ANSWER KEY

1	(c) $3 + \sqrt{5}$
2	(d) $\frac{\sqrt{a}+b}{\sqrt{a}+b}$
3	(b) 0
4	(b) 4
5	(c) 5^{10}
6	(c) $\frac{1}{3}$
7	(c)2
8	(d) $3x^2 + 7$
9	(c)120
10	(d) $\sqrt{2}$
11	$3\sqrt{(343)^{-2}}$ $(7)^{-2}$ $\frac{1}{(7)^2} = \frac{1}{49}$
12(A)	$81y^2 - K = \left(9y + \frac{1}{2}\right)\left(9y - \frac{1}{2}\right)$ $81y^2 - K = (9y)^2 - \left(\frac{1}{2}\right)^2$ $81y^2 - K = 81y^2 - \frac{1}{4}$ Compare $+K = +\frac{1}{4}$ $\boxed{K = \frac{1}{4}}$

	or
12(B)	$p+1=0$, $p=-1$ $q(p) = p^{50}-1$ $q(-1) = (-1)^{50}-1$ $q(-1) = 0$ $\therefore$ $p+1$ is a factor of $p^{50}-1$ $q(p) = p^{51}-1$ $q(-1) = (-1)^{51}-1$ $q(-1) = -2$ $p+1$ is a factor of $p^{50}-1$ because $f(-1)=0$, but it is not factor of $p^{51}-1$ because $q(-1) = -2 \neq 0$.
13	$p(x) = ax^3 - x^2 + x + 4$ $p(-1) = a(-1)^3 - (-1)^2 + (-1) + 4$ $= -a - 1 - 1 + 4$ $= -a + 2$ $= -a+2 \Rightarrow 0$ $\boxed{a=2}$

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14(A)	$x = 1 - \sqrt{2}$ $\frac{1}{x} = \frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$ $= \frac{1+\sqrt{2}}{(1)^2 - (\sqrt{2})^2}$ $\left\{ \text{using} \rightarrow a^2 - b^2 = (a-b)(a+b) \right.$ $= \frac{1+\sqrt{2}}{1-2}$ $= \frac{1+\sqrt{2}}{-1} = \frac{-(1+\sqrt{2})}{1}$ $\left(x + \frac{1}{x}\right)^2$ $\left((1-\sqrt{2}) - (-1-\sqrt{2})\right)^2$ $(1-\sqrt{2})^2 + (-1-\sqrt{2})^2 - 2(1-\sqrt{2})(-1-\sqrt{2})$ $= (1)^2 + (\sqrt{2})^2 - 2(1)(\sqrt{2}) + (1)^2 + (\sqrt{2})^2 - 2(1)(\sqrt{2}) - 2(1-\sqrt{2})(-1-\sqrt{2})$ $= 1 + 2 - \cancel{2\sqrt{2}} + 1 + \cancel{2} + \cancel{2\sqrt{2}} - \cancel{2}$ $= 4$

	or
14(B)	$x - y = 4$ cubing bath side $(x-y)^3 = (4)^3$ $x^3 - y^3 - 3xy(x-y) = 64$ $x^3 - y^3 - 3(21)(4) = 64$ $x^3 - y^3 - 252 = 64$ $x^3 - y^3 = 64 + 252$ $x^3 - y^3 = 316$
15(A)	$\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$ $\frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2 - (1)^2}$ $\left\{ \text{using } (a-b)^2 = a^2 + b^2 - 2ab \right\}$ $\left\{ \text{using } a^2 - b^2 = (a+b)(a+b) \right\}$ $\frac{(\sqrt{3})^2 + (1)^2 - 2(\sqrt{3})(1)}{3-1}$ $\frac{3+1-2\sqrt{3}}{2}$ $\frac{4-2\sqrt{3}}{2}$ $\frac{2(2-\sqrt{3})}{\cancel{2}}$

15(B)

$2-\sqrt{3}$

Comparing both side
$2-\sqrt{3} = \sqrt{a} + b\sqrt{3}$

$a = 2, \quad b = -1$

Or

Step 1: Rewrite using negative exponents

Recall that $x^{-r} = \frac{1}{x^r}$, so:

$$\frac{3}{216^{-\frac{2}{3}}} = 3 \cdot 216^{\frac{2}{3}}$$
$$\frac{1}{256^{-\frac{3}{4}}} = 256^{\frac{3}{4}}$$
$$\frac{2}{243^{-\frac{1}{5}}} = 2 \cdot 243^{\frac{1}{5}}$$

