



Fahaheel Al-Watanieh Indian Private School, Ahmadi, Kuwait
First Term Examination 2025-26
Mathematics (041)

065/F/2

Time: 3 Hours

Class XII

Max. Marks :80

General Instructions:

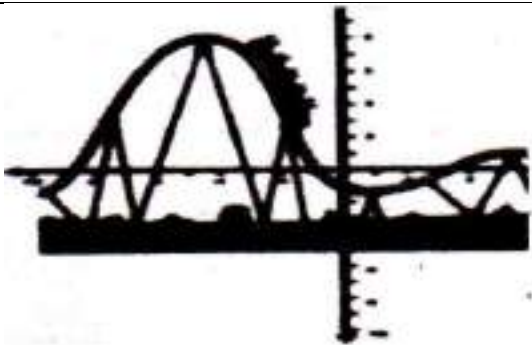
1. This question paper contains five sections – A, B, C, D and E. Each section is compulsory.
2. Section A has **18 MCQs** and **2 Assertion-Reason (A-R)** based questions of **1 mark** each.
3. Section B has **5 questions** of **2 marks** each.
4. Section C has **6 questions** of **3 marks** each.
5. Section D has **4 questions** of **5 marks** each.
6. Section E has **3 case study questions** with **sub parts (4 marks each)**

Qn No.	SECTION A	Marks
	Section A consist of 18 MCQs and 2 Assertion-Reason (A-R) based questions of 1 mark each.	
1	The value of $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$ is a) $\frac{25}{24}$ b) $\frac{25}{7}$ c) $\frac{24}{25}$ d) $\frac{7}{24}$	1
2	If $\sin y = x \cos (a + y)$, then $\frac{dy}{dx}$ is a) $\frac{\cos^2(a+y)}{\cos a}$ b) $-\frac{\cos^2(a+y)}{\cos a}$ c) $\frac{\sin^2 y}{\cos a}$ d) none of these	1
3	If $y^2(2 - x) = x^3$, then $\left(\frac{dy}{dx}\right)_{(1,1)}$ is equal to a) -2 b) 2 c) 3 d) $-\frac{3}{2}$	1
4	$\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to a) $\tan x + \cot x + C$ b) $(\tan x + \cot x)^2 + C$ c) $\tan x - \cot x + C$ d) $(\tan x - \cot x)^2 + C$	1
5	$\int 2^{x+2} dx$ is equal to a) $2^{x+2} + C$ b) $2^{x+2} \log 2 + C$ c) $\frac{2^{x+2}}{\log 2} + C$ d) $2 \cdot \frac{2^{x+2}}{\log 2} + C$	1
6	The value of b for which the function $f(x) = x + \cos x + b$ is decreasing over R is a) $b < 1$ b) No value of b exists c) $b \leq 1$ d) $b \geq 1$	1
7	A wire of length 20 cm is bent in the form of a sector of a circle. The maximum area that can be enclosed by the wire is: a) 10 sq cm b) 20 sq cm c) 25 sq cm d) 30 sq cm	1
8	The value of $\cot\left[\frac{1}{2}\sin^{-1}\frac{\sqrt{3}}{2}\right]$ is: a) 1 b) $\frac{1}{\sqrt{3}}$ c) $\sqrt{3}$ d) 0	1

9	$f(x) = x^x$ has a stationary point at a) $x = e$ b) $x = \frac{1}{e}$ c) $x = 1$ d) $x = \sqrt{e}$	1
10	Let $A = \{1, 2, 3\}$. Then the number of reflexive relations on A is: a) 3 b) 27 c) 64 d) none of these	1
11	The maximum number of equivalence relation on the set $A = \{1, 2, 3\}$ are a) 1 b) 2 c) 3 d) 5	1
12	$\int x^2 e^{x^3} dx$ is equal to a) $\frac{1}{3} e^{x^3} + C$ b) $\frac{1}{3} e^{x^4} + C$ c) $x = 1$ d) $x = \sqrt{e}$	1
13	The domain of the function defined by $f(x) = \sin^{-1} \sqrt{x-1}$ is a) $[1, 2]$ b) $[-1, 1]$ c) $[0, 1]$ d) none of these	1
14	If $3 \tan^{-1} x + \cot^{-1} x = \pi$, the x equals a) 1 b) 2 c) -1 d) 0	1
15	If $y = \sin^{-1} x$, then $(1-x^2) y_2$ is equal to: a) xy_1 b) xy c) xy_2 d) none of these	1
16	If $f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{for } -1 \leq x < 0 \\ 2x^2 + 3x - 2, & \text{for } 0 \leq x \leq 1 \end{cases}$ is continuous at $x = 0$, then k is equal to a) -4 b) -3 c) -2 d) none of these	1
17	For the set $A = \{1, 2, 3\}$, a relation R in the set A is defined as $R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$. Then the ordered pair to be added to R to make it the smallest equivalence relation is: a) (1, 3) b) (3, 1) c) (1, 2) d) (2, 1)	1
18	Let N be the set of natural numbers and the function $f: N \rightarrow N$ be defined by $f(n) = 2n + 3$ for every $n \in N$. Then f is: a) surjective b) injective c) bijective d) none of these	1
	DIRECTION: In question numbers 19 and 20, a statement of assertion (A) is followed by a statement of reason (R) . Choose the correct option: a) Both (A) and (R) are individually true and (R) is the correct explanation of (A) b) Both (A) and (R) are individually true but (R) is not the correct explanation of (A) c) (A) is true but (R) is false d) (A) is false but (R) is true	
19	Assertion (A): $\int \frac{1}{2\sqrt{x}} dx = \sqrt{x} + C$ Reason (R): $\int \cos x dx = \sin x + C$	1
20	Assertion (A): $f(x) = x^4$ is decreasing in the interval $(0, \infty)$ Reason (R): Any function $y = f(x)$ is decreasing if $\frac{dy}{dx} < 0$	1

