

ANSWER KEY

1	(b) Injective
2	(c) 106
3	(b) $-\frac{2\pi}{5}$
4	(b) 4
5	(a) One-one but not onto
6	(a) -1
7	(b) $y = \cos^{-1}x$
8	(d) Continuous as well as differentiable at $x=2$.
9	(d) A is false but R is true
10	(a) Both A and R are true and R is the correct explanation for A
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$$\begin{aligned}
 & \tan^{-1} \left[2 \sin \left(2 \cos^{-1} \frac{\sqrt{3}}{2} \right) \right] \\
 & \tan^{-1} \left[2 \sin \left(2 \cos^{-1} \cos \frac{\pi}{6} \right) \right] \\
 & \tan^{-1} \left[2 \sin \left[2 \times \frac{\pi}{6} \right] \right] \\
 & \tan^{-1} \left[2 \sin \frac{\pi}{3} \right] \\
 & = \tan^{-1} \left[2 \times \frac{\sqrt{3}}{2} \right] \\
 & = \tan^{-1} \sqrt{3} \\
 & = \tan^{-1} \tan \frac{\pi}{3} \\
 & = \frac{\pi}{3}
 \end{aligned}$$

Or

$$\sec^{-1} \left(\frac{1}{2x^2 - 1} \right)$$

$$\text{let } x = \cos \theta$$

$$\theta = \cos^{-1}x$$

$$0 < x < \frac{1}{2}$$

$$0 < \cos \theta < \frac{1}{\sqrt{2}}$$

$$\sec^{-1} \left(\frac{1}{2 \cos^2 \theta - 1} \right)$$

$$\frac{\pi}{2} < \theta < \frac{\pi}{4}$$

$$\sec^{-1} \left(\frac{1}{\cos 2\theta} \right)$$

$$\sec^{-1} (\sec 2\theta)$$

$$2\theta = 2 \cos^{-1}x$$

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$$\begin{aligned}
 & \left[\cos^{-1} \frac{\sqrt{3}}{2} + \cos^{-1} \left(-\frac{1}{2} \right) \right] : \\
 & \left[\cos^{-1} \frac{\sqrt{3}}{2} + \pi - \cos^{-1} \frac{1}{2} \right] \\
 & \left[\cos^{-1} \cos \frac{\pi}{6} + \pi - \cos^{-1} \cos \frac{\pi}{3} \right] \\
 & \left[\frac{\pi}{6} + \pi - \frac{\pi}{3} \right] \\
 & \frac{3\pi + 18\pi - 6\pi}{18} = \frac{15\pi}{18} = \frac{5\pi}{6}
 \end{aligned}$$

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$$\begin{aligned}
 & \sin \left\{ 2 \left(\pi - \cot^{-1} \frac{5}{12} \right) \right\} \\
 & = -\sin \left(2 \tan^{-1} \frac{12}{5} \right) \\
 & \dots = -\sin \left(\sin^{-1} \frac{120}{169} \right) = -\frac{120}{169}
 \end{aligned}$$

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for 1-1

let $x_1, x_2 \in A$
such that $f(x_1) = f(x_2)$

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$x_1 x_2 - 3x_1 - 2x_2 + 6 = x_2 x_1 - 2x_1 - 3x_2 + 6$$

$$-3x_1 - 2x_2 = -2x_1 - 3x_2$$

$$-3x_1 + 2x_1 = -3x_2 + 2x_2$$

$$-x_1 = -x_2$$

$$x_1 = x_2$$

$\therefore f$ is 1-1

Onto

$$\text{let } f(x) = y$$

$$y = \frac{x-2}{x-3}$$

$$yx - 3y = x - 2$$

$$yx - x = 3y - 2$$

$$x(y-1) = 3y-2$$

$$x = \frac{3y-2}{y-1}$$

Range = Co domain

Range = Codomain $y \in \mathbb{R} \setminus \{1\}$ $\therefore f$ is ontoTherefore f is bijective

