केन्द्रीय विद्यालय संगठन, नई दिल्ली
KENDRIYA VIDYALAYA SANGATHAN, NEW DELHI

तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

अध्ययन सामग्री / STUDY MATERIAL

कक्षा / Class : XI

विषय/ Subject : गणित / MATHS

संकलन द्वारा : राजीव रंजन , प्रशिक्षण सहायक, गणित

Compiled by: Rajiv Ranjan, TA, Maths

स्थल : शिक्षा एवं प्रशिक्षण का आंचलिक संस्थान , चंडीगढ़

Venue- Zonal Institute of Education and Research, Chandigarh

INDEX

COURSE STRUCTURE
CLASS –XI (2023-24)

Units	Unit Name	Marks
I	SETS AND FUNCTIONS	23
II	ALGEBRA	25
III	COORDINATE GEOMETRY	12
IV	CALCULUS	08
V	STATISTICS & PROBABILTY	12
	Total	80

UNIT-I: SETS AND FUNCTIONS

1. **Sets**

Sets and their representations, Empty set, Finite and Infinite sets, Equal sets, Subsets, Subsets of a set of real numbers especially intervals (with notations). Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set. Properties of Complement.

2. **Relations & Functions**

Ordered pairs. Cartesian product of sets. Number of elements in the Cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto R × R × R).Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special type of relation. Pictorial representation of a function, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs. Sum, difference, product and quotients of functions.

3. **Trigonometric Functions**

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity
$\sin^2 x + \cos^2 x = 1$, for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing sin (x ± y) and cos (x ± y) in terms of sinx, siny, cosx & cosy and their simple applications. Deducing identities like the following:

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}, \quad \cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$$

$$\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha + \beta)\sin\frac{1}{2}(\alpha - \beta)$$

Identities related to sin2x, cos2x, tan2 x, sin3x, cos3x and tan3x.

UNIT-II: ALGEBRA

1. **Complex Numbers and Quadratic Equations**

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve some of the quadratic equations. Algebraic properties of complex numbers. Argand plane

2. **Linear Inequalities**
Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line.

3. **Permutations and Combinations**
Fundamental principle of counting. Factorial n. (n!) Permutations and combinations, derivation of Formulae for nPr and nCr and their connections, simple applications.

4. **Binomial Theorem**
Historical perspective, statement and proof of the binomial theorem for positive integral indices. Pascal’s triangle, simple applications.

5. **Sequence and Series**
Sequence and Series. Arithmetic Mean (A.M.) Geometric Progression (G.P.), general term of a G.P., sum of n terms of a G.P., infinite G.P. and its sum, geometric mean (G.M.), relation between A.M. and G.M.

Unit-III: Coordinate Geometry

1. Straight Lines
Brief recall of two dimensional geometry from earlier classes. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axis, point -slope form, slope-intercept form, two-point form, intercept form, Distance of a point from a line.

2. Conic Sections
Sections of a cone: circles, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.

3. Introduction to Three-dimensional Geometry
Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points.

Unit-IV: Calculus

1. Limits and Derivatives
Derivative introduced as rate of change both as that of distance function and geometrically. Intuitive idea of limit. Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions. Definition of derivative relate it to scope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

Unit-V Statistics and Probability

1. Statistics
Measures of Dispersion: Range, Mean deviation, variance and standard deviation of ungrouped/grouped data.

2. Probability
Events; occurrence of events, ‘not’, ‘and’ and ‘or’ events, exhaustive events, mutually exclusive events, Axiomatic (set theoretic) probability, connections with other theories of earlier classes. Probability of an event, probability of ‘not’, ‘and’ and ‘or’ events.

MATHEMATICS
QUESTION PAPER DESIGN
CLASS – XI (2023-24)

Time: 3 Hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	**44**	**55**
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	**20**	**25**
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	**16**	**20**
4	**Total**	**80**	**100**

1. No chapter wise weightage. Care to be taken to cover all the chapters
2. Suitable internal variations may be made for generating various templates keeping the overall weightage to different form of questions and typology of questions same.

Choice(s):

There will be no overall choice in the question paper.
However, 33% internal choices will be given in all the sections

INTERNAL ASSESSMENT	**20 MARKS**
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities	10 Marks

SETS
CONCEPTS AND RESULTS

**** Set :** a set is a well-defined collection of objects.

If a is an element of a set A, we say that " a belongs to A" the Greek symbol $\in$(epsilon) is used to denote the phrase 'belongs to'. Thus, we write a $\in$A. If 'b' is not an element of a set A, we write b $\notin$A and read "b does not belong to A".

There are two methods of representing a set :

(i) Roster or tabular form (ii) Set-builder form.

In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within brackets { }. For example, the set of all even positive integers less than 7 is described in roster form as {2, 4, 6}.

In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set. For example, in the set {a, e, i, o, u}, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property. Denoting this set by V, we write V = {x : x is a vowel in English alphabet}

**** Empty Set :** A set which does not contain any element is called the empty set or the null set or the void set. The empty set is denoted by the symbol φ or { }.

**** Finite and Infinite Sets :** A set which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

**** Equal Sets :** Two sets A and B are said to be equal if they have exactly the same elements and we write

A = B. Otherwise, the sets are said to be unequal and we write A $\neq$ B.

**** Subsets :** A set A is said to be a subset of a set B if every element of A is also an element of B.

In other words, A$\subset$B if whenever a $\in$A, then a $\in$B. Thus A $\subset$B if a $\in$A $\Rightarrow$a $\in$B

If A is not a subset of B, we write A $\not\subset$B.

** Every set A is a subset of itself, i.e., A $\subset$A.

** φ is a subset of every set**.**

** If A$\subset$B and A $\neq$ B , then A is called a proper subset of B and B is called superset of A.

** If a set A has only one element, we call it a singleton set. Thus, { a } is a singleton set.

** Closed Interval : $[a , b] = \{x : a \le x \le b\}$

** Open Interval : $(a , b) = \{ x : a < x < b\}$

** Closed open Interval : $[a , b) = \{x : a \le x < b\}$

** Open closed Interval : $(a , b] = \{ x : a < x \le b \}$

**** Power Set :** The collection of all subsets of a set A is called the power set of A. It is denoted by P(A)

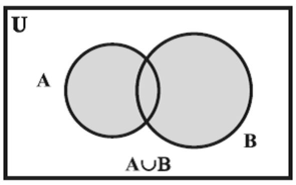
If A is a set with n(A) = m, then it can be shown that n [P(A)] = 2^m.

** **Universal Set :** The largest set under consideration is called Universal set.

** **Union of sets :** The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both). In symbols, we write.

$A \cup B = \{ x : x \in A \text{ or } x \in B \}$.

$x \in A \cup B \Rightarrow x \in A \text{ or } x \in B$



$x \notin A \cup B \Rightarrow x \notin A \text{ and } x \notin B$

** **Some Properties of the Operation of Union**

(i) A $\cup$ B = B $\cup$ A (Commutative law)

(ii) (A $\cup$ B) $\cup$ C = A $\cup$ (B $\cup$ C) (Associative law)

(iii) A $\cup$ φ = A (Law of identity element, φ is the identity of $\cup$)

(iv) $A \cup A = A$ (Idempotent law)

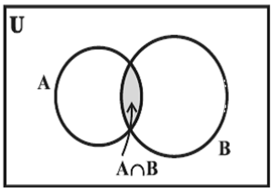
(v) $U \cup A = U$ (Law of U)

** **Intersection of sets :** The intersection of two sets A and B is the set of all those elements which belong to bothA and B.

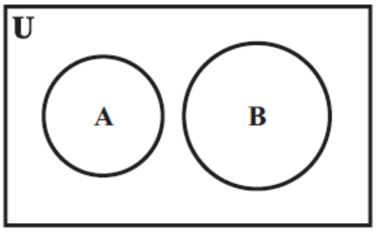
Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$

$x \in A \cap B \Rightarrow x \in A$ and $x \in B$

$x \notin A \cap B \Rightarrow x \notin A$ or $x \notin B$



** **Disjoint sets :** If A and B are two sets such that $A \cap B = \varphi$, then A and B are called disjoint sets.



** **Some Properties of Operation of Intersection**

(i) $A \cap B = B \cap A$ (Commutative law).

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law).

(iii) $\varphi \cap A = \varphi$, $U \cap A = A$ (Law of φ and U).

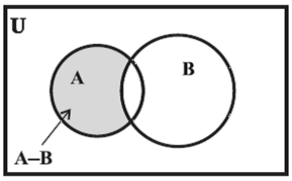
(iv) $A \cap A = A$ (Idempotent law)

(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law) i. e., ∩ distributes over ∪

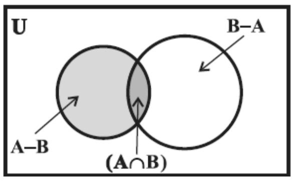
** **Difference of sets :** The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

Symbolically, we write A – B and read as " A minus B".

$A - B = \{ x : x \in A \text{ and } x \notin B \}$.

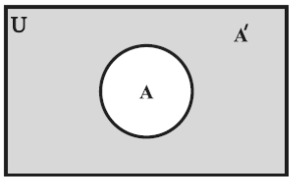


* The sets A – B, A ∩B and B – A are mutually disjoint sets, i.e., the intersection of any of these two sets is the null set.



** **Complement of a Set :** Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write A′ to denote the complement of A with respect to U.

Thus, $A' = \{x : x \in U \text{ and } x \notin A \}$. Obviously $A' = U - A$



** **Some Properties of Complement Sets**

1. Complement laws: (i) $A \cup A' = U$ (ii) $A \cap A' = \varphi$

2. De Morgan's law: (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

3. Law of double complementation : $(A')' = A$

4. Laws of empty set and universal set $\varphi' = U$ and $U' = \varphi$.

** **Practical Problems on Union and Intersection of Two Sets :**

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B)$, if $A \cap B = \varphi$.

(iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.

SOME ILLUTRATIONS/EXAMPLES (WITH SOLUTION)

(A) MCQ.

1.The number of elements in the Power set P(S) of the set S = {1, 2,3} is:
A. 4 B. 8 C. 2 D. None of these

Answer: B. 8
Explanation: Number of elements in the set S = 3
Number of elements in the power set of set S = {1,2,3} = 2^3= 8

2. Empty set is a _______.
A. Infinite set B. Finite set C. Unknown set D. Universal set

Answer: B. Finite set
Explanation: The cardinality of the empty set is zero, since it has no elements. Hence, the size of the empty set is zero.

3. Order of the power set P(A) of a set A of order n is equal to:
A. n B. 2n C. 2^n D. n^2

Answer: C. 2^n
Explanation: The cardinality of the power set is equal to 2^n, where n is the number of elements in a given set.

4. Write X = {1, 4, 9, 16, 25,…} in set builder form.
A. X = {x: x is a set of prime numbers} B. X = {x: x is a set of whole numbers}
C. X = {x: x is a set of natural numbers} D. X = {x: x is a set of square numbers}

Answer: D. X = {x: x is a set of square numbers}
Explanation: Given,
X = {1, 4, 9, 16, 25,…}
X = {1^2, 2^2, 3^2, 4^2, 5^2, …}
Therefore,
X = {x: x is a set of square numbers}

(b) SHORT ANSWER TYPE QUESTION

1.Q. 4: Let X = {1, 2, 3, 4, 5, 6}. If n represent any member of X, express the following assets:
(i) n + 5 = 8
(ii) n is greater than 4
Solution:
(i) Let B = {x | x ∈ X and x + 5 = 8}
Here, B = {3} as x = 3 ∈ X and 3 + 5 = 8 and there is no other element belonging to X such that x + 5 = 8.
(ii) Let C = {x | x ∈ X, x > 4}
Therefore, C = {5, 6}

2. Write the following sets in the roster form.
(i) A = {x | x is a positive integer less than 10 and 2^x – 1 is an odd number}
(ii) C = {x : x^2 + 7x – 8 = 0, x ∈ R}
Solution:

(i) $2^x - 1$ is always an odd number for all positive integral values of x since 2^x is an even number.
In particular, $2^x - 1$ is an odd number for x = 1, 2, … , 9.
Therefore, A = {1, 2, 3, 4, 5, 6, 7, 8, 9}
(ii) $x^2 + 7x - 8 = 0$
(x + 8) (x – 1) = 0
x = – 8 or x = 1
Therefore, C = {– 8, 1}

3. Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}.
Find A′, B′, A′ ∩ B′, A ∪ B and hence show that (A∪B)′ = A′∩ B′.

Solution:
Given,
U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}
A′ = {1, 4, 5, 6}
B′ = {1, 2, 6}.
Hence, A′ ∩ B′ = {1, 6}
Also, A ∪ B = {2, 3, 4, 5}
(A ∪ B)′ = {1, 6}
Therefore, (A∪ B)′ = {1, 6} = A′ ∩ B′

(d) <u>LONG ANSWER TYPE QUESTION</u>

1. Use the properties of sets to prove that for all the sets A and B, A – (A ∩ B) = A – B
Solution:
A – (A ∩ B) = A ∩ (A ∩ B)′ (since A – B = A ∩ B′)
= A ∩ (A′ ∪ B′) [by De Morgan’s law]
= (A∩A′) ∪ (A∩ B′) [by distributive law]
= φ ∪ (A ∩ B′)
= A ∩ B′ = A – B
Hence, proved that A – (A ∩ B) = A – B.

2. Two finite sets have m and n elements. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Find the value of m and n.

Solution:
No. of elements in A=m

No. of elements in B=n

According the question

No. of subsets of set A – No. of subsets off set B = 56

$2^m - 2^n = 56$

$2^n(2^{m-n} - 1) = 56$ **because m> n**

$2^n(2^{m-n} - 1) = 8x7$

$2^n(2^{m-n} - 1) = 2^3x7$

By comparing to both sides we get

$2^n = 2^3$, $2^{m-n} - 1 = 7$

n = 3,$2^{m-n} = 7 + 1$

$2^{m-n} = 8$

m -n = 3

m = n+3

m=6 because n=3

QUESTION FOR PRACTICE

(A)MCQ

1 : How many elements are there in the complement of set A?

A. 0 B. 1 C. All the elements of A D. None of these

2 : Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, P = {1, 2, 5}, Q = {6, 7}. Then P ∩ Q' is :

A. P B. Q C. Q' D. None

3. **The cardinality of the power set of {x: x∈N, x≤10} is ______.**

A. 1024 B. 1023 C. 2048 D. 2043

4. **If A, B and C are any three sets, then A × (B ∪ C) is equal to:**

A. (A × B) ∪ (A × C)
C. (A × B) ∩ (A × C)
B. (A ∪ B) × (A ∪ C)
D. None of the above

Ans:1-D ,2-A , 3-A , 4-A

(B) ASSERTION AND REASON

1. **DIRECTION**: In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as

(a) Both assertion and reason are true and reason is the correct explanation of assertion.
(b) Both assertion and reason are true but reason is not the correct explanation of assertion.
(c) Assertion is true but reason is false.
(d) Assertion is false but reason is true

1. Assertion (A) 'The collection of all natural numbers less than 100' is a set.
Reason (R) :A set is a well-defined collection of the distinct objects.

Ans: 1-A

(C)SHORT ANSWER TYPE

1. Given that N = {1, 2, 3, …,100}, then write the subset A of N, whose elements are odd numbers.
2.**Write the subset B of N, whose elements are represented by x + 2, where x ∈ N.**
3. **Write the following sets in roster form: A = {x: x is an integer and –3 ≤ x < 7}**
4. **B = {x: x is a natural number less than 6}**
5. Write (–5,9] in set-builder form.

6. List all subsets of the set {-1,0,1}
7.Taking the set of natural numbers as the universal set, write down the complements of the set: {x: x is an even natural number}
8. Write the set A = {x: $x^2 + 7x - 8 = 0$, x ∈ R} in roster form.
9. Prove that for all sets A and B, $A - (A - B) = A \cap B$

10. Express the following intervals as a set in set builder form:
(i) (2, 7) (ii) [-4, 4)

ANS:1-A = {1, 3, 5, 7, …, 99} , 2. B = {3, 4, 5, 6, …, 100}3. A = {-3, -2, -1, 0, 1, 2, 3, 4, 5, 6}
4. B = {1, 2, 3, 4, 5} 5. {x: x∈ R, $-5 < x \leq 9$}6. {∅, {1}, {0}, {-1},{-1,1},{-1,0},{-1,1}, {-1, 1, 0}}7. {1, 3, 5, 7.......} 8. {-8, 1}.
9. **(A − B) = Set of those elements which are in A but not in B**
A - (A − B) = Set of those elements which are in A but not in (A − B) = Elements of set B = A ∩ B
10. (i) {x: x∈ R, $2 < x < 9$} (ii){x: x∈ R, $-4 \leq x < 9$}

CLASS: XI
Session: 2023-24
Mathematics (Code-041)
Class Test I
Time Allowed: 45 mints Maximum Marks: 20
General Instructions:

1. **This question paper contains Three sections – A, B and C. Each part is compulsory.**
2. **Section - A has 5 questions of 1 marks each.**
3. **Section – B has 5 questions of 2 marks each.**
4. **Section - C has a question of 5 marks .**

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SECTION – A

Q 1. If A={1,2,3,4,5}, then the number of proper subsets of A is

(a) 120 (b) 30 (c)31 (d)32

Q 2. For any two sets A and B, A∩ (A ∪ B) =

(a)A (b) B (c) ∅ (d) none of these

Q 3. Let A and B be two sets such that n(A)= 16, n(B)= 14, n(A∪B)=25 then n(A∩B) is equal to

(a) 30 (b)50(c)5 (d)none of these

Q 4. For two sets A∪B=A iff

(a)B⊂A (b)A⊂B (c)A ≠B (d)A = B

Q 5. Let A and B be two sets in the same universal set. Then A-B =

(a) A∩B (b)A'∩B (c) A∩B' (d)none of these

SECTION – B

Q 6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60 degree, What is A^i

Q7. Write the given set using set builder method:{1,4,9,16,25,36,49,64,81,100,121,144}

Q8. Write the compliment of the given set A= {x: x is positive multiple of 3, x∈ N}.

Q 9. Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second set. Find the values of m and n.

Q 10. Let U={1,2,3,4,5,6} ,A={2,3} and B={3,4,5} .Show that $(A \cup B)' = A' \cap B'$

SECTION – C

Q 11. Let P be the set of prime numbers and let S={t:$2^t - 1$ is a prime }. Prove that S ⊂ P.

CLASS: XI
Session: 2023-24
Mathematics (Code-041)
Class Test II
Time Allowed: 60 mints Maximum Marks: 30

General Instructions:

1. **This question paper contains Three sections – A, B and C. Each part is compulsory.**
2. **Section – A has 5 questions of 1 mark each.**
3. **Section – B has 5 questions of 2 marks each.**
4. **Section – C has one question of 5 marks.**
5. **Section – D has 2 questions of 5 marks each.**

SECTION – A

Q (1).Set of even prime numbers is

(a) Null set (b) a singleton set (c) a finite set (d) an infinite set

Q (2). The set of circles passing through the origin (0,0)

(a)Finite set (b) infinite set (c) Null set (d) none of these

Q (3).The set A∪ A′

(a) A (b) A’ (c)∅(d) U

Q (4).Two sets A, B are disjoint if :

(a) $A\cup B = \emptyset$ (b) $A\cap B \neq \emptyset$ (c)$A\cap B = \emptyset$ (d)A-B=A

(5).Set A and B have 3 and 6 elements respectively. What can be the minimum number of elements in A∪B?

(a) 3 (b) 6 (c) 9 (d) 18

SECTION – B

Q (6).Write the set { x : x is positive integer and $x^2 < 40$} in the roaster form.

Q (7).Is the set A={x:x^3=8 and 2x+3=0} empty? Justify.

Q (8).Write the following intervals in set- builder form: (i) (-3, 0) (ii) (6, 12].

Q (9).Let V={ a,e,i,o,u } and B={a,i,k,u } . Find V-B and B-V.

Q (10).Show that $(A\cup B) - (A \cap B) = (A - B) \cup (B - A)$

SECTION – C

Q (11) A class teacher Mamta Sharma of class XI write three sets A, B and C are such that A = {1, 3, 5,7,9},B={2,4,6,8} and C={2,3,5,7,11}.

Answer the following questions which are based on above sets.

(i) Find A∩B.

(a) {3,5,7} (b)φ (c) {1,5,7} (d) {2,5,7}

(ii) Find A∩C

(a){3,5,7} (b)φ (c) {1,5,7} (d) {3,4,7}

(iii) Which of the following is correct for two sets A and B to be disjoint?
(a) $A \cap B = \emptyset$ (b) $A \cap B \neq \emptyset$ (c) $A \cup B = \emptyset$ (d) $A \cup B \neq \emptyset$

(iv) Which of the following is correct for two sets A and to be intersecting?
(a) $A \cap C = \emptyset$ (b) $A \cap C \neq \emptyset$ (c) $A \cup C = \emptyset$ (d) $A \cup C \neq \emptyset$

(v) Find $A \cup B$.
(a){1,2,3,4,5,6,7,8,9} (b){1,2,3,4,5,6,7,8} (c){1,2,3,4,5,6,8,9} (d){1,2,3,4,5,6,7,9}

SECTION – D

Q (12). For all sets A, B and C. Is $(A - B) \cap (C - B) = (A \cap C) - B$? Justify your answer.

Q (13). A,B and C are subset of universal set U. If A ={2,4,6,8,12,20} , B={3,6,9,12,15 }, C={ 5,10,15,20} and U is the set of all whole numbers. Draw a Venn diagram showing the relation of U, A,B and C.

…………………………………………………….

RELATIONS & FUNCTIONS
CONCEPTS AND RESULTS

** **Cartesian Products of Sets :** Given two non-empty sets P and Q. The cartesian product P × Q is the set of all ordered pairs of elements from P and Q, i.e., P × Q = { (p , q) : p $\in$ P, q $\in$ Q }

** Two ordered pairs are equal, if and only if the corresponding first elements, are equal and the second elements are also equal.

** If there are p elements in A and q elements in B, then there will be pq elements in A × B, i.e. if n(A) = p and n(B) = q, then n(A × B) = pq.

** If A and B are non-empty sets and either A or B is an infinite set, then so is A × B.

** A × A × A = {(a, b, c) : a, b, c $\in$ A}. Here (a, b, c) is called an ordered triplet.

** **Relation :** A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product A × B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B. The second element is called the image of the first element.

** The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R.

** The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the co-domain of the relation R. Range $\subseteq$ co-domain.

** A relation may be represented algebraically either by the Roster method or by the Set-builder method.

** An arrow diagram is a visual representation of a relation.

** The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A × B. If n(A) = p and n(B) = q, then n (A × B) = pq and the total number of relations is 2^{pq}.

** A relation R from A to A is also stated as a relation on A.

** **Function:** A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

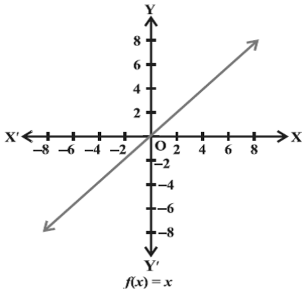
In other words, a function f is a relation from a non-empty set A to a non-empty set B such that the domain of f is A and no two distinct ordered pairs in f have the same first element.

If f is a function from A to B and (a, b) $\in$ f, then f (a) = b, where b is called the image of a under f and a is called the pre-image of b under f.

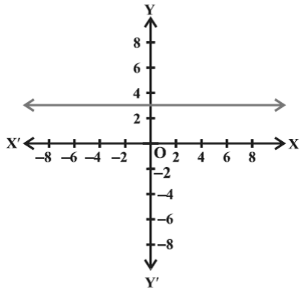
** A function which has either R or one of its subsets as its range is called a real valued function. Further, if its domain is also either R or a subset of R, it is called a real function.

Some functions and their graphs

** **Identity function** Let **R** be the set of real numbers. Define the real valued function f : **R** →**R** by y = f(x) = x for each x $\in$ **R**. Such a function is called the identity function. Here the domain and range of f are **R**.



Constant function : Define the function f: **R** → **R** by y = f (x) = c, $x \in$ **R** where c is a constant and each $x \in$ **R**. Here domain of f is **R** and its range is {c}.

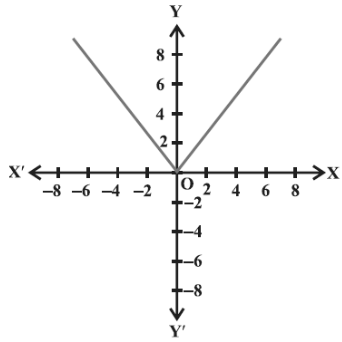


$f(x) = 3$

Polynomial function : A function f : **R**→**R** is said to be polynomial function if for each x in **R**, y = f (x) = $a_0 + a_1x + a_2x^2 + ... + a_n x^n$, where n is a non-negative integer and $a_0, a_1, a_2, ..., a_n \in$ **R**.

** **Rational functions :** are functions of the type $\frac{f(x)}{g(x)}$, where f(x) and g(x) are polynomial functions of x defined in a domain, where $g(x) \neq 0$.

** **The Modulus function :** The function f: **R**→**R** defined by f(x) = |x| for each $x \in$ **R** is called modulus function. For each non-negative value of x, f(x) is equal to x.
But for negative values of x, the value of f(x) is the negative of the value of x, i.e., $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$.

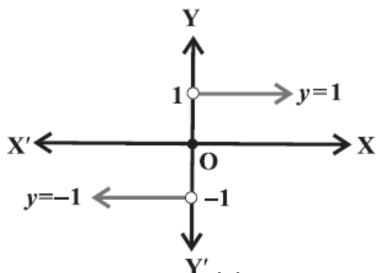


$f(x) = |x|$

** **Signum function :** The function f : **R**→**R** defined by

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

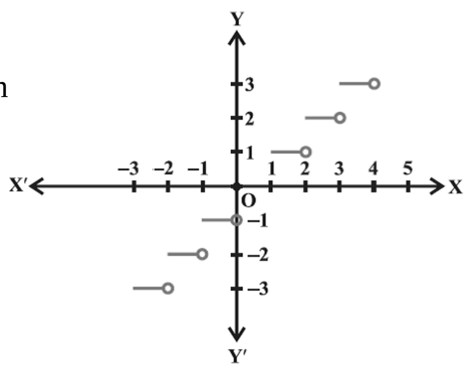
is called the signum function. The domain of the signum function is **R** and the range is the set {–1, 0, 1}.



$f(x) = \frac{|x|}{x}$

** **Greatest integer function :**
The function f : **R** → **R** defined by f(x) = [x], $x \in$ **R** assum the value of the greatest integer, less than or equal to x. Such a function is called the greatest integer function.

$[x] = -1$ for $-1 \leq x < 0$
$[x] = 0$ for $0 \leq x < 1$
$[x] = 1$ for $1 \leq x < 2$
$[x] = 2$ for $2 \leq x < 3$ and so on.



$f(x) = [x]$

Algebra of real functions

** **Addition of two real functions :** Let f : X → **R** and g : X → **R** be any two real functions, where $X \subset$ **R**.
Then, we define (f + g): X → **R** by (f + g) (x) = f (x) + g (x), for all $x \in X$.

** **Subtraction of a real function from another :** Let f :X → **R** and g:X → **R** be any two real functions,

where X ⊂ **R**. Then, we define (f – g) : X→**R** by (f – g) (x) = f(x) – g(x), for all x ∈ X.

** **Multiplication by a scalar :** Let f : X→**R** be a real valued function and α be a scalar. Here by scalar, we mean a real number. Then the product α f is a function from X to **R** defined by

(α f) (x) = α f (x), x ∈ X.

** **Multiplication of two real functions :** The product (or multiplication) of two real functions f :X→**R** and g :X→**R** is a function fg :X→**R** defined by (fg) (x) = f(x) g(x), for all x ∈ X.

** **Quotient of two real functions** Let f and g be two real functions defined from X→**R** where X ⊂ **R**. The quotient of f by g denoted by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided g(x) ≠ 0, x ∈ X

III:- Question for practise:-

MCQs (1 mark each)

Q 1. If A= {a, b} and B = {1, 2} then the number of functions from set A to set B is

(A) 2 (B) 4 (C) 16 (D) None of these

Q 2. A function is defined by f(t) = 2t – 5, then the value of f(-3) is

(A) - 11 (B) 11 (C) 1 (D) -1

Q 3. If f(x) = - |x|. Choose the correct option from the following:

(A) Domain is set of negative real numbers (B) Range is set of real numbers

(C) Range is set of all negative integers (D) Range is (-∞, 0]

Q 4. Let f = {(1, 1), (2, 3), (0, -1), (-1, -3)} be a function from Z to Z defined by f(x) = mx + c.

Determine c.

