ANSWER KEY

1	(b)Injective
2	(c)106
3	(b) $-\frac{2\pi}{5}$
4	(b)4
5	(a) One-one but not onto
6	(a)-1
7	(b) $y=\cos^{-1}x$
8	(d) Continuous as well as differentiable at x=2.
9	(d) A is false but R is true
10	(a) Both A and R are true and R is the correct explanation for A
11	

$$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$$

$$\tan^{-1}\left[2\sin\left(2\cos^{-1}\cos\frac{\pi}{6}\right)\right]$$

$$\tan^{-1}\left[2\sin\left[2\times\frac{\pi}{6}\right]\right]$$

$$\tan^{-1}\left[2\sin\frac{\pi}{3}\right]$$

$$=\tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]$$

$$=\tan^{-1}\sqrt{3}$$

$$=\tan^{-1}\tan\pi/3$$

$$=\pi/3$$

Or

$$\sec^{-1}\left(\frac{1}{2x^2-1}\right)$$

let $x=\cos\theta$ $\qquad 0<x<\frac{1}{2}$

$\theta=\cos^{-1}x$ $\qquad 0<\cos\theta<\frac{1}{\sqrt{2}}$

$\sec^{-1}\left(\frac{1}{2\cos^2\theta-1}\right)$ $\qquad \frac{\pi}{2}<\theta<\frac{\pi}{4}$

$$\sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$$

$$\sec^{-1}(\sec 2\theta)$$

$$2\theta=2\cos^{-1}x$$

12	$\left[\cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\left(\frac{-1}{2}\right)\right]$ $\left[\cos^{-1}\frac{\sqrt{3}}{2} + \pi - \cos^{-1}\frac{1}{2}\right]$ $\left[\cos^{-1}\cos\frac{\pi}{6} + \pi - \cos^{-1}\cos\frac{\pi}{3}\right]$ $\left[\frac{\pi}{6} + \pi - \frac{\pi}{3}\right]$ $\frac{3\pi + 18\pi - 6\pi}{18} = \frac{15\pi}{18} = \frac{5\pi}{6}$
13	$\sin\{2(\pi - cot^{-1}\frac{5}{12})\}$ $= -\sin(2tan^{-1}\frac{12}{5})$ $...= -\sin\left(sin^{-1}\frac{120}{169}\right) = -\frac{120}{169}$
14	for 1-1 let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$ $\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$ $x_1x_2 - 3x_1 - 2x_2 + 6 = x_2x_1 - 2x_1 - 3x_2 + 6$ $-3x_1 - 2x_2 = -2x_1 - 3x_2$ $-3x_1 + 2x_1 = -3x_2 + 2x_2$ $-x_1 = -x_2$ $\boxed{x_1 = x_2}$ $\therefore f$ is 1-1

Onto

let $f(x) = y$

$$y = \frac{x-2}{x-3}$$

$$yx - 3y = x - 2$$

$$yx - x = 3y - 2$$

$$x(y-1) = 3y - 2$$

$$x = \frac{3y-2}{y-1}$$

Range = co domain

Range = Codomain $y \in R - \{1\}$

$\therefore f$ is onto

Therefore f is bijective

or

let (x_1) & $(x_2) \in R$

$$f(x_1) = f(x_2)$$

$$2x_1^2 + 3 = 2x_2^2 + 3$$

$$2x_1^2 - 2x_2^2 = 0$$

$$x_1^2 - x_2^2 = 0$$

$$(x_1 - x_2)(x_1 + x_2)$$

$$x_1 = x_2 \quad ; \quad x_1 = -x_2$$

	$2x_1^2 - 2x_2^2 = 0$ $x_1^2 - x_2^2 =$ $(x_1 - x_2)(x_1 + x_2)$ $x_2 = x_2 \;;\; x_1 = -x_2$ But there will be no unique preimage for -ve no. in the Co-domain Thus, f is not 1-1 Onto $f(x) = y$ $y = 2x^2 + 3$ $2x^2 = 3 - y$ $x^2 = \frac{3-y}{2}$ $x = \sqrt{\frac{3-y}{2}}$ Negative values of y will give irrational which is not in the Co-domain $\therefore$ Thus f is not onto
15	$\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ $\left\{\begin{array}{l}\cos 2\theta = \cos^2\theta - \sin^2\theta \\ 1+\sin 2x = (\cos x - \sin x)\end{array}\right.$ $\tan^{-1}\left(\frac{\cos^2 x/2 - \sin^2 x/2}{(\cos x/2 + \sin x/2)^2}\right)$ $\tan^{-1}\left(\frac{(\cos x/2 - \sin x/2)(\cos x/2 + \sin x/2)}{(\cos x/2 + \sin x/2)^2}\right)$ $\tan^{-1}\left(\frac{\cos x/2 - \sin x/2}{\cos x/2 + \sin x/2}\right)$

	divide by $\cos x/2$ $\tan^{-1}\left(\frac{1-\tan\frac{\pi}{2}\,x/2}{1+\tan\frac{\pi}{2}\,x/2}\right)$ $\left\{\frac{1-\tan\theta}{1+\tan\theta}=\frac{\pi}{4}-\theta\right\}$ $\tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]$ $\boxed{\frac{\pi}{4}-\frac{x}{2}}$
16	(i) $R=\{(1,1)\,(2,2)\,(3,3)\,(4,4)\,(5,5)\,(6,6)\,(2,4)\,(2,6)\,(3,6)\}$ R is reflexive as $(a,a)\in R$ for all $a\in A$. (ii) R is not symmetric as $(2,4)\in R$ but $(4,2)\notin R$ (iv) R is not an equivalence relation as R is not reflexive since $(3,3)\notin R$ R is not transitive since $(4,3)\,(3,4)\in R$ but $(1,4)\notin R$ $\Rightarrow$ R is not an equivalence relation. (ii) No. of reflexive relation : $2^{n(n-1)}$ $=2^{6\times 5}=2^{30}$

17

(i) yes it is symmetric
let $(L_1, L_2) \in R$
$\therefore L_1 \parallel L_2$
$L_2 \parallel L_1$ (True)
$\therefore (L_2, L_1) \in R$.

