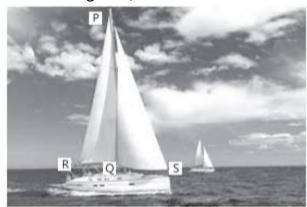
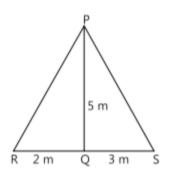
Introduction to Trigonometry

Case Study Based Questions

Case Study 1

A sailing boat with triangular masts is as shown below. Two right triangles can be observed Triangles PQR and PQS, both right-angled at Q. The distance QR = 2 m and QS = 3 m and height PQ = 5 m.





Based on the above information, solve the following questions:

Q1. The value of sec S is:

a.
$$\frac{\sqrt{34}}{5}$$

b.
$$\frac{\sqrt{34}}{3}$$

c.
$$\frac{5}{3}$$

d.
$$\frac{3}{\sqrt{34}}$$

Q 2. The value of cosec R is:

a.
$$\frac{\sqrt{29}}{5}$$

b.
$$\frac{\sqrt{29}}{2}$$

c.
$$\frac{2}{5}$$

d.
$$\frac{5}{\sqrt{29}}$$

a.
$$\frac{9}{4}$$

b.
$$\frac{5}{3}$$

a.
$$\frac{9}{4}$$
 b. $\frac{5}{3}$ c. $\frac{31}{15}$

d.
$$\frac{9}{5}$$

Q 4. The value of $\sin^2 R - \cos^2 S$ is:

c.
$$\frac{97}{85}$$

c.
$$\frac{97}{85}$$
 d. $\frac{589}{986}$

Q 5. The value of sin² S + cos² R is:

c.
$$\frac{97}{85}$$

d.
$$\frac{861}{986}$$

Solutions

1. In right-angled APQS

$$(PS)^2 = (SQ)^2 + (PQ)^2 = (3)^2 + (5)^2 = 9 + 25 = 34$$

(by Pythagoras theorem)

$$\Rightarrow$$
 PS = $\sqrt{34}$ m

∴ In right-angled ∆PQS,

$$\sec S = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{PS}{SQ} = \frac{\sqrt{34}}{3}$$

So, option (b) is correct.

2. In right-angled ∆PQR

$$(PR)^2 = (PQ)^2 + (QR)^2$$

$$^{2} = (PQ)^{2} + (QR)^{2}$$
(by Pythagoras theorem)
$$= (5)^{2} + (2)^{2} = 25 + 4$$

$$= 29$$

$$\Rightarrow$$

$$PR = \sqrt{29} \text{ m}$$

∴ In right-angled ∆PQR,

$$cosec R = \frac{Hypotenuse}{Perpendicular} = \frac{PR}{PQ} = \frac{\sqrt{29}}{5}$$

So, option (a) is correct.

3. Use the identity,

$$1 + \tan^2 S = \sec^2 S$$

$$\Rightarrow \qquad \tan S = \sqrt{\sec^2 S - 1} = \sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - 1}$$

$$=\sqrt{\frac{34}{9}-1}=\sqrt{\frac{25}{9}}=\frac{5}{3}$$

Use the identity, $1 + \cot^2 R = \csc^2 R$

$$\Rightarrow \qquad \cot R = \sqrt{\csc^2 R - 1} = \sqrt{\left(\frac{\sqrt{29}}{5}\right)^2 - 1}$$

(from part 2)

$$=\sqrt{\frac{29}{25}-1}=\sqrt{\frac{4}{25}}=\frac{2}{5}$$

$$\therefore \tan S + \cot R = \frac{5}{3} + \frac{2}{5} = \frac{25+6}{15} = \frac{31}{15}$$

So, option (c) is correct.