(A) 1 (B) 0 (C) - 1 (D) - 3

Assertion and Reason type (1 mark)

Q 5.In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

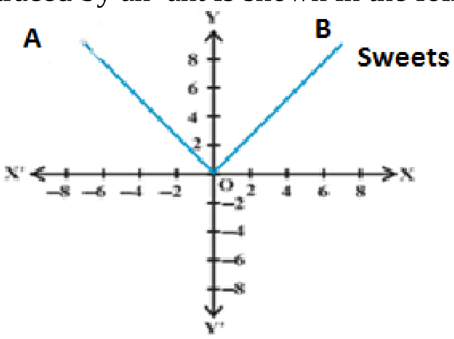
(d) (A) is false but (R) is true.

ASSERTION (A): The function f: A→B defined by f = {(1, x), (2, y), (3, x)}, then its domain is A = {1, 2, 3} and range is {x, y}.

REASON(R) : The range of the function f is always the co-domain set.

Case/Source based questions (1 + 1 + 2 = 4 marks each)

Q 6. A is the anthills of an ant, at B some sweets are there and ant wants to reach at B. The path traced by an ant is shown in the following graph:



On the basis of the above graph find the following:

(i) When ordinate is 6 then find abscissa

(ii) Which axis is line of symmetry for the graph?

(iii) Write the function for the graph along with domain and range.

Q 7. In a school at Chandigarh, students of class XI were discussing about the relations and functions. Two studentsAnkita and Babita form two sets A = {1,2, 3, 4, 5} and B = {2, 4, 6}.

Based on the above information answer the following:

(i) Find n(A×B)

(ii) A correspondence of elements from A to B given as {(1, 2), (2, 2), (3, 4), (3, 6), (4, 4), (5, 6)}.
Is it a function? Justify your answer.

(iii) If the function f: A⟶B such that (a, b) ∈ f and a < b, defined by f = {(1, 2), (x, 4), (2, 4), (4, y), (5, 6)}, then find x and y.

Q 8.

Two persons Ram and Rahim are running in the park and the paths of their running are respectively $f(x) = 1 + 2x$ and $g(x) = x^3 - x$.

(i) find (f + g) (x)

(ii) find (f.g) (x)

(iii) find $\left(\frac{f}{g}\right)(x)$

Short answer type questions (3 marks each)

Q 9. Find the domain and the range of the real function $f(x) = \sqrt{9 - x^2}$

Q 10. Let f, g: R→R be defined by f(x) = x + 1 and g(x) = 2x – 3. Find f – g, f. g and $\frac{f}{g}$.

Q 11.Find the domain and the range of the real function $f(x) = \sqrt{(5 - x)}$.

Q 12. Determine the range of the function f which is defined as $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in R\right\}$

Q 13. Let f be the subset of Z x Z, defined by f = {(ab, a + b): a, b ∈ Z}. Is f a function from Z to Z? Justify your answer.

Answer

Q 1.B Q 2.A Q 3.D Q 4.C Q 5. C

Q 6. (i) ± 6 (ii) y-axis (iii) f(x) = |x|, the domain is R and Range is[0,∞)

Q 7. (i) 15 (ii) No, Element 3 is having two images 4 and 6 (iii) x = 3 , y = 6

Q 8. (i) $x^3 + x + 1$ (ii) $2x^4 + x^3 - 2x^2 - x$ (iii) $\frac{1+2x}{x^3 - x}$

Q 9. Domain = [-3, 3], Range = [0, 3]

Q 10. (f – g) = - x + 4, f. g = $2x^2 – x – 3$, and $\frac{f}{g} = \frac{x+1}{2x-3}$

Q 11. Domain = (- ∞, 5], Range = [0, ∞]

Q 12. Range of f = [0, 1)

Q 13.f is not a function because, if a and b both are positive or both are negative then ab is same but
their images are not same.

Chapter Test 1

Time 1:00 hour **Mathematics – XI**

M.M.: 20

***All the questions are compulsory**

MCQs (1 Mark each)

Q 1. Let R be the relation on set N defined by a + 3b = 12, a and b belong to the set N. Domain of R is

(A) {9, 6} (B) {9, 3} (C) {6, 3, 9} (D) {6, 3}

Q 2. A relation R on set A = {1, 2, 3, 4, 6} defined by R = {(x, y) : y = x + 1. The domain of R is

(A) {1, 2, 4} (B) {1, 2, 3} (C) {1, 2, 3, 4, 6} (D) {2, 3, 4, 5, 7}

Q 3. Range of the function $f(x) = \frac{|x-3|}{x-3}$ is

(A) [-1, 1] (B) (- 1, 1) (C) {- 1, 1} (D) None of these

Q 4. Domain of the function f(x) = $\sqrt{25 - x^2}$ is:

(A) [-5, 1) (B) (- 5, 5) (C) [- 5, 1) (D) [-5, 5]

Q 5. In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

ASSERTION (A): If (x + 1, y – 2) = (3, 1), then x = 2 and y = 3

REASON (R) : Two ordered pairs are equal iff their corresponding elements area equal.

Short Answer type questions (2 Marks each)

Q 6. If $A \times B = \{(p, q), (p, r), (m, q), (m, r)\}$, find A and B

Q 7. Write the relation R = {(x, x^3) : x is a prime number less than 10} in roster form.

Q 8. If X = {1, 2, 3} and Y = {2, 4}and R is a relation from X to Y, (x, y) ∈ R such that x < y. Depict this
relation by an arrow diagram.

Short Answer type questions (3 Marks each)

Q 9. The function 't' which maps temperature in degree Celsius in to temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$. Find t(0), t (- 10) and the value of C, when t (C) = 212.

Q 10. Draw the graph of function f(x) = $\sqrt{4 - x^2}$, Also write the domain and range of f(x).

Q 11. If $f(x) = \frac{x-1}{x+1}$, then show that (i) $f\left(\frac{1}{x}\right) = -f(x)$ (ii) $f\left(-\frac{1}{x}\right) = -\frac{1}{f(x)}$

Chapter Test 2

Time 1:30 hour **Mathematics – XI**

M.M.: 30

***All the questions are compulsory**

MCQs (1 Mark each)

Q 1. Let A = {x, y} and B = {1, 2} then number of non empty relations from set A to set B will be

(A) 64 (B) 32 (C) 16 (D) 15

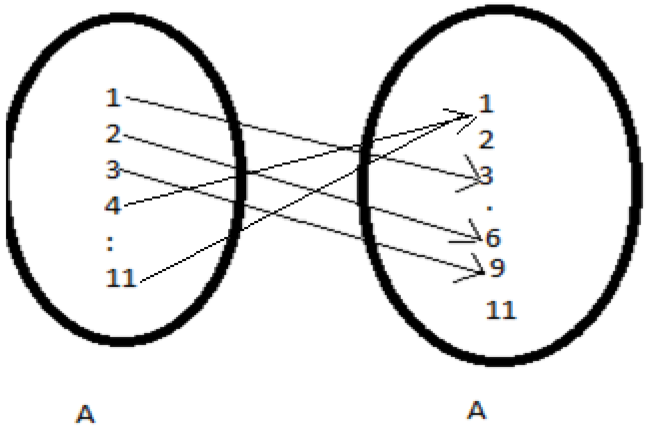
Q 2. The function f: R⟶R, f(x) = [x], x ∈ R, [x] defines the greatest integer less than or equal to x.

Then $f\left(-\frac{3}{2}\right)$ is equal to

(A) – 3 (B) – 2 (C) – 1.5 (D) None of these

Q 3. Domain of the function $f(x) = \frac{x^2 + 3x + 5}{x^2 - 5x + 4}$ is

(A) R – {1, 3} (B) {1, 4} (C) R – {1, 4} (D) Set of all real numbers



Q 4. The figure shows a relation R on A.
Select the corect range from the following:

(A) {1, 2, 3, ..., 14} (B) {1, 3, 6, 9} (C) {1, 2, 3, 4, 11} (D) None of these

Q 5. Which one of the following is given correct values of x and y if $(x + y,\ y - 2x) = (2, -1)$.

(A) x = -1, y = - 1 (B) x = 1, y = - 1 (C) x = - 1, y = 1 (D) x = 1, y = 1

Q 6. Range of a function from set A to set B is........................

Q 7. The domain of f(x) = |x – 1|, x ∈ R..................................

Q 8 If A x B = {(a, x), (b, x), (a, y), (b, y}, find set A and set B.

. In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

Q 9. ASSERTION (A): Let A = {x, y, z} and B = {1, 2}, then 64 relations can be possible from A to B

REASON (R) : Number of relations from set A to set B is given by $2^{(m + n)}$, where m, n are number of elements of set A and set B.

Q 10. ASSERTION (A): Let (x + 1, y - 2) = (3, 1), then x = 2, y = 3

REASON (R) : Two ordered pairs are equal iff their corresponding elements area equal

Short Answer type questions (2 Marks each)

Q 11. Let A = {3, 5} and B = {7, 11. Let R = {(a, b): a∈A and b∈ B, a – b is odd}. Show that R is an empty relation.

Q 12. Let f and g be real functions defined by $f(x) = 2x^2 – 1$ and g(x) = 4x – 7. For what real x, f(x) = g(x)?

Q 13. If $f(x) = x^2$, find $\frac{f(1.1) - f(1)}{(1.1-1)}$

Q 14. If A = {2, 4, 6, 9} and B = {4, 6, 18, 27, 54}, a∈A and b∈ B, find set of ordered pairs such that a is a

factor of b and a < b.

Q 15. The function f is defined by $f(x) = |1 - x|$ where x is a real number.Draw the graph

Short Answer type questions (3 Marks each)

Q 16. If P = {x : x < 3, x∈ N}, Q = {x : x≤ 2, x∈ W}. Find (P∪ Q) × (P ∩ Q).

Q 17. Find the domain and range of the function $f(x) = \sqrt{-16x - x^2}$.

Case/Source based questions (1 + 1 + 2 = 4 marks each)

Q 18. In a school at Chandigarh, students of class XI were discussing about the relations and functions. Three

Students Ankita, Babita and Kavita form three sets A = {1,2, 3, 4, 5}, B = {2, 4, 6} and K = {5, 7}

Based on the above information answer the following:

(i) Find $n(A \times B)$ and $n(K \times B)$

(ii) A correspondence of elements from A to B given as {(1, 2), (2, 2), (3, 4), (3, 6), (4, 4), (5, 6)}.
Is it a function? Justify your answer.

(iii) If the function $f: B \longrightarrow K$ such that $(b, k) \in f$ and $b < k$, defined by f = {(1, 5), (x, 5), (5, y), (2, 7), (4, 7)}, then find x and y.

TRIGONOMETRIC FUNCTIONS
CONCEPTS AND RESULTS

Angles : Angle is a measure of rotation of a given ray about its initial point.

** **Measurement of an angle**.

****English System (Sexagesimal system)**

(i) 1 right angle = 90 degrees = 90^o. (ii) 1^o = 60 minutes = 60'. (iii) 1' = 60 second = 60''.

****French System (Centesimal system)**

(iv) 1 right angle = 100 grades = 100 g. (v) 1 g = 100 minutes = 100 ' (vi) 1' = 100 seconds = 100 ''

****Circular System.**

(vii) $180^o = 200^g = \pi$ radians = 2 right angles, where a radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.

(viii) The circular measure θ of an angle subtended at the centre of a circle by an arc of length **l** is equal to the ratio of the length **l** to the radius r of the circle.

(ix) Each interior angle of a regular polygon of n sides is equal to $\frac{2n-4}{n}$ right angles.

T-ratios	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	n.d	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

** **Formulae for t-ratios of Allied Angles :**

All T-ratio changes in $\frac{\pi}{2}\pm\theta$ and $\frac{3\pi}{2}\pm\theta$ while remains unchanged in $\pi\pm\theta$ and $2\pi\pm\theta$.

$\sin\left(\frac{\pi}{2}\pm\theta\right) = \cos\theta$ $\quad$ $\sin\left(\frac{3\pi}{2}\pm\theta\right) = = \cos\theta$

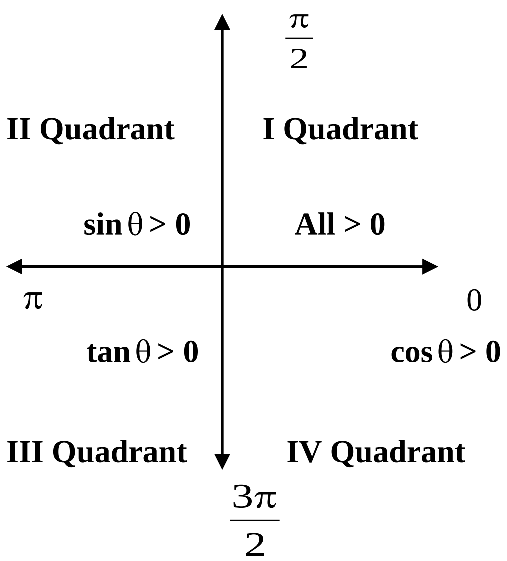
$\cos\left(\frac{\pi}{2}\pm\theta\right) = \mp\sin\theta$ $\quad$ $\cos\left(\frac{3\pi}{2}\pm\theta\right) = \pm\sin\theta$

$\tan\left(\frac{\pi}{2}\pm\theta\right) = \mp\cot\theta$ $\quad$ $\tan\left(\frac{3\pi}{2}\pm\theta\right) = \mp\cot\theta$

$\sin(\pi\pm\theta) = \mp\sin\theta$ $\quad$ $\sin(2\pi\pm\theta) = \pm\sin\theta$

$\cos(\pi\pm\theta) = = \cos\theta$ $\quad$ $\cos(2\pi\pm\theta) = \cos\theta$

$\tan(\pi\pm\theta) = \pm\tan\theta$ $\quad$ $\tan(2\pi\pm\theta) = \pm\tan\theta$



** **Sum and Difference formulae :**

sin(A + B) = sin A cos B + cos A sin B
sin(A – B) = sin A cos B – cos A sin B
cos(A + B) = cos A cos B – sin A sin B
cos(A – B) = cos A cos B + sin A sin B

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$, $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$. $\tan\left(\dfrac{\pi}{4} + A\right) = \dfrac{1 + \tan A}{1 - \tan A}$,

$\tan\left(\dfrac{\pi}{4} - A\right) = \dfrac{1 - \tan A}{1 + \tan A}$ $\cot(A + B) = \dfrac{\cot A.\cot B - 1}{\cot B + \cot A}$ $\cot(A - B) = \dfrac{\cot A.\cot B + 1}{\cot B - \cot A}$

$\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

$\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

Formulae for the transformation of a product of two circular functions into algebraic sum of two circular functions and vice-versa.

$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$

$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

$\sin C + \sin D = 2 \sin \dfrac{C + D}{2} \cos \dfrac{C - D}{2}$, $\sin C - \sin D = 2 \cos \dfrac{C + D}{2} \sin \dfrac{C - D}{2}$.

$\cos C + \cos D = 2 \cos \dfrac{C + D}{2} \cos \dfrac{C - D}{2}$, $\cos C - \cos D = -2 \sin \dfrac{C + D}{2} \sin \dfrac{C - D}{2}$.

** **Formulae for t-ratios of multiple and sub-multiple angles** :

$\sin 2A = 2 \sin A \cos A = \dfrac{2\tan A}{1 + \tan^2 A}$.

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

$1 + \cos 2A = 2\cos^2 A$ $1 - \cos 2A = 2\sin^2 A$ $1 + \cos A = 2\cos^2 \dfrac{A}{2}$ $1 - \cos A = 2\sin^2 \dfrac{A}{2}$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$, $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$.

$\sin 3A = 3 \sin A - 4 \sin^3 A$, $\cos 3A = 4\cos^3 A - 3 \cos A$

$\sin 15^o = \cos 75^o = \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$. & $\cos 15^o = \sin 75^o = \dfrac{\sqrt{3} + 1}{2\sqrt{2}}$,

$\tan 15^o = \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3} = \cot 75^o$ & $\tan 75^o = \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1} = 2 + \sqrt{3} = \cot 15^o$.

$\sin 18^o = \dfrac{\sqrt{5} - 1}{4} = \cos 72^o$ and $\cos 36^o = \dfrac{\sqrt{5} + 1}{4} = \sin 54^o$.

$\sin 36^o = \dfrac{\sqrt{10 - 2\sqrt{5}}}{4} = \cos 54^o$ and $\cos 18^o = \dfrac{\sqrt{10 + 2\sqrt{5}}}{4} = \sin 72^o$.

$\tan\left(22\dfrac{1}{2}\right)^o = \sqrt{2} - 1 = \cot 67\dfrac{1}{2}^o$ and $\tan\left(67\dfrac{1}{2}\right)^o = \sqrt{2} + 1 = \cot\left(22\dfrac{1}{2}\right)^o$.

** **Properties of Triangles** : In any Δ ABC,

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ [Sine Formula]

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, $\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$, $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$.

** **Projection Formulae** : $a = b \cos C + c \cos B$, $b = c \cos A + a \cos C$, $c = a \cos B + b \cos A$

** **Some important trigonometric substitutions :**

$\sqrt{a^2+x^2}$	Put x = a $\tan\theta$ or a $\cot\theta$
$\sqrt{x^2-a^2}$	Put x = a $\sec\theta$ or a $\operatorname{cosec}\theta$
$\sqrt{a+x}$ or $\sqrt{a-x}$ or both	Put x = a $\cos 2\theta$
$\sqrt{a^n+x^n}$ or $\sqrt{a^n-x^n}$ or both	Put $x^n = a^n \cos 2\theta$
$\sqrt{1+\sin 2\theta}$	$=\sin\theta+\cos\theta$
$\sqrt{1-\sin 2\theta}$	$=\cos\theta-\sin\theta,\ 0<\theta<\frac{\pi}{4}$
	$=\sin\theta-\cos\theta,\ \frac{\pi}{4}<\theta<\frac{\pi}{2}$

****General solutions:**

$*\cos\theta = 0 \Rightarrow \theta = n\pi,\ n \in Z$

$*\sin\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2},\ n \in Z$

$*\tan\theta = 0 \Rightarrow \theta = n\pi,\ n \in Z$

$*\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n \alpha,\ n \in Z$

$*\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha,\ n \in Z$

$*\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha,\ n \in Z$

Examples:

1. The value of tan 1° tan 2° tan 3° … tan 89° is:

(a) 0 (b) 1 (c) ½ (d) Not defined

Correct option: (b) 1

Solution: tan 1° tan 2° tan 3° … tan 89°

= [tan 1° tan 2° … tan 44°] tan 45°[tan (90° – 44°) tan (90° – 43°)… tan (90° – 1°)]

= [tan 1° tan 2° … tan 44°] [cot 44° cot 43°……. cot 1°] × [tan 45°]

= [(tan 1° × cot 1°) (tan 2° × cot 2°) …..(tan 44° × cot 44°)] × [tan 45°]

We know that, tan A × cot A =1 and tan 45° = 1

Hence, the equation becomes as;

= 1 × 1 × 1 × 1 × …× 1= 1 {As $1^n = 1$}

2. If α + β = π/4, then the value of (1 + tan α) (1 + tan β) is :

(a) 1 (b) 2 (c) – 2 (d) Not defined

Correct option: (b) 2

Solution: Given, α + β = π/4

Taking "tan" on both sides,

tan(α + β) = tan π/4

We know that, tan(A + B) = (tan A + tan B)/(1 – tan A tan B)

and tan π/4 = 1.

So, (tan α + tan β)/(1 – tan α tan β) = 1

tan α + tan β = 1 – tan α tan β

tan α + tan β + tan α tan β = 1….(i)

(1 + tan α)(1 + tan β) = 1 + tan α + tan β + tan α tan β

= 1 + 1 [From (i)]=2

3. Find the radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm (use π = 22/7).

Solution: Given, Length of the arc = l = 37.4 cm

Central angle = θ = 60° = 60π/180 radian = π/3 radians

We know that,r = l/θ

= (37.4) * (π / 3)= (37.4) / [22 / 7 * 3]= 35.7 cm

Q4.**Find the value of √3 cosec 20° – sec 20°.**
Solution: √3 cosec 20° – sec 20°

$$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} \quad = \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°} = 4\left(\frac{\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°}{2\sin 20° \cos 20°}\right)$$

$$= 4\left(\frac{\sin 60° \cos 20° - \cos 60° \sin 20°}{\sin 40°}\right) \quad = 4\left(\frac{\sin (60° - 20°)}{\sin 40°}\right) = 4$$

QUESTIONS FOR PRACTICE
MULTIPLE CHOICE QUESTIONS

1. The conversion of $40^0 20^/$ into radians is:
   a) π Radians b) $\frac{15}{9}$ π radians c) $\frac{121\pi}{540}$ radians d) None of these.
2. If sinx= $\frac{\sqrt{3}}{2}$ and cosx= $-\frac{1}{2}$ then x lies in:
   a) 1st quadrant b) 2^{nd}quadrant c) 3^{rd}quadrant d) 4^{th}quadrant
3. Value of $\sec^2 x + \cos^2 x$ is:
   a) < 0 $b)$ < 1 $c)$ ≥ 2 d) None of these.
4. If coty $= \frac{7}{24}$ and y lies in the third quadrant then value of cosy-siny is:
   a. $\frac{17}{25}$ b. $\frac{16}{25}$ c. $\frac{14}{25}$ d. $\frac{13}{25}$

5. Value of $2\sin^2\frac{\pi}{6} + \text{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$ is :
   a. $\frac{1}{2}$ b. $-\frac{1}{2}$ c. 1 d. $\frac{3}{2}$
6. The period of function sin3x is:
   a. π b. 2π c. 3 π d. None of thease
7. Value of tan 15^0 is:
a. 0 b. $\frac{1}{2}$ c. $\frac{1}{3}$ d. $2-\sqrt{3}$
8. The minimum value of cosx + sinx is:
a. 0 b. 1 c. -1 d. $-\sqrt{2}$
9. Value of tan (-1575^0) is:
a. 1 b. $\frac{1}{2}$ c.0 d. -1
10. The greatest and least values of sinx , cosx are respectively
a. 1, -1 b. $\frac{1}{2}, -\frac{1}{2}$ c. $\frac{1}{4}, -\frac{1}{4}$ d.2,-2

Short answer type

1. A horse is tied to a post by a ripe. If the horse moves along a circular path always keeping the rope tight and describes 88 m when it has traced out 72^0 at the centre, find the length of the rope.
2. If Cos θ = $-\frac{1}{2}$, $\pi < \theta < \frac{3\pi}{2}$, Evaluate 4 $\tan^2\theta$ – 3 $\text{Cosec}^2\theta$.
3. Show that $\cos 60^0 + \cos 120^0 + \cos 240^0 - \sin 330^0 = 0$
4. Show that $\sqrt{2+\sqrt{2+2\cos 4x}} = 2\,Cosx$
5. Show that $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\left(\frac{x-y}{2}\right)$
6. Show that $\cos 2\theta.\cos\frac{\theta}{2} - \cos 3\theta.\cos\frac{9\theta}{2} = \sin 5\theta.\sin\frac{5\theta}{2}$
7. Show that $\frac{1+Sin\ 2x - Cos\ 2x}{1+Sin\ 2x + Cos\ 2x} = \tan\ x$
8. Show that Cos A. Cos (60 – A). Cos (60 + A) = $\frac{\cos 3A}{4}$

9. If Cos A + Cos B = $\frac{1}{2}$,Sin A + Sin B = $\frac{1}{4}$, Show that tan $\left(\frac{A+B}{2}\right)$ = $\frac{1}{2}$

10. Show that 2 $\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos2(\alpha+\beta) = \cos2\alpha$.

Long answer type

1. Prove that: $\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) + \cos^2\left(x-\frac{\pi}{3}\right) = \frac{3}{2}$

2. If $\tan x = \frac{5}{12}$ and x lies in 2rd quadrant, find the value of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, $\tan\frac{x}{2}$

Ans. $\sin\frac{x}{2} = \frac{5}{\sqrt{26}}$, $\cos\frac{x}{2} = \frac{1}{\sqrt{26}}$, $\tan\frac{x}{2} = 5$

3. Show that $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

4. Show that $\frac{(\cos x - \cos 3x)(\sin 8x + \sin 2x)}{(\sin 5x - \sin x)(\cos 4x - \cos 6x)} = 1$

5. Show that $\text{Cos}20^0 \text{ Cos } 40^0 \text{ Cos } 60^0 \text{ Cos } 80^0 = \frac{1}{16}$.

Answer key of practice questions

MCQ

1. C	2. b	3. c	4. a	5. d
6. d	7. d	8. d	9. d	10 b

Short answer type

1. 70 cm 2. 11

Long answer type 2. $\sin\frac{x}{2} = \frac{5}{\sqrt{26}}$, $\cos\frac{x}{2} = \frac{1}{\sqrt{26}}$, $\tan\frac{x}{2} = 5$

SELF ASSESSMENT TEST - 1

Time: 40 min. **M.M. 20**

Instructions:

- Attempt all questions
- Question 1 to 3 carry one marks each, Question 4to 7 carry three marks each and question 8 carry five marks.

1. Find in degree the angle through which a pendulum, swings if its length is 50 cm and the tip describes an arc of length 10 cm.

. 2. Value of $2\sin^2\frac{\pi}{6} + \text{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$ is:

a. $\frac{1}{2}$ b. $-\frac{1}{2}$ C. 1 d. $\frac{3}{2}$

3. tan $(\frac{\pi}{4} + x)$ tan $(\frac{3\pi}{4} + x)$=…………

.a. 1 b. 2 c. -1 d. -2

4. Prove that $\sin^2 8x - \sin^2 3x = \sin 11x.\sin 5x$

5. Show that $\frac{Sec\,8A-1}{Sec\,4A-1} = \frac{\tan 8A}{\tan 2A}$

6. Show that $\sin x + \sin 2x + \sin 4x + \sin 5x = 4\sin 3x \cos\frac{x}{2}\cos\frac{3x}{2}$

7. Show that $\sqrt{2+\sqrt{2+2\cos4x}} = 2\,Cosx$

8. If $\sin x = -\frac{1}{2}$ and x lies in 4th quadrant, find the value of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, $\tan\frac{x}{2}$

SELF ASSESSMENT TEST - 2

Time: 60min. **M.M. 30**

Instructions:

- Attempt all questions
- From question 1 to 5 carry one marks each, From question 6 to 10 carry three marks each and question 11 to 12 carry five mark each.

1. If sinx = $\frac{1}{3}$ then value of sin3x is:

a.$\frac{23}{27}$ b. $\frac{-23}{27}$ c.$\sqrt{\frac{23}{27}}$ d.$\frac{4\sqrt{2}}{9}$

2. If coty =$\frac{7}{24}$ and y lies in the third quadrant then value of cosy-siny is:

a. $\frac{17}{25}$ b. $\frac{16}{25}$ c. $\frac{14}{25}$ d. $\frac{13}{25}$

3. $\tan\left(\frac{\pi}{4}+x\right)\tan\left(\frac{\pi}{4}-x\right)$ is:

a. 0 b. -1 c.1 d.$\frac{1}{2}$

4. $\sin 10^0 \sin 50^0 \sin 70^0$ =…………….

5. For cyclic quadrilateral ABCD show that Cos A + Cos B + Cos C + Cos D = 0

6. If the arcs of same length in two circles subtend angles 65 degrees and 110 degrees at the Centres , find the ratio of their radii.

7. If x lies in second quadrant, then show that $\sqrt{\frac{1-\sin x}{1+\sin x}}+\sqrt{\frac{1+\sin x}{1-\sin x}}$=-2secx.

8. Show that Cos A + Cos$\left(A+\frac{2\pi}{3}\right)$ + Cos$\left(A+\frac{4\pi}{3}\right)$= 0.

9. Show that $(\cos x-\cos y)^2+(\sin x-\sin y)^2=4\sin^2\left(\frac{x-y}{2}\right)$.

10. Show that $\frac{1+Sin\ 2x-Cos\ 2x}{1+Sin\ 2x+Cos\ 2x}=\tan\ x$

11. If $\cos x=-\frac{3}{5}$ and x lies in third quadrant, find the value of $\sin 2x$, $\cos 2x$ and $\tan 2x$

12.Prove that: $\cos^2 x+\cos^2\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right)=\frac{3}{2}$

ANSWER:

PRACTICE TEST-1

1. $11\frac{5}{11}$, 2. d) 3. c) 8. $\sin\frac{x}{2}=\frac{\sqrt{2-\sqrt{3}}}{2}$, $\cos\frac{x}{2}=\frac{-\sqrt{2+\sqrt{3}}}{2}$, $\tan\frac{x}{2}=\frac{-\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}$

PRACTICE TEST-2

1. a) 2.a) 3. c) 4.$\frac{1}{8}$) 6. 22:13

11. $\sin 2x=\frac{24}{25}$, $\cos 2x=-\frac{7}{25}$, $\tan 2x=-\frac{24}{7}$

COMPLEX NUMBERS
CONCEPTS AND RESULTS

* A number of the form (a + ib) where a b $\in$ R, the set of real numbers, and i = $\sqrt{-1}$ (iota) is called a complex number. It is denoted by z, z = a + ib. "a" is called the real part of complex number z and "b"
is the imaginary part i.e. Re(z) = a and Im(z) = b.
* Two complex numbers are said to be equal i.e. $z_1 = z_2$.

