# Answer key

| | |
|---|---|
| 1 | (d) 25 |
| 2 | (d) 1000 |
| 3 | (b)1/2 |
| 4 | (a)516A |
| 5 | (b)zero square matrix |
| 6 | (b) |
| 7 | (d) |
| 8 | (b) |
| 9 | (c) (A) is true and (R ) is false |
| 10 | (a) Both( A) and (R ) are true and (R ) is the correct explanation of (A) |
| 11 | Given: B is skew symmetric matrix<br>$B^t = -B$<br>To check whether $ABA^t$ is sy. or sk sy<br>Let $P = ABA^t$<br>Take transpose<br>$P^t = (ABA^t)^t$<br>$= (A^t)^t B^t A^t$ $(\because (AB)^t = B^t A^t)$<br>$= AB^t A^t$ $(\because (A^t)^t = A)$<br>$= -ABA^t$ $(\because B^t = -B)$<br>$= -P$<br>$\Rightarrow P$ is skew symmetric.<br>Or<br>$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ ; $A^t = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$<br>$A = \frac{1}{2}(A + A^t) + \frac{1}{2}(A - A^t)$<br>Let $P = \frac{1}{2}(A + A^t)$ and $Q = \frac{1}{2}(A - A^t)$<br>$A = P + Q$ |

| | |
|---|---|
| | $P = \frac{1}{2}(A + A^t)$<br>$P = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$<br>$P = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$<br>$P^t = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$<br>$= P$<br>P is a symmetric matrix.<br>$Q = \frac{1}{2}(A - A^t)$<br>$A = P + Q$<br>$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$<br>$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ |
| 12 | |

$$\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$$

$$A = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$$

13

**Proof:**

Let $A^\top = A$ and $B^\top = B$.

1. **Show $AB + BA$ is symmetric:**

$$\begin{aligned}(AB+BA)^\top &= (AB)^\top + (BA)^\top \\ &= B^\top A^\top + A^\top B^\top \\ &= BA + AB = AB + BA.\end{aligned}$$

2. **Show $AB - BA$ is skew-symmetric:**

$$\begin{aligned}(AB-BA)^\top &= (AB)^\top - (BA)^\top \\ &= B^\top A^\top - A^\top B^\top \\ &= BA - AB = -(AB - BA).\end{aligned}$$



**Conclusion:**

- $AB + BA$ is symmetric.
- $AB - BA$ is skew-symmetric.

OR

| | |
|---|---|
| | $B' = \begin{bmatrix} 4 & 7 & 2 \\ 0 & 1 & 2 \\ 2 & 4 & 6 \end{bmatrix}$ & $A' = \begin{bmatrix} 3 & 1 & 7 \\ 9 & 8 & 5 \\ 0 & -2 & 4 \end{bmatrix}$ <br> $\therefore B'A' = \begin{bmatrix} 4 & 7 & 2 \\ 0 & 1 & 2 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 3 & 1 & 7 \\ 9 & 8 & 5 \\ 0 & -2 & 4 \end{bmatrix}$ <br> $= \begin{bmatrix} 12+63 & 4+56-4 & 28+35+8 \\ 9 & 8-4 & 5+8 \\ 6+36 & 2+32-12 & 14+20+24 \end{bmatrix}$ <br> $= \begin{bmatrix} 75 & 56 & 71 \\ 9 & 4 & 13 \\ 42 & 22 & 58 \end{bmatrix}$ |
| 14 | $$X \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}.$$ <br> Let $X$ be a $2 \times 2$ matrix: <br> $$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$ <br> Multiplying out on the left yields: <br> $$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} a\cdot 1 + b \cdot 4 & a \cdot 2 + b \cdot 5 & a \cdot 3 + b \cdot 6 \\ c \cdot 1 + d \cdot 4 & c \cdot 2 + d \cdot 5 & c \cdot 3 + d \cdot 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}.$$ <br> Equating corresponding entries, we get two separate systems: <br> **First row system (for $a, b$):** <br> $$\begin{cases} a + 4b = -7, \\ 2a + 5b = -8, \\ 3a + 6b = -9. \end{cases}$$ |

