SREE GOKULAM PUBLIC SCHOOL
CHENGALPET

CLASS XII – MATHEMATICS
SUPPORT MATERIAL

2025 – 2026

INDEX

COURSE STRUCTURE

CLASS XII – MATHEMATICS(041)

ASSESSMENT SCHEME

External Assessment	Written Examination	80 Marks
Internal Assessment	Periodic Tests	10 Marks
	Mathematics Activities	10 Marks
Total		100 Marks

UNIT WEIGHTAGE

UNIT NO.	UNIT NAME	CHAPTERS	WEIGHTAGE
I	Relations and Functions	Chapter 1 – Relations and Functions Chapter 2 – Inverse Trigonometric Functions	08
II	Algebra	Chapter 3 – Matrices Chapter 4 – Determinants	10
III	Calculus	Chapter 5 – Continuity and Differentiability Chapter 6 – Applications of Derivative Chapter 7 – Integrals Chapter 8 – Applications of the Integrals Chapter 9 – Differential Equations	35
IV	Vectors and Three-Dimensional Geometry	Chapter 10 – Vectors Chapter 11 – Three-Dimensional Geometry	14
V	Linear Programming	Chapter 12 – Linear Programming	05
VI	Probability	Chapter 13 – Probability	08
Total			**80**

CHAPTER 1 – RELATIONS AND FUNCTIONS

RELATION

A and B are two non-empty sets, the Cartesian product A X B is the set of all ordered pairs of elements from A to B.

$$A \times B = \{(a, b): a \in A, b \in B\}$$

Let A and B be two sets. Then relation R from set A into set B is a subset of A x B. Thus, R is relation from A to B $\Leftrightarrow R \subseteq A \times B$.

If $(a, b) \in R$ then we write aRb which is read as 'a' is related to 'b' by the relation R, if $(a, b) \notin R$, then 'a' is not related to 'b' by the relation R.

DOMAIN

Let R be a relation from a set A to set B. Then, set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R.

RANGE

Let R be a relation from a set A to set B. Then, set of all second components or coordinates of the ordered pairs belonging to R is called the Range of R.

Thus, domain of R = $\{a : (a, b) \in R\}$ and range of R = $\{b : (a, b) \in R\}$

Note: If n(A) = p, n(B) = q from set A to set B, then n(A x B) = p.q and number of relations = 2^{pq}.

TYPES OF RELATION

Empty or Void Relation

A relation R from A to B is called an empty relation or a void relation if R = Φ.

Universal Relation

A relation R from A to B is said to be the universal if R = A x B

Note: Both the empty relation and the universal relation are sometimes called trivial relations.

Reflexive Relation

A relation R defined on a set A is said to be reflexive if aRa, $\forall$ a $\in$ A i.e (a, a) $\in$ R, $\forall$ a $\in$ A.

Example: A = {1, 2} then, R = {(1, 1), (1, 2), (2, 1), (2, 2)} is Reflexive relation.

Symmetric Relation

A relation R defined on a set A is symmetric if (a, b) $\in$ R $\Rightarrow$ (b, a) $\in$ R, $\forall$ a, b $\in$ A. i.e aRb $\Rightarrow$ bRa.

Example: The relation R={(4,5),(5,4),(6,5),(5,6)} on set A={4,5,6} is symmetric.

Transitive Relation

A relation R on a set A is transitive if (a, b) $\in$ R and (b, c) $\in$ R $\Rightarrow$ (a, c) $\in$ R. i.e aRb and bRc $\Rightarrow$ aRc.

Example: Relation R={(1,2),(2,3),(1,3)} on set A={1,2,3} is transitive.

Equivalence Relation

A relation R is said to be an equivalence relation, if it is simultaneously reflexive, symmetric and transitive.

Equivalence Classes

Let R be an equivalence relation in a set A and let a $\in$ A. Then, the set of all those elements of A which are related to 'a' is called equivalence class determined by 'a' and it is denoted by [a]. Thus, $[\mathrm{a}] = \{\mathrm{b} \in \mathrm{A}: (\mathrm{a}, \mathrm{b}) \in \mathrm{A}\}$.

RESULTS

(i) Number of reflexive relations on a set with n elements is $2^{n(n-1)}$.

(ii) Number of symmetric relations on a set with n elements is $2^{\frac{n(n+1)}{2}}$

(iii) Number of Reflexive and Symmetric relations defined on a set of n elements $= 2^{\frac{n(n-1)}{2}}$.

(iv) No. of Transitive relations defined on a set having $0, 1, 2, 3$ and 4 elements are 1, 2, 13, 171 and 3994 respectively.

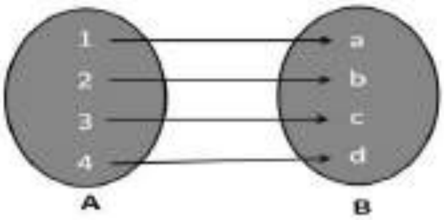
FUNCTION

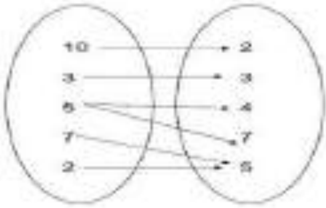
A relation f from a set A to another set B (f: A → B) is said be a function (or mapping) from A to B if with every element (say x) of A, the relation f relates a unique element (say y) of B. This y is called image of x. Also x is called pre-image of y under f.

Set A is Domain and Set B is co-domain.

For any element x ∈ A, function f correlates it to an element in B, which is denoted by f(x) and is called image of x under f. Again y = f(x), then x is called as pre-image of y.

Range = {(f(x): x ∈ A}. Range ⊆ Codomain.

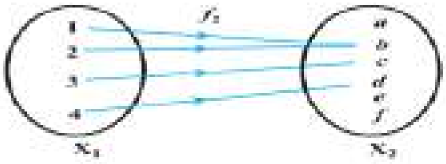




TYPES OF FUNCTION

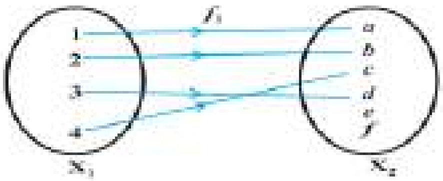
Many to One Function

A function f: A → B which maps two or more elements of A to the same element of B, then f is called many to one function.



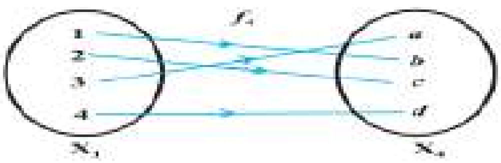
One - One Function or Injective Function

A function f: A → B is one-one function or injective function if distinct elements of A have distinct images in B. Thus, f: A → B is one-one iff f(a) = f(b) ⇒ a = b, ∀a, b ∈ A.



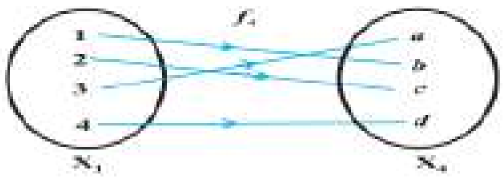
Onto Function or Surjective Function

A function f: A → B is said to be onto or surjective function iff Range = B. i.e ∀ b ∈ B, there exists a ∈ A such that f(a) = b.



Bijective Function

A function which is both Injective and Surjective is called Bijective function.



Working Rule for One-One or Injectivity of a function

(i) Take any two arbitrary elements of x_1, x_2 in domain of f.

(ii) Put $f(x_1) = f(x_2)$.

(iii) Solve $f(x_1) = f(x_2)$, if it gives $x_1 = x_2$, then f is a one-one function.

Working Rule for Onto or Surjectivity of a function

(i) Take an element $y \in B$, where B is the codomain of the function.

(ii) Put $f(x) = y$

(iii) Solve the equation $f(x) = y$ for x and obtain x in terms of y. Let $x = g(y)$.

(iv) If for all values of $y \in B$, the values of x obtained from $x = g(y)$ are in A, then f is onto. If there are some of $y \in B$, for which values of x obtained from $x = g(y)$ is not in A, then f is not onto.

NOTE

If f: A → B is a function such that,

(i) f is one-one $\Rightarrow n(\mathrm{A}) \leq n(\mathrm{B})$.

(ii) f is onto $\Rightarrow n(B) \leq n(\mathrm{A})$.

(iii) f is one-one onto i.e., f is a bijection $\Rightarrow n(\mathrm{A}) = n(\mathrm{B})$.

RESULTS

Let A and B are two nom empty set that $n(A) = p$ and $n(B) = q$. Then

(i) Number of functions from A to B $= q^p$

(ii) Number of one-one functions from A to B $= \begin{cases} {}^{q}P_{p} & , p \leq q \\ 0 & , p > q \end{cases}$

(iii) Number of onto function from A to B $\begin{cases} \sum_{r=1}^{q} (-1)^{q-r} q_{c_r} r^p, & p \geq q \\ 0 & , p < q. \end{cases}$

(iv) Number of bijective functions from A to B $= \begin{cases} p!, p = q \\ 0, p \neq q. \end{cases}$

MULTIPLE CHOICE QUESTIONS

1. Let R be a relation in the set N given by $R = \{(a, b): a = b - 2, b > 6\}$. Then

(a) $(8, 7) \in R$ (b) $(6, 8) \in R$ (c) $(3, 8) \in R$ (d) $(2, 4) \in R$

2. Let A = {3, 5}. Then the number of reflexive relations on A is

(a) 2 (b) 4 (c) 0 (d) 8

3. If the set A contains 5 elements and the set B contains 6 elements, then the number of both one-one and onto mapping from A to B is

(a) 720 (b) 120 (c) 30 (d) 0

4. The number of equivalence relations in the set {1, 2, 3} containing the elements (1, 2) and (2, 1) is

(a) 0 (b) 1 (c) 2 (d) 3

5. If a relation R on the set {1,2,3} be defined by $R = \{(1,2)\}$, then R is

(a) Reflexive (b) Transitive (c) Symmetric (d) None of these

6. Let R be a relation on $A = \{a, b, c\}$ such that $R = \{(a, a), (b, b), (c, c)\}$, then R is

(a) Reflexive (b) Symmetric only (c) Non-Transitive (d) Equivalence

7. Let A and B be finite sets containing m and n elements respectively. The number of relations that can be defined from A to B is

(a) 2^{mn} (b) 2^{m+n} (c) mn (d) 0

8. Let A = {3, 5}. Then number of reflexive relations on A is

(a) 2 (b) 4 (c) 0 (d) 8

9. The relation R in the set {1,2,3} given by $R = \{(1,2), (2,1), (1,1)\}$ is

(a) Symmetric and Transitive but not Reflexive

(b) Reflexive and Symmetric but not Transitive

(c) Symmetric but neither Reflexive nor Transitive

(d) An equivalence relation

10. Set A has 3 elements and the set B has 4 elements. Then the number of injective mappings that can be defined from A to B is

(a) 144 (b) 12 (c) 24 (d) 64

11. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f ={(1, 4), (2, 5), (3, 6)} be a function from A to B. Based on the given information, f is best defined as

(a) Surjective function (b) Injective function

(c) Bijective function (d) Function

12. The number of functions defined from $\{1,2,3,4,5\} \to \{a, b\}$ which are one-one is

(a) 5 (b) 3 (c) 2 (d) 0

13. Let $f: A \to B$ be a one-one function such that range of f is {b}. Then the value of $n(A)$ is

(a) 1 (b) 2 (c) 0 (d) 4

14. Let $A = \{1,2,3,4\}$ and $R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1)\}$ be defined on set A. Then the equivalence classes of [1] is

(a) (1, 2) (b) [1, 2] (c) {1, 2} (d) {1, 2, 3, 4}

15. Let $f: R \to R, f(x) = \sin x$. Then f(x) is

(a) One-one function (b) Onto function (c) Both (d) None of these

16. Let $A = \{1,2,3\}$. Then number of symmetric relations defined on A is

(a) 8 (b) 64 (c) 1 (d) 0

17. Let $A = \{1,2\}$. Then number of reflexive and symmetric relations defined on A is

(a) 8 (b) 4 (c) 2 (d) 1

18. For real numbers x and y define xRy if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation R is

(a) Reflexive (b) Symmetric (c) Transitive (d) Equivalence

19. Let $A = \{1,2,3\}$. Then the smallest equivalence relations defined on set A is

(a) $\{(1,1), (2,2), (3,3)\}$ (b) $\{(1,1), (2,2), (3,3), (1,2), (2,1)\}$

(c) $\{(1,1), (2,2), (3,3), (1,3), (3,1)\}$ (d) $\{(1,1), (2,2), (3,3), (2,3), (3,2)\}$

20. The maximum number of equivalence relations on the set $A = \{1,2,3\}$ are

(a) 1 (b) 2 (c) 3 (d) 5

21. The function $f: [0, \infty) \to R$ given by $f(x) = \frac{x}{x+1}$

(a) f is both one-one and onto (b) f is one-one but not onto

(c) f is onto but not one-one (d) Neither one-one nor onto

22. Let $A = \{1,2,3\}, B = \{1,4,6,9\}$ and R is a relation from A to B define by 'x is greater than y'. Then the range of R is given by

(a) {1, 4, 6, 9} (b) {4, 6, 9} (c) {1} (d) None of these

23. Let N be the set of all natural numbers and let R be a relation in N, defined by R = {(a, b): a is a factor or b}, then R is

(a) Symmetric and transitive but not reflexive (b) Reflexive and symmetric but not transitive

(c) Reflexive and transitive but not symmetric (d) Equivalence

24. Let N be the set of all natural numbers and let R be a relation on N x N defined by (a, b) R (c, d) $\Leftrightarrow$ ad = bc, then R is

(a) Symmetric and transitive but not reflexive (b) Reflexive and symmetric but not transitive

(c) Reflexive and transitive but not symmetric (d) Equivalence

25. Let R be a relation defined on Z as follows: $(x, y) \in R \Leftrightarrow |x - y| \leq 1$. Then R is:

(a) Reflexive and transitive (b) Reflexive and symmetric

(c) Symmetric and transitive (d) An equivalence relation

26. Let $f: R \to R$ be defined as $f(x) = x^4$. Then f is

(a) One-one and onto (b) Many-one and onto

(c) One-one but not onto (d) Neither one-one nor onto

27. Which of the following functions from Z to itself is bijection?

(a) $f(x) = x^3$ (b) $f(x) = x + 2$ (c) $2x + 1$ (d) $f(x) = x^2 + x$

28. A function f from the set of natural numbers to integers $f(n) = \begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ \frac{-n}{2}, & \text{when n is even} \end{cases}$ is

(a) One-one but not onto (b) Onto but not one-one

(c) One-one and onto (d) Neither one-one not onto

29. Let $f: N \to N$ be defined by $f(1) = f(2) = 1$ and $f(x) = x - 2, x > 2$, then f(x) is

(a) One-one and onto (b) Many-one and onto

(c) One-one but not onto (d) Neither one-one nor onto

30. Let $f: N - \{1\} \to N$ be defined by, $f(n) =$ the highest prime factor of n, then f is

(a) One-one but not onto (b) Onto but not one-one

(c) One-one and onto (d) Neither one-one not onto

31. Let A = {3, 5}. Then number of reflexive relation on A is

(a) 2 (b) 4 (c) 0 (d) 8

32. A relation R defined on set $A = \{x: x \in Z \text{ and } 0 \le x \le 10\}$ as $R = \{(x, y): x = y\}$ is given to be an equivalence relation. The number of equivalence classes is

(a) 1 (b) 2 (c) 10 (d) 11

33. A function $\mathrm{f}: \mathrm{R}_+ \to \mathrm{R}$ (where R_+ is the set of all non-negative real numbers) defined by f(x) = 4x + 3 is

(a) one-one but not onto (b) onto but not one-one

(c) both one-one and onto (d) neither one-one nor onto

34. Let $f: R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is

(a) one-one (b) onto (c) bijective (d) neither one-one nor onto

35. Let R_+ denote the set of all non-negative real numbers. Then the function $\mathrm{f}: \mathrm{R}_+ \to \mathrm{R}_+$ defined as $f(x) = x^2 + 1$ is

(a) one-one but not onto (b) onto but not one-one

(c) both one-one and onto (d) neither one-one nor onto

36. A function $\mathrm{f}: \mathbb{R} \to \mathbb{R}$ defined as $\mathrm{f}(x) = x^2 - 4x + 5$ is

(a) injective but not surjective (b) surjective but not injective

(c) both injective and surjective (d) neither injective nor surjective

37. A function $f: R \to A$ defined as $f(x) = x^2 + 1$ is onto, if A is

(a) $(-\infty,\infty)$ (b) $(1, \infty)$ (c) $[1, \infty)$ (d) $[-1, \infty)$

38. Let Z denote the set of integers, then function $\mathrm{f}: \mathrm{Z} \to \mathrm{Z}$ defined as $f(x) = x^3 - 1$ is

(a) both one-one and onto (b) one-one but not onto

(c) onto but not one-one (d) neither one-one nor onto

ASSERTION - REASON TYPE QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

1. Assertion: The relation $f: \{1,2,3,4\} \rightarrow \{x, y, z, p\}$ defined by f = {(1, x), (2, y), (3, z)} is a bijective function.

Reason: The $f: \{1,2,3,4\} \rightarrow \{x, y, z, p\}$ such that f = {(1, x), (2, y), (3, z)} is one-one.

2. Assertion: Let R be the relation on the set of integers Z is given by $R = \{(a, b): 2 \text{ divides } (a - b)\}$ is an equivalence relation.

Reason: A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

3. Assertion: Let $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x$, then f is a one-one function.

Reason: A function $g: A \rightarrow B$ is said to be onto function if for each $b \in B, \exists a \in A$ such that g(a) = b.

4. Assertion: Let function $f: \{1,2,3\} \rightarrow \{1,2,3\}$ be an onto function. Then is must be one-one function.

Reason: A one-one function $g: A \rightarrow B$ where A and B are finite set having same number of elements, then it must be onto and vice versa.

5. Assertion: Let $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \frac{|x|}{x} = \begin{cases} 1 & \text{if } \quad x > 0 \\ 0 & \text{if } \quad x = 0 \\ -1 & \text{if } \quad x < 0 \end{cases}$ is bijection.

Reason: A function $g: A \rightarrow B$ is said to be bijection if it one-one and onto.

6. Assertion: The relation R in the set A = {1, 2, 3, 4, 5, 6} defined as $R = \{(x, y): y \text{ is divisible by } x\}$ is not an equivalence relation.

Reason: The relation R will be an equivalence relation, if its reflexive, symmetric and transitive.

7. Assertion: If R is the relation in the set A = {1, 2, 3, 4, 5} given by $R = \{(a, b): |a - b| \text{ is even }\}$ R is an equivalence relation.

Reason: All elements of {1, 3, 5} are related to all elements of {2, 4}.

8. Assertion: The function $f: R^* \to R^*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where R^* is the set of all non-zero real numbers.

Reason: The function $g: N \to R^*$ defined by $f(x) = \frac{1}{x}$ one-one and onto.

9. Assertion: Let $f: R \to R$ defined by $f(x) = x^2 + 1$. Then pre-images of 17 are ±4.

Reason: A function $f: A \to B$ is called a one-one function if distinct elements of A have distinct images in B.

10. Assertion: The function $f: R \to R, f(x) = |x|$ is not one-one.

Reason: The function $f(x) = |x|$ is not onto.

11. Assertion: The function $f: R \to R$ given by $f(x) = x^3$ is injective.

Reason: A function $f: A \to B$ is injective if $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$.

12. Assertion: Let $A = \{2,4,6\}$ and $B = \{3,5,7,9\}$ and defined a function $f = \{(2,3), (4,5), (6,7)\}$ from A to B. Then f is not onto.

Reason: A function $f: A \to B$ said to be onto if every element of B is the image of some elements of A under f.

13. Assertion: Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2): L_1$, is perpendicular to $L_2\}$ R is not an equivalence relation.

Reason: R is symmetric but neither reflexive nor transitive.

14. Assertion: If R = $\{(T_1, T_2,): T_1$, is congruent to $T_2\}$. Then R is an equivalence relation.

Reason: Any relation R is an equivalence relation if it is reflexive, symmetric and transitive.

15. Assertion: Set A has 3 elements and set B has 5 elements then number of injective function set A to set B will be 60.

Reason: If set A has m elements and set B has n elements where n ≥ m, the number of injective function $^{n}p_{m}$.

16. Assertion: A function $f(x) = \cos x, x \in [0, \frac{3\pi}{2}]$ is bijective.

Reason: For one-one function each elements in domain has unique image in co-domain.

17. Assertion: The function $f: R \to R$ defined by f(x) = [x] is neither one-one nor onto.

Reason: The signum function $f: R \to R$ given by $f(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$ bijective function.

18. Assertion: Domain and Range of relation $R = \{(x, y): x - 2y = 0\}$ defined on A = {1, 2, 3, 4} are respectively {1, 2, 3, 4} and {2, 4, 6, 8}.

Reason: Domain and range of relation R are respectively the sets $\{a: a \in A \text{ and } (a, b) \in R\}$ and $\{b: b \in A \text{ and } (a, b) \in R\}$.

19. Assertion: If X = {0, 1, 2} and the function $f: X \to Y$ defined by $f(x) = x^2 - 2$ is surjective then Y = {-2, -1, 0}.

Reason: If $f: X \to Y$ is surjectiveif $f(X) = Y$.

20. Assertion: Let R be the relation in the set {1, 2, 3, 4} given by $R = \{(1,2), (2,2), (1,1), (4,4), (1,3), (3,3), (3,2)\}$. R is not an equivalence relation.

Reason: R is not reflexive but it is symmetric and transitive.

21. Assertion: Let Z be the set of integers. A function $f: Z \to Z$ defined as f(x) = 3x – 5, ∀x ∈ Z is a bijective.

Reason: A function is a bijective if it is both surjective and injective.

22. Assertion: The number of onto function from a set P containing 5 elements to a set Q containing 2 elements is 30.

Reason: Number of onto function from a set containing m elements to a set containing n elements is n^m.

23. Assertion: The relation R = {(x, y): (x + y) is a prime number and x, y ∈ N} is not reflexive relation.

Reason: The number '2n' is composite for all natural numbers n.

VERY SHORT ANSWERS

1. Let the relation R be given as $R = \{(x, y): x, y \in N \text{ and } x + 3y = 12\}$. Find the domain and range of R.

2. Show that the function $\mathrm{f}: \mathbb{R} \to \mathbb{R}, \mathrm{f(x)} = \mathrm{x}^4$ is neither one-one not onto.

3. Prove that the greatest integer function $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = [x]$ is neither one-one nor onto.

4. Show that the relation S in the set $A = \{x \in Z: 0 \leq x \leq 12\}$ given by $S = \{(a, b): a, b \in Z, |a - b| \text{ is divisible by } 3\}$ is an equivalence relation.

5. Check whether the relation R in the set N of natural numbers given by $R = \{(a, b): a \text{ is divisor of } b\}$ R is an equivalence relation.

6. Show that the modulus function $f: \mathbb{R} \longrightarrow \mathbb{R}$ given by $f(x) = |x|$, is neither one-one nor onto.

7. Prove that the function f is surjective where $f: \mathrm{N} \longrightarrow \mathrm{N}$ such that $f(n) = \begin{cases} \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ \frac{n}{2}, \text{ if } n \text{ is even} \end{cases}$. Is the function injective? Justify your answer.

8. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = $\{(a, b): b = a + 1\}$ is reflexive, symmetric or transitive.

9. A function f: A → B defined as f(x) = 2x is both one-one and onto. If A = {1, 2, 3, 4}, then find the set B.

SHORT ANSWERS

1. Show that function $\mathrm{f}: \mathbb{R} \rightarrow \mathbb{R}$ defined as $\mathrm{f(x)} = \frac{5\mathrm{x}-3}{4}$ is both one-one and onto.

2. Check whether a function $\mathrm{f}: \mathbb{R} \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $\mathrm{f(x)} = \frac{\mathrm{x}}{1+\mathrm{x}^2}$ is one-one and onto or not.

3. Let $\mathrm{f}: \mathbb{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathbb{R}$ be a function defined as $\mathrm{f(x)} = \frac{4\mathrm{x}}{3\mathrm{x}+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

4. Let $\mathrm{A} = \mathrm{R} - \{2\}$ and $\mathrm{B} = \mathrm{R} - \{1\}$. If $\mathrm{f}: \mathrm{A} \rightarrow \mathrm{B}$ is a function defined by $\mathrm{f(x)} = \frac{\mathrm{x}-1}{\mathrm{x}-2}$, show that f is one-one and onto.

5. Prove that a function $\mathrm{f}: [0, \infty) \rightarrow [-5, \infty)$ defined as $\mathrm{f(x)} = 4\mathrm{x}^2 + 4\mathrm{x} - 5$ is both one-one and onto.

6. Show that the function f in $\mathrm{A} = \mathbb{R} - \left\{\frac{2}{3}\right\}$ defined as $\mathrm{f(x)} = \frac{4\mathrm{x}+3}{6\mathrm{x}-4}$ is one-one and onto.

7. Prove that the function $\mathrm{f}: \mathrm{N} \rightarrow \mathrm{N}$, defined by $\mathrm{f(x)} = \mathrm{x}^2 + \mathrm{x} + 1$ is one-one but not onto.

8. Let R be a relation defined on the set of natural numbers N as follows $R = \{(x, y) \mid x \in N, y \in N \text{ and } 2x + y = 24\}$. Find the domain and range of the relation R. Also, verify whether R is equivalence or not.

9. Show that the relation S in the set of R of real numbers defined as $S = \{(a, b): a, b \in R \text{ and } a \leq b^3\}$ is neither reflexive nor symmetric nor transitive.

10. Show that the relation S in the set $A = \{x \in Z: 0 \leq x \leq 12\}$ given by $S = \{(a, b): a, b \in Z, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

11. Let $f: X \to Y$ be a function define a relation R on X given by $R = \{(a, b): f(a) = f(b)\}$. Show that R is an equivalence relation on X.

12. Prove that the relation R in the set A = {1, 2, 3, 4, 5} given by $R = \{(a, b): |a - b| \text{ is even}\}$ is an equivalence relation.

13. Show that $f: N \to N$ given by $f(x) = \begin{cases} x + 1, \text{ if x is odd} \\ x - 1, \text{ if x is even} \end{cases}$ is both one-one and onto.

14. Let $f: N \to N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if n is odd} \\ \frac{n}{2}, & \text{if n is even} \end{cases}$ for all $n \in N$. Check whether the function f is bijective.

15. If $f: R \to R$ be the function defined by $f(x) = 4x^3 + 7$, show that f is a bijection.

16. A relation R on set A = {1, 2, 3, 4, 5} is defined as $R = \{(x, y): |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.

17. Find function f is defined from $R \to R$ as $f(x) = ax + b$, such that f(1) = 1 and f(2) = 3. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.

LONG ANSWERS

1. Show that the function $f: \mathbb{R} \to \{x \in \mathbb{R}: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in \mathbb{R}$ is one-one and onto function.

2. A function $f: R - \{-1,1\} \to R$ is defined by $f(x) = \frac{x}{x^2-1}$. Check (i) f is one-one (ii) f is onto.

3. Consider $f: R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$ where R_+ is the set of all non-negative real numbers. Prove that f is one-one and onto function.

4. Let set A = {1, 2, 3…..,10} and R be a relation in A x A defined by $(a, b)R(c, d) \Leftrightarrow a + d = b + c$ for all (a, b) and (c, d) $\in A \times A$. Prove that R is an equivalence relation.

5. Show that the relation S in set $\mathbb{R}$ of real numbers defined by $S = \{(a, b): a \leq b^3, a \in \mathbb{R}, b \in \mathbb{R}\}$ is neither reflexive nor symmetric nor transitive.

6. Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by $R = \{(a, b)$: both a and b are either odd or even $\}$. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].

7. Show that the function f in A = $R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto.

8. Let N denotes the set of all natural numbers and R be the relation on N x N defined by $(a, b)R(c, d)$ if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

9. Determine whether the relation R defined on the set $\mathbb{R}$ of all real numbers as $R = \{(a, b): a, b \in \mathbb{R}$ and $a - b + \sqrt{3} \in S$, where S is the set of all irrational numbers$\}$, is reflexive, symmetric and transitive.

10. A relation R is defined on a set of real numbers $\mathbb{R}$ as $R = \{(x, y): x.y \text{ is an irrational number}\}$. Check whether R is reflexive, symmetric and transitive or not.

11. Consider $f: R_+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible.

12. Consider $f: R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible.

13. Show that the function $f: (-\infty, 0) \to (-1,0)$ defined by $f(x) = \frac{x}{1+|x|}, x \in (-\infty, 0)$ is one-one and onto.

14. Prove that the greatest integer function $f: R \longrightarrow R$ givne by $f(x) = [x]$ is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

15. Let $f: W \to W$ be defined as $f(n) = \begin{cases} n - 1, \text{if n is odd} \\ n + 1, \text{if n is even} \end{cases}$. Show that f is invertible. Here W is the set of all whole numbers.

16. Consider $f: \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R} - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective.

17. Let $f: \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is one-one function. Also, check whether f is onto function or not.

18. Show that the function $f: \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = \frac{x}{x^2+1}, \forall x \in R$ is neither one-one nor onto.

19. Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b): both a and b are either odd or even}. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].

20. A relation R on set A = {-4, -3, -2, -1, 0, 1, 2, 3, 4} be defined as R = {(x, y): x + y is an integer divisible by 2}. Show that R is an equivalence relation. Also, write the equivalence class [2].

21. A relation R on set $A = \{x: -10 \leq x \leq 10, x \in Z\}$ is defined as $R = \{(x, y): (x - y)$ is divisible by $5\}$. Show that R is an equivalence relation. Also, write the equivalence class [5].

22. A relation R is defined on a set of real numbers $\mathbb{R}$ as $R = \{(x, y): x \cdot y$ is an irrational number$\}$. Check whether R is reflexive, symmetric and transitive or not.

23. If N denotes the set of all natural numbers and R is the relation on N x N defined by (a, b) R (c, d), if ad(b + c) = bc(a + d). Show that R is an equivalence relation.

24. A relation R is defined on N x N (where N is the set of natural number) as $(a, b)R(c, d) \Leftrightarrow a - c = b - d$. Show that R is an equivalence relation.

25. A relation R is defined on N x N (where N is the set of natural numbers) as $(a, b)R(c, d) \Leftrightarrow \frac{a}{c} = \frac{b}{d}$. Show that R is an equivalence relation.

26. Check whether the relation S in the set of real numbers R defined by $S = \{(a, b)$: where $a - b + \sqrt{2}$ is an irrational number $\}$ is reflexive, symmetric or transitive.

27. Let $f: \mathbb{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

28. Let $A = R - \{5\}$ and $B = R - \{1\}$. Consider the function f: A → B, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one-one and onto.

29. Show that a function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = \frac{5x-3}{4}$ is both one-one and onto.

30. Prove that a function $f: [0, \infty) \rightarrow [-5, \infty)$ defined as $f(x) = 4x^2 + 4x - 5$ is both one-one and onto.

31. A function f: $[-4,4] \rightarrow [0,4]$ is given by $\mathrm{f}(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $\mathrm{f(a)} = \sqrt{7}$.

32. Show that a function $f: R \rightarrow R$ defined as $f(x) = x^2 + x + 1$ is neither one-one nor onto. Also, find all the values of x for which f(x) = 3.

33. Check whether a function f: $\mathbb{R} \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $\mathrm{f(x)} = \frac{\mathrm{x}}{1+\mathrm{x}^2}$ is one – one and onto or not.

34. Show that a function $f: R \rightarrow R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f: R \rightarrow A$ becomes an onto function.

35. Let $A = R - \{3\}$ and $B = R - \{a\}$. Find the value of 'a' such that the function $\mathrm{f: A \rightarrow B}$ defined by $f(x) = \frac{x-2}{x-3}$ is onto. Also, check whether the given function is one-one or not.

CASE BASED QUESTIONS

1. The Earth has 24 time zones, defined by dividing the Earth into 24 equal longitudinal segment. These are the regions on Earth that have the same standard time. For example, USA and India fall in different time zones but Sri Lanka and India are same time zone.

A relation R is defined on the set U = {All people on the Earth} such that R = {(x, y)| the time difference between the time zones x and y reside in in 6 hours}

Based on the above information, answer the following:

(i) Check whether the relation R is reflexive, symmetric and transitive.

(ii) Is relation R an equivalence relation?

2. Swathi and Danju are playing ludo at home. While rolling the dice, Swathi's sister Raji observed and noted the possible outcomes of the throw every time belongs to set $\{1, 2, 3, 4, 5, 6\}$. Let A be the set of players while B be the set of all possible outcomes.

$$A = \{S, D\} \text{ and } B = \{1, 2, 3, 4, 5, 6\}$$

(i) Let $R: B \rightarrow B$ be defined by $R = \{(x, y): y \text{ is divisible by } x\}$. Verify that whether R is reflexive, symmetric and transitive.

(ii) Raji wants to know the number of functions from A to B. Find the number of all possible functions.

(iii) Let R be a relation on B defined by $R = \{(1,2), (2,2), (1,3), (3,4), (3,1), (4,3), (5,5)\}$. Then R is which kind of relation?

(Or)

Raji wants to know the number of relations possible from A to B. Find the number of possible relations.

3. Students of Grade 9 planned to plant saplings along straight lines parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = x - 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L.

(i) Let relation R be defined by $R = \{(L_1, L_2): L_1 \parallel L_2 \text{ where } L_1, L_2 \in L\}$. What is the type of relation R?

(ii) Let $R = \{(L_1, L_2): L_1 \perp L_2 \text{ where } L_1, L_2 \in L\}$. What is the type of relation R ?

(iii) Check whether the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x - 4$ is bijective or not.

(Or)

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x + 4$. Find the range of $f(x)$.

4. Read the following passage and answer the following questions:

Dhanush wants take test of his son Amit is a student of class XII. Dhanush said to Amit, "observe the two functions f(x) and g(x) carefully" $f: \mathbb{R} \to \mathbb{R}, g: \mathbb{R} \to \mathbb{R}$ such that $f(x) = x, g(x) = x^2$.

Then, Dhanush asked some questions related to f(x) and g(x) and Amit answered correctly. Write the correct response given by Amit of the following questions.

(i) Check whether f(x) is bijective or not.

(ii) Check whether g(x) is bijective or not.

5. In a school library around 8000 books are there. It is divided and kept in different places in the big library room based on the language – English, Malayalam, Hindi, Sanskrit. It is again divide based on poems, novels, literature, short story, dictionary etc.

Let B is set of all book in the library. A relation R is defined on B is given by $R = \{(x, y); \text{x and y have same number of pages}\}$.

Based on the above information, answer the following questions:

(i) Is R a reflexive relation?

(ii) Is R a symmetric relation?

(iii) Is R a transitive relation?

6. A scout master wants to make different groups of students so that they can be given different tasks. Students started making groups with their friends, then scout master interfere and told them to make groups as per rule "a student will make group with roll number such a way that the difference of roll number is divisible by 3.

Based on the above information, answer the following questions:

(i) Write a relation R in set builder form for the rule told by the scout master.

(ii) Which roll number of students will be in the group of students with roll number 2, if there are 30 students in the class?

(iii) Which roll number of students will be in the group of students with roll number 3, if there are 30 students in the class?

7. Mohan and Swathi were playing a game. The rule for the game is that when Mohan plots a point say P on a cartesian plane, then Swathi has to find points on the plane in such way the distance of points from the origin O is same as that of the distance between the origin and the point P. Mohan's sister Avantika put the rule of the game in set for as $R = \{(P, Q): OP = OQ\}$.

Based on the above information, answer the following:

(i) Is R a reflexive relation?

(ii) Is R a symmetric relation?

(iii) Is R a transitive relation?

8. Abhimanu is having a collection of all lines in XY plane and his friend Arjun made a relation on this collection of lines as $R_1 = \{(L_1, L_2): L_1$ is parallel to $L_2\}$. Arjun also made another relation on the same collection as $R_2 = \{(L_1, L_2): L_1$ is perpendicular to $L_2\}$.

They ask their classmates to check the following:

(i) Is the relation R_1 form and equivalence relation?

(ii) Find the set of lines related to $y = 2x + 4$ with respect to R_1.

(iii) Is the relation R_2 form and equivalence relation?

9. An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of boys and G the set of girls selected for the final race.

Based on the above information, answer the following question:

(i) How many relations possible from B to G?

(ii) Among all the possible relations from B to G, how many functions can be formed from B to G?

(iii) Let $\mathrm{R}: \mathrm{B} \rightarrow \mathrm{B}$ be defined by R = {(x, y): x and y are students of the same sex}. Check R is an equivalence relation.

(iv) A function $\mathrm{f}: \mathrm{B} \rightarrow \mathrm{G}$ be defined by $\mathrm{f} = \{(\mathrm{b}_1, \mathrm{g}_1), (\mathrm{b}_2, \mathrm{g}_2), (\mathrm{b}_3, \mathrm{g}_1)\}$.Check f is bijective. Justify your answer.

10. Students of a school are taken to a railway museum to learn about railways heritage and its history. An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by $\mathrm{R} = \{(l_1, l_2): l_1 \text{ is parallel to } l_2\}$.

On the basis of the above information, answer the following questions:

(i) Find whether the relation R is symmetric or not.

(ii) Find whether the relation R is transitive or not.

(iii) If one of the rail lines on the railway track is represented by the equation y = 3x + 2, then find the set of rail lines in R related to it.

(iv) Let S be the relation defined by $\mathrm{S} = \{(l_1, l_2): l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. A relation R in set A = {1, 2, 3} is defined as $R = \{(1,1), (1,2), (2,2), (3,3)\}$ which of the following ordered pair in R shall be removed to make it an equivalence relation in A.

(a) (1, 1) (b) (1, 2) (c) (2, 2) (d) (3, 3)

2. Let the relation R in the set $A = \{x \in Z: 0 \leq x \leq 12\}$, given $R = \{(a, b): |a - b| \text{ is a multiple of } 4\}$. Then [1], the equivalence class containing 1 is

(a) {1, 5, 9} (b) {0, 1, 2, 5} (c) ϕ (d) A

3. A relation R is defined on N. Which of the following is the reflexive relation?

(a) $R = \{(x, y): x > y, x, y \in N\}$ (b) $R = \{(x, y): x + y = 10, x, y \in N\}$

(c) $R = \{(x, y): xy \text{ is the square number}, x, y \in N\}$ (d) $R = \{(x, y): x + 4y = 10; x, y \in N\}$

4. Assertion: The number of onto functions from a set P containing 5 elements to a set Q containing 2 elements is 30.

Reason: Number of onto functions from set containing m elements to set containing n elements is n^m.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. A relation R is defined on a set of real numbers $\mathbb{R}$ as $R = \{(x, y): x \cdot y \text{ is an irrational number }\}$. Check whether R is reflexive, symmetric and transitive or not.

6. Let $f: \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that in $f: R - \left\{-\frac{4}{3}\right\} \to$ Range,f is one-one and onto.

7. Show that the relation R on the set Z of all integers given by $R = \{(a, b): 2 \text{ divides } (a - b)\}$ is an equivalence relation.

SECTION C

Each carry 3 marks

8. A function $f: [-4,4] \to [0,4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto fucnrtion but not a one-one function. Further, find all possible values of a for which $f(a) = \sqrt{7}$.

9. Let $A = R - \{3\}, B = R - \{1\}$. If $f: A \to B$ be defined by $f(x) = \frac{x-2}{x-3}, \forall x \in A$. Then show that f is bijective.

SECTION D

Each carry 5 marks

10. Prove that a function $f: [0, \infty) \to [-5, \infty)$ defined as $f(x) = 4x^2 + 4x - 5$ is both one-one and onto.

11. Let $\mathbb{N}$ be the set of all natural numbers and R be a relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b)R(c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$. Show that R is an equivalence relation on $\mathbb{N} \times \mathbb{N}$. Also, find the equivalence class of (2, 6). i.e., [(2,6)].

SECTION E

12. An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$ where B represents the set of Boys and G the set of Girls selected for the final race.

Based on the above information, answer the following questions:

(i) How many relations are possible from B to G?

(ii) Among all the possible relations from B to G, how many functions can be formed from B to G?

(iii) Let $R: B \to B$ be defined by R = {(x, y): x and y are students of the same sex}. Check if R is an equivalence relation.

(Or)

A function $f: B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check f is bijective. Justify your answer.

ANSWERS

MCQ	A-R	VSA	SA	LA	CS
1. (b) $(6, 8) \in R$ 2. (b) 4 3. (d) 0 4. (c) 2 5. (b) Transitive 6. (d) Equivalence 7. (a) 2^{mn} 8. (b) 4 9. (a) Symmetric and Transitive but not Reflexive 10. (c) 24 11. (b) Injective	1. (d) 2. (a) 3. (b) 4. (a) 5. (d) 6. (a) 7. (c) 8. (c) 9. (b) 10. (b) 11. (a) 12. (a) 13. (a) 14. (a)	1. Domain = {3, 6, 9} Range = {1, 2, 3} 5. Not an equivalence 7. f is not injective 8. Neither reflexive nor symmetric nor transitive 9. $B = \{2, 4, 6, 8\}$	2. Not one-one but onto 8. Domain of $R = \{1,2,3,4, \dots ,11\}$ Range of R $= \{2,4,6,8,10,12, \dots ,22\}$ R is not an equivalence 10. {5, 9} 14. f is not bijective 16. Reflexive, symmetric but not transitive. 17. a = 2, b = -1 and function is one-one and onto.	2. f is not one-one but f is onto 6. {1,3,5,7} 9. R is reflexive but neither symmetric nor transitive 10. R is symmetric but neither reflexive not transitive 19. $\{1, 3, 5, 7\}$ 20. Equivalence class $[2] = \{-4, -2, 0, 2, 4\}$ 21. Equivalence class $[5] = \{-10, -5, 0, 5, 10\}$ 22. Symmetric but not	1. (i) R is symmetric but not reflexive and transitive (ii) Not an equivalence relation 2. (i) Reflexive, transitive but not symmetric (ii) 36 (iii) R is neither reflexive nor symmetric nor transitive (Or) 2^{12} 3. (i) Equivalence relation (ii) Symmetric but neither reflexive nor transitive (iii) Bijective (Or) $\mathbb{R}$

function 12. (d) 0 13. (a) 1 14. (c) {1, 2} 15. (d) None of these 16. (b) 64 17. (c) 2 18. (d) Equivalence 19. (a) $\{(1,1),(2,2),(3,3)\}$ 20, (d) 5 21. (b) f is one-one but not onto 22. (c) {1}	15. (a) 16. (d) 17. (c) 18. (d) 19. (a) 20. (c) 21. (d) 22. (c) 23. (c)			reflexive and transitive 26. Reflexive But Neither Symmetric nor Tr 27. f is not an onto 31. $a = \pm 3$ 32. -2, 1 33. f is onto. 35. a = 1, f is one-one.	4. (i) f is bijective (ii) g is not bijective. 5. (i) Yes (ii) Yes (iii) Yes 6. (i) $R = \{(a,b): \lvert a-b\rvert \text{ is divisible by } 3\}$ (ii) $\{2,5,8,11,14, \dots\dots\}$ (iii) $\{3,6,9,12,15 \dots\dots\}$ 7. (i) Yes (ii) Yes (iii) Yes 8. (i) Yes (ii) $y = 2x + c$; c is any arbitary constant (iii) No 9. (i) 64 (ii) 8 (iii) Equivalence Relation

23. (d) Equivalence 24. (d) Equivalence 25. (b) Reflexive and symmetric 26. (d) Neither one-one nor onto 27. (b) f(x) = x + 2 28. (c) One-one and onto 29. (b) Many-one and onto 30. (d) Neither one-one not onto					(iv) f is not bijective 10. (i) Symmetric (ii) Transitive (iii) The set is $\{1: 1$ is a line of type y $3x + c, c \in R\}$ (iv) Symmetric but not transitive

31. (b) 4 32. (d) 11 33. (a) one-one but not onto 34. (c) bijective 35. (a) one-one but not onto 36. (d) neither injective nor surjective 37. (c) $[1, \infty)$ 38. (b) one-one but not onto					

UNIT TEST

1. (b) (1, 2)

2. (a) {1, 5, 9}

3. (c) $R = \{(x, y): xy$ is the square number, $x, y \in N\}$

4. (c) Assertion (A) is true but Reason (R) is false.

5. Symmetric but not reflexive and transitive.

6. Proof

7. Proof

8. $a = \pm 3$

9. Proof

10. Proof

11. $\{(1,3), (2,6), (3,9), \ldots \ldots \ldots\}$

12. (i) 64 (ii) 8 (iii) R is an equivalence relation (Or) f is not bijective.

CHAPTER 2 – INVERSE TRIGONOMETRIC FUNCTIONS

INVERSE OF A FUNCTION

Let f: A → B be a function which is one-one and onto, then f^{-1}: B → A is said to be the inverse of the function f if $y = f(x) \Leftrightarrow f^{-1}(y) = x$.

INVERSE TRIGONOMETRIC FUNCTIONS

Trigonometric functions are many-one functions but we know that inverse of function exists if the function is bijective. If we restrict the domain of trigonometric functions, then these functions become bijective and the inverse of trigonometric functions are defined within the restricted domain. The inverse of f is denoted by 'f^{-1}'. Let y = f(x) = sin x, then its inverse is x = $\sin^{-1}$ y.

DOMAIN AND RANGE OF INVERSE TRIGONOMETRIC FUNCTIONS

Function	Domain	Range (Principal Value Branch)
$\sin^{-1} x$	$[-1,1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1,1]$	$[0, \pi]$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1} x$	R	$[0, \pi]$
$\operatorname{cosec}^{-1} x$	R – {-1, 1}	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$\sec^{-1} x$	R – {-1, 1}	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$

NOTE

(i) $\sin^{-1} x$ should not be confused with $(\sin x)^{-1}$. In fact, $(\sin x)^{-1} = \frac{1}{\sin x}$ and similarly for other trigonometric functions.

(ii) The value of inverse trigonometric functions which lies in the range of principal branch is called the principal value of those inverse trigonometric functions.

PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

Property I	**(i)** $\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1} x; x \geq 1 \text{ or } x \leq -1$ **(ii)** $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x; x \geq 1 \text{ or } x \leq -1$ **(iii)** $\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1} x; x > 0 \\ -\pi + \cot^{-1} x; x < 0 \end{cases}$
Property II	(i) $\sin^{-1}(-x) = -\sin^{-1} x; x \in [-1,1]$ (ii) $\tan^{-1}(-x) = -\tan^{-1} x; x \in R$ (iii) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x; \lvert x\rvert \geq 1$
Property III	(i) $\cos^{-1}(-x) = \pi - \cos^{-1} x; x \in [-1,1]$ (ii) $\sec^{-1}(-x) = \pi - \sec^{-1} x; \lvert x\rvert \geq 1$ (iii) $\cot^{-1}(-x) = \pi - \cot^{-1} x; x \in R$
Property IV	(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}; x \in [-1,1]$ (ii) (ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}; x \in R$ (iii) $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}; \lvert x\rvert \geq 1$
Property V	(i) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right); xy < 1$ (ii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right); xy > -1$
Property VI	(i) $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right); \lvert x\rvert \leq 1$ (ii) $2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right); x \geq 0$ (iii) $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right); -1 < x < 1$ (iv) $2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right); \frac{-1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$ (v) $2\cos^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right); \frac{-1}{\sqrt{2}} \leq x \leq 1$ (or) $2\cos^{-1} x = \cos^{-1}(2x^2 - 1); 0 \leq x \leq 1$
Property VII	(i) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$ (ii) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$ (iii) $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$ (iv) $\cos^{-1} x - \cos^{-1} y = \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$

MULTIPLE CHOICE QUESTIONS

1. The domain of the function $\sin^{-1}(2x)$ is

(a) [-1, 1] (b) [0, 1] (c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

2. The principal value of $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$ is

(a) $\frac{2\pi}{5}$ (b) $-\frac{2\pi}{5}$ (c) $\frac{3\pi}{6}$ (d) $-\frac{3\pi}{6}$

3. The principal value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is

(a) $\frac{\pi}{12}$ (b) π (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

4. The principal value of $\cos^{-1}\left\{\sin\left(\cos^{-1}\frac{1}{2}\right)\right\}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

5. The principal value of $\cot[\sin^{-1}\{\cos(\tan^{-1} 1)\}]$ is

(a) $\frac{1}{2}$ (b) ∞ (c) $\frac{1}{\sqrt{2}}$ (d) 1

6. The domain of $\sec^{-1}(2x - 1)$ is

(a) $(-\infty, -1) \cup [0, \infty)$ (b) $(-\infty, -1) \cup (0, \infty)$

(c) $(-\infty, -1] \cup [0, \infty)$ (d) None of these

7. $\sec\left[90° - \cot^{-1}\left(\frac{1}{3}\right)\right]$ is equal to

(a) $\sqrt{10}$ (b) $\frac{1}{3}$ (c) 3 (d) $\frac{\sqrt{10}}{3}$

8. Domain of $f(x) = \sin^{-1}(-x^2)$ is

(a) (-1, 1) (b) $(-\infty, -1]$ (c) $[-1, \infty)$ (d) [-1, 1]

9. The principal value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

10. The value of $\sin^{-1}\left(\cos\frac{13\pi}{5}\right)$ is

(a) $-\frac{3\pi}{5}$ (b) $-\frac{\pi}{10}$ (c) $\frac{3\pi}{5}$ (d) $\frac{\pi}{10}$

11. $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) -1 (d) 1

12. The value of $\cot(\sin^{-1} x)$ is

(a) $\frac{\sqrt{1+x^2}}{x}$ (b) $\frac{x}{\sqrt{1+x^2}}$ (c) $\frac{1}{x}$ (d) $\frac{\sqrt{1-x^2}}{x}$

13. The value of $\sin^{-1}\left(\cos\frac{\pi}{9}\right)$ is

(a) $\frac{\pi}{9}$ (b) $\frac{5\pi}{9}$ (c) $-\frac{5\pi}{9}$ (d) $\frac{7\pi}{18}$

14. Let $\theta = \sin^{-1}(\sin(-600°))$, then value of θ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $-\frac{2\pi}{3}$

15. The value of $\cot\left[\frac{1}{2}\sin^{-1}\frac{\sqrt{3}}{2}\right]$ is

(a) 1 (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 0

16. $\sin(\tan^{-1} x), |x| < 1$ is equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

17. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(a) [1, 2] (b) [-1, 1] (c) [0, 1] (d) None of these

18. Which of the following is the principal value branch of $\text{cosec}^{-1}x$?

(a) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (b) $(0, \pi) - \left[\frac{\pi}{2}\right]$ (c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

19. Let $\cos(2\tan^{-1} x) = \frac{1}{2}$, then the value of x will be

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $1 - \sqrt{3}$ (d) $1 - \frac{1}{\sqrt{3}}$

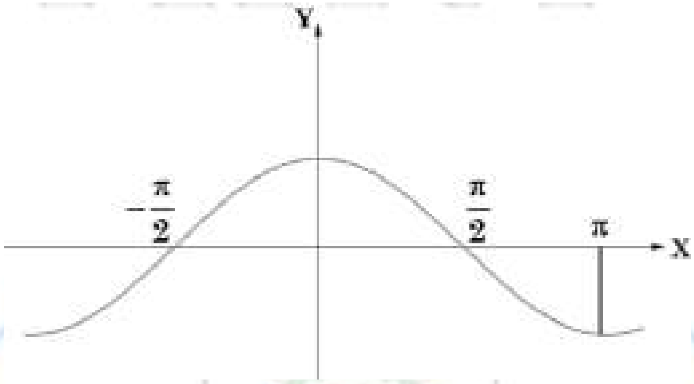
20. If $k \le \sin^{-1} x + \cos^{-1} x + \tan^{-1} x \le m$ then

(a) $k = 0, m = \pi$ (b) $k = 0, m = \frac{\pi}{2}$ (c) $k = \frac{\pi}{2}, m = \pi$ (d) $(k, m) = (0, \pi)$

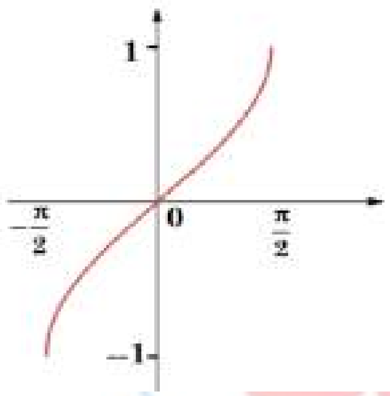
21. $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

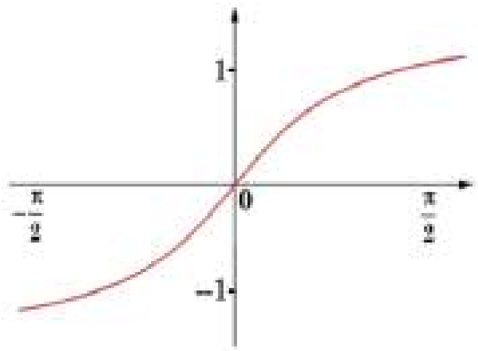
22. The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse?



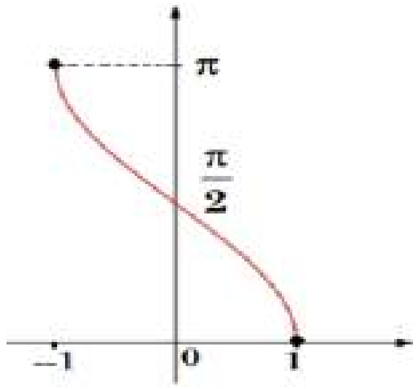
(a)



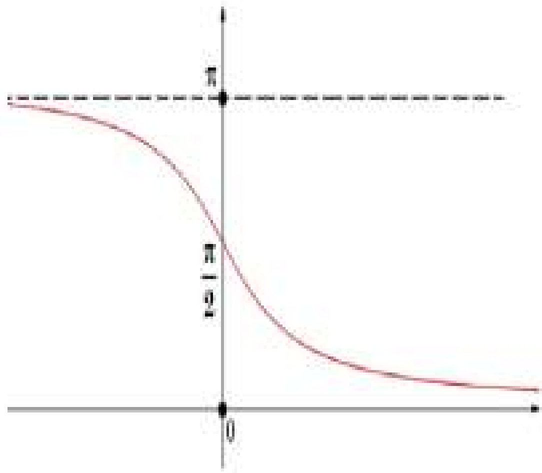
(b)



(c)



(d)



ASSERTION - REASON TYPE QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

1. Assertion: All trigonometric functions have their inverses over their respective domains.

Reason: The inverse of $\tan^{-1}x$ exist for some $x \in R$.

2. Assertion value of $(\cos^{-1} x)^2$ is π^2.

Reason: Range of the principal value branch of $\cos^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

3. Assertion: $[\sin^{-1} x + 2\cos^{-1} x]$ is $[0, \pi]$.

Reason: Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

4. Assertion: The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

Reason: Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1,1\}$.

5. Assertion: Principal value of $\tan^{-1}(-\sqrt{3})$ is $-\frac{\pi}{3}$.

Reason: $\tan^{-1}: \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ so for any $x \in \mathbb{R}, \tan^{-1}(x)$ represents an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

6. Assertion: $\sin^{-1}(-x) = -\sin^{-1} x; x \in [-1,1]$

Reason: $\sin^{-1}: [-1,1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is a bijection map.

7. Assertion: Principal value of $\sin^{-1}\left(\sin\frac{17\pi}{18}\right)$ is $\frac{\pi}{18}$.

Reason: Domain of principal value branch of $\sin^{-1}$ is $[-1,1]$

8. Assertion: The value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is $\frac{-\pi}{2}$.

Reason: The principal branch of $\tan^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $\cot^{-1}(-x) = \pi - \cot^{-1} x$.

9. Assertion: The principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

Reason: The principal branch of $\tan^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $\tan^{-1}(-x) = -\tan^{-1} x$.

10. Assertion: The principal value of $\cos^{-1}\cos(-680) = \frac{2\pi}{9}$.

Reason: $\cos^{-1}(-x) = \pi - \cos^{-1} x$.

11. Assertion: Domain of $y = \cos^{-1}(x)$ is $[-1,1]$.

Reason: The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

12. Assertion: All trigonometric function have their inverses over their respective domains.

Reason: The inverse of $\tan^{-1} x$ exists for some $x \in \mathbb{R}$.

13. Assertion: The range of the function f(x) = $(x) = 2\sin^{-1} x + \frac{3\pi}{2}$ $x \in [-1,1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Reason: The range of the principal value branch of $\sin^{-1} x$ is $[0, \pi]$.

14. Assertion: Maximum value of $(\cos^{-1} x)^2$ is π^2.

Reason: Range of the principal value branch of $\cos^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

15. Assertion: Range of $[\sin^{-1} x + 2\cos^{-1} x]$ is $[0, \pi]$.

Reason: Principal value branch of $\sin^{-1}$ x has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

VERY SHORT ANSWERS

1. Find the domain of $y = \sin^{-1}(x^2 - 4)$.

2. Evaluate: $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$.

3. Evaluate $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}(\cos\pi) + \tan^{-1}(1)$.

4. Find the value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$.

5. Find the domain of the function $y = \cos^{-1}(|x-1|)$. Show your steps.

6. Draw the graph of the function: $y = 2\sin^{-1}(x), -\pi \le y \le \pi$.

7. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then find the value of $\alpha(\beta+\gamma) - \beta(\gamma+\alpha) + \gamma(\alpha+\beta)$.

8. Reduce $\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$ where $\frac{\pi}{2} < x < \pi$ in to simplest form.

9. Simplify: $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$.

10. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$.

11. Write the principal value of $\left[\cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\left(-\frac{1}{2}\right)\right]$.

12. Write the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

13. Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\pi}{4} - \frac{x}{2}, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

14. Prove that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\cos^{-1}x, \frac{1}{\sqrt{2}} \le x \le 1$.

15. Write $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right], x \neq 0$ in the simplest form.

16. Evaluate : $\cos\left[\sin^{-1}\frac{1}{4} + \sec^{-1}\frac{4}{3}\right]$.

17. Express in the simplest form: $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right), -\frac{\pi}{4} < x < \frac{\pi}{4}$.

18. If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, then find the value of x.

19. Draw the graph of $\cos^{-1}x$, where $x \in [-1,0]$. Also, write its range.

20. Draw the graph of $f(x) = \sin^{-1} x, x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of f(x).

21. Evaluate: $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$

22. Draw the graph of $f(x) = \sin^{-1} x, x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of f(x).

23. Evaluate: $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$.

24. Evaluate: $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}(\cos\pi) + \tan^{-1}(1)$.

25. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

26. Draw the graph of $\cos^{-1} x$, where $x \in [-1,0]$. Also, write its range.

27. Find the domain of $y = \sin^{-1}(x^2 - 4)$. Also, find its range.

28. Evaluate: $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$.

29. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] + \tan^{-1} 1$.

30. Simplify: $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$.

31. Find the value of $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cot^{-1}\left(\tan\frac{4\pi}{3}\right)$.

32. Find the domain of $f(x) = \cos^{-1}(1 - x^2)$. Also, find its range.

33. Evaluate: $\sec^2\left(\tan^{-1}\frac{1}{2}\right) + \mathrm{cosec}^2\left(\cot^{-1}\frac{1}{3}\right)$.

34. Find the value of $\left[\sin^2\left\{\cos^{-1}\left(\frac{3}{5}\right)\right\} + \tan^2\{\sec^{-1}(3)\}\right]$.

35. Evaluate: $\cot^2\{\mathrm{cosec}^{-1}3\} + \sin^2\left\{\cos^{-1}\left(\frac{1}{3}\right)\right\}$.

36. Find the value of k if $\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$.

37. Simplify : $\cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right]; \frac{1}{2} \leq x \leq 1$

SHORT ANSWERS

1. Prove that: $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x, -\frac{1}{\sqrt{2}} \leq x \leq 1.$

2. Prove that $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} = \frac{2b}{a}.$

3. Prove that if $\frac{1}{2} \leq x \leq 1$ then $\cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right] = \frac{\pi}{3}.$

4. Show that: $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$

5. Prove the following: $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}.$

6. Solve: $\sin[2\cos^{-1}\{\cot(2\tan^{-1} x)\}] = 0.$

7. Solve: $\tan(\cos^{-1} x) = \sin(\tan^{-1} 2), (x > 0).$

LONG ANSWERS

1. Simplify the following: (i) $\sin^{-1}\left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right)$ (ii) $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right).$

2. Solve the following: (i) $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$ (ii) $\cos^{-1}\frac{8}{x} + \cos^{-1}\frac{15}{x} = \frac{3\pi}{2}.$

3. Simplify the following: (i) $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ (ii) $\tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$

CASE BASED QUESTIONS

1. Read the following passage and answer the following questions:

In a school project Manish was asked to construct a triangle ABC in which two angles B and C are given by $\tan^{-1}\left(\frac{1}{2}\right)$ and $\tan^{-1}\left(\frac{1}{3}\right)$ respectively.

(i) Find the value of sinB

(ii) Find the value of cosC

(iii) Find the value of cos(B + C).

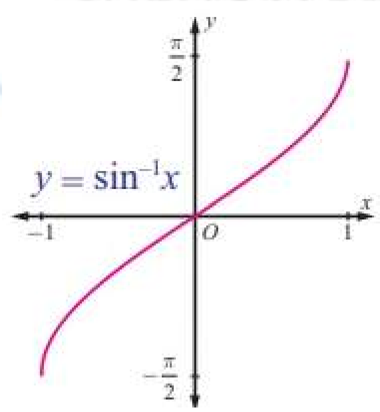
2. Two men on either side of a temple 30 meters high observe its top at the angles of elevation α and β respectively (as shown in the figure). The distance between the two men is $40\sqrt{3}$ meters and the distance between the first person A and the temple is $30\sqrt{3}$ meters.

Based on the above information, answer the following questions:

(i) Find the principal value of $\sin^{-1} \sin\left(\alpha + \frac{\pi}{3}\right)$.

(ii) Find the principal value of $\cos^{-1} \cos\left(\alpha + \frac{\pi}{3}\right)$.

3. Swathi and Uma are students of class XII. Teacher told them about inverse trigonometric functions and he sketched the graph of $\sin^{-1}x$ as follows.



Based on the above information, answer the following questions:

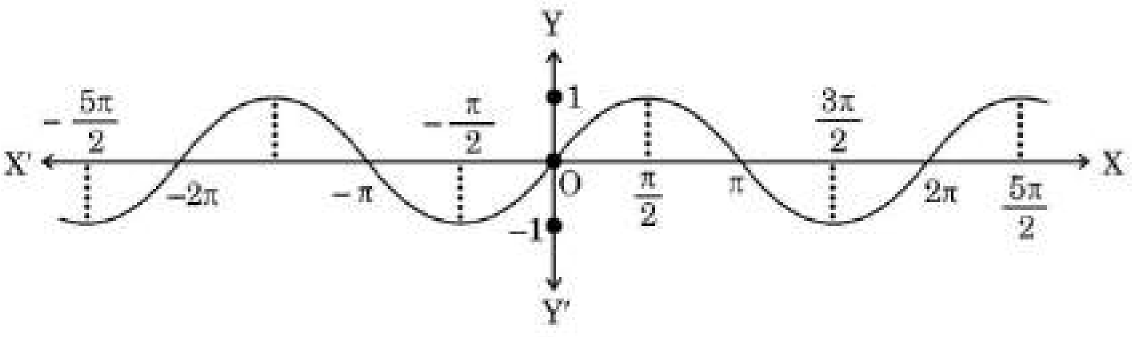
(i) Find the domain of $\sin^{-1}$ x.

(ii) Find the range of principal value branch of $\sin^{-1}$ x.

(iii) Find the domain of $\sin^{-1}(1-\mathrm{x})$.

3. If a function $f: X \rightarrow Y$ defined as f(x) = y is one-one and onto, then we can define a unique function g: Y → X such that g(y) = x, where $x \in X$ and y = f(x), y ∈ Y. Function g is called the inverse of function f.

The domain of sine function is R and function sine: R → R is neither one-one nor onto. The following graph shows the sine function.



y = sin x

Let sine function be defined from set A to [-1, 1] such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from [-1, 1] to A.

On the basis of the above information, answer the following questions:

(i) If A is the interval other that principal value branch, give an example of one such interval.

(ii) If $\sin^{-1}(x)$ is defined from [-1, 1] to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right)-\sin^{-1}(1)$.

(iii) Draw the graph of $\sin^{-1} x$ from [-1, 1] to its principal value branch.

(iv) Find the domain and range of $f(x)=2\sin^{-1}(1-x)$.

UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. What is the domain of the function $\cos^{-1}(2x - 3)$?

(a) [–1, 1] (b) (1, 2) (c) (–1, 1) (d) [1, 2]

2. The principal value of $[\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})]$ is

(a) π (b) $-\frac{\pi}{2}$ (c) 0 (d) $2\sqrt{3}$

3. The domain of $\sin^{-1}x$ + cosx is

(a) [–1, ∞) (b) (–1, 1) (c) [–1, 1] (d) (∞, 1]

4. Assertion: The range of the function $f(x) = 2\sin^{-1}x + \frac{3\pi}{2}$, where x ∈ $[-1,1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Reason: The range of the principal value branch of $\sin^{-1}x$ is [0, π].

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. Evaluate: $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$

6. Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

7. Find the principal value of $\cos^{-1}[\cos(-680°)]$.

SECTION C

Each carry 3 marks

8. Express $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$, where $-\frac{\pi}{4}<x<\frac{\pi}{4}$, in the simplest form.

9. Solve: $\sin^{-1}\frac{2a}{1+a^2}+\sin^{-1}\frac{2b}{1+b^2}=2\tan^{-1}x$

SECTION D

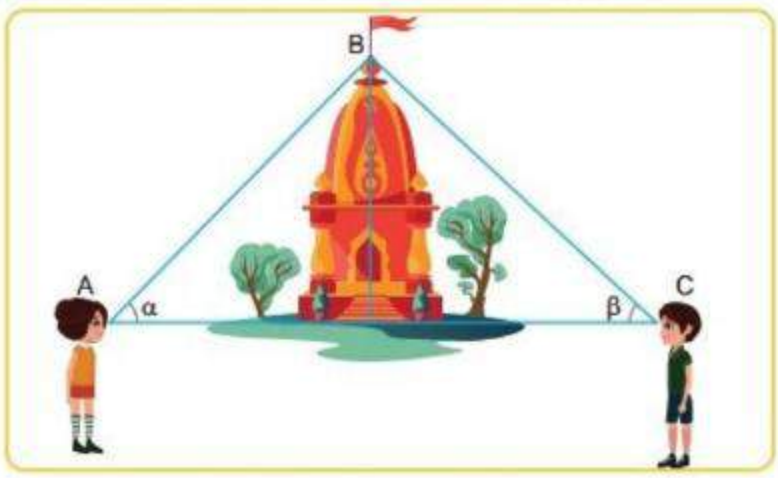
Each carry 5 marks

10. Express the following in simplest form: (i) $\tan^{-1}\left(\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right)$ (ii) $\tan^{-1}\left(\frac{a+bx}{a-bx}\right)$.

11. Express the following in simplest form: (i) $\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$ (ii) $\tan^{-1}\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$

SECTION E

12. Two men on either side of a temple 30 meters high observe its top at the angles of elevation α and β respectively (as shown in the figure). The distance between the two men is $40\sqrt{3}$ meters and the distance between the first person A and the temple is $30\sqrt{3}$ meters.



Based on the above information, answer the following questions:

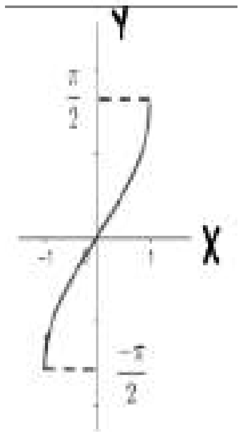
(i) Find $\angle CAB=\alpha$ in terms of $\sin^{-1}$.

(ii) Find $\angle CAB=\alpha$ in terms of $\sin^{-1}$.

(iii) Find $\angle BCA=\beta$ in terms of $\tan^{-1}$.

ANSWERS

MCQ	A-R	VSA	SA II	LA	CS
1. (c) $\left[-\frac{1}{2},\frac{1}{2}\right]$ 2. (b) $-\frac{2\pi}{5}$ 3. (a) $\frac{\pi}{12}$ 4. (a) $\frac{\pi}{6}$ 5. (d) 1 6. (c) $(-\infty,-1]\cup[0,\infty)$ 7. (d) $\frac{\sqrt{10}}{3}$ 8. (d) [-1, 1] 9. (b) $\frac{\pi}{3}$	1. (d) 2. (c) 3. (d) 4. (c) 5. (a) 6. (b) 7. (b) 8. (a) 9. (a) 10. (b) 11. (c)	1. $[-\sqrt{5},-\sqrt{3}]\cup[\sqrt{3},\sqrt{5}]$ 2. $\frac{5\pi}{4}$ 3. $\frac{3\pi}{2}$ 4. $-\frac{\pi}{10}$ 5. [0, 2] 7. 2 8. $\frac{\pi}{2}-\frac{x}{2}$ 9. $\frac{\pi}{4}+\frac{x}{2}$ 10. $\frac{\pi}{3}$	6. $\pm1,-1\pm\sqrt{2},1\pm\sqrt{2}$ 7. $\frac{\sqrt{5}}{3}$	1. (i) $\sin^{-1}x-\sin^{-1}\sqrt{x}$ (ii) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$ 2. (i) 13 (ii) ±17 3. (i) $\frac{x}{2}$ (ii) $\frac{\pi}{4}-\frac{x}{2}$	1. (i) $\frac{1}{\sqrt{5}}$ (ii) $\frac{3}{\sqrt{10}}$ (iii) $\frac{1}{\sqrt{2}}$ 2. (i) $\frac{\pi}{6}$ (ii) $\frac{2\pi}{3}$ 3. (i) [-1, 1] (ii) $[-\frac{\pi}{2},\frac{\pi}{2}]$ (iii) [0, 2] (i) $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ or any other

10. (b) $-\frac{\pi}{10}$ 11. (d) 1 12. (d) $\frac{\sqrt{1-x^2}}{x}$ 13. (d) $\frac{7\pi}{18}$ 14. (a) $\frac{\pi}{3}$ 15. (c) $\sqrt{3}$ 16. (d) $\frac{x}{\sqrt{1+x^2}}$ 17. (a) [1, 2] 18. (d) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ 19. (b) $\frac{1}{\sqrt{3}}$ 20. (d) (k, m) = (0, π) 21. (a) 1	12. (d) 13. (c) 14. (c) 15.(d)	11. $\frac{5\pi}{6}$ 12. $\frac{5\pi}{6}$ 15. $\frac{1}{2}\tan^{-1} x$ 16. $\frac{3\sqrt{15}-\sqrt{7}}{16}$ 17. $\frac{\pi}{4} - x$ 18. $\frac{1}{5}$ 21. $\frac{19\pi}{12}$ 22. Range = $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ 23. $\frac{5\pi}{4}$ 24. $\frac{3\pi}{2}$			interval corresponding to the domain $[-1,1]$ (ii) $\frac{-2\pi}{3}$ (iii)  (iv) Domain = [0, 2] and Range = [-π, π]

22. (b)		25. $\frac{-\pi}{12}$ 26. Range: $\left[\frac{\pi}{2}, \pi\right]$ 27. Domain is $[-\sqrt{5}, -\sqrt{3}$ $[\sqrt{3}, \sqrt{5}]$, Range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. 28. $\frac{\pi}{3}$ 29. $\frac{\pi}{2}$ 30. $\frac{\pi}{4} + \frac{x}{2}$ 31. $\frac{5\pi}{6}$ 32. Domain $=$ $[-\sqrt{2}, \sqrt{2}]$ &			

		Range $= [0, \pi]$ 33. $\frac{85}{36}$ 34. $\frac{216}{25}$ 35. $\frac{80}{9}$ 36. $k = \frac{1}{2}$ 37. $\frac{\pi}{3}$			

UNIT TEST

1. (d) [1, 2]

2. (b) $-\frac{\pi}{2}$

3. (c) [–1, 1]

4. (c) Assertion (A) is true but Reason (R) is false.

5. $\frac{\pi}{3}$

6. $\frac{2\pi}{3}$

7. $40°$ or $\frac{2\pi}{9}$

8. $x+\frac{\pi}{4}$

9. $\frac{a+b}{1-ab}$

10. (i) $\tan^{-1}\left(\frac{a}{b}\right)-x$ (ii) $\frac{\pi}{4}+\tan^{-1}\frac{bx}{a}$

11. (i) $\sqrt{1-x^2}$ (ii) $\frac{1}{2}\cos^{-1}\sqrt{x}$

12. (i) $\sin^{-1}\left(\frac{1}{2}\right)$ (ii) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (iii) $\tan^{-1}(\sqrt{3})$

CHAPTER 3 - MATRICES

MATRIX

Matrix is an ordered rectangular arrangement of numbers (or functions). The numbers (or functions) are called the element of the matrix. Horizontal line of elements is row of matrix and vertical line of elements is column of matrix.

$$
A=[a_{ij}]_{m\times n}=\begin{array}{c}
n \text{ columns} \\
\begin{array}{cccccc} \text{Column 1} & \text{Column 2} & & \text{Column } j & & \text{Column } n \end{array}\\
\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n}\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in}\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn}\end{bmatrix}
\end{array}
\begin{array}{l} \\ \\ \leftarrow \text{Row 1}\\ \leftarrow \text{Row 2}\\ \\ \leftarrow \text{Row } i\\ \\ \leftarrow \text{Row } m\end{array} \; m \text{ rows}
$$

ORDER OF MATRIX

If a matrix has ‘m’ rows and ‘n’ columns, then order of the matrix is m x n (read as m by n). A matrix of order m x n is depicted as $A = [a_{ij}]_{mxn}$; $i, j \in N$.

Note

- In general, a_{ij} means an element lying in the i^{th} row and j^{th} column.
- Number of elements in the matrix $A = [a_{ij}]_{m\ x\ n}$ is given as (m)(n). If number of rows and columns of matrix are 2 and 3 respectively then number of elements in the matrix = (2)(3) = 6 elements.

TYPES OF MATRICES

Column Matrix

A matrix having only one column and any number of rows is called a column matrix or column vector.

General Notation: $A = [a_{ij}]_{mx1}$.

Row Matrix

A matrix having only one row and any number of columns is called a row matrix or row vector.

General Notation: $A = [a_{ij}]_{1xn}$.

Square Matrix

Square matrix is a matrix in which the number of rows is equal to the number of columns.

Diagonal Matrix

A square matrix is said to be a diagonal matrix if all the elements expect those in the leading diagonal are zero. i.e., $a_{ij} = 0$, for all $i \neq j$.

A diagonal matrix is a square matrix with all non-diagonal elements equal to zero and diagonal elements not all zeroes.

Scalar Matrix

A diagonal matrix $A = [a_{ij}]_{mxm}$ is said to be a scalar matrix if its diagonal elements are equal.

i.e., $a_{ij} = \begin{cases} 0, & when\ i \neq j \\ k, & when\ i = j\ for\ some\ constant\ k \end{cases}$

A scalar matrix is a diagonal matrix in which all diagonal elements are equal.

Unit Matrix or Identity Matrix

A square matrix $A = [a_{ij}]_{mxm}$ is said to be an identity matrix if $a_{ij} = \begin{cases} 1, & if\ i = j \\ 0, & if\ i \neq j \end{cases}$

An identity matrix is a scalar matrix in which each diagonal elements is 1(unity).

Zero Matrix or Null Matrix

A matrix is said to be zero matrix or null matrix if each of its elements is 0.

Equality of Matrices

Two matrices A and B are said to be equal and written as A = B, if they are of the same order ans their corresponding elements are identical. i.e., $a_{ij} = b_{ij}$.

OPERATIONS ON MATRICES

(a) Addition of Matrices

Let A and B be two matrices each of order m x n. Then, the sum of matrices A + B is defined only if matrices A and B of same order.

If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ then $A + B = [a_{ij} + b_{ij}]_{m \times n}$.

(b) Subtraction of Matrices

Let A and B be two matrices of the same order, then subtraction of matrices A - B is defined as

If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ then $A - B = [a_{ij} - b_{ij}]_{m \times n}$.

(c) Multiplication of a Matrix by a Scalar

Let $A = [a_{ij}]_{m \times n}$ be a matrix and k be any scalar. Then, the matrix obtained by multiplying each element of A by k is called the scalar multiple of A by k and is denoted by kA is given as $kA = A = [ka_{ij}]_{m \times n}$.

(d) Multiplication of Matrices

The product of two matrices A and B is defined if the number of columns in A is equal to the number of rows in B. Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{jk}]_{n \times q}$ then the product of the matrices A and B is the matrix C of order m x p. To get the $(i, k)^{th}$ element c_{ik} of matrix C, we take the i^{th} row of A and k^{th} column of B, multiply them element wise and take the sum of these products.

NOTE

- The product AB is defined only when the number of columns in matrix A is equal to the number of rows in matrix B.
- In the product AB, A is called the pre-factor and B is called the post-factor.
- If A and B are square matrices of the same order say n, then both the product AB and BA are defined and each is a square matrix of order n.
- If A and B are m x n and n x p matrices respectively, then matrix AB will be m x p matrix i.e., order of matrix AB will be m x p.
- If two matrices A and B are such that AB is possible then it is not necessary, that product BA is also possible.
- If A is m x n matrix and both AB as well as BA is defined, then B will be n x m matrix.

TRANSPOSE OF A MATRIX

If $A = [a_{ij}]$ be an m x n matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. Transpose of the matrix A is denoted by A' or A^T. i.e., $A = [a_{ij}]_{m \times n}$ then $A^T = [a_{ji}]_{n \times m}$.

Properties of Transpose of the Matrices

- $(A')' = A$
- $(kA)' = kA'$
- $(A + B)' = A' + B'$
- $(AB)' = B'A'$.
- $(ABC)' = C'B'A'$.

SYMMETRIC AND SKEW SYMMETRIC MATRICES

Symmetric Matrix

A square matrix $A = [a_{ij}]$ is said to be a symmetric matrix if $A' = A$.

Skew Symmetric Matrix

A square matrix $A = [a_{ij}]$ is said to be a symmetric matrix if $A' = - A$.

Orthogonal Matrix

A matrix A is said to be orthogonal if $AA' = A'A = I$.

RESULTS

- If A is a square matrix, then (a) $A + A^T$ is always symmetric (b) $A - A^T$ is always skew-symmetric matrix.
- For any matrices, AA^T and A^TA are symmetric matrices.
- If A and B are two symmetric (or skew-symmetric) matrices of same order, then A + B is also symmetric (or skew-symmetric).
- If A is symmetric (or skew-symmetric), then kA (k is a scalar) is also symmetric (or skew-symmetric matrix).
- If A and B are symmetric matrices of the same order, then the product AB is symmetric iff AB = BA.
- Every square matrix can be expressed uniquely as the sum of a symmetric and a skew-symmetric matrix. i.e., $A = \frac{1}{2}[P + Q]$, where $P = A + A^T$ is symmetric and $Q = A - A^T$ is a skew-symmetric.
- The matrix B^TAB is symmetric or skew-symmetric according as A is symmetric or skew-symmetric matrix.
- All positive integral powers of a symmetric matrix are symmetric.
- All positive odd integral powers of a skew-symmetric matrix are skew-symmetric and positive even integral powers of a skew-symmetric are symmetric matrix.
- If A and B are symmetric matrices of the same order, then (a) AB – BA is a skew-symmetric (b) AB + BA is symmetric.

MULTIPLE CHOICE QUESTIONS

1. If $A = [a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, \text{when } i \neq j \\ 0, \text{when } i = j \end{cases}$, then A^2 is

(a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. If $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ are equal, then value of $ab - cd$ is:

(a) 4 (b) 16 (c) -4 (d) -16

3. For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ $P - Q$ is:

(a) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 3 \\ 0 & -3 \\ -1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

4. If $X = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ and $3X - \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ then $'a'$ is equal to $-$

(a) 1 (b) 2 (c) 0 (d) -2

5. If A and B are square matrices of the same order, then (A + B)(A – B) is equal to

(a) $A^2 - B^2$ (b) $A^2 - BA - AB - B^2$

(c) $A^2 - B^2 + BA - AB$ (d) $A^2 - BA + B^2 + AB$

6. Total number of possible matrices of order 3 × 3 with each entry 2 or 0 is

(a) 9 (b) 27 (c) 81 (d) 512

7. If A and B are two matrices of the order 3 × m and 3 × n respectively and m = n, then the order of matrix (5A – 2B) is

(a) m × 3 (b) 3 × 3 (c) m × n (d) 3 × n

8. The order of the single matrix obtained from $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \left\{ \begin{bmatrix} -1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 23 \\ 1 & 0 & 21 \end{bmatrix} \right\}$ is

(a) 2 x 3 (b) 2 x 2 (c) 3 x 2 (d) 3 x 3

9. A square matrix $A = [a_{ij}]_{n \times n}$ is called a diagonal matrix of $a_{ij} = 0$ for

(a) i = j (b) i < j (c) i > j (d) $i \neq j$

10. If $P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$, then PQ is equal to

(a) $\begin{bmatrix} -2 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

11. For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, P – Q is

(a) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 3 \\ -0 & -3 \\ -1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

12. A matrix $A = [a_{ij}]_{3\times3}$ is defined by $a_{ij} = \begin{cases} 2i + 3j, & i < j \\ 5, & i = j \\ 3i - 2j, & i > j \end{cases}$. The number of elements in A are more than 5 is

(a) 3 (b) 4 (c) 5 (d) 6

13. If $A = [a_{ij}]$ is a skew-symmetric matrix of order 3 then the value of $5a_{12} + 6a_{22} - 5a_{21}$ is

(a) 1 (b) 0 (c) $-10a_{12}$ (d) $-10a_{21}$

14. If the matrix $B = \begin{bmatrix} 2 & a & 5 \\ -1 & 4 & b \\ c & -4 & 9 \end{bmatrix}$ is a symmetric matrix, then $a + b + c$ is

(a) -1 (b) 0 (c) 1 (d) 2

15. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, then $A^2 - 4A =$

(a) I (b) 3I (c) 5I (d) 0

16. If A is a 2 x 3 matrix and AB is 2 x 5 matrix, then B must be a

(a) 3 x 5 matrix (b) 5 x 3 matrix (c) 3 x 2 matrix (d) 5 x 2 matrix

17. Choose the correct option

(a) Every scalar matrix is an identity matrix

(b) Every square matrix whose each element is 1 is an identity matrix

(c) Every scalar matrix is a diagonal matrix

(d) Every diagonal matrix is a scalar matrix

18. Choose the correct option

(a) If diagonal elements of a square matrix are 0 then it is skew symmetric

(b) If all the elements of a matrix are same then it is a scalar matrix

(c) Matrix denotes a number

(d) None of above

19. If $\begin{bmatrix} 6 & 0 \\ x-3 & x+3 \end{bmatrix}$ is a scalar matrix then value of x is

(a) 3 (b) -3 (c) 0 (d) 2

20. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^T$, then

(a) x = 0, y = 5 (b) x + y = 5 (c) x = y (d) None of these

21. If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ is

(a) 2AB (b) 2 BA (c) A + B (d) AB

22. If $X + \begin{bmatrix} 4 & 6 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 5 & -8 \end{bmatrix}$ then X is

(a) $\begin{bmatrix} -1 & -12 \\ 8 & -15 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -12 \\ -6 & -15 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -12 \\ 8 & -15 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 14 \\ -6 & -15 \end{bmatrix}$

23. If A is a 3 x 4 matrix and B is a matrix such that A^TB and B^TA are both defined then B is of the type

(a) 3 x 4 (b) 3 x 3 (c) 4 x 4 (d) 4 x 3

24. The number of all possible matrix of order 2X2 with entries 0, 1, 2 is

(a) 81 (b) 16 (c) 27 (d) 9

25. If $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ then y is

(a) 5 (b) 4 (c) 2 (d) -3

26. If a matrix has 36 elements, the number of possible orders it can have, is:

(a) 13 (b) 3 (c) 5 (d) 9

27. If the sum of all the elements of a 3 x 3 scalar matrix is 9, then the product of all its elements is

(a) 0 (b) 9 (c) 27 (d) 729

28. The number of all scalar matrices of order 3, with each entry -1, 0 or 1 is

(a) 1 (b) 3 (c) 2 (d) 3^9

29. If $A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is a scalar matrix, then y^x is equal to

(a) 0 (b) 1 (c) 7 (d) ±7

30. If for the matrix $A = \begin{bmatrix} \tan x & 1 \\ -1 & \tan x \end{bmatrix}$, $A + A' = 2\sqrt{3}I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

31. If $\mathrm{A} = [\mathrm{a_{ij}}]$ be a 3 x 3 matrix, where $\mathrm{a_{ij}} = \mathrm{i} - 3\mathrm{j}$, then which of the following is false?

(a) $a_{11} < 0$ (b) $a_{12} + a_{21} = -6$ (c) $a_{13} > a_{31}$ (d) a_{31} = 0

32. If $[x \quad 2 \quad 0]\begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = [3 \quad 1]\begin{bmatrix} -2 \\ x \end{bmatrix}$, then value of x is

(a) -1 (b) 0 (c) 1 (d) 2

33. If A is a square matrix and $A^2 = A$, then $(I + A)^2 - 3A$ is equal to :

(a) I (b) A (c) 2A (d) 3I

34. Find the matrix A^2, where $\mathrm{A} = [\mathrm{a_{ij}}]$ is a 2 x 2 matrix whose elements are given by $\mathrm{a_{ij}}$ = maximum (i, j) – minimum (i, j):

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

35. If a matrix A = [1 2 3], then the matrix AA′(where A′ is the transpose of A) is

(a) 14 (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ (d) [14]

36. If matrices A and B are of order 1 x 3 and 3 x 1 respectively, then the order of A′B′ is

(a) 1 x 1 (b) 3 x 1 (c) 1 x 3 (d) 3 x 3

37. The product of matrix P and Q is equal to a diagonal matrix. If the order of matrix Q is 3 x 2, then order of matrix P is

(a) 2 x 2 (b) 3 x 3 (c) 2 x 3 (d) 3 x 2

38. If A and B are two skew symmetric matrices, then (AB + BA) is

(a) a skew symmetric matrix (b) a symmetric matrix

(c) a null matrix (d) an identity matrix

39. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is

(a) 7 (b) 6 (c) 8 (d) 18

40. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and $2A + B$ is a null matrix, then B is equal to :

(a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

41. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(3I + 4A)(3I - 4A) = x^2 I$, then the value $(s) x$ is/are :

(a) $\pm\sqrt{7}$ (b) 0 (c) ± 5 (d) 25

42. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then A^{2023} is equal to

(a) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2023 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2023 & 0 \\ 0 & 2023 \end{bmatrix}$

43. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$ = P + Q, where P is a symmetric and Q is a skew symmetric matrix, then Q is

(a) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$

44. Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}, C = \begin{bmatrix} 9 & 8 & 7 \end{bmatrix}$, which of the following is defined?

(a) Only AB (b) Only AC (c) Only BA (d) All AB, AC and BA

45. For any square matrix A, $(A - A')$is always

(a) an identity matrix (b) a null matrix

(c) a skew symmetric matrix (d) a symmetric matrix

46. If A is a 2 x 3 matrix such that AB and AB' both are defined, then order of matrix B is

(a) 2 x 2 (b) 2 x 1 (c) 3 x 2 (d) 3 x 3

47. Number of symmetric matrices of order 3 x 3 with each entry 1 or -1 is

(a) 512 (b) 64 (c) 8 (d) 4

48. If $A = \begin{bmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of x + y + z is

(a) 10 (b) 6 (c) 8 (d) 0

49. A and B are skew-symmetric matrices of same order. AB is symmetric, if:

(a) AB = O (b) AB = -BA (c) AB = BA (d) BA = O

50. For what value of $x \in \left[0, \frac{\pi}{2}\right]$, is $A + A' = \sqrt{3}I$, where $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) 0 (d) $\frac{\pi}{2}$

51. A and B are square matrices of same order. If $(A + B)^2 = A^2 + B^2$, then :

(a) AB = BA (b) AB = -BA (c) AB = O (d) BA = O

52. If $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$, then the value of $(2x + y - z)$ is :

(a) 1 (b) 2 (c) 3 (d) 5

53. If A is a 3 x 4 matrix and B is a matrix such that $A'B$ and AB' are both defined, then the order of the matrix B is

(a) 3 x 4 (b) 3 x 3 (c) 4 x 4 (d) 4 x 3

54. If $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $A = B^2$, then x equals

(a) ±1 (b) -1 (c) 1 (d) 2

55. If $A = [a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, & \text{when } i \neq j \\ 0, & \text{when } i = j \end{cases}$ then A^2 is

(a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

56. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is

(a) 1 (b) -2 (c) 2 (d) -1

57. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is

(a) 1 (b) 2 (c) 0 (d) -2

58. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is

(a) 0 (b) 5 (c) 10 (d) 25

59. If $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $2a - (b + c)$ is

(a) 0 (b) 1 (c) -10 (d) 10

ASSERTION - REASON TYPE QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

1. Assertion: The possible dimensions of a matrix containing 32 elements is 6.

Reason: The No. of ways of expressing 32 as a product of two positive integers is 6.

2. Assertion: The matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$ is a skew symmetric matrix.

Reason: All the diagonals elements of a skew symmetric matrix are zero.

3. Assertion: If A and B are skew symmetric matrices then AB – BA is a symmetric matrix.

Reason: For a matrix to be a symmetric $A' = -A$.

4. Assertion: If the order of matrices A, B and C are 2×3, 3×4, 4×2 then order of matrix ABC is 2×2.

Reason: We can multiply two matrices if number of columns of first matrix is equal to number of rows of second matrix.

5. Let A, B, C are three matrices of same order. Now, consider the following statements:

Assertion: If A = B, then AC = BC.

Reason: If AC = BC, then A = B.

6. Assertion: If A = $A = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$, then $A^T A = I$.

Reason: For any square matrix A, $(A^T)^T = A$.

7. Assertion: If A is a symmetric matrix then BAB' is symmetric matrix.

Reason: For any three matrices A, B and C, $(ABC)' = C'B'A'$.

8. Assertion: A and B be two symmetric matrix of order 3, A(BA) and (AB)A are symmetric matrices.

Reason: AB is symmetric matrix if multiplication A and B is commutative.

9. Assertion: If $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $(A + B)^2 = A^2 + B^2 + 2AB$.

Reason: In the above, AB = BA.

10. Assertion: $A = \begin{pmatrix} \cos x & \sin x \\ \cos x & \sin x \end{pmatrix}$ and $B = \begin{pmatrix} \cos x & \cos x \\ \sin x & \sin x \end{pmatrix}$, then $AB \neq I$.

Reason: The product of two matrices can never be equal to an identity matrix.

11. Assertion: The possible dimensions of a matrix containing 32 elements is 6.

Reason: The No. of ways of expressing 32 as a product of two positive integers is 6.