$\Leftrightarrow$ (a + ib) = (c + id)

$\Leftrightarrow$ a = c and b = d

$\Leftrightarrow$ Re (z_1) = Re (z_2) & $Im(z_1) = Im(z_2)$.

* A complex number z is said to be purely real if Im(z) = 0
and is said to be purely imaginary if Re(z) = 0.
* The set R of real numbers is a proper subset of the set of complex number C, because every real number
can be considered as a complex number with imaginary part zero.
* $i^{4n} = (i^4)^n = (1)^n = 1$ $\qquad$ $i^{4n+1} = i^{4n}.i = (1).i = i$
$i^{4n+2} = i^{4n}.\, i^2 = (1)(-1) = -1$ $\qquad$ $i^{4n+3} = i^{4n}.i^3 = (1)(-i) = -i$.

Algebra of Complex Numbers

** **Addition of two complex numbers :** Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers. Then, the sum $z_1 + z_2$ is defined as follows: $z_1 + z_2 = (a + c) + i\,(b + d)$, which is again a complex number.

The addition of complex numbers satisfy the following properties:

(i) The closure law The sum of two complex numbers is a complex number, i.e., $z_1 + z_2$ is a complex number for all complex numbers z_1 and z_2 .

(ii) The commutative law For any two complex numbers z_1 and z_2, $z_1 + z_2 = z_2 + z_1$

(iii) The associative law For any three complex numbers z_1, z_2, z_3, $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$.

(iv) The existence of additive identity There exists the complex number 0 + i 0 (denoted as 0), called the additive identity or the zero complex number, such that, for every complex number z, z + 0 = z.

(v) The existence of additive inverse To every complex number z = a + ib, we have the complex number – a + i(– b) (denoted as – z), called the additive inverse or negative of z. Thus z + (–z) = 0 (the additive identity).

** **Difference of two complex numbers :** Given any two complex numbers z_1 and z_2, the difference $z_1 - z_2$ is defined as follows: $z_1 - z_2 = z_1 + (-z_2)$.

** **Multiplication of two complex numbers :** Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers.
Then, the product $z_1 z_2$ is defined as follows: $z_1 z_2 = (ac - bd) + i(ad + bc)$

**The multiplication of complex numbers possesses the following properties :

(i) **The closure law** The product of two complex numbers is a complex number, the product $z_1 z_2$ is a complex number for all complex numbers z_1 and z_2.

(ii) **The commutative law** For any two complex numbers z_1 and z_2, $z_1 z_2 = z_2 z_1$

(iii) **The associative law** For any three complex numbers z_1, z_2, z_3, $(z_1 z_2) z_3 = z_1 (z_2 z_3)$.

(iv) **The existence of multiplicative identity** There exists the complex number 1 + i 0 (denoted as 1), called the multiplicative identity such that z.1 = z, for every complex number z.

(v) **The existence of multiplicative inverse** For every non-zero complex number z = a + ib or a + bi ($a \neq 0, b \neq 0$), we have the complex number $\frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}$ (denoted by $\frac{1}{z}$ or z^{-1}), called the multiplicative inverse of z such that $z.\frac{1}{z} = 1$ (the multiplicative identity).

(vi) **The distributive law** For any three complex numbers z_1, z_2, z_3,

(a) $z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3$ (b) $(z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3$

****Division of two complex numbers :** Given any two complex numbers z_1 and z_2, where $z_2 \neq 0$, the quotient $\frac{z_1}{z_2}$ is defined by $\frac{z_1}{z_2} = z_1 . \frac{1}{z_2}$.

****Modulus a Complex Number :** Let z = a + ib be a complex number. Then, the modulus of z, denoted by | z |, is defined to be the non-negative real number $\sqrt{a^2 + b^2}$, i.e., $| z | = \sqrt{a^2 + b^2}$

** **Properties of Modulus** :

If z, z_1, z_2 are three complex numbers then

(i) $|z| = 0 \Leftrightarrow z = 0$ i.e., real part and imaginary part are zeroes.

(ii) $|z| = |\bar{z}| = |-z|$

(iii) $z. \bar{z} = |z|^2$

(iv) $|z_1.z_2| = |z_1|.|z_2|$

(v) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, \ z_2 \neq 0$

(vi) $|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2\text{Re}(z_1 \bar{z}_2)$

(vii) $|z_1-z_2|^2 = |z_1|^2 + |z_2|^2 - 2\text{Re}(z_1 \bar{z}_2)$

(viii) $|z_1+z_2|^2 + |z_1-z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

****Conjugate of a Complex Number :** Let z = a + ib then its conjugate is denoted by $\bar{z} = (a - ib)$.

****Properties of conjugates** :

(i) $\overline{(\bar{z})} = z$

(ii) $z + \bar{z} = 2\text{Re}(z)$

(iii) $z - \bar{z} = 2i\text{Im}(z)$

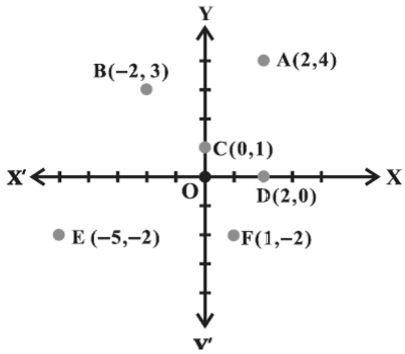
(iv) $z + \bar{z} = 0 \Rightarrow$ z is purely real.

(v) $z. \bar{z} = [\text{Re}(z)]^2 + [\text{Im}(z)]^2$.

(vi) $\overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$

(vii) $\overline{z_1.z_2} = \overline{z_1}.\overline{z_2}$

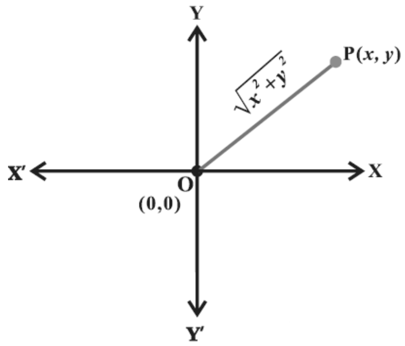
(viii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}, \quad z_2 \neq 0$



****Argand Plane and Polar Representation**

Some complex numbers such as 2 + 4i, – 2 + 3i, 0 + 1i, 2 + 0i, – 5 –2i and 1 – 2i which correspond to the ordered pairs (2, 4), (– 2, 3), (0, 1), (2, 0), (–5, –2), and (1, – 2), respectively, have been represented geometrically by the points A, B, C, D, E, and F, respectively.

The plane having a complex number assigned to each of its point is called the complex plane or the Argand plane.

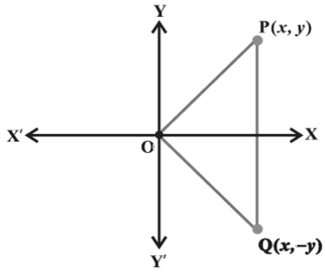


In the Argand plane, the modulus of the complex number $x + iy = \sqrt{x^2 + y^2}$ is the distance between the point P(x, y) to the origin O (0, 0). The points on the x-axis corresponds to the complex numbers of the form a + i 0 and the points on the y-axis corresponds to the complex numbers of the form 0 + i b.

The x-axis and y-axis in the Argand plane are called, respectively, the real axis and the imaginary axis.

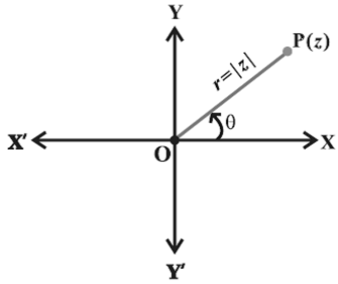
The representation of a complex number $z = x + iy$ and its conjugate $z = x - iy$ in the Argand plane are, respectively, the points P (x, y) and Q (x, – y).

Geometrically, the point (x, – y) is the mirror image of the point (x, y) on the real axis.



** **Polar representation of a complex number :**

Let the point P represent the non zero complex number $z = x + iy$. Let the directed line segment OP be of length r and θ be the angle which OP makes with the positive direction of x-axis .



The point P is uniquely determined by the ordered pair of real numbers (r, θ), called the polar coordinates of the point P. We consider the origin as the pole and the positive direction of the x axis as the initial line.

We have, $x = r\cos\theta$, $y = r\sin\theta$ and therefore,

$z = r(\cos\theta + i\sin\theta)$ is said to be the polar form of the complex number. Here $r = \sqrt{x^2 + y^2} = |z|$ is the modulus of z and θ is called the argument (or amplitude) of z which is denoted by arg z .

For any complex number $z \neq 0$, there corresponds only one value of θ in $0 \leq \theta < 2\pi$. However, any other interval of length 2π, for example $-\pi < \theta \leq \pi$, can be such an interval. We shall take the value of θ such that $-\pi < \theta \leq \pi$, called **principal argument** of z and is denoted by arg z, unless specified otherwise.

** **Cube roots of unity** :

Let $\omega = \sqrt[3]{1}$

$\therefore \quad \omega^3 = 1$

$\Rightarrow \quad \omega^3 - 1 = 0$

$\Rightarrow \quad (\omega - 1)(\omega^2 + \omega + 1) = 0$

$\Rightarrow \quad \omega = 1, \text{ or } \quad \omega = \dfrac{-1 \pm i\sqrt{3}}{2}$.

****Properties of cube roots of unity** :

(i) One of the cube roots of unity is real and the other two are conjugate complex numbers

(ii) Each complex cube root of unity is the square of the other.

(iii) The product of three cube roots of unity is one.

(iv) The sum of the three cube roots of unity is zero

(v) If the equation $x^2 + x + 1 = 0$ has roots ω and ω^2 the roots of equation $x^2 - x + 1 = 0$ has roots $-\omega$ and $-\omega^2$.

(vi) Each complex cube root of unity is the reciprocal of the other.

(vii) Cube roots of 1 are 1, ω, ω^2, then cube roots of –1 are –1, – ω, – ω^2.

****Triangle inequalities** :

(i) $|z_1 + z_2| \leq |z_1| + |z_2|$

(ii) $|z_1 - z_2| \leq |z_1| + |z_2|$

(iii) $|z_1 + z_2| \geq |z_1| - |z_2|$

(iv) $|z_1 - z_2| \geq ||z_1| - |z_2||$

****Quadratic Equations :** Roots of the quadratic equation $ax^2 + bx + c = 0$ with real coefficients a, b, c , $a \neq 0$ and $b^2 - 4ac < 0$ are $\dfrac{-b \pm \sqrt{4ac - b^2}\, i}{2a}$.

II. Some illustrations/Examples (with solution) preferably of different types.

MCQ

1 Multiplicative inverse of complex number (1+i) is ……..

(a)$\frac{1}{2}(1-\mathrm{i})$

(b)$-\frac{1}{2}(1-\mathrm{i})$

(c) $\frac{1}{2}(1+\mathrm{i})$

(d)$-\frac{1}{2}(1+\mathrm{i})$

Sol. Multiplicative inverse of complex number (1+i) is

$z^{-1}=\frac{1}{1+\mathrm{i}} = \frac{1}{1+\mathrm{i}} \times \frac{1-\mathrm{i}}{1-\mathrm{i}} = \frac{1-\mathrm{i}}{1-\mathrm{i}^2} = \frac{1}{2}(1-\mathrm{i})$ **(a)$\frac{1}{2}(1-\mathrm{i})$**

2 If (2+5 i)z=(3-7 i), then z =……

(a)$1+\mathrm{i}$ (b) $1-\mathrm{i}$ (c) $-1+\mathrm{i}$ (d) $-1-\mathrm{i}$

Sol. $(2+5\mathrm{i})z=(3-7\mathrm{i})$

$z=\frac{(3-7\mathrm{i})}{(2+5\mathrm{i})}\times\frac{(2-5\mathrm{i})}{(2-5\mathrm{i})} = \frac{6+35\mathrm{i}^2-15\mathrm{i}-14\mathrm{i}}{4-25\mathrm{i}^2} = \frac{-29-29\mathrm{i}}{29} = -1-\mathrm{i}$ **(d) $-1-\mathrm{i}$**

3 If If $z_{1,}=5-3\mathrm{i}$ and $z_2 = 4+5\mathrm{i}$ then $\overline{z_1}-\overline{z_2}=\ldots$

(a)$3+5\mathrm{i}$ (b) $7-5\mathrm{i}$ (c) $9+2\mathrm{i}$ (d) $1+8\mathrm{i}$

Sol. $\overline{z_1}=5+3\mathrm{i}$, $\overline{z_2}=4-5\mathrm{i}$

$\overline{z_1}-\overline{z_2}=(5+3\mathrm{i})-(4-5\mathrm{i})=1+8\mathrm{i}$ **(d) $1+8\mathrm{i}$**

4 $z=\frac{(1+\mathrm{i})}{(1-\mathrm{i})}$ then $z^4 =$ ...

(a)1 (b) -1 (c) 0 (d) 2

Sol. $z=\frac{(1+\mathrm{i})}{(1-\mathrm{i})}\times\frac{(1+\mathrm{i})}{(1+\mathrm{i})} = \frac{(1+2\mathrm{i}+\mathrm{i}^2)}{(1-\mathrm{i}^2)} = \mathrm{i}$

$z^4=\mathrm{i}^4 = 1$ **(a)1**

Case based study

5 **We have , $\mathrm{i}=\sqrt{-1}$. So, we can write the higher powers of i as follow**
$\mathrm{i}^2=1$, $\mathrm{i}^3=-\mathrm{i}$, $\mathrm{i}^4=1$ ....
In order to compute i^n for n>4, write $\mathrm{i}^n=\mathrm{i}^{4q+r}$ for some $q, r\in N$ and $0\le r\le 3$.
Then $\mathrm{i}^n=\mathrm{i}^{4q}\mathrm{i}^r=(\mathrm{i}^4)^q\,\mathrm{i}^r = (1)^q\,\mathrm{i}^r = \mathrm{i}^r$.
In general for any integer k,
$\mathrm{i}^{4k}=1$, $\mathrm{i}^{4k+1}=\mathrm{i}$, $\mathrm{i}^{4k+2}=-1$ and $\mathrm{i}^{4k+3}-\mathrm{i}$.

On the basis of the above information , answer the following question.

(i) What is the value of i^{-30}.

(ii) If $z=\mathrm{i}^9+\mathrm{i}^{19}$, then write z in the form of $a+ib$

(iii) Find the sum $\mathrm{i}+\mathrm{i}^2+\mathrm{i}^3+\mathrm{i}^4+\cdots$ upto 1000 terms

Sol.

(i) $\mathrm{i}^{-30}=\mathrm{i}^{-32+2}=\mathrm{i}^{-32}\mathrm{i}^2 = (\mathrm{i}^4)^{(-8)}\,\mathrm{i}^2 = (1)^{-8}(-1)=1\times(-1) = -1$

(ii) $z=\mathrm{i}^9+\mathrm{i}^{19} = \mathrm{i}^{4\times2+1}+\mathrm{i}^{4\times4+3} = \mathrm{i}+\mathrm{i}^3=\mathrm{i}-\mathrm{i}=0=0+0\mathrm{i}$

(iii) $\mathrm{i}+\mathrm{i}^2+\mathrm{i}^3+\mathrm{i}^4+\cdots$ upto 1000 terms

	$= (i + i^2 + i^3 + i^4) + (i^5 + i^6 + i^7 + i^8) + \cdots$ upto 1000 terms Sum of four consecutive power power of i is 0 = 0+ 0+ 0+.... upto 250 term =0
Short answer type questions	
6.	if $\left(\frac{1+i}{1-i}\right)^{100} = a + ib$, then find the value of (a, b) **Sol**. $\left(\frac{1+i}{1-i}\right)^{100} = a + ib$, $\left(\frac{1+i}{1-i} \times \frac{(1+i)}{(1+i)}\right)^{100} = a + ib$ $\left(\frac{(1+2i+i^2)}{(1-i^2)}\right)^{100} = a + ib$, $i^{100} = a + ib$, 1= $a + ib$, a=1 and b=0 (a,b)=(1,0)
7.	Show that The set of all the points z in the argand plane for which $\|z + 1\|^2 + \|z - 1\|^2 = 4$ is represent a circle. Sol. $\|z + 1\|^2 + \|z - 1\|^2 = 4$ $z = x + iy$ $\|x + iy + 1\|^2 + \|x + iy - 1\|^2 = 4$ $\left(\sqrt{(x + 1)^2 + y^2}\right)^2 + \left(\sqrt{(x - 1)^2 + y^2}\right)^2 = 4$ $2(x^2 + y^2 + 1) = 4$ $(x^2 + y^2) = 1$ Which represent a circle
8.	**Find the Value of** $\left\|\frac{1+2i+3i^2}{1-2i+3i^2}\right\|$ **Sol.** $\left\|\frac{1+2i+3i^2}{1-2i+3i^2}\right\| = \left\|\frac{1+2i-3}{1-2i-3}\right\| = \left\|\frac{-2+2i}{-2-2i}\right\| = \frac{\|-2+2i\|}{\|-2-2i\|} = \frac{\sqrt{(-2)^2+(2)^2}}{\sqrt{(-2)^2+(-2)^2}} = \frac{\sqrt{4}}{\sqrt{4}} = 1$
Long answer type questions	
9	**Find real θ such that** $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ **is purely real** **Sol.** We have , $\frac{3+2i\sin\theta}{1-2i\sin\theta} = \frac{(3+2i\sin\theta)(1+2i\sin\theta)}{(1-2i\sin\theta)(1+2i\sin\theta)} = \frac{3+6i\sin\theta+2i\sin\theta-4\sin^2\theta}{1+4\sin\theta^2}$ $= \frac{3 - 4\sin^2\theta}{1 + 4\sin^2\theta} + \frac{8i\sin\theta}{1 + 4\sin^2\theta}$ We are given the complex number to be real. Therefore $\frac{8i\sin\theta}{1+4\sin^2\theta} = 0$, i.e. $\sin\theta = 0$. thus $\theta = n\pi$ where n is aninteger
10	**Find the real value of θ for which the expression** $\frac{1+i\cos\theta}{1-2i\cos\theta}$ **is a real number.** **Sol.** $z = \frac{1+i\cos\theta}{1-2i\cos\theta} \times \frac{1+2i\cos\theta}{1+2i\cos\theta} = \frac{1+2i\cos\theta+i\cos\theta+2i^2\cos^2\theta}{1-4i^2\cos^2\theta}$ $= \frac{(1-2\cos^2\theta)+i(3\cos\theta)}{1+4\cos^2\theta}$ The given number is a real number hence the imaginary part of the complex number is zero $\therefore \frac{(3\cos\theta)}{1 + 4\cos^2\theta} = 0$ $\therefore \cos\theta = 0$ $\therefore \theta = 2k\pi \pm \frac{\pi}{2};$ $k\epsilon Z$

III .Questions for Practice	
MCQ	
1	The value of i^{528} (a) 1 (b) -1 (c) i (d) $-$i
2	$\left[i^{19} + \left(\frac{1}{i}\right)^{25}\right]^2$ is equal to (a) 4 (b) -4 (c) i (d) $-i$
3	The conjugate of i^{-35} (a) 1 (b) -1 (c) i (d) –i
4	If $z_1 = 3 + 2i$ and $z_2 = 2 - 4i$ and $\lvert z_1 + z_2\rvert^2 + \lvert z_1 - z_2\rvert^2$ is equal (a) 11 (b) 22 (c) 55 (d) 66
5	The real part of $\frac{(1+i)^2}{3-i}$ is (a) $\frac{1}{3}$ (b) $-\frac{1}{5}$ (c) $\frac{-1}{3}$ (d) None of these
6	If $4x + i(3x - y) = 3 + i(-6)$ then the values of x and y are (a) x =3, y =4 (b) x=3/4, y= 33/4 (c) x =4. y =3 (d) x =33. Y =4
7	If $i^{103} = a + ib$ then a +b is equal to (a) 1 (b) -1 (c) 0 (d) 2
8	Which of the following options defined 'imaginary number'? (a)Square root of any number (b)Square root of positive number (c) Square root of negative number (d)Cube root of number
9	If $z = \frac{7-i}{3-4i}$ then $\lvert z\rvert^{14}$ (a) 2^7

	(b) 2^7 i (c) -2^7 (d) -2^7 i
Assertion- Reason	
10	Assertion (A) if $\mathbf{i = \sqrt{-1}}$ then $\mathbf{i^{4k} = 1}$,$\mathbf{i^{4k+1} = i}$, $\mathbf{i^{4k+2} = -1}$ $\mathbf{i^{4k+3} = -i}$. Reason (R) $\mathbf{i^{4k} + i^{4k+1} + i^{4k+2} + i^{4k+3} = 1}$ (a) A is true, R is true; R is a correct explanation of A. (b) A is true, R is true; R is not a correct explanation of A. (c) A is true; R is false (d) A is false; R is true.
Short answer type	
11	If $(x + iy)^3 = u + iv$, then showthat $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$
12	Simplify $(1 + i)^4(1 + \frac{1}{i})^4$
13	If $z_{1,} = 2 - i$ and $z_2 = -2 + i$ find (i) $\text{Re}\left(\frac{z_1 z_2}{\overline{z_1}}\right)$ (ii) $\text{Im}\left(\frac{1}{z_1 \overline{z_1}}\right)$
14	Find the real values of x and y if $\frac{x-1}{3+i} + \frac{y-1}{3-i} = i$.
15	Show that $\left\lvert\frac{z-2}{z-3}\right\rvert = 2$ represents a circle. Find centre and radius .
16	If a $=\cos\theta + i\sin\theta$, find the value of $\frac{1+a}{1-a}$
17	If$(x + iy)(2 - 3i) = 4 + i$, then find x and y
18	Find x and y if $\left(\frac{1+i}{1-i}\right)^3 + \left(\frac{1-i}{1+i}\right)^3 = x + iy$
19	If $z(2 - i) = (3 + i)$, then z^{20} is equal to
20	Express $z = \left(\frac{3+2i}{2-3i}\right) + \left(\frac{3-2i}{2+3i}\right)$ in the form of a+ib.
Long answer type	
21	If x+iy= $\sqrt{\frac{a+ib}{c+id}}$, then prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$
22	If $z = x + iy$ and $w = \frac{1-iz}{z-i}$ and $\lvert w\rvert = 1$ then show that z is purely real.
23	For what values of x and y are the numbers $3 + ix^2 y$ and $x^2 + y + 4i$ are conjugate of each other.
24	If $\lvert z + 1\rvert = \sqrt{2}\,\lvert z - 1\rvert$ prove that z describes a circle.
25	Find real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real

IV. ANSWERS (Practice Questions)									
Q.1	a	**Q.2**	b	**Q.3**	c	**Q.4**	d	**Q.5**	b
Q.6	b	**Q.7**	b	**Q.8**	c	**Q.9**	a	**Q.10**	c
Q.11	------	**Q.12**	16	**Q.13**	$\frac{-2}{5}, 0$	**Q.14**	X=-4 and y = 6	**Q.15**	$r = \frac{2}{3}$ and cnetre $(\frac{10}{3}, 0)$
Q.16	$i\cot\frac{\theta}{2}$	**Q.17**	x= 5/13, y=14/13	**Q.18**	$x = 0$, $y = 0$	**Q.19**	-2^{10}	**Q.20**	0+i0
Q.21	------	**Q.22**	-----	**Q.23**	$x = \pm 2$, $y = -1$	**Q.24**	------	**Q.25**	$\theta = n\pi$

Complex Number and Quadratic Equation Test-1 (20 Marks)		
1	The value of i^{528} (a) 1 (b) -1 (c) i (d) – i	1
2	The conjugate of i^{-35} (a)1 (b) -1 (c)i (d) –i	1
3	If $i^{103} = a + ib$ then a +b is equal to (a) 1 (b) -1 (c) 0 (d) 2	1
4	If $Z_1 = 1 + i$, $Z_2 = 2 - i$ and $\overline{z_1 z_2}$= a +ib, then a +b is equal to (a) 2 (b) 1 (c) 3 (d) 4	1
5	$(1+i)^8+(1-i)^8$ equal to (a) 1 (b)2 (c) 8 (d) 32	1
6	**Assertion-Reason** Assertion (A) if $\mathbf{i = \sqrt{-1}}$ then $\mathbf{i^{4k} = 1}$,$\mathbf{i^{4k+1} = i}$, $\mathbf{i^{4k+2} = -1 i^{4k+3} = -i}$. Reason (R) $\mathbf{i^{4k} + i^{4k+1} + i^{4k+2} + i^{4k+3} = 1}$ (a) A is true, R is true; R is a correct explanation of A. (b) A is true, R is true; R is not a correct explanation of A. (c) A is true; R is false (d) A is false; R is true.	1
7	**Case study** An ant is moving around a few food pieces scattered on the floor along the curve $\left\|\frac{z-2}{z-3}\right\| = 2.$ (a)What is the shape of the path described by the ant (b) Find the equation of the path described.	4
8	If$(x + iy)(2 - 3i) = 4 + i$, then find x and y	2

9	Find real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real	2
10	If $z_{1,} = 2 - i$ and $z_2 = -2 + i$ find (i)$\text{Re}\left(\frac{z_1 z_2}{\overline{z_1}}\right)$ (ii) $\text{Im}\left(\frac{1}{z_1 \overline{z_1}}\right)$	3
11	Find the modulus and conjugate of $\frac{(2+i)^2}{3+i}$	3

Answers Test-1										
1	2	3	4	5	6	7	8	9	10	11
a	c	b	a	d	c	(i)Circle (ii)$x^2 + y^2 - 8x + 14 = 0$	x= 5/13, y= 14/13	$\theta = n\pi$	$\frac{-2}{5}$, 0	$\|z\|=\frac{\sqrt{10}}{2}$, $\bar{z}=\frac{13}{10} - i\frac{9}{10}$

Complex Number and Quadratic Equation **Test-2 (30 Marks)**		
1	$i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is equal to (a)0 (b) 1 (c) -1 (d) 2	1
2	If $z_1 = 3 + 2i$ and $z_1 = 2 - 4i$ and $\|z_1 + z_2\|^2 + \|z_1 - z_2\|^2$ is equal (a) 11 (b)22 (c) 55 (d)66	1
3	The real part of $\frac{(1+i)^2}{3-i}$ is (a)$\frac{1}{3}$ (b)$-\frac{1}{5}$ (c)$\frac{-1}{3}$ (d) None of these	1
4	If $z = -5i^{-15} - 6i^{-8}$ then $\bar{z}$ is equal to (a) -6-5i (b) -6 +5i (c) 6 -5i (d) 6 +5i	1
5	Multiplicative Inverse of complex number (1-2i)=... (a)$\frac{1}{5} + i\frac{2}{5}$ (b)$\frac{1}{5} - i\frac{2}{5}$ (c)$-\frac{1}{5} + i\frac{2}{5}$ (d) None of these	1
6	**Assertion-Reason** Assertion: The equation $ix^2 - 3ix + 2i = 0$ has non real roots. Reason: If a, b, c are real and $b^2 - 4ac \geq 0$, then the roots of the equation $ax^2 + bx + c = 0$ are real and if $b^2 - 4ac < 0$, then the roots of the equation $ax^2 + bx + c = 0$ are non-real. (a) A is true, R is true; R is a correct explanation of A. (b) A is true, R is true; R is not a correct explanation of A. (c) A is true; R is false (d) A is false; R is true.	1

7	**Case study** The conjugate of a complex number z is the complex number obtained by replacing i with -i number. It is denoted by $\overline{z}$. The modulus of a complex number $z = a + ib$ is defined as the non-negative real number $\sqrt{a^2 + b^2}$. It is denoted by $\|z\|$ i. e $\|z\| = \sqrt{a^2 + b^2}$ (a)If $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$, the find the value of x + y (b) If $f(z) = \frac{7-z}{1-z^2}$,where z = 1 + 2i then find $\|f(z)\|$.	4
8	Express the following complex number in the form $a + ib$ $\frac{3 - \sqrt{-16}}{1 - \sqrt{-9}}$	2
9	Evaluate $1 + i^2 + i^4 + i^6 + \cdots + i^{20}$.	2
10	If $z_{1,} z_2$ are $1 - i$ and $-2 + 4i$ respectively find $Im\left(\frac{z_{1,}z_2}{\overline{Z_1}}\right)$	3
11	Find the value of $(1 + i)^6 + (1 - i)^3$	
12	Solve the equation $\|z + 1\| = z + 2(1 + i)$	3
13	If $z = x + iy$ and $w = \frac{\|1-iz\|}{\|z-i\|}$ and $\|w\| = 1$ then show that z is purely real.	5
14	If x+iy= $\sqrt{\frac{a+ib}{c+id}}$, then prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$	5

Answers Test-2						
Q.1	**Q.2**	**Q.3**	**Q.4**	**Q.5**	**Q.6**	**Q.7**
a	d	d	a	a	d	(i)0, (ii)$\frac{\sqrt{5}}{2}$
Q.8	**Q.9**	**Q.10**	**Q.11**	**Q.12**	**Q.13**	**Q.14**
$\frac{3}{2} + \frac{1}{2}i$	1	2	$-2 - 10i$	$z = \frac{1}{2} - 2i$	------	-------

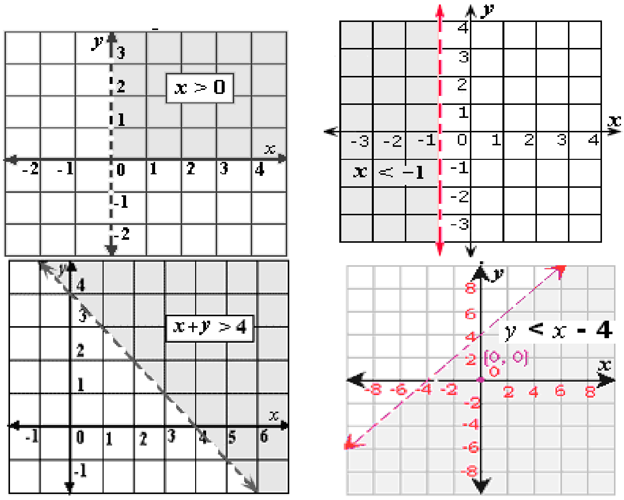
LINEAR INEQUALITIES

MAIN CONCEPTS AND RESULTS

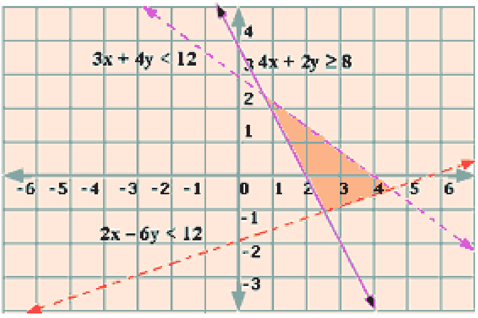
* Two real numbers or two algebraic expressions related by the symbol '<', '>', '≤' or '≥' form an **inequality**.

