DELHI PUBLIC SCHOOL, GBN

SESSION 2025-26

HALF YEARLY EXAMINATION

CLASS XII- MATHEMATICS (041)

SET A

DURATION: 3 Hour **MM: 80**

General Instructions:

- This question paper contains 38 questions. All questions are compulsory.
- The question paper is divided into five Sections A, B, C, D and E.
- In Section A, Question numbers 1 to 18 are multiple choice questions (MCQs), question numbers 19 and 20 are Assertion-Reason based questions of 1 mark each.
- In Section B, Question numbers 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
- In Section C, Question numbers 26 to 31 are short answer (SA) type questions, carrying 3 marks each.
- In Section D, Question numbers 32 to 35 are long answer (LA) type questions carrying 5 marks each.
- In Section E, Question numbers 36 to 38 are case study-based questions carrying 4 marks each.
- There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D.

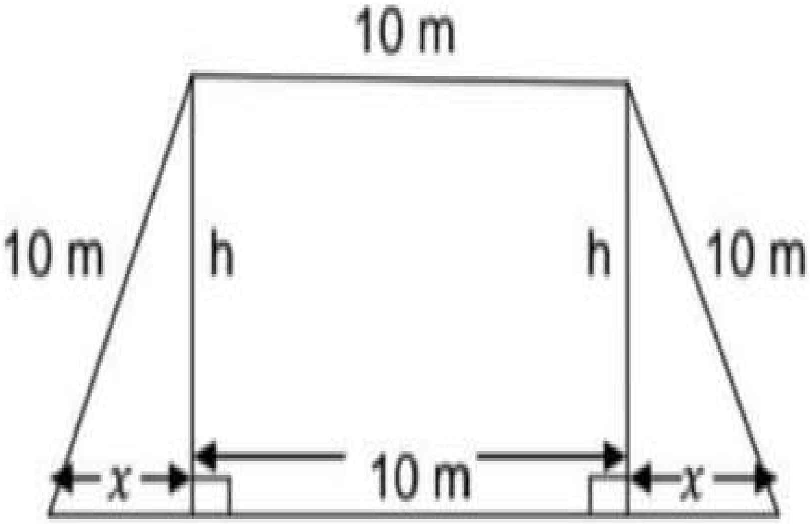
Use of calculators is not allowed.

Q. No	Part No.	Question	Marks
		SECTION A	
1.		If $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, then the value of $\lvert A\,A' \rvert$ is (a) -9 (b) 3 (c) $\frac{1}{9}$ (d) 9	1
2.		If $\int_0^1 \frac{e^x}{1+x} dx = \alpha, then \int_0^1 \frac{e^x}{(1+x)^2} dx\ is\ equal\ to$ (a) $\alpha - 1 + \frac{e}{2}$ (b) $\alpha + 1 - \frac{e}{2}$ (c) $\alpha - 1 - \frac{e}{2}$ (d) $\alpha + 1 + \frac{e}{2}$	1
3.		If A= $[a_{ij}]$ is a scalar matrix of order 3 such that $a_{11} = 2$, then $\lvert A \rvert$ is: (a) 4 (b) 6 (c) 8 (d) none of these	1
4.		A and B are square matrices of order 3, then which of the following is not true? (a) $\lvert A' \rvert = \lvert A \rvert$ (b) $\lvert kA \rvert = k^3 \lvert A \rvert$ (c) $\lvert A + B \rvert = \lvert A \rvert + \lvert B \rvert$ (d) $\lvert AB \rvert = \lvert A \rvert \lvert B \rvert$	1

