Ansneekey

Test 02 XI

Section A

Ans 1) d

Ans 2) d

Ans 3) b

Ans 4) c

Ans 5) a

Ans 6) a

Ans 7) a

Ans 8) a

Ans 9) c

Ans 10) d

Section B

Q-11(A) $(\sin x + \sin 2x) + (\sin 4x + \sin 5x)$

$2\sin\frac{3x}{2}\cos\frac{x}{2} + 2\sin\frac{9x}{2}\cos\frac{x}{2}$

$\left(\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right)$

$2\cos\frac{x}{2}\left(\sin\frac{3x}{2} + \sin\frac{9x}{2}\right)$

$2\cos\frac{x}{2}\left(2\sin 3x \cos\frac{3x}{2}\right)$

$4\sin 3x \cos\frac{x}{2}\cos\frac{3x}{2}$

or

Q-11(B) $\tan\frac{\pi}{8}$ Let $2A = \frac{\pi}{4} \Rightarrow A = \frac{\pi}{8}$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$\tan\frac{\pi}{4} = \frac{2\tan A}{1-\tan^2 A}$

$1-\tan^2 A = 2\tan A$

$\tan^2 A + 2\tan A - 1 = 0$

$u^2 + 2u - 1 = 0$

$u = \frac{-2 \pm \sqrt{4+4}}{2}$

$$u = \frac{-2 \pm \sqrt{8}}{2}$$

$$= \frac{-2 \pm 2\sqrt{2}}{2}$$

$$u = -1 \pm \sqrt{2}$$

but $u = \tan A = \tan\frac{\pi}{8}$

$\Rightarrow \cancel{\tan}\ \pi/8 \in 1^{st}$ quad.

$\therefore$ have positive valu

$\therefore\ u = \sqrt{2} - 1$

$\tan\frac{\pi}{8} = \sqrt{2} - 1.$

Q-12

$$x+iy = \left(\frac{a+ib}{c+id}\right)^{1/2} \quad \text{——— (1)}$$

Take conjugate

$$\overline{x+iy} = \overline{\left(\frac{a+ib}{c+id}\right)^{1/2}}$$

$$x-iy = \overline{\left(\frac{a+ib}{c+id}\right)}^{1/2} \qquad (\bar{z})^n = \overline{(z^n)}$$

$$x - iy = \left(\frac{a-ib}{c-id}\right)^{1/2} \quad \left(\because \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}\right) \quad \text{---(2)}$$

Mult (1) & (2)

$$x^2+y^2 = \left(\frac{a+ib}{c+id} \times \frac{a-ib}{c-id}\right)^{1/2}$$

$$x^2+y^2 = \left(\frac{a^2+b^2}{c^2+d^2}\right)^{1/2}$$

Sq. both sides

$$(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$$

13(A) $\left|\frac{1+2i-3}{1-2i-3}\right|$

$= \left|\frac{-2+2i}{-2-2i}\right|$

$= \left|\frac{(-2+2i)^2}{(-2)^2+(2)^2}\right|$

$= \left|\frac{4+4i^2-8i}{4+4}\right|$

$= \left|\frac{-\cancel{8}i}{\cancel{8}}\right|$

$= |-i|$

$= \sqrt{(0)^2+(-1)^2} = 1$

13(B) $\left(\frac{1}{1-i}+\frac{2}{1+i}\right)\left(\frac{3-i}{5+i}\right)$

$\left(\frac{1+i+2-2i}{1+1}\right)\left(\frac{3-i}{5+i}\right)$

$\left(\frac{3-i}{2}\right)\left(\frac{3-i}{5+i}\right) = \frac{9-1-6i}{2(5+i)}$

$$= \frac{8-6i}{2(5+i)}$$

$$= \frac{4-3i}{5+i} \times \frac{5-i}{5-i}$$

$$= \frac{20-4i-15i-3}{25+1}$$

$$= \frac{17}{26} - \frac{19i}{26}$$

Hence standard faem is $\frac{17}{26} - \frac{19i}{28}$

Ans 14

$$\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|^2$$

$$= \left(\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right)\overline{\left(\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right)}$$

$$= \frac{\beta-\alpha}{1-\overline{\alpha}\beta} \times \frac{\overline{\beta}-\overline{\alpha}}{1-\overline{\alpha}\beta}$$

$$= \frac{\beta\overline{\beta}-\overline{\alpha}\beta-\alpha\overline{\beta}+\alpha\overline{\alpha}}{1-\alpha\overline{\beta}-\overline{\alpha}\beta+\alpha\overline{\alpha}.\beta\overline{\beta}}$$

$$= \frac{|\beta|^2-\overline{\alpha}\beta-\alpha\overline{\beta}+|\alpha|^2}{1-\alpha\overline{\beta}-\overline{\alpha}\beta+|\alpha|^2.|\beta|^2})$$

Given $|\beta| = 1$,

$$= \frac{1+|\alpha|^2-\overline{\alpha}\beta-\alpha\overline{\beta}}{1+|\alpha|^2-\overline{\alpha}\beta-\alpha\overline{\beta}}$$