* **Numerical inequalities :** $3 < 5$; $7 > 5$
* **Literal inequalities :** $x < 5$; $y > 2$; $x \geq 3$, $y \leq 4$
* **Double inequalities :** $3 < 5 < 7$, $2 < y < 4$
* **Strict inequalities :** $ax + b < 0$, $ax + b > 0$, $ax^2 + bx + c > 0$
* **Slack inequalities :** $ax + by \leq c$, $ax + by \geq c$, $ax^2 + bx + c \leq 0$
* **Linear inequalities :** $ax + b < 0$, $ax + b \geq 0$
* **Quadratic inequalities :** $ax^2 + bx + c > 0$, $ax^2 + bx + c \leq 0$

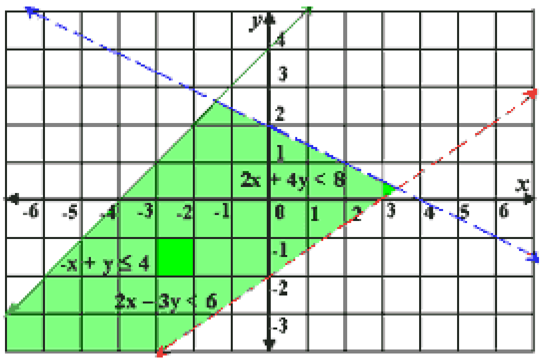
** **Algebraic Solutions of Linear Inequalities in One Variable and their Graphical Representation**



** **Graph of system of linear inequalities,** $2x - 6y < 12$, $3x + 4y < 12$ **and** $4x + 2y \geq 8$.



Graph the system of linear inequalities. $2x - 3y < 6$, $-x + y \leq 4$, $2x + 4y < 8$



II. Some Illustrations/ Examples with solutions

1. The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160 cm, then

(a) breadth > 20 cm (b) length < 20 cm (c) breadth $x \geq 20$ cm (d) length ≤ 20 cm

Correct option: (c) breadth $x \geq 20$ cm

Solution:

Let x be the breadth of a rectangle.

So, length = 3x

Given that the minimum perimeter of a rectangle is 160 cm.

Thus, $2(3x + x) \geq 160$

$\Rightarrow 4x \geq 80$

$\Rightarrow x \geq 20$

2.**If $-3x + 17 < -13$, then**

(a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$ (c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10)$

Correct option: (a) $x \in (10, \infty)$

Solution:

Given,

$-3x + 17 < -13$

Subtracting 17 from both sides,

$-3x + 17 - 17 < -13 - 17$

$\Rightarrow -3x < -30$

$\Rightarrow x > 10$ {since the division by negative number inverts the inequality sign}

$\Rightarrow x \in (10, \infty)$

3. The interval form of $x \leq -2$ is

(a) $x \in (-\infty, -2)$ (b) $x \in (-\infty, -2]$ (c) $x \in (-2, \infty]$ (d) $x \in [-2, \infty)$

Correct option: (b) $x \in (-\infty, -2]$

4.**If $|x - 1| > 5$, then**

(a) **$x \in (-4, 6)$ (b) $x \in [-4, 6]$ (c) $x \in (-\infty, -4) \cup (6, \infty)$ (d) $x \in [-\infty, -4) \cup [6, \infty)$**

Correct option: (c) $x \in (-\infty, -4) \cup (6, \infty)$

Solution:

$|x - 1| > 5$

$x - 1 < -5$ and $x - 1 > 5$

$x < -4$ and $x > 6$

Therefore, $x \in (-\infty, -4) \cup (6, \infty)$

In the following questions ,a statement of Assertion(A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices.

- **(a) Both (A) and(R) are true and (R) is the correct explanation of (A).**
- **(b) Both(A)and(R)are true but (R)is not the correct explanation of (A).**
- **(c) (A) is true but(R) is false.**
- **(d) (A) is false but(R) is true.**

5. Assertion(A) : For $x \in R$, and $x < 2$, then $x-2 < 0$

Reason (R): A number can be added or subtracted from both side of inequality without changing the sign of inequality.

Solution :(a) is correct , If $x < 2$ then subtract 2 from both sides , we get $x-2 < 0$ because we know that if we add or subtract same number both sides of a inequality then the sign of inequality remain unchanged.

Short type Question (Marks 2/ 3)

1. Solve the given linear inequalities $3x-2 < 2x+1$ and show the graph of the solution on the number line.

Solution:

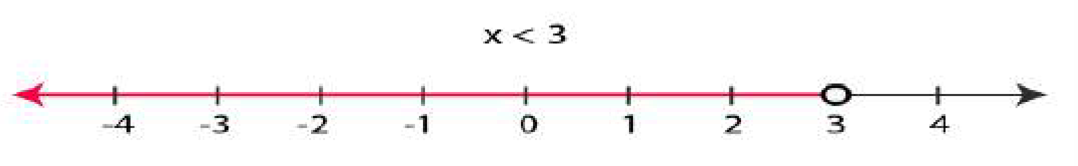
Given linear inequality: $3x-2 < 2x+1$

Bring the x terms on one side and constant terms on another side

$\Rightarrow 3x-2x < 1+2$

$\Rightarrow x < 3$

Therefore, the graphical representation for the solution of a linear inequality in number line is as follows:



2. Ravi scored 70 and 75 marks in the first two-unit test. Calculate the minimum marks he should get in the third test to have an average of at least 60 marks.

Solution:

Assume that x be the marks obtained by Ravi in the third unit test.

It is given that the student should have an average of at least 60 marks.

From the given information, we can write the linear inequality as:

$(70+75+x)/3 \geq 60$

Now, simplify the expression:

$\Rightarrow (145 + x) \geq 180$

$\Rightarrow x \geq 180 - 145$

$\Rightarrow x \geq 35$

Hence, the student should obtain a minimum of 35 marks to have an average of at least 60 marks.

3. The cost and revenue functions of a product are given by C(x) = 20 x + 4000 and R(x) = 60x + 2000, respectively, where x is the number of items produced and sold. How many items must be sold to realise some profit?

Solution:

Given that,

Cost, $C(x) = 20x + 4000$

Revenue, $R(x) = 60x + 2000$

We know that, profit = Revenue – Cost

Now, substitute the given data in the above formula,

Profit = $R(x) - C(x)$

Profit = $(60x + 2000) - (20x + 4000)$

Now, simplify it:

Profit = $60x + 2000 - 20x - 4000$

Profit = $40x - 2000$

To earn some profit, $40x - 2000 > 0$

$\Rightarrow 40x > 2000$

$\Rightarrow x > 2000/40$

$\Rightarrow x > 50$

Thus, the manufacturer should sell more than 50 items to realise some profit.

4. Solve the inequality $\frac{5x-2}{3} - \frac{7x-3}{5} > \frac{x}{4}$

Solution. $\frac{5x-2}{3} - \frac{7x-3}{5} > \frac{x}{4}$

Or $\frac{5(5x-2)-3(7x-3)}{15} > \frac{x}{4}$

Or $\frac{4x-1}{15} > \frac{x}{4}$

Or 4(4x-1) > 15x   Multiply both sides by 60 i.e lcm of 15 and 4

16x-4 > 15x

Or x >4 or $x \in (4, \infty)$

5. Solve the inequality $\frac{x}{2x+1} \geq \frac{1}{4}$.

Solution. $\frac{x}{2x+1} \geq \frac{1}{4}$

Now we have .$\frac{x}{2x+1} - \frac{1}{4} \geq 0$

Or $\frac{4x-(2x+1)}{4(2x+1)} \geq 0$

$$\frac{2x-1}{2x+1} \geq 0$$

Ist case 2x-1≥ 0 and $2x + 1 \geq 0 \Rightarrow x \geq 1/2$

IInd case2x-1 ≤ 0 and $2x + 1 < 0 \Rightarrow$ x <$\frac{-1}{2}$

Hence $x \in (-\infty, \frac{-1}{2}) \cup [\frac{1}{2}, \infty)$

III. Questions for Practice

MCQ (1 Mark Question)

1.Given thatx,y and b are real numbers and x<y, b >0 , then

A. $\frac{x}{b} < \frac{y}{b}$ B. $\frac{x}{b} \leq \frac{y}{b}$ C. $\frac{x}{b} > \frac{y}{b}$ D. $\frac{x}{b} \geq \frac{y}{b}$

2. The solution set of equation $|x + 2| \leq 5$ is

A. (-7,5)   B. [-7.3]   C. [-5,5]   D. (-7,3)

3. The shaded part of a line is in given figure can also be described as

A. $(-\infty, 1) \cup (2, \infty)$ B. $(-\infty, 1] \cup [2, \infty)$ C.(1,2)   D. [1,2]

4. A recharger manufacturing company produces rechargers and its cost function for a week is C(x) $= \frac{1}{10}(4270 + 23x)$ and its revenue function is R(x) = 3x, where x is the number of rechargers produced and sold per week. Number of rechargers must be sold for the company to realize a profit is

A. $x \geq 618$ B. x> 610   C. x> 480   D.None of These

In the following questions, a statement of Assertion(A)is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d)(A) is false but(R) is true.

5. Assertion (A): The solution set of the inequality x-3<2 ,$x \in N$ is {1,2,3,4,5,6,7,8}.

Reason (R) :Solution set of a inequality in x is set of values of x satisfying the inequality .

Answer . 1.A 2.B. 3. A 4. B 5. d

Short type questions (2marks/3 marks)

1. Solve the inequation $3x + 17 \leq 2(1 - x)$

2.Solve the inequality $\frac{x+3}{x-2} \leq 2$

3. Find all pair of consecutive odd integers , both are smaller than 18, such that their sumis more than20.

4.In a game, a person wins if he gets the sum greater than 20 in four throws of a die. In three throws he got numbers 6, 5, 4. What should be number in his fourth throw, so that he wins the game.

5.Solve the inequalities and represent the solution graphically on number line:
$5x + 1 > -24$, $5x - 1 < 24$.

6.Solve 3x-5 < x+1.Show the solution on number line.

7. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 liters of 9% acid solution, how many liters of 3% solution will have to be added?

8. Solve the inequality $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}$.

9.The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm then find the minimum length of the shortest side.

10. Solve the inequation $\frac{2x+4}{x-1} \geq 5$

IV Answer

Answer. 1. $x \leq -3$2. $x \in (-\infty, 2) \cup [7, \infty)$3. 11 and 13, 13 and 15, 15 and 17

4. x>5 i.e. 65. -5 <x <56. x < 2/3 7.More than 230 litres but less than 920 litres

8. $x \leq \frac{63}{10}$ or $x \in (-\infty, \frac{63}{10}]$9. 41 cm10. $1 < x \leq 3$

Class test- I

Time: One Hour **Maximum Marks :20**

1. Solving – 8 ≤ 5x – 3 < 7, we get **(1M)**

(a) –1/2 ≤ x ≤ 2(b) 1 ≤ x < 2(c) –1 ≤ x < 2(d) –1 < x ≤ 2

2. **If 4x + 3 < 6x +7, then x belongs to the interval** **(1M)**

(a) (2, ∞)(b) (-2, ∞)(c) (-∞, 2)(d) (-4, ∞)

3.**If |x −1| > 5, then** **(1M)**

(a) x∈ (– 4, 6)(b) x ∈ [– 4, 6](c) x ∈ (–∞, – 4) U (6, ∞)(d) x ∈ [–∞, – 4) U [6, ∞)

4. Solve 24x< 100 (2M)

(i) When x is a natural number(ii) X is an integer

5.Solve **3(x – 1) ≤ 2 (x – 3)** **(2M)**

6.Solve for x ; x + $\frac{x}{2}$+ $\frac{x}{3}$< 11 **(2M)**

7. To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in the first four examinations are 87, 92, 94 and 95, find the minimum marks that Sunita must obtain in the fifth examination to get Grade 'A' in the course. (2M)

8.The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side. (3M)

9. A man wants to cut three lengths from a single piece of board of length 91cm. The second length is to be 3cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5cm longer than the second? (3M)

10. Solve : -11< 4x-3 < 13 (3M)

Class Test – II

Time 1Hr 30 Minutes Max Marks 30

1.1.Given that x,y and b are real numbers and x<y, b >0 , then (1M)

A. $\frac{x}{b}<\frac{y}{b}$ B. $\frac{x}{b} \le \frac{y}{b}$ C. $\frac{x}{b}>\frac{y}{b}$ D. $\frac{x}{b} \ge \frac{y}{b}$

2.The shaded part of a line is in given figure can also be described as (1M)

A. (−∞ ,1) ∪ (2, ∞) B. (-∞, 1] ∪ [2, ∞) C.(1,2) D. [1,2]

3,If – 3x + 17 < – 13, then (1M)

(a) x∈ (10, ∞)(b) x ∈ [10, ∞)(c) x ∈ (–∞, 10](d) x ∈ [– 10, 10)

In the following questions, a statement of Assertion(A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d)(A) is false but (R) is true.

4, Assertion(A) : For $x \in R$, and $x < 2$, then $x-2 < 0$ (1M)

Reason (R) : A number can be added or subtracted from both side of inequality without changing the sign of inequality.

5.Solve : $\frac{5-2x}{3} \leq \frac{x}{6} - 5$ (2 M)

6. Draw the graphical solution of the following system of inequation : (2M)

$\frac{1}{2}\left(\frac{3}{5}x+4\right) \geq \frac{1}{3}(x-6)$

7.Solve $-12x > 30$, when (i) x is a natural number (ii) x is an integer. (2M)

8. Solve the inequality : $3(1-x) < 2(x+4)$ (2M)

9.Draw the graphical solution of the following system of inequalities : $2x - 7 > 5 - x$

and $11 - 5x \leq 1$ (3M)

10. I.Q. of a person is given by formula I.Q. = $\frac{MA}{CA} \times 100$, where M.A. stands for mental age and C.A., stands for chronological age. If $75 \leq$ I.Q. ≤ 135 for a group of 9 year children. Find the range of their mental age. (3M)

11. Given set $A = \{x \in Z : 0 \leq x \leq 10\}$. Solve the inequation $-2x + 6 \leq 5x - 4$ in set A. (3M)

12. Find all the pairs of consecutive even positive integers, both of which are greater than 10 and their sum is less than 50. (3M)

13. The longest side of a triangle is three times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side. (3M)

14.Solve $\frac{x-2}{x+5} > 2$. (3M)

PERMUTATIONS AND COMBINATIONS
MAIN CONCEPTS AND RESULTS

** Fundamental principle of counting, or(the multiplication principle): "If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrence of the events in the given order is m × n."

****Factorial notation** The notation n! represents the product of first n natural numbers, i.e., the product $1 \times 2 \times 3 \times \ldots \times (n-1) \times n$ is denoted as n!. We read this symbol as 'n factorial'.

Thus, $1 \times 2 \times 3 \times 4 \ldots \times (n-1) \times n = n!$

$n! = n(n-1)!$

$= n(n-1)(n-2)!$ [provided $(n \geq 2)$]

$= n(n-1)(n-2)(n-3)!$ [provided $(n \geq 3)$]

****Permutations** A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

** The number of permutations of n different objects taken r at a time, where $0 < r \leq n$ and the objects do not

repeat is $n(n-1)(n-2)\ldots(n-r+1)$, which is denoted by

$$P(n, r) \quad \text{OR} \quad {}^nP_r = \frac{n!}{(n-r)!}, 0 \leq r \leq n$$

** ${}^nP_0 = 1 = {}^nP_n$

** The number of permutations of n different objects taken r at a time, where repetition is allowed, is n^r.

** The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is $\frac{n!}{p_1!p_2!\ldots p_k!}$.

** The number of permutations of an dissimilar things taken all at a time along a circle is $(n-1)!$.

** The number of ways of arranging a distinct objects along a circle when clockwise and anticlockwise arrangements are considered alike is $\frac{1}{2}(n-1)!$.

** The number of ways in which (m + n) different things can be divided into two groups containing m and n things is $\frac{(m+n)!}{m!\,n!}$.

Combination of n different objects taken r at a time, denoted by ${}^nC_r = \frac{n!}{r!(n-r)!}$.

** ${}^nP_r = {}^nC_r\, r!, \quad 0 \leq r \leq n$

** ${}^nC_0 = 1 = {}^nC_n$

${}^nC_1 = n = {}^nC_{n-1}$

${}^nC_2 = \frac{n(n-1)}{2!} = {}^nC_2$

${}^nC_3 = \frac{n(n-1)(n-3)}{3!} = {}^nC_{n-3}$

** ${}^nC_r = {}^nC_s \Rightarrow r = s \text{ or } r + s = n$

** ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

II. Some illustrations/Examples

MCQs

Q.1 How many three digit numbers are there with all distinct digits1,2,3,4,5,6,7,8,9,0 ?

(a) 458 (b) 568 (c) 648 (d) 748

Sol. (c)9.9.8= 648

9	9	8

Q.2The H.C.F. of 6!, 8!, 9!, 11!. is

(a) 6! (b) 8! (c) 9! (d) 11!

Sol. (a) 6!

Q.3 A polygon has 14 sides then number of its diagonals are:

(a) 91 (b) 77 (c) 28 (d) none of these

Sol. (b) 77

Let the no. of sides is n.

Then no. of its diagonals is = $^{14}C_2 - 14 = 91 - 14 = 77$

Q.4 If $^nC_5 = {}^nC_7$, find n.

(a) 12 (b) 15 (c) 14 (d) 18

Sol. (a) 7 +5 = 12

Case study based

Q.1 A state cricket authority has to choose a team of 11 members, to do it so the authority ask 2 coaches of a government academy to select the team Members that have experience as well as best performer in the last 15 matches. They can make up a team of 11 cricketers amongst 15 possible candidates in which 5 players can bowl.

(i) In how many ways can the final eleven be selected from 15 cricket players ?

(ii) In how many ways can the final eleven be selected if exactly 4 bowlers must be included.

(iii) In how many ways can the final eleven be selected if all bowlers must be included.

Sol. (i) $^{15}C_{11} = 1365$

(ii) 4 bowlers can be select by = $^5C_4$

Remaining 7 players can be select out of 10 (15-5) is = $^{10}C_7$

Total no. of ways is = $^5C_4 . {}^{10}C_7 = 600$

(iii) all 5 bowlers can be select by = $^5C_5$

Remaining 6 players can be select out of 10 (15-5) is = $^{10}C_6$

Total no. of ways is = $^5C_5 . {}^{10}C_6 = 210$

Short answer type question

Q.1 How many number of rectangle forming by 5 different horizontal parallel lines and 7 other different vertical parallel line.

Sol. A rectangle forming by two horizontal and two vertical parallel lines

No. of rectangle = $^5C_2 . {}^7C_2 = 10 . 21 = 210$

Q.2 How many arrangement of the letters of the word APPLICATION.

Sol. Total letters 11, A = 2, P=2, I=2

No. of arrangements = $\frac{11!}{2!.\ 2!.\ 2!} = 4989600$

Q.3There are 12 point on a circle. How many chord can be draw? Find number of intersection of all chord in the circle?

Sol. No. of chord = $^{12}C_2$

No. of intersection of chord = no. of quadrilateral can be made = $^{12}C_4$

Long answer type questions

Q.1Find rank of the word MONDAY in the dictionary wise all arrangements of word MONDAY, also find the word which will on the rank 400.

Sol. Arrange alphabetically A,D,M,N,O,Y

Starting from A _ _ _ _ _ can be arrange by = 5! =120
Starting from D _ _ _ _ _ can be arrange by = 5! =120
Starting from M A _ _ _ _ can be arrange by = 4! =24
Starting from M D _ _ _ _ can be arrange by = 4! =24
Starting from M N _ _ _ _ can be arrange by = 4! =24
Starting from M O A _ _ _ can be arrange by = 3! = 6
Starting from M O D _ _ _ can be arrange by = 3! = 6
Starting from M O N A _ _ can be arrange by = 2! = 2
Starting from M O N D A Y can be arrange by = 1! = 1
Rank =120+120+24+24+24+6+6+2+1 = 327

	rank
Starting from A _ _ _ _ _ can be arrange by = 5! =120	120
Starting from D _ _ _ _ _ can be arrange by = 5! =120	240
Starting from M _ _ _ _ _ can be arrange by = 5! =120	360
Starting from N A _ _ _ _ can be arrange by = 4! =24	384
Starting from N D A _ _ _ can be arrange by = 3! =6	390
Starting from N D M _ _ _ can be arrange by = 3! =6	396
Starting from N D O A _ _ can be arrange by = 2! =2	398
Starting from N D O M _ _ can be arrange by = 2! =2	**400 answer**

Q.2Find the arrangements of the letters of the word VOLUMES. Find the arrangements if:-
(a) all vowels come together.
(b) all vowels don't come together.
(c) vowel don't come together.

Sol. Total alphabets = 7, vowels = 3

Total arrangements = 7! = 5040

(a) All vowels OUE is a one unit and remaining V, L, M, S can be arrange = 5! . 3! = 720
(b) Total arrangements – all vowels come together = 7! – 5! . 3! =5040-720=4320
(c) _V_L_M_S_ three vowels can be on these 5 gaps = $^5C_3$
And 3 vowels can be arrange by = 3!
And 4 consonants can be arrange by = 4!
Then total arrangement = $^5C_3$. 3! . 4! = 10 .6 . 24 = 1440

III .Questions for Practice

MCQs

Q.1 Number of words from the letters of the word **BHARAT** in which B and H will never come together is:

(a) 210 (b) 240 (c) 422 (d) 400

Q.2There are 10 true-false questions in a examination. If all questions are compulsory then these questions can be answered in:

(a) 210 (b) 512 (c) 422 (d) 1024

Q.3If $^{15}C_r : {}^{15}C_{r-1} = 11:5$ then r equals

(a) 15 (b) 11 (c) 5 (d) 4

Q.4 A polygon has 35 diagonals. Find the number of its sides.

(a) 7 (b) 16 (c) 10 (d) 70

Q.5 ASSERTION-REASON

In the following question, a statement of assertion (A) is followed by a statement of Reason(R).

Assertion (A): $^{13}C_9 = {}^{13}C_6$

Reason(R): Selection of the r distinct things out of n is equal to the rejection of the (n-r) distinct things out of n.

Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.

(d) A is false but R is true.

Case study based

Q.1In a company, CEO wants to establish a new branch. New branch required a committee of 5 members is to be formed out of 6 gents and 4 ladies.

In how many ways this can be done, when

i. At least two ladies are included?

ii. ii. At most two ladies are included?

Q.2Read the following passage and answer the questions given below.

Every person has Independence thought.

Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,

(i) Do the words start with P

(ii) Do all the vowels always occur together

(iii) Do the vowels never occur together

OR

Do the words begin with I and end in P?

Q.3Read the following passage and answer the question given below.

The longest river of North America is **Mississippi River**

In how many ways can the letters of the word MISSISSIPPI be arranged Such that

(i) All letters are used

(ii) All I's are together

(iii) All I's are not together

OR

All S 's are not together

Short answer type question

Q.1 If $^{10}P_r=5040$, find the value of r

Q.2 If $5. ^4C_r = 6 . ^5C_{r-1}$, find value of r.

Q.3 How many 6-digit number can be formed from the digits 0,1,3,5,7,9 which are divisible by 10 and no digit is repeated ?

Q.4 How many words can be formed by using the letters of the word ORIENTAL, so that the vowels always occupy the odd places ?

Q.5 How many squares in a chess board?

Q.6 How many palindrome of 5 letters can be made by using letters of the word MATHS?

Q.7 It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible ?

Q.8 Given12 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other?

Q.9 There are four bus routes between A and B; and three bus routes between B and C. A man can travel round-trip in number of ways by bus from A to C via B. If he does not want to use a bus

route more than once, in how many ways can he make round trip?
Q.10 In an examination there are three multiple choice questions and each question has 4 choices. Find the number of ways in which a student can fail to get all answer correct.

Long answer type questions

Q.1 A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) no girl ? (ii) at least one boy and one girl ? (ii) at least 3 girls ?
Q.2 Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, what will be the 50th word?
Q.3 In an examination, a question paper consists of 12 questions divided into two parts i.e., Part I and Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions ?
Q.4 How many number of signals that can be sent by 6 flags of different colours taking one or more at a time?
Q.5 A sports team of 11 students is to be constituted, choosing at least 5 from Class XI and at least 5 from Class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?

