Date......../......../........

Page..........................

PBMT - 01

Ch - 1, 2, 3, 4

Sec B

Q11 $\tan\left(\frac{\pi}{4} + \cos^{-1}\frac{4}{5}\right)$

$\cos^{-1}\frac{4}{5} = \theta$

$\frac{4}{5} = \cos\theta$

$\frac{3}{5} = \sin\theta$

$\tan\theta = \frac{3}{4}$

$\tan^{-1}\frac{3}{4} = \theta$

$\tan\left(\frac{\pi}{4} + \tan^{-1}\frac{3}{4}\right)$ $\left[\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$$\frac{\tan \pi/4 + \tan\left(\tan^{-1}\frac{3}{4}\right)}{1 - \tan\frac{\pi}{4}\tan\left(\tan^{-1}\frac{3}{4}\right)}.$$

$$\frac{1 + \frac{3}{4}}{1 - \frac{3}{4}} = \frac{4+3}{4-3} = 7.$$

Or

$\tan^{-1}y + \tan^{-1}z = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{y+z}{1-yz}\right) = \tan^{-1}1$

$\frac{y+z}{1-yz} = 1$

Date......./......./.......

Page.........................

$y + z = 1 - yz$

$y + z + yz = 1$

Q12 $A^t = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

$A^t A = \begin{bmatrix} 0 + x^2 + x^2 & 0 & 0 \\ 0 & 4y^2 + y^2 + y^2 & 0 \\ 0 & 0 & z^2 + z^2 + z^2 \end{bmatrix}$

$= \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = I$

$\therefore \quad 2x^2 = 1 \qquad 6y^2 = 1 \qquad z = \pm \frac{1}{\sqrt{3}}$

$x^2 = \frac{1}{2} \qquad y = \frac{\pm 1}{\sqrt{6}}$

$x = \frac{\pm 1}{\sqrt{2}}$

Date......./......./

Page.................

Q13 - $f(x) = \dfrac{x}{x^2+1}$

let $f(x_1) = x_1$ & $x_2 \in R$

such that

$$f(x_1) = f(x_2)$$

$$\frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$$

$$x_1(x_2^2+1) = x_2(x_1^2+1)$$

$$x_1 x_2^2 + x_1 = x_2 x_1^2 + x_2.$$

$$x_1 x_2^2 - x_2 x_1^2 = x_2 - x_1$$

$$x_2 x_1(\cancel{x_2 - x_1}) = \cancel{x_2 - x_1}$$

$$x_2 x_1 = 1$$

$$x_2 = \frac{1}{x_1}$$

$\therefore x_1 \neq x_2.$

$\therefore f(x)$ is not one-one fn.

let $f(x) = y$

$$y = \frac{x}{x^2+1}$$

$$yx^2 + y = x$$

$$y = x - yx^2$$

$$y = x(1 - yx)$$

$$y \neq$$

$\therefore f(x)$ is not onto.

Date......../......../........

Page..........................

Sec C

Q 13 :

$$\underset{(1\times 2)}{A} \cdot \begin{bmatrix} 1 & -2 & 5 \\ 4 & -3 & 9 \end{bmatrix}_{2\times 3} = \begin{bmatrix} 14 & -13 & 37 \end{bmatrix}_{1\times 3}$$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & -2 & 5 \\ 4 & -3 & 9 \end{bmatrix}$$

$$\begin{bmatrix} x+4y & -2x-3y & 5x+9y \end{bmatrix} = \begin{bmatrix} 14 & -13 & 37 \end{bmatrix}$$

$$x + 4y = 14 \quad \times 2$$
$$-2x - 3y = -13$$

$$\cancel{2x} + 8y = 28$$
$$-\cancel{2x} - 3y = -13$$

$$x + 12 = 14$$
$$x = 2.$$

$$5y = 15$$
$$y = 3$$

$$A = \begin{bmatrix} 2, & 3 \end{bmatrix}$$

Or.

$$\det(A) = 1(1+\sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(\sin^2\theta + 1)$$
$$= 1 + \sin^2\theta + \sin^2\theta + 1$$
$$= 2 + 2\sin^2\theta = 2(1+\sin^2\theta)$$

$$-1 \le \sin\theta \le 1$$

$\sin\theta = -1$ $\quad$ $\sin\theta = 1$ $\quad$ $\sin\theta = 0$

$\sin^2\theta = 1$ $\quad$ $\sin^2\theta = 1$ $\quad$ $\sin^2\theta = 0$

$$\det(A) = 2(1+\sin^2\theta) \quad [\sin^2\theta = 1]$$

$$= 2(1+1) = 4$$

$$= 2(1+\sin^2\theta) \quad (\sin^2\theta = 0)$$

$$= 2(1+0) = 2$$

$$2 \le |A| \le 4$$

Date....... Page.......