Thus the expression becomes:

$$3 \cdot 216^{\frac{2}{3}} + 256^{\frac{3}{4}} + 2 \cdot 243^{\frac{1}{5}}$$

Step 2: Simplify each part

- **First term:**

$$216 = 6^3 \implies 216^{\frac{2}{3}} = (6^3)^{\frac{2}{3}} = 6^2 = 36$$
$$\Rightarrow 3 \cdot 216^{\frac{2}{3}} = 3 \times 36 = \boxed{108}$$

- **Second term:**

$$256 = 2^8 \implies 256^{\frac{3}{4}} = (2^8)^{\frac{3}{4}} = 2^6 = \boxed{64}$$

- **Third term:**

$$243 = 3^5 \implies 243^{\frac{1}{5}} = (3^5)^{\frac{1}{5}} = 3$$
$$\Rightarrow 2 \cdot 243^{\frac{1}{5}} = 2 \times 3 = \boxed{6}$$

Step 3: Add them all together

$$108 + 64 + 6 = \boxed{178}$$

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$p(t) = t^4 + 3t^2 - 4$

(i) degree of $p(t)$ is 4

(ii) $p(4) = (4)^4 + 3(4)^2 - 4$
$= 256 + 48 - 4$
$= 300$

(iii)

(a) $p(t) = t^4 + 3t^2 - 4$

let $t^2 = x$

$p(x) = x^2 + 3x - 4$
$= x^2 + 4x - x - 4$
$= x(x+4) - (x+4)$
$= (x-1)(x+4)$

$p(t) = (t^2 - 1)(t^2 + 4)$
$= (t-1)(t+1)(t^2+4)$

Or

(iii)(b) Given $t-1$ is a factor

$t-1 \,)\, t^4 + 3t^2 - 4 \,(\, t^3 + t^2 + 4t + 4$
$t^4 - t^3$
$(-)\ (+)$
$t^3 + 3t^2$
$t^3 - t^2$
$(-)\ (+)$
$4t^2 - 4$
$4t^2 - 4t$
$(-)\ (+)$
$4t - 4$
$4t - 4$

Remaining factor

$t^3 + t^2 + 4t + 4$
$t^2(t+1) + 4(t+1)$
$(t^2+4)(t+1)$

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	(i) $p(x) = 6x^2 + 11x - 35$ $= 6x^2 + 21x - 10x - 35$ $3x(2x+7) - 5(2x+7)$ Lav $= 3x-5$, vicky $= 2x+7$ (ii) (a) $3x - 5 = 2x + 7$ $x = 12$ or (ii) (b) $2x + 7 + 3x - 5 = 102$ $5x + 2 = 102$ $x = 100/5 = 20$
18	$p(x) = x^3 + 2x^2 - 5ax - 7$ $q(x) = x^3 + ax^2 - 12x + 6$ $R_1 = p(-1) = (-1)^3 + 2(-1)^2 - 5a(-1) - 7$ $= -1 + 2 + 5a - 7$ $= 5a - 6$ $R_2 = q(2) = (2)^3 + a(2)^2 - 12(2) + 6$ $= 8 + 4a - 24 + 6$ $= 4a - 10$. $2R_1 + R_2 = 6$ $2(5a-6)(4a-10) = 6$ $\Rightarrow 10a - 12 + 4a - 10 = 6$ $14a - 22 = 6$ $\boxed{a = 2}$
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	i) $a^3 - 2\sqrt{2}\, b^3$ $a^3 - \sqrt{8} b^3$ [using $a^3 - b^3 = (a-b)(a^2+ab+b^2)$] $a^3 - (\sqrt{2} b)^3$ $(a - \sqrt{2} b)(a^2 + a\sqrt{2} b + (\sqrt{2} b)^2)$ $(a - \sqrt{2} b)(a^2 + a\sqrt{2} b + 2b^2)$ (ii) $2\sqrt{2}\,a^3 + 8b^3 - 27c^3 + 18\sqrt{2}\,abc$ Rewrite as $(\sqrt{2}a)^3 + (2b)^3 + (-3c)^3 - 3(\sqrt{2}a)(2b)(-3c)$. Use $A = \sqrt{2}a,\ B = 2b,\ C = -3c$ in $A^3 + B^3 + C^3 - 3ABC$: $$(\sqrt{2}a + 2b - 3c)\left(2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}\,ab + 6bc + 3\sqrt{2}\,ac\right).$$ (iii) $a^3 - b^3 + 1 + 3ab$ Group as $a^3 + 1^3 + (-b)^3 - 3a \cdot 1 \cdot (-b)$. Use $A = a,\ B = 1,\ C = -b$ in $A^3 + B^3 + C^3 - 3ABC$: $$(a + 1 - b)\left(a^2 + 1 + b^2 - a + ab - b\right) = (a - b + 1)\left(a^2 + b^2 + 1 - a + b + ab\right).$$

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THINK BEYOND...........
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