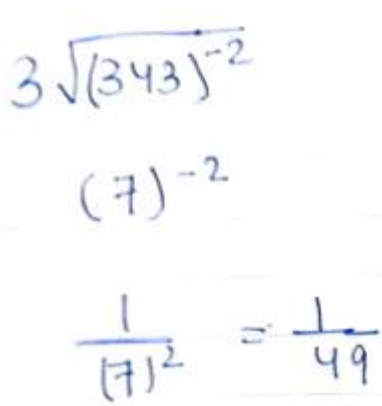
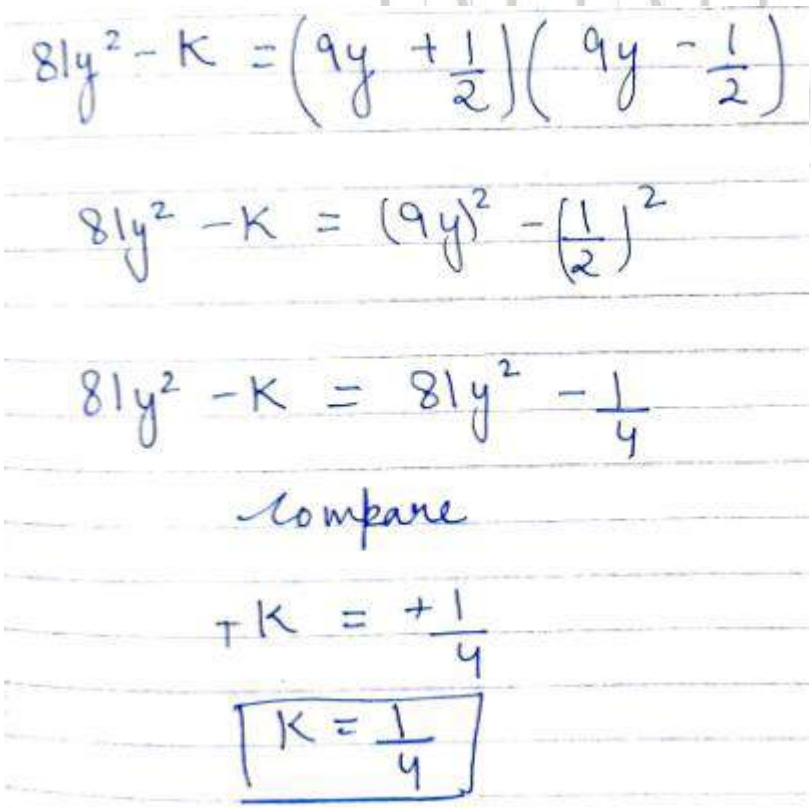


ANSWER KEY

1	(c) $3 + \sqrt{5}$
2	(d) $\frac{\sqrt{a+b}}{\sqrt{a+b}}$
3	(b) 0
4	(b) 4
5	(c) 5^{10}
6	(c) $\frac{1}{3}$
7	(c) 2
8	(d) $3x^2 + 7$
9	(c) 120
10	(d) $\sqrt{2}$
11	
12(A)	

or

12(B)

$$p+1=0, p=-1$$

$$q(p) = p^{50} - 1$$

$$q(-1) = (-1)^{50} - 1$$

$$q(-1) = 0$$

$\therefore p+1$ is a factor of $p^{50}-1$

$$q(p) = p^{51} - 1$$

$$q(-1) = (-1)^{51} - 1$$

$$q(-1) = -2$$

$p+1$ is a factor of $p^{50}-1$
because $f(-1)=0$,
but it is not factor of $p^{51}-1$
because $q(-1) = -2 \neq 0$.