12. Assertion: A and B are symmetric matrices, then AB =BA and AB is also symmetric.

Reason: A and B are symmetric matrices.

13. Assertion: If $\begin{pmatrix} x+y & 2x+z \\ 2x-y & 3z+w \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 0 & 13 \end{pmatrix}$, then x = $\frac{-1}{3}$, y = $\frac{-2}{3}$, z = $\frac{17}{3}$ and w = -4.

Reason: Two matrices $A = [a_{ij}], B = [b_{ij}]$ of the same type are said to be equal if $a_{ij} = b_{ij}$.

14. Assertion: If $\begin{pmatrix} x-y & z \\ 2x-y & w \end{pmatrix} =$ Transpose of $\begin{pmatrix} -1 & 0 \\ 4 & 5 \end{pmatrix}$, then $x = 3, y = 4, z = 5, w = 6$.

Reason: An m x n is called a square matrix if m = n.

15. Assertion: If $Y = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $X + Y = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$, then $X = \begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix}$.

Reason: If A, B and C are three matrices of the same order, then A+ (B + C) = (A + B) + C.

16. Assertion: For matrix A = $\begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0,2\pi]$, $|A| \in [2,4]$.

Reason: $\cos\theta \in [-1,1], \forall\theta \in [0,2\pi]$.

17. Assertion: For any symmetric matrix A, $B'AB$ is a skew-symmetric matrix.

Reason: A square matrix P is skew-symmetric if $P' = -P$.

18. Assertion: Every scalar matrix is a diagonal matrix.

Reason: In a diagonal matrix, all the diagonal elements are 0.

VERY SHORT ANSWERS

1. If $A = \begin{bmatrix} 1 & -2 \\ 3 & 0 \\ 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -3 & 4 \\ 1 & 2 & 0 \end{bmatrix}$, prove that $(AB)' = B'A'$.

2. If $A = \text{diag}[2,5,-3]$ and $B = \text{diag}[-1,6,4]$, find $3A - 2B + 4I$.

3. If $A = \begin{bmatrix} 1 & 3 & 0 \\ -3 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$. Find the matrix C such that A + B + C is a zero matrix.

4. Construct a 4 x 3 matrix whose elements a_{ij} are given by $a_{ij} = \begin{cases} i^2 & \text{if } i < j \\ \frac{i}{j} & \text{if } i = j \\ j^2 & \text{if } i > j \end{cases}$

5. If $A = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$, find the matrix X such that $2A + 3X = 5B$.

6. Find the matrix X such that $\begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} - 3X = \begin{bmatrix} -3 & 4 \\ 5 & -1 \end{bmatrix}$.

7. Express the following as a single matrix: $\begin{bmatrix} 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 & 5 \\ 0 & 2 & 4 \\ -7 & 5 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -5 & 7 \end{bmatrix}$

8. Express the following as a single matrix: $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix} \right\}$.

9. Find a 2 x 2 matrix B such that $\begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix} B = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$

10. If $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} X = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$, find X.

11. If $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and $A^2 + aA - bI = 0$, find the values of a and b.

12. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, where $i = \sqrt{-1}$, verify that $(A + B)^2 = A^2 + B^2$.

13. If A and B are symmetric, then prove that AB + BA is symmetric and AB – BA is skew symmetric.

14. If A and B are square symmetric matrices of same order, then prove that AB is symmetric if and only if A and B commute.

15. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix}$, then show that AA' and $A'A$ are both symmetric.

16. If $A = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$, verify that $(AB)' = B'A'$.

17. Find the values of x, y, z, if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ obeys the law $A'A = I$.

18. Show that the matrix B^TAB is symmetric or skew-symmetric accordingly when A is symmetric or skew-symmetric.

19. If matrix $A = [a_{ij}]$, where $a_{ij} = \begin{cases} 3i - j, \text{if } i \le j \\ |i - 2j|, \text{if } i > j \end{cases}$, find $\frac{a_{12}+a_{21}+a_{23}}{a_{22}}$.

20. Find $2A^2$ when $x = \frac{\pi}{3}$ where $A = \begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$.

SHORT ANSWERS

1. Find A and B, if $2A + 3B = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & -1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 6 & 2 \end{bmatrix}$.

2. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = O$. Use this result to find A^5.

3. Find matrix A such that $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$.

4. Find X and Y if $2X + Y = \begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix}$ and $X - 2Y = \begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$.

5. If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$, verify that (i) (AB)C = A(BC) (ii) A(B + C) = AB + AC.

6. 7. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$, prove that $(A + B)^2 \neq A^2 + 2AB + B^2$.

7. Compute $A^3 - 5A^2 + 4A + 7I$, where I is the unit matrix and $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$.

8. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^3 = a^3I + 3a^2bA$.

9. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$ and I is the unit matrix of order 3, show that $A^3 = pI + qA + rA^2$.

10. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Use this result to find A^4.

11. Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.

12. Find x and y if the matrix $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ may satisfy the condition $AA' = A'A = I_3$.

13. Express the matrix $\begin{bmatrix} 2 & 3 & -2 \\ -1 & 0 & 8 \\ 4 & 0 & 3 \end{bmatrix}$ as the sum of a symmetric and skew-matrices.

14. Find the symmetric and skew − symmetric parts of the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 6 & 8 & 1 \\ 3 & 5 & 7 \end{bmatrix}$.

15. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then find the values of a and b.

LONG ANSWERS

1. If $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $G(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$, show that $[F(\alpha)G(\beta)]' = G(-\beta)F(-\alpha)$.

2. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$.

3. If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ then show that $A^3 - 4A^2 - 3A + 11I = 0$, Hence find A^{-1}.

4. If $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$.

5. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that $A^3 - 6A^2 + 7A + 2I = 0$.

CASE BASED QUESTIONS

1. A manufacture produces three stationary products pencils, eraser and sharpener which he sells in two markets. Annual sales are indicated below:

Market	Product (in numbers)		
	Pencil	**Eraser**	**Sharpener**
A	10000	2000	18000
B	6000	20000	8000

If the unit sale price of pencil, eraser and sharpener are ₹ 2.50, ₹ 1.50 and ₹ 1.00 respectively and unit cost of the above three commodities are ₹ 2.00, ₹ 1.00 and ₹ 0.50 respectively.

Based on the above information answer the following:

(i) Find the total revenue of market A.

(ii) Find the total revenue of market B.

(iii) Find the profit in market A and B respectively.

2. Three students Ram, Mohan and Ankit go to a shop to buy stationary. Ram purchases 2 dozen note books, 1 dozen pens and 4 pencils. Mohan purchases 1 dozen note book , 6 pens and 8 pencils. Ankit purchases 6 note books , 4 pens and 6 pencils. A note book costs ₹ 15, a pen costs ₹4.50 and a pencil costs ₹ 1.50. Let A and B be the matrices representing the number of items purchased by the three students and the prices of items respectively.

Based on the above information answer the following questions.

(i) What is the order of the matrix B representing the prices of the items?

(ii) What is the order of the matrix A representing items purchased by the three students?

(iii) Find the total amount of bill by all the three students.

3. Three schools DPS, CVC and NVS decided to organize a fair for collecting money for helping the food victims. They sold handmade fans, mats and plates from recycled material at a cost of ₹ 25, ₹ 100, and ₹50 each respectively. The numbers of articles sold are given below:

Article \ School	**DPS**	**CVC**	**NVS**
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Based on the above information answer the following questions.

(i) What is the total money collected by all the three schools?

(ii) If the number of handmade fans and plates are interchanged for all the schools, then what is the total money collected by all the three schools.

4. On her birthday, Swathi decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However if there were 16 children more, everyone would have got ₹10 less. Let the number of children be x and the amount distributed by Swathi for one child be ₹ y.

(i) Write the matrix equation to represent the information given above.

(ii) Find the number of children who were given some money by Swathi.

(iii) How much amount Swathi spends in distributing the money to all the children of Orphanage?

5. Gautam buys 5 pens , 3 bags and 1 Instrument box and pays a sum of ₹160 . From the same shop Vikram buys 2 pens , 1 bag and 3 Instrument boxes and pays a sum of ₹190. Ankur buys 1 pen , 2 bags and 4 Instrument boxes and pays a sum of ₹ 250

Based on the above information answer the following questions.

(i) Write the matrix equation to represent the information given above.

(ii) Find $P = A^2 - 5A$.

6. To promote the making of toilets for women, an organization tried to generate awareness through (i) house calls (ii) emails and (iii) announcements.

The cost for each mode per attempt is given below (i) ₹ 50 (ii) ₹ 20 (iii) ₹40

The number of attempts made in the villages X,Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Also the chance of making of toilets corresponding to one attempts of given modes is

(i) 2% (ii) 4% (iii) 20%

Based on the above information answer the following questions :

(i) What is the cost incurred by the organization on village X?

(ii)What is the cost incurred by the village Y?

(iii) What is the total number of toilets that can be expected after the promotion in village X?

7. A trust fund has ₹ 35000 that must be invested in two different types of bonds say X and Y .The first bond pays 10% interest per annum ,which will be given to an old age home and second one pays 8% interest per annum which will be given to women welfare association. Let A be a 1 x 2 and B be a 2 x 1 matrix representing the investment and interest rate on each bond respectively.

Based on the above information answer the following questions

(i) What is the total amount of interest received on both bonds if ₹15000 is invested in bond X ?

(ii)What is the amount of investment in bond Y if the amount of interest given to old age home is ₹500?

UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. If the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ then $A^4 =$

(a) 4A (b) 8A (c) 16A (d) 6A

2. If A is a symmetric matrix and B is a skew symmetric matrix such that $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$,

then AB is

(a) $\begin{bmatrix} 4 & -1 \\ -1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & 2 \\ 1 & 1 \end{bmatrix}$

3. If a matrix A is such that $3A^3 + 2A^2 + 5A + I = 0$, then what is A^{-1} equal to?

(a) $-(3A^2 + 2A + 5)$ (b) $(3A^2 + 2A + 5I)$

(c) $(3A^2 - 2A - 5I)$ (d) $-(3A^2 + 2A + 5I)$

4. Assertion (A): If A is a square matrix such that $A^2 = I$,then $(I + A)^3 - 3A = I$

Reason (R): AI = IA = A, where I is the identity matrix.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. Construct a 3 x 3 matrix $A = [a_{ij}]$ such that $a_{ij} = \begin{cases} 2^i; & i > j \\ i.j; & i = j \\ 3^j; & i < j \end{cases}$

6. Find the values of x and y, if $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ satisfies the equation $A^2 + xA + yI = 0$

7. If $A = \begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$ then prove that $A^3 = \begin{pmatrix} \cos 3x & -\sin 3x \\ \sin 3x & \cos 3x \end{pmatrix}$

SECTION C

Each carry 3 marks

8. If $f(x) = \frac{1}{\sqrt{1-x^2}}\begin{pmatrix} 1 & -x \\ -x & 1 \end{pmatrix}$, prove that f(x).f(y) = $f\left(\frac{x+y}{1+xy}\right)$. Hence show that f(x).f(-x) = 1 where |x| < 1.

9. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = O$. Use this result to find A^5.

SECTION D

Each carry 5 marks

10. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = 0$.

11. (i) If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = 8A + kI$.

(ii) If $A = \begin{bmatrix} 2 & 3 \\ 3 & 10 \end{bmatrix}$ and I is an identity matrix of order 2, show that (2I – A)(10I – A) = 9I.

SECTION E

12. Amit, Biraj and Chirag were given the task of creating square matrix of order 2. Below are the matrices created by them namely A , B and C respectively.

$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

(i) Find A + B + C

(ii) Find AC – BC

(iii) Find the matrix (a+b) B , a = 4, b = -2.

ANSWERS

MCQ	A-R	VSA	SA	LA	CS
1. (d) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ 2. (a) 4 3. (b) $\begin{bmatrix}4 & 3\\-3 & 0\\-1 & -2\end{bmatrix}$ 4. (b) 2 5. (c) $A^2 - B^2 + BA - AB$ 6. (d) 512 7. (d) $3 \times n$ 8. (d) 3 x 3 9. (d) $i \neq j$ 10. (b) $\begin{bmatrix}2 & -2\\-1 & 1\\-1 & 1\end{bmatrix}$ 11. (b) $\begin{bmatrix}4 & 3\\-3 & 0\\-1 & -2\end{bmatrix}$	1. (c) 2. (b) 3. (c) 4. (b) 5. (d) 6. (b) 7. (a) 8. (b) 9. (a) 10. (c) 11. (c) 12. (c) 13. (a) 14. (d)	2. $\mathrm{diag}[12, 7, -13]$ 3. $C = \begin{bmatrix}-2 & -3 & -1\\1 & 1 & -6\end{bmatrix}$ 4. $\begin{bmatrix}1 & 1 & 1\\1 & 1 & 4\\1 & 4 & 1\\1 & 4 & 9\end{bmatrix}$ 5. $\frac{1}{3}\begin{bmatrix}36 & 4\\12 & -14\\25 & 28\end{bmatrix}$ 6. $\begin{bmatrix}\frac{5}{3} & \frac{-7}{3}\\\frac{-1}{3} & \frac{-1}{3}\end{bmatrix}$ 7. $[-21 \quad 15 \quad -10]$ 8. $= \begin{bmatrix}0 & -1 & 1\\2 & 0 & -2\\5 & -2 & -3\end{bmatrix}$ 9. $\begin{bmatrix}6 & -5\\-5 & 6\end{bmatrix}$	1. $A = \frac{1}{7}\begin{bmatrix}11 & -4 & 9\\1 & 18 & 4\end{bmatrix}$ & B = $\frac{1}{7}\begin{bmatrix}-5 & -2 & 1\\4 & -12 & -5\end{bmatrix}$ 2. $A^5 = \begin{bmatrix}-118 & -93\\31 & -118\end{bmatrix}$ 3. $A = \begin{bmatrix}1 & -2\\3 & 4\end{bmatrix}$ 4. $X = \begin{bmatrix}1 & 2 & 3\\3 & 1 & 2\end{bmatrix}$ and Y = $\begin{bmatrix}2 & 0 & 1\\1 & 1 & 0\end{bmatrix}$ 7. $\begin{bmatrix}1 & -12 & 12\\-6 & -5 & 12\\6 & 12 & -17\end{bmatrix}$ 10. $A^4 = \begin{bmatrix}39 & 55\\-55 & -16\end{bmatrix}$ 11. $X = \begin{bmatrix}1 & -2\\2 & 0\end{bmatrix}$ 12. x = 2 and y = -1	2. X = $\begin{bmatrix}1 & 1 & 3\\1 & 3 & 10\\5 & -4 & -2\end{bmatrix}$	1. (i) ₹46000 (ii) ₹53000 (iii) ₹15000 & ₹17000 2. (i) 3x1 (ii) 3x3 (iii) ₹ 756 3. (i) ₹ 21000 (ii) ₹ 21250 4. (i) $\begin{bmatrix}5 & -4\\5 & -8\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} =$ $\begin{bmatrix}40\\-80\end{bmatrix}$ (ii) 32 (iii) ₹ 960

12. (b) 4 13. (d) $-10a_{21}$ 14. (b) 0 15. (c) 5I 16. (a) 3 x 5 matrix 17. (c) Every scalar matrix is a diagonal matrix 18. (d) none of above 19. (a) 3 20. (c) x = y 21. (c) A + B 22. (a) $\begin{bmatrix} -1 & -12 \\ 8 & -15 \end{bmatrix}$ 23. (b) 3 x 3 24. (a) 81 25. (c) 2	15. (b) 16. (a) 17. (d) 18. (c)	10. $X = [-1 \quad 2 \quad 1]$ 11. $a = 2, b = 11$ 17. $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}},$ $z = \pm\frac{1}{\sqrt{3}}$ 19. 1 20. $\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$	13. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 4 \\ 1 & 4 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix}$ 14. $\begin{bmatrix} 1 & 4 & \frac{7}{2} \\ 4 & 8 & 3 \\ \frac{7}{2} & 3 & 7 \end{bmatrix} \begin{bmatrix} 0 & -2 & \frac{1}{2} \\ 2 & 0 & -2 \\ -\frac{1}{2} & 2 & 0 \end{bmatrix}$ 15. a = 1 and b = 4		5. (i) $\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$ (ii) $\begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$ 6. (i) 30000 (ii) 23000 (iii) 40 7. (i) ₹3100 (ii) ₹30000

26. (d) 9 27. (a) 0 28. (b) 3 29. (b) 1 30. (c) $\frac{\pi}{3}$ 31. (c) $a_{13} > a_{31}$ 32. (a) -1 33. (a) I 34. (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 35. (d) [14] 36. (d) 3 x 3					

37. (c) 2 x 3 38. (b) a symmetric matrix 39. (d) 18 40.(b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$ 41. (c) ± 5 42. (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 43. (b) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ 44. (a) Only AB 45. (c) a skew symmetric matrix 46. (d) 3 x 3					

47. (b) 64 48. (d) 0 49. (c) AB = BA 50. (b) $\frac{\pi}{6}$ 51. (b) AB = -BA 52. (d) 5 53. (a) 3 x 4 54. (c) 1 55. (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 56. (c) 2 57. (b) 2					

58. (d) 25 59. (a) 0					

UNIT TEST

1. (c) 16A

2. (b) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

3. (d) $-(3A^2 + 2A + 5I)$

4. (d) Assertion (A) is false but Reason (R) is true.

5. $\begin{pmatrix} 1 & 9 & 27 \\ 4 & 4 & 27 \\ 8 & 8 & 9 \end{pmatrix}$

6. x = -2 and y = 0

7. Proof

8. Proof

9. $\begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$

10. Proof

11. (i) – 7 (ii) Proof

12. (i) $\begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$ (ii) $\begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$

CHAPTER 4 – DETERMINANTS

DETERMINANT

To every square matrix A = [aij] of order n x n, we can associate a number (real or complex) called determinant of the square matrix A, where a_{ij} = (i, j)th element of A. Determinant of A is denoted by det A or |A| or Δ.

Remarks

(a) For matrix A, |A| is read as determinant of A and not modulus of A.

(b) Only square matrixes have determinants.

DETERMINANT OF A MATRIX ORDER ONE

Let A = [a] be the matrix of order 1, then determinant of A is defined to be equal to a.

Example: Let A = [6], then |A| = 6

DETERMINANT OF A MATRIX ORDER TWO

Let A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a matrix of order 2 x 2, then the determinant of A is defined as

det (A) = |A| = Δ = $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ = $a_{11}\, a_{22} - a_{21}\, a_{12}$

DETERMINANT OF A MATRIX OF ORDER 3 × 3

Determinant of a matrix of order three can be determined by expressing it in terms of second order determinants. This is known as expansion of a determinant along a row (or a column). There are six ways of expanding a determinant of order 3 corresponding to each of three rows (R_1, R_2 and R_3) and three columns (C_1 , C_2 and C_3) giving the same value.

Note: For easier calculations, we shall expand the determinant along that row or column which contains maximum number of zeros.

Result: for any matrix A of order n, k is any scalar then $|kA| = k^n\,|A|$

Remarks:

1. The value of determinant is same when expanded by any row or any column.

2. If a row or a column of a determinant consists of all zeros, the value of the determinant is zero.

3. Always expand a determinant along a row or column with maximum number of zeros.

4. If each element above or below the main diagonal of a determinant is zero, then the value of the determinant is the product of elements along the main diagonal.

AREA OF TRIANGLE

Area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Note

- Since area is a positive quantity, we take absolute value of the determinant.
- If area is given, use both positive and negative values of the determinant for calculation.
- If the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear, then $\Delta = 0$.
- The equation of a line passing through (x_1, y_1) and (x_2, y_2) can be obtained by the expression given $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

MINORS AND COFACTORS

Minors

Minor of an element a_{ij} of determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which a_{ij} lies. Minor of an element aij is denoted by M_{ij}.

Cofactors

Cofactor of an element a_{ij} is denoted by A_{ij} and it is defined by $A_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is minor of a_{ij}.

Remark

Δ = sum of the product of elements of any row (or column) with their corresponding cofactors.

i.e., $\Delta = a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$

$\Delta = a_{21} A_{21} + a_{22} A_{22} + a_{23} A_{33}$

$\Delta = a_{31} A_{31} + a_{32} A_{32} + a_{33} A_{33}$

Note:

If elements of a row (or column) are multiplied with cofactors of any other row (or column) then their sum is zero.

Example: $a_{11} A_{21} + a_{12} A_{22} + a_{13} A_{23} = 0$.

ADJOINT OF A MATRIX

The adjoint of a square matrix $A = [a_{ij}]_{n \times n}$ is defined as the transpose of the matrix $[A_{ij}]_{n \times n}$, where A_{ij} is the cofactor of the element a_{ij}. Adjoint of the matrix A is denoted by adjA.

Properties of Adjoint

For any square matrices A and B of order n

(i) $A(\text{adj}A) = (\text{adj}A)A = |A|I$

(ii) $|\text{adj}A| = |A|^{n-1}$.

(iii) $|A.\text{adj}A| = |A|^n$

(iv) $\text{adj}(AB) = (\text{adj}B)(\text{adj}A)$.

(v) $|k\text{adj}A| = k^n|A|^{n-1}$

(vi) $\text{adj}(A^{|}) = (\text{adj}A)^{|}$

(vii) $(\text{adj}A)^{-1} = \text{adj}(A^{-1})$

(viii) $\text{adj}(kA) = k^{n-1}\text{adj}(A)$

(ix) $|\text{adj}(\text{adj}A)| = |A|^{(n-1)^2}$

SINGULAR MATRIX AND NON-SINGULAR MATRIX

Singular Matrix

A square matrix A is said to be singular if $|A| = 0$.

Non-Singular Matrix

A square matrix A is said to be non-singular if $|A| \neq 0$.

INVERSE OF A MATRIX

Inverse of a square matrix A exist if A is non-singular matrix.

$$A^{-1} = \frac{1}{|A|} \text{adj}A.$$

Properties of Inverse of a Square Matrix

(i) $AA^{-1} = A^{-1}A = I$

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

(iii) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

(iv) $(A^{-1})^{-1} = A$

(v) $(A^{|})^{-1} = (A^{-1})^{|}$

(vi) $|A^{-1}| = \frac{1}{|A|}$

Note: $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}^{-1} = \begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{bmatrix}; a \neq 0, b \neq 0, c \neq 0$

SYSTEM OF LINEAR EQUATIONS

Consistent System

A system of equations is said to be consistent if its solutions (one or more) exists.

Inconsistent System

A system of equations is said to be inconsistent if its solution does not exist.

SOLUTION OF SYSTEM OF LINEAR EQUATIONS USING THE INVERSE OF MATRIX.

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$.

Matrix form of the above equations can be written as

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

i.e A X = B

$X = A^{-1}B$, such that $|A| \neq 0$.

Criteria of Consistency

- If $|A| \neq 0$, then the system of equations is said to be consistent and has a unique solution.
- If $|A| = 0$ and $(\text{adj}A)B = 0$, then the system of equations is consistent and has infinitely many solutions.
- If $|A| = 0$ and $(\text{adj}A)B \neq 0$, then the system of equations is inconsistent and has no solution.

MULTIPLE CHOICE QUESTIONS

1. Three points P(2x, x + 3). Q(0, x) and R(x + 3, x + 6) are collinear then value of x is

(a) 0 (b) 2 (c) 3 (d) 1

2. If C_{ij} denotes the cofactor of element P_ij of the matrix $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$, then the value of $C_{31}.C_{23}$ is

(a) 5 (b) 24 (c) -24 (d) -5

3. If A is a square matrix of order 3 and |A| = -5 then |adjA| is

(a) 125 (b) -25 (c) 25 (d) ±25

4. If for the matrix A = $\begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$, $|A^3| = 125$, then value of α is

(a) ±3 (b) -3 (c) ±1 (d) 1

5. The inverse of the matrix X = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is:

(a) $24\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ (b) $\frac{1}{24}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\frac{1}{24}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

6. If A is a 3 x 3 matrix such that |A| = 8, then value of |3A| is

(a) 8 (b) 24 (c) 72 (d) 216

7. If A is a skew symmetric matrix of order 3, then the value of |A| is

(a) 3 (b) 0 (c) 9 (d) 27

8. The area of a triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 square units. Then value of k is

(a) 9 (b) 3 (c) -9 (d) 6

9. The equation of the line joining the points (1, 2) and (3, 6) is

(a) y = 2x (b) x = 3y (c) y = x (d) 4x – y = 5

10. If A and B are matrices of order 3 and |A| = 5, |B| = 3, then value of |3AB| is

(a) 605 (b) 205 (c) 405 (d) 305

11. If $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -1 \end{vmatrix} = 0$, then k is

(a) -1 (b) 0 (c) 1 (d) $\frac{33}{8}$

12. If A = $\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$ and A(adjA) = k $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the value of k is

(a) 0 (b) 1 (c) 3 (d) 2

13. If k is a scalar and I is a unit matrix of order 3, then adj(kI) is

(a) k^3I (b) k^2I (c) $-k^3I$ (d) $-k^2I$

14. If A is a square matrix of order 3 and |A| = 7, then the value of |adjA| is

(a) 45 (b) 48 (c) 49 (d) None of these

15. If A is a square matrix of order 3 such that |adjA| = 64, then |A| is

(a) ±5 (b) ±8 (c) ±6 (d) None of these

16. If A = $\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$ and A-1 = xA, then value of x is

(a) 7 (b) -7 (c) $\frac{1}{7}$ (d) -$\frac{1}{7}$

17. If $\begin{bmatrix} x & y^3 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 2 & 0 \end{bmatrix}$, then $\begin{bmatrix} x & y \\ 2 & 0 \end{bmatrix}^{-1}$ is

(a) $\begin{bmatrix} 0 & -2 \\ -2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & -8 \\ -2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{4} \end{bmatrix}$

18. If A = $\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then $(B^{-1}A^{-1})^{-1}$ is

(a) $\begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -2 \\ 2 & 2 \end{bmatrix}$ (c) $\frac{1}{10}\begin{bmatrix} 2 & 2 \\ -2 & 3 \end{bmatrix}$ (d) $\frac{1}{10}\begin{bmatrix} 3 & 2 \\ -2 & 2 \end{bmatrix}$

19. If A = $\begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$ then A^{-1} exists if

(a) λ = 2 (b) λ ≠ 2 (c) λ ≠ -2 (d) None of these

20. If A is a square matrix of order 3 and |A| = 7 then $|2A^T|$ =

(a) 28 (b) 14 (c) 56 (d) $\frac{8}{7}$

21. If A is non-singular square matrix of order 3 and |A| = 3, then |adj(adjA)| is

(a) 3 (b) 9 (c) 27 (d) 81

22. There are two values of a which makes the determinant $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix}$ equal to 86, then the sum of these two values is

(a) 4 (b) 5 (c) -4 (d) 9

23. For any matrix A, $A^3 = I$, then $A^{-1} =$

(a) A^2 (b) A^3 (c) A (d) I

24. The number of real roots of the equation $x{-}6{-}12{-}3xx{-}3{-}32x$

24. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is

(a) 3 (b) 0 (c) -1 (d) 1

25. If A is a non-singular matrix of order 3 x 3 and $|\text{adj}A| = |A|^k$, then value of k is

(a) -2 (b) 2 (c) 4 (d) 1

26. For the matrix equation AB = AC we say B = C provided A is

(a) Singular matrix (b) Square matrix

(c) Skew-symmetric matrix (d) Non-singular matrix

27. For any 2×2 matrix A, if $A \cdot (\text{adj } A) = \begin{pmatrix} 12 & 0 \\ 0 & 12 \end{pmatrix}$, then $|A|^3$ equal to

(a) 12^2 (b) 12^3 (c) 12^4 (d) None of these

28. Let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$, then $\lim\limits_{t \to 0} \frac{f(t)}{t^2}$

(a) 0 (b) -1 (c) 2 (d) 3

29. If A =diag (1, 2, 3), then $|A^2| =$

(a) 4 (b) 25 (c) 9 (d) 36

30. The value of $\begin{vmatrix} \log_2 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$ is

(a) 7 (b) 10 (c) 13 (d) 17

31. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is

(a) 0 (b) 1 (c) 2 (d) 4

32. For which value of x are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal?

(a) ±3 (b) -3 (c) ± 2 (d) 2

33. The value of the cofactor of the element of second row and third column in the matrix $\begin{bmatrix} 4 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$ is

(a) 5 (b) -5 (c) -11 (d) 11

34. If a_{ij} and A_{ij} represent the $(ij)^{th}$ element and its cofactor of $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ is

(a) 0 (b) -28 (c) 114 (d) -114

35. If A is a square matrix of order 3 and |A| = 6, then the value of |adjA| is

(a) 6 (b) 36 (c) 27 (d) 216

36. If A is a square matrix of order 3 such that the value of |adjA| = 8, then the value of $|A^T|$ is

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 8 (d) $2\sqrt{2}$

37. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is

(a) -4 (b) 1 (c) 3 (d) 4

38. If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is

(a) 1 (b) 2 (c) 3 (d) 4

39. If A is square matrix of order 2 such that det (A) = 4, then det (4 adj.A) is equal to

(a) 16 (b) 64 (c) 256 (d) 512

40. If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals

(a) A (b) A + I (c) I – A (d) A – I

41. If the vertices of a triangle with vertices (2, -6), (5, 4) and (k, 4) is 35 sq.units, then k is

(a) 12 (b) -2 (c) -12, -2 (d) 12, -2

42. The value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ is

(a) 0 (b) 1 (c) x + y + z (d) 2(x + y + z)

43. If for a square matrix A, $\mathrm{A}^2 - 3\mathrm{A} + \mathrm{I} = \mathrm{O}$ and $A^{-1} = xA + yI$, then the value of x + y is

(a) -2 (b) 2 (c) 3 (d) -3

44. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adjA = A. Then (a + b + c + d) is equal to

(a) 2a (b) 2b (c) 2c (d) 0

45. Let |A| = 2, where A is 2 x 2 matrix then $|4\mathrm{A}^{-1}|$ equals

(a) 4 (b) 2 (c) 8 (d) $\frac{1}{32}$

46. If A and B are invertible matrices, then which of the following is not correct?

(a) $(\mathrm{A}+\mathrm{B})^{-1} = \mathrm{B}^{-1} + \mathrm{A}^{-1}$ (b) $(\mathrm{AB})^{-1} = \mathrm{B}^{-1}\mathrm{A}^{-1}$

(c) $\mathrm{adj.}(\mathrm{A}) = |\mathrm{A}|\mathrm{A}^{-1}$ (d) $|A|^{-1} = |A^{-1}|$

47. If A is a square matrix of order 2 and $|A| = -2$, then value of $|5\mathrm{A}'|$ is

(a) -50 (b) -10 (c) 10 (d) 50

48. Let A be a skew-symmetric matrix of order 3. If |A| = x, then $(2023)^x$ is equal to

(a) 2023 (b) $\frac{1}{2023}$ (c) $(2023)^2$ (d) 1

49. Let A be a 3 x 3 matrix such that |adjA| = 64. Then |A| is equal to

(a) 8 only (b) -8 only (c) 64 (d) 8 or -8

50. If $\left|\frac{A^{-1}}{2}\right| = \frac{1}{k|A|}$, where A is a 3×3 matrix, then the value of k is

(a) $\frac{1}{8}$ (b) 8 (c) 2 (d) $\frac{1}{2}$

51. If $\begin{bmatrix} x & 2 \\ 3 & x-1 \end{bmatrix}$ is a singular matrix, then the product of all possible values of x is

(a) 6 (b) -6 (c) 0 (d) -7

52. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and $a \in A$, then the set A is

(a) $\mathbb{R}$ (b) {0} (c) {4} (d) $\mathbb{R}$ - {4}

53. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ to be invertible, the value of λ is

(a) 0 (b) 10 (c) $\mathbb{R} - \{10\}$ (d) $\mathbb{R} - \{-10\}$

54. If |A| = |kA|, where A is a square matrix of order 2, then sum of all possible values of k is

(a) 1 (b) -1 (c) 2 (d) 0

55. If (a, b), (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to

(a) $2\Delta^2$ (b) $4\Delta^2$ (c) 2Δ (d) 4Δ

56. If $A.(\text{adj}A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |\text{adj}A|$ is equal to

(a) 12 (b) 9 (c) 3 (d) 27

57. Let A be the area of a triangle having vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3). Which of the following is correct?

(a) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm A$ (b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$

(c) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{A}{2}$ (d) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = A^2$

58. It is given that X $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Then matrix X is

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

59. Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is

(a) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (c) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (d) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

60. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B = \frac{1}{3}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & \lambda \end{bmatrix}$. If AB = I, then find the value of λ is

(a) $\frac{-9}{4}$ (b) -2 (c) $\frac{-3}{2}$ (d) 0

61. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$, then the value of $|A(\text{adj}.A)|$ is

(a) 100I (b) 10I (c) 10 (d) 1000

ASSERTION - REASON TYPE QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

1. Assertion: The determinant of a skew symmetric matrix of even order is perfect square.

Reason: The determinant of skew symmetric matrix of odd order is equal to zero.

2. Assertion: If $A = \begin{bmatrix} 2 & 1+2i \\ 1-2i & 7 \end{bmatrix}$ then $\det(A)$ is real.

Reason: If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{21} \end{bmatrix}$, a_{ij} being complex numbers, then $|A|$ is always real.

3. Assertion: Minor of an element of a determinant of order $n(n \geq 2)$ is a determinant of order n.

Reason: If A is an invertible matrix of order 2, then $\det(A^{-1})$ is equal to $\frac{1}{|A|}$.

4. Assertion: Determinant of a skew-symmetric matrix of order 3 is zero.

Reason: For any matrix A, $|A^T| = |A|$ and $|-A| = -|A|$.

5. For system of equation AX = B

Assertion: System having unique solution if B is a non-singular matrix and matrix A can be a singular.

Reason: Singular matrix have value of its determinate equal to 0.

6. Assertion: The system of equations: $2x - y = -2$; $3x + 4y = 3$ has unique solution and $x = -\frac{5}{11}$ and $y = \frac{12}{11}$.

Reason: The system of equations AX = B has unique solution, if $|A| \neq 0$.

7. Assertion: If $A = \begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$, then the matrix A is singular if $x = 3$.

Reason: A square matrix is a singular matrix if its determinant is zero.

8. Assertion: If every element of a second order determinant is multiplied by 3, then the value of the new determinant is multiplied by 9 to the actual determinant.

Reason: If k is a scalar and A is n x n matrix then $|kA| = k^n|A|$.

9. Assertion: If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ and $A^{-1} = mA$, then $m = \frac{1}{19}$.

Reason: $|A^{-1}| = \frac{1}{|A|}$.

10. Let $A = [a_{ij}]$ be a matrix of order 3×3.

Assertion: Expansion of determinant of A along second row and first column gives the same value.

Reason: Expanding a determinant along any row or column gives the same value.

11. Assertion: The area of triangle with vertices P(0, 4), Q(-1, 4) and R(2, 4) is zero.

Reason: The area of triangle formed by three collinear points is zero.

12. Assertion: If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} are the cofactors of a_{ij}, then

$a_{11} A_{21} + a_{12} A_{22} + a_{13} A_{23} = 0$.

Reason: Determinant is the sum of product of elements of any row(column) with their corresponding cofactors.

13. Assertion: If $A = \begin{bmatrix} \alpha & 3 \\ 3 & \alpha \end{bmatrix}$ and $|A|^3 = -125$, then $\alpha = \pm 2$.

Reason: Determinant of a square matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given as $a_{11}.a_{22} - a_{12}.a_{21}$.

14. Assertion: The minor of the element of second row and third column in the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & 7 \end{vmatrix}$ is 13.

Reason: The positive value of x, which makes the given pair of determinants $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$ equal is 4.

15. Assertion: A square matrix A has inverse, if and only if A is singular.

Reason: Let A be a square matrix of order 2 x 2 then the value of |kA| is equal to $k^2|A|$.

16. Assertion: If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$, then $|A| = 105$.

Reason: The determinant of a diagonal matrix is equal to the product of the diagonals elements.

17. Assertion: If A and B are square matrices of the same order 3 such that 2AB = I and $|B| = \frac{1}{24}$, then |A| = 3.

Reason: If A and B are square matrices of the same order n and k is a scalar, then $|kA| = (k)^n|A|$ and |AB| = |A||B|.

18. Assertion: If A is a square matrix of order 3 such that |A| = 4, then $|\text{adj}(\text{adj}\,A)| = 4^4$.

Reason: If A is a non-singular square matrix of order n, then $|\text{adj}(\text{adj}\,A)| = |A|^{(n-1)^2}$.

19. Assertion: If A is 3 x 3 non-singular square matrix, then $|A^{-1}\text{adj}\,A| = |A|$.

Reason: If A and B are invertible matrices such that B is inverse of A, then AB = BA = I.

20, Assertion: The system of linear equations 5x + ky = 5 and 3x + 3y = 5 has a unique solution for k ≠ 5.

Reason: The system of linear equations 5x + ky = 5 and 3x + 3y = 5 is inconsistent for k = 5.

VERY SHORT ANSWERS

1. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

2. Solve for x: $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$.

3. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$.

3. Determine the value of k for which the matrix $A = \begin{bmatrix} 2 & -1 & k \\ 1 & -2 & 1 \\ 3 & 1 & -2 \end{bmatrix}$ is singular.

4. Find the minors and the cofactors of each entry of the second row of the matrix A and evlaute det A, where $A = \begin{bmatrix} 2 & 4 & 1 \\ 8 & 5 & 2 \\ -1 & 3 & 7 \end{bmatrix}$.

5. If $A = 2B$, where A and B are square matrices of order 3×3 and $|B| = 5$, what is $|A|$?

6. Find the inverse of the matrix $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$. Hence, find the matrix P satisfying the matrix equation $P\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

7. If A and B are square matrices of order 3 such that |A| = -1, |B| = 3, then find the value of |2AB|.

8. If A is square matrix of order 3 such that A^2 = 2A, then find the value of |A|.

9. If $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$, compute $(AB)^{-1}$.

10. If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

11. If the points (a, 0), (0, b) and (1, 1) are collinear, prove that a + b = ab.

12. For what value of k do the points (-1, 4), (-3, 8) and (1 – k, 3k) lie on a straight line?

13. Find a 2 x 2 matrix X such that $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix} X = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

14. Solve for x and y given that $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

15. If A = $\begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$ and A-1 = $\begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ \frac{-3}{34} & \frac{2}{17} \end{bmatrix}$, find the value of x.

SHORT ANSWERS

1. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$.

2. Find the adjoint of the matrix A = $= \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that A(adjA) = (adjA)A = $|A|I_3$.

3. If A = $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2, then show that A^2 = 4A – 3I, hence find A^{-1}.

4. Find the equation of line joining the P(11, 7) and Q(5, 5) using determinants. Also, find the value of k if R(-1, k) is the point such that the area of $\triangle$ PQR is 9 sq.m.

5. Show that $\begin{bmatrix}1 & -\tan\theta \\ \tan\theta & 1\end{bmatrix} \times \begin{bmatrix}1 & \tan\theta \\ -\tan\theta & 1\end{bmatrix}^{-1} = \begin{bmatrix}\cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta\end{bmatrix}$.

6. If $f(x) = \begin{bmatrix}\cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1\end{bmatrix}$, show that $[f(x)]^{-1} = f(-x)$.

7. If $A = \begin{bmatrix}3 & 8 \\ 2 & 1\end{bmatrix}$, verify that $A^{-1} = \frac{1}{13}A - \frac{4}{13}I$.

8. If $A = \begin{bmatrix}3 & -1 \\ -4 & 0 \\ 2 & 1\end{bmatrix}$ and $B = \begin{bmatrix}2 & 1 \\ -1 & -2 \\ 1 & 1\end{bmatrix}$, find $(A'B)^{-1}$.

9. If $A = \begin{bmatrix}4 & 5 \\ 2 & 1\end{bmatrix}$, show that $A - 3I = 2(I + 3A^{-1})$.

10. Let $A = \begin{bmatrix}1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5\end{bmatrix}$. Verify that $(\operatorname{adj} A)^{-1} = \operatorname{adj}(A^{-1})$.

11. Find the matrix X satisfying the following matrix equation $\begin{bmatrix}2 & 5 \\ -3 & 1\end{bmatrix} X \begin{bmatrix}2 & -2 \\ 1 & 3\end{bmatrix} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$.

12. If $A = \begin{bmatrix}3 & 0 & 2 \\ 1 & 5 & 9 \\ -6 & 4 & 7\end{bmatrix}$ and $AB = BA = I$, find B.

13. If $A = \begin{bmatrix}3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1\end{bmatrix}$, verify that $A^3 = A^{-1}$.

14. If $A = \begin{bmatrix}1 & x \\ x^2 & 4y\end{bmatrix}$, $B = \begin{bmatrix}-3 & 1 \\ 1 & 0\end{bmatrix}$ and $\operatorname{adj}A + B = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$, find the values of x and y.

15. Find the value of the determinant $\begin{vmatrix}1 & \omega^6 & \omega^8 \\ \omega^6 & \omega^3 & \omega^7 \\ \omega^8 & \omega^7 & 1\end{vmatrix}$ where $\omega^3 = 1$.

LONG ANSWERS

1. If $A = = \begin{bmatrix}\cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1\end{bmatrix}$, find adjA and verify that $A(\operatorname{adj} A) = (\operatorname{adj} A)A = |A|I_3$.

2. Using matrices, solve the following system of equations: 2x + 3y + 3z = 5, x – 2y + z = -4 and 3x – y – 2z = 3.

3. Using matrices, solve the following system of equations: 3x + 4y + 7z = 4, 2x – y + 3z = -3 and x + 2y – 3z = 8.

4. If $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations: x + 2y + z = 4, -x + y + z = 0 and x – 3y + z = 4.

5. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence solve the system of equations: x + 2y – 3z = -4, 2x + 3y + 2z = 2 and 3x – 3y – 4z = 11.

6. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of equations: x – y = 3, 2x + 3y + 4z = 17 and y + 2z = 7.

7. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$, find AB. Hence solve the system of equations: x – 2y = 10, 2x + y + 3z = 8 and -2y + z = 7.

8. If $A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$, find A-1. Using A^{-1}solve system of equations: $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5$, $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$.

9. Find the product of matrices $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 4 & -6 \\ 3 & -2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 20 & 2 & 34 \\ 8 & 16 & -32 \\ 22 & -13 & 7 \end{bmatrix}$ and hence solve the system of equations: $\frac{2}{x} + \frac{3}{y} + \frac{4}{z} = -3$, $\frac{5}{x} + \frac{4}{y} - \frac{6}{z} = 4$, $\frac{3}{x} - \frac{2}{y} - \frac{2}{z} = 6$.

10. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it we get 11. By adding first and third numbers we get double of the second number. Represent it algebraically and find the numbers using matrix method.

11. If A = $\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the following system of linear equations:

3x + 4y + 7z = 14, 2x – y + 3z = 4, x + 2y – 3z = 0.

12. If A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, find A^{-1} and use it to solve the system of equations: x + y + 2z = 0, x + 2y – z = 9, x – 3y + 3z + 14 = 0.

13. If A $= \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, find $(AB)^{-1}$. Also, find $|(AB)^{-1}|$.

14. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations:

x + y + z = 5000, 6x + 7y + 8z = 35800, 6x + 7y – 8z = 7000.

15. If matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations: 3x + 2y + z = 2000, 4x + y + 3z = 2500, x + y + z = 900.

16. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse, solve the system of linear equations: x – y + 2z = 1, 2y – 3z = 1, 3x – 2y + 4z = 3.

17. If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$.

18. Use the product of matrices $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{pmatrix}$ to solve the following system of equations: x + 2y – 3z = 6, 3x + 2y – 2z = 3, 2x – y + z = 2.

19. If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations: x – 2y = 3, 2x – y – z = 2, -2y + z = 3.

20. Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence solve the system of linear equations: x + 2y – 3z = -4, 2x + 3y + 2z = 2, 3x – 3y – 4z = 11.

21. If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$, find the value of (a + x) – (b + y).

22. If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that $A'A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$.

CASE BASED QUESTIONS

1. Three shopkeepers Ravi, Raju and Rohit are using polythene bags, handmade bags and newspaper’s envelope as carry bags. It is found that the shopkeepers Ravi, Raju and Rohit are using (20, 30, 40), (30, 40, 20) and (40, 20, 30) polythene bags, handmade bags and newspaper envelopes respectively. The shopkeepers Ravi, Raju and Rohit spent Rs.250, Rs.270 and Rs.200 on these bags respectively.

(i) Represent the above situation algebraically and write the system in the form of matrices.

(ii) Find the cost of each bags respectively.

2. A trust invested some money in two types of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received rupees 2800 as interest. However, if trust had interchanged money in bonds they would have got rupees 100 less as interest.

Based on the information answer the following:

(i) Represent the above situation algebraically and write the system in the form of matrices.

(ii) Find the amount invested in the both bonds.

3. A shopkeeper has 3 varieties of pens A, B and C. Swathi purchased 1 pen of each variety for total of rupees. Ramya purchased 4 pens of variety A, 3 pens of variety B and 2 pens of variety C for rupees 60. While Shikha purchased 6 pens of A variety, 2 pens of B variety and 3 pens of C variety for rupees 70.

(i) Express the above given situation into a matrix equation of the form AX = B.

(ii) Find det A

(iii) Find the cost of each pen.

4. The management committee of a residential colony decided to award some of its members for honesty, some for helping others and some other for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for helping others and supervision added to two times the number of awardees for honesty is 33.If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others.

Based on the above information answer the following:

(i) Find the number of awards for honesty.

(ii) Find the number of awards for helping others.

(iii) Find the number of awards for supervising the workers.

5. The daily income of Sita and Savita are in the ratio 1:2 and their expenditures are in the ratio 2:1. Their saving are Rs.500 and Rs.2500 respectively.

Based on the above information answer the following questions:

(i) If their incomes are x and 2x and their expenditures are 2y and y respectively, then write the linear equations for the above situation.

(ii) Find the income of Sita.

(iii) Find the expenditure of Savita.

6. The upward speed v(t) of a rocket at time t is given by $V(t) = at^2 + bt + c$, $0 \le t \le 100$, where a, b, c are constants. It has been found that the speed at t = 3sec, t = 6sec and t = 9sec are respectively 64, 133 and 208 miles per second.

If $\begin{bmatrix} 9 & 3 & 1 \\ 36 & 6 & 1 \\ 81 & 9 & 1 \end{bmatrix}^{-1} = \frac{1}{18}\begin{bmatrix} 1 & -2 & 1 \\ -15 & 24 & -9 \\ 54 & -54 & 18 \end{bmatrix}$

Based on the above information, answer the following:

(i) Find the value of b + c.

(ii) Find the speed V(t) in terms of t.

7. Two schools P and Q want to award their selected students on the values of discipline, politeness and punctuality. The school P wants to award ₹x each, ₹y each and ₹z each for the three respective values to its 3, 2 and 1 students with total award money of ₹1000. School Q wants to spend ₹1500 to award its 4, 1 and 3 students on the respective values 9by giving the same award money for the three values as before). If the total amount of awards for one prize each value is ₹600.

Based on the above information, answer the following questions:

(i) Represent the above situation algebraically and write the system in the form of matrices.

(ii) Find the award money for each of the value.

8. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instruments boxes and pays a sum of ₹190. Also, Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹250.

Based on the above information, answer the following questions:

(i) Convert the given above situation into a matrix equation of the form AX = B.

(ii) Find |A|

(iii) Find A^{-1}.

(iv) Determine $P = A^2 - 5A$.

9. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.

Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports. In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000.

Based on the above information, answer the following questions:

(i) Express the given information algebraically using matrices.

(ii) Check whether the system of matrix equations so obtained is consistent or not.

(iii) Find the number of scholarships of each kind given by the school, using matrices.

(iv) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school?

UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then det(adjA) is

(a) a^{27} (b) a^9 (c) a^6 (d) a^2

2. If A is non-singular square matrix of order 3 such that $A^2 = 3A$, then the value of |A| is

(a) -3 (b) 3 (c) 9 (d) 27

3. If A is a square matrix such that $A^2 = I$, then A^{-1} is

(a) 2A (b) O (c) A (d) A + I

4. Assertion: The minor of the element a_{32} of the determinant $\begin{vmatrix} 9 & 2 & 3 \\ 2 & 0 & 1 \\ 5 & 3 & 8 \end{vmatrix}$ is 3.

Reason: The minor of an elemet a_{ij} of a determinant is defined as the value of determinant obtained after deleting the i^{th} row and j^{th} column.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. Find the value of x if $\begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix} = 28$

6. If $A = \begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$ and $|A^3| = 125$, then find the values of p.

7. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then find the value of k.

SECTION C

Each carry 3 marks

8. Swathi wants to donate a rectangular plot of land for a school in her village. When she was asked to give dimensions of the plot, she told that if it is length is decreased by 50m and breadth is increased by 50m then its area will remain the same. But if the length is decreased by 10m and breadth is decreased by 20m, then its area will decreased by $5300m^2$. Using matrices, find the dimensions of the plot.

9. Show that A = $= \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $x^2 - 3x - 7 = 0$ and hence find A^{-1}.

SECTION D

Each carry 5 marks

10. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} and hence solve the system of equations: 2x – 3y + 5z = 11, 3x + 2y – 4z = -5 and x + y – 2z = -3.

11. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it solve the system of equations: x – y + z = 4, x – 2y – 2z = 9 and 2x + y + 3z = 1.

SECTION E

12. Two schools A and B decided to award prizes to their students for three games hockey (x), cricket(y) and tennis(z). School A decided to award a total of ₹11000 for the three games to 5, 4

and 3 students respectively, while school B decided to award ₹10700 for the three games to 4, 3 and 5 students respectively. Also, the three prizes together amount to ₹2700.

Using the information given above answer the following:

(i) Represent the above situation using matrix equation.

(ii) Find out the prize amount for hockey, cricket and tennis.

ANSWERS

MCQ	A-R	VSA	SA	LA	CS
1. (d) 1 2. (a) 5 3. (c) 25 4. (a) ±3 5. (d) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ 6. (d) 216 7. (b) 0 8. (b) 3 9. (a) y = 2x	1. (b) 2. (c) 3. (d) 4. (c) 5. (d) 6. (a) 7. (a) 8. (a) 9. (b) 10. (a)	2. x = -1 or 3 3. k = $\frac{-1}{7}$ 4. -145 5. 40 6. P = $\begin{bmatrix} 13 & 8 \\ 1 & 1 \end{bmatrix}$ 7. -24 8. 0 or 8 9. =	1. $\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$ 4. x – 3y + 10 = 0 k = 0 or 6 8. $\begin{bmatrix} 0 & -1 \\ \frac{1}{13} & \frac{12}{13} \end{bmatrix}$ 11. $\frac{1}{136}\begin{bmatrix} 8 & -8 \\ 7 & 10 \end{bmatrix}$ 12. $\frac{1}{65}\begin{bmatrix} -1 & 8 & -10 \\ -61 & 33 & -25 \\ 34 & -12 & 15 \end{bmatrix}$ 14. x = 1 and y = 1.	1. adj(A) = $\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 2. x = 1, y = 2, z = -1 3. x = 1, y = 2, z = -1 4. $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$ x = 2, y = 0, z = 2. 5. $A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ x = 3, y = -2, z = 1 6. AB = $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$	1. (i) 20x + 30y + 40z = 250 30x + 40y + 20z = 270 40x + 20y + 30z = 200 (ii) 1, 5 and 2. 2. (i) $\begin{bmatrix} 10 & 12 \\ 12 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} =$ $\begin{bmatrix} 280000 \\ 270000 \end{bmatrix}$ (ii) Rs.10000 and Rs.15000 3.(i) $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$ (ii) |A| = -5

10. (c) 405	11. (a)	$\begin{bmatrix} -13 & 6 \\ -11 & 5 \end{bmatrix}$	15. 3	x = 2, y = -1, z = 4	(iii) 5, 8, 8.
11. (d) $\frac{33}{8}$	12. (b)	13. $\begin{bmatrix} 6 & 2 \\ \frac{11}{2} & 2 \end{bmatrix}$		7. AB = = $\begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$	4. (i) 3
12. (b) 1	13. (a)				(ii) 4
13. (b) k^2I	14. (b)	14. x = 2, y = 1		x = 4, y = -3, z = 1	(iii) 5
14. (c) 49	15. (d)	15. x = 4		8. $A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$	5. (i) Rs.1500
15. (b) ±8	16. (a)				(ii) Rs.500
	17. (a)			9. AB = $152\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	6. (i) 21
16. (c) $\frac{1}{7}$	18. (a)			x = 1, y = -1 and z = -2	(ii) V(t) = $\frac{1}{3}t^2 + 20t + 1$
17. (d) $\begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{4} \end{bmatrix}$	19. (b)			10. x = 1, y = 2, z = 3	7. (i) 3x + 2y + z = 1000
18. (a) $\begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$	20. (b)			11. x = 1, y = 1, z = 1	4x + y + 3z = 1500
				12. x = 1, y = 3, z = -2	x + y + z = 600
19. (d) None of these				13. $(AB)^{-1} = \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$,	

20. (a) 28 21. (b) 9 22. (c) -4 23. (a) A^2 24. (c) -1 25. (b) 2 26. (d) Non-singular matrix 27. (b) 12^3 28. (a) 0 29. (d) 36 30. (b) 10				$\lvert(AB)^{-1}\rvert = -1$. 14. $A^{-1} = -\frac{1}{16}\begin{bmatrix}-112 & 15 & 1\\ 96 & -14 & -2\\ 0 & -1 & 1\end{bmatrix}$ $x = 1000, y = 2200, z = 1800.$ 15. $A^{-1} = -\frac{1}{5}\begin{bmatrix}-2 & -1 & 5\\ -1 & 2 & -5\\ 3 & -1 & -5\end{bmatrix}$, $x = 400, y = 300$ and $z = 200$. 16. $A^{-1} = \begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$, $x = 1, y = 2, z = 1$ 17. $\begin{bmatrix}10 & 7 & 21\\ -49 & -34 & -103\\ 17 & 12 & 36\end{bmatrix}$ 18. AB = $\begin{pmatrix}7 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 7\end{pmatrix}$,	$\begin{bmatrix}3 & 2 & 1\\ 4 & 1 & 3\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1000\\ 1500\\ 600\end{bmatrix}$ (ii) ₹100, ₹ 200 and ₹300 8. (i) $A = \begin{bmatrix}5 & 3 & 1\\ 2 & 1 & 3\\ 1 & 2 & 4\end{bmatrix}$, $X = \begin{bmatrix}x\\ y\\ z\end{bmatrix}, B = \begin{bmatrix}160\\ 190\\ 250\end{bmatrix}$ (ii) -22 (iii) $A^{-1} = \frac{1}{(-22)}\begin{bmatrix}-2 & -10 & 8\\ -5 & 19 & -13\\ 3 & -7 & -1\end{bmatrix}$ (iv) P = $\begin{bmatrix}7 & 5 & 13\\ 5 & 8 & 2\\ 8 & 3 & 3\end{bmatrix}$

31. (d) 4 32. (c) ± 2 33. (b) -5 34. (a) 0 35. (b) 36 36. (d) $2\sqrt{2}$ 37. (d) 4 38. (d) 4 39. (b) 64 40. (c) I – A 41. (d) 12, -2 42. (a) 0				$x = 1, y = -5, z = -5.$ 19. $AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $x = 1, y = -1, z = 1$ 20. $\begin{bmatrix} 67 & 0 & 0 \\ 0 & 67 & 0 \\ 0 & 0 & 67 \end{bmatrix}$, x = 3, y = -2, z = 1 21. 3	9. (i) $\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 180 \end{bmatrix}$ (ii) Consistent (iii) x = 20, y = 30 (iv) ₹ 1,70,000

43. (b) 2 44. (a) 2a 45. (c) 8 46. (a) $(A+B)^{-1} = B^{-1} + A^{-1}$ 47. (a) -50 48. (d) 1 49. (d) 8 or -8 50. (b) 8 51. (b) -6 52. (d) $\mathbb{R}$ - {4} 53. (d) $\mathbb{R} - \{-10\}$					

54. (d) 0 55. (b) $4\Delta^2$ 56. (a) 12 57. (b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} =$ $\pm 2A$ 58. (c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ 59. (b) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ 60. (b) -2 OR (c) $\frac{-3}{2}$ 61. (d) 1000					

UNIT TEST

1. (c) a^6

2. (d) 27

3. (c) A

4. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

5. $2, -\frac{17}{7}$

6. ±3

7. $k = \frac{1}{19}$

8. 200m and 150m.

9. $\frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

10. $A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ and x = 1, y = 2, z = 3.

11. $AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$, x = 3, y = -2, z = -1

12. (i) $\begin{bmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ x & y & z \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11000 \\ 10700 \\ 2700 \end{bmatrix}$ (ii) Amount of hockey $= ₹1000$, amount of cricket $= ₹900$ and amount of tennis $= ₹800$

CHAPTER 5 – CONTINUITY AND DIFFERENTIABILITY

CONTINUITY

Suppose f is a real function on a subset of the real numbers and let c be a point in the domain of f. Then f is continuous at c if

$$\lim_{x\to c} f(x) = f(c)$$

A function is continuous at x = c if the function is defined at x = c and if the value of the function at x = c equals the limit of the function at x = c. If f is not continuous at c, we say f is discontinuous at c and c is called a point of discontinuity of f.

CONTINUOUS FUNCTION

A real function f is said to be continuous if it is continuous at every point in the domain of f.

Suppose f is a function defined on a closed interval [a, b], then for f to be continuous, it needs to be continuous at every point in [a, b] including the end points a and b. Continuity of f at a means

$$\lim_{x\to a^+} f(x) = f(a)$$

and continuity of f at b means

$$\lim_{x\to b^-} f(x) = f(b)$$

A function f(x) is continuous at a point x = m if

$$\lim_{x\to m^-} f(x) = \lim_{x\to m^+} f(x) = f(m)$$

where $lim f(x)$ is Left Hand Limit of f(x) at x = m and $\lim_{x\to m^+} f(x)$ is Right Hand Limit of f(x) at x = m. Also f(m) is the value of function f(x) at x = m.

REMARKS

- Modulus functions is Continuous on R.
- Trigonometric functions are continuous in their respective domains.
- Exponential function is continuous on R.
- Every identity function is continuous.

- Every constant function is continuous.
- Every polynomial function is continuous on R.
- Every rational function is continuous.
- Greatest integer function is continuous on all non-integral real numbers.

ALGEBRA OF CONTINUOUS FUNCTIONS

Suppose f and g are two real functions, continuous at real number c . Then,

(1) $f + g$ is continuous at x = c

(2) $f - g$ is continuous at x = c

(3) $f \,.\, g$ is continuous at x = c

(4) $\left(\frac{f}{g}\right)$ is continuous at $x = c$, (provided $g(c) \neq 0$).

STANDARD RESULTS OF LIMITS

(i) $\lim\limits_{x\to a} \frac{x^n-a^n}{x-a} = na^{n-1}$

(ii) $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$

(iii) $\lim\limits_{x\to 0} \frac{\tan x}{x} = 1$ (iv) $\lim\limits_{x\to 0} \frac{e^x-1}{x} = 1$

(v) $\lim\limits_{x\to \infty} \frac{1}{x^p} = 0, p \in (0, \infty)$

(vi) $\lim\limits_{x\to 0} \frac{\log(1+x)}{x} = 1$

(vii) $\lim\limits_{x\to 0} \frac{a^x-1}{x} = \log_e a$

(viii) $\lim\limits_{x\to 0} \frac{\sin^{-1} x}{x} = 1$

(ix) $\lim\limits_{x\to 0}(1 + x)^{1/x} = e$

(x) $\lim\limits_{x\to \infty} \frac{\sin x}{x} = 0$

(xi) $\lim\limits_{x\to \infty} \left(1 + \frac{1}{x}\right)^x = e$

(xii) $\lim\limits_{x\to \infty} \sin x = \lim\limits_{x\to \infty} \cos x =$ lies between -1 to 1.

STANDARD DERIVATIVES

Function f(x) or y	Derivative $f^{\prime}(x)$ or $\frac{dy}{dx}$
u ± v	$u^{\prime} \pm v^{\prime}$ (or) $\frac{du}{dx} + \frac{dv}{dx}$
Product Rule: u.v	$u.\ v^{\prime} + v.\ u^{\prime}$ (or) $u\ \frac{dv}{dx} + v\ \frac{du}{dx}$
Quotient Rule: $\frac{u}{v}$	$\frac{vu^{\prime}-uv^{\prime}}{v^2}$ (or) $\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
k(constant)	0
x^n	nx^{n-1}
x	1
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
sinx	cosx
cosx	-sinx
tanx	sec^2x
secx	secx.tanx
cosecx	- cosecx.cotx
cotx	$-cosec^2x$
logx	$\frac{1}{x}$
e^x	e^x
a^x	a^x loga
$sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
$cos^{-1}x$	$\frac{-1}{\sqrt{1-x^2}}$
$tan^{-1}x$	$\frac{1}{1+x^2}$
$cosec^{-1}x$	$\frac{-1}{x\sqrt{x^2-1}}$
$sec^{-1}x$	$\frac{1}{x\sqrt{x^2-1}}$
$cot^{-1}x$	$\frac{-1}{1+x^2}$

CHAIN RULE

Let f be a real valued function which is a composite of two function u and v; i.e., f = v o u. Suppose that t = u(x) and if both $\frac{dt}{dx}$ and $\frac{dv}{dt}$ exist, we have $\frac{df}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx}$

Suppose f is a real valued function which is a composite of three functions u, v and w; i.e., f = (w o u) o v. if t = v(x) and s = u(t), then $\frac{df}{dx} = \frac{dw}{ds} \cdot \frac{ds}{dt} \cdot \frac{dt}{dx}$

DERIVATIVES OF IMPLICIT FUNCTION

Explicit Function

An explicit function is one which is given in terms of the independent variable.

Example: $y = x^2 + 3x - 8$

Implicit Function

Implicit functions, on the other hand, are usually given in terms of both dependent and independent variables.

Example: $y + x^2 - 3x + 8 = 0$

Derivatives of Inverse Trigonometric Functions

f(x)	$\sin^{-1}x$	$\cos^{-1}x$	$\tan^{-1}x$	$\text{cosec}^{-1}x$	$\sec^{-1}x$	$\cot^{-1}x$
$f^{\vert}(x)$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{-1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$	$\frac{-1}{x\sqrt{x^2-1}}$	$\frac{1}{x\sqrt{x^2-1}}$	$\frac{-1}{1+x^2}$

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

f(x)	e^x	logx
$f^{\vert}(x)$	e^x	$\frac{1}{x}$

LOGARITHMIC DIFFERENTIATION

Some Basic Logarithmic Rules

(a) log ab = log a + log b (b) log $(\frac{a}{b})$ = log a – log b (c) log a^n = n.log a

DERIVATIVES OF FUNCTIONS IN PARAMETRIC FORMS

A parametric equation defines a group of quantities as functions of one or more independent variables called parameters. Parametric equations are commonly used to express the coordinates of the points that make up a geometric object such as a curve or surface, in which case the equations are collectively called a parametric representation or parameterization.

A relation expressed between two variables x and y in the form x = f(t), y = g(t) is said to be parametric form with 't' as a parameter.

In order to find derivative of function in such form, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

SECOND ORDER DERIVATIVES

Let y = f(x), then $\frac{dy}{dx} = f'(x)$

If $f'(x)$ is differentiable, we may differentiate the above equation again w.r.to x. Then, the left-hand side becomes $\frac{d}{dx}(\frac{dy}{dx})$ which is called the second order derivative of y w.r.to x and it is denoted by $\frac{d^2y}{dx^2}$.

The second order derivative of f(x) is denoted by $f''(x)$. It is also denoted by D^2y or y'' or y_2.

MULTIPLE CHOICE QUESTIONS

1. The number of points at which the function $f(x) = \frac{1}{x-[x]}$ is not continuous is

(a) 1 (b) 2 (c) 3 (d) None of these

2. The function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

(a) 3 (b) 2 (c) 1 (d) 1.5

3. The function $f(x) = \frac{x-1}{x(x^2-1)}$ is discontinuous at

(a) Exactly one point (b) Exactly two points

(c) Exactly three points (d) No point

4. If $y = \log\sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) Not defined

5. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{\cos x}{2y-1}$ (b) $\frac{\cos x}{1-2y}$ (c) $\frac{\sin x}{1-2y}$ (d) $\frac{\sin x}{2y-1}$

6. For what value of k may the function $f(x) = \begin{cases} k(3x^2-5x), & x \leq 0 \\ \cos x, & x > 0 \end{cases}$ become continuous?

(a) $-\frac{1}{2}$ (b) 0 (c) 1 (d) No value

7. The function f(x) = x|x| is

(a) Continuous and differentiable at x = 0 (b) Continuous and but differentiable at x = 0

(c) Differentiable but not continuous at x = 0 (d) Neither differentiable nor continuous at x = 0

8. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to:

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\frac{1}{2}$

9. If $f(x) = 2|x| + 3|\sin x| + 6$, then the right hand derivative of f(x) at $x = 0$ is

(a) 6 (b) 5 (c) 3 (d) 2

10. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is

(a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (b) $\sec^2\left(\frac{\pi}{4} - x\right)$ (c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$ (d) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

11. If $y = \log(\sin e^x)$, then $\frac{dy}{dx}$ is

(a) $\cot e^x$ (b) $\operatorname{cosec} e^x$ (c) $e^x \cot e^x$ (d) $e^x \operatorname{cosec} e^x$

12. If $y = \sin^2(x^3)$, then $\frac{dy}{dx}$ is equal to

(a) $2\sin x^3 \cos x^3$ (b) $3x^3 \sin x^3 \cos x^3$ (c) $6x^3 \sin x^3 \cos x^3$ (d) $2x^3 \sin^2(x^3)$

13. If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to

(a) x (b) −x (c) 16x (d) -16x

14. If $f(x) = |\cos x|$, then $f\left(\frac{3\pi}{4}\right)$ is

(a) 1 (b) -1 (c) $\frac{-1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

15. If $x = a\cos\theta + b\sin\theta, y = a\sin\theta - b\cos\theta$, then which of the following is true?

(a) $y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ (b) $y^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(c) $y^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ (d) $y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

16. If $y = \log(\log x)$, then the value of $e^y \frac{dy}{dx}$ is

(a) e^y (b) $\frac{1}{x}$ (c) $\frac{1}{\log x}$ (d) $\frac{1}{x\log x}$

17. $\frac{d}{dx}\left[\log\left\{e^x\left(\frac{x-2}{x+2}\right)\right\}^{3/4}\right]$ is equal to

(a) 1 (b) $\frac{x^2+1}{x^2-4}$ (c) $\frac{x^2-1}{x^2-4}$ (d) $e^x \frac{x^2-1}{x^2-4}$

18. If $y = e^{x^x}$, then $\frac{dy}{dx} =$

(a) $y(1 + \log x)$ (b) $yx^x(1 + \log x)$ (c) $ye^x(1 + \log x)$ (d) None of these

19. If $y^x = e^{y-x}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1+\log y}{y\log y}$ (b) $\frac{(1+\log y)^2}{y\log y}$ (c) $\frac{1+\log y}{(\log y)^2}$ (d) $\frac{(1+\log y)^2}{\log y}$

20. If $x^2 + y^2 = 1$, then

(a) $yy'' - (2y')^2 + 1 = 0$ (b) $yy'' - (y')^2 + 1 = 0$

(c) $yy'' - (y')^2 - 1 = 0$ (d) $yy'' - 2(y')^2 + 1 = 0$

21. If $f(x) = \begin{cases} mx + 1, & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then

(a) $m = 1, n = 0$ (b) $m = \frac{n\pi}{2} + 1$ (c) $n = \frac{m\pi}{2}$ (d) $m = n = \frac{\pi}{2}$

22. If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$, is equal to

(a) $\frac{4x^3}{1-x^4}$ (b) $\frac{4x}{1-x^4}$ (c) $\frac{1}{4-x^4}$ (d) $\frac{-4x^3}{1-x^4}$

23. If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) None of these

24. If $y = \cot^{-1}(\cos 2x)^{1/2}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ will be

(a) $\left(\frac{2}{3}\right)^{1/2}$ (b) $\left(\frac{1}{3}\right)^{1/2}$ (c) $(3)^{1/2}$ (d) $(6)^{1/2}$

25. If $y = e^{(1+\log_e x)}$, then $\frac{dy}{dx}$ is equal to

(a) e (b) 1 (c) 0 (d) (d) $\log_e x \cdot x$

26. If $xe^{xy} = y + \sin^2 x$, then at $x = 0, \frac{dy}{dx} =$

(a) -1 (b) -2 (c) 1 (d) 2

27. The value of the derivative of $|x - 1| + |x - 3|$ at $x = 2$ is

(a) 2 (b) 1 (c) 0 (d) -2

28. If $\sin x + y = \log x$, then $\frac{dy}{dx}$ is

(a) $\frac{1-x}{x \sin y}$ (b) $\frac{1-x}{x \cos y}$ (c) $\frac{1+x}{x \cos y}$ (d) None of these

29. The derivative of $\sin^2 x$ w.r.t $e^{\cos x}$ is

(a) $\frac{2\cos x}{e^{\cos x}}$ (b) -$\frac{2\cos x}{e^{\cos x}}$ (c) $\frac{2}{e^{\cos x}}$ (d) None of these

30. If $y = \cos^{-1}x$, then $\frac{d^2y}{dx^2}$ in term of y alone is

(a) $-\cot y.\mathrm{cosec}^2 y$ (b) $\mathrm{cosec}y.\cot^2 y$ (c) $\cot y.\mathrm{cosec}y$ (d) None of these

31. If x = f(t) and y = g(t) then $\frac{d^2y}{dx^2}$ is

(a) $\frac{g'(t)'}{f'(t)}$ (b) $\frac{g''(t)f'(t)-g'(t)f''(t)}{[g'(t)]^3}$ (c) $\frac{g''(t)f'(t)-g'(t)f''(t)}{[g'(t)]^2}$ (d) None of these

32. $y = \cos^2 x + 3x$ then $\frac{d^2y}{dx^2} =$

(a) 3x (b) cos2x (c) 3cos2x (d) -2cos2x

33. If $y = x^3 \log x$, then $\frac{d^2y}{dx^2} =$

(a) x(5 + 6logx) (b) x (c) 5 + 6logx (d) logx.

34. If $y = 4^{x+9}$ then $\frac{dy}{dx}$ is

(a) 4^{x+9} (b) $4^{x+9}\log 4$ (c) log4 (d) logx + 4

35. The point of discontinuity for the function $f(x) = \begin{cases} |x| + 3, \text{ if } x \leq -3 \\ -2x, \text{ if } -3 < x < 3 \\ 6x + 2, \text{ if } x \geq 3 \end{cases}$ is

(a) x = 0 (b) x = -3 (c) x = 3 (d) x = -3, 3

36. For what values of a and b, the function $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, \text{ if } x < \frac{\pi}{2} \\ a, \text{ if } x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2}, \text{ if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$ is

(a) $a = \frac{1}{2}$, b = 4 (b) a = 2, $b = \frac{1}{4}$ (c) a = 1, b = 4 (d) a = 4, $b = \frac{1}{2}$

37. The point of discontinuity for the function $f(x) = \begin{cases} \frac{1-\cos x}{x^2}, \text{ if } x \neq 0 \\ \frac{1}{2}, \text{ if } x = 0 \end{cases}$ is

(a) x = 0 (b) x = 1 (c) x = $\frac{1}{2}$ (d) No point of discontinuity

38. Derivative of $\sqrt{\frac{1-\tan x}{1+\tan x}}$ with respect to x is

(a) $\frac{-\sec^2(\pi/4-x)}{2\sqrt{\tan(\pi/4-x)}}$ (b) $\frac{3}{2}\left(\frac{1-\tan x}{1+\tan x}\right)^{3/2}$ (c) $\frac{1}{2}\left(\frac{1-\tan x}{1+\tan x}\right)^{3/2}$ (d) None of these

39. Derivative of $\sin(\sqrt{\sin\sqrt{x}})$ with respect to x is

(a) $\frac{\cos(\sqrt{\sin\sqrt{x}})(\sin\sqrt{x})}{4\sqrt{\sin\sqrt{x}}}$ (b) $\frac{\sin(\sqrt{\sin\sqrt{x}})(\cos\sqrt{x})}{4\sqrt{x}\sqrt{\cos\sqrt{x}}}$ (c) $\frac{\sin(\sqrt{\sin\sqrt{x}})(\sin\sqrt{x})}{4\sqrt{x}\sqrt{\cos\sqrt{x}}}$ (d) $\frac{\cos(\sqrt{\sin\sqrt{x}})(\cos\sqrt{x})}{4\sqrt{x}\sqrt{\sin\sqrt{x}}}$

40. If $x = \frac{(1+\log t)}{t^2}, y = \frac{3+2\log t}{t}$, then $\frac{dy}{dx}$ is

(a) t (b) t2 (c) $\frac{1}{t}$ (d) -$\frac{1}{t^2}$

41. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$ then $\frac{dy}{dx}$ is equal to

(a) $\frac{y\log x}{x\log y}$ (b) -$\frac{y\log x}{x\log y}$ (c) $\frac{x\log y}{y\log x}$ (d) $\frac{x\log y}{y\log x}$

42. If $x = 2at^2, y = at^3$ then find $\frac{dx}{dy}$ at $t = 2$

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

43. $\frac{d}{dx}(\tan^{-1} x + \cot^{-1} x)$ is

(a) 0 (b) $\frac{1}{1+x^2}$ (c) -$\frac{1}{1+x^2}$ (d) Does not exist

44. The function f:R → R given by f(x) = -|x – 1| is

(a) Continuous as well as differentiable at x = 1

(b) Non continuous but differentiable at x = 1

(c) Continuous but not differentiable at x = 1

(d) Neither continuous nor differentiable at x = 1

45. The function $f(x) = e^{|x|}$| is

(a) Continuous everywhere but not differentiable at x = 0

(b) Continuous and differentiable everywhere

(c) Not continuous at x = 0

(d) None of these

46. The function f(x) = |x| | |x + 1| is

(a) Continuous at x = 0 as well as x = 1

(b) Continuous at x = 1 but not at x = 0

(c) Discontinuous at x = 0 as well as at x = 1

(d) Continuous at x = 0 but not x = 1

47. Let $f(x) = \begin{cases} x + a, & a \geq 1 \\ ax^2 + 1, & x < 1 \end{cases}$, then f is derivable at $x = 1$ if

(a) a = 1 (b) a = 0 (c) a = 2 (d) $a = \frac{1}{2}$

48. If $y = e^{-2x+3}$ and $\frac{dy}{dx} = ky$, then k is

(a) -2 (b) -1 (c) 2 (d) -2x + 3

49. If $y = \sin^{-1} x$, then $(1 - x^2)\frac{d^2y}{dx^2} =$

(a) $x\frac{dy}{dx}$ (b) $y\frac{dy}{dx}$ (c) $\frac{dy}{dx}$ (d) $-x\frac{dy}{dx}$

50. If $e^x(x + 1) = 1$, then

(a) $\frac{d^2y}{dx^2} = \frac{dy}{dx}$ (b) $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ (c) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ (d) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right) = 0$

51. The number of points of discontinuity of $f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$ is

(a) 0 (b) 1 (c) 2 (d) infinite

52. If f (x) = |x| + |x – 1|, then which of the following is correct?

(a) f (x) is both continuous and differentiable, at x = 0 and x = 1

(b) f (x) is differentiable but not continuous, at x = 0 and x = 1

(c) f (x) is continuous but not differentiable, at x = 0 and x = 1

(d) f (x) is neither continuous nor differentiable, at x = 0 and x = 1

53. If f(x) = 2|x| + 3|sinx| + 6, then the fight hand derivative of f(x) at x = 0 is

(a) 6 (b) 5 (c) 3 (d) 2

54. Which of the following statements is true for the function $f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$?

(a) f(x) is continuous and differentiable ∀x ∈ ℝ

(b) f(x) is continuous ∀x ∈ ℝ

(c) f(x) is continuous and differentiable ∀x ∈ ℝ - {0}

(d) f(x) is discontinuous at infinitely many points

55. The number of points, where f(x) = [x]. 0 < x < 3 ([.]) denotes greatest integer function) is not differentiable is

(a) 1 (b) 2 (c) 3 (d) 4

56. A function f(x) = |1 – x + |x|| is

(a) discontinuous at x = 1 only (b) discontinuous at x = 0 only

(c) discontinuous at x = 0, 1 (d) continuous everywhere

57. If $y = \sin^2(x^3)$, then $\frac{dy}{dx}$ is equal to :

(a) $2\sin x^3 \cos x^3$ (b) $3x^3 \sin x^3 \cos x^3$ (c) $6x^2 \sin x^3 \cos x^3$ (d) $2x^2 \sin^2(x^3)$

58. The value of k for which $f(x) = \begin{cases} 3x + 5, & x \geq 2 \\ kx^2, & x < 2 \end{cases}$ is a continuous function is

(a) $-\frac{11}{4}$ (b) $\frac{4}{11}$ (c) 11 (d) $\frac{11}{4}$

59. For what value of k may the function $f(x) = \begin{cases} k(3x^2 - 5x), & x \leq 0 \\ \cos x, & x > 0 \end{cases}$ become continuous?

(a) 0 (b) 1 (c) $-\frac{1}{2}$ (d) No value

60. The function f(x) = |x| - x is

(a) continuous but not differentiable at x = 0

(b) continuous and differentiable at x = 0

(c) neither continuous nor differentiable at x = 0

(d) differentiable but not continuous at x = 0

61. The function f(x) = x|x| is

(a) continuous and differentiable at x = 0

(b) continuous but not differentiable at x = 0

(c) differentiable but not continuous at x = 0

(d) neither differentiable nor continuous at x = 0

62. The function f(x) = |x| + |x – 2| is

(a) continuous, but not differentiable at x = 0 and x = 2

(b) differentiable but not continuous at x = 0 and x =2

(c) continuous but not differentiable at x = 0 only

(d) neither continuous nor differentiable at x = 0 and x = 2

63. If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

(a) -1 (b) 1 (c) –e (d) $-\frac{1}{e}$

64. If $\sin(xy) = 1$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{x}{y}$ (b) $-\frac{x}{y}$ (c) $\frac{y}{x}$ (d) $-\frac{y}{x}$

65. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is

(a) $\sin x e^{\sin^2 x}$ (b) $\cos x e^{\sin^2 x}$ (c) $-2\cos x e^{\sin^2 x}$ (d) $-2\sin^2 x \cos x e^{\sin^2 x}$

66. Let $f(x) = \begin{vmatrix} x^2 & \sin x \\ p & -1 \end{vmatrix}$, where p is a constant. The value of p for which $f'(0) = 1$ is

(a) $\mathbb{R}$ (b) 1 (c) 0 (d) -1

67. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is

(a) $\frac{x}{1+x^4}$ (b) $\frac{2x}{1+x^4}$ (c) -$\frac{2x}{1+x^4}$ (d) $\frac{1}{1+x^4}$

68. $\frac{d}{dx}[\cos(\log x + e^x)]$ at $x = 1$ is

(a) –sin(e) (b) sin(e) (c) –(1 + e) sin(e) (d) (1 + e) sin(e)

69. If $\tan\left(\frac{x+y}{x-y}\right) = \mathrm{k}$, then $\frac{\mathrm{dy}}{\mathrm{dx}}$ is equal to

(a) $\frac{-y}{x}$ (b) $\frac{y}{x}$ (c) $\sec^2\left(\frac{y}{x}\right)$ (d) $-\sec^2\left(\frac{y}{x}\right)$

70. If $y = \log(\sin e^x)$, then $\frac{dy}{dx}$ is

(a) $\cot e^x$ (b) $\operatorname{cosec} e^x$ (c) $e^x \cot e^x$ (d) $e^x \operatorname{cosec} e^x$

71. If $y = \log\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$, then $\frac{dy}{dx}$ is

(a) secx (b) cosecx (c) tanx (d) secx tanx

72. The derivative of $\sin(x^2)$ w.r.t. x, at $x = \sqrt{\pi}$ is

(a) 1 (b) -1 (c) $-2\sqrt{\pi}$ (d) $2\sqrt{\pi}$

73. For what value of l, the function given below is continuous at x = 0?

$$f(x) = \begin{cases} \frac{\sqrt{4+x}-2}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}.$$

(a) 0 (b) $\frac{1}{4}$ (c) 1 (d) 4

74. The value of k for which function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at x = 0 is

(a) 1 (b) 2 (c) any real number (d) 0

75. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is

(a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (b) $\sec^2\left(\frac{\pi}{4} - x\right)$

(c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$ (d) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

76. The derivative of x^{2x} w.r.t. x is

(a) x^{2x-1} (b) $2x^{2x}\log x$ (c) $2x^{2x}(1+\log x)$ (d) $2x^{2x}(1-\log x)$

77. The function f(x) = [x], where [x] denotes the greatest integer less than or equal to x is continuous at

(a) x = 1 (b) x = 1.5 (c) x= -2 (d) x = 4

78. If $x = \mathrm{A}\cos 4\mathrm{t} + \mathrm{B}\sin 4\mathrm{t}$, then $\frac{\mathrm{d}^2x}{\mathrm{dt}^2}$ is equal to

(a) x (b) –x (c) 16x (d) -16x

79. If x = a cosθ + b sinθ, y = a sinθ – b cosθ, then which one of the following is true?

(a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ (b) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(c) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ (d) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

80. Derivative of x^2 with respect to x^3, is

(a) $\frac{2}{3x}$ (b) $\frac{3x}{2}$ (c) $\frac{2x}{3}$ (d) $6x^5$

81. Derivative of e^{2x} with respect to e^x, is

(a) e^x (b) $2e^x$ (c) $2e^{2x}$ (d) $2e^{3x}$

82. Derivative of $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ is

(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 2 (d) $-\frac{1}{2}$

83. If $y = \cos^{-1}(e^x)$, then $\frac{dy}{dx}$ is

(a) $\frac{1}{\sqrt{e^{-2x}+1}}$ (b) $-\frac{1}{\sqrt{e^{-2x}+1}}$ (c) $\frac{1}{\sqrt{e^{-2x}-1}}$ (d) $-\frac{1}{\sqrt{e^{-2x}-1}}$

84. The derivative of 2^x w.r.t. 3^x is

(a) $\left(\frac{3}{2}\right)^x\frac{\log 2}{\log 3}$ (b) $\left(\frac{2}{3}\right)^x\frac{\log 3}{\log 2}$ (c) $\left(\frac{2}{3}\right)^x\frac{\log 2}{\log 3}$ (d) $\left(\frac{3}{2}\right)^x\frac{\log 3}{\log 2}$

85. If $y = \sin^{-1} x$, then $\frac{d^2y}{dx^2}$ is

(a) secy (b) secy.tany (c) $\sec^2 y \tan y$ (d) $\tan^2 y \sec y$

86. If $e^{x^2y} = c$, then $\frac{dy}{dx}$ is

(a) $\frac{xe^{x^2y}}{2y}$ (b) $\frac{-2y}{x}$ (c) $\frac{2y}{x}$ (d) $\frac{x}{2y}$

87. The value of constant c that makes the function f defined by $f(x) = \begin{cases} x^2 - c^2, & \text{if } x < 4 \\ cx + 20, & \text{if } x \geq 4 \end{cases}$

continuous for all real numbers is

(a) -2 (b) -1 (c) 0 (d) 2

ASSERTION - REASON TYPE QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

1. Assertion: If $y = \sin^{-1}\left(6x\sqrt{1-9x^2}\right)$, then $\frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$

Reason: $\sin^{-1}\left(6x\sqrt{1-9x^2}\right) = 3\sin^{-1} 2x$.

2. Assertion: If $x = at^2$ and $y = 2$ at then $\frac{d^2y}{dx^2} | t = 2 = \frac{-1}{16a}$

Reason: $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2 \times \left(\frac{dt}{dx}\right)^2$

3. Assertion: $\frac{d}{dx}(e^{\cos x}) = e^{\cos x}(-\sin x)$

Reason: $\frac{d}{dx}(e^x) = e^x$

4. Assertion: If $xy = e^{x-y}$ then $\frac{dy}{dx} = \frac{y(x-1)}{x(1+y)}$

Reason: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{dv}{dx}$

5. Assertion: If $x = a(\theta + \sin\theta), y = a(1-\cos\theta)$ then $\frac{dy}{dx} = \tan\frac{\theta}{2}$

Reason: $x = f(\theta), y = g(\theta)$ then $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

6. Assertion: If $y = A\sin x + B\cos x$ then $\frac{d^2y}{dx^2} + y = 0$

Reason: $\frac{d^2y}{dx^2} = \frac{d(dy)}{dx(dx)}$

7. Assertion: The function $f(x) = \frac{|x|}{x}$ is continuous at x = 0.

Reason: The left hand limit and right hand limit of the function $f(x) = \frac{|x|}{x}$ are not equal at x = 0.

8. Assertion: Let Let $f(x) = \begin{cases} 3x-4, & 0 \le x \le 2 \\ 2x+k, & 2 < x \le 9 \end{cases}$. If f is continuous at x = 2, then value of k is -2.

Reason: A function is said to be continuous at x = a if LHL = RHL = f(a).

9. Assertion: The number of points of discontinuity of the function f(x) = x – [x] in the interval (0, 7) are 6.

Reason: The greatest integer function [x] is continuous at all integral points.

10. Assertion: f(x) = |x|sinx is differentiable at x = 0.

Reason: If f(x) is not differentiable and g(x) is differentiable at x = a, then f(x).g(x) can still be differentiable at x = a.

11. Consider the function $f(x) = \begin{cases} \frac{x^2-5x+6}{x-3}, & \text{for } x \neq 3 \\ k, & \text{for } x = 3 \end{cases}$ is continuous at x = 3.

Assertion: The value of k is 4

Reason: If f(x) is continuous at point a then $\lim\limits_{x\to a} f(x) = f(a)$.

12. Consider the function f(x) = $\begin{cases} \frac{k|x-3|}{x-3}, & \text{if } x < 3 \\ 5, & \text{if } x \geq 3 \end{cases}$ is continuous at x = 3.

Assertion: The value of k is -5.

Reason: $\frac{|x-3|}{x-3} = \begin{cases} 1, \text{if } x \geq 3 \\ -1, \text{if } x < 3 \end{cases}$

13. Assertion: $\frac{d}{dx}$(sin(cosx)) = cos(sinx).

Reason: Let f be a real valued function which is a composite of two functions u and v; i.e. f = u ○ v. Suppose t = u(x) and if both $\frac{dt}{dx}$ and $\frac{dv}{dt}$ exist, we have $\frac{df}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx}$.

14. Assertion: $\frac{d}{dx}(\sin^{-1}(\cos x)) = -1$.

Reason: $\sin^{-1}(\sin x) = x$, for $|x| \leq 1$ and $\cos x = \sin(\frac{\pi}{2} - x)$.

15. If 2x + 3y = siny.

Assertion: $\frac{dy}{dx} = \frac{-2}{3-\cos y}$

Reason: 2x + 3y = siny is an explicit function.

16. If y = logx

Assertion: $xy'' + y' = 0$

Reason: $\frac{d^2y}{dx^2} = \frac{d(dy)}{dx(dx)}$

17. Assertion: |sinx| is continuous for all x ∈ R.

Reason: sinx and |x| are continuous in R.

18. Let f(x) = [x – 1] + |x – 2|, wher [ ] denotes greatest integer function and | | denotes the modulus function.

Assertion: f(x) is discontinuous at x = 2.

Reason: f(x) is non-derivable at x = 2.

19. Assertion: If f(x).g(x) is continuous at x = a, then f(x) and g(x) are separately continuous at x = a.

Reason: Any function f(x) said to be continuous at x = a if $\lim_{h\to 0} f(a+h) = f(a)$.

20. Assertion: Derivative of x^x with respect of x is $x^x(1 + \log x)$.

Reason: Assertion not true as derivative of x^x with respect of x is xx^{x-1}.

21. Assertion: $f(x) = \begin{cases} 3x-8, x \le 5 \\ 2k, x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$.

Reason: For a function f to be continuous at $x = a$, $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$.

VERY SHORT ANSWERS

1. Find the relationship between a and b so that the function f defined by $f(x) = \begin{cases} ax+1, & \text{if} \quad x \le 3 \\ bx+3, & \text{if} \quad x > 3 \end{cases}$ is continuous at x = 3.

2. Discuss the continuity of the function f(x) at x = $\frac{1}{2}$, when f(x) is defined as follows:

$$f(x) = \begin{cases} \frac{1}{2} + x, & 0 \le x < \frac{1}{2} \\ 1, & x = \frac{1}{2} \\ \frac{3}{2} + x, & \frac{1}{2} < x \le 1 \end{cases}$$

3. For what value of k is the function defined by $f(x) = \begin{cases} k(x^2+2), & \text{if} \quad x \le 0 \\ 3x+1, & \text{if} \quad x > 0 \end{cases}$ continuous at x = 0? Also check whether the function is continuous at x = 1.

4. Find $\frac{dy}{dx}$, if $(x^2+y^2)^2 = xy$.

5. If $xy + y^2 = \tan x + y$, find $\frac{dy}{dx}$.

6. If $y = \cos^{-1}\left[\frac{3x+4\sqrt{1-x^2}}{5}\right]$, find $\frac{dy}{dx}$.

7. Differentiate $\sin^{-1}\left[\frac{5x+12\sqrt{1-x^2}}{13}\right]$ w.r.t. x.

8. If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} + e^{y-x} = 0$.

9. If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, then prove that $\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$.

10. Find $\frac{dy}{dx}$ at $t = \frac{2\pi}{3}$ when $x = 10(t - \sin t)$ and $y = 12(1 - \cos t)$.

11. Find $\frac{dy}{dx}$ at $x = 1, y = \frac{\pi}{4}$ if $\sin^2 y + \cos xy = k$.

12. Differentiate $\sin^{-1}\left(\frac{2^{x+1}\cdot 3x}{1+(36)^x}\right)$ with respect to x.

13. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\frac{dy}{dx} = -\frac{y\log x}{x\log y}$.