IV. ANSWERS of Practice questions :

MCQs

Q.1 B Q.2 D Q.3 C Q.4 C Q.5 A

Case study based

Q.1 (i) 186 (ii) 186
Q.2 (i) 138600 (ii) 16800 (iii) 1663200 – 16800 = 1646400 OR 12600
Q.3 (i) 34650 (ii) 840 (iii) 33810 OR 33810

Short answer type question

Q.1 4 Q.2 3 Q.3 120 Q.4 576
Q.5 204 Q.660 Q.7 2880 Q.8 132
Q.9 72 Q.10 63

Long answer type questions

Q.1 (i) 21 (ii) 441 (iii) 91 Q.2 60, NAAIG Q.3 420
Q.4 1956 Q.5 $2({}^{20}C_5 \cdot {}^{20}C_6)$

V. Two Chapter Test

TEST-1(20 MARKS)

Q.no	Questions	marks
1	Out of 18 points in a plane, no three are in the same line except five points which are collinear. Find the number of lines that can be formed joining the point.	1
2	Ten different letters of alphabet are given. Words with five letters are formed from these given letters. Then find the number of words which have at least one letter repeated.	1
3	If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary. Then what is the rank of the word RACHIT	1
4	Find the number of permutations of n distinct things taken r together, in which 3 particular things must occur together.	1
5	Find the total number of 9 digit numbers which have all different digits	1
6	The number of permutations of n different objects, taken r at a line, when repetitions are allowed, is _______.	1
7	In an examination, a student has to answer 4 questions out of 5 questions; questions 1 and 2 are however compulsory. Determine the number of ways in which the student can make the choice.	2
8	In a certain city, all telephone numbers have six digits, the first two digits always being 24 or 25 or 26 or 28 or 29. How many telephone numbers have all six distinct	2

	digits.	
9	Arrangement of the letters of the word INSTITUTIONS than find:- (a) Total arrangement. (b) If all vowels come together. (c) If no vowels come together. (d) Starting with I and end with S. (e) All vowels come together and all consonant come together.	5
10	Using the digits 1, 2, 3, 4, 5, 6, 7, a number of 4 different digits is formed. Find (a) how many numbers are formed? (b) how many numbers are exactly divisible by 2? (c) how many numbers are exactly divisible by 25? (d) how many of these are exactly divisble by 4? (e) how many even number can be formed?	5

TEST-2(30 MARKS)

Q.no	Questions	marks
1	Find the number of ways in which 5 prizes be distributed among 4 boys, while each boy is capable of having any number of prizes.	1
2	The total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two signs '–' occur together is ______.	1
3	Given 5 different green dyes, four different blue dyes and three different red dyes, find the number of combinations of dyes which can be chosen taking at least one green and one blue dye.	1
4	To fill 12 vacancies there are 25 candidates of which 5 are from scheduled castes. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, find the number of ways in which the selection can be made.	1
5	There are four bus routes between A and B; and three bus routes between B and C. A man can travel round-trip in number of ways by bus from A to C via B. If he does not want to use a bus route more than once, in how many ways can he make round trip?	1
6	In a office party 15 member are there. Find number of handshakes if every member handshake to other?	1
7	Every body in a room shakes hands with everybody else. The total number of hand shakes is 66. Then find total number of persons in the room.	2
8	Find the number of words which can be formed out of the letters of the word ARTICLE, so that vowels occupy the even place.	2
9	How many triangle can be draw in regular octagon. Also find all diagonals.	2
10		2
11	In a cinema hall 4 doors to enter and 3 other doors to exit. (a) How many ways a person can enter? (b) How many ways a person can exit? (c) How many ways a person can enter and then exit? (d) How many ways a person can enter and then exit but he can't enter and exit from the same door.	4
12	There are 3 books on Mathematics, 4 on Physics and 5 on English. How many different collections of 4 books can be made such that each collection consists of : (a) At least one book of each subject (b) At least 3 book of English (c) At least 2 book of physics	4

	(d) At most 2 book of maths	
13	Find the number of permutation of the letters of the word ALLAHABAD. In how many of these permutation (i) All the vowels always occur together (ii) The vowels never occur together	4
14	Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, (a) in how many ways can they be seated? (b) Find also the number of ways of their seating if all the ladies sit together.	4

ANSWERS of Chapter TEST-1

Q.1 144 Q.269760 Q.3 481 Q.4 $^{n-3}C_{r-3}(r-2)!.\ 3!$
Q.59 . 9! Q.6 n^r Q.73 Q.88400
Q.9(a) $\frac{12!}{3!3!2!2!}$ (b) $\frac{5!}{3!}\frac{8!}{3!2!2!}$ (c) $^8C_5\frac{5!}{3!}\frac{7!}{3!2!2!}$ (d) $\frac{10!}{2!3!2!}$ (e) 2! $\frac{5!}{3!}\frac{7!}{3!2!2!}$
Q.10(a) 840 (b) 360 (c) 40 (d) 200 (e) 630

ANSWERS of Chapter TEST-2

Q.1 4^5 Q.235 Q.33720 Q.4 $^5C_3 \times {}^{20}C_9$
Q.572 Q.6105 Q.712 Q.8144
Q.940, 28 Q.10
Q.11 (a) 4 (b) 3 (c) 12(d) 8Q.12(a) 720(b)75 (c) 201 (d) 360
Q.13(a) 360 (b) 7200 Q.14(a) 48(b) 144

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BINOMIAL THEOREM
MAIN CONCEPTS AND RESULTS

** Binomial theorem for any positive integer n

$$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}.b + {}^nC_2a^{n-2}.b^2 + \dots\ {}^nC_{n-1}a.b^{n-1} + {}^nC_nb^n = \sum_{k=0}^{n} {}^nC_ka^{n-k}b^k$$

** The coefficients ${}^nC_r$ occurring in the binomial theorem are known as binomial coefficients.

** There are (n +1) terms in the expansion of $(a+b)^n$, i.e., one more than the index.

** $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \dots + {}^nC_{n-1}x^{n-1} + {}^nC_nx^n$

** $(1-x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 - \dots + (-1)^n\ {}^nC_nx^n$.

** ${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n$.

** ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots + (-1)^{n-1}\ {}^nC_n = 0$.

** General Term in the expansion of $(a+b)^n$ = $t_{r+1} = {}^nC_ra^{n-r}.b^r$

** Middle term in the expansion of $(a+b)^n$

(i) $\left(\frac{n}{2}+1\right)^{th}$ term if n is even (ii) $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+1}{2}+1\right)^{th}$ terms if n is odd.

****Pascal's Triangle**

Index	Coefficients
0	${}^0C_0$ (=1)
1	${}^1C_0$ (=1) ${}^1C_1$ (=1)
2	${}^2C_0$ (=1) ${}^2C_1$ (=2) ${}^2C_2$ (=1)
3	${}^3C_0$ (=1) ${}^3C_1$ (=3) ${}^3C_2$ (=3) ${}^3C_3$ (=1)
4	${}^4C_0$ (=1) ${}^4C_1$ (=4) ${}^4C_2$ (=6) ${}^4C_3$ (=4) ${}^4C_4$ (=1)
5	${}^5C_0$ (=1) ${}^5C_1$ (=5) ${}^5C_2$ (=10) ${}^5C_3$ (=10) ${}^5C_4$ (=5) ${}^5C_5$ (=1)

ILLUSTRATIONS :

EXAMPLE 1:No. of terms in $(1+3x+3x^2+x^3)^6$ is

(A) 7 (B) 6 (C) 17 (D) 19

Ans: D 19
$(1+3x+3x^2+x^3)^6 = ((1+x)^3)^6 = (1+x)^{18}$
n=18 ∴ no. of terms = 19

EXAMPLE 2:$n_{C_0} + n_{C_1} + n_{C_2} + \cdots + n_{C_{n-1}} + n_{C_n} =$

(A)2n (B) 2^n (C) 2^{n-1} (D) none of these

Ans: B 2^n
$(1+a)^n = n_{c_0}a^0 + n_{c_1}a^1 + n_{c_2}a^2 + \cdots + n_{c_{n-1}}a^{n-1} + n_{c_n}a^n$
put a = 1

EXAMPLE 3:In $(1+x^3)^5$ coefficient of x^9 is

(A) 5 (B) 10 (C) 1 (D) NONE

Ans: B 10

$(1+x^3)^5 = 5_{C_0} + 5_{C_1}(x^3) + 5_{C_2}(x^3)^2 + 5_{C_3}(x^3)^3 + 5_{C_4}(x^3)^4 + 5_{C_5}(x^3)^5$

$= 5_{C_0} + 5_{C_1}x^3 + 5_{C_2}x^6 + 5_{C_3}x^9 + 5_{C_4}x^{12} + 5_{C_5}x^{15}$

$\therefore$the coefficient of x^9 is 5_{C_3}i.e. 10

EXAMPLE 4:Number of terms in $\left(1+5\sqrt{2}x\right)^9 + \left(1-5\sqrt{2}x\right)^9$ if $x > 0$ *is* _____

(A) 3 (B) 5 (C) 4 (D) 6

Ans: B 5

EXAMPLE 5:Find 6th term in $\left(3x - \frac{2}{3x}\right)^8$

Solution :$T_{r+1} = n_{C_r} x^{n-r} a^r$

$T_6 = T_{5+1} = 8_{C_5}(3x)^{8-5}(-\frac{2}{3x})^5$

$=56 \times 3^3 \times x^3 \times \left(-\frac{2}{3x}\right)^5 = -\frac{1729}{9}\frac{1}{x^2}$

EXAMPLE 6:Find the value of $(101)^4$ by using Binomial theorem :

Solution :$(1+x)^n = n_{C_0}x^0 + n_{C_1}x^1 + n_{C_2}x^2 + \cdots + n_{C_{n-1}}x^{n-1} + n_{C_n}x^n$

$$(101)^4 = (1+100)^4$$

$$= 4_{C_0}(100)^0 + 4_{C_1}(100)^1 + 4_{C_2}(100)^2 + 4_{C_3}(100)^3 + 4(100)^4$$

$$= 1 + 4(100) + 6(10000) + 4(1000000) + 100000000 = 104060401$$

EXAMPLE 7:Which of the following is larger $99^{50} + 100^{50}$ or 101^{50}

Solution :$(101)^{50} = (1+100)^{50}$

$$= 50_{C_0}(100)^{50} + 50_{C_1}(100)^{49} + 50_{C_2}(100)^{48} + \cdots + 50_{C_{49}}(100)^1 + 50_{C_{50}}(100)^0$$

$$(99)^{50} = (100-1)^{50}$$

$$= 50_{C_0}(100)^{50} - 50_{C_1}(100)^{49} + 50_{C_2}(100)^{48} + \cdots - 50_{C_{49}}(100)^1 + 50_{C_{50}}(100)^0$$

$(101)^{50} - (99)^{50} = 2\{50_{C_1}(100)^{49} + 50_{C_3}(100)^{47} + \cdots + 50_{C_{49}}(100)^1\}$

$(101)^{50} - (99)^{50} = 100 \times (100)^{49} + 2\{50_{C_3}(100)^{47} + \cdots + 50_{C_{49}}(100)^1\}$

$(101)^{50} - (99)^{50} - (100)^{50} = 2\left\{50_{C_3}(100)^{47} + 50_{C_4}(100)^{46} + \cdots + 50_{C_{49}}(100)^1\right\} > 0$

$(101)^{50} > (99)^{50} + (100)^{50}$

Therefore ,$(101)^{50}$ islarger

EXAMPLE 8:ASSERSTION AND REASONING:

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.

(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion

(c) Assertion is correct, Reason is incorrect

(d) Assertion is incorrect, Reason is correct.

ASSERSION(A):$n_{C_0} - n_{C_1} + n_{C_2} - n_{C_{3+}} \ldots + (-1)^{n-1} n_{C_{n-1}} + (-1)^n n_{C_n} = 0$

REASON (R)$(1+x)^n = n_{C_0}x^0 + n_{C_1}x^1 + n_{C_2}x^2 + \cdots + n_{C_{n-1}}x^{n-1} + n_{C_n}x^n$, For all n∈N

Ans: A , here both A and R are true. By Putting, x=-1 in R , A can be obtained.

LARGE ANSWER TYPE:

EXAMPLE 9: Prove that $12^n - 11n - 1$ is divisible by 121 For all n∈N by using binomial theorem.

Solution :

$$12^n = (1+11)^n = n_{C_0}11^0 + n_{C_1}11^1 + n_{C_2}11^2 + \cdots + n_{C_{n-1}}11^{n-1} + n_{C_n}11^n$$

$$= 1 + n11 + n_{C_2}11^2 + \cdots + n_{C_{n-1}}11^{n-1} + 11^n$$

$$12^n - n11 - 1 = 11^2\left(n_{C_2} + n_{C_3}11 \ldots + n_{C_{n-1}}11^{n-3} + 11^{n-2}\right)$$ divisible by 11^2 i.e. 121.

EXAMPLE 10: Expand $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4$, $x \neq 0$ using binomial theorem

Solution :$\left(1+\frac{x}{2}-\frac{2}{x}\right)^4 = \left[\left(1+\frac{x}{2}\right)-\frac{2}{x}\right]^4$

$=4_{C_0}\left(1+\frac{x}{2}\right)^4 + 4_{C_1}\left(1+\frac{x}{2}\right)^3\left(-\frac{2}{x}\right)^1 + 4_{C_2}\left(1+\frac{x}{2}\right)^2\left(-\frac{2}{x}\right)^2 + 4_{C_3}\left(1+\frac{x}{2}\right)^1\left(-\frac{2}{x}\right)^3 + 4_{C_4}\Big(1+$
x20−2x4

$= 1 \times \left[4_{C_0} + 4_{C_1}\left(\frac{x}{2}\right)^1 + 4_{C_2}\left(\frac{x}{2}\right)^2 + 4_{C_3}\left(\frac{x}{2}\right)^3 + 4_{C_4}\left(\frac{x}{2}\right)^4\right] + 4 \times \Big[3_{C_0} + 3_{C_1}\left(\frac{x}{2}\right)^1 + 3_{C_2}\left(\frac{x}{2}\right)^2 +$
3C3x23+4×31×22C0+2C1x21+2C2x224x2+41+x2−8x3+16x4

$$= 1 \times \left[1 + 4 \times \frac{x}{2} + \frac{4\times 3}{1\times 2} \times \frac{x^2}{4} + 4 \times \frac{x^3}{8} + \frac{x}{16}\right] + 4 \times \left[1 + 3 \times \frac{x}{2} + 3 \times \frac{x^2}{4} + \frac{x^3}{8}\right]\left(-\frac{2}{x}\right) + 6 \times \left[1 + 2 \times \frac{x}{2} + \frac{x^2}{4}\right]\left(\frac{4}{x^2}\right) + 4 \times \left(1+\frac{x}{2}\right)\left(-\frac{8}{x^3}\right) + \frac{16}{x^4}$$

$=1 + 2x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4 - \frac{8}{x} - 12 - 6x - x^2 + \frac{24}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} - \frac{16}{x^2} - \frac{16}{x^4}$

$=\frac{16}{x^4} - \frac{32}{x^3} + \frac{8}{x^2} + \frac{16}{x} - 5 - 4x + \frac{x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16}$

PRACTICE QUESTIONS

1	$(1+4x+4x^2)^{10}$ has 10 TERMS B) 11 TERMS C) 20 TERMS D)21 TERMS
2	The ratio of the coefficients of x^r and x^{r-1} in $(1+x)^n$ is A) $\frac{n+r}{r}$ B) $\frac{n-r+1}{r}$ C) $\frac{n+r-1}{r}$ D) NONE
3	If the coefficient of x^2 and x^3 in the expansion of $(3+mx)^9$ are equal , then the value of m is A) $-\frac{9}{7}$ B) $-\frac{7}{9}$ C) $\frac{9}{7}$ D) $\frac{7}{9}$
4	The term independent of x in the expansion of $\left(2x+\frac{1}{3x^2}\right)^6$ is A) 2nd B) 3rd C) 4th D) 5th
5	Using binomial theorem, evaluate $(99)^5$.
6	Expand $\left(x^2+\frac{3}{x}\right)^4$, $x \neq 0$ b using Pascal triangle.
7	Prove that: $(a+b)^6-(a-b)^6$ is aneven number if a and b are integers:
8	Find 4th term of the expansion $\left(3x+\frac{2}{x}\right)^6$
9.	Find 5th term from the end of the expansion $(a+bx)^7$
10.	Find $(a+b)^6-(a-b)^6$hence evaluate$\left(\sqrt{3}+\sqrt{2}\right)^6-\left(\sqrt{3}-\sqrt{2}\right)^6$
11	Find a, if the 4th and 5th term of the expansion $(2 + a)^7$ are equal.
12	Expand: $(x^2+1-2x)^3$
13	Find the value of $\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4$
14	Using binomial theorem prove that $5^{4n}+52n-1$ is divisible by 676 $\forall$ n $\in$ N
15	Find he middle term in the expansion$\left[\frac{x}{7}-\frac{5}{x}\right]^6$
16	Find the coefficient of x^4 in $\left[2x^2-\frac{3}{x}\right]^5$
17	Using the binomial theorem, show that 6^n-5n always leaves remainder 1 when divided by 25
18	Find a if coefficients of x^2 and x^3 in $(3+ax)^9$ are equal.
19	The coefficients of 2nd and 3rdterms in the expansion of $(1+a)^n$are in the ratio 1:2. Find n.
20	Find the middle term(terms) in the expansion of $\left(3x-\frac{x^3}{6}\right)^5$
21	**Case based Question**:In class XI, teacher explained binomial theorem. Two students Shivani and Vishwani trying to solve the exercise. Shivani expanded $(1+x)^6$ by using Binomial theorem, Viswani expanded $(x+1)^6$. **Based on this above information answer the following questions**. 1. According to Shivani find 4th term . (1M) 2. Find the value of $6_{C_0}+6_{C_1}+6_{C_2}+6_{C_3}+\dots\dots+6_{C_6}$ (1M) 3. Find the positive value of x if 3rd terms of Shivani and Vishwani are equal. (2M)

Answers:

1	D	6	$x^5+10x^2+40/x+80/x^4+80/x^7+32/x^{10}$	11	2	16	720
2	B	7	Hint: expand and simplify.	12	$x^6-6x^5+15x^4-20x^3+15x^2-6x+1$	17	Hint: writte $6^n=[1+5]^n$ the expand
3	C	8	4320	13	$2[a^8+6a^6-5a^4-2a^2+1]$	18	9/7
4	C	9	$21a^5b^2x^2$	14	Hint: write $5^{4n}=25^{2n}=(26-1)^{2n}=(1-26)^{2n}$ The expand	19	5

5	9509900499	10	$396\sqrt{6}$	15	$-20\frac{5^3}{7^3}$	20	$\frac{-5x^8}{12}$
21	(1)$20x^3$2)64 3) $6_{C_2}x^2=6_{C_2}x^4$ $\therefore x=\pm 1$ but x is positive $\therefore x=1$						

TEST – 1

CLASS :11 MAX MARKS : 20

MATHEMATICS BINOMIAL THEOREM

1	$(1+4x+4x^2)^{10}$ has A) 10 TERMS B) 11 TERMS C) 20 TERMS D)21 TERMS	1
2	**Value of** $\boldsymbol{n_{C_0}+n_{C_1}+n_{C_2}+\cdots+n_{C_{n-1}}+n_{C_n}}$ **is** A) 2^n B) n C)2^n-1 D) 2n	**1**
3	The ratio of the coefficients of x^r and x^{r-1} in $(1+x)^n$ is A) $\frac{n+r}{r}$ B)$\frac{n-r+1}{r}$ C)$\frac{n+r-1}{r}$ D) NONE	1
4	Expand $\left(x+\frac{1}{x}\right)^6$	2
5	Expand $(99)^4$ by using binomial theorem	2
6	Prove that $\sum_{r=0}^{n} 3^r\ {}^nC_r = 4^n$	2
7	Find 4[th] term of the expansion $\left(2x+\frac{3}{x}\right)^6$	2
8	Find 4[th] term from the end of the expansion $(a+bx)^5$	3
9	Expand $\left(x+\frac{2}{x^2}\right)^5$ by using Pascal's triangle	3
10	Find $(a+b)^6-(a-b)^6$ hence evalute $\left(\sqrt{3}+\sqrt{2}\right)^6-\left(\sqrt{3}-\sqrt{2}\right)^6$	3

ANSWERS:

1.D, 2.A,3.B, 4. $x^6+6x^4+15x^2+20+\frac{15}{x^2}+\frac{6}{x^4}+\frac{1}{x^6}$, 5.96059601, 7.4320, 8.10 $(a)^3(bx)^2$, 9. $x^5+10x^2+\frac{40}{x}+\frac{80}{x^4}+\frac{80}{x^7}+\frac{32}{x^{10}}$, 10.$396\sqrt{6}$

TEST – 2

CLASS :11 MAX MARKS : 30

MATHEMATICS BINOMIAL THEOREM

MCQ **5 X 1 = 5**

1	The number of terms in $((x-5)^2)^5$I A) 8 TERMS B) 9 TERMS C) 10 TERMS D)11 TERMS
2	Which one is True? A) $(1.2)^{4000}>800$ B) $(1.2)^{4000}<800$ C) $(1.2)^{4000}=800$ D)$(1.2)^{4000}=1600$
3	The term independent of x in the expansion of $\left(2x+\frac{1}{3x^2}\right)^9$ is A) 2[nd] B) 3[rd] C) 4[th] D) 5[th]
4	In the expansion of $\left(\frac{x}{3}-\frac{2}{x}\right)^6, x>0$, the constant term is A) $\frac{160}{27}$ B)$\frac{16}{27}$ C) $\frac{-160}{27}$ D) $\frac{120}{27}$
5	If the coefficient of x^2 and x^3 in the expansion of $(3+mx)^9$ are equal , then the value of m is A) $-\frac{9}{7}$ B) $-\frac{7}{9}$ C) $\frac{9}{7}$ D) $\frac{7}{9}$

Short Answer type **5 X 2 = 10**

6	Using binomial theorem, evalute$(101)^4$.
7	Prove that $\sum_{r=0}^{n} 3^r \ {}^nC_r = 4^n$
8	Expand $\left(x^2 + \frac{3}{x}\right)^4$, $x \neq 0$.
9	Expand $(1 + x + x^2)^4$
10	Prove that$(a + b)^4 - (a - b)^4$is an even number if a and b are integers:

Long Answer type **3 X 5 =15**

11	Find the remainder when 3^{3n}-26n is devided by 676, where n∈ N
12	Find the coefficient of a^4 in the product $(1 + 2a)^4(2 - a)^4$ using binomial theorem
13	Show that $2^{4n+4} - 15n - 16$, where n ∈ N is divisible by 225.

ANSWERS:

1	D	6	104060401	11	Write $3^{3n}=27^n=(1+26)^n$ Then expand
2	A	7	Hint: write $4^n=(1+3)^n$ then expand	12	16
3	S	8	$x^8 + 12x^5 + 54x^2 + \frac{108}{x} + \frac{81}{x^4}$		
4	C	9	$x^8 + 4x^7 + 10x^6 + 16x^5 + 19x^4 + 16x^3 + 10x^2 + 4x + 1$		
5	C	10	Hint: expand and simplify.		

SEQUENCE AND SERIES
CONCEPTS AND RESULTS

** **Sequence :** is an arrangement of numbers in a definite order according to some rule. A sequence can also
be defined as a function whose domain is the set of natural numbers or some subsets of the type {1, 2, 3....k). ** A sequence containing a finite number of terms is called a finite sequence. A sequence is called infinite if it is not a finite sequence.

** **Series :** If $a_1, a_2, a_3,\ldots,an$, be a given sequence. Then, the expression $a_1 + a_2 + a_3 +,\ldots+ a_n + ...$

** **Arithmetic Progression (A.P.) :** is a sequence in which terms increase or decrease regularly by the same
constant.

A sequence $a_1, a_2, a_3,\ldots, a_n,\ldots$ is called arithmetic sequence or arithmetic progression if $a_n + 1 = a_n + d$, $n \in N$, where a_1 is called the first term and the constant term d is called the common difference of the A.P.

** The n^{th} term (general term) of the A.P. a, a + d, a + 2d, ... is **$a_n = a + (n – 1) d$.**

** If a, b, c are in A.P. and $k(\neq 0)$ is any constant, then

(i) a + k, b + k, c + k are also in A.P.

(ii) a – k , b – k , c – k are also in A.P.

(iii) ak, bk, ck are also in A.P

(iv) $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in A.P.

** If a, a + d, a + 2d, …, a + (n – 1) d be an A.P. Then l = a + (n – 1) d.

$$\text{Sum to n terms } S_n = \frac{n}{2}[2a + (n-1)d$$

$$= \frac{n}{2}[a + l]$$

** **Arithmetic mean (A.M.)** between two numbers a and b is $\frac{a+b}{2}$.

** **n arithmetic means** between two numbers a and b are

$$a + \frac{(b-a)}{n+1},\ a + \frac{2(b-a)}{n+1},\ a + \frac{3(b-a)}{n+1},\ \ldots\ ,a + \frac{n(b-a)}{n+1}.$$

** Sum of n $A.M.^S$ = n(single A.M.)

** Three consecutive terms in A.P. are a – d , a, a + d.

Four consecutive terms in A.P. are a – 3d , a – d , a + d, a + 3d.

Five consecutive terms in A.P. are a – 2d , a – d , a, a + d, a + 2d.

These results can be used if the sum of the terms is given.

** In an A.P. the sum of terms equidistant from the beginning and end is constant and equal to the sum of first and last terms.

** m^{th} term from end of an A.P. = $(n – m + 1)^{th}$ term from the beginning.

****Geometric Progression (G . P.) :A** sequence is said to be a geometric progression or G.P., if the ratio of any term to its preceding term is same throughout.

A sequence $a_1, a_2, a_3,\ldots, a_n,\ldots$ is called geometric progression, if each term is non-zero and

$$\frac{a_{k+1}}{a_k} = r(\text{constant}),\ \text{for } k \geq 1.$$

By taking $a_1 = a$, we obtain a geometric progression, a, ar, ar^2, ar^3,…., where a is called the first term
and r is called the common ratio of the G.P.

** **General term of a G .P. =** $a_n = ar^{n-1}$.

** **Sum to n terms of a G .P. =** $\frac{a(r^n - 1)}{r-1}$ if $r > 1$ **and** $\frac{a(1-r^n)}{1-r}$ if $r < 1$.

** **Sum of terms of an infinite G.P. =** $\frac{a}{1-r}$.

** **Geometric Mean (G .M.)**: of two positive numbers a and b is the number is $\sqrt{ab}$.

** **n geometric mean** between two numbers a and b are $a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}, a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}, a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}, \ldots\ a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$.

** Three consecutive terms in G.P. are $\frac{a}{r}$, a, ar .

Four consecutive terms in G.P. are $\frac{a}{r^3}, \frac{a}{r}$, ar, ar^3.

Five consecutive terms in G.P. are $\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2.

These results can be used if the product of the terms is given.

** **Harmonic Progression :** A series of quantities is said to be in harmonic progression if their reciprocals are in arithmetic progression.

** n th term of the H.P. $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots$ is $\frac{1}{a+(n-1)d}$.

** **Harmonic Mean** between two quantities a & b is $\frac{2ab}{a+b}$

** Relations b/w **A**(A.M.), **G**(G.M.) & **H**(H.M.)

(i) A, G, H are in G.P.

(ii) A, G, H are in descending order of magnitude i.e. A > G > H.

** **Arithmetico-geometric series** : A type of series in which each term is the product of the corresponding

terms of an A.P. and a G.P.

$a + (a+d)r + (a+2d)r^2 + (a+3d)r^3 + \ldots$ is an arithmetico-geometric series.

** Sum to n terms of the arithmetico-geometric series : $a + (a+d)r + (a+2d)r^2 + (a+3d)r^3 + \ldots$

is $S = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]r^n}{1-r}$

** Sum of an Infinite arithmetico-geometric series = $\frac{a}{1-r} + \frac{dr}{(1-r)^2}$.

** Some useful results

$$\sum n = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

$$\sum n^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum n^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

ILLUSTRATIONS

Example 1: A man saves Rs 135/- in the first year, Rs 150/- in the second year and in this way he increases his savings by Rs 15/- every year. In what time will his total savings be Rs 5550/-?

a) 20 years (b) 25 years (c) 30 years (d) 35 years

Ans: a) 25 years

Example 2: The fourth, seventh and tenth terms of a G.P. are p, q, r respectively, then :
(a) $p^2 = q^2 + r^2$ (b) $q^2 = pr$ (c) $p^2 = qr$ (d) $pqr + pq + 1 = 0$
Ans: (b) $q^2 = p$

Example 3. What is the 20^{th} term of the sequence defined by $a_n = (n-1)(2-n)(3+n)$?
Solution: Put n =20, we get $A_{20} = (20-1)(2-20)(3+20) = 19 .(-18).23 = -7866$
Example 3: In a G.P the 3^{rd} term is 24 and 6^{th} term is 192 . Find the 10^{th} term
Solution : let a be the first term and r be common ratio of given G.P. , therefore

$ar^2 = 24$ (1)

and $ar^5 = 192$ (2)

divide (2) by (1) we get

$\frac{ar^5}{ar^2} = \frac{192}{24}$ => $r^3 = 8$ => r = 2

Putting r=2 is (1) we get ,

a x 4 = 24 or a= 6

$T_{10} = ar^9 = 6 . 2^9 = 6 . 512 = 3072$

Example 4 . Find the sum of the sequence 7, 77, 777, 7777, ... to n terms.
Solution :This is not a G.P., however, we can relate it to a G.P. by writing the terms as
S_n = 7 + 77 + 777 + 7777 + ... to n terms

$= \frac{7}{9}$ [9 +99 + 999+ 9999+…………… to n terms]

$=\frac{7}{9}$ [(10-1) +(10^2 -1) + (10^3 -1)+ (10^4 -1)+…………… to n terms]

$=\frac{7}{9}$ [(10 +10^2 + 10^3 + 10^4 +… to n terms) – (1+1+1+…n terms)]

$=\frac{7}{9}\left[\frac{10(10^n-1)}{10-1}\right] - n = \frac{7}{9}\left[\frac{10(10^n-1)}{9}\right] - n$

Example 5: If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b, then find the value of n.

