

XII Test 03.

Q-1 d)

Q-2 d)

Q-3 a)

Q-4 a)

Q-5 d)

Q-6 c)

Q-7 c)

Q-8 a)

Q-9 a)

Q-10 d)

Q-11(A) $\frac{dx}{d\theta} = a(\sin\theta)$

$$\frac{dy}{d\theta} = a(1 + \cos\theta)$$

$$\frac{dy}{dx} = \frac{a(1 + \cos\theta)}{a \sin\theta}$$

$$\frac{dy}{dn} = \frac{2\cos^2\theta_2}{2\sin\theta_2\cos\theta_2}$$

$$\frac{dy}{dx} = \cot\theta_2$$

diff w.r.t. n

$$\frac{d^2y}{dn^2} = -\operatorname{cosec}^2\theta_2 \times \frac{d\theta}{dn} \times \frac{1}{2}$$

$$= \frac{-\operatorname{cosec}^2\theta_2}{a\sin\theta} \times \frac{1}{2}$$

$$\left. \frac{d^2y}{dn^2} \right|_{\theta=\theta_2} = \frac{-\operatorname{cosec}^2\theta_2}{a\sin\theta_2} \times \frac{1}{2}$$

$$= \frac{-(\sqrt{2})^2}{a(1)} \times \frac{1}{2} = -\frac{1}{a}$$

11 B)

$$\text{Let } u = \sin^2 n \quad v = e^{\cos n}$$

we need to find $\frac{du}{dv}$

$$\frac{du}{dn} = 2\sin n \cos n, \quad \frac{dv}{dn} = e^{\cos n} (-\sin n)$$

$$\frac{du}{dv} = \frac{2\sin n \cos n}{e^{\cos n} (-\sin n)} = \frac{-2\cos n}{e^{\cos n}}$$

$$12(A) \quad f'(x) = 2 \cos 2x$$

$$f'(x) = 0$$

$$2 \cos 2x = 0$$

$$2x = (2n+1)\frac{\pi}{2}$$

$$x = (2n+1)\frac{\pi}{4}$$

$$x = \pi/4, 3\pi/4$$

$$f''(x) = -4 \sin 2x$$

$$f''(\pi/4) < 0 \quad \text{Max}$$

$$f''(3\pi/4) > 0 \quad \text{Min}$$

$$f(\pi/4) = \sin \frac{\pi}{2} + 5 = 1 + 5 = 6$$

Max value = 6 at $x = \pi/4$

$$f(3\pi/4) = \sin 3\frac{\pi}{2} + 5 = -1 + 5 = 4$$

Min value = -4 at $x = 3\pi/4$

12(B)

$$\frac{dV}{dt} = 8 \text{ cm}^3/\text{s}$$

$$V = \frac{4}{3} \pi r^3$$

diff w.r.t. r

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$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{8}{4\pi r^2} = \frac{dr}{dt}$$

$$S.A = 4\pi r^2$$

diff wrt. t

$$\begin{aligned}\frac{ds}{dt} &= 8\pi r \frac{dr}{dt} \\ &= 8\pi r \times \frac{8}{4\pi r^2}\end{aligned}$$

$$= \frac{4}{3} \text{ cm}^2/\text{s}$$

Q13 = $(\cos x)^y = (\sin y)^x$

take log

$$y \log \cos x = x \log \sin y$$

dwr to x

$$y \times \frac{1}{\cos x} (-\sin x) + \frac{dy}{dx} \log \cos x = x \times \frac{1}{\sin y} (\cos y) \frac{dy}{dx}$$

$$-y \tan x + \frac{dy}{dx} \log \cos x = x \cot y \frac{dy}{dx} + \log \sin y$$

$$\frac{dy}{dx} [\log \cos x - x \cot y] = \log \sin y + y \tan x$$

$$\frac{dy}{dx} = \frac{\log \sin y + y \tan x}{\log \cos x - x \cot y}$$

Q14 ~ (A)

$$f(x) = \sin x + \cos x$$

dwrt to x

$$f'(x) = \cos x - \sin x$$

$$f'(x) = 0$$

$$\cos x - \sin x = 0$$

$$\sin x = \cos x$$

$$\sin x = 1$$

$$\cos x$$

$$\tan x = 1$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

Intervals		inc / dec
$(0, \pi/4)$	$f'(x) > 0$	decreasing increasing
$(\frac{\pi}{4}, \frac{5\pi}{4})$	$f'(x) < 0$	dec.
$(\frac{5\pi}{4}, 2\pi)$	$f'(x) > 0$	inc.