4. From part (1), $\sec S = \frac{\sqrt{34}}{3}$

$$\Rightarrow \qquad \cos S = \frac{3}{\sqrt{34}}$$

From part (2), cosec R= $\frac{\sqrt{29}}{5}$

$$\Rightarrow \qquad \sin R = \frac{5}{\sqrt{29}}$$

$$\sin^2 R - \cos^2 S = \left(\frac{5}{\sqrt{29}}\right)^2 - \left(\frac{3}{\sqrt{34}}\right)^2 = \frac{25}{29} - \frac{9}{34}$$

$$= \frac{850 - 261}{986} = \frac{589}{986}$$

So, option (d) is correct.

5. From part (1). $\sec S = \frac{\sqrt{34}}{3}$

$$\Rightarrow \cos 5 = \frac{3}{\sqrt{34}}$$

$$\sin S = \sqrt{1 - \cos^2 S} = \sqrt{1 - \frac{9}{34}} = \sqrt{\frac{25}{34}} = \frac{5}{\sqrt{34}}$$

From part (2), cosec R = $\frac{\sqrt{29}}{5}$

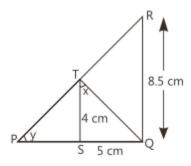
⇒
$$\sin R = \frac{5}{\sqrt{29}}$$

∴ $\cos R = \sqrt{1 - \sin^2 R} = \sqrt{1 - \frac{25}{29}} = \sqrt{\frac{4}{29}} = \frac{2}{\sqrt{29}}$
∴ $\sin^2 S + \cos^2 R = \left(\frac{5}{\sqrt{34}}\right)^2 + \left(\frac{2}{\sqrt{29}}\right)^2 = \frac{25}{34} + \frac{4}{25}$
 $= \frac{725 + 136}{986} = \frac{861}{986}$

So, option (d) is correct.

Case Study 2

Anika is studying in X standard. She is making a figure to understand trigonometric ratio shown as below.



In $\triangle PQR$, <Q is a right angle, $\triangle QTR$ is right- angled at T and $\triangle QST$ is right-angled at S, PQ = 12 cm, QR = 8.5 cm, ST=4 cm, SQ = 5 cm, <QTSx and <TPQ = y.

Based on the given information, solve the following questions:

Q1. The length of PT is:

- a. 8 cm
- b. √65 cm
- c. 7.5 cm
- d. √69 cm

Q2. The value of tan x is:

a.
$$\frac{7.5}{13}$$

b.
$$\frac{5}{4}$$

c.
$$\frac{4}{5}$$

d.
$$\frac{13}{7.5}$$

Q3. The value of sec x is:

a.
$$\frac{\sqrt{91}}{6}$$

b.
$$\frac{\sqrt{71}}{6}$$

c.
$$\frac{\sqrt{41}}{4}$$

d.
$$\frac{\sqrt{31}}{5}$$

Q4. The value of sin y is:

b.
$$\frac{4}{7}$$

c.
$$\frac{7}{4}$$

Q5. The value of cot y is:

a.
$$\frac{7}{4}$$

b.
$$\frac{4}{7}$$

c.
$$\frac{\sqrt{65}}{4}$$

d.
$$\frac{\sqrt{65}}{7}$$

Solutions

1. We have, PSPQ-SQ=12-5=7cm

In right-angled ∆PST,

$$(PT)^2 = (PS)^2 + (ST)^2$$
 (By Pythagoras theorem)

$$=(7)^2+(4)^2=49+16=65$$

$$\Rightarrow$$
 PT = $\sqrt{65}$ cm

So, option (b) is correct.

2. In right-angled ΔTSQ,

$$\tan x = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{SQ}}{\text{TS}} = \frac{5}{4}$$

So, option (b) is correct.

3. We know the identity.

$$\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{5}{4}\right)^2 = 1 + \frac{25}{16} = \frac{41}{16}$$

$$\Rightarrow \qquad \sec x = \sqrt{\frac{41}{16}} = \frac{\sqrt{41}}{4}$$

So, option (c) is correct.

In right-angled ∆TSP,

$$siny = \frac{Perpendicular}{Hypotenuse} = \frac{TS}{PT} = \frac{4}{\sqrt{65}}$$

So, option (a) is correct.