or

$$\text{let } (x_1) \neq (x_2) \in \mathbb{R}$$

$$f(x_1) = f(x_2)$$

$$2x_1^2 + 3 = 2x_2^2 + 3$$

$$2x_1^2 - 2x_2^2 = 0$$

$$x_1^2 - x_2^2 = 0$$

$$(x_1 - x_2)(x_1 + x_2) = 0$$

$$x_1 = x_2 \quad ; \quad x_1 = -x_2$$

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$$2x_1^2 - 2x_2^2 = 0$$

$$x_1^2 - x_2^2 = 0$$

$$(x_1 - x_2)(x_1 + x_2)$$

$$x_1 = x_2 \quad ; \quad x_1 = -x_2$$

But there will be no unique preimage for -ve no. in the co-domain

Thus, f is not 1-1

Onto

$$f(x) = y$$

$$y = 2x^2 + 3$$

$$2x^2 = 3 - y$$

$$x^2 = \frac{3-y}{2}$$

$$x = \sqrt{\frac{3-y}{2}}$$

Negative values of y will give irrational which is not in the co-domain

$\therefore$ Thus f is not onto

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$$\tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right)$$

$$\left. \begin{aligned} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ 1 + \sin 2\theta &= (\cos \theta + \sin \theta)^2 \end{aligned} \right\}$$

$$\tan^{-1} \left(\frac{\cos^2 x/2 - \sin^2 x/2}{(\cos x/2 + \sin x/2)^2} \right)$$

$$\tan^{-1} \left(\frac{(\cos x/2 - \sin x/2)(\cos x/2 + \sin x/2)}{(\cos x/2 + \sin x/2)^2} \right)$$

$$\tan^{-1} \left(\frac{\cos x/2 - \sin x/2}{\cos x/2 + \sin x/2} \right)$$

divide by $\cos x/2$

$$\tan^{-1} \left(\frac{1 - \tan x/2 \tan x/2}{1 + \tan x/2 \tan x/2} \right) = \left\{ \frac{1 - \tan \theta}{1 + \tan \theta} = \tan \frac{\pi - \theta}{4} \right\}$$

$$\tan^{-1} \left[\tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right]$$

$$\left[\frac{\pi}{4} - \frac{x}{2} \right]$$

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(i)

$$R = \{(1,1) (2,2) (3,3) (4,4) (5,5) (6,6) (2,4) (2,6) (3,6)\}$$

R is reflexive as $(a,a) \in R$
for all $a \in A$.

(ii) R is not symmetric
as $(2,4) \in R$ but $(4,2) \notin R$

(iii) R is not an equivalence relation
as R is not reflexive since $(3,3) \notin R$
R is not transitive since $(4,3) (3,6) \in R$
but $(4,6) \notin R$
 $\Rightarrow R$ is not an equivalence relation.

(iv) No. of reflexive relation: $2^{n(n-1)}$
 $= 2^{6 \times 5} = 2^{30}$

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(i) yes it is symmetric
 let $(L_1, L_2) \in R$
 $\therefore L_1 \parallel L_2$
 $L_2 \parallel L_1$ (True)
 $\therefore (L_2, L_1) \in R$.

(ii) let $(L_1, L_2) \in R$ & $(L_2, L_3) \in R$
 $L_1 \parallel L_2$ ①
 $L_2 \parallel L_3$ ②
 from 1 & 2.
 $L_1 \parallel L_3$
 $\therefore (L_1, L_3) \in R$
 Thus R is transitive.

(iii)

This line has slope = 3.

- Lines parallel to it must also have slope = 3, and can have any y-intercept.
- So, the set of rail lines related to it is:

$$\{y = 3x + c \mid c \in \mathbb{R}\}$$

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To prove a relation R is an equivalence relation, it will be sufficient to prove it as a reflexive, symmetric and transitive relation.

i) Reflexivity:

Let (a, b) be an arbitrary element of $\mathbb{N} \times \mathbb{N}$.

