ANSWER KEY

1	(B)2
2	(C)-1
3	(D)c and a have same signs
4	(C)23
5	(B)
6	(D)
7	(C)
8	(D)
9	(A)
10	(D)
11(A) 11(B)	If α *and* β are the zeros of $p(x) = (a^2+16)x^2 + 16x + 8$ then $\alpha\beta = \frac{8a}{a^2+16} = 1 \Longrightarrow a^2 + 16 = 8a \Longrightarrow a^2 - 8a + 16 = 0 \Longrightarrow (a-4)^2 = 0 \Longrightarrow a = 4$ Or $\alpha + \beta = -\frac{b}{a} = \frac{-(-2)}{3} = \frac{2}{3}, \alpha\beta = \frac{c}{a} = -\frac{1}{3}$ $(1-\alpha)(1-\beta) = 1 - (\alpha+\beta) + \alpha\beta = 1 - \frac{2}{3} - \frac{1}{3} = 0$
12	$4x^2 - 4a^2x + a^4 - b^4 = 0$ $4x^2 - 2(a^2-b^2)x - 2(a^2+b^2)x + a^4 - b^4 = 0$ $2x[2x - (a^2-b^2)] - (a^2+b^2)[2x - (a^2-b^2)] = 0$ $[2x - (a^2+b^2)][2x - (a^2-b^2)] = 0$ $x = \frac{a^2+b^2}{2}, \frac{a^2-b^2}{2}$
13(A)	$S_n = 1170$ $\frac{n}{2}(2a + (n-1)d) = 1170$ $\frac{n}{2}(2\times108 + (n-1)2) = 1170$ $n(108 + n - 1) = 1170$ $n^2 + 107n - 1170 = 0$ $n^2 + 117n - 10n - 1170 = 0$ $n(n+117) - 10(n+117) = 0$ $(n-10)(n+117) = 0$ $n = 10, -117$ $\therefore n = 10$

13(B)

OR

Sum of integers between 1 and 200 which any multiply of 3.

Sequence = 3, 6, 9, ... 198

a=3, d=3

$$S = \frac{n}{2}(a_1 + l_1) = \frac{66}{2}(3+198)$$

$$= 33 \times 201 = 6633$$

14(A)

Let the usual speed be x km/h and the usual time be t h. Distance $D = xt$.

From the two altered cases:

$$(x+6)(t-4) = xt \quad \Rightarrow \quad -4x + 6t - 24 = 0 \quad \Rightarrow \quad -2x + 3t - 12 = 0.$$

$$(x-6)(t+6) = xt \quad \Rightarrow \quad 6x - 6t - 36 = 0 \quad \Rightarrow \quad x - t - 6 = 0.$$

From $x - t - 6 = 0$ we get $t = x - 6$. Substitute into $-2x + 3t - 12 = 0$:

$$-2x + 3(x-6) - 12 = 0 \Rightarrow -2x + 3x - 18 - 12 = 0 \Rightarrow x = 30\ \mathrm{km/h}.$$

Then $t = x - 6 = 24$ h, so the distance is

$$D = xt = 30 \times 24 = 720\ \mathrm{km}.$$

Or

14(B)

Let the number of chocolates in lot A be x and in lot B be y.

From the first selling method: ₹2 for 3 chocolates (so price per chocolate = $2/3$ ₹) on lot A, and ₹1 each on lot B, total ₹400:

$$\frac{2}{3}x + 1 \cdot y = 400 \quad \Longrightarrow \quad 2x + 3y = 1200.$$

From the second selling method: ₹1 each on lot A, and ₹4 for 5 chocolates on lot B (price per chocolate = $4/5$ ₹), total ₹460:

