac{2}{3}\times\frac{4}{5}\right)=\frac{-10}{21}-\frac{8}{15}=\frac{-50-56}{105}=\frac{-106}{105}$

Example 6

Simplify : $\left(\frac{-3}{2}\times\frac{4}{5}\right)+\left(\frac{9}{5}\times\frac{-10}{3}\right)-\left(\frac{1}{2}\times\frac{3}{4}\right)$

Solution :

$$\left(\frac{-3}{2}\times\frac{4}{5}\right)+\left(\frac{9}{5}\times\frac{-10}{3}\right)-\left(\frac{1}{2}\times\frac{3}{4}\right)=\frac{-3\times4}{2\times5}+\frac{9\times(-10)}{5\times3}-\frac{1\times3}{2\times4}=\frac{-3\times2}{1\times5}+\frac{3\times(-2)}{1\times1}-\frac{3}{8}=\frac{-6}{5}+\frac{-6}{1}-\frac{3}{8}$$

$$=\frac{-6}{5}+\frac{-6}{1}+\frac{-3}{8}=\frac{(-6)\times8+(-6)\times40+(-3)\times5}{40}=\frac{-48+(-240)+(-15)}{40}=\frac{-303}{40}$$

5. REPRESENTATION OF RATIONAL NUMBERS ON NUMBER LINE

Step – 1 : Draw a line and mark a point 'O' on it to represent 0.

Step – 2 : Mark points at equal distance on the left and right side of 0, such that distance between two adjacent points is 1 unit.

Step – 3 : Then divide each unit into equal parts such that the number of parts is equal to the denominator of rational number.

Step – 4 : The numerator tells the number of parts to be considered.

Example 1

Represent $\frac{5}{3}$ and $\frac{-5}{3}$ on the number line.

Solution :

$\frac{5}{3}$ and $\frac{-5}{3}$ can be written as $1\frac{2}{3}$ and $-1\frac{2}{3}$.

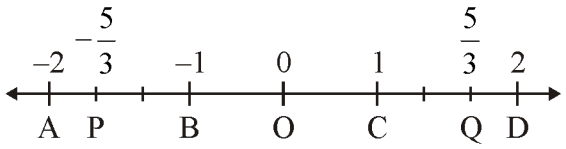
Step – 1 : In order to represent $\frac{5}{3}$ and $\frac{-5}{3}$ on the number line, we draw a number line and mark a point O on it to represent zero.

Step – 2 : Since $1\frac{2}{3}$ and $-1\frac{2}{3}$ lie between 1 and 2 and – 1 and – 2 respectively, therefore mark the points A and B on left and C and D on right side of O, such that A, B, C and D represent – 2, – 1, 1 and 2 respectively.

Step – 3 : Now, the denominator of rational number is 3.

$\therefore$ Divide the intervals in 3 equal parts.

Step – 4 : Since the numerator is 2, then mark second point on the parts. P and Q are the required points.

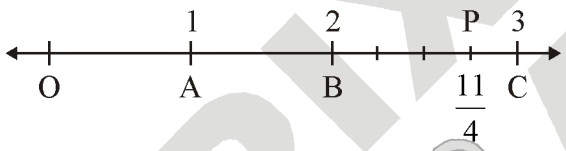


Example 2

Represent $\frac{11}{4}$ on the number line.

Solution :

$\frac{11}{4} = 2\frac{3}{4}$ lies between 2 and 3.



6. RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

Between two rational numbers there are infinitely many rational numbers.

Method – 1 : Let a and b be two rational numbers, then $q_1 = \frac{a+b}{2} \Rightarrow a < q_1 < b$

q_1 is the rational number between a and b.

$q_2 = \frac{a+q_1}{2} \Rightarrow a < q_2 < q_1 < b$

q_2 is the rational number between a and q_1.

$q_3 = \frac{q_1+b}{2} \Rightarrow a < q_2 < q_1 < q_3 < b$

q_3 is the rational number bwtween q_1 and b.

In this manner we can find infinite rational numbers between two given distinct rational numbers.

Method – 2 : $\frac{a}{b}$ and $\frac{c}{d}$ be two rational numbers.

Step – 1 : Make denominators equal in both rational numbers.

Step – 2 : If we have to find n rational numbers between $\frac{ad}{bd}$ and $\frac{cb}{bd}$, then multiply numerators and denominators of both numbers by such a number so that there are n numbers between the numerators.

Example 1

Write three rational numbers lying between $\frac{1}{3}$ and $\frac{1}{2}$.

Solution :

Method – 1 : The rational number between $\frac{1}{3}$ and $\frac{1}{2}$ is $\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{2}{6}+\frac{3}{6}\right)=\frac{1}{2}\times\frac{5}{6}=\frac{5}{12}\Rightarrow\frac{1}{3}<\frac{5}{12}<\frac{1}{2}$

The rational number between $\frac{1}{3}$ and $\frac{5}{12}$ is $\frac{1}{2}\left(\frac{1}{3}+\frac{5}{12}\right)=\frac{1}{2}\left[\frac{4}{12}+\frac{5}{12}\right]=\frac{1}{2}\times\frac{9}{12}=\frac{9}{24}=\frac{3}{8}\Rightarrow\frac{1}{3}<\frac{3}{8}<\frac{5}{12}<\frac{1}{2}$

The rational number between $\frac{5}{12}$ and $\frac{1}{2}$ is $\frac{1}{2}\left(\frac{5}{12}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{5}{12}+\frac{6}{12}\right)=\frac{1}{2}\times\frac{11}{12}=\frac{11}{24}\Rightarrow\frac{1}{3}<\frac{3}{8}<\frac{5}{12}<\frac{11}{24}<\frac{1}{2}$

Hence, $\frac{3}{8},\frac{5}{12},\frac{11}{24}$ are the required three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

Method – 2 : Make the denominator equal by multiplying 2 and 3. $\frac{1}{3}\times\frac{2}{2}=\frac{2}{6}$ and $\frac{1}{2}\times\frac{3}{3}=\frac{3}{6}$

To insert 3 rational numbers we multiply the numerators and denominators by such a number so that difference between the numerators is at least 3. Multiplying the numerators and denominators of both rational numbers by 4, we get $\frac{8}{24}$ and $\frac{12}{24}$. Hence, the required 3 rational numbers are $\frac{9}{24},\frac{10}{24},\frac{11}{24}$.

Example 2

Find three rational numbers between – 2 and 5.

Solution :

Three rational numbers between – 2 and 5 are – 1, 0, 1, $\Rightarrow -2<-1<0<1<5$

SOLVED EXAMPLES

SE. 1

Write the additive inverse of each of the following rational numbers :

(i) $\frac{5}{-11}$ (ii) $\frac{-11}{-14}$

Ans. (i) We have, $\frac{5}{-11}=\frac{-5}{11}$

The additive inverse of $\frac{-5}{11}$ is $-\left(\frac{-5}{11}\right)=\frac{5}{11}$

(ii) We have, $\frac{-11}{-14}=\frac{11}{14}$

The additive inverse of $\frac{11}{14}$ is $-\left(\frac{11}{14}\right)=\frac{-11}{14}$

SE. 2

Verify that – (– x) is the same as x for :

(i) $x=\frac{13}{17}$ (ii) $x=\frac{-21}{31}$

Ans. (i) $x=\frac{13}{17}$

So, $-(-x)=-\left(\frac{-13}{17}\right)=\frac{13}{17}$

(ii) $x=\frac{-21}{31}$

So, $-(-x)=-\left(-\left(\frac{-21}{31}\right)\right)=-\left(\frac{21}{31}\right)=-\frac{21}{31}$

SE. 3

What number should be subtracted from $\frac{27}{13}$ to get $\frac{-3}{7}$?

Ans. We have, difference of the given number and the required number is $\frac{-3}{7}$. Given number = $\frac{27}{13}$.

Other number $=\frac{27}{13}-\frac{-3}{7}=\frac{27}{13}+\frac{3}{7}=\frac{189+39}{13\times 7}$

$=\frac{228}{13\times 7}=\frac{228}{91}$. So, $\frac{228}{91}$ should be subtracted from $\frac{27}{13}$ to get $\frac{-3}{7}$.

SE. 4

The sum of two rational numbers is $\frac{-1}{2}$. If one of them is $\frac{-9}{10}$, find the other.

Ans. Given that the sum of numbers is $\frac{-1}{2}$.

One number $=\frac{-9}{10}$

$\therefore$ Other number = Sum of the numbers – One number

$=\frac{-1}{2}-\left(\frac{-9}{10}\right)=\frac{-1}{2}+\frac{9}{10}=\frac{-5+9}{10}=\frac{4}{10}=\frac{2}{5}$

SE. 5

Using commutativity of addition of rational numbers, express the following $\frac{4}{3}+\frac{-4}{5}+\frac{-2}{3}+\frac{7}{5}-2$ as a rational number.

Ans. Re-arranging and grouping the numbers in such a way that each group contains a pair of rational numbers with equal denominators, we have

$$\frac{4}{3}+\frac{-4}{5}+\frac{-2}{3}+\frac{7}{5}-2=\left(\frac{4}{3}+\frac{-2}{3}\right)+\left(\frac{-4}{5}+\frac{7}{5}\right)-2$$

$$=\frac{4+(-2)}{3}+\frac{(-4)+7}{5}-2=\frac{2}{3}+\frac{3}{5}-2$$

$$=\left(\frac{2}{3}+\frac{3}{5}\right)-2=\frac{2\times 5+3\times 3}{15}+(-2)$$

$$= \frac{10+9}{15} + (-2) = \frac{19}{15} + \frac{(-2)}{1} = \frac{19+(-2)\times 15}{15}$$

$$= \frac{19+(-30)}{15} = \frac{-11}{15}$$

SE. 6

Re-arrange suitably and find the sum of $\frac{-4}{7} + \frac{7}{6} + \frac{2}{7} + 3 + \frac{-11}{6}$.

Ans. Re-arranging and grouping the numbers in such a way that each group contains a pair of rational numbers with equal denominators, we have

$$\frac{-4}{7} + \frac{7}{6} + \frac{2}{7} + 3 + \frac{-11}{6}$$

$$= \left(\frac{-4}{7} + \frac{2}{7}\right) + \left(\frac{7}{6} + \frac{-11}{6}\right) + 3$$

$$= \frac{(-4)+2}{7} + \frac{7+(-11)}{6} + 3 = \frac{-2}{7} + \frac{(-4)}{6} + 3$$

$$= \frac{(-2)}{7} + \frac{(-2)}{3} + 3 = \left(\frac{(-2)}{7} + \frac{(-2)}{3}\right) + 3$$

$$= \frac{(-2)\times 3 + (-2)\times 7}{21} + 3 = \frac{(-6)+(-14)}{21} + 3$$

$$= \frac{(-20)}{21} + \frac{3}{1} = \frac{(-20)+3\times 21}{21} = \frac{-20+63}{21} = \frac{43}{21}$$

SE. 7

What number should be added to $\frac{-5}{8}$ to get $\frac{5}{9}$?

Ans. Let x is the rational number to be added to $\frac{-5}{8}$ to get $\frac{5}{9}$. Then, $\frac{-5}{8} + x = \frac{5}{9}$

$$\Rightarrow x = \frac{5}{9} - \left(\frac{-5}{8}\right)$$

$$\Rightarrow x = \frac{5}{9} + \frac{5}{8} \Rightarrow x = \frac{5\times 8 + 5\times 9}{72} = \frac{40+45}{72} = \frac{85}{72}$$

SE. 8

Subtract $\frac{-8}{33}$ from $\frac{-5}{11}$.

Ans. $\frac{-5}{11} - \left(\frac{-8}{33}\right) = \frac{-5}{11} + \frac{8}{33} = \frac{-5\times 3}{11\times 3} + \frac{8}{33}$

$$= \frac{-15}{33} + \frac{8}{33} = \frac{-15+8}{33} = \frac{-7}{33}$$

SE. 9

Subtract the sum of $\frac{-4}{7}$ and $\frac{5}{14}$ from the sum of $\frac{9}{14}$ and $\frac{23}{14}$.

Ans. Sum of $\frac{-4}{7}$ and $\frac{5}{14} = \frac{-4}{7} + \frac{5}{14} = \frac{-8+5}{14} = \frac{-3}{14}$

Sum of $\frac{9}{14}$ and $\frac{23}{14} = \frac{9}{14} + \frac{23}{14} = \frac{9+23}{14} = \frac{32}{14}$

$= \frac{16}{7}$. Now, subtracting $\frac{-3}{14}$ from $\frac{16}{7}$, we get

$$\frac{16}{7} - \left(\frac{-3}{14}\right) = \frac{16}{7} + \frac{3}{14} = \frac{32+3}{14} = \frac{35}{14} = \frac{5}{2}$$

SE. 10

Evaluate : $\frac{-12}{5} + \frac{-7}{20} + \frac{3}{14} + \frac{1}{7} + \frac{-1}{10}$

Ans. L.C.M. of 5, 20, 14, 7, 10 = 2 × 2 × 5 × 7 = 140

$$\therefore \frac{-12}{5} + \frac{-7}{20} + \frac{3}{14} + \frac{1}{7} + \frac{-1}{10}$$

$$= \frac{(-12)\times(28) + (-7)\times 7 + 3\times 10 + 1\times 20 + (-1)\times 14}{140}$$

$$= \frac{(-336)+(-49)+30+20+(-14)}{140}$$

$$= \frac{(-399)+50}{140} = \frac{-349}{140}$$

SE. 11

Simplify : $\left(\frac{-7}{18}\times\frac{15}{-7}\right)-\left(1\times\frac{1}{4}\right)+\left(\frac{1}{2}\times\frac{1}{4}\right)$

Ans. $\left(\frac{-7}{18}\times\frac{15}{-7}\right)-\left(1\times\frac{1}{4}\right)+\left(\frac{1}{2}\times\frac{1}{4}\right)$

$=\left(\frac{-7}{18}\times\frac{15}{-7}\right)-\left(\frac{1}{1}\times\frac{1}{4}\right)+\left(\frac{1}{2}\times\frac{1}{4}\right)$

$=\frac{1\times5}{6\times1}-\frac{1\times1}{1\times4}+\frac{1\times1}{2\times4}=\frac{5}{6}-\frac{1}{4}+\frac{1}{8}$

$=\frac{5}{6}+\frac{-1}{4}+\frac{1}{8}=\frac{5\times4+(-1)\times6+1\times3}{24}$

$=\frac{20+(-6)+3}{24}=\frac{17}{24}$

SE. 12

Divide :

(i) $\frac{-16}{21}$ by $\frac{-4}{3}$ (ii) $\frac{-8}{13}$ by $\frac{3}{-26}$

Ans. (i) $\frac{-16}{21}\div\frac{-4}{3}=\frac{-16}{21}\times\frac{3}{-4}=\frac{-16\times3}{21\times(-4)}=\frac{4\times1}{7\times1}=\frac{4}{7}$

(ii) $\frac{-8}{13}\div\frac{3}{-26}=\frac{-8}{13}\times\frac{-26}{3}=\frac{(-8)\times(-26)}{13\times3}$

$=\frac{8\times26}{13\times3}=\frac{8\times2}{1\times3}=\frac{16}{3}$

SE. 13

The product of two numbers is $\frac{14}{15}$. If one of the numbers is $\left(\frac{-20}{17}\right)$, then find the other.

Ans. Product of two numbers = $\frac{14}{15}$.

One number = $\frac{-20}{17}$

$\therefore$ Other number $=\frac{14}{15}\div\frac{-20}{17}=\frac{14}{15}\times\frac{17}{-20}$

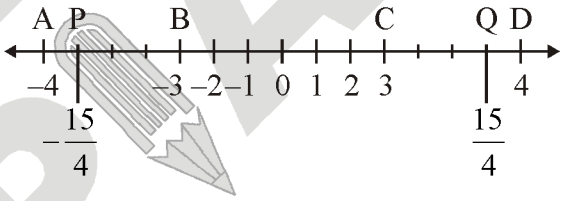
$=\frac{14\times17}{15\times(-20)}=\frac{7\times17}{15\times(-10)}=\frac{119}{-150}=\frac{-119}{150}$

SE. 14

Represent $\frac{15}{4}$ and $\frac{-15}{4}$ on the number line.

Ans. $\frac{15}{4}$ and $\frac{-15}{4}$ can be written as $3\frac{3}{4}$ and $-3\frac{3}{4}$

$\therefore 3\frac{3}{4}$ lies between 3 & 4 and $-3\frac{3}{4}$ lies between – 3 and – 4.



SE. 15

Express $\left(\frac{1}{2}+\frac{3}{4}\right)\div2$ as a rational number and show that it lies between $\frac{1}{2}$ and $\frac{3}{4}$.

Ans. $\left(\frac{1}{2}+\frac{3}{4}\right)\div2=\left(\frac{1\times2}{2\times2}+\frac{3}{4}\right)\div2=\left(\frac{2}{4}+\frac{3}{4}\right)\div2$

$=\frac{5}{4}\div2=\frac{5}{4}\times\frac{1}{2}=\frac{5}{8}$

Let us now arrange the number $\frac{1}{2},\frac{3}{4}$ and $\frac{5}{8}$ in ascending order of magnitude.

L.C.M. of denominators 2, 4 and 8 = 8

Now, $\frac{1}{2}=\frac{1\times4}{2\times4}=\frac{4}{8},\frac{3}{4}=\frac{3\times2}{4\times2}=\frac{6}{8},\frac{5}{8}=\frac{5}{8}$

Since, $4<5<6$, it follows that $\frac{4}{8}<\frac{5}{8}<\frac{6}{8}$

i.e., $\frac{1}{2} < \frac{5}{8} < \frac{3}{4}$.

We find that, $\frac{5}{8}$ lies between $\frac{1}{2}$ and $\frac{3}{4}$.

Therefore, $\left(\frac{1}{2}+\frac{3}{4}\right) \div 2$ lies between $\frac{1}{2}$ and $\frac{3}{4}$.

SE. 16

Find 10 rational numbers between $\frac{-2}{11}$ and $\frac{9}{11}$.

Ans. The given rational numbers are $\frac{-2}{11}$ and $\frac{9}{11}$.

Since the denominators of both the rational numbers are equal and positive. Finding 10 rational numbers between $\frac{-2}{11}$ and $\frac{9}{11}$ is similar to finding 10 integers between – 2 and 9. We know that

$-2 < -1 < 0 < 1 < 2 < 3 < 4 < 5 < 6 < 7 < 8 < 9$

So, $\frac{-1}{11}, \frac{0}{11}, \frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}, \frac{5}{11}, \frac{6}{11}, \frac{7}{11}$ and $\frac{8}{11}$ are the required 10 rational numbers between $\frac{-2}{11}$ and $\frac{9}{11}$.

Space for Notes :

EXERCISE – I

ONLY ONE CORRECT TYPE

1. The standard form of $\frac{192}{-168}$ is :

(A) $\frac{-2}{3}$ (B) $\frac{-8}{7}$

(C) $\frac{-1}{7}$ (D) $\frac{-6}{7}$

2. Standard form of $\frac{-24}{36}$ is a rational number with denominator :

(A) 3 (B) 4

(C) 1 (D) 2

3. 0.75 when represented as rational number, is :

(A) $\frac{75}{99}$ (B) $\frac{75}{90}$

(C) $\frac{3}{4}$ (D) $\frac{5}{4}$

4. The additive inverse of $\frac{-a}{b}$ is :

(A) $\frac{a}{b}$ (B) $\frac{b}{a}$

(C) $\frac{-b}{a}$ (D) $\frac{-a}{b}$

5. The number which is subtracted from $\frac{27}{13}$ to get $\frac{-3}{7}$, is :

(A) $\frac{228}{91}$ (B) $\frac{1}{91}$

(C) $\frac{200}{91}$ (D) $\frac{198}{91}$

6. The sum of the additive inverse and multiplicative inverse of 2 is :

(A) $\frac{3}{2}$ (B) $\frac{-3}{2}$

(C) $\frac{1}{2}$ (D) $\frac{-1}{2}$

7. Which of the following statements is false ?

(A) Every fraction is a rational number.

(B) Every rational number is a fraction.

(C) Every integer is a rational number.

(D) All of these

8. Which of the following statements is true ?

(A) $\frac{5}{7} < \frac{7}{9} < \frac{9}{11} < \frac{11}{13}$ (B) $\frac{11}{13} < \frac{9}{11} < \frac{7}{9} < \frac{5}{7}$

(C) $\frac{5}{7} < \frac{11}{13} < \frac{7}{9} < \frac{9}{11}$ (D) $\frac{5}{7} < \frac{9}{11} < \frac{11}{13} < \frac{7}{9}$

9. Multiplicative inverse of '0' is :

(A) – 1 (B) 0

(C) Does not exist (D) 1

10. The value of x for which the two rational numbers $\frac{3}{7}$ and $\frac{x}{42}$ are equivalent, is :

(A) 18 (B) 15

(C) 12 (D) 10

11. Which of the following illustrates the inverse property of addition ?

(A) 3 + (– 3) = 0 (B) 3 – (– 3) = 6

(C) 3 + 0 = 3 (D) 3 – 0 = 3

12. What number should be added to $\frac{7}{12}$ to get $\frac{4}{15}$?

(A) $-\frac{19}{60}$ (B) $-\frac{11}{30}$

(C) $\frac{51}{60}$ (D) $\frac{1}{20}$

13. The difference between the largest and the smallest of the rationals, $\frac{5}{8}, \frac{7}{12}, \frac{1}{3}, \frac{2}{5}$, is :

(A) $\frac{1}{4}$ (B) $\frac{-5}{24}$

(C) $\frac{7}{24}$ (D) $\frac{13}{21}$

14. The additive inverse of sum of the rational numbers $-\frac{5}{16}$ and $\frac{7}{12}$ is :

(A) $-\frac{7}{48}$ (B) $\frac{1}{24}$

(C) $-\frac{13}{48}$ (D) $\frac{13}{48}$

15. Which of the following rational numbers is the smallest ? $-\frac{5}{16}, \frac{-3}{4}, \frac{-13}{24}$ and $\frac{7}{-12}$

(A) $-\frac{5}{16}$ (B) $\frac{-3}{4}$

(C) $\frac{-13}{24}$ (D) $\frac{-7}{12}$

16. What number should be subtracted from $-\frac{3}{5}$ to get -2 ?

(A) $-\frac{7}{5}$ (B) $-\frac{13}{5}$

(C) $\frac{13}{5}$ (D) $\frac{7}{5}$

17. Name the law of multiplication illustrated by the statement, $\frac{-15}{8} \times \frac{-12}{7} = \frac{-12}{7} \times \frac{-15}{8}$.

(A) Associative law (B) Closure law

(C) Commutative law (D) None of these

18. Which of the following forms a pair of equivalent rational numbers ?

(A) $\frac{14}{35}$ and $\frac{21}{45}$ (B) $\frac{-12}{26}$ and $\frac{18}{39}$

(C) $\frac{-3}{7}$ and $\frac{-21}{56}$ (D) $\frac{-7}{28}$ and $\frac{-5}{20}$

19. The reciprocal of $\left(-\frac{9}{16} \times \frac{8}{15}\right)$ is

(A) $-\frac{3}{10}$ (B) $-\frac{4}{150}$

(C) $-\frac{10}{3}$ (D) $-\frac{2}{50}$

20. By what number should we multiply $\frac{3}{-14}$, so that the product is $\frac{5}{12}$?

(A) $\frac{-35}{18}$ (B) $\frac{34}{19}$

(C) $\frac{35}{18}$ (D) $\frac{-34}{19}$

21. The sum of two rational numbers is $\frac{-3}{5}$. If one of the numbers is $\frac{-9}{20}$, find the other.

(A) $\frac{7}{20}$ (B) $\frac{27}{100}$

(C) $\frac{-21}{20}$ (D) $\frac{-3}{20}$

22. The area of a rectangle is $45\frac{5}{16}$ cm^2. If its length is $7\frac{1}{4}$ cm, then find its breadth.

(A) $6\frac{1}{4}$ cm (B) $4\frac{1}{6}$ cm

(C) $328\frac{33}{64}$ cm (D) $38\frac{1}{16}$ cm

23. If $\frac{3}{7}+x+\left(\frac{-8}{21}\right)+\frac{5}{22}=\frac{-125}{462}$, then x is

(A) $\frac{6}{11}$ (B) $\frac{-5}{11}$

(C) $\frac{-6}{11}$ (D) $\frac{5}{11}$

24. The product of two numbers is $-\frac{16}{35}$. If one of the numbers is $-\frac{15}{14}$, find the additive inverse of other.

(A) $-\frac{2}{5}$ (B) $\frac{-32}{75}$

(C) $\frac{32}{75}$ (D) $-\frac{8}{3}$

25. Simplify : $\left(\frac{3}{5}\times\frac{-15}{21}\right)+\left(\frac{-9}{14}\div\frac{45}{28}\right)-\left(\frac{2}{3}\times\frac{30}{12}\right)$

(A) $-\frac{17}{35}$ (B) $-2\frac{52}{105}$

(C) $-1\frac{4}{11}$ (D) $\frac{-40}{41}$

PARAGRAPH TYPE

PASSAGE # I

If two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ are such that $\frac{c}{d}\neq 0$ and $\frac{a}{b}\times\frac{c}{d}=1$, then $\frac{a}{b}$ and $\frac{c}{d}$ are called multiplicative inverse of each other.

1. $\frac{73}{-29}$ is the multiplicative inverse of _____.

(A) $\frac{29}{73}$ (B) $\frac{-29}{73}$

(C) $\frac{73}{29}$ (D) 1

2. Find the multiplicative inverse of $\frac{1}{2}\left(2+\frac{3}{2}\right)$.

(A) $\frac{4}{7}$ (B) $\frac{8}{3}$

(C) $\frac{7}{2}$ (D) $\frac{2}{3}$

3. If $(x+y)z=1$, then z is a multiplicative inverse of _____.

(A) x (B) y

(C) x + y (D) $\frac{x+y}{2}$

PASSAGE # II

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then

$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\frac{a}{b}\times\frac{c}{d}+\frac{a}{b}\times\frac{e}{f}$.

4. If $\frac{2}{3}\times\frac{-7}{10}+\frac{-2}{3}\times\frac{8}{9}=p\times\left[\frac{-7}{10}+q\right]$, then p and q are

(A) $\frac{2}{3},\frac{8}{9}$ (B) $\frac{-2}{3},\frac{-8}{9}$

(C) $\frac{-2}{3},\frac{8}{9}$ (D) $\frac{2}{3},\frac{-8}{9}$

5. Name the property used above.

(A) Commutativity of multiplication over addition.

(B) Commutativity of addition over multiplication.

(C) Distributivity of multiplication over addition.

(D) Distributivity of addition over multiplication.

6. If $\frac{2}{5}\times\frac{-8}{9}+p\times\frac{5}{9}=\frac{2}{5}\times[q+r]$, then p, q and r are

(A) $\frac{2}{5},\frac{8}{9},\frac{5}{9}$

(B) $\frac{2}{5},\frac{8}{9},\frac{-5}{9}$

(C) $\frac{-2}{5},\frac{-8}{9},\frac{-5}{9}$

(D) $\frac{-2}{5},\frac{-8}{9},\frac{5}{9}$

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the corect combination of elements from List – I and List – II are given as options (A), (B), (C) and (D) out of which one is correct.

1. Match the following :

List – I	**List – II**
(P) Additive identity of a rational number 'a' is	(i) 1/a
(Q) Multiplicative inverse of a rational number 'a' is	(ii) 0
(R) Multiplicative identity of a rational number 'a' is	(iii) – a
(S) Additive inverse of a rational number 'a' is	(iv) 1

(A) (P)→(ii), (Q)→(i), (R)→(iv), (S)→(iii)
(B) (P)→(iii), (Q)→(ii), (R)→(iv), (S)→(i)
(C) (P)→(iii), (Q)→(ii), (R)→(i), (S)→(iv)
(D) (P)→(ii), (Q)→(iv), (R)→(iii), (S)→(i)

2. Match the following :

List – I	**List – II**
(P) Associative law	(i) If a and b are rational numbers, then a + b is a rational number.
(Q) Commutative law	(ii) If a and b are rational numbers, then a + b = b + a
(R) Distributive law	(iii) If a, b and c are rational numbers, then a + (b + c) = (a + b) + c
(S) Closure law	(iv) If a, b and c are rational numbers, then a × (b + c) = ab + ac

(A) (P)→(ii), (Q)→(iii), (R)→(iv), (S)→(i)
(B) (P)→(iii), (Q)→(ii), (R)→(iv), (S)→(i)
(C) (P)→(iii), (Q)→(ii), (R)→(i), (S)→(iv)
(D) (P)→(i), (Q)→(ii), (R)→(iii), (S)→(iv)

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Write additive inverse of the following number.

   (i) $\frac{-11}{-25}$ (ii) 0

2. Fill in blank : $\frac{-9}{14} + ... = -1$.

3. Multiply $\frac{-3}{17}$ by $\frac{-5}{-4}$.

4. Express $\frac{2}{7}$ as a rational number whose numerator is – 6.

5. Express rational number $\frac{4}{-14}$ with positive denominator.

6. Find the value of x, if $\frac{-5}{9} = \frac{10}{x}$.

7. Are $\frac{15}{24}$ and $\frac{45}{48}$ equivalent rational numbers ?

8. Can you write $\frac{1}{2}$ with denominator equal to 5 ?

9. Write the absolute value of $\frac{-9}{-100}$.

10. Can – 2 be the absolute value of any rational number ?

SHORT ANSWER TYPE

1. If $\frac{3}{5}$ of a number exceeds its $\frac{2}{7}$ by 44, find the number.

2. By what number should $\frac{-33}{16}$ be divided to get $\frac{-11}{4}$?

3. Find (x + y) ÷ (x – y), if $x = \frac{5}{4}, y = \frac{-1}{3}$.

4. Find a rational number between $\frac{-2}{3}$ and $\frac{1}{4}$.

5. The product of two rational numbers is $\frac{63}{40}$. If one of the number is $\left(\frac{-7}{5}\right)$, find the other number.

LONG ANSWER TYPE

1. Divide the sum of $\frac{-13}{5}$ and $\frac{12}{7}$ by the product of $\frac{-31}{7}$ and $\frac{-1}{2}$.

2. Simplify : $\left[\frac{3}{11} \times \frac{5}{6}\right] - \left[\frac{9}{12} \times \frac{4}{3}\right] + \left[\frac{5}{13} \times \frac{6}{15}\right]$

3. Verify the property $x \times (y + z) = x \times y + x \times z$ when $x = \frac{-12}{5}, y = \frac{-15}{4}, z = \frac{8}{3}$.

4. Find four rational numbers between $\frac{2}{3}$ and $\frac{4}{5}$.

5. For $x = \frac{1}{2}$ and $y = \frac{2}{3}$, verify that $-(x + y) = (-x) + (-y)$.

TRUE / FALSE TYPE

1. Prime number can also be a negative integer.
2. 2 is the only even prime number.
3. 0 is a rational number.
4. $\frac{3}{5} > \frac{2}{3}$.
5. There exists infinite rational numbers between any 2 integers.

NUMERICAL PROBLEMS

1. Multiplicative identity for rational number is.

2. Find the sum of digits of numerator and denominator of reciprocal of $\left(\frac{2}{5}+\frac{5}{4}\right)$.

3. The product of two numbers is $\frac{45}{56}$. One of them is $\frac{9}{7}$ and the other number is $\frac{m}{n}$. Then m + n is.

4. The sum of two rational numbers is – 3. If one of the numbers is $-\frac{10}{3}$, then numerator of other number is.

5. Find x, if $4\times\frac{7}{9}=\frac{7}{9}\times x$.

ANALYTICAL PROBLEMS & BRAIN TEASER

1. Simplify :

$$\frac{\left(\frac{2}{3}\times\left(-\frac{5}{4}\right)\right)+\left(\left(-\frac{10}{3}\right)\times\frac{5}{2}\right)-\left(\left(-\frac{16}{3}\right)\times\left(-\frac{55}{32}\right)\right)}{\frac{3}{2}\times\left(\left(-\frac{9}{14}\right)\times\left(-\frac{1}{7}\right)\right)}$$

(A) $\frac{1082}{81}$ (B) $-\frac{1082}{81}$

(C) $-133\frac{7}{81}$ (D) $133\frac{7}{81}$

2. To reduce a rational number in its standard form, we divide its numerator and denominator by their _____.

(A) L.C.M. (B) H.C.F.

(C) Product (D) Multiple

3. Which of the following shows distributive property of multiplication over addition for rational numbers ?

(A) $-\frac{3}{4}\times\left\{\frac{1}{3}+\left(-\frac{5}{7}\right)\right\}=\left[-\frac{3}{4}\times\frac{1}{3}\right]+\left[-\frac{3}{4}\times\left(-\frac{5}{7}\right)\right]$

(B) $-\frac{3}{4}\times\left\{\frac{1}{3}+\left(-\frac{5}{7}\right)\right\}=\left[-\frac{3}{4}\times\frac{1}{3}\right]-\left[-\frac{5}{7}\right]$

(C) $-\frac{3}{4}\times\left\{\frac{1}{3}+\left(-\frac{5}{7}\right)\right\}=\frac{1}{3}+\left[-\frac{3}{4}\right]\times\left(-\frac{5}{7}\right)$

(D) $-\frac{3}{4}\times\left\{\frac{1}{3}+\left(-\frac{5}{7}\right)\right\}=\left[\frac{1}{3}+\left(-\frac{5}{7}\right)\right]-\frac{3}{4}$

4. If A : Rational numbers are always closed under division and R : Division by zero is not defined, then _____.

(A) Both A and R are true

(B) Both A and R are false

(C) A is true and R is false

(D) A is false and R is true

5. If a, b, c are rational numbers, then associativity of rational numbers under addition is given by

(A) a + b = b + a

(B) a + (b + c) = (a + b) + c

(C) a × (b × c) = (a × b) × c

(D) a + (b – c) = (a + b) – c

6. Which of the following statements is true ?

(i) $\frac{-5}{0}$ is a negative rational number.

(ii) The reciprocal of $\frac{1}{a}$, if a = 0 is $\frac{1}{0}$.

(iii) $1\div\left(-\frac{1}{4}\right)=-4$

(iv) x ÷ (y + z) = x ÷ y + x ÷ z

(A) Both (i) and (ii)

(B) Only (iii)

(C) (i), (ii) and (iv)

(D) (ii), (iii) and (iv)

7. Zero is _____.
(A) The identity for addition of rational numbers.
(B) The identity for subtraction of rational numbers.
(C) The identity for multiplication of rational numbers.
(D) The identity for division of rational numbers.

8. Which of the following statements is always true ?
(A) $\frac{x-y}{2}$ is a rational number between x and y.
(B) $\frac{x+y}{2}$ is a rational number between x and y.
(C) $\frac{x \times y}{2}$ is a rational number between x and y.
(D) $\frac{x \div y}{2}$ is a rational number between x and y.

9. The number 34 is divided into two parts such that $\frac{4}{7}$ th of the first part is equal to $\frac{2}{5}$ th of the second part. The numbers are respectively
(A) 20, 14 (B) 21, 13
(C) 13, 21 (D) 14, 20

10. Which of the following rational numbers does not lie between $\frac{1}{4}$ and $\frac{2}{3}$?
(A) $\frac{1}{2}$ (B) $\frac{1}{3}$
(C) $\frac{14}{24}$ (D) $\frac{18}{24}$

11. The numerator and the denominator of a rational number are in the ratio 5 : 7. When 6 is added to both the numerator and denominator, the ratio becomes 4 : 5. What is the rational number ?
(A) $\frac{7}{5}$ (B) $\frac{5}{7}$
(C) $\frac{2}{5}$ (D) $\frac{13}{14}$

12. A water pump pumps out $14\frac{1}{6}\,l$ of water per minute from a reservoir. How many litres of water will be pumped out in $1\frac{1}{5}$ of an hour ?
(A) 1125 l (B) 6120 l
(C) 1020 l (D) 1560 l

13. Simplest form of $\dfrac{1}{3-\dfrac{1}{2-\dfrac{1}{7}}}$ is :
(A) $\frac{13}{32}$ (B) $\frac{32}{13}$
(C) $\frac{7}{13}$ (D) $\frac{13}{7}$

14. p : Every fraction is a rational number.
q : Every rational number is a fraction.
Which of the following is correct ?
(A) p is true and q is false.
(B) p is false and q is true.
(C) Both p and q are true.
(D) Both p and q are false.

15. Nine times the reciprocal of a rational number equals 6 times the reciprocal of 17. Find the rational number.
(A) $11\frac{1}{3}$ (B) $25\frac{1}{2}$
(C) $10\frac{1}{3}$ (D) None of these

EXERCISE – III

PREVIOUS YEAR QUESTIONS (NTSE)

1. If a number is divided by 45, then the remainder is 32. If the same number is divided by 15, then the remainder is **(Aryabhatta 2008)**

(A) 2 (B) 3

(C) 16 (D) 4

2. The product of x^2y and $\left(\frac{x}{y}\right)$ is equal to the quotient obtained when x^2 is divided by **(NSTSE 2010)**

(A) 0 (B) 1

(C) x (D) $\frac{1}{x}$

3. Identify a rational number between $\frac{1}{3}$ and $\frac{4}{5}$ **(NSTSE 2012)**

(A) $\frac{1}{4}$

(B) $\frac{9}{10}$

(C) $\frac{17}{30}$

(D) $1\frac{7}{10}$

4. Which of the statements is true about consecutive natural numbers ? **(NSTSE 2012)**

(A) There are 2n + 1 numbers between the difference of squares of consecutive numbers.

(B) There are 2n non-perfect square numbers between the squares.

(C) The sum of the squares of two consecutive numbers is not a perfect square.

(D) $n^2 - 1$ is the standard form of the difference between two consecutive numbers.

5. Identify the ones that is/are greater than 'm' if m $= \frac{9}{11}$. **(NSTSE 2014)**

(i) $\frac{1}{m}$ (ii) $\frac{m+1}{m}$

(iii) $\frac{m+1}{m-1}$

(A) (i) only (B) (ii) and (iii) only

(C) (i) and (iii) only (D) (i) and (ii) only

6. Which number is in the middle if $\frac{-1}{6}, \frac{4}{9}, \frac{6}{-7}, \frac{2}{5}$ and $\frac{-3}{4}$ are arranged in descending order **(NSTSE 2014)**

(A) $\frac{2}{5}$ (B) $\frac{4}{9}$

(C) $\frac{-1}{6}$ (D) $\frac{-6}{7}$

7. If the division N ÷ 5 leaves a remainder of 3, what might be the ones digit of N ? **(NSTSE 2014)**

(A) 2 (B) 3

(C) 4 (D) 6

8. Which of the following numbers does not have a multiplicative inverse ? **(NSTSE 2014)**

(A) $-\frac{1}{3}$ (B) 0

(C) 1 (D) 3

9. The difference between the place value and the face value of 6 in the numeral 856973 is _____. **(NSTSE 2014)**

(A) 973 (B) 6973

(C) 5994 (D) None of these

10. Which of the following expressions is true ? **(NSTSE 2014)**

(A) $0.09 > \frac{7}{8}$ (B) $6\% < 0.09$

(C) $\frac{7}{8} < 8.0 \times 10^{-3}$ (D) $8.0 \times 10^{-3} > 6\%$

11. If x : y = 5 : 2, then (8x + 9y) : (8x + 2y) is : **(NSTSE 2014)**

(A) 22 : 29 (B) 26 : 61

(C) 29 : 22 (D) 61 : 26

12. Which of the following statements is incorrect for rational numbers ? **(NSTSE 2014)**

(A) The rational number 0 is the additive identity for rational numbers.

(B) The rational number 1 is the multiplicative identity for rational numbers.

(C) Subtraction is associative for rational numbers.

(D) There are infinite rational numbers between any two given rational numbers.

Space for Notes :

Answer Key

EXERCISE I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	A	C	A	A	B	B	A	C	A	A	A	C	C	B
16	17	18	19	20	21	22	23	24	25					
D	C	D	C	A	D	A	C	B	B					

PARAGRAPH

1. B **2.** A **3.** C **4.** D **5.** C **6.** A

MATCH THE COLUMN

1. A **2.** B

EXERCISE II

VERY SHORT ANSWER TYPE

1. $-\frac{11}{25}$ **2.** $-\frac{5}{14}$ **3.** $-\frac{15}{68}$ **4.** $\frac{-6}{-21}$ **5.** $-\frac{4}{14}$ **6.** – 18 **7.** No

8. No **9.** $\frac{9}{100}$ **10.** No

SHORT ANSWER TYPE

3. 140 **6.** $\frac{3}{4}$ **7.** $\frac{11}{19}$ **8.** $-\frac{5}{24}$ **10.** $-\frac{9}{8}$

LONG ANSWER TYPE

1. $-\frac{2}{5}$ **2.** $-\frac{177}{286}$ **4.** $\frac{41}{60}, \frac{42}{60}, ---, \frac{47}{60}$

TRUE / FALSE

2. F **3.** T **4.** T **6.** F **10.** T

NUMERICAL PROBLEMS

3. 1 **4.** 8 **5.** 13 **8.** 1 **10.** 4

ANALYTICAL PROBLEMS & BRAIN TEASER

1. C **2.** B **3.** A **4.** D **5.** B **6.** B **7.** A
8. B **9.** A **10.** D **11.** D **12.** C **13.** A **14.** B
15. B

EXERCISE III

1	2	3	4	5	6	7	8	9	10	11	12	
A	D	C	B	D	C	B	B	C	B	C	C	

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : INTEGERS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

LINEAR EQUATIONS IN ONE VARIABLE

2

Concepts

1. *Equation*
2. *linear equation in one variable*
3. *solution of a linear equation*
4. *methods for solving linear equations in one variable*
   - *4.1 Balancing method*
   - *4.2 Transposition method*
   - *4.3 Cross-multiplication method*
5. *Applications of linear equations to practical problems*

Solved Examples

Exercise – I (Competitive Exam Pattern)

Exercise – II (Board Pattern Type)

Answer Key

1. EQUATION

A statement of equality which contains one or more unknown quantity or variable is called an equation.

Example 1

(i) $3x + 7 = 12$, $\frac{5}{2}x - 9 = 1$, $x^2 + 1 = 5$ and $\frac{x}{3} + 5 = \frac{x}{2} - 3$ are equations in one variable x.

(ii) $4x + 3y = 21$, $3x - \frac{y}{5} = 9$ are equations in two variables x and y.

2. LINEAR EQUATION IN ONE VARIABLE

An equation involving only one variable and highest degree of variable is one is called linear equation in one variable.

Example 1

$3x - 2 = 7$, $\frac{3}{2}x + 9 = \frac{1}{2}$, $\frac{y}{3} + \frac{y-2}{4} = 5$ are linear equations in one variable, because the highest power of the variable in each equation is one whereas the equations $3x^2 - 2x + 1 = 0$, $y^2 - 1 = 8$ are not linear equations, because the highest power of the variable in each equation is not one.

3. SOLUTION OF A LINEAR EQUATION

A value of the variable which when substituted for the variable in an equation, makes L.H.S. = R.H.S. is said to satisfy the equation and is called a solution of the equation. In other words, a value of the variable which makes the equation a true statement, is called a solution of the equation.

Example 1

Verify that $x = 4$ is a solution of the equation $2x - 3 = 5$.

Solution :

Substituting $x = 4$ in the given equation, we get

L.H.S. $= 2x - 3 = 2 \times 4 - 3 = 8 - 3 = 5 =$ R.H.S.

Hence, $x = 4$ is the solution of the equation $2x - 3 = 5$.

Example 2

Verify that $x = 8$ is a solution of the equation $\frac{5x-4}{8} - \frac{x-3}{5} = \frac{x+6}{4}$.

Solution :

Substituting $x = 8$ in the given equation, we get

L.H.S. $= \frac{5x-4}{8} - \frac{x-3}{5} = \frac{5\times 8-4}{8} - \frac{8-3}{5} = \frac{36}{8} - \frac{5}{5} = \frac{9}{2} - 1 = \frac{9-2}{2} = \frac{7}{2}$ and R.H.S. $= \frac{x+6}{4} = \frac{8+6}{4} = \frac{14}{4} = \frac{7}{2}$

Thus, for x = 8, we have L.H.S. = R.H.S.

Hence, x = 8 is a solution of the given equation.

4. METHODS FOR SOLVING LINEAR EQUATIONS IN ONE VARIABLE

Solving an equation means determining the value of x i.e., determining the value of the variable which satisfies the equation :

(i) Balancing Method (ii) Transposition Method

(iii) Cross Multiplication Method

4.1 BALANCING METHOD

Following rules can be used for balancing an equation :

Rule – I : Same quantity (number) can be added to both sides of an equation without changing the equality.

Rule – II : Same quantity can be subtracted from both sides of an equation without changing the equality.

Rule – III : Both sides of an equation may be multiplied by the same non-zero number without changing the equality.

Rule – IV : Both sides of an equation may be divided by the same non-zero number without changing the equality.

Example 1

Solve the equation $\frac{x}{5}+11=\frac{1}{15}$ and check the result.

Solution :

We have, $\frac{x}{5}+11=\frac{1}{15} \Rightarrow \frac{x}{5}+11-11=\frac{1}{15}-11$ [Subtracting 11 from both sides]

$\Rightarrow \frac{x}{5}=\frac{1}{15}-11 \Rightarrow \frac{x}{5}=\frac{1-165}{15} \Rightarrow \frac{x}{5}=-\frac{164}{15} \Rightarrow 5\times\frac{x}{5}=5\times\left(-\frac{164}{15}\right)$ [Multiplying both sides by 5]

$\Rightarrow x=-\frac{164}{3}$. Thus, $x=-\frac{164}{3}$ is the solution of the given equation.

