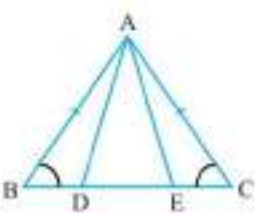
ANSWER KEY

1	d
2	B
3	B
4	A
5	B
6	B
7	B
8	a
9	a
10	d
11	**Given:** In the following figure, BA ⊥ AC, DE ⊥ DF such that BA = DE and BF = EC. **To show:** ΔABC ≅ ΔDEF **Proof:** Since, BF = EC On adding CF both sides, we get BF + CF = EC + CF ⇒ BC = EF ...(i) In ΔABC and ΔDEF, ∠A = ∠D = 90° ...[∵ BA ⊥ AC and DE ⊥ DF] BC = EF ...[From equation (i)] And BA = DE ...[Given] ∴ ΔABC ≅ ΔDEF ...[By RHS congruence rule] Or **SOLUTION** ∠BCD = ∠ADC ∠ACB = ∠BDA ∠BCD + ∠ACB = ∠ADC + ∠BDA ⇒ ∠ACD = ∠BDCACD = BDC In ΔACD and ΔBCD ∠ACD =∠BDCACD = BDC ∠ADC = ∠BCD ADC = BCD CD = CD Therefore, ΔACD ≅ ΔBCD ...(ASA criteria) Hence, AD = BC and ∠A = ∠B.
12	<u>Given:</u> Δ ABC is isosceles, So, AB = AC Also, BE = CD 

To prove: $AD = AE$

Proof:

Since

$AB = AC$

Therefore, $\angle C = \angle B$ *(Angles opposite to equal sides are equal)* ...(1)

In ΔACD and ΔABE,

$AC = AB$ *(Given)*

$\angle C = \angle B$ *(From (1))*

$CD = BE$ *(Given)*

So, $\Delta ACD \cong \Delta ABE$ *(SAS congruence rule)*

$\therefore AD = AE$ *(CPCT)*

Hence proved

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Given:

$AB = AC$

Also, $AD = AB$

i.e. $AC = AB = AD$

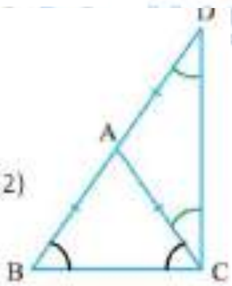
To prove: $\angle BCD = 90°$

Proof:

In ΔABC,

$AB = AC$

$\Rightarrow \angle ACB = \angle ABC$ *(Angles opposite to equal sides are equal)* ...(1)



In ΔACD,

$AC = AD$

$\angle ADC = \angle ACD$ (Angles opposite to equal sides are equal) ...(2)

In ΔBCD,

$\angle ABC + \angle BCD + \angle BDC = 180°$ *(Angle sum property of triangle)*

$\angle ACB + \angle BCD + \angle ACD = 180°$ *(From (1) & (2))*

$(\angle ACB + \angle ACD) + \angle BCD = 180°$

$(\angle BCD) + \angle BCD = 180°$

$2\angle BCD = 180°$

$\angle BCD = \frac{180°}{2}$

$\angle BCD = 90°$

Or

Given:

Given BE is a altitude,

So, $\angle AEB = \angle CEB = 90^\circ$ ...(1)

Also, CF is a altitude,

So, $\angle AFC = \angle BFC = 90^\circ$ ...(2)

Also, BE = CF ...(3)

To prove: Δ ABC is isosceles

Proof:

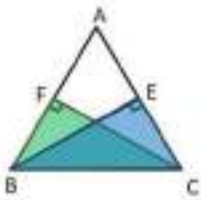
In ΔBCF and ΔCBE

$\angle BFC = \angle CEB = 90^\circ$ (Both 90°)

BC = CB (Common)

FC = EB (From (3))

$\Delta BCF \cong \Delta CBE$ (RHS congruence rule)



$\therefore \angle FBC = \angle ECB$ (CPCT)

So, $\angle ABC = \angle ACB$

AB = AC (Sides opposite to equal angles is equal)

So, ΔABC is an isosceles triangle

14

Given: In a ΔABC, D, E and F are respectively the mid-points of the sides AB, BC and CA.

