ANSWER KEY

1	**a**
2	**b**
3	**c**
4	**a**
5	**b**
6	**c**
7	**a**
8	**b**
9	**a**
10	**b**
11	The first part, $\sum_{r=1}^{11} 3$, represents the sum of the constant 3 for 11 terms. This sum is calculated as $3 \times 11 = 33$. The second part, $\sum_{r=1}^{11} 2^r$, is a geometric series. The first term is $a = 2^1 = 2$. The common ratio is $r = 2$. The number of terms is $n = 11$. The formula for the sum of the first n terms of a geometric series is $S_n = \frac{a(r^n - 1)}{r - 1}$. Substituting the values, the sum is $\frac{2(2^{11} - 1)}{2 - 1}$. This simplifies to $2(2^{11} - 1)$. Hence required sum is $33+2(2^{11}-1)= 31+ 2^{12}$ **Or** It is given that sum of first three terms = $\frac{39}{10}$ i.e. $\frac{a}{r} + a + ar = \frac{39}{10}$ ...(1) product of first three terms is 1 $\frac{a}{r} \times a \times ar = 1$ $a^3 \times \frac{r}{r} = 1$ $a^3 = 1$ $a^3 = (1)^3$ $a = 1$ putting a = 1 in (1) $\frac{a}{r} + a + ar = \frac{39}{10}$

	$10(1 + r + r^2) = 39r$ $10(1 + r + r^2) = 39r$ $10 + 10r + 10r^2 = 39r$ $10 + 10r + 10r^2 - 39r = 0$ $10r^2 + 10r - 39r + 10 = 0$ $10r^2 - 29r + 10 = 0$ $r = \frac{5}{2}$ | $r = \frac{2}{5}$ Hence first three term of G.P are $\frac{2}{5}, 1, \frac{5}{2}$ for $r = \frac{5}{2}$ & $\frac{5}{2}, 1, \frac{2}{5}$ for $r = \frac{2}{5}$
12	Let the geometric series be a, ar, ar^2, ar^3,... Third term = ar^2, first term = a $\therefore ar^2 - a = 9$ ....(i) Second term = ar, fourth term = ar^3 $ar - ar^3 = 18$ ....(ii) Dividing equation (i) by (ii), we get $\frac{a(r^2-1)}{a(r-r^3)}$ $= \frac{9}{18}$ $= \frac{1}{2}$ or $2(r^2 - 1) = r - r^3$ $\therefore r^3 + 2r^2 - r - 2 = 0$ or $(r - 1)(r + 1)(r + 2) = 0$ or $r = 1, -1, -2$ if $r = -2$, From equation (i), $a(4 - 1) = 9$ $\therefore a = 3$ $\therefore$ 4th terms of the geometric progression 3, −6, 12, −24.

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The expansion of $\left(\frac{2}{x}-\frac{x}{2}\right)^6$ is $\frac{64}{x^6}-\frac{96}{x^4}+\frac{60}{x^2}-20+\frac{15x^2}{4}-\frac{3x^4}{8}+\frac{x^6}{64}$.

Or

The evaluation of $(99)^5$ using the binomial theorem is 9509900499.

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We have

$$(1+a)^n = {}^nC_0 + {}^nC_1 a + {}^nC_2 a^2 + \dots + {}^nC_n a^n$$

For $a = 5$, we get

$$(1+5)^n = {}^nC_0 + {}^nC_1 5 + {}^nC_2 5^2 + \dots + {}^nC_n 5^n$$

i.e. $(6)^n = 1 + 5n + 5^2.{}^nC_2 + 5^3.{}^nC_3 + \dots + 5^n$

i.e. $6^n - 5n = 1 + 5^2\,({}^nC_2 + {}^nC_3 5 + \dots + 5^{n-2})$

or $6^n - 5n = 1 + 25\,({}^nC_2 + 5\,.{}^nC_3 + \dots + 5^{n-2})$

or $6^n - 5n = 25k + 1$ where $k = {}^nC_2 + 5\,.{}^nC_3 + \dots + 5^{n-2}$.

This shows that when divided by 25, $6^n - 5n$ leaves remainder 1.

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Let the numbers be a , ar , ar^2 .

Now $a + ar + ar^2 = 13$

$a(1 + r + r^2) = 13$ ---------(1)

$a^2 + a^2r^2 + a^2r^4 = 91$

$a^2 (1 + r^2 + r^4) = 91$ --------(2)

Squaring (1) dividing by (2)

$a^2(1 + r + r^2)^2 / a^2 (1 + r^2 + r^4) = 169 / 91.$

$(1 + r + r^2)^2 / ((1 + r^2)^2 - r^2) = 13 / 7.$

$(1 + r + r^2)^2 / (1 + r^2 + r) (1 + r^2 - r) = 13 / 7.$

$(1 + r^2 + r) / (1 + r^2 - r) = 13 / 7$

$7(1 + r^2 + r) = 13(1 + r^2 - r)$

$7(1 + r^2 + r) = 13(1 + r^2 - r)$

$(7 + 7r^2 + 7r) = (13 + 13r^2 - 13r)$

$6r^2 - 20r + 6 = 0$

$3r^2 - 10r + 3 = 0.$

$3r^2 - 9r - r + 3 = 0.$

$3r(r - 3) - 1(r - 3) = 0.$

$(3r - 1)(r - 3) = 0$

$r = 3, 1/3.$

Substitute r in equ(1) we get

$a(1 + 3 + 9) = 13$ and $a(1 + 1/3 + 1/9) = 13$

$13a = 13$ and $13a/9 = 13$

$a = 1$ and $a = 9.$

Or

Let the geometric progression be $a + ar + ar^2 + ... + ar^{n-1}$

The product of these n terms, $P = a.\ ar\ .\ ar^2 ..... ar^{n-1}$

$= a^n . r^{1+2+......+(n-1)}$

$= a^n r \frac{n(n-1)}{2}$

$\therefore P^2 = a^{2n} . r(n(n-1))$

$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + ... + \frac{1}{ar^{n-1}}$

