First Term Examination (2025 - 26) 360

[redacted]ect - Mathematics (Code - 041)

Time 3 hrs. Class - 12 - C Vanika Sharma Max Marks 80

29

GENERAL INSTRUCTIONS :-

(i) All questions are compulsory.

(ii) This question paper contains 38 questions.

(iii) Question 1- 20 in Section A are multiple choice questions carrying 1 mark each.

(iv) Question 21-25 in Section B are very short-answer type questions carrying 2 marks each.

(v) Question 26-31 in Section C are short answer type questions carrying 3 marks each.

(vi) Question 32-35 in Section D are long answer type questions carrying 5 marks each.

(vii) Question 36-38 in Section E are case study based questions carrying 4 marks each.

[Section – A] **(1 MARK EACH)**

Choose the correct option :-

1. If $3\tan^{-1}x + \cot^{-1}x = \pi$ then x equals :

(a) 0 (b) 1

(c) –1 (d) 12

2. If $A^2 - A + I = O$, then the inverse of A is :

(a) I – A (b) A – I

(c) A (d) A + I

3. $\int(\cos\sqrt{x})/\sqrt{x}\,dx$:

a) $2\cos\sqrt{x} + c$ b) $\sqrt{\cos x}/\sqrt{x} + c$

c) $\sin\sqrt{x} + c$ d) $2\sin\sqrt{x} + c$

4. Let $X = \{x^2 : x \in N\}$ and the relation $f : N \to X$ is defined by $f(x) = x^2$, $x \in N$. Then, this function is :

a) injective only b) not bijective

c) surjective only d) bijective

[P. T. O.]

5. Find the value of $\sin[\pi/3 + \sin^{-1}(1/2)]$

a) 1 b) 1/2
c) 1/3 d) 1/4

6. If A is a 3 x 4 matrix and B is a matrix such that A'B and AB' are both defined, then the order of the matrix B is :

a) 4 x 4 b) 4 x 3
c) 3 x 3 d) 3 x 4

7. $\int \sec x(\sec x + \tan x)dx$:

a) sec x tan x b) sec x + tan x
c) sec x – tan x d) sec x / tan x

8. If A is a square matrix such that $A^2 = I$, then $(A-I)^3 + (A+I)^3 - 7A$ is equal to?

a) I – A b) A
c) I + A d) 3A

9. The set of points of discontinuity of the function f(x) = 2x – [x] is :

a) Q b) R
c) Z d) W

10. Integrate $\int_0^6 x(x+1)+1dx$:

a) 15/2 b) 4
c) 20/3 d) 3/20

11. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to :

a) –6 b) 0
c) 6 d) ±6

12. If $x = t^3$, $y = t^4$ then y'' is :

a) 3/5 b) 3/4t
c) 3/7t d) 4t/3

13. If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx}$ is :

a) e^{x-y} b) e^{x+y}
c) $-e^{x-y}$ d) $2e^{x-y}$

14. The function $f(x) = |x-3|$, $x \in R$ is :

a) Not continuous and not differentiable everywhere

b) Not continuous but differentiable at $x = 3$

c) Is continuous but not differentiable at $x = 3$

d) Is continuous and differentiable everywhere

15. Value of the determinant $\begin{vmatrix} \cos 67^\circ & \sin 67^\circ \\ \sin 23^\circ & \cos 23^\circ \end{vmatrix}$ is :

(A) 0 (B) 1/ 2
(C) $\sqrt{3}/2$ (D) 1

16. If A is a square matrix of order 4 and $|\text{adj } A| = 27$, then A (adj A) is equal to :

(A) 3 (B) 9 (C) 3 I (D) 9 I (I stands for identity matrix)

17. Find the values of k so that the function f is continuous at $x = \pi$ if :

$$f(x) = \begin{cases} kx+1, & x < \pi \\ \cos x, & x \geq \pi \end{cases}$$

a) $-1/\pi$ b) $-2/\pi$ c) $-\pi$ d) -2π

18. Two numbers whose sum is 24 and whose product is as large as possible, then numbers are :

(a) 16, 8 (b) 12, 12

(c) 10, 14 (d) 11, 13

19-20 ASSERTION-REASON BASED QUESTIONS :

In the following questions (19 & 20), a statement of assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices :-

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Q.19. **Assertion :** If A is a square matrix, then A-A' is skew symmetric matrix.

