## **ANSWER KEY**

1	а
2	b
3	С
4	а
5	b
6	С
7	а
8	b
9	а
10	h

11

The first part,  $\sum 3$ , represents the sum of the constant 3 for 11 terms.

This sum is calculated as  $3 \times 11 = 33$ .

The second part,  $\sum 2^r$ , is a geometric series.

The first term is  $a = 2^1 = 2$ .

The common ratio is r=2.

The number of terms is n = 11.

The formula for the sum of the first n terms of a geometric series is

$$S_n = \frac{a(r^n - 1)}{r - 1}.$$

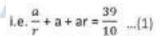
Substituting the values, the sum is  $\frac{2(2^{11}-1)}{2-1}$ .

This simplifies to  $2(2^{11} - 1)$ .

Hence required sum is 33+2(2<sup>11</sup>-1)= 31+ 2 <sup>12</sup> **Or** 

It is given that

sum of first three terms =  $\frac{39}{10}$ 



product of first three terms is 1

$$\frac{a}{r} \times a \times ar = 1$$

$$a^3 \times \frac{r}{r} = 1$$

$$a^3 = 1$$

$$a^3 = (1)^3$$

$$a = 1$$

putting a = 1 in (1)

$$\frac{a}{r} + a + ar = \frac{39}{10}$$



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$$10(1+r+r^2) = 39r$$

$$10(1+r+r^2) = 39r$$

$$10 + 10r + 10r^2 = 39r$$

$$10 + 10r + 10r^{3} - 39r = 0$$

$$10r^2 + 10r - 39r + 10 = 0$$

$$10r^2 - 29r + 10 = 0$$

$$r = \frac{s}{2}$$
  $r =$ 

Hence first three term of G.P are

$$\frac{2}{5}$$
, 1,  $\frac{5}{2}$  for  $r = \frac{5}{2}$ 

$$8.\frac{5}{2}$$
, 1,  $\frac{2}{5}$  for  $r = \frac{2}{5}$ 

Let the geometric series be a, ar, ar<sup>2</sup>, ar<sup>3</sup>,...

Third term =  $ar^2$ , first term = a

$$ar^2 - a = 9$$
 .....(i)

Second term = ar, fourth term = ar3

$$ar - ar^3 = 18$$
 .....(ii)

Dividing equation (i) by (ii), we get

$$\frac{a\left(r^2-1\right)}{a\left(r-r^3\right)}$$

$$=\frac{9}{18}$$

or 
$$2(r^2 - 1) = r - r^3$$

$$r^3 + 2r^2 - r - 2 = 0$$

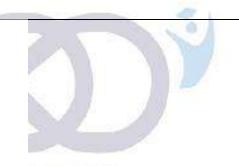
or 
$$(r-1)(r+1)(r+2)=0$$

or 
$$r = 1, -1, -2$$
 if  $r = -2$ ,

From equation (i), a(4-1) = 9

$$\therefore a = 3$$

∴ 4th terms of the geometric progression 3, -6, 12, -24.





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The expansion of  $\left(\frac{2}{x} - \frac{x}{2}\right)^6$  is  $\frac{64}{x^6} - \frac{96}{x^4} + \frac{60}{x^2} - 20 + \frac{15x^2}{4} - \frac{3x^4}{8} + \frac{x^6}{64}$ .

Or

The evaluation of (99)<sup>5</sup> using the binomial theorem is 9509900499.

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We have

$$(1 + a)^n = {}^nC_n + {}^nC_1a + {}^nC_2a^2 + ... + {}^nC_2a^n$$

For a = 5, we get

$$(1+5)^n = {}^nC_0 + {}^nC_15 + {}^nC_25^2 + ... + {}^nC_n5^n$$

i.e. 
$$(6)^n = 1 + 5n + 5^2$$
,  $C_3 + 5^3$ ,  $C_4 + ... + 5^n$ 

i.e. 
$$6^n - 5n = 1 + 5^2 (^nC_1 + ^nC_1 + ... + 5^{n-2})$$

or 
$$6^n - 5n = 1 + 25 (^nC_3 + 5 .^nC_3 + ... + 5^{n-2})$$

or 
$$6^n - 5n = 25k + 1$$
 where  $k = {^nC_2} + 5 {^nC_3} + ... + 5^{n-2}$ .

This shows that when divided by 25,  $6^s - 5n$  leaves remainder 1.

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Let the numbers be a , ar , ar<sup>2</sup> .