$= \frac{\frac{1}{a}\left[\left(\frac{1}{r}\right)^n - 1\right]}{1} / r - 1$

$= \frac{(1 - r^n)r}{ar^n(1 - r)}$

$\therefore R^n = \frac{(1 - r^n)^n}{(a^n r^n (n-1))(1 - r)^n}$

Left Side: $P^2 R^n = a^{2n} r^{n(n-1)} \frac{(1 - r^n)^n}{(a^n r^{n(n-1)} (1 - r)^n)}$

$= \frac{a^n (1 - r^n)^n}{(1 - r)n} = S^n$

	Whereas $S = a + ar + ar^2 + ... + ar^{n-1}$ $= \frac{a(1-r^n)}{1-r}$ Hence, $P^2R^n = S^n$
16	i) Yes, when the mid points of the sides of an equilateral triangle are joined, a new equilateral triangle formed & length is half the side of original triangle. This consistent ratio of $\frac{1}{2}$ between Consecutive terms indicates GP ii $a_1 = 24$ $a_2 = \frac{24}{2} = 12 \quad r = \frac{1}{2}$ $a_4 = ar^3$ $= 24\left(\frac{1}{2}\right)^3 = \frac{24}{8} = 3$ (iii) $a_6 = ar^5 = 24 \times \frac{1}{32} = \frac{3}{4}$ = side of 6^{th} Δ. perimeter of Δ is $3\left(\frac{3}{4}\right) = \frac{9}{4}$ **OR** GP for perimeter $72, 36, \ldots\ldots$ $S_\infty = \frac{a}{1-\frac{1}{2}} = 72 \times 2 = 144$. $\left(\because S_\infty = \frac{a}{1-r}, r<1\right)$
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	17 (i) $r = 1.5 = \frac{3}{2}$ $a_4 = 6750$ $ar^3 = 6750$ $a = 6750 \times \left(\frac{2}{3}\right)^3$ $= 2000$. (ii) $a_2 = ar$ $= 2000\left(\frac{3}{2}\right) = 3000$ $a_3 = ar^2$ $= 2000 \times \frac{3\times3}{2\times2} = 4500$ Together $= 7500$ (iii) $S_4 \geq 40000$ $\frac{a(r^4-1)}{\frac{3}{2}-1} \geq 40000$ $a \geq 4923.08$ (app) $a = 4924$ bill (Minimum in first week) or $S_4 \leq 20000$ $\frac{a(r^4-1)}{r-1} \leq 20000$ $a \leq 2461.54$ $a = 2461$ (maximum in first week)
18	The expression $(x^2+1)^4 + (x^2-1)^4$ can be expanded using the binomial theorem. The binomial expansion of $(A+B)^4$ is given by $A^4 + 4A^3B + 6A^2B^2 + 4AB^3 + B^4$. The binomial expansion of $(A-B)^4$ is given by $A^4 - 4A^3B + 6A^2B^2 - 4AB^3 + B^4$. When these two expansions are added, the terms with odd powers of B cancel out. Therefore, $(A+B)^4 + (A-B)^4 = 2(A^4 + 6A^2B^2 + B^4)$. By substituting $A = x^2$ and $B = 1$, the expression becomes $2((x^2)^4 + 6(x^2)^2(1)^2 + (1)^4)$. This simplifies to $2(x^8 + 6x^4 + 1)$.

	By setting $A = a^2$ and $B = \sqrt{a^2-1}$, the formula $2(A^4 + 6A^2B^2 + B^4)$ can be applied. Substituting the values of A and B, the expression becomes $2((a^2)^4 + 6(a^2)^2(\sqrt{a^2-1})^2 + (\sqrt{a^2-1})^4)$. This simplifies to $2(a^8 + 6a^4(a^2-1) + (a^2-1)^2)$. Further expansion yields $2(a^8 + 6a^6 - 6a^4 + (a^4 - 2a^2 + 1))$. Combining like terms results in $2(a^8 + 6a^6 - 5a^4 - 2a^2 + 1)$.
19	**Ans:** $a + b = 6\sqrt{ab}$ $\frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$ By C and D, we get, $\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$ $\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{2}{1}$ $\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}}{1}$ Again by C and D, we get, $\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ $\Rightarrow \frac{a}{b} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$ $\Rightarrow \frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$ $\Rightarrow a:b = (3+2\sqrt{2}):(3-2\sqrt{2})$ **Or** We have, $a + b = 3$, $ab = p$, $c + d = 12$ and $cd = q$ a, b, c and d form a G.P. $\therefore$ First term = a, $b = ar$, $c = ar^2$ and $d = ar^3$ Then, we have $a + b = 3$ and $c + d = 12$ $\Rightarrow a + ar = 3$ $\Rightarrow a(1+r) = 3\ldots(i)$ Similarly, $ar^2(1+r) = 12\ldots(ii)$

$$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{12}{3}$$

$$\Rightarrow r^2 = 4$$

$$\Rightarrow r = 2$$

$$\therefore a(1+r) = 3$$

$$\Rightarrow a = 1$$

Now , $p = ab$

$$\Rightarrow p = a \times ar = 2$$

And, $q = cd$

$$\Rightarrow q = ar^2 \times ar^3 = 2^5 = 32$$

$$\therefore \frac{q+p}{q-p} = \frac{32+2}{32-2} = \frac{34}{30} = \frac{17}{15}$$