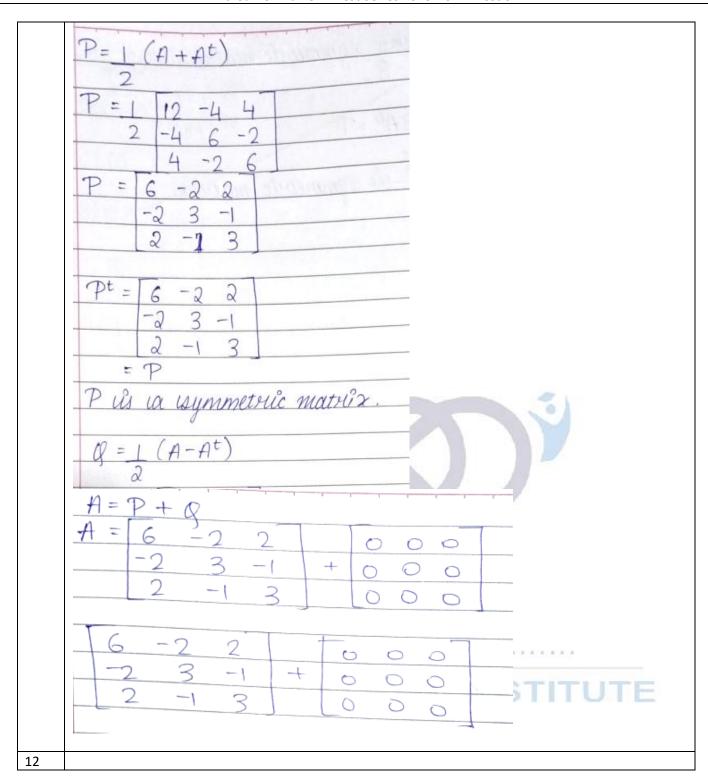
Answer key

1	(d) 25		
2	(d) 1000		
3	(b)1/2		
4	(a)516A		
5	(b)zero square matrix		
6	(b)		
7	(d)		
8	(b)		
9	(c) (A) is true and (R) is false		
10	(a) Both(A) and (R) are true and (R) is the correct explanation of (A)		
11	(a) Both(A) and (R) are true and (R) is the correct explanation of (A) (Linen: B is skend by mmetric matrix Bt=-B. To check whether ABAt is syrorskey Let P=ABAt Take franspose Pt = (ABAt)t = Abt At (: (AB) - BtAt) = ABAt (: (BtB) = ABAt (: BtB) = Symmetric.		
	Or		
	$A = 6 - 2 2$; $A^{t} = 6 - 2 2$ $-2 3 - 1$; $A^{t} = 6 - 2 2$ 2 - 1 3 2 - 1 3 $A = 1 (A + A^{t}) + 1 (A - A^{t})$ 2 2 2 2 2 3 2 3 3		



9 -1 -2 1	
	1 3 [0 4 9]
A = [8	-3 5 -3 -6
THE RESIDENCE OF THE PARTY OF T	-3 5 -3 -6

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Proof:

Let $A^{ op}=A$ and $B^{ op}=B$.

1. Show AB+BA is symmetric:

$$(AB + BA)^{\top} = (AB)^{\top} + (BA)^{\top}$$
$$= B^{\top}A^{\top} + A^{\top}B^{\top}$$
$$= BA + AB = AB + BA.$$

2. Show ${\cal AB}-{\cal BA}$ is skew-symmetric:

$$(AB - BA)^{\top} = (AB)^{\top} - (BA)^{\top}$$
$$= B^{\top}A^{\top} - A^{\top}B^{\top}$$
$$= BA - AB = -(AB - BA).$$



Conclusion:

- $\bullet \quad AB+BA \text{ is symmetric.} \\$
- ullet AB-BA is skew-symmetric.

OR

$$B' = \begin{bmatrix} 4 & 7 & 2 \\ 0 & 1 & 2 \\ 2 & 4 & 6 \end{bmatrix} & & A' = \begin{bmatrix} 3 & 1 & 7 \\ 9 & 8 & 5 \\ 0 & -2 & 4 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} 4 & 7 & 2 \\ 0 & 1 & 2 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 3 & 1 & 7 \\ 9 & 8 & 5 \\ 0 & -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 12 + 63 & 4 + 56 - 4 & 28 + 35 + 8 \\ 9 & 8 - 4 & 5 + 8 \\ 6 + 36 & 2 + 32 - 12 & 14 + 20 + 24 \end{bmatrix}$$

$$= \begin{bmatrix} 75 & 56 & 71 \\ 9 & 4 & 13 \\ 42 & 22 & 58 \end{bmatrix}$$

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$$X \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}.$$

Let X be a 2×2 matrix:

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
.

