CBSE CHAPTER WISE QUESTION CLASS- XII MATHEMATICS 2025

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PREFACE

I am delightful to present this Chapter-wise Collection of Class XII CBSE Previous Year Questions of year 2025. This document covers all the 19 sets of question papers of the year 2025. This document represents the fruits of a collaborative labor of love between myself and my students, as we worked together to compile a comprehensive collection of previous year's questions.

I am convinced that this resource will be a game-changer for CBSE Class XII Mathematics students, offering them a unique window into the exam pattern, question types, and key concepts that are tested in the CBSE board exams. By delving into these questions, students will gain a richer understanding of the subject and develop the skills and confidence needed to excel in their exams.

I hope that this document will be a valuable companion for students, teachers, and educators, and that it will contribute to their academic success. I wish all the students the very best as they embark on their academic journeys, and I have no doubt that they will achieve great things.

JITENDER PRASAD

PGT MATHEMATICS

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CHAPTER -1 RELATION AND FUNCTION

Q.		Code and Marks
1	**Assertion (A)** : Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5$, $\forall x \in Z$ is a bijective. **Reason (R)** : A function is a bijective if it is both surjective and injective. (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	65/1/1 65/1/2 65/1/3 1mark
2	A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$: $R_1 = \{(2, 3), (3, 2)\}$ $R_2 = \{(1, 2), (1, 3), (3, 2)\}$ $R_3 = \{(1, 2), (2, 1), (1, 1)\}$ $R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$ $R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$ The students are asked to answer the following questions about the above relations : (i) Identify the relation which is reflexive, transitive but not symmetric. (ii) Identify the relation which is reflexive and symmetric but not transitive. (iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive. **OR** (iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation ?	65/1/1 65/1/2 65/1/3 4mark
3	Let R be a relation defined over N, where N is set of natural numbers, defined as "mRn if and only if m is a multiple of n, m, n $\in$ N." Find whether R is reflexive, symmetric and transitive or not.	65/2/1 3mark

4	A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.  Based on the above, answer the following : (i) How many relations can be there from S to J ? **1** (ii) A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$ Check if it is bijective. **1** (iii) (a) How many one-one functions can be there from set S to set J ? **2** **OR** (iii) (b) Another student considers a relation $R_1 = \{(S_1, S_2), (S_2, S_4)\}$ in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. **2**	65/2/1 65/2/2 65/2/3 4mark
5	Prove that $f : N \to N$ defined as $f(x) = ax + b$ $(a, b \in N)$ is one-one but not onto.	65/2/2 3mark
6	Let R be a relation on set of real numbers $\mathbb{R}$ defined as $\{(x, y) : x - y + \sqrt{3}$ is an irrational number, $x, y \in \mathbb{R}\}$ Verify R for reflexivity, symmetry and transitivity.	65/2/3 3mark
7	For real x, let $f(x) = x^3 + 5x + 1$. Then : (A) f is one-one but not onto on R (B) f is onto on R but not one-one (C) f is one-one and onto on R (D) f is neither one-one nor onto on R	65/4/1 65/4/2 65/4/3 1 mark

8	(a) If $f : R^+ \to R$ is defined as $f(x) = \log_a x$ ($a > 0$ and $a \neq 1$), prove that f is a bijection. (R^+ is a set of all positive real numbers.) **OR** (b) Let A = {1, 2, 3} and B = {4, 5, 6}. A relation R from A to B is defined as $R = \{(x, y) : x + y = 6, x \in A, y \in B\}$. (i) Write all elements of R. (ii) Is R a function ? Justify. (iii) Determine domain and range of R.	65/4/1 65/4/2 65/4/3 3mark
9	If $f : N \to W$ is defined as $f(n) = \begin{cases} \frac{n}{2}, & \text{if n is even} \\ 0, & \text{if n is odd} \end{cases}$, then f is : (A) injective only (B) surjective only (C) a bijection (D) neither surjective nor injective	65/4/2 65/4/3 1 mark
10	*Assertion (A) :* Let $A = \{x \in R : -1 \le x \le 1\}$. If $f : A \to A$ be defined as $f(x) = x^2$, then f is not an onto function. *Reason (R) :* If $y = -1 \in A$, then $x = \pm\sqrt{-1} \notin A$.	65/5/1 65/5/2 65/5/3 1 mark
11	Let A be the set of 30 students of class XII in a school. Let $f : A \to N$, N is a set of natural numbers such that function f(x) = Roll Number of student x. On the basis of the given information, answer the following : (i) Is f a bijective function ? (ii) Give reasons to support your answer to (i). (iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where R = {(x, y) : x, y are Roll Numbers of students such that y = 3x}. List the elements of R. Is the relation R reflexive, symmetric and transitive ? Justify your answer. **OR** (iii) (b) Let R be a relation defined by R = {(x, y) : x, y are Roll Numbers of students such that $y = x^3$}. List the elements of R. Is R a function ? Justify your answer.	65/5/1 65/5/2 65/5/3 4 mark

12	**20.** *Assertion (A) :* Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f+g)\,x = e^x + \log x$ where domain of $(f+g)$ is R. *Reason (R) :* $\text{Dom}(f+g) = \text{Dom}(f) \cap \text{Dom}(g)$.	65/6/1 65/6/2 1 mark
13	**28.** (a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not. **OR** (b) Show that the function $f : N \to N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n-1, & \text{if n is even} \\ n+1, & \text{if n is odd} \end{cases}$ is a bijection.	65/6/1 65/6/2 65/6/3 3 mark
14	Let $f : A \to B$ be defined by $f(x) = \dfrac{x-2}{x-3}$, where $A = R - \{3\}$ and $B = R - \{1\}$. Discuss the bijectivity of the function.	65/7/1 65/7/2 65/7/3 2mark
15	(a) Show that the function $f : R \to R$ defined by $f(x) = 4x^3 - 5$, $\forall\, x \in R$ is one-one and onto. **OR** (b) Let R be a relation defined on a set N of natural numbers such that $R = \{(x, y) : xy \text{ is a square of a natural number, } x, y \in N\}$. Determine if the relation R is an equivalence relation.	65/7/1 65/7/2 65/7/3 3mark
16	If R be a relation defined as $a R b$ iff $\|a - b\| > 0$ $a, b \in \mathbb{R}$ then R is : (A) reflexive (B) symmetric (C) transitive (D) symmetric and transitive	65/7/2 1mark
17	Which of the following functions from Z to Z is both one-one and onto ? (A) $f(x) = 2x - 1$ (B) $f(x) = 3x^2 + 5$ (C) $f(x) = x + 5$ (D) $f(x) = 5x^3$	65(B) 1 mark
18	Rajesh, a student of Class-XII, visited an exhibition with his family. There he saw a huge swing and found that it traced the path of a parabola $y = x^2$. The following questions came to his mind. Answer the questions : (i) Let $f : R \to R$ be a function defined as $f(x) = x^2$. Find whether f is one-one function.	65(B) 4mark

	(ii) Let $f : R \to R$ be defined as $f(x) = x^2$. Find whether f is an onto function. (iii) (a) Let $f : N \to N$ be defined as $f(x) = x^2$. Find whether f is one-one function. Also, find if it is an onto function. **OR** (iii) (b) Let $f : N \to \{1, 4, 9, 16, ...\}$ defined as $f(x) = x^2$, find where f is one-one function. Also, find if it is an onto function.	

Q.	**ANSWERS**
1	(D) Assertion (A) is false, but Reason (R) is true.
2	**(i) R_4** **(ii) R_5** **(iii)(a) R_1 and R_3** **OR** **(iii)(b) Required pairs to be added to make the relation R_2 as an equivalence relation are:** **$(1,1),(2,2),(3,3),(2,1),(3,1)$ and $(2,3)$**
3	Let $x \in N$. Then we know that x is a multiple of itself. $\Rightarrow xRx$ Hence, R is reflexive. We have $2,8 \in N$ such that 8 is a multiple of 2 $\Rightarrow 8R2$ But, 2 is not a multiple of 8. Hence, 2 is not R-related to 8. Therefore, R is not symmetric. Let $x, y, z \in N$ such that xRy, yRz Then $x=my$, $y=nz$ for some m, $n \in N$ $\Rightarrow x=mnz \Rightarrow x=pz$, where $p=mn \in N$. Hence, xRz Therefore, R is transitive.
4	(i) The number of relations = 24×3=212 (ii) Since, S2 and S3 have been assigned the same judge $J2$, the function is not one-one. Hence, it is not bijective. (iii)(a) There cannot exist any one-one function from S to J as n(S) > n(J). Hence, the number of one-one functions from S to J is 0. OR (b) To make $R1$ reflexive and not symmetric we need to add the following ordered pairs: ($S1$, $S1$), ($S2$, $S2$), ($S3$, $S3$), ($S4$, $S4$)
5	Let x1, x2∈N (Domain) such that f(x1) =f(x2) ⇒ax1+b=ax2+b⇒x1=x2 Therefore, f is one-one. Let y ∈ N (codomain). Then f(x)=y if, ax + b=y i.e., if, x = y−ba, which may not belong to N (domain) Therefore, f is not onto
6	Let x ∈ ℝ. Then we know that x−x+√3=√3, which is an irrational number. ⇒ (x, x ∈R Hence, R is reflexive.

	We have $\sqrt{3},2\in\mathbb{R}$ such that $\sqrt{3}-2+\sqrt{3}=2(\sqrt{3}-1)$, which is an irrational number $\Rightarrow (\sqrt{3},2)\in R$. But, $2-\sqrt{3}+\sqrt{3}=2$, which is a rational number. Hence, $\Rightarrow (2,\sqrt{3})\notin R$. . Therefore, R is not symmetric. Let $-\sqrt{3}, \sqrt{3}, 2\in\mathbb{R}$ such that $(-\sqrt{3},\sqrt{3}), (\sqrt{3},2)\in R$. But, $(-\sqrt{3},2)\notin R$ Therefore, R is not transitive
7	(C) f is one-one and onto on R
8	$f(x)=\log_a x \quad (a>0, a\neq 1)$ Let $x_1, x_2 \in R^+$ such that $f(x_1)=f(x_2)$ $\Rightarrow \log_a x_1 = \log_a x_2$ $\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one. Let $f(x)=y \Rightarrow \log_a x = y \Rightarrow a^y = x$ $\therefore$ for every $y\in R$, there exists $x\in R^+$ $\therefore f$ is onto. f is a bijection. OR $(i)\ R=\{(1,5),(2,4)\}$ $(ii)\ R$ is not a function as $3\in A$ do not have an image in co-domain. (iii) Domain of $R=\{1,2\}$, Range of $R=\{4,5\}$
9	(B) surjective only
10	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**
11	**(i) No, f is not bijective function** **(ii) Range = $\{1,2,3,4,\ldots\ldots\ldots\ldots,30\}$ and codomain= N** **Since, Range $\neq$ codomain $\Rightarrow$ f is not onto and hence f is not bijective.** **(iii) (a)** $R=\{(1,3),(2,6),(3,9),(4,12),(5,15),(6,18),(7,21),(8,24),(9,27),(10,30)\}$ **Since $(1,1)\notin R \implies$ R is not reflexive.** **$(1,3)\in R$ but $(3,1)\notin R \implies$ R is not symmetric** **$(1,3)\in R, (3,9)\in R$ but $(1,9)\notin R \implies$ R is not transitive.** **OR** **(iii) (b) $R=\{(1,1),(2,8),(3,27)\}$** **$\because$ elements 4, 5, 6 … 30 do not have an image. Hence the above relation is not a function.**

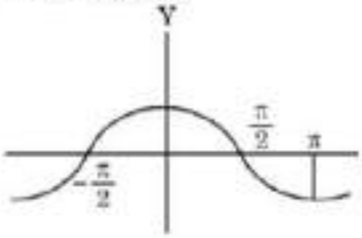
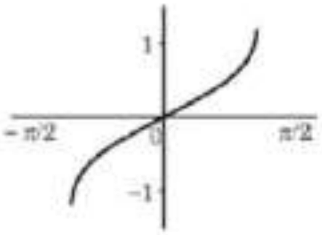
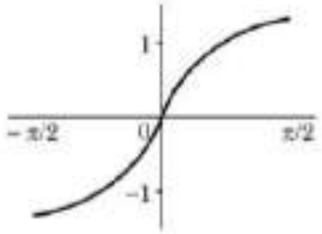
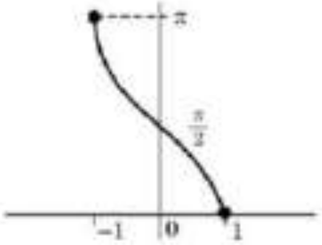
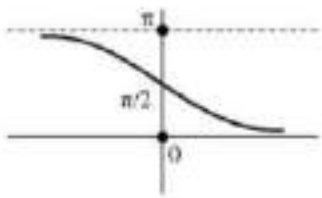
13	(a) R = {(1,39), (2, 37),_ _ _ ,(20, 1)} Domain = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20} Range ={1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39} (1, 1) does not belong to R hence not reflexive (1, 39) belongs to R but (39, 1) does not belong to R hence not symmetric (11, 19) and (19, 3) belong to R but (11, 3) does not belong to R hence not transitive. Hence R is not an equivalence relation. OR (a) Let f(x) = f(y) Let x and y are both odd or both even Then either x+1 = y + 1 or x-1 = y-1 gives x = y x odd and y even is rejected as x + 1 = y -1 gives x – y = -2 not possible as odd number and even number cannot differ by 2 Hence f is one-one For onto: Let f(x) = y gives x = y + 1 or x = y - 1 If y is odd, x is even and if y is even, x is odd. Range = N = co-domain, hence onto As f is both one-one and onto hence bijective
14	Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3} \Rightarrow x_1 = x_2$, $\therefore$ 'f' is one-one. For each $y \in B$, there exists $x = \frac{3y-2}{y-1} \in R - \{3\}$, such that $f(x) = y$, $\therefore$ 'f' is onto $\Rightarrow$ 'f' is one-one & onto, or 'f' is a bijective function.
15	**(a) One-One:** Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2) \Rightarrow 4x_1^3 - 5 = 4x_2^3 - 5 \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$, $\therefore$ 'f' is one-one **Onto:** $x \in R\ (D_f) \Rightarrow x^3 \in R \Rightarrow 4x^3 - 5 \in R \Rightarrow f(x) \in R$, $\therefore$ $R_f = Co-domain(f)$ $\therefore$ 'f' is an onto function $\Rightarrow$ 'f' is one-one & onto both **OR** **(b) Reflexive:** For any $x \in N$, $x \cdot x = x^2$, which is square of the natural number 'x'. $\Rightarrow (x, x) \in R$ $\therefore$ 'R' is a Reflexive relation.

	Symmetric: Let $(x,y) \in R \Rightarrow xy$ is a square of a natural number $\Rightarrow yx$ is a square of a natural number, $\because xy = yx$. $\Rightarrow (y,x) \in R$ $\therefore$ 'R' is a Symmetric relation. Transitive: Let $(x,y),(y,z) \in R \Rightarrow xy = a^2, yz = b^2$ for some $a,b \in N$, $\therefore \frac{a^2}{y} = x, \frac{b^2}{y} = z \in N$ $\Rightarrow xz = \frac{a^2}{y} \cdot \frac{b^2}{y} = \left(\frac{ab}{y}\right)^2, \frac{ab}{y} \in N$ $\Rightarrow (x,z) \in R$ $\therefore$ 'R' is a Transitive relation. Hence, R is an Equivalence relation
16	**(B) symmetric**
17	**(C) f(x)=x+5**
18	(i) $f: R \to R$ $f(x) = x^2$ For $x_1 = -1$ *and* $x_2 = 1$ *we have* $f(x_1) = f(x_2) = 1$ Hence f is not a one -one function (ii) $f: R \to R$ $f(x) = x^2$ *for*, $y = -4 \in R(Codomain)$ there does not exist any $x \in R$ such that f(x) = -4 So f is not onto (iii) (a) $f: N \to N$ $f(x) = x^2$ *Let*, $f(x_1) = f(x_2)$ *for some* $x_1, x_2 \in N$ $\Rightarrow {x_1}^2 = {x_2}^2$ $\Rightarrow x_1 = x_2$ Hence , f is one- one *For*, $y = 5 \in N$ (*codomain*) there does not exist any x(*domain*) such that f(x)=5 So f is not onto

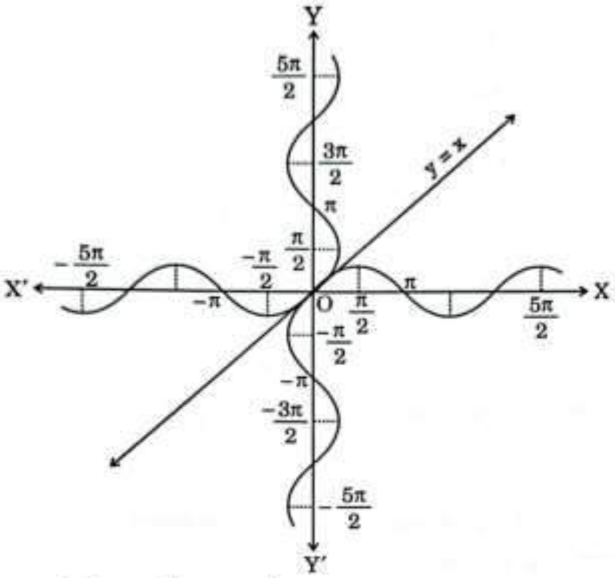
OR

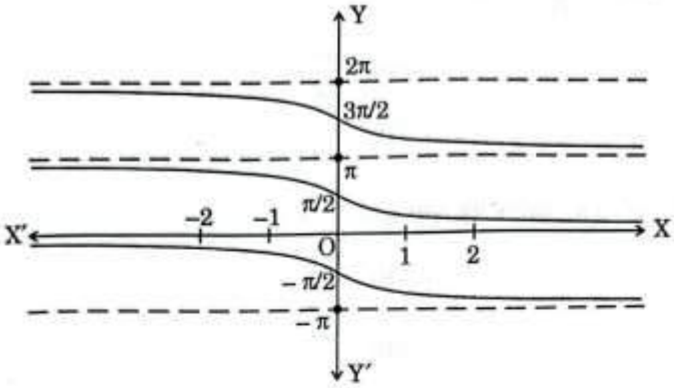
(iii)(b) $f: N \rightarrow \{1, 4, 9, 16 \ldots\}$
$f(x) = x^2$
Let $f(x_1) = f(x_2)$
$\Rightarrow {x_1}^2 = {x_2}^2$
$\Rightarrow x_1 = x_2$
Hence f is one- one
$\forall y \in \{1, 4, 9, 16, \ldots\}$ there exist $x \in N$ such that f(x)=y
$\therefore$ f is onto

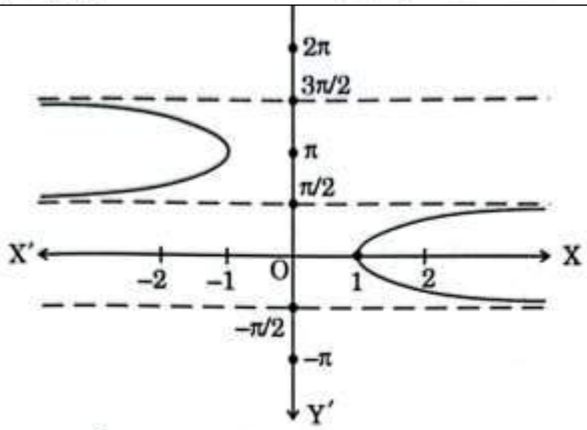
CHAPTER 2 INVERSE TRIGONOMETRIC FUNCTION

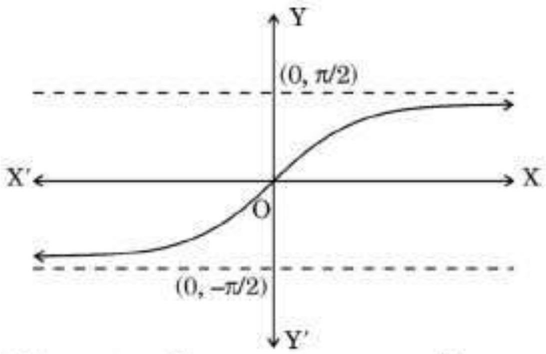
Q.		Code and Marks
1	The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse ?  (A)  (B)  (C)  (D) 	65/1/1 65/1/2 65/1/3 (1m)
2	Evaluate : $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$	65/1/1 (2m)
3	Evaluate : $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$	65/1/2 (2m)
4	Solve for x, $2\tan^{-1}x+\sin^{-1}\left(\frac{2x}{1+x^2}\right)=4\sqrt{3}$	65/1/3 (2m)
5	If $y=\sin^{-1}x$, $-1\le x\le 0$, then the range of y is (A) $\left(\frac{-\pi}{2},0\right)$ (B) $\left[\frac{-\pi}{2},0\right]$ (C) $\left[\frac{-\pi}{2},0\right)$ (D) $\left(\frac{-\pi}{2},0\right]$	65/2/1 (1m)

6	The principal value of $\sin^{-1}\left(\cos\frac{43\pi}{5}\right)$ is (A) $\frac{-7\pi}{5}$ (B) $\frac{-\pi}{10}$ (C) $\frac{\pi}{10}$ (D) $\frac{3\pi}{5}$	65/2/2 (1m)
7	The value of $\cos\left(\frac{\pi}{6}+\cot^{-1}(-\sqrt{3})\right)$ is (A) -1 (B) $\frac{-\sqrt{3}}{2}$ (C) 0 (D) 1	65/2/3 (1m)
8	The principal value of $\sin^{-1}\left(\sin\left(-\frac{10\pi}{3}\right)\right)$ is : (A) $-\frac{2\pi}{3}$ (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$	65/4/1 (1m)
9	(a) Simplify $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$. **OR** (b) Find domain of $\sin^{-1}\sqrt{x-1}$.	65/4/1 65/4/2 65/4/3 (2m)
10	The principal branch of $\cos^{-1}x$ is : (A) $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ (B) $[\pi, 2\pi]$ (C) $[0, \pi]$ (D) $[2\pi, 3\pi]$	65/4/2 (1m)

11	Domain of $\sin^{-1}(2x^2-3)$ is : (A) $(-1, 0) \cup (1, \sqrt{2})$ (B) $(-\sqrt{2}, -1) \cup (0, 1)$ (C) $[-\sqrt{2}, -1] \cup [1, \sqrt{2}]$ (D) $(-\sqrt{2}, -1) \cup (1, \sqrt{2})$	65/4/3 (1m)
12	The principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is : (A) $-\frac{\pi}{3}$ (B) $-\frac{2\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$	65/5/1 65/5/2 65/5/3 (1m)
13	**Find the domain of the function $f(x) = \cos^{-1}(x^2-4)$.**	65/5/1 (2m)
14	Find the domain of $\sec^{-1}(2x+1)$.	65/5/2 (2m)
15	Find the domain of $\sin^{-1}(x^2-3)$.	65/5/3 (2m)
16	**The following graph is a combination of :**  (A) $y = \sin^{-1} x$ and $y = \cos^{-1} x$ (B) $y = \cos^{-1} x$ and $y = \cos x$ (C) $y = \sin^{-1} x$ and $y = \sin x$ (D) $y = \cos^{-1} x$ and $y = \sin x$	65/6/1 (1m)
17	$\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ is equal to : (A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$ (C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$	65/6/1 65/6/2 65/6/3 (1m)
18	Find the domain of $f(x) = \sin^{-1}(-x^2)$.	65/6/1 65/6/2 65/6/3 (2m)

<table>
<tr><td>19</td><td>The graph shown below depicts :


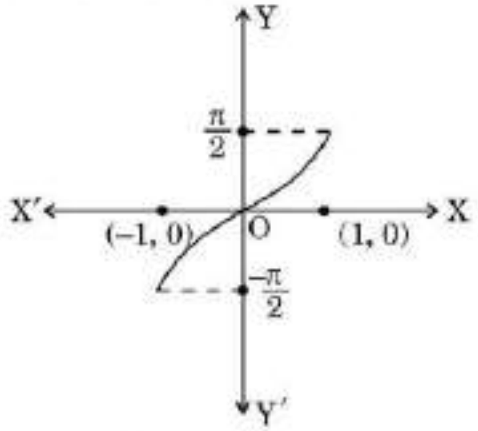
(A) $y = \cot x$ (B) $y = \cot^{-1} x$
(C) $y = \tan x$ (D) $y = \tan^{-1} x$</td><td>65/6/2
(1m)</td></tr>
<tr><td>20</td><td>

(A) $y = \sec^{-1} x$ (B) $y = \sec x$
(C) $y = \mathrm{cosec}^{-1} x$ (D) $y = \mathrm{cosec}\, x$</td><td>65/6/3
(1m)</td></tr>
<tr><td>21</td><td>The given graph illustrates :


(A) $y = \tan^{-1} x$ (B) $y = \mathrm{cosec}^{-1} x$
(C) $y = \cot^{-1} x$ (D) $y = \sec^{-1} x$</td><td>65/7/1
(1m)</td></tr>
<tr><td>22</td><td>Domain of $f(x) = \cos^{-1} x + \sin x$ is :
(A) R (B) $(-1, 1)$
(C) $[-1, 1]$ (D) ϕ</td><td>65/7/1
65/7/2
65/7/3
(1m)</td></tr>
<tr><td>23</td><td>Assertion (A) : Set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set.

Reason (R) : $\sec^{-1} x$ is defined for $x \in R - (-1, 1)$.</td><td>65/7/1
65/7/2
65/7/3
(1m)</td></tr>
</table>

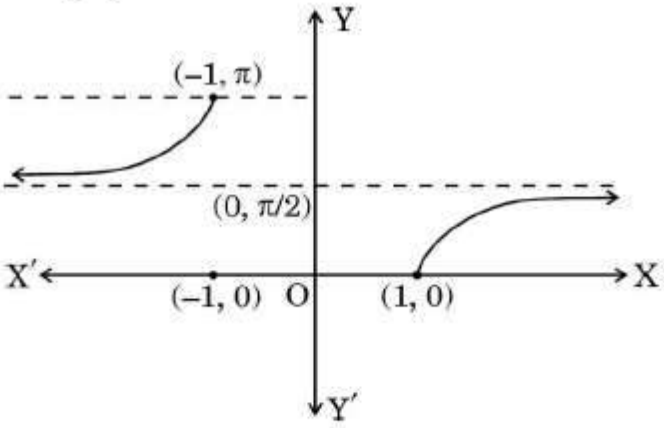
24 Study the given graph. It illustrates : — 65/7/2 (1m)



(A) $y = \tan^{-1} x$ (B) $y = \cos^{-1} x$

(C) $y = \sec^{-1} x$ (D) $y = \sin^{-1} x$

25 The given graph illustrates : — 65/7/3 (1m)



(A) $y = \sec^{-1} x$ (B) $y = \cot^{-1} x$

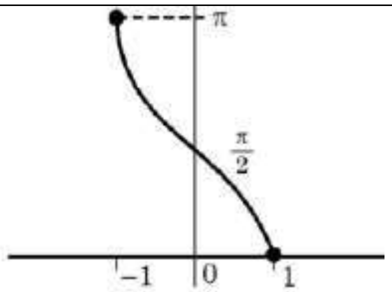
(C) $y = \tan^{-1} x$ (D) $y = \operatorname{cosec}^{-1} x$

26 Value of $4\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$ is — 65(B)

(A) 3 (B) -3

(C) 1 (D) -1

27 Evaluate : $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ — 65(B)

Q.	**ANSWERS**
1	(C) 
2	$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ $=\tan^{-1}\left[2\sin\left(2\times\frac{\pi}{6}\right)\right]=\tan^{-1}\left[2\sin\frac{\pi}{3}\right]$ $=\tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]=\tan^{-1}\sqrt{3}=\frac{\pi}{3}$
3	$\sin^{-1}\left(\sin\frac{3\pi}{5}\right)=\sin^{-1}\left(\sin\left(\pi-\frac{2\pi}{5}\right)\right)$ $=\sin^{-1}\left(\sin\left(\frac{2\pi}{5}\right)\right)$ $=\frac{2\pi}{5}$
4	$2\tan^{-1}x+\sin^{-1}\left(\frac{2x}{1+x^2}\right)=4\sqrt{3}\Rightarrow 2\tan^{-1}x+2\tan^{-1}x=4\sqrt{3}$ $\Rightarrow \tan^{-1}x=\sqrt{3}\notin\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ $\therefore x=\tan\left(\sqrt{3}\right)$ which has no solution.
5	(B) $[-\frac{\pi}{2},0]$
6	(B) $-\frac{\pi}{10}$
7	(A) -1
8	(C) $\frac{\pi}{3}$
9	Put $x=\tan\theta\Rightarrow\theta=\tan^{-1}x$

	$\text{Now} \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$ $=\sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right)=\sin^{-1}(\sin\theta)$ $=\theta=\tan^{-1}x$ $\text{Here} -1\le\sqrt{x-1}\le 1$ $\Rightarrow 0\le x-1\le 1\Rightarrow 1\le x\le 2$ Hence, domain is $x\in[1,2]$
10	(C) $[0, \pi]$
11	(C) $[-\sqrt{2}, -1] \cup [1, \sqrt{2}]$
12	**(D)** $\frac{2\pi}{3}$
13	**Domain of $\cos^{-1}x$ is $[-1, 1]$** $\Rightarrow -1 \le x^2 - 4 \le 1 \Rightarrow 3 \le x^2 \le 5$ $\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$
14	**Domain of $\sec^{-1}x$ is $(-\infty, -1] \cup [1, \infty)$** $\Rightarrow 2x + 1 \le -1$ **or** $2x + 1 \ge 1 \Rightarrow x \le -1$ **or** $x \ge 0$ $\Rightarrow$ **Domain** $= (-\infty, -1] \cup [0, \infty)$
15	**Domain of $\sin^{-1}x$ is $[-1, 1]$** $-1 \le x^2 - 3 \le 1 \Rightarrow 2 \le x^2 \le 4$ $\Rightarrow$ **Domain** $= [-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$
16	(C) $y = \sin^{-1}x$ and $y = \sin x$
17	(D) $\frac{7\pi}{12}$
18	(a) $-1 \le -x^2 \le 1 \Rightarrow -1 \le -x^2 \le 0$ $\Rightarrow 0 \le x^2 \le 1 \Rightarrow -1 \le x \le 1$
19	**(B)** $y = \cot^{-1}x$

20	(A) $y = \sec^{-1}x$
21	(A) $y = \tan^{-1} x$
22	(C) $[-1,1]$
23	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**
24	**(D) $y = \sin^{-1} x$**
25	**(A) $\sec^{-1} x$**
26	**(A) 3**
27	$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ $= \frac{\pi}{3} - \left[\pi - \frac{\pi}{3}\right]$ $= \frac{-\pi}{3}$

CHAPTER 03-MATRICES

Q.		Code and Marks
1	Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$, $C = [9\ 8\ 7]$, which of the following is defined ? (A) Only AB (B) Only AC (C) Only BA (D) All AB, AC and BA	65/1/1 1M
2	If $A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is a scalar matrix, then y^x is equal to (A) 0 (B) 1 (C) 7 (D) ± 7	65/1/1 1M
3	Let A be a matrix of order $m \times n$ and B is a matrix such that A^TB and BA^T are defined. Then, the order of B is : (A) $m \times m$ (B) $n \times n$ (C) $m \times n$ (D) $n \times m$	65/1/2 1M
4	If $\begin{bmatrix} x+y & 3y \\ 3x & x+3 \end{bmatrix} = \begin{bmatrix} 9 & 4x+y \\ x+6 & y \end{bmatrix}$, then $(x-y) = ?$ (A) -7 (B) -3 (C) 3 (D) 7	65/1/3 1M
5	If $A = \begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix}$ is a symmetric matrix, then $(2x + y)$ is (A) -8 (B) 0 (C) 6 (D) 8	65/2/1 1M
6	Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify $4\,AB + 3(AB + BA) - 4\,BA$, where A and B are both matrices of order 2×2. It is known that $A \neq B \neq I$ and $A^{-1} \neq B$. Their answers are given as : Abhay : 6 AB Bina : 7 AB – BA Chhaya : 8 AB Devesh : 7 BA – AB Who answered it correctly ? (A) Abhay (B) Bina (C) Chhaya (D) Devesh	65/2/1 65/2/2 65/2/3 1M

7	Which of the following can be both a symmetric and skew-symmetric matrix ? (A) Unit Matrix (B) Diagonal Matrix (C) Null Matrix (D) Row Matrix	65/2/1 65/2/2 65/2/3 1M
8	If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct ? (A) $A = B$ (B) $AB = BA$ (C) $A = 0$ or $B = 0$ (D) $A = I$ or $B = I$	65/2/1 65/2/2 65/2/3 1M
9	**Assertion (A)** : A = diag [3 5 2] is a scalar matrix of order 3 × 3. **Reason (R)** : If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.	65/2/1 65/2/2 65/2/3 1M
10	Let P be a skew-symmetric matrix of order 3. If $\det(P) = \alpha$, then $(2025)^{\alpha}$ is (A) 0 (B) 1 (C) 2025 (D) $(2025)^3$	65/2/2 1M
11	If A and B are square matrices of same order, then $(AB^T - BA^T)$ is a (A) symmetric matrix (B) skew-symmetric matrix (C) null matrix (D) unit matrix	65/2/3 1M
12	If A and B are square matrices of same order such that $AB = A$ and $BA = B$, then $A^2 + B^2$ is equal to : (A) $A + B$ (B) BA (C) $2(A + B)$ (D) $2BA$	65/4/1 65/4/2 65/4/3 1M
13	The matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -7 \\ 2 & 7 & 0 \end{bmatrix}$ is a : (A) diagonal matrix (B) symmetric matrix (C) skew symmetric matrix (D) scalar matrix	65/4/1 65/4/2 65/4/3 1M
14	If $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = kA$, then find the value of k.	65/4/3 2M
15	If $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^3 is : (A) $3\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$ (C) $\begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$ (D) $\begin{bmatrix} 5^3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$	65/5/1 65/5/2 65/5/3 1M

16	If $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ -1 & 2 \\ 0 & 5 \end{bmatrix}$, then the correct statement is : (A) Only AB is defined. (B) Only BA is defined. (C) AB and BA, both are defined. (D) AB and BA, both are not defined.	65/5/1 1M
17	If $\begin{bmatrix} 4+x & x-1 \\ -2 & 3 \end{bmatrix}$ is a singular matrix, then the value of x is : (A) 0 (B) 1 (C) -2 (D) -4	65/5/1 1M
18	If A and B are two square matrices of the same order, then $(A + B)(A - B)$ is equal to : (A) $A^2 - AB + BA - B^2$ (B) $A^2 + AB - BA - B^2$ (C) $A^2 - AB - BA - B^2$ (D) $A^2 - B^2 + AB + BA$	65/5/2 1M
19	If a matrix A is both symmetric and skew-symmetric, then A is a : (A) diagonal matrix (B) zero matrix (C) non-singular matrix (D) scalar matrix	65/5/2 1M
20	Let A and B be two matrices of suitable orders. Then, which of the following is ***not*** correct ? (A) $(A')' = A$ (B) $(kA)' = kA'$, k is a scalar (C) $(A' + B')' = A + B$ (D) $(AB)' = A'B'$	65/5/3 1M
21	Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is : (A) $n \times n$ (B) $n \times m$ (C) $m \times m$ (D) $m \times n$	65/6/1 65/6/2 65/6/3 1M
22	If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A is a/an : (A) scalar matrix (B) identity matrix (C) symmetric matrix (D) skew-symmetric matrix	65/6/1 1M
23	Sum of two skew-symmetric matrices of same order is always a/an : (A) skew-symmetric matrix (B) symmetric matrix (C) null matrix (D) identity matrix	65/6/1 65/6/2 65/6/3 1M

24	Let $A = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix}$ be two matrices. Then, find the matrix B if AB = C.	65/6/1 3M
25	If $A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 0 & 5 \\ -8 & -5 & 0 \end{bmatrix}$, then A is a : (A) null matrix (B) symmetric matrix (C) skew-symmetric matrix (D) diagonal matrix	65/6/2 1M
26	If $A = [1 \;\; -1 \;\; 0]$, $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 3 & 4 \\ 0 & 5 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, are three matrices, then find ABC.	65/6/2 3M
27	If $A = \begin{bmatrix} 0 & 0 & -5 \\ 0 & 3 & 0 \\ 4\cdot3 & 0 & 0 \end{bmatrix}$, then A is a : (A) skew-symmetric matrix (B) scalar matrix (C) diagonal matrix (D) square matrix	65/6/3 1M
28	Find the value of x, if $[1 \;\; x \;\; 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$.	65/6/3 3M
29	What is the total number of possible matrices of order 3×3 with each entry as $\sqrt{2}$ or $\sqrt{3}$? (A) 9 (B) 512 (C) 615 (D) 64	65/7/1 65/7/2 65/7/3 1M
30	The matrix $A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$ is a/an : (A) scalar matrix (B) identity matrix (C) null matrix (D) symmetric matrix	65/7/1 65/7/2 65/7/3 1M

31	If $\begin{bmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{bmatrix} = \begin{bmatrix} x+3 & 12 \\ 0 & 35 \end{bmatrix}$, then the value of $(x-y)$ is : (A) 2 or 10 (B) -2 or 10 (C) 2 or -10 (D) -2 or -10	65/7/1 65/7/2 65/7/3 1M
32	If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I = 0$.	65/7/1 2M
33	**OR** A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is ₹ 150 (Chemistry), ₹ 175 (Physics) and ₹ 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is ₹ 35,000, what profit did he earn after the sale of two days ?	65/7/1 65/7/2 65/7/3 3M
34	If $\begin{bmatrix} 3 & -1 \\ 0 & 1 \\ 2 & -3 \end{bmatrix} A = \begin{bmatrix} 2 \\ -5 \\ -17 \end{bmatrix}$, then find matrix A.	65/7/2 2M
35	If $A = \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix}$, then find the value of K if $A^2 = 6A + KI_2$, where I_2 is an identity matrix.	65/7/3 2M
36	If $A = \begin{bmatrix} x & 0 & m \\ y & z & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$, where I is a unit matrix, then $x + y + z + m$ is equal to (A) 18 (B) 12 (C) 6 (D) 2	65(B) 1M
37	If $B = \begin{bmatrix} 23 & 41 & 57 \end{bmatrix} \begin{bmatrix} 31 & 42 \\ 53 & 64 \\ 75 & 86 \end{bmatrix}$, then the order of B is : (A) 3×2 (B) 2×2 (C) 1×3 (D) 1×2	65(B) 1M

38	If A and B are square matrices of the same order, then $(A - B)^2 = ?$ (A) $A^2 - 2AB + B^2$ (B) $A^2 - AB - BA + B^2$ (C) $A^2 - 2BA + B^2$ (D) $A^2 - AB + BA + B^2$	65(B) 1M

	ANSWERS
1	(A) Only AB
2	(B) 1
3	(C) $m \times n$
4	(B) -3
5	(D) 8
6	(B) Bina
7	(C) Null Matrix
8	(B) AB = BA
9	(D) Assertion (A) is false and Reason (R) is true.
10	(B) 1
11	(B) skew-symmetric
12	(A) $A + B$
13	(C) skew symmetric matrix
14	$A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$ $A^2 = kA \Rightarrow \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 2k & -2k \\ -2k & 2k \end{bmatrix}$ $\Rightarrow k = 4$
15	(B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$
16	**(C) AB and BA, both are defined.**
17	**(C) -2**
18	**(A)** $\mathbf{A^2 - AB + BA - B^2}$
19	**(B) zero matrix**
20	**(D)** $\mathbf{(AB)' = A'B'}$

21	(B) n × m
22	(C) symmetric matrix
23	(A) skew-symmetric matrix
24	(a) Let B = [x y z] $AB = C \Rightarrow \begin{bmatrix} x & y & z \\ 4x & 4y & 4z \\ -2x & -2y & -2z \end{bmatrix} = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix}$ which gives x = 3, y = 4 and z = 2 B = [3 4 2]
25	(C) skew-symmetric matrix
26	**Required product** $= [2+1+0 \quad 0-3+0 \quad 1-4+0]\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ $= [3 \ -3 \ -3]\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ $= [-15]$
27	(D) square matrix
28	$[1 \quad x \quad 1]\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$ $[1 \quad x \quad 1]\begin{bmatrix} 7+2x \\ 12+x \\ 21+2x \end{bmatrix} = 0$ $x^2 + 16x + 28 = 0$ $\Rightarrow (x+14)(x+2) = 0$ $\Rightarrow x = -14, x = -2$
29	(B) 512
30	(D) symmetric matrix
31	(B)- 2 or 10
32	$A^2 = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$. L.H.S.$= A^2 - 4A + 7I = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O =$R.H.S.

