

# THE INDIAN HIGH SCHOOL, DUBAI

# PERIODIC TEST - 1, 2025-26

DATE: 22.05.2025

GRADE: 12

MATHEMATICS (041)

**DURATION: 3 hours** 

MARKS: 80

### General Instructions:

- 1. This Question paper contains five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
- Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
- Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
- 4. Section C has 6 Short Answer (SA)-type questions of 3 marks each
- Section D has 4 Long Answer (LA)-type questions of 5 marks each.
- Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

#### SECTION - A

This section comprises of multiple-choice questions (MCQs) of 1 mark each  $(20 \times 1 = 20 \text{ Marks})$ 

- 1. Let  $A = \{2, 3, 5\}$  and the relation R on A is defined by  $R = \{(2, 3), (3, 5), (2, 2)\}$ . The minimum number of ordered pairs to be added in R to make it an equivalence relation is:
  - (A) 3
- (B) 4
- (C) 5
- (D) 6
- 2. If a matrix A is both symmetric and skew-symmetric, then A is necessarily a
  - (A) diagonal matrix
- (B) zero square matrix
- (C) square matrix
- (D) identity matrix
- 3. If A =  $[a_{ij}]$  is a matrix of order  $3 \times 3$  such that  $a_{ij} = \begin{cases} 2i+j, & i < j \\ 5, & i = j \\ 3i-2j, & i > j \end{cases}$

The number of elements in A which is more than 5 is:

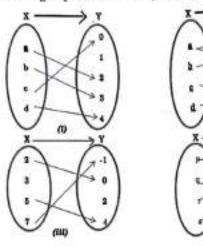
- (A) 3

- (C) 5 (D) 6
- If  $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$  then the value of ac bd is: (A) -16 (B) -4 (C) 4 (D) 16 4.

- 5. The domain of  $f(x) = \sin^{-1}x + \cos^{-1}x$  is [-1, 1] while its range is  $\left[-\frac{3\pi}{2}, \frac{\pi}{2}\right]$ . If g(x) is the inverse of f(x), which of the following is true about the domain of the function g(x)?
  - (A) It is [-1. 1]
- (B) It is independent of the domain and range of f(x)
- (C) It is  $[\frac{-3\pi}{2}, \frac{\pi}{2}]$
- (D) cannot be said without knowing g(x)
- The principal value of  $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$  is: 6.

  - $(A) \frac{\pi}{6} \qquad (B) \frac{\pi}{2}$
- (C)  $\frac{5\pi}{6}$
- (D)  $\frac{2\pi}{3}$

Which of the following represents an injective function? 7.



- (A) (i) only
- (B) (i) and (iv)
- (C) (ii) and (iv)
- (D) (iii) only
- 8. If  $\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$ , then the value of k is:

- (D) 1/3
- Which of the following statements is not correct? 9.
  - (A) A row matrix has only one row.
  - (B) A diagonal matrix has all diagonal elements equal to zero.
  - (C) A symmetric matrix is a square matrix satisfying certain conditions.
  - (D) A skew-symmetric matrix has all diagonal elements equal to zero.
- If M and N are square matrices of order 3 where det(M) = 2 and det(N) = 3 then det(4 MN) is: 10.
  - (A) 24
- (B) 64
- (C) 196
- (D) 384
- If A. (adj A) =  $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ , then the value of |A| |adj A| is equal to: (A) -3 (B) -12 (C) 6 (D) 9 11.

- If  $\begin{bmatrix} 4+x & x-1 \\ -2 & 3 \end{bmatrix}$  is a singular matrix, then the value of x is:
  (A) 0 (B) 1 (C) -2 (D) 12.

- If A is a square matrix of order 2 and |A| = -2, then the value of  $|5 A^T|$  is: 13.
- (B) 10

- If  $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$  and  $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ , then the value of x is: (A) 2 (B)  $-\frac{1}{2}$  (C) 1 14.

- The derivative of  $\log_9(\log_9 x)$  with respect to x is: (A)  $\frac{1}{x \log x}$  (B)  $\frac{1}{x \log_9}$  (C)  $\frac{1}{9 x \log x}$ 15.