To prove: ΔABC is divided into four congruent triangles.

Proof: Since, ABC is a triangle and D, E and F are the mid-points of sides AB, BC and CA, respectively.

Then, $AD = BD = \frac{1}{2}AB$, $BE = EC = \frac{1}{2}BC$

And $AF = CF = \frac{1}{2}AC$

Now, using the mid-point theorem,

$EF \parallel AB$ and $EF = \frac{1}{2}AB = AD = BD$

$ED \parallel AC$ and $ED = \frac{1}{2}AC = AF = CF$

And $DF \parallel BC$ and $DF = \frac{1}{2}BC = BE = CE$

In ΔADF and ΔEFD,

AD = EF

AF = DE

And DF = FD ...[Common]

∴ ΔADF ≅ ΔEFD ...[By SSS congruence rule]

Similarly, ΔDEF ≅ ΔEDB

And ΔDEF ≅ ΔCFE

So, ΔABC is divided into four congruent triangles.

Hence proved.

Or

Given:

AB = AC ...(1)

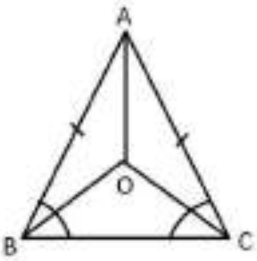
OB is the bisector of ∠B

So, $\angle ABO = \angle OBC = \frac{1}{2}\angle B$ ...(2)

OC is the bisector of ∠C

So, $\angle ACO = \angle OCB = \frac{1}{2}\angle C$ ...(3)

TO Prove:OB=OC



Proof:

Since,

AB = AC *(Angles opposite to equal sides are equal)*

⇒ ∠ACB = ∠ABC

$\frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$ (From (2) & (3))

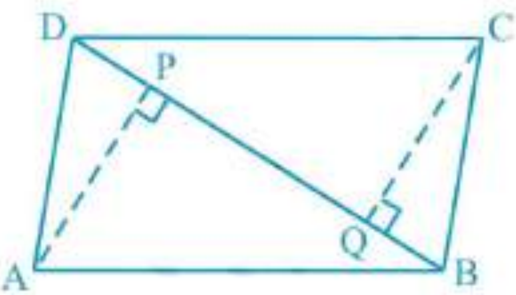
$\angle OCB = \angle OBC$

Hence, *(Sides opposite to equal angles are equal)*

OB = OC

Hence proved

15 **Solution:**



Given: ABCD is a parallelogram and AP ⊥ DB, CQ ⊥ DB

Proof :

(i) In $\triangle APD$ and $\triangle CQB$, we have

AD = BC [Opposite sides of a ||gm]

DP = BQ [Given]

$\angle ADP = \angle CBQ$ [Alternate angles]

$\therefore \triangle APD \cong \triangle CQB$ [SAS congruence]

(ii) $\therefore$ AP = CQ [CPCT]

(iii) In $\triangle AQB$ and $\triangle CPD$, we have

AB = CD [Opposite sides of a ||gm]

DP = BQ [Given]

$\angle ABQ = \angle CDP$ [Alternate angles]

$\therefore \triangle AQB \cong \triangle CPD$ [SAS congruence]

(iv) $\therefore$ AQ = CP [CPCT]

(v) Since in APCQ, opposite sides are equal, therefore it is a parallelogram. **Proved.**

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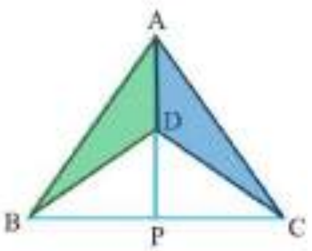
Given 1) $\triangle PQR$, Iso. $\triangle$ such that $PQ = PR$
2) $QT = RS$