33	Let $A=\begin{bmatrix}50 & 60 & 35\\40 & 45 & 50\end{bmatrix}\begin{matrix}\text{Day I}\\\text{Day II}\end{matrix}$, $B=\begin{bmatrix}150\\175\\180\end{bmatrix}$ be the day wise sale and the selling price per subject, matrices respectively. Total sales day wise $=\begin{bmatrix}50 & 60 & 35\\40 & 45 & 50\end{bmatrix}\begin{bmatrix}150\\175\\180\end{bmatrix}=\begin{bmatrix}24,300\\22,875\end{bmatrix}\begin{matrix}\text{Day I}\\\text{Day II}\end{matrix}$ Total sales in two days = ₹ 24,300 + ₹ 22,875 = ₹ 47,175 Profit = ₹ 47,175 – ₹ 35,000 = ₹ 12,175.
34	Let $A=\begin{bmatrix}x\\y\end{bmatrix} \Rightarrow \begin{bmatrix}3 & -1\\0 & 1\\2 & -3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\-5\\-17\end{bmatrix}$ $\Rightarrow 3x-y=2, y=-5$ and $\Rightarrow x=-1$ Put the values in $2x-3y=-17$, L.H.S. $=2(-1)-3(-5)\neq -17=$ R.H.S. $\therefore$ The matrix 'A' does not exist.
35	$A^2=6A+KI_2 \Rightarrow \begin{bmatrix}1 & 0\\-6 & 25\end{bmatrix}=\begin{bmatrix}6+K & 0\\-6 & 30+K\end{bmatrix}$ $\Rightarrow 6+K=1 \Rightarrow K=-5$, also satisfies $30+K=25$.
36	(B) 12
37	(D) 1×2
38	(B) $A^2-AB-BA+B^2$

CHAPTER 4 DETERMINANTS

Q.		Code and Marks
1	If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is (A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	65/1/1 65/1/2 65/1/3 1M
2	If A is a square matrix of order 2 such that det (A) = 4, then det (4 adj A) is equal to : (A) 16 (B) 64 (C) 256 (D) 512	65/1/1 1M
3	If A and B are invertible matrices, then which of the following is <u>not</u> correct ? (A) $(A + B)^{-1} = B^{-1} + A^{-1}$ (B) $(AB)^{-1} = B^{-1}A^{-1}$ (C) $\text{adj}\,(A) = \lvert A \rvert\, A^{-1}$ (D) $\lvert A \rvert^{-1} = \lvert A^{-1} \rvert$	65/1/1 65/1/2 65/1/3 1M
4	A school wants to allocate students into three clubs : Sports, Music and Drama, under following conditions : • The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club. • The number of students in Music club should be 20 more than half the number of students in Sports club. • The total number of students to be allocated in all three clubs are 180. Find the number of students allocated to different clubs, using matrix method.	65/1/1 65/1/2 65/1/3 5M
5	If A is a square matrix of order 3 such that det(A) = 9, then det($9\,A^{-1}$) is equal to (A) 9 (B) 9^2 (C) 9^3 (D) 9^4	65/1/2 1M
6	A system of linear equations is represented as AX = B, where A is coefficient matrix, X is variable matrix and B is the constant matrix. Then the system of equations is (A) Consistent, if $\lvert A \rvert \neq 0$, solution is given by $X = BA^{-1}$. (B) Inconsistent if $\lvert A \rvert = 0$ and (adj A) B = 0 (C) Inconsistent if $\lvert A \rvert \neq 0$ (D) May or may not be consistent if $\lvert A \rvert = 0$ and (adj A) B = 0	65/1/3 1M

7	(a) Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB. Hence, solve the system of linear equations : $x - y + z = 4$ $x - 2y - 2z = 9$ $2x + y + 3z = 1$ **OR** (b) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find A^{-1}. Hence, solve the system of linear equations : $x - 2y = 10$ $2x - y - z = 8$ $-2y + z = 7$	65/2/1 65/2/2 65/2/3 5M
8	If M and N are square matrices of order 3 such that det (M) = m and MN = mI, then det (N) is equal to : (A) -1 (B) 1 (C) $-m^2$ (D) m^2	65/4/1 1M
9	Let A and B be two square matrices of order 3 such that det (A) = 3 and det (B) = – 4. Find the value of det (– 6AB).	65/4/1 2M
10	If A is a 3×3 invertible matrix, show that for any scalar $k \neq 0$, $(kA)^{-1} = \frac{1}{k}A^{-1}$. Hence calculate $(3A)^{-1}$, where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.	65/4/1 65/4/2 65/4/3 5M
11	If $\begin{vmatrix} -1 & 2 & 4 \\ 1 & x & 1 \\ 0 & 3 & 3x \end{vmatrix} = -57$, the product of the possible values of x is : (A) -24 (B) -16 (C) 16 (D) 24	65/4/2 1M
12	Using matrices and determinants, find the value(s) of k for which the pair of equations $5x - ky = 2$; $7x - 5y = 3$ has a unique solution.	65/4/2 2M

13	If A and B are invertible matrices of order 3 × 3 such that det (A) = 4 and det $[(AB)^{-1}] = \frac{1}{20}$, then det (B) is equal to : (A) $\frac{1}{20}$ (B) $\frac{1}{5}$ (C) 20 (D) 5	65/4/3 1M
14	If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of x is : (A) 3 (B) 7 (C) ± 7 (D) ± 3	65/5/1 65/5/2 65/5/3 1M
15	If A = $[a_{ij}]$ is a 3 × 3 diagonal matrix such that $a_{11} = 1$, $a_{22} = 5$ and $a_{33} = -2$, then $\|A\|$ is : (A) 0 (B) –10 (C) 10 (D) 1	65/5/1 65/5/2 65/5/3 1M
16	A furniture workshop produces three types of furniture – chairs, tables and beds each day. On a particular day the total number of furniture pieces produced is 45. It was also found that production of beds exceeds that of chairs by 8, while the total production of beds and chairs together is twice the production of tables. Determine the units produced of each type of furniture, using matrix method.	65/5/1 5M
17	An amount of ₹ 10,000 is put into three investments at the rate of 10%, 12% and 15% per annum. The combined annual income of all three investments is ₹ 1,310, however the combined annual income of the first and the second investments is ₹ 190 short of the income from the third. Use matrix method and find the investment amount in each at the beginning of the year.	65/5/2 5M
18	Let A and B be two matrices of suitable orders. Then, which of the following is ***not*** correct ? (A) $(A')' = A$ (B) $(kA)' = kA'$, k is a scalar (C) $(A' + B')' = A + B$ (D) $(AB)' = A'B'$	65/5/3 1M
19	Three students run on a racing track such that their speeds add up to 6 km/h. However, double the speed of the third runner added to the speed of the first results in 7 km/h. If thrice the speed of the first runner is added to the original speeds of the other two, the result is 12 km/h. Using matrix method, find the original speed of each runner.	65/5/3 5M

20	Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers. Based upon the above information, answer the following questions : (i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form AX = B. *1* (ii) Find $\lvert A \rvert$ and confirm if it is possible to find A^{-1}. *1* (iii) (a) Find A^{-1}, if possible, and write the formula to find X. *2* **OR** (iii) (b) Find $A^2 - 8I$, where I is an identity matrix. *2*	65/6/1 65/6/2 65/6/3 4M
21	If A and B are two square matrices each of order 3 with $\lvert A \rvert = 3$ and $\lvert B \rvert = 5$, then $\lvert 2AB \rvert$ is : (A) 30 (B) 120 (C) 15 (D) 225	65/7/1 65/7/2 65/7/3 1M
22	(a) Let $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.	65/7/1 65/7/2 65/7/3 3M
23	Let A be a square matrix of order 3. If $\lvert A \rvert = 5$, then $\lvert \text{adj } A \rvert$ is : (A) 5 (B) 125 (C) 25 (D) -5	65/7/2 65/7/3 1M
24	If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then the value of x is (A) 0 (B) 9 (C) -6 (D) 6	65(B) 1M
25	If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Using A^{-1}, solve the system of equations : $2x - 3y + 5z = 11$ $3x + 2y - 4z = -5$ $x + y - 2z = -3$	65(B) 5M

	ANSWERS
1	(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2	(B) 64
3	(A) $(A + B)^{-1} = B^{-1} + A^{-1}$
4	**Let x, y and z be the no. of students allocated to Sports, Music and Drama clubs respectively.** **Here,** $x = y + z, y = \frac{x}{2} + 20, x + y + z = 180$ $\Rightarrow x - y - z = 0, x - 2y = -40, x + y + z = 180$ **Given equations can be written as** $AX = B$ **where,** $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\|A\| = -4 \neq 0 \Rightarrow A^{-1}$ **exists.** $adjA = \begin{bmatrix} -2 & 0 & -2 \\ -1 & 2 & -1 \\ 3 & -2 & -1 \end{bmatrix}$ $A^{-1} = \frac{1}{\|A\|} \times adjA = \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}$ $X = A^{-1}B$ $= \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix} = \begin{bmatrix} 90 \\ 65 \\ 25 \end{bmatrix}$ $\therefore x = 90, y = 65, z = 25$ **Number of students allocated in sports, music and drama are 90 , 65 and 25 respectively .**
5	(B) 9^2
6	(D) May or may not be consistent if $\| A \| = 0$ and (adj A) B = 0
7	$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$ The system of equations is equivalent to the matrix equation: $BX = C$, where $C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\Rightarrow X = B^{-1}C$ $AB = 8I$ $\Rightarrow B^{-1} = \frac{1}{8}A$

	$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$ The system of equations is equivalent to the matrix equation: $BX = C$, where $C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\Rightarrow X = B^{-1}C$ $AB = 8I$ $\Rightarrow B^{-1} = \frac{1}{8}A$ $X = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ $\therefore x = 3, y = -2, z = -1$ OR $A^{-1} = \frac{1}{\|A\|}\text{adj}A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ The given system of equations is equivalent to the matrix equation $A^T X = B$, where $B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\Rightarrow X = (A^T)^{-1}B$ $\Rightarrow X = (A^{-1})^T B$ $\Rightarrow X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ $\therefore x = 0, y = -5, z = -3$
8	(D) m^2
9	$\|-6AB\| = (-6)^3\|A\|\|B\|$ $= -216 \times 3 \times (-4) = 2592$
10	Consider $(kA)\left(\frac{1}{k}A^{-1}\right) = k.\frac{1}{k}\left(A.A^{-1}\right) = 1.I = I$ $\Rightarrow kA$ and $\frac{1}{k}A^{-1}$ are inverse of each other. $\therefore (kA)^{-1} = \frac{1}{k}A^{-1}$ $\therefore (3A)^{-1} = \frac{1}{3}A^{-1}$

	Here, $\|A\| = 4 \neq 0 \therefore A^{-1}$ exists. $adjA = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ $\therefore A^{-1} = \frac{1}{\|A\|} . adjA = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ $\therefore (3A)^{-1} = \frac{1}{12} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$
11	(A) -24
12	For unique solution, $\begin{vmatrix} 5 & -k \\ 7 & -5 \end{vmatrix} \neq 0$ $\Rightarrow -25 + 7k \neq 0 \Rightarrow k \neq \frac{25}{7}$ or $R - \left\{ \frac{25}{7} \right\}$
13	(D) 5
14	**(C) ± 7**
15	(B) -10
16	**Let the numbers of chairs, tables and beds produced be x, y and z respectively.** $\therefore\ x + y + z = 45; \quad -x + 0.y + z = 8; \quad x - 2y + z = 0$ **Let** $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}, \ \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$ $\|A\| = 1(0+2) - 1(-1-1) + 1(2-0) = 6 \neq 0$ $\therefore$ A^{-1} **exists** $AX = B \Rightarrow X = A^{-1}B$ $adj(A) = \begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$ $A^{-1} = \frac{1}{6} \begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$ $\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$ **So,** $x = 11, y = 15, z = 19$ **Hence the numbers of chairs, tables and beds produced are 11, 15 and 19 respectively.**

17	**Let x, y and z (in ₹) be three investment amounts.** **Then** $x + y + z = 10,000$ $\frac{10}{100}x + \frac{12}{100}y + \frac{15}{100}z = 1310$ $-\frac{10}{100}x - \frac{12}{100}y + \frac{15}{100}z = 190$ **Let** $A = \begin{bmatrix} 1 & 1 & 1 \\ 10 & 12 & 15 \\ -10 & -12 & 15 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 10,000 \\ 1,31,000 \\ 19,000 \end{bmatrix}$ $\|A\| = 60 \neq 0$ $\therefore A^{-1}$ **exists** $AX = B \Rightarrow X = A^{-1}B$ $adj(A) = \begin{bmatrix} 360 & -27 & 3 \\ -300 & 25 & -5 \\ 0 & 2 & 2 \end{bmatrix}$ $A^{-1} = \frac{1}{60}\begin{bmatrix} 360 & -27 & 3 \\ -300 & 25 & -5 \\ 0 & 2 & 2 \end{bmatrix}$ $\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{60}\begin{bmatrix} 360 & -27 & 3 \\ -300 & 25 & -5 \\ 0 & 2 & 2 \end{bmatrix}\begin{bmatrix} 10,000 \\ 1,31,000 \\ 19,000 \end{bmatrix} = \begin{bmatrix} 2000 \\ 3000 \\ 5000 \end{bmatrix}$ **Hence the investments are ₹ 2000, ₹ 3000 and ₹ 5000 respectively.**
18	**(D)** $(AB)' = A'B'$
19	**Let original speed of three runners be x, y and z respectively.** **Then** $x + y + z = 6$; $x + 2z = 7$; $3x + y + z = 12$ **Let** $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$ $\|A\| = 4 \neq 0 \Rightarrow A^{-1}$ **exists** $AX = B \Rightarrow X = A^{-1}B$ $adj(A) = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$ $A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$ $\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

	Hence the original speed of three runners are 3km/h, 1km/h and 2 km/h respectively.
20	(i)Let the price of each pen, notepad, eraser be ₹x, ₹y and ₹z respectively Given system in the form AX = B is $\begin{pmatrix}4&3&2\\2&4&6\\6&2&3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}60\\90\\70\end{pmatrix}$ (ii)$\|A\| = 50 \neq 0$, hence A^{-1}exists (iii) (a) $A^{-1}=\frac{adjA}{\|A\|} = \frac{1}{50}\begin{pmatrix}0&-5&10\\30&0&-20\\-20&10&10\end{pmatrix}$ $X = A^{-1}B$ **OR** (iii)(b) $A^2 = \begin{pmatrix}4&3&2\\2&4&6\\6&2&3\end{pmatrix}\begin{pmatrix}4&3&2\\2&4&6\\6&2&3\end{pmatrix} = \begin{pmatrix}34&28&32\\52&34&46\\46&32&33\end{pmatrix}$ $A^2 - 8I = \begin{pmatrix}26&28&32\\52&26&46\\46&32&25\end{pmatrix}$
21	(B) 120
22	**(a) The system of equations in matrices is:** $AX = B$, **where** $A=\begin{bmatrix}2&5\\3&2\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}1\\7\end{bmatrix}$ **The solution is given by** $X = A^{-1}B \Rightarrow \begin{bmatrix}x\\y\end{bmatrix}=\frac{-1}{11}\begin{bmatrix}2&-5\\-3&2\end{bmatrix}\begin{bmatrix}1\\7\end{bmatrix}=\begin{bmatrix}3\\-1\end{bmatrix}$ **Point common to paths of the ants is** $(3,-1)$.
23	(C) 25
24	**(C) -6**
25	$adjA = \begin{bmatrix}0&-1&2\\2&-9&23\\1&-5&13\end{bmatrix}$ $\|A\| = 2\times 0 - 3\times 2 + 5\times 1 = -6+5 = -1 \neq 0 \Rightarrow A^{-1}$ **exists** $A^{-1} = \frac{1}{\|A\|}(adjA)=\begin{bmatrix}0&1&-2\\-2&9&-23\\-1&5&-13\end{bmatrix}$ $\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}11\\-5\\-3\end{bmatrix}$ $\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0&1&-2\\-2&9&-23\\-1&5&-13\end{bmatrix}\begin{bmatrix}11\\-5\\-3\end{bmatrix}$ $AX = B \Rightarrow X = A^{-1}B$

	$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ **⇒x=1, y=2, z=3**

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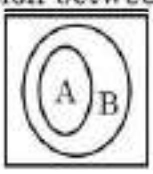
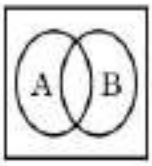
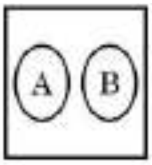
CHAPTER 5 CONTINUITY AND DIFFERENTIABILITY

Q.		Code and Marks
1	If $f(x) = \lvert x \rvert + \lvert x - 1 \rvert$, then which of the following is correct ? (A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$. (B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$. (C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$. (D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.	1 mark 65/1/1
2	**Assertion (A)** : $f(x) = \begin{cases} 3x - 8, & x \le 5 \\ 2k, & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$. **Reason (R)** : For a function f to be continuous at $x = a$, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.	1 mark 65/1/1
3	(a) Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$. **OR** (b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.	2 mark 65/1/2
4	(a) Differentiate $\tan^{-1} \frac{\sqrt{1-x^2}}{x}$ w.r.t. $\cos^{-1}(2x\sqrt{1-x^2})$, $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$ **OR** (b) Find $\frac{dy}{dx}$, if $y = x^{\tan x} + \frac{\sqrt{x^2+1}}{2}$.	5 mark 65/1/2
5	(a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. **OR** (b) If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = \sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.	5 mark 65/1/3
6	If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B ? (A)  (B) 	1 mark 65/2/2 65/2/2 65/2/3

	(C)  (D) 	
7	(a) If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$. **OR** (b) If $f(x) = \begin{cases} 2x-3, & -3 \le x \le -2 \\ x+1, & -2 < x \le 0 \end{cases}$ Check the differentiability of $f(x)$ at $x = -2$.	2 marks 65/2/1 2 mark 65/2/2 65/2/3
8	(a) If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then show that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$. OR (b) If $x\sqrt{1+y} + y\sqrt{1+x} = 0, -1 < x < 1, x \ne y$, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.	3 mark 65/2/1 65/2/2 65/2/3
9	(a) Find k so that $f(x) = \begin{cases} \frac{x^2-2x-3}{x+1}, & x \ne -1 \\ k, & x = -1 \end{cases}$ is continuous at $x = -1$. **OR** (b) Check the differentiability of function $f(x) = x\|x\|$ at $x = 0$.	3 mark 65/4/1 65/4/3
10	If $f(x) = \begin{cases} 3x-2, & 0 < x \le 1 \\ 2x^2+ax, & 1 < x < 2 \end{cases}$ is continuous for $x \in (0, 2)$, then a is equal to : (A) -4 (B) $-\frac{7}{2}$ (C) -2 (D) -1	1 mark 65/4/1 65/4/2 65/4/3

11	The function f defined by $$f(x) = \begin{cases} x, & \text{if } x \le 1 \\ 5, & \text{if } x > 1 \end{cases}$$ is ***not*** continuous at : (A) $x = 0$ (B) $x = 1$ (C) $x = 2$ (D) $x = 5$	1 mark 65/4/1 65/4/3
12	If $f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{for } x \neq \dfrac{\pi}{2} \\ k, & \text{for } x = \dfrac{\pi}{2} \end{cases}$ is continuous at $x = \dfrac{\pi}{2}$, then the value of k is : (A) $\dfrac{3}{2}$ (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{2}$ (D) 1	1 mark 65/4/2
13	If $y = \log_{2x}(\sqrt{2x})$, then $\dfrac{dy}{dx}$ is equal to : (A) 0 (B) 1 (C) $\dfrac{1}{x}$ (D) $\dfrac{1}{\sqrt{2x}}$	1 mark 65/4/3
14	If $f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of a is : (A) 1 (B) -1 (C) ± 1 (D) 0	1 mark 65/5/1 65/5/2 65/5/3

15	If $f(x) = \{[x], x \in R\}$ is the greatest integer function, then the correct statement is : (A) f is continuous but not differentiable at $x = 2$. (B) f is neither continuous nor differentiable at $x = 2$. (C) f is continuous as well as differentiable at $x = 2$. (D) f is not continuous but differentiable at $x = 2$.	1 mark 65/5/1 65/5/2 65/5/3
16	(a) Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x. **OR** (b) If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.	2 marks 65/5/1 65/5/2 65/5/3
17	(a) For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t+\frac{1}{t}\right)^a$, where t is a non-zero real number. **OR** (b) Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$, where a and b are constants.	5 marks 65/5/1 65/5/2 65/5/3
18	If $f(x) = \begin{cases} \frac{\log(1+ax) + \log(1-bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is : (A) a (B) a + b (C) a − b (D) b	1 mark 65/6/1 65/6/2 65/6/3
19	If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\frac{dy}{dx}$ is : (A) $\frac{x}{y}$ (B) $-\frac{x}{y}$ (C) $\frac{a}{x}$ (D) $\frac{a}{y}$	1 mark 65/6/1 65/6/2 65/6/3
20	If $y = a\cos(\log x) + b\sin(\log x)$, then $x^2y_2 + xy_1$ is : (A) cot (log x) (B) y (C) − y (D) tan (log x)	1 mark 65/6/1 65/6/2 65/6/3
21	Let $f(x) = \|x\|$, $x \in R$. Then, which of the following statements is ***incorrect*** ? (A) f has a minimum value at $x = 0$. (B) f has no maximum value in R. (C) f is continuous at $x = 0$. (D) f is differentiable at $x = 0$.	1 mark 65/6/1

22	***Assertion (A) :*** Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)\,x = e^x$ where domain of $(f + g)$ is R. ***Reason (R) :*** $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.	1 mark 65/6/1 65/6/2 65/6/3
23	**(a)** Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$. **OR** **(b)** If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.	2mark 65/6/1 65/6/2 65/6/3
24	**(a)** Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. **OR** **(b)** Differentiate $y = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$ with respect to x, when $x \in (0, 1)$.	3 mark 65/6/1 65/6/2
25	Let $f(x) = x^2$, $x \in R$. Then, which of the following statements is ***incorrect*** ? (A) Minimum value of f does not exist. (B) There is no point of maximum value of f in R. (C) f is continuous at $x = 0$. (D) f is differentiable at $x = 0$.	1 mark 65/6/2
26	If $f(x) = \begin{cases} 1, & \text{if } x \le 3 \\ ax + b, & \text{if } 3 < x < 5 \\ 7, & \text{if } 5 \le x \end{cases}$ is continuous in R, then the values of a and b are : (A) $a = 3,\ b = -8$ (B) $a = 3,\ b = 8$ (C) $a = -3,\ b = -8$ (D) $a = -3,\ b = 8$	1 mark 65/7/1
27	If $f(x) = -2x^8$, then the correct statement is : (A) $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$ (B) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$ (C) $-f'\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$ (D) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$	1 mark 65/7/1 65/7/2 65/7/3
28	*Assertion (A) :* $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$. *Reason (R) :* When $x \to 0$, $\sin \frac{1}{x}$ is a finite value between -1 and 1.	1 mark 65/7/1 65/7/2 65/7/3
29	(a) Differentiate $\left(\frac{5^x}{x^5}\right)$ with respect to x. **OR** (b) If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.	2 mark 65/7/1 65/7/2 65/7/3

30	Differentiate $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3} - 1\right)\right\}}$ with respect to x.	3 mark 65/7/1
31	If $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2} & , \text{if } x \neq 0 \\ 1 & , \text{if } x = 0 \end{cases}$ is continuous at x = 0, then the value of 'a' is : (A) ± 1 (B) -1 (C) 0 (D) 1	1mark 65/7/2
32	Show that the derivative of $\tan^{-1}(\sec x + \tan x)$, $\left[-\frac{\pi}{2} < x < \frac{\pi}{2}\right]$ with respect to x is equal to $\frac{1}{2}$.	3 mark 65/7/2
33	If $f(x) = \begin{cases} 3ax - b & , x > 1 \\ 11 & , x = 1 \\ -5ax - 2b & , x < 1 \end{cases}$ is continuous at $x = 1$, then the values of a and b are : (A) $a = 3,\ b = 5$ (B) $a = 8,\ b = -1$ (C) $a = 1,\ b = -8$ (D) $a = -3,\ b = 5$	1 mark 65/7/3
34	Differentiate $\log(x^x + \text{cosec}^2 x)$ with respect to x.	3 mark 65/7/3
35	If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, then $\frac{dy}{dx}$ is (A) $\frac{-\sqrt{x}}{\sqrt{y}}$ (B) $-\frac{1}{2}\frac{\sqrt{y}}{\sqrt{x}}$ (C) $-\frac{\sqrt{y}}{\sqrt{x}}$ (D) $\frac{-2\sqrt{y}}{\sqrt{x}}$	1 mark 65(B)
36	If $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$, then $\frac{dy}{dx}$ is (A) 1 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) -1	1 mark 65(B)
37	(a) Show that the function $f(x) = (x-1)^{\frac{1}{3}}$ is not differentiable at $x = 1$. **OR** (b) Differentiate $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ w.r.t. x.	2 mark 65(B)
38	(a) Differentiate $x^{\sin x} + (\sin x)^x$ w.r.t. x. **OR** (b) If $y = x + \tan x$, then prove that $\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$	5 mark 65(B)

	ANSWERS
1	(C) f(x) is continuous but not differentiable, at $x = 0$ and $x = 1$.
2	(D) Assertion (A) is false, but Reason (R) is true.
3. (a)	Let $u = 2^{\cos^2 x} \Rightarrow \dfrac{du}{dx} = 2^{\cos^2 x}\left(-2\cos x \sin x\right)\log 2$ Let $v = \cos^2 x \Rightarrow \dfrac{dv}{dx} = -2\cos x \sin x$ Now $\dfrac{du}{dv} = \dfrac{\left(\dfrac{du}{dx}\right)}{\left(\dfrac{dv}{dx}\right)} = 2^{\cos^2 x}\log 2$
(b)	OR $\tan^{-1}\left(x^2 + y^2\right) = a^2 \Rightarrow x^2 + y^2 = \tan a^2$ Differentiate both sides wrt x, $2x + 2y\dfrac{dy}{dx} = 0$ $\Rightarrow \dfrac{dy}{dx} = -\dfrac{x}{y}$
4	(a) Put $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$ ½ Let $u = \tan^{-1}\dfrac{\sqrt{1-x^2}}{x} = \tan^{-1}\left(\dfrac{\sin\theta}{\cos\theta}\right) = \tan^{-1}(\tan\theta) = \theta = \cos^{-1} x$ 1½ $\Rightarrow \dfrac{du}{dx} = -\dfrac{1}{\sqrt{1-x^2}}$ ½ Let $v = \cos^{-1}\left(2x\sqrt{1-x^2}\right) = \cos^{-1}(\sin 2\theta) = \cos^{-1}\left(\cos\left(\dfrac{\pi}{2} - 2\theta\right)\right) = \dfrac{\pi}{2} - 2\cos^{-1} x$ 1½ $\Rightarrow \dfrac{dv}{dx} = \dfrac{2}{\sqrt{1-x^2}}$ ½ $\therefore \dfrac{du}{dv} = \dfrac{du/dx}{dv/dx} = -\dfrac{1}{2}$ ½

	OR	
(b)	Let $y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$, where $u = x^{\tan x}, v = \frac{\sqrt{x^2+1}}{2}$	1
	$u = x^{\tan x} \Rightarrow \log u = \tan x \log x$, differentiating with respect to 'x', we get	
	$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{\tan x}{x} + \sec^2 x \log x$	1½
	$\Rightarrow \frac{du}{dx} = u\left(\frac{\tan x}{x} + \sec^2 x \log x\right) = x^{\tan x}\left(\frac{\tan x}{x} + \sec^2 x \log x\right)$	½
	$v = \frac{\sqrt{x^2+1}}{2} \Rightarrow \frac{dv}{dx} = \frac{2x}{4\sqrt{x^2+1}} = \frac{x}{2\sqrt{x^2+1}}$	1½
	$\Rightarrow \frac{dy}{dx} = x^{\tan x}\left(\frac{\tan x}{x} + \sec^2 x \log x\right) + \frac{x}{2\sqrt{x^2+1}}$	½
5 (a)	Let $x = \sin A, y = \sin B \Rightarrow A = \sin^{-1} x, B = \sin^{-1} y$	1
	$\therefore \sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$	
	$\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$	
	$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$	1
	$\Rightarrow \cot\left(\frac{A-B}{2}\right) = a \Rightarrow A - B = 2\cot^{-1} a$	1
	$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$	½
	differentiate both sides wrt x,	
	$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$	1½
	$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$	
(b)	$x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$	
	$\Rightarrow \frac{dx}{d\theta} = a\left(-\sin\theta + \frac{1}{\tan\frac{\theta}{2}} \times \sec^2\frac{\theta}{2} \times \frac{1}{2}\right)$	½
	$= a\left(-\sin\theta + \frac{1}{\sin\theta}\right) = a\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$	½
	$\frac{dx}{d\theta} = a\cot\theta\cos\theta$	½
	Also, $y = \sin\theta \Rightarrow \frac{dy}{d\theta} = \cos\theta$	½
	$\therefore \frac{dy}{dx} = \frac{\tan\theta}{a}$	
	Differetiating wrt x,	
	$\frac{d^2y}{dx^2} = \frac{\sec^2\theta}{a} \times \frac{d\theta}{dx}$ $= \frac{\sec^3\theta\tan\theta}{a^2}$ $\left.\frac{d^2y}{dx^2}\right]_{at\ \theta=\frac{\pi}{4}} = \frac{2\sqrt{2}}{a^2}$	

6 (a)

B) 

7

(a)

$x = e^{\frac{x}{y}}$

$\Rightarrow \log x = \frac{x}{y}$

$\Rightarrow y \log x = x$ ½

Differentiating both sides w.r.to x, we get

$\frac{y}{x} + \log x \frac{dy}{dx} = 1$ 1

$\Rightarrow \frac{dy}{dx} = \frac{x - y}{x \log x}$ ½

OR

(b)

$Lf'(-2) = \lim_{h\to 0} \frac{f(-2-h)-f(-2)}{-h} \quad (h > 0)$

$= \lim_{h\to 0} \frac{2(-2-h) - 3 - (-7)}{-h}$

$= \lim_{h\to 0} 2 = 2$ 1

$Rf'(-2) = \lim_{h\to 0} \frac{f(-2+h)-f(-2)}{h} \quad (h > 0)$

$= \lim_{h\to 0} \frac{-2 + h + 1 - (-7)}{h}$

$= \lim_{h\to 0} \frac{6+h}{h}$, which does not exist, i.e., RHD does not exist.

Therefore, the function is not differentiable at -2.

Note: (1) If a student finds only RHD and concludes the result, full marks may be awarded.

(2) If a student proves that the function is discontinuous at -2 and hence not differentiable at -2, full marks may be awarded.

