



BAL BHARATI PUBLIC SCHOOL

HALF YEARLY EXAMINATION

CLASS -XII

SUBJECT: MATHS

SET A

Time: 3 hours

Maximum Marks- 80

Date: 22.09.25

Prepared by: Mr. Sanjay Dua

General Instructions:

- The duration of the Examination is 3 hours.
- All the answers should be written on the answer sheet provided separately.
- The intended marks are indicated against each question.
- Answers should be written legibly.
- A neat presentation would be appreciated

This Question paper contains **38** questions. All questions are **compulsory**.

This Question paper is divided into **five** Sections - **A, B, C, D** and **E**.

In **Section A**, Questions no. **1** to **18** are **multiple choice questions (MCQs)** and Questions no. **19** and **20** are **Assertion-Reason** based questions of **1 mark** each.

In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks** each.

In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks** each.

In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks** each.

- Use of calculators is not allowed
- Use of unfair means during exam is strictly prohibited. A candidate found guilty of using unfair means will be disqualified and debarred from appearing for any of the further examinations for a year.

QNO	SECTION A (This section comprises of multiple-choice questions (MCQs) of 1 mark each) Select the correct option (Question 1 - Question 18):	MARKS
1.	For real numbers x and y , define xRy if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation R is (a) only reflexive (b) only symmetric (c) only transitive (d) equivalence	1
2.	The value of $\cos^{-1}(-1) - \sin^{-1}(1)$ is (a) π (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) $-\frac{3\pi}{2}$	1

3	If $\left \frac{A^{-1}}{2} \right = \frac{1}{k A }$, where A is a 3×3 matrix, then the value of k is (a) $\frac{1}{8}$ (b) 8 (c) 2 (d) $\frac{1}{2}$	1
4	If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $A^3 = 27$, then the value of 'α' is : (a) ± 1 (b) ± 2 (c) $\pm \sqrt{5}$ (d) $\pm \sqrt{7}$	1
5	If $\frac{d}{dx}[f(x)] = \log x$, then $f(x) =$ (a) $\frac{-1}{x} + c$ (b) $x(\log x - 1) + c$ (c) $x(\log x + x) + c$ (d) $\frac{1}{x} + c$	1
6	If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$ then the value of x is: (a) 3 (b) 0 (c) -1 (d) 1	1
7	Set A has 3 elements and set B has 4 elements, then the number of injective mapping that can be defined from A to B is: (a) 144 (b) 12 (c) 24 (d) 64	1
8	The function $f(x) = kx - \sin x$ is strictly increasing for : (a) $k > 1$ (b) $k < 1$ (c) $k > -1$ (d) $k < -1$	1
9	Evaluate: $\sin[\cot^{-1}\{\cos(\tan^{-1}1)\}]$ (a) $\frac{\sqrt{2}}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$	1
10	If A & B are two matrices such that $AB=A$ and $BA=B$ then $B^2 =$ (a) B (b) A (c) I (d) O	1
11	$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ is equal to (a) $2(\sin x + x \cos \alpha) + C$ (b) $2(\sin x - x \cos \alpha) + C$ (c) $2(\sin x + 2x \cos \alpha) + C$ (d) $2(\sin x - 2x \cos \alpha) + C$	1
12	If $\int \frac{\cos 8x + 1}{\tan 2x - \cot 2x} dx = a \cos 8x + c$, then $a =$ (a) $\frac{-1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $\frac{-1}{8}$	1
13	If $\frac{d(f(x))}{dx} = 2x + \frac{3}{x}$ and $f(1) = 1$, then $f(x)$ is : (a) $x^2 + 3 \log x + 1$ (b) $x^2 + 3 \log x$ (c) $2 - \frac{3}{x^2}$ (d) $x^2 + 3 \log x - 4$	1

