

ANDHRA EDUCATION SOCIETY SCHOOLS

New Delhi

Midterm 2023 - 24

CLASS 12 - APPLIED MATHEMATICS

Time Allowed : 180 mins

Maximum Marks : 80

**General Instructions:**

1. This Question paper contains - five sections A,B,C,D and E. Each section is compulsory. However, there is some internal choice in some questions.
2. Section A has 18 MCQ's and 02 Assertion Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer(VSA) questions of 2 marks each.
4. Section C has 6 Short Answer(SA) questions of 3 marks each.
5. Section D has 4 Long Answer(LA) questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (04 marks each) with sub parts.
7. Internal Choice is provided in 2 questions in Section - B, 2 questions in Section - C, 2 Questions in Section - D. You have to attempt only one alternatives in all such questions.

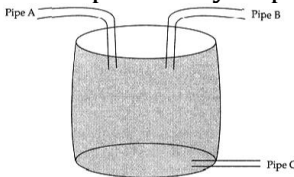
|   | Section A                                                                                                                                                                              |     |
|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 1 | (15 - 53) (mod 4) is<br>a) 1   b) 4   c) 2   d) 3                                                                                                                                      | [1] |
| 2 | The least positive integer x satisfying $28 \equiv x \pmod{6}$ is<br>a) 2   b) 1   c) 3   d) 4                                                                                         | [1] |
| 3 | In what ratio must rice at ₹ 29.30 per kg be mixed with rice at ₹ 30.80 per kg so that the mixture be worth ₹ 30 per kg?<br>a) 7:8   b) 3:8   c) 8:3   d) 8:7                          | [1] |
| 4 | If A is $3 \times 4$ matrix and B is a matrix such that $A^T B$ and $BA^T$ are both defined. Then, B is of the type<br>a) $4 \times 4$ b) $4 \times 3$ c) $3 \times 4$ d) $3 \times 3$ | [1] |
| 5 | If random variable X represents the number of heads when a coin is tossed twice then mathematical expectation of X is                                                                  | [1] |