8 (a)

The given function can be written as

$y = 2\log(x+1) - \log x$

$\Rightarrow y_1 = \frac{2}{x+1} - \frac{1}{x} = \frac{x-1}{x(x+1)}$

$\Rightarrow (x+1)y_1 = \frac{x-1}{x} = 1 - \frac{1}{x}$

$\Rightarrow (x+1)y_2 + y_1 = \frac{1}{x^2}$

$\Rightarrow x(x+1)^2 y_2 + x(x+1)y_1 = 1 + \frac{1}{x}$

$\Rightarrow x(x+1)^2 y_2 + x(x+1)y_1 = 1 + 1 - (x+1)y_1$

$\Rightarrow x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$

OR

(b)

$x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow (x-y)(x+y) + xy(x-y) = 0$

$\Rightarrow (x-y)(x+y+xy) = 0$

$x \neq y \Rightarrow x + y + xy = 0$

$\Rightarrow y = \frac{-x}{1+x}$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+x)^2}$

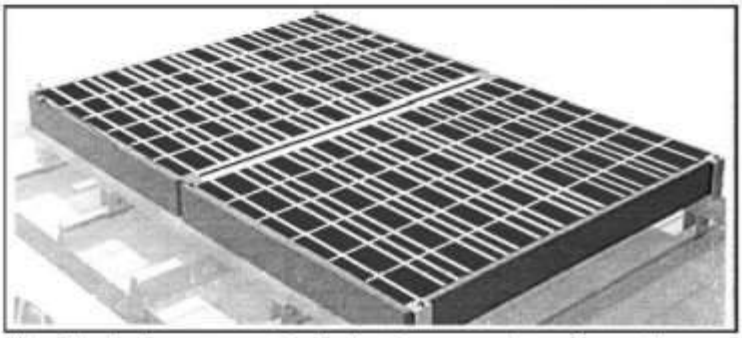
9 (a)	$\lim\limits_{x\to -1}\frac{x^2-2x-3}{x+1}=\lim\limits_{x\to -1}\frac{(x-3)(x+1)}{x+1}=\lim\limits_{x\to -1}(x-3)=-4$ Also, $f(-1)=k$ as f is continuous, $k=-4$	2 ½ ½
	OR	
(b)	$f(x)=x\|x\|=\begin{cases}-x^2 & ,x\le 0\\ x^2 & ,x>0\end{cases}$ $\text{LHD}=\lim\limits_{h\to 0}\frac{f(0-h)-f(0)}{-h}=\lim\limits_{h\to 0}\frac{-h^2-0}{-h}=0$ $\text{RHD}=\lim\limits_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim\limits_{h\to 0}\frac{h^2-0}{h}=0$ Since LHD = RHD, f is differentiable at $x=0$	1 1 ½ ½
10	(D) -1	
11	(B) $x=1$	
12	(C) $\frac{1}{2}$	
13	(A) 0	
14	**(C) ±1**	
15	**(B) f is neither continuous nor differentiable at x=2.**	
16	(a) Let $y=\frac{\sin x}{\sqrt{\cos x}}$ $\frac{dy}{dx}=\frac{\sqrt{\cos x}.\cos x-\sin x.\left(\frac{-\sin x}{2\sqrt{\cos x}}\right)}{\cos x}$ $\Rightarrow \frac{dy}{dx}=\frac{2\cos^2 x+\sin^2 x}{2(\cos x)^{3/2}}$ or $\frac{1+\cos^2 x}{2(\cos x)^{3/2}}$	 1½ ½
	OR	
	(b) $y=5\cos x-3\sin x$, then $\frac{dy}{dx}=-5.\sin x-3.\cos x$ $\Rightarrow \frac{d^2y}{dx^2}=-5.\cos x+3.\sin x=-y$ $\Rightarrow \frac{d^2y}{dx^2}+y=0$	1 ½ ½
17	(a) Let $u=a^{t+\frac{1}{t}} \Rightarrow \frac{du}{dt}=a^{t+\frac{1}{t}}.\log a.\left(1-\frac{1}{t^2}\right)$ $v=\left(t+\frac{1}{t}\right)^a \Rightarrow \frac{dv}{dt}=a\left(t+\frac{1}{t}\right)^{a-1}.\left(1-\frac{1}{t^2}\right)$ $\frac{du}{dv}=\frac{du/dt}{dv/dt}=\frac{a^{t+\frac{1}{t}}.\log a}{a\left(t+\frac{1}{t}\right)^{a-1}}$	
	OR	
	(b) Let $u=y^x$, $v=x^y$ and $w=x^x$ $\Rightarrow \frac{du}{dx}+\frac{dv}{dx}+\frac{dw}{dx}=0$ ....(i) $u=y^x \Rightarrow \log u=x.\log y \Rightarrow \frac{1}{u}.\frac{du}{dx}=\frac{x}{y}.\frac{dy}{dx}+\log y$ $\Rightarrow \frac{du}{dx}=y^x\left(\frac{x}{y}.\frac{dy}{dx}+\log y\right)=xy^{x-1}\frac{dy}{dx}+y^x\log y$	

	$v = x^y \Rightarrow \log v = y.\log x \Rightarrow \frac{1}{v}.\frac{dv}{dx} = \frac{y}{x} + \log x.\frac{dy}{dx}$ $\Rightarrow \frac{dv}{dx} = x^y\left(\frac{y}{x} + \log x.\frac{dy}{dx}\right) = yx^{y-1} + x^y \log x \frac{dy}{dx}$ $w - x^x \rightarrow \log w - x.\log x \rightarrow \frac{1}{w}.\frac{dw}{dx} - 1 + \log x$ $\Rightarrow \frac{dw}{dx} = x^x.(1 + \log x)$ $\therefore$ From (i), we get $xy^{x-1}.\frac{dy}{dx} + y^x.\log y + yx^{y-1} + x^y.\log x.\frac{dy}{dx} + x^x.(1 + \log x) = 0$ $\rightarrow \frac{dy}{dx} - -\frac{x^x.(1+\log x) + y^x.\log y + yx^{y-1}}{x.y^{x-1} + x^y.\log x}$	
18	(C) a – b	
19	(A) x/y	
20	(C) –y	
21	(D) f is differentiable at x = 0	
22	(D) Assertion (A) is false, but Reason (R) is true.	
23	(a) Let u= $\sqrt{e^{\sqrt{2x}}}$ and v = $e^{\sqrt{2x}}$ Derivative of $\sqrt{v}$ w.r.t. v = $\frac{1}{2\sqrt{v}}$. Required derivative = $\frac{1}{2\sqrt{e^{\sqrt{2x}}}}$. OR Taking log on both sides, we get y logx = x logy Differentiating both sides w.r.t. x, we get $\frac{y}{x} + \log x\frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y$ $\Rightarrow \frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$	½ 1 ½ ½ 1 ½
24	(a) x = sint gives y = $\sin^{-1}(\sin 3t) = 3t = 3\sin^{-1}x$ $\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$ Aliter: $\frac{dy}{dx} = \frac{3-12x^2}{\sqrt{1-(3x-4x^3)^2}}$ OR (b) x = tant gives y = $\cos^{-1}(\cos 2t) = 2t = 2\tan^{-1}x$ $\frac{dy}{dx} = \frac{2}{1+x^2}$ Aliter: $\frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}}.\frac{-4x}{(1+x^2)^2}$	$\frac{1}{2}+1+\frac{1}{2}$ 1 3 $\frac{1}{2}+1+\frac{1}{2}$ 1 3
25	**(A) Minimum value of f does not exist**	
26	**(A)** a= 3 , b= −8	
27	(B) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$	
28	**(A)Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	

29	**(a) Let,** $y=\frac{5^x}{x^5}=5^x\cdot x^{-5}\Rightarrow\frac{dy}{dx}=\left(5^x\right)'\cdot x^{-5}+5^x\cdot\left(x^{-5}\right)'$ $=\frac{5^x}{x^5}\log 5-\frac{5^{x+1}}{x^6}$ **OR** **(b) Differentiating** $-2x^2-5xy+y^3=76$**, with respect to 'x'** $-4x-5y-5x\frac{dy}{dx}+3y^2\frac{dy}{dx}=0$ $\Rightarrow\frac{dy}{dx}=\frac{4x+5y}{3y^2-5x}$	
30	$\frac{dy}{dx}=\frac{1}{2\sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}}\cdot\frac{1}{\sin\left(\frac{x^3}{3}-1\right)}\cdot\cos\left(\frac{x^3}{3}-1\right)\cdot\frac{3x^2}{3}$ $=\frac{x^2\cot\left(\frac{x^3}{3}-1\right)}{2\sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}}$	
31	**(A)** ± 1	
32	$\tan^{-1}(\sec x+\tan x)=\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)=\tan^{-1}\left(\frac{1+\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}\right)$ $=\tan^{-1}\left(\frac{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$ $=\tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$ $=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)=\frac{\pi}{4}+\frac{x}{2}$ $\therefore\left(\tan^{-1}(\sec x+\tan x)\right)'=\frac{1}{2}$	½ 1 1 ½
33	**(C)** a= 1, b= - 8	
34	$\frac{d}{dx}\log\left(x^x+\operatorname{cosec}^2x\right)=\frac{1}{x^x+\operatorname{cosec}^2x}\frac{d}{dx}\left(e^{x\log x}+\operatorname{cosec}^2x\right)\quad\left(\because x^x=e^{x\log x}\right)$ $=\frac{1}{x^x+\operatorname{cosec}^2x}\left[e^{x\log x}(1+\log x)-2\operatorname{cosec}^2x\cot x\right]$ $=\frac{1}{x^x+\operatorname{cosec}^2x}\left[x^x(1+\log x)-2\operatorname{cosec}^2x\cot x\right]$	1 1½ ½
35	**(C)** $-\frac{\sqrt{y}}{\sqrt{x}}$	
36	**(B)** $\frac{1}{2}$	

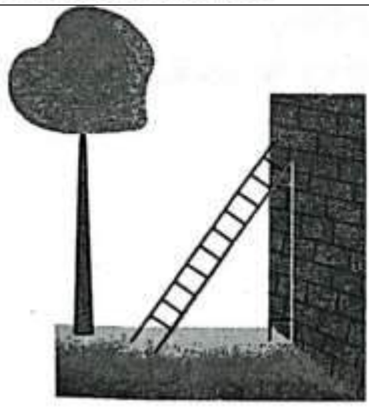
37	(a) $f'(x) = \frac{1}{3}(x-1)^{\frac{-2}{3}}$ $f'(1)$ is not defined OR (b) $y = log\left(x + \sqrt{x^2 + a^2}\right)$ $\frac{dy}{dx} = \frac{1 + \frac{x}{\sqrt{x^2 + a^2}}}{x + \sqrt{x^2 + a^2}}$ $= \frac{1}{\sqrt{x^2+a^2}}$
38	(a) Let $y = x^{sinx} + (sinx)^x = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ $u = x^{sinx}$ $\Rightarrow logu = sinx\, logx$ $\Rightarrow \frac{du}{dx} = x^{sinx}\left[\frac{sinx}{x} + cosx\, logx\right]$ $v = (sinx)^x$ $\Rightarrow logv = x\, log(sinx)$ $\Rightarrow \frac{dv}{dx} = (sinx)^x[xcotx + log(sinx)]$ $\Rightarrow \frac{dy}{dx} = x^{sinx}\left[\frac{sinx}{x} + cosx\, logx\right] + (sinx)^x[xcotx + log(sinx$ OR (b) $y = x + tanx$ $\Rightarrow \frac{dy}{dx} = 1 + sec^2x$ $\Rightarrow \frac{d^2y}{dx^2} = 2sec^2xtanx$ $\Rightarrow cos^2x\frac{d^2y}{dx^2} = 2tanx$ $\Rightarrow cos^2x\frac{d^2y}{dx^2} = 2x + 2tanx - 2x$ $\Rightarrow cos^2x\frac{d^2y}{dx^2} = 2y - 2x$ $\Rightarrow cos^2x\frac{d^2y}{dx^2} - 2y + 2x = 0$

CHAPTER-6 APPLICATION OF DERIVATIVES

Q.		Code and Marks
1	The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is : (A) 0 (B) 2 (C) 4 (D) 5	65/1/1 65/1/2 65/1/3 1M
2	Find the intervals in which function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$ is (i) increasing (ii) decreasing.	65/1/1 2M
3	The side of an equilateral triangle is increasing at the rate of 3 cm/s. At what rate its area increasing when the side of the triangle is 15 cm ?	65/1/1 3M
4	Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$.	65/1/1 65/1/2 5M
5	 A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections. Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres. Based on this information, answer the following questions : (i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y. (ii) Write the area of the solar panel as a function of x. (iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area. **OR** (iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.	65/1/1 65/1/2 65/1/3 4M
6	Find the values of 'a' for which $f(x) = x^2 - 2ax + b$ is an increasing function for $x > 0$.	65/1/2 2M
7	The area of an expanding rectangle is increasing at the rate of 48 cm^2/s. The length of the rectangle is always square of its breadth. At what rate the length of rectangle increasing at an instant, when breadth = 4.5 cm ?	65/1/2 3M
8	Find the values of 'a' for which $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ is decreasing on $\mathbb{R}$.	65/1/3 2M

9	A spherical medicine ball when dropped in water dissolves in such a way that the rate of decrease of volume at any instant is proportional to its surface area. Calculate the rate of decrease of its radius.	65/1/3 3M
10	The function $f(x) = x^2 - 4x + 6$ is increasing in the interval (A) $(0, 2)$ (B) $(-\infty, 2]$ (C) $[1, 2]$ (D) $[2, \infty)$	65/2/1
11	A cylindrical tank of radius 10 cm is being filled with sugar at the rate of 100π cm^3/s. The rate, at which the height of the sugar inside the tank is increasing, is : (A) 0.1 cm/s (B) 0.5 cm/s (C) 1 cm/s (D) 1.1 cm/s	65/2/1 65/2/2 65/2/3 1M
12	Find the values of 'a' for which $f(x) = \sin x - ax + b$ is increasing on R.	65/2/1 65/2/2 65/2/3 2M
13	 A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres. Based on the above, answer the following : (i) Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$. (ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion.	65/2/1 65/2/2 65/2/3 4M
14	The values of λ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real values of x are : (A) $1 < \lambda < \sqrt{2}$ (B) $\lambda \geq 1$ (C) $\lambda \geq \sqrt{2}$ (D) $\lambda < 1$	65/4/1 65/4/2 65/4/3 1M
15	If $f(x) = 2x + \cos x$, then $f(x)$: (A) has a maxima at $x = \pi$ (B) has a minima at $x = \pi$ (C) is an increasing function (D) is a decreasing function	65/4/1 65/4/2 65/4/3 1M
16	(a) Find the least value of 'a' so that $f(x) = 2x^2 - ax + 3$ is an increasin function on [2, 4]. **OR**	65/4/1 65/4/2 65/4/3 2M

	(b) If $f(x) = x + \frac{1}{x}$, $x \geq 1$, show that f is an increasing function.	
17	For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/s, then how fast is the slope of the curve changing when $x = 2$?	65/4/1 2M
18	The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight. (i) Find the rate of growth of the plant with respect to sunlight. (ii) In how many days will the plant attain its maximum height ? What is the maximum height ?	65/4/1 65/4/2 5M
19	Find the local maxima and local minima of the function $f(x) = \frac{8}{3}x^3 - 12x^2 + 18x + 5$.	65/4/2 2M
20	A cylindrical water container has developed a leak at the bottom. The water is leaking at the rate of 5 cm^3/s from the leak. If the radius of the container is 15 cm, find the rate at which the height of water is decreasing inside the container, when the height of water is 2 metres.	65/4/3 2M
21	The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at : (A) (1, –10) (B) (1, 10) (C) (10, 1) (D) (–10, 1)	65/5/1 65/5/2 65/5/3 1M
22	Surface area of a balloon (spherical), when air is blown into it, increases at a rate of 5 mm^2/s. When the radius of the balloon is 8 mm, find the rate at which the volume of the balloon is increasing.	65/5/1 2M
23	Find the value of 'a' for which $f(x) = \sqrt{3}\sin x - \cos x - 2ax + 6$ is decreasing in R.	65/5/1 3M
24	A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum. On the basis of the above information, answer the following questions : (i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant. (ii) Find $\frac{dS}{dx}$.	65/5/1 65/5/2 65/5/3 4M

	(iii) (a) Find a relation between x and y such that the surface area (S) is minimum. **OR** (iii) (b) If surface area (S) is constant, the volume (V) = $\frac{1}{4}(Sx - 2x^3)$, x being the edge of base. Show that volume (V) is maximum for $x = \sqrt{\frac{S}{6}}$.	
25	The radius of a cylinder is decreasing at a rate of 2 cm/s and the altitude is increasing at the rate of 3 cm/s. Find the rate of change of volume of this cylinder when its radius is 4 cm and altitude is 6 cm.	65/5/2 2M
26	Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in $\left[0, \frac{\pi}{4}\right]$.	65/5/2 3M
27	Let the volume of a metallic hollow sphere be constant. If the inner radius increases at the rate of 2 cm/s, find the rate of increase of the outer radius when the radii are 2 cm and 4 cm respectively.	65/5/3 2M
28	Find the interval/intervals in which the function $f(x) = \sin 3x - \cos 3x$, $0 < x < \frac{\pi}{2}$ is strictly increasing.	65/5/3 3M
29	**Determine the values of x for which $f(x) = \frac{x-4}{x+1}$, $x \neq -1$ is an increasing or a decreasing function.**	65/6/1 2M
30	 A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall. Based upon the above information, answer the following questions : (i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. (ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point.	65/6/1 65/6/2 65/6/3 4M

	(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. **OR** (iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall ?	
31	Determine those values of x for which $f(x) = \frac{2}{x} - 5, x \neq 0$ is increasing or decreasing.	65/6/2 2M
32	Let $f(x) = \lvert x \rvert, x \in R$. Then, which of the following statements is ***incorrect*** ? (A) f has a minimum value at $x = 0$. (B) f has no maximum value in R. (C) f is continuous at $x = 0$. (D) f is differentiable at $x = 0$.	65/6/1 1M
33	Let $f(x) = x^2, x \in R$. Then, which of the following statements is ***incorrect*** ? (A) Minimum value of f does not exist. (B) There is no point of maximum value of f in R. (C) f is continuous at $x = 0$. (D) f is differentiable at $x = 0$.	65/6/2 1M
34	$f(x) = x^x$ has a critical point at : (A) $x = e$ (B) $x = e^{-1}$ (C) $x = 0$ (D) $x = 1$	65/6/3 1M
35	Find the interval in which $f(x) = x + \frac{1}{x}$ is always increasing, $x \neq 0$.	65/6/3 2M
36	A spherical ball has a variable diameter $\frac{5}{2}(3x + 1)$. The rate of change of its volume w.r.t. x, when $x = 1$, is : (A) 225π (B) 300π (C) 375π (D) 125π	65/7/1 1M
37	If $f : R \to R$ is defined as $f(x) = 2x - \sin x$, then f is : (A) a decreasing function (B) an increasing function (C) maximum at $x = \frac{\pi}{2}$ (D) maximum at $x = 0$	65/7/1 65/7/2 65/7/3 1M

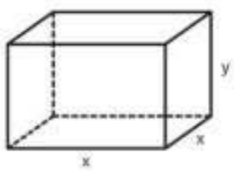
38	Amongst all pairs of positive integers with product as 289, find which of the two numbers add up to the least.	65/7/1 3M
39	If $f(x) = -2x^8$, then the correct statement is : (A) $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$ (B) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$ (C) $-f'\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$ (D) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$	65/7/2 1M
40	Find dimensions of a rectangle of perimeter 12 cm which will generate maximum volume when swept along a circular rotation keeping the shorter side fixed as the axis.	65/7/2 3M
41	Edge of a variable cube increases at the rate of 5 cm/s. The rate at which the surface area of the cube increases when the edge is 2 cm long is : (A) $24\ cm^2/s$ (B) $120\ cm^2/s$ (C) $12\ cm^2/s$ (D) $5\ cm^2/s$	65/7/3 1M
42	Show that of all the rectangles with a fixed perimeter, the square has the greatest area.	65/7/3 3M
43	When x is positive, the minimum value of x^x is (A) e^e (B) $\frac{1}{e}$ (C) $e^{\frac{1}{e}}$ (D) $e^{\frac{-1}{e}}$	65(B) 1M
44	If $y = 7x - x^3$ and x increases at the rate of 2 units per second, then how fast is the slope of the curve changing, when $x = 5$?	65(B) 2M
45	Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is (a) strictly increasing (b) strictly decreasing	65(B) 3M
46	An architect designs a building for a Company. The design of window on the ground floor is proposed to be different than at the other floors. The window is in the shape of a rectangle whose top length is surmounted by a semi-circular opening. This window has a perimeter of 10 m.	65(B) 4M

	Based on the above information, answer the following : (i) If $2x$ and $2y$ represent the length and breadth of the rectangular portion of the window, then establish a relation between x and y. (ii) Find the total area of the window in terms of x. (iii) (a) Find the values of x and y for the maximum area of the window. **OR** (iii) (b) If x and y represent the length and breadth of the rectangle, then establish the expression for the area of the window in terms of x only.
	ANSWERS
1	(C) 4
2	$f(x)=5x^{3/2}-3x^{5/2} \Rightarrow f'(x)=\frac{15}{2}\sqrt{x}(1-x)$ For increasing / decreasing, put $f'(x)=0$ $\Rightarrow x=0,1$ (i) When $x\in[0,1]$, $f'(x)\geq 0$. So, f is increasing when $x\in[0,1]$ (The intervals $(0,1)$, $[0,1)$ or $(0,1]$ can also be considered.) (ii) When $x\in[1,\infty)$, $f'(x)\leq 0$. So, f is decreasing when $x\in[1,\infty)$ (The interval $(1,\infty)$ can also be considered.)
3	Let 'a' be the side of the triangle, so $\frac{da}{dt}=3$ cm/s Now area of an equilateral triangle, $A=\frac{\sqrt{3}}{4}a^2$ $\Rightarrow \frac{dA}{dt}=\frac{\sqrt{3}a}{2}\times\frac{da}{dt}$ $\therefore \left.\frac{dA}{dt}\right]_{a=15\text{cm}}=\frac{\sqrt{3}\times 15}{2}\times 3=\frac{45\sqrt{3}}{2}\text{cm}^2/\text{s}$
4	$f(x)=2x^3-15x^2+36x+1$ $\Rightarrow f'(x)=6(x^2-5x+6)=6(x-2)(x-3)$ $f'(x)=0 \Rightarrow x=2,3\in[1,5]$ Now $f(1)=24, f(2)=29, f(3)=28, f(5)=56$ Hence, the absolute maximum value is 56 and the absolute minimum value is 24.
5	$(i)\ 2x+3y=300$ $(ii)\ A=xy=\frac{x}{3}(300-2x)$ $(iii)(a)\ A=\frac{x}{3}(300-2x)=\frac{1}{3}(300x-2x^2)$ $\Rightarrow \frac{dA}{dx}=\frac{1}{3}(300-4x)$ For critical points, put $\frac{dA}{dx}=0 \Rightarrow x=75$

	Also, $\frac{d^2A}{dx^2}=-\frac{4}{3}<0$. So, A is maximum at $x=75$ Also, maximum area is $A=\frac{75}{3}(300-150)=3750\,\text{m}^2$ OR $(iii)(b)\ A=\frac{x}{3}(300-2x)=\frac{1}{3}(300x-2x^2)$ $\Rightarrow\frac{dA}{dx}=\frac{1}{3}(300-4x)$ For critical points, put $\frac{dA}{dx}=0\Rightarrow x=75$ As $\frac{dA}{dx}$ changes its sign from positive to negative as x passes through $x=75$ from left to right, which means $x=75$ is the point of maximum. Also, maximum area is $A=\frac{75}{3}(300-150)=3750\,\text{m}^2$ Note: Full credit to be given if the student takes equation as $2x+2y=300$ or $2x+4y=300$ or $4x+4y=300$ or $4x+3y=300$ The solutions of sub-parts will differ and marks may be given accordingly.
6	$f'(x)=2x-2a$ $0<x<\infty\Rightarrow-2a<2x-2a<\infty\Rightarrow-2a<f'(x)<\infty$ $f(x)$ is increasing iff $f'(x)\geq0$ $\Rightarrow-2a\in[0,\infty)\Rightarrow a\in(-\infty,0]$ or $(-\infty,0)$
7	Let the length and breadth of the expanding rectangle at any time be 'x' and 'y' respectively. Then, $x=y^2,\ A(\text{Area})=xy=x^{\frac{3}{2}}$ $\frac{dA}{dt}=\frac{3}{2}\sqrt{x}\frac{dx}{dt}$ $\Rightarrow48=\frac{3}{2}\sqrt{(4.5)^2}\frac{dx}{dt}\Rightarrow\frac{dx}{dt}=\frac{64}{9}\text{cm/s}$
8	'f' is a decreasing function iff $f'(x)\leq0\Rightarrow\sqrt{3}\cos x+\sin x-2a\leq0$ $\Rightarrow\sin\left(x+\frac{\pi}{3}\right)\leq a$ We know that for all $x\in R,-1\leq\sin\left(x+\frac{\pi}{3}\right)\leq1$ $\Rightarrow a\in[1,\infty)$ or $(1,\infty)$
9	Let 'V' and 'S' be the volume and surface area of the spherical medicine ball with radius 'r'. $\frac{dV}{dt}=-kS,\ k>0$ $V=\frac{4}{3}\pi r^3\Rightarrow\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}\Rightarrow-kS=4\pi r^2\frac{dr}{dt}$

	$\Rightarrow -k\left(4\pi r^2\right) = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = -k$ **∴ Radius decreases at a constant rate.**
10	(D) $[2, \infty)$
11	(C) 1 cm/s
12	$f'(x) = cosx - a$ For f(x) to be increasing, $f'(x) \geq 0$ $i.e., cosx \geq a$ Since, $-1 \leq cosx \leq 1$ $\Rightarrow a \leq -1$ Hence, $a \in (-\infty, -1]$. (Also, accept $a \in (-\infty, -1)$)
13 (i) (ii)	$f'(x) = e^x(cosx + sinx)$ For critical points, $f'(x) = 0$ $\Rightarrow cosx + sinx = 0$ $\Rightarrow cosx = -sinx$ For x to be a critical point $x \in (0, \pi)$, hence, $x = \frac{3\pi}{4}$ For all $x \in \left[0, \frac{3\pi}{4}\right], f'(x) \geq 0$ Hence, f is increasing in $[0, \frac{3\pi}{4}]$ Note: If a student concludes the answer in any of the following intervals, full marks may be awarded: $\left(0, \frac{3\pi}{4}\right)$ or $\left[0, \frac{3\pi}{4}\right)$ or $(0, \frac{3\pi}{4}]$ For all $x \in \left[\frac{3\pi}{4}, \pi\right], f'(x) \leq 0$ Hence, f is decreasing in $[\frac{3\pi}{4}, \pi]$ Note: If a student concludes the answer in any of the following intervals, full marks may be awarded: $(\frac{3\pi}{4}, \pi)$ or $(\frac{3\pi}{4}, \pi]$ or $[\frac{3\pi}{4}, \pi)$ $x = \frac{3\pi}{4}$ is a critical point $f''(x) = e^x(cosx - sinx) + e^x(cosx + sinx)$ $= 2e^x cosx$ $f''\left(\frac{3\pi}{4}\right) = -ve$ Hence, $\frac{3\pi}{4}$ is a point of local maximum.

14	(C) $\lambda \geq \sqrt{2}$
15	(C) is an increasing function
16 (a) OR (b)	$f(x)=2x^2-ax+3 \Rightarrow f'(x)=4x-a$ Now $2 \leq x \leq 4 \Rightarrow 8-a \leq 4x-a \leq 16-a$ For f to be an increasing function, $f'(x) \geq 0$ $\Rightarrow 8-a \geq 0 \Rightarrow a \leq 8$ $\therefore$ Least value of a does not exist. $f(x)=x+\frac{1}{x} \Rightarrow f'(x)=1-\frac{1}{x^2}=\frac{x^2-1}{x^2}$ Now $\frac{x^2-1}{x^2} \geq 0$ for all $x \geq 1$ $\Rightarrow f'(x) \geq 0 \Rightarrow f$ is an increasing function.
17	$y=5x-2x^3$ Given $\frac{dx}{dt}=2$ units/s slope of the curve $=\frac{dy}{dx}=5-6x^2=m$ $\frac{dm}{dt}=-12x\frac{dx}{dt}=-12x(2)=-24x$ at $x=2, \frac{dm}{dt}=-24(2)=-48$ Hence, slope of curve is decreasing at the rate of 48
18	$(i)\, y=4x-\frac{1}{2}x^2 \Rightarrow \frac{dy}{dx}=(4-x)$ cm/day (ii) For maximum height, $\frac{dy}{dx}=0 \Rightarrow x=4$ days as $\frac{d^2y}{dx^2}<0$, number of days $=4$ Now, Maximum height $=y(4)=16-\frac{1}{2}(16)=8$ cm
19	$f(x)=\frac{8}{3}x^3-12x^2+18x+5$ $\Rightarrow f'(x)=8x^2-24x+18$ $=2(4x^2-12x+9)=2(2x-3)^2$ For critical points, Put $f(x)=0$

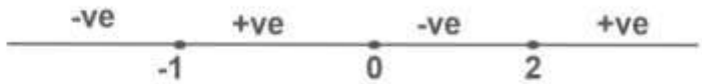
	$\Rightarrow 2(2x-3)^2=0 \Rightarrow x=\frac{3}{2}$ since $f'(x)$ does not change the sign as crosses $x=\frac{3}{2}$ from left to right, f has no local maxima or local minima.
20	Let V, r, h be the volume, radius and height of cylindrical container. Given $\frac{dV}{dt}=-5\,\text{cm}^3/\text{s}$ $V=\pi r^2 h=\pi(15)^2 h=225\pi h$ $\therefore \frac{dV}{dt}=225\pi\frac{dh}{dt} \Rightarrow -5=225\pi\frac{dh}{dt}$ $\Rightarrow \frac{dh}{dt}=-\frac{5}{225\pi}=-\frac{1}{45\pi}$ $\therefore$ Height of the water is decreasing at the rate of $\frac{1}{45\pi}$ cm/s
21	**(A) (1, -10)**
22	$\frac{dS}{dt}=5\,\text{mm}^2/\text{s},\quad \left(\frac{dV}{dt}\right)_{r=8}=?$ $S=4\pi r^2 \Rightarrow \frac{dS}{dt}=8\pi r.\frac{dr}{dt} \Rightarrow \frac{dr}{dt}=\frac{5}{8\pi r}$ $V=\frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt}=4\pi r^2.\frac{dr}{dt} \Rightarrow \frac{dV}{dt}=\frac{5}{2}r$ $\Rightarrow \left(\frac{dV}{dt}\right)_{r=8}=20\,\text{mm}^3/\text{s}$
23	**Since $f(x)$ is a decreasing function $\Rightarrow f'(x)\le 0$** $\Rightarrow \sqrt{3}.\cos x+\sin x-2a\le 0$ $\Rightarrow 2\left(\frac{\sqrt{3}}{2}.\cos x+\frac{1}{2}\sin x\right)-2a\le 0$ $\Rightarrow \cos\left(x-\frac{\pi}{6}\right)\le a$ **Since,** $-1\le\cos\left(x-\frac{\pi}{6}\right)\le 1 \Rightarrow a\ge 1$ **i.e.** $a\in[1,\infty)$ **or** $(1,\infty)$
24	 **(i)** $V=x^2y \Rightarrow y=\frac{V}{x^2}\ldots\ldots\ldots\ldots(i)$ **Hence,** $S=2x^2+4xy=2x^2+\frac{4V}{x}$ **(ii)** $\frac{dS}{dx}=4\left(x-\frac{V}{x^2}\right)$

	(iii) (a) $\frac{dS}{dx} = 0 \Rightarrow V = x^3 \Rightarrow x^2y = x^3 \Rightarrow y = x$ $\frac{d^2S}{dx^2} = 4\left(1 + \frac{2V}{x^3}\right) > 0 \Rightarrow$ S is minimum if $y = x$. **OR** (iii) (b) $V = \frac{1}{4}(Sx - 2x^3) \Rightarrow \frac{dV}{dx} = \frac{1}{4}(S - 6x^2)$ Put $\frac{dV}{dx} = 0 \Rightarrow x = \sqrt{\frac{S}{6}}$ $\left(\frac{d^2V}{dx^2}\right)_{x=\sqrt{\frac{S}{6}}} = -3\sqrt{\frac{S}{6}} < 0 \Rightarrow$ Volume is maximum for $x = \sqrt{\frac{S}{6}}$.
25	$\frac{dr}{dt} = -2$ cm /s, $\frac{dh}{dt} = 3$ cm /s , $\left(\frac{dV}{dt}\right)_{r=4,h=6} = ?$ $V = \pi r^2 h \Rightarrow \frac{dV}{dt} = 2\pi r.\frac{dr}{dt}.h + \pi r^2\frac{dh}{dt}$ When r = 4 cm and h = 6 cm, $\frac{dV}{dt} = 2\pi(4)(-2)(6) + \pi(4)^2(3) = -48\pi$ cm^3/s Volume is decreasing at the rate of 48π cm^3/s
26	$f'(x) = \frac{1}{1 + (\sin x + \cos x)^2}(\cos x - \sin x)$ For $x \in \left[0, \frac{\pi}{4}\right]$, $\cos x \geq \sin x$ $\Rightarrow f'(x) \geq 0$, f is an increasing function in $\left[0, \frac{\pi}{4}\right]$
27	$\frac{dr}{dt} = 2$ cm /s, , $\left(\frac{dR}{dt}\right)_{R=4,r=2} = ?$ $V = \frac{4}{3}\pi(R^3 - r^3) \Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi(3R^2.\frac{dR}{dt} - 3r^2\frac{dr}{dt})$ When R = 4 cm and r = 2 cm, $0 = \frac{4}{3}\pi[3(4)^2.\frac{dR}{dt} - 3(2)^2(2)]$ $\Rightarrow \frac{dR}{dt} = \frac{1}{2}$ cm/s
28	$f'(x) = 3\cos 3x + 3\sin 3x$ $f'(x) = 0 \Rightarrow \sin 3x = -\cos 3x \Rightarrow x = \frac{\pi}{4}$ For $x \in \left(0, \frac{\pi}{4}\right)$, $3\cos 3x + 3\sin 3x > 0$

	$\Rightarrow f'(x) > 0$, **f is strictly increasing function in** $(0, \frac{\pi}{4})$ **or** $(0, \frac{\pi}{4}]$
29	$f'(x) = \frac{x+1-x+4}{(x+1)^2} = \frac{5}{(x+1)^2} > 0$ Hence f is increasing in its domain.
30	(i) $y^2 = h^2 - x^2$ $A = \frac{1}{2}xy = \frac{1}{2}x\sqrt{h^2 - x^2}$ ii) $\frac{dA}{dx} = \frac{1}{2}\sqrt{h^2 - x^2} + \frac{1}{2}x\frac{-x}{\sqrt{h^2 - x^2}}$ $\frac{dA}{dx} = 0$ gives $x = \frac{h}{\sqrt{2}}$ (iii)(a) $A'' = \frac{1}{2}\frac{-4x.\sqrt{h^2 - x^2} - (h^2 - 2x^2)\frac{-x}{\sqrt{h^2 - x^2}}}{h^2 - x^2}$ is < 0 at $x = \frac{h}{\sqrt{2}}$ Hence A is maximum at critical point OR (iii)(b) $y^2 = 25 - x^2$ hence $y = 3$ gives $x = 4$ $2y\frac{dy}{dt} = -2x\frac{dx}{dt}$ $\frac{dx}{dt} = 1.5 m/s$
31	$\mathbf{f'(x) = \frac{-2}{x^2} < 0}$ **Hence f is decreasing in its domain.**
32	(D) f is differentiable at x = 0
33	(A) Minimum value of f does not exist
34	(B) $x = e^{-1}$
35	$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$ $f'(x) = 0$ gives $x = 1, -1$ f is decreasing in (-1, 0) U (0, 1) as $f'(x) < 0$ f is increasing in R - (-1, 1) as $f'(x) > 0$
36	**(C) 375π**
37	(B) an increasing function
38	**Let numbers be 'x' and 'y' such that** $xy = 289 \Rightarrow y = \frac{289}{x}$**, 'S' be their sum, then** $S = x + y = x + \frac{289}{x}$

	$\frac{dS}{dx}=1-\frac{289}{x^2}, \frac{dS}{dx}=0 \Rightarrow x=17$, a positive integer $\left.\frac{d^2S}{dx^2}\right]_{x=17}=289\left.\left(\frac{2}{x^3}\right)\right]_{x=17}>0$, $\therefore$ S is minimum when $x=17, y=17$
39	(B) $f'\left(\frac{1}{2}\right)=-f'\left(-\frac{1}{2}\right)$
40	Let Length and Breadth of the rectangle be 'x' and 'y' respectively. Also 'r' be the radius of the cylinder then, $2(x+y)=12 \Rightarrow x+y=6,\ 2\pi r=x$ $V(\text{Volume of cylinder})=\pi r^2 y \Rightarrow V=\pi\left(\frac{x}{2\pi}\right)^2(6-x)=\frac{1}{4\pi}(6x^2-x^3)$ $V'(x)=0 \Rightarrow \frac{1}{4\pi}(12x-3x^2)=0 \Rightarrow x=4,\ (\because x\neq 0)$ $V''(x)=\frac{1}{4\pi}(12-6x) \Rightarrow V''(4)=-\frac{3}{\pi}<0$ The volume of the cylinder obtained by the rotation will be maximum if the dimensions of the rectangle are $x=4$ cm, $y=2$ cm.
41	(B) 120 cm^2/s
42	Let P be the perimeter of the rectangle, which is a constant. Also assume 'x' and 'y' be the length and breadth of the rectangle, then $2(x+y)=P$ and $A(\text{Area})=xy=\frac{x}{2}(P-2x)=\frac{1}{2}(Px-2x^2)$ $A'(x)=\frac{1}{2}(P-4x),\ \therefore\ A'(x)=0 \Rightarrow x=\frac{P}{4}, y=\frac{P}{4}$ $A''(x)=-2<0$ at $x=\frac{P}{4}$, $\therefore$ Area of the rectangle is max. if it is a square.
43	(D) $e^{-\frac{1}{e}}$
44	$y=7x-x^3$ $m=7-3x^2$ $\frac{dm}{dt}=-6x\frac{dx}{dt}$ $\left[\frac{dm}{dt}\right]_{x=5}=-30(2)=-60$
45	$f(x)=3x^4-4x^3-12x^2+5$ $f'(x)=12x^3-12x^2-24x$ $f'(x)=12x(x-2)(x+1)$

$f'(x) = 0 \Rightarrow x = -1, 0, 2$

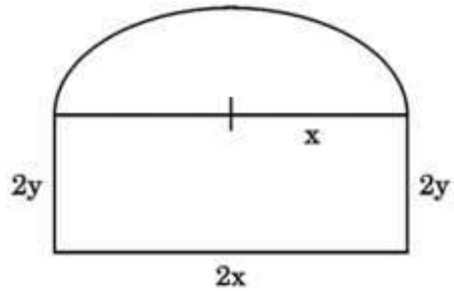


Sign of f'(x)

f is strictly increasing in (-1,0) as well as (2, ∞) or [-1,0] as well as [2, ∞)

and strictly decreasing in (-∞, -1) as well as (0,2) or (-∞, -1] as well as [0,2]

46 **(i)** $2x + 4y + \pi x = 10$



(ii)

$$A = (2x)(2y) + \frac{1}{2}\pi x^2$$

$$y = \frac{10 - 2x - \pi x}{4}$$

$$A = 4x\frac{(10 - 2x - \pi x)}{4} + \frac{1}{2}\pi x^2$$

$$A = 10x - 2x^2 - \frac{1}{2}\pi x^2$$

(iii) (a) $\frac{dA}{dx} = 10 - 4x - \pi x = 0$

$$\Rightarrow 10 = (\pi + 4)x$$

$$\Rightarrow x = \frac{10}{\pi + 4}$$

$$\frac{d^2y}{dx^2} = -4 - \pi < 0$$

$$\Rightarrow y = \frac{1}{4}\left[10 - 2\left(\frac{10}{\pi + 4}\right) - \pi\left(\frac{10}{\pi + 4}\right)\right]$$

$$\Rightarrow y = \frac{1}{4}\frac{[10\pi + 40 - 20 - 10\pi]}{\pi + 4}$$

$$y = \frac{1}{4}\left[\frac{20}{\pi + 4}\right] = \frac{5}{\pi + 4}$$

(iii) (b) $A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$$x + 2y + \pi\left(\frac{x}{2}\right) = 10$$

$$\Rightarrow y = \frac{20 - 2x - \pi x}{4}$$

$$\therefore A = x\left(\frac{20 - 2x - \pi x}{4}\right) + \frac{1}{8}\pi x^2$$

$$A = 5x - \frac{1}{2}x^2 - \frac{\pi}{8}x^2$$

CHAPTER-7 INTEGRALS

Q.		Code and Marks
1	$\int_{-1}^{1} \frac{\lvert x \rvert}{x}\, dx, x \neq 0$ is equal to (A) -1 (B) 0 (C) 1 (D) 2	65/1/1 (1m)
2	If $\int \frac{2^{\frac{1}{x}}}{x^2}\, dx = k \cdot 2^{\frac{1}{x}} + C$, then k is equal to (A) $\frac{-1}{\log 2}$ (B) $-\log 2$ (C) -1 (D) $\frac{1}{2}$	65/1/1 65/1/2 65/1/3 (1m)
3	(a) Find : $\int \frac{x + \sin x}{1 + \cos x}\, dx$ **OR** (b) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$	65/1/1 (3m)
4	$\int \frac{1 - 2 \sin x}{\cos^2 x}\, dx$ is equal to : (A) $\tan x - 2 \sec x + C$ (B) $-\tan x + 2 \sec x + C$ (C) $-\tan x - 2 \sec x + C$ (D) $\tan x + 2 \sec x + C$	65/1/2 (1m)
5	(a) Find : $\int \frac{\cos 2x}{(\sin x + \cos x)^2}\, dx$ **OR** (b) Evaluate : $\int_0^{\frac{\pi}{2}} \frac{5 \sin x + 3 \cos x}{\sin x + \cos x}\, dx$	65/1/2 (3m)

