[illegible] Mahajan

Roll No. 20

BHATNAGAR INTERNATIONAL SCHOOL
PASCHIM VIHAR
Class XII 2025-26

Mid-term Examination

Subject- Mathematics (041)

Set - A

Time: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.

   Section – A:
3. It comprises of 20 MCQs of 1 mark each.

   Section – B:
4. It comprises of 5 VSA type questions of 2 marks each.

   Section – C:
5. It comprises of 6 SA type of questions of 3 marks each.

   Section – D:
6. It comprises of 4 LA type of questions of 5 marks each.

   Section – E:
7. It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks.

SECTION A

1. The number of onto mapping from the set $A = \{1, 2, ...100\}$ to set $B = \{1, 2\}$ is

a) $2^{100} - 2$ b) 2^{100} c) $2^{99} - 2$ d) 2^{99}

2. The domain of the function $f(x) = \sin^{-1}\sqrt{x-1}$ is

a) [1,2] b) [2,1] c) [-1,1] d) [0,1]

3. If p, q, r are 3 real numbers satisfying the matrix equation $[p\ q\ r]\begin{bmatrix}3 & 4 & 1\\ 3 & 2 & 3\\ 2 & 0 & 2\end{bmatrix} = [3\ 0\ 1]$, then $2p + q - r$ is equal to

1×3 3×3

a) -3 b) -1 c) 4 d) 2

4. If A and B are square matrices of order 3 such that $|A| = -1$, $|B| = 3$, then value of $|2AB| =$

a) 6 b) -12 c) -24 d) 12

5. $\int \frac{x-1}{(x-2)(x-3)}dx$ equals

(a) $2\log|x-3| - \log|x-2| + C$ (b) $\log|x-3| - \log|x-2| + C$

(c) $\log|x-3| - 2\log|x-2| + C$ (d) $\log|x-2| - \log|x-3| + C$

6. The value of x for which the polynomial $2x^3 - 9x^2 + 12x + 4$ is a decreasing function of x, is ?

a) $-1 < x < 1$ b) $0 < x < 2$ c) $x > 3$ d) $1 < x < 2$

7. $\int_1^e \frac{\log x}{x}dx$ equals

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $-\frac{3}{2}$ d) $-\frac{1}{2}$

8. Corner points of the feasible region determined by the system of linear constraints are (0,3),(1,1) and (3,0). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that maximum of Z occurs at (3,0) and (1,1) is

(a) $p = 2q$ (b) $p = q/2$ (c) $p = 3q$ (d) $p = q$

9. The points $A\ (a, b + c)$, $B\ (b, c + a)$ and $C\ (c, a + b)$ are

a) vertices of an isosceles triangle
b) vertices of an equilateral triangle
c) collinear
d) none of these

$z = 3q$
$z = p + q$
$z = 3p$

10. If $y = e^{\frac{1}{2}\log(1+\tan^2 x)}$, then $\frac{dy}{dx}$ is equal to

a) $\sec^2 x$ b) $\frac{1}{2}\sec^2 x$ c) $\sec x \tan x$ d) $\tan x$

11. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

$\sec\theta$

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}$ c) 1 d) $\sqrt{2}$

12. If A and B are 3×3 matrices such that $AB = A$ and $BA = B$, then

a) $A^2 = A$ and $B^2 \neq B$ b) $A^2 \neq A$ and $B^2 = B$ c) $A^2 = A$ and $B^2 = B$ d) $A^2 \neq A$ and $B^2 \neq B$

$\tan\theta = x$

13. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then value of x is

a) 3 b) 0 c) 1 d) -1

14. Value of $\int \frac{\cos\sqrt{x}\,dx}{\sqrt{x}}$ is

a) $-2\sin\sqrt{x} + C$ b) $\sin\sqrt{x} + C$ c) $2\cos\sqrt{x} + C$ d) $2\sin\sqrt{x} + C$

15. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x is continuous at

a) $x = 1$ b) $x = 1.5$ c) $x = -2$ d) $x = 4$

16. If the sum of two numbers is 3, then the maximum value of the product of the first and the square of second is

a) 1 b) 4 c) 3 d) 0

17. Let $f: R \rightarrow R$ defined by $f(x) = [x] + x$, then f is

a) one-one onto b) one-one but not onto c) many-one onto d) none of these

18. If $A = \begin{pmatrix} 3 & 5 \\ 7 & 9 \end{pmatrix}$ is written as $A = P + Q$, where P is a symmetric matrix and Q is skew symmetric matrix, then the matrix P will be given by

(a) $\begin{pmatrix} 3 & 3 \\ 6 & 9 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 6 \\ 6 & 9 \end{pmatrix}$ (c) $\begin{pmatrix} 3 & 7 \\ 5 & 9 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$

Followings are Assertion (A) - Reason (R) based questions (from Q19 to 20).

Read the following statements carefully to mark the correct option out of the options given below.