Second row system (for $c, d$):

$$\begin{cases} c + 4d = 2, \\ 2c + 5d = 4, \\ 3c + 6d = 6. \end{cases}$$

Solving these:

- From the first system:

$$a + 4b = -7 \quad (1), \quad 2a + 5b = -8 \quad (2).$$

Multiply (1) by 2: $2a + 8b = -14$. Subtract (2):

$$(2a + 8b) - (2a + 5b) = -14 - (-8) \implies 3b = -6 \implies b = -2.$$

Substitute back into (1): $a + 4(-2) = -7 \implies a - 8 = -7 \implies a = 1.$

- From the second system:

$$c + 4d = 2 \quad (3), \quad 2c + 5d = 4 \quad (4).$$

Multiply (3) by 2: $2c + 8d = 4$. Subtract (4):

$$(2c + 8d) - (2c + 5d) = 4 - 4 \implies 3d = 0 \implies d = 0.$$

Substitute back into (3): $c + 4 \cdot 0 = 2 \implies c = 2.$

Hence:

$$a = 1, \quad b = -2, \quad c = 2, \quad d = 0.$$

So the matrix $X$ is:

$$\boxed{X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}.}$$

or

Given $\cos 2\theta = 0$

$$\text{Let A} = \begin{bmatrix} \theta & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}^2$$

$$\text{A} = \left[\begin{vmatrix} \sin\theta & 0 \\ 0 & \cos\theta \end{vmatrix} - \cos \begin{vmatrix} \cos\theta & 0 \\ \sin\theta & \cos\theta \end{vmatrix} + \sin\theta \begin{vmatrix} \cos\theta & \sin\theta \\ \sin\theta & 0 \end{vmatrix}\right]^2$$

$$\text{A} = \left[0 - \cos\theta\,(\cos\theta - 0) + \sin\theta\,(0 - \sin^2\theta)\right]^2$$

$$\text{A} = [-\cos 3\theta - \sin\theta]\,2$$

$$\text{A} = \left[\cos^3\theta + \sin^3\theta\right]^2 \quad .......(1)$$

| | |
|---|---|
| | $\cos 2\theta = 0$<br>$\Rightarrow 2\theta = \frac{\pi}{2}$<br>$\Rightarrow \theta = \frac{\pi}{4}$<br>Substituting $\theta = \frac{\pi}{4}$ in equation (1) we get<br>$A = \left[\frac{\cos^3 \pi}{4} + \frac{\sin^3 \pi}{4}\right]^2$<br>$= \left[\left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^3\right]^2$<br>$= \left[2 \times \left(\frac{1}{\sqrt{2}}\right)^3\right]^2$<br>$= \left[2 \times \frac{1}{2\sqrt{2}}\right]^2$<br>$= \left[\frac{1}{\sqrt{2}}\right]^2$<br>$A = \frac{1}{2}$ |
| 15 | Area = 16 sq unit<br>Equation of PQ is x-3y=0 |
| 16(i)<br><br>(ii)<br><br>(iii)a | 3x-2y=200<br>x-y=50<br>$\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \end{bmatrix}$<br>$\begin{bmatrix} 13 & 5 \\ 5 & 2 \end{bmatrix}$<br>Or<br>PQ= $\begin{bmatrix} 1 & 0 \\ 4 & 0 \end{bmatrix}$,QP=$\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$ |
| 17 | (i) |

| | |
|---|---|
| | Let $x, y, z$ be the daily production (in tons) of the first, second and third products respectively.<br><br>From the problem:<br><br>• total production $= 45$ gives: $x + y + z = 45$.<br>• "third exceeds first by 8" gives: $z - x = 8$ (or $-x + 0y + z = 8$).<br>• "first + third is twice the second" gives: $x + z = 2y$ (or $x - 2y + z = 0$).<br><br>So the system is<br><br>$$\begin{cases} x + y + z = 45, \\ -x + 0y + z = 8, \\ x - 2y + z = 0. \end{cases}$$<br><br>Matrix form: $AX = B$ where<br><br>$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}.$$<br><br>(ii) x=11 tons, y=15tons& z=19 tons |
| 18 | PROVING |
| 19(A)<br><br>19(B) | X=-21,Y=98,Z=64<br><br>X=2,Y=-1,Z=5 |