Q15 $y = \sin^{-1}(x^2-2)$

Domain of $\sin^{-1} x$ $[-1, 1]$

$-1 \le x^2 - 2 \le 1$

$-1+2 \le x^2 \le 1+2$

$1 \le x^2 \le 3$

$1 \le x \le \pm\sqrt{3}$

Domain of $\sin^{-1}(x^2-2)$ $[1, \sqrt{3}]$

Sec D

Q16 = $|a-b| = 5k$

$a-b = \pm 5k$

for reflexive

$(a,a) \in R$

$a-a = \pm 5k$

$0 = \pm 5k$ (true)

for symetric

$(a,b) \in R$

$a-b = \pm 5k$

$-b+a = \pm 5k$

$b-a = \mp 5k$

$|b-a| = 5k$

$(b,a) \in R$.

for transitive

$|a-b| = 5k$ $|b-c| = 5l$

$a-b = \pm 5k$ —① $b-c = \pm 5l$ —②

add 1 & 2

$a-c = \pm 5(k+l)$

$|a-c| = 5(k+l)$

Date........ Page........

$(a,c) \in R$

$\therefore$ R is an equivalence relation.

Set of all elements related to 1.

$= \{\cancel{(1,1), (1,0), (1,6) (1,11)}$

$= \{1, 0, 6, 11\}$

Or

Q16 $ad(b+c) = bc(a+d)$

$$\frac{b+c}{bc} = \frac{a+d}{ad}$$

$$\frac{1}{b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{d}$$

$$\frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c}$$

for reflexive

Let $(a,b) R (a,b)$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{b} + \frac{1}{a}$$

$$\frac{1}{b} + \frac{1}{a} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{b} + \frac{1}{a}$$ (addition is commutative)

$(a,b) R (a,b)$ is reflexive.

for sym

for

$\frac{1}{a}$

R

Date........./........./.........

Page.............................

for sym.

$(a,b)\ R\ (c,d)$

$$\frac{1}{a}+\frac{1}{d}=\frac{1}{b}+\frac{1}{bc}$$

$$\frac{1}{b}+\frac{1}{bc}=\frac{1}{a}+\frac{1}{d}$$

$$\cancel{\frac{1}{b}+\frac{1}{c}}+\frac{1}{c}+\frac{1}{b}=\frac{1}{d}+\frac{1}{a}$$

$(c,d)\ R\ (a,b)$.

R is sym.

for transitive

$(a,b)\ R\ (c,d)$ $\qquad$ $(c,d)\ R\ (e,f)$

$$\frac{1}{a}+\frac{1}{d}=\frac{1}{b}+\frac{1}{c} \quad -(1) \qquad \frac{1}{c}+\frac{1}{e}=\frac{1}{d}+\frac{1}{f} \quad -(2)$$

add 1 & 2

$$\frac{1}{ac}+\cancel{\frac{1}{d}}+\cancel{\frac{1}{e}}+\frac{1}{e}=\frac{1}{b}+\cancel{\frac{1}{c}}+\cancel{\frac{1}{d}}+\frac{1}{f}$$

$$\frac{1}{a}+\frac{1}{e}=\frac{1}{b}+\frac{1}{f}$$

$(a,b)\ R\ (e,f)$

$\therefore$ R is transitive

R is ~~equa~~ equivalene.

Q17 = $AB = \begin{bmatrix} 2+4+0 & 2-2+0 & -4+4+0 \\ 4-12+8 \ \cancel{4+6} & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I.$$

$AB = 6I$

multipley by A^{-1}

$B = 6A^{-1}$

$\frac{1}{6}B = A^{-1}$

system of eqn

$x - y = 3$

$2x + 3y + 4z = 17$

$y + 2z = 7$

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$AX = C$

multiply by A^{-1}

$IX = A^{-1}C$

$X = \frac{1}{6}BC$

$$x = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

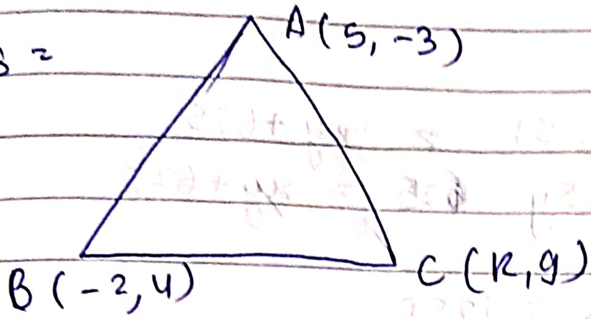
$$= \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ 6 \\ 24 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$x = 2 \quad y = 1 \quad z = 4$.