or

$$(B) \quad f(x) = \sin^4 x + \cos^4 x$$

dwr to x

$$\begin{aligned} f'(x) &= 4\sin^3 x \cos x - 4\cos^3 x \sin x \\ &= 4\sin 2x \cos x [\sin^2 x - \cos^2 x] \\ &= -4\sin 2x \cos x (\cos^2 x - \sin^2 x) \\ &= -4\sin 2x \cos x (\cos 2x) \\ &= -2 [2\sin x \cos x] (\cos 2x) \\ &= -2 [\sin 2x] [\cos 2x] \\ &= -\sin 4x \end{aligned}$$

$$f'(x) = 0$$

$$-\sin 4x = 0$$

$$\sin x = 0$$

$$0, \pi, 2\pi$$

$$\therefore \sin^4 x = 0$$

$$0, \frac{\pi}{4}, \frac{\pi}{2}$$

Intervals	+ve / -ve	dec / inc
$(0, \pi/4)$	$\therefore f'(x) < 0$	decreasing
$(\pi/4, \pi/2)$	$f'(x) > 0$	increasing

Q15=

$$x = a \cos \theta + b \sin \theta$$

dwr to θ

$$\frac{dx}{d\theta} = -a \sin \theta + b \cos \theta$$

$$= b \cos \theta - a \sin \theta$$

$$y = a \sin \theta - b \cos \theta$$

$\frac{dy}{dx} = \text{dwr to } \theta$

$$\frac{dy}{d\theta} = a \cos \theta + b \sin \theta$$

$$\frac{dy}{dx} = \frac{a \cos \theta + b \sin \theta}{b \cos \theta - a \sin \theta}$$

$$= \frac{a \cos \theta + b \sin \theta}{- (a \sin \theta - b \cos \theta)}$$

$$\frac{dy}{dx} = \frac{x}{-y}$$

$$\frac{dy}{dx} = \frac{x}{-y}$$

dwr to x

$$\frac{d^2y}{dx^2} = \frac{(-y) - (x)\left(-\frac{dy}{dx}\right)}{y^2}$$

$$y^2 \frac{d^2y}{dx^2} = -y + x \frac{dy}{dx}$$

$$y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} \neq y = 0$$

Hence, proved.

16.

$$LHL = RHL = f(0)$$

$$\lim_{h \rightarrow 0} f(-h) = \lim_{h \rightarrow 0} f(h) = C$$

$$\lim_{h \rightarrow 0} \frac{-\sin(a+h)h - \sinh}{-h} = \lim_{h \rightarrow 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{bh^{3/2}} = C$$

$$a+1 \lim_{h \rightarrow 0} \frac{\sin(a+1)h}{(a+1)h} + \lim_{h \rightarrow 0} \frac{\sinh}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{h+bh^2} - \sqrt{h} \times \sqrt{h+bh^2}}{bh^{3/2} \sqrt{h+bh^2} + \sqrt{h}}$$

$$a+1+1 = \lim_{h \rightarrow 0} \frac{\cancel{h+bh^2} - \cancel{h}}{\cancel{bh^{3/2}}(\sqrt{h+bh^2} + \sqrt{h})}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{h}}{\sqrt{h}(\sqrt{1+bh} + 1)}$$

$$a+2 = \frac{1}{2} = C$$

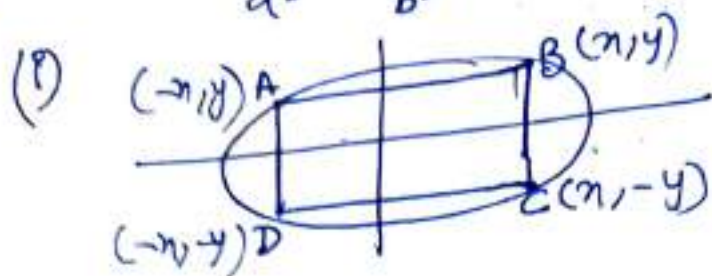
$$a = -\frac{3}{2}, C = \frac{1}{2}$$

a) b can be any Non zero real number

$$(b) a+C = -\frac{3}{2} + \frac{1}{2} = -\frac{2}{2} = -1$$

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$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (1)$$