$$1 \cdot x + \frac{4}{5}y = 460 \quad \Longrightarrow \quad 5x + 4y = 2300.$$

Solve the linear system

$$\begin{cases} 2x + 3y = 1200, \\ 5x + 4y = 2300. \end{cases}$$

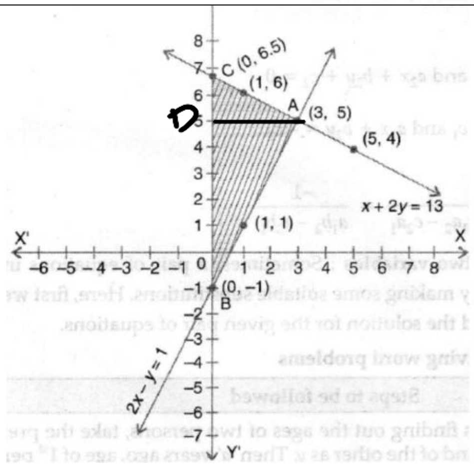
Eliminate x: multiply the first by 5 and the second by 2,

$$10x + 15y = 6000, \qquad 10x + 8y = 4600,$$

subtracting gives $7y = 1400$, so $y = 200$. Substitute in $2x + 3y = 1200$:

$$2x + 600 = 1200 \implies 2x = 600 \implies x = 300.$$

Total chocolates $= x + y = 300 + 200 = \boxed{500}$.

15	$\alpha+\beta=5$, $\alpha\beta=4$ $\frac{1}{\alpha^2}+\frac{1}{\beta^2}-2\alpha\beta$ $\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}-2\alpha\beta$ $\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}-2\alpha\beta=\frac{(5)^2-2(4)}{(4)^2}-2(4)$ $=\frac{25-8}{16}-8$ $=\frac{17}{16}-8=\frac{17-128}{16}=-\frac{111}{16}$
16	(i)40 km/hr (ii)50 km/hr (iii)(A)15hrs (iii)(B)12hrs
17	(i)250 (ii)110 rollers per year (iii) — To find in which year production was 1350: solve $a+(n-1)d=1350$: $250+(n-1)\cdot 110=1350 \Rightarrow (n-1)\cdot 110=1100 \Rightarrow n-1=10 \Rightarrow n=11.$ So production reached **1350 rollers in the 11th year.** — Production in the 8th year: $a_8=a+7d=250+7(110)=250+770=\boxed{1020\text{ rollers}}.$
18	 Area of triangle ABC= ½ x BCx AD= ½ x7.5x 3= 11.25 sq units.
19	

Given $S_4 = 40$ and $S_{14} = 280$:

$$S_4 = \frac{4}{2}(2a + 3d) = 40 \Rightarrow 2a + 3d = 20,$$

$$S_{14} = \frac{14}{2}(2a + 13d) = 280 \Rightarrow 2a + 13d = 40.$$

Subtract the first equation from the second:

$$(2a + 13d) - (2a + 3d) = 40 - 20 \implies 10d = 20 \Rightarrow d = 2.$$

Plug $d = 2$ into $2a + 3d = 20$:

$$2a + 6 = 20 \Rightarrow 2a = 14 \Rightarrow a = 7.$$

Now form S_n:

$$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(14 + 2(n-1)) = \frac{n}{2}(2n + 12) = n(n+6).$$

Quick checks: $S_4 = 4(4+6) = 4 \cdot 10 = 40$ and $S_{14} = 14(14+6) = 14 \cdot 20 = 280$

OR

According to the given condition,

$\Rightarrow mt_m = nt_n$

$\Rightarrow m\{a + (m-1)d\} = n\{a + (n-1)d\}$

$\Rightarrow ma + md(m-1) = na + nd(n-1)$

$\Rightarrow ma + m^2d - md = na + n^2d - nd$

$\Rightarrow ma + m^2d - md - na - n^2d + nd = 0$

$\Rightarrow (ma - na) + (m^2d - n^2d) - (md - nd) = 0$

$\Rightarrow a(m-n) + d(m^2 - n^2) - d(m-n) = 0$

$\Rightarrow a(m-n) + d(m+n)(m-n) - d(m-n) = 0$

$\Rightarrow (m-n)[a + (m+n-1)d] = 0$

$\Rightarrow [a + (m+n-1)d] = 0$ ...[Dividing both sides by $(m-n)$]

$\Rightarrow t(m+n) = 0$

Hence, the $(m+n)^{th}$ term of the given A.P. is zero