Answer key

1	(d) 25
2	(d) 1000
3	(b)1/2
4	(a)516A
5	(b)zero square matrix
6	(b)
7	(d)
8	(b)
9	(c) (A) is true and (R) is false
10	(a) Both(A) and (R) are true and (R) is the correct explanation of (A)
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Given: B is skew symmetric matrix

$B^t = -B$.

To check whether ABA^t is sy. or sk sy.

Let $P = ABA^t$

Take transpose

$P^t = (ABA^t)^t$

$= (A^t)^t B^t A^t \quad (\because (AB)^t = B^t A^t)$

$= AB^t A^t \quad (\because (A^t)^t = A)$

$= -ABA^t \quad (\because B^t = -B)$

$= -P$

$\Rightarrow P$ is skew symmetric.

Or

$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}; \quad A^t = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$A = \frac{1}{2}(A+A^t) + \frac{1}{2}(A-A^t)$

Let $P = \frac{1}{2}(A+A^t)$ and $Q = \frac{1}{2}(A-A^t)$

$A = P + Q$

$$P = \frac{1}{2}(A + A^t)$$

$$P = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$$

$$P = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$P^t = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= P$$

P is a symmetric matrix.

$$Q = \frac{1}{2}(A - A^t)$$

$$A = P + Q$$

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$$

$$A = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$$

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Proof:

Let $A^{\top} = A$ and $B^{\top} = B$.

1. **Show $AB + BA$ is symmetric:**

$$\begin{aligned}(AB+BA)^{\top} &= (AB)^{\top} + (BA)^{\top} \\ &= B^{\top}A^{\top} + A^{\top}B^{\top} \\ &= BA + AB = AB + BA.\end{aligned}$$

2. **Show $AB - BA$ is skew-symmetric:**

$$\begin{aligned}(AB-BA)^{\top} &= (AB)^{\top} - (BA)^{\top} \\ &= B^{\top}A^{\top} - A^{\top}B^{\top} \\ &= BA - AB = -(AB - BA).\end{aligned}$$

Conclusion:

- $AB + BA$ is symmetric.
- $AB - BA$ is skew-symmetric.

OR

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$$B' = \begin{bmatrix} 4 & 7 & 2 \\ 0 & 1 & 2 \\ 2 & 4 & 6 \end{bmatrix} \ \& \ A' = \begin{bmatrix} 3 & 1 & 7 \\ 9 & 8 & 5 \\ 0 & -2 & 4 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} 4 & 7 & 2 \\ 0 & 1 & 2 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 3 & 1 & 7 \\ 9 & 8 & 5 \\ 0 & -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 12+63 & 4+56-4 & 28+35+8 \\ 9 & 8-4 & 5+8 \\ 6+36 & 2+32-12 & 14+20+24 \end{bmatrix}$$

$$= \begin{bmatrix} 75 & 56 & 71 \\ 9 & 4 & 13 \\ 42 & 22 & 58 \end{bmatrix}$$

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$$X \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}.$$

Let X be a 2×2 matrix:

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Multiplying out on the left yields:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} a \cdot 1 + b \cdot 4 & a \cdot 2 + b \cdot 5 & a \cdot 3 + b \cdot 6 \\ c \cdot 1 + d \cdot 4 & c \cdot 2 + d \cdot 5 & c \cdot 3 + d \cdot 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}.$$

Equating corresponding entries, we get two separate systems:

First row system (for a, b):

$$\begin{cases} a + 4b = -7, \\ 2a + 5b = -8, \\ 3a + 6b = -9. \end{cases}$$

Second row system (for c, d):

$$\begin{cases} c + 4d = 2, \\ 2c + 5d = 4, \\ 3c + 6d = 6. \end{cases}$$

Solving these:

- From the first system:

$$a + 4b = -7 \quad (1), \quad 2a + 5b = -8 \quad (2).$$

Multiply (1) by 2: $2a + 8b = -14$. Subtract (2):

$$(2a + 8b) - (2a + 5b) = -14 - (-8) \implies 3b = -6 \implies b = -2.$$

Substitute back into (1): $a + 4(-2) = -7 \implies a - 8 = -7 \implies a = 1.$

- From the second system:

$$c + 4d = 2 \quad (3), \quad 2c + 5d = 4 \quad (4).$$

Multiply (3) by 2: $2c + 8d = 4$. Subtract (4):

$$(2c + 8d) - (2c + 5d) = 4 - 4 \implies 3d = 0 \implies d = 0.$$

Substitute back into (3): $c + 4 \cdot 0 = 2 \implies c = 2.$

Hence:

$$a = 1, \quad b = -2, \quad c = 2, \quad d = 0.$$

So the matrix X is:

$$\boxed{X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}.}$$

or

Given $\cos 2\theta = 0$

$$\text{Let } A = \begin{bmatrix} \theta & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}^2$$

$$A = \left[\begin{vmatrix} \sin\theta & 0 \\ 0 & \cos\theta \end{vmatrix} - \cos \begin{vmatrix} \cos\theta & 0 \\ \sin\theta & \cos\theta \end{vmatrix} + \sin\theta \begin{vmatrix} \cos\theta & \sin\theta \\ \sin\theta & 0 \end{vmatrix}\right]^2$$

$$A = \left[0 - \cos\theta\,(\cos\theta - 0) + \sin\theta\,(0 - \sin^2\theta)\right]^2$$

$$A = [-\cos 3\theta - \sin\theta]\,2$$

$$A = \left[\cos^3\theta + \sin^3\theta\right]^2 \quad ....(1)$$

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$\cos 2\theta = 0$

$\Rightarrow 2\theta = \frac{\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{4}$

Substituting $\theta = \frac{\pi}{4}$ in equation (1) we get

$A = \left[\frac{\cos^3 \pi}{4} + \frac{\sin^3 \pi}{4}\right]^2$

$= \left[\left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^3\right]^2$

$= \left[2 \times \left(\frac{1}{\sqrt{2}}\right)^3\right]^2$

$= \left[2 \times \frac{1}{2\sqrt{2}}\right]^2$

$= \left[\frac{1}{\sqrt{2}}\right]^2$

$A = \frac{1}{2}$

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<tr><td>15</td><td>Area = 16 sq unit
Equation of PQ is x-3y=0</td></tr>
<tr><td>16(i)

(ii)

(iii)a</td><td>

3x-2y=200
x-y=50

$$\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \end{bmatrix}$$

$$\begin{bmatrix} 13 & 5 \\ 5 & 2 \end{bmatrix}$$

Or

PQ= $\begin{bmatrix} 1 & 0 \\ 4 & 0 \end{bmatrix}$,QP= $\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$

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	Let x, y, z be the daily production (in tons) of the first, second and third products respectively. From the problem: • total production $= 45$ gives: $x + y + z = 45$. • "third exceeds first by 8" gives: $z - x = 8$ (or $-x + 0y + z = 8$). • "first + third is twice the second" gives: $x + z = 2y$ (or $x - 2y + z = 0$). So the system is $$\begin{cases} x + y + z = 45, \\ -x + 0y + z = 8, \\ x - 2y + z = 0. \end{cases}$$ Matrix form: $AX = B$ where $$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}.$$ (ii) x=11 tons, y=15tons& z=19 tons
18	PROVING
19(A) 19(B)	X=-21,Y=98,Z=64 X=2,Y=-1,Z=5