6	If $\int_0^1 \frac{e^x}{1+x}\,dx = \alpha$, then $\int_0^1 \frac{e^x}{(1+x)^2}\,dx$ is equal to (A) $\alpha - 1 + \frac{e}{2}$ (B) $\alpha + 1 - \frac{e}{2}$ (C) $\alpha - 1 - \frac{e}{2}$ (D) $\alpha + 1 + \frac{e}{2}$	65/1/3 (1m)
7	(a) Find : $\int \frac{2x-1}{(x-1)(x+2)(x-3)}\,dx$ **OR** (b) Evaluate : $\int_0^5 (\lvert x-1 \rvert + \lvert x-2 \rvert + \lvert x-5 \rvert)\,dx$	65/1/3 (3m)
8	If $f(2a-x) = f(x)$, then $\int_0^{2a} f(x)\,dx$ is (A) $\int_0^{2a} f\left(\frac{x}{2}\right) dx$ (B) $\int_0^{a} f(x)\,dx$ (C) $2\int_a^{0} f(x)\,dx$ (D) $2\int_0^{a} f(x)\,dx$	65/2/1 (1m)
9	Evaluate : $\int_0^{\frac{\pi}{4}} \sqrt{1+\sin 2x}\,dx$	65/2/1 (2m)
10	Find : $\int \frac{1}{x}\sqrt{\frac{x+a}{x-a}}\,dx.$	65/2/1 (3m)
11	$\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to (A) $\tan x + \cot x + C$ (B) $(\tan x + \cot x)^2 + C$ (C) $\tan x - \cot x + C$ (D) $(\tan x - \cot x)^2 + C$	65/2/2 (1m)

12	Evaluate : $\int_0^{\pi} \frac{\sin 2px}{\sin x}\, dx$, $p \in N$.	65/2/2 (2m)
13	f and g are continuous functions on interval [a, b]. Given that $f(a - x) = f(x)$ and $g(x) + g(a - x) = a$, show that $\int_0^a f(x)\, g(x)\, dx = \frac{a}{2} \int_0^a f(x)\, dx$.	65/2/2 (3m)
14	Find : $\int \frac{5x}{(x+1)(x^2+9)}\, dx$.	65/2/2 (5m)
15	If $\int_0^a x\, dx \le \frac{a}{2} + 6$, then (A) $-4 \le a \le 3$ (B) $a \ge 4, a \le -3$ (C) $-3 \le a \le 4$ (D) $-3 \le a \le 0$	65/2/3 (1m)
16	Find : $\int 2x^3 e^{x^2}\, dx$.	65/2/3 (2m)
17	If $\int_a^b x^3\, dx = 0$ and $\int_a^b x^2\, dx = \frac{2}{3}$, then find the values of a and b.	65/2/3 (3m)
18	Find : $\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$.	65/2/3 (5m)
19	$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx$ is equal to : (A) $2(\sin x + x\cos\alpha) + C$ (B) $2(\sin x - x\cos\alpha) + C$ (C) $2(\sin x + 2x\cos\alpha) + C$ (D) $2(\sin x + \sin\alpha) + C$	65/4/1 (1m)

20	The value of $\int_0^1 \frac{dx}{e^x+e^{-x}}$ is : (A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\tan^{-1} e - \frac{\pi}{4}$ (D) $\tan^{-1} e$	65/4/1 (1m)
21	Evaluate : $\int_{\pi/2}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$	65/4/1 (3m)
22	(a) Find : $\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)}\,dx$ **OR** (b) Evaluate : $\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$	65/4/1 (5m)
23	$\int e^x(\cos x - \sin x)\,dx$ is equal to : (A) $e^x \sin x + C$ (B) $-e^x \sin x + C$ (C) $-e^x \cos x + C$ (D) $e^x \cos x + C$	65/4/2 (1m)
24	The value of $\int_0^1 \frac{dx}{e^x+e^{-x}}$ is : (A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\tan^{-1} e - \frac{\pi}{4}$ (D) $\tan^{-1} e$	65/4/2 65/4/3 (1m)

25	**Find** : $$\int \frac{\cos x\, dx}{1+\cos x+\sin x}$$	65/4/2 (3m)
26	(a) Evaluate : $$\int_0^{\pi/4} \frac{\sin x \cos x}{\cos^4 x+\sin^4 x}\, dx$$ **OR** (b) Find : $$\int \frac{\sqrt{x^2+1}\left[\log(x^2+1)-2\log x\right]}{x^2}\, dx$$	65/4/2 (5m)
27	If $\int e^{-3\log x}\, dx = f(x) + C$, then f(x) is : (A) $e^{-3\log x}$ (B) $e^{\log\left(\frac{1}{x^3}\right)}$ (C) $\frac{-1}{2x^2}$ (D) $\frac{-1}{4x^4}$	65/4/3 (1m)
28	Find : $$\int \frac{\sqrt{x}}{1+\sqrt{x^{3/2}}}\, dx$$	65/4/3 (3m)
29	(a) Evaluate : $$\int_0^{3/2} \lvert x\cos \pi x \rvert\, dx$$ **OR** (b) Find : $$\int \frac{dx}{\sin x+\sin 2x}$$	65/4/3 (5m)

30	$\int \sqrt{1+\sin x}\ dx$ is equal to : (A) $2\left(-\sin\frac{x}{2}+\cos\frac{x}{2}\right)+C$ (B) $2\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)+C$ (C) $-2\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)+C$ (D) $2\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)+C$	65/5/1 (1m)
31	$\int_0^{\pi/2} \cos x . e^{\sin x}\ dx$ is equal to : (A) 0 (B) $1-e$ (C) $e-1$ (D) e	65/5/1 (1m)
32	(a) Find : $\int \frac{2x}{(x^2+3)(x^2-5)} dx$ **OR** (b) Evaluate : $\int_1^4 (\lvert x-2\rvert + \lvert x-4\rvert)\ dx$	65/5/1 65/5/2 65/5/3 (3m)
33	$\int \frac{\cos 2x}{\sin^2 x \cos^2 x}\ dx$ is equal to : (A) $\cot x+\tan x+C$ (B) $-(\cot x+\tan x)+C$ (C) $-\cot x+\tan x+C$ (D) $\cot x-\tan x+C$	65/5/2 (1m)
34	If $\int_0^a \frac{1}{1+4x^2}\ dx = \frac{\pi}{8}$, then the value of 'a' is : (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{8}$ (D) 4	65/5/2 (1m)

35	$\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta}\, dx$ is equal to : (A) $2(\sin x + x\cos\theta) + C$ (B) $2(\sin x - x\cos\theta) + C$ (C) $2(\sin x + \sin\theta) + C$ (D) $2(\sin x - x\sin\theta) + C$	65/5/3 (1m)
36	$\int_0^1 \frac{2x}{5x^2+1}\, dx$ is equal to : (A) $\frac{1}{5}\log 6$ (B) $\frac{1}{5}\log 5$ (C) $\frac{1}{2}\log 6$ (D) $\frac{1}{2}\log 5$	65/5/3 (1m)
37	Let $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, $f(1) = 0$. Then, $f(x)$ is : (A) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$ (B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$ (C) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$ (D) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$	65/6/1 65/6/2 65/6/3 (1m)
38	$\int \frac{x+5}{(x+6)^2} e^x\, dx$ is equal to : (A) $\log(x+6) + C$ (B) $e^x + C$ (C) $\frac{e^x}{x+6} + C$ (D) $\frac{-1}{(x+6)^2} + C$	65/6/1 65/6/2 65/6/3 (1m)
39	(a) **Find :** $\int \frac{x^2+1}{(x-1)^2(x+3)}\, dx$ **OR** (b) **Evaluate :** $\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\, dx$	65/6/1 65/6/2 65/6/3 (5m)

40 65/7/1 65/7/3 (1m)

$$\int \frac{e^{9\log x} - e^{8\log x}}{e^{6\log x} - e^{5\log x}}\, dx \text{ is equal to :}$$

(A) $x + C$ (B) $\frac{x^2}{2} + C$

(C) $\frac{x^4}{4} + C$ (D) $\frac{x^3}{3} + C$

41 65/7/1 65/7/2 65/7/3 (1m)

For a function f(x), which of the following holds true ?

(A) $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

(B) $\int_{-a}^{a} f(x)\, dx = 0$, if f is an even function

(C) $\int_{-a}^{a} f(x)\, dx = 2\int_0^a f(x)\, dx$, if f is an odd function

(D) $\int_0^{2a} f(x)\, dx = \int_0^a f(x)\, dx - \int_0^a f(2a+x)\, dx$

42 65/7/1 65/7/2 (1m)

$$\int \frac{e^x}{\sqrt{4 - e^{2x}}}\, dx \text{ is equal to :}$$

(A) $\frac{1}{2}\cos^{-1}(e^x) + C$ (B) $\frac{1}{2}\sin^{-1}(e^x) + C$

(C) $\frac{e^x}{2} + C$ (D) $\sin^{-1}\left(\frac{e^x}{2}\right) + C$

43 65/7/1 (5m)

(a) Find :

$$\int \frac{x^2+1}{(x^2+2)(2x^2+1)}\, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$$

44	$\int \frac{a^x}{\sqrt{1-a^{2x}}}\,dx$ is equal to : (A) $\frac{\sin^{-1}(a^x)}{\log_e a}+C$ (B) $\log_e(1-a^{2x})+C$ (C) $\frac{\cos^{-1}(a^x)}{\log_e a}+C$ (D) $\frac{\sin^{-1}(a^x)}{a^x}+C$	65/7/2 (1m)
45	(a) Find : $\int \frac{x}{(x-1)(x^2+4)}\,dx$ **OR** (b) Evaluate : $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$	65/7/2 (5m)
46	$\int \frac{e^{-x}}{16+9e^{-2x}}\,dx$ is equal to : (A) $\frac{16}{9}\tan^{-1}(e^{-x})+C$ (B) $-\frac{1}{12}\tan^{-1}\left(\frac{3e^{-x}}{4}\right)+C$ (C) $\tan^{-1}\left(\frac{e^{-x}}{4}\right)+C$ (D) $-\frac{1}{3}\tan^{-1}\left(\frac{e^{-x}}{4}\right)+C$	65/7/3 (1m)
47	(a) Find : $\int \frac{3x+1}{(x-2)^2(x+2)}\,dx$ **OR** (b) Evaluate : $\int_0^{\pi/2} \frac{x}{\cos x+\sin x}\,dx$	65/7/3 (5m)
48	$\int \frac{2x^3}{4+x^8}\,dx$ is equal to (A) $\frac{1}{4}\tan^{-1}\frac{x^4}{2}+C$ (B) $\frac{1}{2}\tan^{-1}\frac{x^4}{2}+C$ (C) $\frac{1}{4}\tan^{-1}\frac{x^4}{4}+C$ (D) $\frac{1}{4}\tan^{-1}x^4+C$	65(B) (1m)

49	$\int e^{x} \cdot \frac{x}{(1+x)^{2}}\, dx$ is equal to (A) $e^{x} \cdot \frac{x}{1+x} + C$ (B) $e^{x} \cdot \frac{1}{1+x} + C$ (C) $e^{x} \cdot \frac{1}{x} + C$ (D) $e^{x} \cdot \frac{1}{(1+x)^{2}} + C$	65(B) (1m)
50	Find : $\int \frac{x^{2}-x+1}{(x-1)(x^{2}+1)}\, dx$ **OR** Evaluate : $\int_{1}^{4} (\lvert x \rvert + \lvert 3-x \rvert)\, dx$	65(B) (5m)

	ANSWERS
1	(B) 0
2	(A) $\frac{-1}{\log 2}$
3	$\int \frac{x+\sin x}{1+\cos x} dx$ $= \int \frac{x + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^{2}\frac{x}{2}} dx$ $= \int x\left(\frac{1}{2}\sec^{2}\frac{x}{2}\right)dx + \int \tan\frac{x}{2} dx$ $= x\tan\frac{x}{2} - \int \tan\frac{x}{2} dx + \int \tan\frac{x}{2} dx$ $= x\tan\frac{x}{2} + C$ $\int_{0}^{\pi/4} \frac{dx}{\cos^{3} x\sqrt{2\sin 2x}}$ $= \frac{1}{2}\int_{0}^{\pi/4} \frac{dx}{\cos^{4} x\sqrt{\tan x}}$ $= \frac{1}{2}\int_{0}^{\pi/4} \frac{(1+\tan^{2} x)\sec^{2} x}{\sqrt{\tan x}} dx$ Put $\tan x = t \Rightarrow \sec^{2} x\, dx = dt$ $\therefore I = \frac{1}{2}\int_{0}^{1} \frac{1+t^{2}}{\sqrt{t}} dt$ $= \frac{1}{2}\int_{0}^{1}\left(\frac{1}{\sqrt{t}} + t^{3/2}\right)dt$ $= \frac{1}{2}\left[2\sqrt{t} + \frac{2}{5}t^{5/2}\right]_{0}^{1}$ $= \frac{6}{5}$
4	(A) $\tan x - 2\sec x + C$

5

$$\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx=\int \frac{\cos^2 x-\sin^2 x}{(\sin x+\cos x)^2}dx$$

$$=\int \frac{\cos x-\sin x}{\sin x+\cos x}dx$$

$$=\log|\sin x+\cos x|+C$$

$$I=\int_0^{\pi/2} \frac{5\sin x+3\cos x}{\sin x+\cos x}dx \qquad \text{-- (i)}$$

$$I=\int_0^{\pi/2} \frac{5\sin\left(\frac{\pi}{2}-x\right)+3\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx=\int_0^{\pi/2} \frac{5\cos x+3\sin x}{\cos x+\sin x}dx \qquad \text{-- (ii)}$$

Adding (i) and (ii), we get

$$2I=\int_0^{\pi/2} 8dx \Rightarrow I=4x\Big]_0^{\frac{\pi}{2}}=2\pi$$

6

(B) $\alpha+1-\dfrac{e}{2}$

7

$$\int \frac{2x-1}{(x-1)(x+2)(x-3)}dx=-\frac{1}{6}\int \frac{1}{x-1}dx-\frac{1}{3}\int \frac{1}{x+2}dx+\frac{1}{2}\int \frac{1}{x-3}dx$$

(Using Partial Fraction)

$$=-\frac{1}{6}\log|x-1|-\frac{1}{3}\log|x+2|+\frac{1}{2}\log|x-3|+C$$

$$I=\int_0^6 \left(|x-1|+|x-2|+|x-5|\right)dx$$

$$\therefore\ I=\left[-\int_0^1 (x-1)\,dx+\int_1^6 (x-1)\,dx\right]+\left[-\int_0^2 (x-2)\,dx+\int_2^6 (x-2)\,dx\right]+\left[-\int_0^6 (x-5)\,dx\right]$$

$$=-\frac{(x-1)^2}{2}\Bigg]_0^1+\frac{(x-1)^2}{2}\Bigg]_1^6-\frac{(x-2)^2}{2}\Bigg]_0^2+\frac{(x-2)^2}{2}\Bigg]_2^6-\frac{(x-5)^2}{2}\Bigg]_0^6$$

$$=\frac{17}{2}+\frac{13}{2}+\frac{25}{2}=\frac{55}{2}$$

8

(D) $2\int_0^a f(x)dx$

9

Given definite integral $=\int_0^{\frac{\pi}{4}} \sqrt{(sinx+cosx)^2}dx$

	$= \int_0^{\frac{\pi}{4}} (sinx + cosx)dx$ $= [-cosx + sinx]_0^{\frac{\pi}{4}}$ $= 1$
10	$I = \int \frac{1}{x} \frac{x+a}{\sqrt{x^2-a^2}} dx = \int \frac{1}{\sqrt{x^2-a^2}} dx + a \int \frac{1}{x\sqrt{x^2-a^2}} dx$ $= \log\left\|x + \sqrt{x^2 - a^2}\right\| + \sec^{-1}\left(\frac{x}{a}\right) + C$
11	(C) $\tan x - \cot x + C$
12	$I = \int_0^{\pi} \frac{\sin 2px}{\sin x} dx$ $= \int_0^{\pi} \frac{\sin 2p(\pi - x)}{\sin(\pi - x)} dx$ $I = \int_0^{\pi} \frac{-\sin 2px}{\sin x} dx$ Adding, we get $2I = 0$ $\therefore I = 0$
13	$I = \int_0^a f(x)g(x)dx$ $= \int_0^a f(a-x)g(a-x)dx$ $= \int_0^a f(x)[a - g(x)]dx$ $I = a\int_0^a f(x)dx - \int_0^a f(x)g(x)dx$ Adding, we get $I = \frac{a}{2}\int_0^a f(x)dx$
14	$\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$ $\Rightarrow A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{9}{2}$

	Given integral $= -\frac{1}{2}\int \frac{1}{x+1}dx + \frac{1}{2}\int \frac{x+9}{x^2+9}dx$ $= -\frac{1}{2}\int \frac{1}{x+1}dx + \frac{1}{4}\int \frac{2x}{x^2+9}dx + \frac{1}{4}\int \frac{18}{x^2+9}dx$ $= -\frac{1}{2}\log\lvert x+1\rvert + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\frac{x}{3} + C$
15	(C) $-3 \le a \le 4$
16	$I = \int 2x^3 e^{x^2} dx = \int 2xx^2 e^{x^2} dx$ Put $x^2 = t \Rightarrow 2xdx = dt$ $I = \int te^t dt$ $= te^t - \int e^t dt$ $= te^t - e^t + C$ $= e^{x^2}(x^2 - 1) + C$
17	$\int_a^b x^3 dx = 0 \Rightarrow \frac{b^4 - a^4}{4} = 0$ $\Rightarrow b^4 - a^4 = 0$ $\Rightarrow a = -b \quad (a \neq b)$ $\int_a^b x^2 dx = \frac{2}{3} \Rightarrow \frac{b^3 - a^3}{3} = \frac{2}{3}$ $\Rightarrow b^3 - a^3 = 2$ $\Rightarrow b^3 = 1$ $\Rightarrow b = 1$ $\Rightarrow b = 1, a = -1$
18	$I = \int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx$ $= \sqrt{2}\int \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$ Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x)dx = dt$ On squaring both sides, we get $1 - \sin 2x = t^2$

	$I = \sqrt{2}\int \frac{1}{\sqrt{1-t^2}}dt$ $= \sqrt{2}\sin^{-1}t + C$ $= \sqrt{2}\sin^{-1}(\sin x - \cos x) + C$
19	(A) $2(\sin x + x\cos\alpha) + C$
20	(C) $\tan^{-1} e - \frac{\pi}{4}$
21	$I = \int_{\pi/2}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$ $= \int_{\pi/2}^{\pi} e^x\left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right)dx$ $= \int_{\pi/2}^{\pi} e^x\left(\frac{1}{2}\operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right)dx$ $\therefore I = -\left[e^x \cot\frac{x}{2}\right]_{\pi/2}^{\pi}$ $= -\left[e^{\pi}\cot\frac{\pi}{2} - e^{\frac{\pi}{2}}\cot\frac{\pi}{4}\right]$ $= e^{\frac{\pi}{2}}$
22	$I = \int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)}dx$ $= \int \frac{\cos x}{(4+\sin^2 x)(1+4\sin^2 x)}dx$ $\sin x = t$ gives $I = \int \frac{dt}{(4+t^2)(1+4t^2)}$ $= -\frac{1}{15}\int\frac{dt}{4+t^2} + \frac{4}{15}\int\frac{dt}{1+4t^2}$ $(\because$ using Partial Fraction$)$

	$=-\frac{1}{30}\tan^{-1}\left(\frac{t}{2}\right)+\frac{2}{15}\tan^{-1}(2t)+C$ $=-\frac{1}{30}\tan^{-1}\left(\frac{\sin x}{2}\right)+\frac{2}{15}\tan^{-1}(2\sin x)+C$ $I=\int_0^{\pi}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$ $=2\int_0^{\pi/2}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$ $=2\int_0^{\pi/2}\frac{\sec^2 x}{a^2+b^2\tan^2 x}dx$ $\tan x=t$ gives $I=2\int_0^{\infty}\frac{dt}{a^2+b^2t^2}$ $=\frac{2}{b^2}\cdot\frac{b}{a}\tan^{-1}\left(\frac{bt}{a}\right)\Big]_0^{\infty}$ $=\frac{\pi}{ab}$
23	(D) $e^x \cos x + C$
24	(C) $\tan^{-1} e-\frac{\pi}{4}$
25	$I=\int\frac{\cos x}{1+\cos x+\sin x}dx$ $=\int\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}dx \quad =\int\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{2\cos\frac{x}{2}}dx$ $=\frac{1}{2}\int\left(1-\tan\frac{x}{2}\right)dx \quad =\frac{1}{2}\left[x+2\log\cos\frac{x}{2}\right]+C$

26	$$I=\int_0^{\pi/4}\frac{\sin x\cos x}{\cos^4 x+\sin^4 x}dx$$ dividing numerator and denominator by $\cos^4 x$, $$I=\int_0^{\pi/4}\frac{\tan x\sec^2 x}{1+\tan^4 x}dx$$ Put $\tan^2 x=t \Rightarrow 2\tan x\sec^2 x\,dx=dt$ when $x=0, t=0$; when $x=\frac{\pi}{4}, t=1$ $$\Rightarrow I=\frac{1}{2}\int_0^1\frac{dt}{1+t^2}$$ $$=\frac{1}{2}\left[\tan^{-1}t\right]_0^1$$ $$=\frac{\pi}{8}$$ **Due to printing error, the given function is not integrable.** **So full marks may be given for every attempt.**
27	(C) $\frac{-1}{2x^2}$
28	$$I=\int\frac{\sqrt{x}}{1+\sqrt{x^{3/2}}}dx$$ Put $x^{3/2}=t \Rightarrow \frac{3}{2}\sqrt{x}dx=dt$ $$\Rightarrow I=\frac{2}{3}\int\frac{dt}{1+\sqrt{t}}$$ Put $\sqrt{t}=z \Rightarrow \frac{1}{2\sqrt{t}}dt=dz$ $$\Rightarrow I=\frac{2}{3}\int\frac{2z}{1+z}dz$$ $$=\frac{4}{3}\left[\int 1\,dz-\int\frac{1}{1+z}dz\right]$$ $$=\frac{4}{3}\left[z-\log\lvert 1+z\rvert\right]+C$$ $$=\frac{4}{3}\left[\sqrt{t}-\log\left\lvert 1+\sqrt{t}\right\rvert\right]+C$$ $$=\frac{4}{3}\left[\sqrt{x^{3/2}}-\log\left\lvert 1+\sqrt{x^{3/2}}\right\rvert\right]+C$$

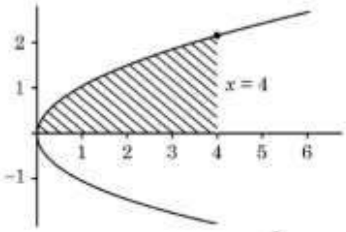
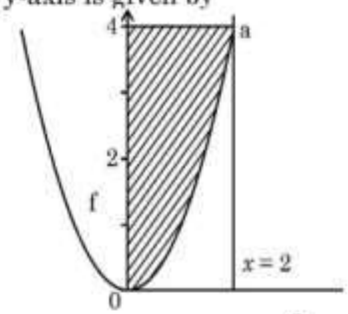
29	$I=\int_0^{3/2}\lvert x\cos\pi x\rvert dx$ $=\int_0^{1/2} x\cos\pi x\,dx-\int_{1/2}^{3/2} x\cos\pi x\,dx \quad \dots(1)$ Consider $\int x\cos\pi x\,dx$ $=\frac{x\sin\pi x}{\pi}-\int\frac{\sin\pi x}{\pi}dx$ $=\frac{x\sin\pi x}{\pi}+\frac{\cos\pi x}{\pi^2} \quad \dots(2)$ using (2) in (1), $\left.\frac{x\sin\pi x}{\pi}+\frac{\cos\pi x}{\pi^2}\right]_0^{1/2}-\left.\frac{x\sin\pi x}{\pi}+\frac{\cos\pi x}{\pi^2}\right]_{1/2}^{3/2}$ $=\left(\frac{1}{2\pi}-\frac{1}{\pi^2}\right)-\left(-\frac{3}{2\pi}-\frac{1}{2\pi}\right)$ $=\frac{5}{2\pi}-\frac{1}{\pi^2}$ $I=\int\frac{dx}{\sin x+\sin 2x}$ $=\int\frac{dx}{\sin x(1+2\cos x)}$ $=\int\frac{\sin x}{\sin^2 x(1+2\cos x)}dx$ $=\int\frac{\sin x}{(1-\cos x)(1+\cos x)(1+2\cos x)}dx$ Put $\cos x=t\Rightarrow\sin x\,dx=dt$ $I=-\int\frac{dt}{(1-t)(1+t)(1+2t)}$ $=-\frac{1}{6}\int\frac{dt}{1-t}+\frac{1}{2}\int\frac{dt}{1+t}-\frac{4}{3}\int\frac{dt}{1+2t}$ $=\frac{1}{6}\log\lvert 1-t\rvert+\frac{1}{2}\log\lvert 1+t\rvert-\frac{2}{3}\log\lvert 1+2t\rvert+C$ $=\frac{1}{6}\log\lvert 1-\cos x\rvert+\frac{1}{2}\log\lvert 1+\cos x\rvert-\frac{2}{3}\log\lvert 1+2\cos x\rvert+C$
30	**(B) 2 $(\sin\frac{x}{2}-\cos\frac{x}{2})$ + C**
31	**(C) e - 1**

32	(a) Let $I = \int \frac{2x}{(x^2+3)(x^2-5)} dx$ Put $x^2 = t \Rightarrow 2x.\,dx = dt$ $\Rightarrow I = \int \frac{dt}{(t+3)(t-5)}$ $= \int \left(-\frac{1}{8\,(t+3)} + \frac{1}{8(t-5)}\right) dt$ $= \frac{1}{8}[\log\|t-5\| - \log\|t+3\|] + c$ $= \frac{1}{8}\log\left\|\frac{x^2-5}{x^2+3}\right\| + c$ OR (b) $\int_1^4 (\|x-2\| + \|x-4\|)\, dx$ $= \int_1^2 (2-x)\, dx + \int_2^4 (x-2) dx - \int_1^4 (x-4)\, dx$ $= \left[\frac{(2-x)^2}{-2}\right]_1^2 + \left[\frac{(x-2)^2}{2}\right]_2^4 - \left[\frac{(x-4)^2}{2}\right]_1^4$ $= \frac{1}{2} + 2 + \frac{9}{2} = 7$
33	(B) $-(\cot x + \tan x) + C$
34	(B) $\frac{1}{2}$
35	(A) $2\,(\sin x + x\cos\theta) + C$
36	(A) $\frac{1}{5}\log 6$
37	(B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$
38	(C) $\frac{e^x}{x+6} + C$
39	(a) $\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3} = \frac{3/8}{x-1} + \frac{1/2}{(x-1)^2} + \frac{5/8}{x+3}$ $I = \frac{3}{8}\log\|x-1\| - \frac{1}{2(x-1)} + \frac{5}{8}\log\|x+3\| + C$ OR (b) Let $I = \int_0^{\frac{\pi}{2}} \frac{x}{\sin x+\cos x}\, dx$ $I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2} - x}{\cos x+\sin x}\, dx$ using property $2I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}}{\sin x+\cos x}\, dx$ $= \frac{\pi}{2}\frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{1}{\sin\left(\frac{\pi}{4}+x\right)} dx$ $2I = \frac{\pi}{2\sqrt{2}} \log\left\|\operatorname{cosec}\left(\frac{\pi}{4}+x\right) - \cot\left(\frac{\pi}{4}+x\right)\right\|_0^{\frac{\pi}{2}}$ $I = \frac{\pi}{4\sqrt{2}} \log\frac{\sqrt{2}+1}{\sqrt{2}-1}$

40	(C) $\dfrac{x^4}{4}+C$
41	(A) $\displaystyle\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx$
42	(D) $\sin^{-1}\left(\dfrac{e^x}{2}\right)+C$
43	(a) $\displaystyle\int\frac{x^2+1}{(x^2+2)(2x^2+1)}dx=\frac{1}{3}\int\frac{1}{x^2+2}dx+\frac{1}{3}\int\frac{1}{2x^2+1}dx$ (Using Partial Fractions) $=\dfrac{1}{3}\cdot\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{x}{\sqrt{2}}\right)+\dfrac{1}{3}\cdot\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}x\right)+C$ or $=\dfrac{1}{3\sqrt{2}}\left(\tan^{-1}\dfrac{x}{\sqrt{2}}+\tan^{-1}\sqrt{2}x\right)+C$ OR (b) Let $I=\displaystyle\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$ – (i) $\Rightarrow I=\displaystyle\int_0^{\pi}\frac{(\pi-x)\tan x}{\sec x+\tan x}dx$ – (ii) Adding (i) & (ii), we get $2I=\pi\displaystyle\int_0^{\pi}\frac{\tan x}{\sec x+\tan x}dx\Rightarrow 2I=\pi\int_0^{\pi}\frac{\tan x(\sec x-\tan x)}{\sec^2 x-\tan^2 x}dx$ $=\pi\displaystyle\int_0^{\pi}(\sec x\tan x-\sec^2 x+1)dx$ $=\pi(\sec x-\tan x+x)\Big]_0^{\pi}$ $=\pi(-1+\pi-1)=\pi(\pi-2)$ $\therefore I=\dfrac{\pi}{2}(\pi-2)$ or $\pi\left(\dfrac{\pi}{2}-1\right)$
44	(A) $\dfrac{\sin^{-1}(a^x)}{\log_e a}+C$
45	(a) $\displaystyle\int\frac{x}{(x-1)(x^2+4)}dx=\frac{1}{5}\int\frac{1}{x-1}dx-\frac{1}{10}\int\frac{2x}{x^2+4}dx+\frac{4}{5}\int\frac{1}{x^2+4}dx$, (By Partial Fractions) $=\dfrac{1}{5}\log\lvert x-1\rvert-\dfrac{1}{10}\log\left(x^2+4\right)+\dfrac{2}{5}\tan^{-1}\dfrac{x}{2}+C$ OR (b) Let $I=\displaystyle\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx$ $\Rightarrow 2I=\pi\displaystyle\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$ Substituting, $\cos x=t, \sin x\,dx=-dt$, we get, $I=\dfrac{\pi}{2}\displaystyle\int_1^{-1}\frac{-dt}{1+t^2}=-\frac{\pi}{2}\tan^{-1}t\Big]_1^{-1}$ $=-\dfrac{\pi}{2}\left(-\dfrac{\pi}{4}-\dfrac{\pi}{4}\right)=\dfrac{\pi^2}{4}$

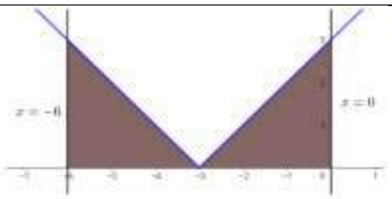
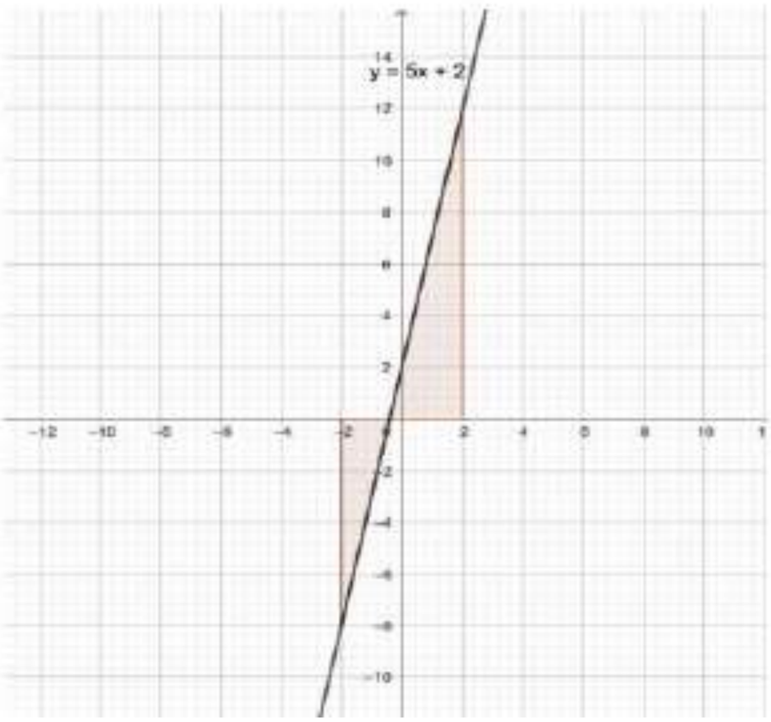
46	(B) $-\frac{1}{12}\tan^{-1}\left(\frac{3e^{-x}}{4}\right)+C$
47	(a) **Using Partial fractions,** $\int \frac{3x+1}{(x-2)^2(x+2)}dx = \frac{5}{16}\int\frac{1}{x-2}dx+\frac{7}{4}\int\frac{1}{(x-2)^2}dx-\frac{5}{16}\int\frac{1}{x+2}dx$ $=\frac{5}{16}\log\lvert x-2\rvert-\frac{7}{4(x-2)}-\frac{5}{16}\log\lvert x+2\rvert+C$ or $=\frac{5}{16}\log\left\lvert\frac{x-2}{x+2}\right\rvert-\frac{7}{4(x-2)}+C$ **Or** (b) Let $I=\int_0^{\pi/2}\frac{x}{\cos x+\sin x}dx \Rightarrow I=\int_0^{\pi/2}\frac{\frac{\pi}{2}-x}{\sin x+\cos x}dx \Rightarrow 2I=\frac{\pi}{2}\int_0^{\pi/2}\frac{1}{\cos x+\sin x}dx$ $\Rightarrow I=\frac{\pi}{4\sqrt{2}}\int_0^{\pi/2}\frac{1}{\cos\left(x-\pi/4\right)}dx=\frac{\pi}{4\sqrt{2}}\int_0^{\pi/2}\sec\left(x-\pi/4\right)dx$ $\Rightarrow I=\frac{\pi}{4\sqrt{2}}\log\left[\sec\left(x-\pi/4\right)+\tan\left(x-\pi/4\right)\right]_0^{\frac{\pi}{2}}=\frac{\pi}{4\sqrt{2}}\left[\log\left(\sqrt{2}+1\right)-\log\left(\sqrt{2}-1\right)\right]$ Or, $I=\frac{\pi}{4\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$
48	(A) $\frac{1}{4}tan^{-1}\frac{x^4}{2}+C$
49	(B) $e^x.\frac{1}{1+x}+C$
50	(a) $\int\frac{x^2-x+1}{(x-1)(x^2+1)}dx$ Let $\frac{x^2-x+1}{(x-1)(x^2+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$ Getting $A=\frac{1}{2}, B=\frac{1}{2}, C=\frac{-1}{2}$ $\frac{1}{2}\int\frac{1}{x-1}dx+\frac{1}{2}\int\frac{x}{x^2+1}dx-\frac{1}{2}\int\frac{1}{x^2+1}dx$ $\frac{1}{2}log\lvert x-1\rvert+\frac{1}{4}log\lvert x^2+1\rvert-\frac{1}{2}tan^{-1}x+C$ OR (b) $\int_1^4(\lvert x\rvert+\lvert 3-x\rvert)\,dx$ $\int_1^4\lvert x\rvert\,dx+\int_1^4\lvert 3-x\rvert\,dx$ $\int_1^4 x\,dx+\int_1^3(3-x)\,dx-\int_3^4(3-x)\,dx$ $= 10$

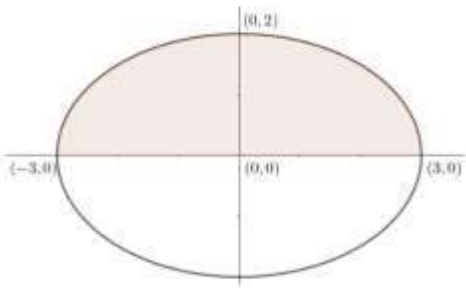
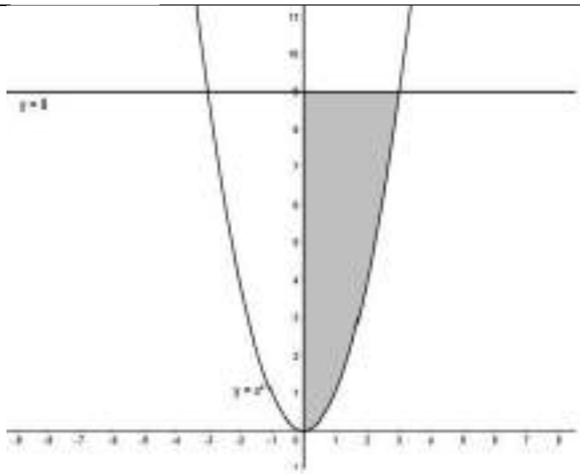
CHAPTER-8 APPLICATION OF INTEGRALS