Let length = $2x$.

breadth = $2y$

Area of rectangle = $4xy$

from (1)

$$y = b \sqrt{1 - \frac{x^2}{a^2}}$$

$$\therefore \text{Area} = 4x b \sqrt{1 - \frac{x^2}{a^2}}$$

$$= 4x \frac{b}{a} \sqrt{a^2 - x^2}$$

$$A(x) = \frac{4b}{a} x \sqrt{a^2 - x^2}$$

$$(ii) \quad A'(x) = \frac{4b}{a} \left(\frac{x}{\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2} \right)$$

$$\frac{4b}{a} \left(\frac{-x^2 + a^2 - x^2}{\sqrt{a^2 - x^2}} \right) = 0$$

$A'(x) = 0$

$$2x^2 = a^2$$

$$\text{iii(a)} \quad x = \frac{a}{\sqrt{2}} \quad \text{length } 2x = 2 \frac{a}{\sqrt{2}} = \sqrt{2}a$$

$$\therefore y = \frac{b}{\sqrt{2}} \quad \text{breadth } 2y = 2 \frac{b}{\sqrt{2}} = \sqrt{2}b$$

$$\text{(ii)b) } A''(x) = \frac{4b}{a} \left(\frac{\sqrt{a^2 - x^2} (-4x) - (a^2 - 2x^2) \frac{-x}{\sqrt{a^2 - x^2}}}{a^2 - x^2} \right)$$

$$= \frac{4b}{a} \left(\frac{(a^2 - x^2)(-4x) + (a^2 - 2x^2)x}{(a^2 - x^2)^{3/2}} \right)$$

$$= \frac{4b}{a} \left(\frac{-4xa^2 + 4x^3 + a^2x - 3x^3}{(a^2 - x^2)^{3/2}} \right)$$

$$= \frac{4b}{a} \left(\frac{x^3 - 3a^2x}{(a^2 - x^2)^{3/2}} \right)$$

$$A''(\sqrt{2}a) = \frac{4b}{a} \frac{2\sqrt{2}a^3 - 3\sqrt{2}a^3}{(a^2 - 2a^2)^{3/2}} < 0$$

pt. of max.

$$\text{Max area} = 4(\sqrt{2}a)(\sqrt{2}b) = 8ab \quad \text{sq. unit}$$

Sec-D

Q162

018

Sec-E.

$$(x-a)^2 + (y-b)^2 = c^2$$

dwr to x

$$2(x-a) + 2(y-b) \frac{dy}{dx} = 0$$

$$x(y-b) \frac{dy}{dx} = -x(x-a)$$

$$y' \frac{dy}{dx} = \frac{-(x-a)}{(y-b)}$$

dwr to x

$$y' = \frac{(y-b)(-1) - [-(x-a)] [dy]}{(y-b)^2}$$
$$= \frac{-(y-b) + (x-a)y'}{(y-b)^2}$$

$$= \frac{-(y-b) + (x-a) \left[\frac{-(x-a)}{y-b} \right]}{(y-b)^2}$$

$$= \frac{-(y-b)^2 - (x-a)^2}{(y-b)^3}$$

$$= \frac{-(y-b)^2 + (x-a)^2}{(y-b)^3}$$

$$y'' = \frac{-c^2}{(y-b)^3}$$

Consider

$$\left[1 + (y')^2 \right]^{3/2}$$

$$y''$$

$$\left[1 + \left[\frac{-(x-a)}{y-b} \right]^2 \right]^{3/2}$$

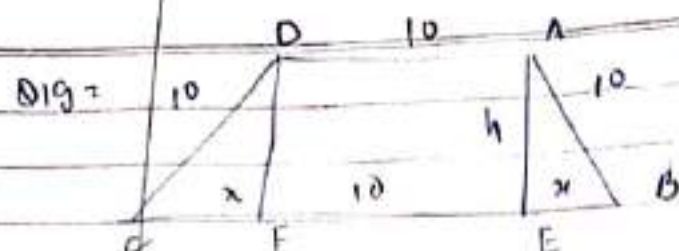
$$\frac{-c^2}{(y-b)^3}$$

$$\left[\frac{(y-b)^2 + (x-a)^2}{(y-b)^2(y-b)^2} \right]^{3/2} \times \frac{(y-b)^3}{-c^2}$$

$$\frac{\left[(y-b)^2 + (x-a)^2 \right]^{3/2}}{\left[(y-b)^2 \right]^{3/2}} \times \frac{(y-b)^3}{-c^2}$$

$$\frac{(c^2)^{3/2}}{(y-b)^3} \times \frac{(y-b)^3}{(-c^2)}$$

$$\frac{c^3}{-c^2} = -c$$



let height of trapezium be h .