Check : Substituting $x=\frac{-164}{3}$ in the given equation, we get

L.H.S. $=\frac{x}{5}+11=\frac{-164}{3}\times\frac{1}{5}+11=\frac{-164}{15}+11=\frac{-164+165}{15}=\frac{1}{15}$ and R.H.S. $=\frac{1}{15}$

$\therefore$ L.H.S. = R.H.S. for $x=\frac{-164}{3}$

Hence, $x=\frac{-164}{3}$ is the solution of the given equation.

Example 2

Solve $\frac{1}{3}x - \frac{5}{2} = 6$ and check the result.

Solution :

We have, $\frac{1}{3}x - \frac{5}{2} = 6$

$\Rightarrow \frac{1}{3}x - \frac{5}{2} + \frac{5}{2} = 6 + \frac{5}{2}$ [Adding $\frac{5}{2}$ on both sides]

$\Rightarrow \frac{1}{3}x = 6 + \frac{5}{2} \Rightarrow \frac{1}{3}x = \frac{12+5}{2} \Rightarrow \frac{1}{3}x = \frac{17}{2} \Rightarrow 3 \times \frac{1}{3}x = 3 \times \frac{17}{2}$ [Multiplying both sides by 3]

$\Rightarrow x = \frac{51}{2}$

Thus, $x = \frac{51}{2}$ is the solution of the given equation.

Check : Substituting $x = \frac{51}{2}$ in the given equation, we get

L.H.S. $= \frac{1}{3} \times \frac{51}{2} - \frac{5}{2} = \frac{17}{2} - \frac{5}{2} = \frac{12}{2} = 6$ and R.H.S. $= 6$

$\therefore$ L.H.S. = R.H.S. for $x = \frac{51}{2}$

Hence, $x = \frac{51}{2}$ is the solution of the given equation.

4.2 TRANSPOSITION METHOD

Any term of an equation may be taken to the other side with a change in its sign. This process is called transposition.

The transposition method involves the following steps :

Step – I : Obtain the linear equation.

Step – II : Identify the variable (unknown quantity) and constants (numbers).

Step – III : Simplify the L.H.S. and R.H.S. to their simplest forms by removing brackets.

Step – IV : Transpose all terms containing variable on L.H.S. and constant terms on R.H.S. The sign of the terms will change in shifting them from L.H.S. to R.H.S. and vice-versa.

Step – V : Simplify L.H.S. and R.H.S. in the simplest form so that each side contains just one term.

Step – VI : Solve the equation obtained in Step V dividing both sides by the coefficient of the variable on L.H.S.

Example 1

Solve for x : $5x - 2 = 3x - 4$

Solution :

$5x - 2 = 3x - 4 \Rightarrow 5x - 3x = -4 + 2$ (Collecting numbers and variables)

$\Rightarrow 2x = -2 \Rightarrow x = \frac{-2}{2} = -1 \therefore x = -1$

Example 2

Solve for y : $2(y + 3) + 3(y + 1) = 4(2y - 3) + 3$

Solution :

$2(y + 3) + 3(y + 1) = 4(2y - 3) + 3$

$\Rightarrow 2y + 6 + 3y + 3 = 8y - 12 + 3$

$\Rightarrow 2y + 3y - 8y = -12 + 3 - 6 - 3$ (Collecting the variables and numbers)

$\Rightarrow -3y = -18$

$\Rightarrow \frac{-3y}{-3} = \frac{-18}{-3} = 6$ [Dividing both sides by – 3]

$\therefore y = 6$

Example 3

Solve for y : $\frac{y+6}{4} + \frac{y-3}{5} = \frac{5y-4}{8}$

Solution :

$\frac{y+6}{4} + \frac{y-3}{5} = \frac{5y-4}{8}$

$\Rightarrow 10(y + 6) + 8(y - 3) = 5(5y - 4)$ [Multiplying throughout by 40, the LCM of 4, 5 and 8]

$\Rightarrow 10y + 60 + 8y - 24 = 25y - 20$

$\Rightarrow 25y - 10y - 8y = 60 - 24 + 20$ [Collecting variables and numbers]

$\Rightarrow 7y = 56 \Rightarrow \frac{7y}{7} = \frac{56}{7} = 8$ [Dividing both sides by 7]

$\therefore y = 8$

Example 4

Solve for y : $0.18(5y - 4) = 0.5y + 0.8$

Solution :

$0.18 \times 5y - 0.18 \times 4 = 0.5y + 0.8$

$\Rightarrow 0.90y - 0.72 = 0.5y + 0.8$

$\Rightarrow 0.90y - 0.5y = 0.8 + 0.72$ [Collecting variables and numbers]

$\Rightarrow 0.4y = 1.52 \Rightarrow \frac{0.4}{0.4}y = \frac{1.52}{0.4} = \frac{15.2}{4} = 3.8$ [Dividing both sides by 0.4]

$\therefore\ y = 3.8$

4.3 CROSS-MULTIPLICATION METHOD

Step – I : Let the equation be of the form : $\frac{ax+b}{cx+d} = \frac{m}{n}$

Consider the equation $\frac{3x+2}{9x+4} = \frac{3}{5}$, where a = 3, b = 2, c = 9, d = 4, m = 3 and n = 5.

Step – II : Equation is obtained directly by equating the product of numerator of L.H.S. and denominator of R.H.S. to the product of denominator of L.H.S. and numerator of R.H.S.

This can be exhibited as follows : $\frac{3x+2}{9x+4} = \frac{3}{5}$

This process of multiplying the numerator of L.H.S. with the denominator of R.H.S. and equating it to the product of the denominator of L.H.S. with the numerator of R.H.S. is called cross-multiplication.

So, we can convert an equation of the form $\frac{ax+b}{cx+d} = \frac{m}{n}$ to a linear equation $n(ax + b) = m(cx + d)$.

Example 1

Solve $\frac{5x-7}{3x} = 2$ and check your solution.

Solution :

$\frac{5x-7}{3x} = 2 \Rightarrow \frac{5x-7}{3x} = \frac{2}{1}$

$\Rightarrow (5x - 7) = 2 \times 3x \Rightarrow 5x - 7 = 6x$

$\Rightarrow 5x - 6x = 7 \Rightarrow -x = 7$ or $x = -7$

Check : When $x = -7$

L.H.S. $= \frac{5(-7)-7}{3(-7)} = \frac{-35-7}{-21} = \frac{-42}{-21} = 2 =$ R.H.S.

Example 2

Solve : $\frac{3x+5}{2x+1} = \frac{1}{3}$.

Solution :

$\frac{3x+5}{2x+1} = \frac{1}{3} \Rightarrow 3(3x + 5) = (2x + 1)$

$\Rightarrow 9x + 15 = 2x + 1 \Rightarrow 9x - 2x = 1 - 15 \Rightarrow 7x = -14 \Rightarrow x = \frac{-14}{7} = -2.$

Hence, $x = -2$.

Example 3

Solve : $\frac{2x-1}{x+3} + \frac{1-2x}{x-3} = \frac{4-3x}{x^2-9}$

Solution :

$\frac{(2x-1)(x-3)+(1-2x)(x+3)}{x^2-9} = \frac{4-3x}{x^2-9}$ [Using the identity $(a + b)(a - b) = a^2 - b^2$]

$\Rightarrow \frac{[2x^2-6x-x+3]+[x+3-2x^2-6x]}{x^2-9} = \frac{4-3x}{x^2-9}$

$\Rightarrow -12x + 3x = 4 - 6 \Rightarrow -9x = -2 \Rightarrow x = \frac{2}{9}$. Hence, $x = \frac{2}{9}$.

$\Rightarrow 2x^2 - 7x + 3 + 3 - 2x^2 - 5x = 4 - 3x \Rightarrow -12x + 6 = 4 - 3x$

5. APPLICATIONS OF LINEAR EQUATIONS TO PRACTICAL PROBLEMS

A word problem is first translated in the form of an equation containing unknown quantities (variables) and known quantities (numbers or constants) and then we solve it by using any one of the methods. The procedure to translate a word problem in the form of an equation is known as the formulation of the problem. Thus, the process of solving a word problem consists of two parts, namely, formulation and solution.

The following steps should be followed to solve a word problem :

Step – I : Read the problem thoroughly.

Step – II : Note what is given and what needs to be find out.

Step – III : Denote the unknown quantity (i.e., the value to be found) with any (variable), say x, y, z etc.

Step – IV : Translate the statements of the given problem into algebraic equation.

Step – V : Solve the equation for the unknown.

Step – VI : The solution of the equation becomes the value of the unknown.

Example 1

The perimeter of a rectangle is 13 cm and its width is $2\frac{3}{4}$ cm. Find its length.

Solution :

Assume the length of the rectangle to be x cm. The perimeter of the rectangle = 2 × (length + width)

$= 2 \times \left(x + 2\frac{3}{4}\right) = 2\left(x + \frac{11}{4}\right)$

The perimeter is given to be 13 cm. Therefore, $2\left(x+\frac{11}{4}\right)=13 \Rightarrow x+\frac{11}{4}=\frac{13}{2}$

$\Rightarrow x=\frac{13}{2}-\frac{11}{4} \Rightarrow x=\frac{26}{4}-\frac{11}{4}=\frac{15}{4}=3\frac{3}{4}$

$\therefore$ The length of the rectangle is $3\frac{3}{4}$ cm.

Example 2

Divide 34 into two parts in such a way that $\left(\frac{4}{7}\right)^{th}$ of one part is equal to $\left(\frac{2}{5}\right)^{th}$ of the other.

Solution :

Let one part be x. Then, other part is (34 – x). It is given that, $\left(\frac{4}{7}\right)^{th}$ of one part = $\left(\frac{2}{5}\right)^{th}$ of the other part

$\Rightarrow \frac{4}{7}x=\frac{2}{5}(34-x) \Rightarrow 20x=14(34-x) \Rightarrow 20x=14\times 34-14x \Rightarrow 20x+14x=14\times 34$

$\Rightarrow 34x=14\times 34 \Rightarrow x=\frac{14\times 34}{34} \Rightarrow x=14$

Hence, the two parts are x = 14 and (34 – x) = 34 – 14 = 20.

SOLVED EXAMPLES

SE. 1

Solve for x : $5x+\frac{7}{2}=\frac{3}{2}x-14$.

Ans. Taking L.C.M. both sides, we get

$\frac{10x+7}{2}=\frac{3x-28}{2} \Rightarrow 10x + 7 = 3x - 28$

$\Rightarrow 10x - 3x = -28 - 7 \Rightarrow 7x = -35$

$\Rightarrow x=\frac{-35}{7} \Rightarrow x = -5$

SE. 2

Solve for x : $5x - 2(2x - 7) = 2(3x - 1) + \frac{7}{2}$

Ans. $5x - 4x + 14 = 6x - 2 + \frac{7}{2}$

$\Rightarrow x + 14 = 6x - 2 + \frac{7}{2}$

$\Rightarrow x - 6x = -2 + \frac{7}{2} - 14$

$\Rightarrow -\frac{5x}{1}=\frac{-16}{1}+\frac{7}{2} \Rightarrow \frac{-5x}{1}=\frac{-32+7}{2}$

$\Rightarrow \frac{-5x}{1}=\frac{-25}{2} \Rightarrow x=\frac{-25}{2\times(-5)}$

$\Rightarrow x=\frac{-25}{-10} \Rightarrow x=\frac{5}{2}$

SE. 3

Solve for x : $6(3x + 2) - 5(6x - 1) = 6(x - 3) - 5(7x - 6) + 12x$.

Ans. $6(3x + 2) - 5(6x - 1)$

$= 6(x - 3) - 5(7x - 6) + 12x$.

$\Rightarrow 18x + 12 - 30x + 5$

$= 6x - 18 - 35x + 30 + 12x$

$\Rightarrow 18x - 30x - 6x + 35x - 12x$

$= -18 + 30 - 12 - 5$

$\Rightarrow 18x + 35x - 30x - 6x - 12x = 30 - 35$

$\Rightarrow 53x - 48x = -5 \Rightarrow 5x = -5 \Rightarrow x = -1$.

SE. 4

Solve for y : $0.3y + 0.4 = 0.28y + 1.16$

Ans. $0.3y + 0.4 = 0.28y + 1.16$

$\Rightarrow 0.30y - 0.28y = 1.16 - 0.40$

$\Rightarrow 0.02y = 0.76 \Rightarrow y=\frac{0.76}{0.02}=\frac{76}{2}=38$

SE. 5

Solve $\frac{\frac{2}{3}x+1}{x+\frac{1}{4}}=\frac{5}{3}$ and check your solution.

Ans. By cross multiplying the given equation, we get

$3\left(\frac{2}{3}x+1\right)=5\left(x+\frac{1}{4}\right) \Rightarrow 2x+3=5x+\frac{5}{4}$

$\Rightarrow 2x-5x=\frac{5}{4}-3 \Rightarrow -3x=\frac{5-12}{4}$

$\Rightarrow -3x=\frac{-7}{4} \Rightarrow x=\frac{7}{4}\times\frac{1}{3} \Rightarrow x=\frac{7}{12}$

Check : At $x=\frac{7}{12}$,

$$\text{L.H.S.}=\frac{\frac{2}{3}\left(\frac{7}{12}\right)+1}{\frac{7}{12}+\frac{1}{4}}=\frac{\frac{7}{18}+1}{\frac{7+3}{12}}=\frac{\frac{25}{18}}{\frac{10}{12}}$$

$=\frac{25}{18}\times\frac{12}{10}=\frac{5}{3}=\text{R.H.S.}$

SE. 6

Solve for x : $\frac{(x+2)(2x-3)-2x^2+6}{x-5}=2$ and check your solution.

Ans. $\dfrac{2x^2-3x+4x-6-2x^2+6}{x-5}=2$

$\Rightarrow \dfrac{x}{x-5}=\dfrac{2}{1}$ .....(i)

By cross multiplying, we get $x = 2(x - 5)$

$\Rightarrow x = 2x - 10 \Rightarrow x - 2x = -10 \Rightarrow x = 10$

Check : Putting $x = 10$ in equation (i), we get

L.H.S. $= \dfrac{10}{10-5}=\dfrac{10}{5}=2$ = R.H.S.

SE. 7

Solve for y : $\dfrac{y-1}{3}-\dfrac{y-2}{4}=1$

Ans. We have, $\dfrac{y-1}{3}-\dfrac{y-2}{4}=1$

L.C.M. of denominators 3 and 4 on L.H.S. is 12.

So, $4(y - 1) - 3(y - 2) = 12$

$\Rightarrow 4y - 4 - 3y + 6 = 12$

$\Rightarrow 4y - 3y - 4 + 6 = 12$

$\Rightarrow y + 2 = 12 \Rightarrow y = 10$

Thus, $y = 10$ is the solution of the given equation.

SE. 8

Solve for x : $\dfrac{2}{5x}-\dfrac{5}{3x}=\dfrac{1}{15}$

Ans. We have, $\dfrac{2}{5x}-\dfrac{5}{3x}=\dfrac{1}{15}$

$\Rightarrow \dfrac{1}{x}\left[\dfrac{2}{5}-\dfrac{5}{3}\right]=\dfrac{1}{15}\Rightarrow \dfrac{6-25}{15}=\dfrac{x}{15}$

($\because$ L.C.M. of 5 and 3 is 15)

$\Rightarrow 6 - 25 = x \Rightarrow x = -19.$

Hence, $x = -19$ is the solution of the given equation.

SE. 9

Solve for x : $\dfrac{17-3x}{5}-\dfrac{4x+2}{3}=5-6x+\dfrac{7x+14}{3}$

Ans. $3(17 - 3x) - 5(4x + 2) = 15(5 - 6x) + 5(7x + 14)$

(Taking L.C.M. of 5 and 3)

$\Rightarrow 51 - 9x - 20x - 10 = 75 - 90x + 35x + 70$

$\Rightarrow 41 - 29x = 145 - 55x$

$\Rightarrow -29x + 55x = 145 - 41$

$\Rightarrow 26x = 104 \Rightarrow x = \dfrac{104}{26} \Rightarrow x = 4$

Thus, $x = 4$ is the solution of the given equation.

SE. 10

Solve for x : $\dfrac{3x+4}{x+1}=\dfrac{3x+2}{x-1}, x\neq 1, x\neq -1.$

Ans. $(x - 1)(3x + 4) = (x + 1)(3x + 2)$

(By cross multiplication)

$\Rightarrow 3x^2 - 3x + 4x - 4 = 3x^2 + 3x + 2x + 2$

$\Rightarrow 3x^2 + x - 4 = 3x^2 + 5x + 2$

$\Rightarrow 3x^2 + x - 3x^2 - 5x = 2 + 4 \Rightarrow -4x = 6$

$\Rightarrow x = \dfrac{-6}{4} \Rightarrow x = \dfrac{-3}{2}$

Hence, $x = \dfrac{-3}{2}$ is the solution of the given linear equation.

SE. 11

Solve for x : $2+\dfrac{2x-3}{2x+3}=\dfrac{3x+4}{x+2}.$

Ans. $\dfrac{2(2x+3)+(2x-3)}{2x+3}=\dfrac{3x+4}{x+2}$

$\Rightarrow \dfrac{4x+6+2x-3}{2x+3}=\dfrac{3x+4}{x+2} \Rightarrow \dfrac{6x+3}{2x+3}=\dfrac{3x+4}{x+2}$

$\Rightarrow (6x + 3)(x + 2) = (3x + 4)(2x + 3)$

(By cross multiplication)

$\Rightarrow 6x^2 + 3x + 12x + 6 = 6x^2 + 8x + 9x + 12$

$\Rightarrow 6x^2 + 15x + 6 = 6x^2 + 17x + 12$

$\Rightarrow 6x^2 + 15x - 6x^2 - 17x = 12 - 6 \Rightarrow -2x = 6$

$\Rightarrow x = -3$

SE. 12

The difference between two positive integers is 50. The ratio of these integers is 1 : 3. Find these integers.

Ans. Let the one integers be x, so the greater integer is x + 50.

According to question, $\frac{x}{x+50} = \frac{1}{3}$

$\Rightarrow 3 \times x = (x + 50)$ (By cross multiplication)

$\Rightarrow 3x = x + 50 \Rightarrow 2x = 50 \Rightarrow x = 25$

Thus, two integers are x = 25 and x + 50 = 75.

SE. 13

The sum of the digits of a two-digit number is 15. If the number formed by reversing the digits is less than the original number by 27. Find the original number.

Ans. Let the digit at unit's place be x. Then, the digit at ten's place = (15 – x)

So, original number = 10 × (15 – x) + x = (150 – 9x). On reversing the digits, we have x at the ten's place and (15 – x) at the unit's palce.

So, the new number = 10x + (15 – x) = (9x + 15).

Now, (original number) – (new number) = 27

$\Rightarrow (150 - 9x) - (9x + 15) = 27$

$\Rightarrow 135 - 18x = 27 \Rightarrow 18x = (135 - 27)$

$\Rightarrow x = \left(\frac{108}{18}\right) = 6$.

$\therefore$ Unit's digit = x = 6 and ten's digit = (15 – x) = (15 – 6) = 9.

Hence, the original number is 96.

SE. 14

The length of a rectangle exceeds its breadth by 9 cm. If the length and breadth are each increased by 3 cm, the area of the new rectangle will be 84 cm^2 more than that of the given rectangle. Find the length and breadth of the given rectangle.

Ans. Let the breadth of the given rectangle be x cm.

Then its length = (x + 9) cm.

Area of the given rectangle = [x(x + 9)] cm^2.

Now, new breadth = (x + 3) cm and new length = [(x + 9) + 3] cm = (x + 12) cm.

$\therefore$ New area = (x + 3) (x + 12) cm^2.

Now, (Area of new rectangle) – (Area of given rectangle) = 84 cm^2

$\Rightarrow (x + 12)(x + 3) - x(x + 9) = 84$

$\Rightarrow (x^2 + 15x + 36) - (x^2 + 9x) = 84$

$\Rightarrow 6x + 36 = 84 \Rightarrow 6x = 48 \Rightarrow x = \left(\frac{48}{6}\right) = 8$

Thus, breadth = 8 cm and length = (8 + 9) cm = 17 cm.

SE. 15

After 12 years I shall be 3 times as old as I was 4 years ago. Find my present age.

Ans. Let my present age be x years.

After 12 years, my age will be (x + 12) years.

4 years ago, my age was (x – 4) years.

It is given that after 12 years I shall be 3 times as old as I was 4 years ago.

$\therefore x + 12 = 3(x - 4) \Rightarrow x + 12 = 3x - 12$

$\Rightarrow x - 3x = -12 - 12 \Rightarrow -2x = -24 \Rightarrow x = 12$

Thus, my present age is 12 years.

EXERCISE – I

ONLY ONE CORRECT TYPE

1. The value of x, if $13x + 7 = -9 - 3x$ is :

(A) 8 (B) $\frac{1}{5}$

(C) $\frac{1}{8}$ (D) – 1

2. If $-8 + n = \frac{5n}{4}$, then the value of n =

(A) – 40 (B) – 32

(C) – 16 (D) – 4

3. Given that $-0.3k + 2.1 = 0.4k$, the value of k =

(A) 21 (B) 7

(C) 3 (D) – 1

4. If $\frac{1}{7} + \frac{x}{7} = 3$, then the value of x =

(A) 20 (B) 7

(C) 3 (D) 1

5. Given that $\frac{-6p-9}{3} = \frac{2p+9}{5}$, the value of p is

(A) – 4 (B) – 2

(C) 3 (D) 5

6. When a certain number m is divided by 5 and added to 8, the result is equal to 3m subtracted from 4. The value of m is

(A) 2 (B) $\frac{4}{3}$

(C) $\frac{-1}{3}$ (D) $\frac{-5}{4}$

7. Kriti scored 80 marks in her Science test which is $\frac{5}{6}$ of the total marks. The total marks of the test are

(A) 96

(B) 100

(C) 104

(D) 108

8. If the total of four consecutive odd numbers is 40, then smallest number is

(A) 7 (B) 9

(C) 11 (D) 13

9. If p is added to – 5, then the result is 7. The value of 3p is

(A) 6 (B) 18

(C) 36 (D) 4

10. The denominator of a rational number is greater than its numerator by 3. If 3 is subtracted from the numerator and 2 is added to its denominator the new number becomes $\frac{1}{5}$. The original number is

(A) $\frac{-5}{8}$ (B) $\frac{5}{8}$

(C) $\frac{3}{8}$ (D) $\frac{-3}{8}$

11. Solve for x : $\frac{6x^2+13x-4}{2x+5} = \frac{12x^2+5x-2}{4x+3}$

(A) 3 (B) 2

(C) 1 (D) 4

12. Twenty years ago, Tina's age was one-third of what it is now. Tina's present age is

(A) 66 years (B) 30 years

(C) 33 years (D) 36 years

13. A number consists of two digits whose sum is 12. If 18 is added to the number, its digits get reversed. The number is

(A) 28 (B) 35

(C) 57 (D) 34

14. The number which should be added to twice the rational number $\frac{-7}{3}$ to get $\frac{3}{7}$ is

(A) $\frac{105}{21}$ (B) $\frac{100}{21}$

(C) $\frac{107}{21}$ (D) $-\frac{89}{21}$

15. Madhulika thought of a number, doubled it and added 20 to it. On dividing the resulting number by 25, she gets 4. What is the number ?
(A) 80 (B) 20
(C) 40 (D) 50

16. Two numbers are in the ratio 3 : 5. If the sum of the numbers is 184, find the difference between the numbers.
(A) 24 (B) 28
(C) 48 (D) 46

17. Divide the share 64 between Seeta and Geeta such that 3 times Seeta's share is greater than 4 times Geeta's share by 10. What is Geeta's share ?
(A) 26 (B) 38
(C) 36 (D) 28

18. Solve for x : $\frac{2x-(7-5x)}{9x-(3+4x)}=\frac{7}{6}$
(A) 3 (B) 2
(C) – 3 (D) $\frac{1}{3}$

19. Three-fourths of a number is 60 more than its one-third. The number is
(A) 108 (B) 84
(C) 144 (D) 116

20. If 20 is added to four times a certain number, the result is 5 less than five times the number. The number is
(A) 10
(B) 15
(C) 20
(D) 25

21. Solve for x : $\frac{2x-7}{4}-\left(\frac{1-x}{6}\right)=17$
(A) $\frac{8}{227}$ (B) $\frac{227}{8}$
(C) 227 (D) 8

22. In a two digit number, the number at ten's place is double of the number at unit's place. If we exchange the numbers mutually then the number decreases by 18, then the number is
(A) 24 (B) 36
(C) 39 (D) 42

23. Find the number whose fifth part increased by 5 is equal to its fourth part diminished by 5.
(A) 200 (B) 300
(C) 100 (D) 400

24. The father is 24 years older than his daughter. In 4 years, he will be thrice as old as his daughter. Find the present age of the daughter.
(A) 10 years (B) 7 years
(C) 9 years (D) 8 years

PARAGRAPH TYPE

PASSAGE # I

A rectangle has a perimeter of 60 cm.

1. If length and breadth of rectangle is (k + 4) cm and (3k – 2) cm respectively, then value of k is
(A) 7 (B) 9
(C) 12 (D) 29

2. The length and breadth of the rectangle is
(A) 11, 7 (B) 11, 19
(C) 19, 20 (D) 23, 11

3. If perimeter of rectangle is equal to perimeter of square, then side of square is
(A) 60 cm (B) 18 cm
(C) 225 cm (D) 15 cm

PASSAGE # II

Linear equation in one variable is an equation of the form ax + b = 0 or ax = – b, where a, b are the real numbers such that a ≠ 0. The value of variable which satisfies a given linear equation is

known as its solution. If $ax = -b$ is linear equation, then $x = \frac{-b}{a}$ is its solution. Geometrically, a linear equation is represented by a straight line. Based on above passage, answer the following questions:

4. $ax + b = 0$ represents an equation where $a \neq 0$. If $a = 0$, then it is
   (A) A linear equation
   (B) An algebraic expression
   (C) No more an equation
   (D) None of these
5. The number of solutions of a linear equation in one variable is/are
   (A) One (B) Three
   (C) Two (D) Cannot be determined
6. Geometrical representation of a linear equation is
   (A) Any curve (B) A straight line
   (C) Circle (D) Triangle

TRUE / FALSE TYPE

1. An equation, in which the maximum degree of a term is one, is called a linear equation.
2. We cannot subtract the same number on both sides of the equation.
3. We can multiply both sides of the equation by same non zero number.
4. We can divide both sides of the equation by same non-zero number.
5. If we transpose any term of the equation from one side to other, its sign gets changed.
6. $8x - 3 = 25 + 174$ then x is a rational number.
7. A linear equation in one variable has two solution.
8. In a linear equation the highest power of the variable appearing in the equation is one.
9. If $\frac{x}{11} = 15$ then $x = \frac{11}{15}$.
10. If $6x = 18$ then $18x = 54$.

MATCH THE COLUMN TYPE

1. Match the following :

List – I	List – II
(P) $\frac{2}{3}$ of a number is 20 less than the original number, then the number is	(i) 30
(Q) Four-fifth of a number is 10 more than two-third of that number, then the number is	(ii) 200
(R) A number whose double is 45 greater than its half, then the number is	(iii) 75
(S) A number whose fifth part increased by 5 is equal to its fourth part diminished by 5, then the number is	(iv) 60

(A) (P)→(iii), (Q)→(iv), (R)→(i), (S)→(ii)
(B) (P)→(iv), (Q)→(iii), (R)→(ii), (S)→(i)
(C) (P)→(iv), (Q)→(iii), (R)→(i), (S)→(ii)
(D) (P)→(i), (Q)→(ii), (R)→(iv), (S)→(iii)

2. Match the following :

List – I	List – II
(P) An expression in variable x is	(i) $2y = 0$
(Q) An equation with negative root is	(ii) $7x + 9 = 23$
(R) An equation with positive root is	(iii) $\frac{x}{5} + \frac{1}{2} = \frac{-2}{3}$
(S) An equation with zero root is	(iv) $\frac{x}{2} - 6$

(A) (P)→(iv), (Q)→(iii), (R)→(ii), (S)→(i)

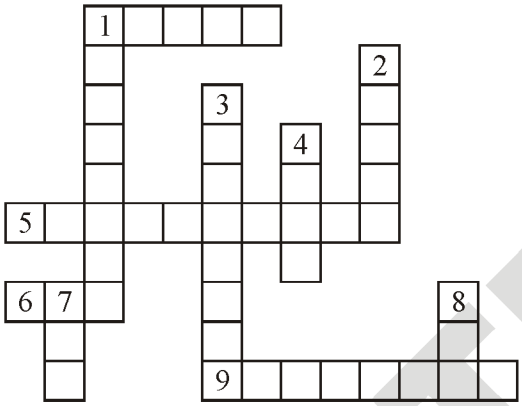
(B) (P)→(i), (Q)→(ii), (R)→(iii), (S)→(iv)

(C) (P)→(iii), (Q)→(i), (R)→(ii), (S)→(iv)

(D) (P)→(iv), (Q)→(ii), (R)→(iii), (S)→(i)

CROSS WORD PUZZLE

Complete the following word puzzle with the help of clues given below :



Across

1. Solving an equation means determining the ____ of the variable which satisfies it. [5]
5. While going upstream in a river by a boat, speed of the stream is _____ from the speed of the boat. [10]
6. In a linear equation, the maximum power of the variable is _____. [3]
9. A mathematical statement of equality which contains one or more variable is called an _____. [8]

Down

1. A linear equation in one _____ has one and only one solution. [8]
2. While going downstream in a river by a boat, the speed of the stream is _____ to the speed of the boat. [5]
3. When we transpose positive quantity from one side of equality to the other side, then the sign of the positive quantity changes to _____. [8]
4. The sign of equality will not change if we add the same quantity to _____ sides of the equation. [4]
7. $3x^3 + 5x^2 + 7x + 3 = 0$ is _____ a linear equation. [3]
8. $3x + 4y = 12$ contains _____ different variables. [3]

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Solve for x : $5x - 17 = 2x - 8$
2. Solve : $7(x + 1) - 9 = 16$.
3. Find the solution of $\frac{2(x+1)}{3} = 18(x-1)$.
4. Solve for x : $\frac{2x+4}{3x+6} = 4$
5. Solve : $\frac{x}{2} + \frac{x}{3} - \frac{x}{4} = 7$
6. If 3 less than a number is 10. Find the number.
7. Find the length of a rectangle having breadth 2 metres and area 24 square metres.
8. Salim thought of a number, doubles it and subtracted 11 from it. The result was 59. Find the number.
9. Two numbers are in the ratio 5 : 8. If the sum of numbers is 182, then find the numbers.
10. Solve : $\frac{2x+3}{3x+4} = \frac{14}{20}$

SHORT ANSWER TYPE

1. Solve for x : $0.16(5x - 2) = 0.4x + 7$
2. Find the value of x if $\frac{x+b}{a-b} = \frac{x-b}{a+b}$.
3. Find the value of x if $\frac{x}{4} + \frac{x}{6} = x - 7$.
4. If $x = p - 2$ and $\frac{x}{p} - \frac{x+1}{p} = 1$, then find p.
5. Solve for x : $\frac{6x+7}{3x+2} = \frac{4x+5}{2x+3}$
6. Two years ago, Dilip was three times as old as his son and two years hence, twice his age will be equal to five times that of his son. Find their present ages.
7. A father is 7 times as old as his son. Two years ago, the father was 13 times as old as his son. What are their present ages ?
8. Raj is three years older than Ravi. Six years ago, Raj's age was four times of Ravi's age. What are their present ages ?

LONG ANSWER TYPE

1. Three prizes are to be distributed in a quiz contest. The value of the second prize is five-sixths the value of the first prize and the value of the third prize is four-fifths that of the second prize. If the total value of three prizes is Rs. 150, then find the value of each prize.
2. Solve for x : $\frac{(4+x)(5-x)}{(2+x)(7-x)} = 1$
3. Solve for x : $\frac{1}{x+1} + \frac{1}{x+2} = \frac{2}{x+10}$
4. Solve for x :

   (i) $\frac{(x+2)(2x-3) - 2x^2 + 6}{x-5} = 2$

   (ii) $\frac{2-7x}{1-5x} = \frac{3+7x}{4+5x}$
5. A number is 56 greater than the average of its third, quarter and one-twelfth. Find the number.

NUMERICAL PROBLEMS

1. The sum of three consecutive integers is 33. The difference between largest and smallest integer is
2. The value of x if $5x - (3x - 1) = x - 4$ is $-k$. The value of k is
3. Twice number increased by 8 is 20. The number is

4. If $\frac{3x-4}{4x+3}=2$, then x is – a. The value of 6a is

5. The value of x, if $\frac{3}{x+8}=\frac{4}{6-x}$ is $-\frac{k}{2}$. The value of k is

6. The value of x if $\left(\frac{2x-3}{4}\right)-\left(\frac{2x-1}{3}\right)=1$ is $\frac{-k}{2}$. The value of k is

7. A is twice as old as B. Three years ago, A's age was three times as that of B. Find the age of B.

8. The difference between two positive integers is 36. The quotient, when one integer is divided by the other, is 4. The largest integer is

9. Find the value of s if 7 (s – 4) = – 3 + 2s.

10. Find the value of m if $\frac{6m+4}{4-m}=8$.

ANALYTICAL PROBLEMS & BRAIN TEASER

1. Solve for x : $\frac{2}{3}(4x-1)-\left(4x-\frac{1-3x}{2}\right)=\frac{x-7}{2}$

(A) 0 (B) 1

(C) 2 (D) 5

2. Solve for x : $\frac{2}{5}(4x-1)-\left[4x-\left(\frac{1-3x}{2}\right)\right]=\frac{x-7}{2}$

(A) 0 (B) $\frac{9}{11}$

(C) $\frac{2}{11}$ (D) – 1

3. The number '51' is divided into two parts such that $\left(\frac{4}{7}\right)^{th}$ of the first part is equal to $\left(\frac{5}{4}\right)^{th}$ of the second part. The numbers are

(A) 18, 33 (B) 29, 22

(C) 36, 15 (D) 35, 16

4. Vidhushi is 8 times as old as her grandson. Four years ago, Vidhushi was 12 times as old as her grandson. Find the difference between their present ages.

(A) 88 years (B) 77 years

(C) 11 years (D) 99 years

5. Sum of the digits of two digit number is 11. The number obtained by interchanging the digits is 27 more than the original number. The original number is :

(A) 74 (B) 47

(C) 56 (D) 65

6. State 'T' for true and 'F' for false.

(i) y^2+9 is a linear equation in one variable.

(ii) The terms linear equation and linear expression are same.

(iii) If x is an even number, then the next odd number is (2x + 1).

(iv) If both sides of an equation is to be divided by the same (non-zero) positive number, then there is a change in equality.

(v) 1 is the solution of $\frac{2x+1}{3x+5}=\frac{3}{8}$.

	(i)	(ii)	(iii)	(iv)	(v)
(A)	T	F	T	F	T
(B)	T	T	T	T	T
(C)	F	F	F	F	T
(D)	F	T	F	F	F

7. Sum of the digits of a two digit number is 9. The number obtained by interchanging the digits is 18 more than twice the original number. The original number is :

(A) 72 (B) 27

(C) 36 (D) 63

8. The tens digit of a two-digit number exceeds its unit digit by 6. The number itself is 10 times the sum of its digits. Find the number.

(A) 82 (B) 90

(C) 93 (D) 60

9. The numerator and the denominator of a rational number are in the ratio 5 : 7. When 6 is added to both the numerator and denominator, the ratio becomes 4 : 5. What is the rational number ?

(A) $\frac{7}{5}$ (B) $\frac{5}{7}$

(C) $\frac{2}{5}$ (D) $\frac{13}{14}$

10. A father is 7 times as old as his son. Two years ago, the father was 13 times as old as his son. Father's present age is :

(A) 24 years (B) 28 years

(C) 30 years (D) 32 years

Space for Notes :

EXERCISE – III

PREVIOUS YEAR QUESTIONS (NTSE)

1. The sum of three numbers is 98. The ratio of the first to the second is $\frac{2}{3}$ and the ratio of the second to the third is $\frac{5}{8}$. The second number is :

(NSTSE – 2010)

(A) 15 (B) 20
(C) 30 (D) 32

2. What is the value of x in the given equation ?

$$\frac{(3x+1)}{16}+\frac{(2x-3)}{7}=\frac{(x+3)}{8}+\frac{(3x-1)}{14}$$

(IMO – 2010)

(A) 2 (B) 4
(C) 3 (D) 5

3. Jasmine is a much better tennis player than Reshma. They decide to have a contest. Every time Jasmine wins a game, she will earn 3 points and every time Reshma wins a game, she will earn 5 points. If they play 48 games and the final score is tied, how many games did Jasmine win ?

(IMO – 2010)

(A) 50 (B) 40
(C) 30 (D) 18

4. Madhuri is on the fourth floor of a building. Her car is in the parking garage three levels below the ground floor. She gets in the elevator and travels from the fourth floor above ground level to the third floor below ground level. How many floors did she travel ?

(IMO – 2010)

(A) 3 (B) 1
(C) 4 (D) 7

5. Mrs. Ravina needs to take a taxi to the doctor's clinic. The taxi ride costs Rs. 13.00 for the first km and Rs. 6 for each km thereafter. How much does Mrs. Ravina pay for a 2.3 km taxi ride ?

(IMO – 2010)

(A) Rs. 25 (B) Rs. 28
(C) Rs. 32 (D) Rs. 30

6. Of the three numbers, the first is twice the second and is half the third. If the average of three numbers is 56, the three numbers in order are

(Aryabhatta – 2011)

(A) 48, 24, 96 (B) 48, 36, 96
(C) 48, 12, 14 (D) 24, 12, 48

7. The sum of a two digit number and the number obtained by interchanging the digits of the number is 121. If the digits differ by 5, then find the number

(Aryabhatta – 2011)

(A) 38, 83 (B) 27, 72
(C) 39, 93 (D) 61, 16

8. The ages of Mira, Tina and Sania are in the ratio 6 : 4 : 7 respectively, if the sum of their ages is 34 years, what is Sania's age ?

(IMO – 2011)

(A) 12 years (B) 10 years
(C) 18 years (D) 14 years

9. $\left(\frac{3}{4}\right)^{th}$ of a number is 20 more than half of the same number. The required number is _____.

(IMO – 2011)

(A) 50 (B) 180
(C) 90 (D) 80

10. Mohan gets 3 marks for each correct answer and loses 2 marks for each wrong answer. He attempts 30 problems and obtains 40 marks. The number of problems solved correctly is.

(IMO – 2011)

(A) 10 (B) 15

(C) 20 (D) 25

11. Neeta's volvo bus takes 50 boys to a field trip. Some of them take Rs. 20 tickets while the rest take Rs. 45 tickets. If the total cost of tickets purchased is Rs. 2000, how many boys took the tickets of Rs. 20 each ?

(NSTSE – 2011)

(A) 7 (B) 10

(C) 12 (D) 15

12. For a journey the cost of a child ticket is $1/3^{rd}$ of the cost of an adult ticket. If the cost of the tickets for 4 adults and 5 children is Rs. 85, the cost of a child ticket is

(IMO – 2012)

(A) Rs. 5

(B) Rs. 6

(C) Rs. 10

(D) Rs. 15

13. The ratio of present ages of Rahul and Deepesh is 3 : 5. 10 years later this ratio becomes 5 : 7. What is the present age of Deepesh ?

(IMO – 2012)

(A) 20 years

(B) 50 years

(C) 25 years

(D) 40 years

14. $\frac{x}{x-a}+\frac{x}{x-b}=2$ find x

(NSTSE – 2013)

(A) $\frac{a}{b}$ (B) ab

(C) $\frac{2ab}{a+b}$ (D) 2ab

15. If x + y = 6 and 3x – y = 4, find the value of x – y.

(NSTSE – 2014)

(A) – 1 (B) 0

(C) 2 (D) 4

Answer Key

EXERCISE I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	B	C	A	B	D	A	A	C	B	C	B	C	C	C
16	17	18	19	20	21	22	23	24						
D	A	A	C	D	B	D	A	D						

PARAGRAPH

1. A **2.** B **3.** D **4.** C **5.** A **6.** B

TRUE / FALSE

1. T **2.** F **3.** T **4.** T **5.** T **6.** T **7.** F
8. T **9.** F **10.** T

MATCH THE COLUMN

1. C **2.** A

CROSSWORD PUZZLE

1. Value **2.** Added **3.** Negative **4.** Both **5.** Subtracted **6.** One
7. Not **8.** Two **9.** Equation

EXERCISE II

VERY SHORT ANSWER TYPE

1. 3 **2.** $\frac{18}{7}$ **3.** $\frac{14}{3}$ **4.** – 2 **5.** 12 **6.** 13 **7.** 12 m

8. 35 **9.** 70, 112 **10.** 2

SHORT ANSWER TYPE

1. 18.3 **2.** – a **3.** 12 **4.** – 1 **5.** $\frac{-11}{9}$ **6.** 14 yrs, 38 yrs

7. 4 yrs, 28 yrs **8.** Raj – 10 yrs, Ravi – 7 yrs

LONG ANSWER TYPE

1. Rs. 60, Rs. 50, Rs. 40 **2.** $\frac{3}{2}$ **3.** $\frac{-26}{17}$ **4.** (i)10,(ii) $\frac{1}{2}$ **5.** 72

NUMERICAL PROBLEMS

1. 2 **2.** 5 **3.** 6 **4.** 12 **5.** 4 **6.** 17 **7.** 6

8. 48 **9.** 5 **10.** 2

ANALYTICAL PROBLEMS & BRAIN TEASER

1. B **2.** B **3.** D **4.** B **5.** B **6.** C **7.** B

8. D **9.** B **10.** B

EXERCISE III

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	D	C	D	A	A	A	D	D	C	B	A	C	C	A

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : INTEGERS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

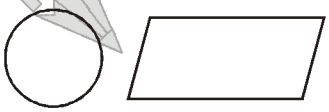
UNDERSTANDING QUADRILATERALS

1 INTRODUCTION

'Poly' means many and 'gon' means sides. So a polygon is a closed figure of many sides. A polygon of 'n' sides is also called n-gon. Polygon can be classified according to the number of sides like triangle (3 sides), Quadrilateral (4 sides). Pentagon (5 sides).

2 CURVE

A plane figure formed by joining a number of points without lifting a pencil from the paper and without retracing any portion of the drawing other than single points.

Open Curves	Closed Curves	Simple Closed Curves
A curve which does not cut itself is called an open curve. 	A figure which begins and ends at the same point is called a closed figure. 	A closed figure which does not intersect itself is called a simple closed figure. 

3 POLYGON

A simple closed curve which is made up of line segments only is called a polygon.

3.1 ELEMENTS OF A POLYGON

(i) Adjacent Sides : Any two sides with a common end-point (vertex) are called adjacent sides of the polygon.

(ii) Adjacent Vertices : The end-points of the same side of a polygon are known as adjacent vertices.

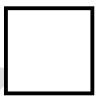
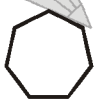
(iii) Diagonals : The line segments obtained by joining two non-adjacent vertices of a polygon are called the diagonals of the polygon.

Focus Point

Note : The number of diagonals of a polygon with n sides is given by $\frac{n(n-3)}{2}$.

3.2 RECOGNISING AND NAMING POLYGON

We classify polygons according to the number of sides (or vertices) they have.

Number of sides	Name of polygon	Number of vertices	Number of diagonals	Figures
3	Triangle	3	0	
4	Quadri-lateral	4	2	
5	Pentagon	5	5	
6	Hexagon	6	9	
7	Heptagon	7	14	
8	Octagon	8	20	
9	Nonagon	9	27	
10	Decagon	10	35	

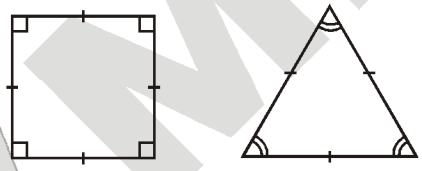
4 CONVEX AND CONCAVE POLYGON

Convex Polygon	Concave Polygon
A convex polygon is a polygon in which no line segment between the points on the boundary ever goes outside the polygon.  Convex polygons	Polygon that have portions of their atleast one diagonal lies in their exteriors is called concave polygon.  Concave polygons

5 REGULAR AND IRREGULAR POLYGONS

5.1 REGULAR POLYGON

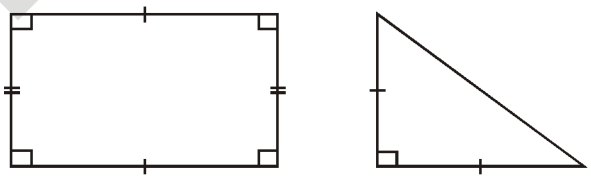
A regular polygon is a polygon whose all sides and all angles are equal.



Ex. 1 A square, an equilateral triangle etc.

5.2 IRREGULAR POLYGON

A polygon which is not a regular polygon is called an irregular polygon.



Ex. 2 Rectangle, isosceles triangle etc.

Focus Point

(1) The sum of the interior angles of an n-sided polygon is $(2n-4) \times 90°$.
(2) The sum of all the exterior angles of a polygon is 360°.

(3) Each interior angle of regular polygon of n-sided is $\left(\frac{(2n-4)90}{n}\right)^{\circ}$.

(4) Each exterior angle of a regular polygon n-sided is $\left(\frac{360}{n}\right)^{\circ}$.

(5) The number of sides n of a regular polygon whose exterior angle x° is $= \left(\frac{360°}{x}\right)$.