(i) As $PQ = PR$
$\Rightarrow \angle PQR = \angle PRQ$ (Iso. $\triangle$ prop)
angle opp to equal sides are equal
$\therefore \angle PQR = 70^\circ$
By ASP $\angle QPR = 180 - 70 - 70 = 40^\circ$

(ii) $\angle PST = 80^\circ$, $\angle PQS = 70^\circ$ (above shown)
By ext. angle prop
$\angle PST = \angle PQS + \angle QPS$
$80^\circ = 70 + \angle QPS$
$\angle QPS = 10^\circ$

(a) In $\triangle PQT$ & $\triangle PRS$
$PQ = PR$ given
$\angle Q = \angle R$ (Iso. prop)
$QS = TR$ given
$QS + ST = TR + ST$
$QT = SR$
By SAS cong rule
$\triangle PQT \cong \triangle PRS$

(b)
$\triangle PQS$ & $\triangle PRT$
$PQ = PR$ given
$\angle Q = \angle R$ (Iso. prop)
$QS = TR$ given
By SAS cong. rule
$\triangle PQS \cong \triangle PRT$.

17	Given: ABCD is ‖gm (i) $\angle D = \angle P = 110$ (opp angle of ‖gm) $\angle x + \angle ADC = 180^\circ$ linear pair. $\Rightarrow \angle x = 180 - 110$ $= 70^\circ$ (ii) $\angle CPQ = 180 - 110^\circ$ $= 70^\circ$ linear pair $\angle Q = 90^\circ$ given. By A.S.P $m = 180 - 90 - 70$ $= 20^\circ$ (iii)(a) ΔAPD & ΔCQB. $AD = BC$ (opp side of ‖gm) $\angle PDA = \angle CBQ = 70^\circ$ (above shown) $\angle P = \angle Q = 90^\circ$ By AAS Congr. rule $\Delta APD \cong \Delta CQB$ or ΔAQC & ΔCPA $\angle QAC = \angle PCA$ (alt angle AB ‖ CD) $AC = AC$ (common side) $\angle P = \angle Q = 90^\circ$ By AAS Cong. rule $\Delta AQC \cong \Delta CPA$.
18	(i) $\Delta ABD \cong \Delta ACD$  Given: ΔABC is isosceles, $AB = AC$ ...(1) Also, ΔDBC is isosceles, $DB = DC$ ...(2) To prove: $\Delta ABD \cong \Delta ACD$

In ΔABD and Δ DBC, we have

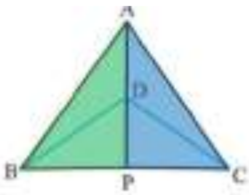
AB = AC *(From (1))*

BD = DC *(From (2))*

AD = AD *(Common)*

Δ ABD ≅ Δ ACD *(SSS congruence rule)*

(ii) Δ ABP ≅ Δ ACP



From part (i),

Δ ABD ≅ Δ ACD

So, ∠BAP = ∠PAC *(CPCT)* ...(1)

In ΔABP and Δ ACP,

AB = AC *(Given)*

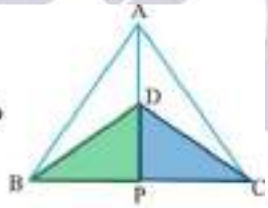
∠BAP = ∠PAC *(From (1))*

AP = AP *(Common)*

Δ ABP ≅ Δ ACP. *(SAS congruence rule)*

(iii)(a)

(iii) AP bisects $\angle A$ as well as $\angle D$.



To prove: $\angle BAD = \angle CAD$ & $\angle BDP = \angle CDP$

Proof:

From part (i) , Δ ABD ≅ Δ ACD

So, $\angle BAD = \angle CAD$. *(CPCT)*

Hence AP bisects ∠ A

For $\angle BDP = \angle CDP$,

we will first prove Δ BDP ≅ Δ CDP

From part(ii) , Δ ABP ≅ Δ ACP

In Δ BDP and CDP, we have

BD = CD *(Given)*

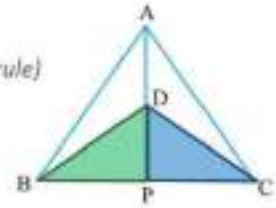
BP = CP *(From (1))*

DP = DP *(Common)*

So, Δ BDP ≅ Δ CDP *(SSS congruence rule)*

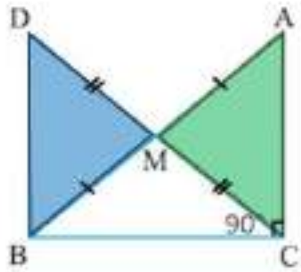
⇒ $\angle BDP = \angle PDC$ *(CPCT)*

Thus, AP bisects ∠D.