|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                |               |    |                |                 |                 |                |               |     |      |   |   |    |    |                |                 |                 |    |     |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|---------------|----|----------------|-----------------|-----------------|----------------|---------------|-----|------|---|---|----|----|----------------|-----------------|-----------------|----|-----|
|      | a) 1            b) 1/2            c) 0            d) 1/4                                                                                                                                                                                                                                                                                                                                                                                                  |                |               |    |                |                 |                 |                |               |     |      |   |   |    |    |                |                 |                 |    |     |
| 6    | The function $f(x) = x^3 - 6x^2 + 15x - 12$ is:<br>a) increasing on $[-\infty, 2]$ and decreasing on $(2, \infty)$<br>b) strictly increasing on R    c) none of these            d) strictly decreasing on R                                                                                                                                                                                                                                              | [1]            |               |    |                |                 |                 |                |               |     |      |   |   |    |    |                |                 |                 |    |     |
| 7    | If the function $f(x) = x^2 - kx + 5$ is increasing on $[2, 4]$ then:<br>a) $k \in (4, \infty)$ b) $k \in (2, \infty)$ c) $k \in (-\infty, 2)$ d) $k \in (-\infty, 4)$                                                                                                                                                                                                                                                                                    | [1]            |               |    |                |                 |                 |                |               |     |      |   |   |    |    |                |                 |                 |    |     |
| 8    | The function $f(x) = 2x^3 - 15x^2 + 36x + 4$ is maximum at $x =$<br>a) 4            b) 3            c) 0            d) 2                                                                                                                                                                                                                                                                                                                                  | [1]            |               |    |                |                 |                 |                |               |     |      |   |   |    |    |                |                 |                 |    |     |
| 9    | A random variable 'X' has the following probability distribution:<br><table border="1"><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>P(X)</td><td>k</td><td>k</td><td>2k</td><td>3k</td><td>k<sup>2</sup></td><td>2k<sup>2</sup></td><td>7k<sup>2</sup></td><td>2k</td></tr></table><br>The value of k is<br>a) - 1            b) 1            c) -1/10            d) 1/10                   | X              | 0             | 1  | 2              | 3               | 4               | 5              | 6             | 7   | P(X) | k | k | 2k | 3k | k <sup>2</sup> | 2k <sup>2</sup> | 7k <sup>2</sup> | 2k | [1] |
| X    | 0                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 1              | 2             | 3  | 4              | 5               | 6               | 7              |               |     |      |   |   |    |    |                |                 |                 |    |     |
| P(X) | k                                                                                                                                                                                                                                                                                                                                                                                                                                                         | k              | 2k            | 3k | k <sup>2</sup> | 2k <sup>2</sup> | 7k <sup>2</sup> | 2k             |               |     |      |   |   |    |    |                |                 |                 |    |     |
| 10   | Let X be a discrete random variable. The probability distribution of X is given<br><table border="1"><tr><td>X</td><td>30</td><td>10</td><td>-10</td></tr><tr><td>P(X)</td><td><math>\frac{1}{5}</math></td><td><math>\frac{3}{10}</math></td><td><math>\frac{1}{2}</math></td></tr></table><br>below:<br>Then E(X) is equal to:<br>a) 3            b) 6            c) -5            d) 4                                                                 | X              | 30            | 10 | -10            | P(X)            | $\frac{1}{5}$   | $\frac{3}{10}$ | $\frac{1}{2}$ | [1] |      |   |   |    |    |                |                 |                 |    |     |
| X    | 30                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 10             | -10           |    |                |                 |                 |                |               |     |      |   |   |    |    |                |                 |                 |    |     |
| P(X) | $\frac{1}{5}$                                                                                                                                                                                                                                                                                                                                                                                                                                             | $\frac{3}{10}$ | $\frac{1}{2}$ |    |                |                 |                 |                |               |     |      |   |   |    |    |                |                 |                 |    |     |
| 11   | For a random variable X, $E(X) = 3$ and $E(X^2) = 11$ . Then variance of X is<br>a) 8            b) 5            c) 2            d) 1                                                                                                                                                                                                                                                                                                                     | [1]            |               |    |                |                 |                 |                |               |     |      |   |   |    |    |                |                 |                 |    |     |
| 12   | The mean and variance of a binomial distribution are 4 and 3 respectively, then the probability of getting exactly six successes in this distribution, is<br>a) ${}^{16}C_6 \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^{10}$ b) ${}^{12}C_6 \left(\frac{1}{20}\right) \left(\frac{3}{4}\right)^6$ c) ${}^{16}C_6 \left(\frac{1}{4}\right)^{10} \left(\frac{3}{4}\right)^6$ d) ${}^{12}C_6 \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^6$ | [1]            |               |    |                |                 |                 |                |               |     |      |   |   |    |    |                |                 |                 |    |     |
| 13   | If X is a Poisson variable such that $P(X = k) = P(X = k + 1)$ , then variance of X is<br>a) k            b) k+1            c) k-1            d) k+2                                                                                                                                                                                                                                                                                                      | [1]            |               |    |                |                 |                 |                |               |     |      |   |   |    |    |                |                 |                 |    |     |
| 14   | An observed set of the population that has been selected for analysis is called<br>a) a forecast    b) a sample    c) a process    d) a parameter                                                                                                                                                                                                                                                                                                         | [1]            |               |    |                |                 |                 |                |               |     |      |   |   |    |    |                |                 |                 |    |     |