5. In right angled ∆TSP

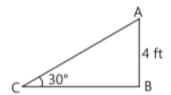
$$\cot y = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\text{PS}}{\text{TS}} = \frac{7}{4}$$

So, option (a) is correct.

Case Study 3

In structural design, a structure is composed of triangles that are interconnecting. A truss a series of triangle in same plane end is one of the major types of engineering structures and is especially used in the design of bridges and buildings. Trusses are designed to support loads, such as the weight of people. A truss is exclusively made of long, straight members connected by joints at the end of each member.





This is a single repeating triangle in a truss system.

Based on the above information, solve the following questions:

- Q1. In the above triangle, what is the length of AC?
- Q2. In the above triangle, what is the length of BC?
- Q3. If sin A = sin C, what will be the length of BC?

0r

If the length of AB doubles, what will happen the Length of AC? Solutions

In right angled ∆ABC,

$$\sin 30^{\circ} = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{4}{AC}$$

$$AC = 8 \text{ ft}$$

2. In right-angled ΔABC,

 \Rightarrow

$$\tan 30^\circ = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{4}{BC} \Rightarrow BC = 4\sqrt{3} \text{ ft}$$

Given, sin A = sin C
 In right-angled Δ ABC,

$$\frac{BC}{AC} = \frac{AB}{AC} \implies BC = AB = 4 \text{ ft}$$

Given, $AB = 2 \times 4 = 8$ ft

∴ In right
$$\triangle ABC$$
, $\sin 30^\circ = \frac{AB}{AC}$

$$\Rightarrow \frac{1}{2} = \frac{8}{AC} \Rightarrow AC = 16 \text{ ft}$$

So, AC doubles the original length.

Case Study 4

Soniya and her father went to her friend Ruhi to enjoy party. When they reached Ruhi's place, Soniya saw the roof of the house, which was triangular in shape. She imagined the dimensions of the roof which is as given in the figure.



Based on the above information, solve the following questions:

- Q1. If D is the mid-point of AC, then find BD.
- Q2. Find the measure of <A and <C.
- Q3. Find the value of sin A + cos C.

Or

Find the value of tan² C + tan² A.

Solutions

1. We have, AB = BC = $6\sqrt{2}$ m and AC = 12 m D is the mid-point of AC.

∴ AD = DC =
$$\frac{12}{2}$$
 = 6 m

In right-angled ΔADB , use Pythagoras theorem

$$AB^{2} = BD^{2} + AD^{2}$$

$$BD^{2} = (6\sqrt{2})^{2} - 6^{2}$$

$$BD^{2} = 72 - 36 = 36$$

2. In right
$$\triangle ADB$$
, $\sin A = \frac{BD}{AB} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$ (from part (1))

$$\Rightarrow$$
 $\sin A = \sin 45^{\circ} \Rightarrow \angle A = 45^{\circ}$

In right
$$\triangle BDC$$
, $\tan C = \frac{BD}{DC} = \frac{6}{6}$
 $\Rightarrow \tan C = 1 = \tan 45^\circ \Rightarrow \angle C = 45^\circ$

3. Here, $\sin A = \frac{1}{\sqrt{2}}$ and $\cos C = \cos 45^\circ = \frac{1}{\sqrt{2}}$
 $\therefore \sin A + \cos C = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

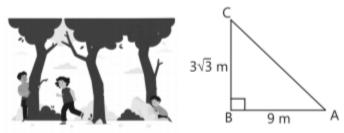
Or

Here, $\tan C = 1$ and $\tan A = \tan 45^\circ = 1$
 $\Rightarrow \tan^2 C + \tan^2 A = 1 + 1 = 2$

Case Study 5

Three friends-Sanjeev, Amit and Digvijay are playing hide and seek in a park. Sanjeev and Amit were supposed to hide and Digvijay had to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure and forms a right-angled triangle such that AB = 9 m,

BC= $3\sqrt{3}$ m and B = 90° .