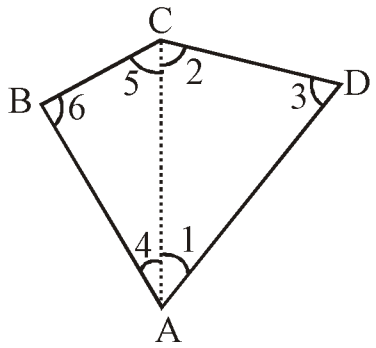
6 ANGLE SUM PROPERTY OF QUADRILATERAL

6.1 SUM OF THE ANGLES OF A QUADRILATERAL IS 360°

Statement : Sum of the angles of a quadrilateral is 360°.

Given : ABCD is a quadrilateral.

To prove : $\angle A + \angle B + \angle C + \angle D = 360°$



Proof : In ΔABC, $m\angle 4 + m\angle 5 + m\angle 6 = 180°$

[Using angle sum property of a triangle]

Also, in ΔADC, $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Sum of the measures of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ of quadrilateral

$m\angle 1 + m\angle 4 + m\angle 6 + m\angle 5 + m\angle 2 + m\angle 3 = 180° + 180°$

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360°$

Thus, sum of the measures of four angles of a quadrilateral is 360°.

Ex. 3 Which of the following groups of angles can be the angle of a quadrilateral ?

(a) 70°, 80°, 90°, 120°

(b) 75°, 65°, 115°, 125°,

Sol. (a) $\because$ Sum of the angles = 70° + 80° + 90° + 120° = 360°

It is a quadrilateral.

(b) $\because$ Sum of the angles = 70° + 65° + 115° + 125° = 380°, which is more than 360°

$\therefore$ It is not a quadrilateral.

Ex. 4 Three angles of a quadrilateral are 30°, 150° and 100°. Find the fourth angle.

Sol. Sum of the angles of a quadrilateral = 360°

Sum of the given angles = 30° + 150° + 100° = 280°

$\therefore$ Fourth angle = 360° – 280° = 80°

Ex. 5 The angles of a quadrilateral are in the ratio 5 : 3 : 9 : 7. Find the angles.

Sol. Let the angles be 5x, 3x, 9x and 7x.

$\therefore$ 5x + 3x + 9x + 7x = 360°

[Angle sum property of a quadrilateral]

$\Rightarrow$ 24x = 360°

$\Rightarrow x = \frac{360°}{24} = 15°$

$\therefore$ Angles are 5x = 5 × 15° = 75°, 3x = 3 × 15° = 45°

9x = 9 × 15° = 135° and 7x = 7 × 15° = 105°.

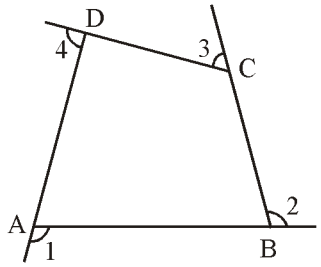
6.2 EXTERIOR ANGLE PROPERTY

Statement : If the sides of a quadrilateral are produced in order, the sum of four exterior angles so formed is 360°.

Proof : Let the sides of a quadrilateral ABCD be produced in order as shown in figure forming exterior angles $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$

Since $\angle 1$ and $\angle DAB$ forms a linear pair and the sum of the angles of a linear pair is 180°.

$\therefore \angle 1 + \angle DAB = 180°$ ... (i)



Similarly, we have

$\angle 2 + \angle CBA = 180°$ ... (ii)

$\angle 3 + \angle DCB = 180°$ ... (iii)

and $\angle 4 + \angle ADC = 180°$ ... (iv)

Adding (i), (ii), (iii) and (iv), we get

$(\angle 1 + \angle 2 + \angle 3 + \angle 4) + (\angle DAB + \angle CBA + \angle DCB + \angle ADC) = 180° + 180° + 180° + 180°$

$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 + 360° = 720°$ [$\therefore \angle DAB + \angle CBA + \angle DCB + \angle ADC = 360°$]

$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 = 720° - 360° = 360°$

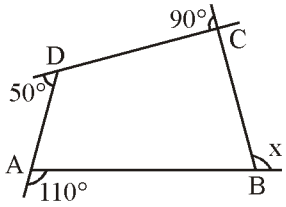
This is true whatever be the number of sides of polygon.

So, the statement can be generalised to :

The sum of the measures of the exterior angles of any polygon is 360°.

Remark : Each exterior angle of a regular polygon of n sides is equal to $\left(\frac{360°}{n}\right)$.

Ex. 6 Find the value of x in the given figure.



Sol. We know that the sum of the measures of exterior angles of a polygon is 360°.

$\therefore$ $x + 90° + 50° + 110° = 360°$

$\Rightarrow x + 250° = 360°$ $\Rightarrow x = 360° - 250°$ $\Rightarrow x = 110°$.

Ex. 7 How many sides does a regular polygon have, if the measure of an exterior angle is 24° ?

Sol. Let there be n sides of the regular polygon. Then, the measure of each exterior angle is $\left(\frac{360°}{n}\right)$.

$\therefore \frac{360°}{n} = 24° \Rightarrow n = \left(\frac{360°}{24°}\right) = 15$. So, the polygon has 15 sides.

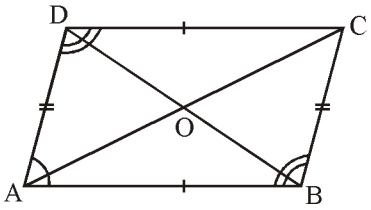
7 DIFFERENT TYPES OF QUADRILATERALS

7.1 PARALLELOGRAM

A quadrilateral is a parallelogram if its both pairs of opposite sides are equal and parallel.

Properties of a Parallelogram

In a parallelogram,



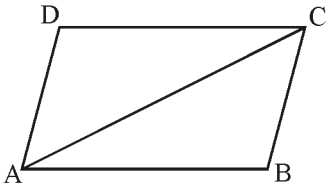
1. The opposite sides are equal i.e., AB = CD and BC = AD
2. The opposite angles are equal i.e., $\angle A = \angle C$ and $\angle B = \angle D$
3. The diagonals bisect each other, i.e., AO = OC and BO = OD
4. Each diagonal divides a parallelogram into two congruent triangles.

i.e., $\Delta ABC \cong \Delta CDA$ and $\Delta BCD \cong \Delta DAB$.

Let us prove all these properties.

1. Opposite sides of a parallelogram are equal.

Let us consider a parallelogram ABCD in which AB || CD and BC || AD. Join AC.

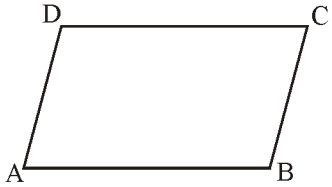


Now, in ΔABC and ΔCDA

$\angle CAB = \angle ACD$	[Alternate angles]
$\angle ACB = \angle CAD$	[Alternate angles]
AC = CA	[Common]
$\therefore \Delta ABC \cong \Delta CDA$	[ASA congruency]
Hence, AB = CD and BC = AD	[C.P.C.T.]

2. Opposite angles of a parallelogram are equal.

Let us consider a parallelogram ABCD in which AB || CD and BC || AD.



Now, AB || DC and AD is a transversal

$\angle A + \angle D = 180°$ ... (i)

[Sum of the measures of interior angles on the same side of a transversal is 180°]

Again, AD || BC and AB is a transversal.

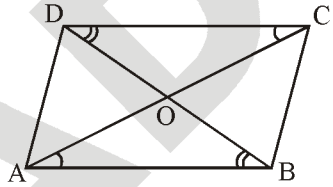
$\therefore \angle A + \angle B = 180°$ ... (ii)

From (i) and (ii), we have

$\therefore \angle D = \angle B$

Similarly, we can prove that $\angle A = \angle C$.

3. Diagonals of a parallelogram bisect each other.



Let us consider a parallelogram ABCD in which AB || CD and AD || BC.

Join AC and BD to intersect at O.

In ΔAOB and ΔCOD, we have

$\angle DCO = \angle OAB$ [Alternate angles]

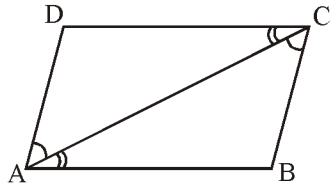
$\angle CDO = \angle OBA$ [Alternate angles]

$AB = CD$ [Opposite sides of a parallelogram]

$\therefore \Delta AOB \cong \Delta COD$ [ASA congruency]

Hence, $AO = OC$ and $BO = OD$ [C.P.C.T.]

4. Each diagonal of a parallelogram divides it into two congruent triangles.



Let us consider a parallelogram ABCD where AB || CD and AD || BC.

Join AC.

In ΔABC and ΔCDA, we have

$\angle DCA = \angle BAC$ [Alternate angles]

$\angle DAC = \angle ACB$ [Alternate angles]

$AC = CA$ [Common]

$\therefore \Delta ABC \cong \Delta CDA$ [ASA congruency]

Similarly, by drawing the other diagonal BD, we can prove that $\Delta ABD \cong \Delta CDB$.

Ex. 8 If two adjacent angles of a parallelogram are equal. What is the measure of each angle ?

Sol. Let the measure of two adjacent and equal angles be x. Since the sum of any two consecutive angles of a parallelogram is 180°.

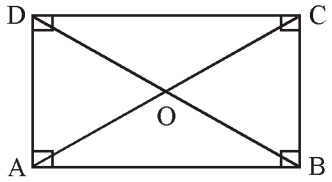
$$\therefore x + x = 180° \Rightarrow 2x = 180° \Rightarrow x = \frac{180°}{2} = 90°$$

Hence, the measure of each angle is 90°.

7.2 RECTANGLE

A parallelogram whose each angle is a right angle is called rectangle.

Properties of a Rectangle.



1. The opposite sides are equal i.e., AB = CD and BC = AD
2. The opposite angles are equal i.e., $\angle A = \angle C$ and $\angle B = \angle D$
3. The diagonals bisect each other, i.e., AO = OC and BO = OD
4. Each diagonal divides a parallelogram into two congruent triangles.

i.e., $\Delta ABC \cong \Delta CDA$ and $\Delta BCD \cong \Delta DAB$.

5. Each angle of a rectangle is a right angle.
6. The diagonals of a rectangle are equal.

1. Each angle of a rectangle is a right angle.

Let us consider a rectangle ABCD is which $\angle A = 90°$ ... (i)

Also, ABCD is a parallelogram.

$\therefore$ AB = DC, BC = AD and $\angle A = \angle C = \angle B = \angle D$

So, $\angle C = 90°$ [From (i)] ... (ii)

Now, AB || DC and AD is the transversal.

$\therefore \angle A + \angle D = 180° \Rightarrow \angle D = 90°$ ... (iii)

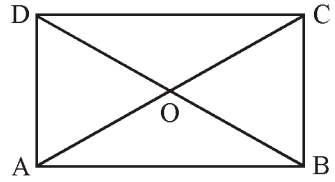
Also, $\angle B = \angle D \Rightarrow \angle B = 90°$ ... (iv)

Hence , $\angle A = \angle B = \angle C = \angle D = 90°$ [From (i),(ii), (iii) & (iv)]

2. The diagonals of a rectangle are equal.

Let us consider a rectangle ABCD with diagonals AC and BD.

In ΔABC and ΔBAD, we have



$\angle ABC = \angle BAD$ [Right angle]

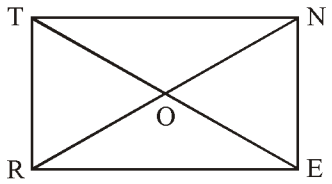
AB = AB [Common]

AD = BC [Opposite sides of rectangle]

So, $\Delta ABC \cong \Delta BAD$ [SAS congruency]

So, AC = BD [C.P.C.T.]

Ex. 9 RENT is a rectangle. Its diagonals meets at O. Find x if OR = 2x + 3 and OT = 3x + 2.



Sol. $OT = \frac{1}{2}$ TE [As diagonals of a rectangle bisect each other]

Also, $OR = \frac{1}{2}$ RN, Now, TE = RN

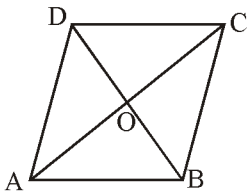
[Diagonals of a rectangle are of equal measure]

So, OT = OR $\Rightarrow 3x + 2 = 2x + 3 \Rightarrow x = 1$

7.3 RHOMBUS

A parallelogram having all sides equal is called a rhombus.

Properties of a Rhombus



1. All sides are equal i.e. AB = BC = CD = DA.
2. The opposite angles are equal i.e., $\angle A = \angle C$ and $\angle B = \angle D$
3. The diagonals bisect each other, i.e., AO = OC and BO = OD
4. Each diagonal divides a parallelogram into two congruent triangles.

i.e., $\Delta ABC \cong \Delta CDA$ and $\Delta BCD \cong \Delta DAB$.

5. Diagonals of a rhombus bisect each other at right angles.

1. Diagonals of a rhombus bisect each other at right angles.

Let us consider a rhombus ABCD with diagonals AC and BD.

In ΔAOB and ΔBOC, we have

AB = BC	[Sides of a rhombus]
BO = OB	[Common]
AO = OC	[Diagonals bisect each other]
$\therefore$ $\Delta AOB \cong \Delta COB$	[SSS congruency]
Hence, $\angle AOB = \angle BOC$	[C.P.C.T.]
But, $\angle AOB + \angle BOC = 180°$	[Linear pair]

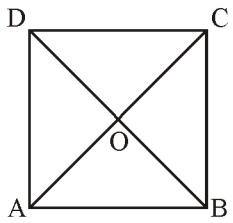
$2\angle AOB = 180°$

$\therefore$ $\angle AOB = \angle BOC = 90°$.

7.4 SQUARE

A square is a rhombus with each of its angles equal to 90°.

Properties of a Square



1. All sides are equal i.e., AB = BC = CD = DA
2. All angles are equal i.e., $\angle A = \angle B = \angle C = \angle D$
3. The diagonals bisect each other, i.e., AO = OC and BO = OD
4. Each diagonal divides a square into two congruent triangles.

i.e., $\Delta ABC \cong \Delta CDA$ and $\Delta BCD \cong \Delta DAB$.

5. Diagonals of a rhombus bisect each other at right angles.
6. Diagonals of a square are equal.

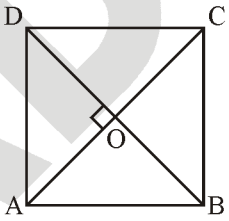
1. Diagonals of a square are equal.

Let us consider a square ABCD with diagonals AC and BD.

In ΔABD and ΔBAC, we have

$AD = BC$	[Sides of square]
$AB = BA$	[Common]
$\angle DAB = \angle ABC$	[Each 90°]
$\therefore\ \Delta ABD \cong \Delta BAC$	[SAS congruency]
$AC = BD$	[C.P.C.T.]

Ex. 10 If the length of a diagonal of a square is 6 cm. Find the length of the side of the square.



Sol. If AC = 6 cm

$\Rightarrow AO = 3$ cm and $DO = 3$ cm

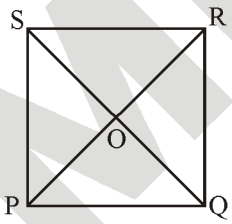
As diagonals of square are perpendicular to each other. In right ΔAOD, we have

$AO^2 + DO^2 = AD^2$

[By Pythagoras Theorem]

$\Rightarrow 3^2 + 3^2 = AD^2 \quad \Rightarrow 18 = AD^2 \quad \Rightarrow \sqrt{18} = AD \quad \therefore\ AD = 3\sqrt{2}$ cm.

Ex. 11 PQRS is a square. PR and SQ interesect at O. State the measure of $\angle POQ$.



Sol. Since the diagonals of a square intersect at right angles. Therefore, $\angle POQ = 90°$

7.5 KITE

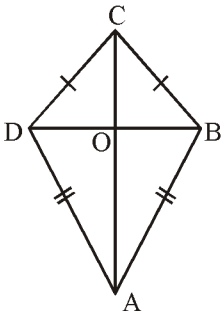
A quadrilateral in which two pairs of adjacent sides are equal is called a kite.

It has unequal opposite sides.

In the adjoining figure, ABCD is a kite.

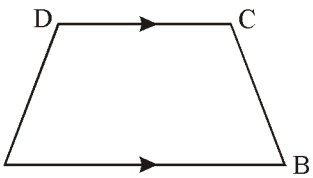
AB = AD and BC = CD

Diagonals AC and BD intersect each other at right angles.



7.6 TRAPEZIUM

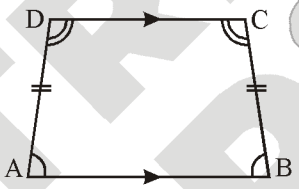
A quadrilateral in which one pair of opposite sides is parallel and the other pair of opposite sides is non-parallel is called trapezium.



In the given figure, ABCD is a trapezium in which AB || CD whereas AD and BC are its non-parallel sides. The trapezium is usually written as Trap.

7.7 ISOSCELES TRAPEZIUM

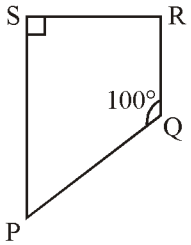
If the non-parallel sides of the trapezium (AD and BC as shown in figure) are equal, then it is called as isosceles trapezium. In the trapezium ABCD, AB || CD; AD and BC are its non-parallel sides such that AD = BC.



So, ABCD is an isosceles trapezium. In an isosceles trapezim, $\angle A = \angle B$ and $\angle C = \angle D$.

Ex. 12 PQRS is a trapezium in which SP || RQ, find the measures of $\angle P$ and $\angle R$.

Sol. Since PQRS is a trapezium in which SP || RQ.



$\therefore \angle P + \angle Q = 180°$ and $\angle S + \angle R = 180°$

$\Rightarrow \angle P + 100° = 180°$ and $90° + \angle R = 180°$

$\Rightarrow \angle P = 180° - 100° = 80°$ and $\angle R = 180° - 90° = 90°$

Ex. 13 ABCD is a trapezium in which AB || DC and $\angle A = \angle B = 40°$.

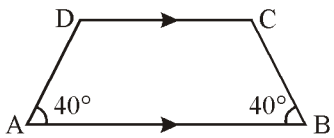
Find $\angle C$ and $\angle D$. Are these angles equal ?

Sol. AB || DC and transversal AD cuts them.

$\therefore \angle A + \angle D = 180°$

$\Rightarrow \angle D = 180° - \angle A = 180° - 40° = 140°$

Similarly, $\angle B + \angle C = 180°$



$\Rightarrow \angle C = 180° - \angle B = 180° - 40° = 140°$

$\therefore \angle C = \angle D = 140°$

Yes, these angles are equal.

$\therefore$ ABCD is an isosceles trapezium.

SOLVED EXAMPLES

SE.1 The angles of a quadrilateral are 100°, 98° and 92° respectively. Find the fourth angle.

Sol. Let the measure of fourth angle be x. We know that the sum of the angles of a quadrilateral is 360°.

$\therefore$ 100° + 98° + 92° + x = 360°

$\Rightarrow$ 290° + x = 360° $\Rightarrow$ x = 360° – 290° = 70°

Hence, the measure of fourth angle is 70°.

SE.2 The measures of two angles of a quadrilateral are 115° and 45°, and the other two angles are equal. Find the measure of each of the equal angles.

Sol. Let the measure of each of the equal angles be x. We know that the sum of all the angles of a quadrilateral is 360°

$\therefore$ 115° + 45° + x + x = 360°

$\Rightarrow$ 160° + 2x = 360°

$\Rightarrow$ 2x = (360° – 160°) = 200° $\Rightarrow$ x = 100°.

Hence, the measure of each of the equal angles is 100°.

SE.3 One angle of a quadrilateral is 108° and the remaining three angles are equal. Find the three equal angles.

Sol. Let ABCD be a quadrilateral such that

$\angle A = 108°$ and $\angle B = \angle C = \angle D$

Further, let $\angle B = \angle C = \angle D = x$

Now, by angle sum property of a quadrilateral, we have

$\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow 108° + x + x + x = 360°$

$\Rightarrow 108° + 3x = 360° \Rightarrow 3x = 360° - 108°$

$\Rightarrow 3x = 252° \Rightarrow x = \dfrac{252°}{3} = 84°$

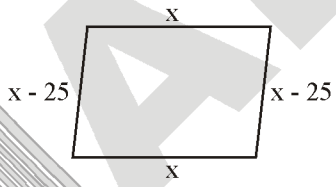
Hence, the measure of each of the remaining three equal angles is 84°.

SE.4 The perimeter of a parallelogram is 150 cm. One of its side is greater than the other by 25 cm. Find the lengths of all the sides of the parallelogram.

Sol. Let one pair of opposite sides of a parallelogram be x cm.

$\therefore$ Other pair of opposite sides be (x – 25) cm.

Perimeter of a parallelogram = 150 cm



$\Rightarrow x + (x - 25) + x + (x - 25) = 150$

$\Rightarrow 4x - 50 = 150 \Rightarrow 4x = 50 + 150$

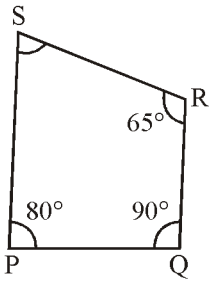
$\Rightarrow 4x = 200 \Rightarrow x = \dfrac{200}{4} \Rightarrow x = 50$

Therefore, x – 25 = 50 – 25 = 25

Hence, lengths of all sides of the parallelogram are 50 cm, 25 cm, 50 cm and 25 cm.

SE.5 In a quadrilateral PQRS; $\angle P = 80°$, $\angle Q = 90°$; $\angle R = 65°$. Find the measure of $\angle S$. Is it a convex or concave quadrilateral ?

Sol. We know, sum of the interior angles of a quadrilateral = 360°



$\therefore \angle P + \angle Q + \angle R + \angle S = 360°$

$\Rightarrow 80° + 90° + 65° + \angle S = 360°$

$\Rightarrow 235° + \angle S = 360°$

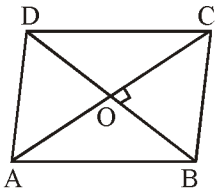
$\Rightarrow \angle S = 360° - 235° = 125°$

$\therefore$ PQRS is a convex quadrilateral.

SE.6 The diagonals of a rhombus are 10 cm and 24 cm. Find the length of a side of the rhombus.

Sol. As AC = 24 cm, so CO = 12 cm

BD = 10 cm, so BO = 5 cm



Now, in right triangle BOC, we have

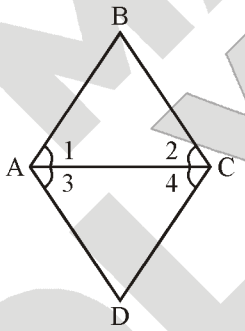
$BC^2 = BO^2 + OC^2$ [By Pythagoras Theorem]

$\Rightarrow BC^2 = 5^2 + 12^2 \Rightarrow BC^2 = 25 + 144 = 169$

$\Rightarrow BC = 13$ cm.

Hence, length of a side of the rhombus is 13cm.

SE.7 Diagonal AC of a rhombus ABCD is equal to one of its sides BC (see fig.) Find all the angles of the rhombus.



Sol. ABCD is a rhombus

$\therefore$ AB = BC = CD = DA .....(i)

Also, given that diagonal

AC = side BC .....(ii)

From (i) and (ii), AB = BC = AC = CD = DA

Now, we have AB = BC = AC

and AC = CD = DA

Therefore ΔABC and ΔADC are equilateral triangles and in an equilateral triangle each angle is of measure 60°.

$\therefore \angle 1 = 60°, \angle 3 = 60°, \angle 2 = 60°$ and

$\angle 4 = 60°, \angle B = 60°$ and $\angle D = 60°$

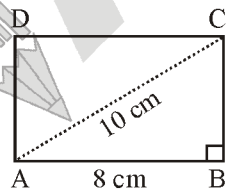
Now, $\angle 1 + \angle 3 = 60° + 60° \Rightarrow \angle A = 120°$

And $\angle 2 + \angle 4 = 60° + 60° \Rightarrow \angle C = 120°$

Hence, required respective angles of rhombus are 120°, 60°, 120° and 60°.

SE.8 The length of a rectangle is 8 cm and each of its diagonals measures 10 cm. Find its breadth.

Sol. Let ABCD be the given rectangle in which length AB = 8 cm and diagonal AC = 10 cm. Since, each angle of a rectangle is a right angle, we have $\angle ABC = 90°$.



$AC^2 = AB^2 + BC^2$ [Pythagoras theorem]

$\Rightarrow BC^2 = (AC^2 - AB^2) = \{(10)^2 - (8)^2\}$

$= (100 - 64) = 36$

$\Rightarrow BC = \sqrt{36} = 6$ cm

Hence, breadth = 6 cm.

SE.9 Prove that the interior angle of a regular pentagon is three times the exterior angle of a regular decagon.

Sol. A pentagon has five sides.

$\therefore$ Each interior angle of a regular pentagon

$$= \left(\frac{2\times 5-4}{5}\times 90\right)^\circ = \left(\frac{6}{5}\times 90\right)^\circ = 108^\circ$$

$$\left[\text{Putting } n = 5 \text{ in } \left(\frac{2n-4}{n}\times 90\right)^\circ\right]$$

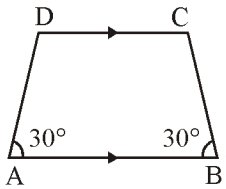
A decagon has 10 sides.
Exterior angle of a regular decagon

$$= \left(\frac{360}{10}\right)^{\circ} = 36^{\circ}$$

$$\left[\text{Putting } n = 10 \text{ in } \left(\frac{360}{n}\right)^{\circ}\right]$$

Clearly, each interior angle of a regular pentagon is three times the exterior angle of a regular decagon.

SE.10 ABCD is a trapezium in which AB || DC and $\angle A = \angle B = 30°$. Find $\angle C$ and $\angle D$. Are these angles equal ?



Sol. AB || DC and a transversal AD cuts them.

$\therefore \angle A + \angle D = 180°$

$\Rightarrow \angle D = 180° - \angle A = 180° - 30° = 150°$

Similarly, $\angle B + \angle C = 180°$

$\Rightarrow \angle C = 180° - \angle B = 180° - 30° = 150°$

$\therefore \angle C = \angle D = 150°$

Yes, they are equal.

EXERCISE-I

ONLY ONE CORRECT TYPE

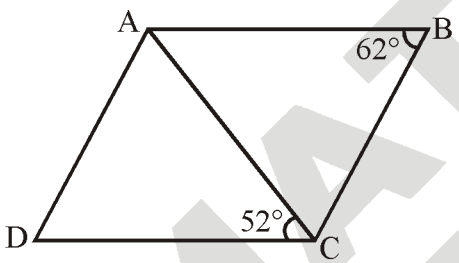
1. Two adjacent angles of a parallelogram are $(2x + 25)°$ and $(3x - 5)°$. The value of x is

(A) 28 (B) 32
(C) 36 (D) 42

2. The diagonals do not necessarily intersect at right angles in a

(A) Parallelogram (B) Square
(C) Rhombus (D) Kite

3. In the given quadrilateral ABCD (not drawn to scale), BC = AC = AD. Find the sum of ∠DAC and ∠ACB.



(A) 76° (B) 132°
(C) 56° (D) 112°

4. If an angle of a parallelogram is two-third of its adjacent angle, then the smallest angle of the parallelogram is

(A) 54° (B) 72°
(C) 81° (D) 108°

5. A quadrilateral can have

(A) 4 acute angles
(B) 4 obtuse angles
(C) 3 obtuse angles
(D) 2 right angles and 2 obtuse angles

6. A quadrilateral, in which only one pair of opposite sides is parallel is a

(A) rectangle (B) kite
(C) trapezium (D) rhombus

7. A quadrilateral, which is both a rectangle and a rhombus is a

(A) square (B) parallelogram
(C) Kite (D) trapezium

8. A regular polygon is

(A) equilateral (B) equiangular
(C) both (A) & (B) (D) none of these

9. Three angles of a quadrilateral are in the ratio 1 : 5 : 6. The mean of these angles is 64°. Find the fourth angle.

(A) 168° (B) 162°
(C) 120° (D) 90°

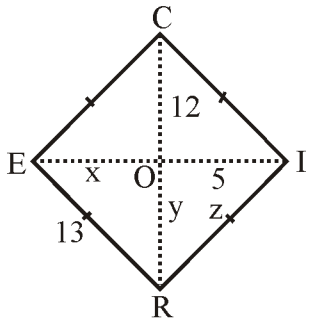
10. If PQRS is a parallelogram, then ∠P – ∠R is

(A) 0° (B) 90°
(C) 180° (D) 360°

11. Whch of the following is a false statement for a parallelogram ?

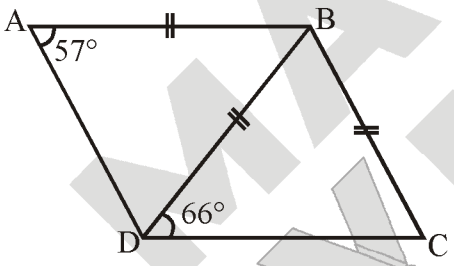
(A) Opposite sides are equal
(B) Opposite angles are equal
(C) Diagonals bisect each other
(D) Diagonals bisect each other at right angles.

12. In the given figure, RICE is a rhomubs. Find x + y + z.



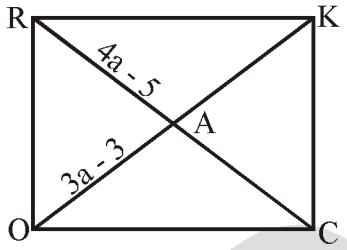
(A) 30 (B) 35
(C) 25 (D) 20

13. In the given figure, ABD and BCD are isosceles triangles, where AB = BC = BD. The special name that is given to quadrilateral ABCD is



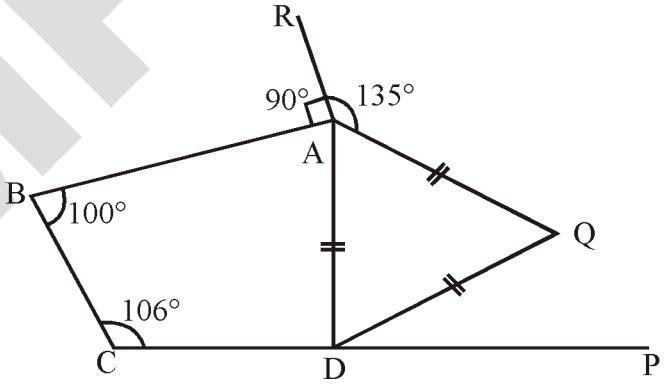
(A) Rectangle (B) Rhombus
(C) Parallelogram (D) Trapezium

14. ROCK is a rectangle. Its diagonals meet at A. Find 'a', if RA = 4a – 5 and OA = 3a + 3.



(A) 3 (B) 5
(C) 8 (D) 4

15. In the given figure, CDP is a straight line, ΔAQD is an equilateral triangle, ∠BAR = 90°, ∠QAR = 135°, ∠BCD = 106° and ∠ABC = 100°. Then, ∠PDQ equals.

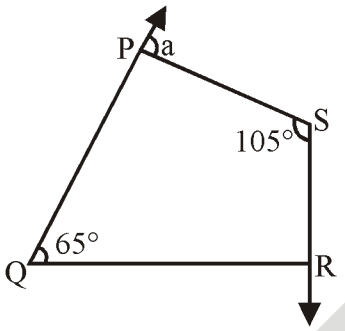


(A) 39° (B) 21°
(C) 41° (D) 53°

16. If one angle of a parallelogram is 24° less than twice the smallest angle, then the largest angle of the parallelogram is

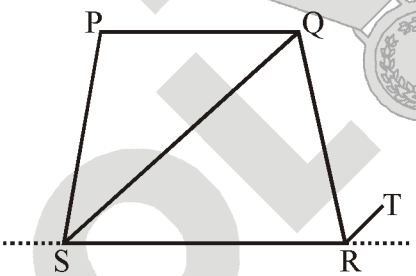
(A) 68° (B) 102°
(C) 112° (D) 176°

17. In the adjacent figure, angle P and angle R are in the ratio of 3 : 7, then the value of a is



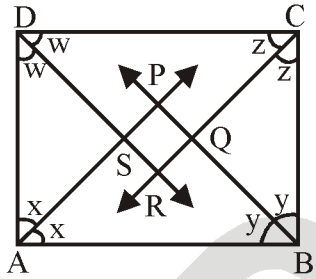
(A) 47° (B) 57°
(C) 123° (D) none of these

18. In the given figure, line RT is drawn parallel to SQ. If ∠QPS = 100°, ∠PQS = 40°, ∠PSR = 85° and ∠QRS = 70°, then ∠QRT =



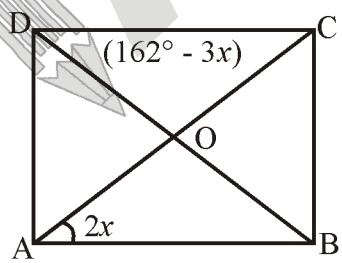
(A) 45° (B) 65°
(C) 85° (D) 90°

19. In the given figure, ABCD is a parallelogram. The quadrilateral PQRS is exactly a



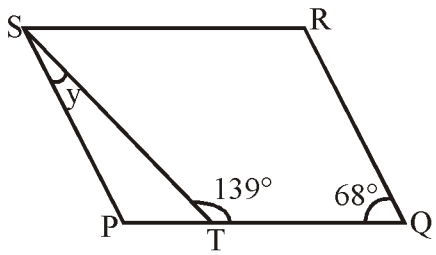
(A) Square (B) Parallelogram
(C) Retangle (D) Rhombus

20. ABCD is a rectangle. Find the value of x.



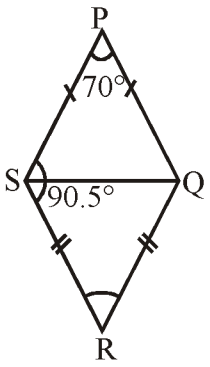
(A) 54° (B) 36°
(C) 24° (D) 18°

21. If PQRS is a parallelogram, then y equals



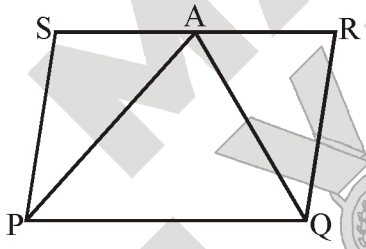
(A) 27° (B) 61°
(C) 41° (D) 28°

22. PQRS is a kite. If $\angle P = 70°$ and $\angle S = 90.5°$, then $\angle R$ equals



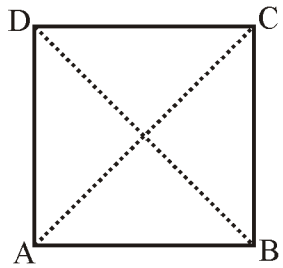
(A) 99° (B) 91°
(C) 111° (D) 109°

23. In the given figure, PQRS is parallelogram and $\angle SPQ = 60°$. If the bisectors of $\angle P$ and $\angle Q$ meet at A on RS, then which of the following is not correct ?



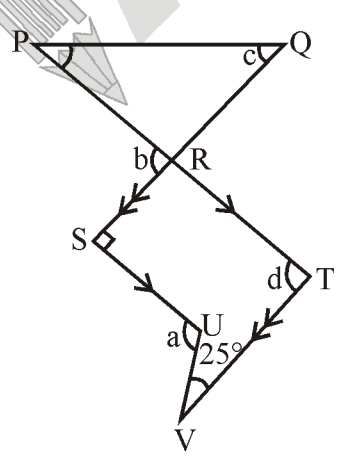
(A) AS = SP (B) AS = AR
(C) AR = SP (D) AQ = PQ

24. ABCD is a square of area of 4 square units which is divided into 4 non-overlapping triangles as shown in figure, then sum of perimeters of the triangles so formed is



(A) $8(2+\sqrt{2})$ units (B) $8(1+\sqrt{2})$ units
(C) $4(1+\sqrt{2})$ units (D) $4(2+\sqrt{2})$ units

25. In the given figure (not drawn to scale), find the value of (b + d) – (a + c).



(A) 115° (B) 40°
(C) 90° (D) 25°

PARAGRAPH TYPE

Passage – I : The angles of a quadrilateral are in the ratio 3 : 5 : 7 : 9.

1. If measure of angle be (3x), (5x), (7x) and (9x), then the value of x is :
(A) 20° (B) 15°
(C) 25° (D) 10°

2. The measure of all angles are :

(A) 45°, 70°, 105°, 140°

(B) 40°, 80°, 105°, 135°

(C) 45°, 75°, 110°, 130°

(D) 45°, 75°, 105°, 135°

3. The sum of the least and greatest angle is :

(A) 175° (B) 180°

(C) 170° (D) 185°

Passage – II : Measure of each exterior angle of a regular polygon of n sides = $\left(\frac{360}{n}\right)^{\circ}$

4. The measure of each exterior angle of a 10 sided regular polygon is :

(A) 36° (B) 30°

(C) 18° (D) 35°

5. If measure of an exterior angle is 45°, the number of sides in a regular polygon is :

(A) 7 (B) 11

(C) 10 (D) 8

6. The polygon with nine sides is called

(A) Octagon (B) Quadrilateral

(C) Nonagon (D) Hexagon

MATCH THE COLUMN TYPE

In this, section, each question has two matching lists. Choices for the correct combination of elements from List - I and List - II are given as options (A), (B), (C) and (D) out of which one is correct.

1. Match the lists :

List – I	**List – II**
(P) Diagonals of a rectangle	(1) Bisect each other at right angles
(Q) Diagonals of a square	(2) Bisect each other
(R) Diagonals of a rhombus	(3) Equal and bisect each other
(S) Diagonals of a parallelogram	(4) Equal and bisect each other at right angles

(A) P – 1, Q – 2, R – 3, S – 4

(B) P – 3, Q – 4, R – 1, S – 2

(C) P – 4, Q – 2, R – 3, S – 1

(D) P – 4, Q – 3, R – 2, S – 1

2. Match the lists :

List – I	**List – II**
(P) ABCDE is a regular pentagon. The bisector of ∠A of the pentagon meets the side CD at M. Then m∠AMC is	(1) 70°
(Q) The angles of a quadrilateral are respectively equal to 110°, 50° and 40°. The fourth angle is	(2) 80°
(R) In a quadrilateral ABCD, the bisector of ∠A and ∠B meet at point P. If ∠C = 100°, ∠D = 60°, then m ∠APB is	(3) 90°
(S) Three angles of a quadrilateral are equal. Fourth angle is of measure 150°. The measure of equal angles is	(4) 160°

(A) P – 1, Q – 2, R – 3, S – 4

(B) P – 2, Q – 4, R – 3, S – 1

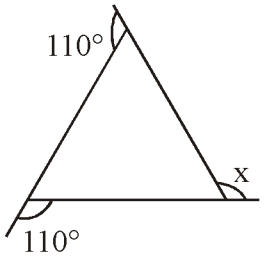
(C) P – 3, Q – 4, R – 2, S – 1

(D) P – 4, Q – 3, R – 2, S – 1

EXERCISE-II

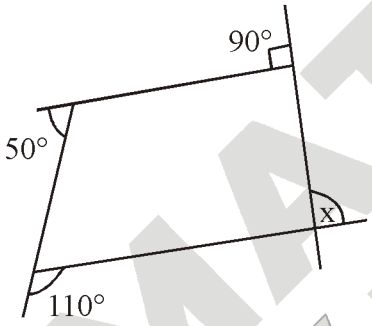
VERY SHORT ANSWER TYPE

1. Find the measure of x in the given figure.



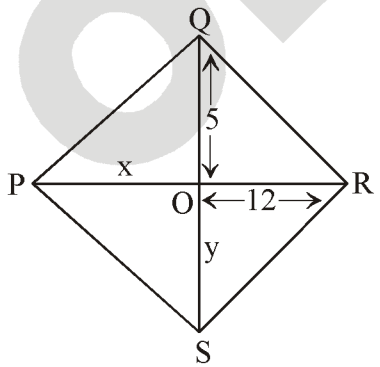
2. In a quadrilateral PQRS ; $\angle P = 70°$, $\angle Q = 90°$; $\angle R = 55°$. Find the measure of $\angle S$. What kind of quadrilateral is it, convex of concave ?

3. Find the measure of x



4. Which of the following groups of angles can be the angles of a quadrilateral ?
(i) 120°, 90°, 75°, 30°
(ii) 100°, 100°, 70°, 90°

5. Let PQRS be a rhombus, find x, y



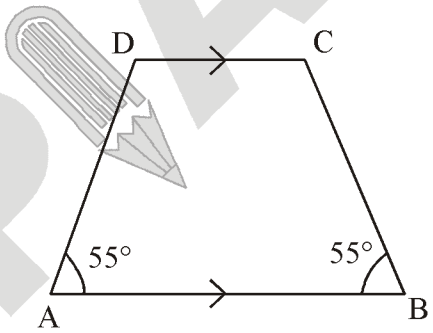
6. In a quadrilateral PQRS, $\angle P = 40°$, $\angle Q = 60°$, $\angle R = 60°$. Find $\angle S$. Is this quadrilateral convex or concave ?

7. Adjacent angles of a parallelogram are in the ratio of 2 : 7. Find their values.

8. In a parallelogram, show that any two adjacent angles are supplementary.

9. The diagonals of a quadrilateral are 8 cm and 6 cm. If the diagonals bisect each other at right angles, find the length of the sides of the quadrilateral.

10. In the given figure, find the measure of $\angle C$.



SHORT ANSWER TYPE

1. Three angles of a quadrilateral are in the ratio 1 : 2 : 3. The mean of these angles is 32°. Find all the four angles.

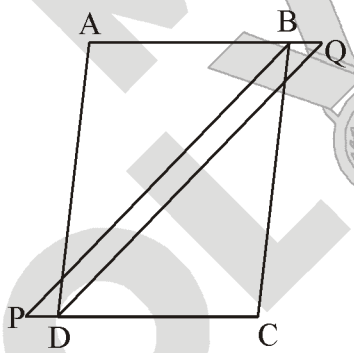
2. Prove that in a rhombus with an angle of 60°, the shorter diagonal divides it into two equilateral triangles.

3. ABCD is a rhombus whose diagonals AC and BD intersect at a point O. If side AB = 10 cm and diagonal BD = 16 cm, find the length of diagonal AC.

4. The ratio of two sides of a rectangle is 3 : 5 and its perimeter is 48 m. Find the sides of the rectangle
5. The interior angle of a regular polygon is 156°. Find the number of sides of the polygon

LONG ANSWER TYPE

1. Two regular polygons are such that the ratio between their number of sides is 1 : 2 and the ratio of measures of their interior angles is 3 : 8. Find the number of sides of each polygon.
2. The exterior angle of a regular polygon is one-third of its interior angle. How many sides the polygon has ?
3. In a quadrilateral ABCD, AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively. Prove that $\angle AOB = \frac{1}{2}(\angle C + \angle D)$.
4. In the given figure, bisectors of $\angle B$ and $\angle D$ of quadrilateral ABCD met CD and AB produced at P and Q respectively. Prove that $\angle P + \angle Q = \frac{1}{2}(\angle ABC + \angle ADC)$



5. The diagonals of a rectangle ABCD meet at O. If $\angle BOC = 44°$, then find $\angle OAD$.

TRUE / FALSE TYPE

1. If all the angles of a quadrilateral are equal, it is a rectangle.
2. The adjacent angles of a parallelogram are equal.
3. The diagonals of a parallelogram bisect each other.
4. In a parallelogram, the diagonals are equal.
5. The diagonals of a rectangle are of equal length.

NUMERICAL PROBLEMS

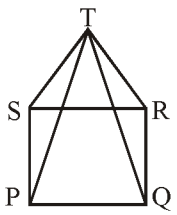
1. The number of sides of a polygon whose sum of exterior and interior angles are in the ratio 1 : 5 is k. The value of k is.
2. The ratio of two sides of a rectangle is 4 : 3 and its perimeter is 56 m. If the sides of rectangle be a and b. Then, a + b (in m) is :
3. The number of sides of a regular polygon when each of its angle has a measure of 135° is
4. Find the number of sides of a regular polygon when each of its angle has a measure of 90°.
5. The measure of angles of a quadrilateral are x, 2x, 3x and 4x. Find the value of $\frac{x}{12°}$

EXERCISE-III

PREVIOUS YEAR QUESTIONS (NTSE)

1. In the figure PQRS is a square and SRT is an equilateral triangle. Then find ∠TQR.

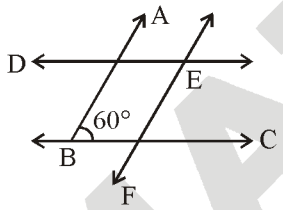
[Aryabhatta 2002]



(A) 60° (B) 90°
(C) 30° (D) 15°

2. In figure AB || EF, DE || BC & ∠ABC = 60°, find ∠DEF

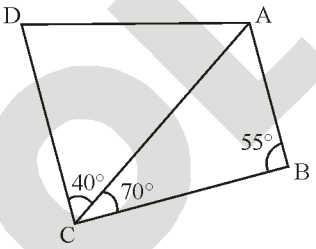
[Aryabhatta 2002]



(A) 60° (B) 120°
(C) 30° (D) 80°

3. In the diagram DA = CB what is the measure of ∠DAC ?

[NSTSE - 2009]



(A) 70° (B) 100°
(C) 95° (D) 125°

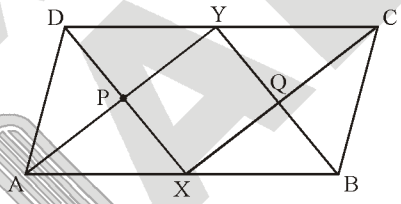
4. Each angle of a rectangle is bisected. Let P, Q, R and S be the points of intersection of the pairs of bisectors adjacent to the same side of the rectangle. Then PQRS is a

[NSTSE - 2009]

(A) rectangle
(B) rhombus
(C) parallelogram with unequal adjacent sides
(D) quadrilateral with no special property

5. X, Y are the mid points of opposite sides AB and DC of a parallelogram ABCD. AY and DX are joined intersecting in P, CX and BY are joined intersecting in Q. The PXQY is

[NSTSE - 2010]



(A) rectangle (B) rhombus
(C) parallelogram (D) square

6. Of all quadrilaterals of a given perimeter, which has the largest area ?

[Aryabhatta 2010]

(A) square (B) rectangle
(C) parallelogram (D) rhombus

7. ABCD is a parallelogram. The angle bisectors of ∠A and ∠D meet at O. The measure of ∠AOD is _____ .

[IMO - 2010]

(A) 45°
(B) 90°
(C) Depends on the angles A and D
(D) Not able to determine from given data

8. In a quadrilateral ABCD, AB || CD and AD = BC = 7 cm. If ∠A = 70° then the measure of ∠C is

[Aryabhatta 2011]

(A) 70° (B) 100°
(C) 80° (D) 110°

9. Smallest angle of a triangle is squal to two-third the smallest angle of a quadrilateral. The ratio of the angle of the quadrilateral is 3 : 4 : 5 : 6. Largest angle of the triangle is twice its smallest angle. What is the sum of second largest angle of the triangle and largest angle of the quadrilateral ?