14. If $y = \cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$, then find $\frac{dy}{dx}$.

15. Differentiate $\sin(\sqrt{\sin x + \cos x})$ with respect to x.

16. Evaluate the derivative of the function $\sqrt{\sin^4 x^2 + \cos^4 x^2}$ at $x = 0$.

17. Differentiate the function $\tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$ with respect to x.

18. Find $\frac{dy}{dx}$, when x and y are connected by the following relation: $xy + xe^{-y} + ye^x = x^2$

19. Find $\frac{dy}{dx}$, when x and y are connected by the following relation: $3\sin(xy) + 4\cos(xy) = 5$.

20. If $3^x + 3^y = 3^{x+y}$, show that $\frac{dy}{dx} = -3^{y-x}$.

21. Differentiate the function $x^{\sin 2x + \cos 2x}$ with respect to x.

22. Find $\frac{dy}{dx}$, when $x = 2\cos^2\theta, y = 2\sin^2\theta$.

23. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\frac{dy}{dx} = -\frac{y\log x}{x\log y}$

24. If $x = 2\cos t - \cos 2t$ and $y = 2\sin t - \sin 2t$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

25. Differentiate $\cos^{-1}(2x^2 - 1)$ with respect to $\cos^{-1} x$.

26. Find the second order derivative of the function $e^{-x}\sin x$.

27. If $y = \tan x + \sec x$, prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.

28. Find $\frac{d^2y}{dx^2}$, where $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ at $\theta = \frac{\pi}{2}$

29. Find $\frac{d^2y}{dx^2}$, where $x = a(1 - \cos\theta), y = a(\theta + \sin\theta)$ at $\theta = \frac{\pi}{2}$

30. If $y = \cot^{-1} x$, show that $(1 + x^2)y_2 - 2xy_1 = 0$.

31. Check for differentiability of the function defined by f(x) = | x – 5| at the point x = 5.

32. Check the differentiability of $f(x) = |\cos x|$ at $x = \frac{\pi}{2}$.

33. Check whether the function f(x) = $x^2|x|$ is differentiable at x = 0 or not.

34. Check the differentiability of $f(x) = \begin{cases} x^2 + 1, & 0 \le x < 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$ at $x = 1$.

35. Function f is defined as $f(x) = \begin{cases} 2x + 2, & \text{if} \quad x < 2 \\ k, & \text{if} \quad x = 2 \\ 3x, & \text{if} \quad x > 2 \end{cases}$. Find the value of k for which the function is continuous at x = 2.

36. If $f(x) = \begin{cases} x^2, & \text{if } x \ge 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.

37. Show that the function $f(x) = |x|^3$ is differentiable at all points of its domain.

38. Find the value(s) of 'λ' if the function $f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & \text{if } x \ne 0 \\ 1, & \text{if } x = 0 \end{cases}$ is continuous at x = 0.

39. Find the value of k for which the function f given as $f(x) = \begin{cases} \frac{1-\cos x}{2x^2}, & \text{if } x \ne 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at x = 0.

40. The function $f(x) = \begin{cases} 3ax + b, & \text{if} \quad x > 1 \\ 11, & \text{if} \quad x = 1 \\ 5ax - 2b, & \text{if} \quad x < 1 \end{cases}$ is continuous at x = 1. Find the values of a and b.

41. If $x^{1/3} + y^{1/3} = 1$, find $\frac{dy}{dx}$ at the point $\left(\frac{1}{8}, \frac{1}{8}\right)$.

42. If $y = \sqrt{\tan\sqrt{x}}$, prove that $\sqrt{x}\frac{dy}{dx} = \frac{1+y^4}{4y}$.

43. If $(x^2 + y^2)^2 = xy$, then find $\frac{dy}{dx}$.

44. If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.

45. If $y = A\sin 2x + B\cos 2x$ and $\frac{d^2y}{dx^2} - ky = 0$, find the value of k.

46. If $y = \left(x + \sqrt{x^2 - 1}\right)^2$, then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.

47. If $y = \sqrt{ax + b}$, prove that $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$.

48. If $f(x) = \begin{cases} ax + b & ; \quad 0 < x \le 1 \\ 2x^2 - x & ; \quad 1 < x < 2 \end{cases}$ is a differentiable function in (0, 2) then find the values of a and b.

49. If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at $x = 1$.

50. If $x = a\sin 2t, y = a(\cos 2t + \log\tan t)$, then find $\frac{dy}{dx}$.

51. If $xy = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$.

52. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

53. If $x = a\cos t$ and $y = b\sin t$, then find $\frac{d^2y}{dx^2}$.

54. If $y = \sqrt{\cos x + y}$, prove that $\frac{dy}{dx} = \frac{\sin x}{1-2y}$.

55. If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.

56. If $y = \operatorname{cosec}(\cot^{-1} x)$, then prove that $\sqrt{1 + x^2}\frac{dy}{dx} - x = 0$.

57. If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$

SHORT ANSWERS

1. Find the values of p and q for which f(x) = $\begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, \text{if } x < \frac{\pi}{2} \\ p, \text{if } x = \frac{\pi}{2} \\ \frac{q(1-\sin x)}{(\pi-2x)^2}, \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at x = $\frac{\pi}{2}$.

2. Find the value of constant k sot that the function f defined below is continuous at x = 0, where

$$f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \leq x < 1 \end{cases}$$

3. If $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & \text{when} \quad x < 0 \\ a, & \text{when} \quad x = 0 \\ \frac{\sqrt{x}}{(\sqrt{16+\sqrt{x}})-4}, & \text{when} \quad x > 0 \end{cases}$ and f is continuous at x = 0, find the value of a.

4. If $f(x) = \begin{cases} 1, & \text{if} \quad x \leq 3 \\ ax+b, & \text{if} \quad 3 < x < 5 \\ 7, & \text{if} \quad x \geq 5 \end{cases}$ find the values of a and b so that f(x) is a continuous function.

5. Find the value of k so that the following function is continuous at x = 2.

$$f(x) = \begin{cases} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2}; & x \neq 2 \\ k; \ x = 2 \end{cases}$$

6. For what value of a is the function f defined by $f(x) = \begin{cases} a\sin\frac{\pi}{2}(x+1), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ is continuous at x = 0.

7. Find all points of discontinuity of f, where f is defined as follows:

$$f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3. \\ 6x+2, & x \geq 3 \end{cases}$$

8. Show that the function f(x) is defined by $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1-\sqrt{1-x})}{x}, & x < 0 \end{cases}$ is continuous at x = 0.

9. If f(x) is defined by the following is continuous at x = 0, find the value of a, b and c.

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & \text{if} \quad x < 0 \\ c, & \text{if} \quad x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}, & \text{if} \quad x > 0 \end{cases}$$

10. Find the values of a and b, if the function f defined by $f(x) = \begin{cases} x^2 + 3x + a, & x \le 1 \\ bx + 2, & x > 1 \end{cases}$ is differentiable at x = 1.

11. Show that the function f(x) = |x – 1| + |x + 1| for all x ∈ R is not differentiable at the points x = -1 and x = 1.

12. Find whether the following function is differentiable at x = 1 and x = 2 or not.

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \le x \le 2. \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

13. For what value of λ of the function defined by $f(x) = \begin{cases} \lambda(x^2 + 2), & \text{if } x \le 0 \\ 4x + 6, & \text{if } x > 0 \end{cases}$ is continuous at x = 0? Hence check the differentiability of f(x) at x = 0.

14. If $y = \sin^{-1}\{x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\}$ and $0 < x < 1$, then find $\frac{dy}{dx}$.

15. If $y = \frac{x\cos^{-1}x}{\sqrt{1-x^2}} - \log\sqrt{1-x^2}$, then prove that $\frac{dy}{dx} = \frac{\cos^{-1}x}{(1-x^2)^{3/2}}$.

16. If $y = \tan^{-1}\left(\frac{a}{x}\right) + \log\sqrt{\frac{x-a}{x+a}}$, prove that $\frac{dy}{dx} = \frac{2a^3}{x^4 - a^4}$.

17. If $y = \sqrt{x^2+1} - \log\left[\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}}\right]$, find $\frac{dy}{dx}$.

18. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if $x = ae^{\theta}(\sin\theta - \cos\theta)$ and $y = ae^{\theta}(\sin\theta + \cos\theta)$.

19. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

20. If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, then show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.

21. If $y = x^3(\cos x)^x + \sin^{-1}\sqrt{x}$, find $\frac{dy}{dx}$.

22. If $y = \sqrt{x^2+1} - \log\left(\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\right)$, then find $\frac{dy}{dx}$.

23. Find $\frac{dy}{dx}$, when x and y are connected by the following relation: $x^3y^2 = \log(x+y) + \sin e^x$

24. If $x = e^{\tan^{-1}[(y-x^2)/x^2]}$, find $\frac{dy}{dx}$.

25. Find $\frac{dy}{dx}$, $x = \frac{a(1-t^2)}{1+t^2}, y = \frac{2bt}{1+t^2}$

26. Find $\frac{dy}{dx}$, $x = e^\theta(2\sin\theta + \sin 2\theta), y = e^\theta(2\cos\theta + \cos 2\theta)$

27. If $x = \sin^{-1}\left(\frac{2t}{1+t^2}\right), y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$, prove that $\frac{dy}{dx} = 1$.

28. If x=cos−1⍰11+t2,=sin−1⍰t1+t2, show that dy28. Show that $\frac{dy}{dx}$ is independent of t if $x = \cos^{-1}\frac{1}{\sqrt{t^2+1}}$ and $y = \sin^{-1}\frac{t}{\sqrt{t^2+1}}$.

29. Differentiate $\tan^{-1}\frac{2\sqrt{x}}{1-x}$ with respect to $\sin^{-1}\frac{2\sqrt{x}}{1+x}$.

30. Differentiate $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$ with respect to $\sin\left(2\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$

31. Differentiate $x^{\sin x}$ with respect to $(\sin x)^x$.

32. If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, prove that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$.

33. If $x = 2\cos t - \cot 2t, y = 2\sin t - \sin 2t$, find the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$.

34. If $y = e^x(\sin x + \cos x)$, prove that $\frac{d^2y}{dx^2} - 2y\frac{dy}{dx} + 2y = 0$.

35. If $y = \tan^{-1}\sqrt{x^2-1}$, show that $(2x^2-1)y_1 + x(x^2-1)y_2 = 0$.

36. Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$.

37. If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.

38. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

39. Given that $y=(\sin x)^x \cdot x^{\sin x}+a^x$, find $\frac{dy}{dx}$.

40. If $y=\sin(\tan^{-1} e^x)$, then find $\frac{dy}{dx}$ at $x=0$.

41. If $y=(\tan x)^x$, then find $\frac{dy}{dx}$.

42. If $x=e^{\cos 3t}$ and $y=e^{\sin 3t}$, prove that $\frac{dy}{dx}=-\frac{y\log x}{x\log y}$.

43. Show that: $\frac{d}{dx}(|x|)=\frac{x}{|x|}, x\neq 0$.

44. If $x^{30}y^{20}=(x+y)^{50}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

45. Find $\frac{dy}{dx}$, if $5^x+5^y=5^{x+y}$.

46. If $y=(\log x)^2$, prove that $x^2y''+xy'=2$.

47. If x cos(p + y) + cosp(p + y) = 0, prove that cosp $\frac{dy}{dx}=-\cos^2(p+y)$, where p is a constant.

48. Find the value of a and b so that function f defined as: $f(x)=\begin{cases}\frac{x-2}{|x-2|}+a, & \text{if } x<2\\ a+b, & \text{if } x=2\\ \frac{x-2}{|x-2|}+b, & \text{if } x>2\end{cases}$ is continuous function.

49. Given that $x^y+y^x=a^b$, where a and b are positive constants, find $\frac{dy}{dx}$.

50. Find $\frac{dy}{dx}$, if $y=(\cos x)^x+\cos^{-1}\sqrt{x}$ is given.

51. If $x=a\sin^3\theta, y=b\cos^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{4}$.

52. Find the values of a and b so that the following function is differentiable for all values of x:

$$f(x) = \begin{cases} ax + b, & x > -1 \\ bx^2 - 3, & x \leq -1 \end{cases}.$$

LONG ANSWERS

1. If $(\tan^{-1} x)^y + y^{\cot x} = 1$, then find $\frac{dy}{dx}$.

2. Differentiate with respect to x: $y = \left(x + \frac{1}{x}\right)^x + x^{\left(x+\frac{1}{x}\right)}$.

3. If $\cos y = x \cos(a + y)$, with $\cos a \neq \pm 1$, then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. Hence show that $\sin a \frac{d^2y}{dx^2} + \sin 2(a + y) \frac{dy}{dx} = 0$.

4. If $x = \cos t(3 - 2\cos^2 t)$ and $y = \sin t(3 - 2\sin^2 t)$, then find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

5. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t), 0 < t < \frac{\pi}{2}$, find $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$.

6. If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then prove that $x(x + 1)^2 y_2 + (x + 1)^2 y_1 = 2$.

7. Differentiate $(\sin 2x)^x + \sin^{-1}\sqrt{3x}$ with respect to x.

8. If $y = \log(x - 1)^{x-1} - \log(x + 1)^{x+1}$, prove that $\frac{dy}{dx} = \log\left(\frac{x-1}{1+x}\right)$.

9. If $\log y = \tan^{-1} x$, prove that $(1 + x^2)y_2 + (2x - 1)y_1 = 0$.

10. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

CASE BASED QUESTIONS

1. A man travel on a path given by $f(x) = \begin{cases} |x| + 3, \text{ if } x \leq -3 \\ -2x, \text{ if } -3 < x < 3. \\ 6x + 2, \text{ if } x \geq 3 \end{cases}$

Depending on the above information, answer the following questions:

(i) Are there any breaks in the path?

(ii) If so where is the break in the path?

2. Let f be a real valued function which is a composite of two functions u and v; i.e. $f = u \circ v$. Suppose $t = u(x)$ and if both $\frac{dt}{dx}$ and $\frac{dv}{dt}$ exist, we have $\frac{df}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx}$. This is called chain rule in the derivatives.

Based on the above information, find the derivatives of the following:

(i) $\sec(\tan\sqrt{x})$

(ii) $2\sqrt{\cot x^2}$

(iii) $\cos(x^3)\sin^2 x^5$

3. The derivative of a function is again differentiable then the derivative of first derivative is called second derivative. If $y = f(x)$ is function then its first derivative is denoted by $f'(x)$ or $\frac{dy}{dx}$ or y_1 and its second derivative is denoted by $f''(x)$ or $\frac{d^2y}{dx^2}$ or y_2.

Basing the above information answer the following:

(i) Find the second derivative of cos(logx)

(ii) Find the second derivative of $\tan^{-1}x$.

(iii) If $y = \sin^{-1} x$, then show that $(1 - x^2)y_2 - xy_1 = 0$.

4. Let f(x) be a real valued function, then its

LEFT HAND DERIVATIVE (LHD): $Lf'(a) = \lim_{h \to 0} \frac{f(a-h)-f(a)}{-h}$

RIGHT HAND DERIVATIVE (LHD): $Rf'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{-h}$

Also, a function f(x) is said to be differentiable at x = a if its LHD and RHD at x = a exist and are equal.

For the function $f(x) = \begin{cases} |x-3|, x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$, answer the following questions.

(i) Find the RHD of f(x) at x = 1.

(ii) Show that f(x) is non-differentiable at x = 3.

(iii) Find the value of $f'(2)$.

5. A pottery made a mud vessel, where the shape of the pot is based on $f(x) = |x - 3| + |x - 2|$ where f(x) represents the height of the pot.

(i) When $x > 4$, what will be the height in term of x?

(ii) Find the derivative of f(x) at x = 3.

(iii) What is the function when the x lies between (2, 3)?

(Or)

If the potter is trying to make a pot using the function $f(x) = [x]$ will he get a pot or not? Why?

6. Read the following passage and answer the questions given below:

The relation between the height of the plant (y cm) with respect to its exposure to the sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to the sunlight.

(i) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight?

(ii) Does the rate of growth of the plant increase or decrease in the first three days?

(iii) What will be the height of the plant after 2 days?

7. Swathi was noticing the path traced by a crawling insect and she observed that the path traced is given by $x = at^2$, $y = 2at$.

Based on the above information, answer the following questions.

(i) Find $\frac{dx}{dt}$

(ii) Find $\frac{dy}{dx}$

(iii) Find $\frac{d^2y}{dx^2}$ at $t = 4$

UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. The value of k for which $f(x) = \begin{cases} 3x+5, & x \geq 2 \\ kx^2, & x < 2 \end{cases}$ is a continuous function, is

(a) $-\frac{11}{4}$ (b) $\frac{4}{11}$ (c) 11 (d) $\frac{11}{4}$

2. If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{-y}{x}$ (b) $\frac{y}{x}$ (c) $\sec^2(\frac{y}{x})$ (d) $-\sec^2(\frac{y}{x})$

3. The derivative of x^{2x} w.r.t. x is

(a) x^{2x-1} (b) $2x^{2x}\log x$ (c) $2x^{2x}(1 + \log x)$ (d) $2x^{2x}(1 - \log x)$

4. Assertion: $f(x) = \frac{x^2+5x+6}{x^2-4x+4}$ is continuous for R.

Reason: A polynomial function is everywhere continuous.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. If $y = \operatorname{cosec}(\cot\sqrt{x})$, then find $\frac{dy}{dx}$.

6. Differentiate the function $y = \log\sqrt{\frac{1-\cos x}{1+\cos x}}$ with respect to x.

7. If $y = \sin(2\sin^{-1} x)$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$.

SECTION C

Each carry 3 marks

8. If $y = x^{\cos x} + (\cos x)^{\sin x}$, find $\frac{dy}{dx}$

9. If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$, prove that $\frac{dy}{dx} = \frac{x-1}{2x(x+1)}$.

SECTION D

Each carry 5 marks

10. Differentiate the function $(\tan^{-1} x)^{\cot x} + (\cot^{-1} x)^{\tan x}$ with respect to x.

11. If $x = \sin t, y = \sin pt$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.

SECTION E

12. Example1: x – y – 6 = 0

Example 2: x + sinxy – y = 0.

When a relationship between x and y is expressed in a way that it is easy to solve for y and write y = f(x), we say that y is given as an explicit function of x. In the second case it is implicit that y is function of x and we say that the relationship of the second type above gives function implicitly. With the above information find the derivative of the following.

(i) y + siny = cosx

(ii) $x^2 + xy + y^2 = 5$.

(iii) $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$

ANSWERS

MCQ	A-R	VSA	SA II	LA	CS
1. (d) None of these 2. (b) 2 3. (c) Exactly three points 4. (b) 1 5. (a) $\frac{\cos x}{2y-1}$ 6. (d) No value 7. (a) Continuous and differentiable at x = 0	1. (c) 2. (c) 3. (b) 4. (b) 5. (a) 6. (a) 7. (d) 8. (a) 9. (c) 10. (a)	1. $a - b = \frac{2}{3}$ 2. f(x) is not continuous at $x = \frac{1}{2}$ 3. $k = \frac{1}{2}$ 4. $\frac{dy}{dx} = \frac{y-4x^3-4xy^2}{4x^2y+4y^3-x}$ 5. $\frac{dy}{dx} = \frac{\sec^2 x - y}{x+2y-1}$ 6. $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$ 7. $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ 10. $\frac{2\sqrt{3}}{5}$	1. $p = \frac{1}{2}$ and q = 4 2. k = -1 3. a = 8 4. a = 3 and b = -8 5. k = 10 6. $a = \frac{1}{2}$ 7. f(x) is discontinuous at x = 3 9. $a = \frac{-3}{2}$ and $c = \frac{1}{2}$, b can be any real	1. $\frac{y^{\cot x}\cdot\operatorname{cosec}^2 x\cdot\log y-(\tan^{-1}x)^{y-1}\cdot\frac{y}{(1+x^2)}}{(\tan^{-1}x)^y\cdot\log(\tan^{-1})x+y^{\cot x-1}\cdot\cot x}$ 2. $\left(x+\frac{1}{x}\right)^x\left[\log\left(x+\frac{1}{x}\right)+\frac{x^2-1}{x^2+1}\right]+x^{\left(x+\frac{1}{x}\right)}\left[\frac{x^2-1}{x^2}\log x+\frac{x^2+1}{x^2}\right]$ 4. 1 5. $\frac{d^2x}{dt^2} = a(\cos t - t\sin t)$, $\frac{d^2y}{dt^2} = a(t\cos t + \sin t)$ $\frac{d^2y}{dx^2} = \frac{\sec^3 t}{at}$ 7. $(\sin 2x)^x\{2x\cot 2x + \log(\sin 2x)\} + \frac{3}{2\sqrt{3x-9x^2}}$	1. (i) yes (ii) At x = 3 2. (i) $\frac{1}{2\sqrt{x}}\sec(\tan\sqrt{x})\tan(\tan\sqrt{x})\sec^2\sqrt{x}$ (ii) $-\frac{2x}{\sqrt{\cot x^2}}\operatorname{cosec}^2(x^2)$ (iii) $-3x^2\sin^2 x^5\cdot\sin x^3 + 10x^4\sin x^5\cos x^5\cos x^3$ 3.(i) $\frac{1}{x^2}\sin\log x - \frac{1}{x^2}\cos(\log x)$ (ii) $-\frac{2x}{(1+x^2)^2}$ 4. (i) -1 (iii) -1

8. (b) $\frac{1}{\sqrt{2}}$ 9. (b) 5 10. $-\sec^2\left(\frac{\pi}{4}-x\right)$ 11. (c) $e^x \cot e^x$ 12. (c) $6x^3 \sin x^3 \cos x^3$ 13. (d) -16x 14. (d) $\frac{1}{\sqrt{2}}$ 15. (a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$	11. (d) 12. (a) 13. (d) 14. (a) 15. (c) 16. (a) 17. (a) 18. (b) 19. (d) 20. (c) 21. (d)	11. $\frac{\pi}{4(\sqrt{2}-1)}$ 13. $\frac{2\cdot 6^x \cdot \log_e 6}{1+36^x}$ 14. $\frac{-2^{x+1}\cdot \log_{e^2} 2}{1+4^x}$ 15. $\cos(\sqrt{\sin x + \cos x}) \cdot \frac{1}{2\sqrt{\sin x+\cos x}} \cdot (\cos x - \sin x)$ 16. 0 17. $\frac{a}{2(1+a^2x^2)}$ 18. $\frac{dy}{dx} = \frac{2x-y-e^{-y}-ye^x}{x-xe^{-y}+e^x}$ 19. $\frac{dy}{dx} = \frac{-y}{x}$ 22. -1	number. 10. a = 3 and b = 5 12. f is not differentiable at x = 1 and differentiable at x = 2. 13. $\lambda = 3$ and f is not differentiable at x = 0. 14. $\frac{1}{\sqrt{1-x^2}} - \frac{1}{2}\cdot\frac{1}{\sqrt{x-x^2}}$ 17. $\frac{\sqrt{x^2+1}}{x}$		5. (i) 2x – 5 (ii) Not differentiable at x = 3 (iii) 1 (Or) No, because greatest integer function is not differentiable 6. (i) 4 – x (ii) Rate of growth of the plant decreases for the first three days. (iii) 6 7. (i) 2at (ii) $\frac{1}{t}$ (iii) $\frac{-1}{128a}$

16. (b) $\frac{1}{x}$ 17. (c) $\frac{x^2-1}{x^2-4}$ 18. (b) $yx^x(1+\log x)$ 19. (d) $\frac{(1+\log y)^2}{\log y}$ 20. (b) $yy'' - (y')^2 + 1 = 0$ 21. (c) $n = \frac{m\pi}{2}$ 22. (b) $\frac{4x}{1-x^4}$ 23. (b) $\frac{3}{4t}$ 24. (a) $\left(\frac{2}{3}\right)^{1/2}$		24. $\sqrt{2}+1$ 25. 2 26. $-2e^{-x}\cos x$ 29. $-\frac{1}{a}$ 31. f is not differentiable at $x = 5$ 32. f is not differentiable at $x = \frac{\pi}{2}$ 33. f(x) is differentiable at x = 0. 34. $f(x)$ is not differentiable	18. 1 21. $x^3(\cos x)^x\left[\frac{3}{x} - x\tan x + \log(\cos x)\right] + \frac{1}{2\sqrt{x-x^2}}$ 22. $\frac{\sqrt{x^2+1}}{x}$ 23. $\frac{dy}{dx} = \frac{1+e^x(x+y)\cos e^x - 3x^2y^2}{2x^3y(x+y)-1}$ 24. $x[2(1+\tan\log x) + \sec^2\log x]$ 25. $\frac{b(t^2-1)}{2at}$		

25. (a) e 26. (c) 1 27. (c) 0 28. (b) $\frac{1-x}{x \cos y}$ 29. (b) -$\frac{2\cos x}{e^{\cos x}}$ 30. (a) – coty.cosec^{2}y 31. (b) $\frac{g''(t)f'(t)-g'(t)f''(t)}{[g'(t)]^3}$ 32. (c) 3cos2x 33. (a) x(5 + 6logx)		at $x = 1$. 35. k = 6 38. $\lambda = \pm 1$ 39. $k = \frac{1}{4}$ 40. a = 3, b = 2 41. -1 43. $\frac{dy}{dx} = \frac{y-4x(x^2+y^2)}{4y(x^2+y^2)-x}$ 44. -8 45. k = -4 48. a = 3 & b = -2 49. 1	26. $\frac{2\cos\theta+\cos 2\theta-2\sin\theta-2}{2\cos\theta+2\cos 2\theta+2\sin\theta+}$ 29. 1 30. $\frac{-1}{2x}$ 31. $\frac{x^{\sin x}\left(\frac{\sin x}{x}+\cos x \log x\right)}{(\sin x)^x(x\cot x+\log \sin x}$ 32. $-\frac{3}{2}$ 36. $\frac{1}{2}$ 39. $(\sin x)^x \cdot x^{\sin x}\left[\log(\sin x) + x\cot x + \frac{\sin x}{x} + \log x \cdot \cos x\right] +$		

34. (b) $4^{x+9}\log 4$ 35. (c) x = 3 36. (a) $a = \frac{1}{2}$, b = 4 37. (d) No point of discontinuity 38. (a) $\frac{-\sec^2(\pi/4-x)}{2\sqrt{\tan(\pi/4-x)}}$ 39. (d) $\frac{\cos(\sqrt{\sin\sqrt{x}})(\cos\sqrt{x})}{4\sqrt{x}\sqrt{\sin\sqrt{x}}}$ 40. (a) t 41. (b) $-\frac{y\log x}{x\log y}$		50. cos2t 53. $-\frac{b}{a^2}\mathrm{cosec}^3 t$ 55. $\frac{dy}{dt} = -12\cos^2(\sec^2 2t) \times \sin(\sec^2 2t) \times \sec^2 2t \times \tan 2t.$	$a^x \log a$ 40. $\frac{1}{2\sqrt{2}}$ 41. $\frac{dy}{dx} = (\tan x)^x \left[\left(\frac{x\sec^2 x}{\tan x}\right) + \log(\tan x)\right]$ 45. -5^{y-x} 48. a = 1, b = -1 49. $\frac{dy}{dx} = \frac{-(x^{y-1}y + y^x\log y)}{(x^y\log x + y^{x-1}x)}$ 50. $(\cos x)^x(-x\tan x + \log(\cos x)) +$		

42. (b) $\frac{2}{3}$ 43. (a) 0 44. (c) Continuous but not differentiable at x = 1 45. (a) Continuous everywhere but not differentiable at x = 0 46. (a) Continuous at x = 0 as well as x			$\frac{-1}{2\sqrt{x-x^2}}$ 51. $\frac{4\sqrt{2}b}{3a^2}$ 52. a = 3, $b = -3/2$		

= 1 47. (b) a = 0 48. (a) -2 49. (d) $-x\frac{dy}{dx}$ 50. (b) $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ 51. (b) 1 52. (c) f (x) is continuous but not differentiable, at x = 0 and x = 1 53. (b) 5					

54. (c) f(x) is continuous and differentiable ∀x ∈ ℝ - {0} 55. (b) 2 56. (d) continuous everywhere 57. (c) $6x^2 \sin x^3 \cos x^3$ 58. (d) $\frac{11}{4}$ 59. (d) No value 60. (a)					

continuous but not differentiable at x = 0 61. (a) continuous and differentiable at x = 0 62. (a) continuous, but not differentiable at x = 0 and x = 2 63. (a) -1 64. (d) $-\frac{y}{x}$ 65. (c) -					

2cosx$e^{\sin^2 x}$ 66. (d) -1 67. (b) $\frac{2x}{1+x^4}$ 68. (c) –(1 + e) sin(e) 69. (b) $\frac{y}{x}$ 70. (c) e^xcote^x 71. (a) secx 72. (c) -2$\sqrt{\pi}$ 73. (b) $\frac{1}{4}$ 74. (d) 0					

75. (a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ 76. (c) $2x^{2x}(1 + \log x)$ 77. (b) x = 1.5 78. (d) -16x 79. (a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ 80. (a) $\frac{2}{3x}$ 81. (b) $2e^{x}$ 82. (b) $\frac{1}{2}$ 83. (d) $-\frac{1}{\sqrt{e^{-2x}-1}}$					

84. (c) $\left(\frac{2}{3}\right)^{x} \frac{\log 2}{\log 3}$ 85. (c) $\sec^2 y \tan y$ 86. (b) $\frac{-2y}{x}$ 87. (a) -2					

UNIT TEST

1. (d) $\frac{11}{4}$

2. (b) $\frac{y}{x}$

3. (c) $2x^{2x}(1 + \log x)$

4. (d) Assertion (A) is false but Reason (R) is true.

5. $\frac{\operatorname{cosec}(\cot\sqrt{x})\cot(\cot\sqrt{x})\times\operatorname{cosec}^2(\sqrt{x})}{2\sqrt{x}}$

6. cosecx

7. Proof

8. $x^{\cos x}\left(\frac{\cos x}{x} - \sin x\log x\right) + (\cos x)^{\sin x}[-\sin x\tan x + \cos x\log\cos x]$

9. Proof

10. $\frac{dy}{dx} = (\tan^{-1} x)^{\cot x}\left[\frac{\cot x}{\tan^{-1} x(1+x^2)} - \operatorname{cosec}^2 \log(\tan^{-1} x)\right] + (\cot^{-1} x)^{\tan x}\left[\sec^2 x\log(\cot^{-1} x) - \frac{\tan x}{\cot^{-1} x(1+x^2)}\right]$

11. Proof

12. (i) $-\frac{\sin x}{1+\cos y}$ (ii) $-\frac{2x+y}{x+2y}$ (iii) $-\frac{2}{1+x^2}$

CHAPTER 6 – APPLICATION OF DERIVATIVES

RATE OF CHANGE OF QUANTITIES

If a quantity y varies with another quantity x, satisfying some rule y = f(x), then $\frac{dy}{dx}$ (or $f'(x)$) represents the rate of change of y with respect to x and $\left.\frac{dy}{dx}\right]_{x=x_0}$ (or $f'(x_0)$) represents the rate of change of y with respect to x at x = x_o.

If two variables x and y are varying with respect to another variable t, i.e. x = f(t) and y = g(t), then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, if $\frac{dx}{dt} \neq 0$.

In other words, the rate of change of y with respect to x can be calculated using the rate of change of y and that of x both with respect to t.

Note: $\frac{dy}{dx}$ is positive if y increases as x increases and is negative if y decreases as x increases.

MARGINAL COST

Marginal cost represents the instantaneous rate change of the total cost at any level of output. If C(x) represents the cost function for x units produced, then the marginal cost denoted by MC is given by

$$MC = \frac{d(C(x))}{dx}$$

MARGINAL REVENUE

Marginal revenue represents the rate of change of total revenue with respect to the number of items sold at any instant. If R(x) is the revenue function for x units sold, then marginal revenue denoted by MR is given by

$$MR = \frac{d(R(x))}{dx}$$

INCREASING AND DECREASING FUNCTION

DEFINITION 1

Let I be an open interval contained in the domain of a real valued function f. Then f is said to be

(i) Increasing on I if $x_1 < x$, in I $\Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x, \in$ I

(ii) Strictly increasing on I if $x_1 < x$, in I $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x, \in I$

(iii) Decreasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in I$

(iv) Strictly decreasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in I$

DERIVATIVE TEST

Let f be a continues function on [a, b] and differentiable on (a, b). Then,

(i) f is increasing on $[a, b]$ if $f'(x) \geq 0$ for each $x \in (a, b)$

(ii) f is strictly increasing on $[a, b]$ if $f'(x) > 0$ for each $x \in (a, b)$

(iii) f is decreasing on $[a, b]$ if $f'(x) \leq 0$ for each $x \in (a, b)$

(iv) f is strictly decreasing on $[a, b]$ if $f'(x) < 0$ for each $x \in (a, b)$

(v) f is constant function on $[a, b]$ if $f'(x) = 0$ for each $x \in (a, b)$

FINDING THE INTERVALS OF INCREASING AND/OR DECREASING OF A FUNCTION

A critical point of a continuous function f is a point at which the derivative is zero or undefined. Critical points are the points on the graph where the function's rate of change is altered - either a change from increasing to decreasing, in concavity, or in some unpredictable fashion. Critical points are useful for determining extrema and solving optimization problems.

Let f be a differentiable function on an interval I. To find intervals on which f is increasing and decreasing:

(i) Consider the function y = f(x)

(ii) Find $f'(x)$

(iii) Put $f'(x) = 0$ and solve to get the critical point(s) or critical numbers

(iv) Use the critical values to divide I into subintervals.

(v) Pick any point x_0 in each subinterval, and find the sign of $f'(x_0)$.

(a) If $f'(x_0) > 0$, then f is increasing on that subinterval.

(b) If $f'(x_0) < 0$, then f is decreasing on that subinterval.

MONOTONIC FUNCTION

A function which is either increasing or decreasing in a given interval I, is called monotonic function.

MAXIMA AND MINIMA

Let *f* be a function defined on an interval I. Then

(a) *f* is said to have a maximum value in I, if there exists a point c in I such that $f(c) \geq f(x)$, for all $x \in$ I

The number *f(c)* is called the maximum value of *f* in I and the point *c* is called a point of maximum value of *f* in I.

(b) *f* is said to have a minimum value in I, if there exists a point *c* in I such that $f(c) \leq f(x)$, for all $x \in$ I.

The number *f (c)*, in this case, is called the minimum value of *f* in I and the point *c*, in this case, is called a point of minimum value of *f* in I.

(c) *f* is said to have an extreme value in I if there exists a point c in I such that *f (c)* is either a maximum value or a minimum value of *f* in I.

The number *f (c)*, in this case, is called an extreme value of *f* in I and the point *c* is called an extreme point.

Note

(i) Every monotonic function assumes its maximum/minimum value at the end points of the domain of definition of the function.(ii) Every continuous function on a closed interval has a maximum and a minimum value.

LOCAL MAXIMA AND LOCAL MINIMA

Let f be a real valued function and let c be an interior point in the domain of f. Then

(a) c is called a point of local maxima if there is an $h > 0$ such that $f(c) \geq f(x)$, for all x in $(c - h, c + h)$

The value $f(c)$ is called the local maximum value of f.

(b) c is called a point of local minima if there is an $h > 0$ such that $f(c) \leq f(x)$, for all x in $(c - h, c + h)$

The value $f(c)$ is called the local minimum value of f.

THEOREM 2

Let f be a function defined on an open interval I. Suppose c ∈ I be any point. If f has a local maxima or a local minima at x = c, then either $f'(c) = 0$ or f is not differentiable at c.

Note

A point c in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable is called a critical point of f. Note that if f is continuous at c and $f'(c) = 0$, then there exists an $h > 0$ such that f is differentiable in the interval $(c - h, c + h)$.

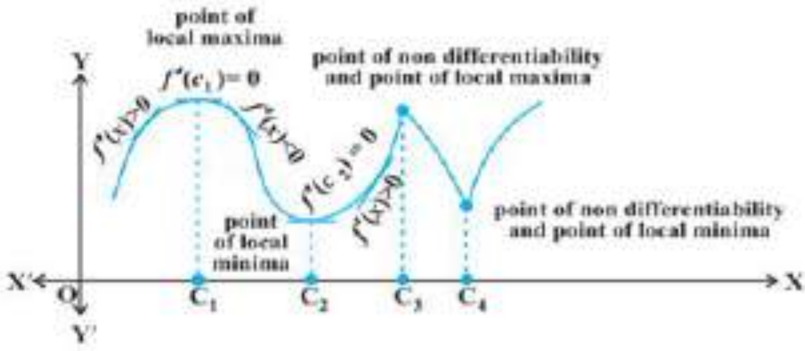
FIRST DERIVATIVE TEST

Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then

(i) If $f'(x)$ changes sign from positive to negative as x increases through c, i.e., if $f'(x) > 0$ at every point sufficiently close to and to the left of c, and $f'(x) < 0$ at every point sufficiently close to and to the right of c, then c is a point of local maxima.

(ii) If $f'(x)$ changes sign from negative to positive as x increases through c, i.e., if $f'(x) < 0$ at every point sufficiently close to and to the left of c, and $f'(x) > 0$ at every point sufficiently close to and to the right of c, then c is a point of local minima.

(iii) If $f'(x)$ does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. Infact, such a point is called point of inflection.



Note

If *c* is a point of local maxima of *f*, then *f(c)* is a local maximum value of *f*. Similarly, if *c* is a point of local minima of *f*, then *f(c)* is a local minimum value of *f*.

SECOND DERIVATIVE TEST

Let *f* be a function defined on an interval I and $c \in I$. Let f be twice differentiable at *c*. Then

(i) $x = c$ is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$

The value $f(c)$ is local maximum value of *f*.

(ii) $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$

In this case, $f(c)$ is local minimum value of *f*.

(iii) The test fails if $f'(c) = 0$ and $f''(c) = 0$.

In this case, we go back to the first derivative test and find whether c is a point of local maxima, local minima or a point of inflexion.

MAXIMUM AND MINIMUM VALUES OF A FUNCTION IN A CLOSED INTERVAL

THEOREM 5

Let f be a continuous function on an interval I = [a, b]. Then f has the absolute maximum value and f attains it at least once in I. Also, f has the absolute minimum value and attains it at least once in I.

THEOREM 6

Let f be a differentiable function on a closed interval I and let c be any interior point of I. Then

(i) $f'(c) = 0$ if f attains its absolute maximum value at c.

(ii) $f'(c) = 0$ if f attains its absolute minimum value at c.

WORKING RULE FOR FINDING ABSOLUTE MAXIMUM AND/OR ABSOLUTE MINIMUM VALUES OF A FUNCTION

Step 1: Find all critical points of f in the interval, i.e., find points x where either $f'(x) = 0$ or f is not differentiable.

Step 2: Take the end points of the interval.

Step 3: At all these points (listed in Step 1 and 2), calculate the values of f.

Step 4: Identify the maximum and minimum values of f out of the values calculated in Step 3. This maximum value will be the absolute maximum (greatest) value of f and the minimum value will be the absolute minimum (least) value of f.

STEPS TO FIND THE POINTS OF LOCAL MAXIMA AND LOCAL MINIMA BY SECOND DERIVATIVE TEST

Step 1: For the function $y = f(x)$, find $f'(x)$.

Step 2: Find $f''(x)$.

Step 3: Put $f'(x) = 0$ and solve this equation for x. Let its roots be a, b, c, etc.

Step 4: At $x = a$,

(i) if $f''(a) < 0$, then $x = a$ is a point of local maxima.

(ii) if $f''(a) > 0$, then $x = a$ is a point of local minima.

(iii) if $f''(a) = 0$, we cannot say anything.

Step 5:

(i) If then x=a is a point of local maxima, f(a) is local maximum value.

(ii) If then x=a is a point of local minimum, f(a) is local minimum value.

Remark

Absolute maximum value is also called as global maximum value or greatest value. Similarly absolute minimum value is called as global minimum value or the least value.

MULTIPLE CHOICE QUESTIONS

1. $f(x) = x^x$ has a stationary point at

(a) x = e (b) x = $\frac{1}{e}$ (c) x = 1 (d) x = $\sqrt{e}$

2. The area of a trapezium is defined by function f and given by f(x) = $(10 + x)\sqrt{100 - x^2}$, then the are when it is maximized is

(a) $75cm^2$ (b) $7\sqrt{3}cm^2$ (c) $75\sqrt{3}cm^2$ (d) $5cm^2$

3. The rate of change of the area of a circle with respect to its radius r at r = 6cm is

(a) 10π (b) 12 π (c) 8 π (d) 11 π

4. The total revenue in rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when x = 15 is

(a) 116 (b) 96 (c) 90 (d) 126

5. The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing in the interval is

(a) $(-\infty, 2) \cup (3, \infty)$ (b) $(-\infty, 2)$ (c) $(-\infty, 2] \cup [3, \infty)$ (d) $[3, \infty)$

6. In a sphere of radius r, a right circular cone of height h having maximum curved surface area is inscribed. The expression for the square of curved surface of cone is

(a) $2\pi r^2 rh(2rh + h^2)$ (b) $\pi^2 hr(2rh + h^2)$ (c) $2\pi^2 r(2rh^2 - h^3)$ (d) $2\pi^2 r^2(2rh - h^2)$

7. The least value of the function $f(x) = ax + \frac{b}{x} (a > 0, b > 0, x > 0)$ is

(a) $\frac{a}{b}$ (b) $2\sqrt{ab}$ (c) 0 (d) None of these

8. Let $f(x) = 2\sin^3 x - 3\sin^2 x + 12\sin x + 5, 0 \leq x \leq \frac{\pi}{2}$. Then f(x) is

(a) Decreasing in $\left[0, \frac{\pi}{2}\right]$ (b) Increasing in $\left[0, \frac{\pi}{2}\right]$

(c) Increasing in $\left[0, \frac{\pi}{4}\right]$ and decreasing in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (d) None of these

9. The value of b for which the function $f(x) = x + \cos x + b$ is strictly decreasing over R is

(a) $b < 1$ (b) No value of b exists (c) $b \leq 1$ (d) $b \geq 1$

10. If $f(x) = a(x - \cos x)$ is strictly decreasing in $\mathbb{R}$, then ' a ' belongs to

(a) {0} (b) $(0, \infty)$ (c) $(-\infty, 0)$ (d) $(-\infty, \infty)$

11. The function f(x) = tanx – 4x is strictly decreasing in

(a) $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$ (b) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ (c) $\left(-\frac{\pi}{3}, \frac{\pi}{2}\right)$ (d) $(\frac{\pi}{2}, \pi)$

12. If If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, where $0 < x \leq 1$, in this interval

(a) Both f(x) and g(x) are increasing functions

(b) Both f(x) and g(x) are decreasing functions

(c) f(x) is an increasing function

(d) g(x) is an increasing function

13. $f(x) = \left(\frac{e^{2x}-1}{c^{2x}+1}\right)$ is

(a) An increasing function (b) A decreasing function (c) An even function (d) None of these

14. The maximum profit that a company can make if the profit function is given by $P(x) = 41 + 24x - 18x^2$ is

(a) 25 (b) 43 (c) 62 (d) 49

15. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ decreasing is

(a) [-1, ∞) (b) [-2, -1] (c) (-∞, -2] (d) [-1, 1]

16. The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly

(a) Increasing in $\left(\pi, \frac{3\pi}{2}\right)$ (b) Decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(c) Decreasing in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (d) Decreasing in $\left[0, \frac{\pi}{2}\right]$

17. The smallest value of the polynomial $x^3 - 18x^2 + 96x$ in [0. 9] is

(a) 126 (b) 0 (c) 135 (d) 160

18. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has

(a) Two points of local maximum (b) Two points of local minimum

(c) One maxima and one minima (d) No maxima or minima

19. The function $f(x) = \cot^{-1} x + x$ increases in the interval

(a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $(-\infty, \infty)$ (d) $(0, \infty)$

20. If $f(x) = \frac{1}{x+1} - \log(1 + x), x > 0$, then f is

(a) An increasing (b) A decreasing function

(c) Both increasing and decreasing function (d) None of these

21. The least value of k for which the function $x^2 + kx + 1$ is an increasing function in the interval $1 < x < 2$ is

(a) -4 (b) -3 (c) -1 (d) -2

22. For the every value of x the function $f(x) = \frac{1}{5^x}$ is

(a) Decreasing (b) Increasing

(c) Neither increasing nor decreasing (d) Increasing for $x > 0$ and decreasing for $x < 0$

23. On the interval (1,3), the function $f(x) = 3x + \frac{2}{x}$ is

(a) Strictly increasing (b) Strictly decreasing

(c) Decreasing in (2, 3) only (d) Neither increasing nor decreasing

24. The function f defined by $f(x) = x^3 - 6x^2 - 36x + 7$ is increasing if

(a) x > 2 and also x > 6 (b) x > 2 and also x < 6

(c) x < -2 and also x < 6 (d) x < -2 and also x > 6

25. The function $f(x) = x^3 - 3x$ is

(a) Increasing on $(-\infty, -1) \cup (1, \infty)$ and decreasing on $(-1, 1)$

(b) Decreasing on $(-\infty, -1) \cup (1, \infty)$ and increasing on $(-1, 1)$

(c) Increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$

(d) Decreasing on $(0, \infty)$ and increasing on $(-\infty, 0)$

26. The minimum value of curve $y = xe^x$ is

(a) $-\frac{1}{e}$ (b) $\frac{1}{e}$ (c) –e (d) e

27. The function $x^2 - 4x, x \in [0,4]$ attains minimum value at

(a) x = 0 (b) x = 2 (c) x = 1 (d) x = 4

28. What is the maximum value of $\sin x + \cos x - \sqrt{2}$?

(a) $-\sqrt{2}$ (b) $\sqrt{2}$ (c) 0 (d) None of these

29. Let the f: R → R be defined by $f(x) = 2x + \cos x$, then f

(a) Has a maximum at x = π (b) Has a maximum at x = 0

(c) Decreasing function (d) Increasing function

30. $y = x(x - 3)^2$ decreases for the values of x given by

(a) 1 < x < 3 (b) x < 0 (c) x > 0 (d) $0 < x < \frac{3}{2}$

31. A spherical balloon is expanding. If at any instant, rate of increase of its volume is 16 times of the increase of its radius, then its radius at that instant is

(a) $\frac{1}{\sqrt{\pi}}$ (b) $\frac{2}{\sqrt{\pi}}$ (c) $\frac{2}{\pi}$ (d) $\frac{4}{3\sqrt{\pi}}$

32. In the interval $x \in (-3,3) - \{0\}$, the function $f(x) = \frac{x}{3} + \frac{3}{x}, x \neq 0$ is

(a) Decreasing (b) Increasing

(c) Neither increasing nor decreasing (d) Partly increasing and partly decreasing

33. For all real values of x, increasing function is

(a) $\frac{1}{x}$ (b) x^2 (c) x^3 (d) x^4

34. The function $f(x) = \tan^{-1} x - \log x$ decreases in

(a) $(-\infty, 0)$ (b) $\left(-\infty, \frac{1}{2}\right)$ (c) $(0, \infty)$ (d) $\left(\frac{1}{2}, \infty\right)$

35. The function $f(x) = xe^{1-x}$

(a) Decrease in (0, 2) (b) Increase in $(0, \infty)$

(c) Strictly decreases in $(1, \infty)$ (d) Strictly increase in $\left(\frac{1}{2}, 2\right)$

36. If $f(x) = x^2 + ax + 5$ is increasing function in [2,3], then the minimum value of a is

(a) -4 (b) -2 (c) 2 (d) 4

37. The value of a, for which $f(x) = -x^3 + 4ax^2 + 2x - 5$ is decreasing $\forall x \in R$, is

(a) (1, 2) (b) (3, 4) (c) Real values of a (d) No value of a

38. The value of a for which the function $(a + 2)x^3 - 3ax^2 + 9ax - 1$ decreases monotonically for all real x is

(a) a < -2 (b) a > -2 (c) -3 < a < 0 (d) $-\infty < a \leq -3$

39. If $f(x) = \frac{1}{4x^2+2x+1}$, then its maximum value is

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) 1 (d) $\frac{4}{3}$

40. The rate of change of the surface area of the sphere of radius r when the radius is increasing at the rate of 2cm/s is proportional to

(a) $\frac{1}{r^2}$ (b) $\frac{1}{r}$ (c) r (d) r^2

41. The rate of change of the volume of sphere with respect to its surface area, when its radius is 2 units is

(a) 1 (b) 2 (c) 3 (d) 4

42. The sides of an equilateral triangle are increasing at the rate of 2cm/sec. The rate at which the area increase, when side is 10cm is

(a) $10\text{cm}^2/\text{s}$ (b) $\sqrt{3}\text{cm}^2/\text{s}$ (c) $10\sqrt{3}\text{cm}^2/\text{s}$ (d) $\frac{10}{3}\text{cm}^2/\text{s}$

43. The critical point(s) of $f(x) = x^2e^{-x}$ is/are

(a) 0 (b) 2 (c) 0 & 2 (d) None of these

44. The intervals in which the function f given by $f(x) = 2x^2 - 3x$ is strictly increasing is

(a) $\left(\frac{3}{4}, \infty\right)$ (b) $\left[\frac{3}{4}, \infty\right)$ (c) $\left(-\infty, \frac{3}{4}\right)$ (d) $\left(-\infty, \frac{3}{4}\right]$

45. For the function $f(x) = x\cos\frac{1}{x}, x \geq 1, f(x)$ is

(a) Strictly increasing in $(1, \infty)$ (b) Strictly decreasing in $(1, \infty)$

(c) Neither increasing nor decreasing in $(1, \infty)$ (d) None of these

46. Local Maximum and Local Minimum values of the function $(x - 1)(x + 2)^2$ are

(a) 4, 0 (b) 0, 4 (c) -4, 0 (d) None of these

47. The sum of two non-zero numbers is 4. The minimum value of the sum of their reciprocal is

(a) $\frac{3}{4}$ (b) $\frac{6}{5}$ (c) 1 (d) None of these

48. The absolute maximum value of $f(x) = \sin\sin x - \cos\cos x$ where $x \in [0, \pi]$ is

(a) 0 (b) 1 (c) -1 (d) $\sqrt{2}$

49. The range of values of x for which the function $y = 5 - 8x - 2x^2$ is increasing is

(a) x < 3 (b) x > 2 (c) x < -2 (d) x > -2

50. The function $f(x) = \frac{2x^2-1}{x^4}, x > 0$, decreases in the interval

(a) $(-\infty, 0)$ (b) $[1, \infty)$ (c) [-1, 1] (d) None of these

51. Let f(x) be a continuous function on [a, b] and differentiable on (a, b). Then, this function f(x) is strictly increasing in (a, b) if

(a) $f'(x) < 0, \forall x \in (a, b)$ (b) $f'(x) > 0, \forall x \in (a, b)$

(c) $f'(x) = 0, \forall x \in (a, b)$ (d) $f(x) > 0, \forall x \in (a, b)$

52. If $f(x) = a(x - \cos x)$ is strictly decreasing in $\mathbb{R}$, then 'a' belongs to

(a) {0} (b) (0, ∞) (c) (-∞, 0) (d) (-∞,∞)

53. The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is

(a) $(-1, \infty)$ (b) (-2, -1) (c) $(-\infty, -2)$ (d) [-1, 1]

54. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is

(a) strictly decreasing on R

(b) strictly increasing on R

(c) neither strictly increasing nor strictly decreasing on R

(d) strictly decreasing on $(-\infty, 0)$

55. The function f(x) = kx – sinx is strictly increasing for

(a) k > 1 (b) k < 1 (c) k > -1 (d) k < -1

56. The rate of change of surface area of a sphere with respect to its radius ‘r’ when r = 4cm is

(a) $64\pi \text{cm}^2/\text{cm}$ (b) $48\pi \text{cm}^2/\text{cm}$ (c) $32\pi \text{cm}^2/\text{cm}$ (d) $16\pi \text{cm}^2/\text{cm}$

57. If the sides of square are decreasing at the rat of 1.5cm/s, the rate of decrease of its perimeter is

(a) 1.5cm/s (b) 6cm/s (c) 3cm/s (d) 2.25cm/s

58. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing when x = 5 is

(a) -60units/sec (b) 60units/sec (c) -70units/sec (d) -140units/sec

59. The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is

(a) 0 (b) 2 (c) 4 (d) 5

60. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has local minima at x equal to

(a) 2 (b) 1 (c) 0 (d) -2

ASSERTION - REASON TYPE QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

1. Assertion: f(x) = tanx – x is always increases.

Reason: Any function $y = f(x)$ is increasing if $\frac{dy}{dx} > 0$.

2. Assertion: $f(x) = x^4$ is decreasing in the interval $(0, \infty)$.

Reason: Any function $y = f(x)$ is decreasing if $\frac{dy}{dx} < 0$.

3. Assertion: The maximum value of $\frac{\log x}{x}$ in $[2, \infty)$ is $\frac{1}{e}$.

Reason: Any function $y = f(x)$ is increasing if $\frac{dy}{dx} > 0$.

4. Assertion: The function $f(x) = x(x + 2)^2$ is increasing in $(0,1) \cup (2, \infty)$.

Reason: $\frac{dy}{dx} = 0$, when $x = 0,1,2$.

5. Assertion: $f(x) = \frac{1}{x-7}$ is decreasing in $x \in R - \{7\}$.

Reason: $f^{|}(x) < 0$, for all $x \neq 7$.

6. Assertion: $f(x) = e^x$ is an increasing function for all $x \in R$.

Reason: $f^{|}(x) \leq 0$, then f(x) is an increasing function.

7. Assertion: If f(x) = log(sinx), x > 0 is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

Reason: $f'(x) \geq 0$ then f(x) is strictly increasing function.

8. Assertion: If f(x) = a(x + sinx) is increasing function if $a \in (0, \infty)$.

Reason: The given function f(x) is increasing only if $a \in (0, \infty)$.

9. Assertion: $f(x) = x - \frac{1}{x}$ is strictly increasing in $R - \{0\}$.

Reason: A function f(x) is called decreasing in I if $f'(x) < 0$ for all x ∈ I.

10. Assertion: $f(x) = (x - 1)e^x + 1$ is an increasing function for all x > 0.

Reason: $f'(x) > 0$ for all $x \in (0, \infty)$.

11. Assertion: $f(x) = x^2e^{-x}$ is increasing in $(0, 2)$.

Reason: $f(x)$ is decreasing in $(2, \infty)$.

12. Assertion: $f(x) = x^2 - x + 1$ is neither increasing nor decreasing on (-1, 1).

Reason: $f'(x) > 0$ then f(x) is strictly increasing function for all x ∈ I.

13. Assertion: If m and M are respectively minimum and maximum values of $f(x) = (x - 1)^2 + 3$ for all x ∈ [-3, 1] then $(m, M) = (f(1), f(-3))$.

Reason: f(x) is strictly increasing on [-3, 1].

14. Assertion: $f(x) = \sin\left(2x + \frac{\pi}{4}\right)$ is strictly decreasing in $x \in \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$.

Reason: The function given above is strictly increasing and decreasing in $x \in \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$.

15. Assertion: The function given as $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$ is always a strictly increasing function in the interval $x \in \left(0, \frac{\pi}{4}\right)$.

Reason: For the given function $f(x), f'(x) > 0$ if $x \in \left(0, \frac{\pi}{4}\right)$.

16. Assertion: For all real values of a, $f(x) = \sin x - ax + b$ is decreasing on $x \in R$.

Reason: Given function f(x) is decreasing only iff $a \in [1, \infty)$.

17. Assertion: Let f(x) = cos3x. Then it will be increasing in an interval iff $f'(x) \geq 0$ in that interval.

Reason: The function $f(x) = \cos 3x$ is increasing as $f'(x) \geq 0$ in $x \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right]$.

18. Assertion: For $f(x) = x + \frac{1}{x}, x \neq 0$, maximum and minimum values both exist.

Reason: Maximum value of f(x) is less than its minimum value.

19. Assertion: f(x) = sin2x + 3 is defined for all real values of x.

Reason: Maximum value of f(x) is 4 and minimum value of f(x) is 2.

20. Assertion: Let $f(x) = e^{1/x}$ is defined for all $x \in R$.

Reason: $f(x) = e^{\frac{1}{x}}$ is always decreasing as $f'(x) < 0$ in $x \in R - \{0\}$.

21. Assertion: An angle $\theta, 0 < \theta < \frac{\pi}{2}$ which increases twice as fast as its sine, is $\frac{\pi}{6}$.

Reason: Rate of change of $3x^2$ with respect to x is 6x.

22. Assertion: The rate of change of the volume of sphere (having radius 'r') with respect to its surface area when r = 2 units is 1 units ^{3}/ units ^{2}.

Reason: The rate of change of the surface area of the sphere of radius r when the radius is increasing at the rate of 2cm/s is proportional to r^2.

23. Assertion: The radius of a balloon is increasing at the rate of 10cm/s. The rate at which surface area of balloon is increasing when radius is 15cm is 1200πcm/s.

Reason: A spherical ice-ball melts uniformly. When its radius is 10cm, then the rate of the change of its volume with respect to the radius is $400\pi cm^3/cm$.

24. Assertion: The length l of a rectangle is decreasing at the rate of 3cm/min and its width b is increasing at the rate of 2cm/min. When l = 7cm and b =3cm, then the rate of change of area of rectangle is $5\text{cm}^2\text{min}^{-1}$.

Reason: For the above data given A, the rate of change of the perimeter of the rectangle is 1cm/min.

25. Let us consider the function $f(x) = \frac{4x^2+1}{x}$

Assertion: f(x) is increasing on $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

Reason: Critical points are -$\frac{1}{2}$ and $\frac{1}{2}$.

VERY SHORT ANSWERS

1. The amount of pollution content added in air in a city due to x-diesel vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the marginal increase in pollution content when 3 diesel vehicles are added.

2. Show then the function f defined by $f(x) = (x - 1)e^x + 1$ is an increasing function for all x > 0.

3. Find the intervals in which the function f is given by $f(x) = \tan x - 4x, x \in \left(0, \frac{\pi}{2}\right)$ is (a) strictly increasing (b) strictly decreasing.

4. Show that the function f is given by $f(x) = x^3 - 3x^2 + 4x, x \in R$ is strictly increasing on R.

5. A man 2m high walks at a uniform speed of 6km/hr away from a lamp post 6m high. Find the rate at which the length of his shadow increases.

6. A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2m/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 away from the wall?

7. At what point of the ellipse $16x^2 + 9y^2 = 400$, does the ordinate decrease at the same rate at which the abscissa increase?

8. The area of an expanding rectangle is increasing at the rate of $48 cm^2/s$. The length of the rectangle always equals the square of the breadth. At what rate the length is increasing at the instant when the breadth is 4.5cm.

9. The radius of a cylinder is increasing at the rate of 2m/sec and its height is decreasing at the rate of 3m/sec. Find the rate of change of volume when radius is 3m and the height is 5m.

10. The diameter and altitude of a right circular cylinder are found at a certain instant to 10cm and 20cm respectively. If the diameter is increasing at the rate of 2cm/s, what change in the altitude will keep the volume constant.

11. A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. Find the points on the curve at which the y-coordinate is changing twice as fast as the x-coordinate.

12. Show that the function $f(x) = \frac{x-2}{x+1}$ is increasing on R.

13. Show that the function $f(x) = 2 - 3x + 3x^2 - x^3$ is decreasing on R.

14. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is strictly increasing function on the interval $\left(0, \frac{\pi}{4}\right)$.

15. Find the absolute maximum value and absolute minimum value of the function $f(x) = 2x^3 - 24x + 107$ in [1,3].

16. Determine two positive numbers whose sum is 15 and sum of whose squares is minimum.

17. Find two positive numbers whose product is 64 having minimum sum.

18. Divide 4 into two positive numbers such that the sum of the squares of one and cube of other is minimum.

19. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is maximum.

20. Divide 64 into two parts such that the sum of the cubes of two parts is minimum.

21. The area of the circle increasing at uniform rate of $2\text{cm}^2/\text{sec}$. How fast is the circumference of the circle increasing when the radius r = 5cm?

22. The surface of a cube increases at the rate of $72\text{cm}^2/\text{sec}$. Find the rate of change of its volume, when the edge of the cube measures 3cm.

23. The volume of a cube is increasing at the rate of $6\text{cm}^3/\text{s}$. How fast is the surface area of cubic increasing, when the length of an edge is 8cm?

24. Find the interval in which the function $x^3 - 12x^2 + 36x + 17$ is strictly increasing.

25. Find the interval in which the function $\text{f}(x) = x^4 - 4x^3 + 10$ is strictly decreasing.

26. Find the points at which the function $f(x) = \frac{4+x^2}{4x-x^3}$ is discontinuous.

27. Find the interval in which the function $f(x) = 2x^3 - 3x$ is strictly increasing.

28. Find the interval in which the function $f(x) = x^3 + \frac{1}{x^3}, x \neq 0$ is decreasing.

29. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}(x \neq 0)$, find the value of (M – m).

30. Show that the function $f(x) = \frac{16\sin x}{4+\cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

31. Find the sub-intervals in which f(x) = $\log(2 + x) - \frac{x}{2+x}, x > -2$ is increasing or decreasing.

32. Find the maximum and minimum values of the function given by f(x) = 5 + sin2x.

33. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima not minima.

34. A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x-coordinate is 1, y-coordinate is changing twice as fast at x-coordinate. Find the value of a.

35. If the circumference of circle is increasing at the constant rate, prove that rate of change of area of circle is directly proportional to its radius.

36. If equal sides of an isosceles triangle with fixed base 10cm are increasing at the rate of 4cm/sec, how fast is the area of triangle increasing at an instant when all sides become equal?

37. If f(x) = a(tanx – cotx), where a > 0, then find whether f(x) is increasing or decreasing function in its domain.

38. Find the intervals in which the function f(x) = $x^4 - 4x^3 + 4x^2 + 15$ is strictly increasing.

39. Find local maximum value and local minimum value (whichever exists) for the function $f(x) = 4x^2 + \frac{1}{x}(x \neq 0)$.

40. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

41. Determine whether the function $f(x) = x^2 - 6x + 3$ is increasing or decreasing in [4, 6].

42. Show that the function f given by f(x) = sinx + cosx is strictly decreasing in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$.

SHORT ANSWERS

1. Show that $y = \log(1 + x) - \frac{2x}{2+x}, x > -1$ is an increasing function of x throughout its domain.

2. Show that $f(x) = 2x + \cot^{-1} x + \log\left(\sqrt{1 + x^2} - x\right)$ is increasing in R.

3. The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}\mathrm{cm/s}$. Find the rate at which its side is increasing,

4. Find the maximum and minimum values of $f(x) = \sec x + \log \cos^2 x, 0 < x < 2\pi$.

5. Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \sin^2 x - \cos x, x \in [0, \pi]$.

6. If the function $f(x) = 2x^3 - 9mx^2 + 12m^2x + 1$, where $m > 0$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then find the value of m.

7. Find the intervals in which the function $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is (a) strictly increasing (b) strictly decreasing.

8. Find the intervals in which the function given by $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ is (a) increasing (b) decreasing.

9. Find the intervals in which the following function is (a) increasing (b) decreasing: $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$.

10. Find the intervals in which the following function is (a) increasing (b) decreasing: $f(x) = 2x^3 - 9x^2 + 12x - 15$.

11. Find the intervals in which the following function $f(x) = (x - 1)^3(x - 2)^2$ is (a) increasing (b) decreasing.

12. Find the intervals in which the following function $x^3 + \frac{1}{x^3}, x \neq 0$ is (a) increasing (b) decreasing.

13. Water is dripping out form a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of $2cm^3/s$ through a tiny hole at the vertex at the bottom. When the slant height of the water is 4cm, find the rate decrease of the slant height of the water.

14. A cone is 10cm in diameter and 10cm deep. Water is poured into it at the rate of 4 cubic cm/min. At what rate is the water level rising at the instant when depth is 6cm?

15. The radius of the base of a cone is increasing at the rate of 3cm/min and altitude is decreasing at the rate of 4cm/min. Find the rate if change of total surface of cine when radius is 7cm and altitude is 24cm.

16. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic meter per hour. Find the rate at which the level of the water is rising at the instant when the depth of the water tank is 4m.

17. Find the intervals in which the following functions are increasing or decreasing: (i) $f(x) = 2x^3 - 8x^2 + 10x + 5$ (ii) $f(x) = -2x^3 - 9x^2 - 12x + 1$.

18. Find the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is increasing or decreasing.

19. Find the local maxima or minima of the function $f(x) = \sin^4 x + \cos^4 x, 0 < x < \frac{\pi}{2}$. Find also the local maxima or local minimum values.

20. Find the local maxima or minima of the function $3x^4 - 2x^3 - 6x^2 + 6x + 1$. Find also the local maxima or local minimum values.

21. Determine the maximum value and minimum value of each of the following in the stated domains: (i) $f(x) = x + \sin 2x$ in $[0, \pi]$ (ii) $f(x) = 2\cos 2x - \cos 4x$ in $[0, \pi]$.

22. Divide a number 15 into two parts such that square of one multiplied with cube of the other is minimum.

23. Find the dimensions of the rectangle of area 96 sq.cm whose perimeter is the least. Also find the perimeter.

24. Show that of all the rectangle of given area, square has the smallest perimeter.

25. Show that the surface area of a closed cuboid with square base and given volume is minimum when it is a cube.

26. Find the intervals in which the function $f(x) = \frac{\log x}{x}$ is strictly increasing or strictly decreasing.

27. Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \frac{x}{2} + \frac{2}{x}$ on the interval [1, 2].

LONG ANSWERS

1. Prove that the surface area of a solid cuboid of square base and given volume is minimum when it is a cube.

2. If the sum of hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

3. Show that the volume of the greatest cylinder that can be inscribed in a cone of height h and semi-vertical angle 'α' is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

4. The sum of the perimeter of a circle and a square is k, where k is some constant. Prove that he sum of their areas is least when the side of the square is double the radius of the circle.

5. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, find the dimensions of the rectangle that will produce the largest area of the window.

6. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

7. Find the maximum area of the isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$withits vertex at one end of the major axis.

8. Show that a cylinder of a given volume which is open at the top has minimum total surface area, when its height is equal to the radius of its base.

9. A window is of the form of a semi-circle with a rectangle on its diameter. The total perimeter of the window is 10m. Find the dimensions of the window to admit the maximum light through the whole opening.

10. If the length of three sides of a trapezium other than base is 10cm each, then find the area of the trapezium when it is maximum.

11. Prove that of all the rectangle inscribed in a given circle, the square has the maximum area.

12. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half that of the cone.

13. An open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

14. A tank with rectangular base and rectangular sides open at the top is to be constructed so that its depth is 2m and volume is $8m^3$. If building of tank costs ₹70 per sq. meter for the base and ₹45 per sq. meter for the sides, what is the cost of least expensive tank?

15. A manufacturer can sell x items at a price of ₹$\left(5 - \frac{x}{100}\right)$. The cost price of x items is ₹$\left(\frac{x}{5} + 500\right)$. Find the number of items he should sell to earn the maximum profit.

16. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be least, when tdepth of the tank is half of its width.

17. Show that a right circular cylinder of given volume, open at the top has minimum total surface area, provided that its height is equal to the radius of the base.

18. Show that a closed right circular cylinder of given surface area and maximum volume is such that its height is equal to the diameter of the base.

19. Show that the rectangle of maximum area that can be inscribed in a circle is square.

20. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius a is square of side $a\sqrt{2}$.

21. Prove that the height of a right circular cylinder of maximum volume that can be inscribed in a sphere of radius a is $\frac{2a}{\sqrt{3}}$.

22. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

23. A wire of length 50m is to be cut into two pieces. One of the two pieces is to be made into a square and other into a circle. What should be length of two pieces so that the combined area of the square and circle is minimum?

24. Show that the height of the cylinder of maximum volume that can be inscribed in a cone of height H is $\frac{H}{3}$.

25. Given the sum of the perimeters of square and circle. Show that the sum of their areas is least when the sides of the square are equal to the diameter of the circle.

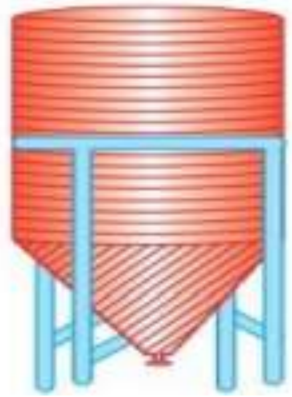
26. It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at x = 1. Find the value of 'a', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.

27. The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

CASE BASED QUESTIONS

1. Read the following passage and answer the following questions:

A tank as shown in the figure below, formed using a combination of a cylinder and a cone offers better drainage as compared to a flat bottomed tank.



A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of $2\text{cm}^3/s$. The semi-vertical angle of the conical tank $45°$.

(i) Find the volume of water in the tank in terms of its radius r.

(ii) Find the rate of change of radius at an instant when $\text{r} = 2\sqrt{2}\text{cm}$.

(iii) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $\text{r} = 2\sqrt{2}\text{cm}$.

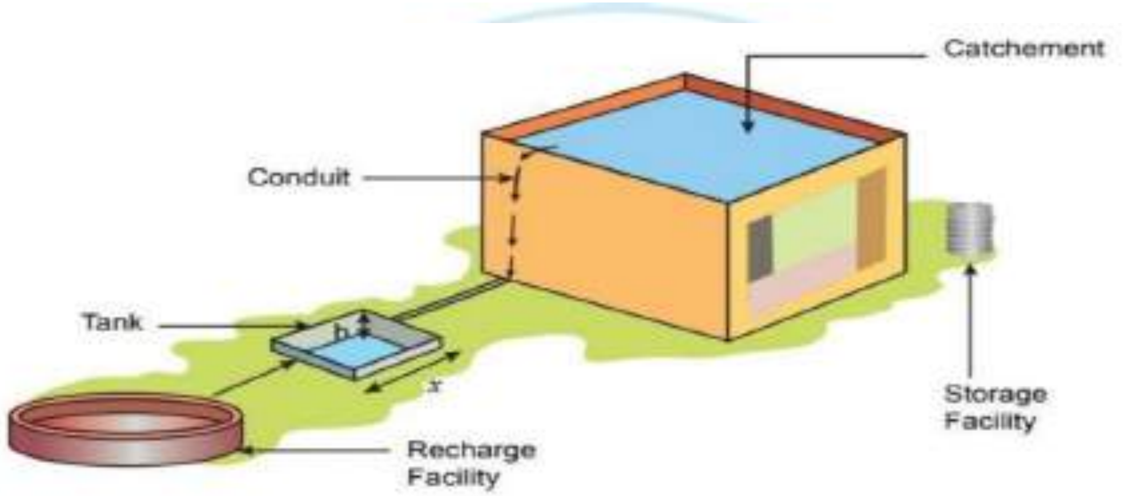
(Or)

Find the rate of change of height h at an instant when slant height is 4cm.

2. Read the following passage and answer the following questions:

In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity 250m^3. The cost of land is ₹5,000per square meter and cost of digging increase with depth and for the whole tank it is ₹$40{,}000\text{h}^2$, where h is the depth of the tank in meters. x is the side of the square base of the tank in meters.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



(i) Find the total cost C of digging the tank in terms of x.

(ii) Find $\frac{dC}{dx}$.

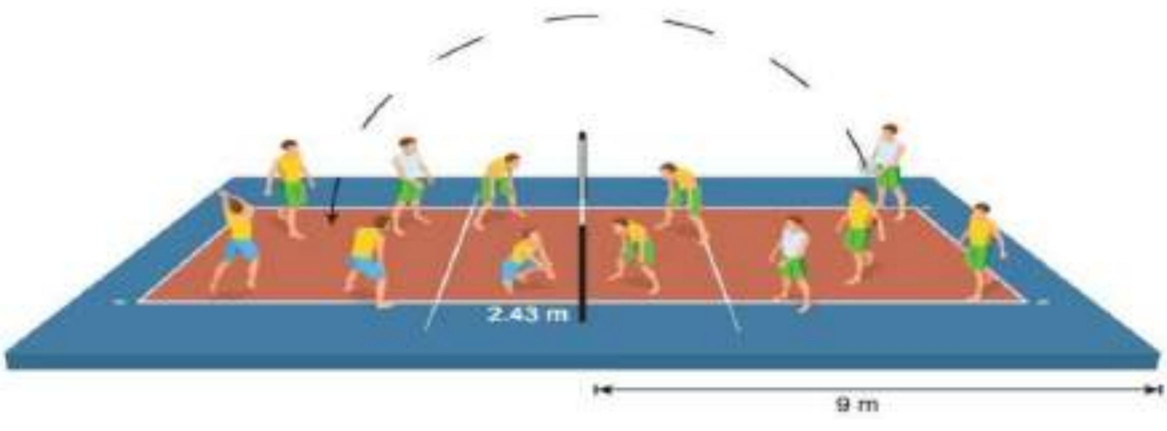
(iii) Find the value of x for which cost C is minimum.

(Or)

Check whether the cost function C(x) expressed in terms of x is increasing or not, where x > 0.

3. Read the following passage and answer the following questions:

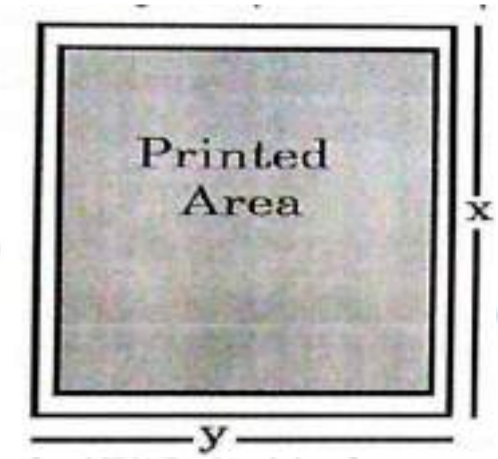
A volleyball player serves the ball which takes a parabolic path is given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where h(t) is the height of ball at any time t (in seconds), (t ≥ 0).



(i) Is h(t) a continuous function? Justify.

(ii) Find the time at which the height of the ball is maximum.

4. Following is the pictorial description for particular page, selected by school administration.



The total area of the page is 150cm^2. The combined width of the margin at the top and bottom is 3cm and the side 2cm.

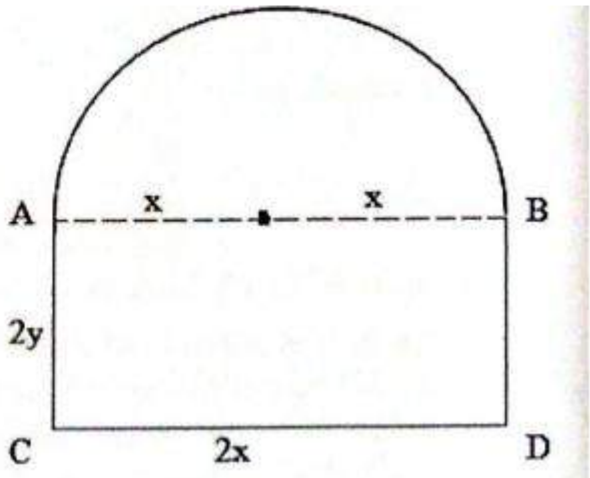
Using the above information, answer the following questions:

(i) Find the relation between x and y.

(ii) Find the area of the printable region of the page in terms of x.

(iii) What should be the dimensions of the page so that it has maximum area to be printed?

5. Mr.Shashi who is an architect designs a building for a small company. The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangles which is surmounted by semi-circular opening. The window is having perimeter 10m as shown below:



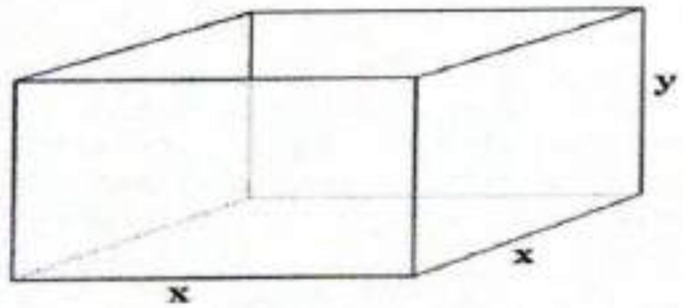
Based on the above information answer the following:

(i) Find the combined area of the rectangular region and semi-circular region of the window expressed as a function of x.

(ii) Find the maximum value of area of the whole window.

(iii) In order to get the maximum light input through the whole window, find the area of only semi-circular opening of the window.

6. A box manufacturing company wanted to hire engineering graduates from an engineering college. The instructor of company told a participant to construct an open box with square base from a given quantity of cardboard having area C sq.units. Consider the figure of the box shown.



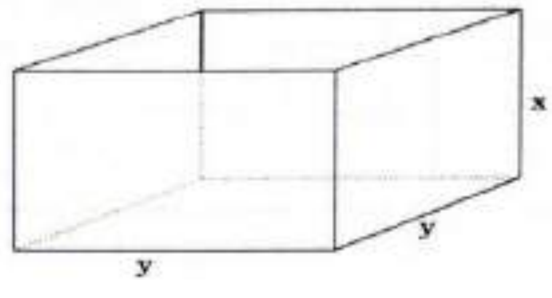
Based on the above information, answer the following:

(i) Obtain the volume of the box in terms of x.

(ii) At what value of x, the volume of box is maximum?

(iii) Find the maximum value of the volume of the box.

7. Rahul owns a box manufacturing company, which is indulged in making steel boxes for packing. The company received an order to construct an open box with square base for given quantity of metallic sheet having an area C^2 sq. units. See the figure given below for the dimensions of the box.



Based on the above information, answer the following:

(i) Find the volume of the box in terms of y.

(ii) For what value of y, the volume of box is maximum?

(iii) Find the maximum value of the volume of the box.

8. The local nursery charges the cost of planting trees by the following formula $C(x) = x^3 - 45x^2 + 600x$, where x is the number of trees and C(x) is the cost of planting x trees in rupees. The owner of local nursery has imposed a restriction that it can plant 10 to 20 trees in community park for a fair distribution.

Keeping the above discussion in mind, answer the following:

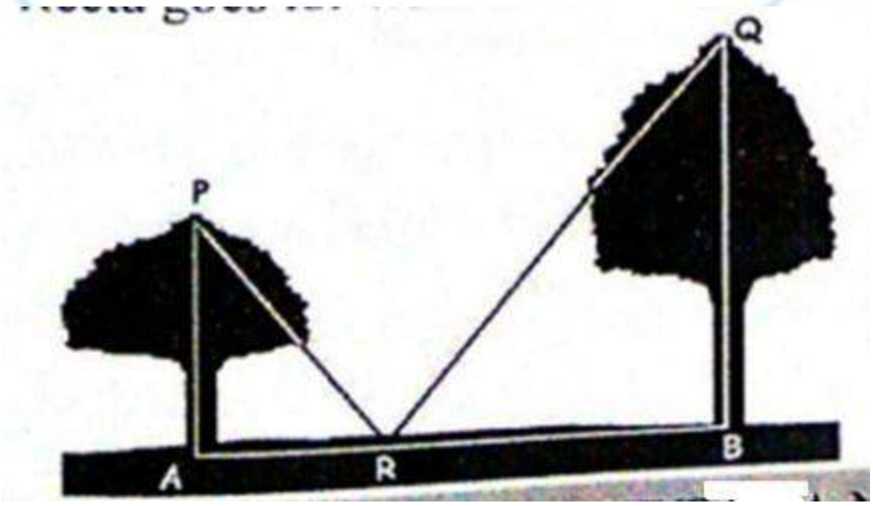
(i) What is the derivative of C(x) with respect to x?

(ii) What are the possible number of trees if $C'(x) = 0$?

(iii) For how many trees should the person place the order, so that his expenses are least?

(iv) How much is the least amount that the person has to spend?

9. Swathi goes for walk in a community park daily. She notices two specific trees in line (as seen in the figure below) whose heights are AP = 16m and BQ = 22m respectively are 20m apart from each other. She stands at a point (say R) in between these trees such AR =x m.

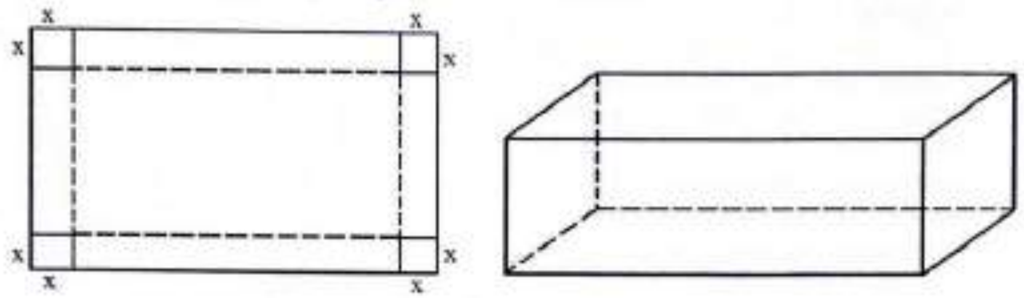


Using the information given above, answer the following:

(i) Find $RP^2 + RQ^2$, in terms of x.

(ii) If $RP^2 + RQ^2$ is minimum then, find the value of x.

10. An online retail company ships its products in the cartons. Each of the cartons is made by a rectangular sheet of fiberboard with dimensions of 8m by 3m. While making carton, equal squares of side length x meters cut-off from each corner of the rectangular sheet of fiberboard. After that, the resulting flaps are folded up to form the carton. See the given figure below to identify the sides of carton.



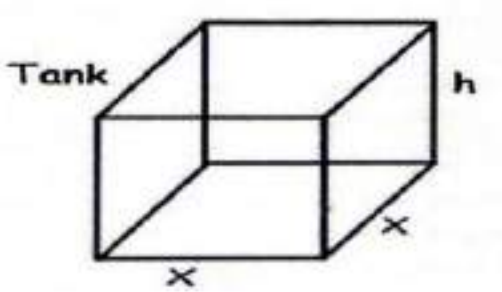
Based on the above information, answer the following:

(i) Find the value of x (in metres) for which the volume V of the carton is maximum.

(ii) What is the length (in metres) of the carton formed, for maximum value of V ?

(iii) What is the maximum volume of the carton formed?

11. A social organization working for welfare of the society, decides to conserve water in village. For this purpose, the organization proposes to construct an open water tank having a square base by using a metal sheet. The area of the metal sheet is to be least and water to be stored in the tank is 4000 cubic meters.



Using the information given above, answer the following:

(i) For what value of x, the area (A) of the tank is least?

(ii) Write the least value of area (A).

12. Vasihnavi is running a small company of manufacturing LED bulbs. She can sell x bulbs at a price of ₹$(250 - x)$ each. The cost price of x bulb is ₹$(2x^2 - 50x + 12)$.

Using the above information, answer the following:

(i) How many bulbs should she sell to earn maximum profit?

(ii) What is the maximum profit (in ₹) earned by her?

13. Mr.Rahul Modi is owner of a drum manufacturing company. He gets an order of manufacturing cylindrical drums of high quality steel without top, with volume of V cubic units.

Using the information given above, answer the following:

(i) Assuming that r and h denote the radius of base and height of the drum respectively, then find the value of h in terms of V and r.

(ii) What is the value of radius r of base of the drum having least surface area?

(iii) What is the height of the drum, for which the drum is having least surface area?

14. A farmer has two small pieces of agricultural land. One is in the shape of square and the other is in the shape of an equilateral triangle. To save his crops from stray animals, he decides to fence his fields. For this purpose, he has a wire of length 40m. This wire has to be cut into two pieces to fence around the fields. The first piece of wire of length x m is used for fencing around the triangular field and the other piece is used for the square field.

Based on the information given above, answer the following:

(i) Let the combined area of square and triangle is denoted by A. Write A, in terms of x.

(ii) For what value of x, the combined area (A), is minimum?

(iii) Find the value of minimum combined area (A) of both the fields.

15. Mr. Narendra decides to build his own house. He purchases a land in a newly developed colony of Chennai. He hires a tanker company to supply water for the construction work. Mr. Narendra uses a cylindrical can of $1000\pi m^3$ to store water for the construction work of his house. Assume that the radius of the base of can is r and height of can is h.

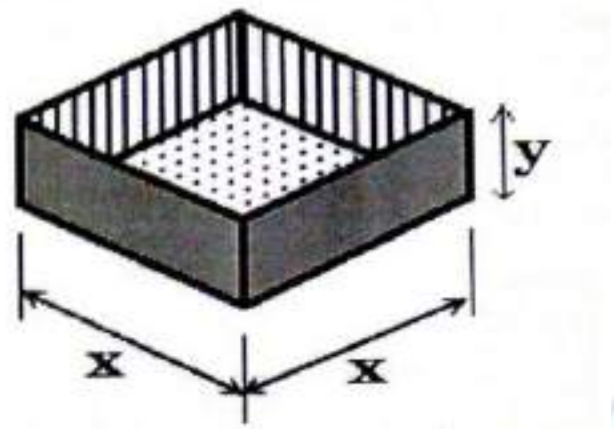
Based on the information given above, answer the following:

(i) Find the surface area S of the can in terms of r.

(ii) For what value of r, the surface area of the can is minimum?

(iii) Find the minimum value of the surface area of the cylindrical can.

16. Bharath is student of class XII. For his Math project, he buys a card-board of are 27 square centimeters. He has to form an open box with squarer base to have maximum capacity and no wastage of the card-board.



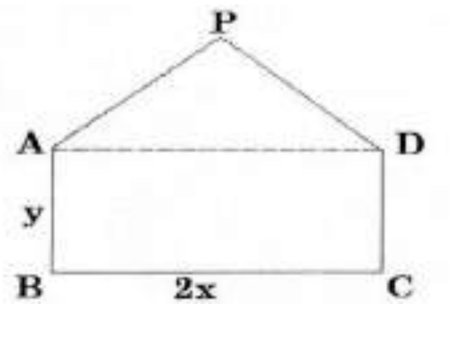
Based on the information given above, answer the following:

(i) Write the volume V of the box in terms of x.

(ii) What will be the value of x, so that the volume of box is maximum?

(iii) Find the maximum volume of the box.

17. Raj Kumar, who is a carpenter, designs a wooden window on order. The window is in the shape of rectangle which is surmounted by an equilateral triangle as show below.



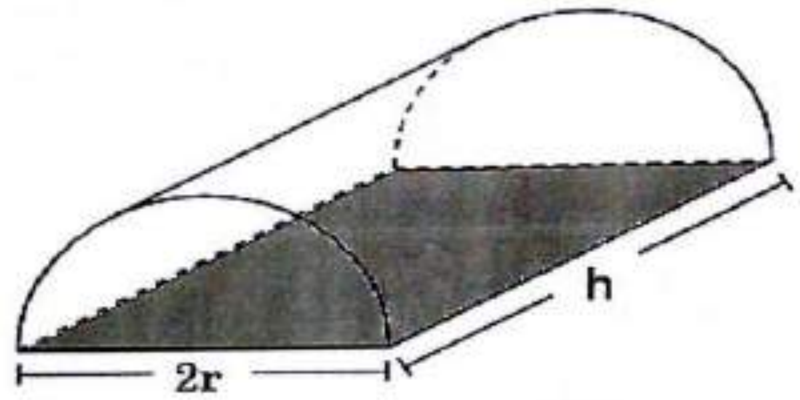
Based on the above information given, answer the following:

(i) Find the combined region A of the rectangular and triangular region of the window expressed as a function of x.

(ii) The carpenter is interested in maximizing the area of the whole window so that maximum light input is possible. For this to happen, determine the length of rectangular portion of the window.

(iii) In order to get the maximum light input through the whole window, find the area on only triangular portion of the window.

18. A company deals in casting and molding of metal on order received from its clients. A given quantity of metal 1000 cubic units is to be cast into half cylinder with rectangular base and semicircular ends.



Using the information given above, answer the following:

(i) Express the total surface area (A) of the half-cylinder, in terms of ' r '.

(ii) For what value of r, the total surface area (A) will be minimum?

(iii) What is the value of h : 2r?

19. Read the passage given below and answer the following questions:

Let $P(x) = 4x^3 - 6x^2 - 72x + 30$ is the total profit function of a company, where x is the production of the company.

(i) Determine the critical points of the profit function?

(ii) Find the interval in the profit function is strictly increasing.

(iii) Find the interval in the profit function is strictly decreasing.

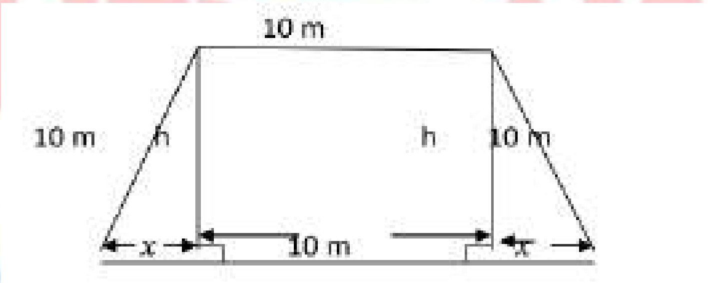
20. Rahul's Mathematics teacher was explaining the topic increasing decreasing functions in the class. He explained about different terms like stationary points, turning points etc. He also explained about the conditions for which a function will be increasing or decreasing. He took examples of different functions to make it more clear to the students. He then took the function $f(x) = (x-1)^3(x-2)^2$ and ask the students to answer the following questions.

(i) Find the stationary points on the curve.

(ii) Find the intervals where the function is increasing.

(iii) Find the interval where the function is decreasing.

21. The bridge is in the shape of a trapezium as shown below. Its three sides other than base are of 10m each. The height of the gate is h meter.



Based on the above information, answer the following:

(i) Express the area function in terms of x.

(ii) Find the value of x, when area is maximum.

(iii) Find the maximum value of A.

22. A balloon is being inflated with the help of an air pump and it remains spherical. Its radius, the surface area and the volume of air in it are all increasing.

Based on the above information, answer the following:

(i) Are the quantities: radius, surface area and volume of the spherical balloon changing at the same rate or different rates, when air is filled in it?

(ii) Write the expressions for the surface area(S) and the volume (v) of the balloon at any time t in terms of radius r at that instant.

(iii) At the instant when the radius of the balloon is 6cm and the radius r is increasing at the rate of 2cm/s, find at what rate the surface area (S) of the balloon is increasing.

(Or)

At the instant when the radius of the balloon is 6cm and the radius r is increasing at the rate of 2cm/s, find at what rate the volume V of the spherical balloon is increasing.

23. A fighter-jet of the enemy is flying along the parabolic path 4y = x2. A soldier is located at the point (0, 5) and is aiming to shoot down the jet when it is nearest to him.

Based on the above, answer the following questions:

(i) Let (x, y) be the position of the jet at any instant. Express the distance between the solider and the jet as the function f(x).

(ii) Taking S = $[f(x)]^2$, find $\frac{dS}{dx}$.

(iii) What will be the position of the jet when the soldier shows it down?

(Or)

What will be the distance between the soldier and the jet at the instant when he shoots it down?

24. The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y-axis at a point (0, -1), answer the following:

(i) Find the value of a.

(ii) Find $f''(x)$ at $x = 1$.

25. In a park, an open tank is to be constructed using metal sheet with a square base and vertical sides so that it contains 500 cubic meters of water.

Using the above information, answer the following:

(i) Find the minimum surface area of the tank.

