XII Test 03

Q-1 d)

Q-2 d)

Q-3 a)

Q-4 a)

Q-5 d)

Q-6 c)

Q-7 c)

Q-8 a)

Q-9 a)

Q-10 d)

Q-11(A)

$$\frac{dx}{d\theta} = a(\sin\theta)$$

$$\frac{dy}{d\theta} = a(1+\cos\theta)$$

$$\frac{dy}{dx} = \frac{\cancel{a}(1+\cos\theta)}{\cancel{a}\sin\theta}$$

$$\frac{dy}{dx} = \frac{2\cos^2\theta/2}{2\sin\theta/2\cos\theta/2}$$

$$\frac{dy}{dx} = \cot\theta/2$$

diff w.r.t. x

$$\frac{d^2y}{dx^2} = -\operatorname{cosec}^2\theta/2 \times \frac{d\theta}{dx} \times \frac{1}{2}$$

$$= \frac{-\operatorname{cosec}^2\theta/2}{a\sin\theta} \times \frac{1}{2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{\theta=\pi/2} = \frac{-\operatorname{cosec}^2\pi/4}{a\sin\pi/2} \times \frac{1}{2}$$

$$= \frac{-(\sqrt{2})^2}{a(1)} \times \frac{1}{2} = \frac{-1}{a}$$

11 B)

Let $u = \sin^2 x$ $\quad v = e^{\cos x}$

we need to find $\frac{du}{dv}$

$$\frac{du}{dx} = 2\sin x\cos x, \quad \frac{dv}{dx} = e^{\cos x}(-\sin x)$$

$$\frac{du}{dv} = \frac{2\cancel{\sin x}\cos x}{e^{\cos x}(-\cancel{\sin x})} = \frac{-2\cos x}{e^{\cos x}}$$

12(A) $f'(x) = 2\cos 2x$

$f'(x) = 0$

$2\cos 2x = 0$

$2x = (2n+1)\frac{\pi}{2}$

$x = (2n+1)\frac{\pi}{4}$

$x = \pi/4, 3\pi/4$

$f''(x) = -4\sin 2x$

$f''(\pi/4) < 0$ Max

$f''(3\pi/4) > 0$ Min

$f(\pi/4) = \sin\frac{\pi}{2} + 5 = 1+5 = 6$

Max value $= 6$ at $x = \pi/4$

$f(3\pi/4) = \sin 3\frac{\pi}{2} + 5 = -1+5 = 4$

Min value $= -4$ at $x = 3\pi/4$

12(B) $\frac{dv}{dt} = 8\,cm^3/s.$

$V = \frac{4}{3}\pi r^3$

diff w.r.t. r

Date

Page No.

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{8}{4\pi r^2} = \frac{dr}{dt}$$

$$S.A = 4\pi r^2$$

diff w.r.t. x

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$= 8\pi r \times \frac{8}{4\pi r^2}$$

$$= \frac{4}{3} \ cm^2/s$$

Date......../......../

Page..........

Q13 = $(\cos x)^y = (\sin y)^x$

take log

$y \log \cos x = x \log \sin y$

dwr to x

$y \times \frac{1}{\cos x}(-\sin x) + \frac{dy}{dx} \log \cos x = x \times \frac{1}{\sin y}(\cos y)\frac{dy}{dx} + \log \sin y$

$-y \tan x + \frac{dy}{dx} \log \cos x = x \cot y \frac{dy}{dx} + \log \sin y$

$\frac{dy}{dx}[\log \cos x - x \cot y] = \log \sin y + y \tan x$

$$\frac{dy}{dx} = \frac{\log \sin y + y \tan x}{\log \cos x - x \cot y}$$

Q14 = (A) $f(x) = \sin x + \cos x$

dwr to x

$f'(x) = \cos x - \sin x$

$f'(x) = 0$

$\cos x - \sin x = 0$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

Intervals		inc / dec
$(0, \pi/4)$	$f'(x) > 0$	~~decreasing~~ increasing
$\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$	$f'(x) < 0$	dec.
$\left(\frac{5\pi}{4}, 2\pi\right)$	$f'(x) > 0$	inc