Now,

$$a, b \in \mathbb{N}$$

$$\Rightarrow ab(a+b) = ba(a+b)$$

$$\Rightarrow (a, b)R(a, b)$$

$$\wedge (a, b)R(a, b) \text{ for all } (a, b) \in \mathbb{N} \times \mathbb{N}$$

Hence, R is reflexive.

ii) Symmetry:

Let $(a, b), (c, d)$ be an arbitrary element of $\mathbb{N} \times \mathbb{N}$ such that $(a, b)R(c, d)$.

$$\therefore ad(b+c) = bc(a+d)$$

$$\Rightarrow cb(d+a) = da(c+b)$$

$$\Rightarrow (c, d)R(a, b)$$

$$\therefore (a, b)R(c, d) \Rightarrow (c, d)R(a, b) \text{ for all } (a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$$

Hence, R is symmetric.

iii) Transitivity:

Let $(a, b), (c, d), (e, f)$ be an arbitrary element of $\mathbb{N} \times \mathbb{N}$ such that $(a, b)R(c, d)$ and $(c, d)R(e, f)$.

$$ad(b+c) = bc(a+d)$$

$$\Rightarrow adb + adc = abc + bcd$$

$$\Rightarrow cd(a-b) = ab(c-d) \quad \dots (1)$$

$$\text{Also, } ce(d+f) = de(c+f)$$

$$\Rightarrow cfd + cfe = dec + def$$

$$\Rightarrow cd(f-e) = ef(d-c) \quad \dots (2)$$

From (1) and (2), we have

$$\frac{a-b}{f-e} = -\frac{ab}{ef}$$

$$\Rightarrow aef - bef = -abf + aeb$$

$$\Rightarrow aef + abf = aeb + bef$$

$$\Rightarrow af(b+c) = be(a+f)$$

$$\Rightarrow (a, b)R(e, f)$$

$$\therefore (a, b)R(c, d) \text{ and } (c, d)R(e, f) \Rightarrow (a, b)R(e, f) \text{ for all } (a, b), (c, d), (e, f) \in \mathbb{N} \times \mathbb{N}$$

Hence, R is transitive.

Thus, R being reflexive, symmetric and transitive, is an equivalence relation on $\mathbb{N} \times \mathbb{N}$.



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one-one

Let $x_1, x_2 \in \mathbb{R}^+$ and $f(x_1) = f(x_2)$

$$9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$$

$$9x_1^2 + 6x_1 = 9x_2^2 + 6x_2 \text{ (removing -5 from both side)}$$

$$3x_1^2 + 2x_1 = 3x_2^2 + 2x_2 \text{ (dividing 3 on both side)}$$

$$3x_1^2 - 3x_2^2 + 2x_1 - 2x_2 = 0$$

$$\Rightarrow 3(x_1^2 - x_2^2) + 2(x_1 - x_2) = 0$$

$$3(x_1 - x_2)(x_1 + x_2) + 2(x_1 - x_2) = 0$$

$$x_1 - x_2 = 0 \text{ (OR) } x_1 + x_2 + 2 = 0$$

$$\text{Consider } x_1 + x_2 + 2 = 0 \Rightarrow x_1 = -x_2 - 2$$

As $x_1, x_2 \in \mathbb{R}^+$ (non-negative real numbers), it $(x_1 + x_2 + 2 = 0)$ can not be possible. So $x_1 - x_2 = 0 \Rightarrow x_1 = x_2$

Hence it is one-one.

THINK BEYOND

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onto

Let $y = f(x)$ and $y \in [-5, \infty]$

$$\text{So } y = 9x^2 + 6x - 5$$

$$9x^2 + 6x - 5 - y = 0$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 9 \times (-y-5)}}{2 \times 9} \quad (\text{by using formula } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ for quadratic equation } ax^2 + bx + c = 0)$$

$$x = \frac{-6 \pm \sqrt{36 + 36(y+5)}}{18}$$

$$\Rightarrow x = \frac{-6 \pm 6\sqrt{(y+6)}}{18}$$

x has two solution $x = \frac{-1 - \sqrt{y+6}}{3}$ and $x = \frac{-1 + \sqrt{y+6}}{3}$. We need to prove either one solution (x) exists for every $y \in [-5, \infty)$. In the solution $x = \frac{-1 - \sqrt{y+6}}{3}$, x is the negative number for any y in the given range. Now we left with $x = \frac{-1 + \sqrt{y+6}}{3}$. For $y = -5$, $x = 0$ and any value greater than -5 x is always non negative real numbers. It proves that every value in the range $y \in [-5, \infty)$ there exists a value in domain $\rightarrow x \in \mathbb{R}^+$. Hence it is onto.

So the function is bijective (one-one and onto).

End