Now 
$$a + ar + ar^2 = 13$$

$$a(1 + r + r^2) = 13$$
 ----(1)

$$a^2 + a^2r^2 + a^2r^4 = 91$$

$$a^{2}(1+r^{2}+r^{4})=91$$
 -----(2)

Squaring (1) dividing by (2)

$$a^{2}(1+r+r^{2})^{2}/a^{2}(1+r^{2}+r^{4}) = 169/91.$$

$$(1+r+r^2)^2/(1+r^2)^2-r^2$$
) = 13/7.

$$(1+r+r^2)^2/(1+r^2+r)(1+r^2-r) = 13/7.$$

$$(1+r^2+r)/(1+r^2-r)=13/7$$

$$7(1+r^2+r) = 13(1+r^2-r)$$







$$7(1+r^2+r) = 13(1+r^2-r)$$

$$(7 + 7r^2 + 7r) = (13 + 13r^2 - 13r)$$

$$6r^2 - 20r + 6 = 0$$

$$3r^2 - 10r + 3 = 0$$

$$3r^2 - 9r - r + 3 = 0$$

$$3r(r-3)-1(r-3)=0.$$

$$(3r-1)(r-3)=0$$

Substitute r in equ(1) we get

$$a(1+3+9) = 13$$
 and  $a(1+1/3+1/9) = 13$ 

$$a = 1$$
 and  $a = 9$ .

Or

Let the geometric progression be a + ar + ar2 + ..... + arn-1

The product of these n terms, P = a. ar .  $ar^2 \dots ar^{n-1}$ 

$$= a^n, r^{1+2+\dots +(n-1)}$$

$$=a^{n}r\frac{n(n-1)}{2}$$

$$\therefore P^2 = a^{2n} \cdot r \left( n \left( n - 1 \right) \right)$$

$$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \dots + \frac{1}{ar^{n-1}}$$

$$=\frac{\frac{1}{a}\left[\left(\frac{1}{r}\right)^{a}-1\right]}{1}/r-1$$

$$\left(1-r^{0}\right)r$$

$$-\frac{\left(1-r^{0}\right)r}{ar^{0}\left(1-r\right)}$$

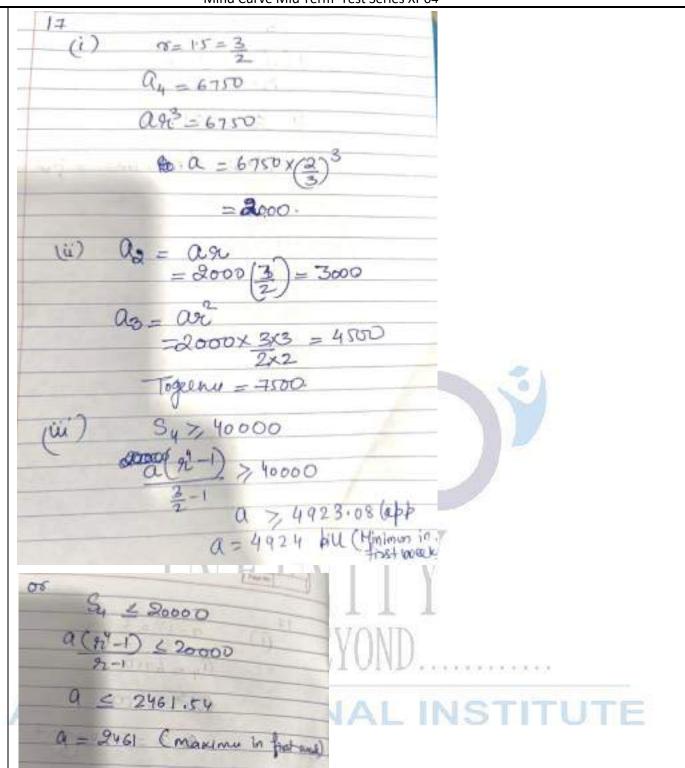
$$\therefore R^{n} = \frac{\left(1-r^{n}\right)^{n}}{\left(a^{n}r^{n}\left(n-1\right)\right)\left(1-r\right)^{n}}$$

$$Left \, Side; \, P^2 \, R^n = a^{2n} r^{n(n-1)} \frac{\left(1-r^n\right)^n}{\left(a^n r^{n(n-1)} (1-r)^n\right)}$$

$$-\frac{a^n(1-r^n)^n}{(1-r)n} = S^n$$



	Mind Curve Mid Term Test Series XI-04
When	reas $S = a + ar + ar^2 + + ar^{n-1}$
_ a (	$(1-r^n)$
3	1-r
Henc	e, $P^2R^n = S^n$
. ()	
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	$ay = ax^3$
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	$S_{\infty} = \frac{\alpha}{1-\frac{1}{2}} = \frac{72\times2}{()} = \frac{9}{1-1} \cdot \frac{1}{1-1}$



The expression  $(x^2 + 1)^4 + (x^2 - 1)^4$  can be expanded using the binomial theorem.

The binomial expansion of  $(A + B)^4$  is given by  $A^4 + 4A^3B + 6A^2B^2 + 4AB^3 + B^4$ .