14	$\int \frac{(x-3) e^x}{(x-1)^3}$ is equal to: (a) $\frac{2 e^x}{(x-1)^3} + C$ (b) $\frac{-2 e^x}{(x-1)^2} + C$ (c) $\frac{e^x}{(x-1)} + C$ (d) $\frac{e^x}{(x-1)^2} + C$	1
15	If A & B are square matrices of order 3×3 such that $ A =5$ & $ B = 3$, then $ 3AB =$ (a) 135 (b) 45 (c) 405 (d) None of these	1
16	If A & B are square matrices of same order, then matrix $(AB^T - BA^T)$ is a (a) Symmetric matrix (b) Skew-symmetric matrix (c) Null matrix (d) Unit matrix	1
17	The function 'f' given by $f(x) = x^3 - 3x^2 + 3x$, $x \in \mathbb{R}$ is increasing on: (a) $[1, \infty)$ (b) $(1, \infty)$ (c) $(-\infty, \infty)$ (d) $(-\infty, 1)$	1
18	If $y = 3x - 4 x $, then $\left(\frac{dy}{dx}\right)_{x=-2} =$ (a) 7 (b) -1 (c) -7 (d) 1	1
	ASSERTION-REASON BASED QUESTIONS (Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.) (A) Both (A) and (R) are true and (R) is the correct explanation of (A). (B) Both (A) and (R) are true but (R) is not the correct explanation of (A). (C) (A) is true but (R) is false. (D) (A) is false but (R) is true.	
19	Assertion (A): If A is a square matrix of order 3×3 and 2 is any scalar then the value of $2A = 8 A$ Reason (R): If 'k' is a scalar and A is a square matrix of order $n \times n$. Then $kA = k^{n-1} A$	1
20	Assertion: A relation $R = \{(1,1), (1,2), (2,2), (2,3), (3,3)\}$ is symmetric Reason: A relation R on the set A is said to be symmetric if $(a,b) \in R$, then $(b,a) \in R$	1
	SECTION B (This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)	
21	Solve: a. $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ b. $\tan^{-1}\left(\tan \frac{7\pi}{6}\right)$	2
22	Find $\int \frac{1}{\sqrt{x^2+2x+3}} dx$. OR Evaluate $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$	2

23	If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then find the value of k.	2
24	If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, then find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$	2
25	The radius of a cylinder is increasing at the rate of 2cm/sec and its altitude is decreasing at the rate of 3cm/sec. Find the rate of change of volume when radius is 3 cm and altitude is 5 cm	2
SECTION C (This section comprises of 6 short answer (SA) type questions of 3 marks each.)		
26	Find the intervals in which the function $f(x) = 8 + 36x + 3x^2 - 2x^3$ is increasing or decreasing	3
27	Discuss the continuity and differentiability of the function $f(x) = x + x - 1 $ in the interval $(-1, 2)$.	3
28	$\int \frac{\sin 2x \cos 2x}{\sqrt{9 - \cos^4(2x)}} dx$	3
29	Find: $\int \frac{x^3 - 1}{x^3 - x} dx$	3
30	A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall? OR A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.	3
31	Integrate: $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$	3
SECTION D (This section comprises of 4 long answer (LA) type questions of 5 marks each)		
32	Find the volume of the largest cylinder that can be inscribed in a sphere of radius r cm (OR) A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is $8 m^3$. If building of tank costs Rs 70 per square metre for the base and Rs 45 square metre for the sides, what is the cost of least expensive tank?	5
33	The sum of three numbers is 6. If we multiply the 3 rd number by 3 and add the second number to it we get 11. By adding 1 st and 3 rd numbers, we get double the 2 nd number. Represent it algebraically and find the numbers using matrix method.	5
34	Differentiate the following function with respect to x: $y = (\sin x)^x + \sin^{-1}\sqrt{x}$	5

35	Let $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$. Show that $R = \{(a,b) : a,b \in A, a - b \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also, write the equivalence class [2].	5
	SECTION- E (This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts.	
36	Case Study-1 Raji visited the exhibition with her family. The exhibition had a huge swing, which attracted many children. Raji found that the swing traced a parabola path given by $y = x^2$ Answer the following questions using the above information 1) Check whether the function $f: R \rightarrow R$ be defined by $f(x) = x^2$ is Injective, Surjective, Bijective or neither of both 2) Check whether the function $f: N \rightarrow N$ be defined by $f(x) = x^2$ is Injective, Surjective, Bijective or neither of both 3) Check whether the function $f: \{1,2,3 \dots \dots\} \rightarrow \{1,4,9 \dots \dots\}$ be defined by $f(x) = x^2$ is Injective, Surjective, Bijective or neither of both 4) Let $f: N \rightarrow N$ be defined by $f(x) = x^2$. Find the range of the function	1 1 1 1
37	Dr. Ritam residing in Delhi went to see an apartment of 3 BHK in Noida. The window of the house was in the form of a rectangle surmounted by a semicircular opening having a perimeter of the window 10 m.  (i) If x any y represents the length and breadth of the rectangular region, then find the relation between the variables. (ii) Express area of the window as a function of x. (iii) Find value of x so that the area of whole window is maximum.	1 1 2

38	A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.  A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of $2 \text{ cm}^3/\text{s}$. The semi-vertical angle of the conical tank is 45°. On the basis of given information, answer the following questions. (i) Find the volume of water in the tank in terms of its radius r. (ii) Find rate of change of radius at an instant when $r = 2\sqrt{2} \text{ cm}$. (iii) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2} \text{ cm}$. (iv) Find the rate of change of height 'h' at an instant when slant height is 4 cm.	1 1 1 1

END OF PAPER.