Based on the above information, solve the following questions:

- Q1. Find the measure of <A by using trigonometric ratio.
- Q2. Find the measure of <C by using trigonometric ratio.
- Q3. Find the length of AC.
- Q4. Find the value of cos 2A.

Or

Find the value of $sin\left(\frac{C}{2}\right)$.

Solutions

1. We have, AB = 9 m, BC = $3\sqrt{3}$ m In right AABC, we have

$$\tan A = \frac{BC}{AB} = \frac{3\sqrt{3}}{9} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \qquad \tan A = \tan 30^{\circ} \Rightarrow \angle A = 30^{\circ}$$

2. In right ∆ABC,

We have,
$$\tan C = \frac{AB}{BC} = \frac{9}{3\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow$$
 tan C = tan 60° $\Rightarrow \angle$ C = 60°

3. In right $\triangle ABC$, $\sin A = \frac{BC}{AC}$

$$\Rightarrow \qquad \sin 30^{\circ} = \frac{BC}{AC} \qquad \qquad \text{(from part (1))}$$

$$\Rightarrow \frac{1}{2} = \frac{3\sqrt{3}}{AC} \Rightarrow AC = 6\sqrt{3} \text{ m}$$

4. :
$$\angle A = 30^{\circ}$$
 [from part (1)]

$$\therefore \qquad \cos 2A = \cos (2 \times 30^\circ) = \cos 60^\circ = \frac{1}{2}$$

Solutions for Questions 6 to 10 are Given Below

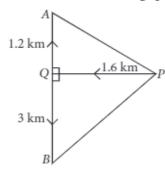
Case Study 6

Case Study 7

Two Flights

Two aeroplanes leave an airport, one after the other. After moving on runway, one flies due North and other flies due South. The speed of two aeroplanes is 400 km/hr and 500 km/hr respectively. Considering PQ as runway and A and B are any two points in the path followed by two planes, then answer the following questions.





- (i) Find $\tan \theta$; if $\angle APQ = \theta$.
 - (a) $\frac{1}{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$

(d) $\frac{3}{4}$

- (ii) Find cotB.
 - (a) $\frac{3}{4}$

(b) $\frac{15}{4}$

(c) $\frac{3}{8}$

(d) $\frac{15}{8}$

- (iii) Find tanA.
 - (a) 2

(b) $\sqrt{2}$

(c) $\frac{4}{3}$

(d) $\frac{2}{\sqrt{3}}$

- (iv) Find secA.
 - (a) 1

(b) $\frac{2}{3}$

(c) $\frac{4}{3}$

(d) $\frac{5}{3}$

- (v) Find cosecB.
 - (a) $\frac{17}{8}$

(b) $\frac{12}{5}$

(c) $\frac{5}{12}$

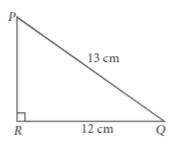
(d) $\frac{8}{17}$

Case Study 8

Bird House

Anita, a student of class 10th, has to made a project on 'Introduction to Trigonometry'. She decides to make a bird house which is triangular in shape. She uses cardboard to make the bird house as shown in the figure. Considering the front side of bird house as right angled triangle *PQR*, right angled at *R*, answer the following questions.





- (i) If $\angle PQR = \theta$, then $\cos\theta =$

- (c) $\frac{12}{13}$
- (d) 13

- (ii) The value of $\sec\theta =$

- (d) $\frac{12}{13}$

- (iii) The value of $\frac{\tan \theta}{1 + \tan^2 \theta} =$ (a) $\frac{5}{12}$

- (iv) The value of $\cot^2\theta = \csc^2\theta =$
 - (a) -1

(c) 1

(d) 2

- (v) The value of $\sin^2\theta + \cos^2\theta =$
 - (a) 0

(c) -1

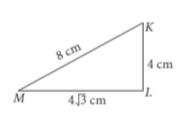
(d) 2

Case Study 9

Sandwich Making

Ritu's daughter is feeling so hungry and so thought to eat something. She looked into the fridge and found some bread pieces. She decided to make a sandwich. She cut the piece of bread diagonally and found that it forms a right angled triangle, with sides 4 cm, $4\sqrt{3}$ cm and 8 cm.