Multiplying out on the left yields:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} a \cdot 1 + b \cdot 4 & a \cdot 2 + b \cdot 5 & a \cdot 3 + b \cdot 6 \\ c \cdot 1 + d \cdot 4 & c \cdot 2 + d \cdot 5 & c \cdot 3 + d \cdot 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}.$$

Equating corresponding entries, we get two separate systems:

First row system (for a, b):

$$\begin{cases} a+4b = -7, \\ 2a+5b = -8, \\ 3a+6b = -9. \end{cases}$$

Second row system (for c, d):

$$\begin{cases} c + 4d = 2, \\ 2c + 5d = 4, \\ 3c + 6d = 6. \end{cases}$$

Solving these:

· From the first system:

$$a+4b=-7$$
 (1), $2a+5b=-8$ (2).

Multiply (1) by 2: 2a + 8b = -14. Subtract (2):

$$(2a+8b)-(2a+5b)=-14-(-8) \implies 3b=-6 \implies b=-2.$$

Substitute back into (1): $a+4(-2)=-7 \implies a-8=-7 \implies a=1$.

· From the second system:

$$c+4d=2$$
 (3), $2c+5d=4$ (4).

Multiply (3) by 2: 2c + 8d = 4. Subtract (4):

$$(2c + 8d) - (2c + 5d) = 4 - 4 \implies 3d = 0 \implies d = 0.$$

Substitute back into (3): $c+4\cdot 0=2\implies c=2$.

Hence:

$$a=1,\quad b=-2,\quad c=2,\quad d=0.$$

So the matrix X is:

$$X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}.$$

Given $\cos 2\theta = 0$

Let
$$A = \begin{bmatrix} \theta & \cos \theta & \sin \theta \\ \cos \theta & \sin \theta & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}^2$$

$$\mathbf{A} = \begin{bmatrix} \left| \sin \theta & 0 \\ 0 & \cos \theta \right| - \cos \left| \cos \theta & 0 \\ \sin \theta & \cos \theta \right| + \sin \theta \left| \cos \theta & \sin \theta \\ \sin \theta & 0 \right| \end{bmatrix}^2$$

$$A = \left[0 - \cos\theta \left(\cos\theta - 0\right) + \sin\theta \left(0 - \sin^2\theta\right)\right]^2$$

$$A = [-\cos 3\theta - \sin \theta] 2$$

$$A = \left[\cos^3\theta + \sin^3\theta\right]^2 \dots (1)$$



 $\cos 2\theta = 0$

$$\Rightarrow 2\theta = \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Substituting $\theta = \frac{\pi}{4}$ in equation (1) we get

$$A = \left[\frac{\cos^3 \pi}{4} + \frac{\sin^3 \pi}{4} \right]^2$$

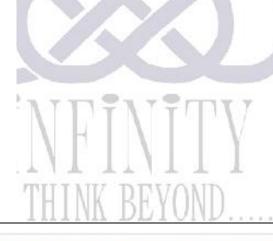
$$= \left[\left(\frac{1}{\sqrt{2}} \right)^3 + \left(\frac{1}{\sqrt{2}} \right)^3 \right]^2$$

$$= \left[2 \times \left(\frac{1}{\sqrt{2}}\right)^3\right]^2$$

$$= \left[2 \times \frac{1}{2\sqrt{2}}\right]^2$$

$$= \left[\frac{1}{\sqrt{2}}\right]^2$$

$$A = \frac{1}{2}$$



15	Area = 16 sg unit

Equation of PQ is x-3y=0

(ii)

$$\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \end{bmatrix}$$

$$\begin{bmatrix} 13 & 5 \\ 5 & 2 \end{bmatrix}$$

$$\begin{array}{c} \text{Or} \\ \text{PQ=} \begin{bmatrix} 1 & 0 \\ 4 & 0 \end{bmatrix} \text{,QP=} \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$$

Let x, y, z be the daily production (in tons) of the first, second and third products respectively.

From the problem:

- total production =45 gives: x+y+z=45.
- "third exceeds first by 8" gives: z x = 8 (or -x + 0y + z = 8).
- "first + third is twice the second" gives: x+z=2y (or x-2y+z=0).

So the system is

$$\begin{cases} x + y + z = 45, \\ -x + 0y + z = 8, \\ x - 2y + z = 0. \end{cases}$$

Matrix form: AX = B where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}, \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \qquad B = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}.$$

(ii) x=11 tons, y=15tons& z=19 tons

18	PROVING
19(A)	X=-21,Y=98,Z=64
19(B)	X=2,Y=-1,Z=5



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