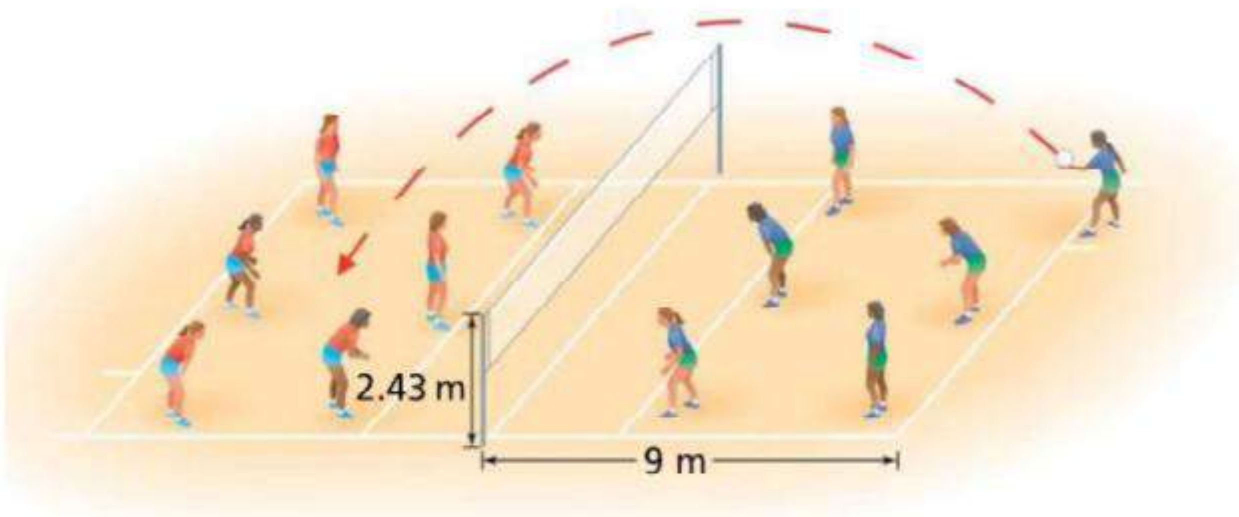
(ii) Find the percentage increase in volume of the tank, if size of square base of tank become twice and height remains same.

26. The equation of the path traced by a roller-coaster is given by the polynomial f(x) = a(x + 9)(x + 1)(x – 3). If the roller-coaster crosses y-axis at a point (0. -1), answer the following:

(i) Find the value of 'a'.

(ii) Find $f''(x)$ at $x = 1$.

27. A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where h(t) is the height of ball at any time t (in seconds), ($t \geq 0$).



Based on the above information, answer the following questions:

(i) Is h(t) a continuous function ? Justify.

(ii) Find the time at which the height of the ball is maximum.

28. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore. The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi \text{cm}^2$..

Based on the above information, answer the following questions:

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r.

(ii) Find $\frac{dV}{dr}$

(iii) Find the radius of cylinder when its volume is maximum.

(iv) For maximum volume, h > r. State true or false and justify.

29. The use of electric vehicles will curb air pollution in the long run. The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V:

V(t) = $\frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$, where t represents the time t = 1, 2, 3,…. corresponds to year 2001, 2002, 2003,… respectively.

Based on the above information, answer the following questions:

(i) Can the above function be used to estimate number of vehicles in the year 2000? Justify.

(ii) Prove that the function V(t) is an increasing function.

30. A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ 4000 $(\text{depth})^2$.

Suppose the side of the square plot is x metres and depth is h metres. On the basis of the above information, answer the following questions:

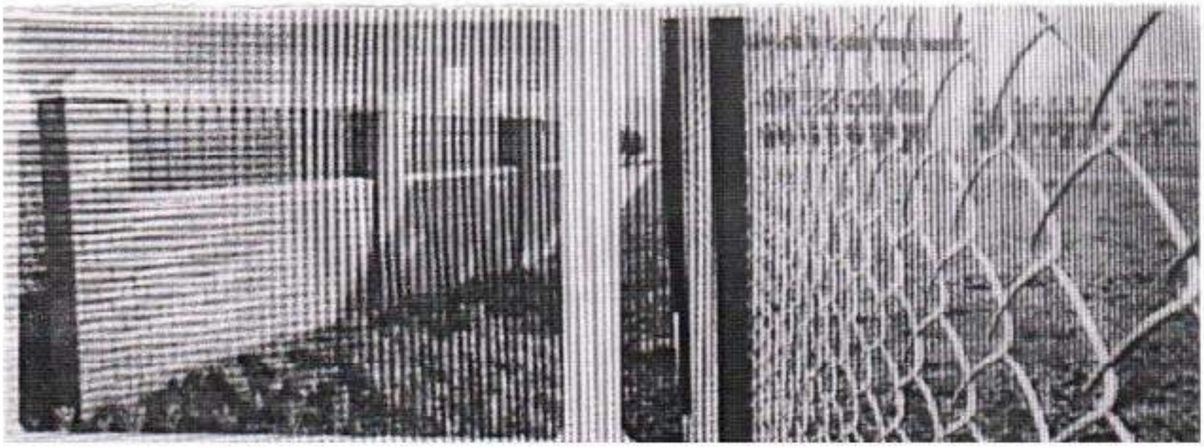
(i) Write cost C(h) as a function in terms of h.

(ii) Find critical point.

(iii) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool?

(iv) Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost.

31, Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as show in the figure. He has 200 metres of fencing wire.



Based on the above information, answer the following questions:

(i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden.

(ii) Determine the maximum value of A(x).

32. In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield.

A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function $f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, 0 \leq x \leq 10$, where x is the number of days the plant is exposed to sunlight.

On the basis of the above information, answer the following questions:

(i) What are the critical points of the function f(x)?

(ii) Using second derivative test, find the minimum value of the function.

33. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.

A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.

On the basis of the above information, answer the following questions:

(i) Express θ in terms of height of the camera installed on the pole and x.

(ii) Find $\frac{d\theta}{dx}$.

(iii) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole.

(iv) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car.

34. Over-speeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.

The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of the above information, answer the following questions:

(i) Find F, when $V = 40\text{km/h}$.

(ii) Find $\frac{dF}{dV}$.

(iii) Find the speed V for which fuel consumption F is minimum.

(iv) Find the quantity of fuel required to travel 600km at the speed V at which $\frac{\mathrm{dF}}{\mathrm{dV}} = -0.01$.

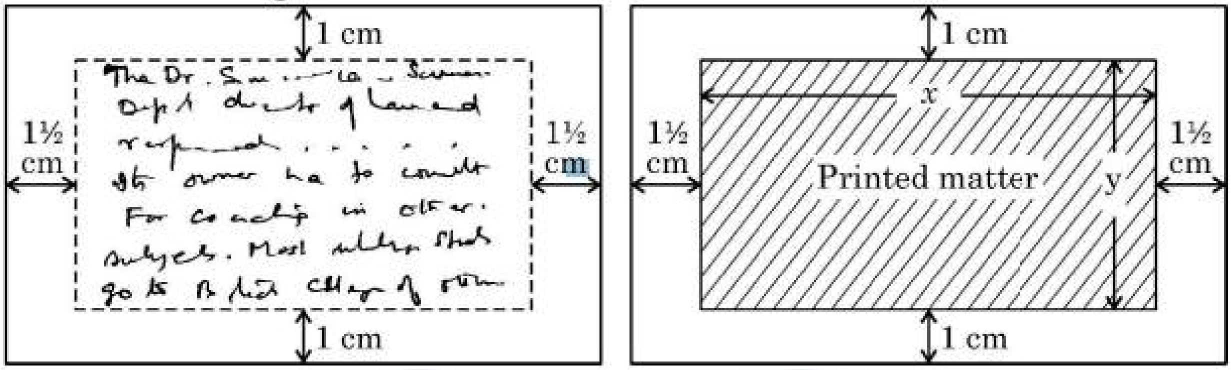
35. A store has been selling calculators at ₹350 each. A market survey indicates that a reduction in price(p) of calculator increases the number of units(x) sold. The relation between the price and quantity sold is given by the demand function $\mathrm{p} = 450 - \frac{1}{2}x$.

Based on the above information, answer the following questions:

(i) Determine the number of units(x) that should be sold to maximise the revenue R(x) = xp(x). Also, verify the result.

(ii) What rebate in price of calculator should the store give to maximise the revenue?

36. A rectangular visiting card is to contain 24sq.cm of printed matter. The margins at the top and bottom of the card are to be 1cm and the margins on the left and right are to be $1\frac{1}{2}$cm as shown below:



On the basis of the above information, answer the following questions:

(i) Write the expression for the area of the visiting card in terms of x.

(ii) Obtain the dimensions of the card of minimum area.

UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. The intervals in which the function f given by f(x) = $x^2 - 4x + 6$ is strictly increasing in

(a) $(-\infty, 2) \cup (2, \infty)$ (b) $(2, \infty)$ (c) $(-\infty, 2)$ (d) $(-\infty, 2] \cup (2, \infty)$

2. The maximum value of $\left(\frac{1}{x}\right)^x$ is

(a) e (b) e^e (c) $e^{1/e}$ (d) $\left(\frac{1}{e}\right)^{1/e}$

3. The maximum value of $[x(x-1)+1]^{\frac{1}{3}}, 0 \leq x \leq 1$ is

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt[3]{\frac{1}{3}}$

4. Assertion: The ratio of change of area of a circle with respect to its radius r when r = 6cm is $12\pi cm^2/cm$.

Reason: Rate of change of area of a circle with respect to its respect to its radius r is $\frac{dA}{dr}$, where A is the area of the circle.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x-coordinate is 1, y-coordinate is changing twice as fast at x-coordinate. Find the value of a.

6. Find whether the function $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$ is increasing or decreasing in the interval $\left(\frac{3\pi}{8}, \frac{7\pi}{8}\right)$.

7. Show that the function $f(x) = \frac{x}{3} + \frac{3}{x}$ decreases in the intervals $(-3,0) \cup (0,3)$.

SECTION C

Each carry 3 marks

8. Find the intervals in which $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$, is strictly increasing or strictly decreasing.

9. Find the minimum value of $(ax + by)$, where $xy = c^2$.

SECTION D

Each carry 5 marks

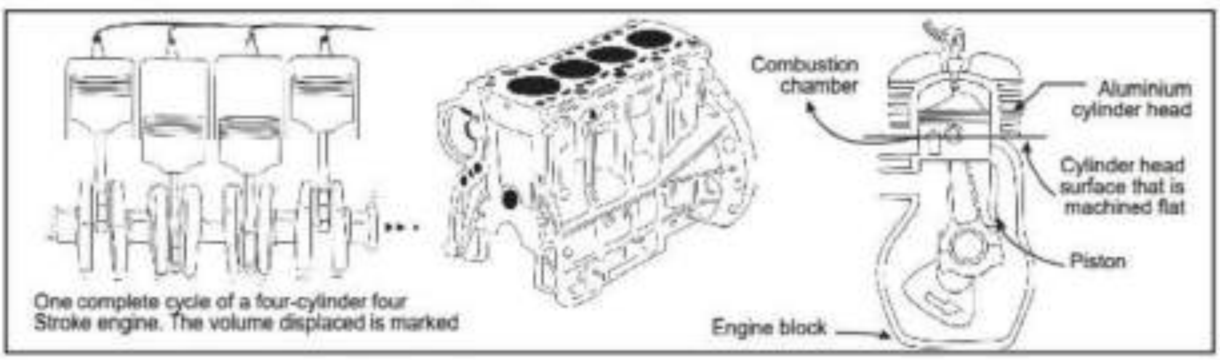
10. Show the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one third of the height of the cone and the greatest volume of the cylinder is $\frac{4}{9}$ times the volume of the cone.

11. Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.

SECTION E

12. Read the following passage and answer the following questions:

Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore.



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of are $75\pi cm^2$.

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of r.

(ii) Find $\frac{dV}{dr}$.

(iii) Find the radius of cylinder when its volume is maximum.

(Or)

For maximum volume, h > r. State true or false and justify.

ANSWERS

MCQ	A-R	VSA	SA	LA	CS
1. (b) x = $\frac{1}{e}$ 2. (c) $75\sqrt{3}\text{cm}^2$ 3. (b) 12 π 4. (d) 126 5. (c) $(-\infty, 2] \cup [3, \infty)$ 6. (c) $2\pi^2 r(2rh^2 - h^3)$ 7. (b) $2\sqrt{ab}$ 8. (b) Increasing in $\left[0, \frac{\pi}{2}\right]$ 9, (b) No value	1. (a) 2. (d) 3. (b) 4. (b) 5. (a) 6. (c) 7. (d) 8. (d) 9. (b) 10. (a) 11. (c) 12. (b) 13. (a) 14. (c)	1. 30.255 3.(a) $\frac{\pi}{3} < x < \frac{\pi}{2}$ (b) $0 < x < \frac{\pi}{3}$ 5. 3km/hr 6. $-\frac{8}{3}\text{m/sec}$ 7. $\left(3, \frac{16}{2}\right)$ and $\left(-3, -\frac{16}{3}\right)$. 8. 7.11cm/s 9. $33\pi\text{m}^3/\text{sec}$. 10. -8cm/s 11. $\left(1, \frac{5}{3}\right)$ and $\left(-1, \frac{1}{3}\right)$ 15. Absolute maximum value = 89 at x = 3 Absolute minimum	3. 4cm/s 4. Maximum value = $2(1 - \log 2)$ Minimum value = -1 5. Absolute Maximum = $\frac{5}{4}$ Absolute Minimum = -1 6. m = 2 7. (a) $(-2,1) \cup (3, \infty)$ (b) $(-\infty, -2) \cup (1,3)$ 8. (a) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{5\pi}{4}, 2\pi\right]$ (b) $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ 9. (a) $[1,2] \cup [3, \infty)$ (b) $(-\infty, 1] \cup [2, 3]$	6. 2ab 7. $\frac{3\sqrt{3}}{4}ab$ sq units. 9. $\left(\frac{20}{4+\pi}\right)$ m and $\left(\frac{10}{4+\pi}\right)$ m 10. $75\sqrt{3}$ sq.cm. 14. ₹5500 15. 240 items 23. $\frac{50\pi}{4+\pi}$ m, $\frac{200}{4+\pi}$ m 26. a = 120, Critical points are -6, 1, 5 and f attains local maximum value at x = 1 and local minimum values at x = -6, 5.	1. (i) $\frac{1}{3}\pi r^3$ (ii) $\frac{1}{4\pi}\text{cm/sec}$ (iii) 2cm(iii) 2/sec(Or) $\frac{1}{4\pi}$ cm/sec 2. (i) $C(x) = 40000 \times \frac{62500}{x^4} + 5000x^2$ (ii) $\frac{-4\times40{,}000\times62500}{x^5} + 10{,}000x$ (iii) $C(x)$ is minimum at $x = 10\text{m}$. (Or) $C(x)$ is not increasing for $x > 0$. 3. (i) Continuous function

of b exists 10. (c) (-∞, 0) 11. (a) $\left(-\frac{\pi}{3},\frac{\pi}{3}\right)$ 12. (c) f(x) is an increasing function 13. (a) An increasing function 14. (d) 49 15. (b) [-2, -1] 16. (b) Decreasing in $\left(\frac{\pi}{2},\pi\right)$ 17. (b) 0	15. (a) 16. (d) 17. (a) 18. (a) 19. (a) 20. (d) 21. (d) 22. (c) 23. (d) 24. (c) 25. (a)	value = 75 at x = 2 16. $\frac{15}{2}$ and $\frac{15}{2}$ 17. 8 and 8 18. $\frac{8}{3}$ and $\frac{4}{3}$ 19. 10 and 25 20. 32 and 32 21. $\frac{2}{5}\,\text{cm/sec}$ 22. Ans: $54\text{cm}^3/\text{sec}$ 23. $3\text{cm}^2/\text{sec}$ 24. $(-\infty,2)\cup(6,\infty)$ 25. $(-\infty,0]\cup[0,3]$ 26. Ans: 0, -2, 2	10. (a) $(-\infty,1]\cup[2,\infty)$ (b) [1, 2] 11. (a) $x\in[2,\infty)$ or $x\in$ $\left(-\infty,\frac{8}{5}\right]$ (b) $x\in\left[\frac{8}{5},2\right]$ 12. (a) $(-\infty,-1]\cup[1,\infty)$ (b) $[-1,0)\cup(0,1]$ 13. $\frac{-\sqrt{2}}{4\pi}\,\text{cm/s}$ 14. $\frac{4}{9\pi}\,\text{cm/min}$ 15. 96π sq.cm/min 16. $\frac{35}{88}\,\text{m/hr}$. 17. (i) Increasing in (-∞, 1] ∪ [$\frac{5}{3}$, ∞] and decreasing in [1, $\frac{5}{3}$]	27. Length of rectangle is 100cm and breadth of rectangle is 50cm.	(ii) $t=\frac{13}{14}$ seconds 4. (i) xy = 150 (ii) $156-2x-3\left(\frac{150}{x}\right)$ (iii) x = 15cm and y = 10cm 5. (i) $A=10x-\left(2+\frac{1}{2}\pi\right)x^2$ (ii) $A=\frac{50}{\pi+4}m^2$ (iii) $\frac{50\pi}{(\pi+4)^2}\,\text{m}^2$ 6. (i) $\frac{Cx-x^3}{4}$ (ii) $x=\sqrt{\frac{C}{3}}$ units (iii) $\frac{C}{6}\times\sqrt{\frac{C}{3}}$ units ^{3} 7. (i) $\frac{C^2y-y^3}{4}$ (ii) $y=\frac{C}{\sqrt{3}}$ units .

18. (c) One maxima and one minima 19. (c) (-∞,∞) 20. (b) A decreasing function 21. (d) -2 22. (a) Decreasing 23. (a) Strictly increasing 24. (d) x < -2 and also x > 6 25. (a) Increasing on		27. $\left(-\infty,-\frac{1}{\sqrt{2}}\right)\cup\left(\frac{1}{\sqrt{2}},\infty\right)$ 28. $[-1,1]-\{0\}$ 29. -4 31. f(x) is decreasing in (– 2, 0) and increasing in (0, ∞). 32. Maximum value is 6 and minimum value is 4 34. a = 2 36. $\frac{40}{\sqrt{3}}\,\text{cm}^2/\text{s}$ 37. $f(x)$ is an increasing function in its domain.	(ii) Increasing in [-2, -1] and decreasing in (-∞, -2] ∪ [-1, ∞) 18. Increasing in [1, 2] ∪ [3, ∞) and decreasing in (-∞, 1] ∪ [2, 3] 19. Local minimum value = $\frac{1}{2}$ at $x=\frac{\pi}{4}$ 20. Local maximum value = $\frac{39}{16}$ at $x=\frac{1}{2}$ Local minimum value = -6 at x = -1 21. (i) Maximum value = 2π Minimum value = 0		(iii) $\frac{C^3}{6\sqrt{3}}$ units ^{3} 8. (i) $3x^2-90x+600$ (ii) x = 10, 20 (iii) 20 trees (iv) ₹2000. 9. (i) $2x^2-40x+1140$ (ii) x = 10m 10. (i) $x=\frac{2}{3}m.$ (ii) $\frac{20}{3}$ m (iii) $\frac{200}{27}\,\text{m}^3$ 11. (i) x = 20m (ii) 1200m^2 12. (i) 50 bulbs (ii) ₹ 7488.

(-∞, -1) ∪ (1, ∞) and decreasing on (-1, 1) 26. (a) $-\frac{1}{e}$ 27. (b) x = 2 28. (c) 0 29. (d) Increasing function 30. (a) $1 < x < 3$ 31. (b) $\frac{2}{\sqrt{\pi}}$ 32. (a) Decreasing 33. (c) x^3		38. $(0,1) \cup (2,\infty)$ 39. Local minimum value $=$ f$\left(\frac{1}{2}\right) = 3$ 40. f is increasing over [4,6].	(ii) Maximum value = $\frac{3}{2}$ Minimum value = -3 22. 6 and 6 23. Dimensions are $\sqrt{96}$ and $\sqrt{96}$ Perimeter = $4\sqrt{96}$ 26. For strictly increasing, $x \in (0, e)$ and for strictly decreasi (e, ∞) 27. Absolute maximum value $= \frac{5}{2}$ absolute minimum value $= 2$		13. (i) $h = \frac{V}{\pi r^2}$ (ii) $r = \left(\frac{V}{\pi}\right)^{1/3}$ (iii) $h = \left(\frac{V}{\pi}\right)^{1/3}$ 14. (i) $A = \left(\frac{40-x}{4}\right)^2 + \frac{x^2}{12\sqrt{3}}$ (ii) $\frac{120\sqrt{3}}{4+3\sqrt{3}}$m. (iii) $\frac{400}{4+3\sqrt{3}}\text{m}^2$ 15. (i) $S = \pi r^2 + \frac{2000\pi}{r}$ (ii) r = 10m (iii) $300\pi\text{m}^2$ 16. (i) $V = \frac{27x}{4} - \frac{x^3}{4}$ (ii) x = 3cm (iii) 13.5cm^3 17. (i) $x^2\sqrt{3} + 6(2x - x^2)$

34. (c) $(0, \infty)$ 35. (c) Strictly decreases in $(1, \infty)$ 36. (a) -4 37. (d) No value of a 38. (d) $-\infty < a \leq -3$ 39. (d) $\frac{4}{3}$ 40. (c) r 41. (b) 2 42. (c) $10\sqrt{3}\text{cm}^2/s$ 43. (c) 0 & 2					(ii) $\frac{12}{6-\sqrt{3}}$ m (iii) $\frac{12\sqrt{3}}{13-4\sqrt{3}}$ m^2 18. (i) $A = \frac{2000}{r} + \pi r^2 + \frac{4000}{\pi r}$. (ii) $r = 10\left[\frac{(\pi+2)}{\pi^2}\right]^{1/3}$ (iii) $\pi : (\pi + 2)$ 19. (i) -2, 3 (ii) $(-\infty, -2) \cup (3, \infty)$ (iii) (-2, 3) 20. (i) 1, 2, $\frac{8}{5}$ (ii) $(-\infty, 1) \cup \left(1, \frac{8}{5}\right) \cup (2, \infty)$ (iii) $\left(\frac{8}{5}, 2\right)$ 21. (i) $(10 + x)\sqrt{100 - x^2}$ (ii) x = 5m

44. (a) $\left(\frac{3}{4}, \infty\right)$ 45. (b) Strictly decreasing 46. (a) 4, 0 47. (c) 1 48. (d) $\sqrt{2}$ 49. (c) x < -2 50. (b) $[1, \infty)$ 51. (b) $f'(x) > 0, \forall x \in (a, b)$ 52. (c) $(-\infty, 0)$ 53. (b) (-2, -1) 54. (b) strictly increasing on R 55. (a) k > 1					(iii) $\frac{75\sqrt{3}}{2}\,m^2$ 22. (i) No, they are changing at the different rates. (ii) $S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$ (iii) $96\pi cm^2/s$ (Or) $288\pi cm^3/s$ 23. (i) Distance $=\sqrt{(x)^2 + \left(\frac{x^2}{4} - 5\right)^2}$ (ii) $\frac{1}{4}x(x^2 - 12)$ (iii) $(\pm 2\sqrt{3}, 3)$ (Or) 4 units 24. (i) $\frac{1}{27}$ (ii) $\frac{20}{27}$ 25. (i) $300m^2$ (ii) 300%

56. (c) $32\pi \text{cm}^2/\text{cm}$ 57. (b) 6cm/s 58. (a) - 60units/sec 59. (c) 4 60. (a) 2					26. Ans: (i) $a = \frac{1}{27}$ (ii) $f''(1) = \frac{20}{27}$ 27. (i) h(t) is continuous function (ii) Height is maximum at $\text{t} = \frac{13}{14}$ 28. (i) V = $\frac{\pi}{2}(75r - r^3)$ (ii) $\frac{dV}{dr} = \frac{\pi}{2}(75 - 3r^2)$ (iii) r = 5 (iv) False 29. (i) For the year 2000, t = 0 & V(0) = - 2 and the number of vehicles cannot be negative, therefore the given function V(t)

					cannot be used. 30. (i) $\frac{125000}{h} + 4000h^2$ (ii) $\frac{5}{2}$ or 2.5 (iii) ₹75000 (iv) x = 4h 31. (i) 2x + y = 200, x(200 – x) (ii) 5000 sq.m 32. (i) 3 & 5 (ii) $\frac{56}{3}$ 33. (i) $\tan^{-1}\left(\frac{5}{x}\right)$ (ii) $\frac{d\theta}{dx} = \frac{-5}{5^2+x^2}$ (iii) $\frac{-4}{101}\,\text{rad/s}$ (iv) -15m/s 34. (i) F = $\frac{36}{5}l$/100km (ii) $\frac{V}{250} - \frac{1}{4}$ (iii) V = 62.5km/hr

					(iv) $37.2l$ 35. (i) Revenue is maximum when $x =$ 450 units are sold (ii) ₹125 36. (i) $2x + \frac{72}{x} + 30$ (ii) Length = 9cm & Breadth = 6cm

UNIT TEST

1. (b) $(2, \infty)$

2. (c) $e^{1/e}$

3. (c) 1

4. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

5. a = 2.

6. f(x) is increasing in $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$.

7. Proof

8. f(x) is strictly increasing in $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right)$ and strictly decreasing $\left(\frac{\pi}{4}, \frac{7\pi}{12}\right) \cup \left(\frac{11\pi}{12}, \pi\right)$.

9. $2c\sqrt{ab}$

10. Proof

11. $\frac{25}{2}$

12. (i) $\frac{\pi}{2} \times (75r - r^3)$ (ii) $\frac{3\pi}{2}(25 - r^2)$ (iii) r = 5cm (Or) h > r is false

CHAPTER 7 – INTEGRALS

INTEGRATION AS AN INVERSE PROCESS OF DIFFERENTIATION

Integration is the inverse process of differentiation. Instead of differentiating a function, we are given the derivative of a function and asked to find its primitive, i.e., the original function. Such a process is called integration or anti differentiation.

SYMBOLS/TERMS/PHRASES WITH THEIR MEANINGS INVOLVED IN INTEGRALS

Symbols/Terms/Phrases	Meaning
$\int f(x)\,dx$	Integral of f with respect to x
$f(x)$ in $\int f(x)\,dx$	Integrand
x in $\int f(x)\,dx$	Variable of integration
Integrate	Find the integral
An integral of f	A function F such that $F'(x) = f(x)$
Integration	The process of finding the integral
Constant of Integration	Any real number C, considered as constant function

FORMULAE

Derivatives	Integrals (Anti-Derivatives)
$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
$\frac{d}{dx}(x) = 1$	$\int dx = x + C$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2\, x$	$\int \mathrm{cosec}^2\, x dx = -\cot x + \mathrm{C}$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x dx = \sec x + \mathrm{C}$
$\frac{d}{dx}(\mathrm{cosec}\, x) = -\,\mathrm{cosec}\, x \cot x$	$\int \mathrm{cosec}\, x \cot x dx = -\mathrm{cosec}\, x + \mathrm{C}$
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + \mathrm{C}$
$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + \mathrm{C}$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\int \frac{dx}{1+x^2} = \tan^{-1} x + \mathrm{C}$
$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$	$\int \frac{dx}{1+x^2} = -\cot^{-1} x + \mathrm{C}$
$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + \mathrm{C}$
$\frac{d}{dx}(\mathrm{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$	$\int \frac{dx}{x\sqrt{x^2-1}} = -\mathrm{cosec}^{-1} x + \mathrm{C}$

$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}(\log\|x\|) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log\|x\| + C$
$\frac{d}{dx}\left(\frac{a^x}{\log a}\right) = a^x$	$\int a^x dx = \frac{a^x}{\log a} + C$

METHODS OF INTEGRATION

- Integration by Substitution
- Integration using Partial Fractions
- Integration by Parts

INTEGRATION BY SUBSTITUTION

The method of evaluating integrals of a function by suitable substitution is called integration by substitution.

Some Important Integrals Involving Trigonometric Functions

(i) $\int \tan x dx = \log|\sec x| + C$

(ii) $\int \cot x dx = \log|\sin x| + C$

(iii) $\int \sec x dx = \log|\sec x + \tan x| + C$

(iv) $\int \operatorname{cosec} x dx = \log|\operatorname{cosec} x - \cot x| + C$

INTEGRATION BY PARTS

To integrate the product of two functions we use the integration by parts. The method is as given below:

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x)\int g(x)dx\right]dx$$

(Or)

$$\int udv = uv - \int vdu$$

NOTE: To integrate the product of two functions we choose the 'u' function according to word ILATE where I stands for inverse function, L stands for logarithmic function, A stands for the algebraic function, T stands for trigonometric function and E stands for exponential function.

Integrals of Some Particular Functions

1. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

3. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

4. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C$

5. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

6. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + C$

7. $\int \sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

8. $\int \sqrt{x^2 - a^2}dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

9. $\int \sqrt{a^2 + x^2}dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$

10. $\int e^x(f(x) + f'(x))dx = f(x) \cdot e^x + C$

INTEGRATION BY PARTIAL FRACTIONS

Rational Function: Rational function is defined as the ratio of two polynomials in the form of $\frac{P(x)}{Q(x)}$ where P(x) and Q(x) are polynomials in x. If the degree of P(x) is less than degree of Q(x) then it is said to be proper, otherwise it is called an improper rational function.

Thus if $\frac{P(x)}{Q(x)}$ is improper, then by long division method it can be reduced to proper function.

$\frac{P(x)}{Q(x)} = T(x) + \frac{P_1(x)}{Q(x)}$, where T(x) is a function of x and $\frac{P_1(x)}{Q(x)}$ is a proper rational function. Such fractions can be evaluated by breaking in factors given as follows:

S. No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{(x-a)} + \frac{B}{(x-b)}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$
5.	$\frac{px^2+qx+r}{(x-a)^3(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-a)^3} + \frac{D}{(x-b)}$
6.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{(x-a)} + \frac{Bx+C}{x^2+bx+c}$, where x^2+bx+c cannot be factored further.

The constants A, B, C etc are obtained by equating coefficients of like terms from both sides or by substituting any value for x on both sides.

DEFINITE INTEGRALS

If F(x) is the integral of f(x) over the interval [a, b] i.e., $\int f(x)dx = F(x)$, then the definite integral of f(x) over the interval [a, b] is denoted by $\int_a^b f(x)$ is defined as $\int_a^b f(x)dx = F(b) - F(a)$ where a is called the lower limit and b is called the upper limit of integration and the interval [a, b] is called the interval of integration.

SOME PROPERTIES OF DEFINITE INTEGRALS

P$_0$: $\int_a^b f(x)dx = \int_a^b f(t)dt$

P$_1$: $\int_a^b f(x)dx = -\int_b^a f(x)dx$. In particular, $\int_a^a f(x)dx = 0$

P$_2$: $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

P$_3$: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

P$_4$: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

P$_5$: $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

P$_6$: $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, \text{ if } f(2a-x) = f(x) \\ \quad 0 \text{ if } f(2a-x) = -f(x) \end{cases}$

P$_7$: (i) $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$, if f is an even function, i.e., if $f(-x) = f(x)$.

(ii) $\int_{-a}^a f(x)dx = 0$, if f is an odd function, i.e., if $f(-x) = -f(x)$.

MULTIPLE CHOICE QUESTIONS

1. $\int e^x(\cos x - \sin x)dx$ is equal to

(a) $e^x \cos x + C$ (b) $e^x \sin x + C$ (c)- $e^x \cos x + C$ (d) $-e^x \sin x + C$

2. If $\int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} dx = ax + b\log|4e^x + 5e^{-x}| + C$ then

(a) $a = \frac{-1}{8}, b = \frac{7}{8}$ (b) $a = \frac{1}{8}, b = \frac{7}{8}$ (c) $a = \frac{-1}{8}, b = \frac{-7}{8}$ (d) $a = \frac{1}{8}, b = \frac{-7}{8}$

3. $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$ is equal to

(a) $\frac{e^x}{1+x^2} + C$ (b) $\frac{-e^x}{1+x^2} + C$ (c) $\frac{e^x}{(1+x^2)^2} + C$ (d) $\frac{-e^x}{(1+x^2)^2} + C$

4. $\int 2^{x+2}dx$ is equal to

(a) $2^{x+2}+C$ (b) $2^{x+2}\log 2+C$ (c) $\frac{2^{x+2}}{\log 2}+C$ (d) $2\cdot\frac{2^{x}}{\log 2}+C$

5. $\int_0^{\pi/8} \tan^2(2x)dx$ is equal to

(a) $\frac{4-\pi}{8}$ (b) $\frac{4+\pi}{8}$ (c) $\frac{4-\pi}{4}$ (d) $\frac{4-\pi}{2}$

6. $\int_0^1 \frac{e^t}{1+t}dt = a$, then $\int_0^1 \frac{e^t}{(1+t)^2}dt$ is equal to

(a) $a-1+\frac{e}{2}$ (b) $a+1-\frac{e}{2}$ (c) $a-1-\frac{e}{2}$ (d) $a+1+\frac{e}{2}$

7. If $f^{|}(x) = x+\frac{1}{x}$ then $f(x)$ is

(a) $x^2+\log|x|+C$ (b) $\frac{x^2}{2}+\log|x|+C$ (c) $\frac{x}{2}+\log|x|+C$ (d) $\frac{x}{2}-\log|x|+C$

8. $\int_{-\pi/4}^{\pi/4} x^3\cos^2 x dx$ is equal to

(a) 0 (b) -1 (c) 1 (d) 2

9. $\int \frac{x-3}{(x-1)^3}e^x dx$ is equal to

(a) $\frac{2e^x}{(x-1)^3}+C$ (b) $\frac{-2e^x}{(x-1)^2}+C$ (c) $\frac{e^x}{(x-1)}+C$ (d) $\frac{e^x}{(x-1)^2}+C$

10. $\int e^{3\log x}(x^4+1)^{-1}dx$ is equal to

(a) $\frac{1}{4}\log(x^4+1)+C$ (b) $\frac{1}{4}\log\left(\frac{1}{x^4+1}\right)+C$ (c) $\frac{x^3}{x^4+1}+C$ (d) $\frac{e^x}{x^4+1}+C$

11. If $\int_0^a \frac{1}{4+x^2}dx = \frac{\pi}{6}$, then the value of ' a ' is

(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

12. The value of $\int_0^{\pi} \tan^2\left(\frac{\theta}{3}\right)d\theta$ is

(a) $\pi+\sqrt{3}$ (b) $3\sqrt{3}-\pi$ (c) $\sqrt{3}-\pi$ (d) $\pi-\sqrt{3}$

13. $\int \frac{1}{x(\log x)^2}dx$ is equal to

(a) $2\log(\log x)+c$ (b) $-\frac{1}{\log x}+c$ (c) $\frac{(\log x)^3}{3}+c$ (d) $\frac{3}{(\log x)^3}+c$

14. The value of $\int_{-1}^{1} x|x|\mathrm{d}x$ is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) - $\frac{1}{6}$ (d) 0

15. $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ is equal to

(a) tanx – cotx + C (b) –cotx – tanx + C

(c) cotx + tanx + C (d) tanx – cotx – C

16. $\int_{-1}^{1} \frac{|x|}{x} dx, x \neq 0$ is equal to

(a) -1 (b) 0 (c) 1 (d) 2

17. The value of $\int_{0}^{\pi/6} \sin 3x dx$ is

(a) -$\frac{\sqrt{3}}{2}$ (b) -$\frac{1}{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$

18. The value of $\int_{0}^{\pi} \tan^2\left(\frac{\theta}{3}\right) d\theta$ is

(a) $\pi + \sqrt{3}$ (b) $3\sqrt{3} - \pi$ (c) $\sqrt{3} - \pi$ (d) $\pi - \sqrt{3}$

19. $\int e^{5\log x} dx$ is equal to:

(a) $\frac{x^5}{5} + C$ (b) $\frac{x^6}{6} + C$ (c) $5x^4 + C$ (d) $6x^5 + C$

20. If $\int \frac{2^{\frac{1}{x}}}{x^2} \mathrm{dx} = \mathrm{k} \cdot 2^{\frac{1}{x}} + \mathrm{C}$, then k is equal to

(a) $-\frac{1}{\log 2}$ (b) –log2 (c) -1 (d) $\frac{1}{2}$

21. If $\frac{d}{dx}(f(x)) = \log x$, then $f(x)$ equals :

(a) $-\frac{1}{x} + C$ (b) $x(\log x - 1) + C$ (c) $x(\log x + x) + C$ (d) $\frac{1}{x} + C$

22. $\int_{0}^{\frac{\pi}{6}} \sec^2\left(x - \frac{\pi}{6}\right) dx$ is equal to :

(a) $\frac{1}{\sqrt{3}}$ (b) -$\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) -$\sqrt{3}$

23. $\int \frac{1+\tan x}{1-\tan x} dx$ is equal to

(a) $\sec^2\left(\frac{\pi}{4}+x\right)+C$ (b) $\sec^2\left(\frac{\pi}{4}-x\right)+C$

(c) $\log\left|\sec\left(\frac{\pi}{4}+x\right)\right|+C$ (d) $\log\left|\sec\left(\frac{\pi}{4}-x\right)\right|+C$

24. The primitive of $\frac{2}{1+\cos 2x}$ is

(a) $\sec^2 x$ (b) $2\sec^2 x\tan x$ (c) tanx (d) –cotx

25. If $\int_0^a 3x^2 dx = 8$, then the value of ' a ' is :

(a) 2 (b) 4 (c) 8 (d) 10

26. If $\frac{d}{dx}[f(x)] = ax + b$ and $f(0) = 0$, then $f(x)$ is equal to

(a) a + b (b) $\frac{ax^2}{2}+bx$ (c) $\frac{ax^2}{2}+bx$ + c (d) b

27. If $\frac{d}{dx}f(x) = 2x + \frac{3}{x}$ and $f(1) = 1$, then $f(x)$ is

(a) $x^2 + 3\log|x| + 1$ (b) $x^2 + 3\log|x|$

(c) $2 - \frac{3}{x^2}$ (d) $x^2 + 3\log|x| - 4$

28. If $\int_0^{2\pi} \cos^2 x dx = k\int_0^{\pi/2} \cos^2 x dx$, then the value of k is

(a) 4 (b) 2 (c) 1 (d) 0

29. $\int 2^{x+2} dx$ is equal to :

(a) $2^{x+2}+C$ (b) $2^{x+2}\log 2 + C$ (c) $\frac{2^{x+2}}{\log 2}+C$ (d) $2\cdot\frac{2^x}{\log 2}+C$

30. $\int \frac{2\cos 2x-1}{1+2\sin x} dx$ is equal to :

(a) x – 2cosx + C (b) x + 2cosx + C (c) –x – 2cosx + C (d) –x + 2cosx + C

31. $\int_0^2 \sqrt{4-x^2}dx$ equals :

(a) 2 log2 (b) -2log2 (c) $\frac{\pi}{2}$ (d) π

32. $\int e^{-x}\left(\frac{x+1}{x^2}\right)dx$ is equal to :

(a) $\frac{e^{-x}}{x}+C$ (b) $\frac{e^{x}}{x}+C$ (c) $\frac{e^{x}}{x^2}+C$ (d) $-\frac{e^{-x}}{x}+C$

33. $\int \frac{\sec x}{\sec x-\tan x}dx$ equals

(a) secx – tanx + C (b) secx + tanx + C (c) tanx – secx + C (d) –(secx + tanx) + C

34. $\int_{-1}^{1}\frac{|x-2|}{x-2}dx, x\neq 2$ is equal to

(a) 1 (b) -1 (c) 2 (d) -2

35. The integral $\int \frac{dx}{\sqrt{9-4x^2}}$ is equal to

(a) $\frac{1}{6}\sin^{-1}\left(\frac{2x}{3}\right)+c$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)+c$ (c) $\sin^{-1}\left(\frac{2x}{3}\right)+c$ (d) $\frac{3}{2}\sin^{-1}\left(\frac{2x}{3}\right)+c$

36. If $\int_0^a \frac{1}{4+x^2}dx=\frac{\pi}{6}$, then the value of ' a ' is :

(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

37. $\int_0^{\pi/2}\frac{\sin x-\cos x}{1+\sin x\cos x}dx$ is equal to

(a) π (b) 0 (c) $\int_0^{\pi/2}\frac{2\sin x}{1+\sin x\cos x}dx$ (d) $\frac{\pi^2}{4}$

38. The value of $\int_{\pi/4}^{\pi/2}\cot\theta\,\mathrm{cosec}^2\theta\,d\theta$ is

(a) $\frac{1}{2}$ (b) -$\frac{1}{2}$ (c) 0 (d) $-\frac{\pi}{8}$

39. If $\int_0^2 2e^{2x}dx=\int_0^a e^x dx$, the value of ' a ' is

(a) 1 (b) 2 (c) 4 (d) $\frac{1}{2}$

40. Anti-derivative of $\sqrt{1+\sin 2x}, x\in\left[0,\frac{\pi}{4}\right]$ is

(a) cosx + sinx (b) –cosx + sinx (c) cosx – sinx (d) –cosx – sinx

41. The value of $\int_{-1}^{1}|x|dx$ is

(a) -2 (b) -1 (c) 1 (d) 2

42. $\int \frac{1}{x(\log x)^2} \mathrm{d}x$ is equal to

(a) 2log(logx) + c (b) $-\frac{1}{\log x}+c$ (c) $\frac{(\log x)^3}{3}+c$ (d) $\frac{3}{(\log x)^3}+\mathrm{c}$

43. The value of $\int_{-1}^{1} x|x|\mathrm{d}x$ is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $-\frac{1}{6}$ (d) 0

44. If $\int_{-2}^{3} x^2\mathrm{d}x = \mathrm{k}\int_{0}^{2} x^2\mathrm{d}x + \int_{2}^{3} x^2\mathrm{d}x$, then the value of k is

(a) 2 (b) 1 (c) 0 (d) $\frac{1}{2}$

45. The value of $\int_{1}^{\mathrm{e}} \log x\mathrm{d}x$ is

(a) 0 (b) 1 (c) e (d) e loge

ASSERTION - REASON TYPE QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

1. Assertion: $\int_{2}^{8} \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx = 3$

Reason: $\int_{a}^{b} f(x)dx = \int_{a}^{b} f(a+b-x)dx$

2. Assertion: $\int_{2}^{8} \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\mathrm{d}x = 3$

Reason: $\int_{\mathrm{a}}^{\mathrm{b}} \mathrm{f}(x)\mathrm{d}x = \int_{\mathrm{a}}^{\mathrm{b}} \mathrm{f}(\mathrm{a}+\mathrm{b}-x)\mathrm{d}x$

VERY SHORT ANSWERS

1. Find: $\int \frac{\tan^3 x}{\cos^3 x} dx$

2. Find $\int \frac{x}{x^2+3x+2} dx.$

3. Evaluate $\int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2}\right] e^{2x} dx$

4. Find the value of $\int_0^1 x(1-x)^n dx$

5. Find: $\int \frac{x-1}{(x-2)(x-3)} dx$

6. Find: $\int_{-\pi/4}^{0} \frac{1+\tan x}{1-\tan x} dx.$

7. Find: $\int \frac{dx}{\sqrt{5-4x-2x^2}}$

8. Find: $\int e^x \frac{\sqrt{1+\sin 2x}}{1+\cos 2x} dx$

9. Evaluate: $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$

10. Evaluate: $\int_0^1 \frac{dx}{\sqrt{2x+3}}$

11. Find: $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$

12. Find: $\int \frac{x^3-1}{x^3-x} dx$

13. Evaluate: $\int_{-4}^{0} |x+2| dx$

14. Find: $\int x\sqrt{1+2x} dx$

15. Evaluate: $\int_0^{\frac{\pi^2}{4}} \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

16. Find: $\int \frac{e^{4x}-1}{e^{4x}+1} dx$

17. Evaluate: $\int_0^{a^3} \frac{x^2}{x^6+a^6} dx$

18. Given $\frac{d}{dx}F(x) = \frac{1}{\sqrt{2x-x^2}}$ and $F(1) = 0$, find $F(x)$.

19. Find: $\int \frac{x^{-1}}{(\log x)^2 - 5\log x + 4} dx$

20. Evaluate: $\int_0^{\pi/4} \frac{1}{\sin x + \cos x} dx$

21. Find: $\int \frac{dx}{x(x^2-1)}$

22. Evaluate: $\int_{\frac{-1}{2}}^{\frac{1}{2}} \cos x \cdot \log\left(\frac{1+x}{1-x}\right) \mathrm{d}x$

23. Find: $\int \cos^3 x \mathrm{e}^{\log \sin x} \mathrm{d}x$

24. Find: $\int \mathrm{x}\sqrt{1 + 2\mathrm{x}}\mathrm{dx}$.

25. Evaluate: $\int_0^{\frac{\pi^2}{4}} \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

26. Find: $\int \frac{\mathrm{e}^{4\mathrm{x}}-1}{\mathrm{e}^{4\mathrm{x}}+1} \mathrm{dx}$

27. Find : $\int \frac{1}{x(x^2-1)} \mathrm{d}x$.

28. Find : $\int \cos^3 x \mathrm{e}^{\log \sin x} \mathrm{d}x$

29. Evaluate: $\int_0^{a^3} \frac{x^2}{x^6 + a^6} dx$

30. Evaluate : $\int_{\frac{-1}{2}}^{\frac{1}{2}} \cos x \cdot \log\left(\frac{1+x}{1-x}\right) \mathrm{d}x$

31. Find : $\int \frac{2x}{(x^2+1)(x^2-4)} \mathrm{d}x$.

SHORT ANSWERS

1. Find: $\int \frac{x^4}{(x-1)(x^2+1)} dx$

2. Find: $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)} dx$

3. Find $\int e^{\cot^{-1} x} \left(\frac{1-x+x^2}{1+x^2}\right) dx$.

4. Find: $\int \frac{\cos x}{\sin 3x} dx$

5. Find: $\int x^2 \log(x^2 + 1) dx$

6. Evaluate: $\int \frac{x^2}{1-x^4} dx$

7. Evaluate: $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cdot \cos^2 x} dx$

8. Evaluate: $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx$

9. Evaluate: $\int e^x \left(\frac{\sin 4x-4}{1-\cos 4x}\right) dx$

10. Evaluate: $\int \frac{(x^2-3x)}{(x-1)(x-2)} dx$

11. Find: $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$

12. Find: $\int \frac{x^2}{x^4-x^2-12} dx$

13. Evaluate: $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$

14. Find: $\int \frac{(3\sin\theta-2)\cos\theta}{5-\cos^2\theta-4\sin\theta} d\theta$

15. Find: $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}} dx$

16. Find: $\int \frac{(2x-5)e^{2x}}{(2x-3)^3} dx$

17. Evaluate: $\int \frac{dx}{\sin(x-a)\cos(x-b)}$

18. Find: $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$

19. Evaluate: $\int_{-1}^{2} |x^3 - x| dx$

20. Evaluate: $\int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx.$

21. Find: $\int \frac{dx}{\cos x\sqrt{\cos 2x}}$

22. Find: $\int \frac{5x-3}{\sqrt{1+4x-2x^2}}dx$

23. Find: $\int \frac{x^2}{(x^2+4)(x^2+9)}dx$

24. Evaluate: $\int_1^3 (|x-1|+|x-2|+|x-3|)dx$

25. Evaluate: $\int_{-2}^{2} \sqrt{\frac{2-x}{2+x}}dx$

26. Find: $\int \frac{1}{x[(\log x)^2-3\log x-4]}dx$

27. Find: $\int x^2 \cdot \sin^{-1}(x^{3/2})dx$

28. Evaluate: $\int_0^{\pi/4} \frac{1}{\sin x+\cos x}dx$

29. Find: $\int \frac{\sqrt{x}}{(x+1)(x-1)}dx$

30. Evaluate : $\int_0^{\frac{\pi}{4}} \frac{x\mathrm{d}x}{1+\cos 2x+\sin 2x}$

31. Find: $\int \mathrm{e}^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}}+\frac{x}{\sqrt{1+x^2}}\right]\mathrm{d}x$

32. Find: $\int \frac{2x+3}{x^2(x+3)}\mathrm{d}x$

33. Evaluate : $\int_0^{\pi} \frac{\mathrm{e}^{\cos x}}{\mathrm{e}^{\cos x}+\mathrm{e}^{-\cos x}}\mathrm{d}x$

34. Find: $\int \frac{\sqrt{x}}{(x+1)(x-1)}dx$

35. Evaluate: $\int_0^{\frac{\pi}{2}} [\log(\sin x)-\log(2\cos x)]dx.$

36. Find: $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)}dx$

37. Evaluate: $\int_0^{\frac{\pi}{2}} e^x \sin x dx$

38. Evaluate: $\int_0^{\frac{\pi}{2}} |\sin x-\cos x|dx.$

39. Find: $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$

40. Evaluate $\int_0^{\pi/4} \log(1+\tan x)\mathrm{d}x$.

41. Evaluate : $\int_0^{\frac{\pi}{4}} \frac{x\mathrm{d}x}{1+\cos 2x+\sin 2x}$

42. Find : $\int \mathrm{e}^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{1+x^2}}\right] \mathrm{d}x$

43. Find : $\int \frac{3x+5}{\sqrt{x^2+2x+4}} \mathrm{d}x$

44. Find $\int \frac{\mathrm{d}x}{\sqrt{\sin^3 x\cos(x-\alpha)}}$.

45. Find $\int e^{\cot^{-1} x}\left(\frac{1-x+x^2}{1+x^2}\right) \mathrm{d}x$.

46. Evaluate $\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{(\mathrm{e}^x+\mathrm{e}^{-x})(\mathrm{e}^x-\mathrm{e}^{-x})} \mathrm{d}x$

47. Evaluate $\int_{-1}^{1} |x^4 - x|\mathrm{d}x$.

48. Find $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} \mathrm{d}x$.

49. Find $\int \mathrm{e}^x \left(\frac{1-\sin x}{1-\cos x}\right) \mathrm{d}x$

50. Find $\int \frac{x+2}{\sqrt{x^2-4x-5}} \mathrm{d}x$.

51. Evaluate $\int_{-a}^{a} f(x)\mathrm{d}x$, where $\mathrm{f}(x) = \frac{9^x}{1+9^x}$.

52. Evaluate: $\int_1^e \frac{1}{\sqrt{4x^2-(x\log x)^2}} dx$

53. Find: $\int \frac{\cos x}{\sin 3x} dx$

54. Find: $\int x^2\log(x^2+1)dx$.

55. Find: $\int \frac{e^x}{\sqrt{e^{2x}-4e^x-5}} dx$

56. Find: $\int_1^4 \frac{1}{\sqrt{2x+1}-\sqrt{2x-1}} dx$.

57. Find: $\int \frac{x^2+x+1}{(x+1)^2(x+2)} dx$

58. Evaluate: $\int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right) dx$

59. Evaluate: $\int_{-2}^{2} \frac{x^2}{1+5^x} dx$

60. Find: $\int \frac{\mathrm{e}^x}{\sqrt{5-4\mathrm{e}^x-\mathrm{e}^{2x}}} dx$

61. Evaluate: $\int_0^{\pi/2} \sqrt{\sin x}\cos^5 x dx$.

62. Evaluate: $\int_0^{2\pi} \frac{1}{1+\mathrm{e}^{\sin x}} \mathrm{d}x$

63. Find: $\int \frac{x^2}{x^2+6x+12} dx$

64. Find: $\int \frac{2}{(1-x)(1+x^2)} dx$

65. Evaluate: $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4} dx$

66. Evaluate: $\int_1^3 \{|(x-1)|+|(x-2)|\}dx$.

67. Find: $\int \frac{x^4}{(x-1)(x^2+1)} \mathrm{d}x$

68. Evaluate : $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cos 2x}{1+\cos 2x} \mathrm{d}x$

69. Find: $\int \mathrm{e}^{x^2}(x^5+2x^3)\mathrm{d}x$

70. Evaluate: $\int_0^{\pi/2} \frac{x\sin x\cos x}{\sin^4 x+\cos^4 x} dx$

71. Evaluate: $\int_1^3 (|x-1|+|x-2|)dx$

72. Find: $\int \frac{\cos\theta}{\sqrt{3-3\sin\theta-\cos^2\theta}} d\theta$

73. Find: $\int \frac{2x^2+1}{x^2(x^2+4)} dx$

74. Evaluate: $\int_1^3 (|x-1|+|x-2|+|x-3|)dx$.

75. Evaluate: $\int_0^{\pi/4} \frac{1}{\sin x+\cos x} dx$

76. Evaluate: $\int_{-2}^{2} \sqrt{\frac{2-x}{2+x}} dx$

77. Find: $\int \frac{1}{x[(\log x)^2-3\log x-4]} dx$

78. Find: $\int x^2 \cdot \sin^{-1}(x^{3/2}) dx$

79. $\int x^2 \log(x^2-1) dx$

80. Given $\frac{\mathrm{d}}{\mathrm{dx}}\mathrm{F(x)} = \frac{1}{\sqrt{2\mathrm{x}-\mathrm{x}^2}}$ and F(1) = 0, find F(x).

81. Find: $\int \frac{x^{-1}}{(\log x)^2-5\log x+4} dx$.

82. Find: $\int \frac{2x+3}{x^2(x+3)} dx$

83. Evaluate : $\int_0^{\pi} \frac{\mathrm{e}^{\cos x}}{\mathrm{e}^{\cos x}+\mathrm{e}^{-\cos x}} \mathrm{d}x$

84. Find : $\int \frac{2x+1}{(x+1)^2(x-1)} \mathrm{d}x$

LONG ANSWERS

1. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x} dx$

2. Evaluate: $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$

3. Find: $\int \frac{(3\cos x-2)\sin x}{5-\sin^2 x-4\cos x} dx$

4. Evaluate: $\int_{-2}^{2} \frac{x^3+|x|+1}{x^2+4|x|+4} dx$

5. Evaluate: $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) \mathrm{d}x$

6. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x} dx$

7. Find: $\int \frac{(3\cos x-2)\sin x}{5-\sin^2 x-4\cos x} dx$

8. Evaluate: $\int_{-2}^{2} \frac{x^3+|x|+1}{x^2+4|x|+4} dx$

UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. $\int x^2 e^{x^3} dx$ is equal to

(a) $\frac{1}{3}e^{x^3} + C$ (b) $\frac{1}{3}e^{x^4} + C$ (c) $\frac{1}{2}e^{x^3} + C$ (d) $\frac{1}{2}e^{x^2} + C$

2. $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ is equal to

(a) $\tan(xe^x) + C$ (b) $\cot(xe^x) + C$ (c) $\cot(e^x) + C$ (d) $\tan[e^x(1+x)] + C$

3. $\int_{-1}^{1} \frac{|x-2|}{x-2} dx, x \neq 2$ is equal to

(a) 1 (b) -1 (c) 2 (d) -2

4. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x dx$ is equal to

(a) -1 (b) 0 (c) 1 (d) 2

5. Assertion: If $f'(x) = x + \frac{1}{1+x^2}$ and $f(0) = 0$ then $f(x) = \frac{x^2}{2} + \tan^{-1} x$.

Reason: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

6. Find $\int \frac{dx}{\sqrt{4x-x^2}}$

7. Find $\int \frac{dx}{x^2-6x+13}$

8. Evaluate: $\int_0^1 x^2e^x dx$

SECTION C

Each carry 3 marks

9. Find: $\int e^x \cdot \sin 2x dx$

10. Find: $\int \frac{2x}{(x^2+1)(x^2+2)} dx$

11. Find $\int \frac{dx}{\sqrt{\sin^3 x \cos(x-\alpha)}}$.

SECTION C

Each carry 5 marks

12. Find: $\int \frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{4} dx$

13. Evaluate: $\int_1^4 (|x-1| + |x-2| + |x-4|) dx$

ANSWERS

MCQ	A-R	VSA	SA	LA
1. (a) $e^x \cos x + C$ 2. (a) $a = \frac{-1}{8}, b = \frac{7}{8}$ 3. (a) $\frac{e^x}{1+x^2} + C$ 4. (c) $\frac{2^{x+2}}{\log 2} + C$ 5. (a) $\frac{4-\pi}{8}$ 6. (b) $a + 1 - \frac{e}{2}$ 7. (b) $\frac{x^2}{2} + \log\|x\| + C$ 8. (a) 0 9. (d) $\frac{e^x}{(x-1)^2} + C$ 10. (a) $\frac{1}{4}\log(x^4 + 1) + \mathrm{C}$	1. (a) 2. (a)	1. $\frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$ 2. $-\log\|x+1\| + 2\log\|x+2\| + C$ 3. $\frac{e^4}{4} - \frac{e^2}{2}$ 4. $\frac{1}{(n+1)(n+2)}$ 5. $\log\frac{(x-3)^2}{(x-2)} + C$ 6. $\log\sqrt{2}$ 7. $\frac{1}{\sqrt{2}}\sin^{-1}\left(\sqrt{\frac{2}{7}}(x+1)\right) + C$ 8. $\frac{1}{2}e^x \cdot \sec x + C$ 9. $\frac{\pi^2}{32}$ 10. 10. $\sqrt{5} - \sqrt{3}$	1. $\frac{x^2}{2} + x + \frac{1}{2}\log\|x-1\| - \frac{1}{4}\log\|x^2+1\| + \frac{1}{2}\tan^{-1} x + C$ 2. 2. $2\log\left\|\frac{\sqrt{x}+1}{\sqrt{x}+2}\right\| + C$ 3. $xe^{\cot^{-1} x} + C$ 4. $-\frac{1}{6}\log\left\|1 - \frac{3}{4\sin^2 x}\right\| + C$ 5. $\frac{2}{3}x^3\log\sqrt{1+x^2} - \frac{2}{9}x^3 + \frac{2}{3}x - \frac{2}{3}\tan^{-1} x + C$ 6. $\frac{1}{4}\log\left\|\frac{1+x}{1-x}\right\| - \frac{1}{2}\tan^{-1} x + C$ 7. $\tan x - \cot x - 3x + C$ 8. $\sin^{-1}\left(\frac{e^x+2}{3}\right) + C$ 9. $e^x \cdot \cot 2x + C$	1. $\frac{1}{40}\log 9$ or $\frac{1}{20}\log 3$ 2. $\frac{\pi}{2} - 1$ 3. $-3\log\|\cos x - 2\| + \frac{4}{\cos x - 2} + C$ 4. $2\log 2 - \frac{1}{2}$ 5. $\frac{\pi}{2} - 1$ 6. $\frac{1}{40}\log 9$ or $\frac{1}{20}\log 3$ 7. $-3\log\|\cos x - 2\| + \frac{4}{\cos x - 2} + C$ 8. $2\log 2 - \frac{1}{2}$

11. (b) $2\sqrt{3}$ 12. (b) $3\sqrt{3}-\pi$ 13. (b) $-\frac{1}{\log x}+c$ 14. (d) 0 15. (b) –cotx – tanx + C 16. (b) 0 17. (d) $\frac{1}{3}$ 18. (b) $3\sqrt{3}-\pi$ 19. (b) $\frac{x^6}{6}+C$ 20. (a) $-\frac{1}{\log 2}$ 21. (b) $x(\log x-1)+C$		11. $2\sin x+2x\cos\alpha+C$ 12. $x+\log\|x\|-\log\|x+1\|+c$ 13. 4 14. $\frac{(1+2x)^{\frac{5}{2}}}{10}-\frac{(1+2x)^{\frac{3}{2}}}{6}+C$ 15. 2 16. $\frac{1}{2}\log\|e^{2x}+e^{-2x}\|+C$ 17. $\frac{1}{3a^3}\tan^{-1}a^6$ 18. $\sin^{-1}(x-1)$ 19. $\frac{1}{3}\log\left\|\frac{\log x-4}{\log x-1}\right\|+C$ 20. $\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$ or $-\frac{1}{\sqrt{2}}\log(\sqrt{2}-1)$ 21. $\frac{1}{2}\log\left\|\left(1-\frac{1}{x^2}\right)\right\|+c$ OR $\frac{1}{2}\log\left\|\left(\frac{x^2-1}{x^2}\right)\right\|+c$ 22. 0	10. $x-2\log\left\|\frac{x-2}{x-1}\right\|+C$ 11. $a\left[\frac{x}{a}\tan^{-1}\sqrt{\frac{x}{a}}-\sqrt{\frac{x}{a}}+\tan^{-1}\sqrt{\frac{x}{a}}\right]+C$ 12. $\frac{1}{7}\log\left\|\frac{x-2}{x+2}\right\|+\frac{\sqrt{3}}{7}\tan^{-1}\frac{x}{\sqrt{3}}+C$ 13. $-\frac{2}{5}\tan^{-1}\frac{x}{2}+\frac{3}{5}\tan^{-1}\frac{x}{3}+C$ 14. $3\log\|\sin\theta-2\|-\frac{4}{\sin\theta-2}+C$ 15. $\frac{2}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}}+C$ 16. $\frac{e^{2x}}{2(2x-3)^2}+C$ 17. $\frac{1}{\cos(a-b)}\log\left\|\frac{\sin(x-a)}{\cos(x-b)}\right\|+C$ 18. $x\cdot\log(\log x)-\frac{x}{\log x}+C$ 19. $\frac{11}{4}$ 20. π 21. $\sin^{-1}(\tan x)+c$	

22. (a) $\frac{1}{\sqrt{3}}$ 23. (c) $\log\left\|\sec\left(\frac{\pi}{4}+x\right)\right\|+C$ 24. (c) tanx 25. (a) 2 26. (b) $\frac{ax^2}{2}+bx$ 27. (b) $x^2+3\log\|x\|$ 28. (a) 4 29. (c) $\frac{2^{x+2}}{\log 2}+C$ 30. (b) x + 2cosx + C 31. (d) π 32. (d) $-\frac{e^{-x}}{x}+C$ 33. (b) secx + tanx + C		23. $-\frac{\cos^4 x}{4}+C$ 24. $\frac{(1+2x)^{\frac{5}{2}}}{10}-\frac{(1+2x)^{\frac{3}{2}}}{6}+C$ 25. 2 26. $\frac{1}{2}\log\|e^{2x}+e^{-2x}\|+C$ 27. $\frac{1}{2}\log\left\|\left(\frac{x^2-1}{x^2}\right)\right\|+c$ 28. $-\frac{\cos^4 x}{4}+C$ 29. $\frac{1}{3a^3}\tan^{-1}a^6$ 30. 0 31. $\frac{1}{5}\log\left\|\frac{x^2-4}{x^2+1}\right\|+c$	22. $-\frac{5}{2}\sqrt{1+4x-2x^2}+\sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}(x-1)\right)+c$ 23. $\frac{-2}{5}\tan^{-1}\left(\frac{x}{2}\right)+\frac{3}{5}\tan^{-1}\left(\frac{x}{3}\right)+C$ 24. 5 25. 2 π 26. $\frac{1}{5}\log\left\|\frac{\log x-4}{\log x+1}\right\|+C$ 27. $\frac{1}{3}\left[x^3\sin^{-1}\left(x^{\frac{3}{2}}\right)+\frac{x^{\frac{3}{2}}}{2}\sqrt{1-x^3}-\frac{1}{2}\sin^{-1}\left(x^{\frac{3}{2}}\right)\right]+C$ 28. $\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$ or $-\frac{1}{\sqrt{2}}\log(\sqrt{2}-1)$ 29. $\tan^{-1}\sqrt{x}+\frac{1}{2}\log\left\|\frac{\sqrt{x}-1}{\sqrt{x}+1}\right\|+c$ 30. $\frac{\pi}{16}\log 2$	

34. (d) -2 35. (b) $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)+c$ 36. (b) $2\sqrt{3}$ 37. (b) 0 38. (a) $\frac{1}{2}$ 39. (c) 4 40. (b) –cosx + sinx 41. (c) 1 42. (b) $-\frac{1}{\log x}+c$ 43. (d) 0 44. (a) 2 45. (b) 1			31. $e^x\frac{x}{\sqrt{1+x^2}}+c$ 32. $\frac{-1}{x}+\frac{1}{3}\log\lvert x\rvert-\frac{1}{3}\log\lvert x+3\rvert+c$ 33. $\frac{\pi}{2}$ 34. $\tan^{-1}\sqrt{x}+\frac{1}{2}\log\left\lvert\frac{\sqrt{x}-1}{\sqrt{x}+1}\right\rvert+c$ 35. $\frac{\pi}{4}\log\frac{1}{4}$ OR $-\frac{\pi}{2}\log 2$ 36. $2[\log(\sqrt{x}+1)-\log(\sqrt{x}+2)]+C$ or $2\log\left(\frac{\sqrt{x}+1}{\sqrt{x}+2}\right)+C$ 37. $\frac{1}{2}e^{\pi/2}+\frac{1}{2}$ or $\frac{1}{2}(e^{\pi/2}+1)$ 38. $2\sqrt{2}-2$ 39. $-\frac{2}{5}\tan^{-1}\frac{x}{2}+\frac{3}{5}\tan^{-1}\frac{x}{3}+C$ 40. $\frac{\pi}{8}\log 2$ 41. $\frac{\pi}{16}\log 2$ 42. $e^x\frac{x}{\sqrt{1+x^2}}+c$	

			43. $3\sqrt{x^2+2x+4}+2\log\left\|(x+1)+\sqrt{x^2+2x+4}\right\|+c$ 44. $-\frac{2}{\cos\alpha}\sqrt{\sin\alpha+\cos\alpha\cot x}+c$ 45. $x^{\cot^{-1}x}+c$ 46. $\frac{1}{4}\log\frac{3}{2}$ 47. 1 48. $\frac{x\sin^{-1}x}{\sqrt{1-x^2}}+\frac{1}{2}\log\|1-x^2\|+c$ 49. $-\mathrm{e}^{\mathrm{x}}\cot\frac{x}{2}+\mathrm{c}$ 50. $\sqrt{x^2-4x-5}+4\log\left\|x-2+\sqrt{x^2-4x-5}\right\|+c$ 51. a 52. $\frac{\pi}{6}$ 53. $-\frac{1}{6}\log\|3\mathrm{cosec}^2x-4\|+C$	

			54. $\frac{x^3}{3}\log(x^2+1)-\frac{2}{3}\left[\frac{x^3}{3}-x+\tan^{-1}x\right]+C$ 55. $\log\left\|e^x-2+\sqrt{e^{2x}-4e^x-5}\right\|+C$ 56. $\frac{26+7^{3/2}-3^{3/2}}{6}$ or $\frac{26+7\sqrt{7}-3\sqrt{3}}{6}$ 57. $-2\log\|x+1\|-\frac{1}{x+1}+3\log\|x+2\|+C$ 58. $\frac{1}{2}e^{\pi/2}$ 59. $\frac{8}{3}$ 60. $\sin^{-1}\left(\frac{e^x+2}{3}\right)+C$ 61. $\frac{64}{231}$ 62. π 63. $x-3\log\|x^2+6x+12\|+2\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right)+C$	

			64. $-\log\|1-x\|+\frac{1}{2}\log(x^2+1)+\tan^{-1}(x)+C$ 65. 6 66. 3 67. $\frac{x^2}{2}+x+\frac{1}{2}\log\|x-1\|-\frac{1}{4}\log\|x^2+1\|-\frac{1}{2}\tan^{-1}x+C$ 68. $\left(\frac{\pi}{2}-1\right)$ 69. $\frac{1}{2}e^{x^2}(x^4)+C$ 70. $\frac{\pi^2}{16}$ 71. 3 72. $\log\left\|\left(\sin\theta-\frac{3}{2}\right)+\sqrt{\sin^2\theta-3\sin\theta+2}\right\|+C$	

			73. $-\frac{1}{4x}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)+C$ 74. 5 75. $\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$ or $-\frac{1}{\sqrt{2}}\log(\sqrt{2}-1)$ 76. 2π 77. $\frac{1}{5}\log\left\|\frac{\log x-4}{\log x+1}\right\|+C$ 78. $\frac{1}{3}\left[x^3\sin^{-1}\left(x^{\frac{3}{2}}\right)+\frac{x^{\frac{3}{2}}}{2}\sqrt{1-x^3}-\frac{1}{2}\sin^{-1}\left(x^{\frac{3}{2}}\right)\right]+C$ 79. $\log(x^2-1)\times\frac{x^3}{3}-\frac{2}{3}\left[\frac{x^3}{3}+x+\frac{1}{2}\log\left\|\frac{x-1}{x+1}\right\|\right]+C$ 80. $F(x)=\sin^{-1}(x-1)$ 81. $\frac{1}{3}\log\left\|\frac{\log x-4}{\log x-1}\right\|+C$	

			82. $\frac{-1}{x}+\frac{1}{3}\log\lvert x\rvert-\frac{1}{3}\log\lvert x+3\rvert+c$ 83. $\frac{\pi}{2}$ 84. $\frac{3}{4}\log\left\lvert\frac{x-1}{x+1}\right\rvert-\frac{1}{2(x+1)}+C$	

UNIT TEST

1. (a) $\frac{1}{3}e^{x^3}+C$

2. (a) $\tan(xe^x)+C$

3. (d) -2

4. (d) 2

5. (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

6. $\sin^{-1}\left(\frac{x-2}{2}\right)+C$

7. $\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right)+C$

8. $e-2$

9. $\frac{e^x}{5}(\sin 2x-2\cos 2x)+C$

10. $\log\left|\frac{x^2+1}{x^2+2}\right| + C$

11. $\frac{-2}{\cos\alpha}\sqrt{\sin\alpha + \cos\alpha\cot x} + C$

12. $= \frac{-1}{3}\left[\left(\frac{x^2+1}{x^2}\right)^{3/2}\log\left(\frac{x^2+1}{x^2}\right) - \frac{2}{3}\left(\frac{x^2+1}{x^2}\right)^{3/2}\right] + C$

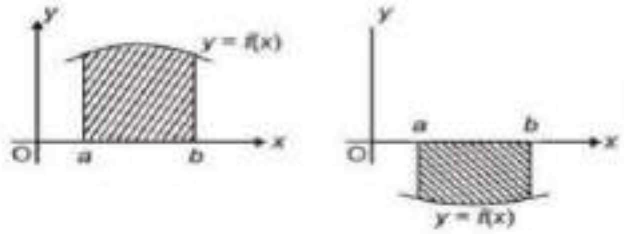
13. $\frac{23}{2}$

CHAPTER 8 – APPLICATION OF INTEGRALS

AREA UNDER SIMPLE CURVES

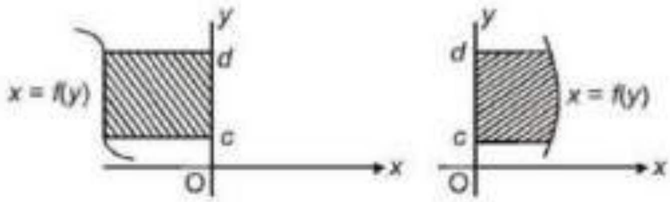
→ Area bounded by the curve y = f(x), the x axis and between the ordinates x = a and x = b is given by

Area $= \int_a^b y dx$ or $\int_a^b f(x) dx$



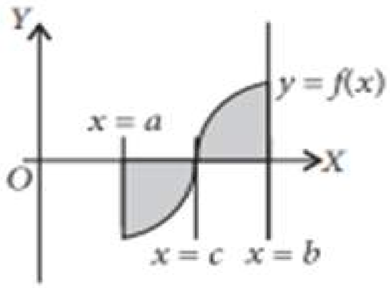
→ Area bounded by the curve x = g(x), the x axis and between the abscissas y = c and y = d is given by

Area $= \int_c^d x dy$ or $\int_c^d g(y) dy$



→ Area of the region bounded by the curve y = f(x) some portion of which is above the x-axis and some below the x-axis is

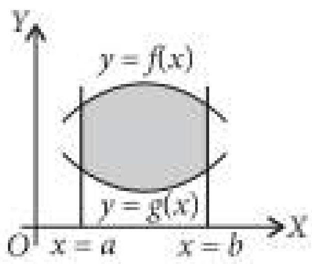
Area $= \left|\int_a^c f(x) dx\right| + \int_c^b f(x) dx$



AREA BETWEEN TWO CURVES

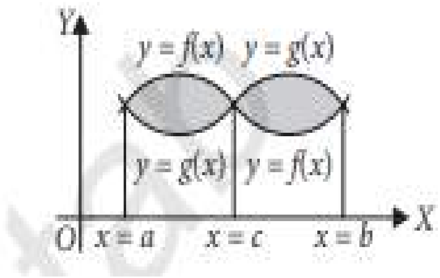
→ Area of the region between two curves y = f(x) and y = g(x) and the lines x = a and y = b is

Area $= \int_a^b [f(x) - g(x)] dx, f(x) \geq g(x)$ in $[a, b]$



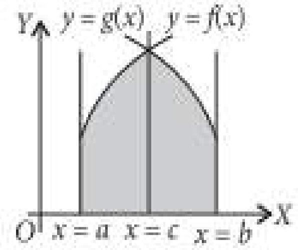
→ If $f(x) \geq g(x)$ in [a, c] and $f(x) \leq g(x)$ in [c, b], where $a < c < b$

Area $= \int_a^c [f(x) - g(x)]dx + \int_c^b [g(x) - f(x)]dx$



→ Area of shaded region as shown in figure

Area $= \int_a^c f(x)dx + \int_c^b g(x)dx$



MULTIPLE CHOICE QUESTIONS

1. The area bounded by the line y = x, x-axis and lines x = -1 and x = 2 is

(a) 0 sq. unit (b) $\frac{1}{2}$ sq. units (c) $\frac{3}{2}$ sq. units (d) $\frac{5}{2}$ sq. units

2. The area of the region bounded by $y = |x - 1|$ and y = 1 is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

3. The area of the region bounded by the curve x = 2y + 3 and lines y = 1 and y = -1 is

(a) 4 sq. units (b) $\frac{3}{2}$ sq. units (c) 6 sq. units (d) 8 sq. units

4. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines x = 0 and x = 2 is

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

5. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line y = 3 is

(a) 2 (b) $\frac{9}{4}$ (c) $\frac{9}{3}$ (d) $\frac{9}{2}$

6. The area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and x-axis is

(a) 8 π sq. units (b) 20 π sq. units (c) 16 π sq. units (d) 256 π sq. units

7. Area of the region bounded by the curve y = cosx between x = 0 and x = π is

(a) 2 sq. units (b) 4 sq. units (c) 3 sq. units (d) 1 sq. unit

8. The area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is

(a) 20π sq. units (b) $20\pi^2$ sq. units (c) $16\pi^2$ sq. units (d) 25π sq. units

9. The area of the region bounded by the curve x = 2y + 3 and the lines y = 1 and y = -1 is

(a) 4 sq. units (b) $\frac{3}{2}$ sq. units (c) 6 sq, units (d) 8 sq, units

10. The area bounded by the curve $y = \frac{3}{2}\sqrt{x}$, the line x = 1 and x-axis is ______ sq. units.

(a) 2 (b) 1 (c) 6 (d) None of these

11. The area of the region bounded by the curves y = |x – 2|, x = 1, x = 3 and x-axis is

(a) 4 (b) 2 (c) 3 (d) 1

12. The area in the first quadrant bounded by $y = 4x^2$, x = 0, y = 1 and y = 4 is

(a) $\frac{7}{3}$ sq. units (b) $\frac{4}{5}$ sq. unit (c) $\frac{3}{4}$ sq. unit (d) None of these

13. The area bounded by the curve $y^2 = x$, line y = 4 and y-axis is

(a) $\frac{16}{3}$ (b) $\frac{64}{3}$ (c) $7\sqrt{2}$ (d) None of these

14. Find the area of the triangle formed by the lines y = 2x, x = 0 and y = 2 is

(a) 1 (b) 2 (c) 3 (d) 4

15. Area bounded by the curve y = sinx and the x-axis between x = 0 and x = 2π is

(a) 2 sq. units (b) 0 sq. unit (c) 3 sq. units (d) 4 sq. units

16. Area of the region bounded by the curve y = cosx between x = 0 and x = 2π is

(a) 4 sq. units (b) 3 sq. units (c) 2 sq. units (d) 1 sq. unit

17. The area of the region bounded by x-axis, the lines x = 2 and y = x is

(a) 2 sq. units (b) 4 sq. units (c) $\sqrt{2}$ sq. units (d) π sq. units

18. The area of the region bounded by the curve $y^2 = 9x, y = 3x$ is

(a) 1 sq. unit (b) $\frac{1}{2}$ sq. unit (c) 4 sq. units (d) 14 sq. units

19. The area of a minor segment of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{2}$ is

(a) $\frac{a^2}{12}(4\pi - 3\sqrt{3})$ sq units (b) $\frac{a^2}{12}(4\pi - 3)$ sq units

(c) $\frac{a^2}{12}(3\pi - 4)$ sq units (d) None of these

20. The area of the region bounded by the line y – 1 = x, the x-axis and the ordinates x = -2 and x = 3 is

(a) $\frac{4}{3}$ sq units (b) $\frac{7}{2}$ sq units (c) $\frac{17}{2}$ sq units (d) $\frac{16}{3}$ sq units

21. Area of the region bounded by curve $y^2 = 4x$ and the x-axis between x = 0 and x = 1 is

(a) $\frac{2}{3}$ (b) $\frac{8}{3}$ (c) 3 (d) $\frac{4}{3}$

22. The area bounded by the curve $y = \sqrt{x}$, y-axis and between the lines y = 0 and y = 3 is

(a) $2\sqrt{3}$ (b) 27 (c) 9 (d) 3

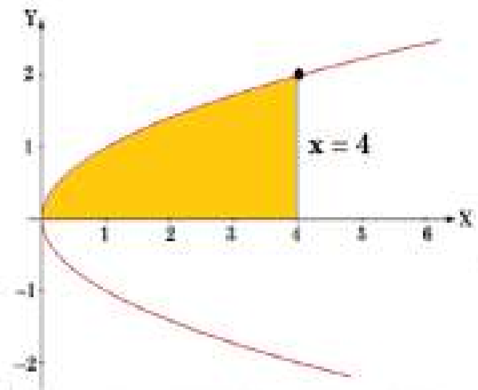
23. The area of the shaded region bounded by the curves $y^2 = x, x = 4$ and the x-axis is given by

(a) $\int_0^4 x dx$

(b) $\int_0^2 y^2 dy$

(c) $2\int_0^4 \sqrt{x} dx$

(d) $\int_0^4 \sqrt{x} dx$



ASSERTION - REASON TYPE QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

1. Assertion: Area enclosed by the circle $x^2 + y^2 = 16$ is 16π sq. units.

Reason: Area enclosed by circle $x^2 + y^2 = r^2$ is πr^2.

2. Assertion: Area bounded by y = |x – 1| from -2 to x = 0 is 4 sq. units.

Reason: y = |x – 1| is differentiable in R.

3. Assertion: Area of the region bounded by the parabola $x^2 = 4y$, y = 1, y = 4 and x = 0 is $\int_1^4 x dy$.

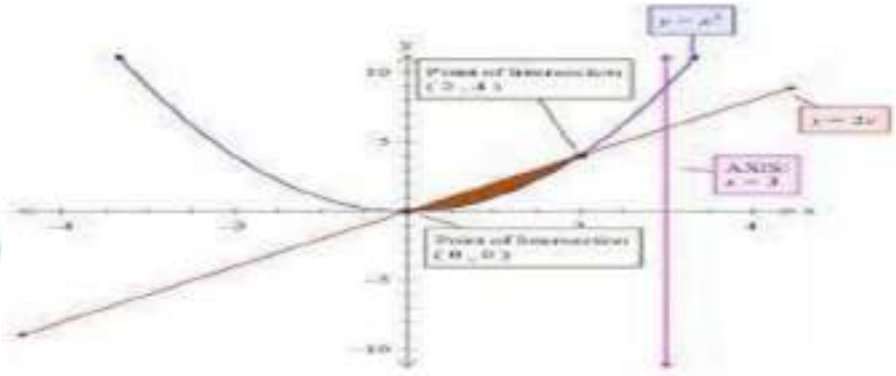
Reason: Area under a curve x = f(y) and right of y-axis lying between the ordinates y = a and y = b is given by $\int_a^b f(y)dy$.

4. Assertion: Area of the region given by $\{(x, y): y^2 \leq 6x, 2 \leq x \leq 5, x, y \geq 0\}$ = $= \int_2^5 \sqrt{6x}$ dy. sq. units.

Reason: Area under a curve x = f(y) and right of y-axis lying between the ordinates y = a and y = b is given by $\int_a^b f(y)dy$.

5. Assertion: Area of the region bounded by the parabola x^2 = y and y = 2x is $\int_0^2 2xdx - \int_0^2 x^2dx$.

Reason:



6. Assertion: Area of the region bounded by the parabola y2 = 4x, x = 1, x = 4 and y = 0 is $\frac{56}{3}$ sq. units.

Reason: Area under a curve y = f(x) and above x-axis lying between the ordinates x = a and x = b is given $\int_a^b f(x)dx$.

7. Assertion: The area of the region bounded by the curve y = x2 and the line y = 4 is $\frac{3}{32}$.

Reason: Area of the bounded by x = f(y) and the line y = 4 is $2\int_0^4$ xdy.

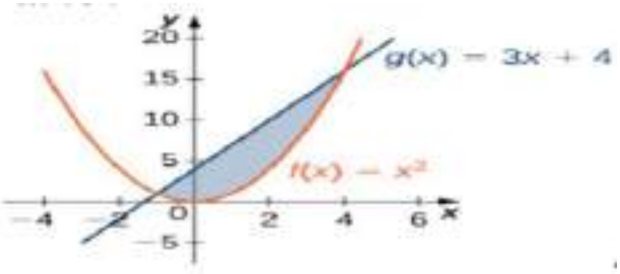
8. Assertion: The area of the region bounded by y = x + 1, x-axis and the lines x = 2 and x = 3 is $\frac{5}{2}$ sq. units.

Reason: The intercept made by the line y = x + 1 on the coordinate axes are 1 unit left and 1 unit above of origin respectively.

9. Assertion: The region bounded by the curve $y = \sqrt{2^2 - x^2}$ is a semicircle above the x-axis.

Reason: Area of the semicircle $y = \sqrt{2^2 - x^2}$ is half of the area bounded by the equation $x^2 + y^2 = 4$.

10. Assertion:



Area is $\int_{-1}^{4} [g(x) - f(x)]dx$

Reason: If f(x) > g(x), for all $x \in (a, b)$ then area bounded by these two curves is $\int_{a}^{b} [f(x) - g(x)]dx$.

VERY SHORT ANSWERS

1. Find the area bounded by the parabola $y^2 = 4ax$, its axis and the two ordinates x = a and x = 2a.

2. Find the area bounded by the by the parabola $y^2 = 4ax$, a > 0 and its latus rectum.

3. Find the area bounded by the parabola $y = x^2 - 1$, x-axis and the line y = 8.

4. Find the area bounded by the curve $y^2 = 4ax$ and the line y = 2a and y-axis.

5. Find the area bounded by the curve $y = \sqrt{4 - x^2}$ and x- axis.

6. Find the area of the region bounded by the parabola $y^2 = 12x$ and its latus rectum.

7. Using integration, find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

8. Sketch the region $\{(x, y): 4x^2 + 9y^2 = 36\}$ and find its area using integration.

9. Using integration, find the area of the region bounded by the line 2y = -x + 6, x-axis and the line x = 2 and x = 4.

10. Using integration, find the area of the region bounded by the line 3y = 2x + 4, x-axis and the line x = 1 and x = 3.

11. Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.

SHORT ANSWERS

1. Using integration, find the area of the region bounded by the curves $y = |x + 1| + 1$, $x = -3$, $x = 3$ and $y = 0$.

2. Using integration, find the area of the region bounded by the curves after making rough sketch of $y = |x + 1| + 1$, $x = -2$, $x = 3$ and $y = 0$.

3. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$.

4. Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$.

5. Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and the x-axis.

6. Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

7. Find the area enclosed between the circle $x^2 + y^2 = 25$ and the line $x + y = 5$.

8. Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y – 2$.

9. Find the area of the region $\{(x, y): x^2 + y^2 \leq 4, x + y \geq 2\}$.

10. Sketch the graph $y = |x + 3|$ and evaluate the area under the curve $y = |x + 3|$ above x-axis between $x = -6$ to $x = 0$.

11. Find the area of the following region using integration: $\{(x, y): y^2 \leq 2x$ and $y \geq x - 4\}$.

12. Using integration, find the area of the region bounded by $y = mx$ $(m > 0)$, $x = 1$, $x = 2$ and the x-axis.

LONG ANSWERS

1. Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

2. Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area.

3. Using integration, find the area of the triangular region whose sides have the equations $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$.

4. Using integration, find the area of the region: $\left\{(x, y): |x| \leq y \leq \sqrt{4 - x^2}\right\}$.

5. Find the area of the region: $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$.

6. Find the area of the region: $\{(x, y): (x, y): x^2 + y^2 \leq 1 \leq x + y\}$.

7. Using the method of integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

8. Using integration, find the area of the region bounded by the triangle whose vertices are (2, 5), (4, 7) and (6, 2).

9. Using integration, find the area of the following region: $\left\{(x, y): \frac{x^2}{9} + \frac{y^2}{4} \leq 1 \leq \frac{x}{3} + \frac{y}{2}\right\}$.

10. Using integration, find the area of the triangle ABC where A(2, 3), B(4, 7) and C(6, 2).

11. Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines $x = -2$ and $x = 2$.

12. Using integration, find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

13. The area of the region bounded by the line y = mx (m > 0), the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.

14. Find the area of the region bounded by the curves $x^2 = y, y = x + 2$ and x-axis using the integration.

15. Sketch the graph of y = x|x| and hence find the area bounded by this curve, x-axis and the ordinates x = -2 and x = 2, using integration.

16. Find the area of the region $\{(x, y): x^2 + y^2 \leq 1 \leq x + y\}$, using integration.

17. Find the area of the region bounded by the lines y = 4x + 5, x + y = 5 and 4y = x + 5 using integration.

18. Using integration, find the area of region bounded by line y $= \sqrt{3}x$, the curve y $= \sqrt{4 - x^2}$ and y-axis in first quadrant.

19. Using integration, find the area of triangle whose vertices are (-1, 1), (0, 5) and (3, 2).

20. Find the area of the smaller region bounded by the curves $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and $\frac{x}{5} + \frac{y}{4} = 1$, using integration.

21. Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines x = -2, x = 2 and the x-axis.

22. Using integration, find the area of the region bounded by the circle $x^2 + y^2 = 16$, line y = x and y-axis but lying in the 1st quadrant.

23. Find the area of the triangle ABC bounded by the lines represented by the equations 5x – 2y – 10 = 0, x – y – 9 = 0 and 3x – 4y – 6 = 0, using integration method.

24. Using integration, find the area of the region bounded by the triangle ABC when its sides are given by the lines 4x – y + 5 = 0, x + y – 5 = 0 and x – 4y + 5 = 0.

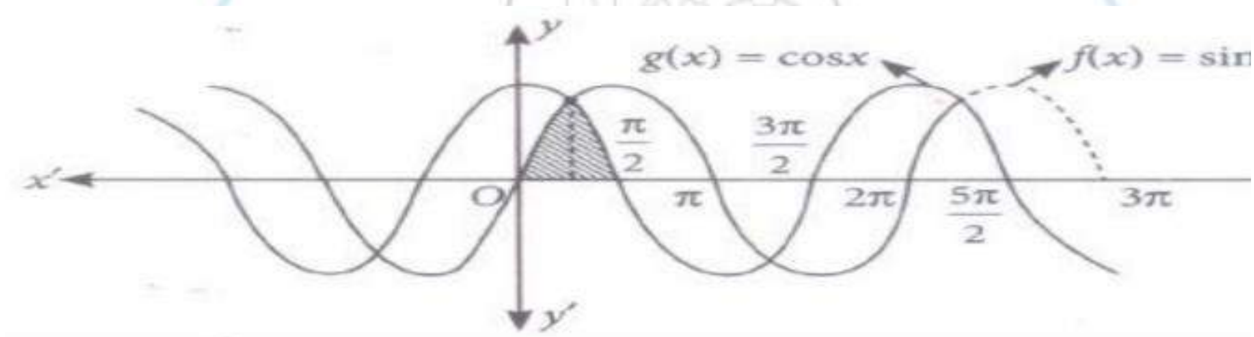
25. If A_1 denotes the area of region bounded by $y^2 = 4x$, x = 1 and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, x = 4, find $A_1 : A_2$.

26. Find the area of the region bounded by the lines x – 2y = 4, x = –1, x = 6 and x-axis, using integration.

27. Using integration, find the area of the region enclosed between the curve $y = \sqrt{4 - x^2}$ and the lines x = -1, x = 1 and the x-axis.

CASE BASED QUESTIONS

1. Graphs of two functions f(x) = sinx and g(x) = cosx is given below:



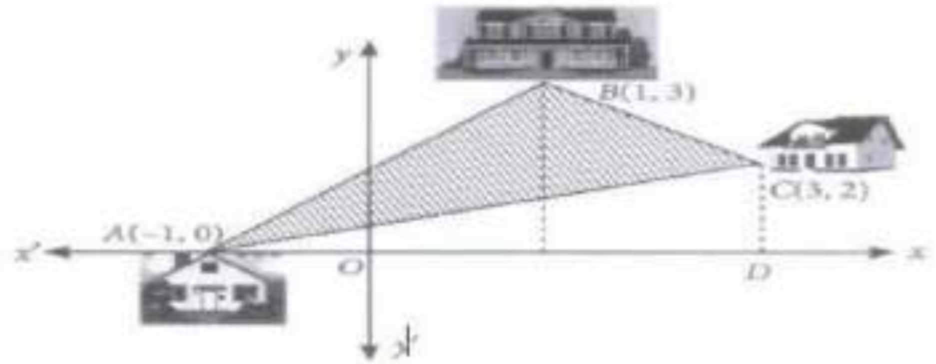
Based on the above information, answer the following questions:

(i) In $[0, \pi]$ the curves $f(x) = \sin x$ and $g(x) = \cos x$ intersect at x = ?

(ii) Find the value $\int_0^{\frac{\pi}{4}} \sin x dx$.

(iii) Find the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x dx$.

2. Location of three houses of a society is represented by the points A(-1, 0), B(1, 3), C(3, 2) and D(3, 0) as shown in the figure.



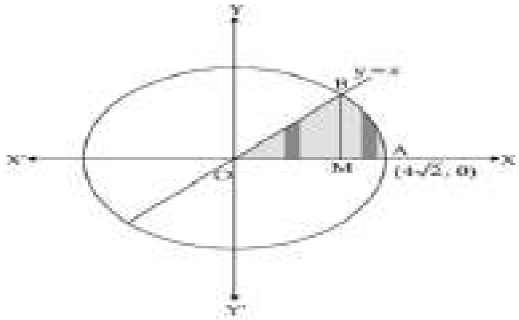
Based on the above information, answer the following questions:

(i) Write an equation of line AB.

(ii) Write an equation of line BC.

(iii) Find area of △ ABC.

3. In the figure O(0, 0) is the center of the circle. The line y = x meets the circle in the first quadrant at the point B.



Based on the above information, answer the following:

(i) Find the equation of the circle.

(ii) Find the coordinates of point B.

(iii) Evaluate the area of the shaded region.

4. A mirror in the shape of an ellipse is represented by $\frac{x^2}{9} + \frac{y^2}{4} = 1$ whas hanging on the wall. Sanjeev and his daughter were playing with football inside the house. All of sudden, football hit the mirror and got a scratch in the shape of line represented by $\frac{x}{3} + \frac{y}{2} = 1$.