|                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |     |
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| 15               | A sampling distribution might be based on which of the following?<br>a) Sample means   b) Sample proportions   c) Sample correlations   d) all of these                                                                                                                                                                                                                                                                                                                                            | [1] |
| 16               | Which of the following symbols represents a population parameter?<br>a) $r$ b) $O$ c) $\sigma$ d) SD                                                                                                                                                                                                                                                                                                                                                                                               | [1] |
| 17               | A boat goes downstream at $u$ km/hr and upstream at $v$ km/hr. The speed of the boat in still water, in km/hr is<br>a) $(u - v)$ b) $u+v$ c) $\frac{1}{2}(u-v)$ d) $\frac{1}{2}(u+v)$                                                                                                                                                                                                                                                                                                              | [1] |
| 18               | If $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} A = \begin{bmatrix} g & hi \\ j & kl \\ m & no \end{bmatrix}$ then order of matrix A is:<br>a) $2 \times 3$ b) $3 \times 3$ c) $3 \times 2$ d) $2 \times 2$                                                                                                                                                                                                                                                                              | [1] |
| 19               | Let A and B be two symmetric matrices of order 3. <b>Assertion (A):</b> $A(BA)$ and $(AB)A$ are symmetric matrices. <b>Reason (R):</b> AB is symmetric matrix, if matrix multiplication of A with B is commutative.<br>a) Both A and R are true and R is the correct explanation of A.<br>b) Both A and R are true but R is not the correct explanation of A.<br>c) A is true but R is false.   d) A is false but R is true.                                                                       | [1] |
| 20               | <b>Assertion (A):</b> The rate of change of area of a circle with respect to its radius $r$ when $r = 6$ cm is $12\pi$ cm <sup>2</sup> /cm. <b>Reason (R):</b> Rate of change of area of a circle with respect to its radius $r$ is $\frac{dA}{dr}$ , where A is the area of the circle.<br>a) Both A and R are true and R is the correct explanation of A.<br>b) Both A and R are true but R is not the correct explanation of A.<br>c) A is true but R is false.   d) A is false but R is true . | [1] |
| <b>Section B</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |     |
| 21               | Find the intervals in which $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing or decreasing.                                                                                                                                                                                                                                                                                                                                                                                                           | [2] |
| 22               | If Z is a standard normal variable, find: $P(-0.78 < Z < 1.87)$<br><b>OR</b><br>If Z is a standard normal variable, find: $P(Z < -1.05)$                                                                                                                                                                                                                                                                                                                                                           | [2] |

|    |                                                                                                                                                                                                                                                                                                                                                                                                                                  |     |
|----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 23 | Find the sample size for the given standard deviation 10 and the standard error with respect of sample mean is 3.<br><br><b>OR</b><br>The following data are from a simple random sample: 5, 8, 10, 7, 10, 14.<br><br>1. What is the point estimate of the population mean?<br><br>2. What is the point estimate of the population standard deviation?                                                                           | [2] |
| 24 | Two pipes can fill a tank in 12 hours and 16 hours respectively. A third pipe can empty the tank in 30 hours. If all three pipes are opened and function simultaneously, how much time will the tank take to be full?                                                                                                                                                                                                            | [2] |
| 25 | In a 10 km race, A, B and C, each running a uniform speed get the gold, silver and bronze medals respectively. If A beats B by 1 km and B beats C by 1 km, then by how much meters does A beat C?                                                                                                                                                                                                                                | [2] |
|    | <b>Section C</b>                                                                                                                                                                                                                                                                                                                                                                                                                 |     |
| 26 | Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.<br><br><b>OR</b><br>If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of a, b and c                                                                                                                                           | [3] |
| 27 | A company sells its product at the rate of ₹ 6 per unit. The variable costs are estimated to run 25% of the total revenue received. If the fixed costs for the product are ₹ 4500, find:<br><br>1. total revenue function<br><br>2. total cost function<br><br>3. profit function<br><br>4. Break even point<br><br><b>OR</b><br>The revenue function is given by $R(x) = 100x - x^2 - x^3$ . Find the marginal revenue function | [3] |
| 28 | A fair coin is tossed four times. Let X denote the number of heads occurring. Find the probability distribution, mean and variance of X.                                                                                                                                                                                                                                                                                         | [3] |
| 29 | Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x - 5$ is increasing or decreasing                                                                                                                                                                                                                                                                                                                              | [3] |