[IMO - 2011]

(A) 160° (B) 180°
(C) 190° (D) 170°

10. Which of the following statements is INCORRECT?

[IMO - 2001]

(A) All rhombuses are parallelograms
(B) All squares are parallelograms.
(C) All rectangles are not squares.
(D) All squares are trapeziums.

11. A quadrilateral that is not a parallelogram but has exactly two equal opposite angles is

[NSTSE - 2012]

(A) a rhombus (B) a trapezium
(C) a square (D) a kite

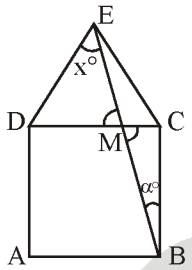
12. Find the measure of largest angle of a quadrilateral if the measures of its interior angles are in the ratio of 3 : 4 : 5 : 6.

[IMO - 2012]

(A) 60° (B) 120°
(C) 90° (D) Can't be determined

13. In the given diagram, equilateral triangle EDC surmounts square ABCD. Find BED represented by x, where EBC = α°.

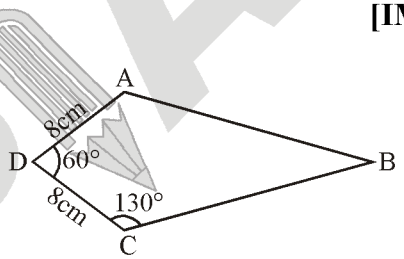
[IMO - 2012]



(A) 45° (B) 60°
(C) 30° (D) None of these

14. In the kite ABCD, AD = CD = 8 cm, ADC = 60°, DCB = 130° and AB = CB. Find ABC.

[IMO - 2012]



(A) 50° (B) 40°
(C) 60° (D) 25°

15. In a parallelogram,

[IMO - 2012]

Statement 1 : Diagonals bisect each other.
Statement 2 : Diagonals divide the parallelogram into two triangles.

(A) Only statement 1 is true.
(B) Only statement 2 is true.
(C) Both statement 1 and 2 are true
(D) Both statement 1 and 2 are false

Answer Key

EXERCISE I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	A	B	B	C	C	A	C	A	A	D	A	D	C	C
16	17	18	19	20	21	22	23	24	25					
C	C	B	C	D	A	D	D	B	D					

PARAGRAPH

1. B **2.** D **3.** B **4.** A **5.** D **6.** C

MATCH THE COLUMN

1. B **2.** C

EXERCISE II

VERY SHORT ANSWER TYPE

1. 140° **2.** 145°, Convex **3.** 110° **4.** (i) No, (ii) Yes **5.** x = 12, y = 5

6. 200°, Concave **7.** 40°, 140° **9.** AB = AD = DC = BC = 5 cm **10.** 125°

SHORT ANSWER TYPE

1. 16°, 32°, 48°, 264° **3.** 12 cm **7.** 9 m, 15 m **8.** 15

LONG ANSWER TYPE

1. 5, 10 **2.** 8 **5.** 68°

TRUE / FALSE

1. True **2.** False **3.** True **4.** False **5.** True

NUMERICAL PROBLEMS

1. 12 **2.** 28 **5.** 8 **7.** 4

10. 3

EXERCISE III

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	A	B	A	C	A	B	D	B	D	D	B	A	B	A

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : UNDERSTANDING QUADRILATERALS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

DATA HANDLING

1 INTRODUCTION

In our day-to-day life we might have come across various informations, such as :

(i) Number of runs made by a batsman in the last 15 test matches.

(ii) Marks obtained by the students of class VIII in the Mathematics unit test.

(iii) Number of cars sold by a car manufacturing company in the months of a particular year.

(iv) The number of novels read by some friends in a particular year.

All the information which is collected in the above cases is called data.

In this chapter, we shall discuss about the data and its representation in different forms.

2 DATA

The word 'data' means the collection of information in the form of numerical figures or set of given facts.

For example :

(i) The marks obtained by 10 students of a class in a test out of 100 marks are

78, 85, 97, 59, 100, 80, 75, 68, 47, 55

(ii) The following data gives the information regarding the favourite game of 110 students of a school.

Sports	Football	Cricket	Tennis	Carrom Board
No. of students	35	45	20	10

When some information is collected and presented in an unorganised manner, then it is called raw data.

And when the data is classified into groups, then it is called grouped data.

Data in raw form can be presented in pictorial form and this makes the data attractive and easy to understand.

Also, we can compare it with other information.

Some commonly known diagrams in which numerical data can be represented are as follows:

(i) Pictograph

(ii) Bar graph

(iii) Double bar graph

(iv) Circle graph or Pie-chart or Pie- diagram

2.1 PICTOGRAPH

It is represents data through appropriate pictures. Generally, same type of symbols or pictures are used to represent data. Each picture or symbol is used to represent a certain value and it is clearly mentioned in the graph. For example, the given pictograph represents the number of tractors produced in the month of April, May and June.

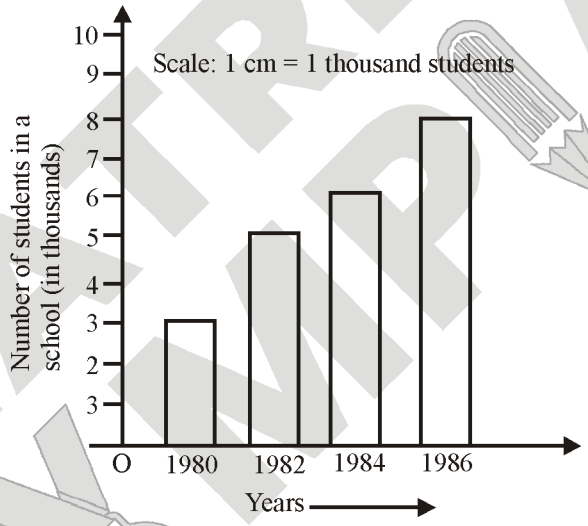
Month	Production of tractors
April	= 350
May	= 400
June	= 450
= 100 tractors; = 50 tractors	

2.2 BAR GRAPH

The representation of data in the form of bars or rectangles in a diagram is called a bar graph or a bar diagram. Here, each bar represents only one value of the data and so, there are as many bars as the number of values in the data.

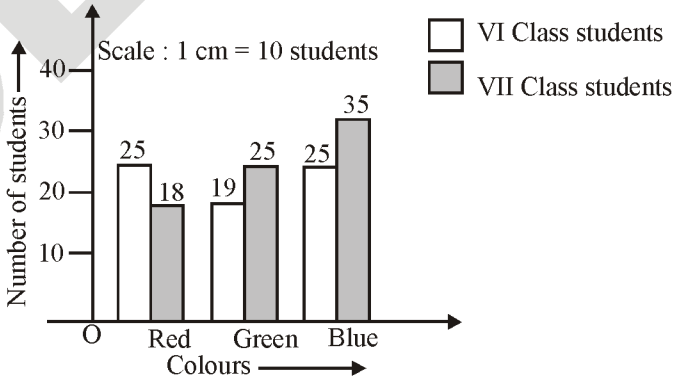
The length of the bar represents the value of the item. The width of the bars should be uniform.

The given bar graph represents the number of students in a school in different years (in thousands).



2.3 DOUBLE BAR GGRAPH

It is a bar graph which shows two sets of data on the same graph. It is useful in the comparison of data.

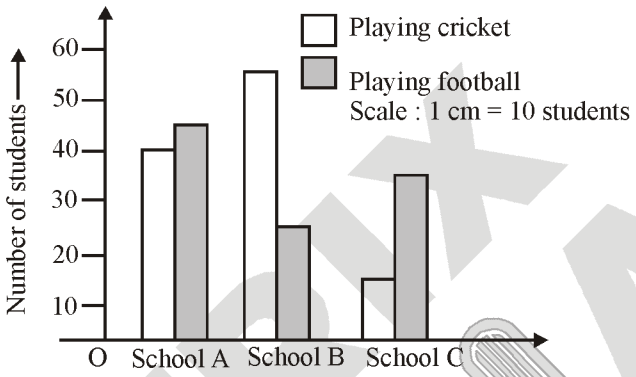


Bar graph above represents liking of different colours by class VI and VII students.

Ex. 1 Draw a double bar graph to represent the information given below :

Children who prefer	School A	School B	School C
Playing cricket	40	55	15
Playing football	45	25	35

Sol.



3 ORGANISING DATA

Generally, the data which we get is in unorganised form (called as raw data). To understand it meaningfully, we must organise it systematically.

For example, a group of children was asked about their favourite game. The results were as given below.

Cricket, Football, Volleyball, Tennis, Cricket, Football, Cricket, Volleyball, Football, Cricket, Football, Cricket, Tennis.

By looking directly to the data, we cannot answer the number of students and their favourite games.

In this case, we arrange data in a table by using tally marks as shown:

Favourite Games	Tally Marks	No. of Students
Cricket	𝍸	5
Football	||||	4
Volleyball	||	2
Tennis	|||	3

Note: We generally use 𝍸 Tally marks to represent value 5. To represent 7, we use 𝍸 || Tally marks.

4 GROUPING DATA

Something organising large data becomes a tedious process. In such cases, we make groups of the raw data and write the groups as intervals. Each such groups is called class-interval. The class intervals have a lower class limit and an upper class limit. The difference between the upper and lower class interval is called class size or width of the class interval. The number of times a particular item appears within a particular class interval is called frequency. We will now fill up the rows with tally marks and count the total number of tally marks in each group.

The total number of tally marks of each group is listed in frequency column. The table so obtained is called frequency distribution table.

Class Interval	Tally Marks	Frequency (No. of Students)
0 - 10	III	3
10 - 20	卌	5
20 -30	卌 II	7
30 - 40	卌 IIII	9
40 - 50	卌 卌 II	12

Frequency Distribution Table

Note: In class inerval 0 - 10, 10 is not included. Similarly, it is true for other intervals also. Now we shall discuss the above mention points in more details.

5 FREQUENCY DISTRIBUTION

Frequency table or Frequency distribution is a method to present raw data in the form from which one can easily understand the information contained in it. Frequency distributions are of two types :

(i) Discrete frequency distribution

(ii) Continuous or grouped frequency distribution

5.1 DISCRETE FREQUENCY DISTRIBUTION

The process of preparing this type of distribution is very simple. The construction of a discrete frequency distribution from the given raw data is done by the method of tally marks. In the first column of the frequency table we write all possible values of the variable from the lowest to the highest.

We now look at the first value in the given raw data and put a bar (vertical line) in the second column opposite to it. Now, we see the second value in the given raw data and put a bar opposite to it in the second column. This process is repeated till all observations in the given data are exhausted. To facilitate counting blocks of five, 卌 are prepared and some space is left in between each block. We finally count the number of bars corresponding to each value of the variable and place it in the third column of frequency. The process will be clear from the following example of the number of children in 20 families

1, 1, 2, 3, 4, 3, 2, 1, 1, 4, 5, 2, 4, 2, 2, 1, 3, 3, 2, 5

The data may be put in the form of discrete frequency distribution as follows :

Number of Children	Tally Marks	Frequency
1.	卌	5
2.	卌 |	6
3.	||||	4
4.	|||	3
5.	||	2

5.2 CONTINUOUS OR GROUPED FREQUENCY DISTRIBUTION

The above method of condensing the raw data is convenient only where the values in the raw data are less repeating and the difference between the greatest and the smallest observation is not very large. If the number of observations in data is large and the difference between the greatest and the smallest observations is large, then we condense the data into classes or groups. For example, let the marks obtained by 30 students of a class in a test be 39, 25, 5, 33, 19, 21, 12, 48, 13, 21, 9, 1, 10, 8, 12, 17, 19, 17, 41, 40, 12, 46, 37, 17, 27, 30, 6, 2, 23, 19. We can arrange these marks as follows :

Marks (Class intervals)	Tally Marks	Number of students (Frequency)
0 - 10	卌 |	6
10 - 20	卌 卌 |	11
20 - 30	卌	5
30 - 40	||||	4
40 - 50	||||	4

Such a presentation of data is known as the grouped frequency distribution.

In the above example, 30 observations have been divided into 5 groups. These groups are called classes. The class 0 - 10 means the marks obtained between 0 and 10 including 0 and excluding 10. The number of observations falling in a particular class is called the frequency of that class or class frequency. Thus, the class 0 -10 has frequency 6 and the class 10 - 20 has 11 as class frequency. In the class 0 - 10, we say that 0 is the lower limit and 10 is the upper limit of the class. Similarly, in the class 10 - 20, 10 is the lower limit and 20 is the upper limit. The span of the class i.e., the difference between the upper limit and the lower limit, is known as the class size. For example, in the class 10 - 20, the class size is 20 -10 = 10.

6 METHODS OF CLASSIFYING DATA ACCORDING TO CLASS INTERVALS

There are two methods of classifiying the data according to the class intervals, viz. (i) exclusive method, and (ii) inclusive method.

6.1 EXLUSIVE METHOD

When the class intervals are so formed that the upper limit of one class is the lower limit of the next class, it is known as the exclusive method of classification. In this method, the upper limit of a class is not include in the class. Thus, in the class 0 - 10 of marks obtained by students, a student who has obtained 10 marks is not included in this class. It is counted in the next class 10 - 20.

6.2 INCLUSIVE METHOD

In this method, the classes are so formed that the upper limit of a class is included in that class. Following example illustrates the method.

By Inclusive Method :

Wages (in Rs.)	No. of workers
1000-1099	125
1100-1199	150
1200-1299	200
1300-1399	250
1400-1499	175
1500-1599	100
	Total = 1000

In the class 1000 - 1099, we include workers having from Rs. 1000 to Rs. 1099 is included. If the income of worker is exactly Rs. 1100, it is included in the next class 1100 - 1199.

By Exclusive Method :

Wages (in Rs.)	No. of workers	Wages (in Rs.)	No. of workers
1000-1100	125	1000-1099	125
1100-1200	150	1100-1199	150
1200-1300	200	1200-1299	200
1300-1400	250	1300-1399	250
1400-1500	175	1400-1499	175
1500-1600	100	1500-1599	100
	Total = 1000		Total = 1000

It is evident from the above example that both the inclusive and exclusive methods give us the same class frequency, although the class intervals are apparently different in the two cases. In the above example, in case of exclusive method the class size is 100. However, 99 is not the correct class size in case of inclusive method. Whenever inclusive method is used it is necessary to make an adjustment to determine the correct class intervals and to have continuity. If a - b is a class in inclusive method, then in exclusive method it becomes.

$\left(a-\frac{h}{2}\right)-\left(b+\frac{h}{2}\right)$, where h = (lower limit of a class) - (upper limit of previous class)

In the above example, in inclusive method, the difference beween the lower limit of a class and the upper limit of the preceding class i.e., h = 1. Therefore, we subtract 1/2 from the lower limit of each class and add 1/2 in the upper limit of each class to make it continuous. The adjusted classes would then be as follows.

Wages (in Rs.)	No. of workers
999.5 - 1099.5	125
1099.5-1199.5	150
1199.5-1299.5	200
1299.5- 1399.5	250
1399.5 - 1499.5	175
1499.5 - 1599.5	100

The mid-value of a class is called the class mark. For example, the class-mark or mid - value of the class 1000-1100 is 1050. In fact

$$\text{Class mark} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

or, Class mark = Lower limit + $\frac{1}{2}$ (Difference between the upper and lower limits).

7 CONSTRUCTION OF A DISCRETE FREQUENCY DISTRIBUTION

To prepare a discrete fequency distribution from the given raw data, we use the following algorithm.

Step I : Obtain the given raw data.

Step II : Prepare a table with three columns : first for variable under study such as marks, weight height etc., second for 'Tally marks' and third for representing corresponding frequency to each value or size of the variable.

Step III : Place all the values of variable in the first column in ascending order.

Step IV : Take the first observation in the raw data and put a bar in the second column opposite to it. Then take the second observation in the given raw data and put a bar opposite to it. Continue this process till all the observations in the given raw data are exhausted. For the sake of convenience, record tally marks in blocks of five, the fifth, is shown by crossing diagonally the other four. Leave some space between each block of bars.

Step V : Count the number of bars (tally marks) in respect of each value of the variable and place it in the third column.

Step VI : Give suitable title to the frequency distribution table so that it conveys exactly what the table is about.

Ex. 2. Given below are the ages of 25 students of class VIII in a school. Prepare a discrete frequency distribution.

15, 16, 16, 14, 17, 17, 16, 15, 15, 16, 16, 17, 15, 16, 16, 14, 16, 15, 14, 15, 16, 16, 15, 14, 15.

Sol. Frequency distribution of ages of 25 students.

Age	Tally	Frequency
14	IIII	4
15	~~IIII~~ IIII	8
16	~~IIII~~ ~~IIII~~	10
17	III	3
	Total	25

8 CONSTRUCTION OF A GROUPED FREQUENCY DISTRIBUTION

Following algorihm is used for the construction of grouped frequency distribution.

Step I : Determine the maximum and minimum value of the variate occuring in the data.

Step II : Decide the number of classes to be formed. Note that the number of classes should be in range of 5 to 15.

Step III : Find the difference between the maximum value and minimum value and divide this difference by the number of classes to be formed to determine the class interval. The difference between the maximum value and minimum value in data is called range.

Step IV : Be sure that there must be classes with us to include minimum and maximum value occuring in the data.

Step V : Take each item from the data, one at time and put a tally mark (|) against the class to which the items belongs. If tally marks are more then 4, then record them in the block of five, the fifth one is marked by crossing diagonally the first four.

Step VI : By counting,.determine the total number of tally marks in each class, which gives us the frequency of the class.

Step VII : Check that the total of all frequencies is same as the total number of observations.

Step VIII : Give a suitable title to the frequency table so that it conveys exactly what the table is about.

Ex. 3 The water tax bills (in rupees) of 30 houses in a locality are given below. Construct a grouped frequency distribution with class size of 10.

30, 32, 45, 54, 74, 78, 108, 112, 66, 76, 88, 40, 14, 20, 15, 35, 44, 66, 75, 84, 95, 95, 102, 110, 88, 74, 112, 14, 34, 44.

Sol. Here, the maximum and minimum values of the variate are 112 and 14 respectively.

$\therefore$ Range = 112 – 14 = 98

It is given that the class size is 10, and $\frac{\text{Range}}{\text{Class size}} = \frac{98}{10} = 9.8$

So, we should have 10 classes each of size 10.

The minimum and maximum values of the variage are 14 and 112 respectively. So, we have to make the classes in such a way that first class includes the minimum value and last class includes the maximum value. If we take the first class as 14-24 it includes the minimum value 14. If the last class is taken as 104-114, then it includes the maximum value 112. Here, we form classes by exlusive method. In the class 14-24, 14 is included but 24 is exluded. Similarly, in other classes, the lower limit is included adn the upper limit is excluded.

In the view of above discussion, we construct the frequency distribution table as follows :

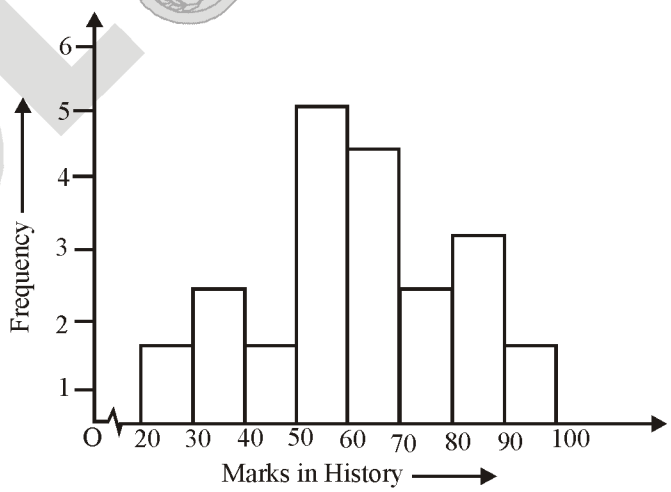
Bill (in rupees)	Tally marks	Frequency
14 - 24	||||	4
24 - 34	||	2
34 - 44	|||	3
44 - 54	|||	3
54 - 64	|	1
64 - 74	||	2
74 - 84	~~||||~~	5
84 - 94	|||	3
94 - 104	|||	3
104 - 114	||||	4
	Total	30

9 GRAPHICAL REPRESENTATION OF FREQUENCY DISTRIBUTION AND HISTOGRAM

In earlier classes, we have learnt how to represent a given infomation by means of bar graphs etc. Now, we will learn how to represent a grouped frequency distribution graphically.

This can be done in a number of ways. Among the commonly used graphical methods, histogram is one. It is a vertical bar graph with no space between bars. To make a histogram, we draw two perpendicular axes. We mark class boundaries of the grouped data on the horizontal axis and the respective class frequencies on the vertical axis, using suitable scale on the axis. In this way, we construct rectangles with respective class intervals as the bases and the corresponding class as heights.

Ex. 4 The figure shows the histogram for the frequency distribution of marks of 21 students in a History test (Out of 100).



Study the histogram and answer the following questions:

(i) What does the longest rectangle depict ?

(ii) What is the class size ?

(iii) How many students got marks between 80 and 90 ?

Sol. (i) The longest rectangle depicts the marks obtained by the students in the interval 50-60 in history is maximum.

(ii) The class intervals are 20 - 30, 30 - 40 etc.

$\therefore$ Class size = 10

(iii) Marks between 80 and 90 means marks in the class interval, the corresponding height of the rectangle is 3 units.

$\therefore$ Number of students getting marks between 80 and 90 = 3

10 CIRCLE GRAPH OR PIE CHART

A circle graph or Pie chart or Pie diagram is a circle which is divided into several sectors. Circle represents the total value of the given data and the sectors represent the proportion of the components of the total.

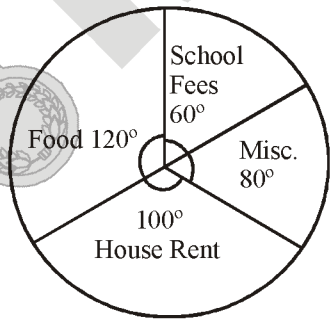
Another name of this diagram is an angular diagram or a circular diagram.

For example, the monthly expenditure on various items of a family is given below:

Item	Food	House Rent	Misc.	School Fees
Amount spent	Rs. 3000	Rs. 2500	Rs. 2000	Rs. 1500

We can represent this infomation on a pie chart as shown below:

Total Expnditure = Rs. 9000



A pie chart

Item	Amount Spent	In Fraction	Central Angle
Food	Rs. 3000	$\frac{3000}{9000} = \frac{1}{3}$	$\frac{1}{3} \times 360^\circ = 120^\circ$
House Rent	Rs. 2500	$\frac{2500}{9000} = \frac{5}{18}$	$\frac{5}{18} \times 360^\circ = 100^\circ$
Misc.	Rs. 2000	$\frac{2000}{9000} = \frac{2}{9}$	$\frac{2}{9} \times 360^\circ = 80^\circ$
School Fees	Rs. 1500	$\frac{1500}{9000} = \frac{1}{6}$	$\frac{2}{6} \times 360^\circ = 60^\circ$

11 DRAWING PIE CHARTS

Let us understand to draw a pie chart with the help of an example.

The favourite games liked by students of a school is given in the percentage form are as follows:

Games	Percentage students who like the games
Cricket	60%
Football	20%
Tennis	20%

Let us represents this data using pie chart.

We know that total angle at the centre of a circle is 360°.

The central angle of the different sectors are fractions of 360°.

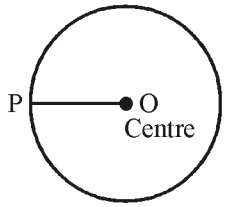
Firstly, we will find the central angle for each game.

Games	Students favouring the games (in %)	In fractions	Central
Cricket	60%	$\frac{60}{100}$	$\frac{60}{100} \times 360^\circ = 216^\circ$
Football	20%	$\frac{20}{100}$	$\frac{20}{100} \times 360^\circ = 72^\circ$
Tennis	20%	$\frac{20}{100}$	$\frac{20}{100} \times 360^\circ = 72^\circ$

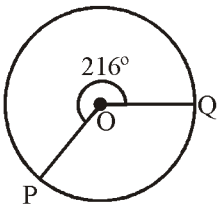
Steps to Draw a Pie Chart :

Step I : Draw a circle of any convenient radius.

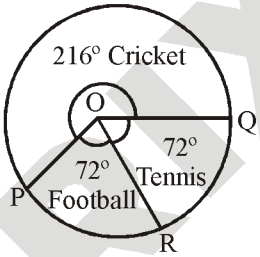
Centre O and radius = OP



Step II : Make the angle of sector for cricket as 216°.



Step III : In the same way, mark the remaining sectors.



Ex. 5 In a motor factory, five varieties of vehicles was manufactured in one year whose data is given below.

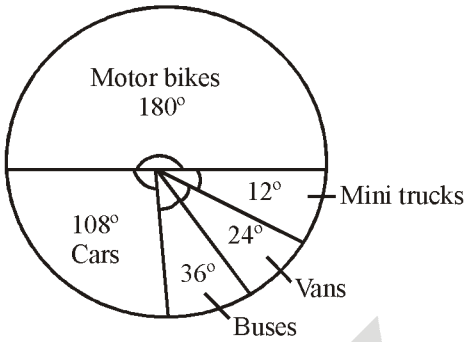
Motor bikes	Cars	Buses	Vans	Mini trucks
6000	3600	1200	800	400

Represent this data as pie chart.

Sol. Total number of vehicles = 12000

Vehicles	Number	In Fraction	Central Angle
Motor bikes	6000	$\frac{6000}{12000} = \frac{1}{2}$	$\frac{1}{2} \times 360 = 180°$
Cars	3600	$\frac{3600}{12000} = \frac{3}{10}$	$\frac{3}{10} \times 360 = 108°$
Buses	1200	$\frac{1200}{12000} = \frac{1}{10}$	$\frac{1}{10} \times 360 = 36°$
Vans	800	$\frac{800}{12000} = \frac{1}{15}$	$\frac{1}{15} \times 360 = 24°$
Mini trucks	400	$\frac{400}{12000} = \frac{1}{30}$	$\frac{1}{30} \times 360 = 12°$

Now, we will represent the above information on a pie chart.



Chance and Probability

(i) Most probably it will rain today.

(ii) Chances are high that the prices of petrol will go up.

(iii) I doubt that he will win the race.

The word 'most probably', 'chances', 'doubt', etc., shows the probablity of occurence of an event. In our fast going life, we come across words like probably, like, chance, may be, hope etc. All these have the same meaning as probablity.

Probability is defined as the numerical method of measuring uncertainty involved in any situation.

It is used very widely in the field of mathematics, statistics, gambling, weather forecasting, physical science, biological science, etc. to draw any conclusion.

12 SOME EXPERIMENTS AND THEIR OUTCOMES

12.1 TOSSING A COIN

Suppose we toss a coin and left it falls flat on the ground. Its upper face will show either Head (H) or Tail (T).

(i) Whatever comes up, is called an outcome.

(ii) Possible outcomes are Head (H) and Tail (T).

12.2 THROWING A DICE

A dice is a solid cube having 6 faces, marked as 1, 2, 3, 4, 5, 6 respectively.

Suppose we throw a dice and let it fall flat on the ground. Its upper face will show one of the numbers 1, 2, 3, 4, 5, 6.

The act if tossing coin or throwing a dice is called an experiment.

Whatever comes up, is called an outcome.

In an experiment, all possible outcomes (i.e., 1, 2, 3, 4, 5, 6) are known.

13 SOME TERMS RELATED TO PROBABILITY

13.1 EXPERIMENT

An operation which can produce some well-defined outcomes is called an experiment. Each outcome is called an event.

13.2 RANDOM EXPERIMENT

An experiment in which all possible outcome are known and the exact outcome cannot be predicted in advance, is called a random experiment. Thus, when we throw a coin we know that all possible outcomes are Head and Tail. But, if we throw a coin at random, we cannot predict in advance whether its upper face will show a head or a tail. So, tossing a coin is a random experiment.

Similarly, throwing a dice is a random experiment.

13.3 TRIAL

By a trial, we mean performing a random experiment.

Probability of occurrence of an event:

$$P(\text{occurrence of an event}) = \frac{\text{Number of trials in which event occured}}{\text{Total number of trials}}.$$

Competition window :

A pack of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each.

Each suit consists of one ace, one king, one queen, one jack and 9 other cards numbered from 2 to 10. Four suits are named as spades (♠), hearts (♥), diamonds (♦), and clubs (♣).

P (not occuring an event) = 1 – Probability of occuring an event.

Ex. 6 An unbised die is thrown. What is the probability of getting :

(i) an even number

(ii) a multiple of 3

(iii) an even number or a multiple of 3

(iv) an even number and a multiple of 3

(v) a number 3 or 4

(vi) an odd number

(vii) a number less than 5

(viii) a number greater than 3

(ix) a number between 3 and 6.

Sol. In a single throw of a die, we can get any one of the six numbers 1, 2,.....,6 marked on its six faces. Therefore, the total number of elementary events associated with the random experiment of throwing a die is 6.

(i) Let A denotes the event 'Getting an even number'.

Clearly event A occurs if we obtain any one of 2, 4, 6 as an outcome.

$\therefore$ Favourable number of outcomes = 3

Hence, $P(A) = \frac{3}{6} = \frac{1}{2}$

(ii) Let A denotes the event 'Getting a multiple of 3'.

We observe that the event A occurs if we obtain either 3 or 6 as an outcome.

$\therefore$ Favourable number of outcomes = 2

Hence, $P(A) = \frac{2}{6} = \frac{1}{3}$

(iii) An even number or a multiple of 3 is obtained if we obtain one of the numbers 2, 3, 4, 6 as an outcome.

$\therefore$ Favourable number of outcomes = 4

Hence, required probability $= \frac{4}{6} = \frac{2}{3}$

(iv) Let A denotes the event 'Getting an even number and multiple 3'.

Clearly, even A happens if we get 6 as an outcome.

$\therefore$ Favourable number of outcomes = 1

Hence, $P(A) = \frac{2}{6}$

(v) Let A deontes the event 'Getting 3 or 4'

Clearly, A occurs when we get either 3 or 4 as an outcome.

$\therefore$ Favourable number of outcomes = 2

Hence, $P(A) = \frac{2}{6} = \frac{1}{3}$

(vi) Let A deontes the event 'Getting an odd number'

We observe that the event A occurs when we get 1 or 3 or 5 as outcome.

$\therefore$ Favourable number of outcomes = 3

Hence, $P(A) = \frac{2}{6} = \frac{1}{3}$

(vii) The event 'Getting a number less than 5' will occur if we get one of the numbers 1,2,3,4 as an outcome.

$\therefore$ Favourable number of outcomes = 4

Hence, $P(A) = \frac{4}{6} = \frac{2}{3}$

(viii) Then event 'Getting a number between 3 will occurs if we obtain one of the number 4,5,6 as an outcome.

$\therefore$ Favourable number of outcomes = 3

Hence, required probability $= \frac{3}{6} = \frac{1}{2}$

(ix) The event 'Getting a number 3 and 6' occurs if we obtain either 4 or 5 as an outcome.

$\therefore$ Favourable number of outcomes = 2

Hence, required probability $= \frac{2}{6} = \frac{1}{3}$

Ex. 7 What is the probability that a number selected from the numbers 1,2,3, ... , 25 is a prime number, when each of the given numbers is equally likely to be selected ?

Sol. Out of 25 numbers 1,2,3,.....25, one number can be chosen in 25 ways.

$\therefore$ Total number of elementary events = 25

The number selected will be a prime number if it is chosen from the numbers 2,3,5,7,11,13, 17, 19, 23.

$\therefore$ Favourable number of outcomes = 9

Hence, required probability $= \frac{9}{25}$.

SOLVED EXAMPLES

SE.1 The given data shows the marks obtained by 25 students in a class test 74, 64, 70, 31, 41, 53, 61, 64, 70, 56, 28, 28, 88, 53, 56, 31, 53, 64, 56, 32, 53, 56, 61, 53, 74, 33 Find:

(i) the maximum marks obtained

(ii) the minimum marks obtained

(iii) the range

(iv) the mean marks

Sol. Arranging the given data in an ascending order, we get marks as 28, 31, 31, 32, 33, 41, 53, 53, 53, 53, 53, 53, 56, 56, 56, 56, 61, 61, 64, 64, 64, 70, 70, 74, 74, 88

Clearly,

(i) the maximum marks obtained is 88.

(ii) the minimum marks obtained is 28.

(iii) Range = (88 – 28) = 60.

(iv) mean marks = $\dfrac{\text{sum of observations}}{\text{number of observations}}$

$= \left(\dfrac{1375}{25}\right) = 55$

SE.2 A selection test was given to a group of 50 students. The test was completed by them in the following times (in minutes):

38, 40, 42, 41, 39, 27, 28, 26, 30, 42, 41, 43, 45, 46, 37, 37, 43, 44, 49, 36, 31, 32, 33, 35, 48, 43, 39, 36, 29, 31, 32, 34, 44, 43, 36, 37, 38, 40, 49, 41, 42, 45, 47, 48, 45, 39, 38, 37, 40, 29.

Prepare a grouped frequency distribution table taking class intervals 25-30, 30-35 etc.

Sol.

Class interval	Tally	Frequency
25-30	卌	5
30-35	卌 ||	7
35-40	卌 卌 ||||	14
40-45	卌 卌 卌	15
45-50	卌 ||||	9
	Total	50

SE.3 Form a discrete frequency distribution from the following scores :

15, 18, 16, 20, 25, 24, 25, 20, 16, 15, 18, 18, 16, 24, 15, 20, 28, 30, 27, 16, 24, 25, 20, 18, 28, 27, 25, 24, 24, 18, 18, 25, 20, 16, 15, 20, 27, 28, 29, 16.

Sol. Frequency distribution of scores.

Variate	Tally Marks	Frequency
15	||||	4
16	卌 |	6
18	卌 |	6
20	卌 |	6
24	卌	5
25	卌	5
27	|||	3
28	|||	3
29	|	1
30		1
	Total	40

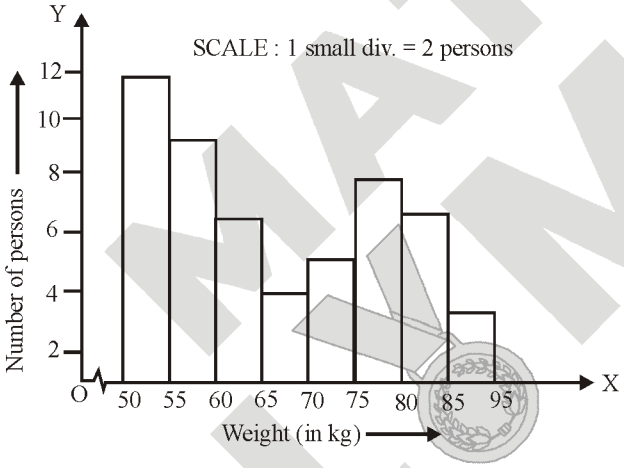
SE.4 The following is the distribution of weights (in kg) of 50 persons:

Weight (in kg)	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90
No. of persons	12	8	5	4	5	7	6	3

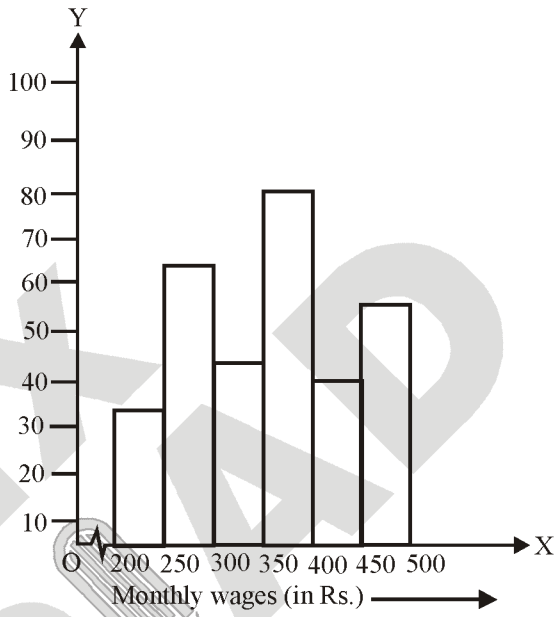
Draw a histogram for the above data.

Sol. We represent the class limits along X-axis using a suitable scale and the frequencies along Y-axis using a suitable scale.

Since the scale on X -axis starts at 50, a kink (break) is indicated near the origin to signify that the graph is drawn to scale beginning at 50, and not at the origin.



SE.5 The following histogram shows the monthly wages (In Rs.) of workers in a factory:

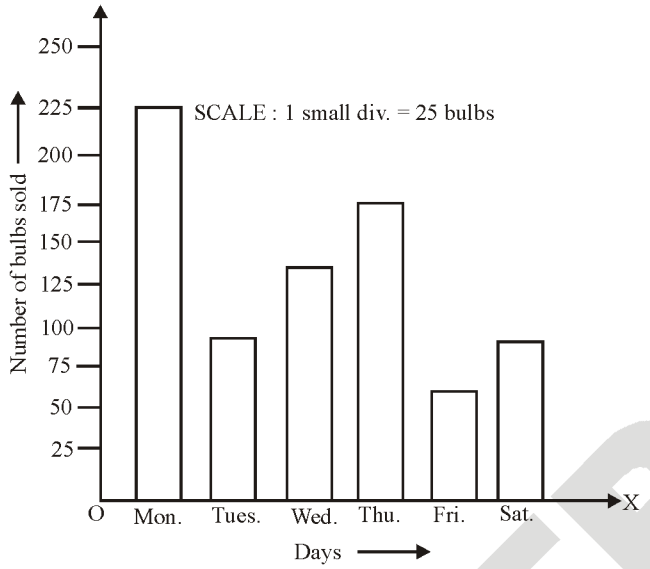


Which groups have the largest and least number or works ? Also find their number.

Sol. In the above histogen, we see that the highest rectangle corresponds to the maximum number of workers, that is 100, and the corresponding group is from Rs. 350 - 400.

The rectangle of minimum height corresponds to the least number of workers, that is 15 and the corresponding group is Rs. 400 -450.

SE.6 Given below is a graph which shows the number of electric bulbs solid in a shop during a week.



Read the the bar graph carefully and answer the questions given below:

(i) On which day of the week was the sale minimum?

(ii) On which day of the week was the sale maximum ?

(iii) What was the total sale during the week ?

(iv) What is the ratio between the minimum sale and the maximum sale ?

Sol. (i) It is clear from the bar graph that the bar of minimum height corresponds to the sale on Friday.

∴ The sale was minimum of Friday.

(ii) From the bar graph, we find that the bar of maximum height corresponds to the sale on Monday.

∴ The sale was maximum of Monday.

(iii) The total sale during the week = (225 + 100 + 150 + 200 + 75 + 100) bulbs = 850 bulbs.

(iv) The minimum sale during the week = 75 bulbs.

The maximum sale during the week = 225 bulbs.

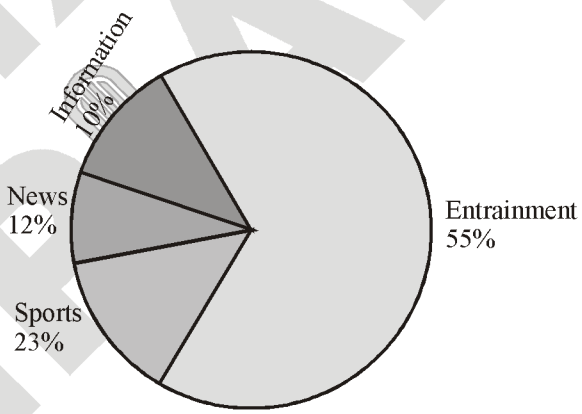
∴ Minimum sale : Maximum sale = 75 : 225

= 1 : 3.

SE.7 Answer the following questions based on the pie chart given below :

(i) Which type of programmes are viewed the most ?

(ii) Which two types of programmes together have viewers almost equal to those watching sports channels ?



Viewers waching different types of channels on TV.

Sol. (i) The entertainment programmers are viewed the most.

(ii) Information and News channels.

SE.8 The following table gives the marks scored by 100 students in an entrance examination.

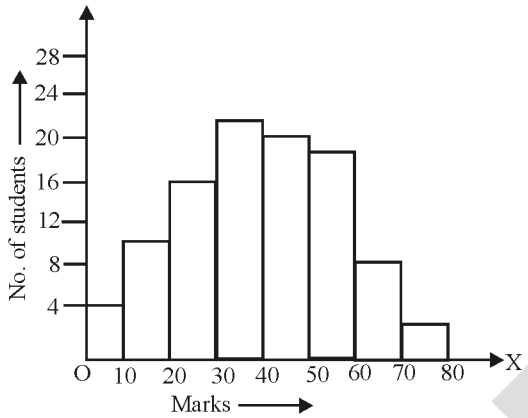
Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students (Frequency)	4	10	16	22	20	18	8	2

Represent this data in the form of a histogram.

Sol. We represent the class intervals along X-axis using a suitable scale and the frequencies along

Y-axis using a suitable scale.

Taking class-intervals as bases and the corresponding frequencies as heights, we construct rectangles to obtain the histogram of the given frequency distribution as shown.



SE.9 The data on the mode of transport used by 720 students are given below:

Mode of Transport	Bus	Cycle	Train	Car	Scooter
No. of students	120	180	240	80	100

Represent the above data by a pie chart.

Sol. Total number of students = 720

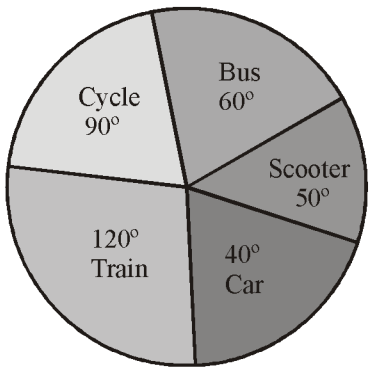
Central angle for a mode of transport

$$= \left(\frac{\text{number of students using that mode}}{\text{total number of students}} \times 360\right)^{\circ}$$

Mode of transport	Number of students	Central angle
Bus	120	$\left(\frac{120}{720} \times 360\right)^{\circ} = 60^{\circ}$
Cycle	180	$\left(\frac{180}{720} \times 360\right)^{\circ} = 90^{\circ}$
Train	240	$\left(\frac{240}{720} \times 360\right)^{\circ} = 120^{\circ}$
Car	80	$\left(\frac{80}{720} \times 360\right)^{\circ} = 40^{\circ}$
Scooter	100	$\left(\frac{100}{720} \times 360\right)^{\circ} = 50^{\circ}$

Steps of construction of pie chart:

1. Draw a circle of any convenient radius.
2. Draw a radius of this circle.
3. Starting with this radius, draw sectors whose central angles are 60°, 90°, 120°, 40°, and 50° respectively.
4. Shade the sectors so obtained different and label each one of them.



Thus, we obtain the required pie chart, as shown in the given figure.

SE.10 Pooja spends different hours of a working day as follows :

Activity	School	Home assignment	Play	Sleep	Other
No. of hours	8	3	2	8	3

Draw a pie chart to represent the above data.

Sol. We know that central angle for a variable

$$= \frac{\text{Frequency of the variable}}{\text{Total frequencies}} \times 360^\circ$$

Total frequencies = 8 + 3 + 2 + 8 + 3 = 24

$\therefore$ Central angle for school = $\frac{8}{24} \times 360^\circ = 120^\circ$

Central angle for home assignments = $\frac{3}{24} \times 360^\circ$

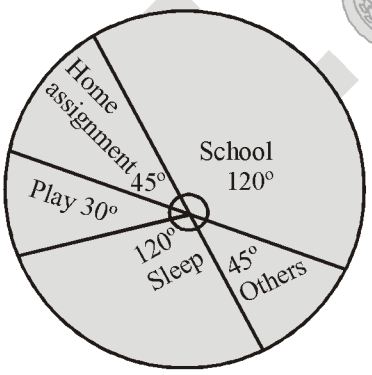
$= 45^\circ$

Central angle for play = $\frac{2}{24} \times 360^\circ = 30^\circ$

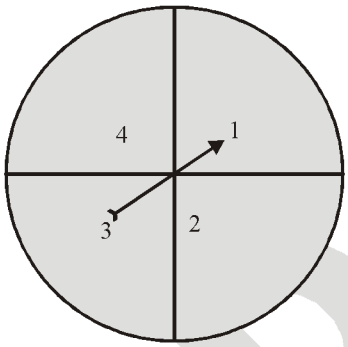
Central angle for sleep = $\frac{8}{24} \times 360^\circ = 120^\circ$

Central angle for other activities = $\frac{3}{24} \times 360^\circ$

$= 45^\circ$



SE.11 A spinner as shown in figure has spun 80 times. How many times do you expect the number '3' ?