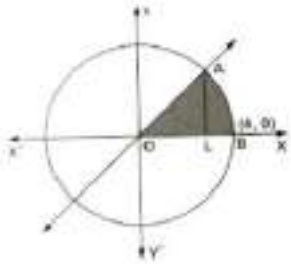
Based on the above information, answer the following:

(i) Sketch the smaller region bounded by the ellipse and straight line.

(ii) Find the points of intersection of ellipse and line.

(iii) Find the value of $\frac{2}{3}\int_0^3 \sqrt{9 - x^2}dx$.

5. The sector of a circle bounded by the circle $x^2 + y^2 = 16$ and the line y = x in the first quadrant is shown in the figure.

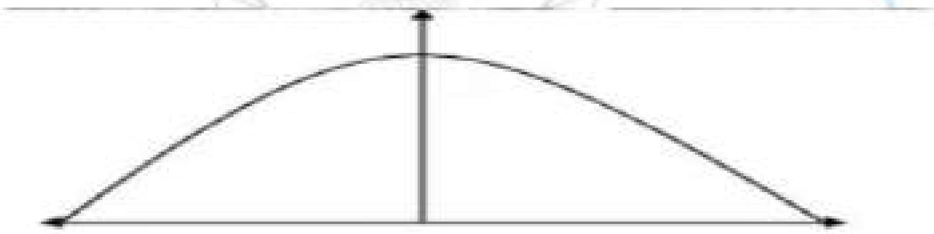


Based on the above information, answer the following:

(i) Find the point of intersection of the both curves.

(ii) Find the area bounded by the two given curves.

6. The bridge connects two hills 100ft apart. The arch on the bridge is in a parabolic form. The highest point on the bridge is 10ft above the road at the middle of the bridge as seen in the figure.



Based on the above information, answer the following:

(i) Find the equation of the parabola designed on the bridge.

(ii) Find the area formed between $x^2 = 250y$, y axis, $y = 0$ and $y = 10$.

7. A student designs an open air honeybee nest on the branch of a tree, whose plane figure parabolic, whose equation is $y^2 = 2x$ and the branch tree is given by a straight line.

Based on the above information, answer the following:

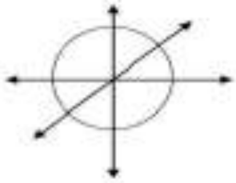
(i) Draw the rough diagram of given parabola and straight line.

(ii) Find the point of intersection of parabola and straight line.

(iii) Find the area enclosed by the two given curves.

8. A child cut a pizza with knife. Pizza is circular in shape which is represented by $x^2 + y^2 = 4$ and knife represents $x = \sqrt{3}y$. On the basis of the above information, answer the questions given below.

PIZZA



(i) Find the point of intersection of circle and straight line.

(ii) Find the area enclosed by the circle, line and x-axis.

UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. The area enclosed by the circle $x^2 + y^2 = 2$ is equal to

(a) 4π sq. units (b) $2\sqrt{2}\pi$ sq. units (c) $4\pi^2$ sq. units (d) 2π sq. units

2. The area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to

(a) π^2 ab sq. units (b) πab sq. units (c) $\pi a^2 b$ sq. units (d) πab^2 sq. units

3. The area of the region bounded by the curve $y = x^2$ and the line $y = 16$ is

(a) $\frac{37}{3}$ sq units (b) $\frac{256}{3}$ sq units (c) $\frac{64}{3}$ sq units (d) $\frac{128}{3}$ sq units

4. Assertion: The area bounded by the curve y = cosx in I quadrant with coordinate axes is 1 square unit.

Reason: $\int_0^{\frac{\pi}{2}} \text{Cos}\, x dx = 1.$

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. Using integration, find the area of the region bounded by the line 2y = 5x + 7, x – axis and the lines x = 2 and x = 8.

6. Find the area of the region $\{(x, y): x^2 + y^2 \leq 1 \leq x + y\}$.

7. Find the area bounded by parabola $y^2 = x$ and straight line $2y = x$.

SECTION C

Each carry 3 marks

8. Sketch the graph of $y = |x + 1|$ and find the area enclosed by this curve $x = -3$, $x = 1$ and x-axis.

9. Find the area of the region enclosed by the parabola $y^2 = 4ax$ and the chord $y = mx$.

SECTION D

Each carry 5 marks

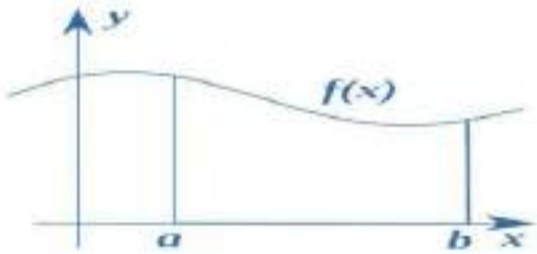
10. Find the area of the region in the first quadrant enclosed by the x-axis the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

11. Using integration, find the area of the region bounded by the triangle whose vertices are (-1, 1), (0, 5) and (3, 2).

SECTION E

12. Area bounded by the curve $y = f(x)$, the x-axis and between the ordinates at $x = a$ and $x = b$ is given by

Area = $= \int_a^b y\,dx = \int_a^b f(x)dx$

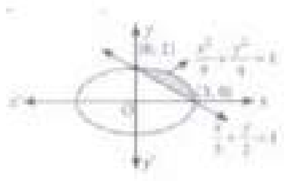


Based on the above information, answer the following:

(i) Draw the graph of $f(x) = |x + 3|$, what does the value of the definite integral on the graph represents?

(ii) Find the value of $\int_{-6}^{0} |x + 3|dx$.

ANSWERS

MCQ	A-R	VSA	SA	LA	CS
1. (d) $\frac{5}{2}$ sq. units	1. (a)	1. $\frac{4}{3}(2\sqrt{2}-1)a^2$ sq.units	1. 16 sq.units	1. 8 sq.units.	1. (i) $\frac{\pi}{4}$
2. (b) 1	2. (c)	2. $\frac{8}{3}a^2$ sq.units	2. $\frac{27}{2}$ sq.units	2. $\left(\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}+\frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}-\frac{1}{2}\right)$ sq.units	(ii) $1-\frac{1}{\sqrt{2}}$
3. (c) 6 sq. units	3. (d)	3. $\frac{104}{3}$ sq.units	3. $\frac{ab}{4}(\pi-2)$ sq.units	3. 6 sq.units	(iii) $1-\frac{1}{\sqrt{2}}$
4. (a) π	4. (a)	4. $\frac{2}{3}a^2$ sq.units	4. $\frac{21}{2}$ sq.units	4. π sq.units	2. (i) $y=\frac{3}{2}(x+1)$
5. (b) $\frac{9}{4}$	5. (a)	5. 2π sq.units	5. $\frac{9}{2}$ sq.units	5. $\frac{23}{6}$ sq.units	(ii) $y=\frac{-1}{2}x+\frac{7}{2}$
6. (a) 8 π sq. units	6. (a)	6. 24 sq.units	6. 27 sq.units	6. $\frac{\pi}{4}-\frac{1}{2}$ sq.units	(iii) 4 sq. units
7. (a) 2 sq. units	7. (d)	7. 6π sq.units	7. $\frac{25}{4}(\pi-2)$.	7. $\frac{7}{2}$ sq. units	3.(i) $x^2+y^2=32$
8. (a) 20π sq. units	8. (d)	8. 2π sq.units	8. $\frac{9}{8}$ sq. units	8. 7 sq. units	(ii) (4, 4)
9. (c) 6 sq, units	9. (a)	9. 3 sq.units	9. $(\pi-2)$ sq,units	9. $\frac{3\pi}{2}-3$ sq. units.	(iii) 4π sq.units
10. (b) 1	10. (a)	10. $\frac{16}{3}$ sq.units	10. 9 sq.units	10. 9 sq.units	4. (i) 
11. (d) 1		11. 5	11. 18	11. $4\sqrt{3}+\frac{8\pi}{3}$	
12. (a) $\frac{7}{3}$ sq. units			12. $\frac{3}{2}$m	12. $\frac{8}{3}a^2$	(ii) (0, 2) and (3, 0)

13. (b) $\frac{64}{3}$ 14. (a) 1 15. (d) 4 sq. units 16. (a) 4 sq. units 17. (a) 2 sq. units 18. (b) $\frac{1}{2}$ sq. unit 19. (a) $\frac{a^2}{12}(4\pi - 3\sqrt{3})$ sq units 20. (c) $\frac{17}{2}$ sq units 21. (d) $\frac{4}{3}$ 22. (c) 9 23. (d) $\int_0^4 \sqrt{x}dx$				13. m = 1 14. $\frac{5}{6}$ 15. $\frac{16}{3}$ 16. $\frac{\pi}{4} - \frac{1}{2}$ 17. $\frac{15}{2}$ 18. $\frac{\pi}{3}$ 19. $\frac{15}{2}$ 20. $5\pi - 10$ 21. $\frac{6\sqrt{21}}{5} + 15\sin^{-1}\left(\frac{2}{5}\right)$ 22. 2π 23. $\frac{343}{3}$ 24. $\frac{15}{2}$ 25. $1:16$ 26. $\frac{29}{4}$	(iii) $\frac{3\pi}{2}$ 5. (i) $(2\sqrt{2}, 2\sqrt{2})$ (ii) 2π sq. units 6. (i) $x^2 = -250y$ (ii) $\frac{2000}{3}$ sq. units 7. (ii) (2. -2) and (8, 4) (iii) 18 sq. units. 8. (i) $(1, \sqrt{3})$ & $(-1, -\sqrt{3})$ (ii) $\frac{2\pi}{3}$ sq. units

				27. $\sqrt{3}+\frac{2\pi}{3}$	

UNIT TEST

1. (d) 2π sq. units

2. (b) πab sq. units

3. (b) $\frac{256}{3}$ sq units

4. $\int_0^{\frac{\pi}{2}} \operatorname{Cos} x dx = 1$

5. 96 sq.units

6. $\frac{1}{2}(\pi - 1)$ sq. units

7. $\frac{4}{3}$ sq. units

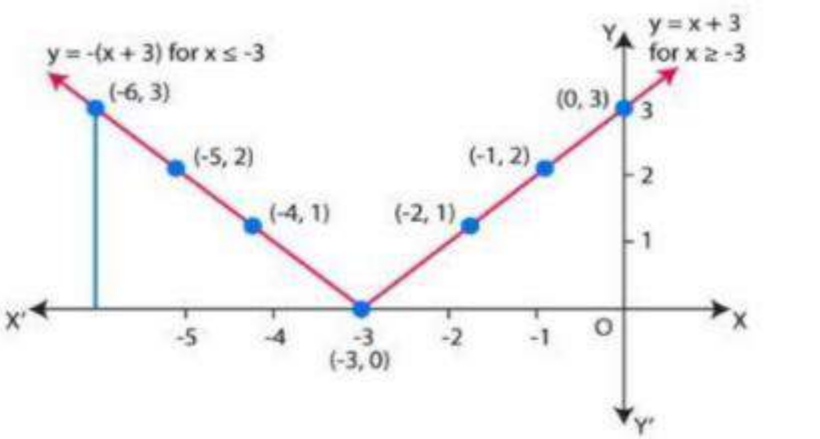
8. 4 sq.units

9. $\frac{8a^2}{3m^3}$ sq.units

10. $\frac{\pi}{3}$ sq.units

11. $\frac{15}{2}$ sq.units

12. (i)

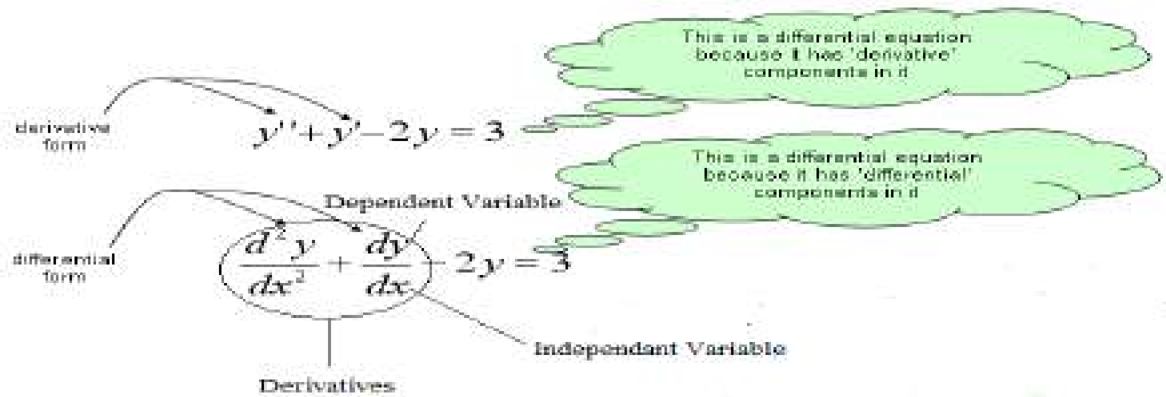


(ii) 9

CHAPTER 9 – DIFFERENTIAL EQUATIONS

DIFFERENTIAL EQUATION

An equation containing an independent variable and a dependent variable and the derivatives of the dependent variable with respect to the independent variable is called an ordinary differential equation.

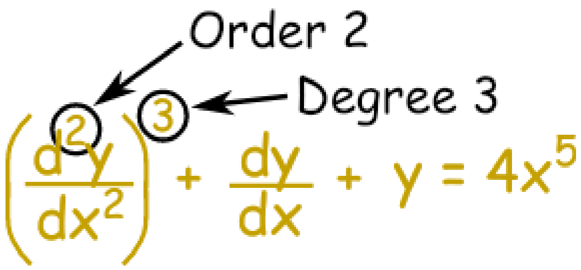


ORDER OF A DIFFERENTIAL EQUATION

Order of a differential equation is defined as the order of the highest order derivative of the dependent variable with respect to the independent variable involved in the given differential equation.

DEGREE OF A DIFFERENTIAL EQUATION

When a differential equation is in a polynomial form in derivatives, the highest power of the highest order derivative occurring in the differential equation is called the degree of the differential equation.



NOTE:

(i) To determine the degree, the differential equation has to be expressed in a polynomial form. If the differential equation cannot be expressed in a polynomial form in the derivatives, the degree of the differential equation is not defined.

(ii) Order and degree (if defined) of a differential equation are always positive integers.

SOLUTIONS OF A DIFFERENTIAL EQUATIONS

Solution of a differential equation is function of the form $y = f(x) + c$ which satisfies the given differential equation.

General Solution: The solution of a differential equation which contains a number of arbitrary constants equal to the order of the differential equation.

Particular Solution: A solution obtained by giving particular values to arbitrary constants in the general solution.

DIFFERENTIAL EQUATION IN VARIABLE SEPARABLE FORM

If the differential equation is of the form $f(x)\,dx = g(y)\,dy$, then the variables are separable and such equations can be solved by integrating on both sides. The solution is given by

$$\int f(x)dx = \int g(y)dy + C, \text{ where C is an arbitrary constant.}$$

HOMOGENEOUS DIFFERENTIAL EQUATIONS

A function $F(x, y)$ is said to be homogeneous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$ for any nonzero constant λ.

PROCEDURE TO SOLVE THE HOMOGENEOUS DIFFERENTIAL EQUATIONS

To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$

We make the substitution y = v.x ------ (1)

Differentiating equation (1) with respect to x, we get $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = g(v)$$

$$x\frac{dv}{dx} = g(v) - v$$

Separating the variable we get, $\frac{dv}{g(v)-v} = \frac{dx}{x}$

Then, integrate on both sides

$$\int \frac{dv}{g(v)-v} = \int \frac{1}{x}dx + C$$

After integration, we get the general solution by replacing v = $\frac{y}{x}$.

Note If the homogeneous differential equation is in the form $\frac{dx}{dy} = F(x, y)$ where, F (x, y) is homogenous function of degree zero, then we make substitution $\frac{x}{y} = v$ i.e., $x = vy$ and we proceed further to find the general solution as discussed above by writing $\frac{dx}{dy} = F(x, y) = h\left(\frac{x}{y}\right)$.

LINEAR DIFFERENTIAL EQUATIONS

A differential equations of the form $\frac{dy}{dx} + Py = Q$ (Or) $\frac{dx}{dy} + Px = Q$ where, P and Q are constants or functions of x only is known as a first order linear differential equation.

STEPS INVOLVED SOLVING FIRST ORDER LINEAR DIFFERENTIAL EQUATION

(i) Write the given differential equation in the form $\frac{dy}{dx} + Py = Q$ ($\frac{dx}{dy} + Px = Q$) where P, Q are constants or functions of x only.

(ii) Find the Integrating Factor (I.F) = $e^{\int Pdx}$ ($e^{\int P \cdot dy}$).

(iii) Write the solution of the given differential equation as

$$y\,(\text{I.F}) = \int (Q \times \text{I.F})dx + C \quad (xe^{\int Pdy} = \int Qe^{\int Pdy}dy + C)$$

MULTIPLE CHOICE QUESTIONS

1. The degree of the differential equation $x^2\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^3$ is

(a) 1 (b) 2 (c) 3 (d) 6

2, The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2\log\left(\frac{d^2y}{dx^2}\right)$ is

(a) 1 (b) 2 (c) 3 (d) Not defined

3. The order and degree of differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^2 = \frac{d^2y}{dx^2}$ respectively are

(a) 1, 2 (b) 2, 2 (c) 2, 1 (d) 4, 2

4. The integrating factor of the differential equation $\frac{dy}{dx}(x\log x) + y = 2\log x$ is

(a) e^x (b) logx (c) log(logx) (d) x

5. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is

(a) y = 2 (b) y = 2x (c) y = 2x – 4 (d) $y = 2x^2 - 4$

6. The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ is

(a) $y = \frac{x^2+C}{4x^2}$ (b) $y = \frac{x^2}{4} + C$ (c) $y = \frac{x^4+C}{x^2}$ (d) $y = \frac{x^4+C}{4x^2}$

7. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ is

(a) 1 (b) 2 (c) 3 (d) Not defined

8. The degree of differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ is

(a) 1 (b) 2 (c) 3 (d) Not defined

9. The order and degree of a differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$, respectively are

(a) 2 and not defined (b) 2 and 2 (c) 2 and 3 (d) 3 and 3

10. Differential equation which has solution of the form $y = A\cos\alpha x + B\sin\alpha x$, where A and B are arbitrary constants is

(a) $\frac{d^2y}{dx^2} - \alpha^2 y = 0$ (b) $\frac{d^2y}{dx^2} - \alpha^2 y = 0$ (c) $\frac{d^2y}{dx^2} + \alpha y = 0$ (d) $\frac{d^2y}{dx^2} - \alpha y = 0$

11. Integrating factor of $x\frac{dy}{dx} - y = x^4 - 3x$ is

(a) x (b) logx (c) $\frac{1}{x}$ (d) –x

12. Solution of $\frac{dy}{dx} - y = 1, y(0) = 1$ is given by

(a) $xy = -e^{x}$ (b) $xy = -e^{-x}$ (c) $xy = -1$ (d) $y = 2e^x - 1$

13. The integrating factor of differential equation $\cos xdydx+y\sin x=1$ is

(a) cosx (b) tanx (c) secx (d) sinx

14. The solution of differential equation cosx.sinydx + sinxcosydy = 0 is

(a) $\frac{\sin x}{\sin y} = C$ (b) sinx.siny = C (c) sinx + siny = C (d) cosx.cosy = C

15. The integrating factor of differential equation $(1 - x^2)\frac{dy}{dx} - xy = 1$ is

(a) $-x$ (b) $\frac{x}{1+x^2}$ (c) $\sqrt{1 - x^2}$ (d) $\frac{1}{2}\log(1 - x^2)$

16. If m and n are respectively the order and the degree of the differential equation $\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)\right]^4 = 0$, then m + n is

(a) 1 (b) 2 (c) 3 (d) 4

17. If p and q are the degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} + \frac{d^3y}{dx^3} = 4$, then value of 2p – 3q is

(a) 7 (b) -7 (c) 3 (d) -3

18. Find the value of m and n, where m and n are order and degree of differential equation $\frac{4\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 - 1$

(a) m = 3, n = 2 (b) m = 2 , n = 3 (c) m = 2, n = 2 (d) m = 3, n = 3

19. Degree of differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} = x$ is

(a) 1 (b) 2 (c) 3 (d) Does not exist

20. Integrating factor of the differential equation $\frac{dy}{dx} = \frac{\cos y}{1 - x\sin y}$ is

(a) cosy (b) –secy (c) secy (d) tany

21. The degree of differential equation $\left(1 + \frac{dy}{dx}\right)^3 = \left(\frac{dy}{dx}\right)^2$ is

(a) 1 (b) 2 (c) 3 (d) 4

22. Integrating factor for the differential equation $\sin^2 x \frac{dy}{dx} + y = \cot x$ is

(a) $e^{-\cot x}$ (b) cotx (c) –cotx (d) $e^{\cot x}$

23. Integrating factor for the solution of differential equation $(x - y^3)dy + ydx = 0$ is

(a) $\frac{1}{y}$ (b) logy (c) y (d) y^2

24. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$?

(a) 3 (b) 2 (c) 6 (d) Not defined

25. The integrating factor of the differential equation $(1 - y^2)\frac{dx}{dy} + yx = ay, (-1 < y < 1)$ is

(a) $\frac{1}{y^2-1}$ (b) $\frac{1}{\sqrt{y^2-1}}$ (c) $\frac{1}{1-y^2}$ (d) $\frac{1}{\sqrt{1-y^2}}$

26. The general solution of the differential equation $xdy - (1 + x^2)dx = dx$

(a) $y = 2x + \frac{x^3}{3} + c$ (b) $y = 2\log x + \frac{x^3}{3} + c$ (c) $y = \frac{x^2}{2} + c$ (d) $y = 2\log x + \frac{x^2}{2} + c$

27. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ is

(a) Order = 3, Degree = 2 (b) Order = 2, Degree = 3

(c) Order = 2, Degree = 2 (d) Order = 3, Degree = 3

28. The degree and order of the differential equation $y = px + 3\sqrt{a^2p^2 + b^2}, p = \frac{dy}{dx}$ are respectively

(a) 3, 1 (b) 1, 3 (c) 1, 1 (d) 3, 3

29. General solution of the differential equation $\frac{dy}{dx} + yg'(x) = g(x) \cdot g'(x)$, where g(x) is a function of x is

(a) g(x) – log[1 – y – g(x)] = C (b) g(x) – log[1 + y – g(x)] = C

(c) g(x) + [1 + y – log g(x)] = C (d) g(x) + log[1 + y – g(x)] = C

30. In order to solve the differential equation $x\cos x \frac{dy}{dy} + y(x\sin x + \cos x) = 1$ the integrating factor is

(a) xcosx (b) xsecx (c) xsinx (d) xcosecx

31. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ respectively are

(a) 1, 2 (b) 2, 3 (c) 2, 1 (d) 2, 6

32. The order of the differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^2y}{dx^2}\right) = 5$ is

(a) 4 (b) 3 (c) 2 (d) not defined

33. The degree of the differential equation $(y'')^2 + (y')^3 = x\sin(y')$ is

(a) 1 (b) 2 (c) 3 (d) not defined

34. The degree and order of differential equation $y''^2 + \log(y') = x^5$ respectively are

(a) not defined, 5 (b) not defined, 2 (c) 5, not defined (d) 2, 2

35. The difference of the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ is

(a) 1 (b) 2 (c) -1 (d) 0

36. The order and the degree of the differential equation $\left(1 + 3\frac{\mathrm{dy}}{\mathrm{d}x}\right)^2 = 4\frac{\mathrm{d}^3\mathrm{y}}{\mathrm{d}x^3}$ respectively are

(a) 1, $\frac{2}{3}$ (b) 3, 1 (c) 3, 3 (d) 1, 2

37. The order of the following differential equation $\frac{\mathrm{d}^3\mathrm{y}}{\mathrm{d}x^3} + x\left(\frac{\mathrm{dy}}{\mathrm{d}x}\right)^5 = 4\log\left(\frac{\mathrm{d}^4\mathrm{y}}{\mathrm{d}x^4}\right)$ is

(a) not defined (b) 3 (c) 4 (d) 5

38. The number of solutions of the differential equation $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{y}+1}{\mathrm{x}-1}$, when y(1) = 2 is

(a) 0 (b) 1 (c) 2 (d) infinite

39. The number of solution of differential equation $\frac{dy}{dx} - y = 1$, given that y(0) = 1 is

(a) 0 (b) 1 (c) 2 (d) infinitely many

40. The general solution of the differential equation x dy + y dx = 0 is

(a) xy = c (b) x + y = c (c) $x^2 + y^2 = c^2$ (d) logy = logx + c

41. The differential equation $\frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{F}(\mathrm{x}, \mathrm{y})$ will not be a homogeneous differential equation if F(x, y) is

(a) $\cos x - \sin\left(\frac{y}{x}\right)$ (b) $\frac{y}{x}$ (c) $\frac{x^2+y^2}{xy}$ (d) $\cos^2\left(\frac{x}{y}\right)$

42. $x\log x\frac{dy}{dx} + y = 2\log x$ is an example of a

(a) variable separable differential equation

(b) homogeneous differential equation

(c) first order linear differential equation

(d) differential equation whose degree is not defined

43. The integrating factor of differential equation $(x+2y^3)\frac{dy}{dx}=2y$ is

(a) $e^{\frac{y^2}{2}}$ (b) $\frac{1}{\sqrt{y}}$ (c) $\frac{1}{y^2}$ (d) $e^{-\frac{1}{y^2}}$

44. The integrating factor of the differential equation $\frac{dy}{dx}+\frac{2}{x}y=0, x\neq 0$ is

(a) $\frac{2}{x}$ (b) x^2 (c) $e^{\frac{2}{x}}$ (d) $e^{\log(2x)}$

45. The general solution of the differential equation $\frac{dy}{dx}=e^{x+y}$ is

(a) $e^x+e^{-y}=c$ (b) $e^{-x}+e^{-y}=c$ (c) $e^{x+y}=c$ (d) $2e^{x+y}=c$

46. The integrating factor for solving the differential equation $x\frac{dy}{dx}-y=2x^2$ is

(a) e^{-y} (b) e^{-x} (c) x (d) $\frac{1}{x}$

47. The integrating factor of the differential equation $(x+2y^2)\frac{dy}{dx}=y(y>0)$ is

(a) $\frac{1}{x}$ (b) x (c) y (d) $\frac{1}{y}$

48. Solution of the differential equation $(1+y^2)(1+\log x)dx+xdy=0$ is

(a) $\tan^{-1}y+\log|x|+\frac{(\log|x|)^2}{2}=C$ (b) $\tan^{-1}y-\log|x|+\frac{(\log|x|)^2}{2}=C$

(c) $\tan^{-1}y-\log|x|-\frac{(\log|x|)^2}{2}=C$ (d) $\tan^{-1}y+\log|x|-\frac{(\log|x|)^2}{2}=C$

49. The integrating factor of the differential equation $(1-x^2)\frac{dy}{dx}+xy=ax$, $-1<x<1$ is

(a) $\frac{1}{x^2-1}$ (b) $\frac{1}{\sqrt{x^2-1}}$ (c) $\frac{1}{1-x^2}$ (d) $\frac{1}{\sqrt{1-x^2}}$

50. The sum of the order and the degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3=\sin y$ is

(a) 5 (b) 2 (c) 3 (d) 4

51. The general solution of the differential equation $xdy-(1+x^2)dx=dx$ is

(a) $y=2x+\frac{x^3}{3}+C$ (b) $y=2\log x+\frac{x^3}{3}+C$

(c) $y=\frac{x^2}{2}+C$ (d) $y=2\log x+\frac{x^2}{2}+C$

52. Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is

(a) 2 (b) 1 (c) Not defined (d) 0

53. The integrating factor of the differential equation $(1 - \mathrm{y}^2)\frac{\mathrm{d}x}{\mathrm{dy}} + \mathrm{y}x = \mathrm{ay}, (-1 < \mathrm{y} < 1)$ is

(a) $\frac{1}{y^2-1}$ (b) $\frac{1}{\sqrt{\mathrm{y}^2-1}}$ (c) $\frac{1}{1-y^2}$ (d) $\frac{1}{\sqrt{1-\mathrm{y}^2}}$

54. The integrating factor of the differential equation $(3x^2 + y)\frac{dx}{dy} = x$ is

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $\frac{2}{x}$ (d) $-\frac{1}{x}$

55. The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is

(a) $\frac{1}{x} + \frac{1}{y} = C$ (b) logx – logy = C (c) xy = C (d) x + y = C

56. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$?

(a) 3 (b) 2 (c) 6 (d) Not defined

57. The order and degree (if defined) of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x\sin\left(\frac{dy}{dx}\right)$ respectively are

(a) 2, 2 (b) 1, 3 (c) 2, 3 (d) 2, degree not defined

58. The solution of the differential equation $\frac{dy}{dx} = \frac{1}{\log y}$ is

(a) logy = x + C (b) y logy – y = x + C (c) log y – y = x + C (d) log y + y = x + C

59. The integrating factor of the differential equation $x\frac{dy}{dx} - y = x^4 - 3x$ is

(a) x (b) –x (c) x^{-1} (d) $\log(\mathrm{x}^{-1})$

ASSERTION - REASON TYPE QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

1. Assertion: The order of the differential equation whose general solution is $y = c_1 \cos x + c_2 \sin^2 x + c_3 e^{2x+c_4}$ is 3.

Reason: The total number of arbitrary parameters in the given general solution in the assertion is 4.

2. Assertion: Order of differential equation $\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2} = 5x$ is 1

Reason: Order of the differential equation is the order of the highest order derivative present in the equation.

3. Assertion: The differential equation $\left(\frac{dy}{dx}\right)^3 + 2y^{\frac{1}{2}} = x$ is of order 1 and degree 3.

Reason: The order and degree of differential equation $\frac{d^3y}{dx^3} = \sqrt{\frac{dy}{dx} + 5}$ are 3 and 1 respectively.

4. Assertion: Degree of differential equation $x - \cos\left(\frac{dy}{dx}\right) = 0$ is 1.

Reason: Differential equation $x - \cos\left(\frac{dy}{dx}\right) = 0$ can be converted in the polynomial equation of derivative.

5. Assertion: The solution of differential equation $\frac{dy}{dx} = \frac{y}{x}$ with x = 1 and y = 1 is x = y.

Reason: Separation of variable method can be used to solve the above given differential equation.

6. Assertion: The differential equation $\frac{dy}{dx} = \frac{x+\sqrt{y^2-x^2}}{x}$ is homogeneous equation.

Reason: If $f(\lambda x, \lambda y) = \lambda^n f(x, y)$ for homogeneous equation.

7. Assertion: $\sin x \frac{d^2y}{dx^2} + \cos x \frac{dy}{dx} + \tan x = 0$ is not a linear differential equation.

Reason: A differential equation is said to be linear if dependent variable and its differential coefficients occur in first degree and are not multiplied together.

8. Assertion: $\frac{dy}{dx} + x^2y = 2x$ is a first order linear differential equation

Reason: If P and Q are functions of x only or constant then differential equation of the form $\frac{dy}{dx} + Py = Q$ is a first order linear differential equation.

9. Assertion: If p and q are the degree and order of differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \frac{d^2y}{dx^2} = 4$ respectively then p = 2 and q = 3.

Reason: 2p – 3q = -5.

10. Assertion: General solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 2x + y$ is $-e^{-y} = \frac{1}{2}e^{2x} + C$.

Reason: Degree of differential equation $\log\left(\frac{dy}{dx}\right) = 2x + y$ is 1.

11. Assertion: $g(x, y) = xy^{\frac{1}{2}} + yx^{\frac{1}{2}}$ is a homogeneous function of degree $\frac{3}{2}$.

Reason: A function is called homogeneous function of degree n if $h(\lambda x, \lambda y) = \lambda^n\ h(x,\ y)$, where $\lambda \neq 0$.

12. Assertion: The solution of the differential equation $\frac{dy}{dx} + y = 1$ with y = 0 at x = 0 is $y = 1 - e^{-x}$.

Reason: The given differential equation is a linear differential equation.

13. Consider the differential equation $(xy - 1)\frac{dy}{dx} + y^2 = 0$

Assertion: The solution of the equation is xy = logy + C.

Reason: The given differential equation can be expressed as $\frac{dx}{dy} + Px = Q$, whose integrating factor is logy.

14. Assertion: The solution of $\mathrm{ydx} - \mathrm{xdy} + \mathrm{y}^2\mathrm{dx} = 0$ is $\frac{\mathrm{x}}{\mathrm{y}} + \mathrm{x} = \mathrm{c}$.

Reason: $\mathrm{d}\left(\frac{\mathrm{x}}{\mathrm{y}}\right) = \frac{(\mathrm{ydx} - \mathrm{x} \cdot \mathrm{dy})}{\mathrm{y}^2}$.

15. Assertion: The differential equation $y^3dy + (x = y^2)dx = 0$ becomes homogeneous if we put $y^2 = t$.

Reason: All differential equation of first order and first degree becomes homogeneous if we put y = tx.

VERY SHORT ANSWERS

1. Solve the following differential equation: $x \cos y\, dy = (x e^x \log x + e^x)dx$.

2. Solve the following differential equation: $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$.

3. Solve the following differential equation: $\mathrm{y}(1 - \mathrm{x}^2)\frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{x}(1 + \mathrm{y}^2)$.

4. Find the general solution of the following differential equation: $\frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{e}^{\mathrm{x}-\mathrm{y}} + \mathrm{x}^2\mathrm{e}^{-\mathrm{y}}$.

5. Find the general solution of the differential equation: $\mathrm{e}^{\mathrm{dy}/\mathrm{dx}} = \mathrm{x}^2$.

6. Find the general solution of the differential equation: $\log\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right) = \mathrm{ax} + \mathrm{by}$.

7. Find the general solution of the differential equation: $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{3\mathrm{e}^{2\mathrm{x}} + 3\mathrm{e}^{4\mathrm{x}}}{\mathrm{e}^{\mathrm{x}} + \mathrm{e}^{-\mathrm{x}}}$.

8. Solve that following differential equation: $\frac{\mathrm{dy}}{\mathrm{dx}} + 2\mathrm{y}\tan \mathrm{x} = \sin \mathrm{x}$ given that y = 0 when x = $\frac{\pi}{3}$.

9. Write the integrating factor of the differential equation $(1 + \mathrm{y}^2) + (2\mathrm{xy} - \cot \mathrm{y})\frac{\mathrm{dy}}{\mathrm{dx}} = 0$.

10. If the solution of the differential equation $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{2\mathrm{xy} - \mathrm{y}^2}{2\mathrm{x}^2}$ is $\frac{\mathrm{ax}}{\mathrm{y}} = \mathrm{b}\log|\mathrm{x}| + \mathrm{c}$. Find the values of a and b.

11. Find the general solution of the differential equation $\mathrm{e}^{\mathrm{y}-\mathrm{x}}\frac{\mathrm{dy}}{\mathrm{dx}} = 1$.

12. Find the solution of the differential equation $\frac{\mathrm{dy}}{\mathrm{dx}} + \frac{1}{\mathrm{y}} + \mathrm{y} = 0$.

13. Find the solution of the following differential equation: $\mathrm{x}\sqrt{1 + \mathrm{y}^2}\mathrm{dx} + \mathrm{y}\sqrt{1 + \mathrm{x}^2}\mathrm{dy} = 0$.

14. Find the particular solution of the differential equation $\frac{dy}{dx} = \sin(x + y) + \sin(x - y)$, given that $y = 0$ when $x = \frac{\pi}{4}$.

15. Find the particular solution of the differential equation: $\frac{dy}{dx} = \frac{x+y}{x}, y(1) = 0$.

SHORT ANSWERS

1. Verify that $y = 3\cos(\log x) + 4\sin(\log x)$ is a solution of the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

2. Find the particular solution of the differential equation $x\frac{dy}{dx} - y = x^2 \cdot e^x$, given $y(1) = 0$.

3. Find the general solution of the differential equation: $x\frac{dy}{dx} = y(\log y - \log x + 1)$.

4. Solve: $x\frac{dy}{dx} + y - x + xy\cot x = 0 (x \neq 0)$

5. Find the particular solution of the differential equation $(1 + e^{2x})dy + (1 + y^2)e^x dx = 0$ and given that $y = 1$ and $x = 0$.

6. Solve that differential equation: $ye^{x/y}dx = (xe^{x/y} + y^2)dy (y \neq 0)$.

7. Solve the following differential equation: $2x^2\frac{dy}{dx} - 2xy + y^2 = 0$.

8. Find the particular solution of the differential equation: $x(x^2 - 1)\frac{dy}{dx} = 1; y = 0$; when $x = 2$

9. Solve the differential equation $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$.

10. Find the particular solution of the differential equation $e^x\sqrt{1 - y^2}dx + \frac{y}{x}dy = 0$ given that $y = 1$ when $x = 0$.

11. Solve the following differential equation: $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1, x \neq 0$.

12. Solve the differential equation: $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$.

13. Find the particular solution of differential equation $\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x}$ given that $y = 1$ when $x = 0$.

14. Solve the differential equation: $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$

15. Solve the differential equation: $\frac{dy}{dx} = -\left[\frac{x+y\cos x}{1+\sin x}\right]$

16. Solve the following differential equation: $(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$.

17. Find the general solution of the differential equation: $\frac{d}{dx}(xy^2) = 2y(1+x^2)$.

18. Find the particular solution of the differential equation $(y+3x^2)\frac{dx}{dy} = x$ given that y = 1 when x = 1.

19. Solve the following differential equation: $(y - \sin^2 x)dx + \tan x dy = 0$.

20. Find the general solution of the differential equation: $(x^3 + y^3)dy = x^2 y dx$.

21. Find the particular solution of the differential equation: $\frac{dy}{dx} = \frac{x+y}{x}, y(1) = 0$.

22. Find the general solution of the differential equation: $e^x \tan y dx + (1 - e^x)\sec^2 y dd = 0$.

23. Find the particular solution of the differential equation $\frac{dy}{dx} + \sec^2 x \cdot y = \tan x \cdot \sec^2 x$, given that y(0) = 0.

24. Solve the differential equation given by $xdy - ydx - \sqrt{x^2+y^2}dx = 0$.

25. Find the general solution of the differential equation: $\frac{dx}{dy} = \frac{e^{x/y}\left(\frac{x}{y}-1\right)}{1+e^{x/y}}$

26. Find the particular solution of the differential equation $\frac{dy}{dx} + \cot x \cdot y = \cos^2 x$, given that when $x = \frac{\pi}{2}, y = 0$.

27. Find the general solution of the differential equation: $\frac{dy}{dx} - \frac{2y}{x} = \sin\frac{1}{x}$.

28. Find the particular solution of the differential equation: $\frac{dy}{dx} = \sin(x+y) + \sin(x-y)$, given that when $x = \frac{\pi}{4}, y = 0$.

29. Find the general solution of the differential equation: $(xy - x^2)dy = y^2dx$.

30. Find the general solution of the differential equation: $(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$.

31. Find the general solution of the differential equation: $\frac{d}{dx}(xy^2) = 2y(1+x^2)$

32. Solve the differential equation: $xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$.

33. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$, given that y = 1 when x = 0.

34. Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$, given that y = 0 when x = 1.

35. Find the particular solution of the differential equation given by $x^2\frac{dy}{dx} - xy = x^2\cos^2\left(\frac{y}{2x}\right)$, given that when x = 1, y = $\frac{\pi}{2}$.

36. Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2\frac{dy}{dx} = 0$; y = 2 when x = 1.

37. Find the general solution of the differential equation: $ydx = (x+2y^2)dy$.

38. Find the particular solution of the differential equation $\frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{y}\cot 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.

39. Find the particular solution of the differential equation: $\left(x\mathrm{e}^{\frac{y}{x}} + \mathrm{y}\right)\mathrm{d}x = x\mathrm{dy}$, given that y = 1 when x = 1.

40. Find the particular solution of the differential equation: $\frac{\mathrm{dy}}{\mathrm{d}x} - 2x\mathrm{y} = 3x^2\mathrm{e}^{x^2}; \mathrm{y}(0) = 5$.

41. Solve the following differential equation: $x^2dy + y(x+y)dx = 0$.

LONG ANSWERS

1. Show that the general solution of the differential equation $\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$ is given by (x + y + 1) = A(1 – x – y – 2xy), where A is a parameter.

2. Solve the differential equation $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$.

3. Solve the following differential equation $3e^x\tan y dx+(2-e^x)\sec^2 y dy=0$, given the $y=\frac{\pi}{4}$ when x = 0.

4. Solve: $xdy-ydx=\sqrt{x^2+y^2}dx$

5. Show that the differential equation $\left[x\sin^2\left(\frac{y}{x}\right)-y\right]dx+xdy=0$ is homogeneous. Find the particular solution of this differential equation given that $y=\frac{\pi}{4}$ when x = 1.

6. Find the particular solution of the differential equation $(x-y)\frac{dy}{dx}=(x+2y)$ given that y = 0 when x = 1.

7. Find the general solution of the differential equation: $x(y^3+x^3)dy=(2y^4+5x^3y)dx$.

8. Show that the differential equation $\frac{dy}{dx}=\frac{y^2}{xy-x^2}$ is homogeneous and also solve it.

9. Solve the differential equation $\left(x\sin^2\left(\frac{y}{x}\right)-y\right)dx+xdy=0$ given $y=\frac{\pi}{4}$ when x = 1.

10. Solve the differential equation $\frac{dy}{dx}-3y\cot x=\sin 2x$ given $y=2$ when $x=\frac{\pi}{2}$.

CASE BASED QUESTIONS

1. A Veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the hospital, it was already dead. The pet lover wanted to find its time of death. He took the

temperature of the cat at 11.30 pm which was 94.6°F. He took the temperature again after one hour; the temperature was lower than the first observation. It was 93.4°F. The room in which the cat was put is always at 70°F. The normal temperature of the cat is taken as 98.6°F when it was alive. The doctor estimated the time of death using Newton law of cooling which is governed by the differential equation: $\frac{\mathrm{dT}}{\mathrm{dt}} \propto (\mathrm{T}-70)$, where 70° F is the room temperature and T is the temperature of the object at time t. Substituting the two different observations of T and t made, in the solution of the differential equation $\frac{\mathrm{dT}}{\mathrm{dt}} = k(\mathrm{T}-70)$, where k is a constant of proportion, time of death is calculated.

Based on the above information, answer the following questions:

(i) Find the solution of the differential equation $\frac{\mathrm{dT}}{\mathrm{dt}} = k(T-70)$.

(ii) If t = 0 when T is 72, then find the value of c.

2. An equation involving derivatives of the dependent variable with respect of the independent variables is called a differential equation. A differential equation is of the form $\frac{\mathrm{dy}}{\mathrm{d}x} = \mathrm{F}(x,\mathrm{y})$ is said to be homogeneous if F(x, y) is a homogeneous function of degree zero, whereas a function F(x, y) is a homogeneous function of degree n if $\mathrm{F}(\lambda x, \lambda \mathrm{y}) = \lambda^{\mathrm{n}}\mathrm{F}(x,\mathrm{y})$. To solve a homogeneous differential equation of the type $\frac{\mathrm{dy}}{\mathrm{d}x} = \mathrm{F}(x,\mathrm{y}) = \mathrm{g}\left(\frac{\mathrm{y}}{x}\right)$,we make the substitution y = vx and then separate the variables.

Based on the above information, answer the following questions:

(i) Show that $(x^2-y^2)\mathrm{d}x + 2xy\mathrm{d}y = 0$ is a differential equation of the type $\frac{\mathrm{dy}}{\mathrm{d}x} = \mathrm{g}\left(\frac{\mathrm{y}}{x}\right)$.

(ii) Solve the above differential equation to find its general solution.

3. A spherical drop of liquid evaporates at a rate proportional to its surface area, if the radius initially is 5mm and 5 minute later the radius is reduced to 2mm.

On the basis of above information, answer the following questions-

(i) Write the differential equation for the above situation.

(ii) Find the rate with which the surface area changes after 5 minutes.

(iii) Find the rate of evaporation after 5 minutes.

4. Differential equation $\frac{dy}{dx} = f(x)g(y)$ can be solved by separating variable $\frac{dy}{g(y)} = f(x)dx$.

On the basis of above information, answer the following questions:

(i) Find the equation of the curve to the point (1, 0) which satisfies the differential equation $(1 + y^2)dx - xydy = 0$.

(ii) Find the solution of the differential equation $\frac{dy}{dx} + \frac{1+y^2}{\sqrt{1-x^2}} = 0$.

(iii) If $\frac{dy}{dx} = 1 + x + y + xy$ and $y(-1) = 0$, then find the equation of the curve.

5. Students of class XII are preparing for their board examination. On revising the topic differential equations, they found the differential equation $(1 + y^2)\mathrm{dx} = (\tan^{-1} y - x)\mathrm{dy}$ and tried to solve.

(i) What is the degree of the differential equation?

(ii) Which is method can be used to solve the above given differential equation?

(iii) Find the solution of the differential equation.

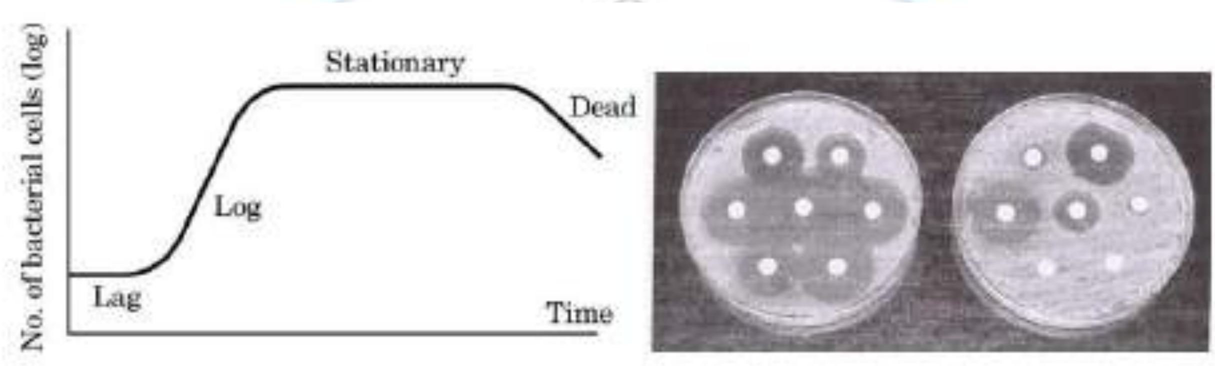
6. An equation involving derivatives of the dependent variable with respect to the independent variables is called differential equation. A differential equation of the form $\frac{\mathrm{dy}}{\mathrm{d}x} = \mathrm{F}(x, y)$ is said to be homogeneous if F(x, y) is a homogeneous function of degree zero, whereas a function F(x, y) is homogeneous function of degree n if $\mathrm{F}(\lambda x, \lambda \mathrm{y}) = \lambda^{\mathrm{n}}\mathrm{F}(x, y)$. To solve a homogeneous differential equation of the type $\frac{\mathrm{dy}}{\mathrm{d}x} = \mathrm{F}(x, y) = \mathrm{g}\left(\frac{\mathrm{y}}{x}\right)$, we make the substitution y = vx and then separate the variables.

Based on the above, answer the following questions:

(i) Show that $(x^2 - y^2)\mathrm{d}x + 2xy\mathrm{d}y = 0$ is a differential equation of the type $\frac{\mathrm{dy}}{\mathrm{d}x} = \mathrm{g}\left(\frac{\mathrm{y}}{x}\right)$.

(ii) Solve the above equation to find its general solution.

7. A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.



The differential equation representing the growth of bacteria is given as: $\frac{\mathrm{dP}}{\mathrm{dt}} = \mathrm{kP}$, where P is the population of bacteria at time ‘t’.

Based on the above information, answer the following questions:

(i) Obtain the general solution of the given differential equation and express it as an exponential function of ‘t’.

(ii) If population of bacteria is 1000 at t = 0, and 2000 at t = 1, find the value of k.

UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. Which of the following is not a homogeneous function of x and y?

(a) $x^2 + 2xy$ (b) $2x - y$ (c) $\cos^2\left(\frac{y}{x}\right) + \frac{y}{x}$ (d) $\sin x - \cos y$

2. The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is

(a) $\frac{1}{x} + \frac{1}{y} = c$ (b) $\log x.\log y = c$ (c) $xy = c$ (d) $x + y = c$

3. The sum of the order and the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + 3x\left(\frac{d^2y}{dx^2}\right)^4 = \log x$ is

(a) 5 (b) 6 (c) 7 (d) 4

4. Assertion: The degree of the differential equation $\frac{d^3y}{dx^3} + 3\left(\frac{dy}{dx}\right) = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is not defined.

Reason: If the differential equation is a polynomial in terms of its derivatives, then its degree is defined.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. Solve the differential equation $(y + 3x^2)\frac{dx}{dy} = x$

6. Find the general solution of the differential equation: $(xy - x^2)\mathrm{dy} = \mathrm{y}^2\mathrm{d}x$.

7. Find the general solution of the following differential equation: $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

SECTION C

Each carry 3 marks

8. Find the particular solution of the differential equation $\frac{dy}{dx} + y\tan x = 3x^2 + x^3 \tan x, x \neq \frac{\pi}{2}$, given that y = 0 when x = $\frac{\pi}{3}$.

9. Show that the differential equation $(x - y)\frac{dy}{dx} = x + 2y$ is homogeneous and solve it.

SECTION D

Each carry 5 marks

10. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$ given that $y = \frac{\pi}{2}$ when x = 1

11. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ given that y = 1 when x = 0.

SECTION E

12. Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administrated the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution of the differential equation $\frac{dy}{dx} = k(50 - y)$, where x denotes the number of weeks and y the number of children who have been given the drops.

Based on the above information answer the following questions:

(i) Which method of solving a differential equation can be used to solve $\frac{dy}{dx} = \mathrm{k}(50 - \mathrm{y})$?

(ii) Find the solution of the differential equation $\frac{dy}{dx} = \mathrm{k}(50 - \mathrm{y})$.

(iii) Determine the value of C in the particular solution given that y(0) and k = 0,049.

ANSWERS

MCQ	A-R	SA I	SA II	LA	CS
1. (a) 1	1. (c)	1. siny = e^x logx + c	2. y = x(e^x – e)	2. $y\log x = -\frac{2}{x}(1+\log x)+C$	(i) $\log\lvert T-70\rvert = kt +$ c
2. (d) Not defined	2. (d)	2. tanx tany = c	3. $\log(\frac{y}{x})$ = cx	3. $\tan y = (2-e^x)^3$	(ii) c = log2
3. (c) 2, 1	3. (c)	3. (1 – x^2)(1 + y^2) = c	4. $y = \frac{1}{x} - \cot x + \frac{c}{x\sin x}$	4. $\left\{y+\sqrt{x^2+y^2}\right\}^2 = C^2x^2$	2. (ii) $x^2+y^2 = Cx$
4. (b) logx	4. (a)	4. $e^y = e^x + \frac{x^3}{3} + c$	5. $\tan^{-1}(e^x) + \tan^{-1} y = \frac{\pi}{2}$	5. $\log x - \cot\left(\frac{y}{x}\right) + 1 = 0$	3. (i) $\frac{dv}{dt} == -k(4\pi r^2)$
5. (c) y = 2x – 4	5. (a)	5. y = 2x(logx – 1) + c	6. $xe^{x/y} = y + c$	7. $y^4 + 4yx^3 = Cx^8$	(ii) 4.8 π sq. mm/min
6. (d) $y = \frac{x^4+C}{4x^2}$	6. (a)	6. $\frac{e^{ax}}{a} - \frac{e^{-by}}{b} + c = 0$	7. $\log\lvert x\rvert + C = \frac{2x}{y}$	8. $y = x(\log y + C)$	(iii) 9.6 π
7. (d) Not defined	7. (d)	7. y = e^{3x} + c	8. $y = \frac{1}{2}\log\left\lvert\frac{x^2-1}{x^2}\right\rvert - \frac{1}{2}\log\frac{3}{4}$	9. $-\cot\left(\frac{y}{x}\right) + \log x + 1 = 0$	4. (i) $x^2 - y^2 = 1$
8. (b) 2	8. (a)	8. $y = \cos x - 2\cos^2 x$	9. $y = \frac{e^{\tan^{-1}x}}{2} + C\cdot e^{-\tan^{-1}x}$	10. $y = 4\sin^3 x - 2\sin^2 x$	(ii) $\tan^{-1} y + \sin^{-1} x =$ c
9. (a) 2 and not defined	9. (b)	9. $1 + y^2$	10. $xe^x - e^x = \sqrt{1-y^2} - 1$		
	10. (b)	10. a = 2 and b = 1			

10. (b) $\frac{d^2y}{dx^2} - \alpha^2 y = 0$ 11. (c) $\frac{1}{x}$ 12. (d) $y = 2e^x - 1$ 13. (c) secx 14. (b) sinx.siny = C 15. (c) $\sqrt{1-x^2}$ 16. (c) 3 17. (b) -7 18. (a) m = 3, n = 2	11. (d) 12. (a) 13. (c) 14. (a) 15. (c)	11. $e^y = e^x + C$ 12. $x + \frac{1}{2}\log\|1 + y^2\| = C$ 13. $\sqrt{1+x^2} + \sqrt{1+y^2} = C$ 14. $\log\|\sec y + \tan y\| = -2\cos x + \sqrt{2}$ 15. $y = x\log\|x\|$	11. $y \cdot e^{2\sqrt{x}} = 2\sqrt{x} + C$ 12. $\sqrt{1+y^2} + \sqrt{1+x^2} + \frac{1}{2}\log\left\|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right\| + C = 0$ 13. $y = \frac{2-x^2}{2(1+\sin x)}$ 14. $x\sin\left(\frac{y}{x}\right) = C$ 15. $y(1+\sin x) = \frac{-x^2}{2} + C$ 16. $y(x^2+1) = \frac{x}{2}\sqrt{x^2+4} + 2\log\left\|x + \sqrt{x^2+4}\right\| + C$ 17. $y\sqrt{x} = 2\sqrt{x} + \frac{2x^{\frac{5}{2}}}{5} + C$ 18. $y = 3x^2 - 2x$		(iii) $e^{\frac{(1-x)^2}{2}} - 1 = 0$ 5. (i) 1 (ii) Method for linear differential equation. (iii) $x = \tan^{-1} y - 1 + C \cdot e^{-\tan^{-1} y}$ 6. (ii) $x^2 + y^2 = Cx$ 7. (i) $P = e^{k+C}$ (ii) $k = \log 2$

19. (b) 2			19. $y\sin x = \frac{\sin^3 x}{3} + C$		
20. (c) secy			20. $\frac{-x^3}{3y^3} + \log\lvert y\rvert = C$		
21. (c) 3			21. $y = x\log\lvert x\rvert$		
22. (a) $e^{-\cot x}$			22. $\tan y = C(1 - e^x)$		
23. (c) y			23. $\mathrm{y}\mathrm{e}^{\tan x} = \mathrm{e}^{\tan x}(\tan x - 1) + 1$ or $\mathrm{y} = \tan x - 1 + \mathrm{e}^{-\tan x}$		
24. (b) 2			24. $y + \sqrt{x^2 + y^2} = Cx^2$		
25. (d) $\frac{1}{\sqrt{1-y^2}}$			25. $e^{x/y} + \frac{x}{y} = \frac{C}{y}$ or $ye^{x/y} + x = C$		
26. (d) $y = 2\log x + \frac{x^2}{2} + c$			26. $y\sin x = -\frac{\cos^3 x}{3}$ or $y =$		
27. (b) Order = 2, Degree = 3					
28. (a) 3, 1					

29. (d) g(x) + log[1 + y – g(x)] = C 30. (b) xsecx 31. (c) 2, 1 32. (a) 4 33. (d) not defined 34. (b) not defined, 2 35. (d) 0 36. (b) 3, 1 37. (c) 4 38. (b) 1			$-\frac{\cos^3 x}{3}\cdot \text{cosec}x$ 27. y. $\frac{1}{x^2} = \cos\frac{1}{x} + C$ 28. $\log\|\sec y + \tan y\| = -2\cos x + \sqrt{2}$ 29. $\frac{y}{x} - \log\frac{y}{x} = \log\|x\| + c$ 30. $y(1+x^2) = \frac{x\sqrt{x^2+4}}{2} + 2\log\left\|x+\sqrt{x^2+4}\right\| + c$ 31. $y\sqrt{x} = 2\sqrt{x} + \frac{2x^{\frac{5}{2}}}{5} +$ C or $y = 2 + \frac{2x^2}{5} + \frac{c}{\sqrt{x}}$ 32. $e^{-\frac{y}{x}} = \log\|x\| + C$		

39. (b) 1 40. (a) xy = c 41. (a) $\cos x - \sin\left(\frac{y}{x}\right)$ 42. (c) first order linear differential equation 43. (b) $\frac{1}{\sqrt{y}}$ 44. (b) x^2 45. (a) $\mathrm{e}^{x} + \mathrm{e}^{-\mathrm{y}} = \mathrm{c}$ 46. (d) $\frac{1}{x}$			33. $\frac{x^2}{2y^2} = \log\|y\|$ 34. $y.(1+x^2) = \tan^{-1} x - \frac{\pi}{4}$ 35. $2\tan\left(\frac{y}{2x}\right) = \log\|x\| + 2$ 36. $y = \frac{2x}{1-\log\|x\|}$ 37. $x = 2y^2 + Cy$ 38. $y = 2\sqrt{\sin 2x}$ 39. $\log\|x\| + e^{\frac{-y}{x}} = e^{-1}$ 40. $y = (x^3+5)e^{x^2}$ 41. $\left\|\frac{y}{y+2x}\right\| = \frac{C^2}{x^2}$ or $x^2 y = k(y+2x)$		

47. (d) $\frac{1}{y}$ 48. (a) $\tan^{-1} y + \log \lvert x \rvert + \frac{(\log \lvert x \rvert)^2}{2} = C$ 49. (d) $\frac{1}{\sqrt{1-x^2}}$ 50. (c) 3 51. (d) $y = 2\log x + \frac{x^2}{2} + C$ 52. (c) Not defined 53. (d) $\frac{1}{\sqrt{1-\mathrm{y}^2}}$ 54. (a) $\frac{1}{x}$					

55. (c) xy = C 56. (b) 2 57. (d) 2, degree not defined 58. (b) y logy – y = x + C 59. (c) x^{-1}					

UNIT TEST

1. (d) sinx – cosy

2. (c) xy = c

3. (a) 5

4. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

5. $y - 3x^2 + Cx = 0$.

6. $\frac{y}{x} - \log\frac{y}{x} = \log|x| + c$

7. $e^y = \frac{x^3}{3} + e^x + C$

8. $y = x^3 - \frac{2\pi^3}{27}\cos x$

9. $-\frac{1}{2}\log|x^2 + xy + y^2| + \sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = C$

10. $x^2\log(x) - y\sin y + \frac{\pi}{2} = 0$

11. $y = e^{\frac{x^2}{2y^2}}$

12. (i) Variable separable method (ii) $-\log|50 - y| = kx + C$ (iii) $C = \log\frac{1}{50}$

CHAPTER 10 – VECTOR ALGEBRA

VECTOR

A quantity that has magnitude as well as direction is called a vector.

Examples: Acceleration, Force, Displacement etc.

Directed line segment is a vector, denoted as $\overrightarrow{AB}$ or simply as $\vec{a}$, and read as "vector AB ' or "vector $\vec{a}$ ".

The point A from where the vector $\overrightarrow{AB}$ starts is called initial point and the point B where it ends is called its terminal point.

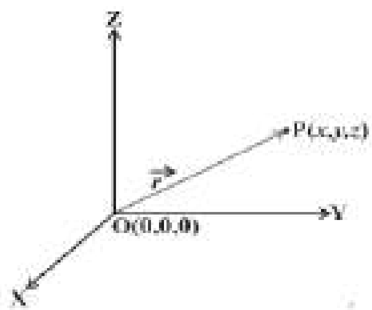
The distance between initial and terminal points of a vector is called magnitude or length of the vector, denoted as $|\overrightarrow{AB}|$, or $|\vec{a}|$, or a.

Note: Since the length is never negative, the notation $|\vec{a}| < 0$ has no meaning.

POSITION VECTOR

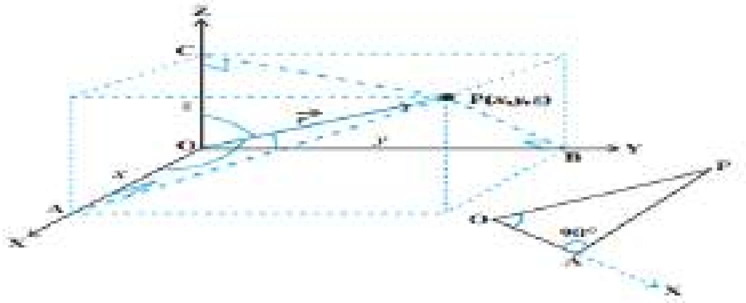
Consider a point P in space, having coordinates (x, y, z) with respect to the origin O(0, 0, 0). Then, the vector $\overrightarrow{OP}$ having O and P as its initial and terminal points, respectively, is called the position vector of the point P with respect to O.

The magnitude of $\overrightarrow{OP}$ or $\vec{r}$ is given by $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$



DIRECTION COSINES

The direction cosines of the vector $\overrightarrow{OP}$ or $\vec{r}$ are the cosines of angles that the vector forms with the coordinate axes.



The angles α, β and γ made by the vector $\vec{r}$ with the positive directions of x, y and z-axes respectively are called its direction angles. The cosine values pf these angles i.e $\cos\alpha, \cos\beta$ and $\cos\gamma$ are called direction cosines of the vector $\vec{r}$. It is denoted by l, m and n.

$$l = \cos\alpha,\ m = \cos\beta \text{ and } n = \cos\gamma$$

Note: $l^2 + m^2 + n^2 = 1$ or $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

DIRECTION RATIOS

$$l = \cos\alpha = \frac{x}{r},\ m = \cos\beta = \frac{y}{r} \text{ and } n = \cos\gamma = \frac{z}{r}$$

Hence the coordinates of the point P(x, y, z) also represented as (lr, mr, nr) The numbers lr, mr and nr, proportional to the direction cosines are called as direction ratios of vector $\vec{r}$, and denoted as a, b and c, respectively.

TYPES OF VECTORS

Zero Vector

A vector whose initial and terminal points coincide, is called a zero vector (or null vector), and denoted as $\vec{0}$. Zero vector cannot be assigned a definite direction as it has zero magnitude.

Unit Vector

A vector whose magnitude is unity (i.e., 1 unit) is called a unit vector. The unit vector in the direction of a given vector $\vec{a}$ is denoted by $\hat{a}$.

Coinitial Vectors

Two or more vectors having the same initial point are called coinitial vectors.

Collinear Vectors

Two or more vectors are said to be collinear if they are parallel to the same line, irrespective of their magnitudes and directions.

Equal Vectors

Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal, if they have the same magnitude and direction regardless of the positions of their initial points, and written as $\vec{a} = \vec{b}$.

Negative of a Vector

A vector whose magnitude is the same as that of a given vector, but direction is opposite to that of it, is called negative of the given vector.

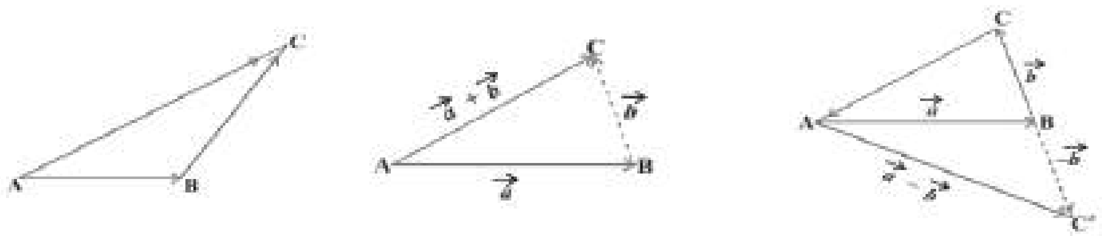
For example, vector BA is negative of the vector $\overrightarrow{\mathrm{AB}}$, and written as $\overrightarrow{\mathrm{BA}} = -\overrightarrow{\mathrm{AB}}$.

Remark

The vectors defined above are such that any of them may be subject to its parallel displacement without changing its magnitude and direction. Such vectors are called free vectors. Throughout this chapter, we will be dealing with free vectors only.

TRIANGLE LAW OF VECTOR ADDITION

Triangle law of vector addition states that when two vectors are represented as two sides of the triangle with the order of magnitude and direction, then the third side of the triangle represents the magnitude and direction of the resultant vector.

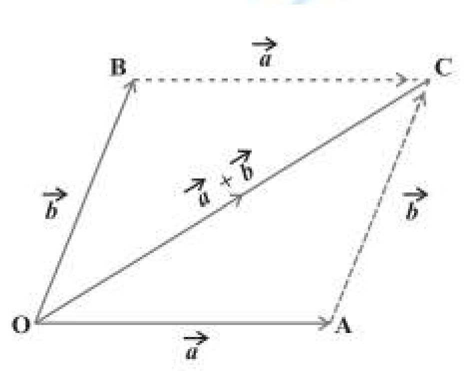


$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AA} = \vec{0} \quad [\text{Since } \overrightarrow{AC} = -\overrightarrow{CA}]$$

PARALLELOGRAM LAW OF VECTOR ADDITION

If we have two vectors $\vec{a}$ and $\vec{b}$represented by the two adjacent sides of a parallelogram in magnitude and direction, then their sum $\vec{a} + \vec{b}$ is represented in magnitude and direction by the diagonal of the parallelogram through their common point. This is known as the parallelogram law of vector addition.



$$\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$$

$$\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC} (\text{ since } \overrightarrow{AC} = \overrightarrow{OB})$$

PROPERTIES OF VECTOR ADDITION

(i) For any two vectors $\vec{a}$ and $\vec{b}$,

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \text{ (Commutative property)}$$

(ii) For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \text{ (Associative property)}$$

(iiii) For any vector $\vec{a}$, we have

$$\vec{a} + \vec{0} = \vec{0} + \vec{a} = \vec{a}$$

Here, the zero vector $\vec{0}$ is called the additive identity for the vector addition.

MULTIPLICATION OF A VECTOR BY A SCALAR

Let $\vec{a}$ be a given vector and λ a scalar, $|\lambda\vec{a}| = |\lambda||\vec{a}|$.

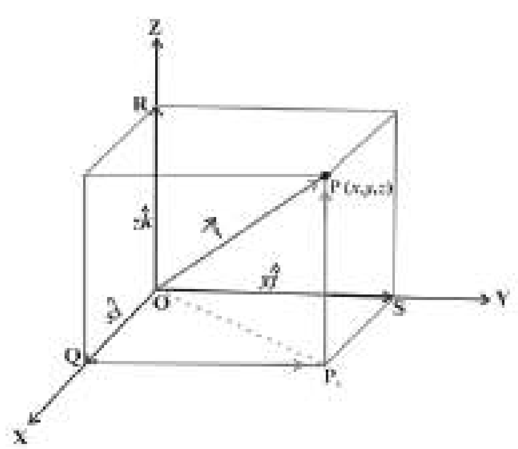
UNIT VECTOR IN THE DIRECTION OF A GIVEN VECTOR

If $\vec{a}$ is any non-zero vector, then whose unit vector in the direction of $\vec{a}$ is given by

$$\hat{a} = \frac{1}{|\vec{a}|}\vec{a}$$

$\hat{a} = \frac{k}{|\vec{a}|}\vec{a}$ has same direction as $\vec{a}$ and has magnitude k.

COMPONENTS OF A VECTOR



$\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are the unit vectors along the x, y and z -axes.

Hence, the position vector of P with reference to O is given by

$$\overrightarrow{\mathbf{OP}}(\text{ or } \vec{\boldsymbol{r}}) = \boldsymbol{x\hat{\imath} + y\hat{\jmath} + z\hat{k}}$$

This form of any vector is called its component form.

Here, x, y and z are called as the scalar components of $\vec{r}$, and $x\hat{\imath}, y\hat{\jmath}$ and $z\hat{k}$ are called the vector components of $\vec{r}$ along the respective axes.

The length (magnitude) of the any vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by

$$|\vec{\boldsymbol{r}}| = |\boldsymbol{x\hat{\imath} + y\hat{\jmath} + z\hat{k}}| = \sqrt{\boldsymbol{x^2 + y^2 + z^2}}$$

RESULTS

If $\vec{a}$ and $\vec{b}$ are any two vectors given in the component form $a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ & $b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$, respectively then

(i) The sum (or resultant) of the vectors $\vec{a}$ and $\vec{b}$ is given by

$$\vec{a} + \vec{b} = (a_1 + b_1)\hat{\imath} + (a_2 + b_2)\hat{\jmath} + (a_3 + b_3)\hat{k}$$

(ii) The difference of the vectors $\vec{a}$ and $\vec{b}$ is given by

$$\vec{a} - \vec{b} = (a_1 - b_1)\hat{\imath} + (a_2 - b_2)\hat{\jmath} + (a_3 - b_3)\hat{k}$$

(iii) The vectors $\vec{a}$ and $\vec{b}$ are equal if and only if

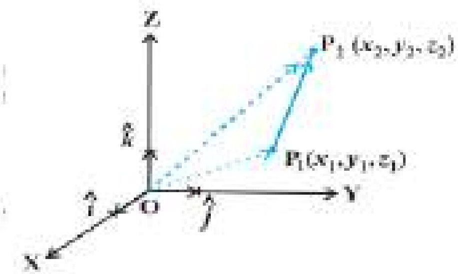
$$a_1 = b_1, a_2 = b_2 \text{ and } a_3 = b_3$$

(iv) The multiplication of vector $\vec{a}$ by any scalar λ is given by

$$\lambda\vec{a} = (\lambda a_1)\hat{\imath} + (\lambda a_2)\hat{\jmath} + (\lambda a_3)\hat{k}$$

VECTOR JOINING TWO POINTS

If P_1 (x_1, y_1, z_1) and P_2 (x_2, y_2, z_2) are any two points, then the vector joining P_1 and P_2 is the vector $\overrightarrow{P_1P_2}$

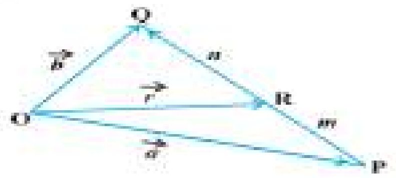


$\overrightarrow{OP_1} + \overrightarrow{P_1P_2} = \overrightarrow{OP_2}$

$\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1}$

SECTION FORMULA

Let P and Q be two points represented by the position vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ respectively, with respect to the origin O. Then the line segment joining the points P and Q may be divided by a third point, say R, in two ways – internally and externally. Here, we intend to find the position vector $\overrightarrow{OR}$ for the point R with respect to the origin O.



Case (i): R divides PQ internally

$$\overrightarrow{\text{OR}} = \frac{m\vec{b}+n\vec{a}}{m+n}$$

Case (ii): R divides PQ externally

$$\overrightarrow{\text{OR}} = \frac{m\vec{b}-n\vec{a}}{m-n}$$

Remark: If R is the midpoint of PQ, then m = n and therefore, from Case I, the midpoint R of $\overrightarrow{\text{PQ}}$, will have its position vector as

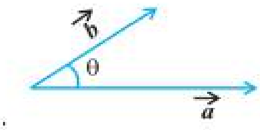
$$\overrightarrow{\text{OR}} = \frac{\vec{a}+\vec{b}}{2}$$

PRODUCT OF TWO VECTORS

SCALAR OR DOT PRODUCT OF TWO VECTORS

The scalar product of two nonzero vectors $\vec{a}$ and $\vec{b}$, denoted by $\vec{a}\cdot\vec{b}$ is defined as

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$



where, θ is the angle between $\vec{a}$ and $b, 0 \le \theta \le \pi$.

Note: If either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then θ is not defined, and in this case, we define $\vec{a}\cdot\vec{b} = 0$.

RESULTS

(i) $\vec{a}\cdot\vec{b}$ is a real number.

(ii) Let $\vec{a}$ and b be two nonzero vectors, then $\vec{a}\cdot b = 0$ if and only if $\vec{a}$ and $\vec{b}$ are perpendicular to each other. $\vec{a}\cdot\vec{b} = 0 \Leftrightarrow \vec{a}\perp\vec{b}$

(iii) If $\theta = 0$, then $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$

Note: $\vec{a}\cdot\vec{a} = |\vec{a}|^2$

(iv) If $\theta = \pi$, then $\vec{a}\cdot\vec{b} = -|\vec{a}||\vec{b}|$

(v) $\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1$ and $\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{k} = \hat{k}\cdot\hat{\imath} = 0$

(vi) The angle between two nonzero vectors $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}\parallel\vec{b}|}, \text{ or } \theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}\parallel\vec{b}|}\right)$$

(vii) The scalar product is commutative. i.e. $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

TWO IMPORTANT PROPERTIES OF SCALAR PRODUCT

(i) Distributivity of scalar product over addition

Let $\vec{a}, \vec{b}$ and $\vec{c}$ be any three vectors, then $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$

(ii) Let $\vec{a}$ and $\vec{b}$ be any two vectors, and λ be any scalar. Then

$$(\lambda\vec{a})\cdot\vec{b} = (\lambda\vec{a})\cdot\vec{b} = \lambda(\vec{a}\cdot\vec{b}) = \vec{a}\cdot(\lambda\vec{b})$$

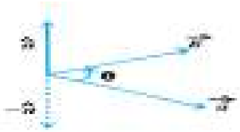
PROJECTION OF A VECTOR ON A LINE

Projection of a vector $\vec{a}$ on other vector $\vec{b}$, is given by $\frac{1}{|\vec{b}|}(\vec{a} \cdot \overrightarrow{b})$.

VECTOR OR CROSS PRODUCT OF TWO VECTORS

The vector product of two nonzero vectors $\vec{a}$ and $\vec{b}$, is denoted by $\vec{a} \times \vec{b}$ and defined as

$$\vec{a} \times \vec{b} = |\vec{a} \parallel \vec{b}| \sin \theta \hat{n}$$



where, θ is the angle between $\vec{a}$ and $\vec{b}, 0 \leq \theta \leq \pi$ and $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, such that $\vec{a}, \vec{b}$ and $\hat{n}$ form a right handed system.

RESULTS

(i) $\vec{a} \times \vec{b}$ is a vector.

(ii) Let $\vec{a}$ and $\vec{b}$ be two nonzero vectors. Then $\vec{a} \times \vec{b} = \vec{0}$ if and only if $\vec{a}$ and $\vec{b}$ are parallel or collinear to each other.

$$\vec{a} \times \vec{b} = \vec{0} \Leftrightarrow \vec{a} \parallel \vec{b}$$

Note: $\vec{a} \times \vec{a} = \vec{0}$ and $\vec{a} \times (-\vec{a}) = \vec{0}$

(iii) If $\theta = \frac{\pi}{2}$ then $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|$.

(iv) For mutually perpendicular unit vectors $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ we have,

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \vec{0}$$

$$\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$$

(v) It is always true that the vector product is not commutative, as $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$.

(vi) $\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}$ and $\hat{\imath} \times \hat{k} = -\hat{\jmath}$

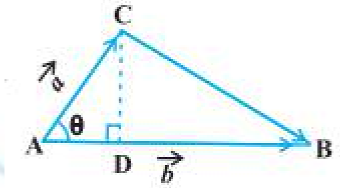
ANGLE BETWEEN TWO VECTORS

The angle between two vectors $\vec{a}$ and $\vec{b}$ is given as $\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

AREA OF TRIANGLE

If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle then its area is given as

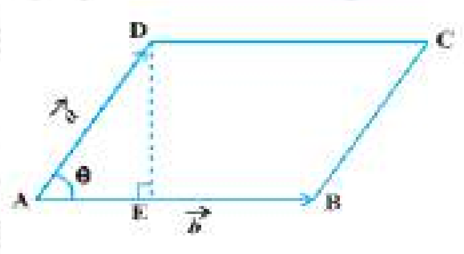
$$\Delta = \frac{1}{2}|\vec{a}\times\vec{b}|$$



AREA OF PARALLELOGRAM

If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a parallelogram, then its area is given as

Area of Parallelogram = $|\vec{a}\times\vec{b}|$.



CROSS OR VECTOR PRODUCT OF TWO VECTORS

Let $\vec{a}$ and $\vec{b}$ be two vectors given in component form as $a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$ and $b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$ respectively. Then their cross product may be given by

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

RESULTS

For any two vectors $\vec{a}$ and $\vec{b}$, we have

(i) $|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2\left[|\vec{a}|^2 + |\vec{b}|^2\right]$

(ii) $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

(iii) $(\vec{a}\cdot\vec{b})^2 + (\vec{a}\times\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$

MULTIPLE CHOICE QUESTIONS

1. The area of a triangle formed by the vertices O, A, B where $\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is

(a) $3\sqrt{5}$ sq. units (b) $5\sqrt{5}$ sq. units (c) $6\sqrt{5}$ sq. units (d) 4 sq. units

2. The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is

(a) 0 (b) – 1 (c) 1 (d) 3

3. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$ then $|\vec{a} - \vec{b}| = |\vec{a} + \vec{b}|$ where θ is equal to

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

4. The value of λ such that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal is

(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{5}{2}$

5. The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{2}{5}$

6. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is

(a) 1 (b) 3 (c) $-\frac{3}{2}$ (d) None of these

7. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 2, |\vec{b}| = 3, |\vec{c}| = 5$, then value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is

(a) 0 (b) 1 (c) -19 (d) 38

8. If $|\vec{a}| = 10, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then value of $|\vec{a} \times \vec{b}|$ is

(a) 5 (b) 10 (c) 14 (d) 16

9. For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to

(a) $\vec{a}^2$ (b) $3\vec{a}^2$ (c) $4\vec{a}^2$ (d) $2\vec{a}^2$

10. If $|\vec{a}| = 3, |\vec{b}| = 4$, then a value of λ for which $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{a} - \lambda\vec{b}$ is

(a) $\frac{9}{16}$ (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$

11. If $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}, |\vec{b}| = 5$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then area of the triangle formed by these two vectors as two sides is

(a) $\frac{15}{4}$ (b) $\frac{15}{2}$ (c) 15 (d) $\frac{15\sqrt{3}}{2}$

12. If $\vec{a}$ and $\vec{b}$ are the two vectors such that $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = 0$, then

(a) $\vec{a}$ is parallel to $\vec{b}$ (b) $\vec{a}$ is perpendicular to $\vec{b}$

(c) Either $\vec{a}$ or $\vec{b}$ is a null vector (d) None of these

13. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then $\vec{b}$ is

(a) $\hat{i} - \hat{j} + \hat{k}$ (b) $2\hat{j} - \hat{k}$ (c) $2\hat{i}$ (d) $\hat{i}$

14. Let the vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

15. If $\vec{p}$ is a unit vector and $(\vec{x} - \vec{p}) \cdot (\vec{x} + \vec{p}) = 80$, then $|\vec{x}|$ is

(a) 9 (b) ±9 (c) -9 (d) $4\sqrt{5}$

16. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them then $\frac{1}{2}(\vec{a} - \vec{b})^2$ is

(a) 1 + cosθ (b) 1 – sinθ (c) 1 + sinθ (d) 1 – cosθ

17. The position vector of a point which divides the join of points with position vectors $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$ in ratio 1:2 internally is

(a) $\frac{(3\vec{a}+2\vec{b})}{3}$ (b) $\vec{a}$ (c) $\frac{(5\vec{a}-\vec{b})}{3}$ (d) $\frac{(4\vec{a}+\vec{b})}{3}$

18. If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} \neq 0$, then

(a) $\vec{b} = \vec{c}$ (b) $\vec{b} = \vec{0}$ (c) $\vec{b} + \vec{c} = \vec{0}$ (d) $\vec{c} = \vec{0}$

19. If the position vectors of two points A and B are $\hat{i} + 2\hat{j} - 3\hat{k}$ and $-\hat{i} - 2\hat{j} + \hat{k}$ respectively then the direction cosines of the vector $\overrightarrow{BA}$ are

(a) $\frac{2}{6}, -\frac{4}{6}, -\frac{4}{6}$ (b) $\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$ (c) $\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}$ (d) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

20. The value of $(\hat{\imath} \times \hat{\jmath}) \cdot \hat{\jmath} + (\hat{\jmath} \times \hat{\imath}) \cdot\ k$ is

(a) 2 (b) 0 (c) 1 (d) -1

21. If in $\triangle ABC, \overrightarrow{BA} = 2\vec{a}$ and $\overrightarrow{BC} = 3\vec{b}$, then $\overrightarrow{AC}$ is

(a) $2\vec{a} + 3\vec{b}$ (b) $2\vec{a} - 3\vec{b}$ (c) $3\vec{b}$ - $2\vec{a}$ (d) -$2\vec{a} - 3\vec{b}$

22. Position vector of the mid-point of line segment AB is $3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$. If position vector of the point A is $2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ then position vector of point B is

(a) $\frac{5\hat{\imath}}{2} + \frac{5\hat{\jmath}}{2} - \frac{7\hat{k}}{2}$ (b) $4\hat{\imath} + \hat{\jmath} - 2\hat{k}$ (c) $5\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$ (d) $\frac{\hat{\imath}}{2} - \frac{\hat{\jmath}}{2} + \frac{\hat{k}}{2}$

23. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ and $|\vec{a} \times \vec{b}| = 3\sqrt{3}$, then the value of $\vec{a} \cdot \vec{b}$ is

(a) 9 (b) 3 (c) $\frac{1}{9}$ (d) $\frac{1}{3}$

24. The position vector of three consecutive vertices of a parallelogram ABCD are $A(4\hat{\imath} + 2\hat{\jmath} - 6\hat{k})$, $B(5\hat{\imath} - 3\hat{\jmath} + \hat{k})$ and $C(12\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$. The position vector of D is given by

(a) $-3\hat{\imath} - 5\hat{\jmath} - 10\hat{k}$ (b) $21\hat{\imath} + 3\hat{\jmath}$ (c) $11\hat{\imath} + 9\hat{\jmath} - 2\hat{k}$ (d) $-11\hat{\imath} - 9\hat{\jmath} + 2\hat{k}$

25. In $\triangle ABC, \overrightarrow{AB} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$. If D is the mid-point of BC, then vector $\overrightarrow{AD}$ is

(a) $4\hat{\imath} + 6\hat{k}$ (b) $2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ (c) $\hat{\imath} - \hat{\jmath} + \hat{k}$ (d) $2\hat{\imath} + 3\hat{k}$

26. The point (x, y, 0) on the xy-plane divides the line segment joining the points (1, 2, 3) and (3, 2, 1) in the ratio is

(a) 1:2 internally (b) 2:1 internally (c) 3:1 internally (d) 3:1 externally

27. A unit vector $\hat{a}$ makes equal but acute angle on the co-ordinate axes. The projection of the vector $\hat{a}$ on the vector $\vec{b} = 5\hat{\imath} + 7\hat{\jmath} - \hat{k}$ is

(a) $\frac{11}{15}$ (b) $\frac{11}{5\sqrt{3}}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5\sqrt{3}}$

28. If $\vec{a} \cdot \hat{\imath} = \vec{a} \cdot (\hat{\imath} + \hat{\jmath}) = \vec{a} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 1$, then $\vec{a}$ is

(a) $\hat{k}$ (b) $\hat{\imath}$ (c) $\hat{\jmath}$ (d) $\hat{\imath} + \hat{\jmath} + \hat{k}$

29. If $|\hat{a}| = 3, |\hat{b}| = 4$ and $|\hat{a} + \hat{b}| = 5$, then $|\hat{a} - \hat{b}|$ is equal to

(a) 6 (b) 5 (c) 4 (d) 3

30. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other, then angle between $\vec{a}$ and $\vec{b}$ is

(a) 45^{o} (b) 60^{o} (c) $\cos^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{2}{7}\right)$

31. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides the line segment PQ in the ratio 3:1 and S is the mid-point of line segment PR. The position vector S is

(a) $\frac{\vec{p}+3\vec{q}}{4}$ (b) $\frac{\vec{p}+3\vec{q}}{8}$ (c) $\frac{5\vec{p}+3\vec{q}}{4}$ (d) $\frac{5\vec{p}+3\vec{q}}{8}$

32. If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are

(a) collinear vectors which are not parallel (b) parallel vectors

(c) perpendicular vectors (d) unit vectors

33. If $\vec{a}, \vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

34. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{5\pi}{6}$ (d) $\frac{11\pi}{6}$

35. The vectors $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $\vec{c} = -3\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$ represents the sides of

(a) an equilateral triangle (b) an obtuse-angled triangle

(c) an isosceles triangle (d) a right angled triangle

36. If $|\vec{a}| = 2$ and $-3 \leq k \leq 2$, then $|k\vec{a}| \in$

(a) [-6, 4] (b) [0, 4] (c) [4, 6] (d) [0, 6]

37. A unit vector $\hat{a}$ makes equal but acute angles on the coordinate axes. The projection of the vector $\hat{a}$ on the vector $\vec{b} = 5\hat{\imath} + 7\hat{\jmath} - \hat{k}$ is

(a) $\frac{11}{15}$ (b) $\frac{11}{5\sqrt{3}}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5\sqrt{3}}$

38. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} \cdot \vec{b} \geq 0$ only when:

(a) $0 < \theta < \frac{\pi}{2}$ (b) $0 \leq \theta \leq \frac{\pi}{2}$ (c) $0 < \theta < \pi$ (d) $0 \leq \theta \leq \pi$

39. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}, |\vec{a}| = \sqrt{37}, |\vec{b}| = 3$, and $|\vec{c}| = 4$, then angle between $\vec{b}$ and $\vec{c}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

40. Two vectors $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ are collinear if

(a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

(c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

41. If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) 0

42. If ABCD is a parallelogram and Ac and BD are its diagonals, then $\overrightarrow{AC} + \overrightarrow{BD}$ is

(a) $2\overrightarrow{DA}$ (b) $2\overrightarrow{AB}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{BD}$

43. If vector $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$, then which of the following is correct?

(a) $\vec{a} \| \vec{b}$ (b) $\vec{a} \perp \vec{b}$ (c) (c) $|\vec{b}| > |\vec{a}|$ (d) (d) $|\vec{a}| = |\vec{b}|$

44. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. Then angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

45. The value of p for which the vectors $2\hat{\imath} + p\hat{\jmath} + \hat{k}$ and $-4\hat{\imath} - 6\hat{\jmath} + 26\hat{k}$ are perpendicular to each other is:

(a) 3 (b) -3 (c) $-\frac{17}{3}$ (d) $\frac{17}{3}$

46. Let $\vec{a}$ be any vector such that $|\vec{a}| = a$. The value of $|\vec{a} \times \hat{\imath}|^2 + |\vec{a} \times \hat{\jmath}|^2 + |\vec{a} \times \hat{k}|^2$ is

(a) a^2 (b) $2a^2$ (c) $3a^2$ (d) 0

47. The value of $(\hat{\imath} \times \hat{\jmath}) \cdot \hat{\jmath} + (\hat{\jmath} \times \hat{\imath}) \cdot \hat{k}$ is

(a) 2 (b) 0 (c) 1 (d) -1

48. If $\vec{a} + \vec{b} = \hat{\imath}$ and $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$, then $|\vec{b}|$ equals :

(a) $\sqrt{14}$ (b) 3 (c) $\sqrt{12}$ (d) $\sqrt{17}$

49. Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin\theta = \frac{3}{5}$. Then, $\hat{a} \cdot \hat{b}$ is

(a) $\pm\frac{3}{5}$ (b) $\pm\frac{3}{4}$ (c) $\pm\frac{4}{5}$ (d) $\pm\frac{4}{3}$

50. If the vector $\hat{\imath} - b\hat{\jmath} + \hat{k}$ is equally inclined to the coordinate axes, then the value of b is

(a) -1 (b) 1 (c) $-\sqrt{3}$ (d) $-\frac{1}{\sqrt{3}}$

51. For what value of λ, the projection of vector $\hat{\imath} + \lambda\hat{\jmath}$ on vector $\hat{\imath} - \hat{\jmath}$ is $\sqrt{2}$?

(a) -1 (b) 1 (c) 0 (d) 3

52. If $\vec{a} \cdot \hat{\imath} = \vec{a} \cdot (\hat{\imath} + \hat{\jmath}) = \vec{a} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 1$, then $\vec{a}$ is

(a) $\hat{k}$ (b) $\hat{\imath}$ (c) $\hat{\jmath}$ (d) $\hat{\imath} + \hat{\jmath} + \hat{k}$

53. Unit vector along $\overrightarrow{PQ}$, where P and Q respectively are (2, 1, -1) and (4, 4, -7) is

(a) $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ (b) $-2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$ (c) $\frac{-2\hat{\imath}}{7} - \frac{3\hat{\jmath}}{7} + \frac{6\hat{k}}{7}$ (d) $\frac{2\hat{\imath}}{7} + \frac{3\hat{\jmath}}{7} - \frac{6\hat{k}}{7}$

54. Position vector of the mid-point of line segment AB is $3\hat{\imath} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$. If position vector of the point A is $2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$, then the position vector of the point B is

(a) $\frac{5\hat{\imath}}{2} + \frac{5\hat{\jmath}}{2} - \frac{7\hat{k}}{2}$ (b) $4\hat{\imath} + \hat{\jmath} - 2\hat{k}$ (c) $5\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$ (d) $\frac{\hat{\imath}}{2} - \frac{\hat{\jmath}}{2} + \frac{\hat{k}}{2}$

55. If in $\triangle ABC, \overrightarrow{BA} = 2\vec{a}$ and $\overrightarrow{BC} = 3\vec{b}$, then $\overrightarrow{AC}$ is

(a) $2\vec{a} + 3\vec{b}$ (b) $2\vec{a} - 3\vec{b}$ (c) $3\vec{b} - 2\vec{a}$ (d) $-2\vec{a} - 3\vec{b}$

56. If $|\vec{a} \times \vec{b}| = \sqrt{3}$ and $\vec{a} \cdot \vec{b} = -3$, then angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{5\pi}{6}$

57. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ and $|\vec{a} \times \vec{b}| = 3\sqrt{3}$, then the value of $\vec{a} \cdot \vec{b}$ is

(a) 9 (b) 3 (c) $\frac{1}{9}$ (d) $\frac{1}{3}$

58. The position vectors of three consecutive vertices of a parallelogram ABCD are $A(4\hat{\imath} + 2\hat{\jmath} - 6\hat{k})$, $B(5\hat{\imath} - 3\hat{\jmath} + \hat{k})$ and $C(12\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$. The position vector of D is given by

(a) $-3\hat{\imath} - 5\hat{\jmath} - 10\hat{k}$ (b) $21\hat{\imath} + 3\hat{\jmath}$ (c) $11\hat{\imath} + 9\hat{\jmath} - 2\hat{k}$ (d) $-11\hat{\imath} - 9\hat{\jmath} + 2\hat{k}$

59. In ΔABC, $\overrightarrow{AB} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$. If D is mid-point of BC, then the vector $\overrightarrow{AD}$ is equal to

(a) $4\hat{\imath} + 6\hat{k}$ (b) $2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ (c) $\hat{\imath} - \hat{\jmath} + \hat{k}$ (d) $2\hat{\imath} + 3\hat{k}$

60. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{a} \times \vec{b}| = 1$, then $\vec{a} \cdot \vec{b}$ is equal to

(a) -1 (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

ASSERTION - REASON TYPE QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

1. Assertion: The point A(1, -2, -8), B(5, 0, -2) and C(11, 3, 7) are collinear.

Reason: The ratio in which B divides AC is 2:3.