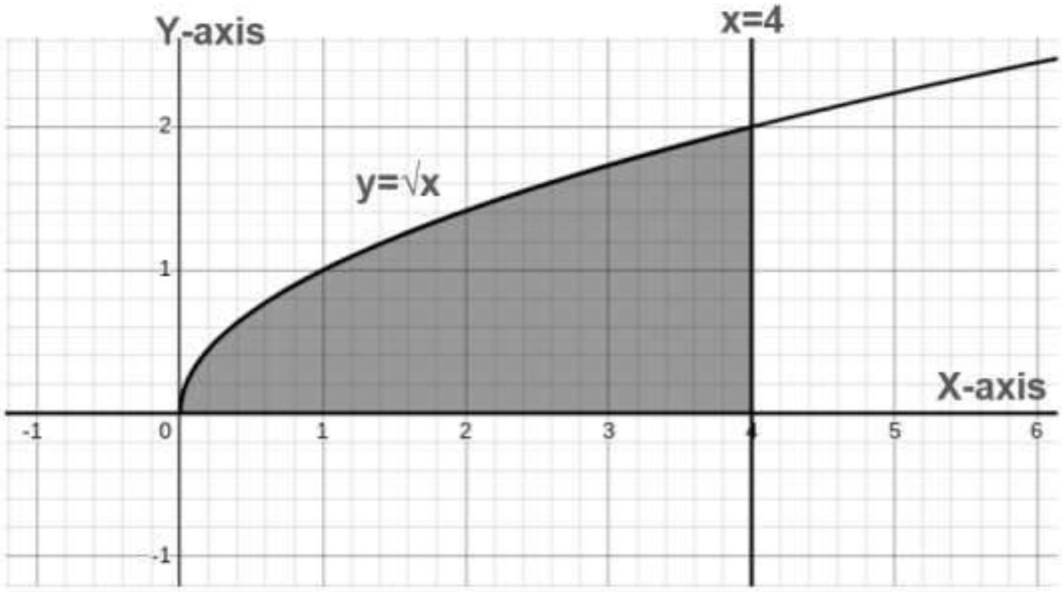
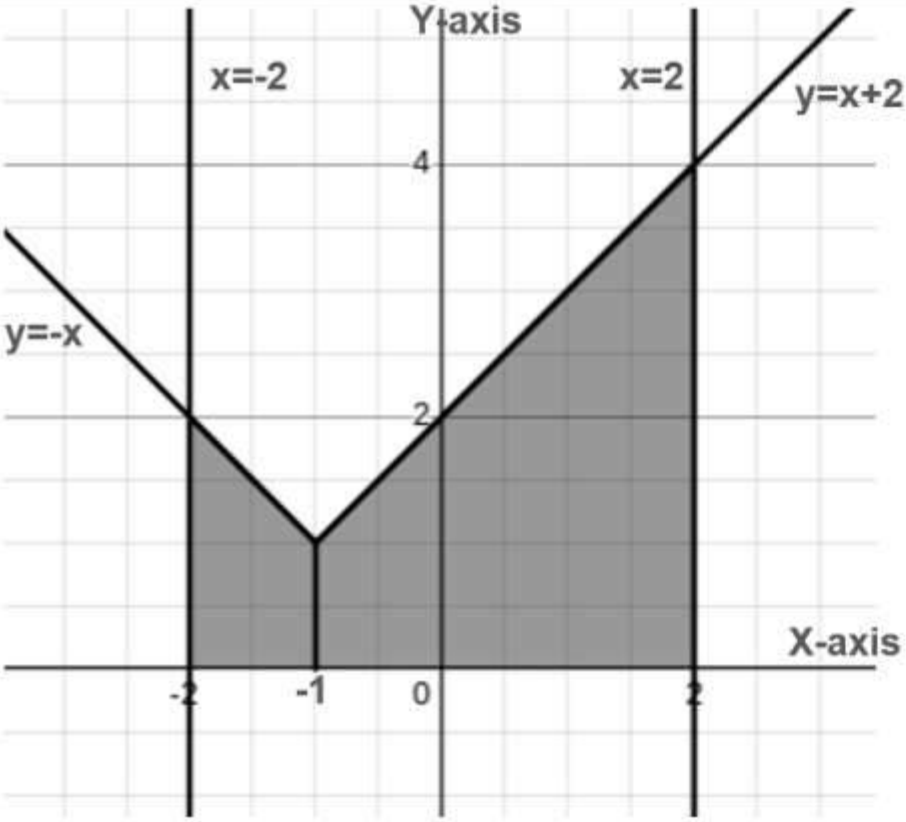
Q.		Code and Marks
1	The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x-axis is given by  (A) $\int_0^4 x\,dx$ (B) $\int_0^2 y^2\,dy$ (C) $2\int_0^4 \sqrt{x}\,dx$ (D) $\int_0^4 \sqrt{x}\,dx$	65/1/1 65/1/2 65/1/3 1M
2	Sketch the graph of $y = \mid x + 3 \mid$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.	65/1/1 65/1/2 65/1/3 3M
3	The area of the shaded region (figure) represented by the curves $y = x^2$, $0 \le x \le 2$ and y-axis is given by  (A) $\int_0^2 x^2\,dx$ (B) $\int_0^2 \sqrt{y}\,dy$ (C) $\int_0^4 x^2\,dx$ (D) $\int_0^4 \sqrt{y}\,dy$	65/2/1 65/2/2 65/2/3 1M
4	Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x – axis and the ordinates $x = -2$ and $x = 2$.	65/2/1 65/2/2 65/2/3 5M
5	The area of the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 0$ and $x = 4$ and x-axis is : (A) $\frac{16}{9}$ sq. units (B) $\frac{32}{9}$ sq. units (C) $\frac{16}{3}$ sq. units (D) $\frac{32}{3}$ sq. units	65/4/1 65/4/2 65/4/3 1M

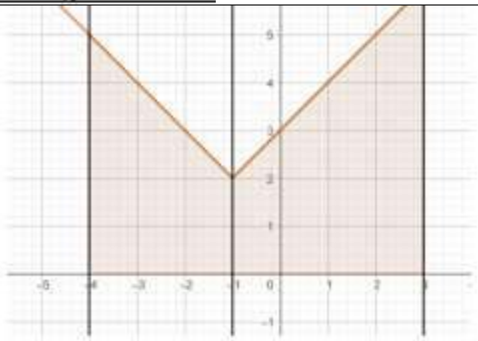
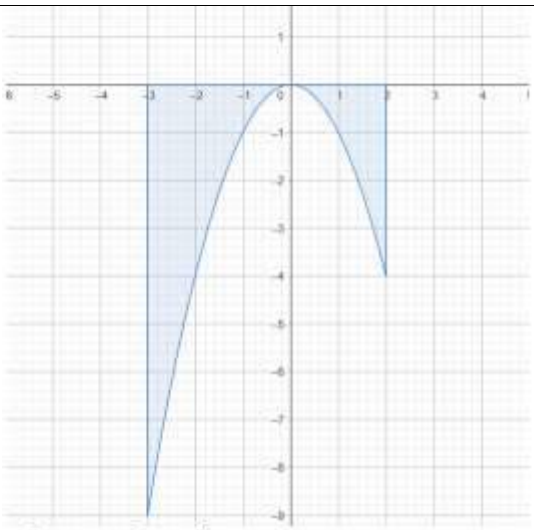
6	Calculate the area of the region bounded by the curve $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the x-axis using integration.	65/4/1 65/4/2 65/4/3 2M
7	The area of the region enclosed between the curve $y = x\|x\|$, x-axis, $x = -2$ and $x = 2$ is : (A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) 0 (D) 8	65/5/1 65/5/2 65/5/3 1M
8	Sketch a graph of $y = x^2$. Using integration, find the area of the region bounded by $y = 9$, $x = 0$ and $y = x^2$.	65/5/1 5M
9	Draw a rough sketch of the curve $y = \sqrt{x}$. Using integration, find the area of the region bounded by the curve $y = \sqrt{x}$, $x = 4$ and x-axis, in the first quadrant.	65/5/2 5M
10	In a rough sketch, mark the region bounded by $y = 1 + \|x + 1\|$, $x = -2$, $x = 2$ and $y = 0$. Using integration, find the area of the marked region.	65/5/3 5M
11	The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is : (A) $\frac{3}{2}$ sq units (B) $\frac{2}{3}$ sq units (C) 3 sq units (D) $\frac{4}{3}$ sq units	65/6/1 65/6/2 65/6/3 1M
12	Draw a rough sketch for the curve $y = 2 + \|x + 1\|$. Using integration, find the area of the region bounded by the curve $y = 2 + \|x + 1\|$, $x = -4$, $x = 3$ and $y = 0$.	65/6/1 5M
13	Use integration to find the area of the region enclosed by curve $y = -x^2$ and the straight lines $x = -3$, $x = 2$ and $y = 0$. Sketch a rough figure to illustrate the bounded region.	65/6/2 5M
14	**Using integration, find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ bounded between the lines $x = -\frac{a}{2}$ to $x = \frac{a}{2}$.**	65/6/3 5M
15	A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.	65/7/1 65/7/2 65/7/3 5M

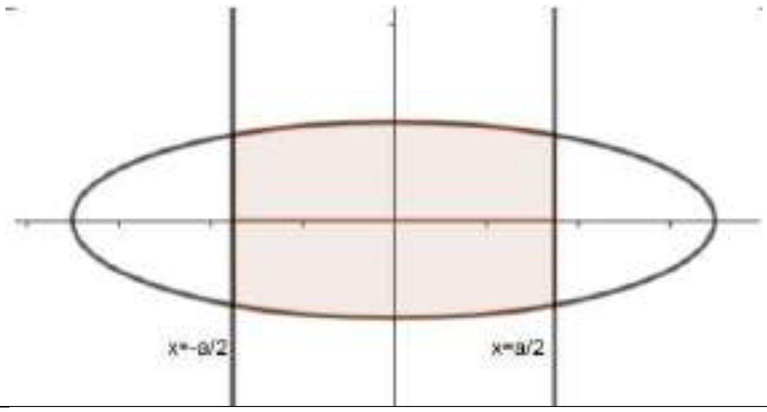
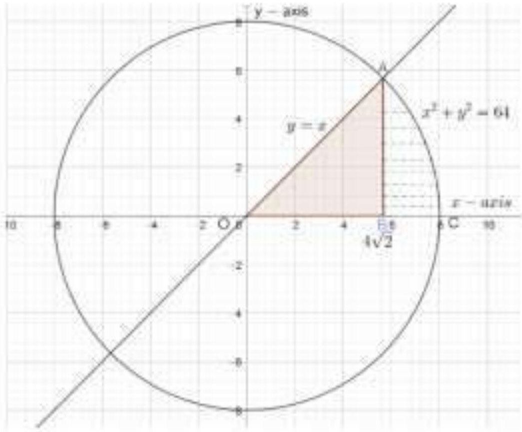
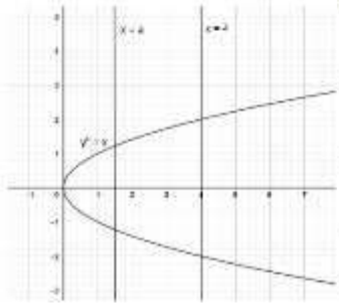
16	The area of the region bounded by the lines $y = x + 1$, $x = 1$, $x = 3$ and x-axis is (A) 6 sq units (B) 8 sq units (C) 7.5 sq units (D) 2 sq units	65(B) 1M
17	The region enclosed between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$. Find the value of a.	65(B) 5M

	ANSWERS
1	(D) $\int_0^4 \sqrt{x}\, dx$
2	 **Required Area** $=\int_{-6}^{0} y\,dx$ $=2\int_{-3}^{0}(x+3)dx$ $=2\left[\frac{(x+3)^2}{2}\right]_{-3}^{0}$ $=9$
3	(D) $\int_0^4 \sqrt{y}dy$
4	

	The required area $= \left\lvert \int_{-2}^{-\frac{2}{5}} (5x+2)dx \right\rvert + \int_{-\frac{2}{5}}^{2} (5x+2)dx$ $= \left\lvert \left[\frac{(5x+2)^2}{10}\right]_{-2}^{-\frac{2}{5}} \right\rvert + \left[\frac{(5x+2)^2}{10}\right]_{-\frac{2}{5}}^{2}$ $= \frac{64}{10} + \frac{144}{10} = \frac{104}{5}$
5	(C) $\frac{16}{3}$ sq. units
6	 $A = 2\times\frac{2}{3}\int_0^3 \sqrt{9-x^2}\,dx$ $= \frac{4}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$ $= \frac{4}{3}\left[\left(0+\frac{9}{2}\sin^{-1}1\right) - 0\right]$ $= 3\pi$
7	**(B)** $\frac{16}{3}$
8	 **Required area** $= \int_0^9 \sqrt{y}\,dy$ $= \frac{2}{3}\left[y^{3/2}\right]_0^9$ $= 18$ **Note: If area is found in second quadrant, may be considered.**

9	 **Required area =** $\int_0^4 \sqrt{x}\ dx$ $= \frac{2}{3}\left[x^{3/2}\right]_0^4$ $= \frac{2}{3} \times 8 = \frac{16}{3}$
10	 **Required area =** $\int_{-2}^{-1}(-x)\ dx + \int_{-1}^{2}(x+2)\, dx$ $= -\frac{1}{2}\left[x^2\right]_{-2}^{-1} + \left[\frac{1}{2}x^2 + 2x\right]_{-1}^{2}$ $= 9$

11	(D) $\frac{4}{3}$ sq units
12	 Correct Graph: Required area = $\int_{-4}^{-1}(2+\lvert x+1\rvert)\,dx + \int_{-1}^{3}(2+\lvert x+1\rvert)\,dx$ $= \int_{-4}^{-1}(1-x)\,dx + \int_{-1}^{3}(3+x)\,dx$ $= -\frac{(1-x)^2}{2}\Big\rvert_{-4}^{-1} + \frac{(3+x)^2}{2}\Big\rvert_{-1}^{3}$ $= \frac{21}{2} + 16 = \frac{53}{2}$
13	 **Correct Graph:** **Required area =** $\left\lvert \int_{-3}^{2} -x^2\,dx \right\rvert$ $= \left\lvert -\frac{1}{3}x^3 \right]_{-3}^{2} \Big\rvert$ $= \left\lvert -\frac{1}{3}(8-(-27)) \right\rvert$ $= \frac{35}{3}$
14	Required area $= \frac{4b}{a}\int_{0}^{\frac{a}{2}}\sqrt{a^2-x^2}\,dx$ $= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{0}^{\frac{a}{2}}$

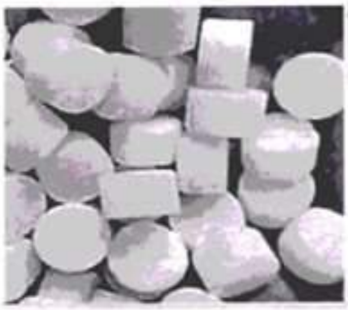
	$= \frac{4b}{a}\left[\frac{a}{4}\cdot\frac{\sqrt{3}a}{2}+\frac{a^2}{2}\cdot\frac{\pi}{6}\right]$ $= ab\left[\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right]$ 
15	 **Correct graph** **Equation of the circular tabletop:** $x^2+y^2=64$ **Equation of line (scratch):** $x=y$ **The line and circle intersect at** $x=4\sqrt{2}$ **Area of the shaded region** $= \int_0^{4\sqrt{2}} x\,dx + \int_{4\sqrt{2}}^{8}\sqrt{64-x^2}\,dx$ $= \left.\frac{x^2}{2}\right]_0^{4\sqrt{2}} + \left[\frac{x}{2}\sqrt{64-x^2}+32\sin^{-1}\frac{x}{8}\right]_{4\sqrt{2}}^{8}$ $= \frac{32}{2}+32\sin^{-1}1-2\sqrt{2}\cdot4\sqrt{2}-32\sin^{-1}\frac{1}{\sqrt{2}}$ $= 16+16\pi-16-8\pi = 8\pi\ \text{cm}^2$
16	**(A) 6 sq units**
17	 $2\int_0^a \sqrt{x}\,dx = \int_0^4 \sqrt{x}\,dx$ $\Rightarrow 2\times\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^a = \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^4$ $\Rightarrow \frac{4}{3}\left[a^{\frac{3}{2}}\right] = \frac{2}{3}[8]$ $\Rightarrow a^{\frac{3}{2}} = 4$ $\Rightarrow a = 4^{\frac{2}{3}}$

CHAPTER-9 DIFFERENTIAL EQUATION

Q.		Code and Marks
1	Which of the following is <u>not</u> a homogeneous function of x and y ? (A) $y^2 - xy$ (B) $x - 3y$ (C) $\sin^2 \frac{y}{x} + \frac{y}{x}$ (D) $\tan x - \sec y$	65/1/1 65/1/2 65/1/3 1M
2	The integrating factor of differential equation $(x + 2y^3)\frac{dy}{dx} = 2y$ is (A) $e^{\frac{y^2}{2}}$ (B) $\frac{1}{\sqrt{y}}$ (C) $\frac{1}{y^2}$ (D) $e^{-\frac{1}{y^2}}$	65/1/1 1M
3	The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is (A) xe^x (B) $\frac{e^x}{x}$ (C) $\frac{x}{e^x}$ (D) $xe^{\frac{1}{x}}$	65/1/2 1M
4	The order and degree of differential function $\left[1+\left(\frac{dy}{dx}\right)^2\right]^5 = \frac{d^2y}{dx^2}$ are (A) order 1, degree 1 (B) order 1, degree 2 (C) order 2, degree 1 (D) order 2, degree 2	65/1/3 1M
5	If p and q are respectively the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$, then (p – q) is (A) 0 (B) 1 (C) 2 (D) 3	65/2/1 65/2/2 65/2/3 1M
6	(a) Solve the differential equation $2(y + 3) - xy\frac{dy}{dx} = 0$; given $y(1) = -2$. **OR** (b) Solve the following differential equation : $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$.	65/2/1 65/2/2 65/2/3 3M

7	The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ are : (A) order 2, degree 2 (B) order 2, degree 1 (C) order 2, degree not defined (D) order 1, degree not defined	65/4/1 1M
8	The integrating factor of the differential equation $\frac{dy}{dx} + y\tan x - \sec x = 0$ is : (A) $-\cos x$ (B) $\sec x$ (C) $\log \sec x$ (D) $e^{\sec x}$	65/4/2 1M
9	The solution of the differential equation $\frac{dy}{dx} = \frac{-x}{y}$ represents family of : (A) parabolas (B) circles (C) ellipses (D) hyperbolas	65/4/3 1M
10	During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature (25°C). Initially the processor's temperature is 85°C. The rate of cooling is defined by the equation $\frac{d}{dt}(T(t)) = -k(T(t) - 25)$, where $T(t)$ represents the temperature of the processor at time t (in minutes) and k is a constant.  Based on the above information, answer the following questions : (i) Find the expression for temperature of processor, $T(t)$ given that $T(0) = 85°C$. (ii) How long will it take for the processor's temperature to reach 40°C ? Given that $k = 0{\cdot}03$, $\log_e 4 = 1{\cdot}3863$.	65/4/1 65/4/2 65/4/3 4M

11	The sum of the order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ is : (A) 2 (B) $\frac{5}{2}$ (C) 3 (D) 4	65/5/1 65/5/2 65/5/3 1M
12	Find the particular solution of the differential equation $\left[x \sin^2\left(\frac{y}{x}\right) - y\right] dx + x\, dy = 0$ given that $y = \frac{\pi}{4}$, when $x = 1$.	65/5/1 3M
13	The integrating factor of the differential equation $\frac{dx}{dy} = \frac{x \log x}{\frac{2}{x} \log x - y}$ is : (A) $\frac{1}{8x}$ (B) e (C) $e^{\log x}$ (D) $\log x$	65/5/2 1M
14	Solve the differential equation $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x.$	65/5/3 3M
15	The integrating factor of the differential equation $\frac{dx}{dy} = \frac{-(1+\sin x)}{x + y \cos x}$ is : (A) $\log \cos x$ (B) $1 + \sin x$ (C) $e^{(1 + \sin x)}$ (D) $e^{\log \cos x}$	65/5/3 1M
16	Solve the differential equation $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x.$	65/5/3 3M
17	The order and degree of the following differential equation are, respectively : $-\frac{d^4y}{dx^4} + 2e^{dy/dx} + y^2 = 0$ (A) $-4, 1$ (B) 4, not defined (C) 1, 1 (D) 4, 1	65/6/1 1M
18	The solution for the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is : (A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$ (C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$	65/6/1 65/6/2 65/6/3 1M

19	(a) Solve the differential equation : $x^2y\,dx - (x^3 + y^3)\,dy = 0$. **OR** (b) Solve the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.	65/6/1 65/6/2 65/6/3 5M
20	The order and degree of the differential equation $\frac{d^2y}{dx^2} + 4\left(\frac{dy}{dx}\right) = x \log\left(\frac{d^2y}{dx^2}\right)$ are respectively : (A) 0, 3 (B) 2, 1 (C) 2, not defined (D) 1, not defined	65/6/2 1M
21	The order and degree of the differential equation $\left[\frac{d^2y}{dx^2} - 1\right]^2 = \frac{dy}{dx}$ are, respectively : (A) 2, 2 (B) 2, not defined (C) 1, 2 (D) 1, not defined	65/6/3 1M
22	Solve the differential equation $\frac{dy}{dx} = \cos x - 2y$.	65/7/1 5M
23	Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.  (Cylindrical-shaped Camphor tablets) A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\frac{dV}{dt} = kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours. Based upon the above information, answer the following questions : (i) Write the order and degree of the given differential equation. (ii) Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that $r(0) = 5$ mm. (iii) (a) If it is given that $r = 3$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm.	65/7/1 65/7/2 65/7/3 4M

	OR (iii) (b) If it is given that r = 1 mm when t = 1 hour, find the value of k. Hence, find t for r = 0 mm.	
24	Solve the differential equation $(x^2 + y^2)\,dx + xy\,dy = 0,\ y(1) = 1.$	65/7/2 5M
25	Solve the differential equation $(x - \sin y)\,dy + (\tan y)\,dx = 0$, given $y(0) = 0$.	65/7/3 5M
26	The integrating factor for solving the differential equation $x \cdot \frac{dy}{dx} - y = 2x^2$ is (A) x (B) $\frac{1}{x}$ (C) e^{-x} (D) $-\log x$	65(B) 1M
27	**Assertion (A) :** $x^2dy = (2xy + y^2)dx$ is a homogeneous differential equation. **Reason (R) :** A differential equation of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ is a homogeneous differential equation.	65(B) 1M
28	(a) Find the particular solution of the differential equation, $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$. **OR** (b) Solve the differential equation : $2xy\frac{dy}{dx} = x^2 + 3y^2$.	65(B) 3M

	ANSWERS
1	(D) $\tan x - \sec y$
2	(B) $\frac{1}{\sqrt{y}}$
3	(B) $\frac{e^x}{x}$
4	(C) order 2, degree 1
5	(B) 1
6 (a)	Given differential equation can be written as $\frac{y}{y+3}dy = \frac{2}{x}dx$ $\Rightarrow \int\left(1 - \frac{3}{y+3}\right)dy = 2\int\frac{1}{x}dx$

(b)	$\Rightarrow y - 3log\|y + 3\| = 2log\|x\| + C$ y = -2, when x = 1 $\Rightarrow C = -2$ Hence, the required particular solution is $\Rightarrow y - 3log\|y + 3\| = 2log\|x\| - 2$ OR Given differential equation can be written as $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$, which is linear in y. I.F. = $e^{\int\frac{2x}{1+x^2}dx} = e^{log(1+x^2)} = 1 + x^2$ The solution is given by $y(1 + x^2) = \int 4x^2 dx$ $\Rightarrow y(1 + x^2) = \frac{4}{3}x^3 + C$ $or\ y = \frac{4x^3}{3(1+x^2)} + C\frac{1}{(1+x^2)}$, which is the required general solution
7	**(C) order 2, degree not defined**
8	**(B) sec x**
9	**(B) circles**
10	$(i)\frac{dT}{dt} = -k(T-25)$ $\Rightarrow \frac{dT}{T-25} = -k\,dt$ $\Rightarrow \int\frac{dT}{T-25} = -k\int dt$ $\Rightarrow \log\|T-25\| = -kt + C \quad ...(a)$ When $t = 0, T = 85$ $\Rightarrow \log 60 = C$ Using in equation (a), $\log\|T-25\| = -kt + \log 60 \quad ...(b)$ (ii) When $k = 0.03$, $\log\|T-25\| = -0.03t + \log 60$ $\Rightarrow \log\left\|\frac{T-25}{60}\right\| = -0.03t$ $\Rightarrow T - 25 = 60.e^{-0.03t}$ When $T = 40, t = t_1$

	$\Rightarrow \frac{15}{60} = e^{-0.03t_1}$ $\Rightarrow e^{-0.03t_1} = \frac{1}{4} \Rightarrow -0.03t_1 = -\log 4$ $\Rightarrow t_1 = \frac{\log 4}{0.03} = \frac{1.3863}{0.03} = 46.21\text{m}$
11	(C) 3
12	$\left[x.\sin^2\frac{y}{x} - y\right].dx + x.dy = 0$ $\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \sin^2\frac{y}{x}$ Put $y = vx \Rightarrow \frac{dy}{dx} = x\frac{dv}{dx} + v$ $\therefore x\frac{dv}{dx} + v = v - \sin^2 v$ $\Rightarrow -\int \operatorname{cosec}^2 v\, dv = \int \frac{dx}{x}$ $\Rightarrow \cot v = \log\lvert x\rvert + c$ $\Rightarrow \cot\frac{y}{x} = \log\lvert x\rvert + c$ $x = 1, y = \frac{\pi}{4} \Rightarrow c = 1$ $\Rightarrow \cot\frac{y}{x} = \log\lvert x\rvert + 1$
13	(D) log x
14	$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sec\frac{y}{x}$ Put $y = vx$ $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\Rightarrow v + x\frac{dv}{dx} = v + \sec v$ $\Rightarrow \int \cos v\, dv = \int \frac{dx}{x}$ $\Rightarrow \sin v = \log\lvert x\rvert + c$ $\Rightarrow \sin\frac{y}{x} = \log\lvert x\rvert + c$
15	(B) 1 + sin x
16	$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sec\frac{y}{x}$ Put $y = vx$ $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\Rightarrow v + x\frac{dv}{dx} = v + \sec v$

	$\Rightarrow \int \cos v \, dv = \int \frac{dx}{x}$ $\Rightarrow \sin v = \log\lvert x\rvert + c$ $\Rightarrow \sin \frac{y}{x} = \log\lvert x\rvert + c$
17	(B) 4, not defined
18	(D) $3e^{-4y} + 4e^{3x} + 12C = 0$
19	(a) Given differential equation can be written as $\frac{dy}{dx} = \frac{yx^2}{x^3+y^3}$ Put $y = vx$, so $\frac{dv}{dx} = v + x\frac{dv}{dx}$ Therefore, $v + x\frac{dv}{dx} = \frac{vx^3}{x^3+v^3x^3} = \frac{v}{1+v^3}$ $x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$ $(\frac{1}{v^4} + \frac{1}{v})dv = \frac{-dx}{x}$ Integrating we get $\frac{-1}{3v^3} + \log\lvert v\rvert = -\log\lvert x\rvert + C$ $\frac{-x^3}{3y^3} + \log\lvert y\rvert = C$ OR (b) Given D.E. is $\frac{dy}{dx} + \frac{2x}{1+x^2}\, y = \frac{4x^2}{1+x^2}$ Integrating factor is $e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$ Solution is $y(1+x^2) = \int 4x^2 dx + C$ $y(1+x^2) = \frac{4x^3}{3} + C$ $y(0) = 0$ gives $C = 0$, hence solution is $y(1+x^2) = \frac{4x^3}{3}$
20	(C)2, not defined
21	(A) 2, 2
22	**The given differential equation can be written as:** $\frac{dy}{dx} + 2y = \cos x$, Taking $P = 2, Q = \cos x$ **Integrating factor is given by,** $I = e^{\int 2dx} = e^{2x}$ $\therefore$ **The solution is,** $y \cdot e^{2x} = \int e^{2x} \cos x dx$ **Let,** $I_1 = \int \cos x \cdot e^{2x} dx = \cos x \frac{e^{2x}}{2} - \int(-\sin x)\frac{e^{2x}}{2}dx$ $= \frac{e^{2x}\cos x}{2} + \frac{1}{2}\left[\sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2}dx\right]$ $\Rightarrow I_1 = \frac{e^{2x}\cos x}{2} + \frac{e^{2x}\sin x}{4} - \frac{1}{4}I_1 \Rightarrow I_1 = \frac{e^{2x}}{5}(2\cos x + \sin x)$

	$\therefore$ The solution of the differential equation is $$y\cdot e^{2x}=\frac{e^{2x}}{5}\left(2\cos x+\sin x\right)+C \Rightarrow y=\frac{1}{5}\left(2\cos x+\sin x\right)+Ce^{-2x}$$
23	(i) Order = 1, Degree = 1 (ii) Separating the variable and integrating, $\int dr=\frac{2k}{3}\int dt\Rightarrow r=\frac{2}{3}kt+C$ Putting $t=0, r=5$, we get $C=5$ $$r=\frac{2}{3}kt+5$$ (iii) (a) Putting $r=3, t=1$, $3=\frac{2}{3}k(1)+5\Rightarrow k=-3$ $r=-2t+5$, For $r=0$, $t=\frac{5}{2}$ hours or 2.5 hours OR (iii) (b) Putting $r=1, t=1$, $1=\frac{2}{3}k+5\Rightarrow k=-6$ $\therefore$ $r=-4t+5$, For $r=0$, $t=\frac{5}{4}$ hours or 1.25 hours
24	$$\frac{dy}{dx}=-\frac{x^2+y^2}{xy}=-\frac{1+\left(\frac{y}{x}\right)^2}{\frac{y}{x}}$$, Put $y=vx$, $\frac{dy}{dx}=v+x\frac{dv}{dx}$ $$\Rightarrow v+x\frac{dv}{dx}=-\frac{1+v^2}{v}\Rightarrow x\frac{dv}{dx}=-\frac{1+2v^2}{v}$$ $$\Rightarrow \frac{1}{4}\int\frac{4v}{1+2v^2}dv=-\int\frac{1}{x}dx$$ $$\Rightarrow \frac{1}{4}\log\left(1+2v^2\right)=-\log\lvert x\rvert+\log C$$ $$\Rightarrow \log\left(1+2v^2\right)=\log\frac{C^4}{x^4}\Rightarrow 1+2\left(\frac{y}{x}\right)^2=\frac{D}{x^4}, D=C^4$$ For $x=1, y=1$, $D=3$, $\therefore$ The solution of the differential equation is, $\left(2y^2+x^2\right)x^2=3$
25	The differential equation can be written as: $\frac{dx}{dy}+\cot y\cdot x=\cos y$, which is a linear order differential equation Here, $P=\cot y$, $Q=\cos y$, I.F. (Integrating Factor) = $e^{\int\cot y\,dy}=e^{\log\sin y}=\sin y$ The solution is , $x(\sin y)=\int\cos y\cdot\sin y\,dy$

	$$\Rightarrow x(\sin y)=\frac{(\sin y)^2}{2}+C, \text{ For } x=0, y=0, C=0.$$ $\therefore$ **The Particular solution is:** $x\sin y=\frac{\sin^2 y}{2}$ or $\sin y=2x$ or $y=\sin^{-1}2x$
26	**(B)** $\frac{1}{x}$
27	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**
28	**(a)** $\frac{dy}{dx}=1+x^2+y^2+x^2y^2$ $\frac{dy}{dx}=(1+x^2)(1+y^2)$ $\int\frac{dy}{1+y^2}=\int(1+x^2)\,dx$ $\Rightarrow \tan^{-1}y=x+\frac{x^3}{3}+C$ **y=1 when x= 0** $\Rightarrow C=\frac{\pi}{4}$ **Particular solution is :** $\tan^{-1}y=x+\frac{x^3}{3}+\frac{\pi}{4}$ OR **(b) The given differential equation can be writ** $\frac{dy}{dx}=\frac{x^2+3y^2}{2xy}$ **putting y= vx , we get** $v+x\frac{dv}{dx}=\frac{x^2+3v^2x^2}{2xvx}$ $\int\frac{2v}{1+v^2}dv=\int\frac{dx}{x}$ $\Rightarrow \log(1+v^2)=\log\lvert x\rvert+\log C$ $\Rightarrow 1+v^2=Cx$ $\Rightarrow 1+\frac{y^2}{x^2}=Cx$ or $x^2+y^2=Cx^3$

CHAPTER 10 VECTOR

Q		Code and Marks
1	If vector $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, then which of the following is correct ? (A) $\vec{a} \parallel \vec{b}$ (B) $\vec{a} \perp \vec{b}$ (C) $\lvert \vec{b} \rvert > \lvert \vec{a} \rvert$ (D) $\lvert \vec{a} \rvert = \lvert \vec{b} \rvert$	65/1/1
2	If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $\lvert \vec{a} \rvert = \sqrt{37}$, $\lvert \vec{b} \rvert = 3$ and $\lvert \vec{c} \rvert = 4$, then angle between $\vec{b}$ and $\vec{c}$ is (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$	65/1/1 65/1/2 65/1/3
3	The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram.	65/1/1 65/2/2
4	(a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings. **OR** (b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2, 1, 3) and B(8, –1, 0) respectively.	65/1/1 65/1/2 65/1/3
5	If $\vec{\alpha} = \hat{i} - 4\hat{j} + 9\hat{k}$ and $\vec{\beta} = 2\hat{i} - 8\hat{j} + \lambda\hat{k}$ are two mutually parallel vectors, then λ is equal to : (A) -18 (B) 18 (C) $\frac{-34}{9}$ (D) $\frac{34}{9}$	65/1/2
6	The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ is (A) $\hat{k}$ (B) $-\hat{k} + \hat{j}$ (C) $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$	65/1/3
7	The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram.	65/1/3

8	The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is (A) $\left(\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert^2}\right)\vec{b}$ (B) $\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert}$ (C) $\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert}$ (D) $\left(\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert^2}\right)\vec{b}$	65/2/1
9	Let $\vec{p}$ and $\vec{q}$ be two unit vectors and α be the angle between them. Then $(\vec{p}+\vec{q})$ will be a unit vector for what value of α ? (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$	65/2/1
10	(a) A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$. **OR** (b) If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $QR = \frac{3}{2}QP$.	65/2/1 65/2/2 65/2/3
11	If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, then find x, such that $\vec{\alpha} = (x-2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3+2x)\vec{a} - 2\vec{b}$ are collinear.	65/2/1
12	The values of x for which the angle between the vectors $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$ is obtuse, is : (A) 0 or $\frac{1}{2}$ (B) $x > \frac{1}{2}$ (C) $\left(0, \frac{1}{2}\right)$ (D) $\left[0, \frac{1}{2}\right]$	65/2/2
13	If $\overrightarrow{PQ} \times \overrightarrow{PR} = 4\hat{i} + 8\hat{j} - 8\hat{k}$, then the area (ΔPQR) is (A) 2 sq units (B) 4 sq units (C) 6 sq units (D) 12 sq units	65/2/2

14	Let $\vec{p} = 2\hat{i} - 3\hat{j} - \hat{k}$, $\vec{q} = -3\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + 2\hat{k}$. Express $\vec{r}$ in the form of $\vec{r} = \lambda\vec{p} + \mu\vec{q}$ and hence find the values of λ and μ.	65/2/2
15	If $\vec{p}$ and $\vec{q}$ are unit vectors, then which of the following values of $\vec{p} \cdot \vec{q}$ is not possible ? (A) $\frac{-1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$	65/2/3
16	If projection of $\vec{a} = \alpha\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units, then α is (A) -13 (B) -5 (C) 13 (D) 5	65/2/3
17	If $\|\vec{a}\| = 2$, $\|\vec{b}\| = 3$ and $\vec{a} \cdot \vec{b} = 4$, then evaluate $\|\vec{a} + 2\vec{b}\|$.	65/2/3
18	If the sides AB and AC of Δ ABC are represented by vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ respectively, then the length of the median through A on BC is : (A) $2\sqrt{2}$ units (B) $\sqrt{18}$ units (C) $\frac{\sqrt{34}}{2}$ units (D) $\frac{\sqrt{48}}{2}$ units	65/4/1 65/4/2 65/4/3
19	(a) Show that the area of a parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ is given by $\frac{1}{2}\|\vec{a} \times \vec{b}\|$. Also find the area of a parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.	65/4/1 65/4/2 65/4/3
20	Let $\vec{a}$ be a position vector whose tip is the point $(2, -3)$. If $\overrightarrow{AB} = \vec{a}$, where coordinates of A are $(-4, 5)$, then the coordinates of B are : (A) $(-2, -2)$ (B) $(2, -2)$ (C) $(-2, 2)$ (D) $(2, 2)$	65/5/1 65/5/2 65/5/3

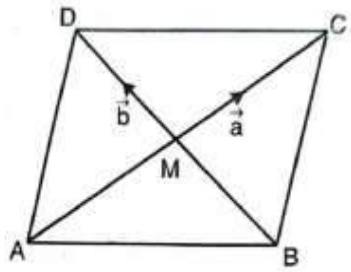
21	The respective values of $\lvert\vec{a}\rvert$ and $\lvert\vec{b}\rvert$, if given $(\vec{a}-\vec{b}).(\vec{a}+\vec{b}) = 512$ and $\lvert\vec{a}\rvert = 3\lvert\vec{b}\rvert$, are : (A) 48 and 16 (B) 3 and 1 (C) 24 and 8 (D) 6 and 2	65/5/1 65/5/2 65/5/3
22	(a) Find a vector of magnitude 5 which is perpendicular to both the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ and $4\hat{i} + 3\hat{j} - 2\hat{k}$. **OR** (b) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ and $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$, $\vec{a} \neq 0$. Show that $\vec{b} = \vec{c}$.	65/5/1 65/5/2 65/5/3
23	A man needs to hang two lanterns on a straight wire whose end points have coordinates A (4, 1, – 2) and B (6, 2, – 3). Find the coordinates of the points where he hangs the lanterns such that these points trisect the wire AB.	65/5/1 65/5/2 65/5/3
24	(a) If $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ such that $\lvert\vec{a}\rvert = 3$, $\lvert\vec{b}\rvert = 5$, $\lvert\vec{c}\rvert = 7$, then find the angle between $\vec{a}$ and $\vec{b}$. **OR** (b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined with each other at an angle θ, then prove that $\frac{1}{2}\lvert\vec{a}-\vec{b}\rvert = \sin\frac{\theta}{2}$.	65/5/1 65/5/2 65/5/3
25	**16.** Let $\lvert\vec{a}\rvert = 5$ and $-2 \leq \lambda \leq 1$. Then, the range of $\lvert\lambda\vec{a}\rvert$ is : (A) [5, 10] (B) [– 2, 5] (C) [– 2, 1] (D) [–10, 5]	65/6/1 65/6/2 65/6/3
26	*Assertion (A) :* If $\lvert\vec{a}\times\vec{b}\rvert^2 + \lvert\vec{a}.\vec{b}\rvert^2 = 256$ and $\lvert\vec{b}\rvert = 8$, then $\lvert\vec{a}\rvert = 2$. *Reason (R) :* $\sin^2\theta + \cos^2\theta = 1$ and $\lvert\vec{a}\times\vec{b}\rvert = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin\theta$ and $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta$.	65/6/1 65/6/2 65/6/3

