**Unit Test 2**

**Class 12 (28/7/2025)**

**MM: 25** **Mathematics** **Time : 50 minutes**

Q1The set of all points where the function $x|x|$ is differentiable is

a) $(-\infty, \infty)$ b) $(-\infty,0) \cup (0, \infty)$ c) $(0, \infty)$ d) $(0,\infty)$ (1)

Q2. The function $f(x) = \tan^{-1}x - x$

a) Always increases b) always decreases c) neither increasing nor decreasing d) none (1)

Q3The rate of change of volume of a sphere with respect to its surface area, when the radius is 4cm

a) $12cm^3/_{cm^2}$ b) $6cm^3/_{cm^2}$ c) $3cm^3/_{cm^2}$ d) $2cm^3/_{cm^2}$ (1)

Q4. In the following question a statement of Assertion(A) is followed by statement of Reason(R), choose the correct answer out of following choices.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true and R is not the correct explanation of A.

(C) A is true but R is false

(D) Ais false but R is true.

Assertion: If $y = f(x)$, then $\frac{d^2y}{dx^2} = \frac{1}{\frac{d^2x}{dy^2}}$

Reason: If $y = f(x)$, then $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ (1)

Q5. Find the interval in which $f(x) = -2x^3 - 9x^2 - 12x + 1$ is strictly increasing. (2)

Q6. Sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand forms acone on the ground in such a way that the height of the cone is always one sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm? (2)

Q7. If $y = (\tan^{-1}x)^2$, show that $(x^2+1)^2y_2 + 2x(x^2+1)y_1 = 2$ (2)

Q8. Find the maximum and minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on $[0,3]$ (3)

OR

Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{\frac{1}{e}}$.

Q9. Find the value of 'a' for which the function f is defined by $f(x) = \begin{cases} a\sin\frac{\pi(x+1)}{2}, & x \le 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ is cts. (3)

Q10. Show that $\frac{dy}{dx} = -\cot 3t$ if $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$ (5)

Q11. CASE BASED QUESTION:

A cable network provider In a small town has 500 subscribers and he used to collect Rs. 300 per month from each subscriber. He proposes to increase the monthly charges and it is believed from past experience that for every increase of Re1, one subscriber will discontinue the service. Based on this information, answer the following questions.

a) Find the expression to find the total revenue 'R' in Rs.

b) Find the amount of maximum revenue generated. (4)