Solution

$$\because \frac{a^n + b^n}{a^{n-1} + b^{n-1}} \; is\ the\ A.M.\ b/w\ a\ and\ b$$

$$\Rightarrow \frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a+b}{2} \Rightarrow 2a^n + 2b^n = a^n + ab^{n-1} + ba^{n-1} + b^n$$

$$\Rightarrow a^n - ba^{n-1} + b^n - ab^{n-1} = 0 \Rightarrow a^{n-1}(a-b) - b^{n-1}(a-b) = 0$$

$$\Rightarrow (a^{n-1} - b^{n-1})(a-b) = 0$$

$$\Rightarrow (a^{n-1} - b^{n-1}) = 0 \; as\ (a-b) \neq 0$$

$$\Rightarrow a^{n-1} = b^{n-1} \Rightarrow \frac{a^{n-1}}{b^{n-1}} = 1 \Rightarrow \left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^0$$

$$\Rightarrow n - 1 = 0 \Rightarrow n = 1$$

Example 6 :The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $(3+2\sqrt{2}):(3-2\sqrt{2})$.
Solution

Let the two numbers be a and b

$\therefore\ a + b = 6\sqrt{ab} \qquad \Rightarrow \dfrac{a+b}{2\sqrt{ab}} = \dfrac{3}{1}$

$\Rightarrow \dfrac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \dfrac{3+1}{3-1}$ [by componendo and dividendo]

$\Rightarrow \dfrac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = 2 \qquad \Rightarrow \dfrac{(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})} = \dfrac{\sqrt{2}}{1}$

$\Rightarrow \dfrac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ [by C and D]

$\Rightarrow \dfrac{2\sqrt{a}}{2\sqrt{b}} = \dfrac{\sqrt{2}+1}{\sqrt{2}-1} \qquad \Rightarrow \dfrac{a}{b} = \dfrac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}} = \dfrac{3+2\sqrt{2}}{3-2\sqrt{2}}$

$\therefore\ a : b = (3+2\sqrt{2}) : (3-2\sqrt{2})$

MULTIPLE CHOICE QUESTIONS :

Q1. The product of 5 terms of a G.P. , whose 3^{rd} term is 2 is

A) 5^2 B) 2^5 C) 3^3 D) 3^5

Q2. The first term of a G.P. is 2 and the sum to infinity is 6. Then the common ratio is

A) 1/2 B) 1/3 C) 1/4 D) 2/3

Q3. $9^{1/3}\ .9^{1/9}\ .9^{1/27}$……… to ∞ equals

A) 1 B) 2 C) 3 D) 9

Q4. If nth term of a sequence is $a_n = (-1)^{n-1}\ n^3$, then its 9^{th} term is

A) 105 B) 177 C) 324 D) 729

Q5. Statement I :The G.P. 2 , 2√2 , 4 , ….. 128 is 13^{th} term of the series.

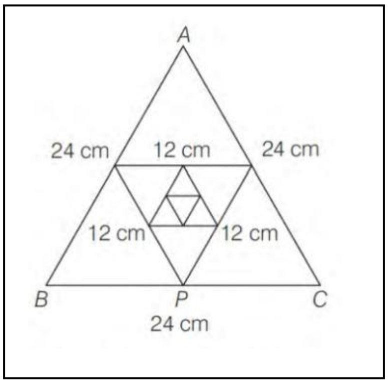
Statement II: A series in which the result of dividing any term by the previous term (if any) is same is called G.P.

a) Both the statement I and Statement II are true and statement II is the correct explanation of Statement I
b) Both the statement I and Statement II are true and statement II is not the correct explanation of Statement I
c) Statement I is true but Statement II is false
d) Statement I is false but Statement II is true

Q6 Case study question:

There is an equilateral triangle with each side of length 24cm. themid point of its sides are joined to form another triangle . This process is going continuously infinite times.

Based on the above information answer the following questions :



(i) The side of the 5^{th} triangle is (in cm)

a) 3 B) 6 c) 1.5 d) 0.75

(ii) The sum of perimeter of first 6 triangles (in cm) is

a) 569/4 B) 567/4 c) 120 d) 144

(iii) The area of all the triangles (in sq cm) is

a) 576 B)192√3 c) 144√3 d) 169√3

(iv) The sum of perimeter of all the triangles (in cm) is

a) 144 B)169 c) 400 d) 625

(v) The perimeter of the 7^{th} triangle (in sq) is

a) 7/8 B)9/8 c) 5/8 d) 3/4

Q 7.A company produces 500 computers in the third year and 600 computers in the seventh year. Assuming that the production increases uniformly by a constant number every year, answer the following questions.

(i) How many computers were produced in the first year?
(ii) By what number does the production increase every year?
(iii) How many computers will be produced in the 21st year?
(iv) Find the total production in 10 years.

Q 8 In a G.P. , the 3rd term is 24 and the 6th term is 192
Based on the above information answer the following questions :
(i) First term of G. P. is
a) 6 b) 8 c) 9 d) 10
(ii) Common ratio of G.P. is
a) 3 b) 2 c) 4 d)6
(iii) 10th term of G.P. is
a) 3150 b) 3060 c) 3072 d) 3070
(iv) Sum of first six terms of G.P. is
a) 360 b) 370 c) 380 d)378

Short answer questions

Q9. Write the first five terms of the sequence whose nth term is given by $a_n = 2^n$.
Q 10. The Fibonacci sequence is defined by $a_1 = a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$, n> 2. Find $\frac{a_{n+1}}{a_n}$ for n= 1, 2, 3, 4 ,5
Q 11.Find the 12th term of a G.P. whose 8th term is 192 and the common ratio is 2.
Q 12.The first term of a geometric progression is 1 . The sum of the third and fifth term is 90. Find the common ratio of the geometric progression .
Q13. For what values of x , the numbers -2/7 , x , -7/2 are in G.P.
Q 14.Given a G.P. with a =729 and 7th term 64, determine S_7 .
Q 15.At the end of each year the value of a certain machine has depreciated by 20% of its value at the beginning of that year. If its initial value was Rs. 1250, find the value at the end of 5 years
Q16. Determine the 3number n of terms in a geometric progression { a_n } , if $a_1 = 3$, $a_n =96$ and $S_n=189$
Q17. Find S for G.P. , $-\frac{3}{4}, \frac{3}{16}, -\frac{3}{64}, \ldots \ldots$
Q18. Insert two numbers 3 and 81 so that the resulting sequence is G.P.

Long answer type questions

Q19. Find sum of 50 terms of the sequence 7 ,7.7, 7.77 ,7.777 ……
Q20. Let the ratio of A.M. and the G.M. of two positive numbers a and b be m:n . Show that

$$a:b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$$

Q21. If the pth, qth and rth terms of a G.P. are a, b and c, respectively. Prove that $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$.
Q22. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.
Q23. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio (3+ 2√2) : (3- 2√2)

ANSWERS :

1. B) 2^5 2. D) 2/3 3. C) 3 4. D) 729 5. B 6. (i) c (ii) b (iii) b (iv) a (v) b
7. (i) 450 (ii) 25 (iii) 950 (iv) 5625 8. (i) a (ii) b (iii) c (iv) d 9. 2 ,4 , 8 ,16 , 32
10. 1, 2, 3/2, 5/3,8/5 11. 3072 12.±3 13.±1 14.2059
15. 409.6 16. 6 17.-3/5 18.9 and 27

19. $\frac{3493}{9} - \frac{7}{81}\left(1 - \frac{1}{10^{49}}\right)$ 20. Prove yourself 21. Prove yourself 22. 16/7 ; 2 ; $16(2^n-1)/7$
23. Prove yourself

MATHS CLASS 11
SEQUENCE AND SERIES
PRACTICE TEST-1(MAX MARKS 20)

Note: Q no. 1 to 5 are 1 marks , Q6 &7 are 2 marks , Q no. 8 &Q no.9 are 3 marks and Q no. 10 is 5 marks

Q1. 4th term from the end of the G.P. 3, 6, 12, 24., ..., 3072 is
(a) 348 (b) 843 (c) 438 (d) 384)

Q2. For a G.P. the ratio of the 7th and the third terms is 16. What is the common ratio?
(a) 2 (b) ±2 (c) 4 (d) ±4

Q3which term of geometric progression : 2 ,8 , 32, …. Is 131072
a) 9^{th} b) 10^{th} c) 11^{th} d) 12^{th}

Q4 $9^{1/3} .9^{1/9} .9^{1/27}$……… to ∞ equals
a) 1 b) 2 c) 3 d) 9

Q5. Statement –I : Sum of the series $-\frac{3}{4}, \frac{3}{16}, -\frac{3}{64}, \ldots\ldots\ldots\ldots....$ is $-\frac{2}{3}$.
Statement –II :$s = \frac{a}{1-r}$ denotes the sum of infinite G.P. with first term a and common ratio r.

a) Both the statement I and Statement II are true and statement II is the correct explanation of Statement I
b) Both the statement I and Statement II are true and statement II is not the correct explanation of Statement I
c) Statement I is true but Statement II is false
d) Statement I is false but Statement II is true

Q 6. A person has 2 parents, 4 grandparents, 8 great grandparents, and so on. Find the number of his ancestors during the ten generations preceding his own.

Q7. Find the sum of 0.15, 0.015, 0.0015, ...upto 20 terms

Q8. Insert three numbers between 1 and 256 so that the resulting sequence is a G.P.

Q9. If a, b, c and d are in G.P. show that $(a^2+b^2+c^2)(b^2+c^2+d^2)=(ab+bc+cd)^2$.

Q10.If A.M. and G.M. of two positive numbers a and b are 10 and 8, respectively, find the numbers.

MATHS CLASS 11
SEQUENCE AND SERIES
PRACTICE TEST-2 (MAX MARKS 30)

Note: Q no. 1 to 5 are 1 marks , Q6 to 8 are 2 marks , Q no. 9 to Q no. 11 are 3 marks and Q no. 12 and Q. no13 are 5 marks

Q1. If 3^{rd} , 8^{th} and 13^{th} terms of G.P. are p,q and r respectively , then which one of the following is correct
a . q^2 =pr b. r^2 = pq c. pq= r d. 2q = p+r

Q2. . If nth term of a sequence is $a_n = (-1)^{n-1} n^3$, then its 9^{th} term is
A) 105 B) 177 C) 324 D) 729

Q3. geometric mean between 1 and 256 IS
A) 8 b. 16 c. 14 d. 12

Q4. If x, 2x+3 ,3x+3 are in G.P. ,then 4^{th} term is
A) -13.5 b. -14.5 c.-15.5 d. -16.5

Q5. Statement I : Four terms of the G.P. 3 ,3^2 , 3^3 ,…. Are needed to give the sum 120
Statement II: T_n $=ar^n$ is n^{th} terms of G.P. whose first term is a and common ratio r.

a) Both the statement I and Statement II are true and statement II is the correct explanation of Statement I
b) Both the statement I and Statement II are true and statement II is not the correct explanation of Statement I
c) Statement I is true but Statement II is false
d) Statement I is false but Statement II is true

Q6 Write first five terms of sequence whose n^{th} term is given by $a_n = (-1)^{n-1}\ 5^{n+1}$

Q7.for what values of x , the numbers -2/7 ,x -7/2 are in G.P.

Q8. Find the 12^{th} term of a G.P. whose 8^{th} term is 192 and the common ratio is 2.

Q9. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Q10. Find the sum to n terms of the sequence, 8, 88, 888, 8888… .

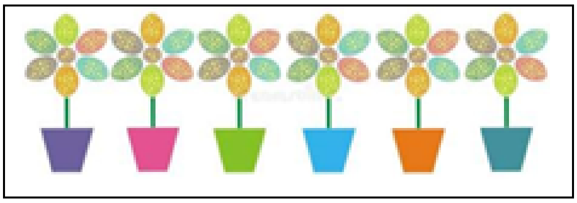
11. Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2R^n = S^n$.

12. If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.

13 **Case study**

Rahul being a plant lover decides toopen a nursery and he bought fewplants with pots. He wants to place potsin such a way that number of pots infirst row is 2, in second row is 4 and in third row is 8 and so on. Answer the following questions based on the above information.

(i) Find the number of pots in the 8th row.
(ii) Find the total number of pots in 10 rows.
(iii) If Rahul wants to place 510 pots in all, how many rows will be formed?



ANSWERS OF PRACTICE TEST-1

Q1. D	Q2.B	Q3.A	Q4. C.	Q5. D	
Q 6 . 2046	Q7 . $(1-(0.1)^{20})/6$	Q8. 4, 16, 64	Q10.4 and 16	Q11. 3072	

ANSWERS OF PRACTICE TEST-2

1. A	2. D	3.B	4.A	5. C
6. 25 , -125, 625 , -3125 , 15625		7. ± 1	8. 3072	9. $\frac{5}{2}$, 1 and $\frac{2}{5}$.

10. $8(10^{n+1}-10-9n)/8$ 13(i) 256 (ii) 2046 (iii) 8

*****

STRAIGHT LINES
CONCEPTS AND RESULTS

****** Any point on the X-axis is (x, 0) and on the Y-axis is (0, y)

****** Distance between two points $A(x_1, y_1)$ & $B(x_2, y_2)$ is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

**** Section formula**

(i) Coordinates of a point dividing the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ internally in the ratio m : n is

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right).$$

(ii) Coordinates of a point dividing the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ externally in the ratio m : n is

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right).$$

** Coordinates of the mid point of the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ is $\left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right)$

****** Centroid of a ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

****** In centre of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ is

$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$ where a = BC, b = AC, c = AB.

****** Area of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3) = \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.

****** Equation of any line parallel to X-axis is y = a, & equation of X-axis is y = 0.

****** Equation of any line parallel to Y-axis is x = b & equation of Y axis is x = 0.

****** Slope of line inclined at an angle θ with the + ve X- axis = tan θ.

****** Slope of a line parallel to X-axis = 0 , slope of a line parallel to Y-axis = undefined.
Slope of a line equally inclined to the coordinate axes is –1 or 1.

****** Slope of a line joining the points $A(x_1, y_1)$, $B(x_2, y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$.

****** Slope of the line ax + by + c = 0, is $-\frac{a}{b}$.

****** If two lines are parallel, then their slopes are equal.

****** If two lines are perpendicular, then the product of their slopes is –1.

****** Any equation of the form Ax + By + C = 0, with A and B are not zero, simultaneously, is called the general linear equation or general equation of a line.

(i) If A = 0, the line is parallel to the x-axis (ii) If B = 0, the line is parallel to the y-axis

(iii) If C = 0, the line passes through origin.

****** Equation of a line having slope = m and cutting off an intercept '**c**' and Y-axis is y = mx + c.

****** Equation of a line through the point (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$.

****** Equation of a line making intercepts of 'a' & 'b' on the respective axes is $\frac{x}{a} + \frac{y}{b} = 1$

****** The equation of the line having normal distance from origin p and angle between normal and the positive x-axis ω is given by x cosω + y sinω = p .

****** Distance of a point $P(x_1, y_1)$ from the line ax + by + c = 0 is $d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.

** Equation of the line parallel to $ax + by + c = 0$ is $ax + by + \lambda = 0$.
** Equation of the line perpendicular to $ax + by + c = 0$ is $bx - ay + \lambda = 0$.
** If two lines are intersecting and θ is the angle between them, then $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$ where m_1 = slope of first line, m_2 = slope of second line and θ = acute angle.
If $\tan\theta$ = negative $\Rightarrow \theta$ = obtuse angle between the intersecting lines.
** Distance between two parallel lines $ax + by + c_1 = 0$ & $ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

CONIC SECTIONS

CONCEPTS AND RESULTS

CIRCLES

A circle is a locus of a point which moves in a plane such that its distance from a fixed point in that plane in always constant. The fixed point is said to be the circle and the constant distance is said to be the radius.
** The equation of a circle with centre (h , k) and the radius r is $(x - h)^2 + (y - k)^2 = r^2$.
** The equation of a circle with centre (0 , 0) and the radius r is $x^2 + y^2 = r^2$.
** General equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ whose centre is $(-g, -f)$ and radius is $\sqrt{g^2 + f^2 - c}$
** Equation of a circle when end points of diameter as $A(x_1, y_1)$, $B(x_2, y_2)$ is given by

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0.$$

** Length of intercepts made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on the X and Y-axes are $2\sqrt{g^2 - c}$ and $2\sqrt{f^2 - c}$.

CONICS

Conic Section or a conic is the locus of a point which moves so that its distance from affixed point bears a constant ratio to its distance from a fixed line.

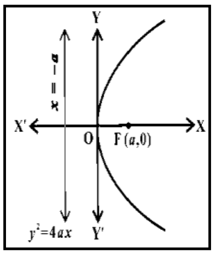
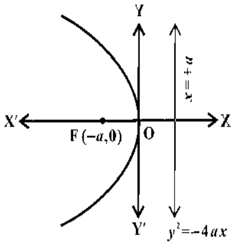
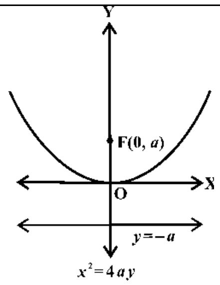
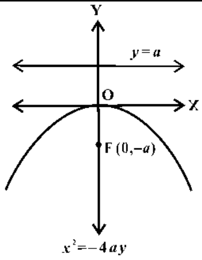
The fixed point is called the **focus**, the straight line the **directrix** and the constant ratio denoted by **e** is called the **eccentricity**.

Eccentricity (e) **e** = $\frac{\text{distance between } P(x,y) \text{ \& Focus}}{\text{distance between } P(x,y) \text{ \& Directrix}}$.

If $e = 1$, then conic is a parabola.
If $e < 1$, then conic is a an ellipse.
If $e > 1$, then conic is a hyperbola.
If $e = 0$, then conic is a circle.

PARABOLA

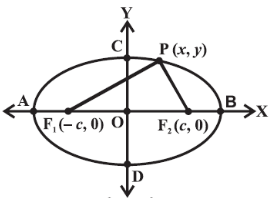
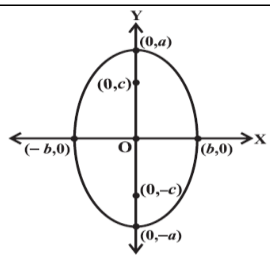
A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point in the plane.

S. No		$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
					
1.	Vertex	(0,0)	(0,0)	(0,0)	(0,0)
2.	Focus	(a,0)	(-a,0)	(0,a)	(0,-a)
3	Equation of directrix	x + a = 0	x – a = 0	y + a = 0	y – a = 0
4.	Equation of Axis	(x-axis), y = 0	(x-axis), y = 0	(y-axis), x = 0	(y-axis), x = 0
5.	Length of Latus Rectum	4a	4a	4a	4a
6.	Focal distance of a p(x, y)	x + a	x – a	y + a	y – a

** Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the hyperbola.

** **Position of a point with respect to a parabola** :The point (x_1, y_1) with respect to the parabola $y^2 = 4ax$ lies (i) outside , if $y_1^2 - 4ax_1 > 0$ (ii) on it if $y_1^2 - 4ax_1 = 0$ (iii)inside if $y_1^2 - 4ax_1 < 0$.

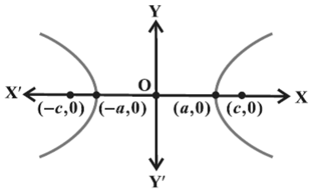
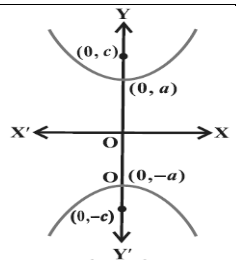
ELLIPSE: ** An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant. Ellipse is also defined as "The ratio of the distances between P(x, y) & Focus and P(x, y) & directrix is always a constant and this constant is said to eccentricity & always less than 1.

S. No.			
1.	Equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
2.	Centre	(0 , 0)	(0 , 0)
3.	Vertices	(± a , 0)	(0 , ± a)
4.	Foci	(± ae, 0) or (± c , 0)where $c^2 = a^2 - b^2$	(0 ± ae) or (0 ± c)where $c^2 = a^2 - b^2$
5.	Equations of irectrices	$x = \pm \frac{a}{e}$.	$y = \pm \frac{a}{e}$
6.	Eccentricity	e = c/a	e = c/a
7.	Length of major axis	2a	2a
8.	Length of minor	2b	2b

	axis		
9.	Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

**** HYPERBOLA**

A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.

S. N			
1	Equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$
2.	Vertices	$(\pm a, 0)$	$(0, \pm a)$
3.	Foci	$(\pm ae, 0)$ or $(\pm c , 0)$ where $c^2 = a^2 + b^2$	$(0 \pm ae)$ or $(0 \pm ae)$ where $c^2 = a^2 + b^2$
4.	Eccentricity	$e = c/a$	$e = c/a$
5.	Equations of directrices	$x = \pm \frac{a}{e}$.	$y = \pm \frac{a}{e}$
6.	Centre	(0, 0)	(0, 0)
7.	Length of Transverse axis	2a	2a
8.	Length of conjugate axis	2b	2b
9.	Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
10.	Equation of Transverse axis	y = 0	x = 0
11.	Equation of conjugate axis	x = 0	y = 0

SOLVED PROBLEMS

1) Slope of a line which cuts off intercepts of equal lengths on the axes is

(a) -1 (b) 0 (c) 2 (d) $\sqrt{3}$

2.The value of y so that the line through (3,y) and (2,7) is parallel to the line through (-1,4) and (0,6) is

a)7 (b) 10 (c) 9 (d) 8

3) The radius of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$ is

a) 8 (b) 10 (c) 9 (d) 7

4) The focus of the parabola $y^2 = - 8x$ is

A) (2,0) b) (-2,0) c) (0,2) d) (0,-2)

5) Assertion (A). The slope of a line passing through two points (-5, 2) and (3,-2) is $\frac{-1}{2}$

Reason (R). The slope of a line passing through two given points (x_1, y_1) and (x_2, y_2) is $\frac{x_{2-x_1}}{y_{2-y_1}}$

a) Both A and R are true and Ris the correct explanation of A.

b) Both Aand R are true but R is not correct explanation of A.

c) A is true but R is false

d) A is false but R is true.

e) Both Aand R are false.

Short Answer questions

6)Find the equation of the straight line making an angle of 135^0 with x- axis and cutting the y-axis at a distance 2 below the origin.

7. Find equation of the line parallel to x-7y+5=0 and having x- intercept 3 .

8..Find the angle between the lines y- √3 x -5 =0 and √3 y –x +6 =0.

LONG ANSWERS.

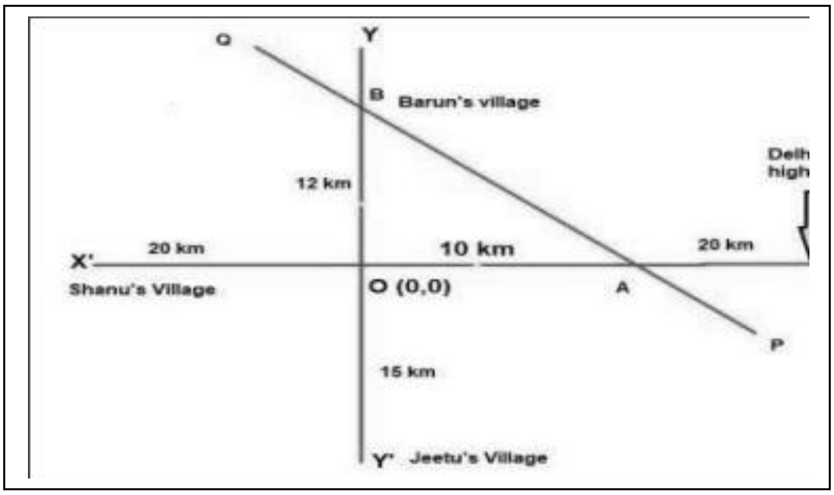
9. Find the equation of the circle passing through the points (4,1), (6,5) and whose centre is on the line 4x+y=16.

10. Find equation of the line passing through the point (2,2) and cutting of intercepts on the axes whose sum is 9.

Case study

11.Villages of Shanu and Arun's are 50km apart and are situated on Delhi Agra highway as shown in the following picture. Another highway YY' crosses Agra Delhi highway at O(0,0). A small local road PQ crosses both the highways at pints A and B such that OA=10 km and OB =12 km. Also, the villages of Barun and Jeetu are on the smaller high way YY'. Barun's village B is 12km from O and that of Jeetu is

15 km from O.



Based on the above information answer the following questions:

i. What are the coordinates of A?

a. (10, 0) b. (10, 12) c. (0,10) d. (0,15)

ii. What is the equation of line AB?

a. 5x + 6y = 60 b. 6x + 5y = 60 c. x = 10 d. y = 12

iii. What is the distance of AB from O(0, 0)?

a. 60 km b. 60/√61 km c. √61 km d. 60 km

iv. What is the slope of line AB?

a) $\frac{6}{5}$. b. $\frac{5}{6}$ c. $\frac{-6}{5}$ d. $\frac{10}{12}$

SOLUTIONS

1.a 2.c 3 .d 4.b 5.c

6.Slope m = tan 135^0 = -1 and c= -2. The equation of the line is x+ y+2 =0.

7. The equation of the line parallel to x-7y+5=0 is x-7y+k=0 . x-intercept is 3 .The line passes through the point (3,0). K= -3. .The equation of the line is x-7y -3=0.

8. slopes of two lines are m_1= √3 and m_2= 1/√3.

Let α be the angle between two lines.

$$\tan \alpha = \left|\frac{m_{2-}m_1}{1+m_1m_2}\right| = \frac{1}{\sqrt{3}}$$

which gives α =30^0 .Hence angle between two lines is either 30^0 or 180^0 -30^0= 150^0.

Long answer.

9. Let the equation of the circle be $(x – h)^2 + (y - k)^2 = r^2$ Since the circle passes through the points (4,1) and (6,5) , we have $(4 – h)^2 + (1 - k)^2 = r^2$

$(6 – h)^2 + (5 - k)^2 = r^2$

Since the centre lies on the line 4x+y =16 we have 4h+ k =16.

Solving the above three equations we get h=3 , k=4 r^2= 10.

Equation of the circle is $x^2+ y^2$ -6x -8y +15 =0.

10. Let the equation of the line in intercept form be $\frac{x}{a} + \frac{y}{b}$ =1.

Give that a+b =9 and the line passes through the point (2,2).

$\frac{2}{a} + \frac{2}{9-a}$ =1. a= 3,6 and b=6,3

The equations of line are x + 2y -6 =0 , 2x + y -6=0.

11. i) a ii) b iii) b iv) c

PRACTICE PROBLEMS

1. The slope of the line 8x- 4y+5 =0 is

a) 2 b) -2 c) $\frac{1}{2}$ d) $-\frac{1}{2}$

2.The equation of the line which is parallel to y-axis and passing through the point (3,-4) is

a) x=-3 b) y=3 c) x= 3 d) y=-3

3. The x- intercept of the line 5x -4y-5=0 is

a) 5 b)1 c)-1 d)4

4.The equation of the parabola with focus (6,0) and vertex (0,0) is

a) $y^2 = 24x$ b) $y^2 = -24x$ c) $x^2 = 24y$ d) $x^2 = -24y$

5. Assertion (A). The length of latus rectum of the parabola $x^2 = -12y$ is 12.

Reason (R). The length of latus rectum of the parabola $x^2 = -4ay$ is 4a.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not correct explanation of A.

c) A is true but R is false

d) A is false but R is true.

e) Both A and R are false.

SHORT ANSWER QUESTIONS

6. . Find the angle between the lines √3 x +y =1 and x+√3 y=1.

7. Let P(a,b) is the mid point of a line segment between axes. Show that the equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$.

8. Convert the equation 2x-3y-5=0 in to a) slope- intercept form b) intercept form.

9. The vertices of a triangle PQR are P(2,1) ,Q(-2,3) and R(4,5) .Find the equation of the median through the vertex R.

10. Find the equation of the right bisector of the line segment joining the points (3,4) and (-1,2).

11.Find the equation of the circle with radius 5 whose centre lies on x – axis and passes through the point (2,3).

12. Find the centre and radius of the circle $x^2 + y^2 + 8x - 4y - 45 = 0$

13. Find the equation of the parabola with vertex (0,0) passing through (2,3) and axis is along x-axis.

14. Find the equation of the Hyperbola with vertices (±2,0) and foci(±3, 0).

15. Find the equation of the ellipse that satisfies the conditions length of major axis 26, foci (±5,0).

LONG ANSWER QUESTIONS

16.If the angle between two lines is $\frac{\pi}{4}$ and slope of one of the lines is $\frac{1}{2}$, find the slope of the other line.

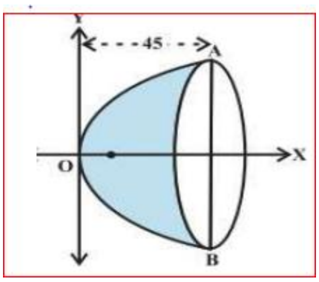
17.. Find the equation of the ellipse with major axis along the X-axis and passing through the points (4,3) and (-1,4).

18)The vertices of a triangle ABC are A(2,3) ,B(4,-1) and C(1,2) .Find the equation and length of the altitude from the vertex A to the side BC.

19.If the slope of a line passing through the point A(3,2) is $\frac{3}{4}$, then find points on the line which are 5 units away from the point A.