On the basis of above information, answer the following questions.

- (i) The value of $\angle M =$
 - (a) 30°

(b) 60°

(c) 45°

(d) None of these

- (ii) The value of $\angle K =$
 - (a) 45°

(b) 30°

(c) 60°

(d) None of these

- (iii) Find the value of tanM.
 - (a) $\sqrt{3}$

(c) 1

(d) None of these

- (iv) $\sec^2 M 1 =$
 - (a) tanM

- (b) tan2M
- (c) tan²M
- (d) None of these

- (v) The value of $\frac{\tan^2 45^\circ 1}{\tan^2 45^\circ + 1}$ is
 - (a) 0

(b) 1

(c) 2

(d) -1

Case Study 10

Roof Top of House

Aanya and her father go to meet her friend Juhi for a party. When they reached to Juhi's place, Aanya saw the roof of the house, which is triangular in shape. If she imagined the dimensions of the roof as given in the figure, then answer the following questions.



- (i) If D is the mid point of AC, then BD =
 - (a) 2 m

(b) 3 m

(c) 4 m

(d) 6 m

- (ii) Measure of $\angle A =$
 - (a) 30°

(b) 60°

(c) 45°

(d) None of these

- (iii) Measure of $\angle C =$
 - (a) 30°

(b) 60°

(c) 45°

(d) None of these

- (iv) Find the value of sinA + cosC.
 - (a) 0

(b) 1

(c) $\frac{1}{\sqrt{2}}$

(d) $\sqrt{2}$

- (v) Find the value of tan²C + tan²A.
 - (a) 0

(b) 1

(c) 2

(d) $\frac{1}{2}$

HINTS & EXPLANATIONS

6. (i) (a): We have, AB = 9 m, $BC = 3\sqrt{3} \text{ m}$ In $\triangle ABC$, we have

$$\tan A = \frac{BC}{AB} = \frac{3\sqrt{3}}{9} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow$$
 tanA = tan30° $\Rightarrow \angle A = 30$ °

- (ii) (c): Similarly, $\tan C = \frac{AB}{BC} = \frac{9}{3\sqrt{3}} = \sqrt{3}$
- ...(2) \Rightarrow tanC = tan60° \Rightarrow $\angle C = 60°$
- (iii) (d): Since, $\sin A = \frac{BC}{AC} \implies \sin 30^\circ = \frac{BC}{AC}$ [Using (1)]
- $\Rightarrow \frac{1}{2} = \frac{3\sqrt{3}}{AC} \Rightarrow AC = 6\sqrt{3} \text{ m}$

(iv) (b): ∴ ∠A = 30°

[From (1)]

- \therefore cos 2A = cos(2 × 30°) = cos 60° = $\frac{1}{2}$
- (v) (b): ∴ ∠C = 60° [Using (2)]
- ...(1) $\therefore \sin\left(\frac{C}{2}\right) = \sin\left(\frac{60^{\circ}}{2}\right) = \sin 30^{\circ} = \frac{1}{2}$
 - 7. (i) (d): $\ln \Delta APQ$, $\tan \theta = \frac{AQ}{PQ} = \frac{1.2}{1.6} = \frac{3}{4}$
 - (ii) (d): In $\triangle PBQ$, $\cot B = \frac{QB}{PQ} = \frac{3}{1.6} = \frac{15}{8}$...(1)
 - (iii) (c): In $\triangle APQ$, $\tan A = \frac{PQ}{AQ} = \frac{1.6}{1.2} = \frac{4}{3}$...(2)
 - (iv) (d): We have, $\tan^2 A + 1 = \sec^2 A$

$$\Rightarrow \sec A = \sqrt{\left(\frac{4}{3}\right)^2 + 1}$$
 [Using (2)]
$$= \sqrt{\frac{16}{9} + 1} = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