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$$p(x) = ax^3 - x^2 + x + 4$$

$$p(-1) = a(-1)^3 - (-1)^2 + (-1) + 4$$

$$= -a - 1 - 1 + 4$$

$$= -a + 2$$

$$= -a + 2 = 0$$

$$\boxed{a=2}$$

14(A)

$$x = 1 - \sqrt{2}$$

$$\frac{1}{x} = \frac{1}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}}$$

$$= \frac{1 + \sqrt{2}}{(1)^2 - (\sqrt{2})^2} \quad \left[\text{using } \rightarrow a^2 - b^2 = (a-b)(a+b) \right]$$

$$= \frac{1 + \sqrt{2}}{1 - 2}$$

$$= \frac{1 + \sqrt{2}}{-1} = \frac{- (1 + \sqrt{2})}{1}$$

$$\left(x + \frac{1}{x} \right)^2$$

$$\left((1 - \sqrt{2}) - (-1 - \sqrt{2}) \right)^2$$

$$(1 - \sqrt{2})^2 + (-1 - \sqrt{2})^2 - 2(1 - \sqrt{2})(-1 - \sqrt{2})$$

$$= (1)^2 + (\sqrt{2})^2 - 2(1)(\sqrt{2}) + (1)^2 + (\sqrt{2})^2 - 2(1)(\sqrt{2}) - 2(1 - \sqrt{2})(-1 - \sqrt{2})$$

$$= 1 + 2 - 2\sqrt{2} + 1 + 2 + 2\sqrt{2} - 2$$

$$= 4$$

14(B)

or

$$x - y = 4$$

cubing both side

$$(x - y)^3 = (4)^3$$

$$x^3 - y^3 - 3xy(x - y) = 64$$

$$x^3 - y^3 - 3(21)(4) = 64$$

$$x^3 - y^3 - 252 = 64$$

$$x^3 - y^3 = 64 + 252$$

$$x^3 - y^3 = 316$$

15(A)

$$\frac{\sqrt{3} - 1}{\sqrt{3} + 1} = a + b\sqrt{3}$$

$$\frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - (1)^2}$$

$$\frac{(\sqrt{3})^2 + (1)^2 - 2(\sqrt{3})(1)}{3 - 1}$$

$$\frac{3 + 1 - 2\sqrt{3}}{2}$$

$$\frac{4 - 2\sqrt{3}}{2}$$

$$\frac{2(2 - \sqrt{3})}{2}$$

$$\left\{ \begin{array}{l} \text{using } (a-b)^2 = \\ a^2 + b^2 - 2ab \\ \text{using } a^2 - b^2 \\ = (a+b)(a-b) \end{array} \right\} \dots\dots\dots$$

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$$2 - \sqrt{3}$$

Comparing both side

$$2 - \sqrt{3} = a + b\sqrt{3}$$

$$a = 2, b = -1$$

Or

Step 1: Rewrite using negative exponents

Recall that $x^{-r} = \frac{1}{x^r}$, so:

$$\frac{3}{216^{-\frac{2}{3}}} = 3 \cdot 216^{\frac{2}{3}}$$

$$\frac{1}{256^{-\frac{3}{4}}} = 256^{\frac{3}{4}}$$

$$\frac{2}{243^{-\frac{1}{5}}} = 2 \cdot 243^{\frac{1}{5}}$$

Thus the expression becomes:

$$3 \cdot 216^{\frac{2}{3}} + 256^{\frac{3}{4}} + 2 \cdot 243^{\frac{1}{5}}$$

Step 2: Simplify each part

- First term:

$$216 = 6^3 \implies 216^{\frac{2}{3}} = (6^3)^{\frac{2}{3}} = 6^2 = 36$$

$$\implies 3 \cdot 216^{\frac{2}{3}} = 3 \times 36 = \boxed{108}$$

- Second term:

$$256 = 2^8 \implies 256^{\frac{3}{4}} = (2^8)^{\frac{3}{4}} = 2^6 = \boxed{64}$$

- Third term:

$$243 = 3^5 \implies 243^{\frac{1}{5}} = (3^5)^{\frac{1}{5}} = 3$$

$$\implies 2 \cdot 243^{\frac{1}{5}} = 2 \times 3 = \boxed{6}$$

Step 3: Add them all together

$$108 + 64 + 6 = \boxed{178}$$

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$$p(t) = t^4 + 3t^2 - 4$$

(i) degree of $p(t)$ is 4

$$\begin{aligned} \text{(ii)} \quad p(4) &= (4)^4 + 3(4)^2 - 4 \\ &= 256 + 48 - 4 \\ &= 300 \end{aligned}$$