Sol. When this spinner spun one time, the possible outcomes are 1, 2, 3, 4.

$$= \frac{\text{Number of favourable outcome}}{\text{Total number of outcome}}$$

$\therefore$ Probability of getting '3' = $\frac{1}{4}$

In 80 times, one can expect '3' to come

$= \frac{1}{4} \times 80 = 20$ times.

SE.12 A suervey of 250 families shows the results given below:

Number of girls in the family	2	1	0
Number of families	45	165	40

Out of these families one is chosen at random. What is the probability that the chosen family has 1 girl?

Sol. Total number of families = 250

Number of families having 1 girl = 165.

Probability of getting a family having 1 girl

$$= \frac{\text{Number of families having 1 girl}}{\text{Total number of families}} = \frac{165}{250} = \frac{33}{50}$$

SE.13 Tickers numbered from 1 to 20 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket has a number which is multiple of 3 or 7?

Sol. Out of 20 tickets numbered from 1 to 20, one can be chosen in 20 ways. So, total number of elementary events associated with the given random experiment is 20. Out of 20 tickets numbered 1 to 20, tickets bearing numbers which are multiple 3 or 7 bear numbers 3, 6, 7, 9, 12, 14, 15 and 18.

Favourable number of elementry events = 8

Hence, required probability $= \frac{8}{20} = \frac{2}{5}$.

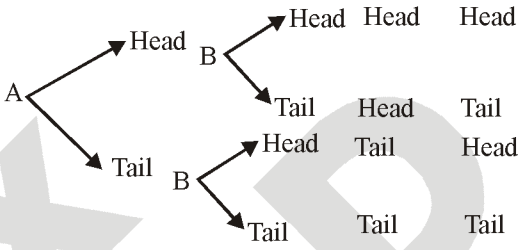
SE.14 A bag has 4 red balls and 2 yellow balls. (The balls are identical in all respects other than colour). A ball is drawn from the bag without looking into the bag. What is the probability of getting red ball ? Is it more or less than getting a yellow ball ?

Sol. There are in all (4 + 2 = 6) outcomes of the event. Getting a red ball consists of 4 outcomes. Therefore, the probability of getting red ball is $\frac{4}{6} = \frac{2}{3}$.

In the same way, probability of getting a yellow ball $= \frac{2}{6} = \frac{1}{3}$. Therefore, the probability of getting a red ball is more than that of getting a yellow ball.

SE.15 If two coins are tossed together, what are the chances of getting 'head, head' ?

Sol. Let us suppose the coins are marked as A and B. Each event of A will be matched with both events of B and we get



∴ All possible outcomes = Head Head, Head Tail, Tail Head, Tail Tail

We can see that out of 4 outcomes, 'Head Head' occurs only once.

∴ Hence, required probability $= \frac{1}{4}$.

EXERCISE-I

ONLY ONE CORRECT TYPE

1. The range of x, 32, 41, 62, 64 and 71 is 45. Which of the following can be the value of x ?

(A) 32 (B) 47

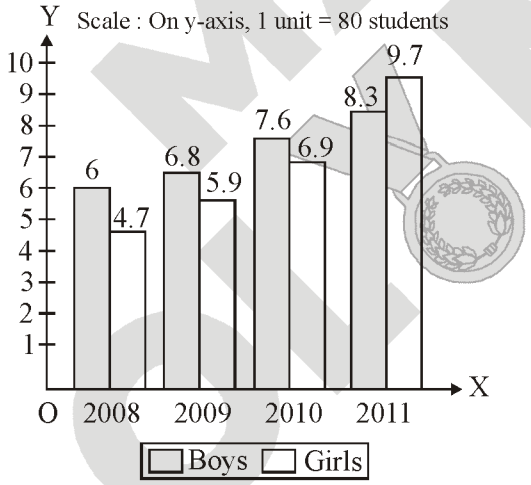
(C) 48 (D) 26

2. In a pie chart representing the percentages of students having interest in reading various kinds of books, the central angle of the sector representing students reading novels is 81°. What is the percentage of students interested in reading novels ?

(A) 15 % (B) 18 %

(C) $22\frac{1}{2}\%$ (D) $27\frac{1}{2}\%$

Directions (4 – 8) : Read the following bar graph and answer the questions.



Bar graph of number of boys and number of girls in a school from 2008 to 2011.

3. In which year the difference between the number of boys and the number of girls is maximum ?

(A) 2008 (B) 2009

(C) 2010 (D) 2011

4. Total number of students in the year 2009 is

(A) 1016 (B) 1270

(C) 1380 (D) 1490

5. Find the minimum difference between the number of boys and girls in any year in the given period.

(A) 90 (B) 70

(C) 56 (D) 30

6. In which year the number of girls is more than the number of boys ?

(A) 2008 (B) 2009

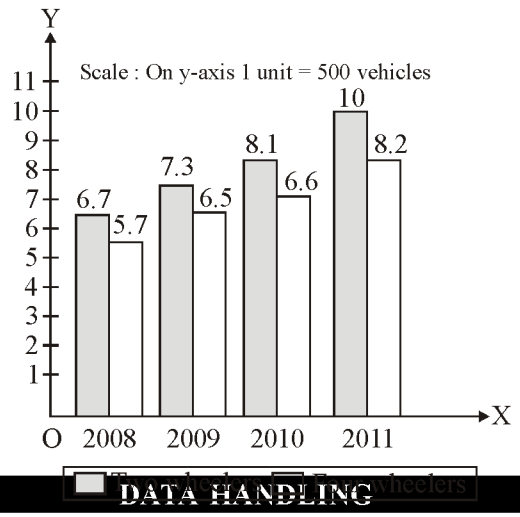
(C) 2010 (D) 2011

7. Find the ratio between the number of students in the year 2008 and in 2009.

(A) 107 : 145 (B) 127 : 145

(C) 29 : 36 (D) 107 : 127

Directions (9 – 13) : Read the bar graph and answer the following questions.



Bar graph represents sale of two wheelers and four wheelers in a city from 2008 to 2011.

8. In which year the difference between sales of two wheelers and four wheelers is minimum ?

(A) 2008 (B) 2009

(C) 2010 (D) 2011

9. Total number of vehicles (two wheelers and four wheelers) sold in the year 2008 and 2009 is

(A) 26100 (B) 28500

(C) 13100 (D) 27500

10. Find the maximum difference between sale of two wheelers and that of four wheelers, in any year, in the given period.

(A) 900 (B) 1700

(C) 1800 (D) 2000

11. Find the total number of two wheelers sold in four years.

(A) 16050 (B) 27000

(C) 31000 (D) 32000

12. Find the ratio between number of vehicles sold in the year 2009 and in the year 2011.

(A) 41 : 46 (B) 69 : 91

(C) 147 : 182 (D) 46 : 49

13. Probability of drawing a consonant randomly from English alphabets is

(A) $\frac{21}{26}$ (B) $\frac{1}{26}$

(C) 0 (D) $\frac{1}{5}$

14. Three coins are tossed simultaneously. What is the probability that head and tail show alternately (i.e., HTH or THT) ?

(A) $\frac{3}{8}$ (B) $\frac{1}{4}$

(C) $\frac{1}{8}$ (D) $\frac{1}{2}$

15. A card is drawn from a well shuffled pack of 52 cards. The probability of drawing a king is

(A) $\frac{1}{52}$ (B) $\frac{2}{13}$

(C) $\frac{1}{26}$ (D) $\frac{1}{13}$

16. A bag contains 5 red balls, 3 blue balls and 1 white ball. A ball is drawn at random without looking into the bag. What is the probability of getting a blue ball ?

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{5}{9}$ (D) $\frac{1}{9}$

17. From the frequency table, the percentage of families with less than 3 children is

Number of children	0	1	2	3	4
Number of families	4	6	12	15	7

(A) 45 % (B) 50 %

(C) 11 % (D) 18 %

18. The difference between upper limit and lower limit is known as

(A) Class size (B) Class mark

(C) Class interval (D) None of these

19. The number of times a particular observation occurs is called its

(A) Range (B) Frequency

(C) Observation (D) None of these

20. The mid-value of a class interval is called its

(A) Class mark (B) Class size

(C) Frequency (D) Range

21. In a single throw of two dice, find the probability of getting a total of 3 or 5.

(A) $\frac{1}{3}$ (B) $\frac{5}{6}$

(C) $\frac{1}{9}$ (D) $\frac{1}{6}$

22. A bag contains x red balls, (x + 5) blue balls and (3x + 10) white balls. If the probability of drawing a white ball is $\frac{11}{18}$, what is the number of blue balls ?

(A) 15 (B) 20

(C) 35 (D) 55

23. The number of students in a hostel speaking different languages is given. The central angle for Tamil is

Language	Number of students
Hindi	40
English	12
Marathi	9
Tamil	7
Bengali	4
Total	72

(A) 75° (B) 45°

(C) 35° (D) 32°

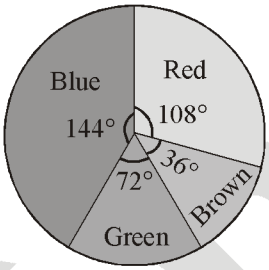
24. Monthly savings (in rupees) of 14 students of class VIII are as follows :

53, 80, 43, 90, 64, 20, 24, 30, 53, 56, 60, 64, 53, 40. The frequency of the observation 53 is

(A) 2 (B) 4

(C) 1 (D) 3

25. Hridya asked 20 of his friends about their favourite colour. The pie chart below shows the data he collected. What percentage of his friends picked blue ?

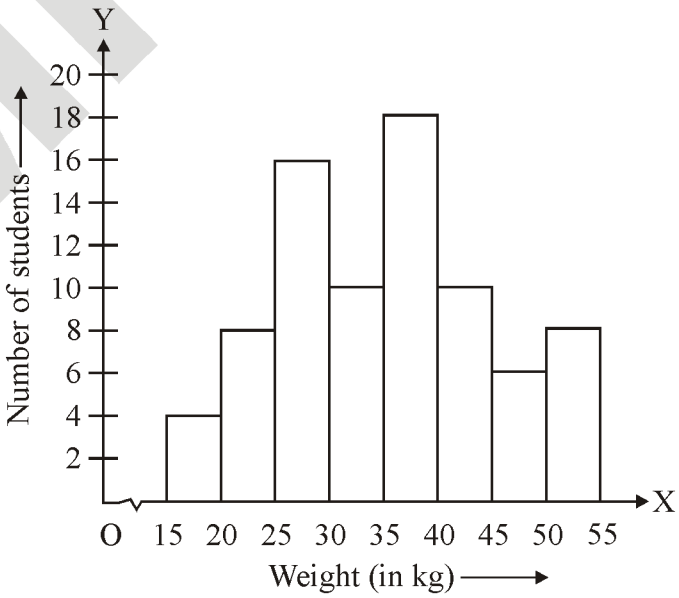


(A) 30 % (B) 10 %

(C) 40 % (D) 20 %

PARAGRAPH TYPE

PASSAGE – I : Given below is the histogram showing weight (in kg) of the students of class VIII in a school.



Study the histogram and answer the following questions :

1. How many students have been observed ?

(A) 20 (B) 55

(C) 40 (D) 80

2. What is the class size ?

(A) 15 (B) 10

(C) 5 (D) 55

3. How many students weigh less than 35 kg ?

(A) 38 (B) 24

(C) 16 (D) 18

PASSAGE – II : In a simultaneous throw of a pair of dice, find the probability of getting :

4. A doublet of prime numbers.

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{5}{12}$ (D) $\frac{1}{12}$

5. An even number on one and a multiple of 3 on the other.

(A) $\frac{11}{36}$ (B) $\frac{12}{35}$

(C) $\frac{1}{6}$ (D) $\frac{5}{12}$

6. Neither 9 nor 11 as the sum of the numbers on the faces.

(A) $\frac{1}{6}$ (B) $\frac{5}{6}$

(C) $\frac{3}{5}$ (D) $\frac{1}{2}$

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from List – I and List – II are given as options (A), (B), (C) and (D) out of which one is correct.

1. If three coins are tossed together. Find the probability of getting

List – I	**List – II**
(P) Exactly two heads	(i) $\frac{1}{2}$
(Q) Atleast two heads	(ii) $\frac{1}{8}$
(R) At least one head	(iii) $\frac{3}{8}$
(S) No tail	(iv) $\frac{7}{8}$

(A) (P) → (i), (Q) → (iii), (R) → (iv), (S) → (ii)

(B) (P) → (iii), (Q) → (i), (R) → (iv), (S) → (ii)

(C) (P) → (ii), (Q) → (i), (R) → (iv), (S) → (iii)

(D) (P) → (ii), (Q) → (iv), (R) → (i), (S) → (iii)

2. Match List – I with List – II.

List – I	**List – II**
(P) The collection of observation is called	(i) Organising data
(Q) Entry in data is called	(ii) Raw Data
(R) The collection of observations collected initially is called	(iii) Data
(S) Data handling means	(iv) Observation

(A) (P) → (iv), (Q) → (iii), (R) → (ii), (S) → (i)

(B) (P) → (iii), (Q) → (ii), (R) → (iv), (S) → (i)

(C) (P) → (iii), (Q) → (iv), (R) → (ii), (S) → (i)

(D) (P) → (i), (Q) → (ii), (R) → (iv), (S) → (iii)

EXERCISE-II

VERY SHORT ANSWER TYPE

1. The rainfall (in mm) in Delhi on 7 days of August 2003 is given below :

Day	Rainfall (in mm)
11th	2.2
12th	21.3
13th	25.6
14th	4.9
15th	0.0
16th	25.6
17th	0.0

(i) What is the highest value of rainfall ?

(ii) What is the range of rainfall in the above data?

2. Given below are the heights (in cm) of 11 boys of a class : 146, 143, 148, 132, 128, 139, 140, 152, 154, 142, 149

Arrange the above data in ascending order and find

(i) The height of the tallest boy.

(ii) The height of the shortest boy.

(iii) The range of the given data.

3. In a study of number of accidents per day, the observations for 10 days were obtained as follows : 0, 1, 3, 4, 0, 3, 2, 2, 3, 0

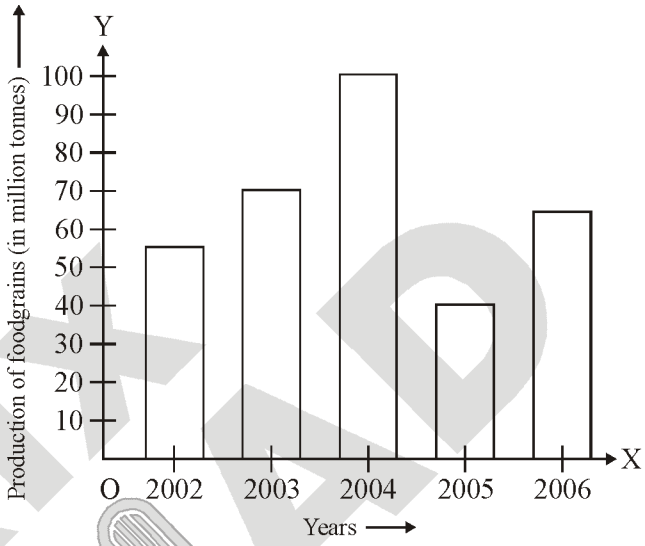
Prepare a frequency table.

4. The marks obtained by Kunal in his annual examination are shown below :

Subjects	Hindi	English	Maths	Sci.	Soc.Stu.
Marks Obtained	60	75	90	70	55

Draw a bar graph to represent the above data.

5. A bar graph is given below :



Production of foodgrains in an Indian state during five consecutive years

Read the bar graph carefully and answer the questions given below :

(i) What information is given by the bar graph ?

(ii) In which year was the production maximum ?

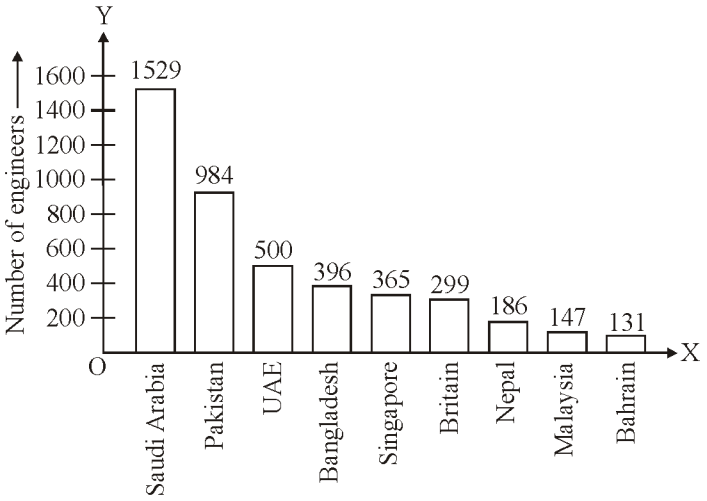
(iii) After which year was there a sudden fall in the production ?

6. Read the given bar graph and answer the following questions :

(i) What information is given by the bar graph ?

(ii) When were the figures taken ?

(iii) Which countries have more than 500 engineers ?

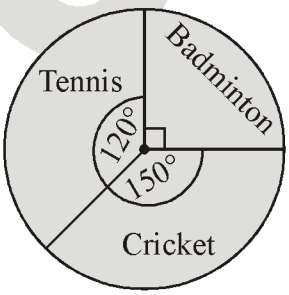


7. A bag contains 3 red and 2 blue marbles. A marble is drawn at random. What is the probability of drawing a blue marble ?

8. The letters of the word 'SWORD' are written on similar cardboards. These cardboards are then mixed and put upside down so that no one can see the letters. Rajni takes one of the cardboards. What is the probability that it has the letter 'R' on it ?

9. It is known that a box of 600 electric bulbs contains 12 defective bulbs. One bulb is taken out at random from this box. What is the probability that it is a non-defective bulb ?

10. A pie chart is shown, represents the games liked by the students of class VIII. A boy is selected at random. What is the probability that he likes cricket ?



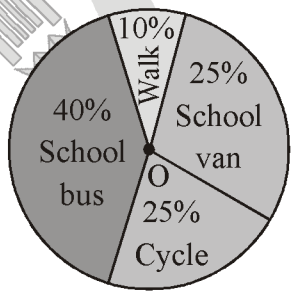
SHORT ANSWER TYPE

1. The frequency distribution of weights (in kg) of 40 persons of a locality is given below :

Weights (in kg)	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65
Freq.	4	12	13	6	5

(i) What is the upper limit of the fourth class interval ?
(ii) Find the class marks of all the classes.
(iii) What is the class size of each class interval ?
(iv) Which class interval has the highest frequency?

2. The given pie chart depicts the modes of transport used by the students to commute to school. Now, answer the following questions.



(i) Which is the most preferred mode of transport ?
(ii) Find out the least preferred mode of transport ?
(iii) If the total number of students are 3600, then find out the number of students who come by cycle ?

3. A coin is tossed 130 times and head is obtained 75 times. Now, if a coin is tossed at random, what is the probability of getting a tail ?

4. A dice is thrown 65 times and 4 appeared 21 times. Now, in a random throw of a dice, what is the probability of getting a 4 ?

5. A bag contains 5 red balls, 8 white balls, 4 green balls and 7 black balls. If one ball is drawn at

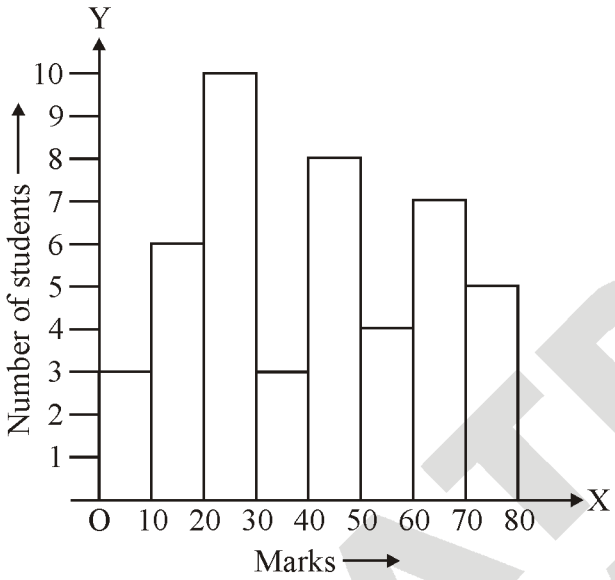
random, find the probability that it is :

(i) black (ii) red

(iii) not green

LONG ANSWER TYPE

1. The following histogram depicts the marks obtained by 45 students of a class :



Look at the histogram and answer the following questions :

(i) What is the class size ?

(ii) How many students obtained less than 10 marks?

(iii) How many students obtained more than 30 marks but less than 40 marks ?

(iv) What is the interval of highest marks and how many students are there in this interval ?

(v) If passing marks are 30, what is the number of failures ?

2. Two dice are thrown simultaneously. Find the probability of getting :

(i) an even number as the sum.

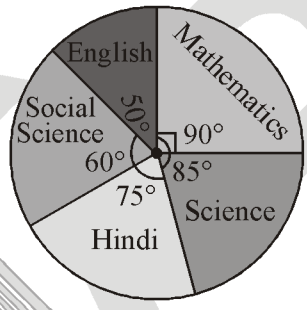
(ii) the sum as a prime number.

(iii) a total of atleast 10.

(iv) a doublet of even number.

(v) a multiple of 2 on one die and a multiple of 3 on the other.

3. The following pie-chart gives the marks scored in an examination by a student in various subjects. If the total marks obtained by the student were 540, answer the following questions :



(i) In which subject did the student score 75 marks?

(ii) How many more marks were obtained by the student in Mathematics than in Hindi ?

(iii) Examine whether the sum of marks obtained in Social Science and Mathematics is more/less than that in Science and Hindi.

4. Cards marked with the number 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from this box. Find the probability that the number on the card is :

(i) an even number.

(ii) a number less than 14.

(iii) a number which is a perfect square.

(iv) a prime number less than 20.

5. The following data relates to the expenditure of the families A, B and C per month :

Items of Expenditure	Family A	Family B	Family C
Food	400	600	1600
Rent	200	400	1500
Clothing	200	300	1000
Education	100	400	800
Litigation	50	100	300
Miscellaneous	50	200	800

Represent this data by pie-diagrams.

TRUE / FALSE TYPE

1. The mean of the observations 7, 8, 9, 11 and 15 is 10.
2. If the range of a data is 9 and its highest value is 81, then its least value is 73.
3. The observation 20 is included in interval 20-30 and not in 10-20.
4. The upper limit of one class interval is always equal to the lower limit of next class interval.
5. The bars in bar graph are always drawn vertically.

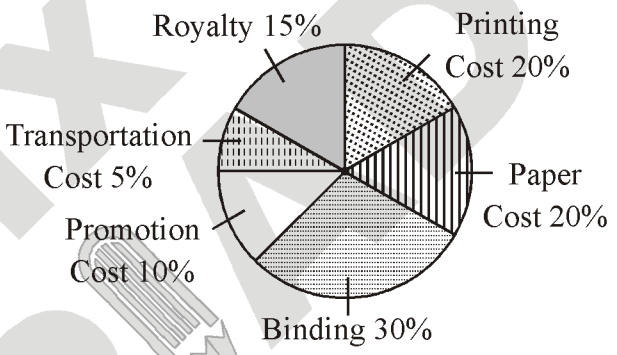
NUMERICAL PROBLEMS

1. Find the range of the data 143, 147, 135, 151, 128, 138, 144, 146, 151 and 153.
2. The probability that a number selected from the numbers 1, 2, 3, ... 15 is a multiple of 4, is.
3. A die is thrown 24 times, 4 is obtained 12 times. The probability of number '4' to come up is k. The value of k is.
4. A pentagonal spinner having numbers 1, 2, 3, 4 and 5 is spun. The probability that the pointer will be at 3 is $\frac{1}{x}$. The value of x is.
5. The probability of selecting a letter 'P' from the letters of the word 'PARKER' is $\frac{k}{m}$. The value of m – k is.

ANALYTICAL PROBLEMS & BRAIN TEASER

1. Study the pie-chart and answer the question based on it.

VARIOUS EXPENDITURES (IN PERCENTAGE INCURRED IN PUBLISHING A BOOK)



Which of the following two expenditures together have a central angle of 108° ?

(A) Binding and Transportation

(B) Printing and Paper

(C) Royalty and Promotion

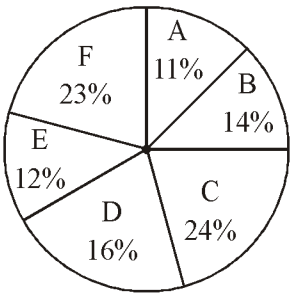
(D) Promotion and Paper

2. Two dice are tossed. The probability that the total score is a prime number is.

(A) $\frac{1}{6}$ (B) $\frac{5}{12}$

(C) $\frac{1}{2}$ (D) $\frac{7}{9}$

Direction (3 – 4) : In the given pie-chart, the percentage of students studying in 6 different schools A, B, C, D, E and F has been shown.



3. What is the average number of students studying in schools B, D and E ?

(A) 11200 (B) 10600

(C) 10000 (D) 10500

4. Ratio of the number of students studying in schools C to that in school E is

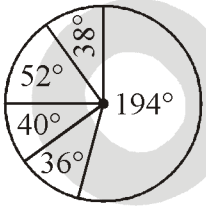
(A) 7 : 3 (B) 5 : 3

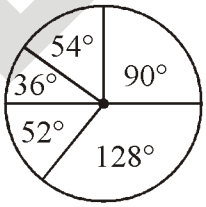
(C) 3 : 5 (D) 2 : 1

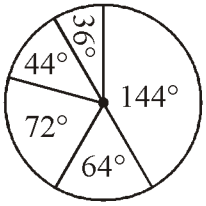
5. The expenditure incurred on various things during the construction of a house are given below.

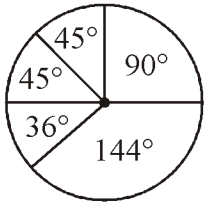
Items	Expenditure (In thousands Rs.)
Bricks	180
Cement	80
Timber	90
Steel	55
Wood	45
Total	450

Which of the following pie charts exhibits the given information ?

(A) 

(B) 

(C) 

(D) 

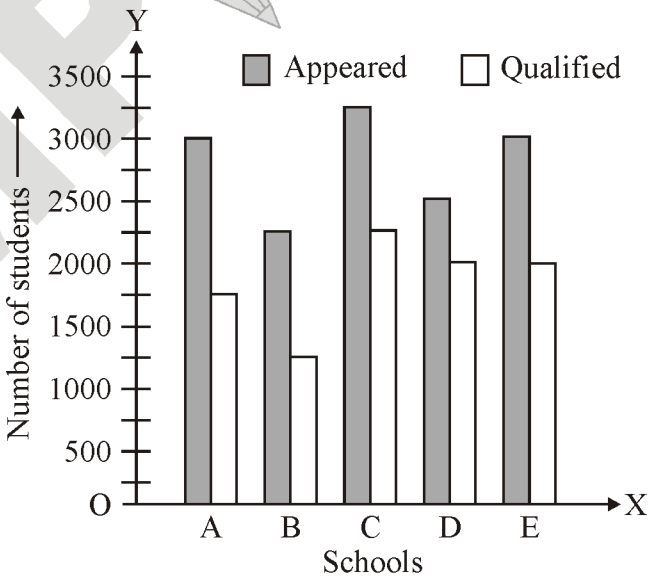
6. One card is drawn from a well-shuffled deck of 52 cards. Find the probability that the number on the card drawn is a multiple of 5.

(A) $\frac{4}{52}$ (B) $\frac{4}{13}$

(C) $\frac{7}{52}$ (D) $\frac{2}{13}$

Direction (7 – 8) : Study the following graph carefully and answer the questions given below :

Total number of Students Appeared and Qualified from Various Schools at a Scholarship Exam



7. The average number of students qualified in the examination from schools C and D is what percent of the average number of students appeared for the examination from the same schools ? (Rounded off to 2 digits after decimal).

(A) 58.62 % (B) 73.91 %

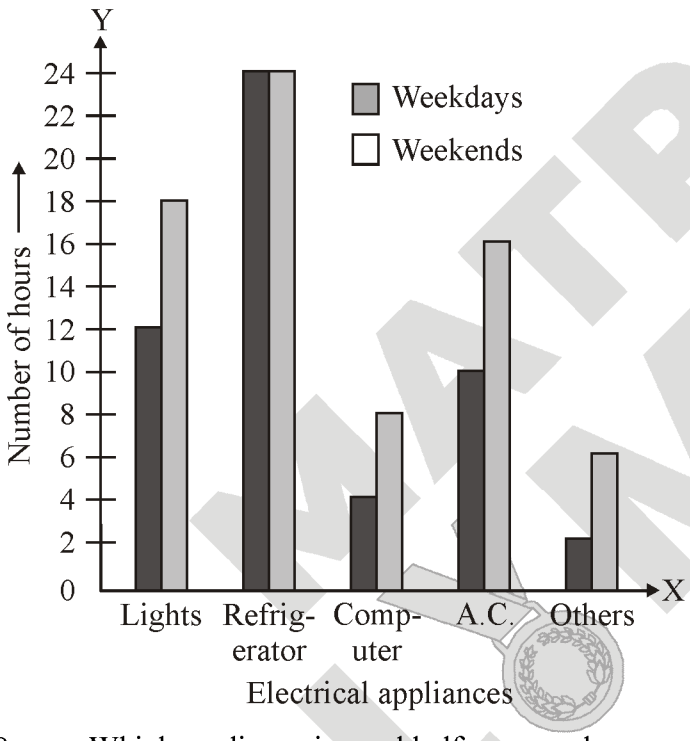
(C) 62.58 % (D) 58.96 %

8. What is the ratio of the number of students qualified in the scholarship examination from school A to the number of students qualified in the examination from school B ?

(A) 8 : 3 (B) 5 : 7

(C) 7 : 3 (D) 7 : 5

Direction (9 – 10) : The given double column graph shows the average number of hours for which an electrical appliance is used on weekdays and wekends. Study the graph carefully and answer the questions that follow :



9. Which appliance is used half as many hours on the weekdays as it is used on the weekends ?

(A) Lights (B) Refrigerator

(C) Computer (D) A.C.

10. Average number of hours, all appliances were used on weekdays is what percent of the average number of hours they were used on weekends ?

(A) 82.35 % (B) 72.22 %

(C) 154 % (D) None of these

EXERCISE-III

PREVIOUS YEAR QUESTIONS (NTSE)

1. The mean of 9 observation is 13. The mean of the first 5 observation is 11 and the mean of last 5 observation is 14, then 5th observation is

(Aryabhatta-2005)

(A) 11 (B) 12

(C) 8 (D) 13

2. Students at a local college were asked to many hours they slept last night. The adjoining chart shows the data. A bar graph of this data will be made on a grid that is 20 units. What scale would be most appropriate for the axis labelled **"Number of Students"** ?

(IMO-2010)

Hours of Sleep	Number of Students
6	14
7	26
8	28
9	15
More than 10	6

(A) 1 unit = 1 student

(B) 1 unit = 2 students

(C) 1 unit = 10 students

(D) 1 unit = 28 students

3. Of the three numbers, the first is twice the second and is half the third. If the average of three numbers is 56, the three numbers in order are :

(Aryabhatta-2011)

(A) 48, 24, 96 (B) 48, 36, 96

(C) 48, 12, 14 (D) 24, 12, 48

4. The class marks of a frequency distribution are given as follow : 15, 20, 25, ... The class corresponding to the class mark 20 is :

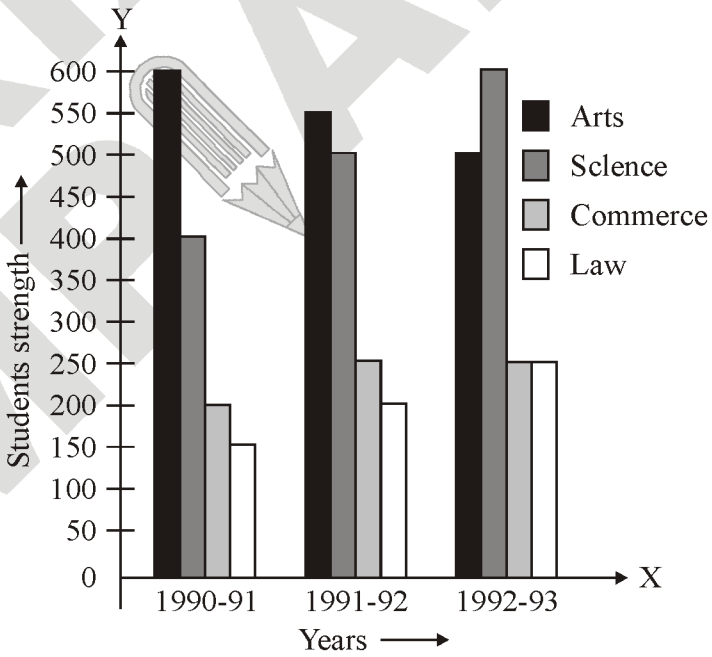
(Aryabhatta-2012)

(A) 12.5 – 17.5 (B) 17.5 – 22.5

(C) 18.5 – 22.5 (D) 19.5 – 20.5

Direction (5 – 6) : A bar diagram depicting the changes in the students strength of a college in four faculties from 1990-91 to 1992-93 is shown.

(IMO-2012)



5. How many times was the total strength of the strength of Commerce students in 1991-92 ?

(A) 3 times (B) 4 times

(C) 5 times (D) 6 times

6. How much percent was the increase in Science students in 1992-93 over 1990-91 ?

(A) 50 % (B) 150 %

(C) $66\frac{2}{3}\%$ (D) 75 %

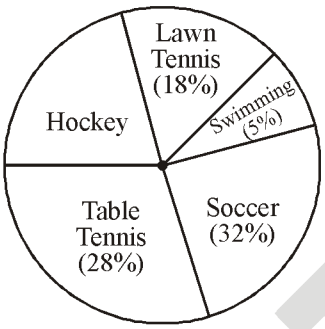
7. M being the mean of $x_1, x_2, x_3, x_4, x_5, x_6$, find the value of $(x_1 - M) + (x_2 - M) + (x_3 - M) + (x_4 - M) + (x_5 - M) + (x_6 - M)$

(Aryabhatta-2012)

(A) 1 (B) 0

(C) 6 M (D) $\frac{M}{6}$

8. The pie-chart shows the favourite sports of a group of students.



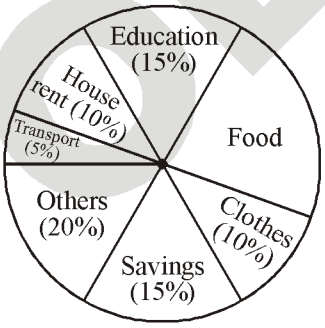
If 34 students like hockey, calculate the total number of students who like lawn tennis and table tennis ?

(NSTSE-2013)

(A) 92 (B) 70

(C) 40 (D) 100

9. The pie-chart shows the expenditure (in percentage) on various items and savings of a family during a month.



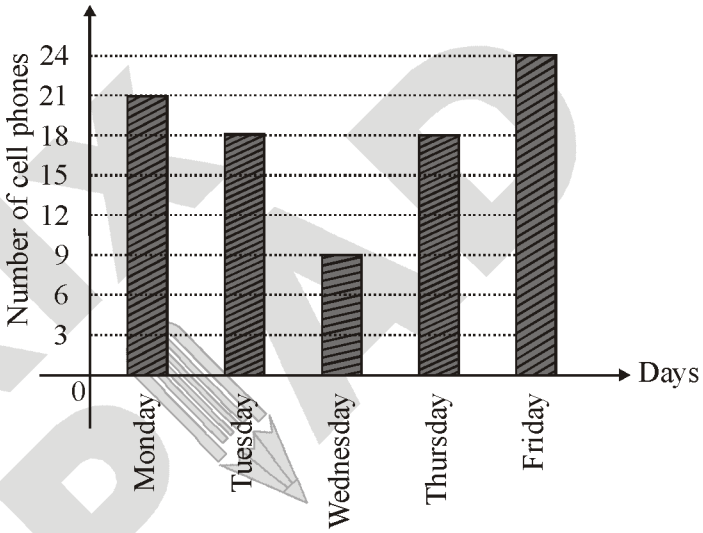
On which item was the expenditure maximum ?

(NSTSE-2014)

(A) Food (B) Clothes

(C) Savings (D) Transport

10. The bar chart shows the number of cell phones sold by a shop on 5 days of a certain week. The difference between the highest number and the lowest number of cell phones sold is 15.



Find the number of cell phones sold on Thursday.

(NSTSE-2015)

(A) 6 (B) 12

(C) 18 (D) 24

Answer Key

EXERCISE I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D		C	D	A	C	D	D	B	C	A	A	B	A	B
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
			D	A	B	A	B			A			D	B
31	32	33	34	35	36	37								
C	D	C												

PARAGRAPH

1. D **2.** C **3.** A **4.** D **5.** A **6.** B

MATCH THE COLUMN

1. B **2.** C

EXERCISE II

VERY SHORT ANSWER TYPE

1. (i) 25.6 mm, (ii) 25.6 **2.** (i) 154 cm, (ii) 128 cm, (iii) 26 cm **5.** (ii) 2004, (iii) 2004

6. (ii) June 30, 1994, (iii) Saudi Arabia and Pakisthan **7.** $\frac{2}{5}$ **8.** $\frac{1}{5}$ **9.** $\frac{49}{50}$

10. $\frac{5}{12}$

SHORT ANSWER TYPE

2. (i) 60, (iii) 5, (iv) 50-55 **3.** (i) School bus, (ii) Walking, (iii) 900

7. $\frac{11}{26}$ **8.** $\frac{11}{26}$

10. (i) $\frac{7}{24}$, (ii) $\frac{5}{24}$, (iii) $\frac{5}{6}$

LONG ANSWER TYPE

1. (i) 10, (ii) 3, (iii) 3, (iv) 70-80, 5, (v) 19 **2.** (i) $\frac{1}{2}$, (ii) $\frac{5}{12}$, (iii) $\frac{1}{6}$, (iv) $\frac{1}{12}$, (v) $\frac{11}{36}$

3. (i) English, (ii) 22.5, (iii) Less **4.** (i) $\frac{1}{2}$, (ii) $\frac{3}{25}$, (iii) $\frac{9}{100}$, (iv) $\frac{2}{25}$

TRUE / FALSE

1. True **2.** False **3.** True **4.** True **5.** False

NUMERICAL PROBLEMS

1. 25 **3.** 0.2 **6.** 0 **7.** 0.5
8. 5 **9.** 0.5 **10.** 5

ANALYTICAL PROBLEMS & BRAIN TEASER

1. D **2.** B **3.** D **4.** D **5.** C **6.** D **7.** B
8. D **9.** C **10.** B

EXERCISE III

1	2	3	4	5	6	7	8	9	10	
C	B	A	B	D	A	B	A	A	C	

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : DATA HANDLING)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

SQUARES AND SQUARE ROOTS

1 INTRODUCTION

We know that the area of a square = side × side (where 'side' means 'the length of a side'). Study the following table.

Side of a square (in cm)	Area of the square (in cm^2)
1	$1 \times 1 = 1 = 1^2$
2	$2 \times 2 = 4 = 2^2$
3	$3 \times 3 = 9 = 3^2$
5	$5 \times 5 = 25 = 5^2$
8	$8 \times 8 = 64 = 8^2$
a	$a \times a = a^2$

What is special about the numbers 4, 9, 25, 64 and other such numbers ? Since, 4 can be expressed as $2 \times 2 = 2^2$, 9 can be expressed as $3 \times 3 = 3^2$, all such numbers can be expressed as the product of the number with itself. Such numbers like 1, 4, 9, 16, 25, ... are known as square numbers.

2 SQUARE OF A NUMBER AND A PERFECT SQUARE

The square of a number is that number raised to the power 2. Thus, if 'a' is a number, then the square of a is written

For example : $2^2 = 2 \times 2 = 4$, so we say that the square of 2 is 4.

$3^2 = 3 \times 3 = 9$, so we say that the square of 3 is 9.

$12^2 = 12 \times 12 = 144$, so we say that the square of 12 is 144.

The following table contains the squares of first thirty natural numbers.

Number	Square	Number	Square
1	$1^2 = 1$	16	$16^2 = 256$
2	$2^2 = 4$	17	$17^2 = 289$
3	$3^2 = 9$	18	$18^2 = 324$
4	$4^2 = 16$	19	$19^2 = 361$
5	$5^2 = 25$	20	$20^2 = 400$
6	$6^2 = 36$	21	$21^2 = 441$
7	$7^2 = 49$	22	$22^2 = 484$
8	$8^2 = 64$	23	$23^2 = 529$
9	$9^2 = 81$	24	$24^2 = 576$
10	$10^2 = 100$	25	$25^2 = 625$
11	$11^2 = 121$	26	$26^2 = 676$
12	$12^2 = 144$	27	$27^2 = 729$
13	$13^2 = 169$	28	$28^2 = 784$
14	$14^2 = 196$	29	$29^2 = 841$
15	$15^2 = 225$	30	$30^2 = 900$

3 PERFECT SQUARE

A natural number is called a perfect square number if it is the square of a natural number i.e., if $n^2 = m$, when n and m are natural numbers, then m is a perfect square.

How to Check a Perfect Square : In order to check whether a given natural number is a perfect square or not, we follow the following steps :

Step – 1 : Obtain the given number.

Step – 2 : Write the number as a product of its prime factors.

Step – 3 : If the prime factors can be grouped into pairs then the number is a perfect square otherwise, the number is not a perfect square.

4 PROPERTIES OF PERFECT SQUARE NUMBERS

(1) A natural number having 2, 3, 7 or 8 at the one's place is never a perfect square.

For example : 12, 23, 37 and 48 are not perfect squares.

(2) A natural number having 0, 1, 4, 5, 6 or 9 at the one's place may or may not be a perfect square.

For example : 100, 121, 144, 256 and 169 are perfect squares but 10, 221, 224, 326 and 389 are not perfect squares.

(3) A number ending with odd number of zeroes is never a perfect square.

For example : 40, 5000, 100000 are not perfect squares.

(4) The square of an odd number is always odd.

For example : $5^2 = 25$ and $9^2 = 81$.

Here, 25 and 81 are odd numbers.

(5) The square of an even number is always even.

For example : 14 is an even number and $14^2 = 196$ is also an even number.

(6) Factors of every square number can be grouped into pairs of equal numbers.

For example : $729 = \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$

(7) (a) The square of a number ending with 1 or 9 ends with 1.

For example : $21^2 = 441$ and $9^2 = 81$.

(b) The square of a number ending with 2 or 8 ends with 4.

For example : $2^2 = 4$, $18^2 = 324$.

(c) The square of a number ending with 3 or 7 ends with 9.

For example : $7^2 = 49$, $13^2 = 169$.

(d) The square of a number ending with 4 or 6 ends with 6.

For example : $4^2 = 16$, $6^2 = 36$.

(e) The square of a number ending with 5 ends with 5.

For example : $5^2 = 25$, $15^2 = 225$.

(f) The square of a number ending with 0 ends with even number of zeroes.

For example : $10^2 = 100$, $20^2 = 400$.

Ex.1 Which of the following are not perfect squares ?

(i) 256 (ii) 289 (iii) 418 (iv) 563

Sol. (i) $256 = 16^2$, so it is a perfect square.

(ii) $289 = 17^2$, so it is a perfect square.

(iii) and (iv) are not perfect square because a number having 2, 3, 7 or 8 at one's place is never a perfect square.

Ex.2 Which of the following numbers are squares of even numbers ?

(i) 625 (ii) 324 (iii) 2401 (iv) 6651

Sol. Only (b) is the square of an even number i.e., $18^2 = 324$.

Ex.3 Determine the digit at the one's place in the square of the following numbers.

(i) 172 (ii) 399

Sol. (i) The square of 172 will have 4 at one's place. [Using property 7(b)]

(ii) The square of 399 will have 1 at one's place. [Using property 7(a)]

5 SOME INTERESTING PATTERNS

(1) The square of any natural number n equals to the sum of first n odd numbers.

For example : $1 = 1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

$1 + 3 + 5 + 7 = 16 = 4^2$

(2) A number with n digits has either $(2n - 1)$ or $2n$ digits in its square.

For example : 1–digit number has 1 or 2 digits in its square i.e., $3^2 = 9$, $9^2 = 81$

A 2–digit number has either 3 or 4 digits in its square, i.e., $12^2 = 144$; $50^2 = 2500$

(3) The square of any odd number can be expressed as the sum of two consecutive positive numbers.

For example : $3^2 = 9 = 4 + 5$

$5^2 = 25 = 12 + 13$

$7^2 = 49 = 24 + 25$

(4) A perfect square number leaves a remainder 0 or 1 when divided by 3.

(5) The square of a natural number greater than 1 can be written as

(i) multiple of 3 or (multiple of 4) + 1

(ii) multiple of 4 or (multiple of 3) + 1.

For example : $3^2 = 9 = 3 \times 3$ or $4 \times 2 + 1$

$4^2 = 16 = 3 \times 5 + 1$ or 4×4

(6) Between the squares of any two consecutive numbers, n and $(n + 1)$ there are 2n non-perfect square numbers.

Between 1^2 and 2^2 i.e., 1 and 4 there are two non-perfect square numbers 2 and 3. It is same as 2×1 numbers.