$= 1$

$$\therefore \left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right| = 1 \text{ or } \left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right| = 1$$

15(A) LHS

$$\sin 20^\circ \sin 40^\circ \sin 80^\circ$$

$$\frac{1}{2}\left(2\sin 20^\circ \sin 40^\circ\right)\sin 80^\circ$$

$$\frac{1}{2}\left(\cos 20 - \cos 60^\circ\right)\sin 80^\circ$$

$$\left(\because 2\sin A \sin B = \cos(A-B) - \cos(A+B)\right.$$

$$= \frac{1}{2}\cos 20^\circ \sin 80^\circ - \frac{1}{2}\times\frac{1}{2}\sin 80^\circ$$

$$= \frac{1}{4}\left(2\cos 20^\circ \sin 80^\circ\right) - \frac{\sin 80^\circ}{4}$$

$$= \frac{1}{4}\left(\sin 100 - \sin(-60^\circ)\right) - \frac{\sin 80^\circ}{4}$$

$$= \frac{\sin(180 - 80^\circ)}{4} + \frac{\sin 60^\circ}{4} - \frac{\sin 80^\circ}{4}$$

$$= \cancel{\frac{\sin 80^\circ}{4}} + \frac{\sqrt{3}}{2} \times \frac{1}{4} - \cancel{\frac{\sin 80^\circ}{4}}$$

$$= \frac{\sqrt{3}}{8} = RHS$$

Hence prove.

(using $2\cos A \sin B = \sin(A+B) - \sin(A-B)$)

15(B) Given: $\sin(x + 3\alpha) = 3\sin(\alpha - x)$

$\sin x \cos 3\alpha + \cos x \sin 3\alpha = 3(\sin\alpha \cos x - \cos\alpha \sin x)$

$\sin x \cos 3\alpha + \cos x \sin 3\alpha = 3\cos x \sin\alpha - 3\cos\alpha \sin x$

$\sin x(\cos 3\alpha + 3\cos\alpha) = \cos x(3\sin\alpha - \sin 3\alpha)$

$\sin x(4\cos^3\alpha - \cancel{3\cos\alpha} + \cancel{3\cos\alpha})$

$= \cos x(\cancel{3\sin\alpha} - \cancel{3\sin\alpha} + 4\sin^3\alpha)$

$(\because \cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

$\frac{\sin x}{\cos x} = \frac{\cancel{4}\sin^3\alpha}{\cancel{4}\cos^3\alpha}$

$\tan x = \tan^3\alpha$.

Q-16

(i)

$3a - 6 = 6b$ & $2b = -6 - a$

$3a - 6b = 6$ $\quad a + 2b = -6$ —(2)

$a - 2b = 2$ —(1)

Solve (1) & (2)

$a - 2b = 2$

$a + 2b = -6$

$2a = -4$

$a = -2$ put in (2)

$b = \frac{-6+2}{2} = -2$

(ii) $$2a+2b=0$$
$$-a+b=4 \quad —(1)$$

$$\therefore \begin{array}{r} 2a+2b=0 \\ -2a+2b=8 \\ \hline 4b=8 \end{array}$$
$$b=2 \quad \text{put in(1)}$$
$$-a=2$$
$$a=-2$$

(iii)(a) $$\left(\frac{1-i}{1+i}\right)^{102}=a+ib$$

$$\left(\frac{(1-i)^2}{1+1}\right)^{102}=a+ib$$

$$\left(\frac{\cancel{1}-\cancel{1}-2i}{2}\right)^{102}=a+ib$$

$$(-i)^{102}=a+ib$$
$$(-1)^{102}(i^2)^{51}=a+ib$$
$$-1=a+ib$$
$$a=-1,\ b=0$$

(iii) (b) $\frac{(1-i)^2}{2-i} = x+iy$

$\frac{(\cancel{1}-\cancel{1}-2i)(2+i)}{4+1} = x+iy$

$\frac{-2i(2+i)}{5} = x+iy$

$\frac{-4i-2i^2}{5} = x+iy$

$\frac{2}{5} - \frac{4}{5}i = x+iy$

$x = \frac{2}{5}$, $y = \frac{-4}{5}$

$\therefore x+y = -\frac{2}{5}$

Q-17

(i) ~~[illegible]~~

$\theta = \angle B = 71\frac{1}{2}^\circ$

$= \left(\frac{143}{2}\right)^\circ = \frac{143\pi}{2\times180} = \frac{143\pi}{360}$

(ii) In one minute 30 rounds

1 sec $\frac{30}{60}$ rounds

$= \frac{1}{2}$ round

In 1 round, it generates 2π angle

$\therefore$ In $\frac{1}{2}$ round, it generates π angle.

iii (a) $\theta = \angle C = 108\frac{1}{2}^\circ$

$= \left(\frac{217}{2}\right)^\circ$

$= \frac{217}{2} \times \frac{\pi}{180}$ radian

$= \frac{217\pi}{360}$ radian

$\therefore\ l = \theta r$

$= \dfrac{217\pi}{\cancel{360}_{\cancel{90}\,45}} \times \cancel{88}^{\cancel{22}\,11}$

$= \dfrac{2387\pi}{45}$ m.