(iii)

$$\text{(a)} \quad p(t) = t^4 + 3t^2 - 4$$

$$\text{let } t^2 = x$$

$$\begin{aligned} p(x) &= x^2 + 3x - 4 \\ &= x^2 + 4x - x - 4 \\ &= x(x+4) - (x+4) \\ &= (x-1)(x+4) \end{aligned}$$

$$\begin{aligned} p(t) &= (t^2-1)(t^2+4) \\ &= (t-1)(t+1)(t^2+4) \end{aligned}$$

Or

(ii)(b) Given $t-1$ is a factor

$$\begin{array}{r} t^3 + t^2 + 4t + 4 \\ t-1 \overline{) t^4 + 3t^2 - 4} \\ \underline{t^4 - t^3} \\ (-) t^3 \end{array}$$

Remaining factor

$$\begin{aligned} t^3 + t^2 + 4t + 4 \\ t^2(t+1) + 4(t+1) \\ (t^2+4)(t+1) \end{aligned}$$

$$\begin{array}{r} t^3 + 3t^2 \\ t^3 - t^2 \\ \underline{(-) (+)} \\ 4t^2 - 4 \\ \underline{4t^2 - 4} \\ (-) (+) \end{array}$$

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$$\begin{aligned} \text{ci) } p(x) &= 6x^2 + 11x - 35 \\ &= 6x^2 + 21x - 10x - 35 \\ &= 3x(2x+7) - 5(2x+7) \end{aligned}$$

$$\text{lav} = 3x - 5, \text{ vicky} = 2x + 7$$

$$\text{cii) a) } 3x - 5 = 2x + 7 \\ x = 12$$

$$\begin{aligned} \text{or} \\ \text{cii) (b) } 2x + 7 + 3x - 5 &= 102 \\ 5x + 2 &= 102 \\ x &= 100/5 = 20 \end{aligned}$$

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$$p(x) = x^3 + 2x^2 - 5ax - 7$$

$$q(x) = x^3 + ax^2 - 12x + 6$$

$$\begin{aligned} R_1 &= p(-1) = (-1)^3 + 2(-1)^2 - 5a(-1) - 7 \\ &= -1 + 2 + 5a - 7 \\ &= 5a - 6 \end{aligned}$$

$$\begin{aligned} R_2 &= q(2) = (2)^3 + a(2)^2 - 12(2) + 6 \\ &= 8 + 4a - 24 + 6 \\ &= 4a - 10 \end{aligned}$$

$$2R_1 + R_2 = 6$$

$$2(5a - 6) + (4a - 10) = 6$$

$$\begin{aligned} \Rightarrow 10a - 12 + 4a - 10 &= 6 \\ 14a - 22 &= 6 \\ \boxed{a = 2} \end{aligned}$$

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i) $a^3 - 2\sqrt{2}b^3$

$$a^3 - \sqrt{8}b^3$$

$$a^3 - (\sqrt{2}b)^3$$

$$\left[\text{using } a^3 - b^3 = (a-b)(a^2 + ab + b^2) \right]$$

$$(a - \sqrt{2}b)(a^2 + a\sqrt{2}b + (\sqrt{2}b)^2)$$

$$(a - \sqrt{2}b)(a^2 + a\sqrt{2}b + 2b^2)$$

(ii) $2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$

Rewrite as $(\sqrt{2}a)^3 + (2b)^3 + (-3c)^3 - 3(\sqrt{2}a)(2b)(-3c)$.

Use $A = \sqrt{2}a$, $B = 2b$, $C = -3c$ in $A^3 + B^3 + C^3 - 3ABC$:

$$(\sqrt{2}a + 2b - 3c)(2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}ab + 6bc + 3\sqrt{2}ac).$$

(iii) $a^3 - b^3 + 1 + 3ab$

Group as $a^3 + 1^3 + (-b)^3 - 3a \cdot 1 \cdot (-b)$.

Use $A = a$, $B = 1$, $C = -b$ in $A^3 + B^3 + C^3 - 3ABC$:

$$(a + 1 - b)(a^2 + 1 + b^2 - a + ab - b) = (a - b + 1)(a^2 + b^2 + 1 - a + b + ab).$$

End

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THINK BEYOND.....

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