FIRST TERM EXAMINATION—2025-26

CLASS–XII

SUBJECT–MATHEMATICS

Time : 3 Hrs. M.M. : 80

General Instructions:

1. *This question paper contains five sections – A, B, C, D and E. Each part is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQs and 2 Assertion-Reasoning based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer-type questions of 2 mark each.*
4. *Section C has 6 Short Answer-type questions of 3 mark each.*
5. *Section D has 4 Long Answer-type questions of 5 mark each.*
6. *Section E has 3 source based/case based questions of 4 mark each with subparts.*

Section A (1 mark each)

1. The value of $\sin^{-1}\left(\cos\left(\frac{43\pi}{3}\right)\right)$ is

   a) $\frac{\pi}{3}$ b) $-\frac{\pi}{3}$

   c) $\frac{\pi}{6}$ d) $-\frac{\pi}{6}$

2. If $f(x) = |\cos x|$, then $f'\left(\frac{3\pi}{4}\right)$ is equal to

   a) $-\frac{1}{\sqrt{2}}$ b) $\frac{1}{\sqrt{2}}$

   c) 1 d) -1

3. The relation $R = \varnothing$ on a set $A = \{1, 2, 3\}$ is

a) Reflexive and Symmetric relation

b) Reflexive and Transitive relation

c) Symmetric and Transitive relation

d) Equivalence relation

4. The value of $\int \frac{(\cos x - \sin x)\, dx}{1 + \sin 2x}$ is

a) $-\frac{1}{(\cos x - \sin x)} + C$

b) $\frac{1}{(\cos x - \sin x)} + C$

c) $\frac{1}{(\sin x + \cos x)} + C$

d) $-\frac{1}{(\sin x + \cos x)} + C$

5. A function $f: R \to R$ *defined by* $f(x) = 1 + |x-2|$ is

a) one one, onto

b) many one, onto

c) many one, into

d) one one, into

6. The derivative of $\frac{1}{x}$ with respect to $\log x$ is

a) $-x$

b) $-\frac{1}{x}$

c) 1

d) $\frac{1}{x}$

7. If the rate of change of area of a circle is equal to the rate of change of its diameter, then its radius is equal to

a) $\frac{2}{\pi}$

b) $\frac{1}{\pi}$

c) $\frac{\pi}{2}$

d) π

8. If $3\tan^{-1}x + \cot^{-1}x = \pi$, then x is

a) 0 b) 1/2

c) −1 d) 1

9. Which of the following function from Z to Z is a bijection

a) $f(x) = x^3$ b) $f(x) = x + 2$

c) $f(x) = 2x + 1$ d) $f(x) = x^2 + 1$

10. Find the value of 'a' if $f(x) = 5 + ax - 6x^2 + x^3$ attains its maximum value at x = 1 in the interval [0, 2]

a) 3 b) 5

c) −5 d) 9

11. The derivative of $y = \sin^{-1}(x\sqrt{x})$ with respect to x is

a) $\frac{3}{2}\sqrt{\frac{x}{1-x^3}}$ b) $\frac{1}{\sqrt{1-x^3}}$

c) $\frac{1}{2x\sqrt{1-x^3}}$ d) $\frac{1}{2\sqrt{x(1-x^3)}}$

12. Let R be reflexive relation having m elements defined on a set A having n elements, then

a) $m \geq n$ b) $m \leq n$

c) $m = n$ d) $m = n + 1$

13. The domain of the function $\tan^{-1}(x-1)$ is

a) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

c) R d) [−1, 1]

14. The value of k for which the function $f(x) = \begin{cases} kx\operatorname{cosec}3x, & x \neq 0 \\ 2 & x = 0 \end{cases}$ is continuous at $x = 0$ is

a) 6 b) 3

c) 0 d) 3/2

15. Given a curve $y = 7x - x^3$ and x decreases at the rate of 2 units per seconds. The rate at which the slope of the curve is changing, when $x = 5$ is

a) –60 units / sec b) 60 units / sec

c) –70 units / sec d) –140 units / sec

16. $\int_{-1}^{1} |x|\, dx$, is equal to

a) 1 b) –1

c) 2 d) –2

17. The value of $\int \frac{dx}{\sin^2 x \cos^2 x}$

a) $\tan x + \cot x + C$ b) $\tan x - \cot x + C$

c) $\tan x \cot x + C$ d) $2\tan x - 2\cot x + C$

18. The value of $\tan^2(\sec^{-1}2) + \cot^2(\operatorname{cosec}^{-1}3)$ is

a) 5 b) 11

c) 13 d) 15

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A

c) A is true but R is false.

d) A is false but R is true.

19. **Assertions (A) :** The function $f(x) = \tan x - 4x$ is strictly decreasing in $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

Reason (R) : A function is strictly decreasing in (a, b) if $f'(x) < 0$, for all $x \in (a, b)$

20. **Assertion (A) :** $\int_0^{2\pi} \cos^{2023} x\, dx = 0$

Reason (R) : $\int_0^{2a} f(x)\, dx = \int_0^{a} [f(x) + f(2a - x)]\, dx$

Section B (2 marks)

21. Find $\frac{dy}{dx}$, if $x = a\left(\cos t + \log\left(\tan\frac{t}{2}\right)\right)$, $y = a \sin t$.

22. The amount of pollution content added in air in a city due to x diesel vehicles is given by p (x) = $0.005x^3 + 0.02x^2 + 30x$. Find the marginal increase in pollution content when 3 diesel vehicles are added.