27	**24.** (a) If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that BC = 3BA. **OR** (b) Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.	65/6/1 65/6/2 65/6/3
28	A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i} + 15\hat{j} + 6\hat{k}$ and the other is along the vector $2\hat{i} + 10\hat{j} + \lambda\hat{k}$, then the value of λ is : (A) 6 (B) 1 (C) $\frac{1}{4}$ (D) 4	65/7/1 65/7/2 65/7/3
29	If $\|\vec{a} + \vec{b}\| = \|\vec{a} - \vec{b}\|$ for any two vectors, then vectors $\vec{a}$ and $\vec{b}$ are : (A) orthogonal vectors (B) parallel to each other (C) unit vectors (D) collinear vectors	65/7/1 65/7/2 65/7/3
30	(a) The scalar product of the vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ with a unit vector along sum of vectors $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$ is equal to 1. Find the value of λ.	65/7/1 65/7/2 65/7/3
31	The number of vector(s) of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is (are) : (A) one (B) two (C) three (D) infinite	65(B)
32	**Assertion (A) :** The vectors $\vec{a} = 4\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + 3\hat{j} - 5\hat{k}$ are mutually perpendicular vectors. **Reason (R) :** Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, if $\vec{a} \cdot \vec{b} = 0$.	65(B)

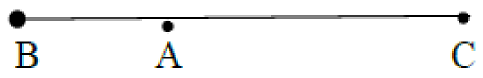
33	(a) If $\|\vec{a}+\vec{b}\| = 60$, $\|\vec{a}-\vec{b}\| = 40$ and $\|\vec{b}\| = 46$, then find $\|\vec{a}\|$. **OR** (b) Using vectors, find the value of K such that the points (K, –11, 2), (0, –2, 2) and (2, 4, 2) are collinear.	65(B)
34	If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{i} - 4\hat{j} - 5\hat{k}$, then find a unit vector perpendicular to both the vectors $(\vec{a}-\vec{b})$ and $(\vec{c}-\vec{b})$.	65(B)

	ANSWERS
1	(B) $\vec{a} \perp \vec{b}$
2	(C) $\frac{\pi}{3}$
3	$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = -2\hat{i} + 3\hat{j} + 7\hat{k}$ **Area of parallelogram** $= \frac{1}{2}\|\vec{a}\times\vec{b}\|$ $= \frac{1}{2}\sqrt{(-2)^2 + 3^2 + 7^2} = \frac{\sqrt{62}}{2}$
4	**Let the required angle between the kite strings be θ.** **Then,** $\cos\theta = \frac{\vec{a}.\vec{b}}{\|\vec{a}\|\|\vec{b}\|}$ $\Rightarrow \cos\theta == \frac{(3\hat{i}+\hat{j}+2\hat{k})(2\hat{i}-2\hat{j}+4\hat{k})}{\sqrt{9+1+4}\sqrt{4+4+16}} = \frac{12}{\sqrt{336}} = \frac{3}{\sqrt{21}}$ $\Rightarrow \theta = \cos^{-1}\left(\frac{12}{\sqrt{336}}\right)$ or $\cos^{-1}\left(\frac{3}{\sqrt{21}}\right)$ **OR** $\overrightarrow{BA} = -6\hat{i} + 2\hat{j} + 3\hat{k}$ **Required unit vector of magnitude 21**

	$=21\times\left(\dfrac{-6\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{36+4+9}}\right)$ $=3\left(-6\hat{i}+2\hat{j}+3\hat{k}\right)$ or $-18\hat{i}+6\hat{j}+9\hat{k}$
5	(B) 18
6	(A) $\hat{k}$
7	$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 3 & -1\end{vmatrix}=-2\hat{i}+3\hat{j}+7\hat{k}$ **Area of parallelogram** $=\dfrac{1}{2}\lvert\vec{a}\times\vec{b}\rvert$ $=\dfrac{1}{2}\sqrt{(-2)^2+3^2+7^2}=\dfrac{\sqrt{62}}{2}$
8	(A) $\left(\dfrac{\vec{a}.\vec{b}}{\lvert\vec{b}\rvert^2}\right)\vec{b}$
9	(D) $\frac{2\pi}{3}$
10	Let α be the angle which the vector $\vec{a}$ makes with all the three axes. Then $3\cos^2\alpha = 1$ $\Rightarrow \cos\alpha = \dfrac{1}{\sqrt{3}}$ The unit vector along the vector $\vec{a} = \frac{1}{\sqrt{3}}(\hat{\imath}+\hat{\jmath}+\hat{k})$ $\vec{a} = 5(\hat{\imath}+\hat{\jmath}+\hat{k})$ OR  $\dfrac{QR}{QP}=\dfrac{3}{2}$ Hence, R divides PQ, externally, in the ratio 1:3. The Position vector of R $= \vec{x} = \frac{\vec{\beta}-3\vec{\alpha}}{1-3} = \frac{3\vec{\alpha}-\vec{\beta}}{2}$

11	$\vec{\alpha}$ and $\vec{\beta}$ are collinear $\Rightarrow \dfrac{x-2}{3+2x} = \dfrac{1}{-2} \quad \Rightarrow x = \dfrac{1}{4}$
12	(C) $\left(0, \frac{1}{2}\right)$
13	(C) 6 sq units
14	$\vec{r} = \lambda\vec{p} + \mu\vec{q}$ $\Rightarrow 1 = 2\lambda - 3\mu, 1 = -3\lambda + 4\mu, 2 = -\lambda + \mu$ $\Rightarrow \lambda = -7, \mu = -5$
15	(D) $\sqrt{3}$
16	(D) 5
17	$(\vec{a} + 2\vec{b})^2 = \lvert\vec{a}\rvert^2 + \lvert 2\vec{b}\rvert^2 + 4\vec{a}.\vec{b}$ $= 56$ $\Rightarrow \lvert\vec{a} + 2\vec{b}\rvert = \sqrt{56}$
18	(C) $\dfrac{\sqrt{34}}{2}$ units
19	Let $ABCD$ be the parallelogram with diagonals $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{BD} = \vec{b}$. $\therefore \overrightarrow{AB} = \frac{1}{2}(\vec{a} - \vec{b})$ and $\overrightarrow{AD} = \frac{1}{2}(\vec{a} + \vec{b})$  Area of $ABCD$ $= \lvert\overrightarrow{AB} \times \overrightarrow{AD}\rvert$ $= \left\lvert\frac{1}{2}(\vec{a} - \vec{b}) \times \frac{1}{2}(\vec{a} + \vec{b})\right\rvert$ $= \frac{1}{4}\lvert\vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}\rvert$ $= \frac{1}{4}\lvert\vec{a} \times \vec{b} + \vec{a} \times \vec{b}\rvert \qquad (\because \vec{a} \times \vec{a} = \vec{0})$ $= \frac{1}{4}\lvert 2(\vec{a} \times \vec{b})\rvert$ $= \frac{1}{2}\lvert\vec{a} \times \vec{b}\rvert$ Given $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$

	Area of parallelogram $=\frac{1}{2}\left\|\vec{a}\times\vec{b}\right\|$ Now $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 3 & -1\end{vmatrix}=-2\hat{i}+3\hat{j}+7\hat{k}$ $\left\|\vec{a}\times\vec{b}\right\|=\sqrt{62}$ Area of parallelogram $=\frac{1}{2}\sqrt{62}$
20	**(C) (−2, 2)**
21	**(C) 24 and 8**
22	Let $\vec{a}=3\hat{i}-2\hat{j}+\hat{k},\quad \vec{b}=4\hat{i}+3\hat{j}-2\hat{k}$ $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -2 & 1\\ 4 & 3 & -2\end{vmatrix}=\hat{i}+10\hat{j}+17\hat{k}$ $\left\|\vec{a}\times\vec{b}\right\|=\sqrt{1^2+10^2+17^2}=\sqrt{390}$ Unit vector $\hat{n}=\frac{\vec{a}\times\vec{b}}{\left\|\vec{a}\times\vec{b}\right\|}=\frac{1}{\sqrt{390}}(\hat{i}+10\hat{j}+17\hat{k})$ $\therefore$ Required vector $=\frac{5}{\sqrt{390}}(\hat{i}+10\hat{j}+17\hat{k})$ **OR** (b) $\vec{a}.\vec{b}=\vec{a}.\vec{c}\Rightarrow\vec{a}.(\vec{b}-\vec{c})=0$ $\Rightarrow$ either $\vec{b}=\vec{c}$ or $\vec{a}\perp(\vec{b}-\vec{c})$, since $\vec{a}\neq 0$ Also, $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}\Rightarrow\vec{a}\times(\vec{b}-\vec{c})=0$ $\Rightarrow$ either $\vec{b}=\vec{c}$ or $\vec{a}\parallel(\vec{b}-\vec{c})$, since $\vec{a}\neq 0$ Since vectors $\vec{a}$ and $(\vec{b}-\vec{c})$ cannot be $\parallel$ and $\perp$ simultaneously Hence $\vec{b}=\vec{c}$
23	 Let P and Q trisect the wire AB. P divides AB in the ratio 1:2 then, coordinate of point P $=\left(\frac{14}{3},\frac{4}{3},-\frac{7}{3}\right)$

	Q divides AB in the ratio 2:1 then, coordinate of point Q = $\left(\frac{16}{3}, \frac{5}{3}, -\frac{8}{3}\right)$
24	Given $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \lvert\vec{a} + \vec{b}\rvert = \lvert-\vec{c}\rvert$ $\Rightarrow \lvert\vec{a} + \vec{b}\rvert^2 = \lvert\vec{c}\rvert^2 \Rightarrow \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + 2\,\vec{a}.\vec{b} = \lvert\vec{c}\rvert^2$ $\Rightarrow 9 + 25 + 2\,\vec{a}.\vec{b} = 49$ $\Rightarrow 2\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta = 15$ $\Rightarrow \cos\theta = \frac{1}{2} \therefore \theta = \frac{\pi}{3}$ **OR** (b) $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = 1$ $\lvert\vec{a} - \vec{b}\rvert^2 = \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 - 2\vec{a}\,.\vec{b}$ $= 1 + 1 - 2\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta$ $= 2 - 2\cos\theta$ $= 2\left(2\sin^2\frac{\theta}{2}\right) = 4\sin^2\frac{\theta}{2}$ $\Rightarrow \sin\frac{\theta}{2} = \frac{1}{2}\lvert\vec{a} - \vec{b}\rvert$
25	1 mark for any attempt as correct answer is not given in any option
26	(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).
27	(a) C divides BA in the ratio 3 : 2 externally Required vector = $\vec{c} = \frac{3\vec{a} - 2\vec{b}}{3-2} = 3\vec{a} - 2\vec{b}$  OR (b) Unit vector equally inclined along coordinate axes is $\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$ $\vec{r} = 5\sqrt{3}\left(\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}\right) = 5\hat{i} + 5\hat{j} + 5\hat{k}$ (or $-5\hat{i} - 5\hat{j} - 5\hat{k}$)
28	**(D) 4**
29	**(A) orthogonal vectors**

30	(a) Let $\vec{d}=\vec{b}+\vec{c}=(2+\lambda)\hat{i}-6\hat{j}+2\hat{k}$ $\hat{d}=\dfrac{(2+\lambda)\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{(2+\lambda)^2+40}}$ $\vec{a}\cdot\hat{d}=(\hat{i}-\hat{j}+2\hat{k})\cdot\dfrac{(2+\lambda)\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{(2+\lambda)^2+40}}=1$ $\Rightarrow(2+\lambda)+6+4=\sqrt{(2+\lambda)^2+40}\Rightarrow\lambda=-5$
31	**(B)two**
32	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**
33	(a) $\lvert\vec{a}+\vec{b}\rvert^2+\lvert\vec{a}-\vec{b}\rvert^2=2\left(\lvert\vec{a}\rvert^2+\lvert\vec{b}\rvert^2\right)$ $(60)^2+(40)^2=2(\lvert\vec{a}\rvert^2+(46)^2)$ $\dfrac{(3600+1600)}{2}=(\lvert\vec{a}\rvert^2+(46)^2)$ $2600-2116=\lvert\vec{a}\rvert^2$ $484=\lvert\vec{a}\rvert^2$ $\lvert\vec{a}\rvert=22$ OR (b)Let A, B, C be the points $(k,-11,2),(0,-2,2)$ *and* $(2,4,2)$*respectively* $\overrightarrow{AB}=-ki+9j$ $\overrightarrow{BC}=2i+6j$ Since $\overrightarrow{AB}$ *is parallel to* $\overrightarrow{BC}$ $\frac{-k}{2}=\frac{9}{6}$ $\Rightarrow k=-3$
34	let $\vec{p}=\vec{a}-\vec{b}=-\hat{\imath}+\hat{\jmath}+\hat{k}$ $\vec{q}=\vec{c}-\vec{b}=\hat{\imath}-5\hat{\jmath}-5\hat{k}$ Vector perpendicular to $\vec{p}$ and $\vec{q}=\vec{p}\times\vec{q}=\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\-1&1&1\\1&-5&-5\end{vmatrix}=-4\hat{\jmath}+4\hat{k}$ Unit Vector perpendicular to $\vec{p}$ and $\vec{q}$ $=\dfrac{-4\hat{\jmath}+4\hat{k}}{\sqrt{4^2+4^2}}$ $=\dfrac{-\hat{\jmath}+\hat{k}}{\sqrt{2}}$ or $\dfrac{-1}{\sqrt{2}}\hat{\jmath}+\dfrac{1}{\sqrt{2}}\hat{k}$

CHAPTER 11 3D

Q.		Code and Marks
1	(a) Verify that lines given by $\vec{r} = (1-\lambda)\hat{i} + (\lambda-2)\hat{j} + (3-2\lambda)\hat{k}$ and $\vec{r} = (\mu+1)\hat{i} + (2\mu-1)\hat{j} - (2\mu+1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines. **OR** (b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.	65/1/1 65/1/2 65/1/3
2	(a) Find the image A' of the point A(1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A'. **OR** (b) Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point Q(2, 4, –1) is 7 units. Also, find the equation of line joining P and Q.	65/1/1 65/1/2 65/1/3
3	If a line makes angles of $\frac{3\pi}{4}, \frac{\pi}{3}$ and θ with the positive directions of x, y and z-axis respectively, then θ is (A) $\frac{-\pi}{3}$ only (B) $\frac{\pi}{3}$ only (C) $\frac{\pi}{6}$ (D) $\pm\frac{\pi}{3}$	65/2/1 65/2/2 65/2/3
4	The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point (4, –3, 7) is : (A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$ (B) $x = 3t + 4, y = t + 3, z = 2t + 7$ (C) $x = 3t + 4, y = t - 3, z = 2t + 7$ (D) $x = 3t + 4, y = -t + 3, z = 2t + 7$	65/2/1 65/2/2 65/2/3

5	The line $x = 1 + 5\mu$, $y = -5 + \mu$, $z = -6 - 3\mu$ passes through which of the following point ? (A) $(1, -5, 6)$ (B) $(1, 5, 6)$ (C) $(1, -5, -6)$ (D) $(-1, -5, 6)$	65/2/1 65/2/2 65/2/3
6	(a) Find the shortest distance between the lines : $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ and $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$. **OR** (b) Find the image A′ of the point A(2, 1, 2) in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA′. Find the foot of perpendicular from point A on the line l.	65/2/1 65/2/2 65/2/3
7	If P is a point on the line segment joining $(3, 6, -1)$ and $(6, 2, -2)$ and y-coordinate of P is 4, then its z-coordinate is : (A) $-\frac{3}{2}$ (B) 0 (C) 1 (D) $\frac{3}{2}$	65/4/1
8	Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.	65/4/1 65/4/2 65/4/3
9	(b) Find the equation of a line in vector and cartesian form which passes through the point $(1, 2, -4)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$, and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.	65/4/1 65/4/2 65/4/3
10	An engineer is designing a new metro rail network in a city. Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by	65/4/1 65/4/2 65/4/3

$l_1: \dfrac{x-2}{3} = \dfrac{y+1}{-2} = \dfrac{z-3}{4}$, while the track for Line B is represented by

$l_2: \dfrac{x-1}{2} = \dfrac{y-3}{1} = \dfrac{z+2}{-3}$.

Based on the above information, answer the following questions :

(i) Find whether the two metro tracks are parallel. *1*

(ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to be positioned parallel to Line A's track (l_1) and pass through the point $(1, -2, -3)$. *1*

(iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point (3, 2, 1). Determine the equation of the pedestrian walkway. *2*

OR

(iii) (b) Find the shortest distance between Line A and Line B. *2*

11	If the direction cosines of a line are $\lambda, \lambda, \lambda$, then λ is equal to : (A) $-\frac{1}{\sqrt{3}}$ (B) 1 (C) $\frac{1}{\sqrt{3}}$ (D) $\pm\frac{1}{\sqrt{3}}$	65/4/2
12	The coordinates of the foot of the perpendicular drawn from the poi A(– 2, 3, 5) on the y-axis is : (A) (0, 0, 5) (B) (0, 3, 0) (C) (– 2, 0, 5) (D) (– 2, 0, 0)	65/4/3
13	(a) Find the foot of the perpendicular drawn from the point (1, 1, 4) on the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$. **OR** (b) Find the point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}$ at a distance of $2\sqrt{2}$ units from the point (–1, –1, 2).	65/5/1 65/5/2 65/5/3
14	**25.** **Determine if the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect with each other.**	65/6/1
15	**30.** (a) Find the distance of the point P(2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. **OR** (b) Let the position vectors of the points A, B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.	65/6/1 65/6/2 65/6/3
16	**35.** Let the polished side of the mirror be along the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{2z-4}{6}$. A point P(1, 6, 3), some distance away from the mirror, has its image	65/6/1

	formed behind the mirror. Find the coordinates of the image point and the distance between the point P and its image.	
17	Find the value of λ if the following lines are perpendicular to each other : $l_1: \dfrac{1-x}{-3} = \dfrac{3y-2}{2\lambda} = \dfrac{z-3}{3}$ $l_2: \dfrac{x-1}{3\lambda} = \dfrac{1-y}{1} = \dfrac{2z-5}{3}$	65/6/2 65/6/3
18	Find the foot of the perpendicular drawn from point (2, –1, 5) to the line $\dfrac{x-11}{10} = \dfrac{y+2}{-4} = \dfrac{z+8}{-11}$. Also, find the length of the perpendicular.	65/6/2
19	**Show that the line passing through the points A (0, –1, –1) and B (4, 5, 1) intersects the line joining points C (3, 9, 4) and D (– 4, 4, 4).**	65/6/3
20	(b) Find the shortest distance between the lines : $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$ $\vec{r} = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k})$.	65/7/1 65/7/2 65/7/3
21	(a) Find the point Q on the line $\dfrac{2x+4}{6} = \dfrac{y+1}{2} = \dfrac{-2z+6}{-4}$ at a distance of $3\sqrt{2}$ from the point P(1, 2, 3). **OR** (b) Find the image of the point (–1, 5, 2) in the line $\dfrac{2x-4}{2} = \dfrac{y}{2} = \dfrac{2-z}{3}$. Find the length of the line segment joining the points (given point and the image point).	65/7/1 65/7/2 65/7/3
22	Find the angle between the two lines whose equations are $2x = 3y = -z$ and $6x = -y = -4z$.	65(B)
23	(a) Find the shortest distance between the lines given by $\vec{r} = (4\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} + 2\hat{j} - 4\hat{k})$ **OR**	65(B)

(b) Find the coordinates of the foot of the perpendicular and the length of the perpendicular drawn from the point P(5, 4, 2) to the line $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$.

ANSWERS

1

(a) Rewriting the lines, we get

$\vec{r} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) + \lambda\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$ and $\vec{r} = \left(\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

Let $\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}$, $\vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$

Note that the dr's of given lines are not proportional so, they are not parallel lines.

The lines will be skew if they do not intersect each other also.

Here $\vec{a}_2 - \vec{a}_1 = \hat{j} - 4\hat{k}$, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}$

Consider $\left(\vec{a}_2 - \vec{a}_1\right).\left(\vec{b}_1 \times \vec{b}_2\right)$

$= \left(\hat{j} - 4\hat{k}\right).\left(2\hat{i} - 4\hat{j} - 3\hat{k}\right) = 8 \neq 0$

Hence lines will not intersect. So the lines are skew.

Shortest Distance $= \dfrac{\left|\left(\vec{a}_2 - \vec{a}_1\right).\left(\vec{b}_1 \times \vec{b}_2\right)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|}$

$= \dfrac{8}{\sqrt{4+16+9}} = \dfrac{8}{\sqrt{29}}$

OR

(b) Let the wicket keeper divides the line segment in ratio $k:1$

$\therefore \vec{W} = \dfrac{k\vec{F} + 1.\vec{B}}{k+1}$

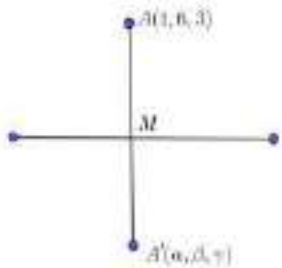
$\Rightarrow 6\hat{i} + 12\hat{j} = \left(\dfrac{12k+2}{k+1}\right)\hat{i} + \left(\dfrac{18k+8}{k+1}\right)\hat{j}$

$\Rightarrow k = \dfrac{2}{3}$

Hence, the required ratio is $2:3$

2

(a) The equation of given line is $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3} = \lambda$



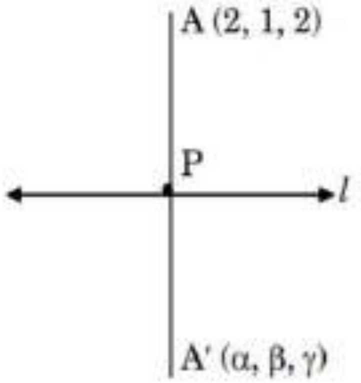
Any arbitrary point on the line is $M\left(\lambda, 2\lambda+1, 3\lambda+2\right)$

dr's of AM are $< \lambda - 1, 2\lambda - 5, 3\lambda - 1 >$

Here $1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$

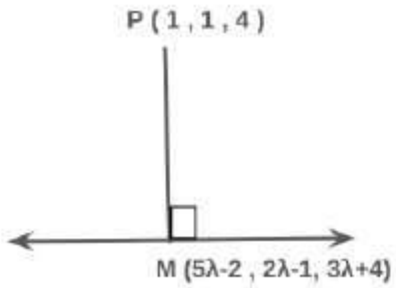
$\Rightarrow \lambda = 1$

	$\therefore M(1,3,5)$ is the foot perpendicular of the point A to the given line. Let image of point A in the line be $A'(\alpha,\beta,\gamma)$ Since M is the mid-point of AA', so $M\left(\frac{1+\alpha}{2},\frac{6+\beta}{2},\frac{3+\gamma}{2}\right)=M(1,3,5)$ $\Rightarrow A'(1,0,7)$ is the image of A. Also, Equation of AA' is $\frac{x-1}{0}=\frac{y-6}{-3}=\frac{z-3}{2}$ **OR** **(b)** **The given line is** $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$ **and** $Q(2,4,-1)$ **Any random point on the line will be given by** $P(\lambda-5,4\lambda-3,-9\lambda+6)$ **Since** $PQ=7\Rightarrow\sqrt{(\lambda-7)^2+(4\lambda-7)^2+(-9\lambda+7)^2}=7$ $\Rightarrow 98(\lambda^2-2\lambda+1)=0\Rightarrow\lambda=1$ **Hence, the required point is** $P(-4,1,-3)$ **The equation of line PQ is** $\frac{x+4}{6}=\frac{y-1}{3}=\frac{z+3}{2}$ **or** $\frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}$
3	No option is correct. Full marks may be awarded for attempting the question.
4	(C) $x=3t+4$, $y=t-3$, $z=2t+7$
5	(C) (1, -5, -6)
6	The vector equations of the lines are $\vec{r}=-\hat{\imath}+\hat{\jmath}+9\hat{k}+\lambda(2\hat{\imath}+\hat{\jmath}-3\hat{k})$ $\vec{r}=3\hat{\imath}-15\hat{\jmath}+9\hat{k}+\mu(2\hat{\imath}-7\hat{\jmath}+5\hat{k})$ $\overrightarrow{a_1}=-\hat{\imath}+\hat{\jmath}+9\hat{k},\ \overrightarrow{a_2}=3\hat{\imath}-15\hat{\jmath}+9\hat{k}$ $\overrightarrow{b_1}=2\hat{\imath}+\hat{\jmath}-3\hat{k},\ \overrightarrow{b_2}=2\hat{\imath}-7\hat{\jmath}+5\hat{k}$ $\overrightarrow{a_2}-\overrightarrow{a_1}=4\hat{\imath}-16\hat{\jmath}$ $\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\2&1&-3\\2&-7&5\end{vmatrix}=-16\hat{\imath}-16\hat{\jmath}-16\hat{k}$ S.D. $=\frac{\lvert(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2})\rvert}{\lvert\overrightarrow{b_1}\times\overrightarrow{b_2}\rvert}=\frac{12}{\sqrt{3}}=4\sqrt{3}$ OR

	 Let the image of A in the line be $A'(\alpha, \beta, \gamma)$ The point P, which is the point of intersection of the lines l and AA', will have coordinates $(\lambda + 4, -\lambda + 2, -\lambda + 2)$ for some λ. Drs of AP are $< \lambda + 2, -\lambda + 1, -\lambda >$ $AP \perp l$ $(\lambda + 2) - (-\lambda + 1) - (-\lambda) = 0$ $\Rightarrow \lambda = -\frac{1}{3}$ Therefore, the coordinates of P are $(\frac{11}{3}, \frac{7}{3}, \frac{7}{3})$ P is the mid-point of AA' $\Rightarrow \frac{2+\alpha}{2} = \frac{11}{3}, \frac{1+\beta}{2} = \frac{7}{3}, \frac{2+\gamma}{2} = \frac{7}{3}$ $\Rightarrow \alpha = \frac{16}{3}, \beta = \frac{11}{3}, \gamma = \frac{8}{3}$ The coordinates of the image are $(\frac{16}{3}, \frac{11}{3}, \frac{8}{3})$ The equation of AA' is $\frac{x-2}{\frac{10}{3}} = \frac{y-1}{\frac{8}{3}} = \frac{z-2}{\frac{2}{3}}$ or, $\frac{3(x-2)}{5} = \frac{3(y-1)}{4} = \frac{3(z-2)}{1}$
7	(A) $-\frac{3}{2}$

8		$l_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$ Any point on l_1 is $(2\lambda+1, 3\lambda+2, 4\lambda+3)$ $l_2: \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$ Any point on l_2 is $(5\mu+4, 2\mu+1, \mu)$ For point of intersection, $2\lambda+1 = 5\mu+4, 3\lambda+2 = 2\mu+1$ Solving, $\lambda = \mu = -1$ Since, $\lambda = \mu = -1$ satisfy $4\lambda+3 = \mu$ $\therefore$ Point of intersection is $(-1,-1,-1)$ Now distance of $(-1,-5,-10)$ from $(-1,-1,-1)$ is: $\sqrt{(-1+1)^2 + (-1+5)^2 + (-1+10)^2} = \sqrt{97}$ units
9	**(b)**	Given lines are $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$ The first line in vector form is $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ $\vec{a}_1 = 8\hat{i} - 19\hat{j} + 10\hat{k}, \vec{a}_2 = 15\hat{i} + 29\hat{j} + 5\hat{k}$ $\vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}, \vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{i} + 36\hat{j} + 72\hat{k}$ $\therefore$ Equation of line passing through $(1,2,-4)$ and parallel to $\vec{b}$ is $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + t(24\hat{i} + 36\hat{j} + 72\hat{k})$ or $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + t'(2\hat{i} + 3\hat{j} + 6\hat{k})$ Cartesian form of line is $\frac{x-1}{24} = \frac{y-2}{36} = \frac{z+4}{72}$ or $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$

10	$l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4}$; $l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$ (i) Drs of l_1 are $<3,-2,4>$, drs of l_2 are $<2,1,-3>$ as drs are not proportional, hence l_1 is not parallel to l_2. (ii) Equations of line parallel to l_1 and passing through $(1,-2,-3)$ is $\frac{x-1}{3} = \frac{y+2}{-2} = \frac{z+3}{4}$ or $\vec{r} = (\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$ $(iii)(a)$ The pathway is perpendicular to l_1 and l_2 $\therefore$ It is parallel to $\vec{b}_1 \times \vec{b}_2$ $\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ 2 & 1 & -3 \end{vmatrix} = 2\hat{i} + 17\hat{j} + 7\hat{k}$ $\therefore$ Equation of pathway is $\vec{r} = (3\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 17\hat{j} + 7\hat{k})$ **OR** $(iii)(b)$ $\vec{a}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{a}_2 = \hat{i} + 3\hat{j} - 2\hat{k}$ $\vec{b}_1 = 3\hat{i} - 2\hat{j} + 4\hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} - 3\hat{k}$ $d = \left\| \frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{\|\vec{b}_1 \times \vec{b}_2\|} \right\|$ $= \left\| \frac{(-\hat{i} + 4\hat{j} - 5\hat{k}).(2\hat{i} + 17\hat{j} + 7\hat{k})}{\sqrt{4 + 289 + 49}} \right\|$ $= \frac{31}{\sqrt{342}}$
11	(D) $\pm\frac{1}{\sqrt{3}}$
12	(B) (0, 3, 0)
13	(a) Let $\frac{x+2}{5} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$ Coordinate of general point on the given line are $M(5\lambda - 2, 2\lambda - 1, 3\lambda + 4)$



Direction Ratios of PM vector are $< 5\lambda - 3, 2\lambda - 2, 3\lambda >$

Since, PM $\perp$ l

$\Rightarrow 5(5\lambda - 3) + 2(2\lambda - 2) + 3(3\lambda) = 0$

$\Rightarrow \lambda = \frac{1}{2}$

Hence, coordinates of M are $\left(\frac{1}{2}, 0, \frac{11}{2}\right)$

OR

(b) Equation of given line be $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$ **(say)**

Coordinate of any general point on the line are $P\ (3\lambda + 1, 2\lambda - 1, 3\lambda + 4)$.

Let distance of point P from $(-1, -1, 2)$ **is** $2\sqrt{2}$.

$\Longrightarrow \sqrt{(3\lambda + 2)^2 + (2\lambda)^2 + (3\lambda + 2)^2} = 2\sqrt{2}$

$\Longrightarrow 22\lambda^2 + 24\lambda = 0$

$\Rightarrow \lambda = 0$ **or** $\lambda = -\frac{12}{11}$

Hence, coordinates of point P are $(1, -1, 4)$ **or** $\left(-\frac{25}{11}, -\frac{35}{11}, \frac{8}{11}\right)$

14 $\vec{b}_1 = 3\hat{i} - \hat{j}$, $\vec{b}_2 = 2\hat{i} + 3\hat{k}$, $\vec{a}_2 = 4\hat{i} - \hat{k}$, $\vec{a}_1 = \hat{i} + \hat{j} - \hat{k}$

$(\vec{a}_2 - \vec{a}_1) = 3\hat{i} - \hat{j}$

$\vec{b}_1 \times \vec{b}_2 = -3\hat{i} - 9\hat{j} + 2\hat{k}$

$(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = (3\hat{i} - \hat{j}).(-3\hat{i} - 9\hat{j} + 2\hat{k}) = 0$, hence lines intersect

15	(a) Let $\vec{a_2} = 2\hat{i} + 4\hat{j} - \hat{k}$, $\vec{a_1} = -5\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$ Distance between point and line is given by $d = \frac{\lvert(\vec{a_2} - \vec{a_1}) \times \vec{b}\rvert}{\lvert\vec{b}\rvert}$ Here $(\vec{a_2} - \vec{a_1}) = 7\hat{i} + 7\hat{j} - 7\hat{k}$ $(\vec{a_2} - \vec{a_1}) \times \vec{b} = -35\hat{i} + 56\hat{j} + 21\hat{k}$ $d = \frac{49\sqrt{2}}{7\sqrt{2}} = 7$ OR (b) Vector parallel to given line $= 3\hat{j} + 4\hat{k}$ Vector equation is $\vec{r} = 3\hat{i} - \hat{j} - 2\hat{k} + \mu(3\hat{j} + 4\hat{k})$ Cartesian equation is $\frac{x-3}{0} = \frac{y+1}{3} = \frac{z+2}{4}$
16	Equation of given line is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ Let coordinates of point on the line be $(\lambda, 2\lambda + 1, 3\lambda + 2)$ for some λ Drs of line perpendicular to line along mirror are $<\lambda-1, 2\lambda - 5, 3\lambda - 1>$ $(\lambda-1).1 + (2\lambda - 5).2 + (3\lambda - 1).3 = 0$ gives $\lambda = 1$ Coordinates of foot of perpendicular are (1,3,5) For image $\frac{x+1}{2} = 1$, $\frac{y+6}{2} = 3$, $\frac{z+3}{2} = 5$ gives image as (1, 0, 7) Required distance $= \sqrt{0 + 36 + 16} = 2\sqrt{13}$
17	$l_1: \frac{x-1}{3} = \frac{y-\frac{2}{3}}{\frac{2}{3}\lambda} = \frac{z-3}{3}$ $l_2: \frac{x-1}{3\lambda} = \frac{y-1}{-1} = \frac{z-\frac{5}{2}}{\frac{3}{2}}$ lines are perpendicular $\Rightarrow 3(3\lambda) + \frac{2}{3}\lambda(-1) + 3.\frac{3}{2} = 0$ $\lambda = \frac{-27}{50}$
18	Let $l: \frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda$ Coordinates of any point on l are $x = 10\lambda + 11, y = -4\lambda - 2, z = -11\lambda - 8$ Drs of perpendicular line are $(10\lambda + 9, -4\lambda - 1, -11\lambda - 13)$ Drs of given line are 10, – 4, – 11 As lines are perpendicular, so

	$(10\lambda + 9)10 + (-4\lambda - 1)(-4) + (-11\lambda - 13) \times (-11) = 0$ $\Rightarrow \lambda = -1$ Hence coordinates of point are (1, 2, 3) which is the foot of the $\perp$ from P to l length of $\perp = \sqrt{(1-2)^2 + (2+1)^2 + (3-5)^2} = \sqrt{1+9+4} = \sqrt{14}$
19	Drs of line passing through points A and B are < 4,6,2> Drs of line passing through C and D are <-7, -5, 0> Consider $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3-0 & 9-(-1) & 4-(-1) \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}$ $= 3(+10) - 10(+14) + 5(22)$ $= 0$ Hence lines intersect each other
20	**(b) The two given lines are parallel with,** $\vec{a}_1 = 2\hat{i} - \hat{j} + 3\hat{k},\ \vec{a}_2 = \hat{i} + 4\hat{k}$ **Then** $\vec{a}_2 - \vec{a}_1 = -\hat{i} + \hat{j} + \hat{k}$ **and the parallel vector is** $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ $\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ -1 & 1 & 1 \end{vmatrix} = -5\hat{i} - 4\hat{j} - \hat{k}$ **Shortest Distance** $= \frac{\lvert \vec{b} \times (\vec{a}_2 - \vec{a}_1) \rvert}{\lvert \vec{b} \rvert} = \frac{\sqrt{42}}{\sqrt{14}} = \sqrt{3}$
21	**(a) The general point on the line** $(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$ **is Q, from some** $\lambda \in R$ $PQ = 3\sqrt{2} \Rightarrow (PQ)^2 = 18 \Rightarrow (3\lambda - 3)^2 + (2\lambda - 3)^2 + (2\lambda)^2 = 18$ $17\lambda^2 - 30\lambda = 0 \Rightarrow \lambda = 0$ or $\lambda = \frac{30}{17}$ **Thus, the point is** $Q(-2, -1, 3)$ **or** $Q\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$ **OR** **(b) Let** $A'(a,b,c)$ **be the image of the point** $A(-1,5,2)$ **in the given line, also assume 'M' as the point of intersection of** AA' **with the given line, then 'M' is the mid-point of the line segment** AA' **The Line in the standard form is:** $\frac{x-2}{1} = \frac{y}{2} = \frac{z-2}{-3}$**, then** **M is the point** $(\lambda + 2, 2\lambda, -3\lambda + 2)$**, for some** $\lambda \in R$

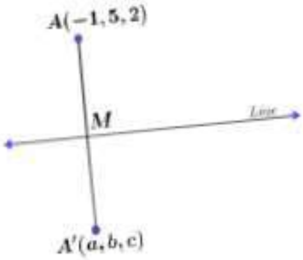
Direction Ratios of AM are $\lambda+3, 2\lambda-5, -3\lambda$

AM $\perp$ Line, $\therefore 1(\lambda+3)+2(2\lambda-5)-3(-3\lambda)=0 \Rightarrow \lambda=\frac{1}{2}$

$M\left(\frac{5}{2},1,\frac{1}{2}\right)=M\left(\frac{a-1}{2},\frac{b+5}{2},\frac{c+2}{2}\right) \Rightarrow a=6, b=-3, c=-1$

$\therefore$ The Image of A in the line is $A'(6,-3,-1)$

And, $AA'=\sqrt{49+64+9}=\sqrt{122}$



22

$$2x=3y=-z \quad \Rightarrow \frac{x}{3}=\frac{y}{2}=\frac{z}{-6}$$

$$6x=-y=-4z \quad \Rightarrow \frac{x}{2}=\frac{y}{-12}=\frac{z}{-3}$$

$$\cos\theta=\frac{3\times 2+2\times(-12)-6\times(-3)}{\sqrt{3^2+2^2+(-6)^2}\,\sqrt{2^2+(-12)^2+(-3)^2}}$$

$$\because\ 3\times 2+2\times(-12)-6\times(-3)=0 \therefore \theta=\frac{\pi}{2}$$

23

(a)

$$\vec{a_1}=4\hat{\imath}-\hat{\jmath}+2\hat{k} \qquad \vec{b_1}=\hat{\imath}+2\hat{\jmath}-3\hat{k}$$
$$\vec{a_2}=2\hat{\imath}+\hat{\jmath}-\hat{k} \qquad \vec{b_2}=3\hat{\imath}+2\hat{\jmath}-4\hat{k}$$

$$\vec{a_2}-\vec{a_1}=-2\hat{\imath}+2\hat{\jmath}-3\hat{k}$$

$$\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 2 & -3\\ 3 & 2 & -4\end{vmatrix}=-2\hat{\imath}-5\hat{\jmath}-4\hat{k}$$