- The derivative of  $\tan^{-1}(x^2)$  with respect to x is: (A)  $\frac{x}{1+x^4}$  (B)  $\frac{2x}{1+x^4}$  (C)  $-\frac{2x}{1+x^4}$  (D)  $\frac{1}{1+x^4}$ 16.
  - (A) x

- If  $x = a \sin \theta$  and  $y = a \cos \theta$  then the value of  $\frac{dy}{dx}$  at  $t = \frac{\pi}{4}$  is:

  (C) 1

  (D)  $-\sqrt{3}$ 17.
  - (A) √3

18. If 
$$f(x) = \begin{cases} \frac{\sin 8x}{3x} + \cos x & x \neq 0 \\ k, & x = 0 \end{cases}$$
 is continuous at  $x = 0$ , then the value of k is:

(A)  $\frac{8}{3}$  (B)  $\frac{5}{3}$  (C)  $\frac{1}{3}$  (D)  $\frac{11}{3}$ 

ASSERTION-REASON BASED QUESTIONS

In the following questions a statement of Assertion (A) is followed by a statement of Reason (R).

- Choose the correct answer out of the following choices.
- (a) Both (A) and (R) are true and (R) is the correct explanation of (A).(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
- (c) (A) is true but (R) is false.
- (d) (A) is false but (R) is true.
- 19. ASSERTION (A): A relation R = {(x, y): |x y| = 0} defined on the set A = {3, 5, 7} is symmetric REASON (R): A relation R on the set A is said to be symmetric if for (a, b) ε R, then (b, a) ε R.
- 20. ASSERTION (A): f(x) = [x], where [x] denotes greatest integer less than or equal to x is not differentiable at x = 2.
  - REASON (R): f(x) = [x], where [x] denotes greatest integer less than or equal to x is not continuous at x = 2.

(This section comprises very short answer (VSA) type questions of 2 marks each)  $(5 \times 2 = 10 \text{ Marks})$ 

- 21. (a) Find the value of  $\tan^{-1}(\frac{1}{\sqrt{3}}) + \cot^{-1}(\frac{1}{\sqrt{3}}) + \tan^{-1}[\sin(\frac{\pi}{2})]$ .
  - (b) Find the domain of the function  $f(x) = \sin^{-1}(x^2 4)$ . Also, find its range.
- 22. If A and B are symmetric matrices such that AB and BA are both defined, then prove that AB + BA is a symmetric matrix and AB - BA is a skew-symmetric matrix.
- 23. If  $A = \begin{bmatrix} 3 & 9 & 0 \\ 1 & 8 & -2 \\ 7 & 5 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 4 & 0 & 2 \\ 7 & 1 & 4 \\ 2 & 2 & 6 \end{bmatrix}$ , then find the matrix  $B^TA^T$ .
- 24. (a). Determine the constant k (if it exists) so that the function f defined by  $f(x) = \begin{cases} k & \text{if } x^2 5x \text{, } x < 0 \\ 1 + \sin x, & x \ge 0 \end{cases}$  may be continuous.
  - (b). Using limits find the left hand and right hand derivatives at x = 2 for the

function 
$$f(x) = \begin{cases} x & , x < 1 \\ 2-x & , 1 \le x \le 2 \text{. Is } f(x) \text{ differentiable? Justify?} \\ -2+3x-x^2 & , x > 2 \end{cases}$$

25. If  $f(x) = |\cos x - \sin x|$ , then find the value of f'(x) at  $x = \frac{\pi}{3}$ .

(This section comprises short answer (SA) type questions of 3 marks each)  $(6 \times 3 = 18 \text{ Marks})$ 

26. If  $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$  and I is the identity matrix of order 2, then show that  $A^2 = 4A - 3I$ . Hence find  $A^{-1}$ .

27. (a) If 
$$X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$
, then find the matrix  $X$ .

(b) If 
$$\cos 2\theta = 0$$
, then prove that  $\begin{vmatrix} 0 & \cos \theta & \sin \theta \\ \cos \theta & \sin \theta & 0 \\ \sin \theta & 0 & \cos \theta \end{vmatrix}^2 = \frac{1}{2}$ .

28. Using determinants, find the area of ΔPQR with vertices P (3, 1), Q (9, 3) and R (5, 7). Also find the equation of line PQ using determinants.