or

(B) $f(x) = \sin^4 x + \cos^4 x$

dwr to x

$f'(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x$

$= 4\sin x \cos x\ [\sin^2 x - \cos^2 x]$

$= -4\sin x \cos x\ (\cos^2 x - \sin^2 x)$

$= -4\sin x \cos x\ (\cos 2x)$

$= -2\,[2\sin x\cos x]\,(\cos 2x)$

$= -2\,[\sin 2x)\,[\cos 2x]$

$= -\sin 4x$

$f'(x) = 0$

$-\sin 4x = 0$

$\sin x = 0$

$0, \pi, 2\pi$

$\therefore \sin 4x = 0$

$0, \frac{\pi}{4}, \frac{\pi}{2}.$

Intervals	~~+ve / -ve~~	dec / inc
$(0, \pi/4)$	$f'(x) < 0$	decreasing
$(\pi/4, \pi/2)$	$f'(x) > 0$	increasing.

Q15 =

$$x = a\cos\theta + b\sin\theta$$

dwr to θ

$$\frac{dx}{d\theta} = -a\sin\theta + b\cos\theta$$

$$= b\cos\theta - a\sin\theta$$

$$y = a\sin\theta - b\cos\theta$$

$\frac{dy}{dx}$ = dwr to θ

$$\frac{dy}{d\theta} = a\cos\theta + b\sin\theta$$

$$\frac{dy}{dx} = \frac{a\cos\theta + b\sin\theta}{b\cos\theta - a\sin\theta}$$

$$= \frac{a\cos\theta + b\sin\theta}{-(a\sin\theta - b\cos\theta)}$$

$$\frac{dy}{dx} = \frac{x}{-y}$$

$$\frac{dy}{dx} = \frac{x}{-y}$$

dwr to x

$$\frac{d^2y}{dx^2} = \frac{(-y) - (x)\left(-\frac{dy}{dx}\right)}{y^2}$$

$$y^2 \frac{d^2y}{dx^2} = -y + x\frac{dy}{dx}$$

$$y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} \mp y = 0$$

Hence, proved.

16.

$$LHL = RHL = f(0)$$

$$\lim_{h\to 0} f(-h) = \lim_{h\to 0} f(h) = c$$

$$\lim_{h\to 0} \frac{-\sin(a+1)h - \sin h}{-h} = \lim_{h\to 0} \frac{\sqrt{h+bh^2}-\sqrt{h}}{b h^{3/2}} = c$$

$$a+1 \lim_{h\to 0} \frac{\sin(a+1)h}{(a+1)h} + \lim_{h\to 0} \frac{\sin h}{h} = \lim_{h\to 0} \frac{\sqrt{h+bh^2}-\sqrt{h}}{b h^{3/2}} \times \frac{\sqrt{h+bh^2}+\sqrt{h}}{\sqrt{h+bh^2}+\sqrt{h}}$$

$$a+1+1 = \lim_{h\to 0} \frac{\cancel{h}+\cancel{b}h^2 \cancel{-h}}{\cancel{b}h^{3/2}(\sqrt{h+bh^2}+\sqrt{h})}$$

$$= \lim_{h\to 0} \frac{\sqrt{h}}{\sqrt{h}(\sqrt{1+bh}+1)}$$

$$a+2 = \frac{1}{2} = c$$

$$a = -\frac{3}{2}, \quad c = \frac{1}{2}$$

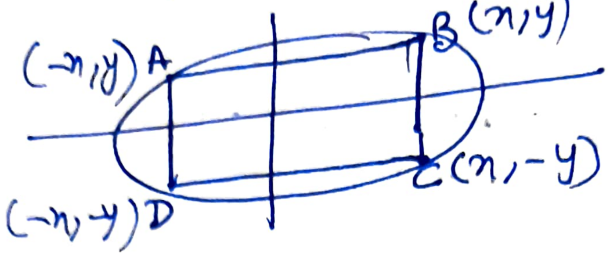
a) b can be any: Non zero real number

(b) $a+c = -\frac{3}{2} + \frac{1}{2} = \frac{-2}{2} = -1$

17

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{---} (1)$$

(i)



Let length $= 2x$.