(ii) let $(L_1, L_2) \in R$ & $(L_2, L_3) \in R$
$L_1 \parallel L_2$ — (1)
$L_2 \parallel L_3$ — (2)
from 1 & 2.
$L_1 \parallel L_3$
$\therefore (L_1, L_3) \in R$
Thus R is transitive.

(iii)
This line has **slope = 3**.

- Lines parallel to it must also have **slope = 3**, and can have **any y-intercept**.
- So, the set of rail lines related to it is:

$$\{y = 3x + c \mid c \in \mathbb{R}\}$$

18

To prove a relation R is an equivalence relation, it will be sufficient to prove it as a reflexive, symmetric and transitive relation.

i) Reflexivity:
Let (a, b) be an arbitrary element of N × N.
Now,
$a, b \in N$

$\Rightarrow ab(a+b)=ba(a+b)$

$\Rightarrow (a,b)R(a,b)$

$\therefore (a, b)R(a, b)$ for all $(a, b) \in N \times N$
Hence, R is reflexive.

ii) Symmetry:
Let (a, b), (c, d) be an arbitrary element of N × N such that $(a, b)R(c, d)$.

	∴ ad(b+c)=bc(a+d) ⇒cb(d+a)=da(c+b) ⇒(c,d)R(a,b) ∴ $(a, b)R(c, d) \Rightarrow (c, d)R(a, b)$ for all $(a, b), (c, d) \in N \times N$ Hence, R is symmetric. iii) Transitivity: Let $(a, b), (c, d), (e, f)$ be an arbitrary element of N × N such that $(a, b)R(c, d)$ and $(c, d)R(e, f)$. ad(b+c)=bc(a+d) ⇒adb+adc=abc+bcd ⇒cd(a−b)=ab(c−d) .....(1) Also,cf(d+e)=de(c+f) ⇒cfd+cfe=dec+def ⇒cd(f−e)=ef(d−c) ....(2) From (1) and (2), we have $\frac{a-b}{f-e} = -\frac{ab}{ef}$ ⇒aef−bef=−abf+aeb ⇒aef+abf=aeb+bef ⇒af(b+e)=be(a+f) ⇒(a, b)R(e, f) ∴$(a, b)R(c, d)$ and $(c, d)R(e, f) \Rightarrow (a, b)R(e, f)$ for all $(a, b), (c, d), (e, f) \in N \times N$ Hence, R is transitive. Thus, R being reflexive, symmetric and transitive, is an equivalence relation on N × N.
19	**<u>one-one</u>** Let $x_1, x_2 \in R+$ and $f(x_1) = f(x_2)$ $9{x_1}^2 + 6x_1 - 5 = 9{x_2}^2 + 6x_2 - 5$ $9{x_1}^2 + 6x_1 = 9{x_2}^2 + 6x_2$ (removing -5 from both side) $3{x_1}^2 + 2x_1 = 3{x_2}^2 + 2x_2$ (dividing 3 on both side) $3{x_1}^2 - 3{x_2}^2 + 2x_1 - 2x_2 = 0$ $\Rightarrow 3({x_1}^2 - {x_2}^2) + 2(x_1 - x_2) = 0$ $3(x_1 - x_2)(x_1 + x_2) + 2(x_1 - x_2) = 0$ $x_1 - x_2 = 0$ (OR) $x_1 + x_2 + 2 = 0$ Consider $x_1 + x_2 + 2 = 0 \Rightarrow x_1 = -x_2 - 2$ As $x_1, x_2 \in R+$ (non-negative real numbers), it $(x_1 + x_2 + 2 = 0)$ can not be possible. So $x_1 - x_2 = 0 \Rightarrow x_1 = x_2$ Hence it is one-one.

onto

Let $y = f(x)$ and $y \in [-5, \infty]$

So $y = 9x^2 + 6x - 5$

$9x^2 + 6x - 5 - y = 0$

$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 9 \times (-y-5)}}{2 \times 9}$ (by using formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for quadratic equation $ax^2 + bx + c = 0$)

$x = \frac{-6 \pm \sqrt{36 + 36(y+5)}}{18}$

$\Rightarrow x = \frac{-6 \pm 6\sqrt{(y+6)}}{18}$

x has two solution $x = \frac{-1-\sqrt{y+6}}{3}$ and $x = \frac{-1+\sqrt{y+6}}{3}$. We need to prove either one solution (x) exists for every $y \in [-5, \infty)$. In the solution $x = \frac{-1-\sqrt{y+6}}{3}$, x is the negative number for any y in the given range. Now we left with $x = \frac{-1+\sqrt{y+6}}{3}$. For y = -5, x = 0 and any value greater then -5 x is always non negative real numbers. It proves that every value in the range $y \in [-5, \infty)$ there exists a value in domain $\rightarrow x \in R+$. Hence it is onto.

So the function is bijective (one-one and onto).

______________End______________