(a) A is true, R is true; R is a correct explanation for A.
(b) A is true, R is true; R is not a correct explanation for A.
(c) A is true, R is false.
(d) A is false, R is true.

$y = 2$

$2\cos A \cos B =$

19. **Assertion (A) :** The domain of the function $f(x) = \cos^{-1}\left(\frac{2x}{3}\right)$ is $x \in \left[-\frac{3}{2}, \frac{3}{2}\right]$.

Reason (R) : $\cos^{-1} x$ is defined when $-1 \le x \le 1$.

20. **Assertion (A) :** The maximum value of the function $y = \sin x$ in $[0, 2\pi]$ is at $x = \frac{\pi}{2}$

$\frac{dy}{dx} = \cos x$

$0 = -\cos x$

Reason (R) : The first derivative of the function is zero at $x = \frac{\pi}{2}$ and second derivative is negative at $x = \frac{\pi}{2}$.

$n + \frac{\pi}{2}$

SECTION B

$n \le \frac{\pi}{2} = 6$

$n = 0$

21. Write the given function in simplest form $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right), \frac{-\pi}{4} < x < \frac{3\pi}{4}$

22. A spherical ice-ball melts uniformly. When its radius is 10 cm, determine the rate of change of its volume with respect to the radius.

23. Differentiate w. r.t. x : $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

24. If the matix $\begin{bmatrix} -2 & x-y & 5 \\ 1 & 0 & 4 \\ x+y & z & 7 \end{bmatrix}$ is symmetric , find the values of x,y and z.

25. Evaluate $\int x^2 e^x dx$

OR

Integrate $\frac{1}{9x^2 + 6x + 5}$

SECTION C

26. Show that the function f: R →R defined by $f(x) = \frac{x}{x^2+1}$ is neither one-one nor onto.

27. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

28. Determine if the given function is continuous or not. If not, find the point of discontinuity.

$$f(x) = \begin{cases} 2x+3, & \text{if } x \le 2 \\ 2x-3, & \text{if } x > 2 \end{cases}$$

OR

Show that the function defined by $f(x) = |\sin x|$ is a continuous function.

29. Find the integral of the function given by $\frac{1}{\cos(x-a)\cos(x-b)}$

30. Evaluate: $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$

31. Find the matrix A satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

SECTION-D

32. Given that $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB and use it to solve the system of equations:

$x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$

33. Find the intervals in which the given function is increasing and decreasing:

$$f(x) = \frac{4\sin x - 2x - x\cos x}{2 + \cos x}$$

OR

Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

34. By using the properties of definite integrals, evaluate the integral $\int_0^{\pi} \log(1 + \cos x)dx$

35. Solve the following Linear Programming Problems graphically:

Minimise and Maximise $Z = x + 2y$

subject to $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$; $x, y \geq 0$.

SECTION-E

36. CASE STUDY-1

Three schools DPS, CVC and KVS decided to organise a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of ₹25, ₹100 and ₹50 each respectively. The number of articles sold are given as

School/Article	DPS	CVC	KVS
Handmade/fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Based on the above information, answer the following questions. Show steps to support your answers.

a) What is total money collected by school DPS? (1)
b) What is total money collected by school CVC and KVS? (1)
c) If the number of handmade fans and plates are interchanged for all the schools then what is the total money collected by all schools? (2)

37. CASE STUDY-2

Arya and Medha are playing snake and ladder game at home during vacation. While rolling the dice Arya 's cousin Anand observed and noted that possible outcomes of the throw every time belongs to set {1,2,3,4,5,6}. Let X be the set of players and Y be the set of all possible outcomes. X= {V, A} Y= {1,2,3,4,5,6}

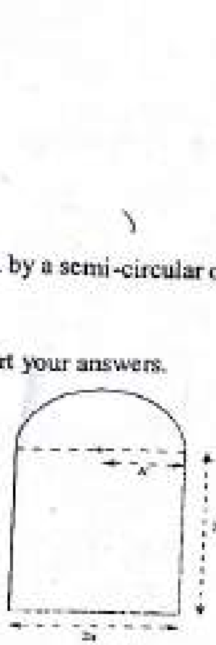
Based on the above information, answer the following questions.

Let R: Y to Y be defined by R= {(a,b) : |a - b| is even}.
a) Is R a reflexive relation? Show steps to support your answers. (1)
b) Is R a symmetric relation? Show steps to support your answers. (1)
c)) Is it an equivalence relation? Show steps to support your answers. (2)

38. CASE STUDY-3

Mr. Dhar is an architect. He designed a window in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 m.

Based on the above information, answer the following questions. Show steps to support your answers.



a) If 2x metres and y metres be the breadth and height of the rectangular part of the window, write the relation between x and y? (1)
b) Write the expression for the area 'A' enclosed by the window in terms of x. (1)
c) Determine the width of the window to allow maximum light through the window. (2)

$r^2 = l^2 - \frac{l^2}{3}$

$\frac{2l^2}{3}$