breadth $= 2y$

Area of rectangle $= 4xy$

from (1)

$$y = b\sqrt{1 - \frac{x^2}{a^2}}$$

$$\therefore \text{Area} = 4xb\sqrt{1 - \frac{x^2}{a^2}}$$

$$= 4x\frac{b}{a}\sqrt{a^2 - x^2}$$

$$A(x) = \frac{4b}{a} x\sqrt{a^2 - x^2}$$

(ii)

$$A'(x) = \frac{4b}{a}\left(\frac{x}{\cancel{2}\sqrt{a^2 - x^2}}(-\cancel{2}x) + \sqrt{a^2 - x^2}\right)$$

$$A'(x) = 0$$

$$\frac{4b}{a}\left(\frac{-x^2 + a^2 - x^2}{\sqrt{a^2 - x^2}}\right) = 0$$

$$2x^2 = a^2$$

iii (a) $\quad x = \dfrac{a}{\sqrt{2}}$ $\quad$ length $2x = 2\dfrac{a}{\sqrt{2}} = \sqrt{2}a$

$\therefore y = \dfrac{b}{\sqrt{2}}$ $\quad$ breadth $2y = 2\dfrac{b}{\sqrt{2}} = \sqrt{2}b$

(iii) b) $$A''(x) = \frac{4b}{a}\left(\frac{\sqrt{a^2-x^2}\,(-4x) - (a^2-2x^2)\times\frac{-\cancel{2}x}{\cancel{2}\sqrt{a^2-x^2}}}{a^2-x^2}\right)$$

$$= \frac{4b}{a}\left(\frac{(a^2-x^2)(-4x) + (a^2-2x^2)\,x}{(a^2-x^2)^{3/2}}\right)$$

$$= \frac{4b}{a}\left(\frac{-4xa^2+4x^3+a^2x-3x^3}{(a^2-x^2)^{3/2}}\right)$$

$$= \frac{4b}{a}\left(\frac{x^3-3a^2x}{(a^2-x^2)^{3/2}}\right)$$

$$A''(\sqrt{2}a) = \frac{4b}{a}\,\frac{2\sqrt{2}a^3-3\sqrt{2}a^3}{(a^2-2a^2)^{3/2}} < 0$$

pt. of max.

Max area $= 4(\sqrt{2}a)(\sqrt{2}b) = 8ab$ sq unit

~~Sec-D~~

~~Q16~~ Sec-E .

Q18 $(x-a)^2 + (y-b)^2 = c^2$

dwr to x

$2(x-a) + 2(y-b)\frac{dy}{dx} = 0$

$\cancel{2}(y-b)\frac{dy}{dx} = -\cancel{2}(x-a)$

$y' \frac{dy}{dx} = \frac{-(x-a)}{(y-b)}$

dwr to x

$y' = \frac{(y-b)(-1) - [-(x-a)][y']}{(y-b)^2}$

$= \frac{-(y-b) + (x-a)y'}{(y-b)^2}$

$= \frac{-(y-b) + (x-a)\left[\frac{-(x-a)}{y-b}\right]}{(y-b)^2}$

$= \frac{-(y-b)^2 - (x-a)^2}{(y-b)^3}$

$= -\frac{[(y-b)^2 + (x-a)^2]}{(y-b)^3}$

$y'' = -\frac{c^2}{(y-b)^3}$

Consider

$$\frac{\left[1+(y')^2\right]^{3/2}}{y''}$$

$$\frac{\left[1+\left[\frac{-(x-a)}{y-b}\right]^2\right]^{3/2}}{\frac{-c^2}{(y-b)^3}}$$

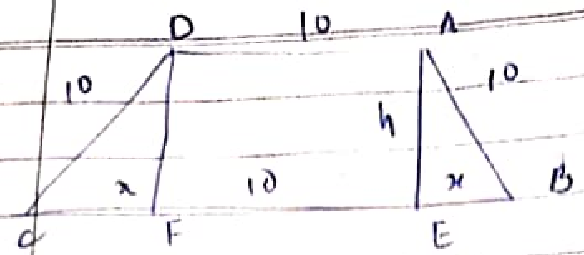
$$\left[\frac{(y-b)^2+(x-a)^2}{\cancel{(y-b)^2}(y-b)^{2}}\right]^{3/2} \times \frac{(y-b)^{\cancel{2}\,3}}{-c^2}$$

$$\frac{\left[(y-b)^2+(x-a)^2\right]^{3/2}}{\left[(y-b)^2\right]^{3/2}} \times \frac{(y-b)^3}{-c^2}$$

$$\frac{(c^{\cancel{2}})^{3/2}}{\cancel{(y-b)^3}} \times \frac{\cancel{(y-b)^3}}{(-c^{\cancel{2}})}$$

$$\frac{c^{\cancel{3}}}{-c^{\cancel{2}}} = -c.$$

Q19 =



Let height of trapezium be h.