Between 2^2 and 3^2 i.e., 4 and 9, there are four non-perfect square numbers 5, 6, 7 and 8. It is same as 2×2 numbers.

(7) Product of two consecutive even or odd natural numbers is $(a + 1)(a - 1) = a^2 - 1$.

For example : $11 \times 13 = (12 - 1)(12 + 1) = 12^2 - 1$

$12 \times 14 = (13 - 1)(13 + 1) = 13^2 - 1$

(8) Squares of natural numbers having all digits 1 follow the following pattern :

$$1^2 = 1$$

$$11^2 = 1\ 2\ 1$$

$$111^2 = 1\ 2\ 3\ 2\ 1$$

$$1111^2 = 1\ 2\ 3\ 4\ 3\ 2\ 1$$

$$11111^2 = 1\ 2\ 3\ 4\ 5\ 4\ 3\ 2\ 1$$

Another interesting pattern

$$7^2 = 49$$

$$67^2 = 4489$$

$$667^2 = 444889$$

$$6667^2 = 44448889$$

$$66667^2 = 4444488889$$

Ex.4 Express 64 as the sum of 8 odd numbers.

Sol. $64 = 8^2 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$

Ex.5 Write $(27)^2$ as the sum of two consecutive positive numbers.

Sol. $(27)^2 = 729 = 364 + 365$

Ex.6 How many numbers lie between the square of the following numbers ?

(i) 4^2 and 5^2 (ii) 7^2 and 8^2

Sol. (i) The number between 4^2 and 5^2 is $2 \times 4 = 8$

(ii) The number between 7^2 and 8^2 is $2 \times 7 = 14$

Ex.7 Express the following as either in terms of multiple of 3 or multiple of 4.

(i) 6^2 (ii) 14^2

Sol. (i) $6^2 = 36 = 3 \times 12$ or 4×9

(ii) $14^2 = 196 = 4 \times 49$ or $3 \times 65 + 1$

6 PYTHAGOREAN TRIPLETS

Three natural numbers m, n, p are said to form a Pythagorean triplet (m, n, p) if $(m^2 + n^2) = p^2$.

Note : For every natural number $m > 1$, we have $(2m, m^2 - 1, m^2 + 1)$ as a Pythagorean triplet.

Ex.8 Find the pythagorean triplet whose smallest number is 12.

Sol. For every natural number $m > 1$, $(2m, m^2 - 1, m^2 + 1)$ is a Pythagorean triplet.

Putting $2m = 12$, i.e., $m = 6$, we get the triplet (12, 35, 37).

7 FINDING THE SQUARE OF A NUMBER

We can also find the square of a number without actual multiplication.

For example : $43^2 = (40 + 3)^2 = (40 + 3)(40 + 3)$

$= 40(40 + 3) + 3(40 + 3)$

$= 1600 + 120 + 120 + 9$

$= 1600 + 240 + 9 = 1849$

8 OTHER PATTERNS IN SQUARE

Consider a number with unit digit 5 i.e., a5

$(a5)^2 = (10a + 5)^2 = (10a + 5)(10a + 5)$

$= 100a^2 + 50a + 50a + 25$

$= 100a(a + 1) + 25$

$= a(a + 1)$ hundred $+ 25$

For example : $95^2 = (90 + 5)^2 = (10 \times 9 + 5)^2$, where a = 9

$= 100 \times 9(9 + 1) + 25$

$= 100 \times 9(10) + 25$

$= 9(9 + 1)$ hundred $+ 25$

$= 9 \times 10$ hundred $+ 25$

$= 9000 + 25 = 9025$

9 SQUARE ROOTS

The square root of a number x is that number which when multiplied by itself gives x as its product. Thus, if b is the square root of a number x, then $b \times b = x \Rightarrow b^2 = x \Rightarrow b = \sqrt{x}$

The square root of a number x is denoted by $\sqrt{x}$.

Now, $\sqrt{9} = 3$

Also, the square of $(-3) = (-3) \times (-3) = 9$

So, 9 has two square roots; + 3 and – 3.

Therefore, every positive number has two square roots; one positive number and other is negative number.

10 METHODS OF FINDING SQUARE ROOTS

10.1 FINDING SQUARE ROOT THROUGH REPEATED SUBTRACTION

Step – 1 : Take the given number whose square root is to be found out.

Step – 2 : Subtract the odd numbers 1,3, 5, ... successively from the given number.

Step – 3 : If the given number is a perfect square, we will get zero at some stage. We stop at the point where we have got zero, and declare the number of times we have performed subtraction as the square root of the given number.

For example : Let us consider the number 49.

(i) 49 – 1 = 48 (ii) 48 – 3 = 45 (iii) 45 – 5 = 40 (iv) 40 – 7 = 33

(v) 33 – 9 = 24 (vi) 24 – 11 = 13 (vii) 13 – 13 = 0

So, we have performed subtraction 7 times. Therefore, $\sqrt{49} = 7$.

10.2 FINDING SQUARE ROOT THROUGH PRIME FACTORISATION

Step – 1 : Find prime factors of the given number.

Step – 2 : Make pairs of same factors obtained in step 1.

Step – 3 : Take one factor from each pair and multiply them together. The product thus obtained will be the required square root.

Ex.9 Find the square root of 1444.

Sol.

2	1444
2	722
19	361
19	19
	1

$1444 = \underline{2 \times 2} \times \underline{19 \times 19}$

$\therefore \sqrt{1444} = 2 \times 19 = 38$

Hence, $\sqrt{1444} = 38$

Ex.10 Is 3528 a perfect square ? If not, find the smallest number by which it must be multiplied so that product becomes a perfect square.

Sol. $3528 = \underline{2 \times 2} \times 2 \times \underline{3 \times 3} \times \underline{7 \times 7}$ We find that the prime factor 2 does not occur in pair. Therefore, 3528 is not a perfect square. To make 3528 a perfect square,

2	3528
2	1764
2	882
3	441
3	147
7	49
7	7
	1

we multiply 3528 by 2 to get $3528 \times 2 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$.

Now, each prime factor is in pair. Therefore, $3528 \times 2 = 7056$ is a perfect square.

Therefore, the required smallest number is 2.

Ex.11 Find the smallest number by which 2352 must be divided so that the quotient is a perfect square. Find the square root of the quotient.

Sol.

2	2352
2	1176
2	588
2	294
7	147
7	21
3	3
	1

We have, $2352 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7} \times 3$

We find that 3 is not forming a pair. So, to make 2352 a perfect square, we have to eliminate 3.

So, we divide 2352 by 3 to get $2352 \div 3 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$

Now, each prime factor is in pair. Therefore $2352 \div 3 = 784$ is a perfect square.

$\therefore \sqrt{784} = 2 \times 2 \times 7 = 28$

10.3 FINDING SQUARE ROOT BY LONG DIVISION METHOD

When the numbers are large and the method of finding square root by prime factorisation becomes inconvenient, then, we use long division method. To understand long division method effectively, it is important to know the number of digits in the square root of a perfect square.

We already know, $1^2 = 1$ and $9^2 = 81$. Also, $10^2 = 100$ and $99^2 = 9801$.

Further $100^2 = 10000$ and $999^2 = 998001$.

Now, we see that square root of a 1-digit or 2-digit number has 1-digit. Square root of a 3-digit or 4-digit has 2-digits and so on.

We can generalise it as : A perfect square number having n-digits, the corresponding square root will have

(a) $\frac{n}{2}$ digits, if n is even. (b) $\frac{(n+1)}{2}$ digits, if n is odd.

Let us consider a number 16641.

Since it is a 5-digit number, so the square root of the number will have 3 digits.

We have the following steps to find the square root by division method.

Step – 1 : Place a bar over every pair of digits starting from the one's digit (from right to left). If the number of digits in it is odd, then the left most single digit too will have a bar i.e., $\bar{1}\ \overline{66}\ \overline{41}$.

Step – 2 : Think of the largest number whose square is less than or equal to the number under the left most bar (in this case, 1).

Step – 3 : Take this number as the divisor and the quotient with the number under the extreme left bar as the dividend (here 1). Divide and get the remainder (in this case, 0).

$$\begin{array}{r|l} & \quad 1 \quad \leftarrow \text{Quotient} \\ \hline \text{Divisor} \rightarrow 1 & \overline{1}\,\overline{66}\,\overline{41} \\ & -1 \\ \hline & \quad 0 \end{array}$$

Step – 4 : Bring down the number under the bar to the right of the remainder and this becomes the new dividend (in this case 66).

$$\begin{array}{r|l} & \quad 1 \\ \hline 1 & \overline{1}\,\overline{66}\,\overline{41} \\ & -1 \\ \hline & \quad 066 \end{array}$$

Step – 5 : Double the quotient and enter it with a blank on its right.

$$\begin{array}{r|l} & \quad 1 \\ \hline 1 & \overline{1}\,\overline{66}\,\overline{41} \\ & -1 \\ \hline 2 & \quad 066 \end{array}$$

Step – 6 : Choose the one's digit of the new divisor in such a way that the product of this digit and new divisor is equal to or less than the new dividend (in this case 22).

$$\begin{array}{r|l} & 12 \\ \hline 1 & \overline{16}\,\overline{64}\,\overline{1} \\ & -1 \\ \hline 22 & 66 \\ & -44 \\ \hline & 22 \end{array}$$

Step – 7 : Again bring down the number under the next bar to the right of the remainder in step 6. Repeat the steps 5 and 6.

$$\begin{array}{r|l} & 129 \\ \hline 1 & \overline{16}\,\overline{64}\,\overline{1} \\ & -1 \\ \hline 22 & 66 \\ & -44 \\ \hline 249 & 2241 \\ & -2241 \\ \hline & 0 \end{array}$$

Step – 8 : Since, the remainder is 0 and no digits are left in the given number, therefore $\sqrt{16641} = 129$.

11 SQUARE ROOTS OF DECIMALS

Let us consider a number 54.76.

We have the following steps to find the square roots of decimals by long division method.

Step – 1 : To find the square root of a decimal number, we put bars on the integral part (i.e., 54) of the number in the usual manner. And place bars on the decimal part (i.e., 76) on every pair of digits beginning with the first decimal place. So, we get $\overline{54}.\overline{76}$

Step – 2 : Now proceed in a similar manner. The left most bar is on 54 and $7^2 < 54 < 8^2$. Take this number as the divisor and number under the left-most bar as the dividend i.e., 54. Divide and get the remainder.

$$\begin{array}{r|l} & 7 \\ \hline 7 & \overline{54}\,.\,\overline{76} \\ & -49 \\ \hline & 5 \end{array}$$

Step – 3 : The remainder is 5. Write the number under the next bar at the right of this remainder, to get 576

$$\begin{array}{r|l} & \quad 7 \\ \hline 7 & \overline{54}\,.\,\overline{76} \\ & -49 \\ \hline 14 & \quad 576 \end{array}$$

Step – 4 : Double the quotient and enter it with a blank on the right. Since, 76 is the decimal part so put a decimal point in the quotient.

$$\begin{array}{r|l} & \quad 7. \\ \hline 7 & \overline{54}\,.\,\overline{76} \\ & -49 \\ \hline 14 & \quad 576 \end{array}$$

Step – 5 : We know that $144 \times 4 = 576$, therefore the new digit is 4. Divide and get the remainder as 0.

$$\begin{array}{r|l} & \quad 7.4 \\ \hline 7 & \overline{54}\,.\,\overline{76} \\ & -49 \\ \hline 144 & \quad 576 \\ & -576 \\ \hline & \quad 0 \end{array}$$

Step – 6 : Since, the remainder is 0 and no bar left, therefore $\sqrt{54.76} = 7.4$.

12 SQUARE ROOT OF PRODUCT OF TWO NUMBERS AND FRACTIONAL NUMBERS

The square root of the product of two numbers is equal to the product of the square root of the one number and square root of the other number. For any two positive numbers a and b, we have $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$. The square root of a fractional number is equal to the quotient of the square root of its numerator and denominator. For any two positive numbers a and b, we have $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Ex.12 Find the square root of $\frac{625}{1296}$.

Sol. We have, $\sqrt{\frac{625}{1296}} = \frac{\sqrt{625}}{\sqrt{1296}}$

Now, we find the square roots of 625 and 1296 separately as shown :

Thus, $\sqrt{625} = 25$ and $\sqrt{1296} = 36$

Hence, $\sqrt{\frac{625}{1296}} = \frac{\sqrt{625}}{\sqrt{1296}} = \frac{25}{36}$

$$\begin{array}{r|l} & 25 \\ \hline 2 & \overline{6}\ \overline{25} \\ & -4 \\ \hline 45 & 225 \\ & -225 \\ \hline & 0 \end{array} \qquad \begin{array}{r|l} & 36 \\ \hline 3 & \overline{12}\ \overline{96} \\ & -9 \\ \hline 66 & 396 \\ & -396 \\ \hline & 0 \end{array}$$

13 ESTIMATING SQUARE ROOTS

We can estimate the square root by following ways. We know that $100 < 150 < 225$.

$\sqrt{100} = 10$ and $\sqrt{225} = 15$

But still we are not close to the square number.

We know that $12^2 = 144$ and $13^2 = 169$.

Therefore, $12 < \sqrt{150} < 13$ and 144 is much closer to 150 than 169.

So, $\sqrt{150}$ is approximately equal to 12.

SOLVED EXAMPLES

SE1. Is 144 a perfect square ? If yes, find the number whose square is 144.

Sol. The given number is 144. Writing 144 as the product of its prime factors, we have

2	144
2	72
2	36
2	18
3	9
3	3
	1

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$= (2 \times 2) \times (2 \times 2) \times (3 \times 3)$

Clearly, the prime factors can be written in pairs as shown here.

Therefore, 144 is a perfect square.

So, $\sqrt{144} = 2 \times 2 \times 3 = 12$

SE2. Find the smallest number by which 25200 should be divided so that the result is a perfect square.

Sol. Resolving 25200 into prime factors, we get

2	25200
2	12600
2	6300
2	3150
3	1575
3	525
5	175
5	35
7	7
	1

$25200 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7$

Now, grouping the factors into prime of equal factors, we get

$25200 = (2 \times 2) \times (2 \times 2) \times (3 \times 3) \times (5 \times 5) \times 7$

We observe that 7 cannot be paired. Thus, if we divide 25200 by 7, the quotient will be a perfect square. Hence, the smallest number by which 25200 must be divide to make the result a perfect square is 7.

SE3. Find whether 55 is a perfect square or not using repeated subtraction.

Sol. In order to check whether 55 is a perfect square or not, we successively subtract 1, 3, 5, 7, 9, ..., from it. We have,

(i) $55 - 1 = 54$ (ii) $54 - 3 = 51$

(iii) $51 - 5 = 46$ (iv) $46 - 7 = 39$

(v) $39 - 9 = 30$ (vi) $30 - 11 = 19$

(vii) $19 - 13 = 6$ (viii) $6 - 15 = -9$

This shows that 55 cannot be expressed as the sum of consecutive odd numbers starting from 1. Hence, it is not a perfect square.

SE4. Find the squares of the following numbers containing 5 in the unit's place :

(i) 15 (ii) 105

Sol. (i) $15^2 = 1 \times (1 + 1)$ hundred + 25

$= 1 \times 2$ hundred + 25

$= 2$ hundred + $25 = 2 \times 100 + 25$

$= 200 + 25 = 225$

(ii) $105^2 = 10 \times (10 + 1)$ hundred + 25

$= (10 \times 11)$ hundred + 25

$= 110 \times 100 + 25 = 11000 + 25 = 11025$

SE5. Write a Pythagorean triplet whose smallest member is 20.

Sol. We can get Pythagorean triplet by using general form $2m, m^2 - 1, m^2 + 1$.

Let $2m = 20 \Rightarrow m = 10$

So, $m^2 - 1 = (10)^2 - 1 = 99$, $m^2 + 1 = (10)^2 + 1 = 101$. The triplet is 20, 99, 101.

SE6. Determine whether squares of the following numbers are even or odd.

(i) 9777 (ii) 40028

Sol. (i) The square of 9777 is an odd number as its one's digit is odd.

(ii) The square of 40028 is an even number as its one's digit is even.

SE7. Which of the following numbers are not perfect squares ?

(i) 697 (ii) 900

(iii) 412 (iv) 1000

Sol. (i) A number having 7 at one's place is never a perfect square. So, 697 is not a perfect square.

(ii) $900 = (30)^2$ is a perfect square.

(iii) A number having 2 at one's place is never a perfect square. So, 412 is not a perfect square.

(iv) A number ending with odd number of zeroes is never a perfect square. So, 1000 is not a perfect square.

SE8. Determine the digit at one's place in the squares of the following numbers.

(i) 1146 (ii) 1955

(iii) 343

Sol. (i) The square of a number ending with 6 ends with 6.

(ii) The square of a number ending with 5 ends with 5.

(iii) The square of a number ending with 3 ends with 9.

SE9. Find the square of 491 using the identity $(a - b)^2 = a^2 - 2ab + b^2$.

Sol. We have $491^2 = (500 - 9)^2$

$= 500^2 - 2 \times 500 \times 9 + 9^2 = 250000 - 9000 + 81 = 241081$.

SE10. Find the square root of 8464 by prime factorization.

Sol. Resolving 8464 into prime factors, we get

$8464 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{23 \times 23}$

2	8464
2	4232
2	2116
2	1058
23	529
23	23
	1

Now, taking one factor from each pair, we obtain

$\sqrt{8464} = 2 \times 2 \times 23 = 92$

$\therefore \sqrt{8464} = 92$

SE11. Find the cost of erecting a fence around a square field whose area is 9 hectares if fencing costs Rs. 3.50 per metre.

Sol. Area of the square field $= (9 \times 10000)m^2$

$= 90000\ m^2$

Length of each side of the field $= \sqrt{90000}$ m

$= 300$ m. Perimeter of field $= (4 \times 300)$ m $= 1200$ m. Cost of fencing $=$ Rs. $\left(1200 \times \frac{7}{2}\right) =$ Rs. 4200.

SE12. Find the greatest 4-digit number which is a perfect square.

Sol. Greatest number of 4-digits = 9999. We find $\sqrt{9999}$ by long division method.

$$\begin{array}{r|l} & 99 \\ 9 & \overline{99}\ \overline{99} \\ & -81 \\ \hline 189 & 1899 \\ & -1701 \\ \hline & 198 \end{array}$$

The remainder is 198. This shows, 99^2 is less than 9999 by 198.

This means if we subtract the remainder from the number, we get a perfect square.

Therefore, the required perfect square is 9999 – 198 = 9801.

And, $\sqrt{9801} = 99$

SE13. Find the square root of 11025 by prime factorization method.

Sol. Resolving 11025 into prime factors, we have

11025 = (3 × 3) × (5 × 5) × (7 × 7)

3	11025
3	3675
5	1225
5	245
7	49
7	7
	1

Taking one factor from each pair, we get

$\sqrt{11025} = 3 \times 5 \times 7 = 105$

SE14. Find the value of $\sqrt{37.0881}$.

Sol. We have, $\sqrt{37.0881} = \sqrt{\dfrac{370881}{10000}}$

$$\begin{array}{r|l} & 609 \\ 6 & \overline{37}\ \overline{08}\ \overline{81} \\ & -36 \\ \hline 1209 & 10881 \\ & -10881 \\ \hline & 0 \end{array}$$

$\therefore \sqrt{37.0881} = \sqrt{\dfrac{370881}{10000}} = \dfrac{609}{100} = 6.09$

SE15. Find the value of $\sqrt{15625}$ and use it to find the value of $\sqrt{156.25} + \sqrt{1.5625}$.

Sol. We first find the value of $\sqrt{15625}$ by long division method as given below :

$$\begin{array}{r|l} & 125 \\ 1 & \overline{1}\ \overline{56}\ \overline{25} \\ & -1 \\ \hline 22 & 56 \\ & -44 \\ \hline 245 & 1225 \\ & -1225 \\ \hline & 0 \end{array}$$

$\therefore \sqrt{15625} = 125$

Now, $\sqrt{156.25} + \sqrt{1.5625}$

$= \sqrt{\dfrac{15625}{100}} + \sqrt{\dfrac{15625}{10000}}$

$= \dfrac{\sqrt{15625}}{\sqrt{100}} + \dfrac{\sqrt{15625}}{\sqrt{10000}} = \dfrac{125}{10} + \dfrac{125}{100}$

= 12.5 + 1.25 = 13.75

EXERCISE-I

ONLY ONE CORRECT TYPE

1. The value of $\sqrt{41+\sqrt{54+\sqrt{88+\sqrt{128+\sqrt{256}}}}}$ is
(A) 7 (B) 6
(C) 8 (D) 10

2. The value of $\left(\sqrt{\frac{225}{729}}-\sqrt{\frac{25}{144}}\right)\div\sqrt{\frac{16}{81}}$ is
(A) $\frac{1}{48}$ (B) $\frac{5}{48}$
(C) $\frac{5}{16}$ (D) $\frac{1}{16}$

3. If $\sqrt{0.04\times0.4\times a}=0.004\times0.4\times\sqrt{b}$, then $\frac{a}{b}$ is
(A) 16×10^{-3} (B) 16×10^{-4}
(C) 16×10^{-5} (D) 16×10^{-6}

4. If $\sqrt{1+\frac{x}{289}}=1\frac{1}{17}$, then x is equal to
(A) 1 (B) 13
(C) 35 (D) 15

5. If a = 0.1039, then the value of $3a-\sqrt{4a^2-4a+1}$ is
(A) 0.1039 (B) 0.2078
(C) 1.1039 (D) 2.1039

6. What is the least number which should be subtracted from 3.26 to make it a perfect square ?
(A) 2 (B) 0.2078
(C) 0.02 (D) 0.04

7. What is the digit at the one's place in the square of the number 2934 ?
(A) 4 (B) 6
(C) 1 (D) 5

8. The sum of 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 is
(A) 64 (B) 85
(C) 81 (D) 100

9. Evaluate : $\sqrt{6084}$
(A) 78 (B) 76
(C) 84 (D) 87

10. The square root of 1471369 is
(A) 1312 (B) 1211
(C) 1219 (D) 1213

11. The value of $\sqrt{1\frac{9}{16}}$ is
(A) $\frac{2}{3}$ (B) $\frac{2}{5}$
(C) $1\frac{1}{4}$ (D) $\frac{3}{4}$

12. What is the square root of 0.0009 ?
(A) 0.3 (B) 0.03
(C) 0.003 (D) 3

13. Evaluate : $\sqrt{175.2976}$
(A) 13.24 (B) 12.14
(C) 12.24 (D) 13.14

14. The value of $\sqrt{\frac{0.289}{0.00121}}$ is
(A) $\frac{17}{11}$ (B) $\frac{170}{11}$
(C) $\frac{70}{11}$ (D) $\frac{17}{110}$

15. The value of $\sqrt{0.121}$ upto three places of decimal is
(A) 0.011 (B) 0.11
(C) 0.347 (D) 1.1

16. The value of $\sqrt{\frac{0.16}{0.4}}$ is
(A) 0.02 (B) 0.2
(C) 0.63 (D) None of these

17. The least perfect square number divisible by 3, 4, 5, 6 and 8 is
(A) 900 (B) 1200
(C) 2500 (D) 3600

18. The least perfect square, which is divisible by each of 21, 36 and 66 is
(A) 213444 (B) 214344
(C) 214434 (D) 231444

19. Which of the following numbers is not a perfect square ?
(A) 612 (B) 529
(C) 900 (D) 100

20. Which of the following numbers is a perfect square ?
(A) 100 (B) 100000
(C) 25000 (D) 40

21. Which of the following numbers is a square of odd number ?
(A) 256 (B) 144
(C) 2601 (D) 400

22. The square of 33 is
(A) 1059 (B) 1069
(C) 1089 (D) 1079

23. A welfare association collected Rs. 52900 as donation from the students. If each paid as many rupees as there were students, find the number of students.
(A) 230 (B) 225
(C) 220 (D) 245

24. Which of the following are pythagorean triplets ?
(A) 2, 3, 5 (B) 5, 7, 9
(C) 6, 9, 11 (D) 8, 15, 17

25. Is 1600 a perfect square ?
(A) No (B) Yes
(C) Can't say (D) None of these

26. Which of the following is not true ?
(A) $\sqrt{ab} = \left(\sqrt{a} \times \sqrt{b}\right)$
(B) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
(C) $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$
(D) $\left(\sqrt{a+b}\right)^2 = a+b$

27. Sum of the first n odd natural numbers is
(A) $2n + 1$ (B) n^2
(C) $n^2 - 1$ (D) $2n^2 + 1$

28. The value of $\sqrt{45} \times \sqrt{20}$ is
(A) 18 (B) 30
(C) 20 (D) 60

29. A perfect square number can never have the digit.....at the units place.
(A) 1 (B) 4
(C) 8 (D) 9

30. What least number must be subtracted from 7250 to get a perfect square ?
(A) 30 (B) 35
(C) 25 (D) 40

31. How many natural numbers lie between squares of 12 and 13 ?
(A) 26 (B) 24
(C) 25 (D) None of these

32. The square root of 1584.04 is
(A) 39.8 (B) 41.8
(C) 43.8 (D) 45.8

33. If $\sqrt{86.49} + \sqrt{5 + k^2} = 12.3$, then k is equal to
(A) $\sqrt{10}$ (B) $2\sqrt{5}$
(C) $3\sqrt{5}$ (D) ± 2

34. Which of the following is not true ?
(A) A square number is never negative.
(B) A square number never ends in 2, 3, 7 or 8.
(C) The number of zeros at the end of a perfect square is always even.
(D) The square of an even number is odd.

35. Which of the following is correct ?
(A) The square of a prime number is prime.
(B) The sum of two perfect squares is a perfect square.
(C) The difference of two perfect square is a perfect square.
(D) The product of two perfect squares is a perfect square.

36. Find the smallest square number divisible by each one of the numbers 8, 9 and 10.
(A) 360 (B) 480
(C) 720 (D) 3600

37. The square of $(111111)^2$ is
(A) 123454321 (B) 1234567123456
(C) 12345654321 (D) 12345612345

38. Find the least number which must be added to 306452 to make it a perfect square.
(A) 643 (B) 554
(C) 464 (D) 644

39. In an auditorium, the number of rows is equal to the number of chairs in each row. If the capacity of the auditorium is 2025, find the number of chairs in each row.
(A) 65 (B) 45
(C) 55 (D) 35

40. The square of 19 as the sum of two consecutive integers is
(A) 179 + 180 (B) 180 + 181
(C) 182 + 183 (D) None of these

PARAGRAPH TYPE

PASSAGE – I :

When a is the side of a square, its area is given by a^2. Area of a square plot is 2304 m^2.

1. The side of the square plot is :
(A) 42 m (B) 48 m
(C) 46 m (D) 50 m

2. If the area of a square is increased by 3172 units, then the new side of the square will be :
(A) 78 m (B) 76 m
(C) 72 m (D) 74 m

3. The area of a square with side 16 m will be :
(A) 96 m^2 (B) 56 m^2
(C) 256 m^2 (D) 196 m^2

PASSAGE – II :

The area of two squares are respectively, 256 and 625 square metres.

4. The side of two squares respectively are :
(A) 16, 25 (B) 25, 16
(C) 17, 25 (D) 16, 5

5. The ratio of their sides are :
(A) 17 : 25 (B) 15 : 16
(C) 16 : 25 (D) 16 : 5

6. If both the squares are joined, then the perimeter of new figure will be :
(A) 140 m (B) 132 m
(C) 123 m (D) 164 m

TRUE / FALSE TYPE

1. 3, 4, 5 are pythagorean triplet.
2. 9 is a perfect square.
3. 80 is a perfect square.
4. A perfect square is always an even number.
5. Square root of 21 is 4.5825.
6. A perfect square number can also be a prime number.
7. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$.
8. Square root of 1.5625 is 1.25.
9. Square root of negative number is real number.
10. Square root of any positive number is always integer.

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from List – I and List – II are given as options (A), (B), (C) and (D) out of which one is correct.

1. Match the following :

List – I	List – II
(P) The least square number which is divisible by each of the numbers 6, 9, 15 and 20 is	(i) 18
(Q) The greatest number of six digits which is a perfect square is	(ii) 9
(R) The least square number which is subtracted from 2509 to make it a perfect square is	(iii) 900
(S) The least square number which must be added to 5607 to obtain a perfect square is	(iv) 998001

(A) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)
(B) (P) → (iii), (Q) → (iv), (R) → (ii), (S) → (i)
(C) (P) → (iii), (Q) → (ii), (R) → (iv), (S) → (i)
(D) (P) → (iv), (Q) → (iii), (R) → (i), (S) → (ii)

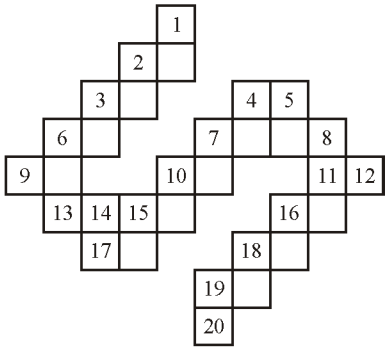
2. Match the numbers in List – I with their square roots in List – II.

List – I	List – II
(P) 1296	(i) 63
(Q) 5476	(ii) 231
(R) 3969	(iii) 36
(S) 53361	(iv) 74

(A) (P) → (ii), (Q) → (iii), (R) → (iv), (S) → (i)
(B) (P) → (i), (Q) → (iv), (R) → (iii), (S) → (ii)
(C) (P) → (iii), (Q) → (iv), (R) → (i), (S) → (ii)
(D) (P) → (iv), (Q) → (iii), (R) → (ii), (S) → (i)

CROSS WORD PUZZLE

Complete the following word puzzle with the help of clues given below :



Across

1. The square of $\sqrt{5}$
2. 3^3
3. The square root of 256
4. $\sqrt{3249}$
6. $\left(\sqrt{8}\right)^4 \div 2$
7. $\sqrt{14} \times 1000$ (round off to nearest integer)
9. 6^2
10. $\sqrt{1156}$
11. $18^2 - 15^2$
13. $\sqrt{2040000}$ (round off to nearest integer)
16. $\sqrt{576} + \sqrt{4096}$
17. $\sqrt{6600 + 625}$
18. $2^6 - 6^2 + 1^2$
19. $\sqrt{85^2 - 77^2}$
20. $\sqrt{4}$

Down

1. $\sqrt{95^2 - 76^2}$
2. $5^2 + 1^2$
3. $6 \div \sqrt{\frac{1}{4}}$
4. $2^5 + 5^2$
5. $\sqrt{5476}$
6. 19^2
7. $16^2 - 15^2 + 3^1$
8. $\sqrt{88804}$
9. $\sqrt{9}$
10. $\sqrt{404 + 1040}$
12. $\sqrt{64} + \sqrt{0.81} + \sqrt{0.01}$
14. $\sqrt{2304}$
15. $\sqrt{24^2 + 7^2}$
16. $\sqrt{80^2 + 39^2}$
18. $\sqrt{441} + \sqrt{25}$
19. $8 \div \left(\frac{1}{2}\right)^2$

EXERCISE-II

VERY SHORT ANSWER TYPE

1. Find the value of $100^2 - 99^2$.
2. Find the squares of the following numbers using a pattern :
   (i) 205 (ii) 515
3. Find the square of the number 509 using the identity $(a + b)^2 = a^2 + 2ab + b^2$.
4. Find the square of 575 using the identity $(a - b)^2 = a^2 - 2ab + b^2$.
5. Find the square root of 11236 by prime factorisation.
6. Find the least square number which is exactly divisible by 10, 12, 15 and 18.
7. Find the value of $\sqrt{147} \times \sqrt{243}$.
8. Evaluate : $\sqrt{1.96}$
9. Find the value of $\sqrt{45} \times \sqrt{20}$.
10. What is the digit at the ones place in the square of 1819 ? Also, determine its square.

SHORT ANSWER TYPE

1. Find the least square number exactly divisible by each of the numbers 8, 12, 15 and 20.
2. A P.T. teacher wants to arrange maximum possible number of 6000 students in a field such that the number of rows is equal to the number of columns. Find the number of rows if 71 were left out after the arrangement.
3. What least number must be subtracted from 7250 to get a perfect square ? Also, find the square root of this perfect square.
4. Find the least number of six digits which is a perfect square. Find the square root of this number.
5. Find the square root of 298116 by prime factorisation.
6. Find the square root of 54756 by long division method.
7. Given that $\sqrt{2} = 1.414$ and $\sqrt{5} = 2.236$, evaluate each of the following :
   (i) $\sqrt{\frac{36}{5}}$ (ii) $\sqrt{\frac{625}{98}}$
8. Find the least square number exactly divisible by each of the numbers 6, 5, 10 and 20.
9. Find the greatest number of five digits which is a perfect square.
10. Find the smallest number that must be added to 1780 to make it a perfect square.

LONG ANSWER TYPE

1. Find the value of $\sqrt{11025 \times 1024}$.
2. The area of a square field is $101\frac{1}{400}$ square metres. Find the length of one side of the field.
3. Find the square root of $52\frac{857}{2116}$.
4. A general wishes to draw up his 36581 soldiers in the form of a solid square. After arranging them , he found that some of them are left over. How many are left ?
5. A group of students decided to collect as many paise from each member of the group as is the number of members. If the total collection amounts to Rs. 59.29. Find the members in the group.

NUMERICAL PROBLEMS

1. Evaluate : $\sqrt{41-\sqrt{21+\sqrt{19-\sqrt{9}}}}$
2. The value of $7\left(\frac{\sqrt{625}}{11}\times\frac{14}{\sqrt{25}}\times\frac{11}{\sqrt{196}}\right)$ is
3. The square root of 15876 is
4. How many two-digit numbers satisfy this property : The last digit (unit digit) of the square of the two-digit number is 8 ?
5. The least number by which 294 must be multiplied to make it a perfect square is
6. Find the smallest number by which 5808 should be multiplied so that the product becomes a perfect square.
7. The tens digit of the least number by which 1470 must be divided to get a perfect square is
8. The value of x if (x, 15, 17) is a pythagorean triplet is
9. 16 (1 + 3 + 5 + 7) is the square of
10. The smallest number by which 12348 must be divided to obtain a perfect square is

ANALYTICAL PROBLEMS & BRAIN TEASER

1. What is the approximate value of $(15.01)^2 \times \sqrt{728}$?
   (A) 6125 (B) 6225
   (C) 6200 (D) 6075
2. What must be subtracted from 16161 to get a perfect square ? Also, find the square root of this perfect square.
   (A) 32, 127 (B) 35, 127
   (C) 32, 128 (D) 35, 130
3. 4096 soldiers are arranged in an auditorium in such a manner that there are as many soldiers in a row as there are rows in the auditorium. How many rows are there in the auditorium ?
   (A) 94 (B) 58
   (C) 44 (D) 64
4. Find the least number which must be added to 304584 to make it a perfect square. Also, find the resulting number and the square root of the resulting number.
   (A) 120, 304704, 554
   (B) 120, 304704, 552
   (C) 115, 304704, 560
   (D) 115, 304704, 565
5. Find the value of $\sqrt{0.09}+\sqrt{0.81}+\sqrt{7.29}+\sqrt{98.01}$.
   (A) 13.7 (B) 13.8
   (C) 13.6 (D) 13.1
6. The number of trees in each row of a garden is equal to the total number of rows in the garden. After 111 trees have been uprooted in a storm, there remain 10914 trees in the garden . The number of rows of trees in the garden is
   (A) 100 (B) 105
   (C) 115 (D) 125
7. Read the statements carefully and select the correct option.
   Statement – I : While solving a mathematical problem, Samidha squared a number and then subtracted 25 from it rather than the required i.e.. first subtracting 25 from the number and then squaring it. But she got the right answer. The given number is 13.
   Statement – II : The smallest natural number which is a perfect square and which ends in 3 identical digits lies between 2000 and 3000.
   (A) Statement – I is true but Statement – II is false.
   (B) Statement – I is false but Statement – II is true.
   (C) Both Statement – I and Statement – II are true.
   (D) Both Statement – I and Statement – II are false.
8. If $\sqrt{(0.16\times0.4x)}=0.4\times0.2\sqrt{y}$. Then, the value of $\frac{x}{y}$ is
   (A) 0.001 (B) 0.1
   (C) 0.01 (D) 1
9. Find the value of $\sqrt{10+\sqrt{25+\sqrt{121}}}$.
   (A) 5 (B) 3
   (C) 4 (D) 6

10. If $\sqrt{13-a\sqrt{10}} = \sqrt{8}+\sqrt{5}$, then the value of a is

(A) – 6 (B) – 4
(C) – 8 (D) – 5

11. Find the square root of $21\frac{2797}{3364}$.

(A) $4\frac{33}{58}$ (B) $4\frac{39}{58}$

(C) $4\frac{29}{57}$ (D) $4\frac{27}{52}$

12. If $(3.9268)^2 = 15.42$, then the value of $\sqrt{154200}$ is

(A) 3926.8 (B) 392.68
(C) 39.268 (D) 3.9268

13. $\sqrt{(0.798)^2 + 0.404 \times 0.798 + (0.202)^2} + 1$ is equal to

(A) 0 (B) 2
(C) 1.596 (D) 0.404

14. If 3a = 4b and a + b = $7\sqrt{29}$, then $\sqrt{a^2+b^2}$ is

(A) $5\sqrt{29}$ (B) 81
(C) 87 (D) 29

EXERCISE-III

PREVIOUS YEAR QUESTIONS (NTSE)

1. If $x+\frac{1}{x}=3$ then $\frac{x}{x^2+1}$ is
(Aryabhatta – 2002)
(A) 9 (B) $\frac{1}{3}$
(C) 13 (D) $\frac{1}{13}$

2. $\sqrt{5+\sqrt{10+\sqrt{x}}}=3$, then the value of x is
(Aryabhatta – 2006)
(A) 49 (B) 36
(C) 16 (D) 25

3. If $\frac{x^2-1}{x}=\frac{1}{2}$, then $4x^2+\frac{4}{x^2}$ is
(Aryabhatta – 2007)
(A) – 7 (B) 7
(C) 9 (D) – 9

4. $\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7}}}}}}$ is equal to
(Aryabhatta – 2007)
(A) 0 (B) 7
(C) $7^{63/64}$ (D) $7^{31/32}$

5. Between which two consecutive whole numbers $\sqrt{1,000}$ lies ?
(IMO – 2010)
(A) 29 and 30 (B) 30 and 31
(C) 31 and 32 (D) 32 and 33

6. The wright brothers had their first successful flight near Kitty Hawk, North Carolina. Mr. Satish finds it easy to remember the year in which the flight occurred because the number $\sqrt{3,621,502}$, the square root of his telephone number is nearest to a whole number. In which year did the flight occur?
(IMO – 2010)
(A) 1902 (B) 1903
(C) 1904 (D) 1905

7. Which of the following is a pythagorean triplet ?
(NSTSE – 2011)
(A) 16, 18, 20 (B) 1, 2, 3
(C) 30, 40, 50 (D) 50, 51, 52

8. Mehul plants 15376 apple trees in his garden and arranges them so that there are as many rows as there are apple trees in each row. The number of rows is.
(IMO – 2011)
(A) 124 (B) 126
(C) 134 (D) 144

9. The square of which number below will contain the first nine natural numbers
(NSTSE – 2012)
(A) 1111111 (B) 11111111
(C) 111111111 (D) 1111111111

10. Which of the following describes a square root of 41 ?
(IMO – 2012)
(A) Between 5 and 6 (B) Between 6 and 7
(C) Between 20 and 21 (D) Between 40 and 42

11. Garima wants to plant 50625 roses and arranges them in such a way that there are as many rows as there are roses in a row. The number of roses in a row are
(IMO – 2012)
(A) 365 (B) 165
(C) 625 (D) 225

12. Identify the value of the given expression
$$\sqrt{\left(\left(\left(\left(\frac{3}{4}\right)^2\div\frac{2}{5}\right)\times\frac{8}{9}\right)\div\frac{5}{16}\right)}$$
(NSTSE – 2013)
(A) $\frac{1}{2\sqrt{2}}$ (B) $\sqrt{10}$
(C) 4 (D) $\frac{1}{\sqrt{5}}$

Answer Key

EXERCISE I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	C	C	C	C	C	B	C	A	D	C	B	A	B	C
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
C	D	A	A	A	C	C	A	D	B	C	B	B	C	C
31	32	33	34	35	36	37	38	39	40					
B	A	D	D	D	D	C	C	B	B					

PARAGRAPH

1. B **2.** D **3.** C **4.** A **5.** C **6.** B

TRUE / FALSE

1. T **2.** T **3.** F **4.** F **5.** T **6.** F **7.** T
8. T **9.** F **10.** F

MATCH THE COLUMN

1. B **2.** C

CROSSWORD PUZZLE

Across

1. 5 **2.** 27 **3.** 16 **4.** 57 **6.** 32 **7.** 3742 **9.** 36
10. 34 **11.** 99 **13.** 1428 **16.** 88 **17.** 85 **18.** 29 **19.** 36
20. 2

Down

1. 57 **2.** 26 **3.** 12 **4.** 57 **5.** 74 **6.** 361 **7.** 34
8. 298 **9.** 3 **10.** 38 **12.** 9 **14.** 48 **15.** 25 **16.** 89
18. 26 **19.** 32

EXERCISE II

VERY SHORT ANSWER TYPE

1. 199 **2.** (i) 42025, (ii) 265225 **3.** 259081 **4.** 330625 **5.** 106 **6.** 900
7. 189 **8.** 1.4 **9.** 30 **10.** 1

SHORT ANSWER TYPE

1. 3600 **2.** 77 **3.** 25, 85 **4.** 100489, 317 **5.** 546 **6.** 234
7. (i) 2.6832, (ii) 2.525 **8.** 900 **9.** 99856 **10.** 69

LONG ANSWER TYPE

1. 3360 **2.** $10\frac{1}{20}$ m **3.** $7\frac{11}{46}$ **4.** 100 **5.** 77

NUMERICAL PROBLEMS

1. 6 **2.** 35 **3.** 126 **4.** 0 **5.** 6 **6.** 3 **7.** 3
8. 8 **9.** 16 **10.** 7

ANALYTICAL PROBLEMS & BRAIN TEASER

1. D **2.** A **3.** D **4.** B **5.** B **6.** B **7.** A
8. B **9.** C **10.** B **11.** B **12.** B **13.** B **14.** A

EXERCISE III

1	2	3	4	5	6	7	8	9	10	11	12	
B	B	C	C	C	B	C	A	C	B	D	C	

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : SQUARES AND SQUARE ROOTS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

CUBES AND CUBE ROOTS

1 INTRODUCTION

This is a story about one of India's great mathematical geniuses, S. Ramanujan. Once another famous mathematician Prof.G.H. Hardy came to visit him in a taxi whose number was 1729. While talking to Ramanujan, Hardy described this number "a dull number". Ramanujan quickly pointed out that 1729 was indeed interesting. He said it is the smallest number that can be expressed as a sum of two cubes in two different ways:

$1729 = 1728 + 1 = 12^3 + 1^3$

$1729 = 1000 + 729 = 10^3 + 9^3$

1729 has since been known as the Hardy – Ramanujan Number, even though this feature of 1729 was known more than 300 years before Ramanujan.

How did Ramanujan know this? Well, he loved numbers. All through his life, he experimented with numbers. He probably found numbers that were expressed as the sum of two squares and sum of two cubes also.

There are many other interesting patterns of cubes. Let us learn about cubes, cube roots and many other interesting facts related to them.

2 CUBE OF A NUMBER

When a number is multiplied three times by itself, we say that the number has been cubed and the product is called cube of that number or the number raised to the power of 3.

For example : 1 is a number and cube of $1 = 1 \times 1 \times 1 = 1^3$

2 is a number and cube of $2 = 2 \times 2 \times 2 = 2^3$

In general x is a number and cube of $x = x \times x \times x = x^3$

The following table gives the cubes'of numbers 1 to 10.

Number	Cube
1	$1^3 = 1\times1\times1 = 1$
2	$2^3 = 2\times2\times2 = 8$
3	$3^3 = 3\times3\times3 = 27$
4	$4^3 = 4\times4\times4 = 64$
5	$5^3 = 5\times5\times5 = 125$
6	$6^3 = 6\times6\times6 = 216$
7	$7^3 = 7\times7\times7 = 343$
8	$8^3 = 8\times8\times8 = 512$
9	$9^3 = 9\times9\times9 = 729$
10	$10^3 = 10\times10\times10 = 1000$

3 PERFECT CUBE NUMBER

A number x is said to be a perfect cube number if there is an integer y such that $x = y \times y \times y = y^3$.

For example : 216 is a perfect cube as there is an integer 6 such that $216 = 6 \times 6 \times 6 = 6^3$

4 CUBES AND THEIR PRIME FACTORS

Each prime factor appears three or multiple of three times in a perfect cube number.

Example : Observe the following table :

Number	Prime Factorisation	Prime Factorisation of its cube
2	2	$2^3 = 8 = 2 \times 2 \times 2 = 2^3$
3	3	$3^3 = 27 = 3 \times 3 \times 3 = 3^3$
4	2×2	$4^3 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3$
6	2×3	$6^3 = 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$
9	3×3	$9^3 = 729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^3 \times 3^3$
15	3×5	$15^3 = 3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 3^3 \times 5^3$

Ex.1 Is 189 a perfect cube or not ?

Sol. Resolving 189 into prime factors, we get $189 = \underline{3 \times 3 \times 3} \times 7$

3	189
3	63
3	21
7	7
	1

Making triplets, we find that one triplet is formed and we are left with one more factor. Thus, 189 cannot be expressed as a product of triplets.

Hence, 189 is not a perfect cube.