Reason : Zero matrix is skew symmetric.

Q.20. **Assertion :** $f(x) = 2x^3 - 9x^2 + 12x - 3$ is increasing outside the interval (1, 2)

Reason : $f'(x) < 0$ for $x \in (1, 2)$.

[Section – B] (2 MARKS EACH)

21. Find the maximum and minimum values of the function f given by $f(x) = \sin x + \cos x$, $x \in (0, \pi/2)$.

OR

Find the intervals in which the function $f(x) = \log x/x$ is strictly increasing or decreasing.

22. Evaluate : $\int \sin x \cdot \log \cos x \, dx$

23. Evaluate : $\int_{\pi/2}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx.$

24. The radius of a balloon is increasing at the rate of 10cm/sec. At what rate is the surface area of the balloon increasing when the radius is 15cm.

OR

Find the absolute maximum and absolute minimum values of a function f given by $f(x)=2x^3 - 15x^2 + 36x + 1$ on the interval [1, 5]

25. Evaluate : $\int \frac{\cos 2x}{\sin x \cos x} dx.$

[Section – C] (3 MARKS EACH)

26. Differentiate $(\log x)^x + (x)^{\log x}$ wrt x.

27. Let A = {1, 2, 3,....9} and R be relation in A × A defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in A × A. Prove that R is an equivalence relation

OR

Let A = R − {2} and B = R − {1}. If f : A → B by $f(x) = \frac{x-1}{x-2}$ show that f is one-one and onto.

28. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}$, $x \in (0, \pi/4)$

29. If $\tan^{-1} x + \tan^{-1} y = \pi/4$, $xy < 1$, then find the value of $x + y + xy$

30. Evaluate : $\int \frac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta} d\theta$

OR

Evaluate : $\int_0^1 |x + 3| dx$

31. Evaluate : $\int_{\frac{\pi}{2}}^{\pi} \frac{\cos x}{1 + e^x} dx$

OR

Evaluate : $\int_0^1 x(1-x)^n dx$

[Section – D] (5 MARKS EACH)

32. Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

$x - y + 2z = 1$, $2y - 3z = 1$, $3x - 2y + 4z = 2$

33. A) If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, show that $A^2 - 5A + 7I = 0$. Hence find A^{-1} (2)

B) Express the matrix as sum of symmetric and skew symmetric. (3)

$$\begin{pmatrix} 0 & 5 & 3 \\ -5 & 0 & -8 \\ -3 & 8 & 0 \end{pmatrix}$$

34. If $f(x) = \begin{cases} \frac{\sin(a+1)x + 2\sin x}{x} & , x<0 \\ 2 & , x=0 \\ \frac{\sqrt{1+bx}-1}{x} & , x>0 \end{cases}$ is continuous at $x = 0$, find values of a and b.

35. Find the following integral $\int (x+3)\sqrt{3-4x-x^2}\,dx$

OR

$$\int_{-1}^{3/2} |x \sin(\pi x)|\,dx$$

[Section – E]

(4 MARKS EACH, CASE BASED QUESTIONS)

36. Two schools A and B want to award their selected students on the values of Honesty, Hard work, and Punctuality. The School A wants to award Rs.x each, Rs.y each, and Rs.z each for the three respective values to its 3, 2 and 1 students respectively with total award money of Rs.2200. School B wants to spend Rs.3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school A). The total amount of award for one prize on each value is Rs.1200.

Based on the above data, answer the following questions :- (2+2)

(i) Using matrices, find the award money for each value.

(ii) a) If a matrix P is both symmetric and skew-symmetric, then |P| is equal to?

b) If P and Q are symmetric matrices of the same order then what can you say about PQ – QP?