| 30             | <p>A manufacturer of a line of patent medicines is preparing a production plan on medicines A and B. There are sufficient ingredients available to make 20,000 bottles of A and 40,000 bottles of B but there are only 45,000 bottles into which either of the medicines can be put. Furthermore, it takes 3 hours to prepare enough material to fill 1000 bottles of A, it takes one hour to prepare enough material to fill 1000 bottles of B and there are 66 hours available for this operation. The profit is ₹ 8 per bottle for A and ₹ 7 per bottle for B. Formulate this problem as a linear programming problem.</p> <p><b>OR</b></p> <p>A firm manufactures 3 products A, B and C. The profits are ₹ 3, ₹ 2 and ₹ 4 respectively. The firm has 2 machines and below is the required processing time in minutes for each machine on each product:</p> <table><tr><th rowspan="2">Machine</th><th colspan="3">Products</th></tr><tr><th>A</th><th>B</th><th>C</th></tr><tr><td>M<sup>1</sup></td><td>4</td><td>3</td><td>5</td></tr><tr><td>M<sup>2</sup></td><td>2</td><td>2</td><td>4</td></tr></table> <p>Machines M<sub>1</sub> and M<sub>2</sub> have 2000 and 2500 machine minutes respectively. The firm must manufacture 100 A's, 200 B's and 50 C's but not more than 150 A's. Set up an LPP to maximize the profit.</p> | Machine | Products |  |  | A | B | C | M <sup>1</sup> | 4 | 3 | 5 | M <sup>2</sup> | 2 | 2 | 4 | [3] |
|----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|----------|--|--|---|---|---|----------------|---|---|---|----------------|---|---|---|-----|
| Machine        | Products                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |         |          |  |  |   |   |   |                |   |   |   |                |   |   |   |     |
|                | A                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | B       | C        |  |  |   |   |   |                |   |   |   |                |   |   |   |     |
| M <sup>1</sup> | 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 3       | 5        |  |  |   |   |   |                |   |   |   |                |   |   |   |     |
| M <sup>2</sup> | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 2       | 4        |  |  |   |   |   |                |   |   |   |                |   |   |   |     |
| 31             | <p>Solve the system of linear inequalities graphically: <math>x + y \geq 1</math>, <math>7x + 9y \leq 63</math>, <math>x \leq 6</math>, <math>y \leq 5</math>, <math>x \geq 0</math>, <math>y \geq 0</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | [3]     |          |  |  |   |   |   |                |   |   |   |                |   |   |   |     |
|                | <p><b>Section D</b></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |         |          |  |  |   |   |   |                |   |   |   |                |   |   |   |     |
| 32             | <p><b>Read the text carefully and answer the questions:</b> The probability distribution of a random variable X is given as</p> <p>under: <math>P(X = x) = \begin{cases} kx^2, &amp; \text{for } x = 1, 2, 3 \\ 2kx, &amp; \text{for } x = 4, 5, 6 \\ 0, &amp; \text{otherwise} \end{cases}</math> where k is a constant.</p> <p>1. What is the value of k? (2)</p> <p>2. P(X) = _____. (1)</p> <p>3. E(X) = (1)</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | [4]     |          |  |  |   |   |   |                |   |   |   |                |   |   |   |     |
| 33             | <p><b>Read the text carefully and answer the questions: Pipes and Cisterns</b> A pipe is connected to a tank or cistern. It is used to fill or empty the cistern. The amount of work done by a pipe is a part of the tank filled or emptied in unit time. Three pipes A, B and C are connected to a tank. A and B fill the tank in 6</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | [4]     |          |  |  |   |   |   |                |   |   |   |                |   |   |   |     |