5.		If $f(x)=\begin{cases}\dfrac{x^2-1}{x-1}, & \text{when } x\neq 1\\ 2k, & \text{when } x=1\end{cases}$ is given to be continuous at $x=1$, then the value of k is (a) -1 (b) -2 (c) 1 (d) 2	1
6.		$\int 4^x.3^x\,dx = f(x)+c$, then $f(x)$ is (a) $\frac{12^x}{log12}$ (b) $\frac{4^x}{log12}$ (c) $\frac{4^x\,3^x}{log3\,.log4}$ (d) $\frac{3^x}{log3}$	1
7.		A set of values of decision variables that satisfies the linear constraints and non-negativity conditions of an L.P.P. is called its: (a) Unbounded solution (b) Optimum solution (c) Feasible solution (d) None of these	1
8.		Let $\sin^{-1}(2x)+\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}$. Then the value of x is (a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) 8 (d) $\frac{1}{4}$	1
9.		Let $f: R \to R$ be defined as $f(x)=\begin{cases}1, & x>0\\ 0, & x=0\\ -1, & x<0\end{cases}$, then $3\,f(\sqrt{2})+f(0)+2\,f(-\pi)=$ (a) 3 (b) 1 (c) 2 (d) Not defined	1
10.		An insect moves along the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$. When the rate of change of abscissa is 4 times that of the ordinate, then the insect lies in the quadrant: (a)*I or II* (b)*II or III* (c)*III or IV* (d) *II or IV*	1
11.		If A is non-singular matrix of order 3× 3 such that $\|A\|$= 2. The value of $\|adj(A)\|$ is: (a) 6 (b) 8 (c) 4 (d) 10	1
12.		The maximum slope of the curve $y=-x^3+3x^2+9x-27$ is: (a)0 (b)12 (c)16 (d)32	1
13.		If $y=5e^{7x}+6e^{-7x}$, such that $y''-ky=0$, then the value of k will be (a) -49 (b) 1 (c) 49 (d) 0	1
14.		Principal value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ is (a) $\frac{7\pi}{6}$ (b) $-\frac{\pi}{6}$ (c) $\frac{\pi}{6}$ (d) $-\frac{7\pi}{6}$	1
15.		The function $f(x)=7x-3$ is strictly ________ on R. (a) Decreasing (b) increasing (c)neither increasing nor decreasing (d) None of these	1
16.		Let the relation R in the set A = {x ∈ Z : 0 ≤ x ≤ 12}, given by R = {(a, b) : |a – b| is a multiple of 4}. Then [1], the equivalence class containing 1, is: (a) {1, 5, 9} (b) {0, 1, 2, 5} (c) ∅ (d)A	1

17.		Derivative of $\tan^{-1}(\frac{1-cosx}{sinx})$ with respect to x is: (a) $\frac{1}{2}$ (b) -1 (c) 1 (d) $-\frac{1}{2}$	1
18.		Let $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx = f(x)+C$. Then $f(x)=$ (a) $\sin^{-1}\left(\frac{e^x+2}{3}\right)$ (b) $\tan^{-1}\left(\frac{e^x+2}{3}\right)$ (c) $\frac{1}{3}\sin^{-1}\left(\frac{e^x+2}{3}\right)$ (d) $\cos^{-1}\left(\frac{e^x+2}{3}\right)$	1
		For question numbers 18 and 19, two statements are given – one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (i), (ii), (iii) and (iv) as given below: (i) Both A and R are true and R is the correct explanation of the assertion (ii) Both A and R are true but R is not the correct explanation of the assertion (iii) A is true, but R is false (iv) A is false, but R is true	
19.		**Assertion (A)**: The relation R = {(a, b): $a \le b^2$}on the set R of real nos. is not reflexive. **Reason (R)**: A relation on a set A is reflexive if (a, a) ∈ R ∀ a ∈ A .	1
20.		**Assertion (A)**: if $\begin{bmatrix} 0 & 7 & 9 \\ -7 & 0 & 6 \\ -9 & -6 & 0 \end{bmatrix}$, then $\|A\| = 0$ **Reason (R)**: The determinant of skew symmetric matrix of order 3×3 is always zero.	1
		SECTION B	
21.		Show that the points $(a+5, a-4), (a-2, a+3)$ and (a, a) do not lie on a straight line for any value of a.	2
22.		Find the value of : $\cos^{-1}\left(\cos\frac{13\pi}{6}\right) + \tan^{-1}\left(\tan\frac{5\pi}{6}\right)$. **OR** Evaluate: $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$.	2
23.		Check if the relation R in the set $A = \{1,2,3,4,5,6\}$ defined as $R = \{(x, y): y \text{ is divisible by } x\}$ is (i)symmetric (ii) transitive.	2
24.		If the circumference of a circle is increasing at the constant rate, then prove that the rate of change of area of circle is directly proportional its radius. **OR** Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?	2
25.		Find: $\int \frac{x+1}{x}(x + logx)$dx.	2
		SECTION C	

