Subject (Mathematics 041)
Half-Yearly Examination
June,2025

Time allowed:3 hours | Maximum Marks: 80

General Instructions

1. This Question paper contains **38 questions. All questions are compulsory.**
2. Question paper is divided into **FIVE Sections - Section A, B, C, D and E.**
3. In **Section A** Question Number **1 to 18** are Multiple Choice Questions **(MCQs)** and Question Number **19 & 20** are **Assertion-Reason** based questions of **1 mark** each.
4. In **Section B** - Question Number 21 to 25 are Very Short Answer (VSA) type questions, carrying **2 marks** each.
5. In **Section C** - Question Number 26 to 31 are Short Answer (SA) type questions, carrying **3 marks** each.
6. In **Section D** - Question Number 32 to 35 are Long Answer (LA) type questions, carrying **5 marks** each.
7. In **Section E** - Question Number 36 to 38 are Case Study-Based questions, carrying **4 marks** each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section - B, 2 questions in Section - C, 2 questions in Section - D and 3 questions in Section - E.
9. Use of a calculator is NOT allowed.

Section A

Questions 1 to 20 carry **1** mark each

1. If $A=\begin{bmatrix}0 & -3 & x\\ 3 & 0 & -5\\ -x & 5 & 0\end{bmatrix}$ then $|A|$ is equal to 1

(a) 15 (b) 0 (c) 1 (d) ± 15

2. Let $A=\{2,4\}$. Then the total number of reflexive relations on A is 1

(a) 2 (b) 4 (c) 0 (d) 8

3. If $tan^{-1}x = y$ then 1

(a) $-1 < y < 1$ (b) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(c) $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (d) $y \in \{-\frac{\pi}{2}, \frac{\pi}{2}\}$

4. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is 1

(a) $\frac{x}{1+x^4}$ (b) $\frac{-2x}{1+x^4}$ (c) $\frac{2x}{1+x^4}$ (d) $\frac{1}{1+x^4}$

5. Let f: $R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set 1

of all non-negative real numbers. Then, f is

(a) one-one (b) onto (c) bijective (d) neither one-one nor onto

6. If the sum of all the elements of a 3 × 3 scalar matrix is 9, then the product of all its elements is 1

(a) 0 (b) 9 (c) 27 (d) 729

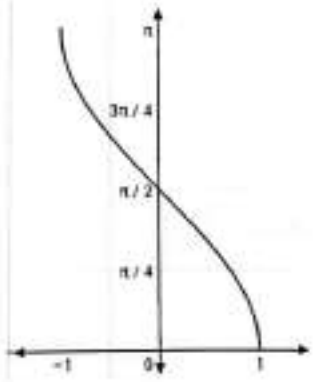
7. If a matrix has 36 elements, the number of possible orders it can have, is 1

(a) 13 (b) 3 (c) 9 (d) 5

8. A function f: $R_+ \to$ R (where R+ is the set of all non-negative real numbers) defined by f(x) = 4x + 3 is 1

(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

9. The graph drawn below depicts 1



(a) $y = \sin^{-1} x$ (b) $y = \cos^{-1} x$ (c) $y = \sec^{-1} x$ (d) $y = \cot^{-1} x$

10. The function $f: R \to Z$ defined by $f(x) = [x]$; where $[x]$ denotes the greatest integer function, is 1

(a) Continuous at $x = 2.5$ but not differentiable at $x = 2.5$

(b) Not Continuous at $x = 2.5$ but differentiable at $x = 2.5$

(c) Not Continuous at $x = 2.5$ and not differentiable at $x = 2.5$

(d) Continuous as well as differentiable at 2.5

11. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is 1

(a) 7 (b) 6 (c) 8 (d) 18

12. If $y = \sin^2(x^3)$, then $\frac{dy}{dx}$ is equal to 1

(a) $3\sin x^3 \cos x^3$ (b) $3x^3 \sin x^3 \cos x^3$

(c) $6x^2 \sin x^3 \cos x^3$ (d) $2x^2 \sin^2 x^3$

13. If $y = \log(\sin e^x)$ then $\frac{dy}{dx}$ is equal to 1

(a) $\cot e^x$ (b) $\text{cosec}\, e^x$

(c) $e^x \cot e^x$ (d) $e^x \text{cosec}\, e^x$

14. If $y = 5\cos x - 3\sin x$ then $\frac{d^2y}{dx^2}$ is equal to 1

(a) y (b) $-y$ (c) $25y$ (d) $9y$

15. The function $f(x) = x^3 + 3x$ is increasing in the interval 1

(a) $(-\infty, 0)$ (b) $(0, \infty)$ (c) R (d) (0,1)

16. If $f(x) = a(x - \cos x)$ is strictly decreasing in R, then the value of a belongs to 1

a) $(-\infty, 0)$ (b) $(0, \infty)$ (c) $\{0\}$ (d) $(-\infty, \infty)$

17. The value of the $\cos^{-1}(\cos\frac{13\pi}{6})$ is 1

(a) $\frac{13\pi}{6}$ (b) $-\frac{\pi}{6}$ (c) $\frac{\pi}{6}$ (d) none of these

18. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is 1

(a) 10π (b) 11π (c) 12π (d) 8π

Directions for Assertion-Reason Based Questions

Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true, and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A)**: Let Z be the set of integers. A function $f: Z \to Z$ defined as $f(x) = 3x - 5, \forall x \epsilon Z$ is a bijective function. 1

Reason (R): A function is bijective if it is both surjective and injective.

20. **Assertion (A)**: If the side of a square is increasing at the rate of 0·2 cm/s, then the rate of increase of its perimeter is 0·8 cm/s. 1

Reason (R): Perimeter of a square = 4 (side).