20. Find the equation of the circle passing through the points (2,-2), (3,4) and whose centre is on the line x+y=2..

21.The focus of a parabolic mirror as shown in Fig is at a distance of 5 cm from its vertex and the mirror is.45 cm deep



Based on the above information answer the following:

I)What is the equation of the above parabolic section:

a) $y^2 = -5x$ b) $y^2 = 45x$ c) $y^2 = 5x$ d) $y^2 = 20x$

II) What is the length of distance AB?

a) 60 cm b) 45 cm c) 30 cm d) 20 cm

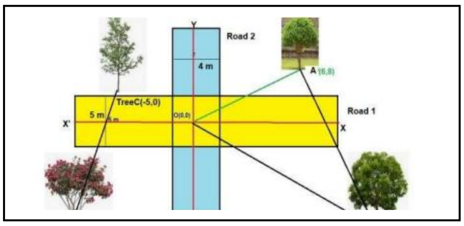
III) What is the coordinate of focus point of the parabolic mirror?

a) (5,0) b) (20,0) c) (45,30) d) (45,15)

IV) What is the coordinate of point A of the parabolic mirror?

a) (5,0) b) (45,5) c) (45,30) d) None of these

22) In a park Road 1 and road 2 of width 5 m and 4 m are crossing at centre point O(0, 0) as shown in the figure .



For trees A, B, C and D are situated in four quadrants of the Cartesian system of coordinate. The coordinates of the trees A, B, C and D are (6, 8), (12, 5), (-5, 0) and (-3, -4) respectively.

Based on the above information answer the following questions:

i. What is the distance of Tree C from the Origin?

a. 5 m b. 10 m c. 15 m d. 25 m

ii. What is the equation of line AB?

a. 2x + y = 22 b. x - 2y = -6 c. x + 2y - 22 = 0 d. x + 2y = 6

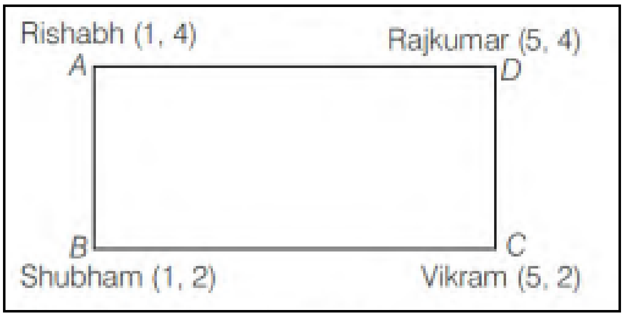
iii. What is the slope of line CD?

a. 2/1 b. -2 c. -1/2 d. 3/2

iv. What is the distance of point B from the origin?

a. 13 m b. 15 m c. 12 m d. 5 m

23) Four friends Rishab,Shubham,Vikram and Raj kumar are sitting on the vertices of a rectangle, whose coordinates are given.



Based on the above information solve the following questions.

i) T he equation formed by Shubham and Rajkumar is

a)x+2y+3=0. B)x-2y-3=0 c)x-2y+3=0 d)x+2y-3=0

ii)pair for the same slope is

a) Rishab , Rajkumar and Shubham,vikram

b) Rishab, Rajkumar and Rajkumar,vikram

c) Rishab, Rajkumar and Shubham,Rishab

d) Rishab, Shubhamr and Shubham,vikram

iii)Slope of the line formed by Shubham and Raj kumar is

a) 0 b) 1 c) 2 d) $\frac{1}{2}$

iv) The distance between Rishab and Shubham is

a) 1 B) 4 c) 2 d) 24

Answers

1)a 2)c 3)b 4) a. 5)a

6)30^0 or 150^0 8) $y = \frac{2}{3}x - \frac{5}{3}$, $\frac{x}{\frac{5}{2}} + \frac{y}{\frac{-5}{3}} = 1$. 9) 3x -4y +8 =0 10) 2x+y =5.

11.x2+y2+ 4x-21=0 and x2+y2+-12x+11=0 12) centre (-4,2) and radius=$\sqrt{65}$

13) $2y^2=9x$. 14) $\frac{x^2}{4} - \frac{y^2}{5} = 1$. 15) $\frac{x^2}{169} + \frac{y^2}{144} = 1$. 16. : m=3 or m= $-\frac{1}{3}$

17) . The equation of ellipse is $7x^2 + 15y^2 = 247$. 18) y – x =1. $\sqrt{2}$ 19) The coordinates of the required points are either (-1, -1) or (7,5). 20) $(x- 0.7)^2 + (y- 1.3)^2 ==12.58$

21) i) d ii)a iii) a iv) c 22) i) a ii)c iii)b iv) a 23) i) c ii)a iii)d iv)c

PRACTICE TEST-1 20MARKS

SECTION-A 1M×5=5M

1. The Y- intercept of the line 4x -5y-5=0 is

a)1 b) 5 c)4 d) -1

2.) The radius of the circle $x^2 + y^2 -10x -8y – 8 = 0$ is

a) 8 (b) 10 (c) 9 (d) 7

3)The focus of the parabola $X^2 = 8Y$ is

A) (2,0) b) (-2,0) c) (0,2) d) (0,-2)

4. Slope of a line which cuts off intercepts of equal lengths on the axes is

(a) -1 (b) 0 (c) 2 (d) √3

5. Assertion (A). The slope of a line passing through two points (-3, 1) and (4,-2) is $\frac{-1}{2}$

Reason

(R). The slope of a line passing through two given points (x_1, y_1) and (x_2, y_2) is $\frac{x_{2-x_1}}{y_{2-y_1}}$

a) Both A and R are true and Ris the correct explanation of A.

b) Both Aand R are true but R is not correct explanation of A.

c) A is true but R is false

d) A is false but R is true.

e) Both Aand R are false.

SECTION-B 2M×2=4M

6. Find the equation of a line that cut off equal intercepts on the coordinate axes and passes through the point (2,3).

7. Find the angle between x-axis and the line joining the points (3,-1)and (4,-2).

SECTION-C 3M×2=6M

8. Find the equation of a circle with centre (2,2) and passes through the point(4,5).

9.Find the equation of parabola with vertex (0,0) and focus at (-2,0).

SECTION-D 5M×1=5M

10.If one diagonal of a square is along the line 8x-15y=0 and one of its vertex is at (1,2), then find the equation of sides of the square passing through its vertex.

PRACTICE TEST-2 30MARKS

SECTION-A 1M×5=5M

1. Find The slope of the line 4x -2y-5=0 is

a)1 b) 2 c)4 d) -1

2.) The centre of the circle $x^2 + y^2 - 10x - 6y – 8 = 0$ is

A) (5, 3) b) (-6,10) c) (-3,5) d) (6,-2)

3)The focus of the parabola $y^2 = -16x$ is

A) (4,0) b) (-4,0) c) (0,4) d) (0,-4)

4.The length of latus rectum of the Hyperbola $3x^2- y^2 =12$ is

(a) 36 (b) 6 (c) 12 (d) 2

5. Assertion (A). The equation of the line which makes equal intercepts on the axes and passes through the point (2,3) is x+y =5.

Reason (R). The equation of the line with intercepts a and b on the axis is $\frac{x}{a}+\frac{y}{b}=1$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not correct explanation of A.

c) A is true but R is false

d) A is false but R is true.

SECTION-B 2M×3=6M

6. Find the equation of the line intersecting x-axis at a distance of 3 units to the left of the origin with slope -2.

7. Find the centre and radius of the circle$x^2 +y^2 + 6x -8 y-45=0$

8. Find the equation of the parabola with vertex (0,0) passing through (5,2) and symmetric with respect to y-axis.

SECTION -C 3M×3=9M

9. the vertices of a triangle PQR are P(10,4) ,Q(-4,9) and R(-2,-1) .Find the equation of the median through the vertex P..

10. Find the equation of the Ellipse with vertices (0,±13) and foci(0,±5).

11. Find the coordinates of the foci , the vertices, the eccentricity and the length of the latus rectum of

Of the Hyperbola $49y^2-16x^2=784$.

SECTION-D

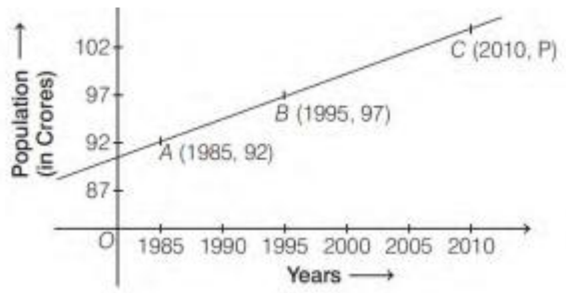
5M×1=5M

12.Find the equation s of the lines which passes through the point (4,5) and makes equal angles with the lines 5x-12y+6=0 and 3x=4y+7.

SECTION-E

5M×1=5M

13.Population Vs Year graph given below.



Based on the above information answer the following questions.

i) The slope of the line AB is 1m

a) 2 b) 1 c) $\frac{1}{2}$ d) $\frac{1}{3}$

ii) The equation of line AB is 1m

a) x +2y =1791 b) x-2y=1801 c) x-2y =1791 d)x-2y+1801=0

ii) The population in year 2010 is (in crores) 1m

a) 104.5 b)119.5 c)109.5 d) None of these

iv) The equation of line perpendicular to AB and passing through ((1995,97) is 2m

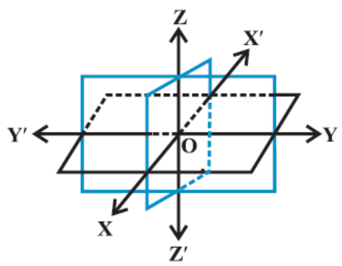
a)2x-y=4087 b) 2x+y=4087 c) 2x+y=1801 d) none of these

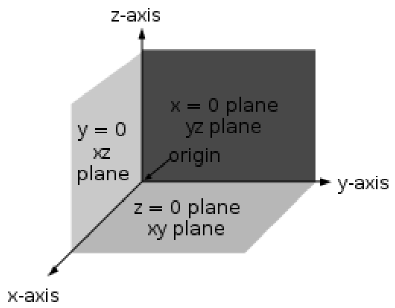
INTRODUCTION TO THREE-DIMENSIONAL GEOMETRY

MAIN CONCEPTS AND RESULTS

****Coordinate Axes and Coordinate Planes in Three Dimensional Space** : In three dimensions, the coordinate axes of a rectangular Cartesian coordinate system are three mutually perpendicular lines. The axes are called the x, y and z-axes.

The three planes determined by the pair of axes are the coordinate planes, called XY, YZ and ZX-planes.





The three coordinate planes divide the space into eight parts known as octants.

** **Coordinates of a Point in Space :** The coordinates of a point P in three dimensional geometry is always written in the form of triplet like (x, y

Here x, y and z are the distances from the YZ, ZX and XY-planes.

Any point (i) on x-axis is of the form (x, 0, 0)

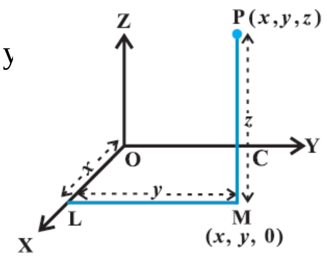
(ii) on y-axis is of the form (0, y, 0)

(iii) on z-axis is of the form (0, 0, z).

Any point (i) in XY-plane is of the form (x, y, 0)

(ii) in YZ-plane is of the form (0, y, z)

(iii) on ZX-plnane is of the form (x, 0, z).



** The three coordinate planes divide the space into eight parts known as octants.

Octants → Coordinates ↓	I XOYZ	II X′OYZ	III X′OY′Z	IV XOY′Z	V XOYZ′	VI X′OYZ′	VII X′OY′Z′	VIII XOY′Z′
x	+	–	–	+	+	–	–	+
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

** Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.

** **Section Formula** : The coordinates of the point R which divides the line segment joining two points

P (x_1, y_1, z_1) and Q (x_1, y_1, z_1) internally and externally in the ratio m : n are given by

$$\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right) , \left(\frac{mx_2-nx_1}{m-n},\frac{my_2-ny_1}{m-n},\frac{mz_2-nz_1}{m-n}\right)$$ respectively.

** The coordinates of the mid point of the line joining P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is

$$\left(\frac{x_1+y_1+z_1}{2},\frac{x_2+y_2+z_2}{2}\right)$$

** The coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) & (x_3, y_3, z_3)

are $$\left(\frac{x_1+y_1+z_1}{3},\frac{x_2+y_2+z_2}{3},\frac{x_3+y_3+z_3}{3}\right).$$

II .Illustrations/Examples:

Example 1: Find the distance between P(2, -3, 4) and Q(– 1, 2, 1).

Solution: By distance formula PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

We get PQ = $\sqrt{((-1)-2))^2 + (2-(-3))^2 + (1-4)^2}$

$= \sqrt{(-3)^2 + (5)^2 + (-3)^2}$

$= \sqrt{43}$ units

Example 2: Determine the point in yz-plane which is equidistant from three points A (2, 0, 3), B (0, 3, 2) and C (0, 0, 1).

Solution: As x-coordinate of every point in yz-plane is zero. So, let P(0, b, c)be any point in yz plane.

The given points are A(2, 0, 3), B (0, 3, 2) and C (0, 0, 1)

From the given condition, we have

PA = PB = PC

Now,

$PA^2 = PB^2$

$(0 – 2)^2 + (b – 0)^2 + (c – 3)^2 = (0 – 0)^2 + (b – 3)^2 + (c – 2)^2$

Therefore,

$4 + b^2 + c^2 – 6c + 9 = b^2 – 6b + 9 + c^2 – 4c + 4$

– 6c + 6b + 4c = 0

3b – c = 0 -----(i)

Again PB = PC

$PB^2 = PC^2$

$(0 – 0)^2 + (b – 3)^2 + (c – 2)^2 = (0 – 0)^2 + (b – 0)^2 + (c – 1)^2$

Therefore, $b^2 – 6b + 9 + c^2 – 4c + 4 = b^2 + c^2 – 2c + 1$

3b + c = 6 ------ (ii)

On solving (i) and (ii) we get,

b = 1 and c =3

Required point is (0, 1, 3).

III:-Practice Questions:

MCQ

Q1 A point lie on the x-axis, then its y coordinate and z –coordinate is

a) b,0 b) 0,c c) b,c d)0, 0

Q2 The point (3, -4, -5) lies in the octant

a) Second Octant b) Fourth Octant c) Sixth Octant d)Eighth Octant

Q3 The locus of a point for which x = 0 is

a) xy-plane b) yz-plane c)zx-plane d)None of these

Q4 y-axis the intersection of two planes

a) xy and yz b)yz and zx plane c) xy and zx d)None of these

Assertion-and-Reason Type

Each question consists of two statements, namely,Assertion (A) and Reason (R).For selecting the correct answer, use the following code:

(a) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

Q5 Statement I: The point A(–2, 3, 5), B(1, 2, 3) and C(7, 0, – 1) are collinear.

Statement II: Three points A, B, C are collinear when AB – BC = AC

Q6 Statement I: The distance between the points (3, 2, – 4) and (1, 2, 3) is 5 unit.

Statement I: The coordinates of points in XY-plane are of the form (a, b, 0).

Short Answer type Questions:

Q7 Show that the points (– 2, 3, 5), (1, 2, 3) and (7, 0, – 1) are collinear.

Q8 Show that the points (0,4,1), (2, 3, -1), (4, 5, 0) and (2, 6, 2) are vertices of a square.

Q9 Find the equation of the set of points P such that $PA^2 + PB^2 = 2k^2$, where A and B are the points (3, 4, 5) and (– 1, 3, – 7) respectively.

Q10 Determine the point on XY-plane which is equidistant from three points A (2, 0, 3), B(0, 3, 2) and C (0, 0, 1).

Q11 Find the point on y axis which is at a distance $\sqrt{10}$ from the point (1, 2, 3).

(IV)Answers

1) (d) 2) (d) 3) (b) 4) (a) 5) (c) 6) (d) 9) $2x^2 + 2y^2 + 2z^2 – 4x – 14y + 4z + 109 – 2k^2 = 0$
10) (3, 2, 0) 11) (0, 2, 0)

PRACTICE TESTI

M. M. 20

Instructions:

(i) Q1 to Q5 carries 1 mark each. (ii)Q6 to Q8 carries 2 mark each.

(iii) Q9 to Q11 carries 3 mark each.

Choose the correct answer:

Q1 A plane is parallel to xy-plane so it is perpendicular to:

a) x –axis b) y-axis c) z-axis d) None of these

Q2 Equation of z-axis is considered as

a) x = 0, y = 0 b) z = 0, y = 0 c)x = 0, z = 0 d) None of these

Q3 The point (3, 7, – 1) lies in the

a) Third Octant b) Fifth Octant c) Sixth Octant d) Seventh Octant

Q4 The length of foot of perpendicular drawn from the point P (3, 4, 5) on y –axis is

a) $\sqrt{41}$ b) $\sqrt{34}$ c)5 d) None of these

Each question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer,use the following code:

(a) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

Q5Statement I :The points (a, b, c), (b, c, a) and (c, a, b) are the vertices of an equilateral triangle.
Statement II: Every point on x – axis has its y – coordinate and z – coordinate zero (0).

Q6 Find the distance of the point (1, – 2, 4) from x, y and z- axis.

Q7 Show that if $x^2 + y^2 = 1$, then the point (x, y, $\sqrt{1-x^2-y^2}$) is at a distance 1 unit from the origin.

Q8 Prove that the points (0, – 1, – 7), (2, 1, – 9) and (6, 5, – 13) are collinear.

Q9 Find a point on ZX-plane which is equidistant from the points (1, – 1, 0), (2, 1, 2) and (3, 2, – 1).

Q10 Are the points (3, 6, 9), (10, 20, 30) and (25, – 41, 5) are the vertices of a right angled triangle?

Q11 Find the equation of path of P which moves so that its distance from points (3, 4, – 5) and (– 2, 1, 4) are equal.

PRACTICE TEST II

M. M. 30

Instructions:

(i) Q1 to Q5 carries 1 mark each. (ii)Q6 to Q8 carries 2 mark each.

iii) Q9 to Q13 carries 3 mark each.(iv) Question carry 4 Marks

Choose the correct answer:

Q1 The x-axis and y-axis taken together determine a plane known as

a)xy-plane b)yz- plane c) zx-plane d) None of these

Q2 The coordinates of points in the XY-plane are of the form

a) (x, y, z) b) (x, 0, 0) c) (x, y, 0) d) None of these Q3 The distance between P (2, 3, 5) and Q (4, 3, 1) is

a) 20 b) $\sqrt{20}$ c)$\sqrt{29}$ d) None of these

Each question consists of two statements, namely, Assertion (A) and Reason (R).For selecting the correct answer,use the following code:

(a) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

Q4Statement I: The distance between the points P (1, 2, 3) and R (7, 0, – 1) is √56.

Statement II:The distance between P(x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Q5 Statement I: The point on x-axis which is equidistant from the point A (3, 2, 2) and B (5, 5, 4) is (49/4, 0, 0).

Statement II: The coordinates of points in YZ-plane are of the form (x, y, 0)

Q6 Show that the triangle ABC, with vertices as A(0, 7, 10), B(– 1, 6, 6) and C(– 4, 9, 6) is an isosceles right angled triangle.

Q7 Show that the points (1, – 2, – 8), (5, 0, – 2) and (11, 3, 7) are collinear.

Q8 Find the point on z-axis which is equidistant from (1, 5, 7) and (5, 1, – 4).

Q9 Show that the points(5, – 1, 1), (7, – 4, 7), (1, – 6, 10) and (– 1, 3, 4)are the vertices of a rhombus.

Q10 Determine the point in YZ-plane which is equidistant from three points A (2, 0, 3), B(0, 3, 2) and C(0, 0, 1)

Q11 Find the coordinates of a point equidistant from the four points O(0, 0, 0), A(a, 0, 0), B(0, b, 0) and C (0, 0, c).

Q12 Find the equation of the set of points P, the sum of whose distances from A(4, 0, 0) and B(– 4, 0, 0) is equal to 10.

Q13 Find the coordinates of a point on y-axis which are at a distance of 5√2 from the point P(3, – 2, 5).

Q14 Find the equation of the set of points which are equidistant from the point (1, 2, 3) and (3, 2, – 1).

Answers

Practice Test I

1) (c) 2) (a) 3) (b) 4) (b) 5) (b) 6) √20, √17, √5

9) (31/10, 0, 1/5)) 10) No 11) 10x + 6y – 18z – 29 = 0

PRACTICE TEST II

1) (a) 2) (c) 3) (b) 4) (a) 5) (c) 8) (0, 0, 3/2)

10) (0, 1, 3) 11) (a/2, b/2, c/2) 12) $9x^2 + 25y^2 + 25z^2 = 225$

13) (0, 2m, 0) or (0, – 6, 0) 14) x – 2z = 0

LIMITS AND DERIVATIVES
MAIN CONCEPTS AND RESULTS

Def : $\lim\limits_{x \to a} f(x) = l$, if to a given $\in > 0$, there exists a +ve number S such that $| f(x) - l | < \in$ for $| x - a| < \delta$.

** **Some Standard Results on Limits** :

** If f(x) = K, a constant function, then $\lim\limits_{x\to a} f(x) = K$.

** $\lim\limits_{x\to a} K.f(x) = K \lim\limits_{x\to a} f(x)$

** $\lim\limits_{x\to a} [f(x) \pm g(x)] = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} g(x)$

** $\lim\limits_{x\to a} f(x).\, g(x) = \lim\limits_{x\to a} f(x).\ \lim\limits_{x\to a} g(x)$

** $\lim\limits_{x\to a} \left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$.

** $\lim\limits_{x\to a} \log f(x) = \log\left(\lim\limits_{x\to a} f(x)\right)$.

** $\lim\limits_{x\to a} [f(x)]^{1/n}. = \left[\lim\limits_{x\to a} f(x)\right]^{1/n}$ Provided $\left[\lim\limits_{x\to a} f(x)\right]^{1/n}$ is a real number.

****Sandwich Theorem** (or squeeze principle).

If f, g, h are functions such that $F(x) \le g(x) \le h(x)$ as $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} h(x) = l$, then $\lim\limits_{x \to a} g(x) = l$

** $\lim\limits_{x\to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$.

** $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$, Also $\lim\limits_{\theta\to 0} \dfrac{\theta}{\sin\theta} = 1$

** $\lim\limits_{\theta\to 0} \dfrac{\tan\theta}{\theta} = 1$, Also $\lim\limits_{\theta\to 0} \dfrac{\theta}{\tan\theta} = 1$

** $\lim\limits_{x\to 0} \dfrac{e^x - 1}{x} = \log e = 1$

** $\lim\limits_{x\to 0} \dfrac{a^x - 1}{x} = \log a$

** $\lim\limits_{x\to 0} (1 + x)^{1/x} = e$

** $\lim\limits_{n\to\infty} \left(1 + \dfrac{1}{n}\right)^n = C$

** $\lim\limits_{x\to 0} \dfrac{\sin^{-1} x}{x} = 1$

** $\lim\limits_{x\to 0} \dfrac{\tan^{-1} x}{x} = 1$

** **Some Standard Results of differentiation**

** $\dfrac{d}{dx}(x^n) = nx^{n-1} (x \in R, n \in R, x > 0)$

** $\dfrac{d}{dx}(x) = 1$

** $\dfrac{d}{dx}(c) = 0$ (where c is a constant)

** $\dfrac{d}{dx}(e^x) = e^x$

** $\dfrac{d}{dx}(a^x) = a^x \log_e a (a \in R, a > 0)$

** $\dfrac{d}{dx}(\log_e x) = \dfrac{1}{x} (x > 0)$

** $\dfrac{d}{dx}(\sin x) = \cos x$

** $\dfrac{d}{dx}(\cos x) = -\sin x$

** $\dfrac{d}{dx}(\tan x) = \sec^2 x$

** $\dfrac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$

** $\dfrac{d}{dx}(\sec x) = \sec x \tan x$

** $\dfrac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x \cot x$

II. Some illustrations/Examples (with solution) preferably of different types.

i) MCQs : 4

Q1. $\lim_{x\to0}\frac{\sin ax}{bx}$ is

(a) 1 (b) 0 (c) a/b (d) b/a

Sol :-. $\lim_{x\to0}\frac{\sin ax}{bx}\left(\frac{0}{0}\ \text{form}\right)$

$= \lim_{x\to0}\frac{\sin ax}{bx}.\frac{a}{a}$

$=\frac{a}{b}\lim_{x\to0}\frac{\sin ax}{ax}$

$=\frac{a}{b}.1$

$=\frac{a}{b}$ Ans[c]

Q2. $y = \frac{x}{\tan x}$, then $\frac{dy}{dx} = \ldots\ldots\ldots\ldots..$

(a) $\cos^2x$ (b) $\sec^2x$(c)$\frac{\tan x - \sec x}{\tan^2 x}$ (d) $\frac{\tan x - x\sec^2 x}{\tan^2 x}$

Solution:

$$\frac{dy}{dx} = \left[\frac{\tan x\frac{d}{dx}(x) - x\frac{d}{dx}(\tan x)}{\tan^2 x}\right]$$

$\frac{dy}{dx} = \left[\frac{\tan x.1 - x.\sec^2 x}{\tan^2 x}\right]$ ans (d)

Q3. Evaluate $\lim_{x\to1}\frac{x^m-1}{x^n-1}$

Solution:

$$\lim_{x\to1}\frac{x^m-1}{x^n-1} = \lim_{x\to1}\frac{x^m-1}{x-1}\times\frac{x-1}{x^n-1}$$

$$= \frac{m\cdot1^{m-1}}{n\cdot1^{n-1}}\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}\right]$$

$$= \frac{m\cdot1}{n\cdot1}$$

$$= \frac{m}{n}$$

(a) 1 (b) 0 (c) m/n (d) n/m

Ans(c)

Q4.$\frac{d}{dx}(2x^2+3x+4)$ is

(a) 4x+3 (b) 4x-3 (c) 3+x (d) x-1

Solution: $\frac{d}{dx}(2x^2+3x+4)$

$=4x+3$

Ans (a)

ii) Short answer type question:

Q5. Evaluate $\lim_{x\to0}\frac{\sin ax+bx}{ax+\sin bx}$, $a, b, a+b \neq 0$

Solution:

$$\lim_{x\to0}\frac{\sin ax+bx}{ax+\sin bx}$$

$$\lim_{x\to0}\frac{\frac{\sin ax}{ax}\cdot ax+bx}{ax+\frac{\sin bx}{bx}\cdot bx}$$

$$\lim_{x\to0}\frac{\frac{\sin ax}{ax}\cdot a+b}{a+\frac{\sin bx}{bx}\cdot b}$$

$$=\frac{1\cdot a+b}{a+1\cdot b}$$

$$=\frac{a+b}{a+b}$$

=1

Q6. Differentiate $\frac{x}{\sin x}$ with respect to x.

Solution:

Let $f(x)=\frac{x}{\sin x}$

$$f'(x)=\frac{d}{dx}\left(\frac{x}{\sin x}\right)$$

$$=\frac{\sin x\frac{d}{dx}x-x\frac{d}{dx}\sin x}{(\sin x)^2}$$

$$=\frac{\sin x\cdot1-x\cdot\cos x}{(\sin x)^2}$$

$$=\operatorname{cosec}x-x\cot x\operatorname{cosec}x$$

$$=\operatorname{cosec}x(1-x\cot x)$$

Q7. Differentiate $e^x\sin x+x^n\cos x$ with respect to x

Solution:

Let $f(x)=e^x\sin x+x^n\cos x$

$$f'(x)=\frac{d}{dx}(e^x\sin x)+\frac{d}{dx}(x^n\cos x)$$

$$=e^x\frac{d}{dx}(\sin x)+\sin x\frac{d}{dx}(e^x)+x^n\frac{d}{dx}(\cos x)+\cos x\frac{d}{dx}(x^n)$$

$$=e^x(\cos x)+\sin x(e^x)+x^n\frac{d}{dx}(-\sin x)+\cos x\cdot n(x^{n-1})$$

$$=e^x(\cos x+\sin x)+x^{n-1}(n\cos x-x\sin x)$$

iii) Long answer type questions: 2

Q8. Find the values of ‘a’ and ‘b’ if $\lim_{x\to 2} f(x)$ and $\lim_{x\to 4} f(x)$exists, where

$$f(x) = \begin{cases} x^2 + ax + b & , \quad 0 \le x < 2 \\ 3x + 2 & , \quad 2 \le x \le 4 \\ 2ax + 5b & , \quad 4 < x \le 8 \end{cases}$$

Solution: To find $\lim_{x\to 2} f(x)$

L.H.L = $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2 + ax + b = 2a + b + 4$

and R.H.L = $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(3x + 2) = (3)(2) + 2 = 8$

since $\lim\limits_{x\to 2} f(x)$ exists therefore $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x)$

so, $2a + b + 4 = 8 \Rightarrow 2a + b = 4$……(1)

To find $\lim_{x\to 4} f(x)$

L.H.L = $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-}(3x + 2) = (3)(4) + 2 = 14$

and R.H.L = $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+}(2ax + 5b) = (2a)(4) + 5b = 8a + 5b$

since $\lim\limits_{x\to 4} f(x)$ exists therefore $\lim\limits_{x\to 4^-} f(x) = \lim\limits_{x\to 4^+} f(x)$

so, $8a + 5b = 14$ …….(2)

From (1)and (2)

a= 3 and b=-2

Q9. Find the derivative of the following function.

$$f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$$

Solution:

$$f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$$

By quotient rule

$$f'(x) = \frac{(\sin x - \cos x)\frac{d}{dx}(\sin x + \cos x) - (\sin x + \cos x)\frac{d}{dx}(\sin x - \cos x)}{(\sin x - \cos x)^2}$$

$$= \frac{-(\sin x - \cos x)^2 - (\sin x + \cos x)^2}{(\sin x - \cos x)^2}$$

$$= \frac{-2(\sin^2 x + \cos^2 x)}{(\sin x - \cos x)^2}$$

$$= \frac{-2}{(\sin x - \cos x)^2}$$

III .Questions for Practice:

MCQ :

Q1. $\lim_{x\to\pi} \frac{\sin x}{x-\pi}$

(a) 1 (b) 2 (c) -1 (d) -2

Q 2. $\lim_{x\to0} \frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$ is

(a) 2 (b) 0 (c) 1 (d) -1

Q3. If $y = \frac{\sin(x+9)}{\cos x}$ then $\frac{dy}{dx}$ at x= 0 is

(a) cos9 (b) sin9 (c) 0 (d) 1

Q4. If $f(x) = \frac{x-4}{2\sqrt{x}}$, then $\frac{dy}{dx}$ at x=0 is

(a) 5/4 (b) 4/5 (c) 1 (d) 0

Assertion Reasoning Based MCQ

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

SHORT ANSWER TYPE QUESTION:

Q5. Assertion (A): If cos y= x cos(a+y), then $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos a}$.