26.		Evaluate: $\int \frac{x}{(x^2+1)\,(x-1)}dx.$ **OR** Evaluate: $\int \left(\frac{1+\sin x}{1+\cos x}\right) e^x dx.$	3
27.		Find the intervals in which the function $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$, is (i) strictly increasing and (ii) strictly decreasing	3
28.		Solve the following linear programming problem graphically. Minimize: $z = 5x + 10y$ Subject to constraints: $x + 2y \le 120, x + y \ge 60, x - 2y \ge 0, x \ge 0, y \ge 0$	3
29.		Find the value of x such that $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$	3
30.		Prove that the relation R on the set N × N defined by (a, b) R (c, d) ⇔ a + d = b + c for all (a, b), (c, d) ∈ N × N is an equivalence relation.	3
31.		Differentiate $(sinx)^{\log x} + x^{\sin x}$ with respect to x. **OR** If $x = a\, sec\theta, y = b\, tan\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \pi/6$	3
		SECTION D	
32.		Prove that the function f : N → N, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.	5
33.		If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$ **OR** If $y\sqrt{x^2+1} = \log(\sqrt{x^2+1} - x)$, show that $(1+x^2)\frac{dy}{dx} + xy + 1 = 0$	5
34.		Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is $\frac{2a}{\sqrt{3}}$.	5
35.		Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$ **OR** Evaluate $\int_1^4 (\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert)dx.$	5
		SECTION E	
36.		The front gate of a building is in the shape of a trapezium as shown below. Its three sides other than base are 10 m each. The height of the gate is h metres. On the basis of this information and figure given below answer the following questions:	4

<table>
<tr><td></td><td></td><td>

(i)Let the area of the gate be A. Write the area of the gate as a function of x.
(ii) Find the critical points of the function.
(iii) Use first derivative test to find the maximum area of the gate of the building in m^2.
OR
Use second derivative test to find the maximum area of the gate of the building in m^2.</td><td></td></tr>
<tr><td>37.</td><td></td><td>If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then $y = f(g(x))$ is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$. This rule is known as CHAIN RULE.
Based on the above information answer the following:
(i) If $y = \log_7(\log x)$, find $\frac{dy}{dx}$.
(ii) If $x = t^2$ and $y = t^3$, then find $\frac{d^2y}{dx^2}$ at $t = 3$.
(iii) If $x = \sqrt{a^{\sin^{-1} t}}$ and $y = \sqrt{a^{\cos^{-1} t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$.
OR
If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, find $\frac{dy}{dx}$.</td><td>4</td></tr>
<tr><td>38.</td><td></td><td>On her birthday Seema decided to donate some money to children of an orphanage home. If there were eight children less everyone would have got Rs 10 more however if there were 16 children more everyone would have got Rs 10 less let the number of children be x and the amount distributed by Seema for one child be y
Based on the given information answer the following questions
(i) Find the equation in terms of x and y and Express them in matrix form
(ii) Find the number of children who were given some money by Seema
(iii) How much amount Seema spends in donation to all the children of the orphanage</td><td>4</td></tr>
</table>