29. If 
$$y = \frac{\sin x}{1 + \frac{\cos x}{1 + \frac{\sin x}{1 + \frac{\cos x}{1 + \frac{\cos x}{1 + \frac{x}{1 + \dots}}}}}$$
, prove that  $\frac{dy}{dx} = \frac{(1+y)\cos x + y\sin x}{1 + 2y + \cos x - \sin x}$ 

30. (a). Differentiate 
$$(\sin x)^{\log x} + x^{\frac{1}{x}}$$
 with respect to x. OR

(b). If 
$$x = e^{\cos 3t}$$
 and  $y = e^{\sin 3t}$ , prove that  $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$ .

31. (a). Differentiate 
$$sin^{-1}\left(\frac{2^{x+1}}{1+(36)^x}\right)$$
 with respect to x.

(b). If 
$$y = a \cos(\log x) + b \sin(\log x)$$
, show that  $x^2 y_2 + x y_1 + y = 0$ .

## SECTION D

(This section comprises long answer (LA) type questions of 5 marks each)

(a). Let A = {1, 2, 3, ..., 9} and R be the relation in A × A defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in A × A. Prove that R is an equivalence relation and also obtain the equivalence class [(2, 5)].

(b). Show that a function  $f: (-\infty, 0) \to (-1, 0)$  defined by  $f(x) = \frac{x}{1+|x|}, x \in (-\infty, 0)$  is both one-one and onto.

33. Solve the following system of equations, using matrices:  $\frac{2}{x} + \frac{3}{y} + \frac{10}{x} = 4, \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{x} = 1, \quad \frac{6}{x} + \frac{9}{y} - \frac{20}{x} = 2$ 

34. If 
$$x = a(\theta - \sin \theta)$$
 and  $y = a(1 - \cos \theta)$ , prove that  $\frac{d^2y}{dx^2} = -\frac{1}{4a} \csc^4 \frac{\theta}{2}$ .

35. (a). If  $y = \cos (m \cos^{-1} x)$ , m is a constant then show that  $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0$ .

(b). Find  $\frac{dy}{dx}$  if  $y^x + x^y + x^x = a^b$ , where a and b are constants.

#### SECTION E

(This section comprises 3 case study based questions of 4 marks each)

- 36. Let A be the set of 30 students of class XII in a school. Let f: A → N, N is a set of natural numbers such that function f(x) = Roll Number of student x. On the basis of the given information, answer the following:
  - (i) Is f a bijective function?

(1)

(ii) Give reasons to support your answer to (i).

Justify your answer.

(1)

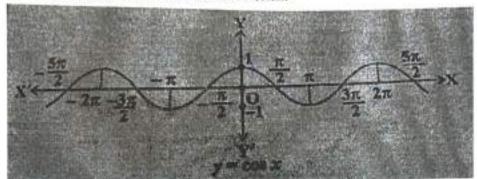
(2)

(iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where R = {(x, y): x, y are Roll Numbers of students such that y = 3x}. List the elements of R. Is the relation R reflexive, symmetric and transitive?

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(b) Let R be a relation defined by R = {(x, y): x, y are Roll Numbers of students such that y = x<sup>3</sup>}. List the elements of R. Is R a function? Justify your answer. (2)

The following graph shows the cosine function.



Let cosine function be defined from set A to [-1, 1] such that inverse of cosine function exists, i.e., cos<sup>-1</sup>x is defined from [-1, 1] to A.

On the basis of the above information, answer the following questions:

- (i) If A is the interval other than principal value branch, give an example of one such interval.
   (1)
- (ii) If  $\cos^{-1}x$  is defined from [-1, 1] to its principal value of branch, find the value of  $\cos^{-1}(-\frac{1}{2}) = \cos^{-1}(\frac{\sqrt{3}}{2})$ .
- (iii) (a) Draw the graph of cos-1 x from [-1, 1] to its principal branch. (2)
  - (b) Find the domain and range of  $f(x) = \cos^{-1}(3x 2)$ .

38. A company produces three products every day. Their production on certain day is 45 tons. It is found that the production of the third product exceeds the production of first product by 8 tons while the total production of first and third product is twice the production of second product.



Based on the above information answer the following questions:

- If x, y and z respectively denote the quantity (in tons) of first, second and third product produced, represent the system of equations.

  (2)
- (ii) If  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix}$  and its inverse is  $\frac{1}{6} \begin{bmatrix} 2 & 2 & 2 \\ 3 & 0 & -3 \\ 1 & -2 & 1 \end{bmatrix}$ , using the result solve x, y and z from the obtained system of equations by matrix method. (2)