$\triangle AEB$

$$10^2 = h^2 + x^2$$

$$10^2 - x^2 = h^2$$

area of trapezium

$$= \frac{1}{2} [20 + 2x] h$$

$$= \frac{1}{2} \times 2(10 + x)h$$

$$= (10 + x)h$$

Square both sides

$$= (10 + x)^2 h^2$$

$$= (10 + x)^2 (10^2 - x^2)$$

$$= (10 + x)^2 (10 + x)(10 - x)$$

$$f(x) = (10 + x)^3 (10 - x)$$

dwrt x

$$f'(x) = 3(10 + x)^2 (10 - x) + (10 + x)^3 (-1)$$

$$= 3(10 + x)^2 (10 - x) - (10 + x)^3$$

$$= (10 + x)^2 [3(10 - x) - 10 - x]$$

$$= (10 + x)^2 [30 - 3x - 10 - x]$$

$$= (10 + x)^2 [20 - 4x]$$

$$f'(x) = 0$$

$$(10+x)^2 (20-4x) = 0$$

$$20-4x=0$$

$$x=5$$

$$(10+x)^2 = 0$$

$$(10+x)(10+x)=0$$

$$x=-10$$

rejected

as length can't be -ve.

now

$$f(x) = (10+x)^2 (20-4x)$$

dwrt to x

$$f'(x) = 2(10+x)(20-4x) + (10+x)^2(-4)$$

$$= 2[200-40x+20x-4x^2] - 4[10+x]^2$$

$$= 2[200-20x-4x^2] - 4[100+x^2+20x]$$

$$= 400-40x-8x^2-400-4x^2-40x$$

$$= -12x^2-80x$$

$$f''(5) = -12(25) - 80(5)$$

$$= -300 - 400$$

$$= -700 < 0$$

$\therefore f(x)$ is ~~max~~

$\therefore$ trapezium has maximum area at $x=5$ cm.

$$\text{maxim area} = \frac{1}{2} [20+2x] h$$

$$= \frac{1}{2} [20+10] 75$$

$$= \frac{30 \times 75}{2}$$

$$= 15 \times 75$$

$$= 1125 \text{ cm}^2$$

$$10^2 - x^2 = h$$

$$(10-5)(10+5) = h$$

$$5 \times 15 = h$$

$$75 = h$$

Or.

Let the radius & height of the cylindrical can be r & h resp.

ATQ.

$$\text{Volume} = \pi r^2 h$$

$$100 = \pi r^2 h$$

$$\frac{100}{\pi r^2} = h$$

$$\text{Surface area} = 2\pi r(r+h)$$

$$= 2\pi r^2 + 2\pi rh$$

$$= 2\pi r^2 + 2\pi r \left[\frac{100}{\pi r^2} \right]$$

$$d f(r) = 2\pi r^2 + 2 \left(\frac{100}{r} \right)$$

dwr to r

$$f'(r) = 4\pi r - \frac{200}{r^2}$$

$$f'(r) = 0$$

$$4\pi r - \frac{200}{r^2} = 0$$

$$\frac{200}{r^2} = 4\pi r$$

$$\frac{200}{4\pi} = r^3$$

$$\frac{50}{\pi} = r^3$$

$$\sqrt[3]{\frac{50}{\pi}} = r$$

now dwr to x

$$f'(x) = 4\pi x - \frac{200}{x^2}$$

dwr to x

$$f''(x) = 4\pi + \frac{200}{x^3}$$

$$\begin{aligned} f''\left[\frac{3\sqrt{50}}{\sqrt{\pi}}\right] &= 4\pi + \frac{200}{\frac{50}{\pi}} \\ &= 4\pi + 200 \times \frac{\pi}{50} \\ &= 8\pi > 0 \end{aligned}$$

$\therefore$ Surface area of cylindrical can is maximum at

$$r = \frac{3\sqrt{50}}{\sqrt{\pi}} \quad r = h =$$

$$\frac{100}{\pi} \left[\frac{50}{\pi} \right]^{3/2}$$