ΔAEB

$10^2 = h^2 + x^2$

$10^2 - x^2 = h^2$

area of trapezium

$= \frac{1}{2}[20 + 2x][h]$

$= \frac{1}{\cancel{2}} \times \cancel{2}(10 + x)(h)$

$= (10 + x)h$

square both sides

$= (10+x)^2 h^2$

$= (10+x)^2 (10^2 - x^2)$

$= (10+x)^2 (10+x)(10-x)$

$f(x) = (10+x)^3 (10-x)$

d w r to x

$f'(x) = 3(10+x)^2(10-x) + (10+x)^3(-1)$

$= 3(10+x)^2(10-x) - (10+x)^3$

$= (10+x)^2[3(10-x) - 10 - x]$

$= (10+x)^2[30 - 3x - 10 - x]$

$= (10+x)^2[20 - 4x]$

$f'(x) = 0$

Date....../....../......

Page.........

$(10+x)^2(20-4x)=0$

$20-4x=0$ $\quad\quad$ $(10+x)^2=0$

$x=5$ $\quad\quad$ $(10+x)(10+x)=0$

$x=-10.$

rejected

as length can't be $-ve$.

now

$f'(x)= (10+x)^2(20-4x)$

dwr to x

$f''(x) = 2(10+x)(20-4x) + (10+x)^2(-4)$

$= 2[200-40x+20x-4x^2] - 4[10+x]^2$

$= 2[200-20x-4x^2) - 4[100+x^2+20x]$

$= \cancel{400} - 40x - 8x^2 - \cancel{400} - 4x^2 - 40x$

$= -12x^2 - 80x$

$f''(5) = -12(25) - 80(5)$

$= -300 - 400$

$= -700 < 0$

$\therefore$ ~~f(x) is m~~

$\therefore$ trapezium haves maximum area at $x = 5$ cm.

maxim area $= \frac{1}{2}[20+2x]h$

$= \frac{1}{2}[20+10]15$

$= \frac{30 \times 15}{2}$

$= 15\times 15$

$= 1125$ cm.

$10^2-x^2=h$

$(10-5)(10+5)=h$

$(5)(15)=h$

$15=h$

Or.

Let the radius & height of the cylindrical can be r & h resp.

ATQ.

$$\text{Volume} = \pi r^2 h$$

$$100 = \pi r^2 h$$

$$\frac{100}{\pi r^2} = h$$

$$\text{Surface area} = 2\pi r(r+h)$$

$$= 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + 2\cancel{\pi} \cancel{r}\left[\frac{100}{\cancel{\pi} r^{\cancel{2}}}\right]$$

$$\cancel{d}\ f(r) = 2\pi r^2 + 2\left(\frac{100}{r}\right)$$

dwr to r

$$f'(x) = 4\pi r \cancel{+} - \frac{200}{r^2}$$

$$f'(x) = 0$$

$$4\pi r - \frac{200}{r^2} = 0$$

$$\frac{200}{r^2} = 4\pi r$$

$$\frac{\cancel{200}^{50}}{\cancel{4}\pi} = r^3$$

$$\frac{50}{\pi} = r^3$$

$$\sqrt[3]{\frac{50}{\pi}} = r$$

now dwr to r

$$f'(r) = 4\pi r - \frac{200}{r^2}$$

dwr to r

$$f''(r) = 4\pi + \frac{200}{r^3}$$

$$f''\left[\sqrt[3]{\frac{50}{\pi}}\right] = 4\pi + \frac{200}{50/\pi}$$

$$= 4\pi + \cancel{200}^{4} \times \frac{\pi}{\cancel{50}}$$

$$= 8\pi > 0$$

$\therefore$ Surface area of cylindrical can is maximum at

$r = \sqrt[3]{\frac{50}{\pi}}$ $\quad$ ~~a~~ = h =

$$\frac{100}{\pi\left[\frac{50}{\pi}\right]^{3/2}}.$$