37. A city's traffic management department is planning to optimize traffic flow by analyzing the connectivity between various traffic signals. The city has five major spots labelled A, B, C, D and E.

The department has collected the following data regarding one-way traffic flow between spots:

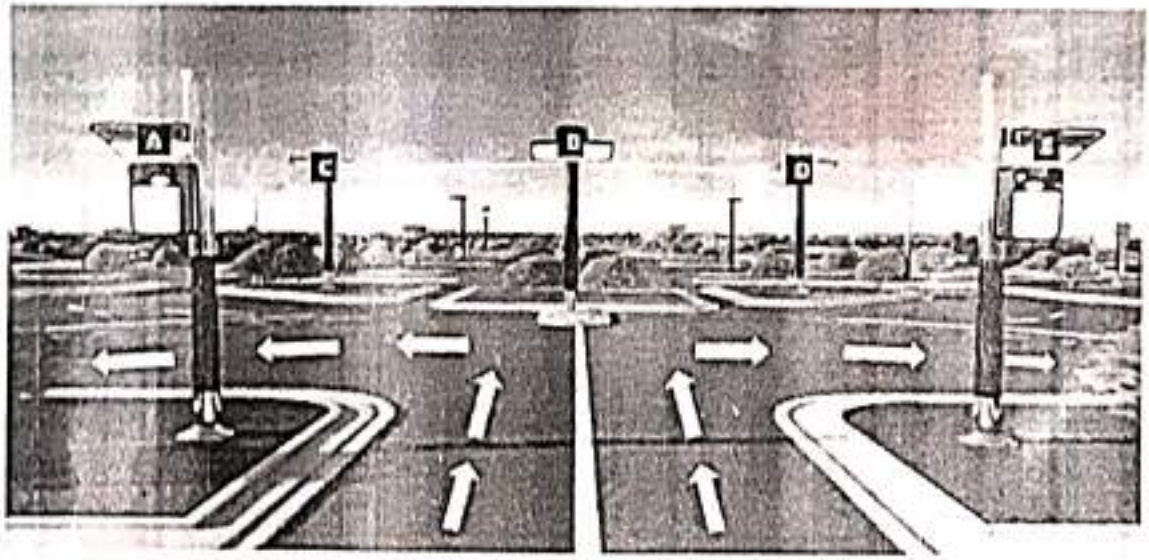
1. Traffic flows from A to B, A to C, and A to D.

2. Traffic flows from B to C and B to E

3. Traffic flows from C to E.

4. Traffic flows from D to E and D to C.

The department wants to represent and analyze this data using relations and functions. Use the given data to answer the following questions :-



I. Is the traffic flow reflexive? Justify. [1]

II. Is the traffic flow transitive? Justify. [1]

III A. Represent the relation describing the traffic flow as a set of ordered pairs. Also state the domain and range of the relation.

OR

III B. Does the traffic flow represent a function? Justify your answer. [2]

38. LED bulbs are energy-efficient because they use significantly less electricity than traditional bulbs while producing the same amount of light. They convert more energy into light rather than heat, reducing waste. Additionally, their long lifespan means fewer replacements, saving resources and money over time. A company manufactures a new type of energy-efficient LED bulb. The cost of production and the revenue generated by selling x bulbs (in an hour) are modelled as

$C(x) = 0.5x^2 - 10x + 150$ and

$R(x) = -0.3x^2 + 20x$ respectively, where

C (x) and R (x) are both in Rs.

To maximize the profit, the company needs to analyze these functions using calculus. Use the given models to answer the following questions:

I. Derive the profit function P(x) [1]

II. Find the critical points of P(x). [1]

III A. Determine whether the critical points correspond to a maximum or a minimum profit by using the second derivative test.

OR

III B. Identify the possible practical value of x (i. e., the number of bulbs that can realistically be produced and sold) that can maximize the profit, if the resources available and the expenditure on machines allows to produce minimum 10 but not more than 18 bulbs per hour. Also calculate the maximum profit. [2]