5 SMALLEST MULTIPLE THAT IS A PERFECT CUBE

To find the smallest number by which a number must be multiplied so that product is a perfect cube and the smallest number by which a number must be divided so that the quotient is a perfect cube.

Ex.2 Is 1296 a perfect cube or not ? If not, find the smallest natural number by which 1296 must be divided so that the quotient is a perfect cube.

Sol. $1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times 2 \times 3$

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

Thus, 1296 is not a perfect cube. The prime factor 6 does not appear in the group of three, since 6 appears only one time, if we divide the number by 6, then we will get a perfect cube. So, $1296 \div 6 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

Hence the smallest number by which 1296 should be divided to make it perfect cube is 6.

Ex.3 Examine if 1512 is a perfect cube. If not, find the smallest number by which it must be multiplied so that the product is a perfect cube. Also find the smallest number by which it must be divided so that the quotient is a perfect cube.

Sol. $1512 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7$

Here the prime factor 7 does not occur in a group of three. Hence, 1512 is not a perfect cube. Further, 7 appears only once. If we multiply the number by 7×7, then in the product, 7 will also appear in a group of three and the product will be a perfect cube. Thus, the smallest number by which the given number should be multiplied is 7×7 i.e., 49. Finally, if we divide the given number 1512 by 7. The resulting number has prime factors in a group of three. In fact, $1512 \div 7 = 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$. Hence, the smallest number by which the given number should be divided so that quotient is a perfect cube is 7.

6 PROPERTIES OF CUBES OF NUMBERS

1. Cubes of all even numbers are even.
2. Cubes of all odd numbers are odd.
3. Cube of a negative number is always negative.

Cubes of the Digits 1 to 9.

x	1	2	3	4	5	6	7	8	9
x^3	1	8	27	64	125	216	343	512	729

From the table we observe that cubes of the digits 1, 4, 5, 6 and 9 are numbers ending in the same digits 1, 4, 5, 6 and 9 respectively. However 2 and 8 make a pair in the sense that the cube of 2 ends in 8 and the cube of 8 ends in 2. Numbers 3 and 7 also make a pair in the same way.

Focus Point

- The sum of the cubes of first n natural numbers is equal to the square of their sum.

  i.e., $1^3 + 2^3 + \ldots\ldots + n^3 = (1 + 2 + \ldots\ldots + n)^2 = \left(\frac{n(n+1)}{2}\right)^2$

Ex.4 Find the cube of (i) 2 (ii) 4 (iii) 6 (iv) 12.

Sol. (i) Cube of 2 : $(2)^3 = 2 \times 2 \times 2 = 8$ (Even)

(ii) Cube of 4: $(4)^3 = 4 \times 4 \times 4 = 64$ (Even)

(iii) Cube of 6 : $(6)^3 = 6 \times 6 \times 6 = 216$ (Even)

(iv) Cube of 12 : $(12)^3 = 12 \times 12 \times 12 = 1728$ (Even)

Ex.5 Find the cube of (i) 3 (ii) 9 (iii) 13.

Sol. (i) Cube of 3: $(3)^3 = 3 \times 3 \times 3 = 27$ (odd)

(ii) Cube of 9 : $(9)^3 = 9 \times 9 \times 9 = 729$ (odd)

(iii) Cube of 13 : $(13)^3 = 13 \times 13 \times 13 = 2197$ (odd)

Ex.6 Find the value of $1^3 + 2^3 + \ldots\ldots + 7^3$.

Sol. $1^3 + 2^3 + \ldots\ldots + 7^3 = (1 + 2 + \ldots + 7)^2$

$\left(\frac{7(7+1)}{2}\right)^2 = (7 \times 4)^2 = (28)^2 = 784$

Ex.7 Write the digits in the unit place for the cube of each of the given numbers :

(i) 21 (ii) 35 (iii) 69 (iv) 22

(v) 98 (vi) 83 (vii) 27

Sol. (i) 21; Unit digit of $2\underline{1}$ is 1.

$\therefore$ The digit in the unit place for its cube is also 1.

$(21)^3 = 21 \times 21 \times 21 = 926\underline{1}$

(ii) 35; Unit digit of $3\underline{5}$ is 5.

$\therefore$ The digit in the unit place for its cube is also 5.

$(35)^3 = 35 \times 35 \times 35 = 4287\underline{5}$.

(iii) 69; Unit digit of $6\underline{9}$ is 9.

$\therefore$ The digit in the unit place for its cube is also 9.

$(69)^3 = 69 \times 69 \times 69 = 328509$

(iv) 22; Unit digit of $2\underline{2}$ is 2.

$\therefore$ The digit in the unit place for its cube is 8.

$(22)^3 = 22 \times 22 \times 22 = 10648$

(v) 98; Unit digit of $9\underline{8}$ is 8.

$\therefore$ The digit in the unit place for its cube is 2.

$(98)^3 = 98 \times 98 \times 98 = 941192$

(vi) 83; The digit in the unit place of $8\underline{3}$ is 3.

$\therefore$ The digit in the unit place for its cube is 7.

$(83)^3 = 83 \times 83 \times 83 = 571787$

(vii) 27; The digit in the unit place of $2\underline{7}$ is 7.

$\therefore$ The digit in the unit place for its cube is 3.

$(27)^3 = 27 \times 27 \times 27 = 19683$

7 CUBE ROOTS

Finding the cube root is the inverse operation of finding cube. The symbol $\sqrt[3]{\ }$ denotes 'cube root'. Consider the following :

Statement : Cube	Inference : Cube root
$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^3 = 125$	$\sqrt[3]{125} = 5$
$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^3 = 512$	$\sqrt[3]{512} = 8$
$9^3 = 729$	$\sqrt[3]{729} = 9$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$

8 METHODS FOR FINDING THE CUBE ROOTS

(i) Using unit digit method (For perfect cube numbers)

(ii) Prime factorisation method

8.1 USING UNIT DIGIT METHOD (FOR PERFECT CUBE NUMBERS)

Step I : Look at the digit at the unit's place and determine the digit at the unit's place in the cube root by using the table given in properties of cubes of numbers.

Step II : Strike out from the right, last three digits (unit's, ten's and hundred's) of the number. If no digit(s) is (are) left, then the digit obtained in Step I is the required cube root. Otherwise go to next step.

Step III : Consider the number left from step II. Find the number whose cube is less than or equal to this left over number. This number is the ten's digit of the cube root.

Step IV : Obtain the required cube root by forming a number whose unit digit is the number obtained in step I and ten's digit is the number obtained in step III.

Ex.8 Find the cube roots of the following numbers :

(i) 4096 (ii) 857375

Sol. (i) **Step 1 :** The unit's digit of 4096 is 6. Therefore, the digit at the unit's place in the cube root is 6.

Step 2 : Group 4096; two groups are 4 and 096.

Step 3 : Choose a number whose cube is less than 4.

$\because\ 1^3 = 1$ and $2^3 = 8$

$\therefore\ 1 < 4 < 8 \Rightarrow 1^3 < 4 < 2^3$

Hence 1 is ten's place digit $\therefore \sqrt[3]{4096} = 16$

(ii) The given number is 857375.

Step 1 : The unit's digit of 857375 is 5. Therefore, unit's digit of its cube root is also 5.

Step 2 : Group 857375; two groups are 857 and 375.

Step 3 : Choose a number whose cube is less than 857.

$\because\ 9^3 = 729$ and $10^3 = 1000$ and $729 < 857 < 1000$

$\Rightarrow 9^3 < 857 < 10^3 \Rightarrow 9$ is ten's place digit.

Hence, $\sqrt[3]{857375} = 95$

8.2 PRIME FACTORISATION METHOD

In order to find the cube root of a perfect cube by factor method we follow the following procedure.

Step I : Obtain the given number.

Step II : Resolve it into prime factors.

Step III : Group the factors into triplets such that all the three factors in each triplet are same.

Step IV : If no factor is left ungrouped, choose one factor from each group and take the product. The product is cube root of number. If some prime factors are left ungrouped, the number is not a perfect cube and the process stops.

Ex.9 Find the cube root of the following :

(a) 4913 (b) 19683

Sol. (a) Resolving the given number into prime factors, we get

$4913 = 17 \times 17 \times 17$

17	4913
17	289
17	17
	1

So, $\sqrt[3]{4913} = 17$

(Taking one factor from triplet)

(b) Resolving 19683 into prime factors, we get

$19683 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$\sqrt[3]{19683} = 3 \times 3 \times 3$

3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

(Taking one factor from each triplet)

$\therefore \sqrt[3]{19683} = 27$

9 CUBE ROOT OF PRODUCT OF INTEGERS

We have, $\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$.

Ex.10 Evaluate : $\sqrt[3]{125 \times 64}$

Sol. We have,

$\sqrt[3]{125 \times 64} = \sqrt[3]{125} \times \sqrt[3]{64}$

$\sqrt[3]{5 \times 5 \times 5} \times \sqrt[3]{4 \times 4 \times 4}$

$= (5 \times 4) = 20.$

Ex.11 Evaluate : $\sqrt[3]{216 \times (-343)}$

Sol. We have,

$\sqrt[3]{216 \times (-343)} = \sqrt[3]{216} \times \sqrt[3]{-343} = \sqrt[3]{216} \times \left(-\sqrt[3]{343}\right).$

$= -\sqrt[3]{6 \times 6 \times 6} \times \sqrt[3]{7 \times 7 \times 7}$

$= -[6 \times 7] = -42.$

10 CUBE ROOT OF A RATIONAL NUMBER

The cube root of a rational number $\frac{a}{b}, (b \neq 0)$, is given by $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$, where a, b are integers.

Ex.12 Evaluate : (i) $\sqrt[3]{\frac{216}{2197}}$ (ii) $\sqrt[3]{\frac{-125}{512}}$

Sol. (i) $\sqrt[3]{\frac{216}{2197}} = \frac{\sqrt[3]{216}}{\sqrt[3]{2197}} = \frac{\sqrt[3]{6 \times 6 \times 6}}{\sqrt[3]{13 \times 13 \times 13}} = \frac{6}{13}$

(ii) $\sqrt[3]{\frac{-125}{512}} = \frac{\sqrt[3]{-125}}{\sqrt[3]{512}} = \frac{-\sqrt[3]{125}}{\sqrt[3]{512}} = \frac{-\sqrt[3]{5 \times 5 \times 5}}{\sqrt[3]{8 \times 8 \times 8}} = \frac{-5}{8}$

Ex.13 Evaluate : $\frac{\sqrt[3]{15625 \times 216}}{\sqrt[3]{3375}}$

Sol. We have,

$\frac{\sqrt[3]{15625} \times \sqrt[3]{216}}{\sqrt[3]{3375}}$

$= \frac{\sqrt[3]{25 \times 25 \times 25} \times \sqrt[3]{6 \times 6 \times 6}}{\sqrt[3]{15 \times 15 \times 15}}$

$= \frac{25 \times 6}{15} = 10$

SOLVED EXAMPLES

SE1. Is 53240 a perfect cube ? If not, then by which smallest natural number should 53240 be divided so that the quotient is a perfect cube ?

Sol. $53240 = 2 \times 2 \times 2 \times 11 \times 11 \times 11 \times 5$

The prime factor 5 does not appear in a group of three. So 53240 is not a perfect cube. To get rid of this extra five we divide the number by 5. So factorisation of the quotient will not contain 5.

So, $53240 \div 5 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11}$

2	53240
2	26620
2	13310
11	6655
11	605
11	55
5	5
	1

Hence the smallest number by which 53240 should be divided to make it a perfect cube is 5. The perfect cube in that case is = 10648.

SE2. Is 246 a perfect cube ?

Sol. Resolving 246 into prime factors, we have

$246 = 2 \times 3 \times 41$

2	246
3	123
3	41
	1

Clearly, we cannot group the factors in the triplets. Therefore, 246 is not a perfect cube.

SE3. Is 1331 a perfect cube ?

Sol. Resolving 1331 into prime factors, we get

$1331 = \underline{11 \times 11 \times 11}$

We find that prime factors of 1331 can be grouped into triplets of equal factors (as shown above) and no factor is left.

$\therefore$ 1331 is a perfect cube.

SE4. Is 68600 a perfect cube ? If not, find the smallest number by which 68600 must be multiplied to get a perfect cube.

Sol. We have, $68600 = \underline{2 \times 2 \times 2} \times 5 \times 5 \times \underline{7 \times 7 \times 7}$.

2	68600
7	34300
7	4900
7	700
2	100
2	50
5	25
5	5
	1

In this factorisation we find that there is no triplet of 5. So, 68600 is not a perfect cube. To make it a perfect cube we multiply it by 5 to complete the triplet of 5. Thus,

$68600 \times 5 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5} \times \underline{7 \times 7 \times 7}$

$= 343000$, which is a perfect cube.

SE5. Prove that if a number is doubled, then its cube is eight times the cube of the given number.

Sol. Let the given number be a. Let b denote the double of a i.e., b = 2a. Then,

$b^3 = b \times b \times b$

$= 2a \times 2a \times 2a$

$= 2 \times 2 \times 2 \times a \times a \times a$

$= 8 \times a \times a \times a = 8 \times a^3$

$\Rightarrow b^3 = 8 \times$ (Cube of a).

SE6. Evaluate : $\{(24^2 + 7^2)^{1/2}\}^3$

Sol. We have,

$$\{(24^2 + 7^2)^{1/2}\}^3 = \{(576 + 49)^{1/2}\}^3 = \left\{\sqrt{625}\right\}^3$$

$$= \left\{\sqrt{25 \times 25}\right\}^3 = 25^3 = 25 \times 25 \times 25 = 15625]$$

SE7. Find the cube root of $\sqrt[3]{392} \times \sqrt[3]{448}$.

Sol. Resolving 392 and 448 into prime factors Now,

$$\sqrt[3]{392} \times \sqrt[3]{448} = \sqrt[3]{392 \times 448}$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7}$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7}$$

$= 2 \times 2 \times 2 \times 7 = 56$

2	392
2	196
2	98
7	49
7	7
	1

2	448
2	224
2	112
2	56
2	28
2	14
7	7
	1

SE8. Is $\frac{27}{125}$ a cube of a rational number ?

Sol. We have,

$$\frac{27}{125} = \frac{3 \times 3 \times 3}{5 \times 5 \times 5}$$

$$\Rightarrow \frac{27}{125} = \frac{3^3}{5^3} = \left(\frac{3}{5}\right)^3$$

$\therefore \frac{27}{125}$ is cube of $\frac{3}{5}$ i.e., a rational number.

SE9. Is 1728 a perfect cube ? If yes, find its cube root.

Sol. Resolving 1728 into prime factors, we have

$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Grouping the factors in triplets of equal factors, we get $1728 = \{2 \times 2 \times 2\} \times \{2 \times 2 \times 2\} \{3 \times 3 \times 3\}$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

We find that the prime factors of 1728 can be grouped into triplets of equal factors and no factor is left over.

$\therefore$ 1728 is a perfect cube.

Taking one factors from each triplet, we obtain

$\sqrt[3]{1728} = 2 \times 2 \times 3 = 12$

Hence, 1728 is the cube of 12.

SE10. Find the cube root of 1.331.

Sol. We have, $1.331 = \frac{1331}{1000}$

$$\therefore \sqrt[3]{1.331} = \sqrt[3]{\frac{1331}{1000}} = \frac{\sqrt[3]{1331}}{\sqrt[3]{1000}} = \frac{11}{10} = 1.1$$

SE11. Observe the following pattern.

$1^3 = 1$

$1^3 + 2^3 = (1 + 2)^2$

$1^3 + 2^3 + 3^3 = (1 + 2 + 3)^2$

Write the next three rows and calculate the value of $1^3 + 2^3 + 3^3 + ... + 9^3 + 10^3$ by above pattern.

Sol. If we observe the above pattern, the next three rows can be written as follows –

$1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1 + 2 + 3 + 4 + 5)^2$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3$

$= (1 + 2 + 3 + 4 + 5 + 6)^2$

The value of

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3$

$= (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10)^2$

$$= \left[\frac{10(10+1)^2}{2}\right]^2 = [5\times 11]^2 = (55)^2 = 3025$$

SE12. Find the volume of a cube whose surface area is 150 m^2.

Sol. Let the length of each edge of the given cube be x metres. Then, Surface area = 150 m^2

$$\Rightarrow 6x^2 = 150 \Rightarrow x^2 = \frac{150}{6} = 25 \Rightarrow x = \sqrt{25} = 5\,m$$

$\therefore$ Volume of the cube = $x^3 = 5^3$ cubic metres

= (5 × 5 × 5) cubic metres = 125 cubic metres.

SE13. For a big icecream of volume 2744 cm^3, Mukti wants to make a box. What should be the edge of box so that the block can be put into it ?

Sol. Volume of box = 2744 cm^3

Let edge of the box = x cm

$\therefore x^3 = 2744 \Rightarrow x = \sqrt[3]{2744}$

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$$\Rightarrow x = \sqrt[3]{2\times 2\times 2\times 7\times 7\times 7} = 2\times 7 = 14$$

$\therefore$ Edge of the box = 14 cm.

SE14. Consider the following pattern :

$2^3 - 1^3 = 1 + 2 \times 1 \times 3$

$3^3 - 2^3 = 1 + 3 \times 2 \times 3$

$4^3 - 3^3 = 1 + 4 \times 3 \times 3.$

Find the value of $12^3 - 11^3$, using the above pattern.

Sol. $12^3 - 11^3 = 1 + 12 \times 11 \times 3 = 1 + 396 = 397.$

SE15. Find the value of $(27 \times 2744)^{1/3}$.

Sol. $(27 \times 2744)^{1/3}$

$= \sqrt[3]{27} \times \sqrt[3]{2744}$

$= 3 \times 14 = 42$

EXERCISE-I

ONLY ONE CORRECT TYPE

1. Which of the following numbers is a perfect cube ?
(A) 1525 (B) 1728
(C) 1458 (D) 3993

2. Which of the following numbers is not a perfect cube ?
(A) 2197 (B) 512
(C) 2916 (D) 343

3. What least number must be multiplied to 3456 so that the product becomes a perfect cube ?
(A) 2 (B) 3
(C) 4 (D) 6

4. $\sqrt[3]{5832} =$
(A) 22 (B) 18
(C) 16 (D) 14

6. $\sqrt[3]{4\frac{12}{125}}$ equals
(A) $1\frac{3}{5}$ (B) $1\frac{2}{5}$
(C) $7\frac{1}{5}$ (D) $7\frac{2}{3}$

7. Possible unit digit of cube root of a number ending with 5 is
(A) 0 (B) 5
(C) 7 (D) 9

8. If $(125)^x = 3125$, then x equals
(A) $\frac{3}{5}$ (B) $\frac{5}{3}$
(C) $\frac{1}{4}$ (D) $\frac{1}{5}$

9. By what least number should 9720 be multiplied to get a perfect cube ?
(A) 15 (B) 25
(C) 5 (D) 75

10. If $\sqrt[3]{(156+x)} = 12$, then the value of x is
(A) 1570 (B) 1572
(C) 1560 (D) 1512

11. The value of $(0-0.4)^3$ is
(A) 0.640 (B) 0.064
(C) – 0.064 (D) – 0.640

12. $3^3 - (-0.6)^3 =$
(A) 27.216 (B) 26.784
(C) – 26.784 (D) – 27.216

13. $\frac{16}{9} \times \left(-1\frac{1}{2}\right)^3 =$
(A) – 12 (B) – 6
(C) $-\frac{8}{3}$ (D) $\frac{8}{9}$

14. $\sqrt[3]{0.125} + 3 =$
(A) 8 (B) 3.5
(C) 2 (D) 0.35

15. Calculate the value of $\sqrt[3]{\frac{192}{81}}$.
(A) $\frac{5}{3}$ (B) $\frac{4}{3}$
(C) $\frac{3}{2}$ (D) $\frac{13}{9}$

16. Which of the following is the cube of a negative number ?
(A) – 396 (B) 4096
(C) – 81 (D) – 2744

17. Which of the following numbers is the cube of an odd number ?
(A) 79507 (B) 2744
(C) 32768 (D) 1728

18. Which of the following numbers is the cube of an even number ?
(A) 6859 (B) 648
(C) 13824 (D) 42875

19. The symbol $\sqrt[3]{\ }$ denotes
(A) square (B) cube
(C) square root (D) cube root

20. The cube root of 97336 is
(A) 17 (B) 18

(C) 46 (D) 23

21. What is the least number by which 13720 must be divided so that the quotient is a perfect cube ?
(A) 2 (B) 3
(C) 5 (D) 6

22. The value of $\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$ is
(A) 2 (B) 2.273
(C) 2.327 (D) None of these

23. The value of $\frac{\sqrt[3]{8} + \sqrt[3]{27} - \sqrt[3]{343}}{(2)^2 - 3}$ is
(A) 7 (B) – 2
(C) 8 (D) – 5

24. Observe the pattern given below
$1^3 = 1$
$2^3 = 3 + 5$
$3^3 = 7 + 9 + 11$
$4^3 = 13 + 15 + 17 + 19$
$5^3 = 21 + 23 + 25 + 27 + 29$
According to this pattern, the number of consecutive odd numbers whose sum equals 9^3 is
(A) 3 (B) 9
(C) 12 (D) 15

25. Evaluate : $\sqrt[3]{27} + \sqrt[3]{0.008}$
(A) 3.4 (B) 3.1
(C) 3.3 (D) 3.2

PARAGRAPH TYPE

PASSAGE – I : Three numbers are in the ratio 2 : 3 : 4.

1. The sum of their cubes is 33957. Find the numbers.
(A) 2, 4, 8 (B) 14, 21, 28
(C) 6, 9, 18 (D) 5, 10, 15

2. The cubes of these numbers are
(A) 8, 64, 512
(B) 2744, 7261, 8849
(C) 2744, 9261, 21952
(D) 7261, 125, 1000

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from List – I and List – II are given as options (A), (B), (C) and (D) out of which one is correct.

1. Match the numbers given in List – I with their cube roots given in List – II.

List – I	**List – II**
(P) 148877	(i) 24
(Q) 13824	(ii) 53
(R) 35937	(iii) 33
(S) 17576	(iv) 26

(A) (P)→(ii), (Q)→(iii), (R)→(iv), (S)→(i)
(B) (P)→(ii), (Q)→(i), (R)→(iii), (S)→(iv)
(C) (P)→(i), (Q)→(ii), (R)→(iii), (S)→(iv)
(D) (P)→(i), (Q)→(iii), (R)→(ii), (S)→(iv)

2. Match the following :

List – I	**List – II**
(P) The smallest number by which 392 must be multiplied so that the product is a perfect cube is	(i) 5
(Q) The smallest number by which 8640 must be divided so that the quotient is a perfect cube is	(ii) 3
(R) The smallest number by which 3087 must be multiplied so that product is a perfect cube is	(iii) 25
(S) The smallest number by which 33275 must be divided so that the quotient becomes a perfect cube is	(iv) 7

(A) (P)→(iv), (Q)→(i), (R)→(ii), (S)→(iii)
(B) (P)→(i), (Q)→(ii), (R)→(iii), (S)→(iv)
(C) (P)→(ii), (Q)→(iii), (R)→(iv), (S)→(i)
(D) (P)→(iv), (Q)→(ii), (R)→(i), (S)→(iii)

EXERCISE-II

VERY SHORT ANSWER TYPE

1. Is 1024 a perfect cube ?
2. Find the unit digit of cube of 8888.
3. Is 196 a perfect cube ?
4. Find the smallest number by which 96 must be multiplied so that the product is a perfect cube.
5. Find the smallest number by which 3087 must be divided so that the quotient is a perfect cube.
6. Show that 1331 is a pefect cube.
7. Consider the following pattern.
   $2^3 - 1^3 = 1 + 2 \times 1 \times 3$
   $3^3 - 2^3 = 1 + 3 \times 2 \times 3$
   $4^3 - 3^3 = 1 + 4 \times 3 \times 3$
   Using the above pattern find the value of $7^3 - 6^3$.
8. Find the cube root of 17576.
9. Find the cube root of 0.003375.
10. Evaluate the following : $\left\{\sqrt{15^2 + 8^2}\right\}^3$.

SHORT ANSWER TYPE

1. Is 8000 a pefect cube ? What is the number whose cube is 8000 ?
2. Find the cube root of 389017.
3. Find the smallest number which when multiplied with 3600 make the product a perfect cube. Further, find the cube root of the product.
4. Find the smallest number by which 8192 must be divided so that quotient is a perfect cube. Also, find the cube root of the quotient so obtained.
5. Three numbers are in the ratio 1 : 2 : 3. The sum of their cubes is 98784. Find the numbers.

LONG ANSWER TYPE

1. Find the cube root of 85184 by prime factorisation method.
2. Find the cube root of 2300×5290.
3. Evaluate : $\sqrt[3]{1372} \times \sqrt[3]{1458}$.
4. Find the cube root of $\frac{3375}{2744}$.
5. Find the cube root of 110592.

TRUE / FALSE TYPE

1. A perfect cube can end with even number of zeroes.
2. The cube of a 2 digit number may be a 3 digit number.
3. The cube of a single digit number may be a single digit number.
4. If n ends in 3, then n^3 ends in 7.
5. The cube root of a negative perfect cube is negative.

NUMERICAL PROBLEMS

1. If $\sqrt[3]{\frac{a^6 \times b^3 \times c^{21}}{c^9 \times a^{12}}} = \frac{bc^k}{a^{k/2}}$, then k = _____.
2. The cube of the number p is 16 times the number. Then find p where $p \neq 0$ and $p \neq -4$.
3. A number is multiplied 3 times by itself and then 61 is subtracted from the product obtained. If the final result is 9200, then the number is.
4. $\sqrt[3]{0.125} + \sqrt[3]{0.729} = \frac{n}{10}$. Find n.
5. If a = 2b and b = 4c, then $\sqrt[3]{\frac{a^2}{16bc}}$ = _____.

ANALYTICAL PROBLEMS & BRAIN TEASER

1. Simplify : $\left(\sqrt[6]{27} - \sqrt{6\frac{3}{4}}\right)^2$

   (A) $\frac{3}{4}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{3\sqrt{3}}{4}$ (D) $\frac{3}{2}$

2. By what least number 3600 must be divided to make it a perfect cube ?
(A) 9 (B) 50
(C) 300 (D) 450

3. The value of $\sqrt{\frac{0.00001225}{0.00005329}} - \sqrt[3]{\sqrt{0.000064}}$ is
(A) 0.2 (B) 0.279
(C) 0.479 (D) 0

4. Evaluate : $\sqrt[3]{0.008} - \sqrt[3]{-512} + \sqrt[3]{2.197}$
(A) 9.3 (B) – 6.5
(C) 9.5 (D) 6.5

5. $\frac{3\sqrt[3]{13824}}{2\sqrt[3]{-15625}} \div \frac{2\sqrt[3]{-13824}}{\sqrt[3]{5832}} =$
(A) $\frac{50}{27}$
(B) $\frac{-50}{27}$
(C) $\frac{27}{50}$
(D) $\frac{-27}{50}$

6. Cube root of a number when divided by the smallest prime number gives square of the smallest prime number. Find the number.
(A) 512 (B) 8
(C) 64 (D) 125

7. If a number has digit 2 at unit place, then its cube has digit _____ at its unit place.
(A) 1 (B) 2
(C) 8 (D) 4

8. Which of the following is incorrect ?
(A) The cube of an even natural number is always even.
(B) The cube root of a rational number $\frac{x}{y}$ is $\frac{\sqrt[3]{x}}{\sqrt[3]{y}}$.
(C) The cube of a negative number is always positive.
(D) 2197 is a perfect cube.

9. $\left(\sqrt[3]{3}+\sqrt[3]{2}\right)\left(\sqrt[3]{9}+\sqrt[3]{4}-\sqrt[3]{6}\right) =$
(A) 5
(B) $\sqrt[9]{5}$
(C) $\sqrt[6]{5}$
(D) $\sqrt[3]{5}$

10. $\frac{\sqrt[3]{1.728}-\sqrt[3]{0.216}}{\sqrt[3]{2.197}-\sqrt[3]{0.343}} =$
(A) 1
(B) – 1
(C) 2
(D) – 2

11. If $3^9 + 3^{12} + 3^{15} + 3^n$ is a perfect cube, $n \in N$, then the value of n is
(A) 18
(B) 17
(C) 14
(D) 16

12. If $\sqrt{\sqrt[3]{x \times 0.000009}} = 0.3$, then the value of $\sqrt{x}$ is :
(A) 27
(B) 81
(C) 9
(D) 18

13. $\frac{\sqrt[3]{1.728}}{\sqrt[3]{13.824}} \times \frac{\sqrt[3]{4.096}}{\sqrt[3]{216}} =$
(A) $\frac{15}{8}$
(B) $\frac{4}{15}$

(C) $\frac{2}{15}$

(D) $\frac{15}{4}$

14. If $\sqrt[3]{\frac{x}{729}}+\sqrt[3]{\frac{27x}{3375}}=1$, then find the value of x.

(A) $\frac{79507}{3375}$

(B) $\frac{91125}{2744}$

(C) $\frac{2025}{196}$

(D) $\frac{443}{125}$

15. Evaluate : $\sqrt[3]{\frac{4096}{64}}+2\sqrt[3]{\frac{5832}{216}}-3\sqrt[3]{\frac{3375}{125}}$

$+4\sqrt[3]{\frac{1728}{64}}$

(A) 12

(B) 13

(C) – 13

(D) – 12

EXERCISE-III

PREVIOUS YEAR QUESTIONS (NTSE)

1. $\dfrac{10+\sqrt[3]{8}}{98.4+\sqrt{2.56}}$ is equal to

(Aryabhatta – 2006)

(A) 0.12 (B) 0.012
(C) 1.2 (D) 11.2

2. Cube root of $\dfrac{0.216}{27}$ is

(Aryabhatta – 2006)

(A) 0.2 (B) 0.02
(C) 0.002 (D) 0.06

3. $6^3 - 5^3$ is always divisible by

(Aryabhatta – 2008)

(A) 9 (B) 8
(C) 1 (D) 5

4. $10 + 10^3$ equals

(NSTSE – 2009)

(A) 2.0×10^3 (B) 8.0×10^3
(C) 4.0×10^3 (D) 1.01×10^3

5. By what least number 3600 be divided to make it a perfect cube ?

(NSTSE – 2010)

(A) 9 (B) 50
(C) 300 (D) 450

6. Find the value of $\sqrt[3]{\sqrt{0.000064}}$.

(IMO – 2011)

(A) 002 (B) 0.2
(C) 2 (D) None of these

7. The largest four-digit number which is a perfect cube, is _____.

(IMO – 2012)

(A) 8000 (B) 9261
(C) 9999 (D) None of these

8. The cube of a number is 8 times the cube of another number. If the sum of the cubes of numbers is 243, then what is the difference of the numbers ?

(NSTSE – 2013)

(A) 3 (B) 4
(C) 6 (D) – 6

9. The product 864 × n is a perfect cube. What is the smallest value of 'n' ?

(NSTSE – 2014)

(A) 2 (B) 1
(C) 4 (D) 3

Answer Key

EXERCISE I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	C	C	B		A		B	B	D	B		C	A	B
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
B		B		D	A	C	D		C	C	A		B	B
31	32	33	34	35	36									
	D			B										

PARAGRAPH

1. B **2.** C

MATCH THE COLUMN

1. B **2.** A

EXERCISE II

VERY SHORT ANSWER TYPE

1. No **2.** 2 **3.** No **4.** 18 **5.** 9 **6.** 11

7. $1 + 7 \times 6 \times 3 = 127$ **8.** 26 **9.** 0.15 **10.** 4913

SHORT ANSWER TYPE

1. 20 **2.** 73 **3.** **4.** 60 **5.** 16 **6.** 14, 28, 42 **7.**

8. **9.** **10.**

LONG ANSWER TYPE

1. 44 **2.** 230 **3.** 126 **4.** $\frac{15}{14}$ **5.** 48

TRUE / FALSE

1. T **2.** **3.** F **4.** T **5.** **6.** T **7.**
8. T **9.** **10.**

NUMERICAL PROBLEMS

1. 4 **2.** 4 **3.** **4.** **5.** 27 **6.** 14 **7.** 1
8. **9.** **10.**

ANALYTICAL PROBLEMS & BRAIN TEASER

1. A **2.** D **3.** B **4.** C **5.** C **6.** A **7.** C
8. C **9.** A **10.** A **11.** C **12.** B **13.** C **14.** B
15. B

EXERCISE III									
1	2	3	4	5	6	7	8	9	
A	A	C	D	D	B	B	A	A	

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : CUBES AND CUBE ROOTS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

COMPARING QUANTITIES

1 INTRODUCTION

When we compare the quantities using concepts of ratio, proportions, percentage, etc. that is called comparing quantities.

2 RATIO

The quotient of two numbers or quantities indicating their relative sizes.

The ratio of a to b is written as a : b or $\frac{a}{b}$. The first term is antecedent and the second term is consequent.

Ex. 1 If $\frac{x}{5} = \frac{y}{8}$, then find $(x + 5) : (y + 8)$.

Sol. Let $\frac{x}{5} = \frac{y}{8} = k$. Then $x = 5k$ and $y = 8k$

$$\therefore \frac{x+5}{y+8} = \frac{5k+5}{8k+8} = \frac{5(k+1)}{8(k+1)} = \frac{5}{8}$$

$\Rightarrow x + 5 : y + 8 = 5 : 8$

Ex. 2 If $2A = 3B$ and $4B = 5C$, then find A : C.

Sol. $2A = 3B$ and $4B = 5C$

$$\Rightarrow \frac{A}{B} = \frac{3}{2} \text{ and } \frac{B}{C} = \frac{5}{4}$$

$$\Rightarrow \frac{A}{C} = \left(\frac{A}{B} \times \frac{B}{C}\right) = \left(\frac{3}{2} \times \frac{5}{4}\right) = \frac{15}{8}$$

$\Rightarrow A : C = 15 : 8$

3 PERCENTAGE

Percent : Indicating hundredths. A fraction can be expressed as a percentage by multiplying it by 100. A change in a quantity from a to b is a change of $\frac{100(b-a)}{a}$ percent.

Finding a percentage of a number : To find a percent of a given number, we proceed as follows :

Step – 1 : Obtain the number, say x.

Step – 2 : Obtain the required percent, say P %.

Step – 3 : Multiply x by P and divide by 100 to obtain the required P % of x i.e., P % of x = $\frac{P}{100} \times x$.

Ex. 3 Find 30 % of Rs. 150.

Sol. 30 % of Rs. 150 $= \frac{30}{100} \times 150$ = Rs. 45

Ex. 4 Find x, if x % of 250 + 25 % of 68 = 67.

Sol. $\frac{x}{100} \times 250 + \frac{25}{100} \times 68 = 67$

$\Rightarrow \frac{25x}{10} = 67 - 17 \Rightarrow x = \frac{50 \times 10}{25} \Rightarrow x = 20$

Finding how much percent one quantity is of another quantity :

To find : What percent is x of y, where x and y are any two numbers ?

Since, 'is' is placed before 'of', then required percent = $\frac{x}{y} \times 100\,\%$

To find : What percent of a is b, where a and b are any two numbers ?

Since, 'of' is placed before 'is', then required percent = $\frac{b}{a} \times 100\,\%$

Ex. 5 What percent of 35 kg is 2.5 kg ?

Sol. Here 'of' is before 'is', so

Required percent = $\frac{2.5}{35} \times 100\%$

$= \frac{25}{35 \times 10} \times 100\% = \frac{50}{7}\% = 7\frac{1}{7}\%$

Thus, 2.5 kg is $7\frac{1}{7}\%$ of 35 kg.

Ex. 6 What percent is 18 hours of 3 days ?

Sol. 3 days = 3 × 24 hours = 72 hours.

[To find percentage both quantities should be in same unit]

Here 'is' is before 'of', so

Required percent = $\frac{18}{72} \times 100\% = 25\,\%$

Thus, 18 hours is 25 % of 3 days.

4 INCREASE OR DECREASE PERCENT

We calculate increase or decrease percent as follows :

$$\text{Increase \%} = \frac{\text{Increase}}{\text{Original Value}} \times 100\%; \ \text{Decrease \%} = \frac{\text{Decrease}}{\text{Original Value}} \times 100\%$$

Ex. 7 The price of a scooter which was Rs. 34000 last year increased by 20 % this year. What is the price now ?

Sol. Let original price be Rs. 100.

Increase in price = Rs. 20

$\therefore$ Increased price = 100 + 20 = Rs. 120

If original price is Rs. 100, increased price = Rs. 120.

If original price is Rs. 34000, increased price $= \frac{120}{100} \times 34000 = \text{Rs. } 40800$.

5 PROFIT AND LOSS

Cost Price : The amount paid to purchase an article or the price at which an article is made is known as its cost price. The cost price is abbrevited as C.P.

The overhead expenses like taxes, labour charges, etc. are included in the cost price. If overhead expenses are not included in the cost price, then

Effective Cost Price = Pyment made while purchasing the goods + Overhead expenses

Selling Price : The price at which an article is sold is known as its selling price.

The selling price is abbreviated as S.P.

Profit : If the selling price (S.P.) of an article is greater than the cost price (C.P.), then the diffrence between the selling price and cost price is called profit.

Profit = S.P. – C.P.

Profit Percentage : The profit percent is the profit that would be obtained for a C.P. of Rs. 100 i.e.,

$$\text{Profit percent} = \left(\frac{\text{Profit}}{\text{C.P.}} \times 100\right)\%.$$

Formulas for calculating S.P. and C.P., when profit % is given.

$$\text{S.P.} = \left(\frac{100 + \text{Profit \%}}{100}\right) \times \text{C.P.}; \ \text{C.P.} = \frac{100 \times \text{S.P.}}{(100 + \text{Profit \%})}$$

Loss : If the selling price (S.P.) of an article is less than the cost price (C.P.) then the difference between the cost price (C.P.) and the selling price (S.P.) is called loss.

Loss = C.P. – S.P.

Loss Percentage : The loss percent is the loss that would be made for a C.P. of Rs. 100.

That is, loss percent $= \left(\frac{\text{Loss}}{\text{C.P.}} \times 100\right)\%$.

Formulas for calculating S.P. and C.P. when loss % is given :

$$\text{S.P.} = \left(\frac{100 - \text{Loss}\,\%}{100}\right) \times \text{C.P.};\ \text{C.P.} = \frac{100 \times \text{S.P.}}{(100 - \text{Loss}\,\%)}$$

Ex. 8 Rishi bought a wrist watch for Rs. 2200 and sold it for Rs. 1980. Find loss and loss percent.

Sol. We have, C.P. of watch = Rs. 2200

S.P. of watch = Rs. 1980

Since S.P. < C.P. So, there is loss given by

Loss = C.P. – S.P. = 2200 – 1980 = Rs. 220

Now, loss % $= \left(\frac{\text{Loss}}{\text{C.P.}} \times 100\right)\% = \left(\frac{220}{2200} \times 100\right)\% = 10\%$

Hence, loss = Rs. 220 and loss % = 10 %.

Ex. 9 Raghu bought an almirah for Rs. 6250 and spent Rs. 375 on its repairs. Then, he sold it for Rs. 6890. Find his gain or loss percent.

Sol. C.P. of the almirah = Rs. 6250, overheads = Rs. 375.

Total cost price = (6250 + 375) = Rs. 6625.

Selling price = Rs. 6890.

Since (S.P.) > (C.P.), Raghu gains.

Gain = (6890 – 6625) = Rs. 265.

Gain % $= \left(\frac{\text{Gain}}{\text{Total C.P.}} \times 100\right)\% = \frac{265}{6625} \times 100\ \% = 4\%$

Ex. 10 On selling a fan for Rs. 810, Samrat gains 8 %. For how much did he purchase it ?

Sol. S.P. of the fan = Rs. 810, Gain % = 8 %.

$\therefore$ C.P. of the fan $= \left\{\frac{100}{(100 + \text{gain}\ \%)} \times \text{S.P.}\right\}$

$= \left\{\frac{100}{(100+8)} \times 810\right\} = \left(\frac{100}{108} \times 810\right) = \text{Rs.}750.$

Hence, Samrat purchased the fan for Rs. 750.

Ex. 11 Mayank lost 20% by selling a bicyle for Rs. 1536. What percent shall he gain or loss by selling it for Rs. 2000 ?

Sol. S.P. = Rs. 1536, Loss % = 20 %

$\therefore$ C.P. = $\left(\frac{100}{80}\times 1536\right)$ = Rs.1920

New S.P. = Rs. 2000.

Gain = (2000 – 1920) = Rs. 80.

$\therefore$ Gain % = $\frac{80}{1920}\times 100 = \frac{25}{6}$ %

6 DISCOUNT

Market price or list price : The price written on the article or tagged with the article is called the marked price (M.P.) or list price (L.P.).

Discount : The deduction made on the marked price is called discount. Discount is generally given as a certain percent of the marked price. It is always calculated on the marked price or list price.

Selling Price : The difference between the marked price (M.P.) and discount is called the selling price (S.P.) of the article.

- Discount = M.P. – S.P.
- Rate of discount = Discount % = $\left(\frac{\text{Discount}}{\text{M.P.}}\times 100\right)$%
- S.P. = M.P. – Discount

$\Rightarrow$ S.P. = M.P. – $\frac{\text{Discount \%}\times\text{M.P.}}{100}$ $\Rightarrow$ S.P. = M.P. × $\left(1-\frac{\text{Discount \%}}{100}\right)$

$\Rightarrow$ S.P. = M.P. × $\left(\frac{100-\text{Discount \%}}{100}\right)$

- M.P. = $\frac{100\times\text{S.P.}}{100-\text{Discount \%}}$

Ex. 12 The marked price of a pen set is Rs. 450. The shopkeeper gives a discount of 20%. What is his selling price ?

Sol. Given that M.P. = Rs. 450 and discount % = 20 %

Discount = 20 % of Rs. 450 = $\frac{20}{100}$ × Rs. 450 = Rs. 90

$\therefore$ S.P. = M.P. – Discount = Rs. 450 – Rs. 90 = Rs. 360

Thus, the pen set was sold for Rs. 360.

Ex. 13 Which is better offer : two successive discounts of 10% and 8% or a single discount of 18% ?

Sol. Let the marked price = Rs. 100

First discount = Rs. 10

First reduced price = (100 – 10) = Rs. 90. Second discount = $\left(\frac{8}{100}\times 90\right)$ = Rs. 7.20

Net selling price = Rs. (90 – 7.20) = Rs. 82.80

Total discount allowed = Rs. (100 – 82.80) = Rs. 17.20

∴ Single discount equivalent to two successive discounts of 10% and 8% is 17.2%.

∴ Single discount of 18% is better.

7 SALES TAX/VALUE ADDED TAX

Tax is the money collected by the government from the citizens of the country in order to provide them services. There are various types of taxes – Income Tax, Sales Tax, Service Tax and Value Added Tax etc.

Sales tax or Value added tax : It is collected by the government on the sale of different commodities. It is charged by the shopkeeper from the customer and given to the government. Therefore, this is always on the selling price of an article and is added to the value of the bill (known as VAT).

- The rate of this tax depends upon the item sold.
- When C.P. is given exclusive of tax. Tax to be paid = $\frac{\text{Tax \%}}{100}\times$ C.P. So, bill amount = C.P. + Sales Tax
- When C.P. is given inclusive of tax or VAT. Original Price = $\frac{100}{100+\text{Tax \%}}\times$ C.P.

Note

Goods and Services Tax : Goods and Services Tax (G.S.T.) is an Indirect Tax, which has replaced many Indirect taxes like VAT, Central Excise duty, Sales Tax, Service Tax, etc. It is described as one tax for one nation. G.S.T. is classified under three categories :

♦ **C.G.S.T. :** The revenue collected under "Central Goods and Services Tax". (C.G.S.T.) is for centre government.

♦ **S.G.S.T. :** The revenue collected under "State Goods and Services Tax". (S.G.S.T.) is for state government.

♦ **I.G.S.T. :** "Integrated Goods and Services Tax" (I.G.S.T.) is charged when movement of goods and services from one state to another. The revenue collected out of I.G.S.T. is shared by state government and central government as per the rates fixed by the authorities.

Ex. 14 The cost of a pair of shoes at a shop was Rs. 450. The sales tax charged was 5%. Find the bill amount.

Sol. On Rs. 100, the tax paid was Rs. 5.

On Rs. 450, the tax paid would be = $\frac{5}{100} \times 450$ = Rs. 22.50

Bill amount = Cost of item + Sales tax = Rs. 450 + Rs. 22.50 = Rs. 472.50

Ex. 15 Sanjana bought a sofa for Rs. 13500. If 8% VAT is included in the price, find the original price of sofa.

Sol. The price includes the VAT, i.e., the value added tax. Thus, 8% VAT means if the price without VAT is Rs. 100, then price including VAT is Rs. 108.

Now, when price including VAT is Rs. 108, the original price is Rs. 100.

Hence, when price including tax is Rs. 13500,

The original price = $\frac{100}{108} \times 13500$ = Rs. 12500

Ex. 16 Reena goes to a shop to buy a washing machine, costing Rs. 11800. The rate of G.S.T. is 18%. She tells the shopkeeper to reduce the price of the washing machine to such an extent that she has to pay Rs. 11800, inclusive of G.S.T. Find the reduction needed in the price of the washing machine.

Sol. Let the reduced price, excluding the G.S.T., of the washing machine be x. Then,

G.S.T. = 18% of x = Rs. $\frac{18x}{100}$

$\therefore$ Selling price of the washing machine = Rs. $\left(x + \frac{18x}{100}\right)$ = Rs. $\frac{118x}{100}$

But, the selling price of the washing machine is Rs. 11800.