	SECTION B Section B consists of 5 questions of 2 marks each Internal choice is provided in 2 questions. All questions are compulsory. In case of internal choices, attempt anyone.	
21	If $f(x) = \sqrt{\tan \sqrt{x}}$, then find $f'\left(\frac{\pi^2}{16}\right)$	2
22	A particle moves along the curve $3y = ax^3 + 1$ such that a point with x coordinate 1, y-coordinate is changing twice as fast as x-coordinate. Find the value of a. OR An edge of a variable cube is increasing at the rate of 5 cm per second. How fast is the volume increasing when the side is 15 cm?	2
23	Find: $\int \frac{x^2+2}{x+1} dx$	2
24	Solve for x: $\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$	2
25	Find the domain of $y = \sin^{-1}(x^2 - 4)$ OR Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also write its range	2
	SECTION C Section C consists of 6 questions of 3 marks each. Internal choice is provided in 2 questions. All questions are compulsory. In case of internal choices, attempt anyone.	
26	If $y = (x + \sqrt{1+x^2})^m$ then show that $(x^2 + 1) y_2 + x y_1 = m^2 y$	3
27	Find: $\int \frac{\sin^3 x}{\cos^6 x} dx$	3
28	Show that the relation R in a set $A = \{x: x \in \mathbb{Z}, 0 \leq x \leq 12\}$, given by $R = \{(a, b) : a - b \text{ is a multiple of } 4\}$, is an equivalence relation. Also find the equivalence class [1]. OR Check whether the function $f: \mathbb{R} \rightarrow \{x \in \mathbb{R} : 0 < x < 1\}$ defined by $f(x) = \frac{x}{1+ x }$, $x \in \mathbb{R}$ is Bijective or not.	3
29	Sand is pouring from a pipe at the rate of $12 \text{ cm}^3/\text{s}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?	3

30	Find the intervals in which the function f given by $f(x) = \frac{4 \sin x - 2x - x \cos x}{2 + \cos x}$, $0 < x < 2\pi$, is (i) increasing (ii) decreasing	3
31	Prove that: $\tan\left(\frac{1}{2} \sin^{-1} \frac{3}{4}\right) = \frac{4 - \sqrt{7}}{3}$ OR Prove that: $\tan \frac{1}{2} \left[\sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-y^2}{1+y^2} \right] = \frac{x+y}{1-xy}$, $ x < 1$, $y > 0$ and $xy < 1$	3
	SECTION D Section D consists of 4 questions of 5 marks each. Internal choice is provided in 2 questions. All questions are compulsory. In case of internal choices, attempt anyone.	
32	Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$. Also prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. OR Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.	5
33	Find: $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$ (2 ½) Find: $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ (2 ½)	5
34	Let N denotes the set of all natural numbers and R is a relation on $N \times N$ defined by $(a, b) R (c, d)$ if $ad(b+c) = bc(a+d)$. Show that R is an equivalence relation.	5
35	If $x = \cos t + \log \tan\left(\frac{t}{2}\right)$, $y = \sin t$, then find the values of $\frac{d^2y}{dt^2}$ at $t = \frac{\pi}{4}$ and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$ OR Differentiate $\tan^{-1} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}$ with respect to $\cos^{-1} x^2$.	5
	Section E This section comprises of 3 case study questions of 4 marks each. All questions are compulsory.	
36	Case study-1 The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x+9)(x+1)(x-3)$. If the roller-coaster crosses y-axis at a point (0, -1), answer the following:	

		
	Based on the above data, answer the following questions.	
(1)	Find the value of 'a'.	1 ½
(2)	Find $f''(x)$ at $x=1$.	2 ½
37	Case study-2 In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ $40,000 h^2$, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres. ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM 	
	Based on the above data answer the following questions	
(1)	Find the total cost C of digging the tank in terms of x .	1
(2)	Find $\frac{dC}{dx}$.	1
(3)	(a) Find the value of x for which cost C is minimum. OR (b) Check whether the cost function $C(x)$ expressed in terms of x is increasing or not, where $x > 0$.	2
38	Case study-3 An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.	



Based on the above data answer the following questions

(1)	How many relations are possible from B to G?	1
(2)	Among all the possible relations from B to G, how many functions can be formed from B to G?	1
(3)	Let $R : B \rightarrow B$ be defined by $R = \{(x, y) : x \text{ and } y \text{ are students of the same sex}\}$. Check if R is an equivalence relation. OR A function $f : B \rightarrow G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective. Justify your answer.	2