Reason (B): $\frac{d}{dx}(\sin x) = \cos x$

SHORT ANSWER TYPE:

Q6. Evaluate the left hand and right hand limits of the following functions at x=2. Does $\lim_{x\to2} f(x)$ exists?

$$f(x) = \begin{cases} 2x+3, & \text{if } x \le 2 \\ x+5, & \text{if } x > 2 \end{cases}$$

Q7. Find the derivative of sin2x by first principle.

Q8. Evaluate $\lim_{x\to1} \frac{(2x+3)(\sqrt{x}-1)}{2x^2+x+3}$

Q9. Evaluate $\lim_{x\to\frac{\pi}{6}} \frac{\sqrt{3}\sin x - \cos x}{x-\frac{\pi}{6}}$

Q10. Differentiate $\frac{x^2\cos\frac{\pi}{4}}{\sin x}$ with respect to x.

Q11. Show that $\lim_{x\to4}\frac{|x-4|}{x-4}$ does not exist.

Q12. Find the derivative of (secx-1)(secx+1)

Q13. Evaluate $\lim_{x\to0}$ f(x), where f(x) = $\begin{cases}\frac{|x|}{x}, & x \neq 0\\ 0, & x = 0\end{cases}$.

Q14. Evaluate $\lim_{x\to1}\frac{\sqrt{1+x}-\sqrt{1-x}}{1+x}$

Q15. Find n, if $\lim_{x\to2}\frac{x^n-2^n}{x-2} = 80$, $n \in N$

Q16. Find f'(x) , if $f(x) = (x-2)^2(2x-3)$

Answers:

TYPE OF Q.	QUESTION	ANSWER	QUESTION	ANSWER	QUESTION	ANSWER	QUESTION	ANSWER
MCQ	Q1.	(c)	Q2.	(c)	Q3.	(a)	Q4.	(a)
AR	Q5.	(d)						
SHORT ANSWER TYPE	Q6.	7	Q7.	2COS 2X	Q8.	0	Q9.	2
	Q10.	$\frac{x}{\sqrt{2}}$[2cosecx - xcotx.cosecx]	Q11.	does not exist	Q12.	2 tanx. $\sec^2x$	Q13.	Does not exist
	Q14.	$\frac{1}{\sqrt{2}}$	Q15.	5	Q16.	$6x^2$-22x+20		

CHAPTER TEST -1

CHAPTER- LIMITS AND DERIVATIVES MAX MARKS – 20

SECTION-A (1 MARK EACH)

Q1. If $y=\sqrt{x}+\frac{1}{\sqrt{x}}$, then $\frac{dy}{dx}$ at x= 1 is

(b) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 0

Q2. $\lim_{x\to\pi}\frac{\sin x}{x-\pi}$ is

(a) 1 (b) 2 (c) -1 (d) -2

Q3. $\lim_{x\to0}\frac{x}{\sin x}$ is equal to

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 0

Q4. The derivative of $\frac{1}{x}$ is

(a) 1 (b) $\frac{-1}{x^2}$ (c) $\frac{1}{x^2}$ (d) $\frac{1}{x}$

Q5. $\lim_{x \to 3^+} \frac{x}{[x]}$is equal to

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 0

SECTION-A (2 MARKS EACH)

Q6. If y = $\frac{\text{sinx} + \text{cosx}}{\text{sinx} - \text{cosx}}$, then $\frac{dy}{dx}$ at x= 0 is

(a) -2 (b) $\frac{1}{2}$ (c) 0 (d) does not exist

Q7. $\lim_{x \to 2} \frac{3X^2 - X - 10}{X^2 - 4}$is equal to

(a) -2 (b) $\frac{11}{4}$(c) 0 (d) None of these

SECTION-A (3 MARKS EACH)

Q8. Suppose f(x) = $\begin{cases} a + bx, x < 1 \\ 4 \quad\quad , x = 1 \\ b - ax, x > 1 \end{cases}$ and if .$\lim_{x \to 1} f(x)$ = f (1) what are the

the possible values of a and b ?

Q9. Evaluate $\lim_{x \to 0} \frac{ax + x\cos\ x}{b \sin\ x}$

SECTION-A(5 MARKS EACH)

Q10. Compute the derivative of $x \sin x$ by using first principle.

CHAPTER TEST -2

CHAPTER- LIMITS AND DERIVATIVES MAX MARKS – 30

SECTION-A (1 MARK EACH)

Q1. $\lim_{x \to 3^+} \frac{[x]}{x}$is equal to

(b) 1 (b) $\frac{1}{2}$ (c) 2 (d) 0

Q2. $\lim_{x \to 0} \frac{(x+1)^5 - 1}{x}$isequal to

(b) $\frac{1}{2}$ (b) 5 (c) $\frac{1}{3}$ (d) -2

Q3. $\lim_{x \to 0} \frac{1 - \cos 2x}{2x}$is equal to

(b) 1 (b) $\frac{1}{2}$ (c) 2 (d) 0

SECTION-B (2 MARK EACH)

Q4. The derivative of cosxtanx is

(b) 1 (b) cos x (c) -sinx (d) sinx $\sec^2$x

Q5. If y= $\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$, then $\frac{dy}{dx}$ is

(c) $\frac{-2}{(x+1)^2}$ (b) $\frac{2}{(x-1)^2}$(c) $\frac{1}{(x-1)^2}$(d) $\frac{-2}{(x-1)^2}$

Q3. If y = e^{x^2} , then the value of $\frac{dy}{dx}$ at x= 0 is

(b) -2 (b) $\frac{1}{2}$ (c) 0 (d) does not exist

Q4. For what value of k so that $\lim_{x\to 2}$ f(x) may exist, where f(x) = $\begin{cases} 2x+3, x \le 2 \\ x+k, \ x > 2 \end{cases}$.

(b) 5 (b) $\frac{1}{2}$ (c) 0 (d) does not exist

SECTION-A (3 MARKS EACH)

Q5. Compute the derivative of sinx+cosxby using first principle.

Or

Compute the derivative of $\frac{x+1}{x-1}$by using first principle.

Q6. Evaluate $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$.

Q7. If the function f(x) satisfies $\lim_{x\to 0} \frac{f(x)-2}{x^2-1} = \pi$, evaluate $\lim_{x\to 0} f(x)$.

SECTION-A (5 MARKS EACH)

Q8. Find the derivative of the following functions:-

(a) $\frac{x}{1+\tan x}$ (b) $(5x^3+3x-1)(x-1)$

Q9. Suppose f(x) = $\begin{cases} mx^2+n, x<0 \\ nx+m, 0 \le x \le 1 \\ nx^3+m, x>1 \end{cases}$. For what integers m and n does both $\lim_{x\to 0}$ f(x) and $\lim_{x\to 1}$ f(x) exist?

STATISTICS
CONCEPTS AND RESULTS

** Mean $\bar{x} = \frac{\sum x_i}{n}$

$$= \frac{\sum f_i x_i}{\sum f_i}$$

$$= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) h, \quad \text{where a is the assumed mean , h is the class size and } u_i = \frac{x_i - a}{h}$$

** Median $= \left(\frac{n+1}{2} \right)^{th}$ observations (arranged in ascending or descending order) & the number of observations is odd.

$= \text{mean of } \left(\frac{n}{2} \right)^{th} \& \left(\frac{n}{2} + 1 \right)^{th}$ observations (arranged in ascending or descending order) & the number of observations is odd.

$= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$ where, l = lower limit of median class, n = number of observations,

cf = cumulative frequency of class preceding the median class, f = frequency of median class, h = class size.

**Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$, where l = lower limit of the modal class, h = size of the class interval,

f_1 = frequency of the modal class, f_0 = frequency of the class preceding the modal class,

f_2 = frequency of the class succeeding the modal class.

** **Measures of Dispersion:** The dispersion or scatter in a data is measured on the basis of the observations

and the types of the measure of central tendency, used there. There are following measures of dispersion:

(i) Range, (ii) Quartile deviation, (iii) Mean deviation, (iv) Standard deviation.

** **Range:** Range of a series = Maximum value – Minimum value.

** **Mean Deviation :** The mean deviation about a central value 'a' is the mean of the absolute values of the

deviations of the observations from 'a'. The mean deviation from 'a' is denoted as M.D. (a).

(i) For ungrouped data

$$\text{M.D.}(\bar{x}) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|, \text{ where } \bar{x} = \text{Mean} \qquad \text{M.D.}(M) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|, \text{ where } M = \text{Median}$$

(ii) For grouped data

(a) Discrete frequency distribution

$$\text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{\sum_{i=1}^{n} f_i} = \frac{1}{N}\sum_{i=1}^{n} f_i |x_i - \bar{x}| \qquad \text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{n}|x_i - M|$$

(b) Continuous frequency distribution

$$M.D.(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|, \text{ using } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right)h$$

$$M.D.(M) = \frac{1}{N}\sum_{i=1}^{n}|x_i - M|, \text{ using } M = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

** **Limitations of mean deviation :** The sum of the deviations from the mean (minus signs ignored) is more than the sum of the deviations from median. Therefore, the mean deviation about the mean is not very scientific.Thus, in many cases, mean deviation may give unsatisfactory results. Also mean deviation is calculated on the basis of absolute values of the deviations and therefore, cannot be subjected to further algebraic treatment.

Chapter Test

General Instructions: All questions are compulsory.

Section A (MCQ)

Question 1. Find the mean and variance for each of the data

6, 7, 10, 12, 13, 4, 8, 12

A) 9 and 9.25
B) 19 and 9.25
C) 3and 19.25
D) 29 and 6.25

Question 2. Find the mean deviation about the mean for the data

4, 7, 8, 9, 10, 12, 13, 17

A) 4
B) 6
C) 3
D) 8

Question 3. Find the mean deviation about the median for the data

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

A) 9
B) 1.2
C) 4
D) 2.33

Question 4. Find the mean and variance for each of the data

First 10 multiples of 3

A) 9 and 9.25
B) 16.5 and 74.25
C) 32 and 19.25
D) 19 and 61.25

DIRECTION:- In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as:

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true

Question 5: Assertion (A) Consider the following data :

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Then, the mean deviation about the mean is 6.32

Reason: (R) Consider the following data :

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Then, the mean deviation about the mean is 15

Ans- (c) Assertion is true but reason is false.

Section B Short Answer Type Question

Question 6. Find the mean deviation about the mean.

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Find the mean deviation about the median for the data in Exercises 7 and 8.

Question 7.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Question 8.

x_i	15	21	27	30	35	
f_i	3	5	6	7	8	

Find the mean deviation about the mean for the data in Exercises 9 and 10.

Question 9.

Income per day	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800
Number of persons	4	8	9	10	7	5	4	3

Question 10.

Heights (in cm)	95-105	105-115	115-125	125-135	135-145	145-155
Number of boys	9	13	26	30	12	10

Question 11.

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

Question 12

Find the mean and standard deviation using short-cut method

x_i	60	61	62	63	64	65	66	67	68

f_i	2	1	12	29	25	12	10	4	5

Find the mean and variance for the following frequency distributions in Exercises 13and 14.

Question 13

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

Question 14.

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Question 15.

Find the mean, variance and standard deviation using short-cut method.

Heights in cms	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of children	3	4	7	7	15	9	6	6	3

Section C (Long Answer Type Question)

Question 16.

Find the mean deviation about median for the following data:

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of girls	6	8	14	16	4	2

Question 17

Calculate the mean deviation about median age for the age distribution of 100 persons given below:

Age	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55
Number	5	6	12	14	26	12	16	9

[Hint: Convert the given data into continuous frequency distribution by subtracting 0.5 from lower limit and adding 0.5 to the upper limit of each class interval]

Question 18.

The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of Circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

[Hint: First make the data continuous by making the classes as 32.5 – 36.5, 36.5 – 40.5, 40.5 – 44.5, 44.5 – 48.5, 48.5 – 52.5 and then proceed.]

Question 19: The mean and variance of 8 observations are 9 and 9.25 respectively. If six of the observations are 6 , 7 , 10, 12 , 12 and 13 . Find the remaining two observations

Question 20 : The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are 2 , 4 , 10 , 12 , and 14 . Find the remaining two observations .

Section D (Case Study)

21 You are gfiven the following data

Xi	2	5	6	8	10	12
fi	2	8	10	7	8	5

Based on the following data answer the following questions

I) Mean of the grouped data is (a) 7 (b) 7.5 (c) 8 (d) 8.5

II)Mean deviation about the mean is (a) 2.1 (b) 2.2 (c) 2.3 (d) 2.4

III) The value of median is (A) 5 (b) 6 (c) 7 (d) 8

IV) The mean deviation about mean is (a) 1.9 (2.0 (c) 2.2 (d)2.3

V) The difference between mean and median is (a)0 .9 (b) 0.7 (c) 0.5 (d) 0.3

PROBABILITY

MAIN CONCEPTS AND RESULTS

** **Random Experiments :** An experiment is called random experiment if it satisfies the following two
conditions:
(i) It has more than one possible outcome.
(ii) It is not possible to predict the outcome in advance.

** **Outcomes and sample space :** A possible result of a random experiment is called its outcome.
The set of all possible outcomes of a random experiment is called the sample space associated with the experiment. Each element of the sample space is called a sample point. Any subset E of a sample space S is called an event.

** **Impossible and Sure Events :** The empty set φ and the sample space S describe events. φ is called an
impossible event and S, i.e., the whole sample space is called the sure event.

** **Compound Event :** If an event has more than one sample point, it is called a Compound event.

** **Complementary Event** : For every event A, there corresponds another event A' called the complementary event to A. It is also called the event 'not A'.

** **The Event 'A or B' :** When the sets A and B are two events associated with a sample space, then '$A \cup B$' is the event 'either A or B or both'. This event '$A \cup B$' is also called 'A or B'.

** **The Event 'A and B' :** If A and B are two events, then the set $A \cap B$ denotes the event 'A and B'.

** **The Event 'A but not B' :** the set A – B denotes the event 'A but not B'. $A - B = A \cap B'$

** **Mutually exclusive events :** two events A and B are called mutually exclusive events if the occurrence of
any one of them excludes the occurrence of the other event, i.e., if they can not occur simultaneously. In
this case the sets A and B are disjoint i.e. $A \cap B = \varphi$.

** **Exhaustive events :** if E1, E2, ..., En are n events of a sample space S and if

$E1 \cup E2 \cup E3 \cup \ldots \cup En = \bigcup_{i=1}^{n} E_i = S$, then E1, E2, ..., En are called exhaustive events.

if $Ei \cap Ej = \varphi$ for $i \neq j$ i.e., events Ei and Ej are pairwise disjoint and $\bigcup_{i=1}^{n} E_i = S$, then events E1, E2, ..., En are
called **mutually exclusive and exhaustive events.**

** **Axiomatic Approach to Probability :** Let S be the sample space of a random experiment. The probability P
is a real valued function whose domain is the power set of S and range is the interval [0,1] satisfying the
following axioms
(i) For any event E, $P(E) \geq 0$
(ii) P (S) = 1
(iii) If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$.
From the axiomatic definition of probability it follows that
(i) $0 \leq P(\omega i) \leq 1$ for each $\omega i \in S$
(ii) $P(\omega 1) + P(\omega 2) + ... + P(\omega n) = 1$
(iii) For any event A, $P(A) = \Sigma P(\omega i)$, $\omega i \in A$.

** **Equally likely outcomes** : All outcomes with equal probability.

** **Probability of an event**: For a finite sample space with equally likely outcomes

Probability of an event $P(A) = \frac{n(A)}{n(S)}$ (A) □ , where n(A) = number of elements in the set A,

n(S) = number of elements in the set S.

** **Probability of the event 'A or B' :** $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

For mutually exclusive events A and B, we have $P(A \cup B)$ □ □ $P(A)$ □ □ $P(B)$

** **Probability of event 'not A' =** $P(A') = P(\text{not } A) = 1 - P(A)$.

Example 1:A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.

Solution: Let R denotes the red ball and W denotes the white ball.

Given that a box contains 1 red and 3 identical white ball.

To draw two balls at random in succession without replacement, the sample space can be written as:

S = {RW, WR, WW}= {(R,W), (W,R),(W,W)}

Example2: An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

A: the sum is greater than 8.

B: 2 occurs on either die

C: the sum is at least 7 and a multiple of 3.

Which pairs of these events are mutually exclusive?

Solution: Given that a pair of dice rolled.

Sample space = S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3),

(5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

n(S) = 36

Event A: The sum is greater than 8

A = {(3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)}

Event B: 2 occurs on either die

B = {(1, 2), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (4, 2), (5, 2), (6, 2)}

Event C: The sum is at least 7 and a multiple of 3

C = {(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)}

Here,

$A \cap B = \Phi$

$B \cap C = \Phi$

$A \cap C \neq \Phi$

Therefore, the pair of events A, B and B, C are mutually exclusive.

Examples3: What is the probability of getting the number 6 at least once in a regular die if it can roll it 6 times?

Let A be the event that 6 does not occur at all.

Now, the probability of at least one 6 occurs = 1 – P(A)

$= 1 - \left(\frac{5}{6}\right)^6$

Example 4: A couple has two children,

(i) Find the probability that both children are males if it is known that at least one of the children is male.

(ii) Find the probability that both children are females if it is known that the elder child is a female.

Solution:

A couple has two children,

Let, the boy be denoted by b & girl be denoted by g

So, S = {(b, b),(b, g),(g, b), (g, g)}

To find probability that both children are males, if known that at least one of children is male

Let E: Both children are males
F : At least one child is male

To find P(E/F)

E: Both children are males

E = {(b, b)}

P(E) = $\frac{1}{4}$

F: At least one child is male

F = {(b, g), (g, b), (b, b)}

P(F) = $\frac{3}{4}$

E ∩ F = {(b, b)}

P (E ∩ F) = $\frac{1}{4}$

P (E/F) = $\frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

∴ Required Probability is $\frac{1}{3}$

(ii) Find the probability that both children are females if it is known that the elder child is a female.

S = {(b, b),(b, g),(g, b), (g, g)}

To find the probability that both children are females, if from that the elder child is a female.

Let E: both children are females
F : elder child is a female

To find P (E/F)

E: both children are females

E = {(g, g)}

P(E) = $\frac{1}{4}$

F : elder child is a female

F = {(g, b), (g, g)}

P(F) = $\frac{2}{4} = \frac{1}{2}$

Also, $E \cap F = \{(g, g)\}$

So, $P(E \cap F) = \frac{1}{4}$

$P(E/F) = \frac{P(E \cap F)}{P(F)}$

$= \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$

Examples 5:

A pack of 50 tickets is numbered from 1 to 50 and is shuffled. Two tickets are drawn at random. Find the probability that (i) both the tickets drawn bear prime numbers (ii) Neither of the tickets drawn bear prime numbers.

Solution:

The total number of tickets = 50

Prime numbers from 1 to 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47.

The total number of prime numbers between 1 and 50 is 15.

(i) Probability that both tickets are drawn bears prime numbers:

P(Both tickets bearing prime numbers) = ${}^{15}C_2 / {}^{50}C_2$

$= \frac{3}{35}$

Hence, the probability that both tickets are drawn bear prime numbers is $\frac{3}{35}$

(ii) Probability that neither of the tickets drawn bears prime numbers:

P (Neither of the tickets bearing prime numbers) = ${}^{35}C_2 / {}^{50}C_2$

$= \frac{17}{35}$

Therefore, the probability that neither of the tickets drawn bears a prime number is $\frac{17}{35}$

Practice Questions

Q.1 What is the total number of elements in sample spaces when a coin is tossed and a die is thrown?

a)12 b)10 c)11 d)13

Q.2 A bag contains 5 brown and 4 white socks. Ram pulls out two socks. What is the probability that both the socks are of the same colour?

a)$\frac{4}{5}$ b)$\frac{4}{9}$ c)$\frac{4}{3}$ d)$\frac{4}{7}$

Q.3: What is the probability of selecting a vowel in the word "ZIET"?

a)1 b)2 c) 0.5 d)None of the above

Q.4 An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. What is the probability that they are of different colours?

a) $\frac{4}{15}$ b) $\frac{2}{15}$ c)$\frac{1}{15}$ d) $\frac{8}{15}$

Q5.Assertion : Two dice are thrown simultaneously. There are 11 possible outcomes and each of them has a probability $\frac{1}{11}$

Reason : Probability of an event E is defined as P(E) =Number of favourable outcomes/total number of outcomes

A)Both assertion and reason are true and reason is the correct explanation of assertion.

B) Assertion and reason are true but reason is not the correct explanation of assertion.

C)Assertion is true but reason is false.

D) Assertion is false but reason is true.

Case Study

6) Two students Rohan and Soham appeared in an examination. The probability that Rohan will qualify the examination is 0.05 and that Soham will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Based on the above information, answer the following questions:

(i) Find the probability that both Rohan and Soham will not qualify the examination.

(ii) Find the probability that at least one of them will not qualify the examination.

(iii) only one of them will qualify the exam

Q7. 20 cards are numbered from 1 to 20. One card is drawn at random what is the prob. that the number on the card drawn is
(i) A prime Number (ii) An odd Number (iii) A multiple of 5 OR (iii) Not divisible by 3

8. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students what is the probability that
(a) You both enter the same section
(b) You both enter the different section.

SHORT TYPE QUESTIONS

9. If the letters of the word ALGORITHM are arranged at random in a row what is the probability the letters GOR must remain together as a unit?

10.A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P(G), where G is the event that a number greater than 3 occurs on a single roll of the die.

11. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is a consonant

12. 4 cards are drawn from a well snuffled deck of 52 cards what is the probability of obtaining 3 diamonds and one spade.

13.If P (A) = 0.6, P(B) =0.4 and P $(A \cap B)$ = 0, then the events are?

14. Three coins are tossed once. Find the probability at most two heads.

15.From a deck of 52 cards four cards are accidently dropped. Find the chance that the missing cards should be one from each type.

16.What is the probability that an ordinary year has 53 Sundays?

17.A book contains 100 pages. A page is chosen at random. What is the chance that the sum of the digit on the page is equal to9

18.In a single throw of two dice, find the prob. that neither a doublet nor a total of 10 will appear.

LONG TYPE QUESTION

19. In a class of 60 students, 30 opted for NCC, 32 opted for NSS, 24 opted for both NCC and NSS. If one of these students is selected at random. Find the probability that
(i) The student opted for NCC or NSS.
(ii) The student has opted neither NCC nor NSS.
(iii) The student has opted NSS but not NCC

20.In a town, there are 6000 people of which 1200 are over 50 years old and 2000 are females. It is said that 30% of females are over 50 years. Find the probability that an individual chosen randomly from the town is either female or over 50 years.

21.There are 20 cards that are numbered from 1 to 20. If a card is withdrawn randomly, then find the probability that a number on the card will be:

(i) Multiple of 4

(ii) Even number

(iii) Not divided by 5

(iv) Prime Number

22. If an entrance exam that is graded based on two exams, the probability of chosen at random, students clearing the 1st exam is 0.8 and the probability of passing the 2nd exam is 0.7. The probability of clearing at least one of them is 0. 95. Find the probability of clearing both.

23. A card has been drawn from a well-shuffled deck of 52 cards. What will be the probability that a card will be an

(i) Diamond

(ii) Black card

(iii) Not an ace

(iv) Not a diamond

ANSWERS

1. a 2. b 3. c 4. d 5.d 6. i) 0.87 ii)0.98 iii)0.11 (7) i) 2/5 ii)1/2 iii)1/5 OR 7/10 (8) (i) 17/33 (ii) 16/33

Short type

9) 1/72 10) 4/9 11)7/13 12) 286/20825 13.EXCLUSIVE AND EXHAUSTIVE 14.7/8

15. 2197/20825 (16).1/7 17.1/10 18.7/9

LONG ANSWER TYPE QUESTION

19 i. 19/30 ii 11/30 iii.4/30 20.13/20 21. (i) ¼ (ii) ½ (iii) 4/5 (iv) 2/5

22 0.55 23 (i) ¼ (ii) ½ (iii) 12/13 (iv) 3/4

CLASS TEST 1

MM -20 **TT- 40 MIN**

SECTION A (1X2=2)

1.In a single throw of two die, number of outcomes is ……………..

2.A die is thrown then probability that a prime number will appear…….

SECTION B (2X2=4)

3.If leap year selected random, what is the chance that will contain 53 Tuesdays?

4. A card is drawn from a deck of 52 cards. Find the probability of getting a king or a heart or a red.

SECTION C (3X3=9)

5. A bag contains 5 red, 6 white and 7 black balls. Two balls are drawn at random. What is the

probability that both balls are red or both are black?

6. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination 0.75, what is the probability of passing the Hindi examination?

7. Probability that a truck stopped at a roadblock will have faulty brakes or badly worn tires are 0.23 and 0.24, respectively. Also, the probability is 0.38 that a truck stopped at the roadblock will have faulty brakes and /or badly working tires. What is the probability that a truck stopped at this roadblock will have faulty breaks are well as badly worn tires.

SECTION D (5 × 1=5)

8.If a person visits his dentist, suppose the probability that he will have his teeth cleaned is 0.48, the probability that he will have a cavity filled is 0.25, the probability that he will have a tooth extracted is 0.20, the probability that he will have a teeth cleaned and a cavity filled is 0.09, the probability that he will have his teeth cleaned and a tooth extracted is 0.12, the probability that he will have a cavity filled and a tooth extracted is 0.07 and the probability that he will have his teeth cleaned, a cavity filled, and a tooth

Answers

1. 36 2. ½ 3. 2/7 (4) 7/13 5) 31/153 (6) 0.65 (7) 0.098) (8) 0.68

CLASS TEST 2

MM -30 **TT- 60 MIN**

SECTION A (1×2=2)

1. When a coin is tossed three times, then the number of possible outcomes ………………

2. A coin is tossed twice, what is probability that at least one tail occurs……………

SECTION B (2 ×2=4)

3.Consider the experiment of rolling a die. Let A be the event 'getting a prime number', B be the event 'getting odd number'. Write the sets representing the events (1) A or B (ii) A and B (iii) A but not (iv) "not A'.

4. Coin tossed twice, what is the probability that at least one tail occurs?

SECTION C (3 × 3=9)

5. A die tossed once. What is the probability of the number 8' coming up? What is the probability of number "less than 8" coming up?

6. What probability that randomly chosen two-digit positive integer is multiple of 3?

7. A letter chosen random from the word 'ASSASSINATION". Find the probability that letter

(i) a vowel (ii) a consonant.

SECTION D (5 × 3=15)

8. Suppose that each child born is equally likely to be boy or a girl. Consider a family with exactly three children. (a) List the eight elements in the sample space whose outcomes are all possible genders of the Three children.

(b) Write each of the following events as set and find its probability: (i) The even that exactly one child is girl (ii) The event that least two children are girls (iii) The even that no child is girl

9. A typical PIN (personal identification number) is sequence of any four symbols chosen from the 26 letters in the alphabet and the ten digits. If all PINs are equally likely, what the probability that randomly chosen PIN contains repeated symbol?

10. In class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% class study both Mathematics and Biology. If a student is selected at random from the c the probability that he will be studying Mathematics or Biology.

Answers

1. 8 2. ¾ 3. (i){1,2,3,5} (ii) {3,5} (iii) {2} (iv){ 1,4,6} (4) ¾ 5) 0 ,1 (6) 1/3 (7) 6/13, 7/13

8) {bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg} (b) i) 3/8 ii)1/2 iii) 1/89) 0.1583 10) 3/5