$\therefore \frac{118x}{100} = 11800 \Rightarrow x = \frac{11800 \times 100}{118} \Rightarrow x =$ Rs.10,000

Hence, the reduction needed in the price of the washing machine = Rs. (11800 – 10000) = Rs. 1800

8 INTEREST

(i) Simple Interest : If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called simple interest.

(ii) Compound Interest : Sometimes it happens that the borrower and the lender agree to fix up a certain unit of time, say yearly or half-yearly or quarterly to settle the account. In such case, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on. Difference between the amount and the money borrowed is called the compound interest (abbreviated as C.I. for that period). [When interest is calculated on the amount of previous year, it is compound interest].

Ex. 17 Find the amount of Rs. 8000 for 3 years, compounded annually at 5% per annum. Also, find the compound interest.

Sol. Here, P = Rs. 8000, R = 5% per annum and n = 3 years. Using the formula $A = P\left(1+\frac{R}{100}\right)^n$, we get

Amount after 3 years $= 8000\times\left(1+\frac{5}{100}\right)^3 = \left(8000\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}\right) = \text{Rs.}9261.$

Thus, amount after 3 years = Rs. 9261. And, compound interest = Rs. (9261 – 8000) = Rs. 1261.

When the interest is compounded annually but rates are different for different years :

Let P = principal, time = 2 years, and let the rates of interest be p% p.a. during the first year and q% p.a. during the second year.

Then, amount after 2 years $= \left\{P\times\left(1+\frac{p}{100}\right)\times\left(1+\frac{q}{100}\right)\right\}$

This formula may similarly be extended for any number of years.

Ex. 18 Find the amount of Rs. 12000 after 2 years, compounded annually; the rate of interest being 5% p.a. during the first year and 6% p.a. during the second year. Also, find the compound interest.

Sol. Here, P = Rs. 12000, p = 5% p.a. and q = 6% p.a.

Using the formula, $A = P\times\left(1+\frac{p}{100}\right)\times\left(1+\frac{q}{100}\right)$,

Amount after 2 years $= \left\{12000\times\left(1+\frac{5}{100}\right)\times\left(1+\frac{6}{100}\right)\right\} = \left(12000\times\frac{21}{20}\times\frac{53}{50}\right) = \text{Rs.}13356$

And, compound interest = (13356 – 12000) = Rs. 1356.

When interest is compounded annually but time is a fraction :

For example, P = principal, R = rate of interest and suppose time is $2\frac{3}{5}$ years. Then,

$$\text{Amount} = P\times\left(1+\frac{R}{100}\right)^2\times\left(1+\frac{3/5}{100}\times R\right)$$

Ex. 19 Find the compound interest on Rs. 31250 at 8% per annum for $2\frac{3}{4}$ years.

Sol. Here P = Rs. 31250, R = 8%, n = $2\frac{3}{4}$ years. $A = \left[31250\times\left(1+\frac{8}{100}\right)^2\times\left\{1+\frac{3/4}{100}\right\}\times 8\right]$

$$=\left\{31250\times\left(\frac{27}{25}\right)^2\times\left(\frac{53}{50}\right)\right\}=\left(31250\times\frac{27}{25}\times\frac{27}{25}\times\frac{53}{50}\right)=\text{Rs.}38637$$

Hence, compound interest = (38637 – 31250) = Rs. 7387.

When interest is componded half yearly :

Let Principal = P, Rate = R% per annum, Time = n years.

Then, rate = $\left(\frac{R}{2}\right)\%$ per half-year, time = (2n) half-years, and amount = $P\times\left(1+\frac{R}{2\times100}\right)^{2n}$

Compound interest = (amount) – (principal).

Ex. 20 Find the compound interest on Rs. 15625 for $1\frac{1}{2}$ years, at 8% per annum when compounded half-yearly.

Sol. Here, principal = Rs. 15625, Rate = 8% per annum = 4% per half-year,

Time = $1\frac{1}{2}$ years = 3 half-years.

$$\therefore \text{Amount}=\left[15625\times\left(1+\frac{4}{100}\right)^3\right]=\left(15625\times\frac{26}{25}\times\frac{26}{25}\times\frac{26}{25}\right)=\text{Rs.}17576$$

∴ Compound interest = (17576 – 15625) = Rs. 1951.

When interest is compounded quarterly :

Let P = principal, Rate = R% per annum, Time = n years. Suppose that the interest is componded quarterly.

Then, rate = $\left(\frac{R}{4}\right)\%$ per quarter, Time = (4n) quarters, ∴ Amount = $P\times\left(1+\frac{R}{4\times100}\right)^{4n}$

Compound interest = (amount) – (principal).

Ex. 21 Find compound interest on Rs. 125000 for 9 months 8% per annum, compounded quarterly.

Sol. Here, principal = Rs. 125000, Rate = 8% p.a. = $\left(\frac{8}{4}\right)\%$ = 2% per quarter, Time = 9 months = 3 quarters

$$\therefore \text{Amount}=\left\{125000\times\left(1+\frac{2}{100}\right)^3\right\}=\left(125000\times\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}\right)=\text{Rs.}132651$$

∴ Compound interest = (132651 – 125000) = Rs. 7651.

SOLVED EXAMPLES

SE.1 Find the ratio of :

(i) 75 paise to Rs. 3

(ii) 3 weeks to 3 days

Sol. (i) Rs. 3 = 3 × 100 paise = 300 paise

Ratio of 75 paise to Rs. 3

= Ratio of 75 paise to 300 paise

= 75 : 300 = 1 : 4

(ii) 1 week = 7 days

So, 3 weeks = 3 × 7 days = 21 days

Ratio of 3 weeks to 3 days

= Ratio of 21 days to 3 days

= 21 : 3 = 7 : 1

SE.2 The ratio of the sum of money in three bags A, B and C is 4 : 3 : 2. If Rs. 50 is added to each of the bags, the ratio becomes 14 : 13 : 12. Find the sum of money in each of the bags.

Sol. Let the sum of money in the bags be Rs. x, Rs. y, Rs. z respectively.

As these are in the ratio 4 : 3 : 2, we get

x = 4k, y = 3k, z = 2k.

We are given that Rs. 50 is added to each bag.

So, the sums are now 4k + 50, 3k + 50, 2k + 50 respectively. But these are in the ratio 14 : 13 : 12

∴ 4k + 50 = 14p .....(i)

3k + 50 = 13p .....(ii)

2k + 50 = 12p .....(iii)

Subtracting equation (ii) from equation (i)

[or equations (iii) from (ii) or (iii) from (i)]

We get, k = p

∴ 4k + 50 = 14p ⇒ 4k + 50 = 14k

⇒ 10k = 50 ⇒ k = 5

∴ The sum of money in the bags are,

Rs. 4 × 5, Rs. 3 × 5, Rs. 2 × 5

i.e., Rs. 20, Rs. 15, Rs. 10

SE.3 If 23% of a is 46, then find a.

Sol. We have, 23% of a = $\frac{23}{100}$ × a.

But, 23% of a is given as 46.

$\therefore \frac{23}{100} \times a = 46 \Rightarrow a = 46 \times \frac{100}{23} \Rightarrow a = 200$

SE.4 A football team loss 10 games from the total they played. This was 20% of the total. How many games were played in all ?

Sol. Let total games played be x.

So, 20% of x = 10 or, x × $\frac{20}{100}$ = 10

or, x = $\frac{10 \times 100}{20} = 50$

Thus, total games were 50.

SE.5 Convert the following into percent.

(i) $12\frac{1}{2}$ (ii) 4 : 5

Sol. (i) $12\frac{1}{2} = \frac{25}{2} = \frac{25}{2} \times 100\% = 1250\%$

(ii) $4 : 5 = \frac{4}{5} = \frac{4}{5} \times 100\% = (4 \times 20)\% = 80\%$

SE.6 Find :

(i) 75% of 500 (ii) $4\frac{1}{2}\%$ of 1800 m

Sol. (i) 75% of 500 = $\frac{75}{100} \times 500 = 375$

(ii) $4\frac{1}{2}\%$ of 1800 m = $\frac{9}{2}\%$ of 1800 m

$= \frac{9}{2} \times \frac{1}{100} \times 1800\,\text{m} = 81\,\text{m}$

SE.7 If Kamini had Rs. 500 left after spending 50% of her money, how much did she have in the beginning ?

Sol. Let the money in the beginning be Rs. 100.

Now, money spent = Rs. 50

$\therefore$ Money left = 100 – 50 = Rs. 50.

If money left is Rs. 50, then amount in the beginning = Rs. 100

If money left is Rs. 500, then amount in the beginning $= \frac{100}{50} \times 500 = \text{Rs.}1000$

SE.8 The population of a town increases by 6% every year. If the present population is 16000, find its population after a year.

Sol. Original (present) population = 16000

$\therefore$ Increase in population = 6% of original

$= \frac{6}{100} \times 16000 = 960$

Thus, population after a year

= Present population + Increase in population

= 16000 + 960 = 16960

SE.9 A number is increased by 10% and then it is decreased by 10%. Find the net increase or decrease percent.

Sol. Let the number be = 100

Increase in the number = 10% of 100 = 10

$\therefore$ Increased number = 100 + 10 = 110

This number is decreased by 10%.

$\therefore$ Decrease in the number = 10% of 110

$= \left(\frac{10}{100} \times 110\right) = 11$

$\therefore$ New number = 110 – 11 = 99

Thus, net decrease = 100 – 99 = 1

Hence, net percentage decrease

$= \left(\frac{1}{100} \times 100\right)\% = 1\%$

SE.10 x is 5% of y, y is 24% of z. If x = 480, find the values of y and z.

Sol. $x = 5\% \text{ of } y \Rightarrow x = \frac{5}{100} y$ ...(i)

and $y = 24\% \text{ of } z \Rightarrow y = \frac{24}{100} z$ ...(ii)

Now, x = 480 (Given)

Substituting the value of x in (i), we get

$480 = \frac{5y}{100} \Rightarrow 480 \times 100 = 5y$

$\Rightarrow y = \frac{480 \times 100}{5} = 9600 \Rightarrow y = 9600$ ...(iii)

Substituting the value of y from (iii) in (ii),

we get $9600 = \frac{24}{100} z \Rightarrow 9600 \times 100 = 24z$

$\Rightarrow z = \frac{9600 \times 100}{24} \Rightarrow z = 40000$

SE.11 A bookseller bought a book for Rs. 20 and sold it for Rs. 24. Find his gain percent.

Sol. Cost price of the book = Rs. 20

Selling price of the book = Rs. 24

Gain = S.P. – C.P. = (24 – 20) = Rs. 4

$\text{Gain\%} = \frac{\text{Total gain}}{\text{C.P.}} \times 100$

$= \left(\frac{4}{20} \times 100\right) = 20\%$

SE.12 An electronic goods dealer purchased a second hand colour T.V. for Rs. 7250. He spent Rs. 250 on its repairs and sold it for Rs. 7800. Find his gain or loss percent.

Sol. C.P. of the television = Rs. 7250

Overhead (repair) charges = Rs. 250

Total cost of the T.V. = Rs. 7500

S.P. of the T.V. = Rs. 7800

$\therefore$ Gain = 7800 – 7500 = Rs. 300

$$\therefore \text{Gain\%} = \frac{\text{Total gain}}{\text{C.P.}} \times 100 = \frac{300 \times 100}{7500} = 4\%$$

SE.13 The cost price of 16 articles is equal to the selling price of 12 article. Find the gain or loss percent.

Sol. L.C.M. of 16 and 12 = 48

Let C.P. of 16 articles = Rs. 48

$\therefore$ C.P. of 1 article = 48 ÷ 16 = Rs. 3

S.P. of 12 articles = Rs. 48

$\therefore$ S.P. of 1 article = 48 ÷ 12 = Rs. 4

$\therefore$ Gain on one article = 4 – 3 = Rs. 1

$$\therefore \text{Gain percent} = \frac{\text{Total gain}}{\text{C.P.}} \times 100$$

$$= \frac{1}{3} \times 100 = 33\frac{1}{3}\%$$

SE.14 A man purchases two fans for Rs. 2160. By selling one fan at a profit of 15% and the other at a loss of 9% he neither gains nor loss in the whole transaction. Find the cost price of each fan.

Sol. Let the cost price of first fan be Rs. x. Then,

Cost price of second fan = Rs. (2160 – x)

It is given that in the whole transaction, the man neither gains nor losses.

$\therefore$ Gain on the sale of first fan = Loss in the sale of second fan

$\Rightarrow$ 15% of Rs. x = 9% of Rs. (2160 – x)

$$\Rightarrow \frac{15}{100} \times x = \frac{9}{100} \times (2160 - x)$$

$\Rightarrow 15x = 9(2160 - x)$

$\Rightarrow 5x = 3(2160 - x) \Rightarrow 5x = 6480 - 3x$

$\Rightarrow 5x + 3x = 6480 \Rightarrow 8x = 6480$

$$\Rightarrow x = \frac{6480}{8} \Rightarrow x = 810$$

$\therefore$ C.P. of first fan = Rs. 810

C.P. of second fan = (2160 – x) = (2160 – 810) = Rs. 1350.

SE.15 By selling a towel for Rs. 126.90, a draper loses 6%. For how much should he sell the towel to gain 4% ?

Sol. We have, C.P. of the towel

$$= \left(\frac{100}{100 - \text{Loss\%}} \times \text{S.P.}\right) = \left(\frac{100}{100 - 6} \times 126.90\right)$$

$$= \left(\frac{100}{94} \times 126.90\right) = \text{Rs.}135$$

Now, C.P. of the towel = Rs. 135 and

Required gain% = 4%

$$\therefore \text{S.P.} = \left(\frac{100 + \text{Gain\%}}{100}\right) \times \text{C.P.}$$

$$\Rightarrow \text{S.P.} = \left(\frac{100 + 4}{100} \times 135\right) = \left(\frac{104}{100} \times 135\right)$$

= Rs. 140.40

Hence, the draper should sell the towel for Rs. 140.40

SE.16 A dealer buys 50 chairs for Rs. 50,000 but 20 of them are damaged. He decides to sell each damaged one at three fourths the price of the normal one. What should this price be in order that he may make a profit of 35% on the whole transaction ?

Sol. We have, C.P. of 50 chairs = Rs. 50000

Required profit percent = 35%

$\therefore$ Overall profit = 35% of Rs. 50000

$= \left(\frac{35}{100} \times 50000\right) = \text{Rs.}17500.$

$\therefore$ Desired S.P. of 50 chairs

= (50000 + 17500) = Rs. 67500

$\Rightarrow$ S.P. of one chair $= \left(\frac{67500}{50}\right) = \text{Rs.}1350$

$\Rightarrow$ S.P. of one damaged chair $= \frac{3}{4}$ of 1350

$= \left(\frac{3}{4} \times 1350\right) = \text{Rs.}1012.50$

SE.17 The marked price of a ceiling fan is Rs. 1250 and the shopkeeper allows a discount of 6% on it. Find the selling price of the fan.

Sol. Marked price = Rs. 1250 and discount = 6%

Discount = 6% of M.P. = (6% of Rs. 1250)

$= \left(1250 \times \frac{6}{100}\right) = \text{Rs.}75.$

Selling price = (M.P.) – (discount)

= (1250 – 75) = Rs. 1175.

Hence, the selling price of the fan is Rs. 1175.

SE.18 How much percent above the cost price should a shopkeeper mark his goods so that after allowing a discount of 25% on the marked price, he gains 20% ?

Sol. Let the cost price be Rs. 100.

Gain% = 20%.

$\therefore$ Selling price = Rs. 120.

Let the marked price be Rs. x.

Then, discount = 25% of Rs. x

$= \left(x \times \frac{25}{100}\right) = \text{Rs.}\frac{x}{4}.$

$\therefore$ Selling price = (M.P.) – (discount)

$= \left(x - \frac{x}{4}\right) = \text{Rs.}\frac{3x}{4}.$

$\therefore \frac{3x}{4} = 120 \Rightarrow x = \left(120 \times \frac{4}{3}\right) = 160.$

$\therefore$ Marked price = Rs. 160.

Hence, the marked price is 60% above cost price

SE.19 If 8% VAT is included in the prices, find the original price of

(i) a TV bought for Rs. 14,500.

(ii) a shampoo bottle bought for Rs. 180.

Sol. (i) Let original price be Rs. 100.

Then VAT = 8% of Rs. 100 = Rs. 8

$\therefore$ C.P. = Rs. 108

Here, C.P. = Rs. 108, original price = Rs. 100

then for C.P. = Rs. 1, original price = Rs. $\frac{100}{108}$

So, for C.P. = Rs. 14,500, original price

$= \frac{100}{108} \times 14,500 = \text{Rs.}13425.90$ (approx)

(ii) Let original price be Rs. 100.

Then, VAT = 8% of Rs. 100 = Rs. 8

So, C.P. = Rs. 108

For C.P. = Rs. 108, original price = Rs. 100

Then for C.P. = Rs. 180,

original price $= \frac{100}{108} \times 180$

$= \frac{18000}{108} = \text{Rs.}166.70$ (approx).

EXERCISE-I

ONLY ONE CORRECT TYPE

1. If x is 90% of y, then what percent of x is y ?
(A) 90 (B) 190
(C) 101.1 (D) 111.1

2. A number exceeds 20% of itself by 40. The number is :
(A) 50 (B) 60
(C) 80 (D) 320

3. 12.5% of 192 = 50% of x, then x =
(A) 48 (B) 96
(C) 24 (D) None of these

4. One-third of 1206 is what percent of 134 ?
(A) 3 (B) 30
(C) 300 (D) None of these

5. 8/40 is equivalent to :
(A) 20% (B) 40%
(C) 25% (D) 8%

6. $\sqrt{(3.6\% \text{ of } 40)}$ is equal to :
(A) 2.8 (B) 1.8
(C) 1.2 (D) None of these

7. 75% of a number when added to 75 is equal to the number. The number is :
(A) 150 (B) 200
(C) 225 (D) 300

8. If 4.6% of x is 23, find x :
(A) 400 (B) 200
(C) 100 (D) 500

9. If A's salary is 30% more than B's, then how much percent is B's salary less than A's ?
(A) 30% (B) 25%
(C) $23\frac{1}{13}\%$ (D) $33\frac{1}{3}\%$

10. Bananas are bought at 15 for a rupee and sold at the rate of 9 for a rupee. The gain percent is :
(A) 30% (B) 60%
(C) $66\frac{2}{3}\%$ (D) $33\frac{1}{3}\%$

11. A trader allows a trade discount of 20% and a cash discount of $6\frac{1}{4}\%$ on the marked price of the goods and gets a net gain of 20% on the cost. By how much above the cost should the goods be marked for sale ?
(A) 40% (B) 50%
(C) 60% (D) 70%

12. A shopkeeper marks his goods 20% higher than the cost price and allows a discount of 5%. The percentage of his profit is :
(A) 10% (B) 14%
(C) 15% (D) 20%

13. By selling a vehicle for Rs. 455000, Sumant suffers 25% loss, what was his loss ?
(A) Rs. 115370 (B) Rs. 113570
(C) Rs. 113750 (D) None of these

14. If the difference between selling a shirt at a profit of 10% and 15% is Rs. 10, then the cost price is :
(A) Rs. 110 (B) Rs. 115
(C) Rs. 150 (D) Rs. 200

15. By selling an article for Rs. 100, one gains Rs. 10. Then, the gain percent is :
(A) 9% (B) 10%
(C) $11\frac{1}{9}\%$ (D) None of these

16. A man sold a radio for Rs. 1980 and gained 10%. The radio was bought for :
(A) Rs. 1782 (B) Rs. 1800
(C) Rs. 2178 (D) None of these

17. By selling an article for Rs. 247.50, we get a profit of $12\frac{1}{2}\%$. The cost of the article is :
(A) Rs. 210 (B) Rs. 220
(C) Rs. 224 (D) Rs. 225

18. What is the cost price of an article which is sold at a loss of 25% for Rs. 150 ?
(A) Rs. 125 (B) Rs. 175
(C) Rs. 200 (D) Rs. 225

19. What single discount is equivalent to two successive discounts of 25% and 5% ?
(A) 28% (B) 27%
(C) 28.75% (D) 28.2%

20. By selling a watch for Rs. 1140, a man loses 5%. In order to gain 5%, the watch must be sold for :
(A) Rs. 1311 (B) Rs. 1197
(C) Rs. 1254 (D) Rs. 1260

21. There would be 10% loss if a toy is sold at Rs. 10.80 per piece. At what price should it be sold to earn a profit of 20% ?
(A) Rs. 12 (B) Rs. 12.96
(C) Rs. 14.40 (D) None of these

22. A fruit seller had some apples. He sells 40% and still has 420 apples. Originally, he had :
(A) 588 apples (B) 600 apples
(C) 672 apples (D) 700 apples

23. At what rate of interest will a sum of money amounts to 7/4 of itself in 8 years and 4 months ?
(A) 9% (B) 8%
(C) 7% (D) 10%

24. If every side of a right angled triangle is doubled, then percentage increase in the area of the triangle is
(A) 100% (B) 200%
(C) 300% (D) 400%

25. Joseph bought a scooter for Rs. 8400 and sold it at a gain of 6%. At what price did he sell the scooter ?
(A) Rs. 8204 (B) Rs. 8601
(C) Rs. 8904 (D) Rs. 8900

26. A man buys 10 articles for Rs. 8 and sells them at the rate of Rs. 1.25 per article. His gain is :
(A) 20% (B) 50%
(C) $19\frac{1}{2}\%$ (D) $56\frac{1}{4}\%$

27. If 70% of the students in a school are boys and the number of girls is 504, then number of boys is :
(A) 1176 (B) 1008
(C) 1208 (D) 3024

28. In an examination, 42% students failed in Hindi and 52% failed in English. If 17% failed in both the subjects, the percentage of those who passed in both the subjects, is :
(A) 23% (B) 27%
(C) 34% (D) 40%

29. Selling price of one thing is 3/2 times of its cost price. What will be the percentage of profit ?

(A) $20\frac{1}{2}\%$ (B) $25\frac{1}{4}\%$

(C) $33\frac{1}{3}\%$ (D) 50%

30. 25% of a number is more than 18% of 650 by 19. The number is :

(A) 380.8 (B) 450

(C) 544 (D) None of these

31. A man donated 5% of his income to a charitable organisation and deposited 20% of the remaining in a bank. Now if he has Rs. 1919 left, what is his income ?

(A) Rs. 2558.50 (B) Rs. 2525

(C) Rs. 2500 (D) Rs. 2300

32. If the cost price of 21 copies of a book, are the same as the selling price of 18 copies of the book, then gain percent is :

(A) $14\frac{2}{7}\%$ (B) $16\frac{2}{3}\%$

(C) $33\frac{1}{3}\%$ (D) $23\frac{1}{3}\%$

33. A merchant has 120 kg of rice. He sells a part of it at a profit of 10% and the rest at a profit of 25%. He gains 15% on the whole. Quantity of rice he sold at 25% gain is :

(A) 30 kg (B) 40 kg

(C) 50 kg (D) 55 kg

34. Mr. Raman purchase sewing machine for Rs. 8850 with G.S.T. and sold it to customer for Rs. 10300 with G.S.T. The rate of G.S.T. is 18%. Find the C.G.S.T. and S.G.S.T. paid by Mr. Raman :

(A) Rs. 675 each (B) Rs. 500, Rs. 850

(C) Rs. 1350 each (D) Rs. 400, Rs. 950

35. Rs. 2400 becomes Rs. 2976 in 3 years at a certain rate of simple interest. What sum of money will become Rs. 5016 in 4 years at the same rate of interest ?

(A) Rs. 3820 (B) Rs. 3800

(C) Rs. 3700 (D) Rs. 3905

36. Three radio sets were purchased by a shopkeeper at Rs. 2275 each. He spent Rs. 225 on each for repairing. He sold one of them at a loss of 5%. At what price should the other two be sold, so as to gain 10% on the sale of the three sets ?

(A) Rs. 5870 (B) Rs. 5877

(C) Rs. 5858 (D) Rs. 5875

37. Rahim sells apples to his customers at the cost price itself but uses a weight of 800 g instead of 1 kg weight. Find his profit %

(A) 25% (B) 20%

(C) 15% (D) 30%

38. If a : b = 3 : 5 and a : c = 5 : 7, what (b – c) : (b + c) equal to ?

(A) 4 : 46 (B) 49 : 4

(C) 5 : 48 (D) 48 : 5

39. The price of a house is Rs. 676000. If the price increases every year by 4%, then two years back the price of the house was :

(A) Rs. 600000 (B) Rs. 625000

(C) Rs. 650000 (D) Rs. 575000

40. At what rate percent per annum will Rs. 32000 yield a compound interest of Rs. 5044 in 9 months, interest being compounded quarterly ?

(A) 20% (B) 12%
(C) 15% (D) 18%

PARAGRAPH TYPE

PASSAGE – I : P% of x = $\frac{P}{100} \times x$

1. Find 135% of 80 cm.

(A) 10800 m (B) 10.8 m
(C) 1.08 m (D) 108 m

2. Find a if 8.4% of a is 42

(A) 500 (B) 50
(C) 5 (D) 5000

3. What percent of 24m is 6m ?

(A) 60% (B) 25%
(C) 5% (D) 6.25%

4. Ajit lost 20% of his money after which he was left with Rs. 6400. The amount he had in the beginning with him is

(A) Rs. 8000 (B) Rs. 9000
(C) Rs. 7000 (D) Rs. 1000

PASSAGE – II : If the selling price of an article is greater than the cost price, the difference between selling price and cost price is called profit :

5. C.P. = Rs. 620 and S.P. = Rs. 713, then profit is :

(A) Rs. 73 (B) Rs. 93
(C) Rs. 1333 (D) Rs. 13

6. C.P. = Rs. 345, Profit = Rs. 98, then S.P. =

(A) Rs. 443 (B) Rs. 247
(C) Rs. 434 (D) Rs. 742

7. If S.P. = Rs. 840, Profit = Rs. 62 then C.P. =

(A) Rs. 247 (B) Rs. 877
(C) Rs. 902 (D) Rs. 778

TRUE / FALSE TYPE

1. To convert a fraction into a percent multiply by 100.
2. The additional money paid by the borrower to the lender after a specified period of time is called amount.
3. 40 % of 60 is greater than 25 % of 90.
4. If C.P. = Rs. 400 and loss = 15 %, then SP = Rs. 400 – 15 % of 400.
5. Profit and loss are always calculated on SP.

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from List – I and List – II are given as options (A), (B), (C) and (D) out of which one is correct.

1. Match the quantities given in List I to the respective ratios given in List II

List – I	**List – II**
(P) 36 to 64	(i) 2 : 5
(Q) 40 paise to Rs. 3	(ii) 9 : 16
(R) 24 minutes to an hour	(iii) 1 : 16
(S) 125 ml to 2 litres	(iv) 2 : 15

(A) (P) → (ii), (Q) → (iv), (R) → (i), (S) → (iii)
(B) (P) → (ii), (Q) → (i), (R) → (iv), (S) → (iii)
(C) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)
(D) (P) → (i), (Q) → (iii), (R) → (ii), (S) → (iv)

2. Match the ratios given in List I to the respective percentage given in List II

List – I	List – II
(P) 2 : 5	(i) 28.57%
(Q) 2 : 7	(ii) 17.33%
(R) 13 : 20	(iii) 65%
(S) 13 : 75	(iv) 40%

(A) (P)→(iv), (Q)→(iii), (R)→(i), (S)→(ii)
(B) (P)→(iv), (Q)→(i), (R)→(iii), (S)→(ii)
(C) (P)→(i), (Q)→(ii), (R)→(iii), (S)→(iv)
(D) (P)→(iii), (Q)→(iv), (R)→(i), (S)→(ii)

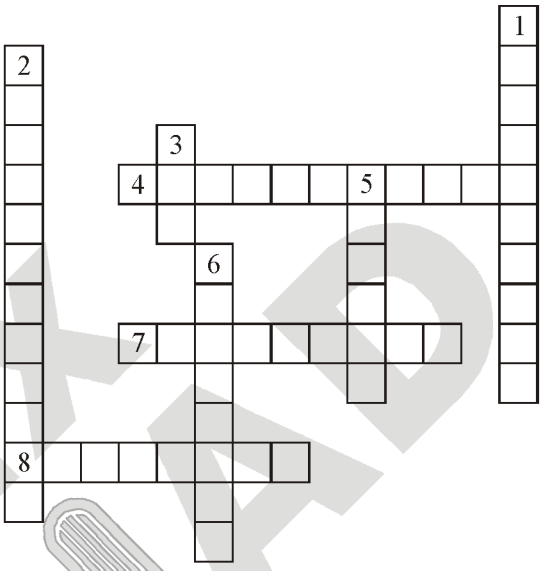
3. Match the statements given in List-I with the corresponding amount and compound interest given in List II

List – I	List – II
(P) Rs. 12600 for 2 years at 10% p.a. compounded annually	(i) Rs. 22781.25 Rs. 6781.25
(Q) Rs. 18000 for 2 years at 10% p.a. compounded annually	(ii) Rs. 21780 Rs. 3780
(R) Rs. 16000 for 3 years at $12\frac{1}{2}$% p.a. compounded annually	(iii) Rs. 15246 Rs. 2646
(S) Rs. 1500 for $2\frac{1}{2}$ years at 10% p.a.compounded annually	(iv) Rs. 1905.75, Rs. 405.75

(A) (P)→(i), (Q)→(ii), (R)→(iii), (S)→(iv)
(B) (P)→(iii), (Q)→(i), (R)→(ii), (S)→(iv)
(C) (P)→(iii), (Q)→(ii), (R)→(i), (S)→(iv)
(D) (P)→(ii), (Q)→(iii), (R)→(i), (S)→(iv)

CROSS WORD PUZZLE

Complete the following word puzzle with the help of clues given below :



Across

4. Shopkeeper fixes _____ over the cost price which is greater than the cost price and then, he allows us discount at that price. [6, 5]
7. Profit and loss are always calculated on the _____. [4, 5]
8. When the interest due at the end of a certain period is added to the principal of the previous period to get the principle of the next period, then such an interest is called _____ interest. [8]

Down

1. If two or more discounts are allowed one after the other, then such discounts are known as _____ discounts. [10]
2. Sales tax is always calculated on the ____. [7, 5]
3. Money is collected by the government from the citizens in the form of _____. [3]
5. When selling price is greater than the cost price, then we will have a _____. [6]
6. Reduction in the cost price of an article by the shopkeeper is called a _____. [8]

EXERCISE-II

VERY SHORT ANSWER TYPE

1. Find the ratio compounded of the three ratios $2a : 3b$, $6ab : 5c^2$, $c : a$.
2. Find the ratio of :
   (i) 35 minutes of 1 hour
   (ii) 8 kg to 400 gms.
3. Find the ratio of the following :
   (i) 5m to 10 km
   (ii) 50 paise to Rs. 5
   (iii) Speed of cycle which is 15 km per hour to the speed of the scooter which is 30 km per hour.
4. A certain company has 80 employees who are engineers. In this company engineers constitute 40% of its work force. How many people are employed in the company ?
5. During one year, the population of a town increased by 5% and during the next year, the population decreased by 5%. If the total population is 9975 at the end of the second year, then what was the population size in the beginning of the first year ?
6. Express 75 paise as a percent of Rs. 8
7. The ratio of bus and train fares from Calcutta to a certain place is 3 : 4. If the train fare increases by 20% and bus fare by 10% then what will be new ratio of bus and train fares ?
8. Rakesh purchased a car for Rs. 135000 and soon after he sold it for Rs. 142500. Find his gain percent.
9. A dishonest dealer professes to sell his goods at cost price, but he uses a weight of 960 grams for 1 kg. Find gain percent
10. Find the buying price of a towel when 5% sales tax is added on the purchase of it at Rs. 50.

SHORT ANSWER TYPE

1. Malvika gets 98 marks in her exams. This amounts to 56% of the total marks. What are the maximum marks ?
2. A nursery has 5000 plants. 5% of the plants are roses and 1% are mango plants. What is the total number of other plants ?
3. The price of a Maruti car rises by 30%, while the sales of the car goes down by 20%. What is the percentage change in the total revenue ?
4. By selling 144 pens, Jyoti lost the S.P. of 6 pens. Find her loss percent.
5. Mohit bought a CD for Rs. 750 and sold it for Rs. 875, find his gain percent.
6. Ishaan purchased an old scooter for Rs. 12000 and spent Rs. 2850 on its overhauling. Then, he sold it to his friend Sohan for Rs. 13860. How much percent did he gain or lose ?
7. 200 kg of sugar was purchased at the rate of Rs. 15 per kg and sold at a profit of 5%. Compute the profit and the selling price per kg.
8. Calculate amount and compound interest on Rs. 9600 for 3 years at $12\frac{1}{2}$% p.a. compounded annually.
9. Simple interest on a sum of money for 2 years at $6\frac{1}{2}$% per annum is Rs. 5200. What will be the compound interest on that sum at the same rate for the same time period ?

10. Find the compound interest on Rs. 160000 for 2 years at 10% per annum when compounded semi-annually.

LONG ANSWER TYPE

1. Ruchi's weight is 25% that of Sneha and 40% that of Tameena. What percentage of Tameena's weight is Sneha's weight ?
2. A man buys a plot of agricultural land for Rs. 300000. He sells one-third at a loss of 20% and two-fifths at a gain of 25%. At what price must he sell the remaining land so as to make an over-all profit of 10% ?
3. A man sold two articles at Rs. 25920 each. These were sold at 8% gain and 4% loss respectively. Find the gain or loss percent in the whole transaction.
4. A man bought two T.V. sets for Rs. 42500. He sold one at a loss of 10% and other at a profit of 10%. If the selling price of each T.V. set is same, determine the C.P. of each set.
5. At what rate, will a sum of Rs. 1000 amount to Rs. 1102.50 in 6 months at compound interest, if the interest is compounded quarterly ?

NUMERICAL PROBLEMS

1. 75% of 20 students are not good at English. How many of them are good in English ?
2. If 50% of $(x - y) = 30\%$ of $(x + y)$, then $x = ky$, find the value of 10k.
3. A shopkeeper buys 50 articles for 2000 and sells them for a profit of 10%. Find the selling price of one article.
4. The value of a T.V. depreciates every year by 25%. The value after a year if its present value is Rs. 45,000 will be 3375×10^n. Find n.
5. A person sells an article for Rs. 550, gaining 1/10 of its C.P. If gain % is 10^n. Find n.
6. Shobha's Mathematics test had 75 problems i.e., 10 arithmetic, 30 algebra and 35 geometry problems. Although she answered 70% of the arithmetic, 40% of the algebra and 60 % of the geometry problems correctly, she did not pass the test because in algebra she got less than 60% of the problems right. How many more questions she would have needed to answer correctly to score a 60% passing grade ?
7. If $n\frac{n}{3}$ kg of pure salt must be added to 30 kg of 2% solution of salt and water to increase it to a 10% solution, then find n.
8. Tanuj sold a sofa set for Rs. 8800, thereby losing 12%. Had he sold it for Rs. 9600, what percent profit or loss would he have made ?
9. In what time will Rs. 1000 amount to Rs. 1331 at 10% per annum compounded annually ?
10. The present population of a town is 16000. If it increases at rate of 5% p.a., what will be the population after 2 years ?

ANALYTICAL PROBLEMS & BRAIN TEASER

1. A sum of money fetches a certain interest when it is invested for 4 years at the rate of 8 % per annum. If it is invested for 6 years, then the interest increases by Rs. 640. Find the sum of money.

   (A) Rs. 4050 (B) Rs. 2000

   (C) Rs. 2050 (D) Rs. 4000

2. The compound interest earned by Suresh on a certain amount at the end of two years at the rate of 8 % per annum was Rs. 1414.40. What was the total amount that Suresh got back at the end of two years ?
(A) Rs. 9414.40
(B) Rs. 9914.40
(C) Rs. 9014.40
(D) Rs. 8914.40

3. Amar wrote exams in four subject – Maths, Science, Hindi and Social Studies. The ratio of marks he got in these exams was 2 : 3 : 4 : 5. He got an aggregate of 70 % in these exams. Each exam had the same maximum marks. In how many of these subjects did he get more than 50 % ?
(A) 1 (B) 2
(C) 3 (D) 4

4. If 540 is 10 % of y and z % of y is 16200, then find the values of y and z respectively.
(A) 5400, 20 (B) 5400, 300
(C) 300, 5400 (D) 20, 5800

5. Two successive discounts of x % and y % on a western gown is same as the single discount of :
(A) $\left(x+y+\frac{xy}{100}\right)\%$ (B) $\left(x-y-\frac{xy}{100}\right)\%$
(C) $\left(x+y-\frac{xy}{100}\right)\%$ (D) $\left(y-x-\frac{xy}{100}\right)\%$

6. If A : B = 5 : 6 and B : C = 7 : 8, then by approximately what percent is C more than A ?
(A) 36 % (B) 40 %
(C) 37.14 % (D) 48.23 %

7. Sonika spent Rs. 45760 on the interior decoration for her home, Rs. 27896 on buying air conditioner and the remaining 28 % of the total amount she had as cash with her. What was the total amount?
(A) Rs. 98540 (B) Rs. 102300
(C) Rs. 134560 (D) Rs. 97500

8. Simple interest on a sum of money for 1 year at 12 % per annum is Rs. 1200. What will be the compound interest when compounded half-yearly on that sum at the same rate for the same period?
(A) Rs. 1236 (B) Rs. 1326
(C) Rs. 11236 (D) Rs. 13260

9. A man borrowed Rs. 4000 at 10 % per annum at compound interest. At the end of each year he has repaid Rs. 1000. The amount of money he still incurs after the third year is :
(A) Rs. 2740 (B) Rs. 2104
(C) Rs. 2014 (D) Rs. 3400

10. A shopkeeper fixes the marked price of a pair of shoes 45 % above its cost price. What is the percentage of discount allowed to gain 16 % ?
(A) 30 % (B) 25 %
(C) 15 % (D) 20 %

11. A man sells two articles each at Rs. 198. He makes a profit of 10% on one article and a loss of 10% on the other. Net profit or loss of the person is :
(A) 2% profit (B) 2% loss
(C) 1% profit (D) 1% loss

12. The present population of a city is 8000. If it increases by 10 % during the first year and by 20% during the second year, then population after two years will be :

(A) 12400 (B) 14400

(C) 10560 (D) None of these

13. A businessman fixed the selling price of an article after increasing the cost price by 40 %. Then he allowed his customers a discount of 20 % and gained Rs. 48. The cost price of the article is :

(A) Rs. 200 (B) Rs. 248

(C) Rs. 400 (D) Rs. 448

14. The compound interest on Rs. 1000 in 2 years at 4 % per annum, the interest being compounded half yearly, is :

(A) Rs. 6360.80 (B) Rs. 82.43

(C) Rs. 912.86 (D) Rs. 828.82

15. The difference between the compound interest (compounded annually) and simple interest on a certain sum at 10 % per annum for 2 years is Rs. 631. Find the sum.

(A) Rs. 80100 (B) Rs. 63100

(C) Rs. 60000 (D) Rs. 73100

EXERCISE-III

PREVIOUS YEAR QUESTIONS (NTSE)

1. Anil owns a plot worth Rs. 10000. He sells it to Mukesh at a profit of 10 %. After sometimes, Mukesh sells it back to Anil at a loss of 10 %, then Anil
(NSTSE – 2010)
(A) Loses Rs. 100 (B) Losses Rs. 900
(C) Gains Rs. 100 (D) Gains Rs. 1100

2. The difference between a discount of 40 % and two successive discounts of 36 % and 4 % for Rs. 10000 is :
(NSTSE – 2010)
(A) Rs. 0 (B) Rs. 144
(C) Rs. 256 (D) Rs. 400

3. Sushmita used 200 square tiles to tile her kitchen floor. Of these, 66 tiles were green. What percent of the tiles were green ?
(IMO – 2010)
(A) 134 % (B) 66 %
(C) 33 % (D) 3.03 %

4. Shreya bought an article an sold it for 125 % per cent of its cost price. What was the cost price of the article, it Shreya sold it for Rs. 30,750 ?
(IMO – 2011)
(A) Rs. 24,600 (B) Rs. 25,640
(C) Rs. 24,250 (D) Rs. 23,200

5. If 35 % of a number is 12 less than 50 % of that number, then the number is _____.
(IMO – 2011)
(A) 40 (B) 50
(C) 60 (D) 80

6. Out of a number of electronic items, Rohit purchases 60 % coloured T.Vs. 5 % of these are found to be defective. The percentage of defective T.Vs. in all is _____.
(IMO – 2011)
(A) 3 % (B) 6 %
(C) 12 % (D) Can't be determined

7. Rajesh bought an arm chair for Rs. 2200 and sold it for Rs. 2420. Find his profit percentage ?
(NSTSE – 2012)
(A) 5 % (B) 10 %
(C) 12 % (D) 15 %

8. Latika owns a mobile worth Rs.10000. She sells it to Priya at a profit of 10 % based on the worth of the mobile. Priya sells the mobile back to Latika at a loss of 10 %. In this transaction Latika gets ____.
(IMO – 2012)
(A) No profit Rs.110 loss
(B) Profit of Rs.1000
(C) Profit of Rs.1100
(D) Loss of Rs.1100

9. If 5 % more is gained by selling an article for Rs. 350 than by selling it for Rs. 340. What is the cost of the article
(NSTSE – 2013)
(A) Rs. 50 (B) Rs. 160
(C) Rs. 200 (D) Rs. 225

10. A candidate get 20 % marks and fails by 10 marks but another candidate who gets 42 % marks gets 12 marks more than the passing marks. Find the maximum marks.
(Aryabhatta – 2013)
(A) 100 (B) 200
(C) 50 (D) 150

11. What will be the difference between simple and compound interest at the rate of 10 % per annum on a sum of Rs. 1000 after 4 years ?
(NSTSE – 2009)
(A) Rs. 64.10 (B) Rs. 40.40
(C) Rs. 32.10 (D) Rs. 31

12. The compound interest on a certain sum of money for 2 years at 5 % per annum is Rs. 102.50. Find the compound interest on the same for the same period at 4 % per annum.
(Aryabhatta – 2011)
(A) Rs. 81.60 (B) Rs. 85.70
(C) Rs. 82.50 (D) Rs. 79.60

13. A sum of money invested at compound interest amounts to Rs. 800 in 3 years and to Rs. 840 in 4 years. The rate of interest per annum is _____.
(IMO – 2011)
(A) $2\frac{1}{2}\%$ (B) 4 %
(C) 5 % (D) $6\frac{2}{3}\%$

14. What is the present worth of Rs. 169 due in 2 years at 4 % per annum compound interest ?
(NSTSE – 2013)
(A) Rs. 150.50 (B) Rs. 154.75
(C) Rs. 156.25 (D) Rs. 158

15. A man borrowed Rs. 3125 on C.I. which amounted to Rs. 4500 in 2 years. What was the rate of interest per annum ?
(NSTSE – 2014)
(A) 30 % (B) 25 %
(C) 20 % (D) 15 %

Answer Key

EXERCISE I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	A	A	C	A	C	D	D	C	C	C	B	D	D	C
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
B	B	C	C	D	C	D	A	C	C	D	A	A	D	C
31	32	33	34	35	36	37	38	39	40					
B	B	B	C	B	D	A	A	B	A					

PARAGRAPH

1. C **2.** A **3.** B **4.** A **5.** B **6.** A **7.** D

TRUE / FALSE

1. True **2.** False **3.** True **4.** True **5.** False

MATCH THE COLUMN

1. A **2.** B **3.** C

CROSSWORD PUZZLE

Across

4. Marked price **7.** Cost price **8.** Compound

Down

1. Successive **2.** Selling price **3.** Tax **5.** Profit **6.** Discount

EXERCISE II

VERY SHORT ANSWER TYPE

1. $\frac{4a}{5c}$ **2.** (i) 7 : 12, (ii) 20 : 1 **3.** (i) 1 : 2000, (ii) 1 : 10, (iii) 1 : 2 **4.** 200

5. 10000 **6.** $\frac{75}{8}\%$ **7.** 11 : 16 **8.** $\frac{50}{9}\%$ **9.** $4\frac{1}{6}\%$ **10.** Rs. 52.5

SHORT ANSWER TYPE

1. 175 **2.** 4700 **3.** 4% **4.** 4% **5.** $16\frac{2}{3}\%$ **6.** Loss = $6\frac{2}{3}\%$

7. Rs. 15.75 **8.** Rs. 4068.75 **9.** Rs. 5369 **10.** Rs. 34481

LONG ANSWER TYPE

1. 160% **2.** Rs. 100000 **3.** Gain% = $1\frac{11}{17}\%$ **4.** Rs. 19125 **5.** 20% per annum

NUMERICAL PROBLEMS

1. 5 **2.** 40 **3.** 44 **4.** 1 **5.** 1 **6.** 5 **7.** 2
8. 4 **9.** 3 **10.** 17640

ANALYTICAL PROBLEMS & BRAIN TEASER

1. D **2.** B **3.** C **4.** B **5.** C **6.** C **7.** B
8. A **9.** C **10.** D **11.** D **12.** C **13.** C **14.** B
15. B

EXERCISE III

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	B	C	A	D	A	B	C	C	A	A	A	C	C	C

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : COMPARING QUANTITIES)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

MATRIX

JEE Division | NEET Division

Piprali Road, Sikar, Rajasthan 332001 | 01572-241911, 01572-243911

MATRIX HIGH SCHOOL

Pre-foundation & Schooling Division

Piprali Road, Sikar, (Raj.) 332001 | Bikaner Bypass Road, Near Gokulpura Village, Sikar (Raj.) 332021 | 01572-242911

Scan QR to Register

Visit us www.mof.matrixedu.in

@matrixsikar @matrix_sikar @MatrixSikar /c/matrixacademy

COMPUTER SCIENCE DISCUSSION 1.0#