Sec E

Q18 =



(i) $\frac{1}{2}\begin{vmatrix} 5 & -3 & 1 \\ -2 & 4 & 1 \\ k & 9 & 1 \end{vmatrix} = 14$

(i) $\frac{1}{2}\left[5(4-9) - (-3)(-2-k) + 1(-18-4k)\right] = 14$

$5(-5) + 3(-2-k) + (-18-4k) = 28$

$-25 - 6 - 3k - 18 - 4k = 28$

$-49 - 7k = 28$

$-7k = 28 + 49$

$-7k = 77$

$k = -11$

(ii) $\begin{bmatrix} -11+a & 0 & 0 \\ 0 & -11+2b & 0 \\ 0 & 0 & -11+3c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$-11 + a = 1$ $\quad$ $-11 + 2b = 1$ $\quad$ $-11 + 3c = 1$

$a = 12$ $\quad$ $2b = 12$ $\quad$ $3c = 12$

$b = 6$ $\quad$ $c = 4$

$a + b + c = 12 + 6 + 4 = 22$

Q19 = let length be x & breadth be y.

$$\begin{bmatrix} l-25 \\ l-20 \end{bmatrix}_{2\times1} \begin{bmatrix} b+25 & b+10 \end{bmatrix}_{1\times2} = \begin{bmatrix} lb+625 \\ lb-200 \end{bmatrix}$$

(i) $(x-25)(y+25) = xy+625$

$\cancel{xy} + 25x - 25y - 625 = \cancel{xy} + 625$

$25(x-y) = 1250$

$x-y = \frac{1250}{25}$

$x-y = 50$

$(x-20)(y+10) = xy-200$

$\cancel{xy} + 10x - 20y - \cancel{200} = \cancel{xy} - \cancel{200}$

$10x - 20y = 0$

$x - 2y = 0$

system $x - y = 50$

$x - 2y = 0$

$$\begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \end{bmatrix}$$

(ii) $AX = B$

$X = A^{-1}B$

$A^{-1} = \frac{1}{|A|}(\text{Adj } A)$

$|A| = -2-(-1) = -2+1 = -1$

Date....../....../......

Pago..........

adj A.

$$\begin{bmatrix} -2 & 1 \\ -1 & 1 \end{bmatrix}$$

$$X = \frac{-1}{1}\begin{bmatrix} -2 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 50 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 50 \\ 0 \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \end{bmatrix}$$

$x = 100 \qquad y = 50.$

Sec A

Q1 = $\sec^{-1}\left(\sec \frac{\pi}{3}\right) + \sin^{-1}\left(\sin \frac{\pi}{6}\right) - \tan^{-1}\left(\tan \frac{\pi}{3}\right)$

$$\cancel{\frac{\pi}{3}} + \frac{\pi}{6} - \cancel{\frac{\pi}{3}} = \frac{\pi}{6} \quad (d).$$

Q2 = $A^2 = 2A$ $\qquad |2A|$

$|A|^2 = 8|A|$ $\qquad 2^3 \cdot |A| -$

$\cancel{\frac{|A|^2}{8}}$ $\qquad 8|A|$

Q3 = $\cos^{-1}\cos\left(\frac{13\pi}{6}\right) = \cos^{-1}\cos\left(2\pi + \frac{\pi}{6}\right)$

$$\cos^{-1}\cos\frac{\pi}{6}.$$

(d) $\pi/6$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Date......
Page......

Q4 (d) bijective

Q5: (a)

Q6: $a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32}$

(b) -7

Q7: $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\beta = \pi + \pi + \pi$

$\cos^{-1}\alpha = \pi \Rightarrow -1$

$\cos^{-1}\beta = \pi \quad = -1$

$\cos^{-1}\gamma = \pi \quad = -1$

$\alpha + \beta + \gamma = -3$

(c) -3

Q8 (a) A skew sym matrix

Q9: $|A| = 3(9) - 0 - 0$

$= 27$.

(d).

$|A|\,|adjA|$

Q10: (A).