2. Assertion: If $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = 0$ then $\vec{b} = \lambda(\vec{a} \times \vec{c})$.

Reason: The cross product of two vectors is the third vector that is parallel to the two original vectors.

3. Assertion: The vector $\frac{\hat{i}-2\hat{j}-2\hat{k}}{3}$ is a unit vector in the direction of the vector $\hat{i} - 2\hat{j} - 2\hat{k}$.

Reason: The unit vector in the direction of $\vec{a}$ is given by $\frac{\vec{a}}{|\vec{a}|}$.

4. Assertion: The direction cosines of the vector $\hat{i} - 2\hat{j} - 2\hat{k}$ are $\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$.

Reason: The magnitude r, direction ratios a, b, c and direction cosines l, m, n of any vector are related as $l = \frac{a}{r}, m = \frac{b}{r}, n = \frac{c}{r}$.

5. Assertion: The vector joining the points P(2, 3, 0) and Q(– 1, – 2, – 4) is $\overrightarrow{PQ} = -3\hat{i} - 5\hat{j} - 4\hat{k}$.

Reason: If P (x_1, y_1, z_1) and Q(x_2, y_2, z_2) are any two points, then magnitude of the vector $\overrightarrow{PQ}$ is given by $|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

6. Assertion: The mid-point of the vector joining the points P(2, 3, 4) and Q(4, 1, –2) is R(3,2,1).

Reason: The position vector of the point R which divides P and Q internally in the ratio of m:n is given by $\overrightarrow{OR} = \frac{m\vec{b}+n\vec{a}}{m+n}$.

7. Assertion: $\vec{a} = \hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ are perpendicular to each other.

Reason: Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other if $\vec{a} \times \vec{b} = \vec{0}$.

8. Assertion: The vector in the direction $\vec{a} = \hat{i} - 2\hat{j}$ with magnitude 7 units is $\frac{7}{\sqrt{5}}\hat{i} - \frac{14}{\sqrt{5}}\hat{j}$.

Reason: The vector in the direction of $\vec{r}$, which has magnitude d units is d. $\hat{r}$.

9. Assertion: For any two vectors $\vec{a}$ and $\vec{b}$ with $|\vec{a}| \neq 0 \neq |\vec{b}|$ we always have $|\vec{a}+\vec{b}| \leq |\vec{a}||\vec{b}|$.

Reason: $\vec{a} \cdot \vec{a} = |\vec{a}|^2$

10. Assertion: The unit vector in XY-plane making an angle of of 30^{o} with positive direction of x-axis is $\frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}$.

Reason: $\hat{r} = \cos 30^0\hat{\imath} + \sin 30^0\hat{\jmath}$.

11. Assertion: $(\vec{b} \cdot \vec{c})\vec{a}$ is a scalar quantity.

Reason: Dot product of two vectors is a scalar quantity.

12. Assertion: Projection of $\vec{a}$ on $\vec{b}$ is same as projection of $\vec{b}$ on $\vec{a}$.

Reason: Angle between $\vec{a}$ and $\vec{b}$ is same as angle between $\vec{b}$ and $\vec{a}$ numerically.

13. Assertion: The vectors $\vec{a} = 6\hat{\imath} + 2\hat{\jmath} - 8\hat{k}$, $\vec{b} = 10\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$, $\vec{c} = 4\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$ represents the sides of a right angled triangle.

Reason: Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vectors or sum of any two vectors is equal to the third.

VERY SHORT ANSWERS

1. The x-coordinate of a point on the line joining the point P(2, 2, 1) and Q(5, 1, –2) is 4. Find its z-coordinate.

2. Find ' λ ' when the projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units.

3. What are the direction cosines of a line, which makes equal angles with the co-ordinate axes?

4. The vectors $\vec{a} = 3\hat{\imath} + x\hat{\jmath}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} + y\hat{k}$ are mutually perpendicular. If $|\vec{a}| = |\vec{b}|$, then find the value of y.

5. Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$ be two vectors. Show that the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other.

6. For any two vectors $\vec{a}$ and $\vec{b}$, prove that $(\vec{a} \times \vec{b})^2 = \vec{a}^2\vec{b}^2 - (\vec{a} \cdot \vec{b})^2$

7. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$ then show that the angle between $\vec{a}$ and $\vec{b}$ is $60°$.

8. Find the area of a parallelogram ABCD whose side AB and the diagonal AC are given by the vectors $3\hat{i} + \hat{j} + 4\hat{k}$ and $4\hat{i} + 5\hat{k}$ respectively.

9. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\vec{a} - \sqrt{2}\vec{b}$ to be a unit vector.

10. For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$, find the value of $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b})$.

11. Find a vector $\vec{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units.

12. Show that the three points A(1, -2, -8), B(5, 0, -2) and C(11, 3, 7) are collinear and find the ratio in which B divides AC.

13. Vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are given by $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j}$, $\vec{c} = 3\hat{i} + 5\hat{j} - 2\hat{k}$ and $\vec{d} = \hat{k} - \hat{j}$. Prove that the vectors $\vec{b} - \vec{a}$ and $\vec{d} - \vec{c}$ are parallel. Also find the ratio of their magnitudes.

14. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

15. Show that the ΔABC whose vertices are $7\hat{j} + 10\hat{k}, -\hat{i} + 6\hat{j} + 6\hat{k}, -4\hat{i} + 9\hat{j} + 6\hat{k}$ is isosceles and right angled.

16. If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$, find the angle between the vectors $2\vec{a} + \vec{b}$ and $\vec{a} + 2\vec{b}$.

17. Find the projection of $\overrightarrow{PQ}$ on $\overrightarrow{AB}$ where P, Q, A, B are the points (-2, 1, 3), (3, 2, 5), (4, -3, 5) and (7, -5, -1) respectively.

18. If $\vec{a}$ makes equal angles with $\hat{i}, \hat{j}$ and $\hat{k}$ and has magnitude 3, then prove that the angle between $\vec{a}$ and each of $\hat{i}, \hat{j}$ and $\hat{k}$ is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

19. Find the relation between λ and μ such that $\lambda\vec{a} + \mu\vec{b}$ is orthogonal to $\vec{c}$, where $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = -\hat{i} + \hat{j} + 2\hat{k}$.

20. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$, determine a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} + \vec{c}$.

21. If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3, |\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

22. For the vectors $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$, verify that the angle between $\vec{a}$ and $\vec{a} \times \vec{b}$ is $\frac{\pi}{2}$.

23. $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors. If θ is the angle between $\vec{a}$ and $(2\vec{a} + 3\vec{b} + 6\vec{c})$, find the value of cosθ.

24. If the projection of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ on the vector $p\hat{\imath} + \hat{\jmath} - 2\hat{k}$ is $\frac{1}{3}$, then find the value(s) of p.

25. Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $-\hat{\imath} + \hat{\jmath} + \hat{k}$ respectively in the ratio 4:1 externally. Further, find $|\overrightarrow{AB}| : |\overrightarrow{BC}|$.

26. If three non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$.

27. If $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{\jmath}) \cdot (\vec{r} \times \hat{k}) - 12$.

28. Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{\imath} + \hat{\jmath} + \hat{k}$.

29. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a} + \vec{b}) \perp \vec{a}$ and $(2\vec{a} + \vec{b}) \perp \vec{b}$, then prove that $|\vec{b}| = \sqrt{2}|\vec{a}|$.

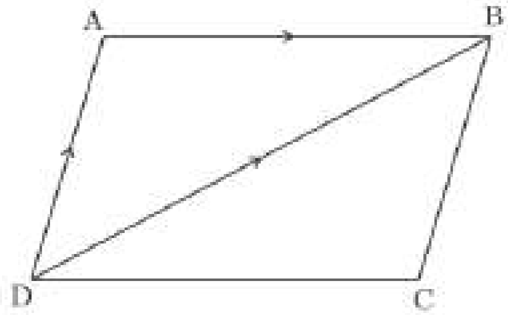
30. Position vectors of the points A, B and C as shown in the figure below $\vec{a}, \vec{b}$ and $\vec{c}$ respectively.

If $\overrightarrow{AC} = \frac{5}{4}\overrightarrow{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$.

31. For two non-zero vectors $\vec{a}$ and $\vec{b}$, if $|\vec{a} - \vec{b}| = |\vec{a} + \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

32. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} + \vec{b} - \vec{c} = \vec{0}$, find the angle between vectors $\vec{a}$ and $\vec{c}$.

33. In the given figure, ABCD is a parallelogram. If $\overrightarrow{AB} = 2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\overrightarrow{DB} = 3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$, then find $\overrightarrow{AD}$ and hence find the area of parallelogram.



34. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then find the angle between $\vec{a}$ and $\vec{b} - \vec{c}$.

35. If points A, B and C have position vectors $2\hat{\imath}, \hat{\jmath}$ and $2\hat{k}$ respectively, then show that ΔABC is an isosceles triangle.

36. Find a vector of magnitude 6, which is perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{b}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$.

SHORT ANSWERS

1. The scalar product of the vector $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ with a unit vector along the sum of the vectors $\vec{b}=2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\vec{c}=\lambda\hat{\imath}+2\hat{\jmath}+3\hat{k}$is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b}+\vec{c}$.

2. Let $\vec{a},\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}|=1,|\vec{b}|=2$ and $|\vec{c}|=3$. If projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$ and $\vec{b}$ and $\vec{c}$ are perpendicular to each other, then find the value of $|3\vec{a}-2\vec{b}+2\vec{c}|$.

3. If $\vec{a},\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitude, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a},\vec{b}$ and $\vec{c}$. Also find the angle which $\vec{a}+\vec{b}+\vec{c}$ with $\vec{a}$ or $\vec{b}$ or $\vec{c}$.

4. If $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{b}=\hat{\jmath}-\hat{k}$, then find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=3$.

5. Let $\vec{a}=\hat{\imath}+4\hat{\jmath}+2\hat{k},\vec{b}=3\hat{\imath}-2\hat{\jmath}+7\hat{k}$ and $\vec{c}=2\hat{\imath}-\hat{\jmath}+4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}\cdot\vec{c}=18$.

6. The magnitude of the vector product of the vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\lambda\hat{\imath}+2\hat{\jmath}+3\hat{k}$ is equal to $\sqrt{2}$. Find the value of λ.

7. Show that the points A, B, C with position vectors $2\hat{\imath}-\hat{\jmath}+\hat{k},\hat{\imath}-3\hat{\jmath}-5\hat{k}$ and $3\hat{\imath}-4\hat{\jmath}-4\hat{k}$ respectively are the vertices of a right-angled triangle. Hence find the area of the triangle.

8. Find a unit vector perpendicular to the plane of triangle ABC, where the coordinates of its vertices are A(3, –1, 2), B(1, –1, –3) and C(4, –3, 1).

9. If $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 2\hat{j} - 3\hat{k}$ then express $\vec{b}$ in the from of $\vec{b} = \vec{b}_1 + \vec{b}_2$, where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$.

10. Given that $\vec{a}, \vec{b}, \vec{c}$ form a triangle such that $\vec{a} = \vec{b} + \vec{c}$. Find p, q, r, s such that the area of the triangle is $5\sqrt{6}$ where $\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}, \vec{b} = s\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$.

11. The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represents the two sides vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle ABC. Find the length of the median through A.

12. If $\vec{a}, \vec{b}$ are unit vectors such that the vector $\vec{a} + 3\vec{b}$ is perpendicular to $7\vec{a} - 5\vec{b}$ and $\vec{a} - 4\vec{b}$ is perpendicular to $7\vec{a} - 2\vec{b}$, then find the angle between $\vec{a}$ and $\vec{b}$.

13. Find a unit vector perpendicular to both of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

14. If $\vec{p} = 5\hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{q} = \hat{i} + 3\hat{j} - 5\hat{k}$ then find the value of λ, so that $\vec{p} + \vec{q}$ and $\vec{p} - \vec{q}$ are perpendicular vectors.

15. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 27$.

16. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the two unit vector parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.

17. Given that $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = 3\hat{i} + 2\hat{j} - 7\hat{k}$ and $\vec{c} = 5\hat{i} + 6\hat{j} - 5\hat{k}$, verify that
$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$.

18. Prove that the points $2\vec{i} - \vec{j} + \vec{k}, \vec{i} - 3\vec{j} - 5\vec{k}, 3\vec{i} - 4\vec{j} - 4\vec{k}$ are the vertices of a right angled triangle. Find also the other two angles of the triangle.

19. Dot product of a vector with vectors $3\hat{i} - 5\hat{k}, 2\hat{i} + 7\hat{j}$ and $\hat{i} + \hat{j} + \hat{k}$ are respectively -1, 6 and 5. Find the vector.

20. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and $\vec{a}, \vec{b}, \vec{c}$ are perpendicular to $\vec{b} + \vec{c}, \vec{c} + \vec{a}, \vec{a} + \vec{b}$ respectively show that $|\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$.

21. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ then find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

22. If vectors $\vec{a}, \vec{b}$ and $2\vec{a} + 3\vec{b}$ are unit vectors, then find the angle between $\vec{a}$ and $\vec{b}$.

23. The position vectors of vertices of ΔABC are $A(2\hat{\imath} - \hat{\jmath} + \hat{k})$, $B(\hat{\imath} - 3\hat{\jmath} - 5\hat{k})$ and $C(3\hat{\imath} - 4\hat{\jmath} - 4\hat{k})$. Find all the angles of ΔABC.

24. Find a vector of magnitude 4 units perpendicular to each of the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath} - \hat{k}$ and hence verify your answer.

25. Given: $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = 3\hat{i} - \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 3$.

LONG ANSWERS

1. Show that the four points A, B, C, D with position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively, such that $3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = \vec{0}$ are coplanar. Also find the position vector of the point of intersection of lines AC and BD.

2. (i) If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$, then prove that the vector $2\vec{a} + \vec{b}$ is perpendicular to vector $\vec{b}$.

(ii) Find $|\vec{a} - \vec{b}|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$.

3. (i) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors of magnitude 3, 2, 5 respectively. If each one is perpendicular to the sum of other two vectors, then find the value of $|\vec{a} + \vec{b} + \vec{c}|$.

(ii) If $\hat{a}$ and $\hat{b}$ are unit vectors and θ is the angle between them, prove that $\cos\frac{\theta}{2} = \frac{1}{2}|\hat{a} + \hat{b}|$.

4. (i) Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$, making an angle $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{2}$ with y-axis and an angle θ with z-axis.

(ii) If $\vec{a}$ is any vector in space, show that $\vec{a} = (\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}$.

5. (i) Find λ and μ if $(\hat{i} + 3\hat{j} + 9\hat{k}) \times (3\hat{i} - \lambda\hat{j} + \mu\hat{k}) = \vec{0}$.

(ii) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$ find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

CASE BASED QUESTIONS

1. Solar panels to be installed carefully so that the tilt of the roof and direction to the sun produce the largest possible electrical power in the solar panels. A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. Suppose the points are labeled counter clockwise from the roof corner nearest to the camera in units of meters $P_1(6, 8, 4)$, $P_2(21, 8, 4)$, $P_3(21, 16, 10)$ and $P_4(6, 16, 10)$.

Based on the above information, answer the following questions:

(i) Find the components to the two edge vectors defined by $\vec{A}$ = PV of P_2 – PV of P_1 and $\vec{B}$ = PV of P_4 – PV of P_1 (where PV stands for position vector).

(ii) Find the magnitude of the vectors $\vec{A}$ and $\vec{B}$.

(iii) Find the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof.

2. A monkey starts moving from a point A(0,0,0) to 5 meters away a point B in east direction then 6 meters in north direction to a point C and then climbs a tree of 10 meters to a point D.

From above information answer the questions given below:

(i) Find position vector of monkey.

(ii) Find displace of monkey.

(iii) Find unit vector of vector $\overrightarrow{AD}$.

3. Two toddlers are playing seesaw game in a garden. A student of XII class observing the positions of toddlers and finds the position of first toddler (3,4,2) and position of second toddler (5,6,0).

From above information answer the questions given below:

(i) Find position vector of midpoint of seesaw.

(ii) Find distance between both toddlers

(iii) Find position vector of point on seesaw which divides seesaw beam in 2:1.

4. Three birds are sitting on tree at positions A(4,6,8) , B (6, 7 ,7) , C(5,6, 9). A student of class XII wants apply vector algebra concept to find different component of triangle.

Solve the problem that he finds in following questions.

(i) Find vector $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(ii) Find centroid of triangle.

(iii) Find angle between vector $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

5. There is a 9 meters tree in a field. Shape of field is in parallelogram whose two adjacent sides are given $\vec{a} = 4\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = -2\hat{\imath} + \hat{\jmath} - 2\hat{k}$.

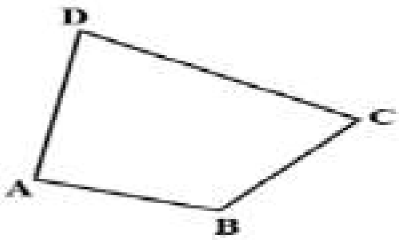
From above information solve following questions

(i) Find area of field.

(ii) Find height in vector form.

6. A garden is in shape of quadrilateral. A student wants to finds its area by using vector algebra. points are given as A(0,1,1), B(2,3,-2), C(22,19,-5), and D(1,-2,1).

From above information solve following questions

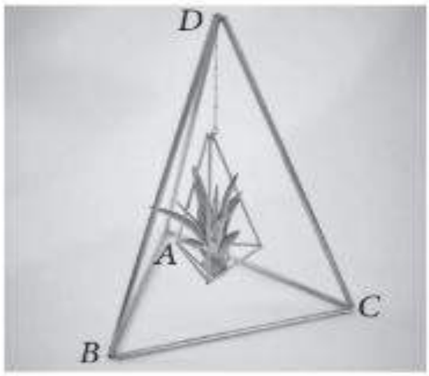


(i) Find $\overrightarrow{AC}$.

(ii) Find $\overrightarrow{BD}$.

(iii) Using this find area of quadrilateral.

7. Swathi purchased an air plant holder which is in the shape of a tetrahedron. Let A, B, C and D are the coordinates of the air plant holder where A(1, 1, 1), B(2, 1, 3), C(3, 2, 2) and D(3, 3, 4).



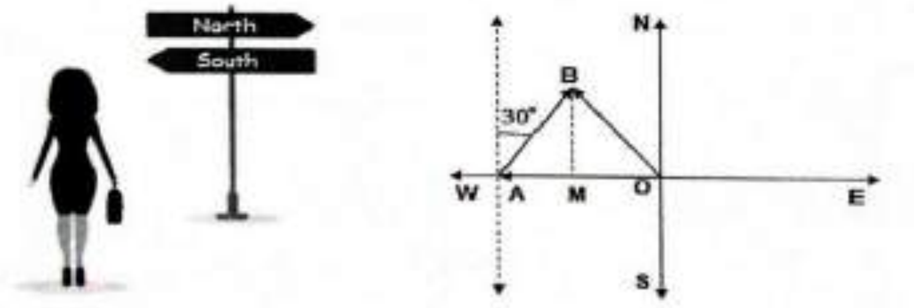
Based on the above information, answer the following questions:

(i) Find the position vector $\overrightarrow{AB}$.

(ii) Find the unit vector along $\overrightarrow{AD}$.

(iii) Find the area of the $\triangle$ ABC.

8. Swathi walks 4 km towards west, then 3km in a direction 30° east of north and then she stops. The situation above has been depicted in the diagram as shown below, assuming that the girl starts her walk from O.



In the diagram, ON represents positive y-axis and North direction, OE represents positive x-axis and East direction. Similarly, OW is representing negative x-axis and West direction, whereas OS represents negative y-axis and South direction. Let OA = 4km and AB = 3km.

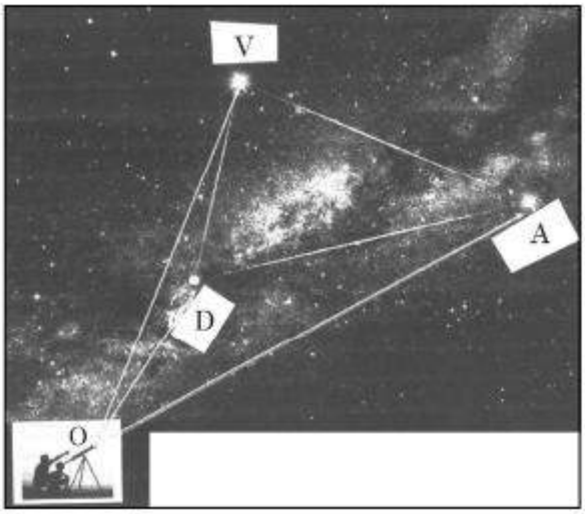
Using the above information, answer the following:

(i) What is the vector $\overrightarrow{AB}$?

(ii) Find $\overrightarrow{AB} \times \overrightarrow{OA}$.

(iii) Find area of ΔOAB.

9. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0, 0, 0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, $7\hat{\imath} + 5\hat{\jmath} + 8\hat{k}$ and $-3\hat{\imath} + 7\hat{\jmath} + 11\hat{k}$ respectively.



Based on the above information, answer the following questions:

(i) How far is the star V from star A?

(ii) Find a unit vector in the direction of $\overrightarrow{DA}$.

(iii) Find the measure of $\angle VDA$.

(iv) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$?

UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is

(a) 0 (b) 1 (c) $-\frac{2}{3}$ (d) $-\frac{3}{2}$

2. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4 respectively and $\vec{a} \cdot \vec{b} = 2\sqrt{3}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{5\pi}{2}$

3. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is

(a) One (b) Two (c) Three (d) Infinite

4. Assertion: If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}| = 9$.

Reason: If $\vec{a}$ and $\vec{b}$ are any two vectors, then $(\vec{a} \times \vec{b})^2$ is equal to $(\vec{a})^2(\vec{b})^2 - (\vec{a} \cdot \vec{b})^2$.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. If $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $|2\hat{a} + \hat{b} + \hat{c}|$.

6. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 400$ and $|\vec{a}| = 5$, then find the value of $|\vec{b}|$.

7. Find the projection of $\vec{b} + \vec{c}$ on $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ & $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

SECTION C

Each carry 3 marks

8. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

9. If $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}, \vec{b} = -\hat{\imath} + \hat{k}, \vec{c} = 2\hat{\jmath} - \hat{k}$, find the area of the parallelogram having diagonals $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$.

SECTION D

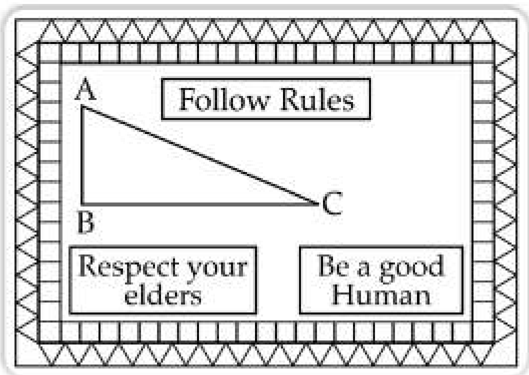
Each carry 5 marks

10. If $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, find the angles which the vector $2\vec{a} + \vec{b} + 2\vec{c}$ makes with the $\vec{a}, \vec{b}$ and $\vec{c}$.

11. Three vertices A, B, C and D of a parallelogram ABCD are given by, A(0, –3, 3), B(–5, m – 3, 0) and D(1, –3, 4). The area of the parallelogram ABCD is 6 sq. units. Using vector method, find the value(s) of m.

SECTION E

12. The slogans on chart papers are to be place on a school bulletin board at the points A, B and C displaying A (follow Rules), B (Respect your elders) and C (Be a good human). The coordinates of these points are (1, 4, 2), (3, –3, –2) and (–2, 2, 6), respectively.



(i) If $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively, then find $|\vec{a} + \vec{b} + \vec{c}|$.

(ii) Find area of $\triangle$ ABC.

ANSWERS

MCQ	A-R	VSA	SA	LA	CS
1. (a) $3\sqrt{5}$ sq. units 2. (b) – 1 3. (c) $\frac{\pi}{2}$ 4. (d) $-\frac{5}{2}$ 5. (a) $\frac{2}{3}$ 6. (c) $-\frac{3}{2}$ 7. (c) -19 8. (d) 16 9. (d) $2\vec{a}^2$ 10. (b) $\frac{3}{4}$ 11. (a) $\frac{15}{4}$ 12. (c) Either $\vec{a}$ or $\vec{b}$ is a null	1. (b) 2. (c) 3. (a) 4. (a) 5. (b) 6. (a) 7. (c) 8. (a) 9. (d) 10. (a) 11. (d) 12.	1. -1 2. 5 3. $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$ 4. $\pm 2\sqrt{10}$ 8. $\sqrt{42}$ sq. units. 9. $\frac{\pi}{4}$ 10. 0 11. $\vec{r} = \pm 3(\hat{\imath} + \hat{\jmath} + \hat{k})$ 12. 2:3 13. 1:3	1. 1 & $\frac{3}{7}\hat{\imath} + \frac{6}{7}\hat{\jmath} - \frac{2}{7}\hat{k}$ 2. $\sqrt{61}$ 3. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 4. $\vec{c} = \left(\frac{5}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}\right)$. 5. $64\hat{\imath} - 2\hat{\jmath} - 28\hat{k}$ 6. 1 7. $\frac{\sqrt{210}}{2}$ sq. units 8. $\frac{-10}{\sqrt{165}}\hat{\imath} - \frac{7}{\sqrt{165}}\hat{\jmath} + \frac{4}{\sqrt{165}}\hat{k}$ 9. $\vec{b}_1 = 4\hat{\imath} - 2\hat{\jmath} - 4\hat{k}$ &	2. (ii) $\sqrt{5}$. 3. (i) $\sqrt{38}$ 4. (i) $5\hat{\imath} \pm \frac{1}{\sqrt{2}}\hat{k}$ 5. (i) -9 & 27 (ii) $\frac{1}{3}(5\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$	1. (i) 15, 0, 0 and 0, 8, 5 (ii) 15 units and 10 units (iii) 0, -90, 120 2. (i) $5\hat{\imath} + 6\hat{\jmath} + 10\hat{k}$ (ii) $\sqrt{161}$ meter (iii) $\widehat{AD} = \frac{5\hat{\imath}+6\hat{\jmath}+10\hat{k}}{\sqrt{161}}$. 3. (i) $4\hat{\imath} + 5\hat{\jmath} + \hat{k}$ (ii) $2\sqrt{3}$ units

vector 13. (d) $\hat{\imath}$ 14. (b) $\frac{\pi}{4}$ 15. (a) 9 16. (d) 1 – cosθ 17. (d) $\frac{(4\vec{a}+\vec{b})}{3}$ 18. (a) $\vec{b}=\vec{c}$ 19. (b) $\frac{1}{3},\frac{2}{3},-\frac{2}{3}$ 20. (d) -1 21. (c) $3\vec{b}$ - $2\vec{a}$ 22. (b) $4\hat{\imath}+\hat{\jmath}-2\hat{k}$ 23. (b) 3 24. (c) $11\hat{\imath}+9\hat{\jmath}-2\hat{k}$ 25. (d) $2\hat{\imath}+3\hat{k}$	(d) 13. (b)	14. $\frac{3}{\sqrt{22}}\hat{\imath}-\frac{3}{\sqrt{22}}\hat{\jmath}+\frac{2}{\sqrt{22}}\hat{k}$ 16. $\cos^{-1}\left(\frac{31}{50}\right)$ 17. $\frac{3}{49}\hat{\imath}-\frac{2}{49}\hat{\jmath}-\frac{6}{49}\hat{k}$ 19. $3\lambda+5\mu=0.$ 20. $\frac{-3\vec{\imath}-6\vec{k}}{\sqrt{45}}$ 21. $\frac{\pi}{6}$ 22. $\theta=\frac{\pi}{2}$ 23. $\frac{2}{7}$ 24. p = 2 or p = $\frac{1}{4}$ 25. $\frac{1}{3}(-5\hat{\imath}+2\hat{\jmath}+5\hat{k})$ and 3:1	$\vec{b}_2=3\hat{\imath}+4\hat{\jmath}+\hat{k}$ 10. p = – 8, 8; q = 4; r = 2 & s = –11, 5 11. $\frac{\sqrt{34}}{2}$ units 12. $\theta=\frac{\pi}{3}$ 13. $-\frac{1}{\sqrt{6}}\hat{\imath}+\frac{2}{\sqrt{6}}\hat{\jmath}-\frac{1}{\sqrt{6}}\hat{k}$ 14. $\lambda=\pm1$ 15. $\vec{d}=96\hat{\imath}-3\hat{\jmath}-42\hat{k}$ 16. $\frac{1}{2\sqrt{6}}(4\hat{\imath}-2\hat{\jmath}-2\hat{k})$ $\frac{1}{10}(6\hat{\imath}+8\hat{k})$ $2\sqrt{101}$ sq. units		(iii) $\vec{r}=\frac{13\hat{\imath}+16\hat{\jmath}+2\hat{k}}{3}$ 4. (i) $2\hat{\imath}+\hat{\jmath}-\hat{k},\hat{\imath}+\hat{k}$ (ii) $(5,\frac{19}{3},8)$ 5. (i) 3 sq. units (ii) $9\frac{(-\hat{\imath}+2\hat{\jmath}+2\hat{k})}{3}$ 6.(i) $22\hat{\imath}+18\hat{\jmath}-6\hat{k}$ (ii) $-\hat{\imath}-5\hat{\jmath}+3\hat{k}$ (iii) $\frac{1}{2}\sqrt{12940}$ sq. units 7. (i) $\hat{\imath}+2\hat{k}$ (ii) $\frac{1}{\sqrt{17}}(2\hat{\imath}+2\hat{\jmath}+$

26. (d) 3:1 externally 27. (a) $\frac{11}{15}$ 28. (b) $\hat{\imath}$ 29. (b) 5 30. (b) 60^{o} 31. (d) $\frac{5\vec{p}+3\vec{q}}{8}$ 32. (c) perpendicular vectors 33. (a) $\frac{2\pi}{3}$ 34. (c) $\frac{5\pi}{6}$ 35. (d) a right angled triangle 36. (d) [0, 6] 37. (a) $\frac{11}{15}$ 38. (b) $0 \leq \theta \leq \frac{\pi}{2}$ 39. (c) $\frac{\pi}{3}$		27. 0 28. $3\hat{\imath}+3\hat{\jmath}+3\hat{k}$ and $-3\hat{\imath}-3\hat{\jmath}-3\hat{k}$ 30. $\vec{c}=\frac{5\vec{b}}{4}-\frac{\vec{a}}{4}$ 31. 90° 32. $\frac{\pi}{3}$ 33. $\overrightarrow{AD}$ $-\hat{\imath}+2\hat{\jmath}+3\hat{k}$ and Area = $11\sqrt{5}$ 34. $\frac{\pi}{2}$ 36. $\frac{6}{\sqrt{24}}(-2\hat{\imath}+4\hat{\jmath}-2\hat{k})$ or $\sqrt{6}(-\hat{\imath}+2\hat{\jmath}-\hat{k})$	18. $\cos^{-1}\sqrt{\frac{6}{41}}$ & $\cos^{-1}\sqrt{\frac{35}{41}}$ 19. $3\hat{\imath}+2\hat{k}$ 21. $-\frac{1}{\sqrt{6}}\hat{\imath}+\frac{2}{\sqrt{6}}\hat{\jmath}-\frac{1}{\sqrt{6}}\hat{k}$ 22. π 23. $\cos^{-1}\left(\frac{\sqrt{35}}{\sqrt{41}}\right)$, $\cos^{-1}\left(\frac{\sqrt{6}}{\sqrt{41}}\right)$, $\frac{\pi}{2}$ 24. $2\sqrt{2}\hat{\jmath}+2\sqrt{2}\hat{k}$ (or $-2\sqrt{2}\hat{\jmath}-2\sqrt{2}\hat{k}$) 25. $\vec{d}=3\hat{\imath}+15\hat{\jmath}+9\hat{k}$		$3\hat{k}$) (iii) $\frac{1}{2}\sqrt{14}$ sq. units 8. (i) $\frac{3}{2}\hat{\imath}+\frac{3\sqrt{3}}{2}\hat{\jmath}$ (ii) $6\sqrt{3}\hat{k}$ (iii) $3\sqrt{3}$ Sq.units 9. (i) $\sqrt{113}$ units (ii) $\frac{5\hat{\imath}+2\hat{\jmath}+4\hat{k}}{3\sqrt{5}}$ (iii) $\cos^{-1}\left(\frac{11\sqrt{2}}{90}\right)$ (iv) $\frac{11\sqrt{5}}{15}$

40. (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$ 41. (c) $\frac{\pi}{2}$ 42. (c) $2\overrightarrow{BC}$ 43. (b) $\vec{a} \perp \vec{b}$ 44. (a) $\frac{\pi}{2}$ 45. (a) 3 46. (b) $2a^2$ 47. (d) -1 48. (b) 3 49. (c) $\pm\frac{4}{5}$ 50. (a) -1 51. (a) -1 52. (b) $\hat{\imath}$ 53. (d) $\frac{2\hat{\imath}}{7} + \frac{3\hat{\jmath}}{7} - \frac{6\hat{k}}{7}$					

54. (b) $4\hat{\imath} + \hat{\jmath} - 2\hat{k}$ 55. (c) $3\vec{b} - 2\vec{a}$ 56. (d) $\frac{5\pi}{6}$ 57. (b) 3 58. (c) $11\hat{\imath} + 9\hat{\jmath} - 2\hat{\text{k}}$ 59. (d) $2\hat{\imath} + 3\hat{k}$ 60. (b) 1					

UNIT TEST

1. (c) $-\frac{2}{3}$

2. (b) $\frac{\pi}{3}$

3. (b) Two

4. (d) Assertion (A) is false but Reason (R) is true.

5. $\sqrt{6}$.

6. 4

7. 2

8. Proof

9. $\frac{1}{2}\sqrt{21}$ sq. units.

10. $\cos^{-1}\frac{2}{3}, \cos^{-1}\frac{1}{3}, \cos^{-1}\frac{2}{3}$

11. ±4

12. (i) $\sqrt{29}$ (ii) $\frac{1}{2}\sqrt{1937}$ sq. units

CHAPTER 11 – THREE DIMENSIONAL GEOMETRY

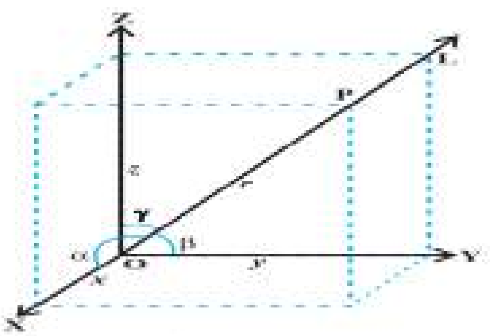
DIRECTION COSINES OF A LINE

The direction of line OP is determined by the angles α, β, γ which it makes with OX, OY and OZ respectively. These angles are called the direction angles and their cosines are called the the direction cosines.

Direction cosines of a line are denoted by l, m, n such that $l = \cos\alpha, m = \cos\beta, n = \cos\gamma$.

Sum of squares of direction cosines of a line is always 1.

$$l^2 + m^2 + n^2 = 1, \textit{i.e.}\ , cos^2\,\alpha + cos^2\,\beta + cos^2\,\gamma = 1$$



DIRECTION RATIOS OF A LINE

Numbers proportional to the direction cosines of a line are called direction ratios of a line. If a, b, c are direction ratios of a line then $\frac{1}{a} = \frac{m}{b} = \frac{n}{c}$.

If a, b, c are direction ratios of a line, then its direction cosines are

$$l = \pm\frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \pm\frac{b}{\sqrt{a^2 + b^2 + c^2}}, n = \pm\frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

DIRECTION COSINES OF A LINE PASSING THROUGH TWO POINTS

We can determine the direction cosines of a line passing through the given points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ as follows

$$\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$$

Where PQ $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

The direction ratios of the line segment joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ may be taken as

$$x_2 - x_1, y_2 - y_1, z_2 - z_1 \text{ or } x_1 - x_2, y_1 - y_2, z_1 - z_2$$

EQUATION OF A LINE IN SPACE

A line is uniquely determined if

(i) it passes through a given point and has given direction

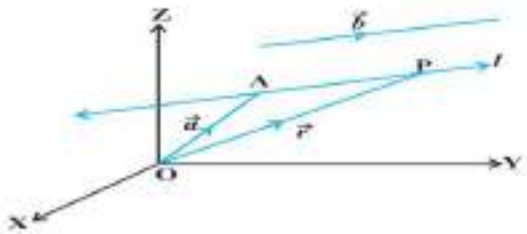
(ii) it passes through two given points

EQUATION OF A LINE THROUGH A GIVEN POINT AND PARALLEL TO A GIVEN VECTOR $\vec{b}$

Vector Form

Let $\vec{a}$ be the position vector of the given point A with respect to the origin O and l be the line which passes through the point A and is parallel to a given vector $\vec{b}$. Hence, the vector equation of line is given by

$$\vec{r} = \vec{a} + \lambda\vec{b}$$



Note: If $\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$, then a, b, c are direction ratios of the line and conversely, if a, b, c are direction ratios of a line, then $\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$ will be the parallel to the line.

Cartesian Form

Equation of the line in the Cartesian form is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Note: If l, m, n are the direction cosines of the line, the equation of the line is

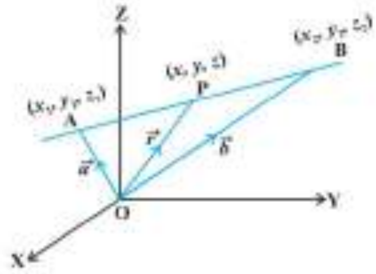
$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

EQUATION OF A LINE PASSING THROUGH TWO GIVEN POINTS

Vector Form

Let $\vec{a}$ and $\vec{b}$ be the position vectors of two points A(x_1, y_1, z_1) and B(x_2, y_2, z_2) respectively that are lying on the line. Then the vector equation of line is given by

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}), \lambda \in \mathrm{R}$$



Cartesian Form

Equation of the line in the Cartesian form is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

ANGLE BETWEEN TWO LINES

$\cos\theta = \left\lvert\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right\rvert$ (If the lines are given in cartesian form)	$\cos\theta = \left\lvert\frac{\vec{b}_1 \cdot \vec{b}_2}{\lvert\vec{b}_1\rvert\lvert\vec{b}_2\rvert}\right\rvert$ (If the lines are given in vector form)

PERPENDICULAR LINES

Two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular, then

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

PARALLEL LINES

Two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are parallel, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

SKEW LINES

In three-dimensional geometry, skew lines are two lines that do not intersect and are not parallel.

DISTANCE BETWEEN TWO SKEW LINES

Vector Form	Cartesian Form
Let l_1 and l_2 be two skew lines with equations $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$. Hence, the shortest distance is $d = \left\lvert \dfrac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{\lvert \vec{b}_1 \times \vec{b}_2 \rvert} \right\rvert$	The shortest distance between the lines $l_1 : \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $l_2 : \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is $d = \left\lvert \dfrac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}} \right\rvert$

NOTE: If the shortest distance between two lines is zero, then the lines are intersecting.

DISTANCE BETWEEN PARALLEL LINES

If two lines l_1 and l_2 are parallel whose equation is given by $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$. Hence, the distance between the given parallel lines is

$$d = \left\lvert \frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{\lvert \vec{b} \rvert} \right\rvert$$

COPLANAR LINES

If the lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplanar lines iff

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

MULTIPLE CHOICE QUESTIONS

1. What is the value of λ for which the lines $\frac{x-1}{2}=\frac{y-3}{5}=\frac{z-1}{\lambda}$ and $\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z}{2}$ are perpendicular to each other?

(a) 2 (b) – 2 (c) ±2 (d) 0

2. If the direction ratios of a line are 1, −2, 2 then what are the direction cosines of the line?

(a) $\frac{-1}{3},\frac{2}{3},\frac{-2}{3}$ (b) $\frac{-1}{\sqrt{8}},\frac{2}{\sqrt{8}},\frac{-2}{\sqrt{8}}$ (c) $\frac{1}{3},\frac{-2}{3},\frac{2}{3}$ (d) $\frac{1}{\sqrt{8}},\frac{-2}{\sqrt{8}},\frac{2}{\sqrt{8}}$

3. The equation of a line passing through (0, 1, 2) and equally inclined to co-ordinate axes is

(a) x = y −1 = z – 2 (b) x = y = z (c) x = y + 1 = z + 2 (d) x – 1 = y + 2 = z + 3

4. The direction ratios of a line joining the points A(0, 4, 1) and B(2, 3, -1) is

(a) 2, -1, 2 (b) $\frac{2}{3},-\frac{1}{3},-\frac{2}{3}$ (c) -2, 1, -2 (d) -2, -1, 2

5. The coordinates of the foot of the perpendicular drawn from the point (2, 5, 7) on the x-axis are

(a) (2, 0, 0) (b) (0, 5, 0) (c) (0, 0, 7) (d) (0, 5, 70

6. The equation of line passing through the origin and (0, 0, -1) is

(a) $\frac{x}{1}=\frac{y}{0}=\frac{z}{0}$ (b) $\frac{x}{0}=\frac{y}{1}=\frac{z}{-1}$ (c) $\frac{x}{0}=\frac{y}{0}=\frac{z-1}{1}$ (d) $\frac{x}{0}=\frac{y}{0}=\frac{z+1}{-1}$

7. The angle between the lines whose d.r.s are a, b, c and b – c, c – a, a – b is

(a) 30° (b) 60° (c) 90° (d) 180°

8. For the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$, the point of intersection is

(a) (1, -1, -1) (b) (-1, 1, -1) (c) (-1, -1, -1) (d) (-1, -1, 1)

9. A line passes through the point with position vector $2\hat{i}-3\hat{j}+4\hat{k}$ and makes angles $60°, 120°$ and $45°$ with x, y and z axes respectively. Then the equation of the line in cartesian form is

(a) $2x-4=2y-6=\sqrt{2}z-4\sqrt{2}$ (b) $2x-4=-2y-6=\sqrt{2}z-4\sqrt{2}$

(c) $2x-4=2y+6=\sqrt{2}z-4\sqrt{2}$ (d) $\vec{r}=2\hat{i}-3\hat{j}+4\hat{k}+\lambda(\hat{i}-\hat{j}+\sqrt{2}\hat{k})$

10. The x-coordinate of a point on the line joining the points P(2, 2, 1) and Q(5, 1, -2) is 4. Then its z-coordinate is

(a) 1 (b) -1 (c) 2 (d) 0

11. The equation of the line joining (1, 2, 3) and (-3, 4, 3) is

(a) Perpendicular to the z-axis (b) Perpendicular to the x-axis

(c) Perpendicular to the y-axis (d) Parallel to z-axis

12. Direction cosines of a line perpendicular to both x-axis and z-axis are:

(a) 1, 0, 1 (b) 1, 1, 1 (c) 0, 0, 1 (d) 0, 1, 0

13. The direction ratios of a line parallel to z-axis are

(a) <1, 1, 0> (b) <1, 1, 1> (c) <0, 0, 0> (d) <0, 0, 1>

14. Equation of a line passing through point (1, 1, 1) and parallel to z-axis is

(a) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ (b) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$ (c) $\frac{x}{0} = \frac{y}{0} = \frac{z-1}{1}$ (d) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$

15. Equation of line passing through origin and making $30°, 60°$ and $90°$ with x, y, z axes respectively is

(a) $\frac{2x}{\sqrt{3}} = \frac{y}{2} = \frac{z}{0}$ (b) $\frac{2x}{\sqrt{3}} = \frac{y}{1} = \frac{z}{0}$ (c) $2x = \frac{2y}{\sqrt{3}} = \frac{z}{1}$ (d) $\frac{2x}{\sqrt{3}} = \frac{y}{1} = \frac{z}{1}$

16. The value of λ for which the lines $\frac{x-5}{7} = \frac{2-y}{5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{2y-1}{\lambda} = \frac{z}{3}$ are at right angle is

(a) 2 (b) 4 (c) -4 (d) -2

17. Equation of a line passing through point (1, 2, 3) and equally inclined to the coordinate axes is

(a) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ (b) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ (c) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ (d) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$

18. The value of λ for which the angle between lines $\vec{r} = \hat{i} + \hat{j} + \hat{k} + p(2\hat{i} + \hat{j} + 2\hat{k})$ and $\vec{r} = (1+q)\hat{i} + (1+q\lambda)\hat{j} + (1+q)\hat{k}$ is $\frac{\pi}{2}$ is

(a) -4 (b) 4 (c) 2 (d) -2

19. The lines $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(6\hat{i} + 9\hat{j} - 18\hat{k})$ where λ and μ are scalars are

(a) Coincident (b) Skew (c) Intersecting (d) Parallel

20. Under what condition do $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{2}, k \right\rangle$ represent direction cosines of a line?

(a) $k = \frac{1}{2}$ (b) $k = -\frac{1}{2}$ (c) $k = \pm\frac{1}{2}$ (d) k can take any value

21. If the direction cosines of a line are $\left(\frac{1}{c}, \frac{1}{c}, \frac{1}{c}\right)$ then

(a) $0 < c < 1$ (b) $c > 2$ (c) $c > 0$ (d) $c = \pm\sqrt{3}$

22. The ordered pair (λ, μ) such that the points (λ, μ, -6), (3, 2, -4) and (9, 8, -10) become collinear is

(a) (3, 4) (b) (5, 4) (c) (4, 5) (d) (4, 3)

23. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point then the value of k is

(a) -5 (b) 5 (c) 2 (d) -2

24. The direction cosines l, m, n of one of the two lines connected by the relations $l - 5m + 3n = 0$, $7l^2 + 5m^2 - 3n^2 = 0$ are

(a) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (b) $\frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (c) $\frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (d) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}$

25. Direction ratios of two lines are a, b, c and $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ then the lines are

(a) Perpendicular (b) Parallel (c) Coincident (d) None of these

26. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if

(a) k = 3 or -2 (b) k = 0 (c) k = 1 or -1 (d) k = 0 or -3

27. If the straight lines $x = 1 + s$, $y = -3 - \lambda s$, $z = 1 + \lambda s$ and $x = \frac{t}{2}$, $y = 1 + t$, $z = 2 - t$ with parameters s and t respectively are coplanar, then λ is

(a) 0 (b) -1 (c) $-\frac{1}{2}$ (d) -2

28. The distance of the point (1,6,3) to the line $\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ is

(a) $\sqrt{13}$ (b) 13 (c) $2\sqrt{13}$ (d) None of these

29. If the point $P(a, b, 0)$ lies on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$, then (a, b) is

(a) (1, 2) (b) $\left(\frac{1}{2}, \frac{2}{3}\right)$ (c) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (d) (0, 0)

30. The shortest distance between the lines given by $\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$ and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$ is

(a) 7 units (b) 2 units (c) 14 units (d) 3 units

31. If the direction cosines of a line are $\sqrt{3}k, \sqrt{3}k, \sqrt{3}k$, then the value of k is

(a) ±1 (b) $\pm\sqrt{3}$ (c) ±3 (d) $\pm\frac{1}{3}$

32. The value of λ for which the angle between the lines $\vec{r} = \hat{\imath} + \hat{\jmath} + \hat{k} + p(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ and $\vec{r} = (1 + q)\hat{\imath} + (1 + q\lambda)\hat{\jmath} + (1 + q)\hat{k}$ is $\frac{\pi}{2}$ is

(a) -4 (b) 4 (c) 2 (d) -2

33. Direction cosines of the line $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$ are :

(a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ (b) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$ (c) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$ (d) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

34. Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are

(a) 2, -1, 6 (b) 2, 1, 6 (c) 2, 1, 3 (d) 2, -1, 3

35. Direction cosines of a line perpendicular to both x-axis and z-axis are

(a) 1, 0, 1 (b) 1, 1, 1 (c) 0, 1, 1 (d) 0, 1, 0

36. The cartesian equation of a line passing through the point with position vector $\vec{a} = \hat{\imath} - \hat{\jmath}$ and parallel to the line $\vec{r} = \hat{\imath} + \hat{k} + \mu(2\hat{\imath} - \hat{\jmath})$ is

(a) $\frac{x-2}{1} = \frac{y+1}{0} = \frac{z}{1}$ (b) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$ (c) $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z}{0}$ (d) $\frac{x-1}{2} = \frac{y}{-1} = \frac{z-1}{0}$

37. The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by:

(a) (1, 0, 0) (b) (2, 0, 0) (c) $(\sqrt{5}, 0,0)$ (d) (0, 0, 0)

38. The vector equation of a line passing through the point (1, -1, 0) and parallel to Y-axis is

(a) $\vec{r} = \hat{\imath} - \hat{\jmath} + \lambda(\hat{\imath} - \hat{\jmath})$ (b) $\vec{r} = \hat{\imath} - \hat{\jmath} + \lambda\hat{\jmath}$ (c) $\vec{r} = \hat{\imath} - \hat{\jmath} + \lambda\hat{k}$ (d) $\vec{r} = \lambda\hat{\jmath}$

39. Equation of line passing through point (1, 1, 1) and parallel to z-axis is

(a) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ (b) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$ (c) $\frac{x}{0} = \frac{y}{0} = \frac{z-1}{1}$ (d) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$

40. If a line makes angles of $90°, 135°$ and $45°$ with the x, y and z axes respectively, then its direction cosines are

(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

41. Equation of line passing through origin and making $30°, 60°$ and $90°$ with x, y, z axes respectively is

(a) $\frac{2x}{\sqrt{3}} = \frac{y}{2} = \frac{z}{0}$ (b) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$ (c) $2x = \frac{2y}{\sqrt{3}} = \frac{z}{1}$ (d) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{1}$

42. The angle which the $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of y-axis is

(a) $\frac{5\pi}{6}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{4}$ (d) $\frac{7\pi}{4}$

43. The angle between the lines 2x = 3y = -z and 6x = -y = -4z is

(a) 0° (b) 30° (c) 45° (d) 90°

44. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is

(a) 90° (b) 120° (c) 60° (d) 0°

45. The direction cosines of vector $\overrightarrow{BA}$, where coordinates A and B are (1, 2, -1) and (3, 4, 0) respectively are

(a) -2, -2, 1 (b) $-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$ (c) 2, 2, 1 (d) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

46. If the point P(a, b, 0) lies on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$, then (a, b) is

(a) (1, 2) (b) $\left(\frac{1}{2}, \frac{2}{3}\right)$ (c) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (d) (0, 0)

47. The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 2 (d) 3

48. The cartesian equation of the line passing through the point (1, -3, 2) and parallel to the line $\vec{r} = (2+\lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is

(a) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$ (b) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

(c) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$ (d) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

ASSERTION - REASON TYPE QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

1. A line makes angle α, βand γ and with the X, Y & Z axes respectively.

Assertion: $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Reason: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

2. Assertion: The distance of the point P(a, b, c)` from the x-axis is $\sqrt{b^2+c^2}$.

Reason: Any point in the Y- axis is of the form (0, y, 0).

3. Assertion: If the x-coordinate of a point P on the join of Q(2, 2, 1) and R(5, 1, -2) is 4, then its z-coordinate is -1.

Reason: Equation of a line joining 2 points $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$ is $\frac{x+x_1}{x_1+x_2} = \frac{y+y_1}{y_1+y_2} = \frac{z-z_1}{z_1-z_2}$.

4. A line makes angle α, $\beta$and γ and with the X, Y & Z axes respectively.

Assertion: The direction ratios of x- axis are 0, 0, 1.

Reason: The X- axis makes angles with coordinate axes are 0^o, 90^o & 90^o respectively.

5. A line makes the same angle θ with each of the x and z-axes and β with y-axis.

Assertion: If $\sin^2\beta = 3\sin^2\theta$ then $\cos^2\theta = \frac{3}{5}$.

Reason: $\cos^2\theta + \cos^2\beta + \cos^2\theta = 1$.

6. Assertion: If a line has direction ratios 2, – 1, – 2, its direction cosines are $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$.

Reason: If l, m, n are direction cosines and a, b, c are direction ratios of a line, then $a = \lambda\, l$, $b= \lambda\, m$ and $c = \lambda\, n$, for any nonzero $\lambda \in R$.

7. Assertion: The direction cosines of the line passing through the two points P(– 2, 4, – 5) and Q(1, 2, 3) are $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{-8}{\sqrt{77}}$. .

Reason: The direction ratios of the line segment joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ may be taken as $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$.

8. Assertion: If a line makes angles $90°, 135°, 45°$with the x, y and z-axes respectively, then its direction cosines are $0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$.

Reason: If a line makes angle α, β and γ with the X, Y and Z axes respectively then its direction cosines are $\sin\alpha, \sin\beta$ and $\sin\gamma$.

9. Assertion: The acute angle between the line $\vec{r} = \hat{i} + \hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j})$ and X-axis is $\frac{\pi}{4}$.

Reason: If θ is the acute angle between $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$, then $\cos\theta = \left|\frac{\overrightarrow{b_1}\cdot\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}\right|$.

10. Assertion: The vector form of the line through the point (5, 2, −4) and which is parallel to the vector $2\hat{i} + \hat{j} - 6\hat{k}$ is $2\hat{i} + \hat{j} - 6\hat{k} + \lambda(5\hat{i} + 2\hat{j} - 4\hat{k})$.

Reason: Vector equation of a line that passes thorough the given point whose position vector is $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

11. Assertion: If a line passes through a point whose position vector is $\vec{a} = \hat{\imath} - 4\hat{\jmath} - 2\hat{k}$ and is parallel to the vector $\vec{b} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ then its equation is $\vec{r} = \hat{\imath} - 4\hat{\jmath} - 2\hat{k} + \lambda(2\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$.

Reason: If a line passes through a point whose position vector is $\vec{a}$ and is parallel to the vector $\vec{b}$, then its equation is $\vec{r} = \vec{a} + \lambda\vec{b}$.

12. Assertion: If a line passes through a point (4, -2, 3) having the direction ratios 1, 5, -1 then its equation is $\frac{x-4}{1} = \frac{y+2}{5} = \frac{z-3}{-1}$.

Reason: $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ represents the cartesian equation of a line which passes through the point (x_1, y_1, z_1) having its direction ratios a, b, c.

13. Assertion: $\frac{x+1}{1} = \frac{y-2}{1} = \frac{z-5}{-6}$ is a line passing through the points $(-1, 2, 5)$ and $(0, 3, -1)$.

Reason: $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$ represents the cartesian equation of a line which passes through the points (x_1, y_1, z_1) and (x_2, y_2, z_2).

14. Assertion: $\frac{x-1}{1} = \frac{y-2}{-2} = \frac{z+7}{4}$ is a line which is passes through the point (1, 2, -7) and is parallel to the line $\frac{x+1}{1} = \frac{y}{-2} = \frac{z-3}{4}$.

Reason: The direction ratios of the line $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - 6\hat{\jmath} + 5\hat{k})$ are given by $2, -6, 5$.

15. Assertion: $\frac{x}{1} = \frac{y-2}{-2} = \frac{z-1}{4}$ and $\frac{x+1}{2} = \frac{y-1}{-4} = \frac{z-3}{8}$ are parallel lines.

Reason: The direction ratios of parallel lines are proportional.

16. Assertion: $\frac{x}{1} = \frac{y-3}{-2} = \frac{z}{2}$ and $\frac{x-4}{2} = \frac{y+7}{2} = \frac{z-1}{1}$ are perpendicular lines.

Reason: The direction ratios of parallel lines are proportional.

17. Assertion: $\vec{r} = \hat{\imath} + \lambda(2\hat{\imath} + 3\hat{k})$ and $\vec{r} = 2\hat{\imath} - \hat{\jmath} + \hat{k} + \lambda(3\hat{\imath} + \hat{\jmath} - 2\hat{k})$ are perpendicular lines.

Reason: For two perpendicular lines whose d.r.s are a_1, b_1, c_1 and a_2, b_2, c_2 we must have $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

18. Assertion: $\vec{r} = \hat{i} - \hat{j} + \lambda(2\hat{i} + 3\hat{k})$ is a line which is passes through the points with position vectors $\hat{i} - \hat{j}$ and $3\hat{i} - \hat{j} + 3\hat{k}$.

Reason: For two parallel lines whose d.r.s are a_1, b_1, c_1 and a_2, b_2, c_2 we must have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

19. Assertion: (0,11,9) is a point, where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cuts the yz-plane.

Reason: $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$ represents the cartesian equation of a line which passes through the points (x_1, y_1, z_1) and (x_2, y_2, z_2).

20. Assertion: For the line $\frac{x+3}{1} = \frac{y-4}{3} = \frac{z-2}{0}$, the direction cosines are $\pm\frac{1}{\sqrt{10}}, \pm\frac{3}{\sqrt{10}}, 0$.

Reason: Angle between the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is given by $\cos^{-1}\left(\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right)$.

21. Assertion: A line in space cannot be drawn perpendicular to x, y and z axes simultaneously.

Reason: For any line making angles α, β, γ with the positive directions of x, y and z axes respectively, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

22. Assertion: Equation of a line passing through the points (1, 2, 3) and (3, -1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$.

Reason: Equation of a line passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

23. Assertion: If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

Reason: The sum of squares of the direction cosines of a line is 1.

24. Assertion: A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (-1, -2, 1) and (1, 2, 5).

Reason: Lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ are parallel if $\overrightarrow{b_1} \cdot \overrightarrow{b_2} = 0$.

25. Assertion: The lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ are perpendicular when $\overrightarrow{b_1} \cdot \overrightarrow{b_2} = 0$.

Reason: The angle θ between the lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ is given by $\cos\theta = \frac{\overrightarrow{b_1} \cdot \overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$.

VERY SHORT ANSWERS

1. The equation of a line are 5x – 3 = 15y + 7 = 3 – 10z. Find the direction cosines of the line.

2. Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

3. If the cartesian equation of a line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, write the vector equation of the line.

4. Find the equation of the line which is parallel to $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and which passes through the point (5, -2, 4).

5. Find the cartesian equation of the line which passes through the point (-2, 4, -5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

6. Find the angle between the following pair of lines $\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$.

7. Find the value of λ, so that the lines following lines are perpendicular to each other $\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}$ and $\frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$.

8. Find the equation of a line which is passes through the point (1, 2, 3) and is parallel to the line $\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$.

9. Determine the value of m so that the line joining the points A(m, 1, -1), B(2, 0, 2m) is perpendicular to the line joining the points C(4, 2m, 1) and D(2, 3, 2).

10. Find the angle between the lines whose direction ratios are 1, 1, 2 and $\sqrt{3} - 1, -\sqrt{3} - 1, 4$.

11. Find the angle between the lines joining the points (1, 4, 2), (-2, 1, 2) and (1, 2, 3), (2, 3, 1).

12. Show that the line joining the points (1, 2, 3) and (-1, -2, -3) is perpendicular to the line joining the points (-2, 1, 5) and (3, 3, 2).

13. Find the values of a and b so that the line joining the points P(2, 4, -5) and Q(-1, 2, -1) will be parallel to the line joining the points R(a, 3, 0) and S(5, 9, b).

14. Find the direction cosines of the sides of the triangle whose vertices are (3, 5, -4), (-1, 1, 2) and (-5, -5, -2).

15. If α, β, γ be the angles which a line makes with the coordinate axes, then show that (i) $\sin^2\gamma + \sin^2\beta + \sin^2\gamma = 2$ (ii) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

16. A line passes through the point with position vector $2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and is in the direction of $\hat{\imath} + \hat{\jmath} - 2\hat{k}$. Find equations for the line in vector and in cartesian form.

17. Find the vector and cartesian equation of the line passing through the points (2, 1, -3) and (5, -4, 1).

18. Find the coordinates of the point where the line through A(3, 4, 1) and B(5, 1, 6) crosses the xy-plane.

19. Find the angle between the pair of lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

20. Find the angle between the following pair of lines: a line with direction ratios 2:2:1 and a line joining (3, 1, 4) and (7, 2, 12).

21. Find the angle between the following pair of lines: $\vec{r} = (1-t)\hat{\imath} + (t-2)\hat{\jmath} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}$.

22. Find the angle between the following pair of lines: x = 2 + 3t, y = 1 + 5t, z = 2 – 2t and x = -1 + 2t, y = 3 – t, z = 5 – 3t.

23. Find the shortest distance between the parallel lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$.

24. Find the shortest distance between the parallel lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(-\hat{i} + \hat{j} - \hat{k})$.

25. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar.

26. Find the vector equation of the line passing through the point (2, 3, -5) and making equal angles with co-ordinate axes.

27. Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$; $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.

28. If the angle between the lines $\frac{x-5}{\alpha} = \frac{y+2}{-5} = \frac{z+\frac{24}{5}}{\beta}$ and $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$ is $\frac{\pi}{4}$, find the relation between α and β.

29. Check whether the lines given by equations x = 2λ + 2, y = 7λ + 1, z = -3λ – 3 and x = -μ – 2, y = 2μ + 8, z = 4μ + 5 are perpendicular to each other or not.

30. The equations of a line are 5x - 3 = 15y + 7 = 3 - 10z. Write the direction cosines of the line and find the coordinates of a point through which it passes.

31. Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$units from origin.

32. If the equation of a line is x = ay + b, z = cy + d, then find the direction ratios of the line and a point on the line.

33. Find the value of p, so that lines $\frac{x-1}{-2} = \frac{y-4}{3p} = \frac{z-3}{4}$ and $\frac{x-2}{4p} = \frac{y-5}{2} = \frac{1-z}{7}$ are perpendicular to each other.

34. Find the vector and the cartesian equation of a line that passes through the point A(1, 2, -1) and parallel to the line 5x – 25 = 14 – 7y = 35z.

SHORT ANSWERS

1. Prove that the line through A(0, -1, 1) and B(4, 5, 1) intersects the line through C(3, 9, 4) and D(-4, 4, 4).

2. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1, 2, 3).

3. The points A(4, 5, 10), B(2, 3, 4) and C(1, 2, -10 are three vertices of a parallelogram ABCD. Find the vector equations of sides AB and BC. Also find the coordinates of point D.

4. Find the vector and cartesian equations of the line through the point (1, 2, -4) and perpendicular to the two lines $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

5. Find the equation of the line passing through the point (-1, 3, -2) and perpendicular to the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.

6. Find the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$.

7. Find the shortest distance between the following lines $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \vec{\lambda}(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$.

8. Find the shortest distance between the two lines whose vector equations are $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and $\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$.

9. Find the shortest distance between the two lines whose vector equations are $\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and $\vec{r} = (-4\hat{i} - \hat{k}) + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$.

10. Find the shortest distance between the following lines: $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}, \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

11. Find the shortest distance between the following lines: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}; \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$.

12. The cartesian equation of a line 6x - 2 = 3y + 1 = 2z – 2. Find the direction ratios of the line and write down the vector equation of the line through (2, -1, -1) which is parallel to the given line.

13. Find the vector equation of the line passes through a point with position vector $2\hat{i} - \hat{j} + \hat{k}$ and parallel to the line joining the points with position vectors $-\hat{i} + \hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$. Also, find the cartesian equation of the line.

14. The vector equation of two lines are $\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$ and $\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$. Find the shortest distance between them.

15. Find the shortest distance between the lines $\vec{r} = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (1 - \lambda)\hat{k}$ and $\vec{r} = 2(1 + \mu)\hat{i} - (1 - \mu)\hat{j} + (-1 + \mu)\hat{k}$.

16. Find the shortest distance between the pair of lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}; z = 2$.

17. Find the coordinates of the foot of the perpendicular drawn from point (5, 7, 3) to the line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

18. Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

LONG ANSWERS

1. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.

2. Show that the lines $\vec{r} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ and $\vec{r} = 5\hat{\imath} - 2\hat{\jmath} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$ are intersecting. Hence, find their point of intersection.

3. Find the image of the point (2, -1, 5) in the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$. Also find the equation of the line joining the given point and its image. Find the length of that line segment.

4. Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. Also find the coordinates of the foot of the perpendicular and the equation of the perpendicular.

5. Find the equation of the perpendicular from the point (3, -1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also, find the coordinates of the foot of the perpendicular and the length of the perpendicular.

6. Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point P(5, 4, 2) to the line $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + \hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$. Also, find the image of the line.

7. Find the equation of a line passing through the points A(0, 6, -9) and B(-3, -6, 3). If D is the foot of perpendicular drawn from a point C(7, 4, -1) on the line AB, then find the coordinates of the point D and the equation of line CD.

8. Find the perpendicular distance of the point (2, 3, 4) from the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also, find the coordinates of the foot of the perpendicular.

9. Find the foot of the perpendicular drawn from the point $2\hat{\imath} - \hat{\jmath} + 5\hat{k}$ to the line $\vec{r} = 11\hat{\imath} - 2\hat{\jmath} - 8\hat{k} + \lambda(10\hat{\imath} - 4\hat{\jmath} - 11\hat{k})$. Also, find the length of the perpendicular.

10. Find the coordinates of the foot of the perpendicular from P(1, 8, 4) on the line joining the points A(0, -1, 3) and B(2, -3, -1).

11. Find the length and the equation of the line of the shortest distance between the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.

12. Find the length and the equations of the line of the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. Also, find the points where the line of the shortest distance meets the given lines.

13. Find the length and the equations of the line of the shortest distance between the lines $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ and $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$.

14. Find the shortest distance and the vector equations of the line of the shortest distance between the lines given by $\vec{r}=(3\hat{i}+8\hat{j}+3\hat{k})+\lambda(3\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=(-3\hat{i}-7\hat{j}+6\hat{k})+\mu(-3\hat{i}+2\hat{j}+4\hat{k})$.

15. Find the magnitude and equations of the line of the shortest distance between the lines $\vec{r}=3\hat{i}+5\hat{j}+7\hat{k}+\lambda(\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r}=(-\hat{i}-\hat{j}-\hat{k})+\mu(7\hat{i}-6\hat{j}+\hat{k})$.

16. Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also, find the point of intersection.

17. Find the value of b so that the lines $\frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are intersecting lines. Also, find the point of intersection of these given lines.

18. Find the vector and the Cartesian equations of a line passing through the point (1, 2, -4) and parallel to the line joining the points A(3, 3, -5) and B(1, 0, -11). Hence, find the distance between the two lines.

19. Find the coordinates of the foot of perpendicular drawn from the point (2, 3, -8) to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also, find the perpendicular distance of the given point from the line.

20. Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4), C(1, 2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.

21. Find the equations of the line passing through the points A(1, 2, 3) and B(3, 5, 9). Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B.

22. Find the equations of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, -6), Q(5, -3, 1), R(12, 4, 5) and S(11, 9, -2). Use these to find the point of intersection of diagonals.

23. A line *l* passes through point (-1, 3, -2) and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line *l*. Hence, obtain its distance.

24. Vertices B and C of ΔABC lie on the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z}{4}$. Find the area of ΔABC given that point A has coordinates (1, -1, 2) and the line segment BC has length of 5 units.

25. The vertices of Δ ABC are A(1, 1, 0), B(1, 2, 1) and C(– 2, 2, – 1). Find the equations of the medians through A and B. Use the equations so obtained to find the coordinates of the centroid.

26. The image of point P(x, y, z) with respect to line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is $P'(1,0,7)$. Find the coordinates of the point P.

27. Equations of sides of a parallelogram ABCD are as follows:

AB: $\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-1}{2}$, BC: $\frac{x-1}{3} = \frac{y+2}{-5} = \frac{z-5}{3}$

CD: $\frac{x-4}{1}=\frac{y+7}{-2}=\frac{z-8}{2}$, DA: $\frac{x-2}{3}=\frac{y+3}{-5}=\frac{z-4}{3}$

Find the equation of diagonal BD.

28. Find the equation of the line which bisects the line segment joining points A(2, 3, 4) and B(4, 5, 8) and is perpendicular to the lines $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.

29. Find the value of p for which the lines $\vec{r}=\lambda\hat{\imath}+(2\lambda+1)\hat{\jmath}+(3\lambda+2)\hat{k}$ and $\vec{r}=\hat{\imath}-3\mu\hat{\jmath}+(p\mu+7)\hat{k}$ are perpendicular to each other and also intersect. Also, find the point of intersection of the given lines.

30. Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and $\frac{x-1}{0}=\frac{y}{-3}=\frac{z-7}{2}$ and perpendicular to these lines.

31. Two vertices of the parallelogram ABCD are given as A(-1, 2, 1) and B(1, -2, 5). If the equation of the line passing through C and D is $\frac{x-4}{1}=\frac{y+7}{-2}=\frac{z-8}{2}$, then find the distance between sides AB and CD. Hence, find the area of the parallelogram.

32. Find the distance between the line $\frac{x}{2}=\frac{2y-6}{4}=\frac{1-z}{-1}$ and another line parallel to it passing through the point (4, 0, -5).

33. If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of line perpendicular to these lines and passing through the point (3, -4, 7).

34. Find the shortest distance between the lines L_1 & L_2 given below:

L_1 : The line passing through $(2,-1,1)$ and parallel to $\frac{x}{1}=\frac{y}{1}=\frac{z}{3}$

L_2: $\vec{r}=\hat{\imath}+(2\mu+1)\hat{\jmath}-(\mu+2)\hat{k}$.

CASE BASED QUESTIONS

1. The Indian Coast Guard (lCG) while patrolling, saw a suspicious boat with four men. They were nowhere looking like fishermen. The soldiers were closely observing the movement of the boat for an opportunity to seize the boat. They observe that the boat is moving along a path L: $\frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-8}$. At an instant of time, the coordinates of the position of coast guard helicopter and boat are H(2, 3, 5) and B(1, 4, 2) respectively.

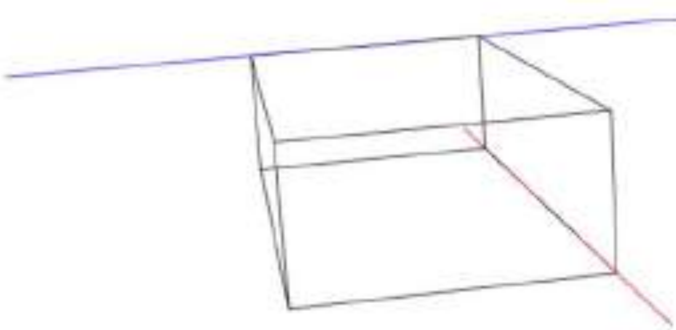
(i) Find the equation of a line joining the points H(2, 3, 5) and B(1, 4, 2)

(ii) Find the vector and cartesian equation of the line passing through the point H(2, 3, 5) and is in the direction of the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.

(iii) Find the equation of the line passing through the point B(1, 4, 2) and parallel to the line L: $\frac{3x-5}{3} = \frac{-y+2}{-2} = \frac{2z+4}{-8}$.

2. Given two lines in the two-dimensional plane, the lines are equal, they are parallel but not equal, or they intersect in a single point. In three dimensions, a fourth case is possible. If two lines in space are not parallel, but do not intersect, then the lines are said to be skew lines

(See the Figure).



Based on the above information, answer the following questions:

The equations of given lines are: L_1: x = 2s – 1, y = s – 1, z = s – 4 and L_2: x = t – 3, y = 3t + 8, z = 5 – 2t.

(i) Write the above equations in standard form.

(ii) Are the lines are perpendicular?

(iii) What are the direction ratios of L_1 and L_2?

3. Two motorcycles A and B are running at the speed more that allowed speed on the road along the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} + \mu(2\hat{i} + \hat{j} + \hat{k})$, respectively.

Based on the above information, answer the following questions:

(i) Find the shortest distance between the given lines.

(ii) Find the point at which the motorcycles may collide.

4. Read the following passage and answer the question given below:

Electrical transmission wires which laid down in winters are stretched tightly to accommodate expansion in summers.

Two such wires lie along the following lines: L_1: $\frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$ and L_2: $\frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2}$.

(i) Are the lines L_1 and L_2 are coplanar?

(ii) Find the point of intersection of the lines L_1 and L_2.

5. An insect crawling along the line $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$ and another insect crawling along the line $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} + 2\hat{k})$.

Based on the above information answer the following questions:

(i) At what points on the lines should they reach so that the distance between them is shortest?

(ii) Find the shortest possible between them.

UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. What is the distance of point (a, b, c) from x −axis?

(a) $\sqrt{b^2 + c^2}$ (b) $\sqrt{a^2 + c^2}$ (c) $\sqrt{a^2 + b^2}$ (d) 0

2. What is the angle between the lines 2x = 3y = −z and 6x = −y = −4z?

(a) 0° (b) 30° (c) 45° (d) 90°

3. The cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$, then its vector form is

(a) $\vec{r} = 5\hat{\imath} + 4\hat{\jmath} + 6\hat{k} + \lambda(3\hat{\imath} + 7\hat{\jmath} - 2\hat{k})$ (b) $\vec{r} = \hat{\imath} - 2\hat{\jmath} + 4\hat{k} + \lambda(3\hat{\imath} - 7\hat{\jmath} - 2\hat{k})$

(c) $\vec{r} = 4\hat{\imath} - 5\hat{\jmath} + 6\hat{k} + \lambda(3\hat{\imath} - 7\hat{\jmath} + 2\hat{k})$ (d) $\vec{r} = 5\hat{\imath} - 4\hat{\jmath} + 6\hat{k} + \lambda(3\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$

4. Assertion: If a line makes an angle of $\frac{\pi}{4}$ with each of y and z-axes, then it makes a right angle with x-axis.

Reason: The sum of the angles made by a line with the coordinate axes is 180^o.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 2 marks

5. Find the angle between the lines $\vec{r} = 2\hat{\imath} - 5\hat{\jmath} + \hat{k} + \lambda(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$ and $\vec{r} = 7\hat{\imath} - 6\hat{k} + \mu(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$.

6. Find the vector equation of a line passing through the point (1, -1, 2) and parallel to the line whose equation is $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.

7. Show that the line joining the points (1, 2, 3) and (4, 5, 7) is parallel to the line joining the points (-4, 3, -6) and (2, 9, 2).

SECTION C

Each carry 3 marks

8. Find the vector and cartesian equations of the line passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$.

9. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point P(1, 3, 3).

SECTION D

Each carry 5 marks

10. Show that the lines $\vec{r}=(\hat{\imath}+\hat{\jmath}-\hat{k})+\lambda(3\hat{\imath}-\hat{\jmath})$ and $\vec{r}=(4\hat{\imath}-\hat{k})+\mu(2\hat{\imath}+3\hat{k})$ intersect. Also, find their point of intersection.

11. Find the coordinates of the points where the line of the shortest distance between the lines $\frac{x-12}{-9}=\frac{y-1}{4}=\frac{z-5}{2}$ and $\frac{x-23}{-6}=\frac{y-19}{-4}=\frac{z-25}{3}$ meets them.

SECTION E

12. A football match is organized between students of class XII of two schools, say school A and school B. For which a team from each school is chosen. Remaining students of class XII of school A and B are respectively standing in the lines L_1 and L_2 respectively represented by the equations $\vec{r}=3\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(\hat{\imath}+2\hat{\jmath}+2\hat{k})$ and $\vec{r}=5\hat{\imath}-2\hat{\jmath}+0\hat{k}+\mu(3\hat{\imath}+2\hat{\jmath}+6\hat{k})$.

Based on the above information, answer the following:

(i) Write the cartesian equations of the lines L_1 and L_2.

(ii) Write the direction ratios of the lines L_1 and L_2.

(iii) Find the angle between the lines L_1 and L_2.

ANSWERS

MCQ	A-R	VSA	SA	LA	CS
1. (a) 2 2. (c) $\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}$ 3. (a) x = y −1 = z – 2 4. (a) 2, -1, 2 5. (a) (2, 0, 0) 6. (d) $\frac{x}{0} = \frac{y}{0} = \frac{z+1}{-1}$ 7. (c) 90° 8. (c) (-1, -1,	1. (a) 2. (b) 3. (c) 4. (d) 5. (a) 6. (a) 7. (b) 8. (c) 9. (a) 10. (d)	1. $\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}$ 2. $\frac{2}{7}, \frac{-6}{7}, \frac{3}{7}$ 3. $\vec{r} = (3\hat{\imath} - 4\hat{\jmath} + 3\hat{k}) + \lambda(-5\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$ 4. $\vec{r} = (5\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) + \lambda(2\hat{\imath} - \hat{\jmath} + 3\hat{k})$ 5. $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$ 6. θ = $\frac{\pi}{2}$ 7. 1 8. $\frac{x-1}{-2} = \frac{y-2}{14} = \frac{z-3}{3}$ 9. 1	2. $\left(\frac{56}{17}, \frac{43}{17}, \frac{11}{17}\right)$ 3. $\vec{r} = 4\hat{\imath} + 5\hat{\jmath} + 10\hat{k} + \lambda(-2\hat{\imath} - 2\hat{\jmath} - 6\hat{k})$ $\vec{r} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k} - \mu(\hat{\imath} + \hat{\jmath} + 5\hat{k})$ (3, 4, 5) 4. $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + t(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ 5. $(-\hat{\imath} + 3\hat{\jmath} - 2\hat{k}) + \lambda(2\hat{\imath} -$	1. $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$ 2. (-1. -6, -12) 3. (0, 5, 1) and = $2\sqrt{14}$ units 4. (3, -4, -2), $2\sqrt{6}$ units, $\frac{x-1}{2} = \frac{y}{-4} = \frac{z}{-2}$ 5. (2, 5, 7), $\sqrt{53}$ units, $\frac{x-3}{-1} = \frac{y+1}{6} = \frac{z-11}{-4}$ 6. (1, 6, 0), $2\sqrt{6}$ units, (-3,	1. (i) $\frac{x-2}{-1} = \frac{y-3}{1} = \frac{z-5}{-3}$ (ii) $\vec{r} = 2\hat{\imath} + 3\hat{\jmath} + 5\hat{k} + \lambda(3\hat{\imath} + 2\hat{\jmath} - 8\hat{k})$ & $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-5}{-8}$ (iii) $\frac{x-1}{1} = \frac{y-4}{2} = \frac{z-2}{-4}$

-1) 9. (b) $2x-4=-2y-6=\sqrt{2}z-4\sqrt{2}$ 10. (b) -1 11. (a) Perpendicular to the z-axis 12. (d) 0, 1, 0 13. (d) <0, 0, 1> 14. (d)	11. (a) 12. (a) 13. (a) 14. (b) 15. (a) 16. (b) 17. (a) 18. (b) 19. (b) 20. (b) 21. (a) 22. (a)	10. $\frac{\pi}{3}$ 11. $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ 13. a = -4 and b = -12 14. $\frac{-2}{\sqrt{17}},\frac{-2}{\sqrt{17}},\frac{3}{\sqrt{17}};\frac{-2}{\sqrt{17}},\frac{-3}{\sqrt{17}},\frac{-2}{\sqrt{17}}$ and $\frac{4}{\sqrt{42}},\frac{5}{\sqrt{42}},\frac{-1}{\sqrt{42}}$ 16. $\vec{r}=(2\hat{i}-\hat{j}+4\hat{k})+\lambda(\hat{i}+\hat{j}-2\hat{k})$ $x-2=y+1=\frac{z-4}{-2}$ 17. $\vec{r}=2\hat{i}+\hat{j}-3\hat{k}+\lambda(3\hat{i}-5\hat{j}+4\hat{k})$ $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+3}{4}$ 18. $\left(\frac{13}{5},\frac{23}{5},0\right)$	$7\hat{j}+4\hat{k})$ and $\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$. 6. $=\frac{3}{2}\sqrt{2}$ units 7. $\frac{17\sqrt{5}}{5}$ units 8. $\frac{3}{\sqrt{19}}$ unit 9. 9 units 10. $2\sqrt{29}$ units 11. $\frac{1}{\sqrt{6}}$ unit. 12. $\frac{1}{6},\frac{1}{3},\frac{1}{2}$ or 1,2,3 $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\lambda(\hat{i}+2\hat{j}+3\hat{k})$	8, -2) 7. (-1, 2, -5), $\frac{x-7}{4}=\frac{y-4}{1}=\frac{z+1}{2}$ 8. $\left(\frac{170}{49},\frac{78}{49},\frac{10}{49}\right)$, $\frac{3}{7}\sqrt{101}$ units. 9. $\hat{i}+2\hat{j}+3\hat{k},\sqrt{14}$units 10. $\left(-\frac{5}{3},\frac{2}{3},\frac{19}{3}\right)$ 11. $\frac{x-5}{2}=\frac{y-7}{3}=\frac{z-3}{6}$, 14 units 12. (3, 8, 3), (-3, -7, 6), $3\sqrt{30}$ units, $\frac{x-3}{2}=\frac{y-8}{5}=\frac{z-3}{-1}$	2. (i) $L_1:\frac{x+1}{2}=\frac{y+1}{1}=\frac{z+4}{1}=s$ $L_2:\frac{x+3}{1}=\frac{y-8}{3}=\frac{-z+5}{2}=t$ (ii) No (iii) 2, 1, 1 and 1, 3, 2 3. (i) 0 (ii) (1, -2, -1) 4. (i) Yes (ii) (2, 1, -3) 5. (i) $5\hat{i}+4\hat{j}$,

$\frac{x-1}{0}=\frac{y-1}{0}=$ $\frac{z-1}{1}$ 15. (b) $\frac{2x}{\sqrt{3}}=\frac{y}{1}=\frac{z}{0}$ 16. (b) 4 17. (d) $\frac{x-1}{1}=\frac{y-1}{1}=$ $\frac{z-1}{1}$ 18. (a) -4 19. (d) Parallel 20. (c)	23. (a) 24. (c) 25. (a)	19. $\cos^{-1}\frac{8\sqrt{3}}{15}$ 20. $\cos^{-1}\left(\frac{2}{3}\right)$ 21. $\cos^{-1}\left(\frac{5\sqrt{6}}{18}\right)$ 22. $\cos^{-1}\left(\frac{\sqrt{7}}{2\sqrt{19}}\right)$ 23. $\sqrt{\frac{11}{6}}$ 24. $\sqrt{26}$. 26. $\vec{r}=2\hat{\imath}+3\hat{\jmath}-5\hat{k}+\lambda(\hat{\imath}+\hat{\jmath}+\hat{k})$ 27. $\vec{r}=(2\hat{\imath}+\hat{\jmath}+3\hat{k})+\lambda(2\hat{\imath}-7\hat{\jmath}+4\hat{k})$ 28. $2\alpha\beta=25$ 29. Perpendicular Lines	13. $\vec{r}=\left(2\hat{\imath}-\hat{\jmath}+\hat{k}\right)+$ $\lambda\left(-2\hat{\imath}-2\hat{\jmath}-\hat{k}\right)$, $\frac{x-2}{2}=\frac{y+1}{2}=\frac{z-1}{1}$ 14. $\frac{8}{\sqrt{29}}$ units 15. $\frac{3}{\sqrt{2}}$ units 16. $\frac{9}{\sqrt{195}}$ 17. (9, 13, 15) 18. (2, 3, -1)	13. $4\sqrt{3}; x=y=z$ 14. $3\sqrt{30}$; $\vec{r}=(3\hat{\imath}+8\hat{\jmath}+3\hat{k})+\lambda(2\hat{\imath}+5\hat{\jmath}-\hat{k})$ 15. $2\sqrt{29}$; $\vec{r}=(3\hat{\imath}+5\hat{\jmath}+7\hat{k})+$ $\lambda(2\hat{\imath}+3\hat{\jmath}+4\hat{k})$ 16. $\left(\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right)$ 17. b = 2, point of intersection: (-1, -1, -1) 18, $\vec{r}=3\hat{\imath}+3\hat{\jmath}-5\hat{k}+$ $\mu(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ &	$-\hat{\imath}-2\hat{\jmath}-3\hat{k}$ (ii) 9 units

$k = \pm\frac{1}{2}$ 21. (d) $c = \pm\sqrt{3}$ 22. (b) (5, 4) 23. (a) -5 24. (a) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ 25. (b) Parallel 26. (d) k = 0 or -3 27. (d) -2		30. Direction cosines are $\left\langle \pm\frac{6}{7}, \pm\frac{2}{7}, \mp\frac{3}{7} \right\rangle$ and the point through which it passes is $\left(\frac{3}{5}, \frac{-7}{15}, \frac{3}{10}\right)$ 31. (1, 3, 1) and (-1, -1, -3) 32. d.r.s: a, 1, c and point on the line is (b, 0, d) 33. p = -14 34. $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 5\hat{\jmath} + \hat{k})$, $\frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1}$		$d = \frac{\sqrt{293}}{7}$ 19. (2, 6, -2) and $3\sqrt{5}$ 20. Equation of the line AB: $\frac{x-4}{2} = \frac{y-7}{4} = \frac{z-8}{4}$, Equation of the line BC : $\frac{x-2}{3} = \frac{y-3}{5} = \frac{z-4}{3}$, Equation of the line CD : $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z-1}{2}$, Equation of the line DA : $\frac{x-4}{3} = \frac{y-7}{5} = \frac{z-8}{3}$. Coordinates of foot of perpendicular: $\left(\frac{28}{9}, \frac{56}{9}, \frac{83}{9}\right)$	

28. (a) $\sqrt{13}$ 29. (c) $\left(\frac{1}{2}, \frac{1}{4}\right)$ 30. (c) 14 units 31. (b) $\pm\sqrt{3}$ 32. (a) -4 33. (d) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$ 34. (d) 2, -1, 3 35. (d) 0, 1, 0 36. (b)				21. (7, 11, 21) and (– 1, – 1, – 3) 22. Equations of diagonals: $\frac{x-4}{8} = \frac{y-2}{2} = \frac{z+6}{11}$ & $\frac{x-5}{6} = \frac{y+3}{12} = \frac{z-1}{-3}$, Point of intersection: $\left(8,3,-\frac{1}{2}\right)$ 23. $\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$ (or) $\vec{r} = -\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$, $\sqrt{\frac{5}{69}}$ 24. $\frac{5\sqrt{497}}{14}$ sq. units. 25. Equations of medians:	

$\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{0}$ 37. (d) (0, 0, 0) 38. (b) $\vec{r}=\hat{\imath}-\hat{\jmath}+\lambda\hat{\jmath}$ 39. (d) $\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{1}$ 40. (a) $0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}$ 41. (b)				$\frac{x-1}{-3}=\frac{y-1}{2}=\frac{z}{0}$ & $\frac{x-1}{-3}=\frac{y-2}{-1}=\frac{z-1}{-3}$, centroid: $\left(0,\frac{5}{3},0\right)$. 26. P(1, 6, 3) 27. $\frac{x-1}{1}=\frac{y+2}{-1}=\frac{z-5}{-1}$ 28. $\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-6}{6}$ 29. p = 2, point of intersection: (1, 3, 5) 30. $\frac{x-1}{13}=\frac{y-3}{-2}=\frac{z-5}{-3}$ 31. $\frac{\sqrt{26}}{3}$, $2\sqrt{26}$ 32. $\frac{\sqrt{533}}{3}$ units	

$\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$ 42. Ans: (b) $\frac{3\pi}{4}$ 43. (d) 90° 44. (a) 90° 45. (b) $-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$ 46. (c) $\left(\frac{1}{2}, \frac{1}{4}\right)$ 47. (c) 2 48. (d) $\frac{x-1}{1} = \frac{y+3}{1} =$ $\frac{z-2}{2}$				33. k = -2, $\vec{r} = (3\hat{\imath} - 4\hat{\jmath} + 7\hat{k}) +$ $\lambda(26\hat{\imath} - 33\hat{\jmath} - 27\hat{k})$ 34. $\frac{1}{\sqrt{6}}$	

UNIT TEST

1. (a) $\sqrt{b^2 + c^2}$

2. (d) 90°

3. (d) $\vec{r} = 5\hat{\imath} - 4\hat{\jmath} + 6\hat{k} + \lambda(3\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$

4. (c) Assertion (A) is true but Reason (R) is false.

5. $\theta = \cos^{-1}\left(\frac{19}{21}\right)$

6. $\vec{r} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$

7. Proof

8. $\vec{r} = (2\hat{\imath} + \hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} - 7\hat{\jmath} + 4\hat{k})$ and $\frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$

9. (-2, -1, -3) and (4, 3, 7)

10. (4, 0, -1)

11. (3, 5, 7), (11, 11, 31)

12. (i) $\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2}$ & $\frac{x-5}{3} = \frac{y+2}{2} = \frac{z}{6}$ (ii) 1, 2, 2 & 3, 2, 6 (iii) $\cos^{-1}\left(\frac{19}{21}\right)$

CHAPTER 12 - LINEAR PROGRAMMING

OPTIMIZATION PROBLEM

A problem which seeks to maximize or minimize a function is called an optimization problem.

An optimization problem may involve maximization of profit, production etc or minimization of cost from available resources etc.

LINEAR PROGRAMMING PROBLEM

A linear programming problem is one in which we have to find optimal value (maximum or minimum) of a linear function of several variables (called objective function) subject to certain conditions that the variables are non-negative and satisfying by a set of linear inequalities with variables, are sometimes called division variables.

TERMS RELATED TO LINEAR PROGRAMMING

Objective Function: A linear function $z = px + qy$ (p and q are constants) which has to be maximized or minimized, is called an objective function.

Constraints: The linear inequalities or equations or restrictions on the variables of the linear programming problem are called constraints. The conditions $x \geq 0$, $y \geq 0$ are called non-negative restrictions.

Optimal Value: The maximum or minimum value of an objective function is known as its optimal value.

Feasible Region: The common region determined by all the constraints including non-negative constraints $x, y > 0$ of a linear programming problem is called the feasible region for the problem. The region other than the feasible region is called an infeasible region. The feasible region is always a convex polygon.

Feasible Solutions: Points within and on the boundary of the feasible region represent feasible solutions of the constraints. Any point outside the feasible region is called an infeasible solution.

Optimal Feasible Solution: Any point in the feasible region that gives the optimal value of the objective function is called the optimal feasible solution.

THEOREM 1

Let R be the feasible region (convex polygon) for a linear programming problem and let $Z = ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

THEOREM 2

Let R be the feasible region for a linear programming problem and let $Z = ax + by$ be the objective function. If R is bounded, then the objective function Z has both a maximum and a minimum value on R and each of these occurs at a corner point (vertex) of R.

CORNER POINT METHOD

1. Find the feasible region of the linear programming problem and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.

2. Evaluate the objective function $Z = ax + by$ at each corner point. Let M and m, respectively denote the largest and smallest values of these points.

3. When the feasible region is bounded, M and m are the maximum and minimum values of Z.

WORKING RULE TO SOLVE AN LPP GRAPHICALLY (CORNER POINT METHOD)

(a) Formulate the given linear programming problem.

(b) Convert the linear constraints into equalities and the draw their graphs which will be straight lines.

(c) Find the feasible region of the linear programming problem and determine its corner points by solving the equations of lines taken two at a time.