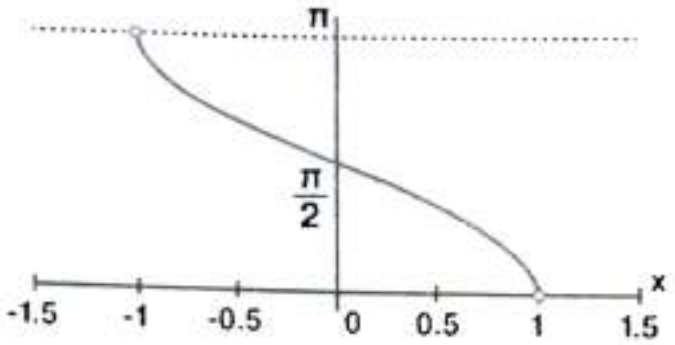
OR

A man of height 2m walks at a uniform speed of 5 km/h away from a lamp post which is 6m high. Find the rate at which the length of his shadow increases.

23. If $\tan^{-1}\left(\frac{y}{x}\right) = \log\sqrt{x^2 + y^2}$, show that $\frac{dy}{dx} = \frac{x + y}{x - y}$

24. Show that the function f $(x) = \tan^{-1}(\sin x + \cos x)$, $x > 0$ is always strictly increasing in $\left(0, \frac{\pi}{4}\right)$.

25. Identify the function from its graph given below. Also write its range and domain.



OR

Prove that $\cos^{-1}x + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right) = \frac{\pi}{3}$

Section C (3 marks)

26. Find the value of k for which the function given as $f(x) = \begin{cases} \frac{1-\cos 2kx}{x^2}, & \text{if } x \le 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & \text{if } x > 0 \end{cases}$

is continuous at x = 0

27. Check whether $f : R_* \to A$ *defined by* $f(x) = x^2 - 4x + 5$ is one one function. Redefine set A to make f an onto function.

28. If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$.

OR

If $y = e^{a\cos^{-1}x}, -1 \le x \le 1$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

29. If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \theta$, then prove that $\frac{x^2}{a^2} - 2\frac{xy}{ab}\cos\theta + \frac{y^2}{b^2} = \sin^2\theta$

30. Evaluate : $\int e^{3x}\cos 2x\,dx$ **OR** Evaluate : $\int \frac{2+\sin 2x}{1+\cos 2x}e^x\,dx$

31. Evaluate : $\int \frac{dx}{\cos(x-a)\cos(x-b)}$

Section D (5 marks)

32. Show that semi vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$

OR

If length of three sides of a trapezium other than base are equal to 10cm, then find the area of trapezium when it is maximum.

33. Evaluate : $\int_0^{\pi} \log(1+\cos x)\,dx$

34. Differentiate with respect to x : $x^{x\cos x} + \left(x+\frac{1}{x}\right)^x$

OR

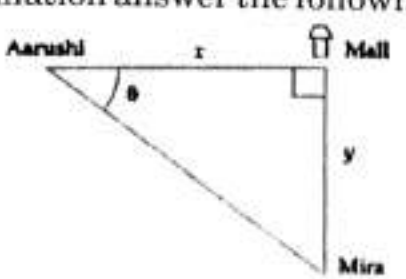
If $(x-a)^2 + (y-b)^2 = c^2$, *for some* $c > 0$, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is the constant independent of a and b.

35. Evaluate : $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx$

Section E (4 marks) - Case Study Questions

36. Arushi is rollerblading due east towards a Mall at a speed of 10 km/hr. Mira is biking due south away from the mall at a speed of 15km/hr. Let x be the distance between Arushi and the Mall at time t and let y be the distance between Mira and the Mall at any time t. Let S be the distance between Mira and Arushi at a time t, when Arushi and Mira are respectively 5km and 12km away from the mall.

Based on the above information answer the following questions:



i) Find the rate of change of S at a time t

ii) Let θ be the angle as shown in figure. Find the rate of change of θ in radian per hour at time t. (2 + 2)

37. The Apache helicopter is a highly advanced military aircraft known for its speed and maneuverability. Suppose an Apache helicopter is flying along the curve defined by the equation $y = x^2 + 7$. A soldier is located at radar station at the point (3, 7). The soldier wants to shoot down the helicopter when it is closest to him.

Based on this information, answer the following:

i) What is the point on the path when the distance between helicopter and radar station is minimum

ii) What is the minimum distance? (3 + 1)

38. On a World Environment Day, children of Navyug society planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = 3x + 2$. Let L be the set of all lines on the ground and R be the relation defined on L as $R = \{(l_1, l_2) : l_1 \| l_2, \text{where } l_1, l_2 \in L\}$

i) Show that R is an equivalence relation.

ii) Find the equation of straight line related to $y = 3x + 2$. (3+1)

–x–x–