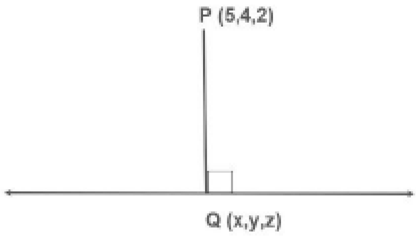
$$\left|\vec{b_1}\times\vec{b_2}\right|=\sqrt{4+25+16}=\sqrt{45}=3\sqrt{5}$$

$$(\vec{a_2}-\vec{a_1}).(\vec{b_1}\times\vec{b_2})=4-10+12=6$$

$$SD=\left|\frac{(\vec{a_2}-\vec{a_1}).(\vec{b_1}\times\vec{b_2})}{\left|\vec{b_1}\times\vec{b_2}\right|}\right|=\frac{6}{3\sqrt{5}}=\frac{2}{\sqrt{5}}$$

OR

(b)



Let coordinates of Q be $(2\lambda - 1, 3\lambda + 3, 1 - \lambda)$

D. Ratios of PQ $\langle 2\lambda - 6, 3\lambda - 1, -1 - \lambda \rangle$

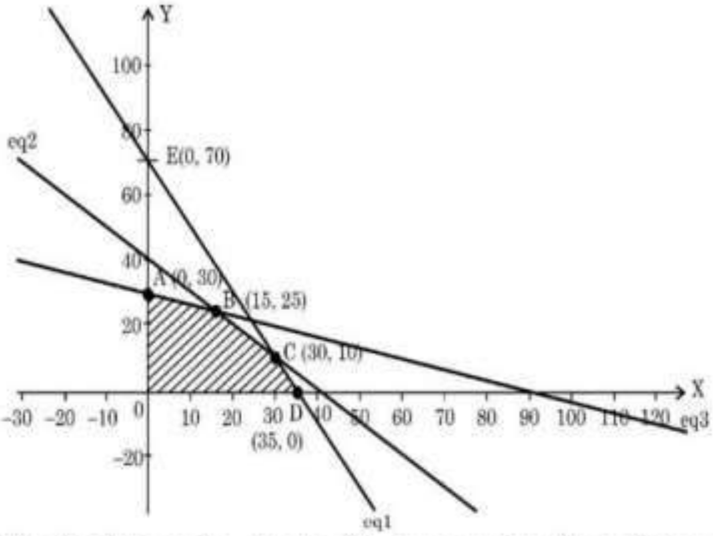
PQ $\perp$ line
$\Rightarrow 2(2\lambda - 6) + 3(3\lambda - 1) - (-1 - \lambda) = 0$

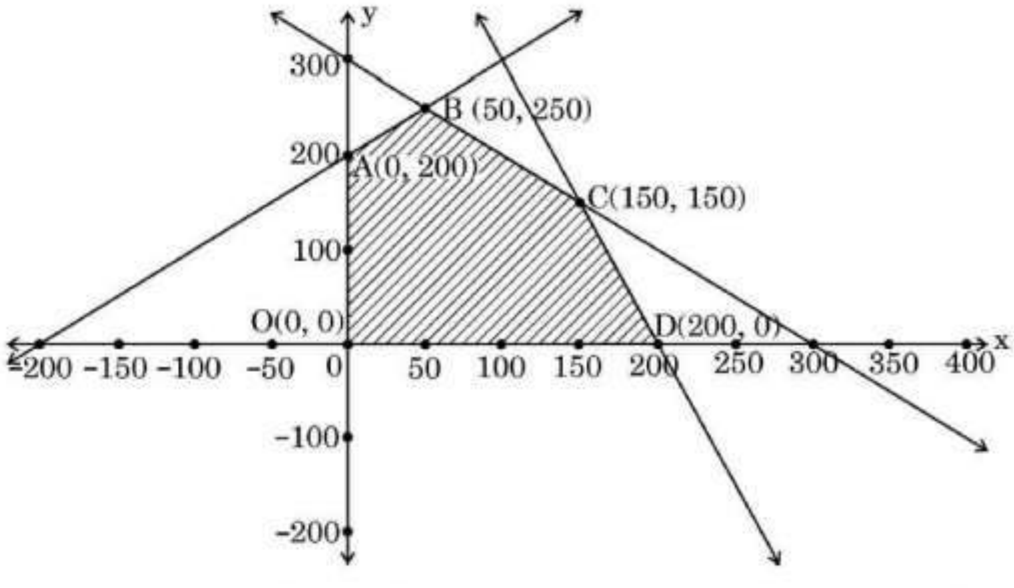
$\Rightarrow \lambda = 1$

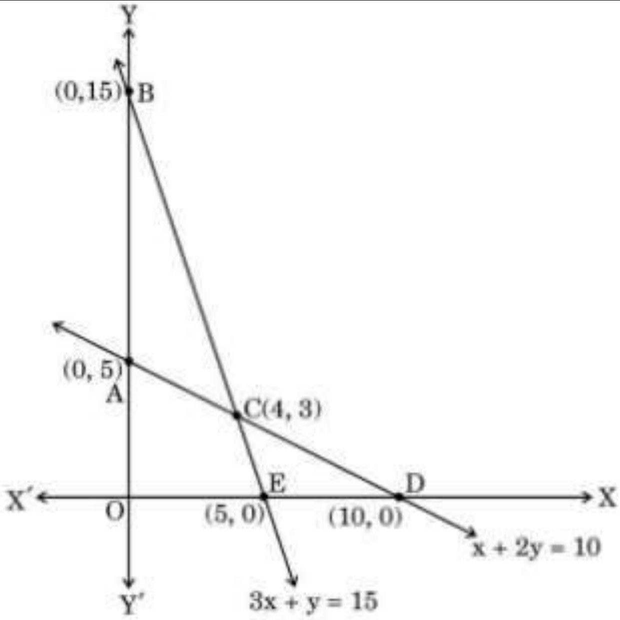
Foot of perpendicular is Q (1, 6, 0)
Length of perpendicular $= \sqrt{(4)^2 + (-2)^2 + (2)^2}$
$= \sqrt{16 + 4 + 4} = \sqrt{24} = 2\sqrt{6}$

CHAPTER 12 LINEAR PROGRAMMING PROBLEM

Q		Code and Marks
1	The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, 15). If $Z = 18x + 9y$ be the objective function, then (A) Z is maximum at (2, 72), minimum at (15, 20) (B) Z is maximum at (15, 20) minimum at (40, 15) (C) Z is maximum at (40, 15), minimum at (15, 20) (D) Z is maximum at (40, 15), minimum at (2, 72)	65/1/1 65/1/2 65/1/3 1M
2	If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct ? (A) It will only have a maximum value. (B) It will only have a minimum value. (C) It will have both maximum and minimum values. (D) It will have neither maximum nor minimum value.	65/1/1 65/1/2 65/1/3 1M
3	Solve the following linear programming problem graphically : Maximise $Z = x + 2y$ Subject to the constraints : $x - y \geq 0$ $x - 2y \geq -2$ $x \geq 0, y \geq 0$	65/1/1 3M
4	Solve the following linear programming problem graphically : Maximise $Z = 20x + 30y$ Subject to the constraints : $x + y \leq 80$ $2x + 3y \geq 100$ $x \geq 14$ $y \geq 14$	65/1/2 3M
5	Solve the following linear programming problem graphically : Maximize $Z = 8x + 9y$ Subject to the constraints : $2x + 3y \leq 6$ $3x - 2y \leq 6$ $y \leq 1$ $x \geq 0, y \geq 0$	65/1/3 3M
6	A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct ? (A) The objective function maximizes the difference of the profit earned from products X and Y. (B) The objective function measures the total production of products X and Y.	65/2/1 65/2/2 65/2/3 1M

	(C) The objective function maximizes the combined profit earned from selling X and Y. (D) The objective function ensures the company produces more of product X than product Y.	
7	**Assertion (A) :** Every point of the feasible region of a Linear Programming Problem is an optimal solution. **Reason (R) :** The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region.	65/2/1 65/2/2 65/2/3 1M
8	Solve the following linear programming problem graphically : Minimise $Z = x - 5y$ subject to the constraints : $x - y \geq 0$ $-x + 2y \geq 2$ $x \geq 3, y \leq 4, y \geq 0$	65/2/1 3M
9	 The feasible region along with corner points for a linear programming problem are shown in the graph. Write all the constraints for the given linear programing problem.	65/2/2 3M
10	Solve the following linear programming problem graphically : Minimise $Z = 2x + y$ subject to the constraints : $3x + y \geq 9$ $x + y \geq 7$ $x + 2y \geq 8$ $x, y \geq 0$	65/2/3 3M
11	The corner points of the feasible region of a Linear Programming Problem are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). If $Z = ax + by$; $(a, b > 0)$ be the objective function, and maximum value of Z is obtained at (0, 2) and (3, 0), then the relation between a and b is : (A) $a = b$ (B) $a = 3b$ (C) $b = 6a$ (D) $3a = 2b$	65/4/1 65/4/2 65/4/3 1M

12	*Assertion (A) :* In a Linear Programming Problem, if the feasible region is empty, then the Linear Programming Problem has no solution. *Reason (R) :* A feasible region is defined as the region that satisfies all the constraints.	65/4/1 65/4/2 65/4/3 1M
13	Solve the following Linear Programming Problem using graphical method : Maximise Z = 100x + 50y subject to the constraints $3x + y \leq 600$ $x + y \leq 300$ $y \leq x + 200$ $x \geq 0, y \geq 0$	65/4/1 3M
14	Solve the following Linear Programming Problem graphically : Minimise Z = 3x + 5y subject to the constraints $x + 2y \geq 10$ $x + y \geq 6$ $3x + y \geq 8$ $x, y \geq 0$	65/4/2 3M
15	 For the given graph of a Linear Programming Problem, write all the constraints satisfying the given feasible region.	65/4/3 3M
16	For a Linear Programming Problem (LPP), the given objective function Z = 3x + 2y is subject to constraints : $x + 2y \leq 10$ $3x + y \leq 15$ $x, y \geq 0$	65/5/1 65/5/2 65/5/3 1M



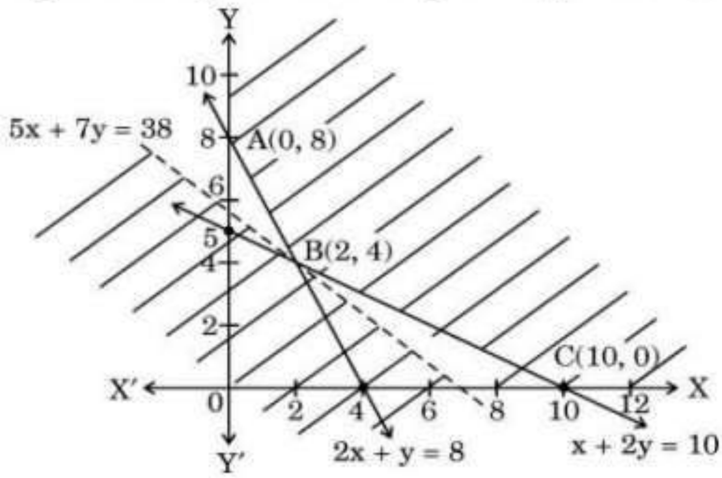
The correct feasible region is :

(A) ABC (B) AOEC

(C) CED (D) Open unbounded region BCD

17 — 65/5/1, 65/5/2, 65/5/3 — 1M

Assertion (A) : The shaded portion of the graph represents the feasible region for the given Linear Programming Problem (LPP).



Min $Z = 50x + 70y$

subject to constraints

$2x + y \geq 8,\ x + 2y \geq 10,\ x, y \geq 0$

$Z = 50x + 70y$ has a minimum value = 380 at B(2, 4).

Reason (R) : The region representing $50x + 70y < 380$ does not have any point common with the feasible region.

18 — 65/5/1, 65/5/2, 65/5/3 — 3M

In the Linear Programming Problem (LPP), find the point/points giving maximum value for $Z = 5x + 10y$

subject to constraints

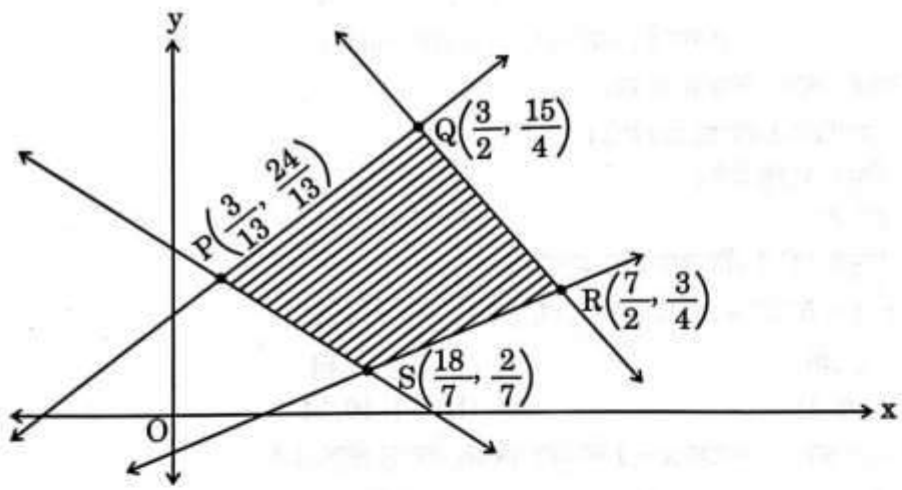
$x + 2y \leq 120$ $\quad$ $x - 2y \geq 0$

$x + y \geq 60$ $\quad$ $x, y \geq 0$

19 For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.

65/6/1
65/6/2
65/6/3

1M



(Note : The figure is not to scale)

$$P \equiv \left(\frac{3}{13}, \frac{24}{13}\right),\ Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right),\ R \equiv \left(\frac{7}{2}, \frac{3}{4}\right),\ S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$$

Which of the following statements is correct ?

(A) Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$

(B) Z is maximum at $R\left(\frac{7}{2}, \frac{3}{4}\right)$

(C) (Value of Z at P) > (Value of Z at Q)

(D) (Value of Z at Q) < (Value of Z at R)

---

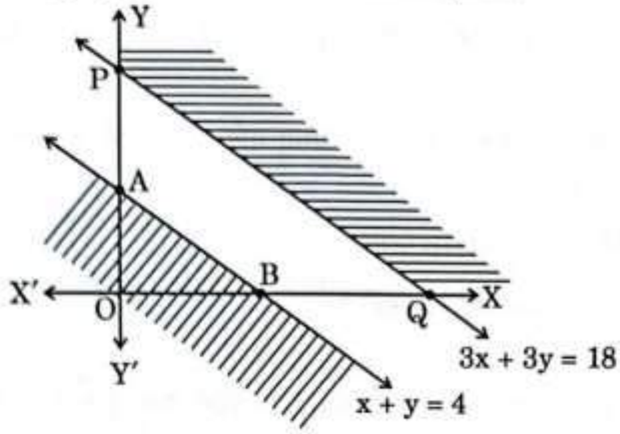
20 In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints :

$$x + y \le 4,\ 3x + 3y \ge 18,\ x, y \ge 0$$

Study the graph and select the correct option.

65/6/1
65/6/2
65/6/3

1M



(Note : The figure is not to scale)

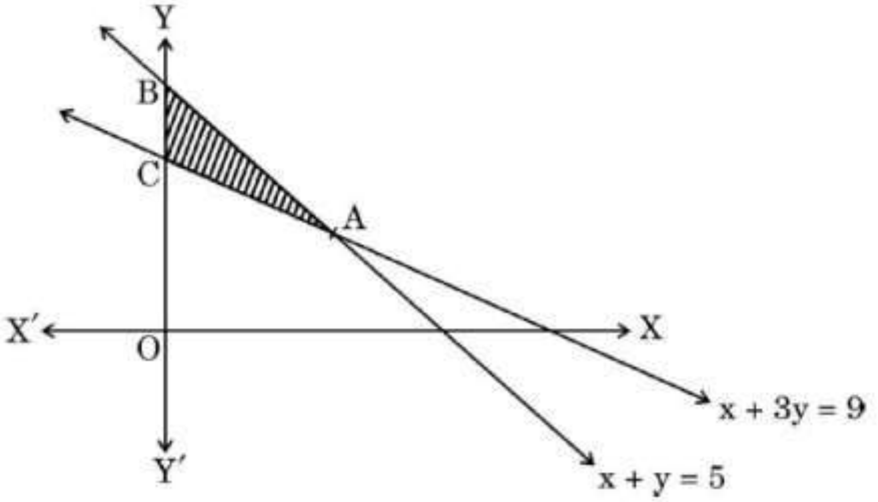
The solution of the given LPP :

(A) lies in the shaded unbounded region.

(B) lies in Δ AOB.

(C) does not exist.

(D) lies in the combined region of Δ AOB and unbounded shaded region.

21	Consider the Linear Programming Problem, where the objective function $Z = (x + 4y)$ needs to be minimized subject to constraints $2x + y \geq 1000$ $x + 2y \geq 800$ $x, y \geq 0.$ Draw a neat graph of the feasible region and find the minimum value of Z.	65/6/1 65/6/2 65/6/3 3M
22	In a Linear Programming Problem, the objective function $Z = 5x + 4y$ needs to be maximised under constraints $3x + y \leq 6$, $x \leq 1$, $x, y \geq 0$. Express the LPP on the graph and shade the feasible region and mark the corner points.	65/7/1 2M
23	In the Linear Programming Problem for objective function $Z = 18x + 10y$ subject to constraints $4x + y \geq 20$ $2x + 3y \geq 30$ $x, y \geq 0$ find the minimum value of Z.	65/7/1 65/7/2 65/7/3 3M
24	In a Linear Programming Program (LPP) for objective function $Z = 14x - 10y$ subject to constraints $x + y \leq 8$ $3x - 2y \geq -6$ $x, y \geq 0$ shade the feasible region and mark the corner points in a neatly drawn graph.	65/7/2 2M
25	For a Linear Programming Problem, find min $Z = 5x + 3y$ (where Z is the objective function) for the feasible region shaded in the given figure.  (Note : The figure is not to scale)	65/7/3 2M

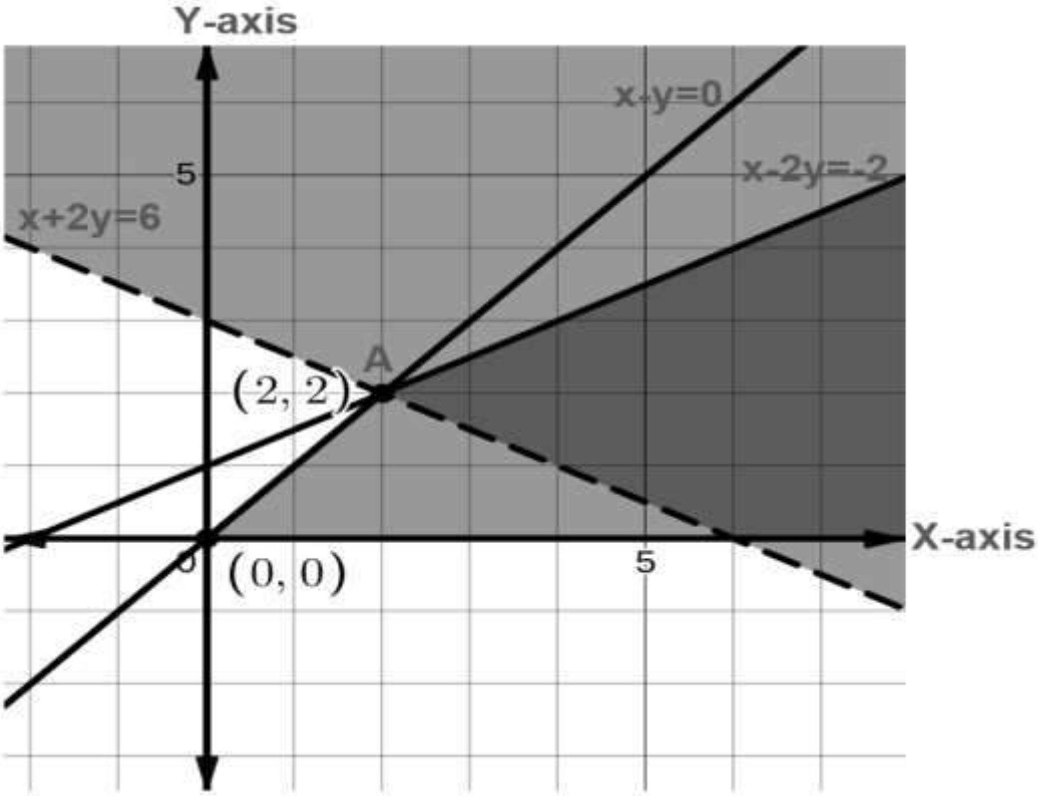
26	Of all the points of the feasible region, for maximum or minimum values of the objective function, the point lies : (A) inside the feasible region (B) at the boundary line of the feasible region (C) at the corner points of the feasible region (D) at the coordinate axes	65(B) 1M
27	The common region for the inequalities $x \geq 0$, $x + y \leq 1$ and $y \geq 0$, lies in (A) IV Quadrant (B) II Quadrant (C) III Quadrant (D) I Quadrant	65(B) 1M
28	The corner points of the feasible region determined by some system of linear inequations, are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = ax + by$, where $a, b > 0$. Find the condition on a and b so that the maximum of Z occurs at both points (3, 4) and (0, 5).	65(B) 3M

	ANSWERS
1	(C) Z is maximum at (40, 15), minimum at (15, 20)
2	(C) It will have both maximum and minimum values.
3	see below

3

Corner Point	Value of $Z = x + 2y$
$O(0,0)$	0
$A(2,2)$	6

Since feasible region is unbounded. Plot $x + 2y > 6$ which has common region with feasible region, thus Z has no maximum value.



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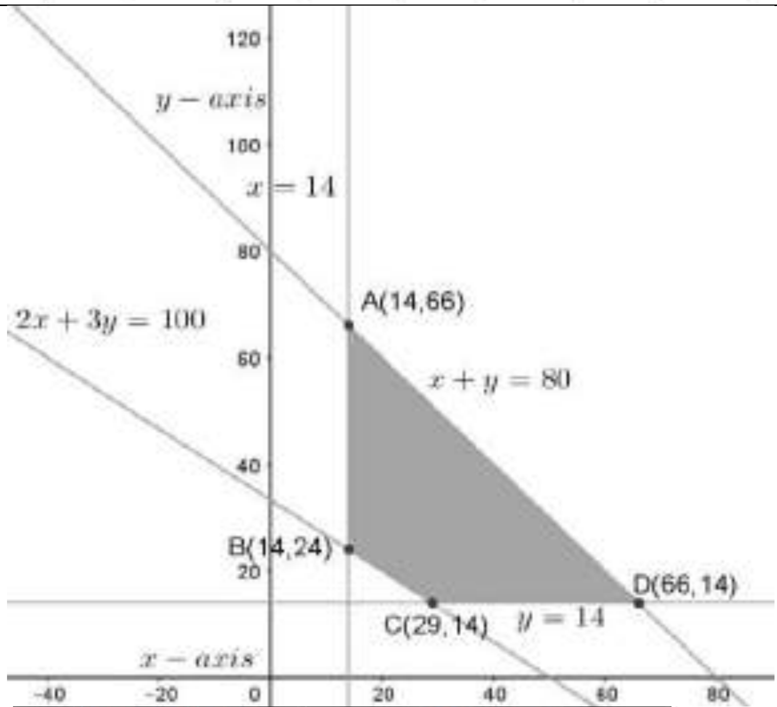
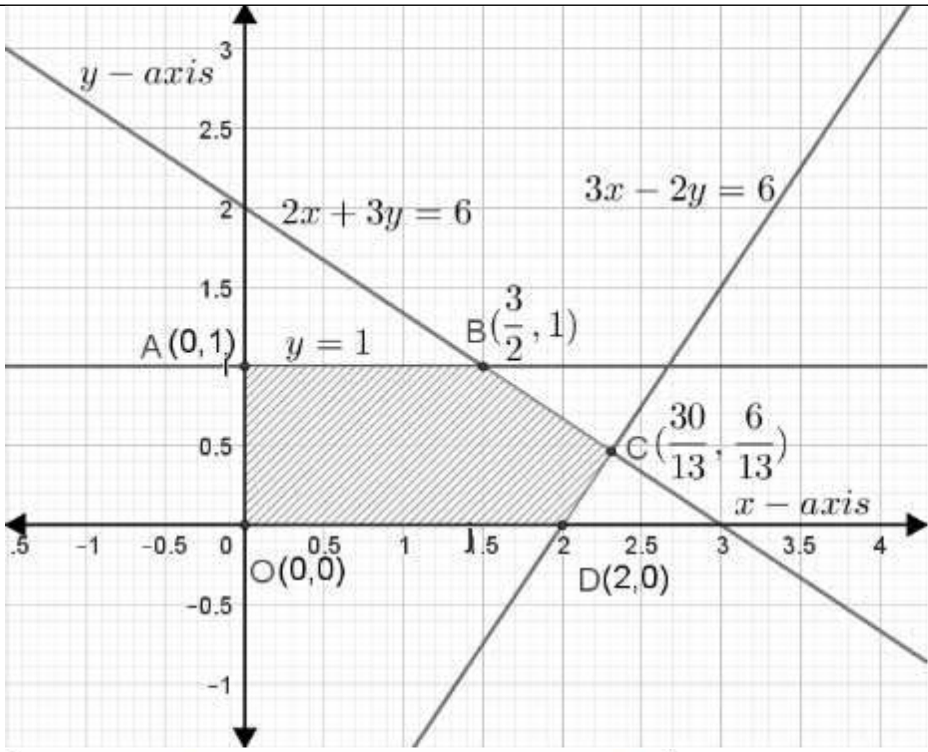


Corner Point	Value of $Z = 20x + 30y$
$A(14,66)$	2260
$B(14,24)$	1000
$C(66,14)$	1740
$D(29,14)$	1000

Max(Z) = 2260

5	

Corner Point	Value of $Z = 8x + 9y$
$A(0,1)$	9
$B\left(\frac{3}{2}, 1\right)$	21
$C\left(\frac{30}{13}, \frac{6}{13}\right)$	$\frac{294}{13}$
$D(2,0)$	16
$O(0,0)$	0

$$\text{Max}(Z) = \frac{294}{13}$$

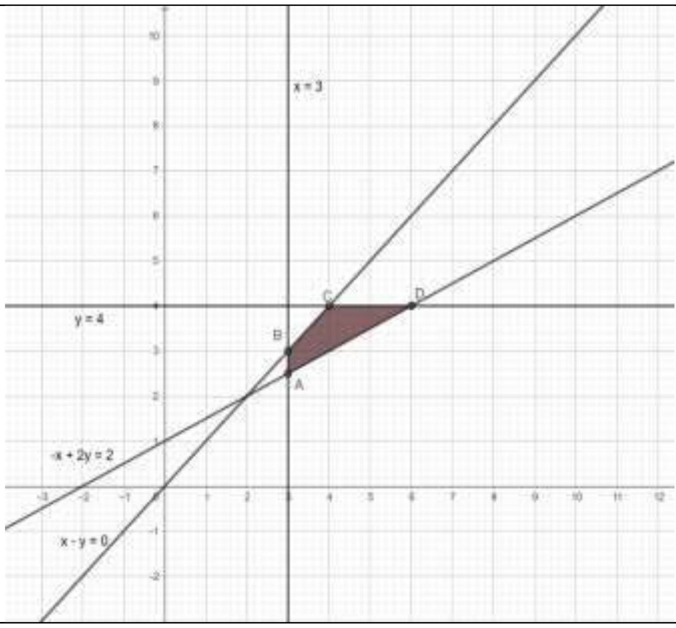
6 (C) The objective function maximizes the combined profit earned from selling X and Y

7 (D) Assertion (A) is false and Reason (R) is true.

8

Corner point	Value of Z = x – 5y
A (3, 2.5)	-9.5
B (3, 3)	-12
C (4, 4)	-16
D (6, 4)	-14

The minimum value of Z is -16, which is attained at x = 4, y = 4.



9

The equation of the line AB is

$y - 30 = \frac{25-30}{15-0}(x - 0)$

$\Rightarrow x + 3y = 90$

The equation of the line BC is

$y - 10 = \frac{10-25}{30-15}(x - 30)$

$\Rightarrow x + y = 40$

The equation of the line CD is

$y - 10 = \frac{70-10}{0-30}(x - 30)$

$\Rightarrow 2x + y = 70$

Hence, the constraints are

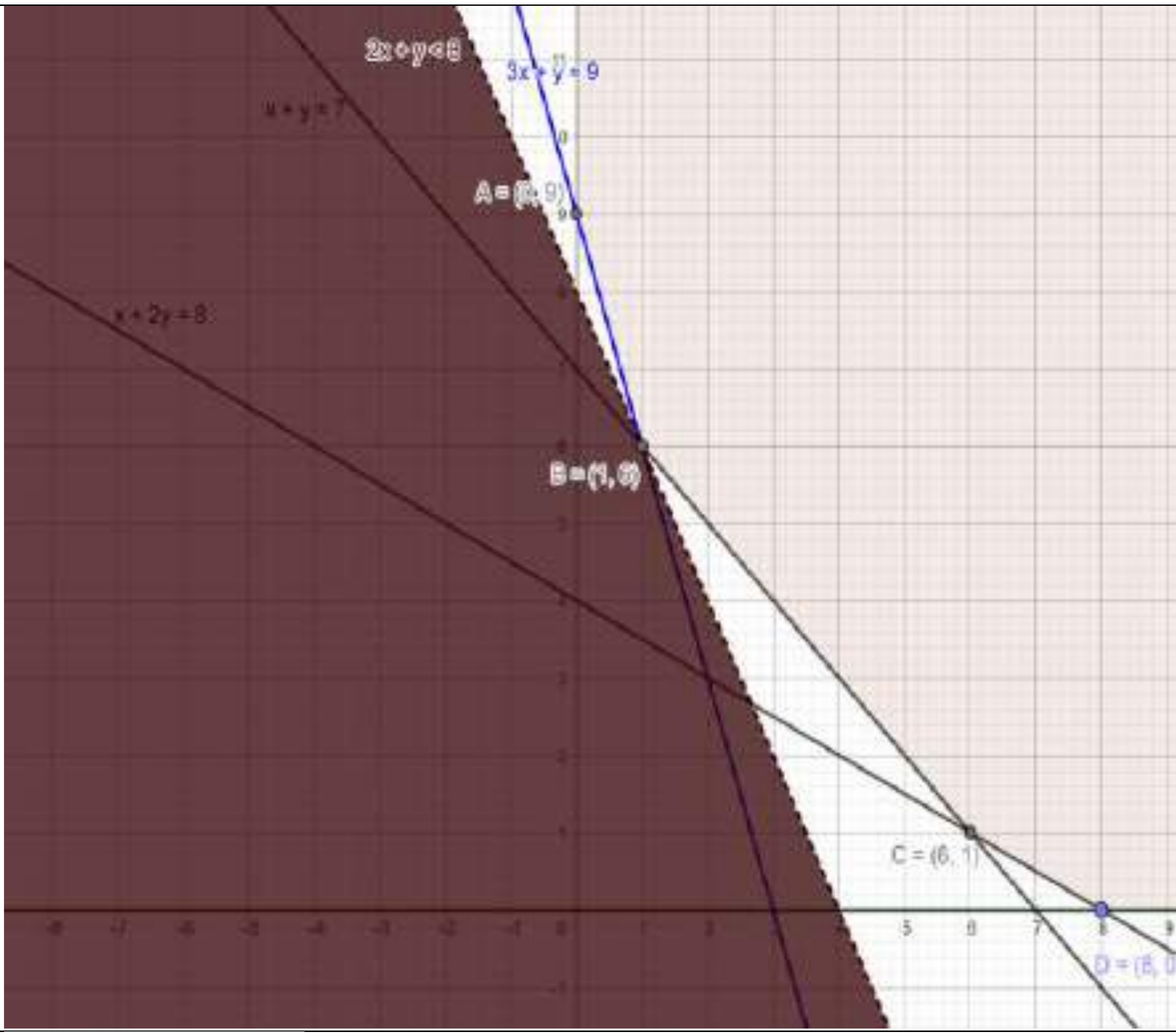
$x + 3y \leq 90, x + y \leq 40, 2x + y \leq 70$

$x \geq 0, y \geq 0$

10

Corner point	Value of Z = 2x + y
A(0, 9)	9
B(1, 6)	8
C(6, 1)	13
D(8, 0)	16

In the half-plane $2x + y < 8$, there is no point in common with the feasible region. Hence, the minimum value of Z is 8, which is attained at $x = 1$, $y = 6$.



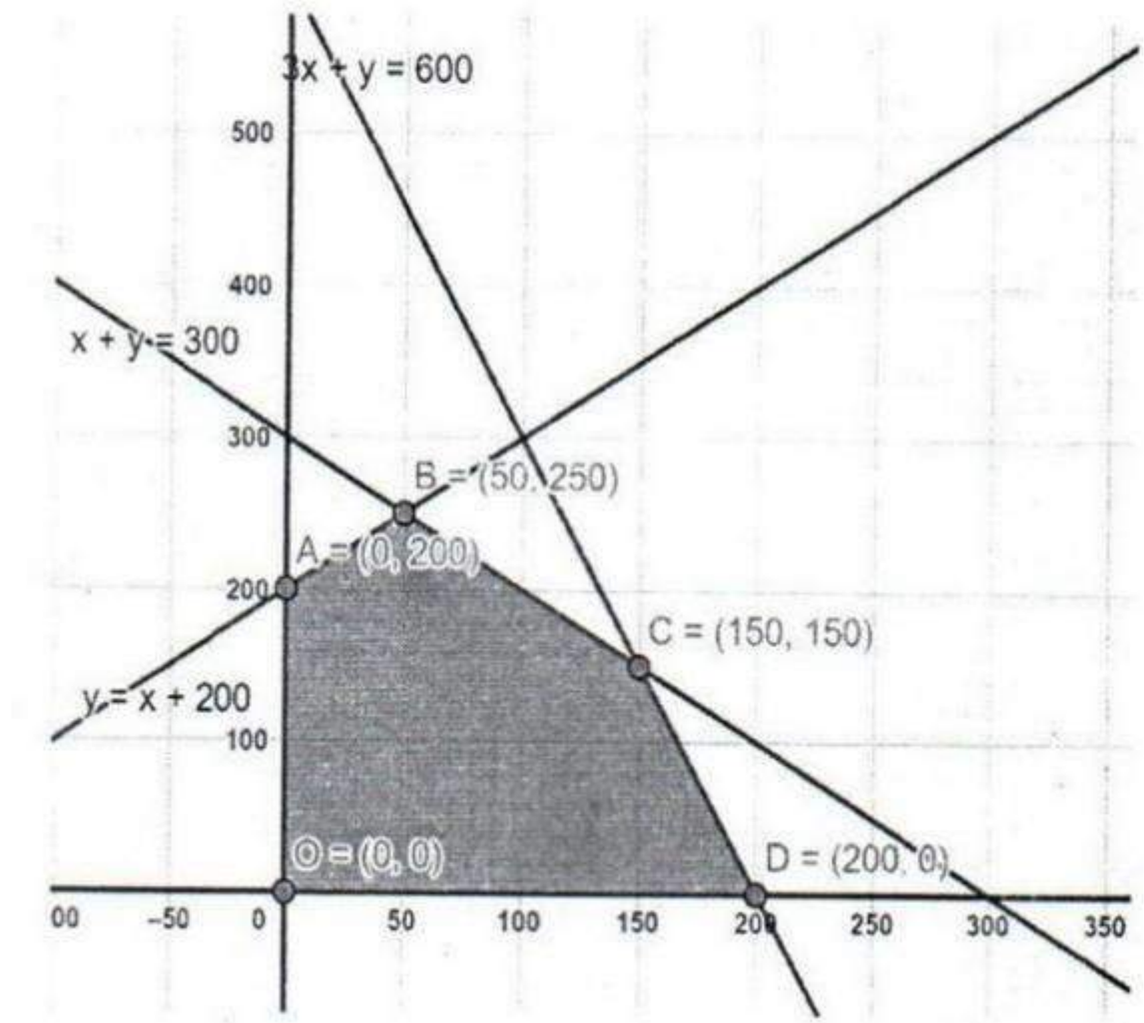
11 **(D)** $3a = 2b$

12 (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

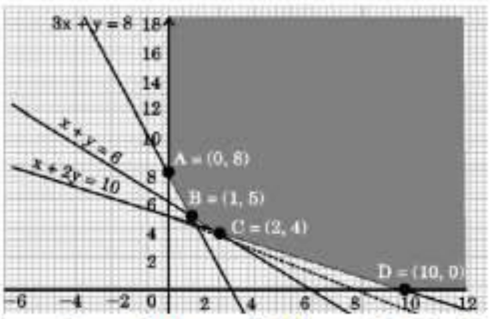
13

Corner Point	Value of $Z = 100x + 50y$
$O(0,0)$	0
$A(0,200)$	10000
$B(50,250)$	17500
$C(150,150)$	22500
$D(200,0)$	20000

$Z_{\max} = 22500$ when $x = 150$, $y = 150$



14



Corner Point	Value of $Z = 3x + 5y$
$A(0,8)$	40
$B(1,5)$	28
$C(2,4)$	26
$D(10,0)$	30

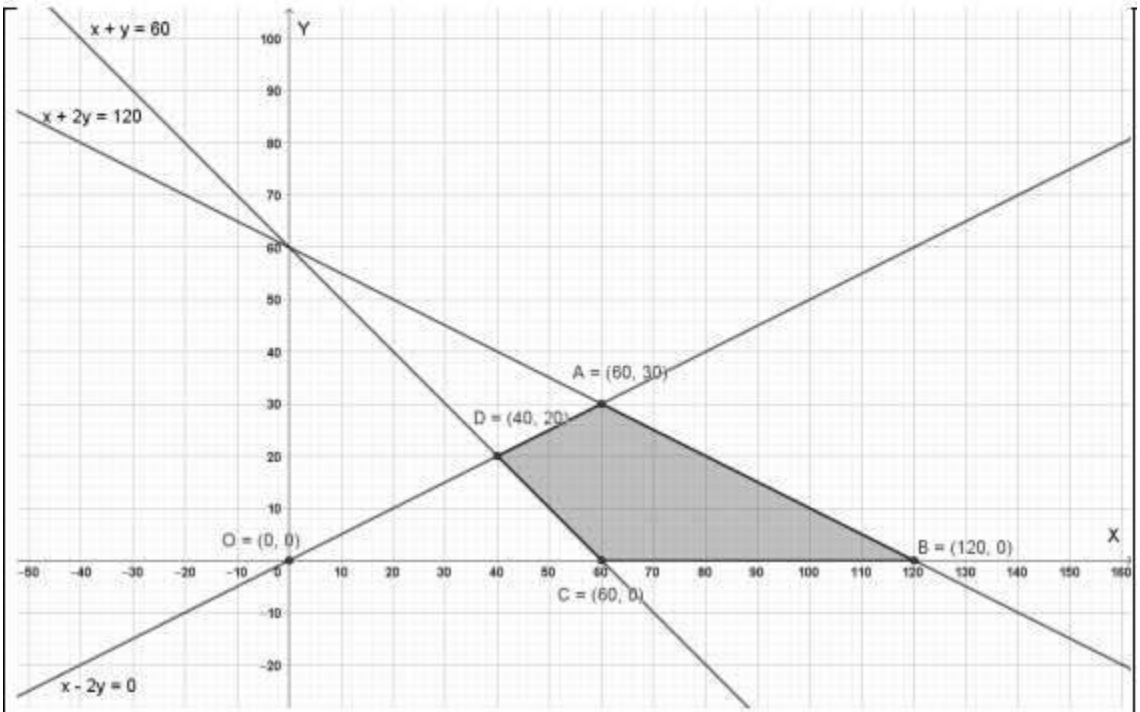
$3x + 5y < 26$ has no common region with the feasible region.