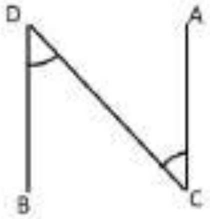


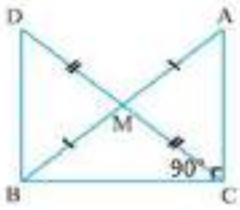
Or

(iii)(b)

	Proof From part(ii), Δ ABP $\cong$ Δ ACP BP = CP (CPCT) $\angle APB = \angle APC$ (CPCT) Since BC is a line, $\therefore \angle APB + \angle APC = 180^\circ$ (Linear Pair) $\angle APB + \angle APB = 180^\circ$ $2\angle APB = 180^\circ$ $\angle APB = \frac{180^\circ}{2}$ $\angle APB = 90^\circ$ So, $\angle APB = \angle APC = 90^\circ$ Since, BP = CP & $\angle APB = \angle APC = 90^\circ$ $\Rightarrow$ AP is perpendicular bisector of BC.
19	(i) ΔAMC $\cong$ ΔBMD  Given: $\angle$ ACB = 90° M is the mid-point of AB So, AM = BM ...(1) Also, DM = CM ...(2) To prove: ΔAMC $\cong$ ΔBMD Proof: Lines CD & AB intersect... So, $\angle$AMC = $\angle$BMD (Vertically opposite angles) ...(3) In ΔAMC and ΔBMD, AM = BM (From (1)) $\angle$AMC = $\angle$BMD (From (3)) CM = DM (From (2)) $\therefore$ ΔAMC $\cong$ ΔBMD (SAS congruence

(ii) ∠DBC is a right angle.





From part 1,

$\Delta AMC \cong \Delta BMD$

$\therefore \angle ACM = \angle BDM$ (CPCT)

But $\angle ACM$ and $\angle BDM$ are alternate interior angles

for lines AC & BD

If a transversal intersects two lines such that pair of alternate interior angles is equal, then lines are parallel.

So, BD || AC

Now, Since

DB || AC and Considering BC as transversal

, $\Rightarrow \angle DBC + \angle ACB = 180°$ *(interior angles on the same side of transversal are supplementary)*

$\Rightarrow \angle DBC + 90° = 180°$

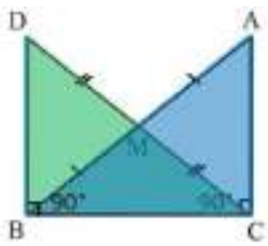
$\Rightarrow \angle DBC = 180° - 90°$

$\Rightarrow \angle DBC = 90°$

Hence, ∠ DBC is a right angle

Hence proved

(iii) $\Delta DBC \cong \Delta ACB$



From part 1,

$\Delta AMC \cong \Delta BMD$

AC = BD *(CPCT)* ...(1)

In ΔDBC and ΔACB,

DB = AC *(From (1))*

$\angle DBC = \angle ACB$ *(Both 90°)*

BC = CB *(Common)*

$\therefore \Delta DBC \cong \Delta ACB$ *(SAS congruence rule)*

Or

	From part 3, $\Delta DBC \cong \Delta ACB$ $\therefore DC = AB$ (CPCT) $\frac{1}{2} DC = \frac{1}{2} AB$ $\boxed{\begin{array}{l} \text{As } DM = CM \\ \therefore DM = CM = \frac{1}{2} DC \end{array}}$ $CM = \frac{1}{2} AB$ Hence proved

____________End_____________