OR

$\theta = 90^\circ = \dfrac{\pi}{2}$

Area $= \dfrac{\pi r^2}{4} = \dfrac{\pi}{4}(88)^2$

length of arc $= \theta r$

$= \dfrac{\pi}{2} \times 88$

Ratio $= \dfrac{\frac{\cancel{\pi}}{\cancel{4}_2}(88)^{\cancel{2}}}{\frac{\cancel{\pi}}{\cancel{2}}\cancel{(88)}}$

$= \dfrac{88}{2} = \dfrac{44}{1}$

Ratio 44:1.

Ans -18(A)

$$\text{LHS} = \cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) + \cos^2\left(x-\frac{\pi}{3}\right)$$

$$= \left(\frac{1+\cos 2x}{2}\right) + \left[\frac{1+\cos 2\left(x+\frac{\pi}{3}\right)}{2}\right] + \left[\frac{1+\cos 2\left(x-\frac{\pi}{3}\right)}{2}\right]$$

$$\left[\because \cos 2x = 2\cos^2 x - 1 \quad \text{or} \quad \cos^2 x = \frac{1+\cos 2x}{2}\right]$$

$$= \frac{1}{2}\left[1+\cos 2x + 1 + \cos 2\left(x+\frac{\pi}{3}\right)\right] + 1 + \cos 2\left(x-\frac{\pi}{3}\right)$$

$$= \frac{1}{2}\left[3+\cos 2x + \cos 2\left(x+\frac{\pi}{3}\right) + \cos 2\left(x-\frac{\pi}{3}\right)\right]$$

Using $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

Replace x by $\left(2x+\frac{2\pi}{3}\right)$ and y by $2x-\frac{2\pi}{3}$

$$= \frac{1}{2}\left[3+\cos 2x + 2\cos\left(\frac{2x+\frac{2\pi}{3}+2x-\frac{2\pi}{3}}{2}\right)\cos\left(\frac{2x+\frac{2\pi}{3}-\left(2x-\frac{2\pi}{3}\right)}{2}\right)\right]$$

$$= \frac{1}{2}\left[3 + \cos 2x + \cos\left(\frac{4x}{2}\right)\cos\left(\frac{\frac{4\pi}{3}}{2}\right)\right]$$

$$= \frac{1}{2}\left[3 + \cos 2x + 2\cos 2x \cos\left(\frac{2\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3 + \cos 2x + 2\cos 2x \cos\left(\pi - \frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3 + \cos 2x + 2\cos 2x \left(-\cos\left(\frac{\pi}{3}\right)\right)\right]$$

$$\left[\text{using } \cos(\pi - \theta) = -\cos\theta\right]$$

$$= \frac{1}{2}\left[3 + \cos 2x + 2\cos 2x\left(-\frac{1}{2}\right)\right]$$

$$= \frac{1}{2}[3 + \cos 2x - \cos 2x]$$

$$= \frac{3}{2} = \text{RHS}$$

Ans 18(B)

L.H.S. $= \cos 6x = \cos 3(2x)$ [$\because \cos 3x = 4\cos^3 x - 3\cos x$]

$= 4\cos^3 2x - 3\cos 2x$

$= \cos 2x\,(4\cos^2 2x - 3)$

$= \cos 2x\,[4(\cos 2x)^2 - 3]$ [$\because \cos 2x = 2\cos^2 x - 1$]

$= \cos 2x\,[4(2\cos^2 x - 1)^2 - 3]$

$= \cos 2x\,[4(4\cos^4 x + 1 - 4\cos^2 x) - 3]$

$= \cos 2x\,(16\cos^4 x + 4 - 16\cos^2 x - 3)$

$= (2\cos^2 x - 1)\,(16\cos^4 x + 4 - 16\cos^2 x - 3)$

$= 32\cos^6 x + 8\cos^2 x - 32\cos^4 x - 6\cos^2 x - 16\cos^4 x - 4 + 16\cos^2 x + 3$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

= R.H.S. Hence Proved.

Ans 19

Given tanx = 3/4

$$\sec^2 x = 1 + \tan^2 x = 1 + \frac{9}{16} = \frac{25}{16}$$

$$\sec x = \pm\frac{5}{4} \text{ or } \cos x = -\frac{4}{5} \quad \because \pi < x < \frac{3\pi}{2}$$

We have $2\sin^2\frac{x}{2} = 1 - \cos x = 1 - \left(-\frac{4}{5}\right) = \frac{9}{5}$

$$\Rightarrow \quad \sin^2\frac{x}{2} = \frac{9}{10}$$

$$\Rightarrow \quad \sin\frac{x}{2} = \frac{3}{\sqrt{10}} \quad \because \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

Again $2\cos^2\frac{x}{2} = 1 + \cos x = 1 + \left(-\frac{4}{5}\right) = \frac{1}{5}$

$$\cos^2\frac{x}{2} = \frac{1}{10}$$

or $\quad \cos\frac{x}{2} = -\frac{1}{\sqrt{10}} \quad \because \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\frac{3}{\sqrt{10}}}{-\frac{1}{\sqrt{10}}} = -3$$