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|    | <p>hours and 8 hours respectively when operated independently. Pipe C empty</p>  <p>the full tank in 12 hours when opened alone.</p> <ol style="list-style-type: none"> <li>1. If both pipes A and B are opened together, then how long will it take for the tank to fill? (1)</li> <li>2. If pipes A and C are opened together, then how long will it take for the tank to fill? (1)</li> <li>3. If all three pipes A, B and C are opened together, then how long will it take for the tank to fill? (2)</li> </ol>                                                                                                                                                                                                                                                                                |     |
| 34 | <p><b>Read the text carefully and answer the questions:</b> Two schools Oxford and Navdeep want to award their selected students on the values of sincerity, truthfulness and helpfulness. Oxford wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with a total award money of 1600. Navdeep wants to spend 2300 to award its 4, 1 and 3 students on the respective values (by giving the same amount to the three values as before). The total amount of the award for one prize on each is ₹ 900.</p>  <ol style="list-style-type: none"> <li>1. What is the value of x, y and z? (2)</li> <li>2. What is the value of <math>x + y + z</math>? (1)</li> <li>3. What are the values of <math>4x + y + 3z</math>? (1)</li> </ol> | [4] |
|    | <b>Section E</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |     |
| r  | <p>Two farmers Ram Kishan and Gurcharan Singh cultivate only three varieties of rice namely Basmati, Permal, and Naura. The sale (in ₹ ) of these varieties of</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | [5] |

|    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |     |
|----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
|    | <p>rice by both the farmers in the month of September and October are given by</p> <p style="text-align: center;">September Sales (in Rupees)</p> $A = \begin{matrix} & \begin{matrix} \text{Basmati} & \text{Permal} & \text{Naura} \end{matrix} \\ \begin{matrix} \text{Ramkishan} \\ \text{Gurcharan Singh} \end{matrix} & \begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix} \end{matrix}$ <p style="text-align: center;">October Sales (in Rupees)</p> $B = \begin{matrix} & \begin{matrix} \text{Basmati} & \text{Permal} & \text{Naura} \end{matrix} \\ \begin{matrix} \text{Ramkishan} \\ \text{Gurcharan Singh} \end{matrix} & \begin{bmatrix} 5000 & 10,000 & 6000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix} \end{matrix}$ <p>the following matrices A and B.</p> <p>Find:</p> <ol style="list-style-type: none"> <li>What were the combined sales in September and October for each farmer in each variety?</li> <li>What was the change in sales from September to October?</li> <li>If both farmers receive 2% profit on gross rupees sales, compute the profit for each farmer and for each variety sold in October.</li> </ol> |     |
| 36 | Find the equation of the normal to the curve $x^2 + 2y^2 - 4x - 6y + 8 = 0$ at the point whose abscissa is 2.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | [5] |
| 37 | <p>A simple random sample of 400 individuals provides 100 yes responses</p> <ol style="list-style-type: none"> <li>What is the point estimate of the population proportion that would provide yes responses?</li> <li>Compute the 95% confidence interval for the population proportion</li> </ol> <p style="text-align: center;">OR</p> <p>Suppose a student measuring the boiling temperature of a certain liquid observes the reading (in degree Celsius) 102.5, 101.7, 103.1, 100.9, 100.5 and 102.2 on 6 different samples of the liquid. If he knows that the standard deviation for this procedure is <math>1.2^\circ</math> Celsius, what is the interval estimation for the population mean at a 95% confidence level?</p>                                                                                                                                                                                                                                                                                                                                                                                                                                           | [5] |
| 38 | <p>A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an LPP and solve it graphically.</p> <p style="text-align: center;">OR</p> <p>Solve the following linear programming problem graphically: Maximize <math>Z = 50x + 15y</math> Subject to <math>5x + y \leq 100</math> <math>x + y \leq 60</math> <math>x, y \geq 0</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                            | [5] |