Section B

Questions 21 to 25 carry **2** marks each.

21. Find the value of 2

$\tan^{-1}(1) + \sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$

OR

Find the domain of the function $y = \cos^{-1}(x^2 - 4)$.

22. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate. 2

23. Find the equation of the line joining (3,1) and (9,3) using determinants. 2

OR

If the area of triangle is 35 $units^2$ with the vertices $(2, -6), (5,4)$ and $(k,4)$, then find the value of k.

24. Show that the modulus function $f: R \to R$, given by $f(x) = |x|$, is neither one-one nor onto. 2

25. Differentiate $e^{\sin^2 x}$ with respect to $log|\sin^2 x|$. 2

Section C

Questions 26 to 31 carry **3** marks each.

26. Find the intervals in which the function $f(x) = \frac{\log x}{x}$ is strictly increasing or strictly decreasing. 3

OR

A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

27. If $y = Ae^{mx} + Be^{nx}$, show that: 3

$$\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$$

28. Find the value of a and b so that the function f(x) defined as: 3

$$f(x) = \begin{cases} \frac{x-2}{|x-2|} + a, & if\ x < 2 \\ a + b, & if\ x = 2 \\ \frac{x-2}{|x-2|} + b, & if\ x > 2 \end{cases}$$

is a continuous function.

29. If $(\cos x)^y = (\cos y)^x$, find $\frac{dx}{dy}$. 3

30. Simplify: 3

$$\cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right], \quad if\ \frac{1}{2} \le x \le 1$$

OR

Find the domain and principal range of the function $y = \cos^{-1}(x^2 - 4)$.

31. Find the values of *a* and *b* such that the function defined by. 3

$$f(x) = \begin{cases} 5, & if\ x \le 2 \\ ax + b & if\ 2 < x < 10 \\ 21, & if\ x \ge 10 \end{cases}$$

is a continuous function.

Section D

*Questions 16 to 17 carry **5** marks each*

32. If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that: 5

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$ is a constant independent of *a* and *b*.

33. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of linear equations: 5

$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

OR

Find the product of matrices AB, $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$ and use the result to solve the following system of linear equations:

$$x - 2y - 3z = 1$$
$$-2x + 4y + 5z = -1$$
$$-3x + 7y + 9z = -4$$

34. Find $\frac{dy}{dx}$ when $y = \left(x + \frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$. 5

35. Show that function $f: R \to (-1,1)$ defined by $f(x) = \frac{x}{1+|x|}$, $x \epsilon R$ is one-one and onto function. 5

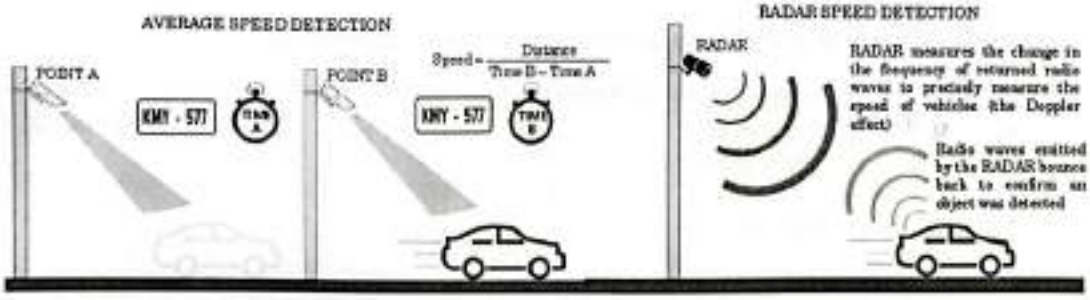
OR

Show that function $f: R \to R$ defined by $f(x) = \frac{x}{1+x^2}$, $x \epsilon R$ is neither one-one nor onto function.

Section E

Questions 36 to 38 carry 4 marks each

36. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.



A camera is installed on a pole at a height of 5 m. It detects a car travelling away from the pole at the speed of $20\ m/s$. At any point, $x\ m$ away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.

On the basis of the above information, answer the following questions:

(i) Express θ in terms of the height of the camera installed on the pole and x. 1

1

6

$-2 \times 1 + 4 \times 1 + 5x - 4 \quad |x - 1 + -6 \quad -3$

$-2 + 4 - 20 =$ $\quad -1 - 6$

$-7 + 6$

(ii) Find $\frac{d\theta}{dx}$.

(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. 2

OR

(b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car.

37. Amit, Biraj and Chirag were given the task of creating a square matrix of order 2. Below are the matrices created by them. A, B, and C are the matrices created by Amit, Biraj and Chirag, respectively.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

If a = 4 and b = −2, based on the above information answer the following:

(i) Find: A+$(B+C)$. 1

(ii) Find: $(bA)^T$. 1

(iii) (a) Find: $(AC - BA)^T$. 2

OR

(b) Find: $aA + bB - (a + b)C$

38. A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set $A = \{1,2,3\}$:

$R_1 = \{(2,3),(3,2)\}$

$R_2 = \{(1,2),(1,3),(3,2)\}$

$R_3 = \{(1,2),(2,1),(1,1)\}$

$R_4 = \{(1,1),(1,2),(3,3),(2,2)\}$

$R_5 = \{(1,1),(1,2),(3,3),(2,2),(2,1),(2,3),(3,2)\}$

The students are asked to answer the following questions about the above relation:

(i) Identify the relation, which is reflexive, transitive but not symmetric. 1

(ii) Identify the relation which is reflexive and symmetric but not transitive. 1

(iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive. 2

OR

(b) What pairs should be added to the relation R_2 to make it an equivalence relation?

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