(v) (a): Since,
$$\csc B = \sqrt{\cot^2 B + 1}$$

= $\sqrt{\left(\frac{15}{8}\right)^2 + 1}$ [Using (1)]
= $\frac{17}{8}$

8. $\therefore \Delta PQR$ is a right angled triangle.

$$\therefore PR^2 + RQ^2 = PQ^2$$

$$\Rightarrow PR^2 = (13)^2 - (12)^2 = 25 \Rightarrow PR = 5 \text{ cm}$$

(i) (c):
$$\cos \theta = \frac{QR}{PQ} = \frac{12}{13}$$

(ii) (c):
$$\sec \theta = \frac{1}{\cos \theta} = \frac{13}{12}$$

(iii) (c):
$$\tan \theta = \frac{PR}{RO} = \frac{5}{12}$$
 ...(1)

$$\therefore \frac{\tan \theta}{1 + \tan^2 \theta} = \frac{\frac{5}{12}}{1 + \frac{25}{144}} = \frac{\frac{5}{12}}{\frac{169}{144}} = \frac{60}{169}$$

(iv) (a):
$$\cot \theta = \frac{1}{\tan \theta} = \frac{12}{5}$$
 [Using (1)]
 $\csc \theta = \frac{PQ}{PR} = \frac{13}{5}$

$$\therefore \cot^2 \theta - \csc^2 \theta = \frac{144}{25} - \frac{169}{25} = -1$$

(v) (b):
$$\sin^2\theta + \cos^2\theta = 1$$
 (Using identity)

9. We have, KL = 4 cm, $ML = 4\sqrt{3} \text{ cm}$, KM = 8 cm

(i) (a):
$$\tan M = \frac{KL}{LM} = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow$$
 tanM = tan30° \Rightarrow $\angle M = 30°$

(ii) (c):
$$\tan K = \frac{ML}{KL} = \frac{4\sqrt{3}}{4} = \sqrt{3} = \tan 60^{\circ}$$

 $\Rightarrow \angle K = 60^{\circ}$

(v) (a):
$$\frac{\tan^2 45^\circ - 1}{\tan^2 45^\circ + 1} = \frac{(1)^2 - 1}{1^2 + 1} = \frac{0}{2} = 0$$

10. We have, $AB = BC = 6\sqrt{2}$ m and AC = 12 m.

(i) (d): :: D is mid point of AC.

$$\therefore AD = DC = 6 \text{ m}$$

Now, $AB^2 = BD^2 + AD^2$ (: $\triangle ABD$ is a right triangle)

$$\Rightarrow BD^2 = (6\sqrt{2})^2 - 6^2 = 72 - 36 = 36$$

$$\Rightarrow BD = 6 \text{ m}$$
 ...(1)

(ii) (c): In
$$\triangle ABD$$
, $\sin A = \frac{BD}{AB} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$ [Using (1)]

$$\Rightarrow$$
 $\sin A = \sin 45^{\circ} \Rightarrow \angle A = 45^{\circ}$

(iii) (c): In
$$\triangle BDC$$
, $\tan C = \frac{BD}{DC} = \frac{6}{6}$ [Using (1)]

$$\Rightarrow$$
 tan $C = 1 = \tan 45^{\circ} \Rightarrow \angle C = 45^{\circ}$

(iv) (d):
$$\sin A = \frac{1}{\sqrt{2}}$$
, $\cos C = \cos 45^\circ = \frac{1}{\sqrt{2}}$

$$\therefore \quad \sin A + \cos C = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

(v) (c):
$$\tan C = 1$$
, $\tan A = \tan 45^{\circ} = 1$

$$\Rightarrow$$
 $\tan^2 C + \tan^2 A = 1 + 1 = 2$