(d) Determine the value of the objective function Z at each corner point. Also, find the least value m and greatest value M of the values of objective functions at corner points.

(e) If the feasible region R is bounded, then M and m will be the maximum and minimum values of Z, respectively.

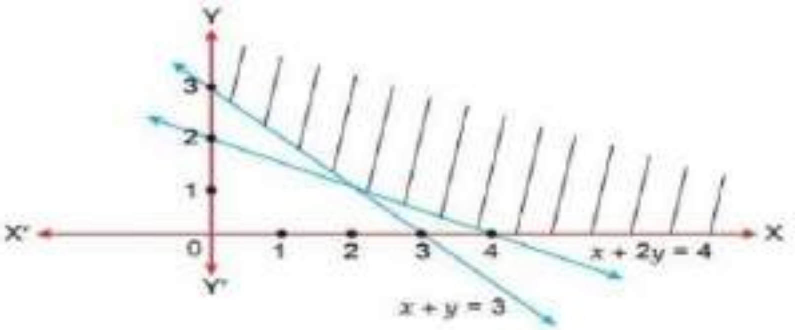
(f) If the feasible region R is unbounded, then

(i) M is the maximum value of the objective function Z when the open half plane determined by $ax + by > M$ has no point in common with feasible region R. Otherwise Z will have no maximum value.

(ii) m is the minimum value of the objective function Z if the open half plane determined by $ax + by < m$ has no point in common with the feasible region R. Otherwise Z has no minimum value.

MULTIPLE CHOICE QUESTIONS

1. The feasible region of a linear programming problem is shown in the figure below:



Which of the following are the possible constraints?

(a) $x + 2y \geq 4, x + y \leq 3, x \geq 0, y \geq 0$ (b) $x + 2y \leq 4, x + y \leq 3, x \geq 0, y \geq 0$

(c) $x + 2y \geq 4, x + y \geq 3, x \geq 0, y \geq 0$ (d) $x + 2y \geq 4, x + y \geq 3, x \leq 0, y \leq 0$

2. The objective function Z =ax + by of an LPP has maximum value at 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true?

(a) a = 9 and b = 1 (b) a = 9, b = 2 (c) a = 3, b = 5 (d) a = 5, b = 3

3. The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If Z = 18x + 9y be the objective function, then

(a) Z is maximum at (2, 72), minimum at (15, 20)
(b) Z is maximum at (15, 20), minimum at (40, 15)
(c) Z is maximum at (40, 15), minimum at (15, 20)
(d) Z is maximum at (40, 15), minimum at (2, 72)

4. The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is

(a) 2 (b) 3 (c) 4 (d) 5

5. If the corner points of the feasible region of an LPP are (0, 3), (3, 2) and (0, 5) then the minimum value of Z = 11x + 7y is

(a) 21 (b) 33 (c) 14 (d) 35

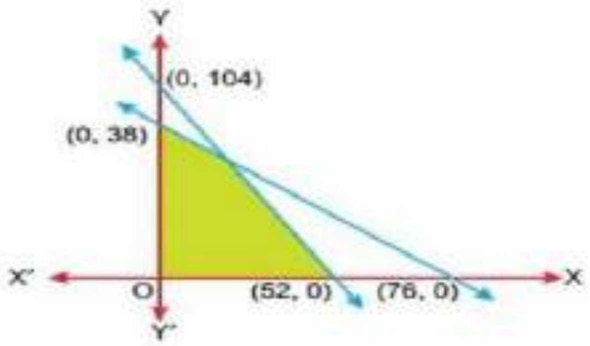
6. The number of solution of the system of inequtions $x + 2y \leq 3$, $3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$ is

(a) 0 (b) 2 (c) Finite (d) Infinite

7. The maximum value of $Z = 3x + 4y$ subject to the constraints $x \geq 0$, $y \geq 0$ and $x + y \leq 1$ is

(a) 7 (b) 4 (c) 3 (d) 10

8. The feasible region of an LPP is given in the following figure:



Then, the constraints of the LPP are $x \geq 0$, $y \geq 0$ and

(a) $2x + y \leq 52$ and $x + 2y \leq 76$ (b) $2x + y \leq 104$ and $x + 2y \leq 76$

(c) $x + 2y \leq 104$ and $2x + y \leq 76$ (d) $x + 2y \leq 104$ and $2x + y \leq 38$

9. For the following LPP: Max $Z = 3x + 4y$; subject to constraints $x - y \geq -1$, $x \leq 3$, $x \geq 0$, $y \geq 0$, then the maximum value is

(a) 0 (b) 4 (c) 25 (d) 30

10. In an LPP, if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points of which Z_{max} occurs is

(a) 0 (b) 2 (c) Finite (d) Infinite

11. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $Z = px + qy$ where $p, q > 0$. Condition on p and Q so that the minimum of Z occurs are (3, 0) and (1, 1) is

(a) $p = 2q$ (b) $p = \frac{q}{2}$ (c) $p = 3q$ (d) $p = q$

12. The optimal value of the objective function is attained at the points

(a) Given by intersection of inequation with y-axis only

(b) Given by intersection of inequation with x-axis only

(c) Given by corner points of the feasible region

(d) None of these

13. The corner points of the feasible region for a LPP are P(0, 5), Q(1, 5), R(4, 2) and S(12, 0). The minimum value of objective function Z = 2x + 5y is at the point

(a) P (b) Q (c) R (d) S

14. The point which doesn't lie in the half plane $2x + 3y - 12 \leq 0$ is

(a) (1, 2) (b) (2, 1) (c) (2, 3) (d) (-3, 2)

15. The objective function of an LPP is

(a) A constraint (b) A linear function to be optimized

(c) A relation between the variables (d) None of these

16. The set of all feasible solutions of a LPP is a

(a) Concave set (b) Convex set (c) Feasible set (d) None of these

17. Corner points of the feasible region for an LPP are (0,2), (3,0), (6,0), (6,8) and (0,5). Let F = 4x + 6y be the objective function. Maximum of F – Minimum of F =

(a) 60 (b) 48 (c) 42 (d) 18

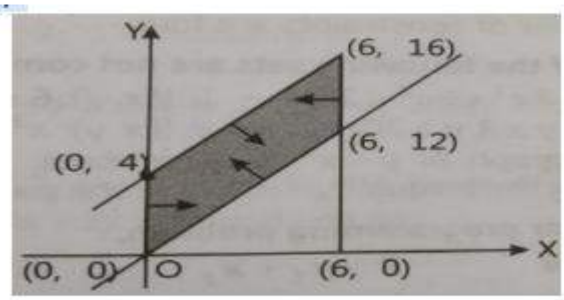
18. In a LPP, if the objective function Z = ax+by has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same..........value.

(a) Minimum (b) Maximum (c) Zero (d) None of these

19. The maximum value of the object function Z = 5x + 10 y subject to the constraints $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$ is

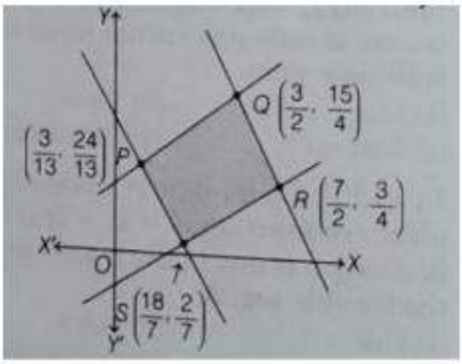
(a) 300 (b) 600 (c) 400 (d) 800

20. The feasible region for an LPP is shown shaded in the figure. Let Z = 3x - 4y be objective function. Maximum value of Z is



(a) 0 (b) 8 (c) 12 (d) -18

21. In the given figure, the feasible region for a LPP is shown. The maximum and minimum value of Z = x + 2y is



(a) 8, 3.2 (b) 9, 3.14 (c) 9, 4 (d) None of these

22. Shape of the feasible region formed by the following constraints $x + y \le 2$, $x + y \ge 5$, $x \ge 0$, $y \ge 0$ is

(a) No feasible region (b) Triangular region (c) Unbounded solution (d) Trapezium

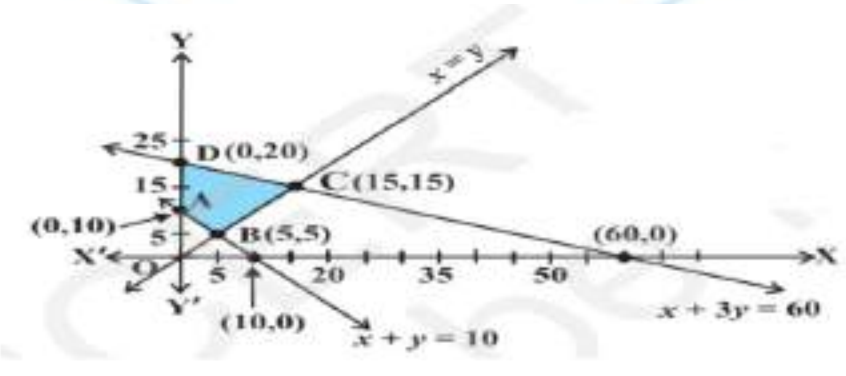
23. The region represented by the inequalities $x \ge 6$, $y \ge 2$, $2x + y \le 0$, $x \ge 0$, $y \ge 0$ is

(a) Unbounded (b) A polygon (c) Exterior of a triangle (d) None of these

24. The maximum value of $z = 2x + 5y$ Subject to constraints given below $2x + 4y \le 8$, $3x + y \le 6$, $x + y \le 4$, $x, y \ge 0$ is

(a) 10 (b) 25 (c) 20 (d) 100

25. Based on the given shaded region as the feasible region in the graph, at which point(s) is the objective function Z = 3x + 9y maximum?



(a) Point B (b) Point C (c) Point D (d) Every point on the line segment CD

26. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 8) and (0, 5). Let Z = 4x + 6y be the objective function. The minimum value of Z occurs at

(a) (0, 2) only

(b)(3,0)

(c) Any point on the line segment joining the point (0, 2) and (3,0)

(d) (6,8)

27. A Linear function, which is minimized or maximized is called

(a) an objective function (b) an optimal function (c) A feasible function (d) None of these

28. Maximize $Z = 4x + 6y$, subject to $3x + 2y \leq 12$, $x + y \geq 4$, $x, y \geq 0$.

(a) 16 at (4, 0) (b) 24 at (0, 4) (c) 24 at (6, 0) (d) 36 at (0, 6)

29. The minimum value of $Z = 4x + 3y$ subjected to the constraints $3x + 2y \geq 160$, $5 + 2y \geq 200$, $2y \geq 80$; $x, y \geq 0$ is

(a) 220 (b) 300 (c) 230 (d) None of these

30. $Z = 7x + y$, subject to $5x + y \geq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$. The minimum value of Z occurs at

(a) (3, 0) (b) (12,52) (c) (7, 0) (d) (0, 5)

31. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called:

(a) feasible solutions (b) constraints

(c) optimal solutions (d) infeasible solutions

32. The common region determined by all the constraints of a linear programming problem is called:

(a) an unbounded region (b) an optimal region

(c) a bounded region (d) a feasible region

33. The number of corner points of the feasible region determined by constraints $x \geq 0$, $y \geq 0$, $x + y \geq 4$ is

(a) 0 (b) 1 (c) 2 (d) 3

34. The point which lies in the half-plane $2x + y - 4 \leq 0$ is

(a) (0, 8) (b) (1, 1) (c) (5, 5) (d) (2, 2)

35. The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then:

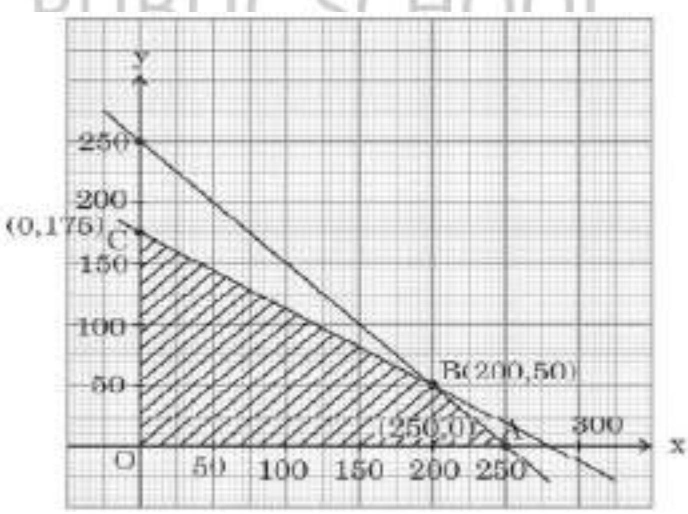
(a) z is maximum at (2, 72), minimum at (15, 20)

(b) z is maximum at (15, 20), minimum at (40, 15)

(c) z is maximum at (40, 15), minimum at (15, 20)

(d) z is maximum at (40, 15), minimum at (2, 72)

36. The corner points of the bounded feasible region of an LPP are O(0, 0), A(250, 0), B(200, 50) and C(0, 175). If the maximum value of the objective function $Z = 2ax + by$ occurs at the points A(250, 0) and B(200, 50), then the relation between a and b is :



(a) 2a = b (b) 2a = 3b (c) a = b (d) a = 2b

37. If the feasible region of a linear programming problem with objective function Z = ax + b y, is bounded, then which of the following is correct?

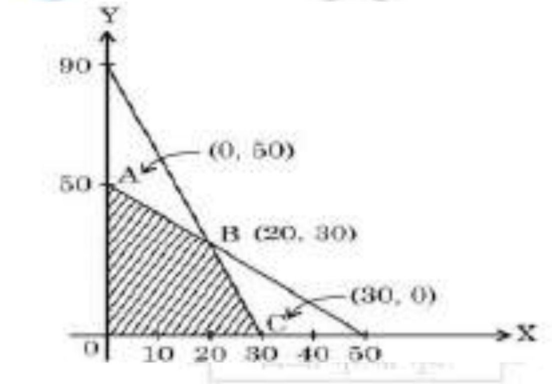
(a) It will only have a maximum value

(b) It will only have a minimum value

(c) It will have both maximum and minimum values

(d) It will have neither maximum nor minimum value

38. The maximum value of Z = 4x + y for a LPP whose region is given below:



(a) 50 (b) 110 (c) 120 (d) 170

39. The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is

(a) 2 (b) 3 (c) 4 (d) 5

40. The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, 15). If Z = 18x + 9y be the objective function, then

(a) Z is maximum at (2, 72), minimum at (15, 20)

(b) Z is maximum at (15, 20), minimum at (40, 15)

(c) Z is maximum at (40, 15), minimum at (15, 20)

(d) Z is maximum at (40, 15), minimum at (2, 72)

41. The solution set of the inequation 3x + 5y < 7 is:

(a) whole xy-plane except the points lying on the line 3x + 5y = 7

(b) whole xy-plane along with the points lying on the line 3x + 5y = 7

(c) open half plane containing the origin except the points of line 3x + 5y = 7

(d) open half plane not containing the origin.

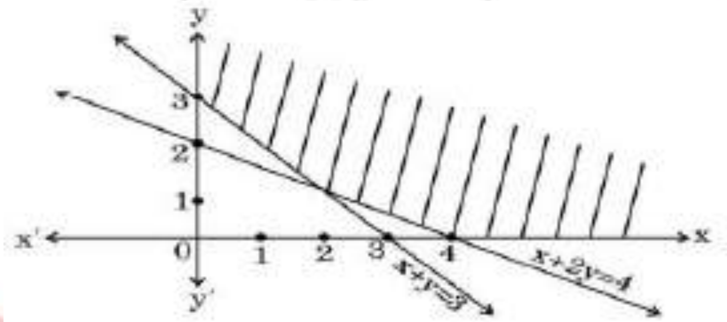
42. The objective function Z = ax + by of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true?

(a) a = 9, b = 1 (b) a = 5, b = 2 (c) a = 3, b = 5 (d) a = 5, b = 3

43. The number of feasible solutions of the linear programming problem given as Maximize z = 15x + 30y subject to constraints : 3x + y ≤ 12, x + 2y ≤ 10, x ≥ 0, y ≥ 0 is

(a) 1 (b) 2 (c) 3 (d) infinite

44. The feasible region of a linear programming problem is shown in the figure below:

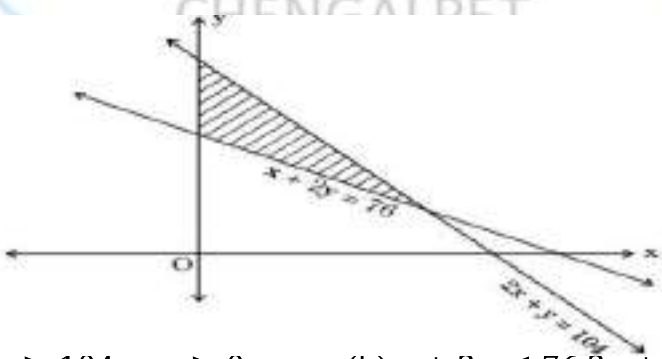


Which of the following are the possible constraints?

(a) $x + 2y \geq 4, x + y \leq 3, x \geq 0, y \geq 0$ (b) $x + 2y \leq 4, x + y \leq 3, x \geq 0, y \geq 0$

(c) $x + 2y \geq 4, x + y \geq 3, x \geq 0, y \geq 0$ (d) $x + 2y \geq 4, x + y \geq 3, x \leq 0, y \leq 0$

45. Of the following, which group of constraints represents the feasible region given below?



(a) $x + 2y \leq 76, 2x + y \geq 104, x, y \geq 0$ (b) $x + 2y \leq 76, 2x + y \leq 104, x, y \geq 0$

(c) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$ (d) $x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0$

ASSERTION - REASON TYPE QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

1. Assertion: Feasible region is the set of points which satisfy all of the given constraints.

Reason: The optimal value of the objective function is attained at the points on X-axis only.

2. Assertion: It is necessary to find objective function value at every point in the feasible region to find optimum value of the objective function.

Reason: For the constrains2x+3y 6, 5x+3y 15, x 0 and y 0 corner points of the feasible region are (0,2), (0,0) and (3,0).

3. Assertion: If an LPP attains its maximum value at two corner points of the feasible region then it attains maximum value at infinitely many points.

Reason: If the value of the objective function of a LPP is same at two corners then it is same at every point on the line joining two corner points

4. Assertion: Bounded region of constraint lies in the first quadrant of $x + y \leq 20$, $3x + 2y \leq 48$, $x \geq 0$, $y \geq 0$.

Reason: $x \geq 0$, $y \geq 0$ are non-negative constraints.

5. Assertion: The set of all feasible solution of a LPP is a convex set.

Reason: Bounded region form a polygon whose each interior angle would be less than 180^o.

6. Assertion: Maximum value of $Z = 11x + 7y$, subject to constraints $2x + y \leq 6$, $x \leq 2$, $x \geq 0$, $y \geq 0$ will be obtained at (0,6) .

Reason: In a bounded feasible region, it always exist a maximum and minimum value.

7. Assertion: The linear programming problem, maximize Z = 2x + 3y subject to constraints $x + y \le 4, x \ge 0, y \ge 0$. It gives the maximum value of Z as 8.

Reason: To obtain maximum value of Z, we need to compare value of Z at all the corner points of the feasible region.

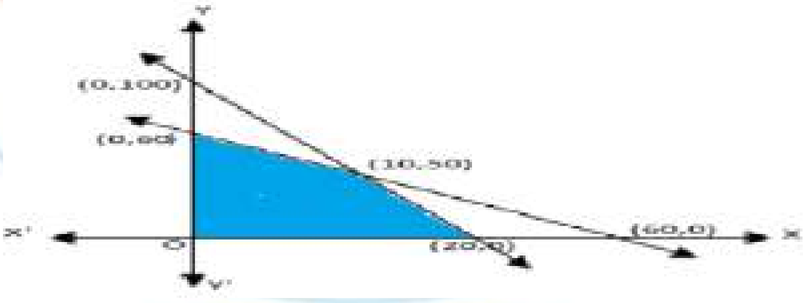
8. Assertion: Objective function $Z = 13x - 15y$, is minimized subject to constraints $x + y \le 7$, $2x - 3y + 6 \ge 0$, $x \ge 0$, $y \ge 0$ occur at corner point (0, 2) .

Reason: If the feasible region of the given LPP is bounded, then the maximum or minimum values of an objective function occur at corner points.

9. Assertion: For an objective function = 4x + 3y , corner points are (0,0), (25,0) , (16,16) and (0,24) . Then optimal values are 112 and 0 respectively.

Reason: The maximum or minimum values of an objective function is known as optimal value of LPP. These values are obtained at corner points.

10. Consider the graph of the constraints stated by linear inequations $5x + y \le 100$, $y + x \le 60$, and x, $y \ge 0$.



Assertion: (25, 40) is infeasible solution of the problem.

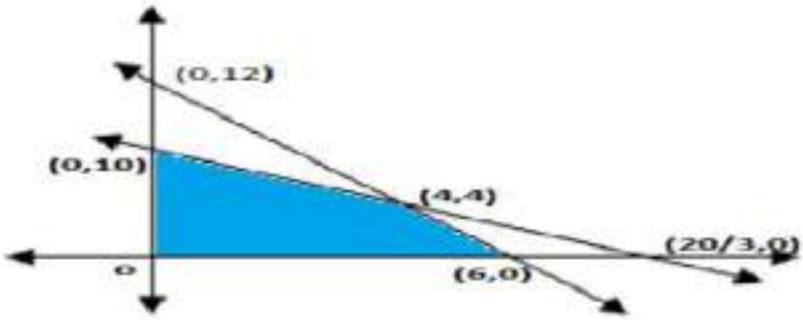
Reason: Any point which lies in feasible region is infeasible solution.

11. Assertion: The Maximum value of Z = 11x +7y, subject to the constraints $2x + y \le 6$, $x \le 2$, x, $y \ge 0$, occurs at corner point (0,6)

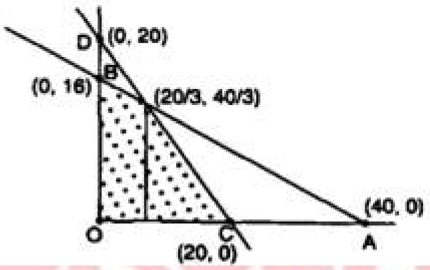
Reason: If the feasible region of given LPP is bounded then the maximum and minimum value occurs at the corner points.

12. Assertion: The maximum value of Z = 5x + 3y subject to the constraints stated by linear inequations $2x + y \leq 12$, $2y + 3x \leq 20$, and $x, y \geq 0$ is at (4,4)

Reason: If the feasible region of given LPP is bounded then the maximum and minimum value occurs at the corner points



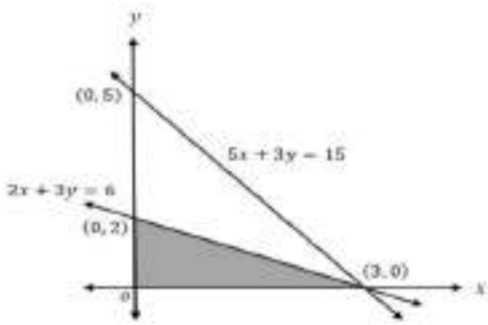
13. Assertion: Shaded region represented by $2x + 5y \geq 80$, $x + y \leq 20$, $x \geq 0$, $y \geq 0$ is



Reason: A region or a set of points is said to be convex if the line joining any two of its points lies completely in the region.

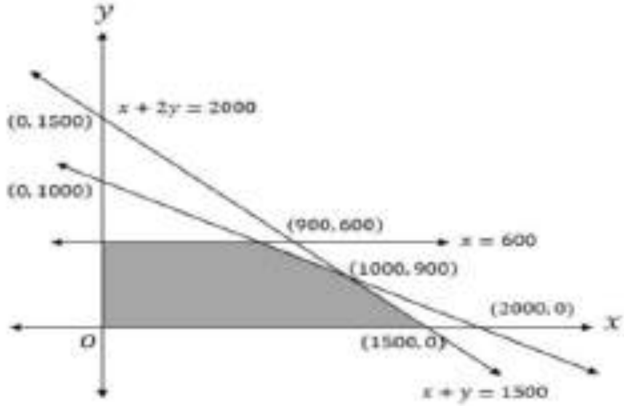
14. Assertion: The solutions of constraints must be checked by substituting them back into objective function.

Reason(R): Here, (0, 2), (0, 0) and (3, 0) all are vertices of feasible region.



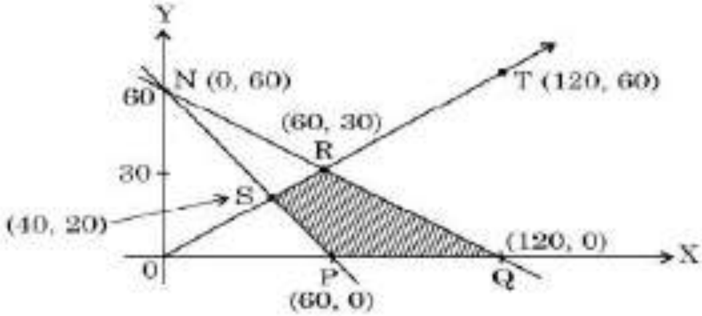
15. Assertion: For the constraints of an LPP given by x + 2y ≤ 2000, x + y ≤ 1500, y ≤ 600 and x, y ≥ 0, the points (1000, 0), (0, 500), (2, 0) lie in the positive bounded region, but point (2000, 0) does not lie in the positive bounded region.

Reason:



From the graph, it is clear that the point (2000, 0) is outside of the feasible region.

16. Assertion: The corner points of the bounded feasible region of a LPP are shown below. The maximum value of Z = x + 2y occurs at infinite points.



Reason: The optimal solution of a LPP having bounded feasible region must occur at corner points.

SHORT ANSWERS

1. Maximize Z = 80x + 120y subject to the constraints 3x + 4y ≤ 60, x + 3y ≤ 30, x ≥ 0, y ≥ 0.

2. Maximize Z = 100x + 120y subject to the constraints 5x + 8y ≤ 200, 5x + 4y ≤ 30, x, y ≥ 0.

3. Maximize Z = 300x + 190y subject to constraints $x + y \le 24, x + \frac{1}{2}y \le 16, x, y \ge 0$.

4. Maximize Z = 5x + 8y subject to the constraints: x + y = 5, x ≤ 4, y ≥ 2, x, y ≥ 0.

5. Maximize Z = 10500x + 9000y subject to constraints $x + y \leq 50$, $2x + y \leq 80$, $x, y \geq 0$.

6. Solve the following linear programming problem graphically: Maximize Z = 34x + 45y under the following constraints $x + y < 300$, $2x + 3y < 70$, $x > 0$, $y > 0$.

7. Solve the following L.P.P. graphically: Minimize Z = 5x + 10y, subject to constraints $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$.

8. Solve the following Linear Programming problem graphically: Minimize: Z = 6x + 3y, subject to the constraints $4x + y \geq 80$, $x + 5y \geq 115$, $3x + 2y \leq 150$, $x, y \geq 0$.

9. Solve the following Linear Programming problem graphically: Minimize Z = 3x + 3.5y, subject to the constraints $x + 2y \geq 240$, $3x + 1.5y \geq 270$, $1.5x + 2y \leq 310$, $x, y \geq 0$.

10. Graphically find x and y to Maximize Z = 4x + 3y, subject to constraints $3x + 4y \leq 24$, $8x + 6y \leq 48$, $x \leq 5$, $y \leq 6$, $x, y \geq 0$.

11. Solve the following Linear Programming Problem graphically: Maximize z = 10x + 15y subject to constraints: $3x + 2y \leq 50$, $x + 4y \geq 20$, $x \geq 8$, $y \geq 0$.

12. Determine graphically the minimum value of the following objective function: z = 500x + 400y, subject to constraints: $x + y \leq 200$, $x \geq 20$, $y \geq 4x$, $y \geq 0$.

13. Solve graphically the following linear programming problem: Maximise Z = 6x + 3y, subject to the constraints $4x + y \geq 80$, $3x + 2y \leq 150$, $x + 5y \geq 115$, $x \geq 0$, $y \geq 0$.

14. Solve the following linear programming problem graphically: Maximize P = 100x + 5y, subject to the constraints: $x + y \leq 300$, $3x + y \leq 600$, $y \leq x + 200$, $x, y \geq 0$.

15. Solve the following linear programming problem graphically: Maximize z = 600x + 400y subject to the constraints: $x + 2y \leq 12$, $2x + y \leq 12$, $x + 1.25 \geq 5$, $x, y \geq 0$.

16. Solve the following linear programming problem graphically: Minimize $Z = 5x + 10y$, subject to constraints: $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$.

17. Solve the following linear programming problem graphically: Maximize $Z = x + 2y$, subject to constraints: $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, $y \geq 0$.

18. Solve the following Linear programming problem graphically: Maximize $Z = 3x + 3.5y$ subject to constraints: $x + 2y \geq 240$, $3x + 1.5y \geq 270$, $1.5x + 2y \leq 310$, $x \geq 0$, $y \geq 0$.

19. Solve the following linear programming problem graphically: Maximize $z = 3x + 9y$ subject to constraints: $x + y \geq 10$, $x + 3y \leq 60$, $x \leq y$, $x \geq 0$, $y \geq 0$.

20. Solve the following linear programming problem graphically: Minimize $z = x + 2y$, subject to the constraints: $2x + y \geq 3$, $x + 2y \geq 6$, $x \geq 0$, $y \geq 0$.

21. Solve the following linear programming problem graphically: Minimise $z = -3x + 4y$ subject to the constraints: $x + 2y \leq 8$, $3x + 2y \leq 12$, $x, y \geq 0$.

22. Solve the following linear programming problem graphically: Maximise $z = -3x - 5y$ subject to the constraints $-2x + y \leq 4$, $x + y \geq 3$, $x - 2y \leq 2$, $x \geq 0$, $y \geq 0$.

23. Solve the following linear programming problem graphically: Maximise $z = 5x + 3y$ subject to the constraints: $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x, y \geq 0$.

24. Solve the following linear programming problem graphically: Maximise $z = 500x + 300y$, subject to constraints: $x + 2y \leq 12$, $2x + y \leq 12$, $4x + 5y \geq 20$, $x \geq 0$, $y \geq 0$.

25. Solve the following Linear Programming Problem graphically: Minimize $Z = 60x + 80y$ subject to constraints: $3x + 4y \geq 8$, $5x + 2y \geq 11$, $x, y \geq 0$.

26. Solve the following Linear Programming Problem graphically: Minimise z = 3x + 8y subject to the constraints $3x + 4y \geq 8$, $5x + 2y \geq 11$, $x \geq 0$, $y \geq 0$.

27. Solve the following linear programming problem graphically: Maximise z = 5x – 2y, subject to the constraints: $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$.

$\text{MinZ} = 160$ at $x = 40, y = 20$

28. Solve the following linear programming problem graphically: Maximise z = 4x + 3y, subject to the constraints: $x + y \leq 800$, $2x + y \leq 1000$, $x \leq 400$, $x, y \geq 0$.

29. Solve the following linear programming problem graphically: Minimize z = 600x + 400y subject to constraints: $x + y \geq 8$, $x + 2y \leq 16$, $4x + y \leq 29$, $x, y \geq 0$.

30. Solve the following linear programming problem graphically: Maximize z = x + y subject to constraints $2x + 5y \leq 100$, $8x + 5y \leq 200$, $x \geq 0$, $y \geq 0$.

31. Solve that following linear programming problem graphically: Maximise Z = 2x + 3y subject to constraints: $x + y \leq 6$, $x \geq 2$, $y \leq 3$, $x, y \geq 0$.

32. Solve the following LPP: Maximise Z = x + 3y, subject to the constraints: $x + 2y \leq 200$, $x + y \leq 150$, $y \leq 75$, $x, y \geq 0$.

33. Solve the following LPP graphically: Maximize Z = 60x + 40y, subject to the constraints: $x + 2y \leq 12$, $2x + y \leq 12$, $4x + 5y \geq 20$, $x, y \geq 0$.

34. Solve the following LPP graphically: Minimise Z = 6x + 3y subject to constraints: $4x + y \geq 80$, $x + 5y \geq 115$, $3x + 2y \leq 150$, $x, y \geq 0$.

LONG ANSWERS

1. Find graphically, the maximum value of Z = 2x + 5y, subject to constraints given below: $2x + 4y \leq 8$, $3x + y \leq 6$, $x + y \leq 4$, $x \geq 0$, $y \geq 0$.

2. Maximize Z = 8x + 9y subject to the constraints given below: $2x + 3y \le 6$, $3x - 2y \le 6$, $y \le 1$, $x \ge 0$, $y \ge 0$.

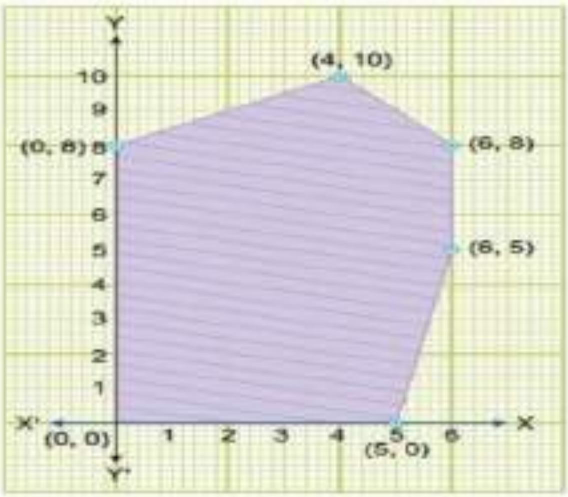
3. Solve the following problem graphically. Maximize Z = 3x + 9y. Subject to the constraints : $x + 3y \le 60$, $x + y \ge 10$, $x \le y$, $x \ge 0$, $y \ge 0$.

4. Solve the following linear programming problem (L.P.P) graphically. Maximize Z = x + 2y subject to constraints; $x + 2y \ge 100$, $2x - y \le 0$, $2x + y \le 200$, $x, y \ge 0$.

5. Solve the following LPP graphically: Maximize Z = 3x + 2y subject to $5x + y \ge 10$, $x + y \ge 6$, $x + 4y \ge 12$, $x, y \ge 0$.

CASE BASED QUESTIONS

1. The corner points of the feasible region determined by the system of linear constraints are as shown below:

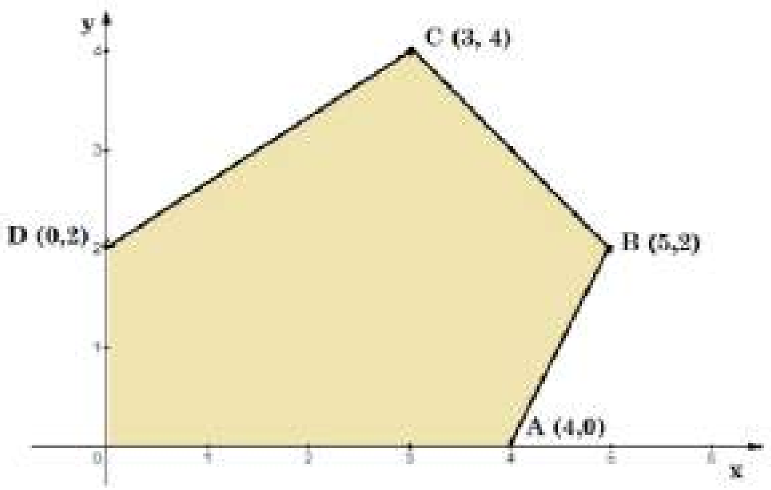


Based on the above information, answer the following:

(i) Let Z = 3x – 4y be the objective function. Find the maximum and minimum value of Z and, also the corresponding points at which the maximum and minimum value occurs.

(ii) Let Z = px + qy where p, q > 0 be the objective function. Find the condition on p and q so that the maximum value of Z occurs at B(4,10) and C(6, 8) . Also mention the number of optimal solutions in this case.

2. The corner points of the feasible region determined by the system of linear constraints are as shown below:

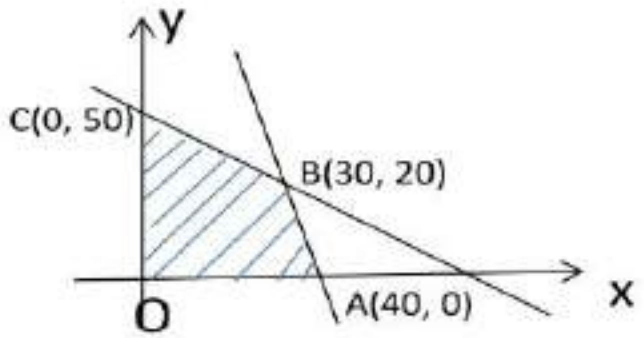


Based on the above information, answer the following:

(i) Let Z = 13x – 15y be the objective function. Find the maximum and minimum values of z and, also the corresponding points at which the maximum and minimum values occur.

(ii) Let Z = kx + y be the objective function. Find k, if the value of z at A is same as the value of z at B.

3. The feasible region of an LPP is shown in the figure.



(i) Find the equation of AB.

(ii) Find equation of BC.

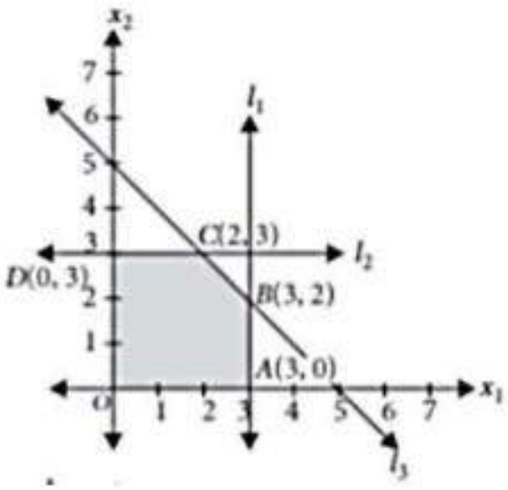
(iii) Determine at which point the objective function Z = 10500x + 9000y is maximum.

4. Corner points of the feasible, region for an LPP are (0, 3), (5, 0), (6, 8), (0, 8). Let Z = 4x – 6y be the objective function.

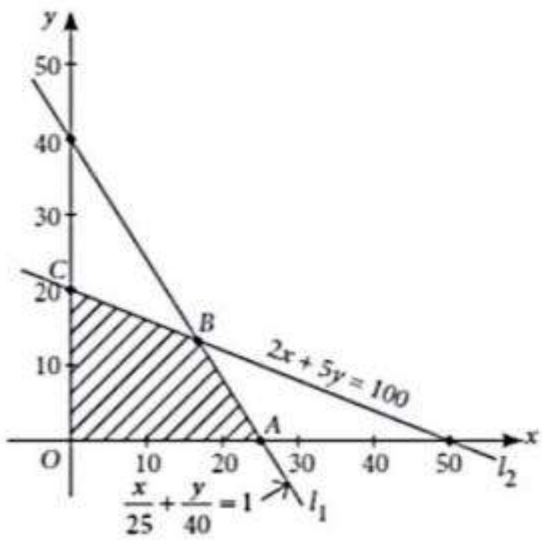
Based on the above information, answer the following questions.

(i) Find the value of Max Z – Min Z.

(ii) Find the corner points of the feasible region determined by the system of linear inequalities in the graph given below.



5. The feasible region for the LPP is shown below:

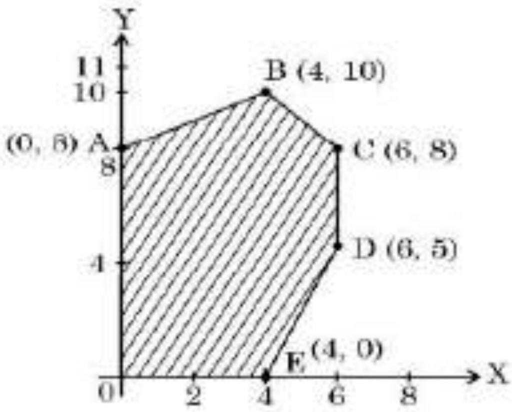


(i) What is the point of intersection of line l_1 and l_2?

(ii) If Z = x + y be the objective function and max Z = 30. Find the point where maximum value occurs.

(iii) If Z = 6x + 3y be the objective function, then what is the minimum value of Z?

6. The corner points of the feasible region determined by the system of linear constraints are as shown in the following figure:



(i) If Z = 3x – 4y be the objective function, then find the maximum value of Z.

(ii) If Z = px + qy where p, q > 0 be the objective function. Find the condition of p and q so that maximum value of Z occurs at B(4, 10) and C(6, 8).

7. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

Figure-1

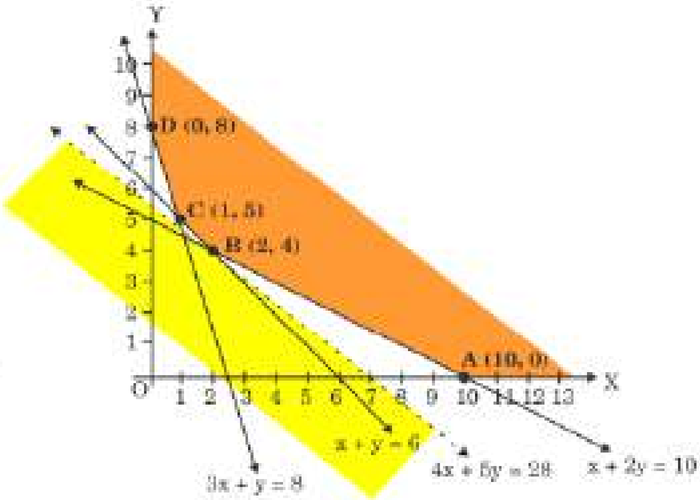


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of ₹16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

On the basis of the above information, answer the following questions:

(i) Identify and write all the constraints which determine the given feasible region in Figure-2.

(ii) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region.

UNIT TEST

Duration: 1 hour **Marks: 30**

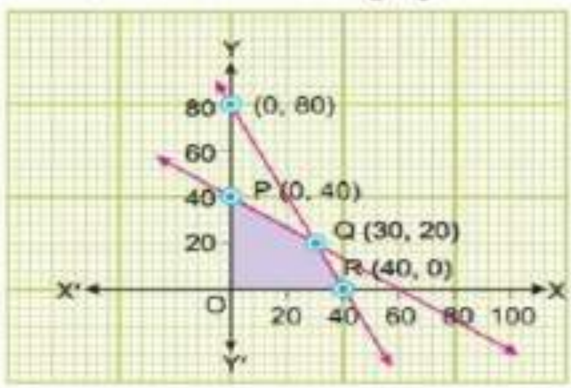
SECTION A

Each carry 1 mark

1. The corner points of the feasible region determined by the set of constraints are P(0, 5), Q(3, 5), R(5, 00 and S(4, 1) and the objective function Z = ax + 2by where a, b > 0. The condition on a and b such that the maximum Z occurs at Q and S is

(a) a – 5b = 0 (b) a – 3b = 0 (c) a – 2b = 0 (d) a – 8b = 0

2. For a LPP the objective function is Z = 4x + 3y and the feasible region determined by a set of constraints is shown in the graph



Which of the following statement is true?

(a) Maximum value of Z is at R (b) Maximum value of Z is at Q

(c) Value of Z at R is less than the value at P (d) Value of Z at Q is less than the value of R

3. The graph of the inequality $2x + 3y > 6$ is

(a) Half plane that contains the origin

(b) Half plane that neither contains the origin nor the points of the line 2x + 3y = 6

(c) Whole XOY plane excluding the points on the line 2x + 3y = 6

(d) Entire XOY plane

4. Assertion: A LPP admits unique optimal solution.

Reason: The solution set of the inequation $2x+y> 5$ is open half plane not containing the origin.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 3 marks

5. Solve the following linear programming problem graphically: Maximum $Z = 3x + 9y$ subject to the constraints $x + y \geq 10$, $x + 3y \leq 60$, $x \leq y$, $x \geq 0$, $y \geq 0$.

6. Solve the following linear programming problem graphically: Minimize $Z = 5x + 10y$ subject to constraints: $x + 2y \leq 120$, $x + y \geq 60$, $x – 2y \geq 0$, $x \geq 0$, $y \geq 0$.

7. Solve graphically the following linear programming problem: Maximize $Z = 6x + 3y$, subject to the constraints $4x + y \geq 80$, $3x + 2y \leq 150$, $x + 5y \geq 115$, $x \geq 0$, $y \geq 0$.

8. Minimize $Z = 10x + 4y$ subject to constraints $4x + y \geq 80$, $2x + y \geq 60$, $x \geq 0$, $y \geq 0$.

SECTION D

Each carry 5 marks

9. Solve the following LPP graphically: Minimize $Z = 5x + 7y$ subject to the constraints $2x + y \geq 8$, $x + 2y \geq 10$, $x \geq 0$, $y \geq 0$.

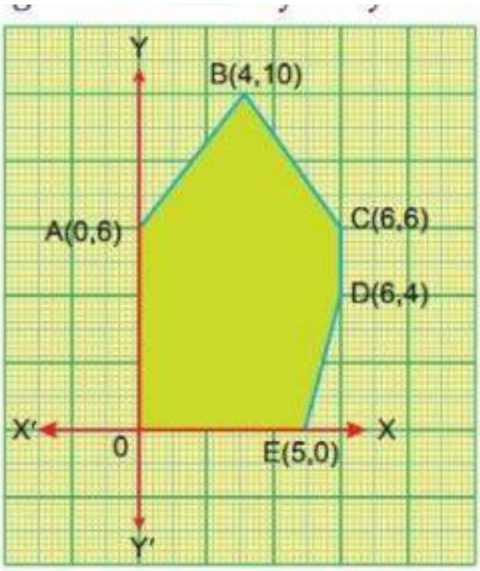
10. Minimize and Maximize $Z = 5x + 2y$ subject to the following constraints $x – 2y \leq 2$, $3x + 2y \leq 12$, $-3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$.

SECTION E

11. Read the following passage and answer the following questions:

Linear programming problem is method of or finding the optimal values (maximum or minimum) of quantities subject to the constraints when relationship is expressed as a linear equations or linear inequations.

The corner points of a feasible region determined by the system of linear constraints are as shown below.



(i) Is this feasible region bounded?

(ii) Write the number of corner points in the feasible region.

(iii) If Z = ax + by has maximum value at C(6, 6) and B(4, 10). Find the relationship between a and b.

ANSWERS

MCQ	A-R	SA	LA	CS
1. (c) $x + 2y \geq 4, x + y \geq 3, x \geq 0, y \geq 0$ 2. (c) a = 3, b = 5 3. (c) Z is maximum at (40, 15), minimum at (15, 20) 4. (a) 2 5. (a) 21 6. (a) 0 7. (b) 4 8. (b) 2x + y ≤ 104 and x + 2y ≤ 76	1. (c) 2. (d) 3. (a) 4. (a) 5. (a) 6. (b) 7. (d) 8. (a) 9. (a) 10. (c)	1. Max Z = 1680 at (12, 6) 2. Max Z = 3200 at (8, 20) 3. Max Z = 5440 at (8, 16) 4. Max Z = 40 at (0, 5) 5. Max Z = 495000 at (30, 20) 6. Max Z = 1190 at (35, 0) 7. Min Z = 300 at (60, 0) 8. Min Z = 150 at (15, 20) 9. Min z = 470 at (40, 100)	1. Max z = 10 at (0, 2) 2. Max z = $\frac{294}{13}$ at $\left(\frac{30}{13}, \frac{6}{13}\right)$ 3. Max Z = 180 at every point on the line segment joining the points (15, 15) & (0, 20) 4. Max Z = 250 at (50, 100) 5. Max Z = 36 at	1. (i) Min Z = -32 at (0, 8) and Max Z = 12 at (4, 0) (ii) p = q 2. (i) Min Z = -30 at (0, 2) and Max Z = 52 at (4, 0) (ii) k = -2 3. (i) 2x + y = 80 (ii) x + y = 50 (iii) B 4. (i) 68

9. (c) 25 10. (d) Infinite 11. (b) p = $\frac{q}{2}$ 12. (c) Given by corner points of the feasible region 13. (c) R 14. (c) (2, 3) 15. (b) A linear function to be optimized 16. (a) Concave set 17. (a) 60 18. (b) Maximum 19. (b) 600	11. (a) 12. (a) 13. (d) 14. (d) 15. (a) 16. (b)	10. Max Z = 24 at every point on the line segment joining the points $(\frac{24}{7}, \frac{24}{7})$ & $(5, \frac{4}{3})$ 11. z_{max} is 275 when $x = 8, y = 13$ 12. Minimum z = 42000 at (20, 80) 13. (40, 15) & Max Z = 285 14. (200, 0), Max P = 20000 15. (4, 4), Max z = 4000 16. Min(Z) = 300 at	(12, 0)	(ii) (0, 0), (3, 0), (3, 2), (2, 3), (0, 3) 5. (i) $\left(\frac{50}{3}, \frac{40}{3}\right)$ (ii) $\left(\frac{50}{3}, \frac{40}{3}\right)$ (iii) 0 6. (i) Maximum value of Z is 12 at E when $x = 4, y = 0$ (ii) p = q 7. (i) x + 2y ≥ 10, x + y ≥ 6, 3x + y ≥ 8, x ≥ 0, y ≥ 0. (ii) The minimum cost is ₹ 112

20. (a) 0 21. (b) 9, 3.14 22. (a) No feasible region 23. (d) None of these 24. (a) 10 25. (d) Every point on the line segment CD 26. (c) Any point on the line segment joining the point (0, 2) and (3,0) 27. (a) an objective function 28. (d) 36 at (0, 6) 29. (a) 220		x = 60; y = 0 17. $\text{Min}(Z) = 400$ at $x = 0; y = 200$ 18. $\text{Max}(Z) = 595$ at $x = 140; y = 50$ 19. Maximum lies at every point on the line segment AD. 20. Minimum z = 6 at all points on line segment AB 21. Minimum z = -12 at (4, 0) 22. Max $z = -\frac{29}{3}$ at $\left(\frac{8}{3}, \frac{1}{3}\right)$		

30. (d) (0, 5) 31. (b) constraints 32. (d) a feasible region 33. (c) 2 34. (b) (1, 1) 35. (c) z is maximum at (40, 15), minimum at (15, 20) 36. (a) 2a = b 37. (c) It will have both maximum and minimum values 38. (c) 120 39. (a) 2 40. (c) Z is maximum at (40, 15),		23. Max z = $\frac{235}{19}$ at $(\frac{20}{19}, \frac{45}{19})$ 24. Max z = 3200 at x = 4, y = 4 25. Minimum Z = 160 at all points of the line AC 26. $z_{min} = 8$ when $x = \frac{8}{3}, y = 0$ 27. MinZ = 160 at x = 40, y = 20 28. $z_{max} = 2600$ when $x = 200, y = 600$ 29. $z_{min} = 3200$ when $x = 0, y = 8$ 30. $z_{max} = 30$ when		

minimum at (15, 20) 41. (c) open half plane containing the origin except the points of line 3x + 5y = 7 42. (c) a = 3, b = 5 43. (d) infinite 44. (c) $x + 2y \geq 4, x + y \geq 3, x \geq 0,$ $y \geq 0$ 45. (c) $x + 2y \geq 76, 2x + y \leq 104,$ $x, y \geq 0$		$x = \frac{50}{3}, y = \frac{40}{3}$ 31. Maximum Value of Z is 15 at $x = 3, y = 3.$ 32. Maximum Value of Z is 275 at $x = 50 \& y = 75$ 33. $\text{Max}(Z) = 400$ at $x = 4, y = 4$ 34. $\text{Min}(Z) = 150$ at $x = 15, y = 20$		

UNIT TEST

1. (d) a – 8b = 0

2. (b) Maximum value of Z is at Q

3. (b) Half plane that neither contains the origin nor the points of the line 2x + 3y = 6

4. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

5. $Z_{Max} = 180$ at infinitely many points lying on the line joining points $(0,20)$ and $(15,15)$.

6. Min Z = 300 at (60, 0)

7. Max Z = 285 at (40, 15)

8. Min Z = 260 at (10, 40)

9. Min Z = 38 at (2, 4)

10. Min Z = 0 at (0, 0) and Max Z = 19 at $\left(\frac{7}{2}, \frac{3}{4}\right)$

11. (i) Yes (ii) 6 (iii) a = 2b

CHAPTER 13 – PROBABILITY

CONDITIONAL PROBABILITY

DEFINITION 1

If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e. $P(E \mid F)$ is given by

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} \text{ provided } P(F) \neq 0$$

PROPERTIES OF CONDITIONAL PROBABILITY

Property 1

Let E and F be events of a sample space S of an experiment, then we have

$$P(S \mid F) = P(F \mid F) = 1$$

Property 2

If A and B are any two events of a sample space S and F is an event of S such that $P(F) \neq 0$, then

$$P((A \cup B)|F) = P(A|F) + P(B|F) - P((A \cap B)|F)$$

Property 3

$$P(E' \mid F) = 1 - P(E \mid F)$$

MULTIPLICATION THEOREM ON PROBABILITY

Let E and F are two events associated with a sample space S and then $E \cap F$ denotes the event that both E and F have occurred. The event $E \cap F$ is also written as EF.

$$P(E \cap F) \text{ or } (EF) = P(E)P(F \mid E) = P(F)P(E \mid F) \text{ provided } P(E) \neq 0 \text{ and } P(F) \neq 0.$$

The above result is known as the multiplication rule of probability.

MULTIPLICATION RULE OF PROBABILITY FOR MORE THAN TWO EVENTS

If E, F and G are three events of sample space, we have

$$P(E \cap F \cap G) = P(E)P(F \mid E)P(G \mid (E \cap F)) = P(E)P(F \mid E)P(G \mid EF)$$

Note: Multiplication rule of probability can be extended to four or more events.

INDEPENDENT EVENTS

Two events E and F are said to be independent events if the occurrence (or non-occurrence) of one does not affect the probability of the occurrence (and hence non-occurrence) of the other.

Illustration 1: In the simultaneous throw of two coins 'getting a head' on first coin and 'getting a tail' on the second coin are independent events.

Illustration 2: When a card is drawn from a pack of well shuffled cards and replaced before the second card is drawn, the result of second draw is independent of first draw.

Let E and F be two events associated with the same random experiment, then E and F are said to be independent if

$$P(E \cap F) = P(E).P(F).$$

RESULTS

(i) Two events E and F are said to be dependent if they are not independent, i.e. if

$$P(E \cap F) \neq P(E).P(F)$$

(ii) Three events A, B and C are said to be mutually independent, if

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cap C) = P(A)P(C)$$

$$P(B \cap C) = P(B)P(C)$$

$$P(A \cap B \cap C) = P(A)P(B)P(C)$$

If at least one of the above is not true for three given events, we say that the events are not independent.

(iii) If E and F are independent events, then

(a) E and F' are independent events

(b) E' and F are independent

(c) E' and F' are independent

PARTITION OF A SAMPLE SPACE

A set of events $E_1, E_2, \ldots, E_n$ is said to represent a partition of the sample space S if

(a) $E_i \cap E_j = \phi, i \neq j, i, j = 1,2,3, \ldots, n$

(b) $E_1 \cup E_2 \cup \ldots \cup E_n = S$ and

(c) $P(E_j) > 0$ for all $i = 1,2, \ldots, n$.

In other words, the events $E_1, E_2, \ldots, E_n$ represents a partition of the sample space S if they are pairwise disjoint, exhaustive and have non-zero probabilities.

THEOREM OF TOTAL PROBABILITY

Let $\{E_1, E_2, \ldots, E_n\}$ be a partition of the sample space S and suppose that each of the events $E_1, E_2, \ldots, E_n$ has non-zero probability of occurrence. Let A be an event associated with S, then

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P(A_| \mid E_2) + \cdots + P(E_n)P(A \mid E_n)$$

$$P(A) = \sum_{j=1}^{n} P(E_j)P(A \mid E_j)$$

BAYE'S THEOREM

If $E_1, E_2, \ldots, E_n$ are n non empty events which constitute a partition of sample space S, i.e. $E_1, E_2, \ldots, E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup \ldots \cup E_n = S$ and A is any event of non-zero probability, then

$$P(E_i \mid A) = \frac{P(E_i)P(A \mid E_i)}{\sum_{j=1}^{n} P(E_j)P(A \mid E_j)} \text{ for any } i = 1,2,3, \ldots, n$$

MULTIPLE CHOICE QUESTIONS

1. If $P(A \mid B) = P(A' \mid B)$, then which of the following statements is true?

(a) $P(A) = P(A')$ (b) P(A) = 2P(B) (c) $P(A \cap B) = \frac{1}{2}P(B)$ (d) $P(A \cap B)$ = 2 P(B)

2. Let E and F be two events such that P(E) = 0.1, P(F) = 0.3, $P(E \cup F) = 0 \cdot 4$, then P(F|E) is

(a) 0.6 (b) 0.4 (c) 0.5 (d) 0

3. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then:

(a) $A \subset B$, but $A \neq B$ (b) A = B (c) $A \cap B = \emptyset$ (d) P(A) = P(B)

4. If A and B are two events such that $P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$ and $P(B') = \frac{1}{2}$, then A and B are

(a) independent (b) dependent (c) mutually exclusive (d) None of these

5. Two numbers are selected at random from integers 1 to 9. If the sum is even, the probability that both numbers being odd is:

(a) $\frac{1}{6}$ (b) $\frac{2}{3}$ (c) $\frac{4}{9}$ (d) $\frac{5}{8}$

6. If one ball is drawn at random from each of three boxes containing 3 white and 1 black, 2 white and 2 black. 1 white and 3 black balls, then the probability that 2 white and 1black balls will be drawn is

(a) $\frac{13}{32}$ (b) $\frac{1}{4}$ (c) $\frac{1}{32}$ (d) $\frac{3}{16}$

7. If $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then $P\left(\frac{B}{A}\right) + P\left(\frac{A}{B}\right)$ equals

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{5}{12}$ (d) $\frac{7}{12}$

8. If $P\left(\frac{A}{B}\right) = 0.3, P(A) = 0.4$ and $P(B) = 0.8$, then $P\left(\frac{B}{A}\right)$ is equal to:

(a) 0.6 (b) 0.3 (c) 0.06 (d) 0.4

9. For two events A and B, if $\mathrm{P(A)} = 0.4, \mathrm{P(B)} = 0.8$ and $\mathrm{P(B/A)} = 0.6$, then $P(A \cup B)$ is:

(a) 0.24 (b) 0.3 (c) 0.48 (d) 0.96

10. If for two events A and $B, P(A - B) = \frac{1}{5}$ and $P(A) = \frac{3}{5}$, then $P\left(\frac{B}{A}\right)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{2}{3}$

11. If A and B are two events such that $\mathrm{P(A \mid B)} = 2\mathrm{P(B \mid A)}$ and $\mathrm{P(A)} + \mathrm{P(B)} = \frac{2}{3}$, then $\mathrm{P(B)} =$

(a) $\frac{2}{9}$ (b) $\frac{7}{9}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

12. For any two events A and B if $P(\overline{A}) = \frac{1}{2}, P(\overline{B}) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P\left(\frac{\overline{A}}{\overline{B}}\right) =$

(a) $\frac{5}{8}$ (b) $\frac{8}{9}$ (c) $\frac{1}{8}$ (d) $\frac{1}{4}$

13. The probability that A speaks the truth is $\frac{4}{5}$ and that of B speaking the truth is $\frac{3}{4}$. The probability that they contradict each other in stating same fact is:

(a) $\frac{7}{20}$ (b) $\frac{1}{5}$ (c) $\frac{3}{20}$ (d) $\frac{4}{5}$

14. Let E be an event of a sample space S of an experiment, then P(S|E) =

(a) $P(S \cap E)$ (b) P(E) (c) 1 (d) 0

15. Ashima can hit a target 2 out of 3 times. She tried to hit the target twice. The probability that she missed the target exactly once is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{4}{9}$ (d) $\frac{1}{9}$

16. X and Y are independent events such that $P(X \cap \overline{Y}) = \frac{2}{5}$ and $P(X) = \frac{3}{5}$. Then P(Y) is equal to

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{3}$ (d) $\frac{1}{5}$

17. If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4 is

(a) $\frac{1}{9}$ (b) $\frac{4}{9}$ (c) $\frac{1}{18}$ (d) $\frac{1}{2}$

18. If A and B are two events such that $P(A/B) = 2 \times P(B/A)$ and $P(A) + P(B) = \frac{2}{3}$, then P(B) is equal to

(a) $\frac{2}{9}$ (b) $\frac{7}{9}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

19. The events E and F are independent. If P(E) = 0.3 and P(EUF) = 0.5, then P(E|F) – P(F|E) equals:

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{35}$ (d) $\frac{1}{70}$

20. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is

(a) $\frac{27}{32}$ (b) $\frac{5}{32}$ (c) $\frac{31}{32}$ (d) $\frac{1}{32}$

21. A fair die is rolled. Events E and F are E = {1, 3, 5} and F = {2, 3} respectively. Value of P(E|F) is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{2}$

ASSERTION - REASON TYPE QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

1. Assertion: If R and S are two events such that P(R | S) = 1 and P(S) > 0, then

Reason: If two events A and B are such that $P(A \cap B) = P(B)$, then $A \subset B$.

2. Assertion: For an event A of the sample space S of a random experiment, P(S|A) = P(A|A) = 1.

Reason: For three events A, B and C of the sample space S of a random.

3. Assertion: Probability of drawing a red ball from any of the bag I or II, when bag I contains 3 red and 5 blue balls whereas bag II contains 4 red and 4 blue balls is $\frac{7}{16}$.

Reason: There are two bags I and II , bag I contains 3 red and 5 blue balls whereas bag II contains 4 red and 4 blue balls . A ball is drawn from either of bag and is found red . Probability of it to be from bag I is $\frac{3}{7}$.

4. Assertion(A): Two coins are tossed simultaneously. The probability of getting two heads, if it is known that atleast one head comes up is $\frac{1}{3}$.

Reason: Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

VERY SHORT ANSWERS

SHORT ANSWERS

1. It is known that 20% of the students in a school have above 90% attendance and 80% of the students are irregular. Past year results show that 80% of students who have above 90% attendance and 20% of irregular students get 'A' grade in their annual examination. At the end of a year, a student is chosen at random from the school and is found to have an 'A' grade. What is the probability that the student is irregular?

2. E and F are two independent events such that $P(\bar{E}) = 0.6, P(E \cup F) = 0 \cdot 6$. Find P(F) and $P(\bar{E} \cup \bar{F})$.

3. The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0·3, 0·8 and 0·5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

4. A card from well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a king. Find the probability of the lost card being a king.

5. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

6. A bag contains 7 red, 4 white and 5 black balls. Two balls are drawn at random and are found to be white. What is the probability that both the balls are white?

7. Assume that each born child is equally likely to be a boy or a girl. If family has two children, what is the conditional probability that both are girls? Given that (i) the youngest is a girl (ii) atleast one is a girl.

8. 12 cards numbered 1 to 2 are placed in a box, mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more that 3, then find the probability that it is an even number.

9. A bag contains 3 red and 7 black balls. Two balls are selected at random one-by-one without replacement. If the second selected ball happens to be red, what is the probability that the first selected ball is also red?

10. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectviely. If both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solved the problem.

11. A speaks truth in 75% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact? Do you think that statement of B is true?

12. Three persons A, B and C apply for a job of manager in private company. Chances of their selection are in the ration 1:2:4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the changes do not take place, find the probability that it is due to appointment of C.

13. Three machines E_1, E_2 and E_3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines E_1 and E_2 are defective and that 5% of those produced by machine E_3 are defective. If one bulb is picked up at random from a day's production, calculate the probability that is defective.

14. There are two bags, bag I and bag II. Bag I contains 4 white and 3 red balls while another bag II contains 3 white and 7 red balls. One ball is drawn at random from one of the bags and it found it to be white. Find the probability that it was drawn from bag I.

15. There are two coins. One of them is a biased coin such that P (head):P (tail) is 1:3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

16. A and B are independent events such that $P(A \cap \overline{B}) = \frac{1}{4}$ and $P(\overline{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).

17. A fair coin and an unbiased die are tossed. Let A be the event, "Head appears on the coin" and B be the event, "3 comes on the die". Find whether A and B are independent events or not.

18. Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.

19. Bag I contains 3 red and 4 black balls, Bag II contains 5 red and 2 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. Find the probability that the drawn ball is red in colour.

LONG ANSWERS

1. Consider the experiment of tossing a coin, If the coin shows head, toss it again, but if it shows tail, then throw a die. Find the conditional probability of event that "the die shows a number greater than 4" given that there is atleast one tail.

2. If A and B are two independent events such that $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$, then find P(A) and P(B).

3. Bag A contains 3 red and 5 black balls, while bag B contains 4 red and 4 black balls. Two balls are transferred at random from bag A to bag B and then a ball is drawn from B at random. If the ball the bag is drawn from bag B is found to be red find the probability that two red balls were transferred from A to B.

4. In a factory which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs, 3, 4 and 1 percent respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.

5. In answering a question on a MCQ, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability $\frac{1}{3}$, what is the probability that the student knows the answer given that he answered it correctly?

6. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.

7. Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces it chance by 25%. At time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. Interpret the result and state which of the above stated methods is more beneficial for the patient.

8. In a group of 400 people. 160 are smokers and non-vegetarian, 100 are smokers and vegetarian and the remaining are non-smokers and vegetarian. The probabilities of getting a special chest disease are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the disease. What is the probability that the selected person is smoker and non-vegetarian?

9. Three bags contains balls as shown in the table below:

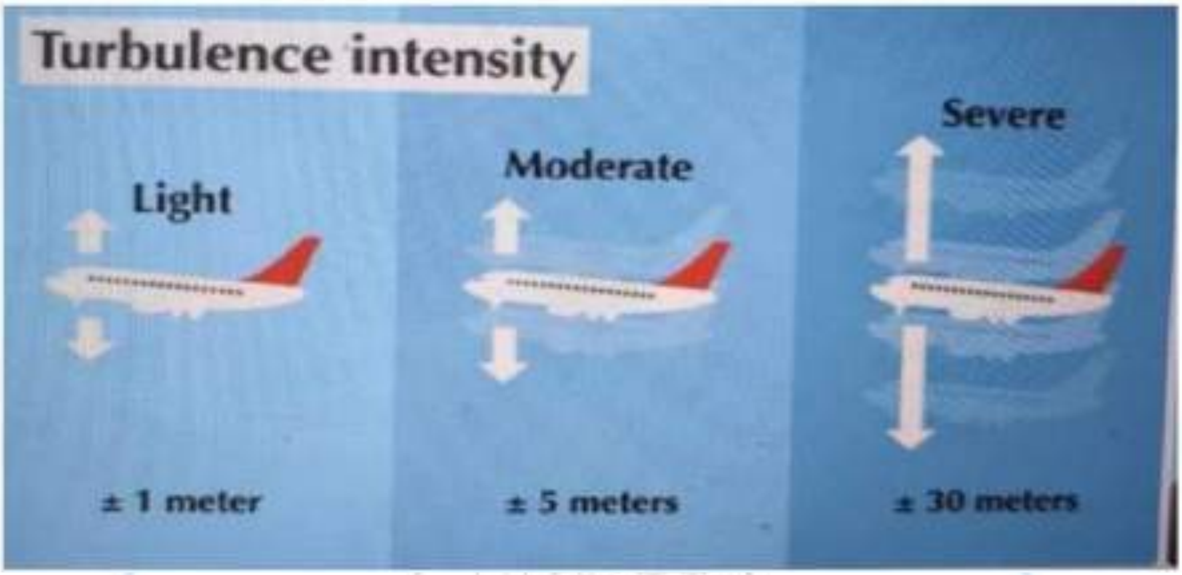
Bag	No. of white balls	No. of black balls	No. of red balls
I	1	2	3
II	2	1	1
III	4	3	2

A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from the III bag?

CASE BASED QUESTIONS

1. According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights. Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and

17% due to severe, moderate and light turbulence respectively. On the basis of the above information, answer the following questions:



(i) Find the probability that an airplane reached its destination late.

(ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence.

2. Airplanes are by far the safest mode of transportation when the number of transported passengers is measured against personal injuries and fatality totals.

Previous records state that the probability of an airplane crash is 0·00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions:

(i) Find the probability that the airplane will not crash.

(ii) Find $P(A \mid E_1) + P(A \mid E_2)$.

(iii) Find $P(A)$.

(iv) Find $P(E_2 \mid A)$.

3. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. . The event selection is independent of each other.

Based on the above information, answer the following questions:

(i) What is the probability that at least one of them is selected?

(ii) Find $P(G \mid \overline{H})$ where G is the event of Jaspreet's selection and denotes $\overline{H}$ the event that Rohit is not selected.

(iii) Find the probability that exactly one of them is selected.

(iv) Find the probability that exactly two of them are selected.

4. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions:

(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time. Find $P(E_1), P(E_2)$.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A \mid E_1)$ and $P(A \mid E_2)$.

(iii) Find the probability of customer paying second month's bill in time

(iv) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

5. Recent studies suggest that roughly 12% of the world population is left handed. Depending upon the parents, the chances of having a left handed child are as follows:

A: When both father and mother are left handed: Chances of left handed child is 24% .

B: When father is right handed and mother is left handed: Chances of left handed child is 22%.

C: When father is left handed and mother is right handed: Chances of left handed is 17%.

D: When both father and mother are right handed: Chances of left handed child is 9%.

Assuming that P(A) = P(B) = P(C) = P(D) = $\frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions:

(i) P(L/C)

(ii) Find $P(\overline{L}/A)$

(iii) Find (A/L)

(iv) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

6. A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0·65. The probability that many are not present and still the work gets completed on time is 0·35. The probability that work will be completed on time when all workers are present is 0·80.

Let: E_1: represent the event when many workers were not present for the job;

E_2: represent the event when all workers were present; and

E: represent completing the construction work on time.

Based on the above information, answer the following questions:

(i) What is the probability that all the workers are present for the job?

(ii) What is the probability that construction will be completed on time?

(iii) What is the probability that many workers are not present given that the construction work is completed on time?

(iv) What is the probability that all workers were present given that the construction job was completed on time?

7. A company conducts a mandatory health check-up for all newly hired employees, to check for infections that could affect other regular employees. A blood infection affects roughly 5% of the population. The probability of a false positive on the test for this infection is 4%, while the probability of a false negative on the test is 3%.

(Note: A false positive on a test refers to a case when a person is not infected, but tests positive for the infection. A false negative on a test refers to a case when a person is infected, but tests negative for the infection.)

(i) What is the probability that a person tests positive given that he is actually infected?

What is the probability that the employee tests positive for the infection?

(ii) If a person tests positive for the infection,

(a) what is the probability that the employee is infected?

(b) What is the probability that employee is not infected?

(iii) What is the probability that a person tests negative for the infection? If a person tests negative for the infection, what is the probability that the employee is actually infected?

UNIT TEST

Duration: 1 hour **Marks: 30**

SECTION A

Each carry 1 mark

1. Two events A and B will be independent if

(a) A and B are mutually exclusive events (b) P(A) = P(B)

(c) $P(A'B') = (1 - P(A)(1 - P(B)$ (d) $P(A) + P(B) = 1$

2. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then:

(a) $A \subset B$, but $A \neq B$ (b) P(A) = P(B) (c) $A \cap B = \phi$ (d) A = B

3. If the sum of numbers obtained on throwing a pair of dice is 9 , then the probability that number obtained on one of the dice is 4 is:

(a) $\frac{1}{9}$ (b) $\frac{4}{9}$ (c) $\frac{1}{18}$ (d) $\frac{1}{2}$

4. Assertion (A): Given two independent events A and B such that P(A) = 0.3 and P(B) = 0.6 , then P(A and not B)= 0.12.

Reason(R): For two independent events A and B , P (A and B)=P(A).P(B).

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION B

Each carry 3 marks

5. Out of outstanding students of a school, there are 4 boys and 5 girls. A team of 4 students is to be selected for quiz competition. Find the probability that two boys and two girls are selected.

6. A couple has 2 children. Find the probability that both are boys, if it is known that (i) one of them is a boy (ii) the older child is a boy.

7. P speaks truth in 70% of the cases and Q in 80% of the cases. In what percent of cases are they likely to agree in stating the same fact? Do you think, when they agree, means both are speaking truth?

8. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events A coming in time and B coming in time are independent, find the probability of only one of them coming to the school in time.

SECTION C

Each carry 5 marks

9. An urn contains 7 red and 4 blue balls. Two balls are drawn at random with replacement, Find the probability of getting (a) 2 red balls (b) 2 blue balls (c) one red and one blue ball.

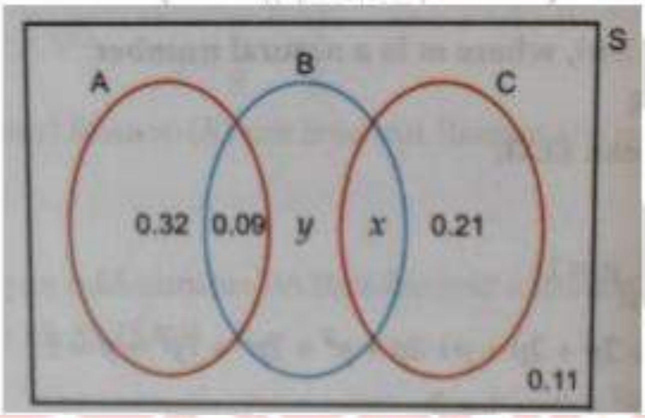
10. In a factory which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4, and 1 percent respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.

SECTION E

11. Read the following passage and answer the following questions:

There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below:

The Venn diagram below represents the probabilities of three different types of Yoga A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44.



(i) Find the value of x

(ii) Find the value of y.

(iii) Find $P\left(\frac{C}{B}\right)$.

(OR)

Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.

ANSWERS

MCQ	A-R	SA	LA	CS
1. (c) $P(A \cap B) = \frac{1}{2} P(B)$	1. (c)	1. $\frac{1}{2}$	1. $\frac{2}{9}$	1. (i) $\frac{109}{300}$ (ii) $\frac{37}{109}$
2. (d) 0	2. (b)	2. $\frac{1}{3}, \frac{13}{15}$	2. $P(A) = \frac{5}{6}, P(B) = \frac{4}{5}$ or $P(A) = \frac{1}{5}, P(B) = \frac{1}{6}$	2. (i) 0.9999999 (ii) $\frac{195}{100}$ (iii) $\frac{999999995}{1000000000}$ (iv) $\frac{999999995}{1000000000}$
3. (d) P(A) = P(B)	3. (b)	3. $\frac{1}{3}$	3. $\frac{18}{133}$	3. (i) $\frac{3}{5}$ (ii) $\frac{1}{3}$ (iii) $\frac{13}{30}$ (iv) $\frac{3}{20}$
4. (a) independent	4. (a)	4. $\frac{1}{17}$	4. $\frac{11}{31}$	4. (i) $P(E_1) = \frac{7}{10} = 0.7$, $P(E_2) = \frac{3}{10} = 0.3$ (ii) $P(A \mid E_1) = 0.8$, $P(A \mid E_2) = 0.4$ (iii) 0.68 or $\frac{17}{25}$ (iv) $\frac{14}{17}$
5. (d) $\frac{5}{8}$		5. $\frac{22}{45}$	5. $\frac{9}{11}$	
6. (a) $\frac{13}{32}$		6. $\frac{1}{20}$	6. $\frac{10}{49}$	
7. (d) $\frac{7}{12}$		7. (i) $\frac{1}{2}$ (ii) $\frac{1}{3}$	7. $\frac{15}{29}$	
8. (a) 0.6		8. $\frac{5}{9}$	8. $\frac{28}{45}$	
9. (d) 0.96		9. $\frac{1}{15}$		
10. (d) $\frac{2}{3}$		10. (i) $\frac{2}{3}$ (ii) $\frac{1}{2}$		
11. (a) $\frac{2}{9}$		11. 30%		

12. (a) $\frac{5}{8}$ 13. (a) $\frac{7}{20}$ 14. (c) 1 15. (c) $\frac{4}{9}$ 16. (c) $\frac{1}{3}$ 17. (d) $\frac{1}{2}$ 18. (a) $\frac{2}{9}$ 19. (d) $\frac{1}{70}$ 20. (c) $\frac{31}{32}$ 21. (d) $\frac{1}{2}$		12. 0.7 13. $\frac{17}{400}$ 14. $\frac{40}{61}$ 15. $\frac{1}{3}$ 16. P(A) $=\frac{1}{3}$ & $P(B)=\frac{1}{4}$ OR $\frac{3}{4}$ & $\frac{2}{3}$ 17. A & B are independent events. 18. $\frac{25}{52}$ 19. $\frac{41}{63}$	9. $\frac{5}{17}$	5. (i) $\frac{17}{100}$ (ii) $\frac{19}{25}$ (iii) $\frac{1}{3}$ (iv) $\frac{39}{100}$ 6. (i) 0.35 (ii) 0.51 (iii) 0.45 (iv) 0.55 7. (i) 0.97, 0.0865 (ii) (a) $\frac{97}{173}$ (b) $\frac{76}{173}$ (iii) 0.9135, $\frac{1}{609}$

UNIT TEST

1. (c) $P(A'B') = (1 - P(A))(1 - P(B))$

2. (b) P(A) = P(B)

3. (d) $\frac{1}{2}$

4. (a)

5. $\frac{10}{21}$

6. (i) $\frac{1}{3}$ (ii) $\frac{1}{2}$

7. $\frac{62}{100}$

8. $\frac{26}{49}$

9. (a) $\frac{49}{121}$ (b) $\frac{16}{121}$ (c) $\frac{56}{121}$

10. $\frac{11}{31}$

11. (i) x = 0.23 (ii) y = 0.04 (iii) $\frac{23}{36}$ (OR) 0.45

MATHEMATICS – Code No. 041
SAMPLE QUESTION PAPER
CLASS - XII (2025-26)

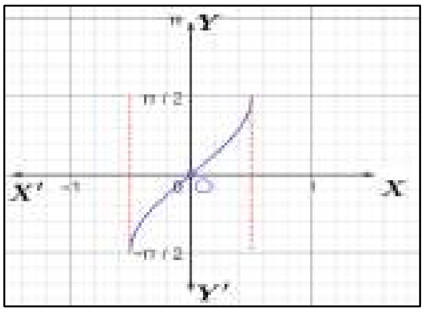
Maximum Marks: 80 **Time: 3 hours**

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) with only one correct option and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculator is not allowed.

SECTION-A
This section comprises of multiple choice questions (MCQs) of 1 mark each.
Select the correct option (Question 1 - Question 18)

Q.No.	Questions	Marks
1.	Identify the function shown in the graph  (A) $\sin^{-1} x$ (B) $\sin^{-1}(2x)$ (C) $\sin^{-1}\left(\frac{x}{2}\right)$ (D) $2\sin^{-1} x$	1
	For Visually Impaired:	
1.	Inverse Trigonometric Function, whose domain is $\left[-\frac{1}{3}, \frac{1}{3}\right]$, is ... (A) $\cos^{-1} x$ (B) $\cos^{-1}\left(\frac{x}{3}\right)$ (C) $\cos^{-1}(3x)$ (D) $3\cos^{-1} x$	

*Please note that the assessment scheme of the Academic Session 2024-25 will continue in the current session i.e. 2025-26.

2.	If for three matrices $A=[a_{ij}]_{m\times 4}$, $\mathrm{B}=[b_{ij}]_{n\times 3}$ and $\mathrm{C}=[c_{ij}]_{p\times q}$ products AB and AC both are defined and are square matrices of same order, then value of m,n,p and q are: (A) $m=q=3\ \ and\, n=p=4$ (B) $m=2,q=3\ \ and\, n=p=4$ (C) $m=q=4\ \ and\, n=p=3$ (D) $m=4,p=2\ \ and\, n=q=3$	1
3.	If the matrix $A=\begin{bmatrix}0 & r & -2\\ 3 & p & t\\ q & -4 & 0\end{bmatrix}$ is skew-symmetric, then value of $\frac{q+t}{p+r}$ is…. (A)-2 (B) 0 (C) 1 (D) 2	1
4.	If A is a square matrix of order 4 and $\lvert adj\ A\rvert=27$, then $A\ (adj\ A)$ is equal to (A) 3 (B) 9 (C) $3\ I$ (D) $9\ I$	1
5.	The inverse of the matrix $\begin{bmatrix}3 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 5\end{bmatrix}$ is… (A) $\begin{bmatrix}0 & 0 & 3\\ 0 & 2 & 0\\ 5 & 0 & 0\end{bmatrix}$ (B) $\begin{bmatrix}\frac{1}{3} & 0 & 0\\ 0 & \frac{1}{2} & 0\\ 0 & 0 & \frac{1}{5}\end{bmatrix}$ (C) $\begin{bmatrix}-\frac{1}{3} & 0 & 0\\ 0 & -\frac{1}{2} & 0\\ 0 & 0 & -\frac{1}{5}\end{bmatrix}$ (D) $\begin{bmatrix}-3 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -5\end{bmatrix}$	1
6.	Value of the determinant $\begin{vmatrix}\cos 67^o & \sin 67^o\\ \sin 23^o & \cos 23^o\end{vmatrix}$ is (A) 0 (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) 1	1
7.	If a function defined by $f(x)=\begin{cases}kx+1, & x\leq \pi\\ \cos x & , x>\pi\end{cases}$ is continuous at $x=\pi$, then the value of k is (A) π (B) $\frac{-1}{\pi}$ (C) 0 (D) $\frac{-2}{\pi}$	1
8.	If $f(x)=x\tan^{-1}x$, then $f'(1)$is equal to (A)$\frac{\pi}{4}-\frac{1}{2}$ (B)$\frac{\pi}{4}+\frac{1}{2}$ (C)$-\frac{\pi}{4}-\frac{1}{2}$ (D)$-\frac{\pi}{4}+\frac{1}{2}$	1
9.	A function $f(x)=10-x-2x^2$ is increasing on the interval (A) $\left(-\infty,-\frac{1}{4}\right]$ (B) $\left(-\infty,\frac{1}{4}\right)$ (C) $\left[-\frac{1}{4},\infty\right)$ (D)$\left[-\frac{1}{4},\frac{1}{4}\right]$	1
10.	The solution of the differential equation $xdx+ydy=0$ represents a family of (A) straight lines (B) parabolas (C) Circles (D) Ellipses	1

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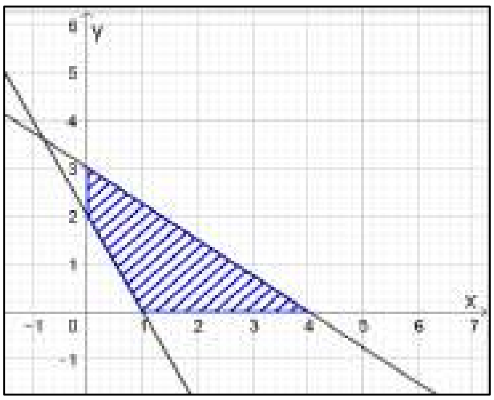
11.	If $f(a+b-x)=f(x)$, then $\int_a^b x\, f(x)dx$ is equal to (A) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (B) $\frac{a+b}{2}\int_a^b f(a-x)dx$ (C) $\frac{b-a}{2}\int_a^b f(x)\ dx$ (D) $\frac{a+b}{2}\int_a^b f(x)dx$	1
12.	If $\int x^3\, sin^4(x^4)\cos(x^4)\ dx = a\ sin^5(x^4)$ + C, then a is equal to (A) $-\frac{1}{10}$ (B) $\frac{1}{20}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$	1
13.	A bird flies through a distance in a straight line given by the vector $\hat{\imath}+2\hat{\jmath}+\hat{k}$. A man standing beside a straight metro rail track given by $\vec{r}=(3+\lambda)\hat{\imath}+(2\lambda-1)\hat{\jmath}+3\lambda\hat{k}$ is observing the bird. The projected length of its flight on the metro track is (A) $\frac{6}{\sqrt{14}}$ units (B) $\frac{14}{\sqrt{6}}$ units (C) $\frac{8}{\sqrt{14}}$ units (D) $\frac{5}{\sqrt{6}}$ units	1
14.	The distance of the point with position vector $3\hat{\imath}+4\hat{\jmath}+5\hat{k}$ from the y-axis is (A) 4 units (B) $\sqrt{34}$ units (C) 5 units (D) $5\sqrt{2}$ units	1
15.	If $\vec{a}=3\hat{\imath}+2\hat{\jmath}+4\hat{k}$, $\vec{b}=\hat{\imath}+\hat{\jmath}-3\hat{k}$ and $\vec{c}=6\hat{\imath}-\hat{\jmath}+2\hat{k}$ are three given vectors, then $(2\vec{a}.\hat{\imath})\hat{\imath}-(\vec{b}.\hat{\jmath})\hat{\jmath}+(\vec{c}.\hat{k})\hat{k}$ is same as the vector (A) $\vec{a}$ (B) $\vec{b}+\vec{c}$ (C) $\vec{a}-\vec{b}$ (D) $\vec{c}$	1
16.	A student of class XII studying Mathematics comes across an incomplete question in a book. Maximise $Z=3x+2y+1$ Subject to the constraints $x\geq 0, y\geq 0, 3x+4y\leq 12$, He/ She notices the below shown graph for the said LPP problem, and finds that a constraint is missing in it: Help him/her choose the required constraint from the graph.  The missing constraint is (A) $x+2y\leq 2$ (B) $2x+y\geq 2$ (C) $2x+y\leq 2$ (D) $x+2y\geq 2$	1

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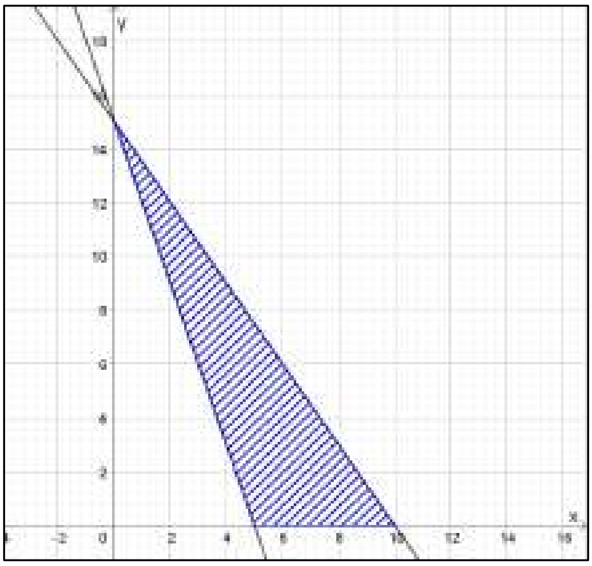
16. **For Visually Impaired:**

If $Z = ax + by + c$, where $a, b, c > 0$, attains its maximum value at two of its corner points (4,0) and (0,3) of the feasible region determined by the system of linear inequalities, then

(A) $4a = 3b$ (B) $3a = 4b$ (C) $4a + c = 3b$ (D) $3a + c = 4b$

17. The feasible region of a linear programming problem is bounded but the objective function attains its minimum value at more than one point. One of the points is (5,0). — 1



Then one of the other possible points at which the objective function attains its minimum value is

(A) (2,9) (B) (6,6) (C) (4,7) (D) (0,0)

For Visually Impaired:

The graph of the inequality $3x + 5y < 10$ is the

(A) Entire XY −plane
(B) Open Half plane that doesn't contain origin
(C) Open Half plane that contains origin, but not the points of the line $3x + 5y = 10$
(D) Half plane that contains origin and the points of the line $3x + 5y = 10$

18. A person observed the first 4 digits of your 6-digit PIN. What is the probability that the person can guess your PIN? — 1

(A) $\frac{1}{81}$ (B) $\frac{1}{100}$ (C) $\frac{1}{90}$ (D) 1

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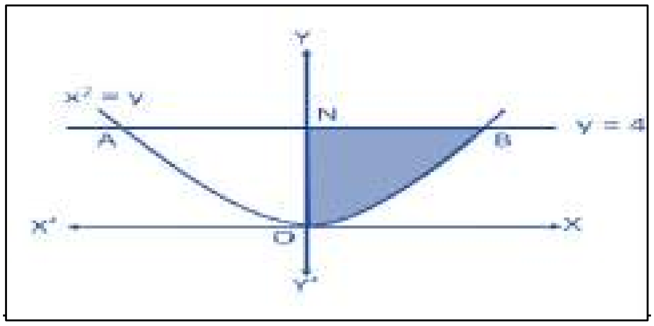
	ASSERTION-REASON BASED QUESTIONS **(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)** (A) Both (A) and (R) are true and (R) is the correct explanation of (A). (B) Both (A) and (R) are true but (R) is not the correct explanation of (A). (C) (A) is true but (R) is false. (D) (A) is false but (R) is true.	
19.	**Assertion (A):** Value of the expression $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1} 1 - \sec^{-1}(\sqrt{2})$ is $\frac{\pi}{4}$. **Reason (R):** Principal value branch of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and that of $sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.	1
20.	**Assertion(A):** Given two non-zero vectors $\vec{a}$ and $\vec{b}$. If $\vec{r}$ is another non-zero vector such that $\vec{r} \times (\vec{a} + \vec{b}) = \vec{0}$. Then $\vec{r}$ is perpendicular to $\vec{a} \times \vec{b}$. **Reason (R):** The vector $(\vec{a} + \vec{b})$ is perpendicular to the plane of $\vec{a}$ and $\vec{b}$	1
	SECTION B **This section comprises of 5 very short answer (VSA) type questions of 2 marks each.**	
21A 21B	Evaluate $\tan\left(\tan^{-1}(-1) + \frac{\pi}{3}\right)$ **OR** Find the domain of $\cos^{-1}(3x - 2)$	2
22	If $y = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, then prove that $\boldsymbol{\frac{dy}{dx} - \sec x = 0}$	2
23A 23B	Find: $\int \frac{(x-3)}{(x-1)^3} e^x \, dx$ **OR** Find out the area of shaded region in the enclosed figure. 	2

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23 B	**For Visually Impaired:** Find out the area of the region enclosed by the curve $y^2 = x$, $x = 3$ and x-axis in the first quadrant.	
24.	If $f(x+y) = f(x)f(y)$ for all $x, y \in$ R and $f(5)$= 2, $f'(0) = 3,$ then using the definition of derivatives, find $f'(5)$.	2
25.	The two vectors $\hat{\imath} + \hat{\jmath} + \hat{k}$ and $\widehat{3\imath} - \hat{\jmath} + 3\hat{k}$ represent the two sides OA and OB, respectively of a ΔOAB, where O is the origin. The point P lies on AB such that OP is a median. Find the area of the parallelogram formed by the two adjacent sides as OA and OP.	2
SECTION C **This section comprises of 6 short answer (SA) type questions of 3 marks each.**		
26A. 26B.	If $x^y = e^{x-y}$ prove that $\frac{dy}{dx} = \frac{\log x}{(\log(xe))^2}$ and hence find its value at $x = e$. **OR** If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ find $\frac{d^2y}{dx^2}$.	3
27	A spherical ball of ice melts in such a way that the rate at which its volume decreases at any instant is directly proportional to its surface area. Prove that the radius of the ice ball decreases at a constant rate.	3
28A 28B 28A 28B	Sketch the graph $y = \|x+1\|$. Evaluate $\int_{-4}^{2} \|x+1\| dx$. What does the value of this integral represent on the graph? **OR** Using integration find the area of the region $\{(x, y) : x^2 - 4y \leq 0, y - x \leq 0\}$ **For Visually Impaired**: Define the function $y = \|x+1\|$. Evaluate $\int_{-4}^{2} \|x+1\| dx$. What does the value of this integral represent? **OR** Using integration find the area enclosed within the curve: $25x^2 + 16y^2 = 400$	3
29A 29B	Find the distance of the point $(2, -1, 3)$ from the line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \mu(3\hat{\imath} + 6\hat{\jmath} + 2\hat{k})$ measured parallel to the z-axis. **OR** Find the point of intersection of the line $\vec{r} = (3\hat{\imath} + \hat{k}) + \mu(\hat{\imath} + \hat{\jmath} + \hat{k})$ and the line through $(2, -1, 1)$ parallel to the z-axis. How far is this point from the z-axis?	3

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30.	Solve graphically: Maximise $Z = 2x + y$ subject to $x + y \leq 1200$ $x + y \geq 600$ $y \leq \frac{x}{2}$ $x \geq 0, y \geq 0$.	3
30	**For Visually Impaired:** The objective function $Z = 3x + 2y$ of a linear programming problem under some constraints is to be maximized and minimized. The corner points of the feasible region are $A(600,0), B(1200,0), C(800,400)$ and $D(400,200)$. Find the point at which Z is maximum and the point at which Z is minimum. Also, find the corresponding maximum and minimum values of Z.)	
31.	Two students Mehul and Rashi are seeking admission in a college. The probability that Mehul is selected is 0.4 and the probability of selection of exactly one of the them is 0.5. Chances of selection of them is independent of each other. Find the chances of selection of Rashi. Also find the probability of selection of at least one of them.	3
	SECTION D **This section comprises of 4 long answer (LA) type questions of 5 marks each**	
32.	For two matrices $A = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ -2 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$, find the product AB and hence solve the system of equations: $3x - 6y - z = 3$ $2x - 5y - z + 2 = 0$ $-2x + 4y + z = 5$	5
33A	Evaluate: $\int_0^1 \frac{\log(1+x)}{1+x^2}\, dx$	5
	OR	
33B	Find $\int \frac{(3\sin\theta - 2)\cos\theta}{5 - cos^2\theta - 4\sin\theta}\, d\theta$	
34A	Solve the differential equation: $y + \frac{d}{dx}(xy) = x(\sin x + x)$	5
	OR	
34B	Find the particular solution of the differential equation: $2\, y\, e^{x/y} dx + (y - 2\, x\, e^{x/y})\, dy = 0$ given that $y(0) = 1$	

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35. The two lines $\frac{x-1}{3} = -y\,, z+1=0$ and $\frac{-x}{2} = \frac{y+1}{2} = z+2$ intersect at a point whose y-coordinate is 1. Find the co-ordinates of their point of intersection. Find the vector equation of a line perpendicular to both the given lines and passing through this point of intersection. **5**

SECTION- E

This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each

36. **Case Study -1** **4**

A city's traffic management department is planning to optimize traffic flow by analyzing the connectivity between various traffic signals. The city has five major spots labelled $A, B, C, D, and\ E$.