$\therefore Z_{\min} = 26$

15

Equation of AB:

$$y - 200 = \frac{250 - 200}{50 - 0}(x - 0) \text{ i.e. } -x + y = 200$$

	Equation of BC: $y-250=\frac{150-250}{150-50}(x-50)$ i.e. $x+y=300$ Equation of CD: $y-0=\frac{0-150}{200-150}(x-200)$ i.e. $3x+y=600$ Required constraints are: $-x+y\le 200$ $x+y\le 300$ $3x+y\le 600$ $x\ge 0, y\ge 0$
16	(B) AOEC
17	(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A)
18	

Corner Points	Value of Z
A (60, 30)	600
B (120, 0)	600
C (60, 0)	300
D (40, 20)	400

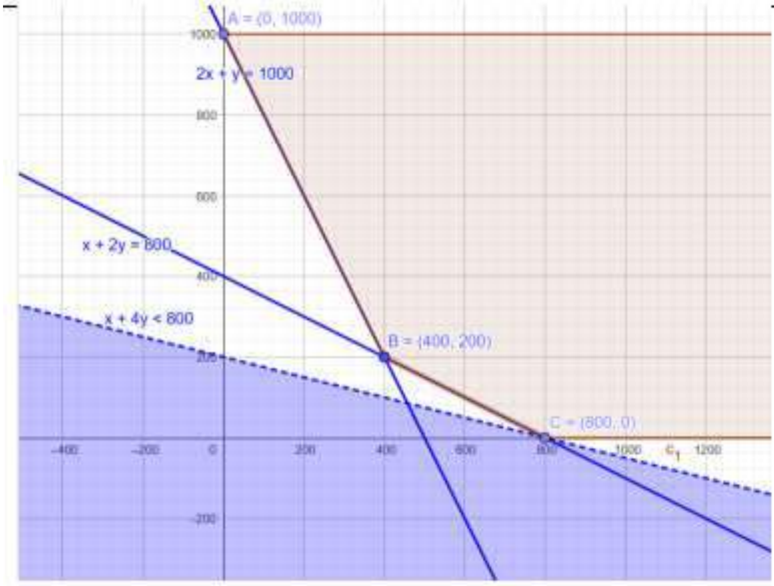
Since Z is maximum on points A and B

Hence all points lying on segment AB give maximum Z.

19	(A) Z is minimum at S $(\frac{18}{7}, \frac{2}{7})$

20 (C) does not exist

21

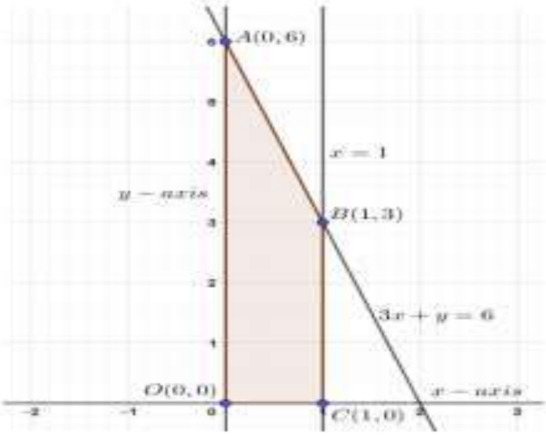


Correct Graph and shading:

Corner points	Value of Z
(800, 0)	800
(400, 200)	1200
(0, 1000)	4000

$x + 4y < 800$ has no region common with feasible region, hence 800 is minimum

22



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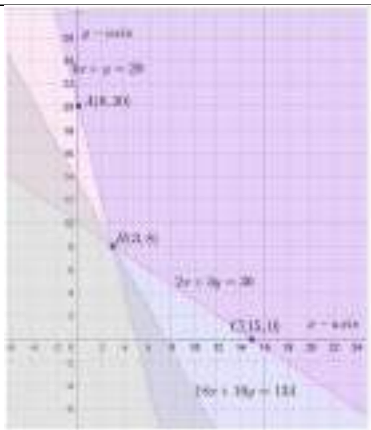
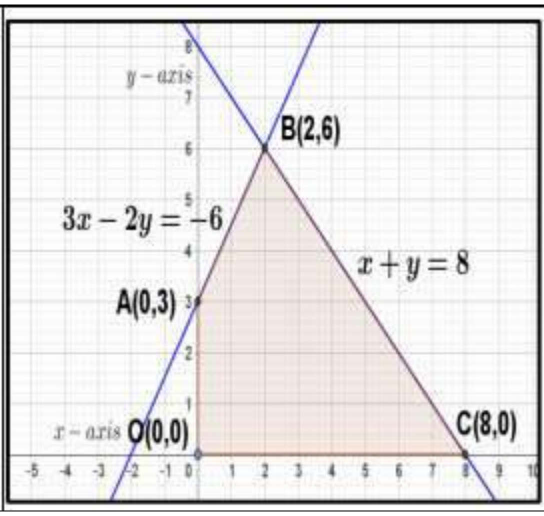


Correct Fig.

Corner points	Value of Z = 18x + 10y
A (0, 20)	200
B (3, 8)	134
C (15, 0)	270

Also, $Z < 134$, does not have any common point with the feasible region,
$\therefore \text{Min}(Z) = 134$ at B (3, 8)

24	
25	Corner Points / Value of Z = 5x + 3y: A (3,2) → 21; B (0,5) → 15; C (0,3) → 9 Min (Z) = 9
26	(C) at the corner points of the feasible region
27	(D) I Quadrant
28	**Z= ax + by** **At (3 ,4), Z=3a+4b** **At (0 ,5), Z=5b** **3a+4b=0+5b** **3a=b**

Corner Points	Value of Z = 5x + 3y
A (3,2)	21
B (0,5)	15
C (0,3)	9

CHAPTER 13 PROBABILITY

Q.		Code and Marks
1	If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E/\overline{F})$ is equal to : (A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{7}{9}$	65/1/1 65/1/2 1M
2	(a) The probability distribution for the number of students being absent in a class on a Saturday is as follows : **X**: 0, 2, 4, 5 **P(X)**: p, 2p, 3p, p Where X is the number of students absent. (i) Calculate p. **1** (ii) Calculate the mean of the number of absent students on Saturday. **2** **OR** (b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.	65/1/1 65/1/2 65/1/3 3M
3	 A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively. Based on the above information, answer the following :	65/1/1 65/1/2 65/1/3 4M

(i) What is the probability that a customer after availing the loan will default on the loan repayment ?

(ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest ?

---

4 Let M and N be two events such that $P(M) = 0.6$, $P(N) = 0.2$ and $P(M \cap N) = 0.5$, then $P(M'/N')$ is

(A) $\frac{7}{8}$ (B) $\frac{2}{5}$

(C) $\frac{1}{2}$ (D) $\frac{2}{3}$

65/1/3
1M

---

5 If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\overline{E}/\overline{F})$ is

(A) $\frac{P(\overline{E})}{P(\overline{F})}$ (B) $1 - P(\overline{E}/F)$

(C) $1 - P(E/F)$ (D) $\frac{1 - P(E \cup F)}{P(\overline{F})}$

65/2/1
65/2/2
65/2/3
1M

---

6 (a) A die with number 1 to 6 is biased such that $P(2) = \frac{3}{10}$ and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice.

OR

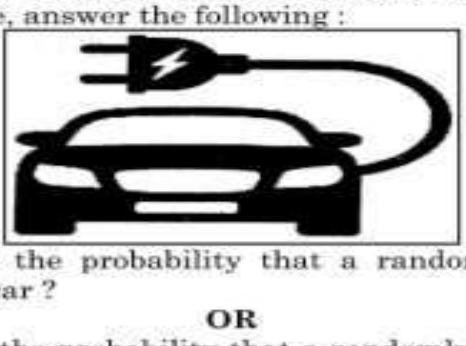
(b) Two dice are thrown. Defined are the following two events A and B : $A = \{(x, y) : x + y = 9\}$, $B = \{(x, y) : x \neq 3\}$, where (x, y) denote a point in the sample space.

Check if events A and B are independent or mutually exclusive.

65/2/1
65/2/2
65/2/3
3M

---

7 Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated).
Based on the above, answer the following :



(i) (a) What is the probability that a randomly selected car is an electric car ?

OR

(i) (b) What is the probability that a randomly selected car is a petrol car ?

(ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet ?

(iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi ?

65/2/1
65/2/2
65/2/3
4M

8 In the following probability distribution, the value of p is :

X	0	1	2	3
P(X)	p	p	0.3	2p

(A) $\frac{7}{40}$ (B) $\frac{1}{10}$

(C) $\frac{9}{35}$ (D) $\frac{1}{4}$

65/2/2
65/2/3
1M

9 *Assertion (A) :* If A and B are two events such that $P(A \cap B) = 0$, then A and B are independent events.

Reason (R) : Two events are independent if the occurrence of one does not effect the occurrence of the other.

65/4/1
65/4/2
65/4/2
1M

10 (a) Find the probability distribution of the number of boys in families having three children, assuming equal probability for a boy and a girl.

OR

(b) A coin is tossed twice. Let X be a random variable defined as number of heads minus number of tails. Obtain the probability distribution of X and also find its mean.

65/4/1
65/4/2

3M

11 Some students are having a misconception while comparing decimals. For example, a student may mention that 78·56 > 78·9 as 7856 > 789. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question : In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table :

Name of student	Distance of javelin (in meters)
Ajay	47·7
Bijoy	47·07
Kartik	43·09
Dinesh	43·9
Devesh	45·2

The students were asked to identify who has thrown the javelin the farthest.
Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions :

(i) What is the probability of a student not having misconception but still answers Bijoy in the test ? *1*

65/4/1
65/4/2
65/4/3

4M

	(ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test ? *1* (iii) (a) What is the probability that a student who answered as Bijoy is having misconception ? *2* **OR** (iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception ? *2*	
12	(a) Consider the experiment of tossing a coin. If the coin shows head, toss it again; but if it shows a tail, then throw a die. Find the conditional probability of the event A : 'the die shows a number greater than 3' given that B : 'there is at least one tail'. **OR** (b) The probability distribution of a random variable X is given as : X: 1, 2, 3, 2λ, 3λ, 4λ P(X): $\frac{11}{30}$, $\frac{1}{15}$, $\frac{1}{10}$, $\frac{3}{10}$, $\frac{1}{15}$, $\frac{1}{10}$ (i) Calculate λ, if $E(X) = 3{\cdot}2$. (ii) Find $P(X > 1)$.	65/4/2 3M
13	(a) The probability distribution of a random variable X is given by : X: 0, 1, 2, 3 P(X): p, $\frac{p}{3}$, $\frac{p}{6}$, $\frac{p}{12}$ (i) Determine the value of p. (ii) Calculate $P(X \geq 1)$. (iii) Calculate expectation of X, i.e. E(X) **OR** (b) In a city, a survey was conducted among residents about their preferred mode of commuting. It was found that 50% people preferred using public transport, 35% preferred using a bicycle and 20% use both public transport and a bicycle. If a person is selected at random, find the probability that : (i) The person uses only public transport. (ii) The person uses a bicycle, given that they also use the public transport. (iii) The person uses neither public transport nor a bicycle.	65/4/3 3M
14	If $P(A \cup B) = 0{\cdot}9$ and $P(A \cap B) = 0{\cdot}4$, then $P(\overline{A}) + P(\overline{B})$ is : (A) 0·3 (B) 1 (C) 1·3 (D) 0·7	65/5/1 65/5/2 65/5/3 1M
15	(a) The probability that a student buys a colouring book is 0·7 and that she buys a box of colours is 0·2. The probability that she buys a colouring book, given that she buys a box of colours, is 0·3. Find the probability that the student : (i) Buys both the colouring book and the box of colours.	65/5/1 65/5/2 65/5/3 3M

(ii) Buys a box of colours given that she buys the colouring book.

OR

(b) A person has a fruit box that contains 6 apples and 4 oranges. He picks out a fruit three times, one after the other, after replacing the previous one in the box. Find :

(i) The probability distribution of the number of oranges he draws.

(ii) The expectation of the random variable (number of oranges).

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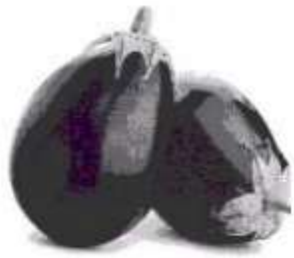
16 (65/5/1, 65/5/2, 65/5/3 — 4M)

A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.

Radish

Cabbage



Brinjal

Based upon the above information, answer the following questions :

(i) Calculate the probability of a randomly chosen seed to germinate.

(ii) What is the probability that it is a cabbage seed, given that the chosen seed germinates ?

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17 (65/6/1 — 1M)

A box has 4 green, 8 blue and 3 red pens. A student picks up a pen at random, checks its colour and replaces it in the box. He repeats this process 3 times. The probability that at least one pen picked was red is :

(A) $\frac{124}{125}$ (B) $\frac{1}{125}$

(C) $\frac{61}{125}$ (D) $\frac{64}{125}$

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18 (65/6/1 — 3M)

A person is Head of two independent selection committees I and II. If the probability of making a wrong selection in committee I is 0·03 and that in committee II is 0·01, then find the probability that the person makes the correct decision of selection :

(i) in both committees

(ii) in only one committee

19	A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C. A person buys a smartphone from this shop. (i) Find the probability that it was defective. (ii) What is the probability that this defective smartphone was manufactured by company B ?	65/6/1 65/6/2 65/6/3 4M
20	Chances that three persons A, B, and C go to the market are 30%, 60% and 50% respectively. The probability that at least one will go to the market is : (A) $\frac{14}{10}$ (B) $\frac{43}{50}$ (C) $\frac{9}{100}$ (D) $\frac{7}{50}$	65/6/2 1M
21	A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed three times, find the probability distribution of number of tails. Hence, find the mean of the distribution.	65/6/2 3M
22	A meeting will be held only if all three members A, B and C are present. The probability that member A does not turn up is 0·10, member B does not turn up is 0·20 and member C does not turn up is 0·05. The probability of the meeting being cancelled is : (A) 0·35 (B) 0·316 (C) 0·001 (D) 0·65	65/6/3 1M
23	Bag I contains 4 white and 5 black balls. Bag II contains 6 white and 7 black balls. A ball drawn randomly by from bag I is transferred to bag II and then a ball is drawn randomly from bag II. Find the probability that the ball drawn is white.	65/6/3 3M
24	If $P(A) = \frac{1}{7}$, $P(B) = \frac{5}{7}$ and $P(A \cap B) = \frac{4}{7}$, then $P(\bar{A} \mid B)$ is : (A) $\frac{6}{7}$ (B) $\frac{3}{4}$ (C) $\frac{4}{5}$ (D) $\frac{1}{5}$	65/7/1 1M
25	A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is : (A) $\frac{2}{13}$ (B) $\frac{3}{26}$ (C) $\frac{19}{26}$ (D) $\frac{3}{13}$	65/7/1 65/7/2 65/7/3 1M

26	(a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, '2' on four of them and '3' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean. **OR** (b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village ?	65/7/1 65/7/2 65/7/3 2M
27	Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record. Let A_1 : People with good health, A_2 : People with average health, and A_3 : People with poor health. During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50%, respectively. Based upon the above information, answer the following questions : (i) A person was tested randomly. What is the probability that he/she has contracted the disease ? (ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2 ?	65/7/1 65/7/2 65/7/3 4M
28	If $P(A) = \frac{1}{5}$, $P(B) = \frac{3}{5}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$, then $P(A' \cap B')$ is : (A) $\frac{11}{25}$ (B) $\frac{19}{25}$ (C) $\frac{8}{25}$ (D) $\frac{6}{25}$	65/7/2 1M
29	If A and B are two events such that $P(B) = \frac{1}{5}$, $P(A \mid B) = \frac{2}{3}$ and $P(A \cup B) = \frac{3}{5}$, then P(A) is : (A) $\frac{10}{15}$ (B) $\frac{2}{15}$ (C) $\frac{1}{5}$ (D) $\frac{8}{15}$	65/7/3 1M

30	A and B appeared for an interview for two vacancies. The probability of A's selection is $\frac{1}{5}$ and that of B's selection is $\frac{1}{3}$. The probability that none of them is selected is : (A) $\frac{11}{15}$ (B) $\frac{7}{15}$ (C) $\frac{8}{15}$ (D) $\frac{1}{5}$	65(B) 1M
31	(a) Find the probability distribution of the number of doublets in three throws of a pair of dice. **OR** (b) If E and F are two independent events with P(E) = p, P(F) = 2p and P(exactly one of E, F) = $\frac{5}{9}$, then find the value of p.	65(B) 3M
32	There are three categories of students in a class of 60 students : A : Very hardworking students, B : Regular but not so hard working, C : Careless and irregular students. It is known that 6 students in category A, 26 in category B and the rest in category C. It is also found that probability of students of category A, unable to get good marks in the final year examination, is 0.002, of category B it is 0.02 and of category C, this probability is 0.20. Based on the above information, answer the following : (i) Find the probability that a student selected at random is unable to get good marks in the final examination. (ii) A student selected at random was found to be one who could not get good marks in the final examination. Find the probability, that this student is NOT of category A.	65(B) 4M

	ANSWERS
1	(C) $\frac{2}{3}$
2 (a)	(i) Since $\sum P(X)=1 \Rightarrow p+2p+3p+p=1$ $\Rightarrow p=\frac{1}{7}$ (ii) Mean $=\sum X.P(X)=0(p)+2(2p)+4(3p)+5(p)$ $=21p=21\left(\frac{1}{7}\right)=3$ OR
(b)	Let E_1 : The applicant is a male E_2 : The applicant is a female A : The candidate chosen will have distinction in the written test. $P(E_1)=\frac{1}{3}, P(E_2)=\frac{2}{3}, P(A\mid E_1)=0.4, P(A\mid E_2)=0.35$ $\therefore P(A)=P(E_1)P(A\mid E_1)+P(E_2)P(A\mid E_2)$ $=\frac{1}{3}\times 0.4+\frac{2}{3}\times 0.35$ $=\frac{11}{30}$

3	E_1 :customer avails loan on fixed rate E_2 :customer avails loan on floating rate E_3 :customer avails loan on variable rate A:the person defaults on the loan $P(E_1)=\frac{1}{10}, P(E_2)=\frac{2}{10}, P(E_3)=\frac{7}{10}$ $P(A\mid E_1)=\frac{5}{100}, P(A\mid E_2)=\frac{3}{100}, P(A\mid E_3)=\frac{1}{100}$ $(i) P(A)=P(E_1).P(A\mid E_1)+P(E_2).P(A\mid E_2)+P(E_3).P(A\mid E_3)$ $=\frac{1}{10}\times\frac{5}{100}+\frac{2}{10}\times\frac{3}{100}+\frac{7}{10}\times\frac{1}{100}$ $=\frac{18}{1000}$ or $\frac{9}{500}$ $(ii) P(E_3\mid A)=\frac{P(E_3).P(A\mid E_3)}{P(E_1).P(A\mid E_1)+P(E_2).P(A\mid E_2)+P(E_3).P(A\mid E_3)}$ $=\frac{\frac{7}{10}\times\frac{1}{100}}{\frac{18}{1000}}$ $=\frac{7}{18}$
4	(A) $\frac{7}{8}$
5	(D) $\frac{1-P(E\cup F)}{P(\bar{F})}$
6 (a)	$P(2)=\frac{3}{10}$, P(any other number) $=1-\frac{3}{10}=\frac{7}{10}$ Let X represent the Random Variable "the number of 2's". Then X = 0, 1, 2 The probability distribution is X: 0, 1, 2; P(X): $\frac{7}{10}\times\frac{7}{10}=\frac{49}{100}$, $\frac{3}{10}\times\frac{7}{10}\times 2=\frac{42}{100}$, $\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$; XP(X): 0, $\frac{42}{100}$, $\frac{18}{100}$ Mean = $\sum XP(X)=\frac{60}{100}=0.6$ OR
(b)	$A=\{(3,6),(4,5),(5,4),(6,3)$ $P(A)=\frac{4}{36}=\frac{1}{9}, P(B)=\frac{30}{36}=\frac{5}{6}$ $P(A\cap B)=\frac{3}{36}=\frac{1}{12}$ $P(A)\times P(B)=\frac{5}{54}\neq P(A\cap B)$ Therefore, A and B are not independent.

Distribution table for 6 (a):

X	P(X)	XP(X)
0	$\frac{7}{10}\times\frac{7}{10}=\frac{49}{100}$	0
1	$\frac{3}{10}\times\frac{7}{10}\times 2=\frac{42}{100}$	$\frac{42}{100}$
2	$\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$	$\frac{18}{100}$

	A and B are not mutually exclusive as $A \cap B \neq \emptyset$
7. 1(A)(I) 1(II) 1(B) 1(C)	Let A = Amber manufactures the car B = Bonzi manufactures the car C = Comet manufactures the car E = The selected car is electric $P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$ $P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P(\frac{E}{C})$ $= \frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}$ $= \frac{155}{1000}$ *or* $\frac{31}{200}$ OR Let A = Amber manufactures the car B = Bonzi manufactures the car C = Comet manufactures the car E = The selected car is a petrol car $P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$ $P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P(\frac{E}{C})$ $= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{90}{100} + \frac{10}{100} \times \frac{95}{100}$ $= \frac{845}{1000}$ *or* $\frac{169}{200}$ $P\left(\frac{C}{E}\right) = \frac{P(C) \times P(\frac{E}{C})}{P(E)}$ $= \frac{\frac{10}{100} \times \frac{5}{100}}{\frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}}$ $= \frac{\frac{50}{10000}}{\frac{1550}{10000}} = \frac{1}{31}$ $\quad$ $P\left(\frac{A \text{ or } B}{E}\right) = 1 - P\left(\frac{C}{E}\right) = 1 - \frac{1}{31} = \frac{30}{31}$
8	A) $\frac{7}{40}$
9	(D) Assertion (A) is false, but Reason (R) is true.
10 (a)	Let X denote the random variable which counts the number of boys. $X = 0, 1, 2, 3$ $P(\text{Boy}) = P(\text{Girl}) = \frac{1}{2}$ Required Probability Distribution

X	0	1	2	3
$P(X)$	$\left(\frac{1}{2}\right)^3=\frac{1}{8}$	$3\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2=\frac{3}{8}$	$3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)=\frac{3}{8}$	$\left(\frac{1}{2}\right)^3=\frac{1}{8}$

OR

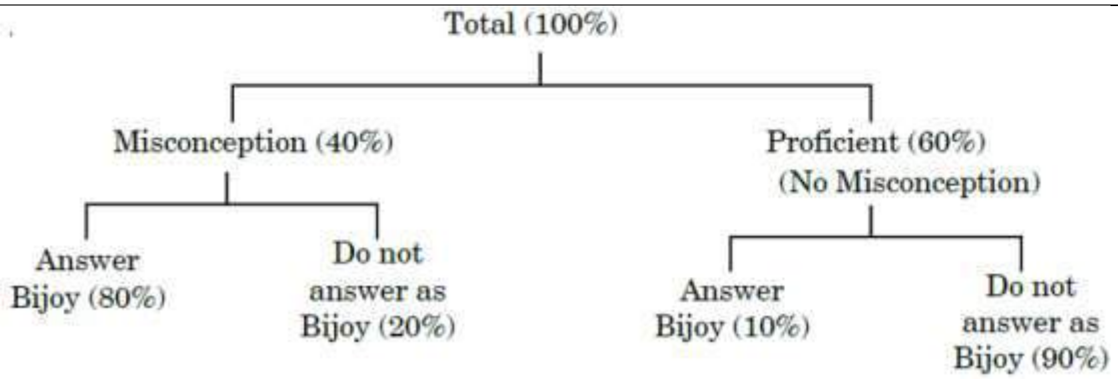
(b)

Possible values of X are $-2,0,2$

X	-2	0	2
$P(X)$	$\frac{1}{4}$	$\frac{2}{4}=\frac{1}{2}$	$\frac{1}{4}$

$$\text{Mean}=\sum XP(X)=-2\left(\frac{1}{4}\right)+0\left(\frac{1}{2}\right)+2\left(\frac{1}{4}\right)=0$$

11



Let E_1 : Student has a misconception

E_2 :Student does not have misconception

A: Student answered Bijoy as correct

$\therefore P(E_1)=\frac{40}{100}, P(E_2)=\frac{60}{100}$

$P(A|E_1)=\frac{80}{100}, P(A|E_2)=\frac{10}{100}$

$P(\bar{A}|E_1)=\frac{20}{100}, P(\bar{A}|E_2)=\frac{90}{100}$

$(i)\ P(A|E_2)=\frac{10}{100}$ or $\frac{1}{10}$

$$(ii)\ P(A)=P(E_1)P(A|E_1)+P(E_2)P(A|E_2)$$

$$=\frac{40}{100}\times\frac{80}{100}+\frac{60}{100}\times\frac{10}{100}$$

$$=\frac{38}{100} \text{ or } \frac{19}{50}$$

	$(iii)(a)P(E_1 \mid A)=\dfrac{P(E_1)P(A\mid E_1)}{P(A)}$ $=\dfrac{\frac{40}{100}\times\frac{80}{100}}{\frac{38}{100}}=\dfrac{16}{19}$ **OR** $(iii)(b)P(E_2 \mid A)=\dfrac{P(E_2)P(A\mid E_2)}{P(A)}$ $=\dfrac{\frac{60}{100}\times\frac{10}{100}}{\frac{38}{100}}=\dfrac{3}{19}$
12 (a)	Let A: The die shows a number greater than 3 and B: There is at least one tail $P(B\cap A)=P(T4,T5,T6)=\frac{3}{12}=\frac{1}{4}$ $P(B)=P(HT,T1,T2,T3,T4,T5,T6)=\frac{1}{4}+\frac{6}{12}=\frac{3}{4}$ $P(A\mid B)=\dfrac{P(B\cap A)}{P(B)}=\dfrac{1/4}{3/4}=\dfrac{1}{3}$
(b)	**OR** $(i)E(X)=\sum X.P(X)$ $=1\left(\frac{11}{30}\right)+2\left(\frac{1}{15}\right)+3\left(\frac{1}{10}\right)+2\lambda\left(\frac{3}{10}\right)+3\lambda\left(\frac{1}{15}\right)+4\lambda\left(\frac{1}{10}\right)$ Given $\sum X.P(X)=3.2$ $\therefore 24+36\lambda=96$ $\Rightarrow \lambda=2$ $(ii)P(X>1)=1-P(X=1)$ $=1-\frac{11}{30}=\frac{19}{30}$
13 (a)	$(i)\ p+\frac{p}{3}+\frac{p}{6}+\frac{p}{12}=1$ $\Rightarrow p=\frac{12}{19}$ $(ii)P(X\geq 1)=1-P(X=1)$ $=1-\frac{p}{3}=1-\frac{12}{19}=\frac{7}{19}$

	$(iii)E(X)=\sum X.P(X)$ $=0(p)+1\left(\frac{p}{3}\right)+2\left(\frac{p}{6}\right)+3\left(\frac{p}{12}\right)$ $=\frac{11}{12}p=\frac{11}{12}\times\frac{12}{19}=\frac{11}{19}$
(b)	Let T :Person uses public transport B :Person uses a bicycle Given $P(T)=\frac{50}{100}, P(B)=\frac{35}{100}, P(T\cap B)=\frac{20}{100}$ $(i)P(\text{only}\,T)=P(T)-P(T\cap B)$ $=\frac{50}{100}-\frac{20}{100}=\frac{30}{100}$ $(ii)P(B\mid T)=\frac{P(T\cap B)}{P(T)}$ $=\frac{\frac{20}{100}}{\frac{50}{100}}=\frac{2}{5}$ $(iii)P(T'\cap B')=1-P(T\cup B)$ $=1-[P(T)+P(B)-P(T\cap B)]$ $=1-\left[\frac{50}{100}+\frac{35}{100}-\frac{20}{100}\right]$ $=1-\frac{65}{100}=\frac{35}{100}$
14	**(D) 0.7**
15	**(a) Let A be the event of buying colouring book and** **B be the event of buying coloured box.** $P(A)=0.7,\quad P(B)=0.2,\quad P(A/B)=0.3$ **(i)** $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)} \Rightarrow 0.3=\frac{P(A\cap B)}{0.2}$ $\Rightarrow P(A\cap B)=0.06$ **or** $\frac{3}{50}$ **(ii)** $P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$ $=\frac{0.06}{0.7}=\frac{3}{35}$ **or** 0.086 **OR** **(b) Let X be random variable for number of oranges.** **X = 0, 1, 2, 3**

	Let A be the event that orange is drawn. $P(A) = \frac{4}{10} = \frac{2}{5}, \quad P(\bar{A}) = 1 - \frac{2}{5} = \frac{3}{5}$ **(i)** <table><tr><th>X</th><th>0</th><th>1</th><th>2</th><th>3</th></tr><tr><td>P(X)</td><td>$\frac{27}{125}$</td><td>$\frac{54}{125}$</td><td>$\frac{36}{125}$</td><td>$\frac{8}{125}$</td></tr></table> **(ii)** $E(X) = \sum p_i x_i = 0 \times \frac{27}{125} + 1 \times \frac{54}{125} + 2 \times \frac{36}{125} + 3 \times \frac{8}{125}$ $= \frac{150}{125}$ or $\frac{6}{5}$
16	**Let A: Event that chosen seed germinates.** **B: Event that Brinjal seed is chosen.** **C: Event that Cabbage seed is chosen.** **R: Event that Radish seed is chosen.** $P(B) = \frac{10}{30};\ P(C) = \frac{12}{30};\ P(R) = \frac{8}{30};$ $P\left(\frac{A}{B}\right) = \frac{25}{100};\ P\left(\frac{A}{C}\right) = \frac{35}{100};\ P\left(\frac{A}{R}\right) = \frac{40}{100}$ **(i)** $P(A) = P(B).P\left(\frac{A}{B}\right) + P(C).P\left(\frac{A}{C}\right) + P(R).P\left(\frac{A}{R}\right)$ $= \frac{10}{30} \times \frac{25}{100} + \frac{12}{30} \times \frac{35}{100} + \frac{8}{30} \times \frac{40}{100}$ $= \frac{990}{3000}$ or $\frac{33}{100}$ **(ii)** (a) $P\left(\frac{C}{A}\right) = \frac{P(C).P\left(\frac{A}{C}\right)}{P(B).P\left(\frac{A}{B}\right)+P(C).P\left(\frac{A}{C}\right)+P(R).P\left(\frac{A}{R}\right)}$ $= \frac{\frac{12}{30} \times \frac{35}{100}}{\frac{990}{3000}}$ $= \frac{42}{99}$ or $\frac{14}{33}$
17	(C) $\frac{61}{125}$
18	(i) P(correct decision in both committees) =(1- 0.03) . (1- 0.01) = 0.9603 (ii) P(correct decision in one committee) = 0.03 . (1- 0.01) + (1- 0.03). 0.01 = 0.0394
19	(i) P(defective smartphone) = 0.25×0.05 + 0.35× 0.04 + 0.40×0.02 = 0.0345 (ii) P(B/ Defective) = $\frac{0.35\times 0.04}{0.25\times 0.05 + 0.35\times 0.04 + 0.40\times 0.02}$ $= \frac{140}{345}$ or $\frac{28}{69}$

20	**(B)** $\frac{43}{50}$
21	$P(H)=\frac{3}{4}, P(T)=\frac{1}{4}$ X: 0, 1, 2, 3 $P(X)$: $\frac{27}{64}$, $\frac{27}{64}$, $\frac{9}{64}$, $\frac{1}{64}$ $XP(X)$: 0, $\frac{27}{64}$, $\frac{18}{64}$, $\frac{3}{64}$ **Mean** $=\frac{3}{4}$
22	(B) 0.316
23	P(white ball transferred) $=\frac{4}{9}$, Probability(black ball transferred) $=\frac{5}{9}$ P(white ball drawn from bag II) $=\frac{4}{9}\cdot\frac{7}{14}+\frac{5}{9}\cdot\frac{6}{14}$ $=\frac{29}{63}$
24	**(D)** $\frac{1}{5}$
25	**(B)** $\frac{3}{26}$
26	**(a) Probability distribution table is:** X: 0, 1, 2, 3 P(X): $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{1}{10}$ **Mean** $=E(X)=\sum p_i x_i = 0\cdot\frac{2}{10}+1\cdot\frac{3}{10}+2\cdot\frac{4}{10}+3\cdot\frac{1}{10}=\frac{14}{10}=\frac{7}{5}$ **(or 1.4)** **OR** **(b) A = A randomly chosen person is a woman** **B = A randomly chosen person works outside village.** $P(A)=\frac{4000}{8000}=\frac{1}{2},\ P(B)=\frac{3000}{8000}=\frac{3}{8},\ P(A\cap B)=\frac{1200}{8000}=\frac{3}{20}$ **Required probability** $= P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{1}{2}+\frac{3}{8}-\frac{3}{20}=\frac{29}{40}$
27	**(i) Let A: Person contracted the disease** $P(A)=P(A_1)\cdot P(A\mid A_1)+P(A_2)\cdot P(A\mid A_2)+P(A_3)\cdot P(A\mid A_3)$ $=\frac{7}{10}\left(\frac{25}{100}\right)+\frac{2}{10}\left(\frac{35}{100}\right)+\frac{1}{10}\left(\frac{50}{100}\right)$ $=\frac{295}{1000}=0.295$ or $\left(\frac{59}{200}\right)$

	(ii) $P\left(A_2 \mid \bar{A}\right)=\dfrac{P\left(A_2\right)\cdot P\left(\bar{A}/A_2\right)}{P\left(A_1\right)\cdot P\left(\bar{A}/A_1\right)+P\left(A_2\right)\cdot P\left(\bar{A}/A_2\right)}$ $=\dfrac{\frac{2}{10}\times\frac{65}{100}}{\frac{7}{10}\times\frac{75}{100}+\frac{2}{10}\times\frac{65}{100}+\frac{1}{10}\times\frac{50}{100}}$ $=\dfrac{2\times13}{7\times15+2\times13+1\times10}=\dfrac{26}{141}$
28	(A) $\dfrac{11}{25}$
29	(D) $\dfrac{8}{15}$
30	(C) $\frac{8}{15}$
31	**(a) Let X denote the number of doublets** **X= 0, 1, 2, 3** **P(doublet) =** $\frac{1}{6}$ **, P(not a doublet) =** $\frac{5}{6}$

X	0	1	2	3
P(X)	$\left(\frac{5}{6}\right)^3 = \frac{125}{216}$	$3\times\frac{1}{6}\times\left(\frac{5}{6}\right)^2 = \frac{75}{216}$	$3\times\left(\frac{1}{6}\right)^2\times\left(\frac{5}{6}\right) = \frac{15}{216}$	$\left(\frac{1}{6}\right)^3 = \frac{1}{216}$

OR

OR

(b) $P(E\cup F)=P(E)+P(F)-P(E\cap F)$
$= p+2p-p\times 2p$

$P(E\cup F)=P(exactly\ one\ of E,F)+P(E\cap F)$
$=\frac{5}{9}+p\times 2p$
$\Rightarrow 3p-2p^2=\frac{5}{9}+2p^2$
$\Rightarrow 36p^2-27p+5=0$
$\Rightarrow (3p-1)(12p-5)=0$
$\Rightarrow p=\frac{1}{3}\ or\ \frac{5}{12}$

32	**Let G be an event that student is not able to get good marks** **We have** $P(A)=\frac{6}{60}$ $\qquad P(G\mid A)=0.002$

$P(B) = \frac{26}{60}$ $\qquad$ $P(G|B) = 0.02$

$P(C) = \frac{28}{60}$ $\qquad$ $P(G|C) = 0.20$

(i) $P(G) = P(A)P(G|A) + P(B)P(G|B) + P(C)P(G|C)$

$$= \frac{6}{60} \times 0.002 + \frac{26}{60} \times 0.02 + \frac{28}{60} \times 0.2$$

$$\frac{511}{5000}$$

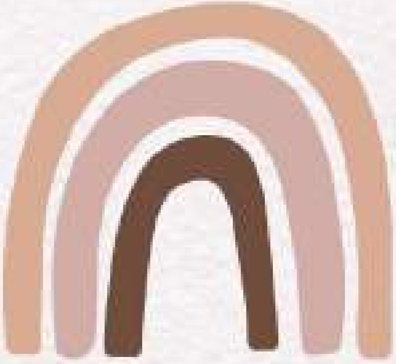
(ii) $1 - P(A|G)$

$$= 1 - \frac{P(A)P(G|A)}{P(A)P(G|A) + P(B)P(G|B) + P(C)P(G|C)}$$

$$= 1 - \frac{\frac{1}{10} \times 0.002}{\frac{511}{5000}}$$

$$= 1 - \frac{1}{511}$$

$$= \frac{510}{511}$$



The only way to learn mathematics is to do mathematics