The binomial expansion of  $(A - B)^4$  is given by  $A^4 - 4A^3B + 6A^2B^2 - 4AB^3 + B^4$ .

When these two expansions are added, the terms with odd powers of B cancel out.

Therefore,  $(A + B)^4 + (A - B)^4 = 2(A^4 + 6A^2B^2 + B^4)$ .

By substituting  $A = x^2$  and B = 1, the expression becomes  $2((x^2)^4 + 6(x^2)^2(1)^2 + (1)^4)$ .

This simplifies to  $2(x^8 + 6x^4 + 1)$ .

By setting  $A = a^2$  and  $B = \sqrt{a^2 - 1}$ , the formula  $2(A^4 + 6A^2B^2 + B^4)$  can be applied.

Substituting the values of A and B, the expression becomes

$$2((a^2)^4 + 6(a^2)^2(\sqrt{a^2 - 1})^2 + (\sqrt{a^2 - 1})^4).$$

This simplifies to  $2(a^8 + 6a^4(a^2 - 1) + (a^2 - 1)^2)$ .

Further expansion yields  $2(a^8 + 6a^6 - 6a^4 + (a^4 - 2a^2 + 1))$ .

Combining like terms results in  $2(a^8 + 6a^6 - 5a^4 - 2a^2 + 1)$ .

Ans:  $a + b = 6\sqrt{ab}$ 

$$\frac{a + b}{2\sqrt{ab}} = \frac{3}{1}$$

By C and D, we get,

$$\frac{\mathbf{a} + \mathbf{b} + 2\sqrt{\mathbf{a}\mathbf{b}}}{\mathbf{a} + \mathbf{b} - 2\sqrt{\mathbf{a}\mathbf{b}}} = \frac{3+1}{3-1}$$

$$\Rightarrow \frac{(\sqrt{\mathbf{a}} + \sqrt{\mathbf{b}})^2}{(\sqrt{\mathbf{a}} - \sqrt{\mathbf{b}})^2} = \frac{2}{1}$$

$$\Rightarrow \frac{\sqrt{\mathbf{a}} + \sqrt{\mathbf{b}}}{\sqrt{\mathbf{a}} - \sqrt{\mathbf{b}}} = \frac{\sqrt{2}}{1}$$

Again by C and D, we get,

$$\frac{\sqrt{a} + \sqrt{b} + \sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b} - \sqrt{a} + \sqrt{b}} = \frac{\sqrt{2} + 1}{\sqrt{2} - 1}$$

$$\Rightarrow \frac{\mathbf{a}}{\mathbf{b}} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$$

$$\Rightarrow \frac{\mathbf{a}}{\mathbf{b}} = \frac{\overset{2}{3} + \overset{1}{2}\sqrt{2}}{\overset{2}{3}\sqrt{2}}$$

$$\Rightarrow$$
 a:b =  $(3 + 2\sqrt{2})$ : $(3 - 2\sqrt{2})$ 



## Or

We have,

$$a + b = 3$$
,  $ab = p$ ,  $c + d = 12$  and  $cd = q$ 

a, b, c and d form a G.P.

$$\therefore$$
 First term = a, b = ar, c = ar<sup>2</sup> and d = ar<sup>3</sup>

Then, we have

$$a + b = 3$$
 and  $c + d = 12$ 

$$\Rightarrow a + ar = 3$$

$$\Rightarrow a(1+r) = 3...(i)$$

Similarly,  $ar^{2}(1+r) = 12...(ii)$ 

$$\Rightarrow \frac{ar^{2}\left(1+r\right)}{a\left(1+r\right)} = \frac{12}{3}$$

$$\Rightarrow r^2 = 4$$

$$\Rightarrow r = 2$$

$$a(1+r)=3$$

$$\Rightarrow a = 1$$

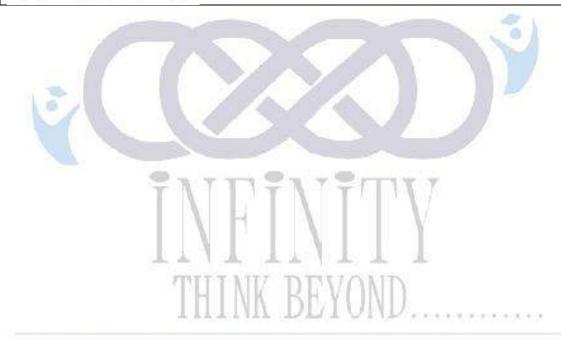
Now, 
$$p = ab$$

$$\Rightarrow p = a \times ar = 2$$

And, 
$$q = cd$$

$$\Rightarrow q = ar^2 \times ar^3 = 2^5 = 32$$

$$\therefore \frac{q+p}{q-p} = \frac{32+2}{32-2} = \frac{34}{30} = \frac{17}{15}$$



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