The department has collected the following data regarding one-way traffic flow between spots:

1. Traffic flows from A to B, A to C, and A to D.
2. Traffic flows from B to C and B to E.
3. Traffic flows from C to E.
4. Traffic flows from D to E and D to C.

The department wants to represent and analyze this data using relations and functions. Use the given data to answer the following questions:

I. Is the traffic flow reflexive? Justify. [1]

II. Is the traffic flow transitive? Justify. [1]

III A. Represent the relation describing the traffic flow as a set of ordered pairs. Also state the domain and range of the relation.

OR

III B. Does the traffic flow represent a function? Justify your answer. [2]

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37.	**Case Study -2** LED bulbs are energy-efficient because they use significantly less electricity than traditional bulbs while producing the same amount of light. They convert more energy into light rather than heat, reducing waste. Additionally, their long lifespan means fewer replacements, saving resources and money over time. A company manufactures a new type of energy-efficient LED bulb. The cost of production and the revenue generated by selling x bulbs (in an hour) are modelled as $C(x) = 0.5x^2 - 10x + 150$ and $R(x) = -0.3x^2 + 20x$ respectively, where $C(x)$ and $R(x)$ are both in ₹.  To maximize the profit, the company needs to analyze these functions using calculus. Use the given models to answer the following questions: I. Derive the profit function $P(x)$ [1] II. Find the critical points of $P(x)$. [1] III A. Determine whether the critical points correspond to a maximum or a minimum profit by using the second derivative test. **OR** III B. Identify the possible practical value of x (i.e., the number of bulbs that can realistically be produced and sold) that can maximize the profit, if the resources available and the expenditure on machines allows to produce minimum 10 but not more than 18 bulbs per hour. Also calculate the maximum profit. [2]	4

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38. **Case Study -3** 4

Excessive use of screens can result in vision problems, obesity, sleep disorders, anxiety, low retention problems and can impede social and emotional comprehension and expression. It is essential to be mindful of the amount of time we spend on screens and to reduce our screen-time by taking regular breaks, setting time limits, and engaging in non-screen-based activities.

In a class of students of the age group 14 to 17, the students were categorised into three groups according to a feedback form filled by them. The first group constituted of the students who spent more than 4 hours per day on the mobile screen or the gaming screens, while the second group spent 2 to 4 hours /day on the same activities. The third group spent less than 2 hours /day on the same. The first group with the high screen time is 60% of all the students, whereas the second group with moderate screen time is 30% and the third group with low screen time is only 10% of the total number of students. It was observed that 80% students of first group faced severe anxiety and low retention issues, with 70% of second group, and 30% of third group having the same symptoms.

I. What is the total percentage of students who suffer from anxiety and low retention issues in the class? [2]

II. A student is selected at random, and he is found to suffer from anxiety and low retention issues. What is the probability that he/she spends screen time more than 4 hours per day? [2]

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MATHEMATICS – Code No. 041
MARKING SCHEME
CLASS – XII (2025-26)

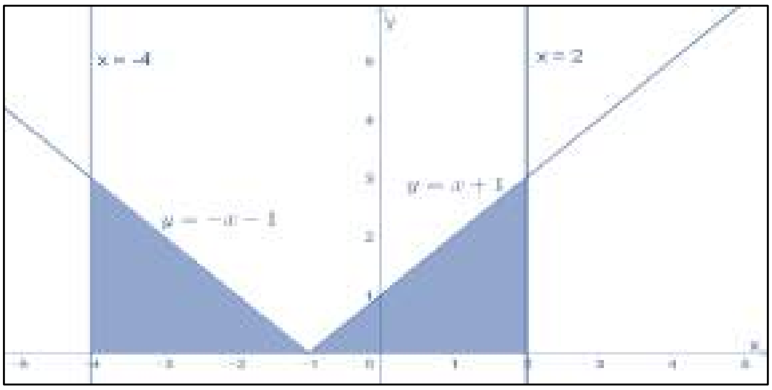
SECTION-A (MCQs of 1 mark each)		
Sol. N.	**Hint / Solution**	**Marks**
1	Clearly from the graph Domain is $\left[-\frac{1}{2},\frac{1}{2}\right]$ So graph is of the function $\sin^{-1}(2x)$ **Answer is (B)** $\sin^{-1}(2x)$	**1**
1 (V.I.)	Domain is $\left[-\frac{1}{3},\frac{1}{3}\right]$ So the function is $\cos^{-1}(3x)$ **Answer is (C)** $\cos^{-1}(3x)$	**1**
2	AB is defined so n=4 AC is defined so p=4 AB and AC are square matrices of same order so $m \times 3 = m \times q \Rightarrow q = 3 = m$ **Answer is (A)** $m = q = 3 \quad and\; n = p = 4$	**1**
3	As A is skew symmetric So $p = 0, q = 2, r = -3, t = 4$ So $\frac{q+t}{p+r} = \frac{6}{-3} = -2$ **Answer is (A) -2**	**1**
4	$\lvert adj\ A\rvert = 27 \Rightarrow \lvert A\rvert^3 = 27 = 3^3 \Rightarrow \lvert A\rvert = 3$ $A\,(adj\ A) = \lvert A\rvert\, I = 3\, I$ **Answer is (C) 3 $\boldsymbol{I}$**	**1**
5	Inverse of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ **Answer is (B)**	**1**
6	$\begin{vmatrix} \cos 67^o & \sin 67^o \\ \sin 23^o & \cos 23^o \end{vmatrix} = \cos 67^o \cos 23^o - \sin 67^o \sin 23^o = \cos(67^o + 23^o) =$ $\cos 90^o = 0$ **Answer is (A) 0**	**1**
7	$f(x)$ is continuous at $x = \pi$ $\Rightarrow \lim_{x\to\pi^-}(kx+1) = \lim_{x\to\pi^+} cos\ x = f(\pi)$ $\Rightarrow \lim_{h\to 0}[k(\pi - h) + 1] = \lim_{h\to 0} cos(\pi + h) = k\pi + 1$ $\Rightarrow k\pi + 1 = -1 \qquad \Rightarrow k = \frac{-2}{\pi}$ **Answer is (D)** $\boldsymbol{\frac{-2}{\pi}}$	**1**

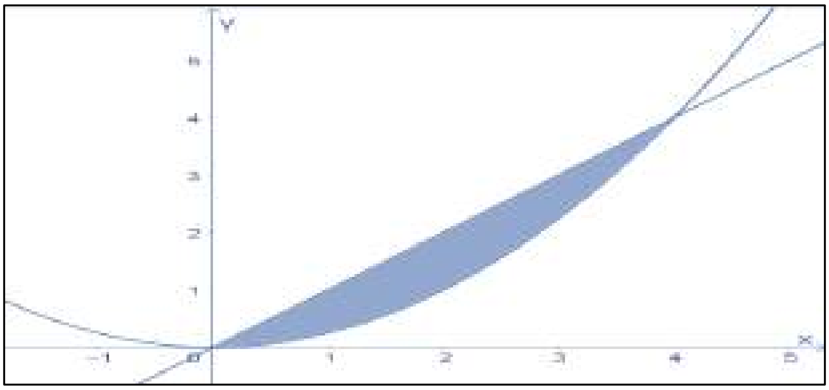
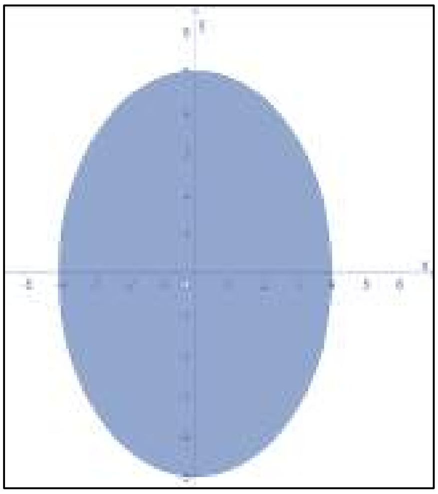
8	$f(x) = x\tan^{-1}x$ $f'(x) = 1.\tan^{-1}x + x.\frac{1}{1+x^2}$ $f'(1) = 1.\tan^{-1}1 + \frac{1}{1+1} = \frac{\pi}{4} + \frac{1}{2}$ **Answer is (B)** $\boldsymbol{\frac{\pi}{4} + \frac{1}{2}}$	1
9	$f(x) = 10 - x - 2x^2$ $\Rightarrow f'(x) = -1 - 4x$ For increasing function $f'(x) \geq 0$ $\Rightarrow -(1+4x) \geq 0$ $\Rightarrow (1+4x) \leq 0$ $\Rightarrow x \leq -{}^{1}/_{4}$ $\Rightarrow x \in \left(-\infty, -\frac{1}{4}\right]$ **Answer is (A)** $\left(-\infty, -\boldsymbol{\frac{1}{4}}\right]$	1
10	$xdx + ydy = 0$ $\Rightarrow \int xdx = -\int ydy$ $\Rightarrow \frac{x^2}{2} = -\frac{y^2}{2} + k$ $\Rightarrow x^2 + y^2 = 2k$ Solution is $x^2 + y^2 = 2k, k$ being an arbitrary constant. **Answer is (C) Circles**	1
11	$I = \int_a^b x\,f(x)dx = \int_a^b (a+b-x)f(a+b-x)dx$ $\Rightarrow I = \int_a^b (a+b-x)f(x)dx$ (given $f(a+b-x) = f(x)$) $\Rightarrow I = \int_a^b (a+b)\,f(x)dx - \int_a^b x\,f(x)dx$ $\Rightarrow 2\,I = (a+b)\int_a^b f(x)dx$ $\Rightarrow I = \frac{1}{2}(a+b)\int_a^b f(x)dx$ **Answer is (D)** $\boldsymbol{\frac{a+b}{2}\int_a^b f(x)dx}$	1
12	Let $I = \int x^3\,sin^4(x^4)\cos(x^4)\,dx$ Let $\sin(x^4) = t \Rightarrow 4x^3\cos(x^4)\,dx = dt \Rightarrow x^3\cos(x^4) = \frac{1}{4}dt$ Thus $I = \int t^4\,(\frac{1}{4}dt) = \frac{1}{20}t^5 + C = \frac{1}{20}sin^5(x^4) + C$ $\Rightarrow I = \frac{1}{20}sin^5(x^4) + C = a\ sin^5(x^4) + C$ So, $a = \frac{1}{20}$ **Answer is (B)** $\boldsymbol{\frac{1}{20}}$	1
13	The projection of the vector $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ on the line $\vec{r} = (3\hat{\imath} - \hat{\jmath}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ is $\frac{1x1+2x2+1x3}{\sqrt{1^2+2^2+3^2}} = \frac{8}{\sqrt{14}}$ units **Answer is (C)** $\boldsymbol{\frac{8}{\sqrt{14}}}$ **units**	1

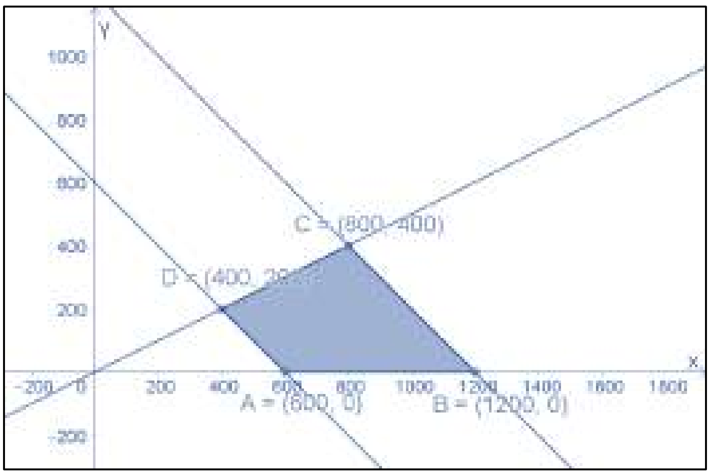
14	The distance of the point (a, b, c) from the y-axis is $\sqrt{a^2+c^2}$ So, the distance is $\sqrt{3^2+5^2}$ = $\sqrt{34}$ units. **Answer is (B) $\sqrt{34}$ units**	**1**
15	$(2\vec{a}.\hat{\imath})\hat{\imath} - (\vec{b}.\hat{\jmath})\hat{\jmath} + (\vec{c}.\hat{k})\hat{k} = (2\times 3)\hat{\imath} - (1)\hat{\jmath} + (2)\hat{k}$ $= 6\hat{\imath} - \hat{\jmath} + 2\hat{k} = \vec{c}$ **Answer is (D) $\vec{c}$**	**1**
16	The points (1,0) and (0,2) satisfy the equation $2x + y = 2$ And shaded region shows that (0,0) doesn't lie in the feasible solution region So, the inequality is $2x + y \geq 2$ **Answer is (B) $2x + y \geq 2$**	**1**
16 (V.I.)	(4,0) and (0,3) gives maximum value so $Z_{(4,0)} = Z_{(0,3)} \Rightarrow 4a + c = 3b + c \Rightarrow 4a = 3b$ **Answer is (A) $4a = 3b$**	**1**
17	The student may read the point (2,9) from the line on the graph. The student may find the equation $3x + y = 15$ joining (5,0) and (0,15) and then verify the point (2,9) satisfies it. **Answer is (A) (2,9)**	**1**
17 (V.I.)	**Answer is (C)** Open Half plane that contains origin, but not the points of the line $3x + 5y = 10$	**1**
18	**Answer is (B) $\frac{1}{100}$** The person knows the first 4 digits. So the person has to guess the remaining two digits. P (guessing the PIN)=1×1×1×1×$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$	**1**
19	$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1} 1 - \sec^{-1}(\sqrt{2}) = \frac{\pi}{3} + \frac{\pi}{4} - \frac{\pi}{4} = \frac{\pi}{3} \neq \frac{\pi}{4}$ So, A is false. Principal Value branch of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and that of $sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$. So, R is true **Answer is (D)Assertion is false, but Reason is true**	**1**
20	C. $\vec{r} \times (\vec{a} + \vec{b}) = \vec{0} \Rightarrow \vec{r}$ is parallel to $(\vec{a} + \vec{b})$ and $(\vec{a} + \vec{b})$ lies on the plane of $\vec{a}$ and $\vec{b}$. So, $\vec{r}$ is parallel to the plane of $\vec{a}$ and $\vec{b} \Rightarrow \vec{r}$ is perpendicular to $(\vec{a} \times \vec{b})$. So, Assertion is true But $(\vec{a} + \vec{b})$ lies on the plane of $\vec{a}$ and $\vec{b}$, so $(\vec{a} + \vec{b})$ is not perpendicular to the plane of $\vec{a}$ and $\vec{b}$ Therefore, Reason is false. **Answer is (C) Assertion is true, but Reason is false**	**1**

SECTION B **(VSA type questions of 2 marks each)**		
21A	$\tan\left(\tan^{-1}(-1)+\frac{\pi}{3}\right)=\tan\left(-\frac{\pi}{4}+\frac{\pi}{3}\right)$ $=\frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$ $=\frac{\sqrt{3}-1}{1+\sqrt{3}}\ or\ 2-\sqrt{3}$ **OR**	½ 1 ½ OR
21B	For domain, $-1\le 3x-2\le 1$ $\Rightarrow 1\le 3x\le 3$ $\Rightarrow \frac{1}{3}\le x\le 1$ So, domain of $\cos^{-1}(3x-2)$ is $\left[\frac{1}{3},1\right]$	½ ½ ½ ½
22	$y=\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$ Differentiating with respect to x $\frac{dy}{dx}=\frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}.sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right).\frac{1}{2}$ $=\frac{\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}.\frac{1}{cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}.\frac{1}{2}$ $=\frac{1}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}=\frac{1}{\sin\left(\frac{\pi}{2}+x\right)}=\frac{1}{\cos x}$ $\Longrightarrow \frac{dy}{dx}-\sec x=0$	½ 1 ½
23A	$\int\frac{(x-3)e^x}{(x-1)^3}\,dx=\int\frac{(x-1-2)e^x}{(x-1)^3}\,dx$ $=\int\left(\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right)e^x\,dx=\int\left(\frac{1}{(x-1)^2}+\frac{d}{dx}\left(\frac{1}{(x-1)^2}\right)\right)e^x\,dx$ $=\frac{e^x}{(x-1)^2}$ + c (as $\int(f(x)+f'(x))e^x\,dx=e^x\,f(x)$ + c) **OR**	1 1 OR
23B	A = $\int_0^4 x\,dy$ = $\int_0^4\sqrt{y}\,dy$ = $\frac{2}{3}\times y^{3/2}\Big]_{y=0}^{y=4}$ = $\frac{16}{3}$ sq. units	1 1
23B	**For Visually Impaired:** A = $\int_0^3 y\,dx$ = $\int_0^3\sqrt{x}\,dx$ = $\frac{2}{3}\times x^{3/2}\Big]_{x=0}^{x=3}$ = $2\sqrt{3}$ sq. units	1 1

24	Given $f(x+y)=f(x)f(y)$ $f(0+5)=f(0)f(5)$ $\Longrightarrow \quad f(0)=1$ $f'(5)=\lim_{h\to 0}\frac{f(5+h)-f(5)}{h}=\lim_{h\to 0}\frac{f(5)f(h)-f(5)}{h}$ $[\because f(x+y)=f(x)f(y)]$ $=\lim_{h\to 0}\frac{2f(h)-2}{h}$ $[\because f(5)=2]$ $=2\lim_{h\to 0}\frac{f(h)-1}{h}=2\lim_{h\to 0}\frac{f(h)-f(0)}{h}=2f'(0)$ $=2(3)$ $[\because f'(0)=3]$ $=6$	 ½ 1 ½
25	The vector $\overrightarrow{OP}=\frac{1}{2}(4\hat{\imath}+4\hat{k})=2\hat{\imath}+2\hat{k}$ Area of the parallelogram formed by the two adjacent sides as OA and OP $=\left\|(\overrightarrow{OA}\times\overrightarrow{OP})\right\|=\left\|\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 1 & 1\\ 2 & 0 & 2\end{vmatrix}\right\|$ $=\left\|2\hat{\imath}-2\hat{k}\right\|$ $=2\sqrt{2}$ square units.	½ ½ ½ ½
SECTION C **(SA type questions of 3 marks each)**		
26A	$x^y=e^{x-y}$ Taking log of both sides $y\log x=(x-y)\log e$ $y\log x+y=x$ (since $\log e=1$) $\Longrightarrow \quad y=\frac{x}{1+\log x}$ Differentiating with respect to x $\frac{dy}{dx}=\frac{(1+\log x).1-x.\frac{1}{x}}{(1+\log x)^2}$ $=\frac{\log x}{(\log e+\log x)^2}$ $=\frac{\log x}{(\log(x\,e))^2}$ Now $\left.\frac{dy}{dx}\right]_{x=e}=\frac{\log e}{(\log e^2)^2}=\frac{1}{(2\log e)^2}=\frac{1}{2^2}=\frac{1}{4}$ (as $\log e=1$) **Alternative Solution:** $x^y=e^{x-y}$ Taking log of both sides $y\log x=(x-y)\log e$ $y\log x+y=x$ (since $\log e=1$) Differentiating both sides w.r.t. x $\log x\frac{dy}{dx}+\frac{y}{x}+\frac{dy}{dx}=1$ $\Rightarrow\frac{dy}{dx}(1+\log x)=1-\frac{y}{x}$ $\Rightarrow\frac{dy}{dx}=\frac{x-y}{x(1+\log x)}=\frac{x-\frac{x}{1+\log x}}{x(1+\log x)}=\frac{x(1+\log x)-x}{x(1+\log x)^2}=\frac{x(1+\log x-1)}{x(\log e+\log x)^2}=\frac{\log x}{(\log(xe))^2}$ Now $\left.\frac{dy}{dx}\right]_{x=e}=\frac{\log e}{(\log e^2)^2}=\frac{1}{(2\log e)^2}=\frac{1}{2^2}=\frac{1}{4}$ (as $\log e=1$)	 1 1 1

	OR	OR
26B	$\frac{dx}{d\theta} = a(1-\cos\theta)$, $\frac{dy}{d\theta} = a(0+\sin\theta)$, $\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1-\cos\theta)}$ $= \frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2sin^2\left(\frac{\theta}{2}\right)} = \cot\frac{\theta}{2}$ $\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2}\,cosec^2\left(\frac{\theta}{2}\right)\frac{d\theta}{dx}$ $= -\frac{1}{2a}\,cosec^2\left(\frac{\theta}{2}\right)\frac{1}{2sin^2\left(\frac{\theta}{2}\right)}$ $= -\frac{1}{4a}cosec^4\left(\frac{\theta}{2}\right)$	1 1 1
27	Let r be the radius of ice ball at time t . $V = \frac{4}{3}\pi r^3$………… (1) S = $4\pi r^2$…………..(2) Given $\frac{dV}{dt} \propto$ S $\Longrightarrow \frac{dV}{dt} = -$ k S (where k is some positive constant) ………(3) Differentiating (1) w.r.t. t, we get $\frac{dV}{dt} = \frac{4}{3}\pi.(3r^2)\frac{dr}{dt}$ $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$ ………. (4) $\Longrightarrow$ - k S = $4\pi r^2\frac{dr}{dt}$ (from (3) and (4)) $\Longrightarrow$ - k S = S $\frac{dr}{dt}$ (using (2)) $\Longrightarrow \frac{dr}{dt} = -$ k ⇒ radius of the ice-ball decreases at a constant rate	½ ½ 1 ½ ½
28A	 $\int_{-4}^{2}\lvert x+1\rvert dx = \int_{-4}^{-1}(-x-1)\,dx + \int_{-1}^{2}(x+1)\,dx$ $= -\frac{(x+1)^2}{2}\Big]_{-4}^{-1} + \frac{(x+1)^2}{2}\Big]_{-1}^{2}$ $= -\left(0-\frac{9}{2}\right) + \left(\frac{9}{2}-0\right) = 9$ It represent the area of shaded region bounded by the curve $y = \lvert x+1\rvert$, $x -$ axis and the lines $x = -4$ and $x = 2$	1 ½ ½ ½ ½

	OR	
28B		1
	Required Area = $\int_0^4 x\,dx - \int_0^4 \frac{x^2}{4}\,dx$	1
	$= \left.\frac{x^2}{2}\right]_0^4 - \frac{1}{12}[x^3]_0^4$	½
	$= \frac{1}{2}(16-0) - \frac{1}{12}(64-0) = 8 - \frac{16}{3} = \frac{8}{3}$ sq. units	½
	For Visually Impaired:	
	$y = \lvert x+1 \rvert = f(x) = \begin{cases} -x-1, & x < -1 \\ x+1, & x \geq -1 \end{cases}$	1
	$\int_{-4}^{2} \lvert x+1 \rvert dx = \int_{-4}^{-1}(-x-1)\,dx + \int_{-1}^{2}(x+1)\,dx$	
	$= -\left.\frac{(x+1)^2}{2}\right]_{-4}^{-1} + \left.\frac{(x+1)^2}{2}\right]_{-1}^{2}$	1
	$= -\left(0 - \frac{9}{2}\right) + \left(\frac{9}{2} - 0\right) = 9$	
	It represent the area of shaded region bounded by the curve $y = \lvert x+1 \rvert$, $x -$ axis and the lines $x = -4$ *and* $x = 2$	1
	OR	
		

	$25x^2 + 16y^2 = 400 \Rightarrow \frac{x^2}{16} + \frac{y^2}{25} = 1 \Rightarrow \frac{x^2}{4^2} + \frac{y^2}{5^2} = 1 \Rightarrow y = \frac{5}{4}\sqrt{4^2 - x^2}$ Required Area = $4\int_0^4 \frac{5}{4}\sqrt{4^2 - x^2}\,dx$ $= 5\left[\frac{x\sqrt{4^2-x^2}}{2} + \frac{4^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$ $= 5[0 + 8\sin^{-1}(1) - 0]$ $= 40 \times \frac{\pi}{2} = 20\pi$ sq. units	1 1 1
29A 29B	The line through $(2,-1,3)$ parallel to the z-axis is given by $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(\hat{k})$ Any point on this line is P $(2,-1,3+\lambda)$ Any point on the given line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \mu(3\hat{\imath} + 6\hat{\jmath} + 2\hat{k})$ is Q $(2+3\mu, -1+6\mu, 2+2\mu)$ For the intersection point Q $(2+3\mu, -1+6\mu, 2+2\mu)$ = P (2, -1, 3+ λ) ⇒ $2 = 2 + 3\mu \Rightarrow \mu = 0$ The point of intersection is $(2,-1,2)$ The distance from $(2,-1,3)$ to $(2,-1,2)$ is clearly 1 unit. **Alternative Solution:** Any point on the line through $(2,-1,3)$ parallel to the z-axis is $(2,-1,\lambda)$ Any point on the given line is $(2+3\mu, -1+6\mu, 2+2\mu)$ Therefore, $2 = 2 + 3\mu \Rightarrow \mu = 0$ The point of intersection is $(2,-1,2)$ The distance from $(2,-1,3)$ to $(2,-1,2)$ is clearly 1 unit. **OR** The line through $(2,-1,1)$ parallel to the z-axis is $\vec{r} = (2\hat{\imath} - \hat{\jmath} + \hat{k}) + \lambda(\hat{k})$ Any point on this line is P (2, −1, 1+ λ) Any point on the given line is A (3+ μ, μ, 1+ μ) A (3+ μ, μ, 1+ μ) = P (2, −1, 1+ λ) $\Rightarrow \mu = -1$ The point of intersection is (2, −1,0) The distance of $(2,-1,0)$ from the z-axis is $\sqrt{2^2 + (-1)^2} = \sqrt{5}$ units.	1 ½ ½ ½ ½ 1 1 ½ ½ 1 1 ½ ½
30	Sketching the graph  	$1\frac{1}{2}$

30	Corner points A(600,0), B(1200,0), C(800,400), D(400,200) Values of Z: $Z_A = 1200, Z_B = 2400, Z_C = 2000, Z_D = 1000$ Maximum $Z = 2400$ when $x = 1200$ and $y = 0$ **For Visually Impaired:** At Corner points A(600,0), B(1200,0), C(800,400), D(400,200) Values of Z are $Z_A = 1800, Z_B = 3600, Z_C = 3200, Z_D = 1600$ Maximum Value of Z = 3600 at B(1200,0) And Minimum Value of Z= 1600 at D(400,200)	½ ½ ½ 1 1 1
31	Let the events be: A: Mehul is selected B: Rashi is selected Then according to the question, A and B are independent events and $P(A) = 0.4, \quad P(A \cap \bar{B}) + P(B \cap \bar{A}) = 0.5$ Let $P(B) = x$ Then $P(A \cap \bar{B}) + P(B \cap \bar{A}) = 0.5$ $\Rightarrow P(A)P(\bar{B}) + P(B)P(\bar{A}) = 0.5$ $\Rightarrow 0.4(1-x) + x(1-0.4) = 0.5$ $\Rightarrow 0.4 - 0.4x + 0.6x = 0.5$ $\Rightarrow 0.2x = 0.5 - 0.4 = 0.1$ $\Rightarrow x = \frac{0.1}{0.2} = \frac{1}{2} = 0.5$ So, probability of selection of Rashi = 0.5 Probability of selection of at least one of them $= 1 - P(\bar{A} \cap \bar{B})$ $= 1 - P(\bar{A})P(\bar{B})$ $= 1 - 0.6 \times 0.5$ $= 1 - 0.3 = 0.7$	1 1 1
	SECTION D **(LA type questions of 5 marks each)**	
32	$AB = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ -2 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ So, $A^{-1} = B \text{ and } B^{-1} = A$ Given system of equations is $3x - 6y - z = 3, \ 2x - 5y - z + 2 = 0, \ -2x + 4y + z = 5$ In matrix form it can be written as: $AX = C$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$ Here $\|A\| = -3 - 0 + 2 = -1 \neq 0$ So, the system is consistent and has unique solution given by the expression $X = A^{-1}C = BC$ $\Rightarrow X = \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 21 \end{bmatrix}$ Thus $x = 2, y = -3, z = 21$	1 ½ ½ ½ ½ ½ 1 ½

33A	Let $x = \tan\theta \Rightarrow dx = sec^2\theta\, d\theta$ $I = \int_0^{\frac{\pi}{4}} \frac{\log(1+tan\theta)}{1+tan^2\theta} . sec^2\theta\ d\theta$ $I = \int_0^{\frac{\pi}{4}} log\ (1+tan\theta)d\theta = \int_0^{\frac{\pi}{4}} log\ \left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right] d\theta$ $=\int_0^{\frac{\pi}{4}} \log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right] d\theta$ $=\int_0^{\frac{\pi}{4}} \log\left[\frac{1+\tan\theta+1-\tan\theta}{1+\tan\theta}\right] d\theta$ $=\int_0^{\frac{\pi}{4}} \log\left[\frac{2}{1+\tan\theta}\right] d\theta$ $=\int_0^{\frac{\pi}{4}} \log 2\ d\theta - \int_0^{\frac{\pi}{4}} \log[1+tan\theta]\ d\theta$ $= \log 2 \times x]_0^{\frac{\pi}{4}} - I$ $\Rightarrow 2I = \frac{\pi}{4}\log 2$ $\Rightarrow I = \frac{\pi}{8}\log 2$	½ 1 1 1 1 ½
33B	**OR**	OR
	I = $\int \frac{(3\ \sin\theta-2)\cos\theta}{5-cos^2\theta-4\sin\theta} d\theta = \int \frac{(3\ sin\,\theta-2)\,cos\,\theta}{5-(1-sin^2\theta)-4\,sin\,\theta}\ d\theta$ $Let\ \ sin\,\theta = t \implies cos\,\theta\ d\theta = dt$ $I = \int \frac{(3t-2)}{5-(1-t^2)-4t} dt$ $=\int \frac{(3t-2)}{t^2-4t+4} dt = \int \frac{3t-2}{(t-2)^2} dt$ Let $\frac{3t-2}{(t-2)^2} = \frac{A}{(t-2)} + \frac{B}{(t-2)^2}$ $3t-2 = A(t-2)+B$ Comparing the coefficients of t and constant terms on both sides $A = 3, -2A+B = -2, B = 4$ $\int \frac{(3\ \sin\theta-2)\cos\theta}{5-cos^2\theta-4\sin\theta} d\theta = \int \frac{3}{t-2}\ dt + \int \frac{4}{(t-2)^2}$ dt $= 3\log\|t-2\| - \frac{4}{t-2} + C$ $= 3\log\|sin\,\theta-2\| - \frac{4}{\sin\theta-2} + C$	½ 1 ½ +½ 1+1 ½
34A	$y + \frac{d}{dx}(xy) = x(\sin x + x)$ $\Longrightarrow y + (x\frac{dy}{dx} + y) = x(\sin x + x)$ $\Longrightarrow 2y + x\frac{dy}{dx} = x(\sin x + x)$ $\Longrightarrow \frac{dy}{dx} + \frac{2y}{x} = (\sin x + x)$ This a linear differential equation of the form $\frac{dy}{dx} + Py = Q$ P= $\frac{2}{x}$,Q= $(\sin x + x)$ I.F $= e^{\int\frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$ Solution will be y . I.F $=\int Q.IF\ dx$ $yx^2 = \int(sin\,x + x)\,x^2$ dx $yx^2 = \int sinx.x^2\,dx + \int x^3\,dx$	 1 1 1

	$\Rightarrow \quad yx^2 = -x^2\cos x + 2\int x\cos x\, dx\ + \frac{x^4}{4}$ + C	1
	$\Rightarrow yx^2 = -x^2 cos\ x$ +2 $(x\ sinx + cos\ x)$ + $\frac{x^4}{4}$ + C	
	Which is the required solution	1
	OR	
34B	$2y\ e^{x/y}dx + (\ y - 2x\ e^{x/y}\ )\ dy$ = 0	
	$\Rightarrow \quad \frac{dx}{dy} = \frac{2x\ e^{x/y} - y}{2y\ e^{x/y}} = \frac{2\frac{x}{y}e^{\frac{x}{y}} - 1}{2\ e^{\frac{x}{y}}}$	1
	It is a homogeneous differential equation.	
	Let $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$	1
	$v + y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v}$	
	$\Rightarrow y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v = \frac{2ve^v - 1 - 2ve^v}{2e^v}$	
	$\Rightarrow \quad y\frac{dv}{dy} = \frac{-1}{2e^v}$	
	$\Rightarrow 2e^v\ dv = -\frac{dy}{y}$	1
	$\int 2e^v\ dv = -\int \frac{dy}{y}$	
	$\Rightarrow 2e^v = -\log\|y\|$ + C	
	$\Rightarrow 2e^{\frac{x}{y}}$ +" $log\|y\|$ = C	1
	When $x = 0\ , y = 1$, C = 2	
	Required solution 2 $e^{\frac{x}{y}}$ + $\log\|y\|$ = 2	1
35	Let $\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0} = \lambda \quad \Rightarrow$ Any point on it is $(3\lambda + 1, -\lambda, -1)$	½
	For the point where $y = 1 \quad \Rightarrow \quad \lambda = -1$	1
	$\Rightarrow$ The point is $(-2, 1, -1)$	½
	The directions of the two lines are $\vec{m} = 3\hat{\imath} - \hat{\jmath}$	1
	and $\vec{n} = -2\hat{\imath} + 2\hat{\jmath} + \hat{k}$	½
	$\vec{m} \times \vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -1 & 0 \\ -2 & 2 & 1 \end{vmatrix} = -\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$	1
	The equation of the required line is	
	$\vec{r} = (-2\hat{\imath} + \hat{\jmath} - \hat{k}) + \mu(-\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$	½
	Alternative Solution:	
	Let $\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0} = \lambda \ \Rightarrow$ Any point on it is $(3\lambda + 1, -\lambda, -1)$	½
	For the point where $y = 1 \quad \Rightarrow \quad \lambda = -1$	1
	$\Rightarrow$ The point is $(-2, 1, -1)$	½
	Let the direction ratios of the required line be a, b, c	
	Then $3a - b = 0$	
	And $-2a + 2b + c = 0$	1
	Solving we get $\frac{a}{-1} = \frac{-b}{3} = \frac{c}{4} \Rightarrow \frac{a}{-1} = \frac{b}{-3} = \frac{c}{4}$	1
	The required line is $\frac{x+2}{-1} = \frac{y-1}{-3} = \frac{z+1}{4} = \mu$	½
	In vector form $\vec{r} = (-2\hat{\imath} + \hat{\jmath} - \hat{k}) + \mu(-\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$	½

SECTION- E (3 case-study/passage-based questions of 4 marks each)		
36	I. Traffic flow is not reflexive as $(A, A) \notin R$ (or no major spot is connected with itself) II. Traffic flow is not transitive as $(A, B) \in R$ and $(B, E) \in R, but$ $(A, E) \notin R$ III A. $R = \{(A, B), (A, C), (A, D), (B, C), (B, E), (C, E), (D, E), (D, C)\}$ $\text{Domain} = \{A, B, C, D\}$ $\text{Range} = \{B, C, D, E\}$ **OR** III B. No, the traffic flow doesn't represent a function as A has three images.	1 1 1 ½ + ½ 1+1
37	I. $P(x) = R(x) - C(x) = -0.3x^2 + 20x - (0.5x^2 - 10x + 150)$ $= -0.8x^2 + 30x - 150$ II. For critical points $P'(x) = 0 \Rightarrow -1.6x + 30 = 0$ $\Rightarrow x = \frac{30}{1.6} = \frac{300}{16} = 18.75$ III A. Now $P''(x) = -1.6$ In particular $P''(18.75) = -1.6 < 0$ So, critical value $x = 18.75$ corresponds to a maximum profit. **OR** III B. As x is the number of bulbs, so practically 18 bulbs correspond to a maximum profit. Maximum profit is $P(18) = -0.8 \times 18^2 + 30 \times 18 - 150$ $= -259.2 + 540 - 150$ $= 540 - 409.2 = ₹130.80$	1 1 1 1 1 1
38	Let the events be E$_1$: the student is in the first group (time spent on screen is more than 4 hours) E$_2$: the student is in the second group (time spent on screen is 2 to 4 hours) E$_3$: the student is in third group (time spent on screen is less than 2 hours) A: the event of the student showing symptoms of anxiety and low retention $P(E_1) = \frac{60}{100}$ $P(E_2) = \frac{30}{100}$ and $P(E_3) = \frac{10}{100}$ $P(A\|E_1) = \frac{80}{100}$ $P(A\|E_2) = \frac{70}{100}$ and $P(A\|E_3) = \frac{30}{100}$ I. $P(A) = P(E_1) \times P(A\|E_1) + P(E_2) \times P(A\|E_2) + P(E_3) \times P(A\|E_3)$ $= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{70}{100} + \frac{10}{100} \times \frac{30}{100} = \frac{72}{100} = 72\%$ II. $P(E_1\|A) = \frac{P(E_1 \cap A)}{P(A)}$ $= \frac{\left(\frac{60}{100} \times \frac{80}{100}\right)}{\left(\frac{72}{100}\right)} = \frac{48}{72} = \frac{2}{3}$	 2 2

CLASS XII – MATHEMATICS(041)

2025 – 2026 SAMPLE PAPER ANALYSIS

Unit No.	Unit	Chapter	Section A		Section B	Section C	Section D	Section E	Total
			MCQ	**AR**					
I	Relations and Functions	Relations & Functions						1(4)	**1(4)**
		Inverse Trigonometric Functions	1(1)	1(1)	1*(2)				**3(4)**
II	Algebra	Matrices	2(2)						**2(2)**
		Determinants	3(3)				1(5)		
III	Calculus	Continuity and Differentiability	2(2)		2(4)	1*(3)			**5(9)**
		Applications of Derivatives	1(1)			1(3)		1(4)	**3(8)**
		Integrals	2(2)		1#(2)		1*(5)		**4(7)**
		Application of Integrals				1*(3)			**2(5)**
		Differential Equations	1(1)				1*(5)		**2(6)**
IV	Vectors & Three Dimensional Geometry	Vector Algebra	1(1)	1(1)	1(2)				**3(4)**
		Three Dimensional Geometry	2(2)			1*(3)	1(5)		**4(10)**
V	Linear Programming	Linear Programming	2(2)			1(3)			**3(5)**
VI	Probability	Probability	1(1)			1(3)		1(4)	**3(8)**
Total			**18(18)**	**2(2)**	**5(10)**	**6(18)**	**4(20)**	**3(12)**	**38(80)**

Note: (i) Numeric outside the bracket indicates the number of questions **(ii) Numeric inside the bracket indicates the marks**

*** Indicates that question has an internal choice**

 # Internal Choice is given between two Chapters Questions

PRACTICE PAPER I

CLASS XII - MATHEMATICS

MARKS: 80 **DURATION: 3 HOURS**

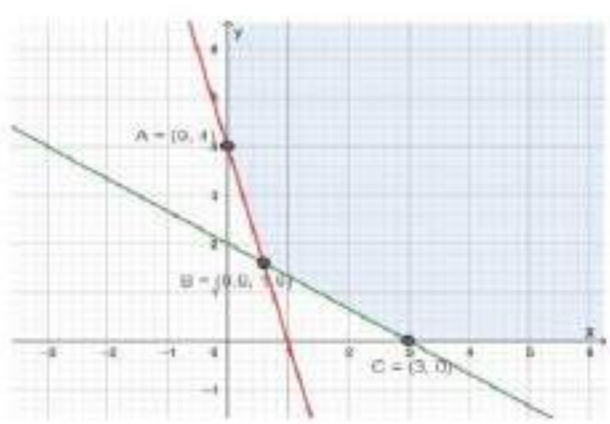
GENERAL INSTRUCTIONS:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

SECTION A

Section A consists of 20 questions of 1 mark each.

S.No		Marks
1	If AB = C, then matrices A, B, C are (a) $A_{2\times3}, B_{3\times2}, C_{2\times3}$ (b) $A_{3\times2}, B_{2\times3}, C_{3\times2}$ (c) $A_{3\times3}, B_{2\times3}, C_{3\times3}$ (d) $A_{3\times2}, B_{2\times3}, C_{3\times3}$	1
2	If A and B are invertible matrices of the same order such that $\lvert(AB)^{-1}\rvert = 8$ and $\lvert A\rvert = 2$, then $\lvert B\rvert$ is (a) 16 (b) 4 (c) 6 (d) $\frac{1}{16}$	1
3	If A is a square matrix of order 3 x 3 such that $\lvert adjA\rvert = 16$, also $(\lvert 2A\rvert)^2 = 2^p$, then the value of p is (a) 4 (b) 5 (c) 8 (d) 10	1
4	The value of $\lvert A\rvert\lvert adj\, A\rvert$ if $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, is (a) -2 (b) 1 (c) -1 (d) -3	1
5	If A and B are invertible matrices, then which of the following is not correct? (a) $adj\, A = \lvert A\rvert A^{-1}$ (b) $\det(A)^{-1} = [\det(A)]^{-1}$ (c) $(AB)^{-1} = B^{-1}A^{-1}$ (d) $(A + B)^{-1} = B^{-1} + A^{-1}$	1
6	The function f(x) = |x| + |x - 1| is (a) Continuous at x = 0 as well as at x = 1 (b) Continuous at x = 1 but not at x = 0 (c) Discontinuous at x = 0 and at x = 1 (d) Continuous at x = 0 but not at x = 1	1
7	If $x = 10(t - \sin t)$ and $y = 12(1 - \cos t)$ then, value of $\frac{dy}{dx}$, at $t = \frac{2\pi}{3}$ is (a) $\frac{2\sqrt{3}}{5}$ (b) $-\frac{6}{5\sqrt{3}}$ (c) $\frac{3\sqrt{2}}{5}$ (d) $\frac{2\sqrt{5}}{3}$	1
8	$\int \frac{e^{\tan^{-1}\sqrt{x}}}{\sqrt{x}+x\sqrt{x}}dx =$ (a) $\log\left\lvert\tan^{-1}\sqrt{x}\right\rvert + C$ (b) $\frac{1}{2}e^{\tan^{-1}\sqrt{x}} + C$ (c) $\frac{3}{2}e^{\tan^{-1}\sqrt{x}} + C$ (d) $2e^{\tan^{-1}\sqrt{x}} + C$	1

9	The solution of $\frac{dy}{dx} - 1 = e^{x-y}$ is (a) $(x + c)e^{y-x} = 1$ (b) $(x - c)e^{y+x} + 1 = 0$ (c) $(x + c)e^{x-y} = 1$ (d) $(x + c)e^{x-y} + 1 = 0$	1
10	The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^2 = \frac{d^2y}{dx^2}$ respectively are (a) 1, 2 (b) 2, 2 (c) 2, 1 (d) 4, 2	1
11	If $\lvert\vec{a}\rvert = 8, \lvert\vec{b}\rvert = 3$ and $\lvert\vec{a} \times \vec{b}\rvert = 12$, then value of $\vec{a} \cdot \vec{b}$ is (a) $6\sqrt{3}$ (b) $8\sqrt{3}$ (c) $12\sqrt{3}$ (d) $21\sqrt{3}$	1
12	The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel, is (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{2}{5}$	1
13	The domain of the function y = $\sin^{-1}$(- x2) is (a) [0,1] (b) (0,1) (c)[-1,1] (d)(-1,1)	1
14	The direction ratios of a line joining the points A(0, 4, 1) and B(2, 3, -1) is (a) 2, -1, -2 (b) $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$ (c) -2, 1, -2 (d) -2, -1, 2	1
15	The angle between the lines whose d.r.s are a, b, c and b – c, c – a and a – b is (a) 30^o (b) 60^o (c)90^o (d) 180^o	1
16	The corner points of the shaded unbounded feasible region of an LPP are (0, 4), (0.6, 1.6) and (3, 0) as shown in the figure. The minimum value of the objective function Z = 4x + 6y occurs at  (a) (0.6, 1.6) only (b) (3, 0) only (c) (0.6, 1.6) and (3, 0) only (d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)	1
17	Corner points of the feasible region for a LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let 4x + 6y be the objective function. The minimum value of occurs at (a) Only (0,2) (b) only (3,0) (c) The mid-point of the line segment joining the points (0,2) and (3,0) (d) Any point on the line segment joining the points (0,2) and (3,0).	1

18	Let A and B be two events. If P(A) = 0.2, P(B) = 0.4 and P(A U B) = 0.6, then P(A|B) is (a) 0.8 (b) 0.5 (c) 0.3 (d) 0	1

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

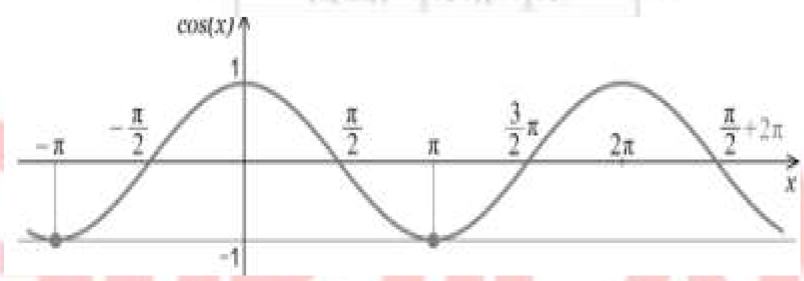
(d) Assertion (A) is false but reason (R) is true.

19	Assertion (A) : Set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set. Reason (R) : $\sec^{-1} x$ is defined for $x \in R - (-1,1)$.	1
20	Consider the function $f(x) = \begin{cases} \frac{k\|x-3\|}{x-3}, & \text{if } x < 3 \\ 5, & \text{if } x \geq 3 \end{cases}$ is continuous at x = 3. Assertion (A): The value of k is -5. Reason (R): $\frac{\|x-3\|}{x-3} = \begin{cases} 1, \text{if } x \geq 3 \\ -1, \text{if } x < 3 \end{cases}$	1

SECTION B

Section B consists of 5 questions of 2 marks each.

21	Evaluate: $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$. (OR) Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0,1]$	2
22	Evaluate $\frac{dy}{dx}$ at $x = 1$ and $y = \frac{\pi}{4}$ if $\sin^2 y + \cos(xy) = k$.	2
23	$\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $\|\vec{a}\| = 10, \|\vec{b}\| = 2$ and $\vec{a} \cdot \vec{b} = 12$. Find the area of the parallelogram.	2
24	Show that the function f is given by $f(x) = \tan^{-1}(\sin x + \cos x)$ is decreasing for all $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. (OR) If $f(x) = \frac{1}{4x^2+2x+1}; x \in \mathbb{R}$, then find the maximum value of f(x).	2
25	Evaluate: $\int_{-5}^{5} \|x+2\| dx$. (OR) Find the area bounded by the curves $y = x^2$ and the line $y = 4$.	2

SECTION C **Section C consists of 6 questions of 3 marks each.**		
26	If $(a+bx)e^{\frac{y}{x}} = x$ then prove that $x\frac{d^2y}{dx^2} = \left(\frac{a}{a+bx}\right)^2$. (OR) Find the value of k so that the following function is continuous at $x = \frac{\pi}{2}$: $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 5, & \text{if } x = \frac{\pi}{2} \end{cases}$	3
27	Find the intervals in which the function $f(x) = (x-1)^3(x-2)^2$ is (a) increasing (b) decreasing.	3
28	Find the area bounded by the curve y = cos x, x-axis and the ordinates $x = -\frac{5\pi}{6}$ and $x = \pi$ (OR) In a classroom, the teacher explains the properties of a particular curve by saying that this particular curve has beautiful ups and downs. It starts at 1 and heads down until π radian, and then heads up again as shown in the figure  Then find the area enclosed by the curve, x = -π and x = π.	3
29	Find the cartesian equation of the line passing through the point (-1, 3, -2) and perpendicular to the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. (OR) Find the coordinates of the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at the distance of 5 units from the point (1,3,3).	3
30	Solve the following equation graphically: Minimum Z = 5x + 10y Subject to: $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$; $x, y \geq 0$.	3
31	Four students of class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem.	3

SECTION D **Section D consists of 4 questions of 5 marks each.**		
32	Evaluate: $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$ (OR) Find: $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx, x \in [0,1]$	5
33	If A = $\begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$ find A^{-1}. Using A^{-1}, solve the following system of linear equations: 3x + 3y + 2z = 1; x + 2y = 4 ; 2x – 3y – z = 5.	5
34	Solve the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x,$ given that $y = 1$ when $x = \frac{\pi}{2}$. (OR) Solve the differential equation $\left(\sqrt{1 + x^2 + y^2 + x^2y^2}\right)dx + xydy = 0$.	5
35	Find the image of the point (1, 6, 3) in the line $x = \frac{y-1}{2} = \frac{z-2}{3}$. Also write the equation of the line joining the given point and its image and find the length of segment joining the given point and its image. (OR) A bird flying along the line $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and another bird flying along the line $\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$. At what points on the lines they reach so that the distance between them is the least? Find the shortest possible distance between them.	5
SECTION E **Case study-based questions are compulsory.**		
36	During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$. Based on the above information, answer the following questions: (i) Let $f: N \longrightarrow R$ is defined by $f(x) = x^2$. What will be the range of the function? (ii) Let $f: N \longrightarrow N$ is defined by $f(x) = x^2$. Check if the function is injective or not. (iii) Let $f: \{1,2,3,4 \ldots\ldots\} \longrightarrow \{1,4,9,16 \ldots\ldots\}$ be defined by $f(x) = x^2$. Prove that the	4

	function is bijective. (OR) Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective.	
37	A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively. Based on the given information, answer the following questions: (i) If the perimeter of the window is 12 m, find the relation between x and y. (ii) Using the expression obtained in (i), write an expression for the area of the window as a function of x only. (iii) Find the dimensions of the rectangle that will allow maximum light through the window. (use expression obtained in (ii)) (OR) (iii) If it is given that the area of the window is 50 m^2, find an expression for its perimeter in terms of x.	4
38	A shopkeeper sells three types of flower seed A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4:4:2 respectively. The germination rates of three types of seeds are $45\%, 60\%$ and 35% respectively. Based on the above information, answer the following questions: (i) Calculate the probability that a randomly chosen seed will germinate. (ii) Calculate the probability that the seed is of type A_2, given that a randomly chosen seed germinates.	4

PRACTICE PAPER II
CLASS XII - MATHEMATICS

MARKS: 80 **DURATION: 3 HOURS**

GENERAL INSTRUCTIONS:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.

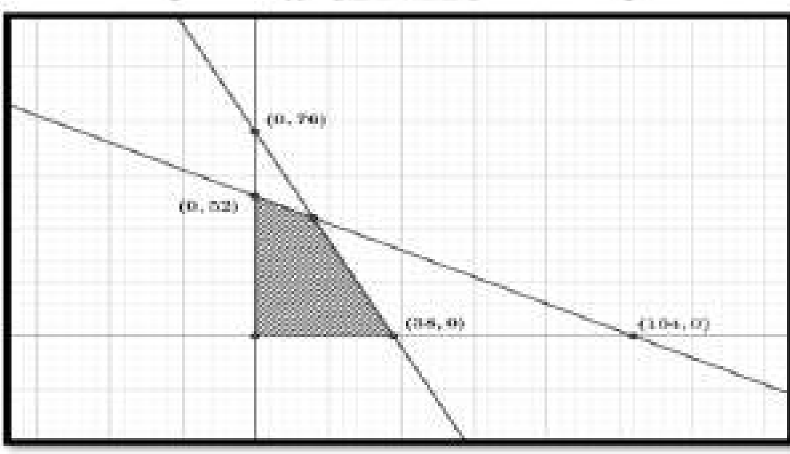
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.

6. **Section E** has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

SECTION A

Section A consists of 20 questions of 1 mark each.

S.No		Marks
1	If $f(x) = \|x\| + \|x-1\|$, then which of the following is correct? (a) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$. (b) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$. (c) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$. (d) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.	1
2	The sum of the degree and the order of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$ is (a) 2 (b) 3 (c) 4 (d) 5	1
3	In $\triangle ABC, \overrightarrow{AB} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$. If D is the midpoint of BC, then $\overrightarrow{AD}$ is equal to (a) $4\hat{\imath} + 6\hat{k}$ (b) $2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ (c) $\hat{\imath} - \hat{\jmath} + \hat{k}$ (d) $2\hat{\imath} + 3\hat{k}$	1
4	The value of $\|A\|$, if $A = \begin{bmatrix} 0 & 1-2x^2 & 2\sqrt{x} \\ 2x^2-1 & 0 & -\sqrt{x} \\ -2\sqrt{x} & \sqrt{x} & 0 \end{bmatrix}$, where $x \in R^+$ is: (a) $(2x^2-1)^2$ (b) 0 (c) $2x(1-2x^2)$ (d) $2x(2x^2-1)$	1
5	A and B are two events such that $P(A \cap B) = 0$. Then $P(A/A \cup B)$ is: (a) 0 (b) 1 (c) $\frac{P(A)}{P(A)+P(B)}$ (d) $\frac{P(A)}{P(A)XP(B)}$	1
6	If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of $x + y$ is (a) -2 (b) 2 (c) 3 (d) -3	1
7	The general solution of the differential equation $xdy - (1+x^2)dx = dx$ is: (a) $y = 2x + \frac{x^3}{3} + C$ (b) $y = 2\log x + \frac{x^3}{3} + C$ (c) $y = 2\log x - \frac{x^2}{2} + C$ (d) $y = 2\log x + \frac{x^2}{2} + C$	1
8	The value of λ for which the angle between the lines $\vec{r} = \hat{\imath} + \hat{\jmath} + \hat{k} + p(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ and $\vec{r} = (1+q)\hat{\imath} + (1+q\lambda)\hat{\jmath} + (1+q)\hat{k}$ is $\frac{\pi}{2}$ is (a) -4 (b) 4 (c) 2 (d) -2	1

9	Let A be the area of a triangle with vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3). Which of the following is correct? (a) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm A$ (b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$ (c) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{A}{2}$ (d) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = A^2$	1
10	The function $f(x) = x^3 - 3x^2 + 12x - 18$ is (a) Strictly decreasing on R (b) Strictly increasing R (c) neither strictly increasing nor strictly decreasing on R (d) Strictly decreasing on $(-\infty, 0)$	1
11	The value of $\int_{-\pi/4}^{\pi/4} \sec^2 x dx$ (a) -1 (b) 0 (c) 1 (d) 2	1
12	The feasible region of a linear programing problem given in the following figure is determined by the system of constraints:  (a) $2x + y \le 52$ and $x + 2y \le 76$, x≥ 0, y≥0 (b) $2x + y \ge 104$ and $x + 2y \ge 76$, x≥ 0, y≥0 (c) $x + 2y \le 104$ and $2x + y \le 76$, x≥ 0, y≥0 (d) $2x + y \le 104$ and $x + 2y \le 76$, x≥ 0, y≥ 0	1
13	The direction cosines of a line are c, c, c. Then: (a) $c > 0$ (b) $0 < c < 1$ (c) $c = 1$ (d) $c = \pm\frac{1}{\sqrt{3}}$	1
14	If $y = \log\sqrt{\sec\sqrt{x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}$ is (a) $\frac{1}{\pi}$ (b) π (c) $\frac{1}{2}$ (d) $\frac{1}{4}$	1
15	If $A \cdot (\text{adj}A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $\|A\| + \|\text{adj}A\|$ is equal to (a) 12 (b) 9 (c) 3 (d) 27	1
16	$\int_e^{e^2} \frac{\log x}{x} dx$ equals (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) -$\frac{3}{2}$ (d) -$\frac{1}{2}$	1

17	If A is square matrix of order 2 and $\|A\| = 2$, then $\|4A^{-1}\|$ equals: (a) 4 (b) 2 (c) 8 (d) $\frac{1}{32}$	1
18	Of all the points of the feasible region of an LPP, for maximum or minimum values of objective function, the point lie: (a) inside the feasible region (b) at the boundary line of the feasible region (c) at the corners of the feasible region (d) at the points of intersection of the feasible region with x-axis	1

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19	Assertion (A): $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$ is equal to $\frac{2\pi}{5}$ Reason(R): The range of principle value branch of the function f(x) = $\sin^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{-\pi}{2}\right]$	1
20	Assertion (A): $(\vec{b} \cdot \vec{c})\vec{a}$ is a scalar quantity. Reason (R) : Dot product of two vectors is a scalar quantity.	1

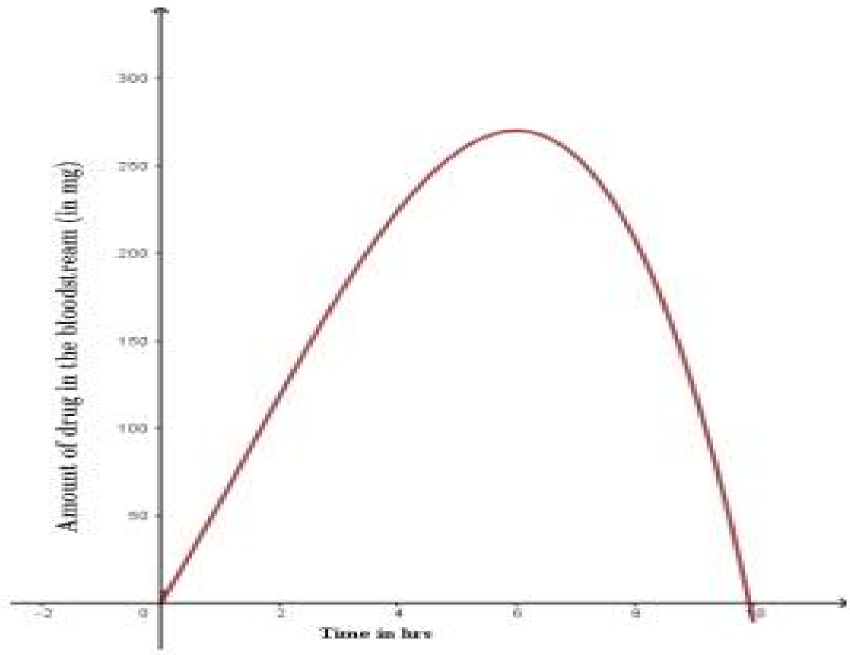
SECTION B

Section B consists of 5 questions of 2 marks each.

21	Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$.	2
22	Evaluate $\int_0^{\pi} \frac{1}{1+e^{\cos x}} dx$ (OR) Find the area of the curve $y = \sin x$ between 0 and π.	2
23	Find the value of k, if $\sin^{-1}\left[\text{k}\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$. (OR) Evaluate: $\sec^2(\tan^{-1} 3) + \text{cosec}^2(\cot^{-1} 2)$.	2

24	Check whether the function $f{:}R \rightarrow R$ defined by $(x) = x^3 + x$ has any critical point/s or not. If yes, then find the points.	2
25	If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{c} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$, find λ such that $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$.	2
SECTION C **Section C consists of 6 questions of 3 marks each.**		
26	Find the area of the region bounded by the lines x – 2y = 4, x = -1, x = 6 and x-axis using integration. (OR) Using integration, find the area of the region bounded by the line y = | x – 2 |, the x-axis and the ordinates x = 1 and x = 4.	3
27	If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$ (OR) If $x = a\left(\cos t + \log \tan \frac{t}{2}\right), y = a\sin t$, evaluate $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$	3
28	The cartesian equation of a line is 6x – 2 = 3y + 1 = 2z – 2. Find the direction cosines of the line. Write down the cartesian and vector equations of a line passing through (2, – 1, – 1) which are parallel to the given line. (OR) Find the value(s) of a so that the following lines are skew: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-a}{4}, \frac{x-4}{5} = \frac{y-1}{2} = z$	3
29	Solve the following linear programing problem graphically: Maximize $Z = 100x + 5y$, Subject to the constraints $x + y \leq 300$; $3x + y \leq 600$; $y \leq x + 200$; $x, y \geq 0$.	3
30	A, B and C shot to hit a target. If A hits the target 4 times in 5 trails; B hits it 3 times in 4 trails; and C hits the target 2 times in 3 trails; What is the probability that the target is hit by at least 2 persons.	3
31	A kite is flying at a height of 3 metres and 5 metres of string is out. If the kite is moving away horizontally at the rate of 200 cm/s, find the rate at which the string is being released.	3
SECTION D **Section D consists of 4 questions of 5 marks each.**		
32	Find the value of b so that the lines $\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are	5

	intersecting lines. Also, find the point of intersection of these lines. (Or) Find the equations of the line passing through the points A(1, 2, 3) and B(3, 5, 9). Hence, find the coordinates of the points on this line which are at a distance 14 units from point B.	
33	If $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations: y + 2z = 8, x – y = -1, 2x + 3y + 4z = 20.	5
34	Find: $\int \frac{(3\cos x - 2)\sin x}{5 - \sin^2 x - 4\cos x} dx$ (OR) Evaluate: $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$	5
35	Solve the differential equation: $y + \frac{d}{dx}(xy) = x(\sin x + x)$ (OR) Find the particular solution of the differential equation: $xe^{\frac{y}{x}} - y\sin\left(\frac{y}{x}\right) + x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) = 0$ for $y(1) = 0$.	5
	SECTION E **Case study-based questions are compulsory.**	
36	A classroom teacher is keen to assess the learning of her students the concept of ‘relation’ taught to them. He writes the following five relation each defined on the set A = {1, 2, 3}. $R_1 = \{(2,3), (3,2)\}$ $R_2 = \{(1,2), (1,3), (3,2)\}$ $R_3 = \{(1,2), (2,1), (1,1)\}$ $R_4 = \{(1,1), (1,2), (3,3), (2,2)\}$ $R_5 = \{(1,1), (1,2), (3,3), (2,2), (2,1), (2,3), (3,2)\}$ The students are asked to answer the following question about the above relations: (i) Identify the relation which is reflexive, transitive but not symmetric. (ii) Identify the relation which is reflexive and symmetric but not transitive. (iii) Identify the relations which are symmetric but not neither reflexive nor transitive. (OR) What pairs should be added to the relation R_2 to make it an equivalence relation?	4

37	A medicinal drug administered into a human body requires some time to produce its effect on the body. The amount (in mg) of a certain medicinal drug in the bloodstream at t hours after administering the drug to an individual is given by the function: $C(t) = -t^3 + 4.5t\ 2 + 54t$, $0 \le t \le 10$ Shown below is the graph of $C(t)$ in the interval [0, 10].  Based on the above information, answer the following questions: (i) Find the rate at which the amount of drug is changing in the bloodstream at 5 hours after administering the drug. Show your work. (ii) Show that the function C(t) is strictly increasing in the interval (3, 4).	4
38	In a school, teacher asks a question to three students Ravi, Mohit, and Sonia. The probability the question by Ravi, Mohit, and Sonia 30%, 25% and 45% respectively. The Probability of making error by Ravi, Mohit, and Sonia are 1%, 1.2% and 2% respectively. Based on the above information answer the following questions: (i) Find the conditional probability that an error is committed in solving question given that question is solved by Sonia. (ii) Find the probability that Sonia solved the question and committed an error. (iii) Find the total probability of committing an error in solving the question. (Or) If the solution of question is checked by teacher and has some error, then find the probability that the question is not solved by Ravi.	4

PRACTICE PAPER III
CLASS XII - MATHEMATICS

MARKS: 80 **DURATION: 3 HOURS**

GENERAL INSTRUCTIONS:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.

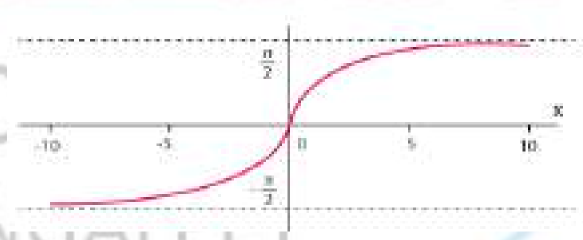
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.

6. **Section E** has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

SECTION A

Section A consists of 20 questions of 1 mark each.

S.No		Marks
1	If $f(x) = \log\sqrt{\tan x}$, then the value of $f'(x)$ at $x = \frac{\pi}{4}$ is (a) 1 (b) 0 (c) ∞ (d) $\frac{1}{2}$	1
2	The value of λ when the projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units is (a) 5 (b) 7 (c) ± 5 (d) ± 7	1
3	If $\|\vec{a}\| = 8, \|\vec{b}\| = 3, \|\vec{a}\cdot\vec{b}\| = 12\sqrt{3}$, then find the value of $\|\vec{a}\times\vec{b}\|$ (a) $4\sqrt{3}$ (b) $12\sqrt{3}$ (c) 12 (d) 6	1
4	If P(A) = 0.4, P(B) = 0.8 and P(B/A) = 0.6, then P(A∪B) is (a) 0.96 (b) 1.44 (c) 1.04 (d) 0.24	1
5	The value of k for which the function f(x) = $\begin{cases} \frac{x^3-8}{x-2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$ is continuous at x = 2 is (a) 4 (b) -4 (c) 12 (d) 0	1
6	$\int \frac{e^x}{x+1}\{1 + (x+1)\log(x+1)\}dx$ (a) $e^x\log(x+1) + e^x + C$ (b) $e^x\log(x+1) + C$ (c) $\frac{e^x}{x+1} + C$ (d) $\frac{e^x x}{x+1} + C$	1
7	The degree of the differential equation: $4\frac{\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 - 1$ is (a) 2 (b) 3 (c) 1 (d) Not defined	1
8	If $\begin{bmatrix} 3a+6 & 2 \\ -8 & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$, then the value of ab is (a) 16 (b) -16 (c) 4 (d) -4	1

9	If A is a square matrix of order 3 and |A| = 12, then the value of |A adjA| is (a) 12 (b) 144 (c) 1728 (d) 27	1
10	Sam plotted three points A (2, -3), B (x, -1) and C (0, 4) on a graph sheet. The value of x that makes the points collinear is (a) -10 (b) 10 (c) $\frac{10}{7}$ (d) $\frac{-10}{7}$	1
11	The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is (a) $(-1,\infty)$ (b) (-2, -1) (c) $(-\infty, -2)$ (d) [-1, 1]	1
12	If A is a square matrix such that $A^2 = I$, then find the value of $(A - I)^3 + (A + I)^3 - 7A$ is (a) A (b) I – A (c) I + A (d) 3A	1
13	$\int \frac{1}{x(x^5+1)} dx$ (a) $\frac{1}{5}\log\left\|\frac{x^5}{x^5+1}\right\| + c$ (b) $\frac{1}{5}\log\|x^5(x^5+1)\| + c$ (c) $\frac{1}{5}\log\left\|\frac{x^5+1}{x^5}\right\| + c$ (d) None of these	1
14	The corner points of the feasible region determined by a system of linear inequalities with Z = 3x + 9y as objective function are A(0, 20), B(15, 15), C(5, 5) and D(0, 10). The maximum of Z is (a) occurs only at A (b) occurs only at B (c) occurs at A and B (d) occurs at every point on AB	1
15	The value of x + y for which the matrix A = $\begin{bmatrix} 0 & -3 & 1 \\ 3 & 0 & -5 \\ x & y & 0 \end{bmatrix}$ is skew-symmetric is (a) 4 (b) -4 (c) 6 (d) -6	1
16	Identify the function from the graph. (a) $\sec^{-1}x$ (b) $\operatorname{cosec}^{-1}x$ (c) $\tan^{-1}x$ (d) $\cot^{-1}x$ 	1
17	The corner points of the feasible region determined by a system of linear equations with Z = ax + by where a, b > 0 are (0, 0), (2, 4), (4, 0) and (0, 5). The relation between a and b so that the maximum of Z occurs at both (2,4) and (4,0) is (a) a = 2b (b) 2a = b (c) a = b (d) 3a = b	1
18	The positions of a kite at two different timings were noted and the equation of the line joining these two points was given as $x = -3, \frac{2y+4}{6} = 4z - 12$. The direction ratios of the line are: (a) (1, 6, 1) (b) (1, 3,4) (c) (0, 6, 1) (d) $\left(0,3,\frac{1}{4}\right)$	1

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A) (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A) (c) Assertion (A) is true but reason (R) is false. (d) Assertion (A) is false but reason (R) is true.			
19	Assertion(A): If a unit vector makes angles $60°$ with $\hat{\imath}$, $45°$ with $\hat{\jmath}$ and an acute angle θ with $\hat{k}$ then the value of θ is $60°$. Reason(R): If l, m, n are the direction cosines of a vector, then $l^2 + m^2 + n^2 = 1$.	1	
20	Assertion (A): f : R → R defined by f(x) = x, then f is an injective function. Reason (R): A function g: A → B is said to be onto function if for each b ∈ B, there exists a ∈ A such that g(a) = b.	1	
SECTION B **Section B consists of 5 questions of 2 marks each.**			
21	Find the domain of the function $y = \cos^{-1}(x^2 - 4)$. (Or) Find the value of $\sin^{-1}\left[\cos\left(-\frac{17\pi}{8}\right)\right]$	2	
22	If $y = \sin(\log x)$, then show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.	2	
23	Find the length of the sum of the three mutually perpendicular unit vectors.	2	
24	A man whose height is 2m walks at a uniform speed of 5m/min away from a lamp post which is 6m high. Find the rate at which the length of his shadow increases.	2	
25	Evaluate $\int \frac{\cos 2x - \cos 2\emptyset}{\cos x - \cos \emptyset} dx$ (OR) Find the area bounded between $y = \sin^{-1}x$ and y-axis between y = 0 and $y = \frac{\pi}{2}$.	2	
SECTION C **Section C consists of 6 questions of 3 marks each.**			
26	Discuss the continuity and differentiability of the function $f(x) = \lvert x\rvert + \lvert x - 1\rvert$ in the interval (-1, 2). (OR) Given $e^x + e^y = e^{x+y}$. Show that $\frac{dy}{dx} + e^{y-x} = 0$	3	

27	Find the area bounded by the curve y = cosx, x-axis and the ordinates x = $\frac{-5\pi}{6}$ and x = π. (OR) Find the area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and x − axis.	3
28	Find the intervals in which the function $f(x) = 8 + 36x + 3x^2 - 2x^3$ is increasing or decreasing.	3
29	If A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).	3
30	Solve the following LPP graphically. Maximise: Z = 100x + 120y, Subject to constraints: 2x + 3y ≤ 30; 3x + y ≤ 17; x, y ≥ 0.	3
31	Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from $P(1,3,3)$. (OR) Find the equation of a line passing through (1, 2, -4) and perpendicular to the lines $\vec{r} = 8\hat{\imath} + 2\hat{\jmath} - 5k + \lambda(3\hat{\imath} - 16\hat{\jmath} + 7k)$ and $\vec{r} = 3\hat{\imath} - \hat{\jmath} + 5\hat{k} + \mu(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$,	3
SECTION D **Section D consists of 4 questions of 5 marks each.**		
32	10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises of hard workers, the second group has honest and law abiding students and the third group contains obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of the first and second group is four times that of the third group. Using matrix method, find the number of students in each category.	5
33	Evaluate: $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ (OR) Evaluate: $\int (x-3)\sqrt{x^2 + 3x - 18}dx$	5
34	Find the coordinates of the foot of the perpendicular drawn from the point (1, 2, 1) to the line joining the points (1, 4, 6) and (5, 4, 4). Also find the image.	5
35	Find the particular solution of the following differential equation given that y = 0 when x = 1; $(x^2 + xy)\, dy = (x^2 + y^2)\, dx$. (OR) Solve the differential equation: $(\tan^{-1} y - x)dy = (1 + y^2)dx$.	5

SECTION E **Case study-based questions are compulsory.**		
36	Indian railways is the largest rail network in Asia and the world's second largest. Hence the government pays a huge amount as fuel cost. Decreasing fuel cost can increase railway profit and hence will improve the economy. The fuel cost for running a train is proportional to the square of the speed generated (v) in km/hr. The fuel costs Rs 48 per hour at speed 16 km/hr and the fixed charges amount to Rs 1200 per hour. (i) If C is the total cost for covering the distance S km, then express C as a function of v. (ii) Find the critical point for C. (iii) Use first derivative test to find the most economical speed of the train. (OR) Use second derivative test to find the point of local minimum and minimum cost if distance is 100 km.	4
37	A coach is training 3 players. He observes that player A can hit a target 4 times in 5 shots, player B can hit a target 3 times in 4 shots and player C can hit a target 2 times in 3 shots. Based on this situation answer the following questions. (i) Find the probability that exactly one hits the target. (ii) Find the probability that exactly two of them hit the target. (iii) Find the probability that at most one of them hit the target. (OR) Find the probability that at least two of them hit the target	4
38	Swathi and Mani are playing Ludo at home while it was raining outside. While rolling the dice Swathi 's brother Varun observed and noted the possible outcomes of the throw every time belongs to the set {1, 2, 3, 4, 5, 6}. Let A be the set of players while B be the set of all possible outcomes. A = {Swathi, Mani}, B = {1, 2, 3, 4, 5, 6}. Based on the above, answer the following questions: (i) Let R: B → B, be defined by R = {(x, y): y is divisible by x}.Verify that whether R is reflexive, symmetric and transitive.	4

	(ii) Is it possible to define an onto function from A to B? Justify. (iii) Which kind of relation is R defined on B given by R= {(1,2), (2,2), (1,3), (3,4), (3,1), (4,3), (5,5)}? (OR) Find the number of possible relations from A to B .	

PRACTICE PAPER I – ANSWER KEY

CLASS XII - MATHEMATICS

S.No		Marks
	SECTION A	
	Section A consists of 20 questions of 1 mark each.	
S.No		**Marks**
1	(d) $A_{3\times2}$, $B_{2\times3}$, $C_{3\times3}$	1
2	(d) $\frac{1}{16}$	1
3	(d) 10	1
4	(c) -1	1
5	(d) $(A + B)^{-1} = B^{-1} + A^{-1}$	1
6	(a) Continuous at x = 0 as well as at x = 1	1
7	(a) $\frac{2\sqrt{3}}{5}$	1
8	(d) $2e^{\tan^{-1}\sqrt{x}} + C$	1
9	(c) $(x + c)e^{x-y} = 1$	1
10	(c) 2, 1	1
11	(c) $12\sqrt{3}$	1
12	(b) $\frac{3}{2}$	1
13	(c)[-1,1]	1
14	(a) 2, -1, -2	1
15	(c)90^{o}	1
16	(d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)	1
17	(d) Any point on the line segment joining the points (0,2) and (3,0).	1
18	(d) 0	1
19	(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)	1
20	(d) Assertion (A) is false but reason (R) is true.	1
	SECTION B	
	Section B consists of 5 questions of 2 marks each.	
21	$-\frac{\pi}{12}$ (Or) -1	2
22	$\frac{\pi}{4(\sqrt{2}-1)}$	2

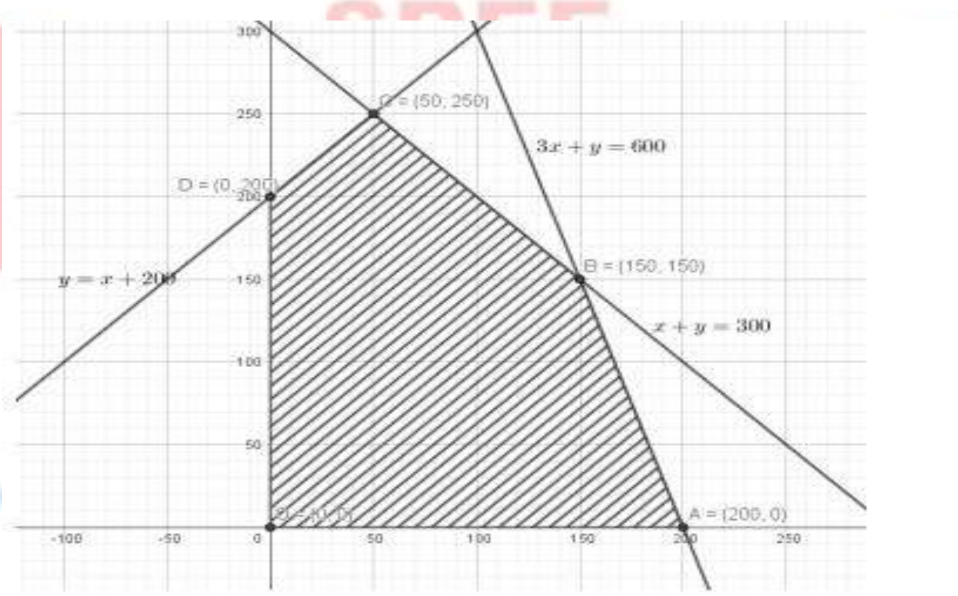
23	16	2
24	Proof (Or) $\frac{4}{3}$	2
25	29 (OR) $\frac{32}{3}$ Square units	2
SECTION C **Section C consists of 6 questions of 3 marks each.**		
26	Proof (OR) k = 10	3
27	(a) $(2, \infty)$ (b) $\left(\frac{8}{5}, 2\right)$	3
28	$\frac{7}{2}$ (OR) 4	3
29	$\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$ (OR) (-2, -1, 3) and (4, 3, 7)	3
30	Minimum value Z is 300, when $x = 60$ and $y = 0$.	3
31	$\frac{19}{45}$	3
SECTION D **Section D consists of 4 questions of 5 marks each.**		
32	$\frac{3}{8}\log\lvert x-1\rvert - \frac{1}{2}\cdot\frac{1}{(x-1)} + \frac{5}{8}\log\lvert x+3\rvert + C$ (OR) $x - \frac{4}{\pi}x\cos^{-1}\sqrt{x} - \frac{2}{\pi}\left[\sin^{-1}\sqrt{x} - \sqrt{x}\sqrt{1-x}\right] + C$	5
33	$\mathrm{A}^{-1} = \frac{-1}{17}\begin{bmatrix} -2 & 1 & -7 \\ -3 & -7 & 15 \\ -4 & 2 & 3 \end{bmatrix}$, $\mathrm{x} = 2, \mathrm{y} = 1, \mathrm{z} = -4$	5
34	$y = \sin x$ (OR) $\sqrt{1+y^2} + \sqrt{1+x^2} + \frac{1}{2}\log\left\lvert\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right\rvert = C$	5

35	(1, 0, 7) and $2\sqrt{13}$ (Or) $\frac{3}{\sqrt{19}}$	5
SECTION E **Case study-based questions are compulsory.**		
36	(i) $R_f = \{1,4,9,16, \ldots\}$ i.e.set of perfect squares of natural numbers. (ii) f is injective (iii) f is bijective	4
37	(i) Perimeter $(\mathrm{P}) = 3\mathrm{x} + 2\mathrm{y} = 12$ (ii) Area(A) = $6\mathrm{x} - \frac{3}{2}\mathrm{x}^2 + \frac{\sqrt{3}}{4}\mathrm{x}^2$ (iii) $\frac{18-6\sqrt{3}}{6-\sqrt{3}}$ m (OR) $3\mathrm{x} + 2\left(\frac{50}{\mathrm{x}} - \frac{\sqrt{3}}{4}\mathrm{x}\right)$ m	4
38	(i) 0.49 (ii) $\frac{24}{49}$	4

PRACTICE PAPER II – ANSWER KEY

CLASS XII - MATHEMATICS

SECTION A **Section A consists of 20 questions of 1 mark each.**		
S.No		**Marks**
1	(c) f(x) is continuous but not differentiable, at $x = 0$ and $x = 1$.	1
2	(c) 4	1
3	(d) $2\hat{i} + 3\hat{k}$	1
4	(b) 0	1
5	(c) $\frac{P(A)}{P(A)+P(B)}$	1
6	(b) 2	1
7	(d) $y = 2\log x + \frac{x^2}{2} + C$	1
8	(a) -4	1
9	(b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$	1
10	(b) Strictly increasing R	1
11	(d) 2	1
12	(c) $x + 2y \leq 104$ and $2x + y \leq 76$, $x \geq 0$, $y \geq 0$	1
13	(d) $c = \pm \frac{1}{\sqrt{3}}$	1
14	(a) $\frac{1}{\pi}$	1
15	(a) 12	1
16	(b) $\frac{3}{2}$	1
17	(c) 8	1
18	(c) at the corners of the feasible region	1
19	(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)	1
20	(d) Assertion (A) is false but reason (R) is true.	1
SECTION B **Section B consists of 5 questions of 2 marks each.**		
21	$\frac{1}{2}$	2
22	$\frac{\pi}{2}$ (OR) 2 square units	2

<table>
<tr><td>23</td><td>k $= \frac{1}{2}$
(OR)
15</td><td>2</td></tr>
<tr><td>24</td><td>$f(x)$ has no critical points.</td><td>2</td></tr>
<tr><td>25</td><td>-2</td><td>2</td></tr>
<tr><td colspan="3">SECTION C
Section C consists of 6 questions of 3 marks each.</td></tr>
<tr><td>26</td><td>$\frac{29}{4}$ square units
(OR)
$\frac{5}{2}$ square units</td><td>3</td></tr>
<tr><td>27</td><td>Proof
(Or)
$\frac{8\sqrt{3}}{a}$</td><td>3</td></tr>
<tr><td>28</td><td>$r = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}), \frac{x-2}{1} = \frac{y+1}{2} = \frac{z+1}{3}$
(OR)
$a \in R - \{3\}$</td><td>3</td></tr>
<tr><td>29</td><td>

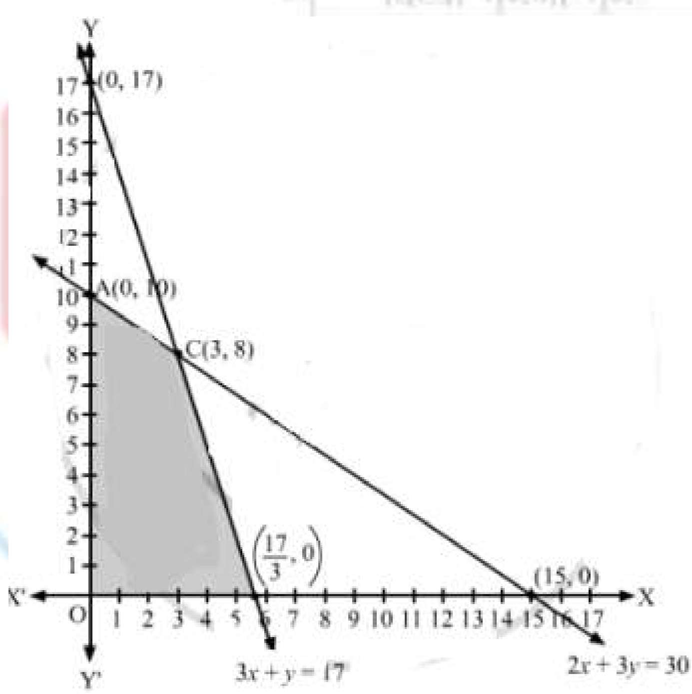
<table>
<tr><th>Corner points</th><th>Value of $Z = 100x + 5y$</th></tr>
<tr><td>$(0,0)$</td><td>0</td></tr>
<tr><td>$(200,0)$</td><td>$20000 \rightarrow Maximum$</td></tr>
<tr><td>$(150,150)$</td><td>15750</td></tr>
<tr><td>$(50,250)$</td><td>6250</td></tr>
<tr><td>$(0,200)$</td><td>1000</td></tr>
</table></td><td>3</td></tr>
<tr><td>30</td><td>$\frac{5}{6}$</td><td>3</td></tr>
<tr><td>31</td><td>160cm/s</td><td>3</td></tr>
</table>

SECTION D **Section D consists of 4 questions of 5 marks each.**		
32	b = 2 & (-1, -1, -1) (Or) $\frac{x-3}{2}=\frac{y-5}{3}=\frac{z-9}{6}$ and $Q(7,11,21)$ or, $Q(-1,-1,-3)$	5
33	$x=1, y=2, z=3$	5
34	$\frac{4}{\cos x-2}-\frac{3}{2}\log\lvert 5-\sin^2 x-4\cos x\rvert+c$ (OR) $\sqrt{2}\sin^{-1}(\sin x-\cos x)+C$	5
35	$\mathrm{y}=-\cos \mathrm{x}+\frac{2\sin \mathrm{x}}{\mathrm{x}}+\frac{2\cos \mathrm{x}}{\mathrm{x}^2}+\frac{\mathrm{x}^2}{4}+\frac{\mathrm{C}}{\mathrm{x}^2}$ (OR) $\mathrm{e}^{-\frac{y}{x}}\left(\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right)=2\log\lvert x\rvert+1.$	5
SECTION E **Case study-based questions are compulsory.**		
36	(i)R_4 (ii)R_5 (iii) R_1 and R_3 (OR) (1,1), (2,2), (3,3), (2,1), (3,1) and (2,3)	4
37	(i) = 24mg/hr (ii) Proof	4
38	(i) $\frac{2}{100}$ or 0.02 (ii) 0.009 (iii) 0.015 (Or) 0.8	4

PRACTICE PAPER III – ANSWER KEY

CLASS XII - MATHEMATICS

SECTION A **Section A consists of 20 questions of 1 mark each.**		
S.No		**Marks**
1	(a) 1	1
2	(a) 5	1
3	(c) 12	1
4	(a) 0.96	1
5	(c) 12	1
6	(b) $e^x \log(x+1) + C$	1
7	(a) 2	1
8	(c) 4	1
9	(c) 1728	1
10	(c) $\frac{10}{7}$	1
11	(b) (-2, -1)	1
12	(a) A	1
13	(a) $\frac{1}{5}\log\left\lvert\frac{x^5}{x^5+1}\right\rvert + c$	1
14	(d) occurs at every point on AB	1
15	(a) 4	1
16	(c) $\tan^{-1}x$	1
17	(a) a = 2b	1
18	(d) $\left(0,3,\frac{1}{4}\right)$	1
19	(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)	1
20	(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)	1
SECTION B **Section B consists of 5 questions of 2 marks each.**		
21	$[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$ (Or) $\frac{3\pi}{8}$	2

22	Proof	2
23	$\lvert\vec{a}+\vec{b}+\vec{c}\rvert=\sqrt{3}$	2
24	$\frac{5}{2}$, m/min	2
25	$2(\sin x + x\cos \emptyset) + C$ (OR) 1	2
	SECTION C **Section C consists of 6 questions of 3 marks each.**	
26	f(x) is continuous in the interval (-1, 2) and not differentiable at x = 0 and x = 1.	3
27	$\frac{7}{2}$ (OR) 8π square units	3
28	Increasing in (-2, 3) and decreasing in $(-\infty,-2)$ U $(3,\infty)$.	3
29	$P(A)=\frac{3}{4}$ or $\frac{1}{3}$ and $P(B)=\frac{2}{3}$ or $\frac{1}{4}$	3
30	 The maximum value of Z is 1260 at $x=3, y=8$.	3
31	$(-2,-1,3)$ or $(4,3,7)$ (OR) $\vec{r}=\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda_1(24\hat{\imath}+36\hat{\jmath}+72\hat{k})$	3
	SECTION D **Section D consists of 4 questions of 5 marks each.**	

32	$x = 5, y = 3, z = 2$	5
33	$\frac{\pi^2}{16}$ (OR) $\frac{1}{3}(x^2+3x-18)^{3/2} - \frac{9}{8}(2x+3)\sqrt{x^2+3x-18} + \frac{729}{16}\log\left\|\frac{(2x+3)}{2} + \sqrt{x^2+3x-18}\right\| + C$	5
34	(3, 4, 5) and (5, 6, 9)	5
35	$\frac{-y}{x} - 2\log(x-y) + \log x = 0$ (OR) $x = \tan^{-1} y - 1 + Ce^{-\tan^{-1} y}$.	5
SECTION E **Case study-based questions are compulsory.**		
36	(i) $C = 1200\left(\frac{S}{v}\right) + \frac{3vS}{16}$ (ii) v = 80 (iii) 80km/hr (OR) Rs.3000	4
37	(i) $\frac{3}{20}$ (ii) $\frac{13}{30}$ (iii) $\frac{1}{6}$ (OR) $\frac{5}{6}$	4
38	(i) R is reflexive and transitive but not symmetric. (ii) No. Because n(B) is greater than n(A) (iii) R is neither reflexive nor symmetric nor transitive (OR) 2^{12}	4